APPLICATION OF THE RAM JET
TO VERTICAL ASCENT

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James Victor Rowney

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SUMMARY

The purpose of this paper is to investigate the applicability of the ramjet to replace the first step of a two-step or multi-step rocket, with the hope of increasing the efficiency and performance of vertical ascent missiles of the present day. The ramjet is considered as a booster motor to boost the primary missile through the atmosphere. It is to be discarded from the primary missile after reaching its maximum velocity. To boost the ramjet to operating speed the second stage rocket must be operated for the first few seconds as a ducted rocket.

General ramjet performance is calculated graphically by using a step-by-step integration process to solve the differential equation of motion. The resulting flight velocity, fuel consumption per initial weight, and altitude are presented graphically in terms of time after launching the ramjet. It is assumed throughout the problem that gravity is constant and that the altitude necessary to start the ramjet is negligible. The acceleration of the missile is limited to 25 g's.

Important results present in this paper are: The most important factor that limits the performance of the ramjet is the air density ratio. The greatest increase in second-step launching altitude, by improved thrust and drag coefficients and increased ramjet cross-section area, is achieved at low ramjet launching velocities. The performance of the ramjet missile operating at

a specific fuel consumption of .0007 $\frac{\text{lbs/sec}}{\text{lbs thrust}}$ shows a marked increase of efficiency over a missile using a higher fuel consumption value. Missiles using a lower specific fuel consumption value, of the same order as the increased value, show negligible improvement in efficiency. When ramjet performance is compared to rocket performance; specifically, maximum velocity, altitude of maximum speed and altitude attained, the ramjet missile will burn only one third of the fuel required by a two-step rocket. Conversely, for the same fuel consumed, the ramjet missile will attain 65 % more altitude than the two-step rocket.

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LIST OF SYMBOLS

A = cross-sectional area of the missile

C = exhaust velocity of a rocket

C_D = coefficient of drag

C_F = coefficient of thrust

C_f = specific fuel consumption

D = drag of missile

F = thrust of missile

g = acceleration of gravity

h = altitude

k = any constant

M_o = initial mass of rocket

M_f = final mass of rocket

M_p = mass of rocket propellant

t = time of flight

t_p = burning time of a rocket or time for ramjet to reach maximum velocity

V = flight velocity of the missile

V_{max} = maximum flight velocity of the missile

W = instantaneous weight of the missile

W_{BF} = weight of booster fuel for ramjet missile

W_f = weight of ramjet missile fuel

W_m = weight of ramjet missile structure

ϵ = parameter defined by initial conditions, $\frac{1}{2} \rho_o V_o^2 \frac{A}{W_o}$

$\frac{T.F.C.}{W_o}$ = total fuel consumption per pound of missile initial weight up to time t.

LIST OF SYMBOLS (Cont'd)

$\frac{dv}{dt}$ = acceleration of the missile

$\frac{W_o}{A}$ = initial weight per square foot of missile cross-sectional area

Δ = an increment

ρ = density of standard air

σ = ρ/ρ_o , density ratio of standard atmosphere

ζ = loading factor of a rocket, M_P/M_o

λ = payload ratio = $\frac{\text{weight of second step missile}}{\text{initial weight of missile at sea level}}$

optimum altitude = the altitude at which the flight velocity of the missile is a maximum

zero subscript = initial condition

APPLICATION OF THE RAMJET TO VERTICAL ASCENT

I. INTRODUCTION

This paper is an investigation of the applicability of the ramjet to replace the first step of a two-step or multi-step rocket, with the hope of increasing the efficiency and performance of the vertical ascending missiles of the present day.

Since the ramjet is an air breathing engine, it seems only natural that it would operate more efficiently from the standpoint of fuel consumption and payload ratio, than a primary-step rocket which is forced to carry its own oxygen through the range of altitudes within the atmosphere.

The ramjet is considered as a booster motor to boost the primary missile through the atmosphere, or it may be used as an "atmospheric-step" of a multi-step rocket operation. It is to be discarded after reaching its maximum velocity so that the second stage may use this ramjet velocity as an initial velocity. The primary missile, or the second stage, as the case may be, can then be launched at high altitude with a high initial velocity and a substantial fuel saving. To boost the ramjet to operating speed the second step rocket must be operated for the first few seconds as a ducted rocket.

A preliminary study entitled "Application of the Ramjet to

High Altitude Sounding Vehicles" was made by L. H. Schindel in his thesis written at M.I.T. in 1948. This paper, also suggested by Dr. H. S. Tsien, is an analytic study of the ramjet in vertical ascent but rather limited to the specific ramjet with which to boost a V-2 rocket. The results obtained by Schindel indicate that only one third as much fuel is necessary for the ramjet rocket combination as compared to the two-step rocket. The ramjet rocket therefore allows the second-step rocket to carry larger payloads or attain higher altitudes. Schindel's paper appears as Ref. 1.

The method of obtaining general ramjet performance in this paper is to express the differential equation of vertical motion in terms of ramjet coefficients of thrust, drag, and specific fuel consumption based on the maximum cross-sectional area of the ramjet. These coefficients together with initial conditions are grouped into two parameters which are made to vary well beyond the present day range of fuels and materials. In obtaining the continuously varying velocities, accelerations, and fuel consumption rates over the vertical flight path, a step-by-step integration process was used. Calculations were made using time intervals of one second and prescribing the initial velocity. Only vertical flight is considered and this is justified since the problem is calculated entirely within the powered flight trajectory. Throughout the problem gravity is assumed constant, the altitude necessary to start the ramjet is neglected, accelerations of the ramjet missile are limited to 25 g's, and the time rate of fuel consumption of the ramjet is assumed proportional to the net thrust and inversely proportional to the flight velocity. The ramjet is also assumed to be capable of burning fuel in the rarefied atmosphere.

The resulting flight velocity, fuel consumption per initial weight, and altitude presented graphically in terms of time after launching the ramjet show several important results: The most important factor that limits the performance of the ramjet is the air density ratio. The greatest increase in second-step launching altitude, by improved thrust and drag coefficients and increased ramjet cross-sectional area, is achieved at low ramjet launching velocities rather than at high launching velocities. The performance of the ramjet missile operating at a specific fuel consumption of $.0007 \frac{\text{lbs/sec}}{\text{lbs thrust}}$ shows a marked increase of efficiency over a missile using a higher fuel consumption value. Missiles using a lower specific fuel consumption value, of the same order as the increased value, show negligible improvement in efficiency. When the ramjet missile is compared to a two-step rocket in Section V it is compared on the basis of equal maximum velocity and altitude attained at this velocity for the first step-rocket and the ramjet-step. The fuel used in the ducted rocket operation necessary to boost the ramjet is charged against the ramjet in comparing fuel consumption of the two missiles. It is found, as a result, that the ramjet missile will burn only one third of the fuel required by the two-step rocket and, therefore, the ramjet missile will have a payload ratio of three times that of the two-step rocket. Conversely, for the same fuel consumed, the ramjet missile will attain 65% more altitude than the two-step rocket.

II. GENERAL METHOD OF RAMJET PERFORMANCE

Newton's First Law of Motion is the fundamental equation used to develop the general equation of ramjet performance. This equation of motion is applied to the vertical direction only and the sum of the vertical forces present is equated to the product of the instantaneous mass and the acceleration in differential form. The forces of thrust and drag are changed to coefficient form since the forces are functions of flight velocity. The ratio of flight velocity to initial velocity and the ratio of air density to initial air density are then created and the initial functions grouped together to form the non-dimensional parameter $\epsilon = \frac{1}{2}\rho_o V_o^2 \frac{A}{W_o}$ which may also be described as the initial force per pound of weight needed to accelerate the missile.

The instantaneous weight is changed into the sum of the structural and payload weight and the weight of ramjet fuel. The weight of the ramjet fuel is, of course, decreased by the fuel flow which is equal to the specific fuel consumption times the thrust. The ratio of instantaneous weight to initial weight then becomes

$$1 - \frac{\text{Total Fuel Consumption}}{\text{Initial Weight}} = 1 - \frac{T.F.C.}{W_o}$$

where the total fuel consumption per pound of initial weight is expressed as $\epsilon \int_0^t c_f C_F \sigma \left(\frac{V}{V_o}\right)^2 dt$

The time rate of fuel consumption of the ramjet is, however, assumed proportional to the net thrust and inversely proportional to the flight velocity. The thrust coefficient is also considered a mean value that is constant for a given flight. This leads to a group of terms considered as the second parameter, $\frac{C_{f_0} C_F}{C_F - C_D}$, which has the dimensions of $\frac{1}{\text{sec.}}$ due to the initial specific fuel consumption. The general equation of ramjet performance then becomes

$$\frac{1}{g}\frac{dv}{dt} = \epsilon\left(C_F - C_D\right) \frac{\sigma\left(\frac{V}{V_0}\right)^2}{1 - \epsilon\left(C_F - C_D\right)\frac{C_{f_0} C_F}{C_F - C_D}\int_0^{\tau} \sigma \frac{V}{V_0}\, dt} - 1$$

which may also be described as

$$\frac{\text{The change of flight velocity}}{\text{Unit time}} = \left[\frac{\frac{\text{initial force}}{W_0}\,\sigma\left(\frac{V}{V_0}\right)^2}{1 - \frac{T.F.C.}{W_0}} - 1\right] g$$

With this formula ramjet performance is calculated as described in section III of this paper.

A. Analytical Development

The equation of motion in the vertical direction is:

$$F - D - W = \frac{W}{g}\frac{dV}{dt} \qquad (1)$$

where F = net thrust of the missile

D = drag of the missile

W = instantaneous weight of the missile

g = gravity constant, 32.2 feet per second squared

$\frac{dv}{dt}$ = rate of change of velocity with time, acceleration, feet per second squared

Dividing by instantaneous weight, the following is obtained:

$$\frac{F - D}{W} - 1 = \frac{1}{g}\frac{dv}{dt} \qquad (2)$$

The thrust and drag are now put into coefficient form based on the maximum cross-sectional area of the missile

$$F = C_F \frac{\rho}{2} AV^2 \qquad D = C_D \frac{\rho}{2} AV^2 \qquad (3)$$

where C_F = thrust coefficient, non-dimensional

C_D = drag coefficient, non-dimensional

ρ = density of air in slugs per cubic foot

A = maximum cross-sectional area of missile in square feet

V = flight velocity feet per second

The instantaneous weight is equal to the weight of the missile, W_m, which is constant throughout the flight; plus the weight of the fuel, W_f, which is decreasing during the flight at the rate of the fuel flow.

$$W = W_m + W_f$$
$$W_o = W_m + W_{f_o} \qquad (4)$$

where the zero subscript denotes the initial condition.

Then $$\frac{W_m}{W_o} + \frac{W_{f_o}}{W_o} = 1 \qquad (5)$$

Substituting equations (3) and (4) into equation (2) the following results:

$$\frac{1}{g}\frac{dv}{dt} = \left(\frac{\frac{1}{2}A\rho_o V_o^2}{W_o}\right)\frac{\sigma\left(\frac{V}{V_o}\right)^2(C_F - C_D)}{\frac{W_m}{W_o} + \frac{W_f}{W_o}} - 1 \tag{6}$$

where σ = ρ/ρ_o , density ratio

V_o = initial flight velocity of the missile, feet per second

V = flight velocity at any time, feet per second

For initial conditions, starting at sea level

$$\left(\frac{V}{V_o}\right)^2 = 1 \qquad \sigma = 1 \qquad \frac{W_m}{W_o} + \frac{W_{f_o}}{W_o} = 1$$

$$\therefore \frac{1}{g}\left(\frac{dv}{dt}\right)_{t=o} = \left(\frac{\frac{1}{2}A\rho_o V_o^2}{W_o}\right)(C_F - C_D) - 1 \tag{7}$$

Let one parameter be ϵ (non-dimensional) defined as:

$$\epsilon = \frac{\frac{1}{2}A\rho_o V_o^2}{W_o} = \left[\frac{1}{g}\left(\frac{dv}{dt}\right)_{t=o} + 1\right]\left[\frac{1}{C_{F_o} - C_{D_o}}\right] \tag{8}$$

Substituting (8) into (6) results in:

$$\frac{1}{g}\frac{dv}{dt} = \epsilon(C_F - C_D)\frac{\sigma\left(\frac{V}{V_o}\right)^2}{\frac{W_m}{W_o} + \frac{W_f}{W_o}} - 1 \tag{9}$$

Specific Fuel Consumption, C_f of most thermal jet engines is defined as the rate of fuel consumption in pounds per second divided by the thrust in pounds:

$$C_f = \frac{\text{Fuel Flow}}{F} \qquad 1/\text{sec.}$$

Fuel flow = rate of fuel consumption

Fuel consumption to time t = $\int_0^t C_f F \, dt$ lbs.

The instantaneous weight of the fuel is equal to the initial weight of the fuel minus the fuel consumed.

$$W_f = W_{f_0} - \int_0^t C_f F \, dt$$

$$W_f = W_{f_0} - \frac{1}{2} \rho_0 A V_0^2 \int_0^t \sigma \left(\frac{V}{V_0}\right)^2 C_f C_F \, dt$$

$$\frac{W_f}{W_0} = \frac{W_{f_0}}{W_0} - \epsilon \int_0^t C_f C_F \, \sigma \left(\frac{V}{V_0}\right)^2 dt$$

$$\frac{W_f}{W_0} + \frac{W_m}{W_0} = \frac{W_m}{W_0} + \frac{W_{f_0}}{W_0} - \epsilon \int_0^t C_f C_F \, \sigma \left(\frac{V}{V_0}\right)^2 dt$$

$$\frac{W_m}{W_0} + \frac{W_{f_0}}{W_0} = 1 \qquad \text{by equation} \quad (4)$$

$$\therefore \frac{W_f}{W_0} + \frac{W_m}{W_0} = 1 - \epsilon \int_0^t C_f C_F \, \sigma \left(\frac{V}{V_0}\right)^2 dt \qquad (10)$$

where $\epsilon \int_0^t C_f C_F \, \sigma \left(\frac{V}{V_0}\right)^2 dt$ is the fuel consumption of the missile per pound of initial weight.

