

A PHOTOELASTIC INVESTIGATION
OF THE EFFECT OF ELLIPTICAL AND MODIFIED CUTOUTS
IN FLAT PANELS SUBJECTED TO COMBINED
BENDING AND SHEAR

Thesis by

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INTRODUCTION

A. Thesis Problem

There are many problems in the field of stress analysis which are not amenable to practical analysis due to their complexity. Often cases arise which although they might be solved by analytical methods are so long and involved that they sorely tax the most diligent and persevering efforts of even a devoted mathematician--far less the imagination of a practical engineer. In this group fall many of the present day problems in structural design. Fortunately many of these cases can be solved by means of photoelasticity in a relatively simple manner, providing a properly equipped laboratory is available. If one desires to delve further into the subject, even the most intricate stress problems can be solved by use of a combination of photoelastic and mathematical methods.

The investigation which has been carried out here is only one of an infinite number of combinations which might be considered. The general problem is the determination of stress distribution around openings or cutouts due to various types of loadings. Obviously this could continue ad infinitum.

It was thought the problem of stress concentrations might be of general interest in all fields of structural design. The object of this investigation has been to

determine the magnitude of the concentrations under various combinations of loading and to present the results in a somewhat practical manner so that they might be readily applied to problems falling into the category under consideration.

B. Previous Investigations

There have been many investigations carried out on flat panels or plates with various types of cutouts subjected to pure shear, bending, tension or compression, but little work has been done with combined loadings. Many people have investigated stresses around circular, square, elliptical and modified cutouts subjected to simple types of loading. Recently aircraft companies have become interested in the use of photoelastic methods for investigating stresses around various types of lightening holes in ribs and spar webs.

The problem attempted here is more or less a continuation of the work conducted last year by Tyra and Hollister at the California Institute of Technology. The results are presented in their paper entitled "A Photoelastic Investigation of the Effect of Cutouts in Panels Subjected to Combined Bending and Shear". This work concerned itself with circular cutouts and various modifications of these.

The present investigation was carried out for an elliptical cutout and modifications of this until a rectangle was finally reached. Obviously the scope of this type of investigation is very limited, since there is an infinite number of combinations of sizes and panel cross sections. However, it is hoped that the results may be of some aid in determining the magnitude of stresses due to these types of cutouts in various separate structural members. Some of the results might for instance be applied to wing ribs with lightening holes. It is doubtful if they can be used for cutouts in metal monocoque structures of aircraft since the latter is a thin plate construction reinforced with stringers and subjected to buckling phenomena.

THEORY

A. Elasticity

In this problem, as in almost any other, the question of variation of the several variables involved arises. Obviously the size of the specimen and the basic dimensions of the cutout must remain fixed for any limited investigation such as presented here. This leaves the variation of the loading versus stresses around the cutouts as the basic problem.

Since this is a cantilever type of beam, variation of the ratio of bending stresses to shear stresses is relatively simple. It is apparent that this ratio is proportional to the ratio of bending moment to shear load, which in turn is proportional to the distance from the section under investigation to the point of application of the external load. This then appeared to be the simplest and most desirable parameter against which to calculate the stress variations around the cutouts.

At first it was decided to attempt to keep the bending moment constant as the position of the loads was varied. However, starting with a load of reasonable magnitude at the extreme outboard position required loadings directly adjacent to the cutout which were of such magnitude as to be impractical. Next the idea of maintaining constant

shear was entertained. However, the maximum bending moment for the extreme outboard position of loading which was limited by the physical properties of the material did not give as high a shear load as was desired at the extreme inboard position. Therefore, on the basis of the above considerations, maximum bending moment permitted by the material was maintained for the extreme outboard loading position and the maximum load which could be obtained with the equipment available was applied at the extreme inboard position. Loads at intermediate positions were varied approximately linearly between the aforementioned limits.

As may be seen from the initial part of this discussion, the ratio of bending moment to shear is a function only of the distance from the section under investigation to the point of the applied load, and the only reason for attempting to get maximum loads was in order to obtain higher values of fringe orders in the stress pattern and thereby arrive at more accurate results.

B. Photoelasticity.

Throughout this experiment circularly polarized light with a monochromatic source was used. The polaroscope was set for "light field", that is, the light is maximum for the unloaded position which corresponds to the zero

fringe order and also to all integral fringe orders.

This may best be shown by the equation:

$$I = \frac{1}{2} I_m \left[1 + \cos w (t_2 - t_1) \right]$$

where

I = intensity of the light transmitted

$I_m = I_{\max}$

$w = 2\pi f$;

f = frequency of light vibrations.

$(t_2 - t_1)$ = relative retardation of velocity of passage of light through the specimen.

The term $w (t_2 - t_1)$ may better be written as $2\pi N$ where N = fringe order. The original equation then becomes:

$$I = \frac{1}{2} I_m (1 + \cos 2\pi N).$$

Inspection of this shows that for integral values of "N" the intensity is a maximum and for values corresponding to $1/2$, $3/2$, etc., it is zero. For intermediate values of "N" the intensity varies sinusoidally. In other words bands of brightest light on the loaded specimen correspond to integral values of "N", while black bands correspond to values of $1/2$, $3/2$, etc.

It can be shown experimentally that for a doubly refracting material such as bakelite that:

$$N = \frac{(\sigma_1 - \sigma_2) b}{C}$$

where

σ_1 & σ_2 = principal stresses

b = thickness of specimen, i.e. dimension parallel to incident light rays.

C = stress optical coefficient of specimen.

Therefore, if "N" is known ($\sigma_1 - \sigma_2$) is always known providing C has been previously determined. The latter may be determined at a free boundary from a known condition of loading, observation of the fringe order and thickness of the specimen. Since τ_{max} (maximum shearing stress) = $(\frac{\sigma_1 - \sigma_2}{2})$ the various white and dark lines really indicate lines of constant maximum shearing stress.

Obviously to determine principal stresses throughout a specimen another relationship between σ_1 and σ_2 is necessary. There are several methods for obtaining this relationship, but considerable work, far beyond the time available for this experiment, would be required. Actually such determination in this case serves no useful purpose since here we are primarily interested in the stresses around the boundary where they are a maximum.

Fortunately these can be readily found since, if the boundary is free, there can be no shearing stress parallel to it, nor any stress normal to it. The remaining stress is therefore a principal stress, and readily calculated from the relationship $\sigma_1 = 2 \tau_{max}$.

In cases where τ_{max} is the design criteria for a given material this can be immediately found at any point in the specimen.

The C for this specimen was found by applying a bending moment and calculating $\sigma_y = \frac{My}{I}$ where y is the distance to the outer fibers and consequently σ_y is a principal stress. σ_y was then calculated at various points around the various cutouts by observing the fringe orders. Instead of plotting these values of σ_y a stress ratio was plotted, referred to a basic stress which was taken to be the principal stress at the outermost fibers of the contemplated cutout.

Complete details of these calculations are discussed and illustrated in the appendix.

LABORATORY PROCEDURE

A. Equipment.

The polariscope used for this work was of the conventional type employing a sodium vapor lamp of sixty watt intensity. Circularly polarized light was used since only the isochromatics were investigated. The apparatus is illustrated in figure I. Between the light source and the polarizer a condensing system of two plano-convex lenses was placed with the polarizer at the focal point of the condensing system. The polarizer and the analyzer were both placed at the focal point of their respective mirrors and specimen placed in the beam of light about midway between the mirrors. This gives a fairly wide field but reverses the image which, however, entails no additional difficulties. However, in the interest of those concerned with photoelasticity, it must be said that this setup for an optical bench is not recommended unless the available space for a laboratory is very restricted and even then a very complete focusing system is highly recommended.

