DIMENSIONAL ANALYSIS OF
THE INFLATION PROCESS OF PARACHUTE CANOPIES

Thesis by

Saul Kaplun

In Partial Fulfillment of the Requirements

for the Degree of

Aeronautical Engineer

California Institute of Technology

Pasadena, California

1951

ACKNOWLEDGEMENTS

The author is indebted to Dr. H. J. Stewart who suggested that the possibility of a systematic approach based on dimensional analysis be investigated, and who supervised the thesis. Gratitute is expressed to Mr. G. E. Eichinger of Army Materiel Command and to Mr. Edwin Pounder of GALCIT for many informative discussions and the use of experimental data. All illustrations are based on or reproduced from U.S.A.F. Report No. 6117.

ABSTRACT

The possibility of an experimental approach based on dimensional analysis is considered. Dimensional analysis of the filling of a parachute tied to a fixed support is given. The physical variables of the problem are discussed, and an attempt is made to retain for subsequent analysis all those variables for which the a priori assumption of negligibility is not warranted. The variables are reduced to dimensionless form, leading to tables of model rules which must be obeyed for various purposes. The possibility of a rigorous experimental procedure and, especially, of separating the effects of the several parameters is discussed in light of the model rules.

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I. INTRODUCTION

The subject of this thesis arose in connection with the study of the filling process of parachute canopies conducted at the GALCIT 10-ft. Wind Tunnel (Ref. 1).

The filling of a parachute canopy is one of a number of phenomena occurring during the descent of a parachute; the canopy is first extended from the pack until the shroudlines become taut, then it becomes inflated and, finally, the parachute decelerates to its limiting velocity and descends, usually in an oscillatory fashion. While the descent of an already inflated parachute is reasonably tractable, the canopy filling process presents a challenge. The hope of elucidating this problem on a rational basis must be equivalent to the hope that some particularly simple type of flow occurs during the inflation and that not all of the large number of variables involved bring their effect to bear. There exist some indications that this might, in fact, be the case. For instance, there are indications that the filling process might be governed by a flow resembling the familiar <u>hydraulic jump</u>, in which the upstream flow is "unaware" of the inflated portion of the canopy downstream — the two regions being separated by a more or less discrete dissipation region which progresses toward the skirt.

Unfortunately, the experimental information which is available does not seem sufficiently significant or extensive to form a convincing basis for hypotheses of this kind. It is especially true that the knowledge of the role played by the design parameters is to a large extent based on opinion. It is therefore felt, that while experiments of a heuristic nature (such as making the flow field during the inflation visible) would be useful in suggesting ideas, what is needed to evaluate such ideas is a more definite empirical picture of the effects of the variables involved.

The dimensional analysis given here is intended merely to show what the variables of the problem are, and to give the model rules which would be necessary for rigorous experimentation.

Since our interest lies in the fundamental mechanics of canopy inflation, the case of a parachute tied to a fixed support is taken. Consequently, Froude number* is always zero and will not appear explicitly. After the effects of the basic variables are understood, it will be possible to extend the discussion to the so-called "free flight" cases in which the Froude number is greater than zero. It should be noted that Froude number has a very definite effect, leading eventually to the inversion of the canopy for sufficiently high values.

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* Froude number is the ratio of inertia to gravity forces. In the present application it is the ratio of $1/2\ \rho v^2 D^2$ to the weight of the load carried by the parachute.

II. THE PHYSICAL VARIABLES

The fundamental assumption of dimensional analysis is that a given set of physical variables determines the problem completely. This implies that:

a) there is no guarantee that the list of variables is complete, and

b) that the entire configuration, geometric, elastic, and inertial, is given by a finite number of characteristic values under the condition of similarity. Comparison of models of dissimilar design, on a dimensional basis, is therefore impossible.

The aerodynamic variables will be taken as v, ρ, μ, and a. The turbulence level of the wind tunnel will not be considered.

The effect of gravity in the case of a fixed support would be noticed only at very low dynamic pressures, and could be immediately detected as the sagging of the canopy. This is also omitted.

The design parameters which will be considered here are those which specify the geometric configuration of the parachute, its mass distribution, and its elastic properties.

To specify the geometric configuration, we will use two characteristic lengths, the over-all diameter and a characteristic dimension of the fabric thread. The thread dimension is introduced to make a distinction between the over-all Reynolds number, and the Reynolds number based on the thread dimension which governs the seepage through the canopy and may well be in the laminar regime (Fig. 1).