Equation (9) then becomes

$$\frac{1}{g} \frac{dV}{dt} = \frac{\epsilon (C_F - C_D) \quad \sigma \left(\frac{V}{V_0}\right)^2}{1 - \epsilon \int_0^t C_f C_F \, \sigma \left(\frac{V}{V_0}\right)^2 dt} - 1 \qquad (11)$$

but from assumption (5) the time rate of fuel consumption of the ramjet is proportional to the net thrust and inversely proportional to the flight velocity.

$$\text{Fuel flow} = \frac{KF}{V} \quad \text{lbs/sec}$$

and $$C_f = \frac{K}{V} \quad \text{1/sec}$$

where K = a constant in ft/sec^2

$$\therefore \quad \frac{C_f}{C_{f_o}} = \frac{V_o}{V}$$

Equation (11) becomes

$$\frac{1}{g}\frac{dv}{dt} = \frac{\epsilon\,(C_F - C_D)\;\sigma\left(\frac{V}{V_o}\right)^2}{1 - \epsilon(C_F - C_D)\frac{C_{f_o} C_F}{C_F - C_D}\int_o^\tau \sigma \frac{V}{V_o}\,dt} - 1 \qquad (12)$$

where C_F is considered a mean value, constant with respect to time and velocity over a given flight.

B. Parameters and Range of Parameters

Consider the two parameters $\epsilon\left(C_F - C_D\right)$ and $\frac{C_{f_o} C_F}{C_F - C_D}$ which may also be described as functions of the initial force per unit weight to accelerate the missile and the initial specific fuel consumption. These parameters must be varied through a range of values so that, when substituted into equation (12), they will give ramjet performance comparable to that which may be developed within the next decade.

Substituting initial conditions into equation (12), i.e.,

$\sigma = 1$, $\left(\frac{V}{V_0}\right)^2 = 1$, $dt = 0$ it is seen that for initial zero acceleration, $\epsilon(C_F - C_D) = 1$. Therefore unity is the lower limit of this parameter. To find the upper limit of this parameter proved to be a little more complicated and a few trial calculations were necessary in order to establish the values. Values of $\frac{W_0}{A}$ were selected from 100 to 1100 since $\frac{W_0}{A}$ = 1000 is approximately that of a V-2 rocket. Ramjet areas are much larger than comparable rockets and so $\frac{W_0}{A}$ = 100 was selected in the belief that the area was large enough to provide excessive thrust for the given weight. This proved to be the case as shown in Fig. 1. The ϵ was calculated for different initial velocities as shown in the table below:

	ϵ for			
$\frac{W_0}{A}$	$V_0 = 894$	$V_0 = 1230$	$V_0 = 1675$	$V_0 = 2230$
100	9.5	-	-	-
300	-	7.1	11.1	-
500	-	-	6.7	11.8
700	-	-	4.8	8.44
900	-	-	3.7	6.56
1100	-	-	3.0	5.36

where $$\epsilon = \frac{1}{2} \rho_0 V_0^2 \frac{A}{W_0}$$

Blanks in the lower V_0 columns under the numbers 9.5 and 7.1 show the area where $\epsilon(C_F - C_D) \lesssim 1$ and therefore, the area where the acceleration is zero or negative. Blanks before the numbers 7.1, 11.1, and 11.8 in the second third and fourth columns, respectively, show the area where the acceleration is expected to be too high for our arbitrary limit of 25 g's. The acceleration can be expected to be high where the W_o is low and where the 'A' is high which indicates excessive thrust for the initial weight, specifically, when this low initial weight is launched at high initial velocities. This proved to be the case when several trial calculations were made with the above values of ϵ substituted into equation (12). Values of ϵ = 12 gave tremendous accelerations and velocities, and so it was at this point that accelerations were arbitrarily limited to 25 g's (see Fig. 1) and $\epsilon(C_F - C_D)$ fell into a range of numbers which increased with the initial velocity as follows:

V_o	$\epsilon(C_F - C_D)$
894	1 to 2.5
1230	1 to 4
1790	1 to 6
2230	1 to 8

To calculate the range of the parameter $\frac{C_{f_o} C_F}{C_F - C_D}$ the ramjet performance coefficients were estimated from Fig. 2 as follows:

C_F ranges from 0.5 to 1.5

C_D ranges from 0.1 to 0.4

$C_F - C_D$ ranges from 0.1 to 1.4

C_{f_o} ranges from 1 $\frac{lb/hr}{lb}$ to 18 $\frac{lb/hr}{lb}$

or .0002 $\frac{lb/sec}{lb}$ to .005 $\frac{lb/sec}{lb}$

$\frac{C_{f_o} C_F}{C_F - C_D}$ was first varied between

$$\frac{.0002 \times .5}{1.4} \cong .0001 \quad \text{and} \quad \frac{.005 \times 1.5}{.1} \cong 0.1$$

and held constant for the first set of curves (Figs. 3 to 6) at its normal operating value of $\frac{.0011 \times 1.0}{1.0 - .25} \cong .001$. However, it was found in trail calculations that as $\frac{C_{f_o} C_F}{C_F - C_D}$ increased to 0.1 the acceleration of the normal missile increased by a factor of 20. This increase of acceleration is due to the fact that the fuel is now such a sizeable quantity as to affect the decrease in weight very markedly and therefore to increase the acceleration. This increase in acceleration is so tremendous, however, that it was thought more prudent to decrease the value of $\frac{C_{f_o} C_F}{C_F - C_D}$ to values of a rocket operation with high drag, i.e., C_{f_o} = .005, C_F = 2.0, C_D = 1.0 to give $\frac{C_{f_o} C_F}{C_F - C_D}$ = .01 as the top limit. Even this value gives accelerations up to 60 g's in some cases.

III. METHOD OF CALCULATION

The method of calculating equation (12) is a graphical step-by-step process which solves the equation for each one second interval using the result for the next second solution. The equation may also be solved analytically, but the graphical solution lends itself better to the boundary conditions. The first parameter is varied through its range of values for one set of calculations while the second parameter is held constant at its normal value; and vice versa, for the second set of calculations. Commencing with an assumed value of V_0 and varying values of the two parameters as noted above, the velocity ratio is found. The altitude is solved for by $h_2 = h_1 + \frac{V_1+V_2}{2} \Delta t$ and the σ is picked from an altitude table for each altitude. σ and $\frac{V}{V_0}$ are averaged with each preceding second and the product summed to evaluate the integral $\int_0^t \sigma \frac{V}{V_0} dt$ over each one second range. The product of this integral and the two parameters gives the total fuel consumption per pound of missile initial weight. When the acceleration is found in feet per second square, it is multiplied by the time interval and added to the velocity for that second to give the velocity for the next second. This process is repeated until the acceleration is zero, and at this point maximum flight velocity is attained. To simplify the problem the altitude is started at zero, neglecting the altitude necessary to start the ramjet. Gravity is also considered a constant throughout the problem.

Figs. 3 to 6 show the first set of calculations with $\epsilon(C_F - C_D)$ varying and $\frac{C_{f_o} C_F}{C_F - C_D}$ held constant. These curves are plotted for flight velocity and fuel consumption per pound of initial weight versus time after launching the ramjet. $\epsilon(C_F - C_D)$ is varied between 1.5 and 8 and $\frac{C_{f_o} C_F}{C_F - C_D}$ is constant at the normal value of .001 for these sets of curves. Each Fig. represents the different initial velocities of 894, 1230, 1790, 2230 feet per second which correspond respectively to Mach numbers 0.8, 1.2, 1.6, and 2.0 at sea level. Fig. 7 shows the altitudes attained by the missile flights represented in Fig. 3 to 6 and Fig. 8 shows the optimum altitudes of these flights plotted against the parameter $\epsilon(C_F - C_D)$.

Figs. 9 to 12 show the second set of calculations with the parameter $\frac{C_{f_o} C_F}{C_F - C_D}$ varying between .0001 and .01 and with $\epsilon(C_F - C_D)$ constant for each V_o. $\epsilon(C_F - C_D)$, however, cannot be taken as the same constant throughout this set of calculations due to its dependence on V_o. Therefore, $\epsilon(C_F - C_D)$ is taken as 2, 4, 6, 8 for each initial velocity of 894, 1230, 1790, and 2230 feet per second, respectively. Altitude versus time is shown in Fig. 13 while Fig. 14 shows an approximation of the amount of fuel necessary to reach a given altitude with the ramjet missile defined by the parameter $\epsilon(C_F - C_D)$ and using the normal values of $\frac{C_{f_o} C_F}{C_F - C_D} = .001$.

Fig. 15 shows the effect of the different launching velocities on two configurations of ramjet missiles defined by $\frac{W_o}{A} = 500$ and 1000. These Figs. are discussed in section IV.

A numerical procedure of the calculations may be represented as follows:

Given values of V_o, $\epsilon(C_F - C_D)$, $\frac{C_{f_o} C_F}{C_F - C_D}$ and $\sigma(h)$.

Divide the time scale into intervals such as:

$$t = 0, \; t_1, \; t_2, \; t_3, \; t_4$$

or

$$0, \; \Delta t, \; 2\Delta t, \; 3\Delta t, \; 4\Delta t$$

For $0 \leq t \geq \Delta t$ or the first step, equation (12) is

$$\frac{1}{g}\frac{\Delta V_1}{\Delta t} = \frac{\epsilon(C_F - C_D)\,\sigma(h_o)}{1 - 0} - 1$$

or

$$\Delta V_1 = g\left[\epsilon(C_F - C_D) - 1\right]\Delta t$$

Then the velocity of the missile at time equal t_1 is: $V_o + \Delta V_1$

The altitude at this time is $h_1 = h_o + (V_o + \frac{1}{2}\Delta V_1)\Delta t$

For the second step compute $\sigma(h_1)$, $\left(\frac{V_o + \Delta V_1}{V_o}\right)^2$, $\frac{\sigma(h_o) + \sigma(h_1)}{2}$

and $\dfrac{\frac{V_o}{V_o} + \frac{V_o + \Delta V_1}{V_o}}{2}$ then substitute again into equation (12):

$$\frac{1}{g}\frac{\Delta V_2}{\Delta t} = \frac{\epsilon(C_F - C_D)\,\sigma(h_1)\left(\frac{V_o + \Delta V_1}{V_o}\right)^2}{1 - \epsilon(C_F - C_D)\frac{C_{f_o} C_F}{C_F - C_D}\left[\frac{\sigma(h_o) + \sigma(h_1)}{2} \times \frac{\frac{V_o}{V_o} + \frac{V_o + \Delta V_1}{V_o}}{2}\right]\Delta t} - 1$$

$$\Delta V_2 = g\left[\frac{\epsilon(C_F - C_D)\ \sigma(h_1)\left(\frac{V_0 + \Delta V_1}{V_0}\right)^2}{1 - \epsilon(C_F - C_D)\frac{C_{F_0} C_F}{C_F - C_D}\left\{\frac{\sigma(h_0) + \sigma(h_1)}{2} \times \frac{\frac{V_0}{V_0} + \frac{V_0 + \Delta V_1}{V_0}}{2}\right\}\Delta t} - 1 \right]\Delta t$$

The velocity of the missile at time t_2 is $V_0 + \Delta V_1 + \Delta V_2$

Altitude at this time is $h_2 = h_0 + (V_0 + \frac{1}{2}\Delta V_1)\Delta t + (V_0 + \Delta V_1 + \frac{1}{2}\Delta V_2)\Delta t$

In general for the n^{th} step, equation (12) is:

$$\frac{1}{g}\frac{\Delta V_n}{\Delta t} = \frac{\epsilon(C_F - C_D)\ \sigma(h_{n-1})\left(1 + \sum_{i=1}^{n-1}\frac{\Delta V_i}{V_0}\right)^2}{1 - \epsilon(C_F - C_D)\frac{C_{F_0} C_F}{C_F - C_D}\sum_{j=1}^{n-1}\left\{\left[\frac{\sigma(h_{j-1}) + \sigma(h_j)}{2}\right]\left[\frac{V(t_{j-1}) + V(t_j)}{2V_0}\right]\right\}\Delta t} - 1$$

Low initial weights correspond to high values of the parameter $\epsilon(C_F - C_D)$; therefore, the missile with the higher $\epsilon(C_F - C_D)$, at the same V_0 , will have the higher V_{max} and higher optimum altitude. If $\epsilon(C_F - C_D)$ is too high for the specific launching velocity at which it is used, excessive acceleration above the 25g limit will result. The reduction of ramjet cross-sectional area for the same initial weight missile, i.e., the increase of the $\frac{W_0}{A}$ ratio, will reduce the maximum velocity and optimum altitude, or require a higher booster velocity for the same maximum velocity and optimum altitude. On Fig. 3 the $\frac{W_0}{A}$ = 350 curve shows a V_{max} = 4700 ft/sec. If $\frac{W_0}{A}$ is increased to 475, V_{max} is reduced to 1400 ft/sec. This is at a launching velocity of 894 ft/sec. If this launching velocity is increased to 1230 ft/sec as on Fig. 4 the same $\frac{W_0}{A}$ = 350 shows a V_{max} increased to 10,000 ft/sec. In general, the effect of an increase in V_0 increases the V_{max} and optimum altitude to the same magnitude for the same $\frac{W_0}{A}$ missile. However, in Figs. 3 to 6 the allowable values of $\frac{W_0}{A}$ as well as the range of values, which keep the accelerations in practical limits, increase as V_0 increases.