In this particular setup the polarizer and quarter wave plate are secured to the same mount and the analyzer and quarter wave plate similarly. There is provision for relative rotation of the polaroid and wave plate to obtain either a light or dark field and also three set screws to permit easy removal of the wave plate for investigation of isoclinics.

The camera equipment used was rather simple, consisting only of a black box with an opening in the front end and another in the rear end, the latter having provision for receiving a 5" x 7" film holder. To obtain proper focus before taking a picture, a glass plate with a transparent sheet of paper attached was used to view the picture and then the box was slid back and forth to obtain proper focus. In order to more closely investigate the fringe orders and sketch the pattern, a large screen was used upon which the image was projected before actually taking the pictures. Ortho X Eastman film was used. The first two sets of pictures taken could not be properly focused due to many difficulties which may be attributed to primarily the inexperience of the authors in the field of photography, the inadequate optical equipment, the type of light source (sodium vapor has more than one line) and the fact that a relatively thick specimen was used.

In the three latter sets of pictures the amount of light was reduced by stopping down the light just before it went through the analyzer. This resulted in obtaining a clearer picture and a better focus, but the effect produced was rather undesirable as may be seen, since only one plane of the specimen was brought into focus and then the errors of all the work were amplified.

However, the photographs are not exactly a criteria

for the results achieved, since in each case the images were thrown upon a large screen and the fringe orders thoroughly investigated.

The method of loading is shown in figure 2. The loading frame was mounted upon one of the optical benches and the specimen firmly bolted to it. The beam was then attached to the specimen in a similar manner and loaded as a cantilever with various loads at several positions as shown in figure 2. This permitted variation of the ratio of bending moment to shear load.

B. Specimen.

The specimen used was bakelite Bt-61-893. The particular supply available was all one-half inch in thickness, which is rather thick for most work of this type, but it seemed rather desirable in view of the type of loading applied in order to avoid the objectional results of torsional instability.

Considerable difficulty was encountered in finding a piece which did not contain residual stresses. As a matter of fact the whole supply was investigated and finally one piece was found which was stress free except for a slight edge effect. Prior to this, attempts were made to apply a stress relief anneal to others of this batch, but with no success, probably due to the fact that an annealing furnace with a very fine rate of temperature control in the lower

range was not available.

The specimen shown in figure 3 is 11" x 4" x $\frac{1}{2}$ ". An elliptical hole 2" x 1" was first cut and was then modified to a $\frac{1}{2}$ " fillet, and then 1/4" and finally the fillets were removed altogether. The cutting operations were carried out by the machine shop which had had previous experience with cutting this type of material and the results they achieved were very gratifying in that no edge stresses were set up around the boundary of the hole. It will be noticed on the photographs that at the bottom edge of the specimen there are indications of residual stresses. However these were in the original piece of bakelite as received, and are sufficiently far from the region being investigated so as to have no effect on the pattern around the cutout.

The main points to be remembered in eliminating cutting stresses are:

1. Be sure the cutter is very sharp.
2. Take very thin cuts.
3. Have the specimen held securely.
4. Avoid any vibrations of the machinery.
5. Keep cutting speeds moderate.
6. Make finishing cuts not more than 0.005".

C. Loading Procedure

For mounting the specimen eight 3/16" holes spaced $\frac{1}{2}$ " apart and $\frac{1}{2}$ " from the edge of the specimen were drilled.

It was then secured to the loading frame and beam by $3/16$ " x $1\ 1/4$ " machine bolts and nuts. The number of bolts used was the maximum deemed possible and this was desired in order to distribute the load into the specimen as uniformly as possible. Actually a type of clamped end fitting would be theoretically the best, but the physical difficulties involved are evident. The stress concentrations resulting from the bolts, however, are sufficiently far removed from the cutout that they produce no effect on the stress distribution around it.

The load itself consisted of a large, specially made can, which was filled with water. The amount of load was measured by the height of water, using a calibration curve obtained by actual weight-water level measurements. The can was suspended from the loading arm by means of an "S" hook.

The actual procedure followed was to:

1. Suspend the required load at the particular station.
2. Adjust the optical system to throw the image on a large translucent screen.
3. Determine the exact fringe orders represented by the system of observed iso-chromatic lines, which was done by manually removing the load and applying it gradually.

4. Readjust the geometry of the optical system so that the image was thrown on the camera screen, and set the marker number to correspond to that particular load position as shown in figure 2. (these are the numbers shown under the specimen in the photographs).
5. Expose the plate.

USE OF THE CURVES

It is expected that one who wished to determine the stress ratios, or more specifically, the actual stress in a panel around a cutout, will first of all know the general dimensions and type of loading of the panel under investigation and also the contemplated size and shape of cutout.

With this information the ratio of the bending moment to the shear load at the section in question may be calculated. Next the shear stress and normal stress should be calculated at the point corresponding to the center line of the section in question at the outermost fibers of the proposed cut out by the usual elementary beam formulae: $\bar{T} = \frac{V}{Ib} \int_{y_1}^{y_2} y^2 dA$ & $\sigma = \frac{My_1}{I}$

where

\bar{T} = Shearing stress

V = Shear Load

y_2 = one-half the height of beam cross section.

y_1 = distance from vertical axis to outermost fiber
of contemplated cutout.

I = moment of inertia of cross section (without
cutout) about beam neutral axis.

b = width of beam cross section

σ = normal stress

M = bending moment

With the values of $\bar{\tau}$ and $\bar{\sigma}$ the maximum principal stress, $\bar{\sigma}_{prmax}$, may be calculated from the formula:

$$\bar{\sigma}_{prmax} = \frac{\bar{\sigma}}{2} + \sqrt{\left(\frac{\bar{\sigma}}{2}\right)^2 + \bar{\tau}^2}$$

The stress ratio for any point around the cutout may then be picked from the curve corresponding to the M/V and shape of cutout. (See figures 4, 5, 6, and 7.) Multiplication of this ratio by $\bar{\sigma}_{prmax}$ then gives the actual value of the stress.

If only the maximum stress is desired, figure 8 may be entered with the same values and the maximum stress ratio obtained immediately.

There remains a question of the application of these results to panels which do not have the same dimensions as the one used in this investigation. It is fairly clear that variation of thickness should have no bearing on the results. However, extension of the use of the curves to geometrically similar panels is open to argument. This could be checked rather simply by investigating a geometrically similar panel for say one cutout shape and one loading by photoelastic methods.

CONCLUSIONS

Several interesting results are revealed upon study of the curves shown in figures 4 to 7 inclusive. In all cases the highest boundary stress occurred at the boundary adjacent to the load where the bending moment is lower than on the boundary adjacent to the support. According to theory, if no concentrations existed the calculated combined stress should be less since the stress due to bending is less and the shear of course remains constant. This result is somewhat mystifying and there is no apparent explanation for it.

In all types of cutouts as would be expected, the stress at the neutral axis of the beam at the boundary of the cutout is zero. The bending stress here is theoretically zero, while the shear would be maximum if there were no cutout. However, due to the free boundary where no parallel shear stress exists there can be no perpendicular shear stress and therefore these points are unstressed.

In connection with this same vertical boundary it will be noticed that as the fillet radius is decreased the stress along this boundary becomes less, that is the material adjacent to it is less effective. Therefore, part of this material along the neutral axis might conceivably be removed if the primary purpose of the cutout is for lightening.

A study of figure 8 shows that for higher values of M/V , say greater than 10, the type of cutout has practically no effect on the maximum stress ratio. Therefore, if the load is reasonably far removed from the cutout it appears that any convenient shape of cutout will suffice.

