CORRELATION OF EXISTING DESIGN INFORMATION ON SANDWICH CONSTRUCTION

Thesis by

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ABSTRACT

The problem considered is one of survey and correlation of existing design information on sandwich construction. The subject matter deals with the types of sandwiches, advantages, buckling and stress analysis, published test data, design methods, core materials, weight comparisons, current applications and the manufacture, repair and inspection of sandwich construction.

Suggestions for extensions needed are given and where obvious improvements in analysis could be seen, these modified methods were suggested.

Suggested further work - Every phase of sandwich construction considered in the present thesis shows need for further development. Particularly useful fields for further work are the development of more adequate stress analysis methods for the core, more general correlation of existing formulas and the development of better means of attachment, inspection and repair.

DEFINITION OF SYMBOLS

Basic symbols

A	Area of the cross section
$A*$	Cross-sectional area of the core
a	Beam span (In the application suggested $a = L$)
B	Flexural stiffness of the column per unit width
b	Width of the column or beam
c	Core thickness
$(E_y)_c$	Young's modulus of the core parallel to the loading
$(E_y)_f$	Young's modulus of the face parallel to the loading
E_t	Tangent modulus of the face material
E_c^*	Young's modulus of the core in the transverse direction
e_y	Strain in the skin corresponding to the compressive yield stress
e	Shear factor defined by equation 54, page 11 of reference 7
e_1	Critical strain at wrinkling in the symmetrical mode
E_1	Longitudinal Young's modulus of the core
e_2	Critical strain at wrinkling in the asymmetrical mode
E_1I	Flexural stiffness defined by equation 39a, page 9 of reference 7
$(E_1I)_{eff}$	Effective flexural stiffness of a beam including the effects of shear deformation in the core. Defined in reference 7 by equation 56 on page 12.
F_{cr}	Critical wrinkling stress of the skins
F_y	Compressive yield stress of the face material
f	Thickness of one face
f_E	Critical buckling stress in the skins

G	Shear modulus of the core in the longitudinal and transverse directions
h	Total sandwich thickness
L	Length of column
n	A/A^*
P_{m_b}	Total load at failure
P_{cr}	Total load at which buckling of the column takes place
P_e	Euler critical buckling load (Equivalent to equation 4-4)
P_e^*	Euler critical buckling load defined on page 10 of reference 55
P_E	Load per unit width at failure
p_f	Allowable compressive stress of the face material
p_c	Core compression stress
r	Shear deformation correction factor defined on page 10 of reference 55
T	Sandwich thickness between the face centerlines
α	π/half wave length of wrinkle
σ	Poisson's ratio of the core

Equivalent Symbols (The original report notations were retained)

d	c
E	$(E_y)_f$ below the elastic limit
E_d	E_c^*
E_c	$(E_y)_c$
E_p	$(E_y)_f$
E_s	$(E_y)_f$
E_f	$(E_y)_f$

e_w	e_2
f_p	p_f
G_c	G
G_d	G
P_y	P_{m_b}
P	P_{m_b}
P_c^*	P_{cr}
P_w	P_E
s	f
t	f
w	b

I. INTRODUCTION TO SANDWICH CONSTRUCTION

1.1 Description

Sandwich construction as applied to aircraft is a type of construction consisting of two strong, stiff sheets of material bonded to a light weight core material. The definition given, however, will be further restricted for purposes of this thesis. The type of sandwich construction considered here is restricted to sandwiches in which either continuous or nearly continuous support is given to the face materials by the core. Specifically this means that sandwiches having balsa, cellular cellulose acetate and other continuous materials will be considered. In addition, core materials consisting of a honeycomb type of construction in which the cells of the honeycomb are small will be considered since nearly continuous stabilization of the faces is given by this type of core. Materials consisting of some type of stringer arrangement with two faces will not be considered since this can be termed semi-conventional or semi-sandwich construction. Some reference will be made to where information can be found on semi-conventional materials.

The reason for the distinction between semi-conventional and sandwich construction becomes apparent when an attempt is made to analyse them. The types of failure and therefore, the methods of analysis differ between these materials. It appears from a limited amount of research on the semi-conventional construction that the methods of analysis are not as adequately developed as they are for sandwich construction although it may be an improvement over conventional construction. Even from theory, however, the semi-conventional construction may prove advantageous for heavier loaded parts requiring thick skins where local panel buckling is not a problem. For use on ordinary sizes of aircraft the sandwich type of construction should prove superior to both conventional and semi-conventional construction.

1.2 Development

The disadvantages and inefficiencies of conventional sheet stringer construction are well known. The elastic instability, large deflections and general lack of stiffness of this type of construction is not a desirable feature for aircraft structures. The skin material due to its low rigidity buckles at a small fraction of its ultimate strength. The buckling of the skin under low load not only indicates inefficient use of material but also leads to aerodynamic inefficiencies. The choice of materials for sheet-stringer construction is extremely limited. Where loads do not require the use of materials available for sheet stringer construction, a weight penalty must be accepted since gages cannot be reduced below thicknesses which can be riveted satisfactorily. A structure made up of conventional construction requires thousands of parts including rivets and small fittings. This, of course, increases engineering, manufacturing, tooling, assembly and repair costs. The method of design and analysis of conventional sheet-stringer construction is as a general rule a trial and error procedure. The large structural deflections often lead to unpredictable secondary effects and increased vibration and flutter problems. Examination of the loads causing easily manufactured stringers to fail indicates that even the stringers themselves do not develop the available strength of the materials used.

There are many more disadvantages but these are sufficient to justify a search for a more efficient type of construction. The saving in weight and convenience of sheet-stringer construction have so far led to its general use.

A more effective type of construction is under development and may lead to substantial improvement in aircraft structures. This reasonably new material is called sandwich construction.

1.3 Current Applications

Sandwich construction is in current use in applications varying from minor experimental panel installations to most of the primary and secondary structure of an entire airplane. The most extensive current use of this material is on the Chance Vought F5U and F6U experimental Navy fighters. About 95% of the structure of the F6U is "Metalite" which is an aluminum face, balsa core sandwich. Further mention will be made of the applications of sandwich construction but needless to say, it is desirable that the application of sandwich construction be made as efficiently as possible.

Examination of current results obtained indicates that this material is still in the development stage with far greater potentialities than have been advantageously utilized. However, even in its present stage of development this type of construction has proven competitive with the best sheet stringer construction available. A brief examination of the individual applications is sufficient to show several interesting facts. The first thing that is observed is that the weight figures from present use indicate a variation of from 40% weight saved to a weight penalty of 10% without any obvious differences in the applications.

Further it is found that extensive tests have been made where the material has been used efficiently and tests rather than theoretical analyses have been the basis for the sandwich core and face dimensions. Extensive tests are costly and highly undesirable if theoretical analyses supplemented by a limited number of tests can provide an optimum design. Further, it has been noted that in many applications the sandwich construction resulted in approximately the same weight but was overstrength from 40 to 80 per cent. In these cases a properly designed sandwich would have resulted in considerable weight saving.

1.4 Analysis

It is apparent that an examination into the analysis of sandwich construction is in order. It was at first thought that ordinary strength of materials methods would give reasonable results for this type of construction, but test results proved that this is not the case. Several unexpected types of failure were soon discovered and no readily available methods were found which would predict these failing loads. It was also discovered that shear deformations which were considered negligible in the original analyses are not negligible.

The core materials available for use in sandwich construction have different densities and properties varying with the density. This, of course, makes it desirable to have theory available for determining what core material and properties will produce the most efficient sandwich.

A large amount of research has been done on the problem of analysis, but there does not appear to be any comprehensive correlation of the work. As a result, the methods used in analysing the material are inferior to the methods developed in research papers, but not used because of the difficulty in locating them and putting them into a useable form. It is in the interests of developing adequate methods of analysis and design that have been checked by test data that this thesis is written.

1.5 Continued Development

Before considering the theoretical work done on sandwich construction it would be well to consider the reason for the continuing development of sandwich construction. With increasing speeds of aircraft, the maintenance of aerodynamic shapes become more cirtical. In order to accomplish this with conventional construction, severe weight penalties must be taken. However, with sandwich construction buckling does not occur until failure so the aerodynamic shape and smoothness is maintained at all loads up to failure. This advantage can be obtained with a decrease in weight of the structure due to the more efficient use of the skin material. Sandwich construction is particularly efficient where high torsion and bending loads are encountered such as are often experienced with aerodynamic surfaces. An additional saving in weight is found from the rigidity of the sandwich which allows increased rib and frame spacings and complete elimination of stringers. There is a considerable reduction in the number of parts used in structures of sandwich construction which leads to simplified design, fabrication, assembly and repair problems.

Recent trends have been toward the development of structures with high rigidity due to aero-elastic problems in high performance aircraft. Sandwich construction offers a possible solution to this problem. Problems of control reversal, high speed flutter and dynamic loads induced by gusts and landing impact may be reduced through the use of sandwich construction. Improved thermal and sound insulation properties as well as appearance are possible additional advantages.

From the above considerations sandwich material would appear to be ideal for aircraft uses but these advantages are not obtained without new problems. After considerable research on the subject these problems do not appear to be basic limitations on this type of structure but rather, development problems which can be solved through experience and research. Only a few of the potentialities of this type of construction have been noted, but even these will dictate the continued development and increased use of sandwich construction.

1.6 Scope of the Thesis

This thesis is written as a basis for further correlation work on the subject. Due to the almost inexhaustable amount of information available only a start on this work could be accomplished. It was first planned that this thesis would consist of checking the methods of analysis and design equations with the available data. In addition it was desired to set up design charts and establish the limits of accuracy expected in the analysis of sandwich construction. Another objective was to establish the best methods of analysing each particular type of sandwich. It was soon discovered that all of this is not possible at the present time for the following reasons. No coordinated work on this subject could be found. The references located were usually technical reports dealing with one type of failure or one particular phase of the work and the full significance of the work done in the reports could only be obtained through considerable study. As a further complication, the equations obtained by reasonably exact methods are often quite formidable due to the effects of shear deformation in the core in addition to the necessity of finding two minumum values of buckling load. A further difficulty was encountered when it was discovered that the skin material is often stressed above the elastic limit requiring the use of a reduced modulus. This requires a trial and error procedure which is quite time consuming unless design charts are prepared. Further complication was introduced when it was found that some of the expressions must be minimized but this had not been done in the original reports. For these reasons a slightly different objective was adopted. If the data correlation were undertaken, this thesis would simply become another detailed report so that there still would be no correlated work. The objective adopted is then one of gathering as much information as possible on all phases of sandwich construction and establishing this information as the basis for further correlation. The information gathered and presented is primarily with reference to analysis of this type of construction since this phase of the work appears to be needed most for more efficient application of the material. However, some information is given on all the different aspects that could be considered in the available time and reference given to where further work can be found on the subjects considered. An extensive reference list is presented at the end of the thesis which can form a basis for further correlation. All of the material could not be investigated, but the availability of an adequate reference list is considered to be of value for future work.

In the thesis itself there has been no attempt to reproduce the curves in the various reports but they will be referred to specifically. Since it will serve primarily as a reference for data and design method correlation, the notation of the original sources was left unchanged. Although this leads to the necessity of studying the notation given for each equation, it will at the same time allow easier reference to the source material. A fair amount of data has been published on the test results from numerous types of sandwiches. Reference is given to these data which can be used to check the equations presented as well as to give an idea from theoretical work whether or not the test specimens represent the optimum sandwich or relatively inefficient specimens. It is believed that through this cross check of experiment and theory the methods of designing sandwiches can be considerably improved.

Mention should be made of a new book called Engineering Laminates which is reference 117. This book was published near the conclusion of this thesis work. Some of the information contained in that book is presented here but the book also contains several lengthy reference lists on sandwich construction. The material presented there does not duplicate the work of this thesis but will be found to supplement it. Where certain information is presented there in very good form, it did not seem desirable to repeat it in the thesis. Reference is made to this material at several points in the thesis.

1.7 Thesis Plan

The plan followed in the thesis is to present first a discussion of various types of sandwich construction and the advantages of sandwich construction. This is followed by an analysis section and then a design section discussing the material available on the different subjects. The general applicability of the material is then considered in the design section which includes information on construction, repair, fatigue and weathering of this type of construction. Some mention is also made of service tests on current applications and tentative conclusions are drawn. A reference list is presented.

There is also much additional information in restricted and confidential reprots both British and American which is not considered. It is felt that the test of the formulae developed is the agreement with the available test data. Since much test data and probably adequate formulae have been developed in many cases, the problem resolves itself into data correlation rather than additional methods of analysis. During the correlation the need for further development of theory has become apparent but duplication of effort should be avoided.