The fuel consumption curves of Figs. 3 to 6 show an increase in the rate of fuel used as velocity and altitude are increased up to a point and then as maximum velocity is approached, the rate of fuel consumption shows a marked decrease. This is due to the factor in the denominator of equation (12) which factor is:

$$1 - \epsilon(C_F - C_D)\,\frac{C_{f_0} C_F}{C_F - C_D}\int_0^{\tau} \sigma\,\frac{V}{V_0}\,dt$$

and which reduces to

$$1 - \frac{1}{2}\rho_0 V_0^2\,\frac{A}{W_0}\int_0^{\tau} C_f C_F\,\sigma\left(\frac{V}{V_0}\right)^2 dt$$

where $\frac{1}{2}\rho_0 A\int_0^{\tau} C_f C_F\,\sigma V^2\,dt$ is the fuel consumption to time t. It is apparent, then, that for a given missile the fuel consumption varies with the altitude, the flight velocity, and the density ratio, σ. While σ is close to unity, at low altitude, the increase of velocity and altitude increases the fuel consumption. However, at higher altitudes, as σ decreases, decreasing density counteracts the effect of increasing velocity and altitude to rapidly decrease the rate of fuel consumption.

The results of Figs. 3 to 6 are cross plotted in Fig. 14 to show the optimum altitude reached for any configuration as depending on total fuel consumption per pound of initial weight and the parameter $\epsilon(C_F - C_D)$. Consequently, Fig. 14 may be used to give a quick approximation of the total fuel necessary for any configuration to reach a desired optimum altitude. For any given optimum altitude it can be seen that the total fuel consumption per pound increases with $\epsilon(C_F - C_D)$.

Fig. 7 shows the optimum altitudes attained for the missile flights corresponding to Figs. 3 to 6. Fig. 8 is a cross plot of

Fig. 7 and shows the dependence of optimum altitude on initial velocity and $\epsilon(C_F - C_D)$. The curves of Fig. 8 are loci of optimum altitude points attained for different configurations and launching velocities of ramjet missiles. Fig. 8 points the way for future research upon the ramjet missile in vertical flight. It shows that the greatest performance in optimum altitude will not be developed by increasing the initial launching velocity but will be attained by improved configurations, i.e. improved C_F, C_D and A launched at lower initial velocities. For instance, $\epsilon(C_F - C_D)$ varies directly as the cross-sectional area of the missile; therefore, for the same increase in area, i.e., increase in $\epsilon(C_F - C_D)$, the optimum altitude will increase four times the value for a missile launched at 894 ft/sec as for one launched at 2230 ft/sec.

Figs. 9 to 12 show the calculations similar to those of Figs. 3 to 6 with the exception that the parameter $\frac{C_{f_0} C_F}{C_F - C_D}$ is varied through its range of values and $\epsilon(C_F - C_D)$ is held constant for each value of V_0. As a result, the general shape and appearance of the curves is the same as in Figs. 3 to 6. Velocities, accelerations, and altitudes show an increase in magnitude, however, for values of $\frac{C_{f_0} C_F}{C_F - C_D} > .001$ while for values of $\frac{C_{f_0} C_F}{C_F - C_D} < .001$ the decrease in these performances are negligible. These variations are due to the term $1 - \frac{T.F.C}{W_0}$ in the denominator of equation (12). The $1 - \frac{T.F.C.}{W_0}$ varies from 1.00 to .900 in normal operation, $\frac{C_{f_0} C_F}{C_F - C_D} = .001$, and therefore influences the acceleration only slightly. If the specific fuel consumption is increased as shown in Figs. 9 through 12 the factor $1-\frac{T.F.C.}{W_o}$ will vary from 1.00 to .38. This factor, of course, increases the acceleration tremendously. Physically this means simply that the large fuel consumption has reduced the missile weight (for a given initial weight) appreciably by the time the high acceleration period is reached. Since more fuel must be carried, for this case, the payload of the missile will be decreased.

Fig. 12, for instance, shows the previous variations when fuel consumption is increased or decreased by a factor of ten from the normal operating value of $\frac{C_{f_o} C_F}{C_F - C_D}$ = .001 for the same missile of a weight-area ratio equal to 890 lbs/ft^2. The altitude-velocity performance of the missile using increased fuel is one half again as great as the normal performance, but the payload is one half less than the normal performance. On the other hand, the performance of the missile using less than normal fuel is approximately equal to normal performance while the payload is increased about 6%. This clearly shows the necessity of low fuel consumption from an economical standpoint. Efficiency and economy will suffer greatly if $\frac{C_{f_o} C_F}{C_F - C_D}$ is allowed to increase much above .001 although a decrease in this parameter does not substantially improve these performances.

V. SPECIFIC EXAMPLES

The specific examples of this problem, divided into three sections, are used to compare the ramjet missile with the two-step rocket and to obtain a preliminary approximation of the increased efficiency in fuel consumption. The first example shows the method of obtaining ramjet performance for an assumed missile configuration. The second example is a comparison between the ramjet missile and a comparable two-step rocket on the basis of fuel consumption for the total flight. It is assumed that the two missiles are comparable if each has the same maximum velocity and altitude at the end of the first step. It is further assumed that the rocket carried by the ramjet will be used as a ducted rocket to boost the ramjet and this fuel is charged against the ramjet. It is found in this example that the ramjet missile uses only one-third of the fuel used by a two-step rocket and thus the payload ratio of the ramjet missile is tripled compared to the two-step rocket. The third example is a comparison between the same type of missiles for an equal amount of fuel consumed. It is found that for the same fuel consumed the ramjet missile shows a 65% gain in altitude.

A. Performance for an assumed ramjet missile

Given a ramjet missile of the following configuration:

Ramjet structural weight ----------	2400 lbs
Ramjet fuel weight----------------	800 lbs
Rocket structural weight-----------	3200 lbs
Rocket fuel weight-----------------	2000 lbs
Total initial weight,	8400 lbs

Cross-sectional area of missile is 10 sq. ft.

Assumed coefficients are:

$$C_F = 1.0 \qquad C_F - C_D = .75$$

$$C_D = .25$$

$$C_{f_o} = 8.0 \times 10^{-4} \ \frac{lb/sec}{lb}$$

To find the maximum velocity, fuel consumption, altitude of maximum velocity, and duration of powered flight of the above described ramjet missile, the following calculations are made:

Calculate: $$\frac{C_{f_o} C_F}{C_F - C_D} = \frac{8 \times 10^{-4} \times 1.0}{1.0 - .25} = .001 \ {}^{1}/_{sec}.$$

$$\frac{W_o}{A} = \frac{8400}{10} = 840 \ {}^{lbs}/_{ft^2}$$

Enter Fig. 2 with $\frac{W_o}{A}$ = 840, pick off highest $\epsilon(C_F - C_D)$ for best performance. Enter Fig. 5 for $\epsilon(C_F - C_D) = 3.25$

$V_o = 1790 \ {}^{ft}/_{sec}.$

then: $V_{max} = 3600 \ {}^{ft}/_{sec} \qquad h = 70,000 \ ft. \qquad t_p = 20 \ sec.$

$$\frac{TFC}{W_o} = .047 \qquad TFC = 400 \ lbs \ fuel$$

or

Fig. 6 for $\epsilon(C_F - C_D) = 5.25 \qquad V_o = 2230 \ {}^{ft}/_{sec}$

$V_{max} = 5000 \ {}^{ft}/_{sec} \qquad h = 80,000 \ ft. \qquad T.F.C. = 540 \ lbs. \ fuel$

B. Performance Comparison with Two Step Rocket

Assume such a configuration as a V-2 rocket mounted on a super-rocket. This is compared with a ramjet boosted rocket on the basis of fuel consumption for the initial step for the same maximum velocity and altitude for the initial step. From Fig. 2 pick a $\frac{W_0}{A}$ and $\epsilon(C_F - C_D)$ which will give good performance; say, $\frac{W_0}{A} = 635$, $\epsilon(C_F - C_D) = 7$. This gives a launching velocity of 2230 ft/sec. Entering Fig. 6 the performance of the ramjet missile is obtained:

$$V_{max} = 7100 \text{ ft/sec}$$

$$h = 100,000 \text{ ft.}$$

$$\frac{T.F.C.}{W_0} = .086 \qquad t_p = 20 \text{ sec.}$$

Assuming that the cross-sectional area of the ramjet is four times that of the V-2 rocket, it follows that

$$W_0 = 635 \times 96 = 61,000 \text{ lbs.}$$

$$T.F.C. = 5250 \text{ lbs}$$

Booster fuel necessary to obtain 2230 ft/sec on the ramjet missile may be calculated as follows:

The mass ratio required during boosting operation, using the formula for short burning time, is:

$$\frac{\text{initial mass}}{\text{final mass}} = e^{V_0/c}$$

where V_o = boosted velocity, or initial velocity of the ramjet

C = exhaust velocity of the booster rocket

Since it is planned to operate the rocket as a ducted rocket for this boost operation, C will be taken as 6880 ft/sec, the exhaust velocity of the V-2.

The final weight is 61,000 lbs and the final weight plus the booster fuel weight is equal to the initial weight.

Therefore:

$$1 + \frac{W_{B.F.}}{61{,}000} = e^{\frac{2230}{6880}} = 1.383$$

$$W_{B.F.} = 14{,}300 \text{ lbs.}$$

Total weight of fuel consumed for this flight is:	Booster	14,300 lbs
	Ramjet	5,250 lbs
	Rocket	19,000 lbs
		38,500 lbs

Calculation for the fuel required for a two-step rocket, super plus V-2, which will meet the same performance as the ramjet missile is as follows:

The super rocket must attain 100,000 feet at a burn out velocity of 7100 ft/sec, the same as the ramjet in order to compare fuel consumption. The mass ratio required formula, although an expression used for flights of short burning time, may be used here for expedience. This expression favors the rocket in this comparison since the assumption of longer burning time will mean an increase of the mass ratio.

Therefore: $$\frac{M_o}{M_f} = e^{\frac{V}{c}}$$

where M_o = initial mass

M_f = final mass = $M_o - M_p$

M_p = mass of propellant

V = 7100 ft/sec, velocity required at burnout

c = 6880 ft/sec, exhaust velocity of V-2 is used

ζ = loading factor, M_p/M_o

then $$1 - \zeta = e^{-\frac{7100}{6880}} = \frac{1}{2.8} = 0.357$$

$$\zeta = 0.643$$

To find the mass of the first step:
(See Ref. (2), PP. 263)

$$M_o = \Lambda^{-N} M_\ell$$

where Λ = payload ratio

N = no. of steps = 2

M_ℓ = payload of 2nd step

M_o = mass of 1st step

and $$\eta = (1-\lambda)(1-\epsilon)$$

where ϵ is a structural factor and the lowest value yet achieved is .24

$\therefore \quad \epsilon = .24$

But $$\lambda_1 = \lambda_2 = \lambda_{V-2} = \frac{\text{payload } V\text{-}2}{\text{Gross wt. } V\text{-}2}$$

where the payload of the V-2 = 28,000 x .155 = 4340 lbs.

Then $$W_o = (.155)^{-1}\, 4340 = 180,000 \text{ lbs.}$$

and W_p = weight of propellant = .643 x 180,000 = 116,000 lbs.
for 1st step

Total fuel used for this flight is: super rocket = 116,000

V-2 = 19,000

135,000 lbs.

$$\frac{\text{Fuel used by ramjet missile}}{\text{Fuel used by two-step rocket}} = \frac{38,550}{135,000} = 0.286$$

It is assumed in this problem that the ramjet will carry a rocket of 33,300 lbs. fuel capacity, 14,300 lbs. to be used for the booster operation and 19,000 lbs to be used in the third stage operation. The third stage operation of the ramjet missile and the second stage of the two-step rocket (V-2 stage) are assumed to be of equal performance.

Altitude attained by each of these missiles is computed by equation 104 in Ref. (2):

$$h = \frac{c^2}{2g}\left[\ln(1-\zeta)\right]^2 + c t_p\left[1 + \frac{1}{\zeta}\ln(1-\zeta)\right] + \frac{V_0^2}{2g} + h_0 - \frac{V_0 c}{g}\ln(1-\zeta)$$

where

$C_D = 0$

$\zeta = .70$ for V-2

$c = 6880$ ft/sec.

$t_p = 70$ sec.

$V_0 = 7100$ ft/sec.

$h_0 = 100{,}000$ ft.

Substituting these values into this equation altitude is obtained as:

h = 3,395,000 feet = 644 miles

Thus for the same altitude the ramjet missile uses only one-third of the fuel used by a two step rocket.

Payload ratio of these two missiles may also be compared. Payload ratio, λ, may be defined as follows:

$$\lambda = \frac{\text{weight of payload}}{\text{gross weight of that step}}$$

or

$$\lambda = \frac{\text{weight of 2nd step missile}}{\text{initial weight at sea level}}$$

In a multi-step rocket the payload ratio is usually equal for all steps. This will not be true if a ramjet is used for one step since a ramjet will deliver more weight of the next step than a comparable rocket due to better fuel consumption.