In connection with the stress ratio values in the region of large M/V 's it will be noted that there is only slight variation in these which can be attributed to experimental scatter. Although this statement assumes them to be the same it is felt that this is reasonable in view of the results obtained. This constancy of the stress ratios may possibly be explained by the principle of Saint-Venant.

In the region of low values of M/V the effect of a small fillet radius is quite evident. The largest radius possible in the cutout, i.e., one-half inch, gives the lowest stress ratio while of course the square corners give the highest. Theoretically for the latter case the values should be infinite but based on visual observations of fringe orders this is impossible.

The elliptical cutout was taken as the starting point of this investigation in contrast to the circle which has been previously investigated to determine whether it would have a radically different effect on stress concentrations. However, it is seen that the curve for the ellipse follows substantially the trend of the modified cutout curves. In the

lower region of M/V it falls in between the curves for cut-outs with one-half and one-fourth inch fillets. In other words the governing factor is the smallest radius of curvature which in the case of the ellipse is somewhere in between the one-half and one-fourth of an inch radii.

At the beginning of this investigation a rather ambitious program was contemplated in that nine different loading positions or values of M/V were to be used. However, this was cut down to five after the first series of photographs were taken and finally results for only four positions were plotted. The reasons for this are readily apparent if one studies the photographs of group II, Page 35. There is practically no change in the isochromatic values for the larger values of M/V . This fact is even more revealing in the plot of the curves. The stress ratios spread out in the region of lower values of M/V but at the higher ones they are almost constant. To attempt to plot the values for a wide range of M/V would require a tremendously large scale and even then it is very doubtful if the variation of fringe orders would be sufficient to be discernible.

All this may be summed up by stating that in the higher ranges of M/V the stress ratios are substantially constant which in itself is sufficient information.

Much of the purpose of this thesis would be lost if an attempt was not made to compare and correlate these

results with the previous work of Tyra and Hollister which was concerned with circular and modified cutouts. Unfortunately the parameters used in their work were different from the ones used here. Also the ratio of their cutouts to beam height was 1 to 3.3, whereas in this case it was 1 to 2. The difficulties of comparison are readily apparent.

Since in each case the cutouts with the largest fillet radius possible proved best from the standpoint of stress concentrations it was decided to attempt to compare these types of cutouts, i.e., the rectangular one with the 1/2" radius, presented in this report, and the circular one.

Parameters of the two reports were reduced to a common basis as were the maximum stress ratios. A comparison of the common maximum stress ratios was then made. It must be remembered that in the case of the circle 1/3.3 or about 30 per cent of the material was removed and in the case of the rectangle 1/2 or 50 per cent of the material was removed. With this in mind the following conclusions were made:

- I. For M/V's above about 15 the rectangular cutout showed an increase in the maximum stress ratio of about 25 per cent over that for the circle.
- II. For lesser M/V values the percentage rise was increasingly rapid until at M/V of about 6 it was almost 100 per cent greater.

From the above the general conclusion may be reached that the more material that is removed perpendicular to the neutral axis of the beam the higher are the stress ratios produced. This is not so critical if the load is reasonably far removed from the section at the cutout but increases very rapidly as the load is applied closer to the cutout and more and more material is removed.

This indicates that if cutouts are desired for lightening purposes it would be better to keep the height of the cutout small compared to the height of the beam and increase the length parallel to the neutral axis of the beam.

APPENDIX

Refer to figures 2 and 3 on pages 55 and 56 for dimensions and loads in the following calculations.

Let V_s = static load, i.e. weight of loading beam

V_a = applied load

M_s = static moment

M_a = applied moment

$$V = V_a + V_s$$

$$M = M_a + M_s$$

M/V = ratio of bending moment to shear load

$$V_s = 2.70 + 2.02 = 4.72 \text{ lbs.}$$

$$M_s = 2.70 \times 16.64 + 2.02 \times 2.34 = 49.63 \text{ in. lb.}$$

Load Position 2

$$V = 4.72 + 51.7 = 56.42 \text{ lbs.}$$

$$M = 49.63 + 51.7 \times 29.5 = 1573 \text{ in. lb.}$$

$$M/V = 1573/56.42 = 27.9$$

Load Position 4

$$V = 4.72 + 69.0 = 73.72 \text{ lbs.}$$

$$M = 49.63 + 69.0 \times 21.5 = 1534 \text{ in. lb.}$$

$$M/V = 1534/73.72 = 20.8$$

Load Position 5

$$V = 4.72 + 79.49 = 84.21 \text{ lbs.}$$

$$M = 49.63 + 79.49 \times 13.5 = 1122 \text{ in. lb.}$$

$$M/V = 1122/84.21 = 13.3$$

Load Position 7

$$V = 4.72 + 100.03 = 104.8 \text{ lbs.}$$

$$M = 49.6 + 100.03 \times 5.5 = 601.3 \text{ in. lb.}$$

$$M/V = 601.3/104.8 = 5.74$$

Load Position 8

$$V = 4.72 + 111.53 = 116.3 \text{ lbs.}$$

$$M = 49.6 + 111.53 \times 2 = 272.6$$

$$M/V = 272.6/116.3 = 2.34$$

The stress optical coefficient of this specimen was found by calculating the principal stress at the outer boundary of the specimen with no cutout for a particular point and noting the fringe order at this same point. The formula $C = \frac{(\sigma_1 - \sigma_2) b}{N}$ was then used

where

C = stress optical coefficient for the model in
lbs. per in. per fringe order.

σ_1 = maximum principal stress in lbs. per in.²

σ_2 = minimum " " " " " "

b = thickness of the model in in.

N = fringe order.

C was determined for several points along the boundary, and an average value of 87.3 used for subsequent calculations.

Since at a boundary no shear stress parallel to the boundary can exist unless externally applied and no tensile

or compressive stress can exist perpendicular to the boundary then principal stresses exist here, one of which is parallel to the boundary and the other of which must be zero. In this case we will take σ_2 to be zero. Then rearranging the above equation it becomes $\sigma_1 = \frac{CN}{d}$. Substituting $C = 87.3$ and $b = \frac{1}{2}$, $\sigma_1 = 174.6 \text{ N}$.

This last relationship now permits us to obtain σ_1 at any free boundary say the cutout by noting the value of N at any point and applying this relationship.

Since we are interested in the values of the stress ratios around the cutout the various values of σ_1 must be compared with some basic or standard value. This will be taken to be the maximum principal stress existing at the vertical centerline of the proposed position of the cutout and one inch from the neutral axis for the load under consideration.

Let x-x coincide with the neutral axis of the beam.

y-y be vertical and perpendicular to x-x.

σ_x = tensile or compressive stress in x-x direction.

τ_{xy} = shearing stress in the x-x direction
perpendicular to y-y.

h = height of cross section

I = moment of inertia of cross section about
its own horizontal neutral axis.

$$I = \frac{bh^3}{12} = \frac{1}{2} \times (4)^3 / 12 = 8/3 \text{ in.}^4$$

$$\sigma_x = Mc/l = Mx/l / 8/3 = 3M/8$$

$$\tau_{xy} = V/Ib \int_y^2 dA = V \left(\frac{8}{3} \times \frac{1}{2} \right) \times 1.5 \times \frac{1}{2} = 9V/16$$

The values of M and V previously calculated have been substituted in the above formulas and the results are tabulated below.

| Load Position | <u>M</u> | <u>V</u> | <u>σ_x</u> | <u>τ_{xy}</u> |
|---------------|----------|----------|------------------------------|-------------------------------|
| 2 | 1573 | 56.4 | 590 | 31.7 |
| 4 | 1534 | 73.7 | 575 | 41.4 |
| 5 | 1122 | 84.2 | 421 | 47.3 |
| 7 | 601.3 | 104.8 | 225 | 59.0 |
| 8 | 272.6 | 116.3 | 102.3 | 65.5 |

In finding the maximum principal stress, σ_1 , the following formula was used:

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sigma_{1o}$$

This σ_{1o} was then taken as the basic stress to which all the various stresses around the various cuts were compared. Curves of these stress ratios for various M/V ratios were then plotted as shown in figures 4, 5, 6, and 7.