II. TYPES OF SANDWICH CONSTRUCTION

2.1 Continuous Core

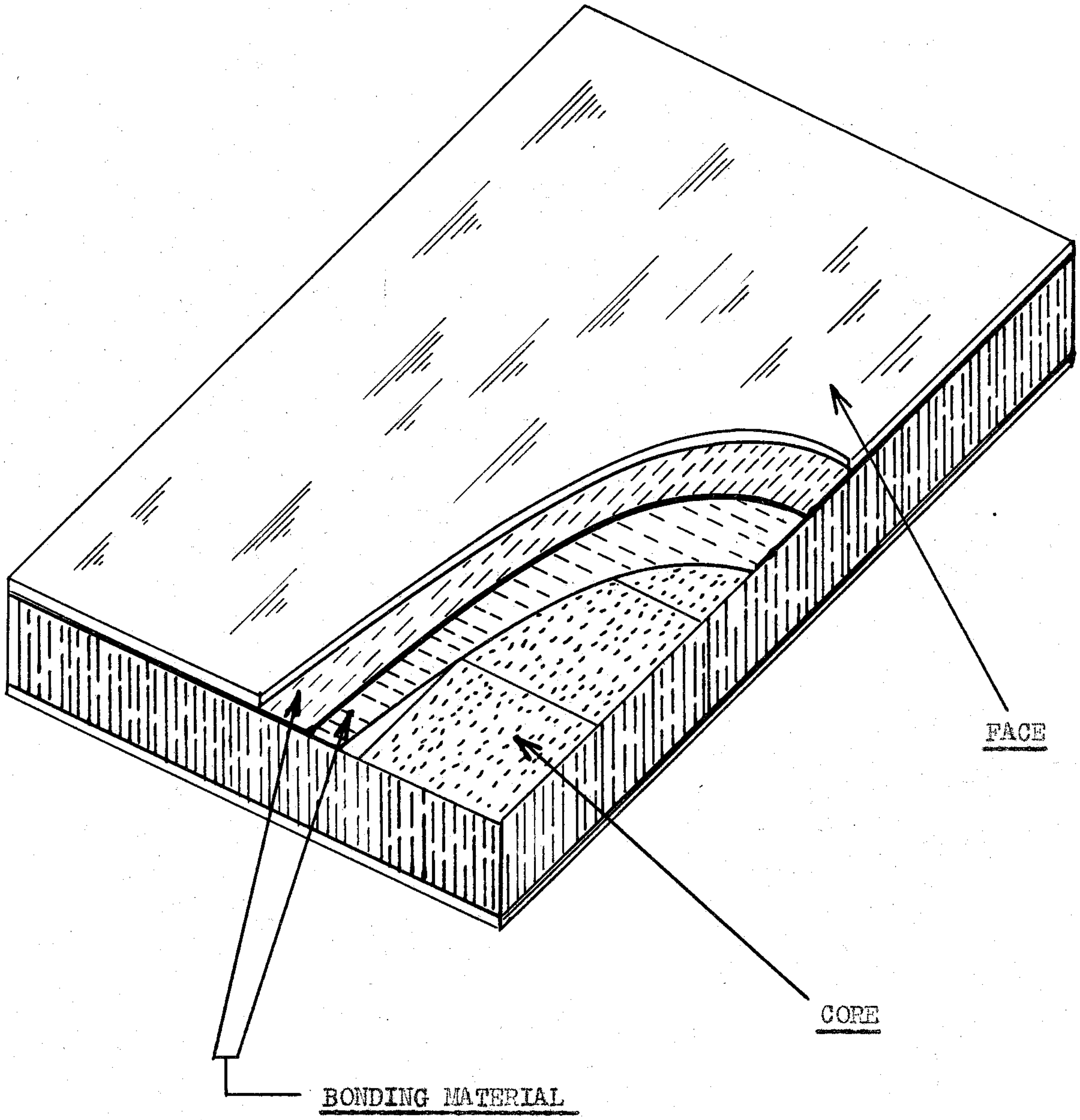
There are three types of construction that have been given the name of sandwich construction. The first type of sandwich is one consisting of two faces of strong, stiff material bonded to a light weight but continuous core to form a laminated material having three layers. An example of this type of construction is a material formed by gluing two sheet aluminum faces to a balsa wood core. A sandwich of this type is shown in figure 1.

2.2 Honeycomb Core

The second type of sandwich consists of two faces of strong, stiff material bonded to a light weight core which is not continuous but for all practical purposes may be considered to be continuous. An example of this type of construction is a material formed by bonding two sheet aluminum faces to a honeycomb core made up of impregnated paper having the axis of the cells perpendicular to the faces. See figure 2.

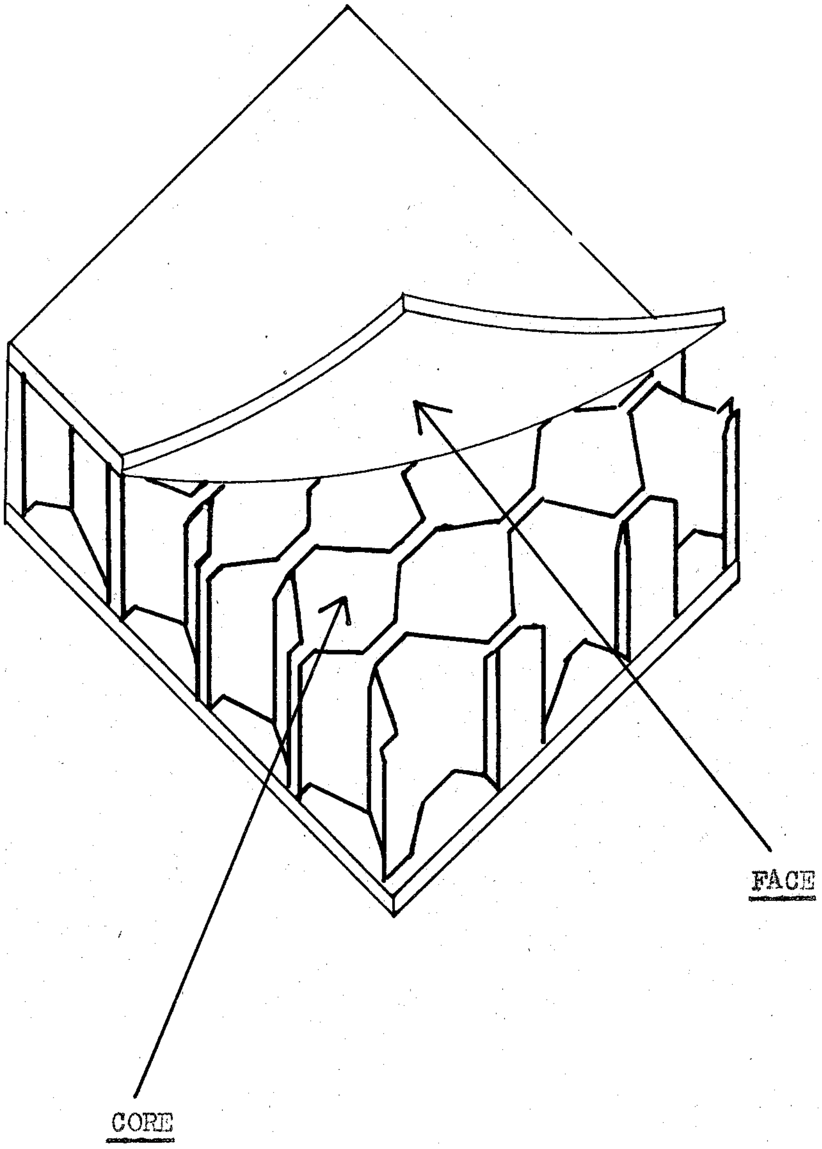
2.3 Grid Core

The third type of sandwich construction consists of two strong face materials bonded to a discontinuous core. The core usually consists of a crossed framework of stringers made from strong, dense materials such as hard woods. An example of this type of construction is shown in figure 3. This type of sandwich is sometimes referred to as a grid type of sandwich construction although for purposes of this thesis it is preferable to term it semi-conventional rather than sandwich construction.



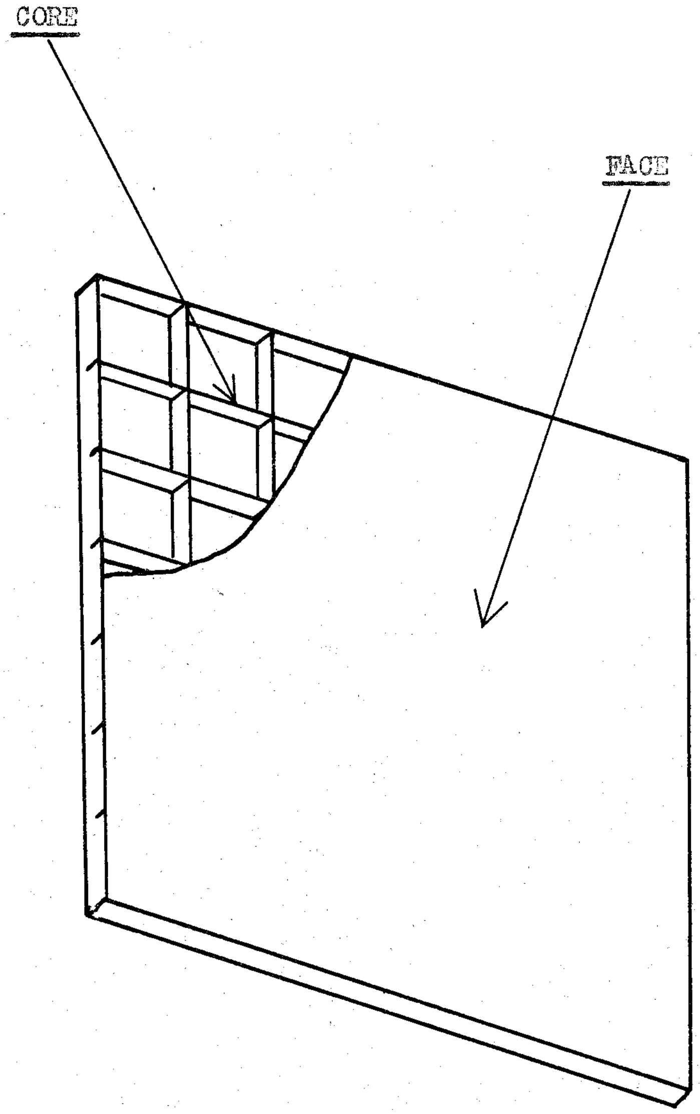
BALSA CORE SANDWICH

FIGURE 1



HONEYCOMB SANDWICH CONSTRUCTION

FIGURE 2



GRID SANDWICH CONSTRUCTION

FIGURE 3

The basic differences in these types of sandwiches are considered later in the analysis and design discussion of sandwiches. The available core and face materials are discussed in the design section.

III. THEORETICAL ADVANTAGES OF SANDWICH CONSTRUCTION

3.1 Theory

The current applications of sandwich construction have been made in an effort to take advantage of the desirable properties of this material. Theory indicates that the superior properties of sandwich construction come from a more efficient distribution of the material in the sandwich as compared to the material used in sheet stringer construction. In sheet stringer construction the face material buckles at low load due to its low flexural rigidity. Even the stringers suffer from buckling so that the entire sheet stringer panel must work at stresses considerably below the compressive strength of the material used. By placing the face material far from the neutral axis, the flexural rigidity of the panel as a whole is increased so that buckling will not take place until the allowable compressive strength of the face material is reached. However, buckling of the faces individually would occur unless this is prevented. The core material serves the double purpose of spacing the faces far from the neutral axis and, at the same time, prevents buckling of the faces individually. Since in most cases the core material is not useful for load carrying its presence must be justified by the higher stresses at which the face materials are allowed to work. Since the face material of sheet stringer construction buckles at a negligibly low load and the stringers buckle in many cases well below the compressive strength of the material, it is not difficult to justify the presence of the core material in sandwich construction. The face materials of an efficiently designed sandwich will work close to the compressive strength of the material.

It has often been found that the sandwich panel in shear will allow face stresses in the panel to exceed the normally allowed shear stress of the face material. Since core materials are usually of very low density some loss of efficiency is experienced from shear deformations in the core which in most cases are not negligible. In spite of this feature the use of sandwich construction can result in considerable weight saving even in its present stage of development.

Theoretical considerations then would indicate two important advantages from sandwich construction. The first is weight saving and the second is increased flexural and torsional rigidity with resultant decreases in deflections.

3.2 Application

Aerodynamic surfaces appear to be an ideal application for this material. In addition to a saving in weight, the aerodynamic surface shapes can be maintained without distortion throughout the load range since sandwich construction will not buckle until the failing load is actually reached. Increased performance of aircraft requires this characteristic which can only be obtained with use of sheet stringer construction by taking prohibitive weight penalties.

As is often the case where more efficient types of construction are found, there are additional advantages not related to the theoretical advantages. This is certainly the case with sandwich construction. As a result of the increased flexural rigidity of the structure, it is found that ribs and frames may be spaced at much larger intervals resulting in substantial weight saving from these items. In addition, the number of parts used in construction is greatly reduced which should result in reduced engineering, construction assembly and repairs costs.