The payload ratio of the super rocket and V-2 rocket is

$$\lambda = \frac{\text{weight of V-2 rocket}}{\text{initial weight}} = \frac{28,000}{180,000} = .155$$

and the ramjet missile payload ratio is

$$\lambda = \frac{\text{weight of rocket - booster fuel}}{\text{+ booster fuel}} = \frac{49,000 - 14,300}{61,000 + 14,300} = .460$$

where 49,000 lbs. is the estimated gross weight of a rocket which carries 33,300 lbs. of fuel.

Thus the payload ratio of a ramjet missile is increased by a factor of three over a two-step rocket payload ratio.

C. Altitude comparison to a two-step rocket with equal fuel consumption.

From Ref. (2) pp. 265 two-step rocket performance is listed as follows:

W_o	= 9800 lbs,	initial weight
ζ	= .457	propellant mass ratio
W_p	= 4480 lbs,	initial weight of propellant
t_p	= 37 sec.	burning time of each step
ϵ	= .33	structural factor

λ = .318 payload ratio

c = 12,500 ft/sec, exhaust velocity

From

$$v = -c \ln\left[\epsilon(1-\lambda) + \lambda\right] - g t_p$$

$$h = c t_p \left[1 + \frac{1-\zeta}{\zeta} \ln(1-\zeta)\right] - \frac{1}{2} g t_p^2$$

the velocity and altitude at end of burning of the first step are calculated:

v = 6440 ft/sec   h = 105,500 feet

The altitude at the top of the trajectory is from eq. 104:

$$h = \frac{c^2}{2g}\left[\ln(1-\zeta)\right]^2 + c t_p\left[1 + \frac{1}{\zeta}\ln(1-\zeta)\right] + \frac{V_0^2}{g} + h_0 - \frac{V_0 c}{g}\ln(1-\zeta)$$

where C_D = 0

ζ = .457 t_p = 37 sec.

c = 12,500 ft/sec V_0 = 6440 ft/sec

h_0 = 105,500 feet

h = 3,016,000 feet = 570 mi.

Substitute a ramjet that burns 4480 lbs of fuel (including booster fuel) for the first step rocket.

Selecting $\epsilon(C_F - C_D)$ = 4 and V_0 = 1230 ft/sec.

arbitrarily, Fig. 3 gives V_{max} = 10,600 ft/sec. at 126,000 feet and $\frac{T.F.C.}{W_0}$ = .089

Booster fuel for 1230 ft/sec is obtained by:

$$1 + \frac{W_{B.F.}}{W_o} = e^{\frac{1230}{6880}} = 1.196$$

$$.089\ W_o = 4480 - W_{B.F.}$$

Solving simultaneously for $W_{B.F.}$ and W_o, one obtains

W_o = 15,700 lbs

$W_{B.F.}$ = 3,080 lbs

Ramjet fuel = 1400 lbs

The altitude at the top of the second-step trajectory, using the same second-step rocket is:

$$h = \frac{c^2}{2g}\left[\ln(1-\zeta)\right]^2 + ct_p\left[1 + \frac{1}{\zeta}\ln(1-\zeta)\right] + \frac{V_o^2}{2g} + h_o - \frac{V_o c}{g}\ln(1-\zeta)$$

where C_D = 0 $\quad$ t_p = 37 sec

ζ = .457 $\quad$ V_o = 10,600 ft/sec

c = 12,500 ft/sec $\quad$ h_o = 126,000 ft

h = 4,983,000 ft = 943 miles.

Thus for the same fuel consumption the ramjet missile shows a 65% gain in altitude.

VI. SUMMARY OF RESULTS

1. With normal specific fuel consumption, i.e., $\frac{c_{f_o} C_F}{C_F - C_D} = .001$ the performance of a ramjet powered missile in vertical flight is limited by loss of thrust with increase of altitude. The reduction of weight resulting from the fuel consumed is not sufficient to compensate for the decrease of thrust. This decrease of thrust is directly proportional to the air density ratio, σ ; and this factor, then, has the most pronounced effect on limiting the vertical flight of the ramjet missile.

2. The reduction of ramjet cross-sectional area for the same initial weight missile; i.e., the increase of the $\frac{W_o}{A}$ ratio, will reduce the maximum velocity and optimum altitude, or require a higher booster velocity for the same maximum velocity and altitude.

3. Maximum velocity and optimum altitude for a specific configuration of missile increase in the same order as the increase of launching velocity.

4. Ramjet improvements in performance, i.e., C_F and C_D, show more effect in increasing the optimum altitude when the missile is launched at low initial velocities than if it is launched at high initial velocities.

5. The greatest increase of optimum altitude reached by a ramjet missile will be achieved by increasing the ramjet cross-sectional

area so as to launch the missile at lower initial velocities.

6. An increase in specific fuel consumption of ten times that of normal operation will give an increase in acceleration of the same order since weight is reduced quicker by burning the fuel faster. The payload is reduced 50%, however, since the extra fuel will replace payload. In the same manner a decrease of one tenth in fuel consumption as compared with normal operation will only slightly influence the acceleration and will also allow 6% more payload to be carried in place of the fuel saved.

7. For the same altitude and velocity the ramjet missile uses only one-third the fuel used by a two-step rocket. Payload ratios for this case is tripled for the ramjet missile. These calculations use assumptions favoring the rocket.

8. For the same fuel consumed the ramjet missile shows a 65% increase in altitude over a two-step rocket.

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TABLE I

Weight Area Ratio vs. Initial Velocity

Let $C_F - C_D = .75$

$$\frac{W_a}{A} = \frac{.00119 \times .75\ V_o^2}{\epsilon\,(C_F - C_D)} = \frac{.00089\ V_o^2}{\epsilon(C_F - C_D)}$$

		$\frac{W_o}{A}$ for values of $\epsilon(C_F - C_D)$								
V_o	V_o^2	1	2	3	4	5	6	7	8	9
800	$.64 \times 10^6$	570	285	190	142	114	95	81	71	63
1000	1×10^6	890	445	297	222	178	148	127	111	99
1200	1.44×10^6	1280	640	427	320	254	213	183	160	142
1400	1.96×10^6	1725	862	575	431	345	287	246	215	192
1600	2.56×10^6	2280	1140	760	570	456	380	326	285	253
1800	3.24×10^6	2880	1440	927	720	576	463	411	360	309
2000	4×10^6	3560	1780	1187	890	712	593	508	445	396
2200	4.84×10^6	4300	2150	1433	1075	860	716	614	537	478
2400	5.76×10^6	5120	2560	1707	1280	1024	853	731	640	569

TABLE II

SAMPLE CALCULATIONS FOR FIGS. 3 THRU 6.

$\epsilon(C_F - C_D) = 7$ $V_o = 2230$ $\frac{C_{F_o} C_F}{C_F - C_D} = .001$

t	V	$\frac{V}{V_o}$	$\left(\frac{V}{V_o}\right)^2$	$\left(\frac{V}{V_o}\right)_{AV.}$	h	σ	$\sigma_{AV.}$	$\frac{T.F.C.}{W_o}$	$\frac{dv}{dt}$
0	2230	1	1	1	0	1	1	0	225
1	2455	1.10	1.21	1.05	2342	.937	.968	.0069	225
2	2680	1.20	1.44	1.15	4909	.860	.899	.0141	250
3	2930	1.31	1.72	1.25	7714	.792	.826	.0213	281
4	3211	1.44	2.07	1.38	10785	.723	.757	.0286	315
5	3526	1.58	2.50	1.51	14163	.647	.685	.0358	346
6	3872	1.73	3.00	1.65	17862	.574	.610	.0428	373
7	4245	1.90	3.61	1.81	21920	.501	.537	.0496	396
8	4641	2.08	4.33	1.99	26363	.429	.465	.0561	411
9	5052	2.26	5.12	2.17	31310	.359	.394	.0621	409
10	5461	2.45	6.00	2.36	36466	.292	.325	.0675	391
11	5852	2.62	6.87	2.54	42122	.225	.258	.0721	343
12	6195	2.78	7.73	2.70	48156	.168	.197	.0758	284
13	6479	2.91	8.45	2.84	54493	.124	.146	.0787	224
14	6703	3.00	9.00	2.95	61084	.094	.109	.0809	175
15	6878	3.08	9.50	3.04	67874	.065	.080	.0833	120
16	6998	3.13	9.80	3.10	74812	.041	.053	.0845	67
17	7065	3.16	10.00	3.14	81844	.029	.035	.0853	39

18	7104	3.19	10.17	3.18	88929	.018	.023	.0858	13
19	7117	3.19	10.17	3.19	96039	.015	.017	.0861	5
20	7122	3.19	10.17	3.19	103159	.012	.014	.0864	-2

Formulae:

$$\left(\frac{V}{V_0}\right)_{AV.} = \frac{\left(\frac{V}{V_0}\right)_1 + \left(\frac{V}{V_0}\right)_2}{2}$$

$$\sigma_{AV.} = \frac{\sigma_1 + \sigma_2}{2}$$

$$h_2 = h_1 + \frac{V_1 + V_2}{2} \Delta t$$

$$\frac{T.F.C.}{W_0} = \epsilon (C_F - C_D) \frac{C_{F_0} C_F}{C_F - C_D} \int_0^{\tau} \sigma \frac{V}{V_0} dt$$

$$\text{where} \int_0^{\tau} \sigma \frac{V}{V_0} dt = \Delta t \sum_0^{h} \sigma_{AV.} \left(\frac{V}{V_0}\right)_{AV.}$$

$$\frac{dv}{dt} = \left[\frac{\epsilon (C_F - C_D) \sigma \left(\frac{V}{V_0}\right)^2}{1 - \frac{T.F.C.}{W_0}} - 1 \right] g$$

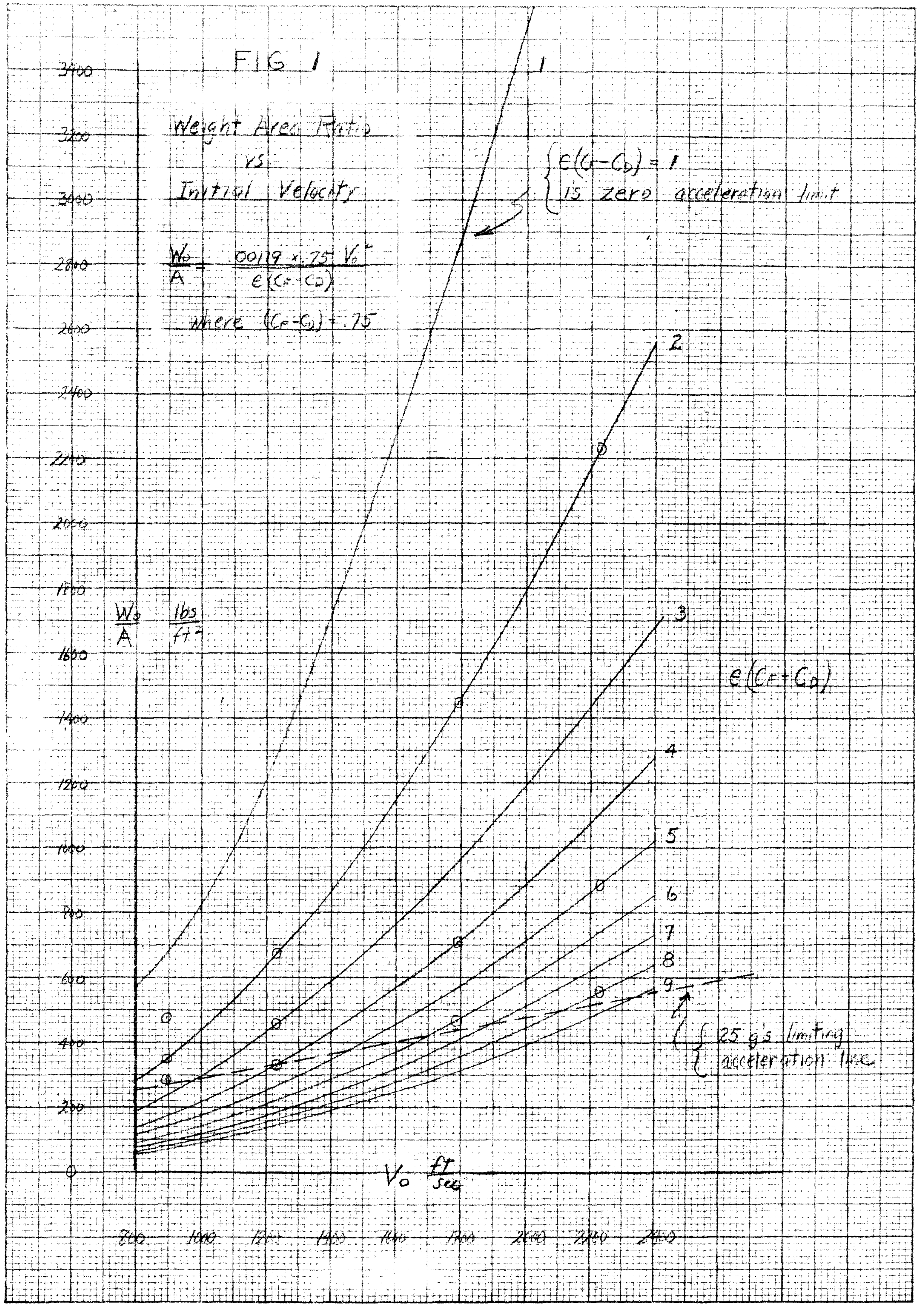


FIG 1
Weight Area Ratio
vs
Initial Velocity
Wo/A = .00119 × .75 Vo² / ε(CF-CD)
where (CF-CD) = .75
ε(CF-CD) = 1
is zero acceleration limit
Wo/A lbs/ft²
ε(CF+CD)
25 g's limiting
acceleration line
Vo ft/sec
1
2
3
4
5
6
7
8
9
3400
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
800
1000
1200
1400
1600
1800
2000
2200
2400