Calculated values of σ_{1o} and a simplified general form of the stress ratio are tabulated as follows:

| <u>Load Position</u> | <u>M/V</u> | <u> </u> | Stress Ratio |
|----------------------|------------|-----------------------------|---|
| | | | $\frac{\sigma_i}{\sigma_{i_0}} = \frac{174.6N}{\sigma_{i_0}}$ |
| 2 | 27.9 | 592.0 | 0.296 N |
| 4 | 20.8 | 588.3 | 0.297 N |
| 5 | 13.3 | 426.0 | 0.410 N |
| 7 | 5.74 | 239.3 | 0.730 N |
| 8 | 2.34 | 134.4 | 1.298 N |

The last column above was used to facilitate calculation of the various stress ratios from the fringe orders observed for plotting the curves.

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PHOTOGRAPHS

This part contains photographs of the various types of cutouts for the various loadings. They are arranged first according to types of cutouts as follows:

- | | |
|---------|---------------------------------------|
| Group 1 | No cutout. |
| Group 2 | Elliptical Cutout. |
| Group 3 | Rectangular cutout with 1/2" fillets. |
| Group 4 | Rectangular cutout with 1/4" fillets. |
| Group 5 | Rectangular Cutout. |

Within each group the photographs are arranged according to the numbers appearing underneath the model, starting with the lower numbers and progressing upward. The lowest number corresponds to the highest ratio of bending moment to shear loads while the highest number corresponds to the lowest ratio. It will be noticed that the first two groups of pictures are numbered from one to nine, while the remaining groups have certain numbers missing. These latter numbers were omitted because the stress pattern changed so little from one successive loading position to the next as to make it practically impossible to discern any change in fringe orders. However, the omission of these numbers need cause no confusion since the remaining numbers are arranged in order of magnitude.