Aeroelastic problems are very critical in high performance aircraft and have led to development of structures with higher rigidity. Sandwich construction shows considerable promise in this field since rigidity is a characteristic feature. It may serve to reduce the dangers of control reversal, high speed flutter and dynamic loads induced by gusts and landing impact.

The thermal and sound insulation properties of sandwich construction should not be overlooked.

An additional characteristic feature of sandwich construction is its adaptability to construction from different materials. Sheet stringer construction is practically limited to the aluminum alloys and further restricted to such gages as can be readily riveted. On light planes this means weight penalties due to minimum gages. On heavier aircraft, lightly loaded fuselage structure may be the cause of weight penalties from use of minumum gages. With sandwich construction it is possible to go to materials such as impregnated fiberglas faces with wrapped cellular cellulose acetate core. There appears to be no reason for taking weight penalties due to light loads since sandwich materials can be chosen which will operate efficiently in any load range.

There is the possibility with sandwich construction of gradually reducing the strength from the wing root to tip as the loads decrease by reducing the core thickness. This variation in load with span presents an important structural problem with sheet stringer construction while it should not cause any particular difficulty with sandwich construction.

The problem of forming ordinary wing tips or even metal sandwich wing tips and parts with considerable double curvature is a difficult one. However a plastic sandwich that can be molded presents a possible solution to this problem. Radome applications are of this type and sandwich construction has found effective use particularly due to its electrical properties as well as structural. It may be possible to house radio antenna in plastic sandwich vertical fins which would allow a reduction in drag.

Space is often at a premium in aircraft. With the reduction of internal structure and elimination of stringers by use of sandwich construction more space should become available. All the internal volume out to the inner sandwich face should be available for fuel storage and other purposes.

On the whole, it appears that the design of aircraft should be considerably simplified by use of sandwich construction. Many of the advantages noted have not been obtained due to the fact that this type of construction is in an early development stage. With further develop- and experience, all of the advantages noted should be obtained. This material definitely appears to be ideal for aircraft application. There are still many development problems, however, that must be investigated before its general use.

For further information see references 53, 61, 92 and 167. Most of the above information was obtained from these sources.

IV. THE ANALYSIS OF SANDWICH CONSTRUCTION

4.1 Introduction

Considerable research has been done into the analysis of sandwich type construction. However, much of the work done has been duplication while at the same time the accuracy of the results has not been established.

Simple analyses neglecting shear deformation of the sandwich core have proven inadequate in the intermediate column range where much aircraft work is done. In addition to the fact that shear deformations must be taken into account, several unexpected types of failure have occurred requiring more extensive analysis than was originally thought necessary. Since the stresses in the skins are often above the elastic limit of the material this factor must be taken into account.

The analysis of sandwich type construction must be undertaken with the viewpoint that the analysis involved is that of structure, not a material.

There are three primary types of failure for a continuous core sandwich column. However, for these types of failure to occur, there must be no premature shear, tension, or compression failure in the core or failure of the bonding material between the faces and the core. Furthermore, most of the available methods of analysis assume geometrically perfect component parts. Any stresses due to manufacture, assembly and initial waviness have been neglected. These can cause early wrinkling failure. If the theoretical work is done correctly, the test points should in general lie below the theoretical curves. (See reference 51). These formulae are unconservative which must be taken into account in the analysis.

It is at this point that the reason for the distinction between a continuous core, nearly continuous core and a grid type construction becomes apparent. With the continuous core sandwich type compression columns there are three types of failure which can be called crushing of the skin or core, modified Euler buckling and wrinkling failure. For the honeycomb core sandwich there are the three types of failure mentioned plus an additional one which could be called local honeycomb buckling. It should be noted that in both the continuous core and nearly continuous core construction complete failure occurs at buckling.

An analysis of the characteristics of the grid type construction shows a radical difference between the continuous core and grid constructions. With the grid core, wrinkling and generally crushing of the face material cannot occur. In addition, buckling of the individual face panels can occur at low loads while complete failure does not occur until the compressive strength of the longitudinal grid members is reached unless Euler buckling of the panel as a whole takes place. These actually are the characteristics of sheet stringer construction rather than sandwich construction so it would seem reasonable to term it semi-conventional rather than sandwich construction. The analysis methods to be applied in this case seem to be the methods developed for sheet stringer construction rather than sandwich construction. For that reason the methods presented in this thesis will be for continuous or nearly continuous core sandwiches rather than the grid type of construction.

This subject is divided into two parts. The first part considers the primary methods of failure for the continuous and nearly continuous core sandwiches. The second part gives methods for stress analysing sandwich construction to determine whether the component part strengths are adequate to permit the primary types of failure to occur.

As stated in reference 53, only shear and compression forces are liable to cause instability. Since much of the structure of aircraft acts under a compression load at various times, the analysis of sandwich columns loaded in axial compression forms a basis for further analysis and evaluation of sandwich construction. Other applications and loading conditions are also considered in this section.

The problem of tension presents no difficulty since in any case the ultimate tensile strength of the faces is available to resist these loads.

The principal application of sandwich construction is in replacing structure which is subject to elastic instability in compression and shear as noted in reference 53.

4.2 Crushing of Sandwich Columns

The crushing of a sandwich column will cause failure if the other two types of failure do not occur before the critical crushing load of the sandwich is reached.

The general method of determining the critical load on a sandwich column due to compression failure of the skin is given in reference 37 as

$$P_{m_b} = p_f \left(2f + \frac{(E_y)_c}{(E_y)_f} c\right) b \qquad (4\text{-}1)$$

This equation applies only where the allowable compressive stress of the face material is below or equal to the elastic limit and where the core does not fail before the faces.

A similar equation for this case is given in reference 167 as

$$P_y = w(2tF_y + TE_c e_y) \qquad (4\text{-}2)$$

The two expressions are identical, but the second one brings out the fact that for adequate use of the skin material the actual strain of the core at the crushing strength of the core material should be equal to or greater than the strain of the face material at failure. This eliminates crushing of the core parallel to the loading as a possible mode of failure. That is usually the case with most core materials since they have relatively high strains at failure. However, this point must be checked for any set of face and core materials under consideration.

A possible simplification can be noted from expression (4-1) since $(E_y)_c/(E_y)_f$ is usually an extremely small number. This second term can be neglected in most cases so the load at failure due to compression becomes the expression given in reference 51 as

$$P = 2f_p s \qquad (4\text{-}3)$$

This simplified equation can be applied to honeycomb sandwiches and metal-balsa sandwiches having the grain of the balsa perpendicular to the faces, but cannot be applied where the balsa is arranged parallel to the load or if wood faces are used.

It should be noted that the allowable compressive stress of the faces from test have consistently proved to be above the yield stress of the face material as was noted in reference 92. In that case the formula given is conservative if the yield stress of the face material is used and should be modified for the higher allowable compressive face stresses as well as stressing the material above the yield point of the material.

4.3 Modified Euler Buckling of Sandwich Columns

There are a number of methods of calculating the critical load in modified Euler buckling of sandwich columns. The term modified Euler buckling is appropriate since shear deformations in the core are important in sandwich construction. See figure 4.

The Euler equation was first applied to the problem of buckling of sandwich construction. This equation neglects shear deformation of the core, but was found adequate in the long column range. In the ranges in which it is applicable, there is no reason for going to more complicated solution. The limits of accuracy and range in which the Euler formula applies have not been established.

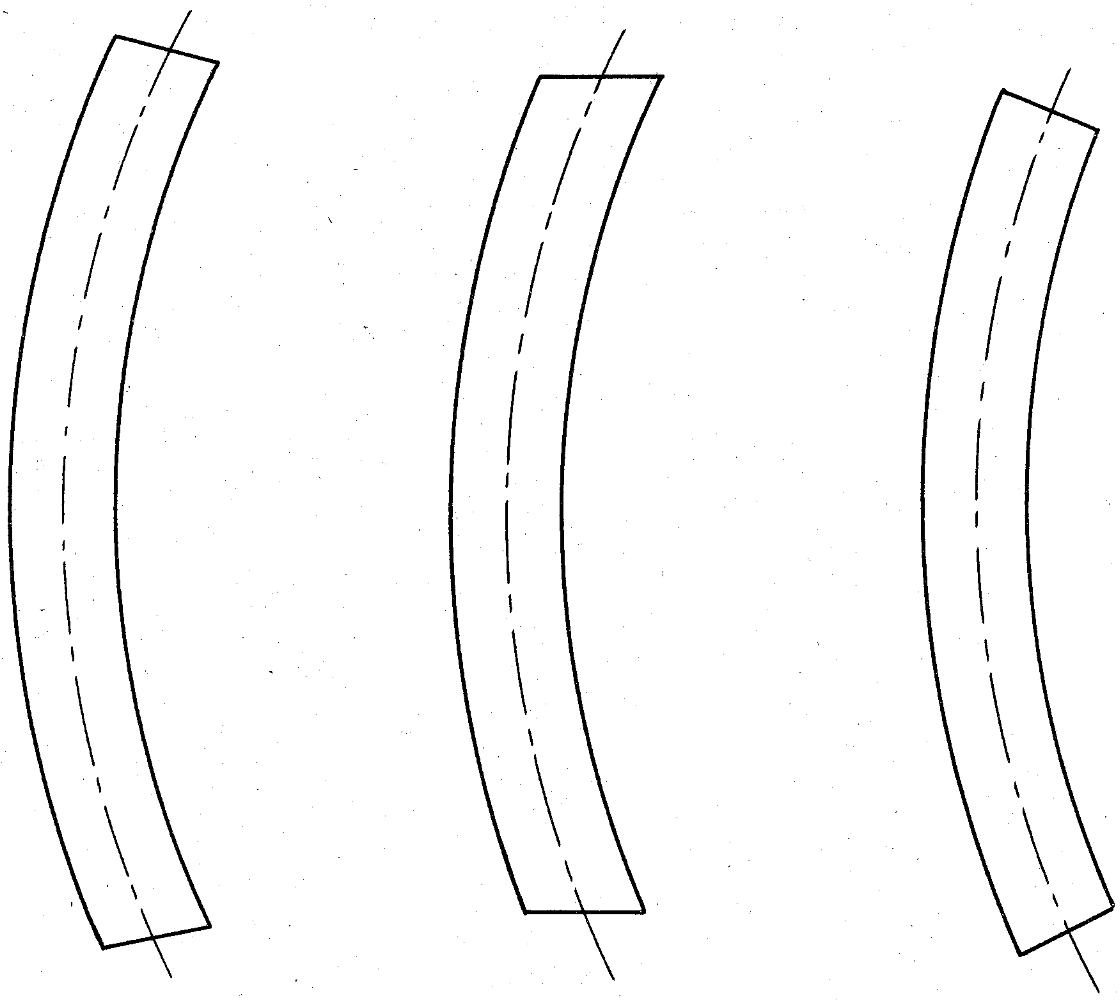
The Euler critical load is given for pin ended columns in reference 167 as

$$P_{cr} = \frac{\pi^2 B\,w}{L^2} \tag{4-4}$$

An approximate formula for the flexural stiffness is given in the same reference as

$$B = .5E_p t T^2 + .0833E_c T^3 \tag{4-5}$$

based on an equivalent cross section of the beam taking into account



MODIFIED
EULER BUCKLING

BUCKLING
NO SHEAR RIGIDITY

EULER
BUCKLING

FIGURE 4

the elastic properties of the core and faces. This neglects the effect of shear deformation in the core.

Equations (4-4) and (4-5) were applied to several long sandwich columns and the results are given in reference 167. They are definitely inadequate for the intermediate column range.

Although the equation (4-4) is given for pin ended columns, it is applicable to ones with other end conditions provided the proper length factor is used. If the critical load gives face stresses above the elastic limit of the material, a reduced modulus of elasticity must be used. The Engesser-von Karman or the tangent modulus theory can be used.

In reference 7 the effective stiffness of a simply supported sandwich beam including the effects of shear deformation in the core has been developed from an elasticity solution of the problem. The results should be directly applicable to the Euler equation so that modified for shear, the equation would become for a pin ended column

$$P_{cr} = \frac{\pi^2 \, (E_1 I)_{eff}}{L^2} \qquad (4\text{-}6)$$

A test program was carried out and reported in reference 8 to determine the adequacy of the shear deformation correction factor

$$\left[1 + \left(\frac{h}{a}\right)^2 e\right] \qquad (4\text{-}7)$$

in the effective stiffness equation for bending

$$(E_1 I)_{eff} = \frac{E_1 I}{\left[1 + \left(\frac{h}{a}\right)^2\right]} \qquad (4\text{-}8)$$

given as equation 56, page 12 of reference 7. The experimental work indicated reasonable agreement between test results and the theoretical equation. The results of one group of tests showed an average of 95% for the ratio of computed to observed effective modulus of elasticity in bending for a range of shear correction factor from 1.00 to 1.32. In these tests the properties of the materials used were carefully matched to the specimen tested. Difficulty was encountered where values were assumed for material of the density used. With this careful matching of properties the ratio of computed to observed was about 92% for shear correction factors from 1.4 to 3.8.