FIG 2

Variation of Ramjet Coefficients with Mach Number - Invariant with Altitude

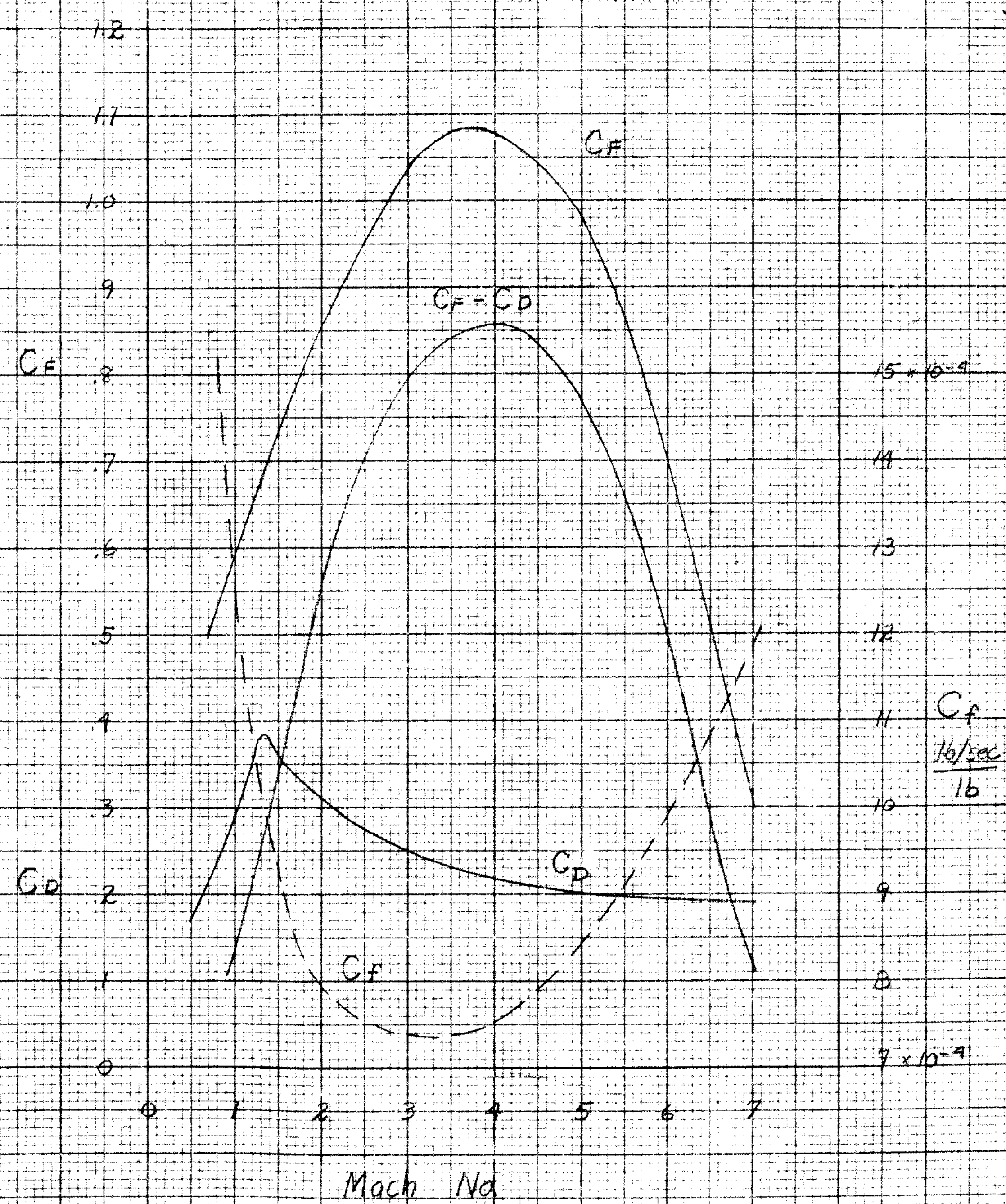


FIG 3

Flight Velocity and Total Fuel Consumption per Pound vs Time after Launching – $V_0 = 894$, $\frac{C_{F_0} C_F}{C_F - C_D} = .001$

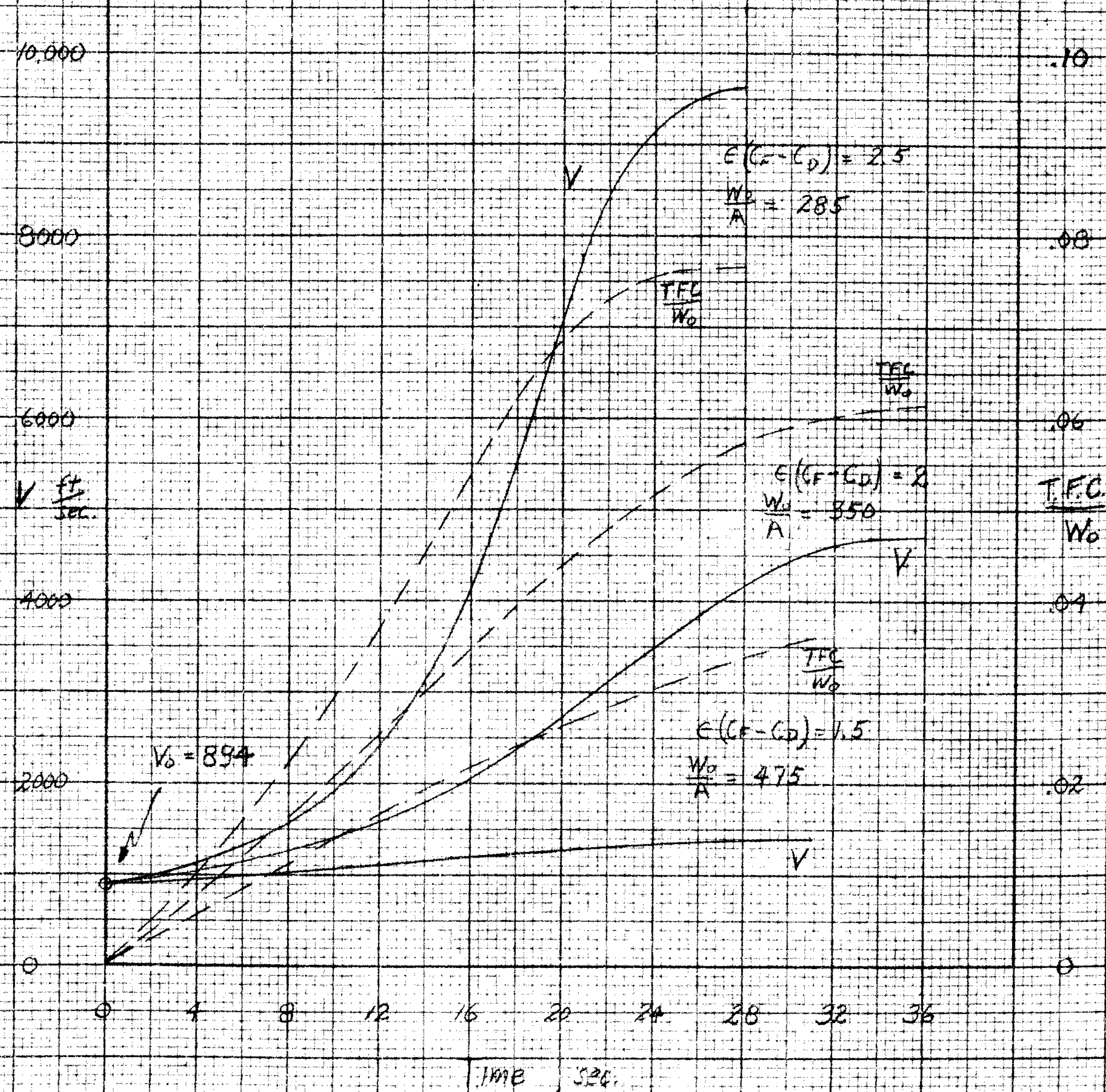


FIG 4

Flight Velocity and Total Fuel Consumption per Pound

vs.

Time After Launching – $V_0 = 1230$, $\frac{C_{F_0} C_F}{C_F - C_D} = .001$

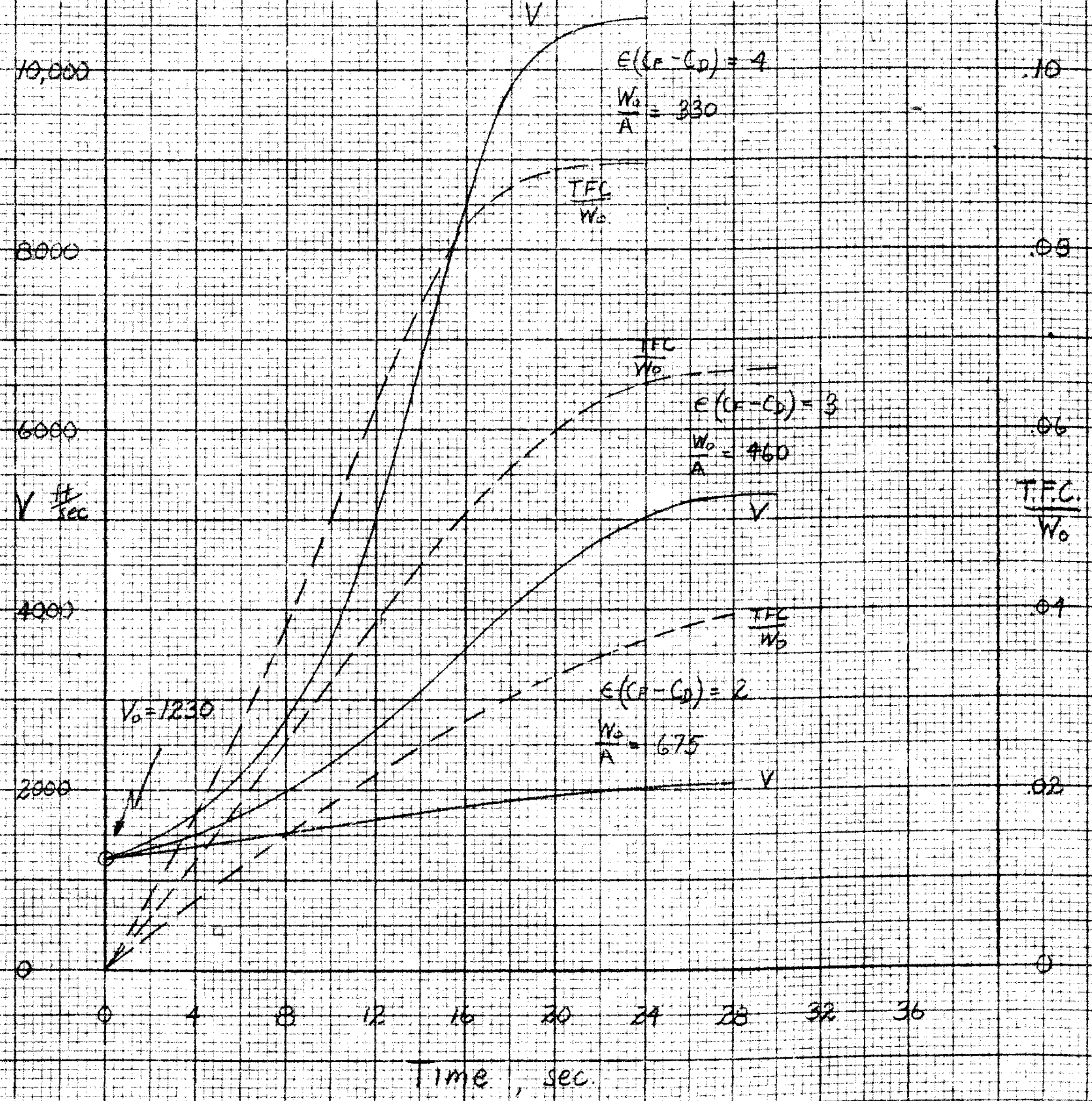


FIG 5

Flight Velocity and Total Fuel Consumption per Pound

vs.