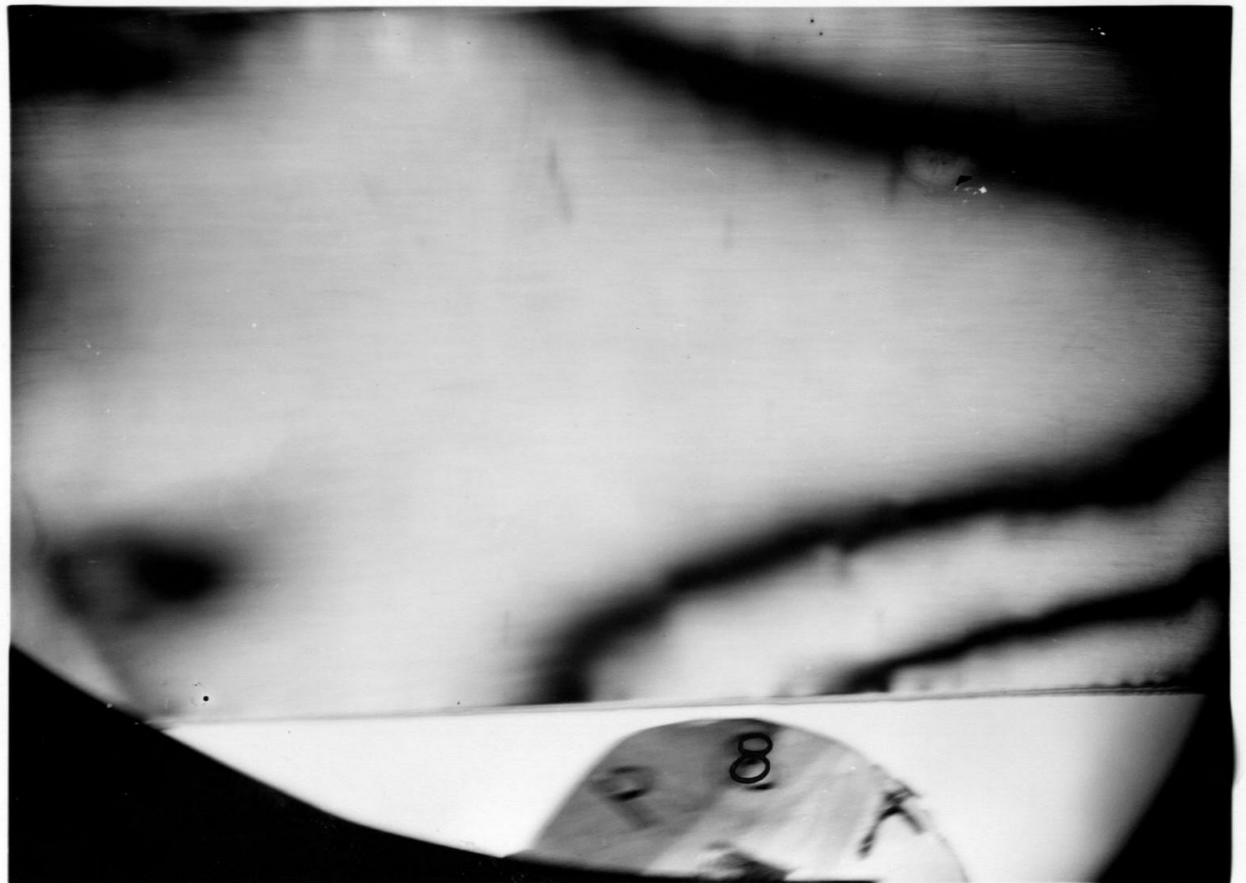
GROUP 1

No Cutout in Panel



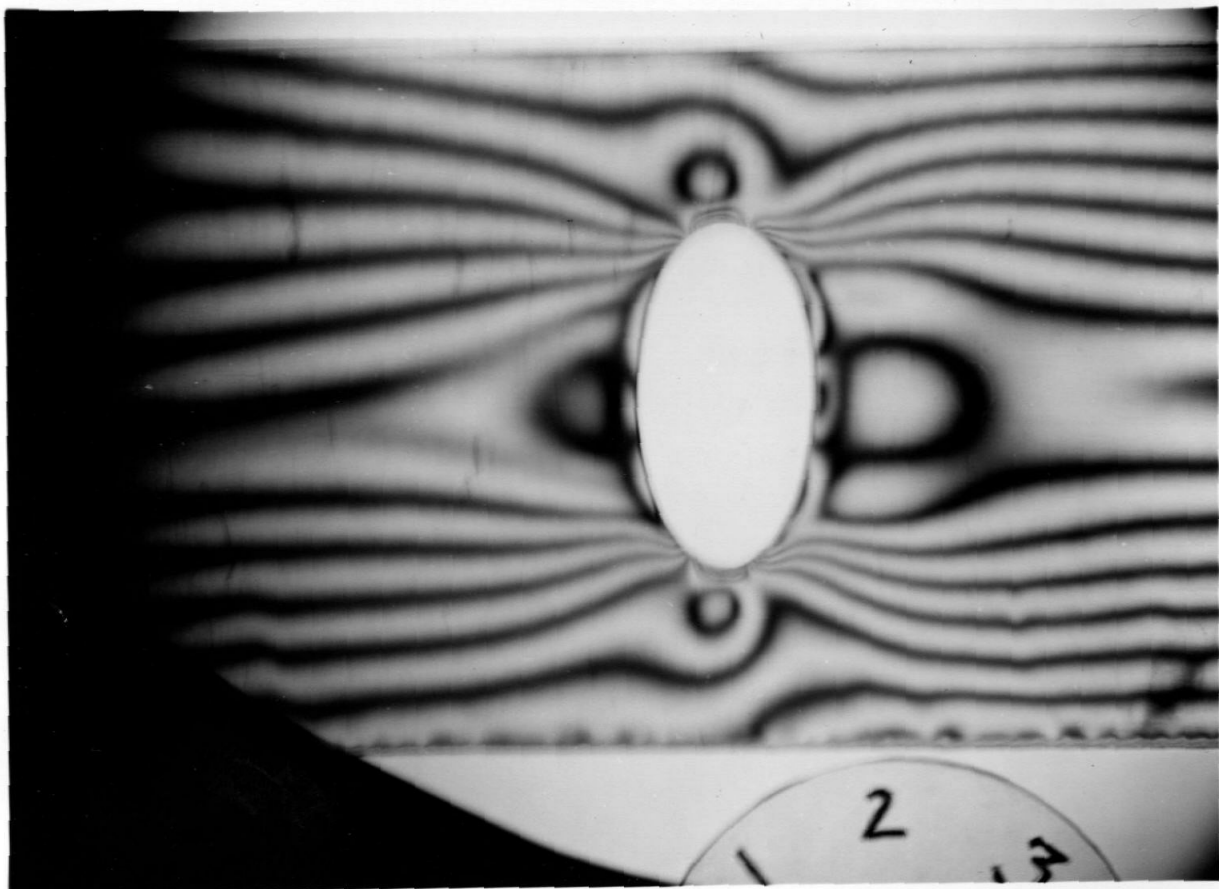
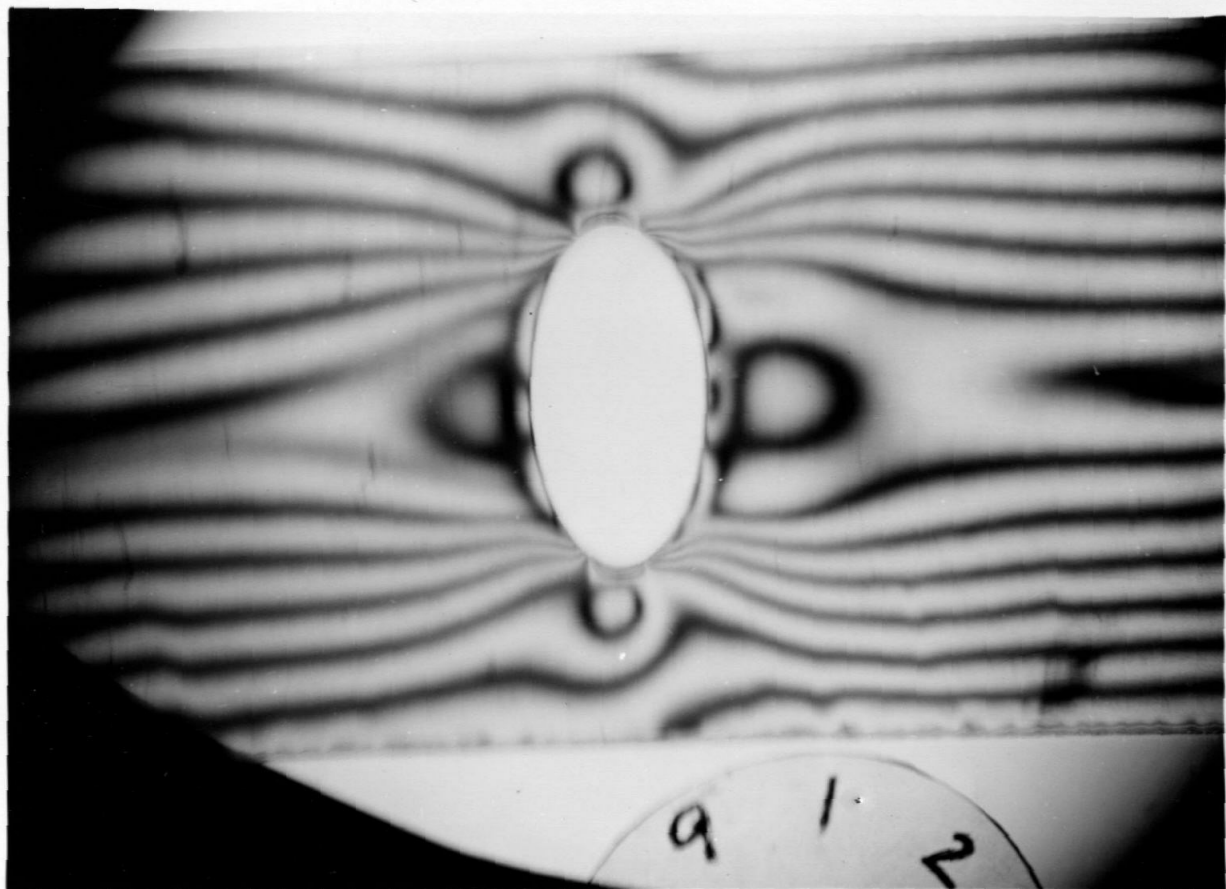