It would be very desirable also to consider porosity, but, unfortunately, a change in porosity in general requires an essential change in the fabric geometry. Conversely, experiments show that different fabrics of the same porosity are not equivalent (Fig. 2).

The experimenter would have to ascertain whether the two characteristic lengths do in fact determine the geometric configuration. It has been noticed, for instance, that the cloth porosity changes after a number of openings. It is also impossible to reproduce the folds and wrinkles of the canopy each time the parachute is released, which causes an experimental scatter. The results should therefore be interpreted statistically. Fig. 3 shows the nature of this scatter: the release occurs at different points of nearly the same opening curve, giving good agreement in the neighborhood of the force peaks, but showing that the opening time is sensitive to the initial entrance area at instant of release.

The specification of the elastic properties of the parachute is so complicated that, at least, some departures from rigor will be unavoidable. Properly, considering the fabric to be a plastic, anisotropic medium, it would be necessary to specify twenty-one elastic parameters, and, in addition to the similarity of distribution, require that the stress-strain diagrams remain similar. Let us tentatively take the other extreme by assuming that the spring constant of the shroudlines and the flexural rigidity of the canopy are the only important factors. Whether this assumption is actually justified can be decided only on the basis of consistency of the experimental data derived from it. However, it should be mentioned that rigidity, which varies inversely as the fourth power of the diameter, is especially important in small models. It may also be important in full scale parachutes, in the neighborhood of the vent*.

To complete the list of physical variables, we specify the distribution of mass in the parachute by the mass of the canopy.

Thus, it is assumed that the flow is completely described by:

velocity	V	LT^{-1}
density	ρ	ML^{-3}
absolute viscosity	μ	$ML^{-1}T^{-1}$
speed of sound	a	LT^{-1}

and the parachute by: (1)

design inflated diameter	D	L
thread dimension or ribbon width	d	L
spring constant of the shroudlines	k	MT^{-2}
characteristic canopy rigidity	(EI)	$ML^{3}T^{-2}$
mass of the canopy	m	M

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* This information is due to Mr. Eichinger.

III. THE DIMENSIONLESS PARAMETERS

To the nine physical variables, there corresponds a set of six dimensionless parameters, which may be chosen in an infinite number of ways. Since the choices are all dimensionally equivalent, the choice made here is based on physical significance:

$$
\begin{aligned}
&\text{Reynolds number} && R = \frac{VD\rho}{\mu} \\
&\text{Reynolds number of the fabric} && R_f = \frac{Vd\rho}{\mu} \\
&\text{Mach number} && M = \frac{V}{a} \\
&\text{Shroudline elasticity} && e_s = \frac{k}{\rho V^2 D} \qquad (2)\\
&\text{Canopy rigidity} && e_c = \frac{(EI)}{\rho V^2 D^4} \\
&\text{Canopy inertia number*} && I = \frac{m}{\rho D^3}
\end{aligned}
$$

These are the <u>independent parameters</u> which, it is assumed, determine the performance of the parachute. We can now introduce, in dimensionless form, any of the performance variables which may be of interest. For instance,

$$
\begin{aligned}
&\text{Dimensionless time} && \tau = t\,\frac{V}{D} \qquad (3)\\
&\text{Dimensionless force} && f = \frac{F}{\frac{1}{2}\rho V^2 D^2} \times \frac{4}{\pi}
\end{aligned}
$$

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* The canopy inertia number is proportional to the ratio of the mass of the canopy to the "apparent additional mass" of the parachute. It also determines the ratio of the inertia force in a canopy element to that in a fluid element.

Thus, the expressions for the total filling time, T, and the maximum force, F_{max}, are of the form:

$$\frac{TV}{D} = f_1(R, R_f, M, e_s, e_c, I) \tag{4}$$

$$\frac{F_{max}}{\frac{1}{2}\rho V^2 D^2} = f_2(R, R_f, M, e_s, e_c, I) \tag{5}$$

Also, if we are interested in squidding, we have:

$$f_3(R_{cr}, R_{f_{cr}}, M_{cr}, e_{s_{cr}}, e_{c_{cr}}, I_{cr}) = 0 \tag{6}$$

where the subscript $(\ )_{cr}$ denotes critical values.