This indicates the requirement with sandwich construction that the material properties must be matched to the test specimens if reasonable accuracy is desired. The assumption of average values for material having the same average density in the case of balsa core is not satisfactory.

Equation (4-7) could be used to obtain approximate limits for the range in which shear deformation is important. The core material was not assumed isotropic so the expressions given in reference 7 have general validity.

There have been a number of solutions dealing directly with the problem of buckling of sandwich columns in which shear deformations have been taken into account and the core was not assumed isotropic.

The Engesser formula is valid for columns weak in shear because of their material or the shape of their cross-sections. It is only approximately correct for sandwich construction in which the faces are strong, but the core is weak in shear as stated in reference 88. The difference between the Engesser formula and more accurate expressions is believed to be small in cases where the face thickness is small compared to the core thickness. It should be satisfactory for ordinary sandwich construction as noted in reference 88. From reference 114 the Engesser equation is

$$P_{cr} = \frac{P_e}{1 + \frac{n\,P_e}{AG}} \qquad (4\text{-}9)$$

More accurate equations for the critical buckling loads of sandwich columns have been developed in references 69, 84, and 76. As stated in reference 117, these results are all equivalent and differ little from the Engesser equation (4-9) when the ratio of face to core thickness is small.

The expression developed in reference 69 is given by reference 117 as

$$P_{cr} = \frac{G_c cw(P_1 + P_2 + P_c) + P_2\left[4P_c + (c/c + t)^2 P_1\right]}{P_c + (c/c + t)^2 P_1 + G_c cw} \qquad (4\text{-}10)$$

$$P_1 = \frac{\pi^2 (c + t)^2 twE_f}{(2)L^2} \qquad (4\text{-}11)$$

$$P_2 = \frac{\pi^2 t^3 wE_f}{6L^2} \qquad (4\text{-}12)$$

$$P_c = \frac{\pi^2 c^3 w\, E_c}{12L^2} \qquad (4\text{-}13)$$

The significance of the P_1, P_2, and P_c can be given as follows. P_1 is the critical load for the column if it had perfect shear rigidity and t^3 were neglected. P_2 is the buckling load of the faces acting independently. P_c is the buckling load of a column of the core only.

Equation (4-10) is equivalent to the equation given in reference 84 which is expressed in reference 117 as

$$P_{cr} = \frac{P_1P_2 + G_c cwP_1 + G_c cwP_2}{P_1 + G_c cw} \qquad (4\text{-}14)$$

if P_c can be assumed to be negligible. For balsa with the grain perpendicular to the faces this is justified, but with grain parallel to the load P_c is not negligible. Equations 4-10 and 4-14 hold over a wider range of core to skin thicknesses than the Engesser formula. If P_c is assumed negligible and the buckling load of the faces acting independently can be neglected so that $P_2 = 0$ both equations 4-10 and 4-14 reduce to equation 4-9.

Reasonable agreement was found experimentally as stated in reference 117 for equation 4-14 although there was considerable scatter. The average experimental values were about 80% of those predicted by theory and the maximum values by experiment were generally quite close to the theoretical values. The experimental results are expected to fall below the theoretical values.

As is pointed out in reference 93, when the buckling stress obtained by these formulae exceeds the proportional limit of the face material, the value of the Young's modulus in the equations for P_1 and P_2 must be replaced by the tangent modulus value corresponding to the buckling stress obtained from the formula for the critical load. A trial and error process is necessary to find the appropriate value of the Young's modulus of the faces beyond the elastic limit.

A limited number of tests to correlate equation 4-14 with experiment are reported in reference 93. It is well to point out certain conclusions drawn from the test results. The agreement between theory and experiment was good for balsa core specimens. However, when the core was cellular cellulose acetate the scatter of the experimental points was considerable. From bending tests, the value of G used in the theoretical formula was 5000 psi. A second theoretical curve in which G was assumed 2500 psi helped to explain the scatter.

Expanded core materials are never completely uniform. When the shear modulus is determined from beam tests every cross section is of equal importance and the modulus obtained is the arithmetic mean value. In column tests regions of large shearing deformation can shift to regions of small shearing rigidity so the apparent shearing rigidity of the core in a column test is smaller than in a beam test, the difference depending on the random variations of the rigidity from one cross section to the other. From this consideration and the test results, the shearing rigidity of expanded cores should be estimated conservatively.

There are two British methods in which the core was assumed aeolotropic which should be noted. The exact equivalence of the two results was not established since several simplifying assumptions were made in one solution which were not made in the other. These are both elasticity solutions of the problem.

The first of the elasticity solutions is a by-product of the analysis of another problem. This solution is given in reference 55 as

$$1/P_c^* = 1/P_e^* + r \tag{4-15}$$

which can be put into the form

$$P_c^* = \frac{P_e^*}{1 + rP_e^*} \tag{4-16}$$

which is the same form as the Engesser expression 4-9. This equation is limited to d/s greater than 40. An examination of equation 4-9 will show that

$$r = \frac{n}{A\ G} + \frac{1}{G(\frac{E_1\ d^2}{6E_s\ s})} \tag{4-17}$$

which indicates that the only difference between equation 4-16 and equation 4-9 is in the term $GE_1d^2/6E_ss$.

A more exact but extremely lengthy elasticity solution to the problem of modified Euler buckling of sandwich columns with aeolotropic cores is given in Appendix B, page 16 of reference 50. Due to its length and complexity, it will not be reproduced here. Application of the resulting expression would be very difficult.

Comparison of the above two methods with experiment could not be located. Another solution to the problem is given in reference 76 but will not be given here since it is equivalent to equation 4-14 except for a factor c/c+t which is almost 1 for most sandwich construction.

There is a large amount of data published on column tests that have been made so that the adequacy of these equations can be determined by actual check with experimental results. Time limitations prevented this from being done but reference is made later in the thesis to where some of the reported test data is available.

The exact elasticity solution appears to be too cumbersome to be valuable for practical applications. The practical modifications necessary to make these solutions apply accurately can only be determined after data correlation.

Where face stresses above the elastic limit are found, a reduced modulus of the form

$$E_r = \frac{2\ E\ E_t}{E\ + E_t} \qquad (4\text{-}18)$$

can be used as stated in reference 93. Values of E_t are given in reference 175. It should be noted that the direct application of E_t instead of E_r gives better agreement with test in many cases.

Isotropy was not assumed in the above equations but they are, of course, all applicable to sandwiches with isotropic cores. A number of special solutions have been worked out for isotropic core materials or special cores. A limited number of these solutions will be referred to.

The first two isotropic core solutions to be mentioned are given in reference 50, Appendix A and C. Appendix A gives an exact elasticity solution to the problem while Appendix C gives an approximate solution. Both solutions result in extremely cumbersome expressions. However, there is a further simplification made to these expressions and the resulting formula is used in reference 52 as the basis of a design method to be considered later. The resulting expression used is for L/d greater than 10. The equation given in reference 52 is

$$\frac{1}{P_E} = \frac{2}{\pi^2\ E_s}\left(\frac{L}{d}\right)^2\left(\frac{L}{s}\right) + \frac{}{G_d}\left(\frac{L}{d}\right) \qquad (4\text{-}19)$$

The core material used was calcium alginate for the design method mentioned.

A second design method reported in reference 51 for isotropic core uses the modified Euler failure equation

$$\frac{f_E}{E_s} = \frac{\pi^2 d}{4L} \left[\frac{1}{3\left(\frac{d}{s}\right)^2} + \frac{1}{1 + \frac{\pi^2}{2}\left(\frac{E_s}{G_d}\right) \frac{d}{s}\left(\frac{s}{L}\right)^2} \right] \qquad (4\text{-}20)$$

The last special solution is a form of the Engesser expression modified for application to honeycomb core sandwiches. This solution is given in reference 88 and consists of a long and a short column curve. The equation for the long column curve is a modified form of the Engesser equation. However, in order to avoid the use of the tangent modulus for stresses above the proportional limit a straight line short column curve is constructed. The equation for the short column curve is included in that reference and both column curves shown. This method of solution has been checked experimentally and gives reasonably good agreement. The results are shown in reference 88.

The subject of sandwich columns under axial load appears to have been adequately investigated theoretically. In some cases empirical modifications may be required to obtain adequate agreement between theory and tests. The validity of all the formulae given here can be established only through extensive correlation with test data.

4.4 Wrinkling of Sandwich Columns and Panels

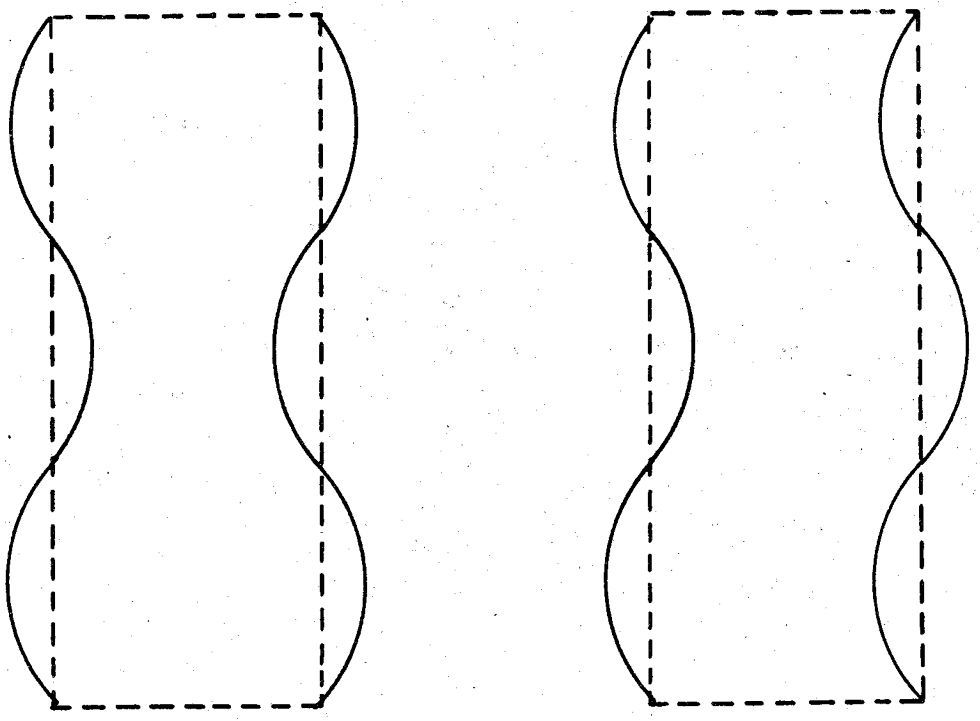
The methods of calculating wrinkling of the faces in sandwich columns were developed after this type of failure was discovered during tests. Wrinkling failure is shown in figure 5. The original solutions given in reference 94 were based on isotropic core materials, but several solutions have been worked out for aeolotropic core materials. The extent to which isotropic formulae can be applied to aeolotropic core sandwiches both for Euler buckling and wrinkling has not been adequately checked experimentally. In many cases such applications are believed suitable for engineering calculations.

The general elasticity solution for the buckling of sandwich columns presented in reference 50 gives two minimum points. One of these corresponds to modified Euler buckling while the other gives the wrinkling mode of failure. Since the expressions are very lengthy and cumbersome to use, they will not be presented here. However, they can be found in reference 50, Appendices A, B, and C. The minimum points have not been worked out generally for these solutions so that is necessary in order to determine the minimum wrinkling load.

As a by-product of an analysis of another problem, the wrinkling load is given in reference 55 for aeolotropic core materials. The results given are entirely equivalent to the ones obtained in reference 50. Here again, the minimum values of the expression have not been determined generally. The expressions given are also very cumbersome. Equation 1 in reference 56 is a summary of the resulting expression where from reference 55 the critical load P_c^* is given as

$$P_c^* = (2E_s s + E_1 d)e_2 \qquad (4\text{-}21)$$

where e_2 is given by equation 1 in reference 56. The expressions given in references 55 and 56 are in terms of the critical strain e_2 which



SYMMETRIC WRINKLING

ANTISYMMETRIC WRINKLING

FIGURE 5

is given for antisymmetrical wrinkling.

Several simplified expressions have been obtained from the solution which are of interest. There are two modes of buckling essentially. These are the symmetrical and the asymmetrical modes. For the wrinkling form of buckling the two modes of wrinkle occur at approximately the same load. The first simplified expression is a sufficiently accurate approximation of the exact solution if the ratio of the core to skin thickness is greater than 50. In this case from reference 55

$$e_w = (.6)(E_1)^{\frac{1}{6}}(E_d)^{\frac{1}{2}}/(E_s)^{\frac{2}{3}} \qquad (4\text{-}22)$$

and

$$\alpha s = 1.48(E_1E_d^3/E_s^4)^{\frac{1}{12}} \qquad (4\text{-}23)$$

The wrinkling critical load is obtained by substituting the value for e_w from equation 4-22 into equation 4-21 for e_2. For lower values of d/s the exact expressions must be used. However, even in these cases the form of equation 4-22 is still valid if e_1 and e_2 are substituted for e_w and k_1 and k_2 substituted for the value .6 respectively. In this case k_1 and k_2 are functions principally of the ratio of s/d and depend only to a small extent on the values of the elastic constants. In most normal sandwiches e_2 is less than e_1 so the wrinkling should occur in the asymmetrical mode.