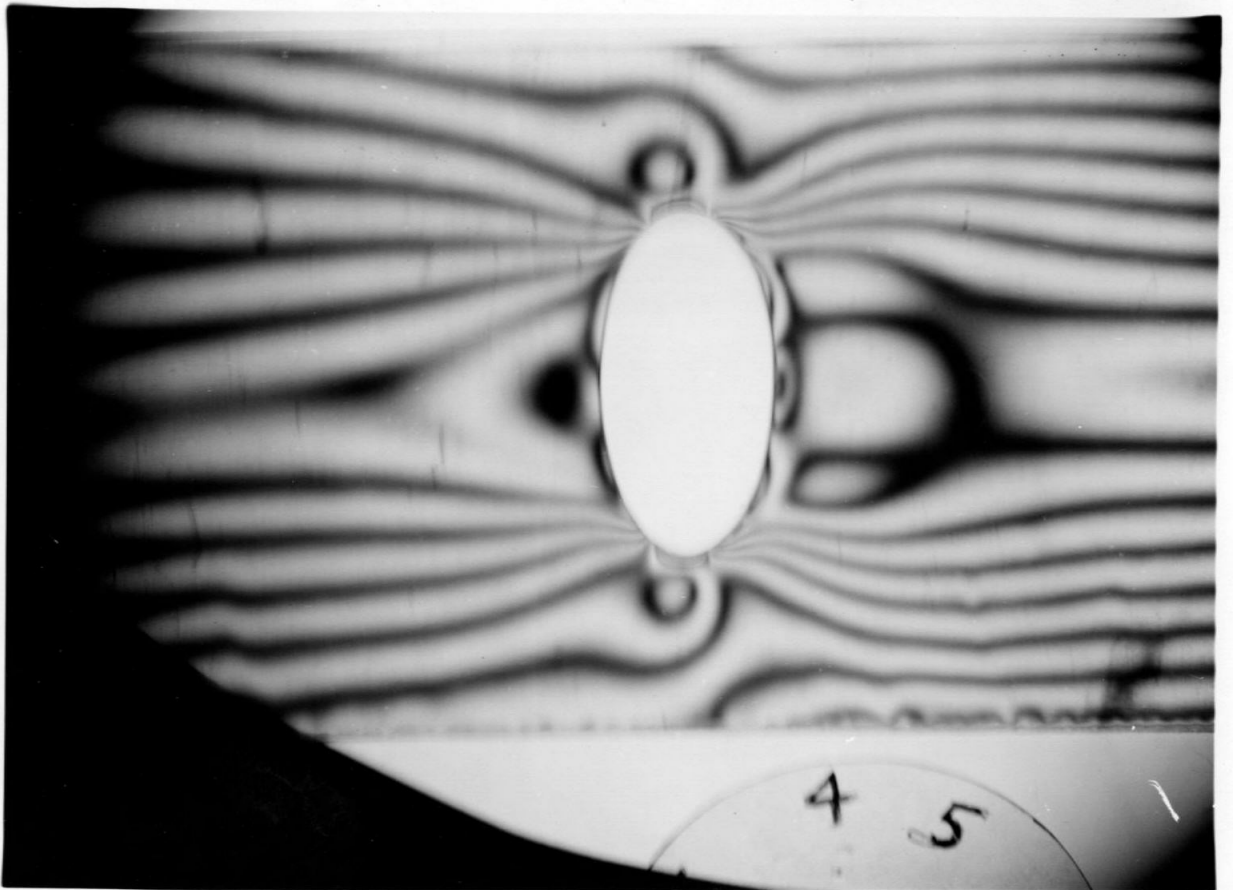
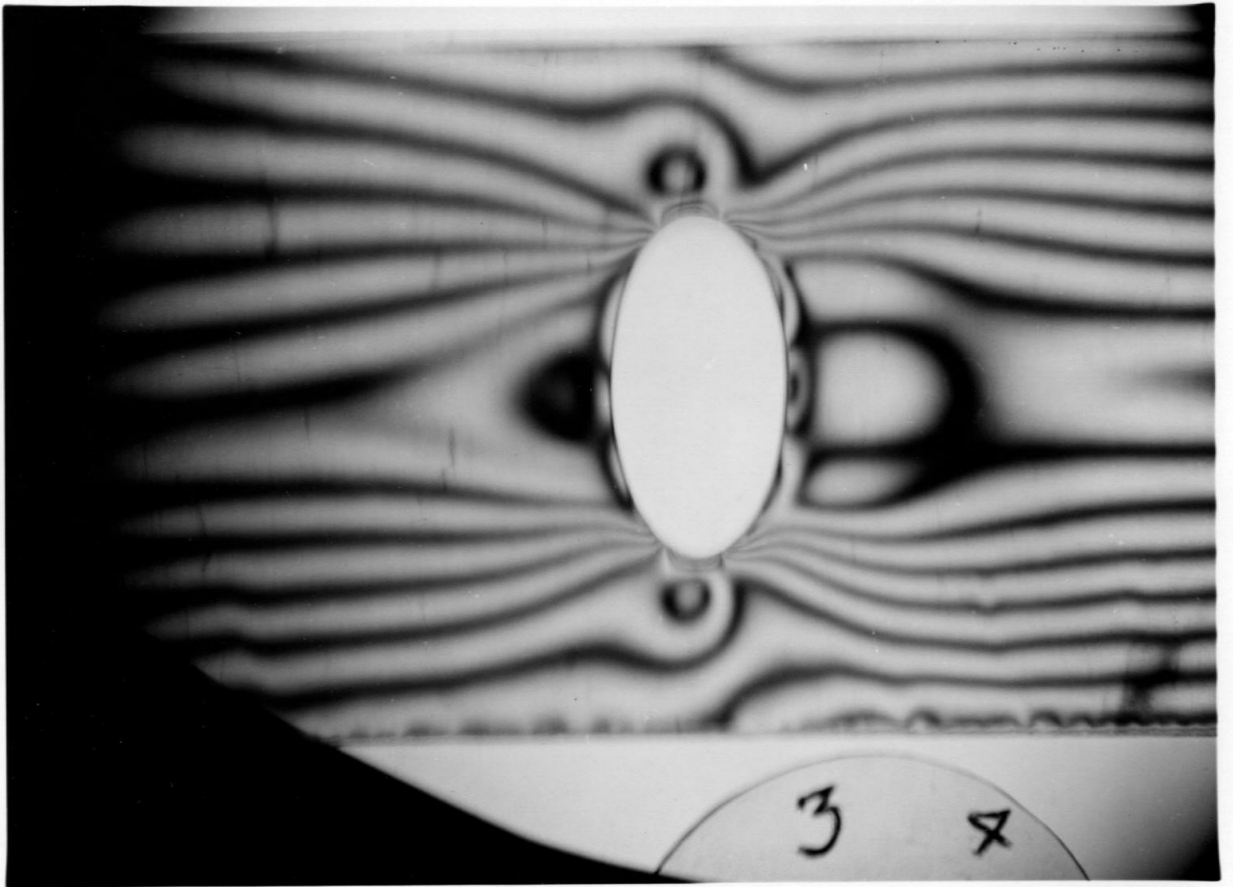