It should be noted that, if it is assumed that the parameters have no effect, then the expressions for the performance variables (2) become determined up to a constant of proportionality. For instance, the opening time and shock become*:

$$T = \frac{D}{V} \text{ x constant} \tag{7}$$

$$F_{max} = \frac{1}{2}\rho V^2 D^2 \text{ x constant} \tag{8}$$

In general, the approximation given by expressions of the type of (7) and (8) must, for our purposes, be considered inadequate since

* An expression of this type, for the breathing frequency of a flat circular parachute, $\omega D/V$ = constant, was found to be accurate to within 1%, however the amplitudes of the modes excited were completely altered by the parameters.

it can be seen that they must break down completely as squidding conditions are approached.

IV. MODEL RULES

Figs. 2 and 5 show typical variation of force-time diagrams for two different kinds of parachute. These curves are based on simple experiments where all the parameters, except the inertia number, are varied at the same time over a limited range.

This type of data is characteristic of the experimental information available at the present time. Both for accurate extrapolation of measurements conducted on models to full-size parachute, and for the rational analysis of the filling process, it is necessary to understand the effect of each of the parameters, separately. In the author's opinion it is particularly important to establish the effect of the inertia number, since an opening governed by canopy inertia forces would be characteristically different from one in which the mass of the canopy has no effect. In the former case the rate of opening would be determined by the dynamic pressure and the inertia of the canopy, while in the latter case by the volume-inflow of air. It is the latter, flow-determined case which has been assumed by O'Hara* and by W. Müller** in the rational analyses which they respectively suggested.

Table I, on the following page, gives the model rules which must be observed if we wish to vary only one parameter at a time. This table was constructed on the assumption that the absolute viscosity of the fluid and the speed of sound will not be varied. The free physical variables are encircled, and the others must be changed in the ratios given. A study of Table I will reveal the

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* Ref. 2.

** Ref. 3.

experimental difficulties involved in keeping the five parameters constant at the same time.

Parameter to be varied	ρ	V	D	d	m	k	(EI)	In order to get a variation	
R_f	$\frac{1}{D}$	const.	(D)	(d)	D^2	const.	D^3	$\frac{d}{D}$	$\neq$ const.
R	$\frac{1}{d}$	const.	(D)	(d)	$\frac{D^3}{d}$	$\frac{D}{d}$	$\frac{D^4}{d}$	$\frac{d}{D}$	$\neq$ const.
M	$\frac{1}{VD}$	(V)	(D)	D	$\frac{D^2}{V}$	V	VD^3	V	$\neq$ const.
I	$\frac{1}{D}$	const.	(D)	D	(m)	const.	D^3	$\frac{m}{D^2}$	$\neq$ const.
e_s	$\frac{1}{D}$	const.	(D)	D	D^2	(k)	D^3	k	$\neq$ const.
e_c	$\frac{1}{D}$	const.	(D)	D	D^2	const.	(EI)	$\frac{(EI)}{D^3}$	$\neq$ const.

TABLE I

Considerable simplification of the model rules can be introduced by holding a smaller number of parameters constant. For instance, if we neglect Mach number a priori, the following model rules apply:

Parameter to be varied	ρ	V	D	d	m	k	(EI)	In order to get a variation
R_f	$\frac{1}{VD}$	(V)	(D)	(d)	$\frac{D^2}{V}$	V	VD^3	$\frac{d}{D} \neq$ const.
R	$\frac{1}{Vd}$	(V)	(D)	(d)	$\frac{D^3}{Vd}$	$\frac{VD}{d}$	$\frac{VD^3}{d}$	$\frac{d}{D} \neq$ const.
I	$\frac{1}{VD}$	(V)	(D)	D	(m)	V	VD^3	$\frac{mV}{D^2} \neq$ const.
e_s	$\frac{1}{VD}$	(V)	(D)	D	$\frac{D^2}{V}$	(k)	VD^3	$\frac{k}{V} \neq$ const.
e_c	$\frac{1}{VD}$	(V)	(D)	D	$\frac{D^2}{V}$	V	((EI))	$\frac{(EI)}{VD^3} \neq$ const.

TABLE II

Mach Number Neglected

Further simplification may be introduced if a number of parameters is varied simultaneously. Such experiment, however, could only serve to put an upper limit on their cumulative effect. An example of particularly simple experiment which calls only for model of different diameter is given below:

Parameter to be varied	ρ	V	D	d	m	k	(EI)	
R and I	const.	const.	(D)	const.	(m)	D	D^4	$R \propto D$ $I \propto \frac{m}{D^3}$

TABLE III

Combined Effect of R and I

V. PRACTICAL SIGNIFICANCE OF THE MODEL RULES

It is not intended to give here a rigid outline of an experimental program. A study of the model rules, in particular of Table II, will show that complicacy of the situation requires that considerable amount of thought be given to the individual experiments. It becomes necessary to consider the use of all possible experimental facilities, including

1) variable density wind tunnel
2) medium other than air
3) models of different sizes.