Time After Launching – $V_0 = 1790$, $\frac{C_E C_F}{C_F - C_D} = .001$

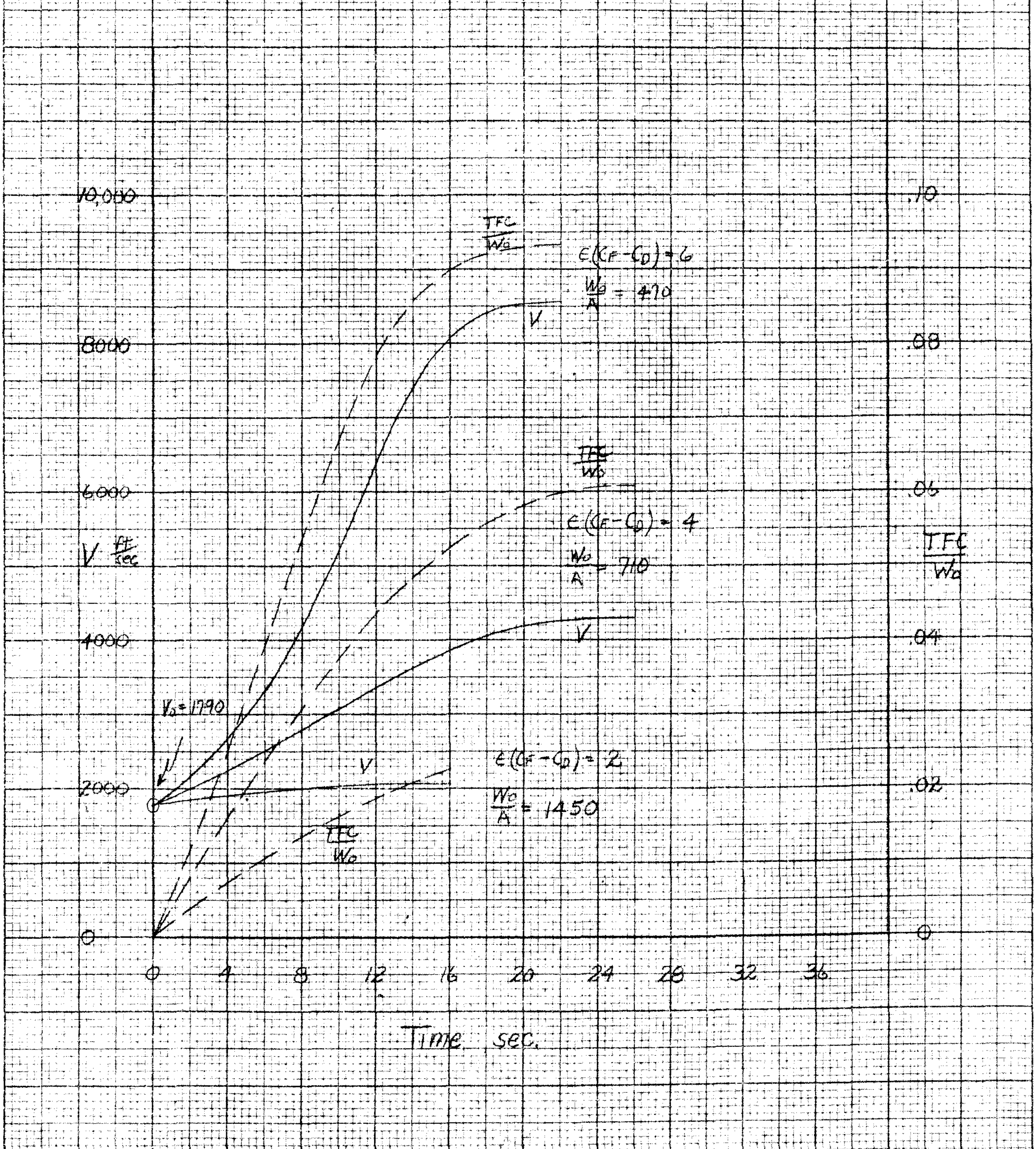
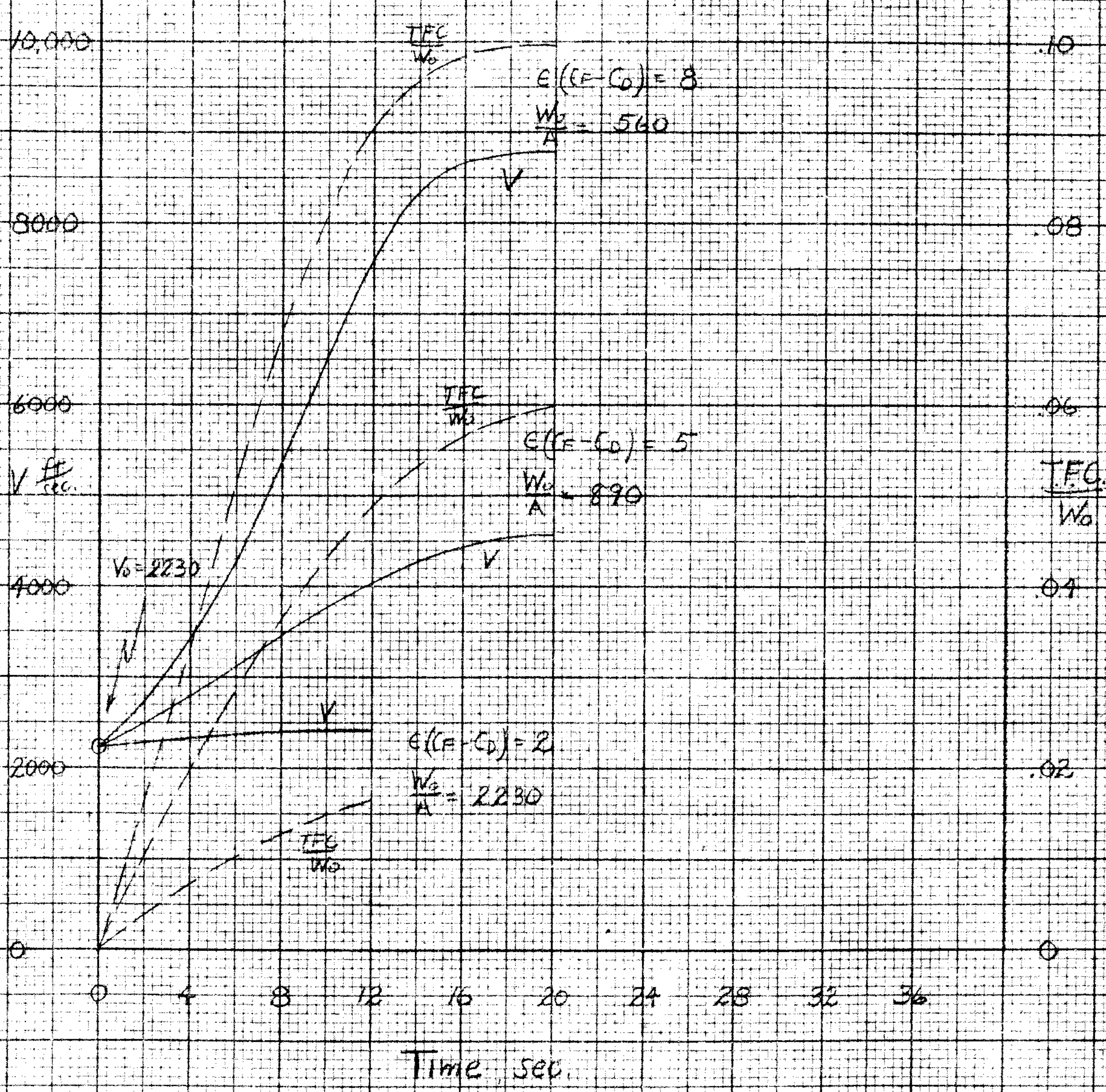


FIG 6

Flight Velocity and Total Fuel Consumption per Pound vs Time After Launching - $V_0 = 2230$, $\frac{C_{f_0} C_F}{C_F - C_D} = .001$