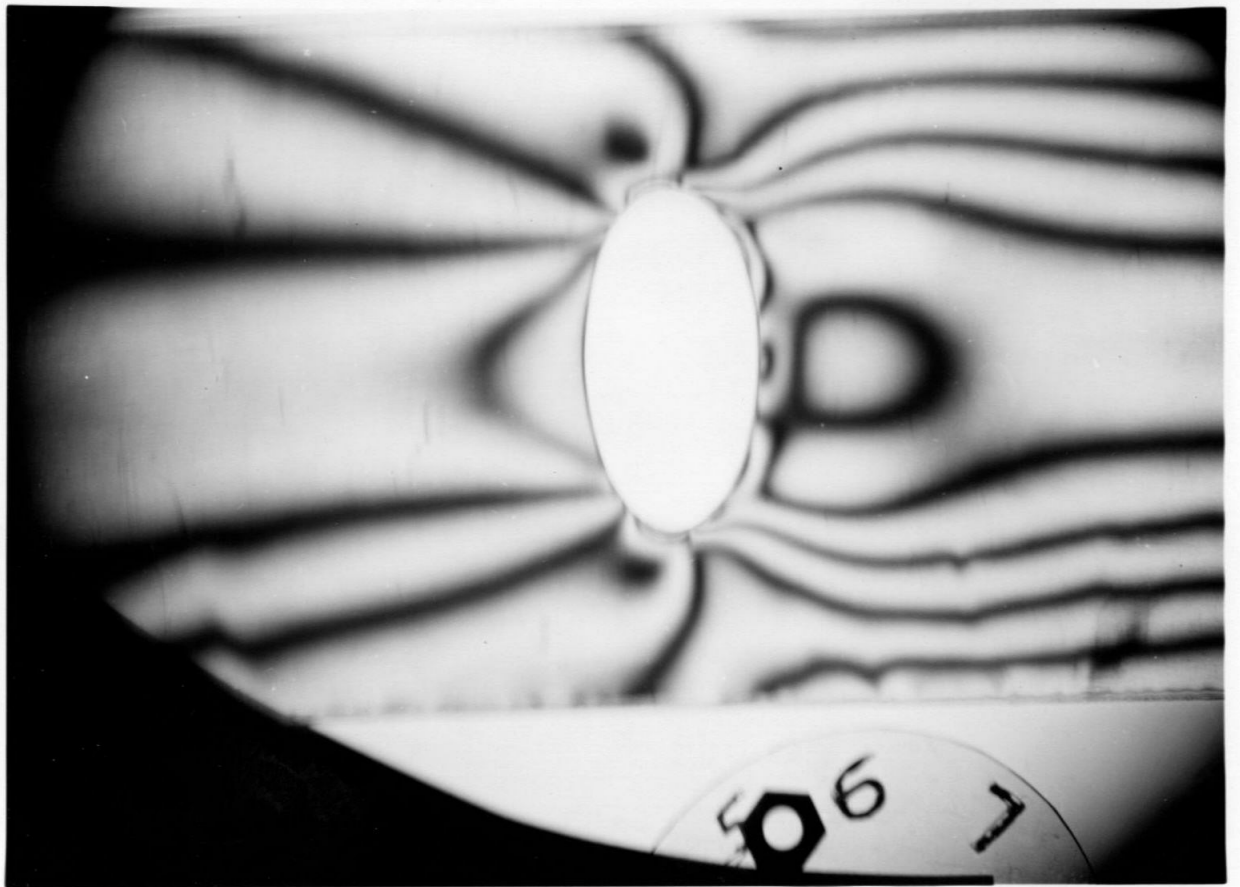
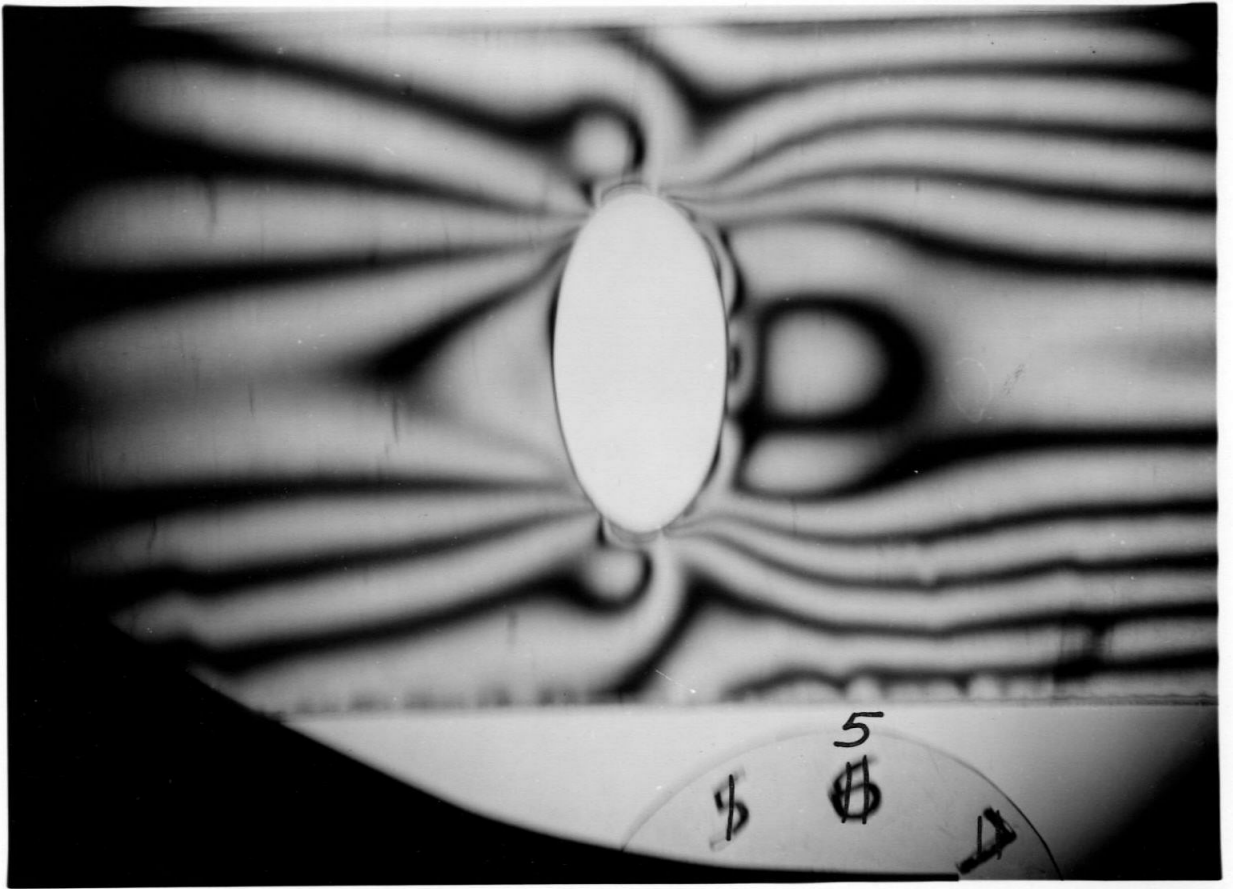