Particular difficulties are connected with the building of precision parachute models. For example, true geometric scaling of the fabric, in three dimensions, would require the scaling of each component fiber of which the threads are composed, since otherwise the viscous and elastic properties, and the mass distribution of the parachute are disturbed. This is, of course, not feasible. For accurate work, therefore, it would be advisable to keep as far away from changing the scale as possible at first, and to turn to it gradually, as various judicious violations of the similarity rule become justified in light of more exact experiments. Such cautious violations of the similarity rule, seem to offer the easiest way out of the experimental difficulties. As an illustration we might mention

1) use of shroudlines of different material, disregarding the change of mass distribution and rigidity which is involved.

2) use of a light spring suspension between the parachute and its support, which changes the distribution of the elastic

properties, in addition to the overall spring constant.

3) manipulation of the canopy reinforcements so as to obtain the required rigidity. This presupposes that the rigidity of the parachute is primarily due to the reinforcements, which may well be found justifiable if the models are not too small. However, changes of mass distribution of the canopy are involved.

It appears, therefore, that the initial experimentation should be conducted with the explicit purpose of showing how good the models actually are, and conversely, how much tampering with the inertia number and other parameters can be allowed for any required degree of accuracy. The experiments which serve this purpose are those which put an upper limit on the combined effect of several parameters. To see what the possibilities are, we might consider the following procedure:

1. Putting an upper limit on the combined effect of rigidity and inertia to see, conservatively, how much tampering with the mass of the canopy can be tolerated.

Exp. 1a Analysis of reproducibility of results, consisting of the study of porosity changes and wind tunnel dust collection after a large number of openings, a statistical analysis of the scatter, and so on, with the purpose of establishing definitely the validity of the results, and improving the accuracy to the point where small variations in trend can be distinguished from the scatter.

Exp. 1b Combined effect of canopy rigidity and inertia, over a wide range. Use single model, variable density tunnel, and constant Reynolds number. Speed should be low enough to neglect gravity (canopy must not sag). Plot performance vs rigidity (const./V) and inertia (const. x V).

Exp. 1c <u>Effect of altering the mass distribution</u>, small range. Use single model, constant density, and constant speed. Small weights are attached on the inside of the canopy along the reinforcing tapes in such a manner that rigidity is not affected. The weights must be very small so that no local geometric changes result.

2. Improvising rigidity by means of re-enforcements, to put an upper limit on the effects of rigidity and its distribution in various vital spots of the parachute. A series of such experiments may show that canopy rigidity is unimportant for certain models over a working range of aerodynamic variables.

3. If rigidity and Mach number can be neglected, the exact determination of the effects of the remaining variables becomes possible. Effects of inertia are determined by repeating Exp. 1b, and the two Reynolds number effects are measured by using parachutes of the same fabric but different diameters. The following model rules apply:

Parameter to be varied	ρ	V	D	d	m	k	In order to get a variation
R_f	$\frac{m}{D^3}$	$\frac{D^2}{m}$	(D)	const.	(m)	$\frac{D^2}{m}$	$D \neq$ const.
R	$\frac{m}{D^3}$	$\frac{D^3}{m}$	(D)	const.	(m)	$\frac{D^4}{m}$	$D \neq$ const.
I	(ρ)	$\frac{1}{\rho}$	const.	const.	const.	$\frac{1}{\rho}$	$\rho \neq$ const.
e_s	constant					(k)	$k \neq$ const.

TABLE IV

Mach Number and Rigidity Neglected

REFERENCES

1. "Parachute Inflation Process, Wind Tunnel Study", U.S.A.F. Report No. 6117.

2. O'Hara, F., "Notes on the Opening Behaviour and Opening Forces of Parachutes", Journal of the Royal Aeronautical Society, November 1949.