An experimental investigation of wrinkling of sandwiches using steel skins and expanded formvar core material was carried out and reported in reference 56 for a limited number of tests. The shear modulus was determined by bending test of a sample sandwich. The experimental results were compared with the exact expression given in reference 55 and the stress at wrinkling from tests were between 9 and 30% higher than the theoretical values. The average error was 16%. To account for this, it was noted that the core material was not uniform across the cross section. As a result of having softer core material near the center with the stiff material available to locally support the faces the actual wrinkling stress is increased above the theoretical value. An unusually low value of transverse modulus of elasticity for the core was found experimentally so the proper value may not have been obtained when considering variation of the material across the cross section.

It should be noted that the experimental values should have fallen below the theoretical ones in all cases due to the assumption of geometric perfection of parts.

The approximate expression 4-22 gave almost identical results to the exact expressions in the range considered.

Reference 90 contains a discussion of the elasticity solution of the problem of buckling of sandwich columns. As is pointed out, the elasticity solution for wrinkling stress assumes that every component of the sandwich is homogeneous and geometrically perfect and that the core material possesses a finite stiffness, but infinite strength. The wrinkling stress developed by this solution is the upper limit.

A considerable amount of information on wrinkling is given by Hoff in reference 117. He reviews the earlier work done on wrinkling and points out that the work in reference 94 made an erroneous assumption which was later corrected in reference 50 and in reference 76. However, the corrections are not significant for sandwich beams, columns and panels of conventional design failing by wrinkling. Hoff points out that all the theoretical work predicts asymmetrical buckling but experiments carried out by Hoff and Mautner gave wrinkled shapes differing from the theoretical form. Papreg face sandwiches with cellular cellulose acetate cores gave buckling which could be classified as symmetric when the ratio of core to face thickness was greater than 20 and skew for the ratio smaller than 20.

Hoff and Mautner developed a simple approximate formula for critical wrinkling from strain energy considerations in reference 87. From reference 117 the expression is

$$F_{cr} = \frac{1}{2}\sqrt[3]{E_f E_c^* G_c} \qquad (4\text{-}24)$$

It was assumed that the entire compressive load is carried by the faces. The equation is valid for symmetric and skew wrinkles as well as anisotropic cores. It is almost identical to the results obtained in reference 94 for a face stabilized by an isotropic, semi-infinite core. From this expression wrinkling is predicted solely from the mechanical properties of the face and core materials. This equation was checked by experiment as noted in reference 87 but proof of the formula is not complete since there was considerable scatter in all the tests. The expression does give a conservative approximation to the correct value which is accurate enough for selecting the core material to use with a given face material.

Several important problems have not been solved regarding the wrinkling type of failure. No rigorous theory has yet been devised for the skew wrinkle and no theoretical explanation is available to show why test panels buckle according to the symmetric and skew rather than the antisymmetric wrinkle pattern according to Hoff in reference 117.

It was concluded in reference 117 that (a) the agreement between the wrinkling stresses according to the rigorous and approximate solutions of the symmetric case is satisfactory, (b) the symmetric wrinkle corresponds to the smaller wrinkling stress when the core to face thickness ratio is large. (c) the skew wrinkle corresponds to the smaller wrinkling stress when the core to thickness ratio is small and (d) the minimum wrinkling stress is sufficiently constant over the entire practical range of core to face thickness ratios to warrant the use of equation 4-24. It is valid for orthotropic core and faces since in the strain energy theory isotropy was not assumed.

There are several solutions to wrinkling stress in columns which have been derived on the assumption that the core is isotropic. The pioneer work in reference 94 was done under this assumption.

The results of references 50 and 94 as used in reference 52 can be given as

$$P_w = (2s) \left\{ 9E_s E_d^2 / 4(1 + \ )^2 (3 - \sigma)^2 \right\}^{\frac{1}{3}} \qquad (4\text{-}25)$$

Since the core is assumed isotropic the expression can also be written in terms of the shear modulus of the core. This expression has been modified for stresses above the proportional limit of the material and the result presented in reference 88.

Another formula for calculating the wrinkling stress with an isotropic core is given by equation 3, page 2 of reference 51. It is used in that reference as the basis for a design method for sandwich columns.

The wavelength of wrinkle for an isotropic core sandwich is given in reference 117 by equation 1-87 on page 70.

Equation 4-25 has been checked by experiment as reported in reference 117 and reasonable agreement found. This has also been done for the wavelength equation noted.

An entirely different viewpoint of wrinkling than all of the previous methods was adopted by Wan as reported in reference 90. According to Hoff in reference 92, Wan indicates that the critical stress in ripple type buckling depends not only on the Young's modulus of the material but also on the strength of the core. It is known that structures do not fail because of instability, but because of excessive stresses that may be induced by instability. Wan's work is an attempt to stress analyse the core in order to find the loads under which the stress in some portion of the column reaches the yield point. In Hoff's opinion Wan's own experimental results disprove his theory at least for aluminum alloy cross grained balsa specimens tested. According to Hoff, Wan's reasoning is correct so the discrepancy may be due to the particular law assumed to govern the initial deflections. A summary of the work done by Wan on wrinkling of sandwich columns is presented in reference 90. It is contended there that the actual wrinkling stress in sandwiches is closely related to the initial waviness present in the facings. Most facings are made from commercial sheet stock which in general is not truly flat. The core material is not perfectly uniform in thickness nor is the bonding material. These initial irregularities may be partly corrected for in the fabrication process, but any initial stress set up in the bonding materials has the same effect as initial waviness. Due to these considerations the theoretical wrinkling stress may or may not be achieved.

In reference 90 a reasonable form for the initial waviness in a sandwich column is assumed. For balsa only, an approximate solution is carried out for the sandwich wrinkling in the symmetrical form with a tension failure in the core. This assumption is based on numerous tests on sandwiches with aluminum alloy faces and end-grained balsa. It has been confirmed by comparison with the rigorous solution that for balsa core tension failure occurs at almost the same facing stress for both symmetrical and antisymmetrical attitude while shear failure in the core corresponds to a much higher facing stress.

The analysis is carried out in reference 90 and the results presented there. Charts of the solution are also presented. It should be noted that tension or compression failure in the core or bond was assumed. However, shear failure in the core is not probable if the ultimate shear strength of the core is greater than 12% of its ultimate tension or compression strength. In order to apply the results obtained statistical surveys of particular test specimens are required to determine an average value of initial waviness.

Using the results of reference 90 it is possible to estimate the value of the ultimate tensile strength of the core bond combination required for a given stress in the facings to prevent core failure due to initial waviness in the facings. A rigorous solution to the problem of relating the core failing stress to the critical wrinkling stress of the faces including the effects of initial surface waviness is possible. The work presented in reference 50 has been extended to this problem and the solution presented in reference 90. Due to the length and complexity of the results they are not presented here.

In addition, in the same reference the problem of wrinkling under biaxial stresses including the effects of initial surface waviness is considered and an approximate solution presented. The results have not been checked by experiment.

The adequacy of all of the work presented on wrinkling can only be determined through extensive correlation with available test data. As an indication of the need for this correlation a check was made of equations 4-22 and 4-25 with the results of reference 56. In that report the adequacy of the expression 4-22 was tested and found to give results 9 to 30% too low. By comparing the theoretical values from equation 4-25 with the results of tests presented in reference 56 it was found to give ultra-conservative values for the tests made.

4.5 Local Honeycomb Buckling

With a honeycomb core material there is one additional type of failure which does not exist with continuous core materials. This failure consists of local buckling of the faces into the cells of the honeycomb.

Consideration of this type of failure is given in reference 63, 88, and 117. In reference 63 it was noted that with thin faces local buckling into the cells of the honeycomb was possible. The limiting compressive strength due to this type of failure is a function of the face thickness for a given cell size. This local buckling can be estimated using the assumption that the surface sheets are simply supported by a series of longitudinal walls such that the distance between supports is equal to the cell height. This method of calculation was checked and reported in reference 63. The test points are fairly well scattered probably because of variations in the geometry of the honeycomb as prepared in the laboratory.

4.6 Buckling of Sandwich Beams

The problem of buckling of sandwich type beams has been considered in two reports. The most readily used form of the results is given in reference 117 reporting the work of Van der Neut in Holland. An approximate expression is given in reference 117, page 78 and equation 1-93 for solution of this problem. The expression given holds provided that $(\pi c/b)^2$ is considerably less than unity which should be the case with normal sandwich panels.

The more general problem of combined bending and compression buckling of a beam or column is treated in reference 55. The analysis was carried out in terms of strains by an elasticity solution which resulted in very complex expressions. The conclusions and summary of the work give an indication to the more useful parts of the analysis. They present exact solutions to the problem which are so complex that every attempt is made to obtain simplified expressions. However, where the assumptions made in the simplifications made are not justified the exact expressions must be used. A practical conclusion was reached that when the strain under combined bending and compression reached the value for wrinkling under direct compression, wrinkling would occur.

4.7 Buckling Load of Flat Sandwich Panels in Single Axial Compression

The buckling of panels in aircraft construction is a subject of considerable importance. It is therefore desirable to have theoretical methods for determining the buckling loads for sandwich panels since the analysis differs considerably from other types of construction. This subject appears to have been considered adequately by the Forest Products Laboratory.

An energy method was applied to this problem for sandwich construction in order to obtain the approximate formulas presented in references 11-18. The method applied usually results in buckling loads that are too high by 8% or less. Three simplifying assumptions were made in the solution to the problem. It was assumed in reference 11 that the materials of the faces and core are stressed below the proportional limit and that the effect of shear deformation in the core may be neglected. These two assumptions were both corrected for in reports 12-15 since they were found to be inadequate. The third assumption is that the faces of the panels do not wrinkle before buckling. Since methods for determining the wrinkling of sandwich construction have been established it is possible to make a separate analysis to determine whether or not this assumption is justified.

The formula developed in reference 11 is given as equation 1 of reference 12. A test program was set up to test the validity of this formula but it was found that in each case the formula had to be modified to include shear effects and the effect of stresses above the proportional limit of the face material. This correction work was carried out in references 12 to 16.

A theoretical and experimental investigation of the buckling load of sandwich panels simply supported on all edges and loaded in compression on two of them is reported in reference 12. This work was done to test the validity of the equation obtained in reference 11 as well as to provide the necessary modifications for applications involving the simply supported edge conditions. As has been noted, wrinkling was assumed not to occur before buckling of the panels. The modifications required for shear effects and stresses above the proportional limit of the face materials are given in reference 12. When the shear corrections were applied to the theoretical work and compared to the experimental values the agreement between theory and test was improved 0 to 12%.

The actual shear correction factor used in references 11 to 16 was developed in reference 44 and checked experimentally in reference 45. The method of varification was to calculate the deflection of a simply supported panel due to a uniform transverse load neglecting shear and then apply the shear correction factor for comparison with the experimental values. The general expression for the shear correction factor is given in reference 44 on page 19, equation 90 which was modified to equation 144, page 8 of reference 45 for comparison with experiment. The experimental values checked the calculated values within normal experimental error and scatter. Further justification for the use of the shear correction factor developed is established in references 12 to 16.

In order to modify the buckling load for stresses above the proportional limit of the face material, a reduced modulus was used which was found satisfactory when expressed in the form for three layer sandwich construction as equation 4-18. The buckling stress corrected only for stresses above the proportional limit of the material is given by equation 21, page 18 of reference 12. Since the reduced critical buckling stress and the reduced modulus are both unknown a trial and error procedure is required. However, by use of the stress strain curve of the face material and a simplified method given in reference 12 the solution can be accomplished conveniently graphically. Where the stress strain curve of the face material is not available an approximate solution is possible by using equation 23 of reference 12.

It is usually desired to correct for both shear deformation and stresses above the proportional limit in the faces. This can be accomplished by use of equation 26 of the noted reference. A simplified graphical procedure for the trial and error solution is given there. If the stress strain curve of the face material is not known or the increased accuracy is not desired another approximate method of correction is given.

The methods given in reference 12 have been experimentally tested and proved satisfactory within the range of the tests.

A solution to the buckling load on flat sandwich panels in compression with the loaded edges simply supported and remaining edges clamped is given in reference 13. The expressions developed are similar to the ones obtained in the previous report and the methods of application are the same. Shear deformations and stresses above the proportional limit had to be corrected for. The experimental results averaged about 92% of the values obtained by the corrected formula. These values were believed to be lower than the calculated values because the edges were not secured as firmly as assumed in the formula. The formula was derived by energy methods and can lead to values as much as 8% too great.