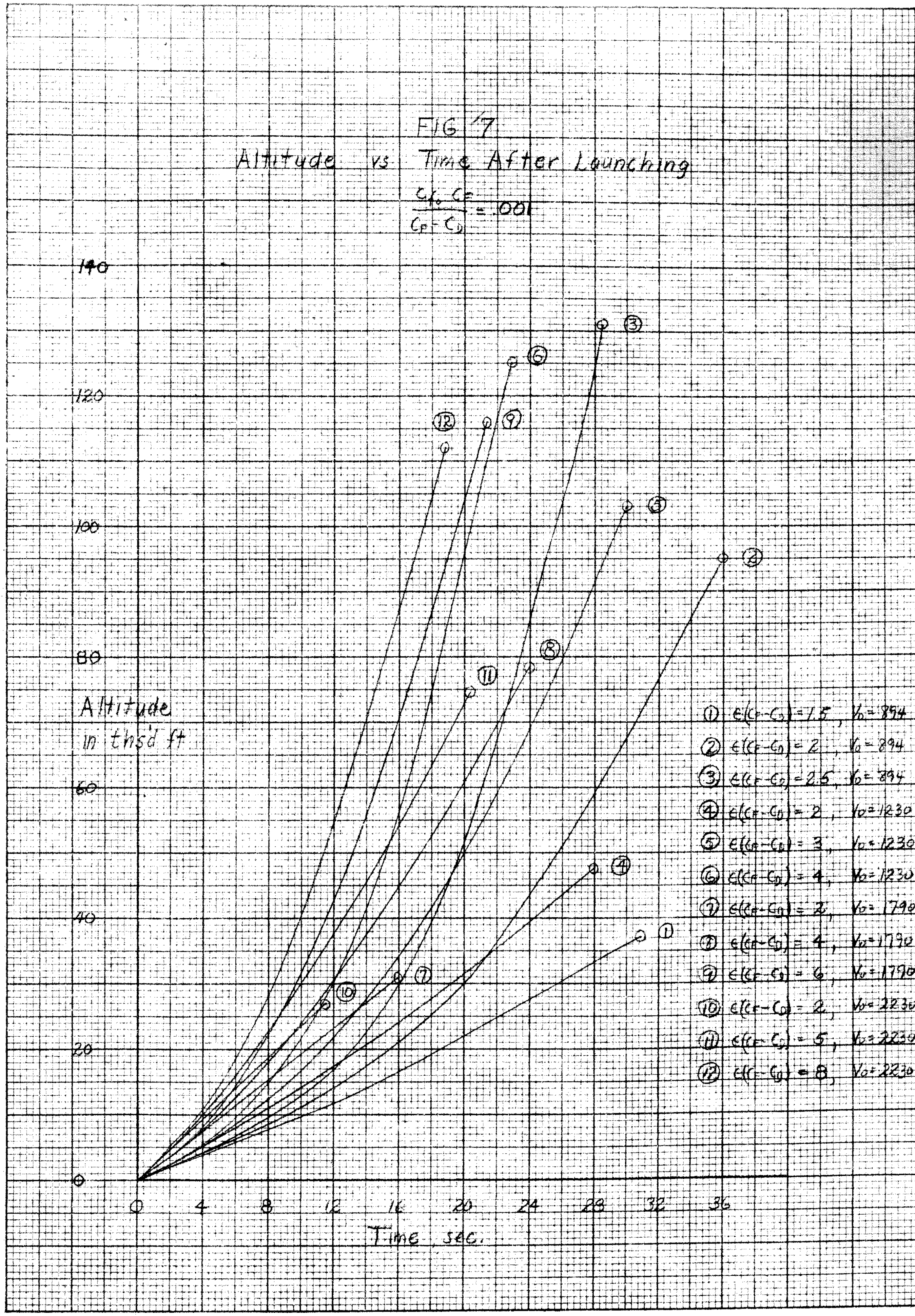


FIG 7
Altitude vs Time After Launching

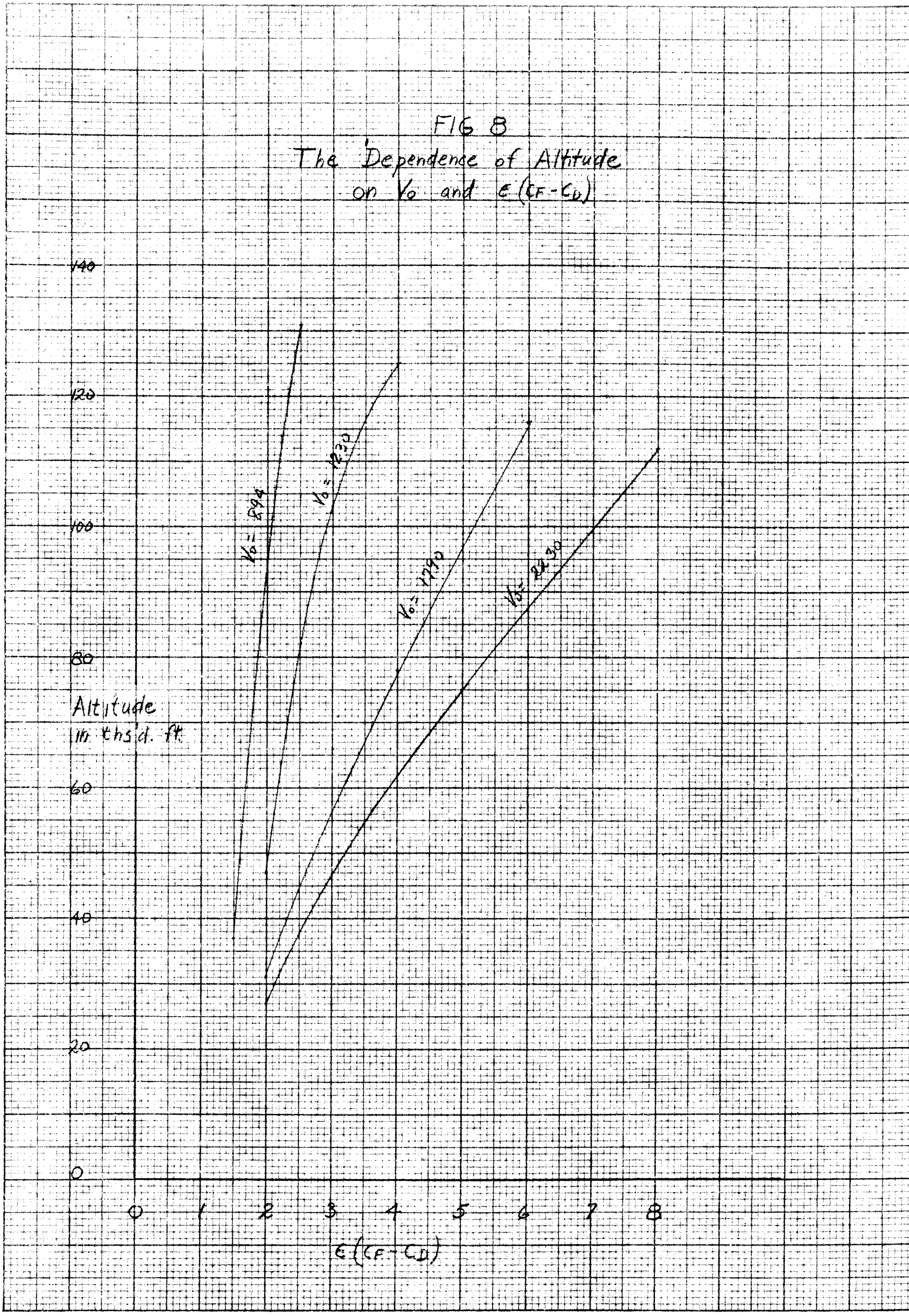
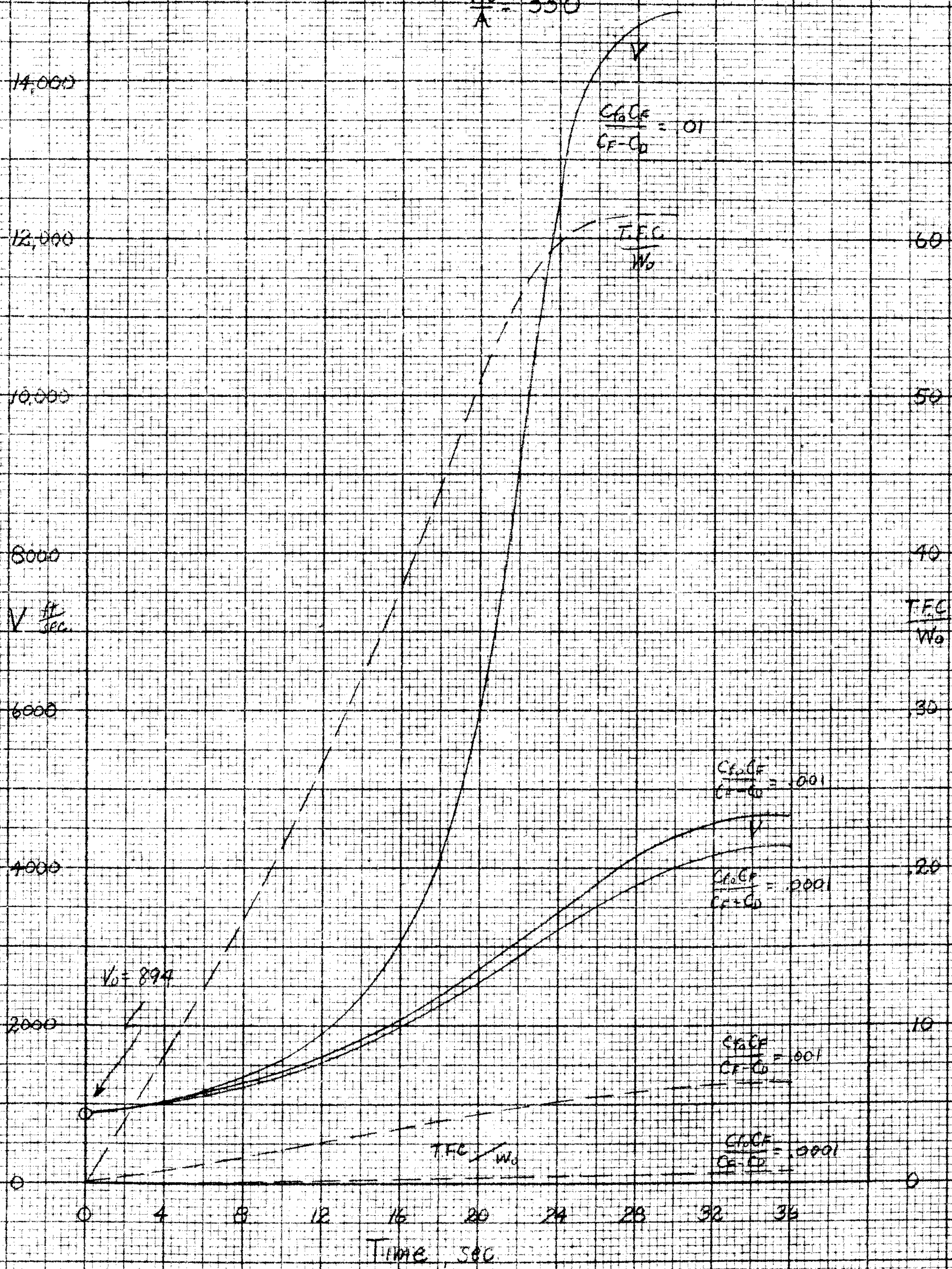


FIG 8
The Dependence of Altitude on V_0 and $\epsilon(C_F - C_D)$
140
120
100
80
Altitude in ths'd. ft.
60
40
20
0
0
1
2
3
4
5
6
7
8
$\epsilon(C_F - C_D)$
V_0 = 894
V_0 = 1230
V_0 = 1790
V_0 = 2230

FIG 9

Flight Velocity and Total Fuel Consumption per Pound vs Time After Launching - $V_o = 894$, $\epsilon(C_F - C_D) = 2$, $\frac{W_o}{A} = 350$



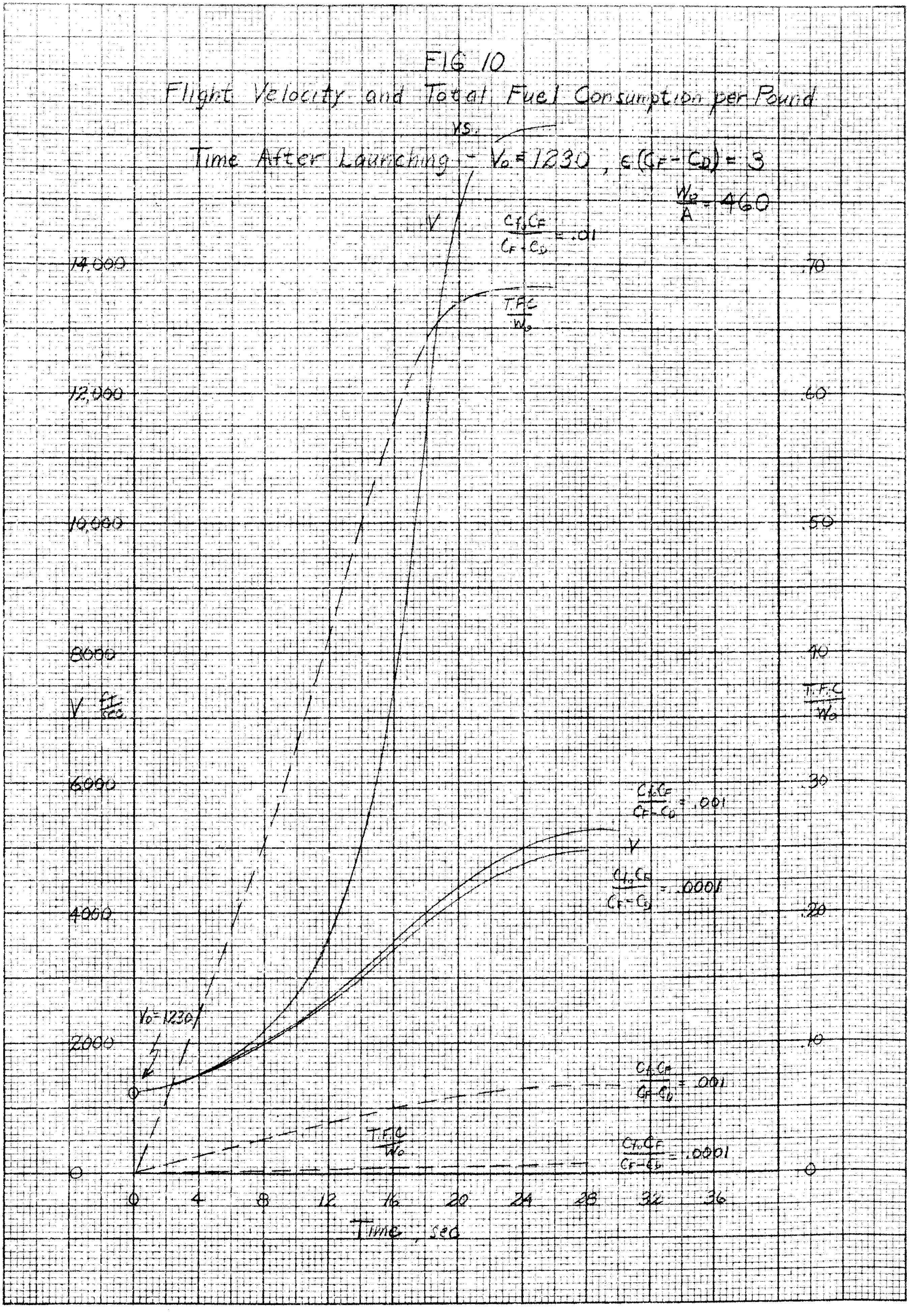


FIG 10
Flight Velocity and Total Fuel Consumption per Pound
vs.
Time After Launching - V₀ = 1230 , ε(C_F - C_D) = 3
W₀/A = 460
V
C_f C_F/(C_F - C_D) = .01
T.F.C./W₀
14,000
12,000
10,000
8000
6000
4000
2000
0
V ft/sec
.70
.60
.50
.40
.30
.20
.10
0
T.F.C./W₀
C_f C_F/(C_F - C_D) = .001
V
C_f C_F/(C_F - C_D) = .0001
V₀ = 1230
C_f C_F/(C_F - C_D) = .001
T.F.C./W₀
C_f C_F/(C_F - C_D) = .0001
0
4
8
12
16
20
24
28
32
36
Time, sec

FIG III

Flight Velocity and Total Fuel Consumption per Pound vs. Time After Launching – $V_0 = 1790$, $\epsilon(C_F - C_D) = 4$, $\frac{W_0}{A} = 710$