3. Müller, W., "Parachutes for Aircraft", N.A.C.A. Technical Memorandum 450.

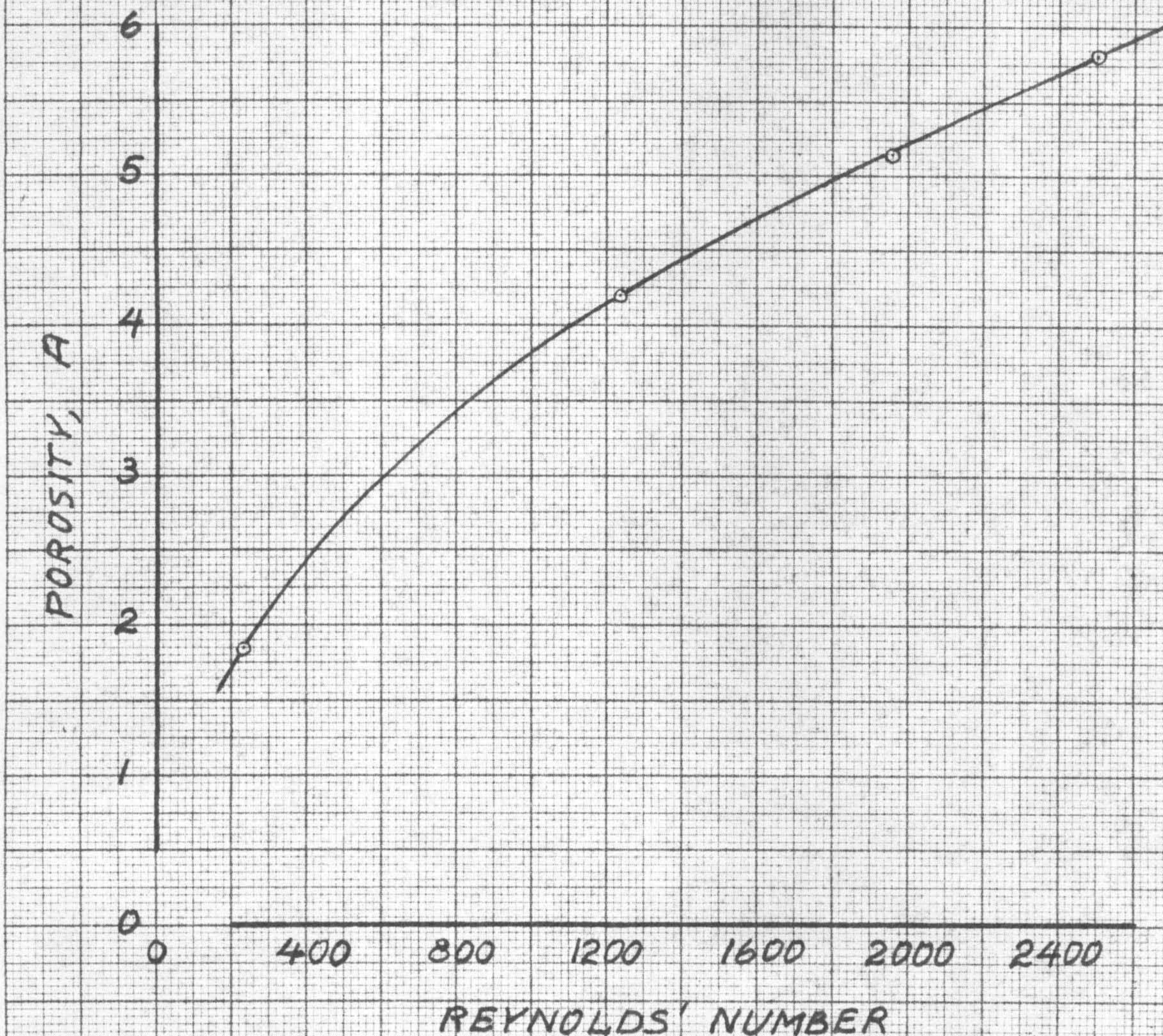


FIG. 1 SHOWING SIGNIFICANT VARIATION OF POROSITY WITH REYNOLDS' NUMBER.

POROSITY $A = \frac{1}{2}\rho Q^2/\Delta p$, WHERE Q = RATE OF VOLUME FLOW PER UNIT AREA AND Δp = STATIC PRESSURE DIFFERENCE ACROSS THE FABRIC.

REYNOLDS' NUMBER IS HERE REFERRED TO A THREAD DIMENSION OF THE FABRIC (1/NUMBER OF THREADS PER UNIT LENGTH) AND THE SPEED Q.