The buckling loads on flat sandwich panels in compression with the loaded edges clamped and remaining edges simply supported are given in reference 14 by similar methods. The same corrections as before were found necessary and the results agreed approximately with the computed values.

The solution is given in reference 15 for the buckling loads on flat sandwich panels in compression with all edges clamped. The same methods as noted were used and the same corrections were found necessary. The experimental values were about 82% of the computed values. These values were believed to be lower because the testing apparatus at the unloaded edges did not provide the theoretically assumed clamping and the method used often gives values as much as 8% too high.

Further varification work was carried out experimentally and reported in reference 16. It should be noted that with glass cloth laminate face material there is no proportional limit which eliminates one of the corrections. Previous verification before reference 16 was for continuous core materials whereas the tests reported in this case were made using honeycomb cores. The method was concluded to be probably suitable for this type of panel.

In all cases wrinkling was assumed not to occur and in the case of honeycomb core buckling into the honeycomb cells was assumed not to occur.

Additional work on buckling of flat sandwich panels was done by Leggett and Hopkins and reported in reference 117. The buckling stress of rectangular sandwich panels in edgewise compression is given there by equation (1-92) on page 77. Date to check this equation are available in references 12 to 16.

Due to the stiffness properties of honeycomb core materials an exact elasticity solution for the buckling of that type of panel under compressive end load can be developed as was done in reference 107. The buckling formula developed is for a plate of infinite length simply supported at two of the edges. The exact expressions developed are somewhat lengthy, but under certain simplifying assumptions the critical buckling load is reduced to equation 19 on page 5 of that reference. This is for the wrinkle type buckling while the overall buckling expression obtained is given by equation 24 of that report.

It is discovered in the paper that local wrinkling is governed in this case by symmetrical buckling while the modified Euler buckling is anti-symmetrical. These expressions apparently have not been checked experimentally and the original paper should be checked also for the simplifying assumptions. The exact expressions are also contained in reference 107. It should be noted that flexural failure of relatively long honeycomb sandwich panels generally results from buckling of the face on the compression side of the specimen while very short specimens produce shear failure which is not predicted by these equations.

4.8 Buckling of Flat Sandwich Panels in Shear

The subject of buckling of flat sandwich panels in shear is treated in references 36, 61, and 63. The most adequate treatment of this problem has probably been given in reference 36. However, none of these sources take into account the shear deformations in the core. The theoretical analysis used was developed for plywood in reference 3 and experimentally checked and reported in a supplementary report of that series.

The formula developed by the Forest Products Laboratory is given by equation 20 of reference 3 for plates with all edges simply supported. For sandwich construction having isotropic faces and all edges simply supported this equation reduces to an expression given in reference 36 but reference 3 must be used in the evaluation of coefficients in the equation. A test program carried out to determine the adequacy of these equations is reported in reference 36. Test results closest to the theoretical values were obtained when the specimens buckled at low stresses. When the facing thickness was increased, the deviation from the theoretical values was increased. By reinforcing the loading frames the buckling stress was increased to 15% above the theoretical value for .020 inch aluminum alloy faced specimens. Heavier loading frames increased the buckling stresses of .032 inch facings from 55% to 80 or 90% of the computed values. The light frames used in some of the tests apparently did not provide simply supported edge conditions.

Panels in the tests which failed by a method of buckling called crimping, failed at about 40% lower than the computed values. Larger specimens of the same type of construction failed by buckling nearer to the theoretical values. Crimping occurs at the buckling stress.

Tests carried out on a sandwich panel acting as the shear web of a plate girder gave an experimental buckling stress about 20% lower than the computed value indicating that the heavy flanges did not provide simply supported edge conditions.

To summarize the experimental results, the panels tested in frames having adequate stiffness to provide simply supported edges agreed well with theoretically predicted values. The actual test value of the buckling stress is dependent upon the relative stiffness of the loading frame. A panel that buckles at a stress below the computed value will buckle at a higher stress if the loading frame is reinforced. Because of this, according to reference 36 it is questionable if the designer can predict the edge conditions sufficiently accurately for adequate use of the theoretical work.

The analysis of metalite panels in shear by Chance Vought is discussed in reference 61. Their analysis is based on the standard formula for the stability of plates and shells in which the values used are equivalent thicknesses and equivalent moduli of elasticity. The equations for the equivalence are given in that reference. By substitution of the equivalent expressions given into standard formulas the expressions used by Vought are obtained.

It is pointed out in reference 61 that much of the theory of plates and shells has been found inadequate and in need of extension. The effects of large deflections normal to the plate, of combined loadings and of concentrated loads cannot be predicted accurately with present theory. The accurate prediction of the transverse shear stress distribution in a panel cannot be made under all conditions, yet this stress is critical for many panels.

Reference 63 considers the problem of buckling of a sandwich web in shear. They use equation 216 on page 361 of reference 114 for determining the critical shear flow causing buckling. Expressions are given in reference 63 for adaptation of that formula to sandwich construction. If the material is stressed above the proportional limit of the face material a reduced modulus is used. Tests to varify this equation gave values below the theoretical curve. In all cases this appeared due to the fact that failure took place above the elastic limit of the material. Another factor not accounted for was the low core rigidity allowing shear deformations that are not negligible.

4.9 Flat Sandwich Panels Under Combined Loading

The analysis of flat sandwich panels under combined loading is an important problem in aircraft structures. The most extensive work on this subject located during this research was done in Holland. The results would be extremely difficult to summarize in this thesis, but a brief review of the contents of two reports, references 78 and 81, is in order. The first of these references contains expressions for antisymmetric buckling of a simply supported panel compressed in two perpendicular directions. It gives the complete expressions as well as approximate ones for the buckling stress. The antisymmetric buckling of a simply supported, infinitely long panel compressed in two perpendicular directions and loaded by shearing forces is also considered. Curves of the results are presented in the report. In reference 81 the problem of the buckling of sandwich plates due to compression along two edges combined with shear is considered. Also previous theoretical work on sandwich panel analysis is reviewed there. Curves of the results are presented in the report. The adequacy is the results is discussed there. This is the only theoretical work of this type located during the research and should provide a basis for further work on the more complicated loading conditions. A number of other important Dutch reports on the subject of sandwich construction are listed in the references to this thesis.

An approximate solution to the problem of wrinkling under biaxial stresses is given in reference 90. The analysis resulted in a trial and error procedure for solution which was not considered justified due to the approximate nature of the solution. As a result, a highly simplified method is given for obtaining a conservative estimate of the wrinkling of sandwich panels subjected to biaxial stresses.

4.10 Curved Sandwich Panels and Cylinders Under Shear and Compressive Loads

The problem of compression buckling of curved sandwich panels has not been solved rigorously. There are, however, several approximate solutions available.

The most extensive work on this subject located in the research was reported in referance 29. A rather complete discussion of the problem is given there.

An approximate expression for the critical buckling stress of a long curved sandwich plate loaded in axial compression if the stresses are below the proportional limit is given in reference 29 by equation 1 on page 7 of that report. The curves of reference 11 are required for solution of the problem. For stresses above the proportional limit, the critical stress must be obtained by use of a reduced moduli. Using equations 6 and 7 of reference 29 and the procedure outlined on pages 7 and 10 of that report, the critical buckling stress above the proportional limit may be obtained.

It should be pointed out that this is only one type of failure for curved sandwich panels. There is also the possibility of wrinkling of the faces as well as compression failure of the core and faces or shear failure in the core alone. In addition, the method has been restricted to long panels since shear deformation is unimportant in these. The method given does not take into account shear deformation in the core.

Of both theoretical and practical interest, the Forest Products Laboratory discovered that the axial buckling strength of a well made long, curved plate of sandwich material may be compured by adding the critical stress of a complete cylinder of which the plate may be considered a part to the critical stress of a flat plate having the same dimensions as the curved plate.

A test program was carried out and reported in reference 29 for the buckling of curved sandwich panels. The panels tested were constructed so that wrinkling of the faces as well as crushing could not occur. By the method given in that reference and under these restrictions, the theoretical and experimental values agreed satisfactorily. The test data showed considerable scatter usually due to small surface irregularities. The scatter was different for different types of sandwiches. It was found that for cores 1/2 inch thick or more, the shear deformation correction amounts to over 20%. Incipient dents or buckles have a considerable effect on failure loads.

A formula for calculating the buckling load of slightly curved sandwich panels is given in reference 117 by equation 1-94 on page 79. The critical buckling stress of a complete sandwich cylinder is also given in that reference by equation 1-95 on page 79 by a rather lengthy expression.

The problem of wrinkling of curved sandwich cylinders has been considered in reference 52 by equation 4 on page 7 of that report. It was derived for an isotropic core material.

The crushing type of failure can be predicted from equations 4-1 to 4-3 of this thesis depending on the type of core material used. If the load taken by the core can be considered negligible equation 4-3 should be used.

One special solution has been carried out in reference 107 for the buckling of circular cylinders having a honeycomb core material. The solution is appropriate as long as the wavelength of buckling is long enough to justify the use of bending theory. The limitations are stated in a useful form in that report. There apparently has been no experimental check of the results obtained in this analysis.

The problem of curved sandwich panels in shear has been considered in reference 41. It was found that the buckling stress of a curved plate in shear is approximately equal to the sum of the buckling stress of a complete cylinder subjected to torsion and the buckling stress of a flat plate with the same dimensions as the curved plate subjected to shear. Equation 2 of that reference gives the critical buckling stress for a sandwich cylinder in torsion. An equation for shear buckling of flat plates in shear is given by equation 3 which is modified to give the shear buckling of a flat sandwich with isotropic core. The buckling stress of the curved sandwich plate in shear is obtained as described by the sum of these. The method of application of these equations is described in reference 41 with charts given for the appropriate coefficients. The experimental work was done primarily on plywood plates but one large sandwich panel was tested as a preliminary check on the extension to sandwich construction. Reasonable agreement was found, but the experimental values for specimens that were complete cylinders were lower than the computed values. The difference probably arises from the plates not being subjected to pure shear experimentally. The test value of the buckling stress for the one sandwich panel agreed well with the theoretical value. The specimen was large and the effects of shear deformation in the core was small. However, for smaller plates, the effects of shear deformation may become important and have not been accounted for in this analysis. In panels stressed above the proportional limit of the skin material an additional correction is necessary.

V SANDWICH CONSTRUCTION STRESS ANALYSIS METHODS

5.1 Introduction

The methods of stress analysis of sandwich construction are only approximate as a general rule. In many cases not even an approximate method is available for determining the core stresses. Usually the application of strength of materials methods is non-conservative due to the effects of shear deformation in the core.

5.2 Face Compression Stresses

Equation 4-1 can be used to determine the stress in the face material under direct compression by solving for p_f.

5.3 Core Compression Stresses

The stress in the core material under direct compression can be obtained from the following formula

$$p_c = p_f (E_y)_c / (E_y)_f \tag{5-1}$$

where p_f is obtained from equation 4-1.

5.4 Face Compression Stresses If the Core Strength Is Negligible

If the load carried by the core is negligible the face stresses can be calculated from equation 4-3. This assumption is justified in most practical cases except for balsa core material with the grain parallel to the load. This assumption also allows easy calculation of the face stresses under bending loads.

5.5 Normal Stresses Under Bending

The normal stress in a sandwich beam can be calculated by normal strength of materials methods as described in reference 167 based on an equivalent cross section of the beam. Also equation 1-37 of reference 117 gives the same expression expressed a little differently. However, these methods will be found to give non-conservative results. A slightly better method would appear to be to use equation 1-37 of reference 117 in which the value of EI is replaced by $(E_1I)_{eff}$ from equation 4-8 of this thesis. The resulting expression would take into account shear deformations in the core. Where shear deformation are not important, the strength of materials methods would appear to apply.

5.6 Beam Shear Stresses

Under beam shear loads reference 167 gives the standard strenght of materials expression for calculation of shear stresses. Equation 1-38 in reference 117 gives the same equation expressed a little differently. However, as is pointed out in reference 167 this equation will give values that are too low as noted in tests reported there.

A slightly better method would appear to be to use equation 1-38 in which the value of EI is replaced by $(E_1I)_{eff}$ from equation 4-8 of this thesis. The resulting expression would take into account shear deformations in the core. Where shear deformations are not important, the strength of materials methods give satisfactory results as noted in tests reported in reference 167.

5.7 Stress Analysis of Plates and Shells

In reference 61 the analysis of metalite by Chance Vought is described. Use is made of an equivalent plate and the theory of plates and shells to make the stress analysis. However, in calculation of the stresses it is necessary to apply the factors given in that reference. As mentioned in reference 92 this method is generally satisfactory for calculating face stresses when the deflections are low. However, they are inadequate when deflections are large. These theories do not take into account the shear deformations in the core. The distribution of stresses in both the face and core in regions of high concentrated loads as well as the maximum core shear and tension stresses in various types of bonded joints have not been adequately determined by analysis according to reference 92.