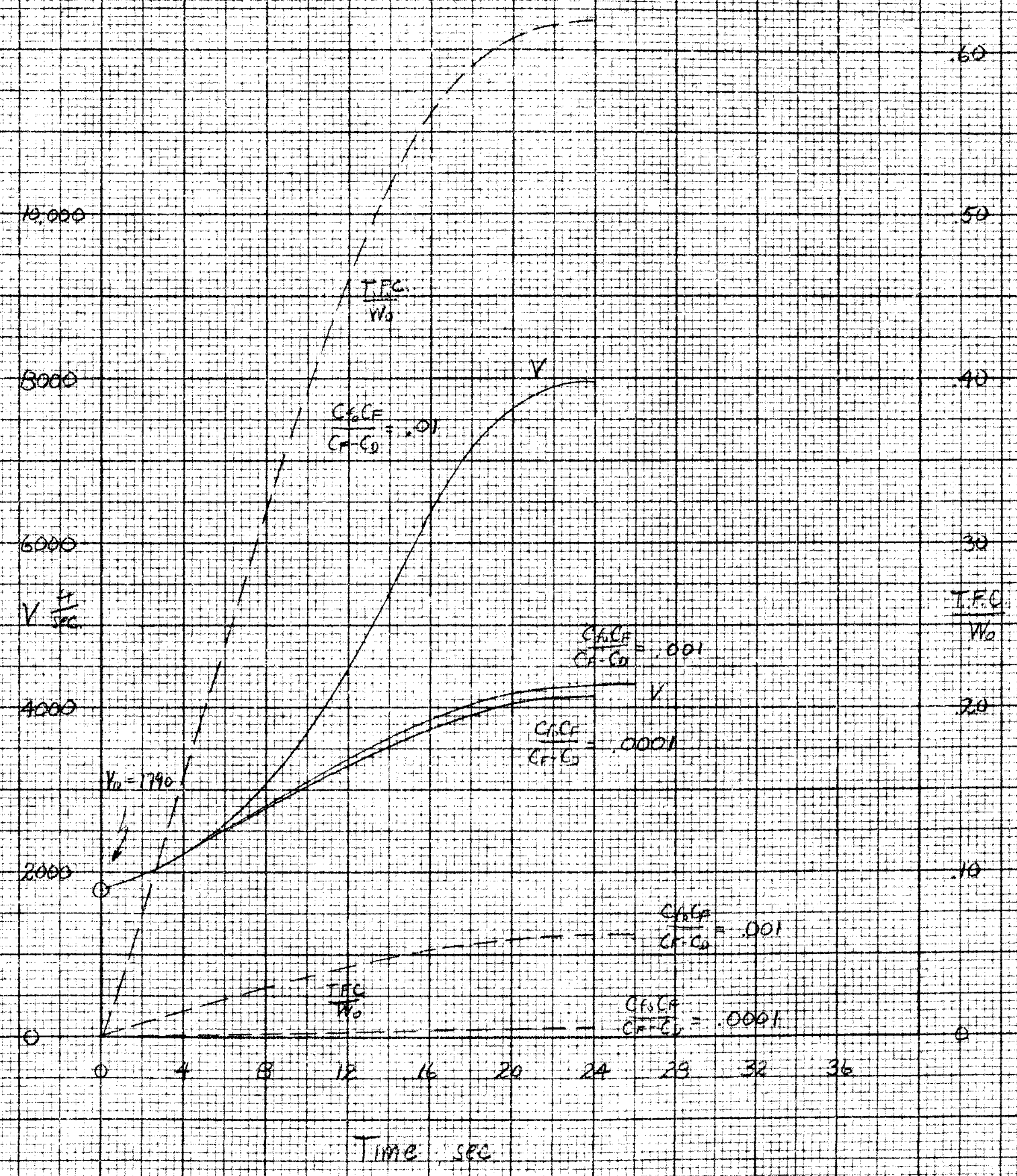
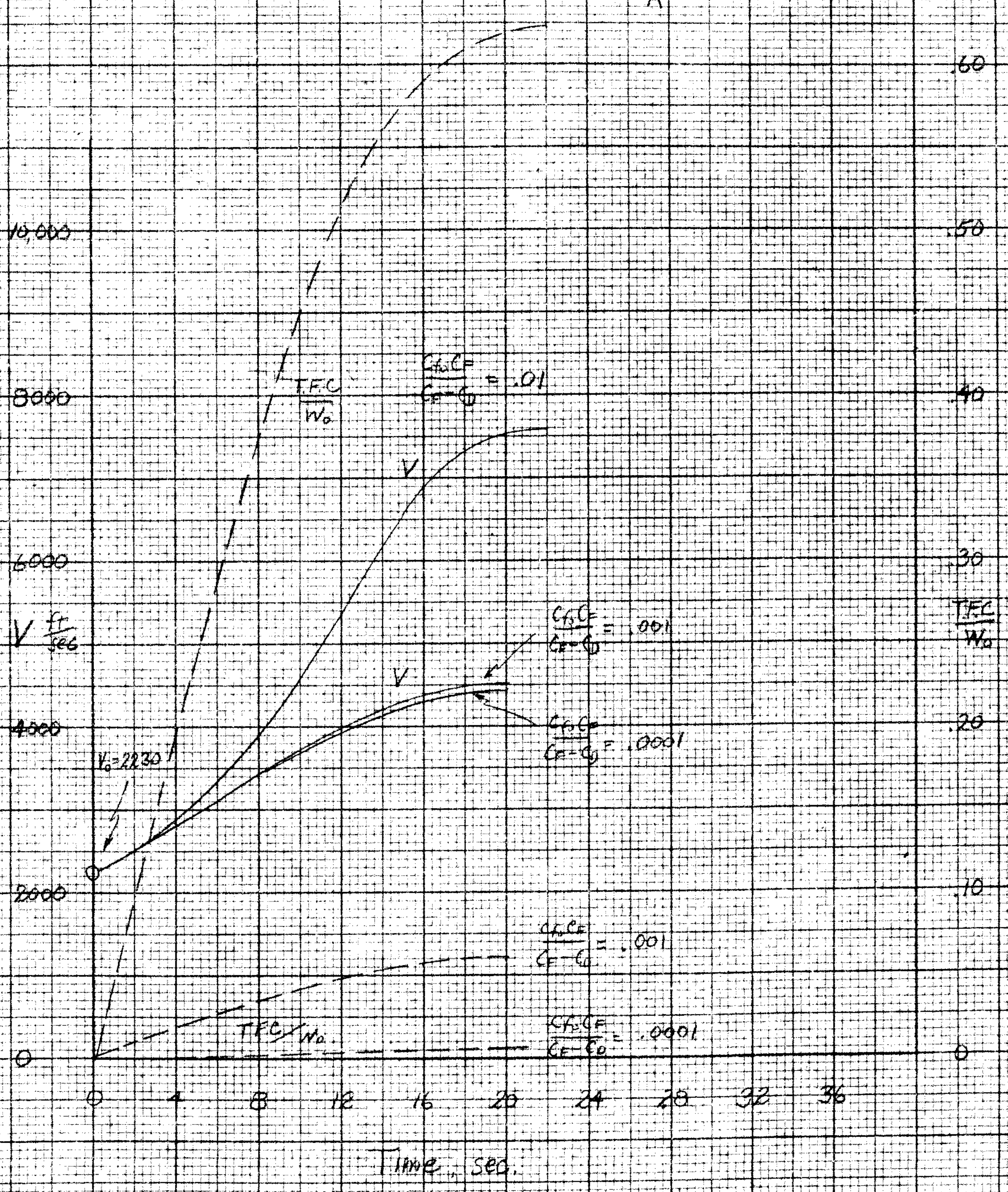


FIG 12

Flight Velocity and Total Fuel Consumption per Pound vs. Time After Launching – $V_0 = 2230$, $\epsilon(C_F - C_D) = 5$, $\frac{W_0}{A} = 890$



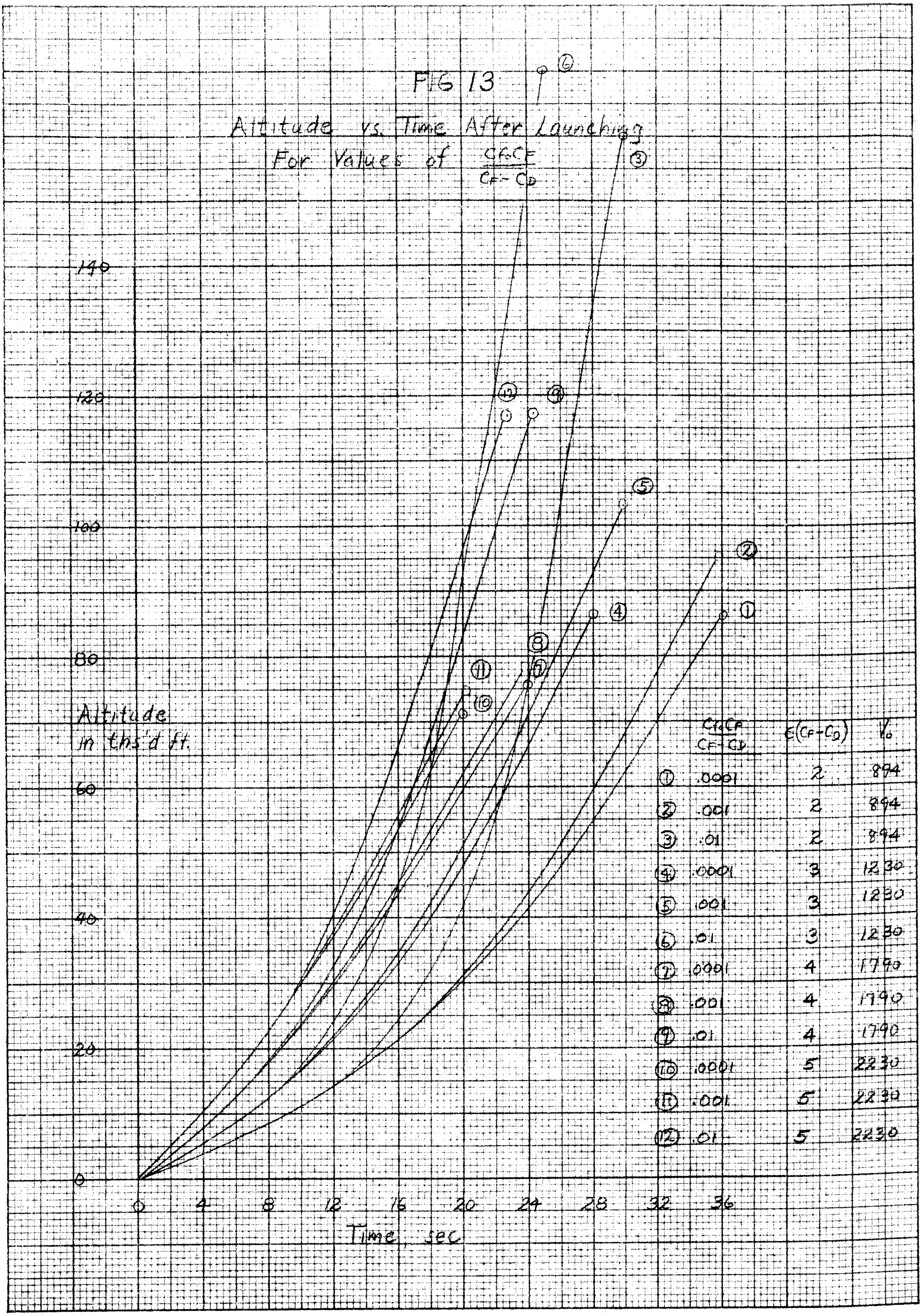


FIG 13
Altitude vs. Time After Launching
For Values of Cf·CF / CF − CD
Altitude in ths'd ft.
0
20
40
60
80
100
120
140
Time, sec
0
4
8
12
16
20
24
28
32
36
Cf·CF / CF − CD | ϵ(CF − CD) | V0
① .0001 2 894
② .001 2 894
③ .01 2 894
④ .0001 3 1230
⑤ .001 3 1230
⑥ .01 3 1230
⑦ .0001 4 1790
⑧ .001 4 1790
⑨ .01 4 1790
⑩ .0001 5 2230
⑪ .001 5 2230
⑫ .01 5 2230

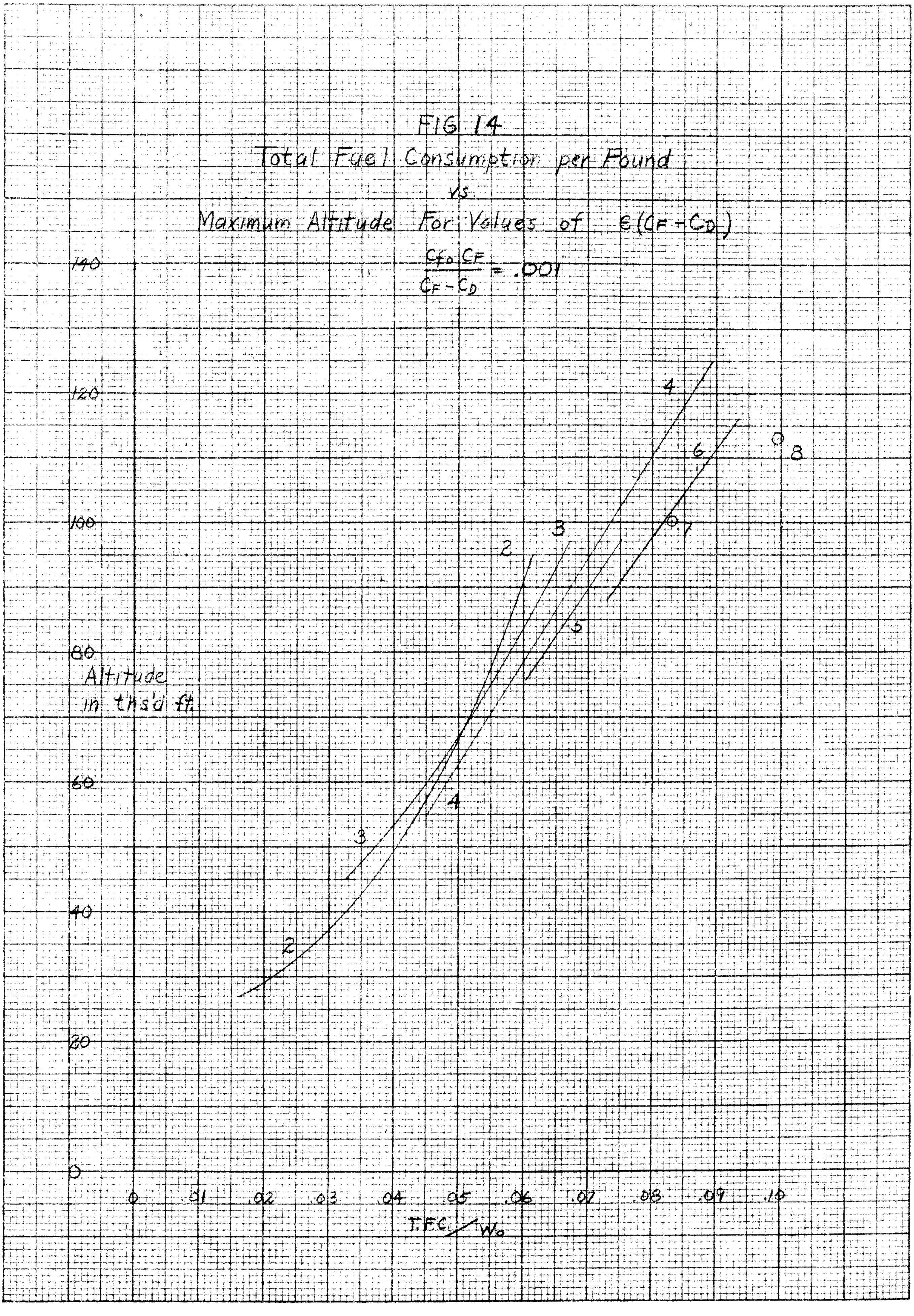


FIG 14
Total Fuel Consumption per Pound
vs
Maximum Altitude For Values of ε(CF - CD)
Cfo CF / CF - CD = .001
140
120
100
80
60
40
20
0
Altitude in thsd ft.
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
T.F.C./Wo
2
3
4
5
6
7
8

FIG 15

Flight Velocity vs Time After Launching For Two Weight-Area Ratios At Launching Velocities of 894, 1230, 1790, 2230 ft/sec

$$\frac{C_{F_0} C_F}{C_F - C_D} = .001$$