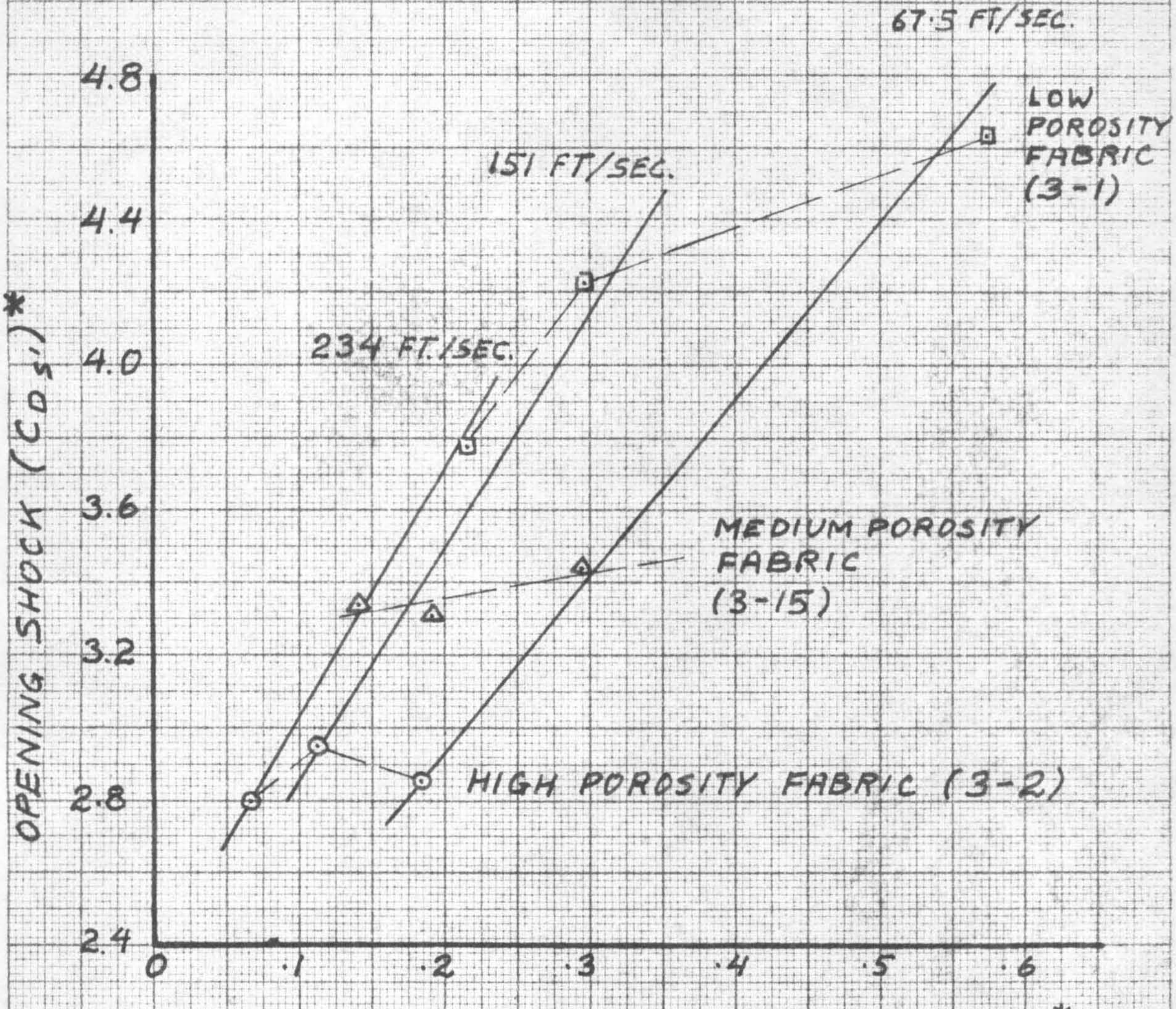


FIG. 2. EFFECT OF POROSITY ON THE OPENING SHOCK. ST'D. FLAT-CIRCULAR PARACHUTE.

CHANGES OF POROSITY ARE OBTAINED IN TWO WAYS: BY VARYING THE WINDTUNNEL AIR SPEED (THEREFORE R_F AND ϱ_C), AND BY USING FABRICS OF DIFFERENT PERMEABILITY.

* REF. 1.

FIG. 3.

REPRODUCIBILITY OF THE FORCE-TIME DIAGRAMS UNDER FIXED EXPERIMENTAL CONDITIONS.