A complicated expression for the shear stress in the core of a cantilever sandwich plate is given in reference 92 in terms of the average shear stress. The solutions for various types of loads are being derived by the Forest Products Laboratory but as yet have not been published.

A solution of the small deflection theory of Reissner is given on page 66 of reference 117 which also analyses for the shear stress in the core. This solution is for a cantilever plate with a special end load distribution which for most practical panels is a uniformly distributed load. The solution includes the effects of shear deformation.

The only investigation of the transverse core compressive strength requirement of sandwich construction is given in references 61 and 90. The method given in reference 90 can be used for stress analysis purposes although the adequacy of the results has not been established.

It should be pointed out that initial imperfections and waviness have a considerable effect on the failing load in the core so these loads causing core failure cannot be adequately predicted unless something is known regarding the flatness and eccentricities of sandwich plates. The Forest Products Laboratory is obtaining this information as is stated in reference 92.

A suggested design procedure in reference 92 gives a graphical method which involves the stress analysis of sandwich panels. In this method the shear stress in the core which can limit the design load is determined using the load deflection curve of the sandwich panel. This load deflection curve can in some cases be determined theoretically. Use can possibly be made of the work done in reference 45 for this purpose. The effects of shear deformation can be taken into account by this method.

Further work is necessary in the field of sandwich construction stress analysis before the required core strengths can be determined for any given design.

VI DEFLECTIONS OF SANDWICH CONSTRUCTION STRUCTURES

6.1 Introduction

The analysis of the deflections of sandwich construction is in general the study of the effects of shear deformation. Where shear deformations are not important the standard methods can be applied to sandwich construction.

The specific work done on deflections of sandwich construction will be indicated for reference purposes.

6.2 Deflection of a Simply Supported Beam Under Concentrated Load

Probably the most useful deflection formula developed is the deflection at midspan of a sandwich beam with a concentrated load at the center. The expression is given in reference 88 including the effects of shear deformation. It was derived by considering the total deflection as the sum of a bending and a shearing deflection. As has been noted in reference 63 this equation is usually used for determining the shear modulus of the core of sandwich material by test.

6.3 Special Deflection Equations

A summary is given in reference 93 of the deflection equations for a cantilever beam of sandwich material under concentrated load. The resulting expressions have been checked by experiment and found to give adequate results.

In references 7 and 8 an equivalent flexural rigidity of sandwich beams is worked out and tested by deflection tests. The method gives very good results for deflections of simply supported beams.

6.4 Deflections of Sandwich Panels

There is considerable need for panel deflections in aircraft and a method of approach used in reference 45 to check the shear correction factor developed in reference 44 may be useful. In this report the center deflection of a sandwich panel was calculated neglecting shear deformations and then the shear correction factor developed in reference 44 was applied to obtain the total deflections including shear effects. This may prove to be the most useful method to apply to panel deflections.

In reference 117 the deflection expression for a cantilever sandwich panel under a special load condition at the end is given on page 66. By use of St. Venant's principle the results may be applied to other similar types of load conditions.

6.5 Deflections of Circular Sandwich Cylinder

Reference 117 treats the problem of the relationship between the maximum deflection and the normal stress in a circular cylinder on page 66. An expression for this relationship is presented there including several simplified forms.

6.6 General Deflection Theory of Sandwich Construction

The more general problem of sandwich construction deflections has been treated in several places. Eric Reissner in references 92 and 102 gives a small deflection theory for sandwich plates and shells. Integration of the differential equations has proved difficult and the only readily available useful solution is noted above under 6.4. More solutions of these equations, however, may prove useful. Shear deformations are considered which leads to the difficulties in solution.

Another small deflection theory is given in reference 98 in which the solutions are based on a knowledge of various stiffnesses of the plates in various directions. It is possible as noted in reference 92 to evaluate these from the properties and dimensions of the sandwich plates. Lockheed Aircraft Corporation has done some work along this line.

There is need for a large deflection theory of sandwich construction as stated in reference 92, but at present this has not been developed.

VII GRID TYPE SANDWICH CONSTRUCTION

As stated in reference 53, a system with a lattice work of stringers and cross-pieces of high density material for a core and two strong, stiff faces is intermediate between the conventional construction and sandwich construction. This type of construction is not analysed by the methods considered in the present thesis, but some reference will be made to where material is given on this subject.

A general description of grid type sandwich construction is given in reference 53. A design methods, test data and an attempt to analyse this type of construction is given in reference 58. The analysis method used does not appear adequate, but the design method gave reasonably efficient panels.

Test data on compression and shear of this type of panel is given in reference 48, but no attempt is made to analyse the panels tested.

A brief summary of the work done on this subject is given in reference 117.

Work is being done on a construction of the semi-conventional type at the California Institute of Technology but is not covered in this thesis.

VIII PUBLISHED TEST DATA FROM SANDWICH CONSTRUCTION TESTS

The formulas given in this thesis and referred to require extensive correlation with test data before the general accuracy can be established. In some cases the equations have not been checked with any data while in others a limited correlation was carried out.

The work of gathering information for this thesis has prevented the data correlation. However, it is anticipated that the work started here will be continued in the future.

A large amount of test data is available and was located in the current research in published form. A limited amount of data is presented in the following references which may be of use in the correlation work.

The references are as follows:

(i) Of the Forest Products Laboratory reports, references 8, 9, 11-16, 19, 20, 22-26, 17, 18, 29, 31-35, 36, 37, 38, 40, 41, 42, 46, 47.

(ii) R and M reports, references: 48, 51, 52, 53, 54, 56, 57, 188.

(iii) Australian Council for Aeronautics reports, references 58, and 59.

(iv) Additional references 63, 87, 88, 90, 92, 93, 95, 96, 97, 99. All of the company reports listed and the most of the magazine references contain data of useful nature. In addition, the Army and Navy reports are useful for this sort of data.

A further list of references to test data is given on page 683 of reference 117 in addition to the actual data in Chapter 20 and pages 690 to 701.

Not all of the references at the end of this thesis could be explored but it is expected that more data is available in them.

IX INTRODUCTION TO THE PROBLEM OF DESIGN WITH SANDWICH CONSTRUCTION

The problem of design with sandwich construction is considered in this section. The methods of analysis have been indicated in the previous part of the thesis while in the present section they will be used to help design installations where sandwich construction is found advantageous for aircraft. The problem of design is too complex to give in detail here but the methods that have been located can be indicated by general description and referred to for more detailed study.

The design of sandwich construction requires a knowledge of the properties and characteristics of the materials available for use in sandwich construction. Much material on this phase of the design has been located and will be indicated.

Of additional interest is weight comparison between this type of construction and other types and the particular applications sandwich construction is suitable for. An additional factor to consider in design of structures is the manufacture and repair of structures which are to use this type of construction. A survey of the material covered in the research is given as well as reference to additional material on these subjects.

X SANDWICH CONSTRUCTION DESIGN

10.1 Design Methods

The design of sandwich construction for minimum weight is complex, but subject to solutions in closed form. The important variables in the problem for a given face and core material are the load, length, face and core thicknesses, transverse modulus of elasticity of the core, transverse and longitudinal shear modulus of the core and the allowable strength of the core material. This applies to a continuous core material whereas there are several additional variables with honeycomb core sandwiches. The density of the core is a variable in the problem since the elastic moduli are functions of the core density.

There are two important design methods devised by the British which would appear to give more efficient columns than the methods generally used in this country for design purposes. These methods are developed in references 51 and 52 for Steel face material and Calcium Alginate core. However, these methods could be applied to any face material and any isotropic core. The only differences between the two methods are the expressions used for the different types of failure. It has been substantiated in these reports that the most efficient column or structure will be one in which all the modes of failure possible occur simultaneously at the given design load and length.

There are three types of failure to consider for sandwich columns. These failures are crushing of the faces, wrinkling failure and modified Euler buckling of the panel as noted in reference 53. The different variables in the problem are determined so that these three types of failure occur simultaneously to give the minimum total column weight. This procedure is carried out in references 51 and 52. The results are presented in the form of graphs giving E_d, d/L, s/d, and the efficiency as functions of P/L where P is the load to be carried. The conditions usually imposed on a designer are the load P and length L over which it must be carried. From the design conditions the value of P/L can be determined. This value is then used on the charts to obtain the values of the transverse Young's modulus of the core E_d, the core thickness d, the skin thickness s and the weight from a plot of the efficiency. From a chart for the core material, the density required to give the required E_d can be read. This procedure is possible because most core materials used in sandwiches have a wide range of density and the material properties vary with density. The core material properties should be such that they allow simultaneous failure in the three modes for the face material chosen.

Where the mechanical properties of the core do not permit simultaneous failure in all three modes the procedure called for in reference 52 is to make wrinkling and modified Euler failure coincide and then differentiate with respect to the efficiency to obtain the optimum.

Reference 52 clearly shows the fact that a given core and face material has a certain range in which its efficiency is best and that type of sandwich is most applicable in that range.

Most of the work done by the British is based on a number called the structural loading coefficient. A derivation of the structural loading coefficient for a column is given in reference 53. This coefficient is independent of the scale of the structure. Similar coefficients can be derived for other types of structure from the principle of geometric similarity.

A design method similar to the one presented for struts is also presented in reference 52 for sandwich cylinders. This design is based on the fact stated in the report that only wrinkling and compression failure occur in sandwich cylinders. However, this is not true for curved sandwich panels as has been shown in reference 29.

A discussion of the general characteristics and conflicting design conditions for sandwich columns is given in reference 50. This and reference 94 are two of the most basic papers on the subject of the analysis of sandwich construction. Reference 50 gives a discussion of the method of investigating the probable behavior of a particular column once it is designed. For theoretical comparison purposes, the behavior of steel faced sandwiches with onazote, balsa and plywood cores is given over a wide range of structural loadings coefficients. The curves and discussion show effectively the basis for the design method given in references 51 and 52. The effects of choosing too weak and too strong a core are clearly shown and discussed.

Quantitatively the design methods given in references 51 and 52 do not agree due to the fact that different formulas have been used for predicting the failing loads. As stated in the reports, a test program is being carried out to determine which method gives the best results.

Several comments regarding this work should be made. The method given appears to be the best theoretical work on the subject of determining the optimum design of sandwich columns. Once the load to be designed for and the length over which it must be carried are determined, the optimum design column can be read directly from the charts. The shear deformations in the core have been taken into account and the yield stress was chosen as the crushing strength of the material so stresses above the proportional limit need not be considered. Actually as stated in reference 92 this is a desirable choice of the crushing strength. Further, this method and the equations used can be applied to other isotropic sets of face and core materials. The substitution of aluminum for steel would increase the overall efficiencies obtained. There are a number of isotropic core materials that can be used as well as other face materials. Although more complex, this method can be extended to aeolotropic cores and faces with the desirable feature that the most efficient design for a sandwich strut with the given materials has been obtained under all conditions of load and length. Only a limited experimental investigation of the type suggested in reference 53 would be necessary to check the validity of the design curves.

From a research into the optimum design methods suggested by papers written in this country the design variable most consistently neglected has been the core density. Most of the core materials used for sandwich construction can be obtained with a number of different densities and properties that vary with density. The British theoretical work has taken this factor into account to make as many types of failure occur simultaneously as possible with a given core material. Although it is impractical to obtain core materials in all possible densities, it is practical to stock the core materials with several different densities if an adequate saving in weight is possible by doing so. To some extent this has been done.

Several limitations of the method noted should also be stated. The method is based on geometrically perfect component parts. The effect of initial stresses and variation in properties will cause the actual column to fail below the theoretical value. As noted previously, data on this factor are being obtained at the Forest Products Laboratory and this effect can be taken into account when sufficient data become available. In addition, the strength of the core is assumed to be sufficient to allow these types of failure to occur. Thus, a stress analysis of the core is necessary to see that shear, tension or compression failure in the core do not occur first. The load carried by the core has been neglected which is satisfactory for most core material. However, this is not satisfactory for balsa wood core with grain parallel to the load.

With the honeycomb type of core material, an additional type of failure occurs. That is, buckling of the faces into the cells of the honeycomb. However, there is an additional design variable, the cell size. Since the wall thickness of the honeycomb can be varied, the same elastic modulus can be obtained for a wide range of cell sizes. The criterion for optimum design in this case is that all four methods of failure occur simultaneously. The additional design variables are available to make this possible. The distinct advantage of honeycomb cores is the wide range of densities and properties that can be obtained. If this factor is neglected, the optimum design is not obtainable. Though from practical considerations, it may not be possible to obtain the optimum design.

Efficient, light core sandwiches are in some cases very thick. Usually due to the elimination of stringers and increased rib and frame spacing, the thickness is not objectionable. The actual limitations depend on the particular application.

The analysis of panels under various load conditions is difficult. However, the general principle of obtaining as many types of failure as possible simultaneously with efficient core and face materials is applicable in any case. A trial and error procedure of guessing at the core and face dimensions and then modifying them until reasonable agreement for the failing loads is obtained will be useful in many cases as suggested in reference 117.