(HEMISPHERICAL PARACHUTE, $q = 60$ LB/FT^2)

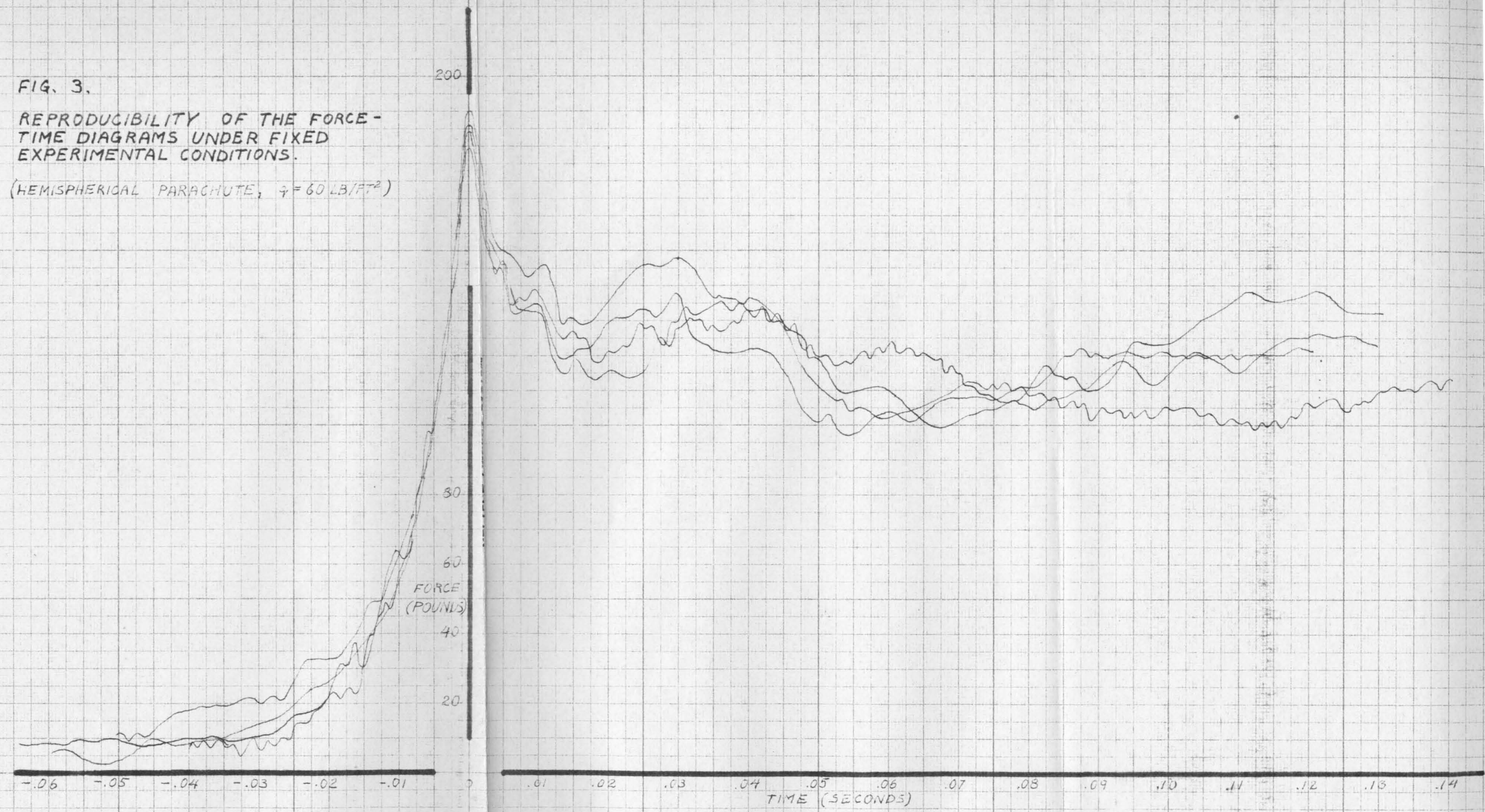
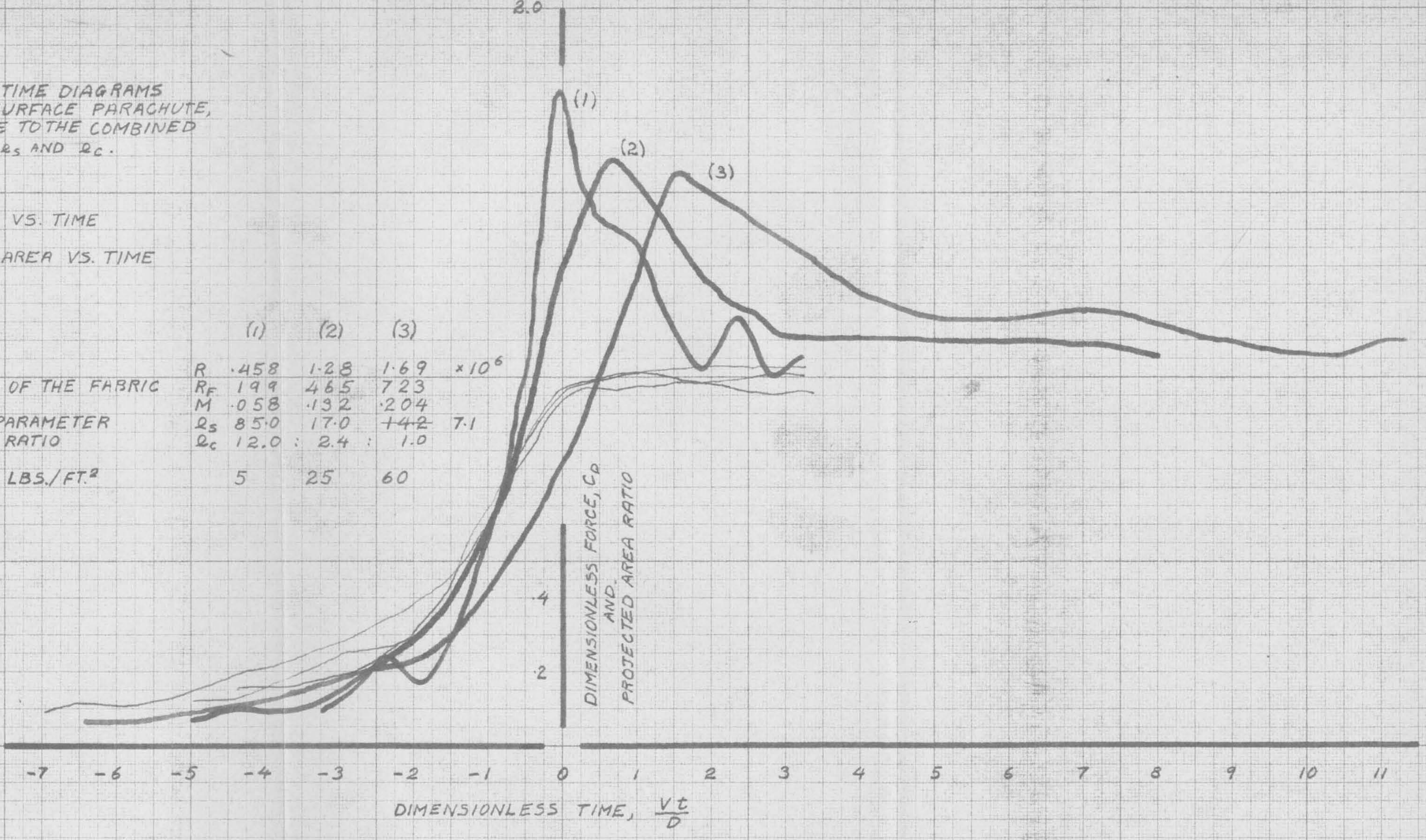


FIG. 4

DIMENSIONLESS FORCE-TIME DIAGRAMS FOR A RIBLESS GUIDE SURFACE PARACHUTE, SHOWING VARIATION DUE TO THE COMBINED EFFECT OF R, R_F, M, ϱ_s AND ϱ_c.



DRAG FORCE VS. TIME (thick line)

PROJECTED AREA VS. TIME (thin line)

		(1)	(2)	(3)	
REYNOLDS' NUMBER	R	.458	1.28	1.69	$\times 10^6$
" " OF THE FABRIC	R_F	199	465	723	
MACH NUMBER	M	.058	.132	.204	
SHROUDLINE ELASTIC PARAMETER	ϱ_s	85.0	17.0	~~14.2~~	7.1
RIGIDITY PARAMETER RATIO	ϱ_c	12.0 :	2.4 :	1.0	
DYNAMIC PRESSURE, LBS./FT.^{2}		5	25	60	

FIG. 5

DIMENSIONLESS FORCE-TIME DIAGRAMS FOR A STAND'D. FLAT CIRCULAR PARACHUTE, SHOWING VARIATION DUE TO THE COMBINED EFFECT OF R, R_F, M, ϱ_s AND ϱ_c.