A design procedure is suggested in reference 92 for sandwich panels which may prove of value. The methods developed by the Forest Products Laboratory will eventually be published as ANC-23.

There are several additional design methods available for determining the dimensions of efficient panels. This problem is considered from the flexural stiffness standpoint neglecting shear deformation in the core in reference 167. It is pointed out there that the methods used indicate that maximum flexural stiffness and strength cannot be obtained simultaneously. This method, of course, can only obtain approximate results, but it would be of interest to compare these results with the more exact solutions to see if the more exact solutions of the problem give appreciably different results. Actual test results on various core materials using aluminum faces are presented in reference 167.

Other approaches to the problem are presented in references 88, 63 and 117. However, in some of these cases although shear deformation is considered, the simultaneous failure principle of obtaining the maximum efficiency is not considered. In the case of reference 88, wrinkling is said to occur only with impractical slenderness ratios and face thicknesses are taken above a minimum value at which the face will buckle into the honeycomb cells. However, these statements are true only if the variable properties of the core and cell sizes are neglected. If these variables are introduced it is quite probable that more efficient columns could be obtained than indicated by the method given in reference 88.

From the standpoint of design it must be possible to determine the required characteristics of the core material so that failures will not occur in the core. The minimum shear modulus required for a panel to support a given load can be determined approximately by the method proposed in reference 92. A method is also suggested there and a discussion given on how to obtain the core thickness for the lightest sandwich panel. It is suggested that the face dimensions be determined from the normal stresses. Wrinkling failure and shear failure in the core may limit the design, but these have not been considered in this design method. It is stated in reference 92 that shear stresses in the core may limit the design load as well as initial eccentricities.

The face to core bond must have adequate strength in any design so this is a factor which must be checked by stress analysis.

Referemce 117 presents a method of obtaining the minimum required core modulus of elasticity for a given face material based on an approximate expression for the wrinkle type failure.

10.2 Detail Design

The detail design of aircraft incorporating sandwich construction presents problems peculiar to this type of construction. There are no basic difficulties, but experience and development is required to solve many of the problems. The references that discuss this phase of design work are references 60, 61, 62, 63 and 92.

10.3 Allowable Stresses

Information on the allowable stresses for sandwich construction is available in references 61, 63, 92 and 37. The allowable strengths have, as a general rule, been determined by test. Exceedingly high values have been obtained. Additional information in connection with allowable stresses obtained from actual manufactured structures is given in references 123, 124 and 156.

A limited amount of information is available on the creep behavior of sandwich construction in reference 63. This may prove to be a limiting design condition at high temperatures.

10.4 Fatigue Strength

Information of fatigue of sandwich construction can be found in references 30 to 35, 62 and 63. The fatigue properties of sandwich construction appear to be more than adequate.

10.5 Additional Factual Design Information

Further discussions regarding the suitability of sandwich construction including the general durability of cores and sandwich panels, fatigue properties, crushing and surface damage, accelerated weathering and water immersion tests, service experience, gunfire tests, vibration, deterioration and erosion properties are given in references 60 to 63, 20, 38, 40 and 42.

Additional information on an extensive investigation of sandwich construction being carried on by the Army Air Forces is published in reports from Wright Field. The Navy has also done a considerable amount of work with sandwich construction.

XI FACE, CORE AND BONDING MATERIALS USED IN SANDWICH CONSTRUCTION

11.1 Face Materials

There is a vast amount of data available on the properties of various materials used for sandwich faces. The material selection should depend on the particular application.

Aluminum alloy has proved to be the most efficient face material in the majority of applications as is stated in reference 61 while glass cloth laminate is a promising face material and suitable for molding where aluminum alloy cannot be used.

References 51 and 53 which are British reports give the properties of some materials and consider the problem of selection of face materiall from the standpoint of coefficients of merit for the materials depending on the type of failure. From these reports, the desirable qualities of face materials are high allowable stress to weight ratio for crushing, high ratio of Young's modulus to the cube of the density for wrinkling, and a high Young's modulus to density ratio for modified Euler type failure.

For further information of face materials references 51, 53 and 61 should be consulted. The properties of suitable face materials are given by many of the references at the end of this thesis.

11.2 Core Materials

Similar to face materials, there is also a vast amount of information on core materials.

The generally desired range of properties of core materials and general specifications are given by the Air Materiel Command in reference 129.

The best general discussion of core materials is given in reference 53 where it is pointed out that for efficient use of sandwich construction, the total weight of the composite structure must be a minimum. Only when simultaneous failure in the three modes is approached is there complete evidence that the materials have been used to the best advantage. This requires that the core material be available over a wide range of densities. When the core material is available only in one form so that no saving will result from reducing core stiffness to the minimum necessary, the criterion of efficiency is that the two types of failure that can be adjusted by changing other variables occur simultaneously.

Actually the various core strengths required for specific uses in sandwich construction have never been established. The reason for this is the fact that no really accurate method of completely stress analysing sandwich construction is available.

Providing the strength requirements are satisfied, the most important deciding factors are the shear modulus, the Young's modulus and the density. For a given density a high shear modulus is desired to limit shear deformations in the sandwich in the modified Euler type of buckling. A high Young's modulus is desired to prevent local wrinkling. The lowest density consistent with the required properties is desired to reduce the total weight of the sandwich.

General discussion of the factors affecting the core selection are given in references 9, 17, 18, 19, 60 to 63, 88, 50, 53, 54, 51, 92, 96, 99, 117 and 129.

A few of the most generally used or suitable core materials are balsa wood, paper and glass cloth honeycombs, cellular cellulose acetate, aluminum foil honeycomb, and strips of cellular cellulose acetate wrapped in glass cloth laminate face material.

An example of what is desired in synthetic core materials is calcium alginate. Its mechanical properties for sandwich construction purposes are the best so far obtained and it is available over a wide density range. The reasons for its lack of general use are the facts that it is costly, subject to degradation if subjected to moderate temperatures in the presence of moisture and difficult to manufacture.

There are a number of other core materials available and for certain applications may be superior to the others mentioned.

Balsa and paper honeycomb core materials are the only ones that have found widespread general use. British practice is to place the balsa with the grain parallel to the load in order to take advantage of its load carrying ability in that direction while American practice is to place the grain perpendicular to the faces in order to prevent local instability of the face material and take advantage of a number of practical features to be gained by that arrangement as discussed in reference 61. The paper honeycomb has the advantage of variable density over an unlimited range and is more uniform than balsa although the gluing area available is considerably less than with balsa.

A relatively complete picture of the aspects of using balsa for a core material can be obtained from references 61, 63, 53, 92 and 167. The actual properties of the material are given rather completely in reference 10 as well as several other Forest Products Laboratory reports.

The various aspects of using honeycomb core material can be obtained from references 53, 63, 97, 99 and 167.

Cellular cellulose acetate is considered in references 53, 93 and several magazine references.

Calcium alginate is described and the properties given in references 53, 54, 51 and 52.

Information and results obtained from various kinds of woods are given in reference 167 while various other types of core materials are considered in references 53, 54 and 47.

11.3 Bonding Materials

The subject of bonding materials and adhesives is considered in references 61 and 63. Additional information is available in references 20, 38, 39, 40, 95 and 117.

Of particular interest to the future of sandwich construction is the investigation made in reference 20 of the tensile strength at elevated temperatures of glued joints between aluminum and end grain balsa. The tests were carried out up to 190°F and it was found that the glue and process had a material effect on the strength of the bond at elevated temperatures. Several processes were found which gave bonds that showed little or no change in strength up to 190°F.

XII WEIGHT COMPARISON BETWEEN SANDWICH AND CONVENTIONAL SHEET STRINGER CONSTRUCTION

The conclusion of Chance Vought as given in reference 61 is that their particular balsa aluminum sandwich construction is competitive in weight for nearly all aircraft structural applications and more economical in weight for most of them as compared to conventional sheet stringer construction.

It was noted from tests reported in reference 57 that within the ranges considered in the report, the sandwich construction was 20 to 30% more efficient than the corresponding sheet stringer construction. Curved sandwich panels are considered to have a much greater weight advantage over conventional construction than flat panels.

In reference 88 a weight comparison was made between the sheet stringer construction and 24ST Honeycomb struts and it was noted that the results slightly favored the sheet stringer construction. However, as stated, the comparison neglects weight of transverse supporting members, connections and other details. Weight saving due to elimination of supporting ribs or transverse formers is a large factor for sandwich construction. It should probably be noted that the design method used in this case was probably based on modified Euler failure rather than the optimum design method of simultaneous failure in the four possible modes for honeycomb sandwiches.

Tests shown in reference 52 indicate that balsa aluminum sandwiches are more efficient as compression columns than the conventional construction.

Reference 61 states that fair weight comparisons can be made only on actual parts designed and fabricated with the two different types of construction and service tested under comparable consitions. A metalite stabilizer on the Chance Vought F4U replaced the corresponding sheet stringer construction with a saving of 5 to 10% in weight even though the sandwich stabilizer was overstrength 65%. If the sandwich stabilizer had been redesigned for just adequate strength, a considerably greater saving in weight would have been realized. This undoubtedly was not done due to lack of experience with sandwich construction.

A fuselage section and a wing panel to replace the conventional constructions were produced by the Army Air Forces and complete reports given in references 123 and 124. In reference 156 a summary of the fuselage design is given. Two sandwich construction fuselages had total weights of 75 and 78 as compared to 70 pounds for the conventional one but the sandwich sections were both 75% stronger than the conventional construction. By redesign a substantial weight saving could be obtained.

A model size wing section was manufactured from sandwich construction as noted in reference 62 with the result that the sandwich wing was 40% lighter than the conventional construction. This part had severe second degree curvature and relatively small thickness ratio which were largely responsible for the superiority of the sandwich construction. The construction was made with glass fabric skins and cellular cellulose acetate core.

Any comparison on the basis of weight neglects the added aerodynamic efficiency and other advantages of sandwich construction previously noted. In the outer wing panel designed by the Army Air Forces very little metal of any kind was incorporated, yet the wing tip deflection was reduced as compared to the conventional outer wing installation.

The above comparisons indicate that even in its present crude stage of development sandwich construction is an effective type of construction for aircraft.

XIII CURRENT APPLICATIONS OF SANDWICH CONSTRUCTION

The most important current applications of sandwich construction located in this research have been on the Chance Vought F4U, F5U, F6U and F7U airplanes. These applications with the exception of the F7U are discussed in reference 61. About 75% of the aerodynamic surfaces of the F5U and about 95% of the structure of the F6U are of sandwich construction.

Boeing aircraft has made major applications of sandwich construction on their model M-337. Martin has made similar applications to a twin engine transport plane.

The general application of sandwich construction to radomes has proved advantageous.

If further details are desired on the applications of sandwich construction references 61, 62, 63, 123, 124, 60 and 92 should be consulted.

An extensive investigation of cargo flooring has been made and reported in references 21 to 26. Sandwich construction has proved very effective in this application.

Three difficulties with applying sandwich construction have been noted. Two major problems encountered by Boeing in using sandwich construction have been the lack of efficient means of making attachments and the inability to inspect the material other than by destructive tests of samples which makes extremely close quality control necessary. This difficulty has also been noted in reference 61. The problem of field repair has largely precluded the use of sandwich construction at the Douglas Aircraft Company, Inc. El Segundo plant since primarily military aircraft are constructed at that plant. However, work has been done on this problem both by the army and by Chance Vought since Chance Vought is using this type of construction on military ships.

It is interesting to note that the Douglas Aircraft Company, Inc. has found good use for sandwich construction in the construction of Arctic huts which must be transported by air.

XIV MANUFACTURE, REPAIR AND INSPECTION OF SANDWICH CONSTRUCTION

A brief discussion of the methods of manufacture of sandwich construction is given in reference 53. However, more extensive and useful information is avialable in references 43, 60, 61 and 117. References 123 and 124 give additional information of the subject.

The problem of inspection of sandwich construction is investigated and the results reported in reference 39. A further discussion is also given in reference 61 on this problem.

The problem of repair of sandwich construction is considered in references 46, 61 and 92. According to these sources of information adequate methods of repair have been developed for this type of construction which would permit more extended use of this type of construction in military aircraft.

XV CONCLUSIONS

The general purpose of this thesis has been to provide a survey and preliminary correlation of the existing design information of sandwich construction. It was also intended that it should provide a basis for further correlation work on this subject.

It is felt that this general purpose has been accomplished and proper reference made for guidance to further research work to be done on the subject under consideration.

Although no final conclusions can be drawn from this thesis work, several preliminary conclusions are in order. The first conclusion to be drawn is that to the extent of the research done, no basic limitations to the application of sandwich construction could be found in any phase of the work. The second is that considerable weight saving and other advantages will become possible with the further development of this type of construction. It is assumed however that considerable development work will be required by new users of this type of construction before the full benefits can be obtained.

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