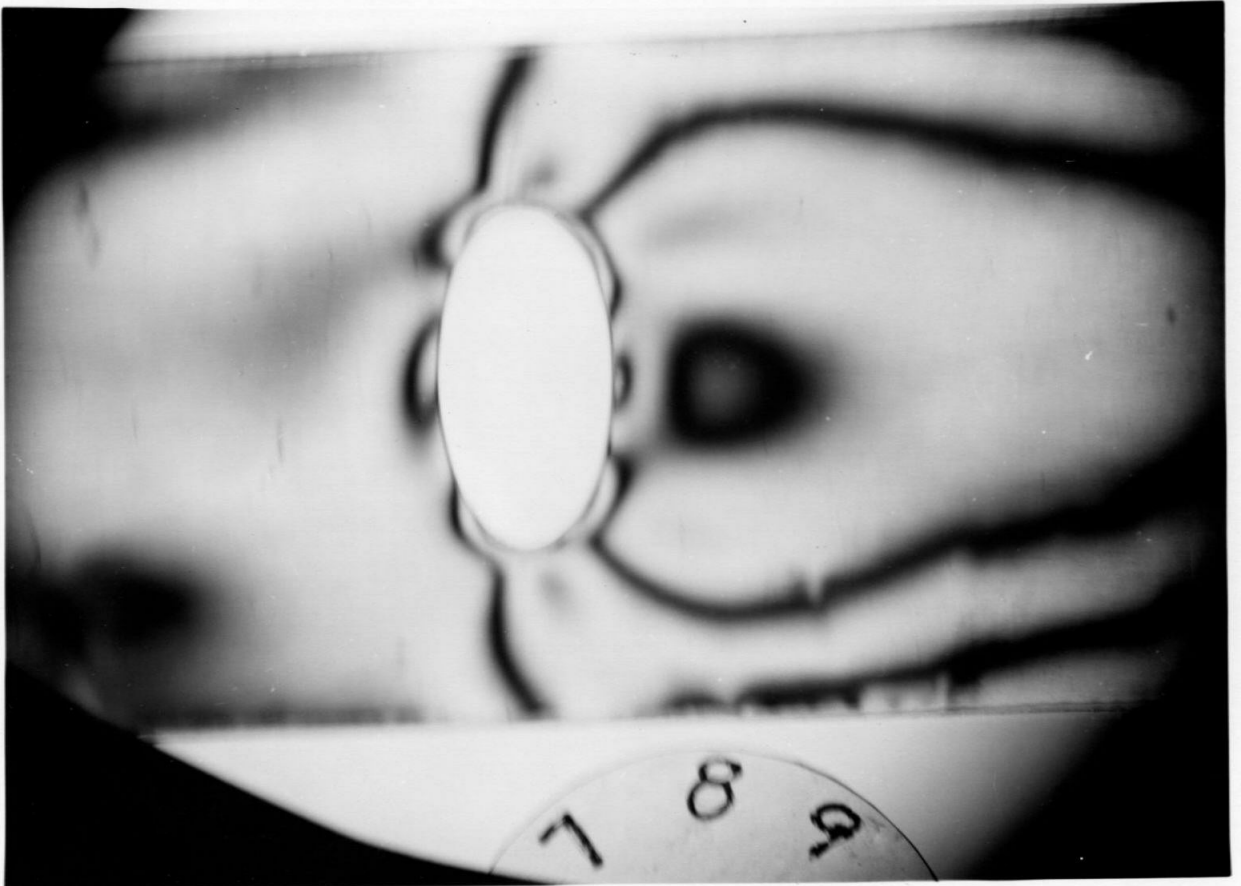
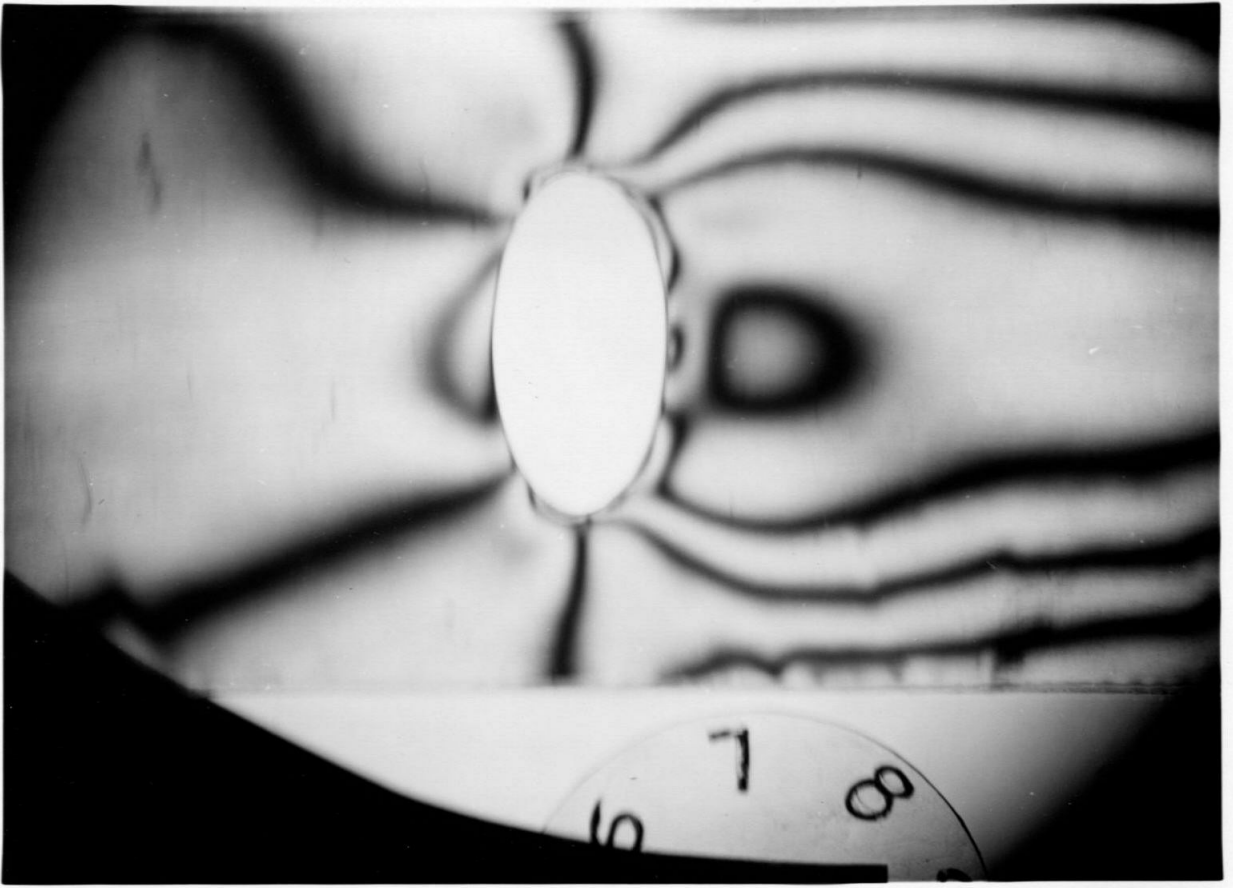


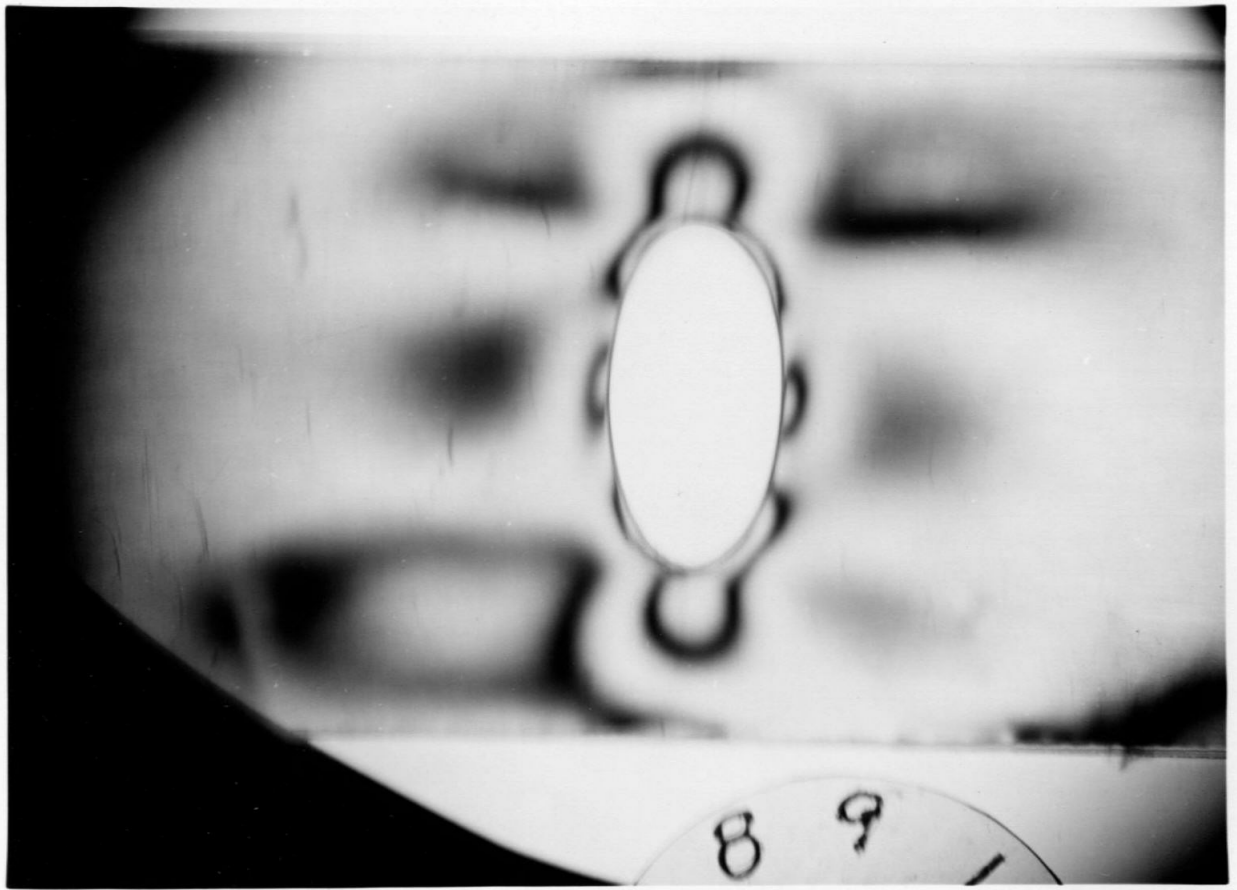
GROUP 2
Elliptical Cutout





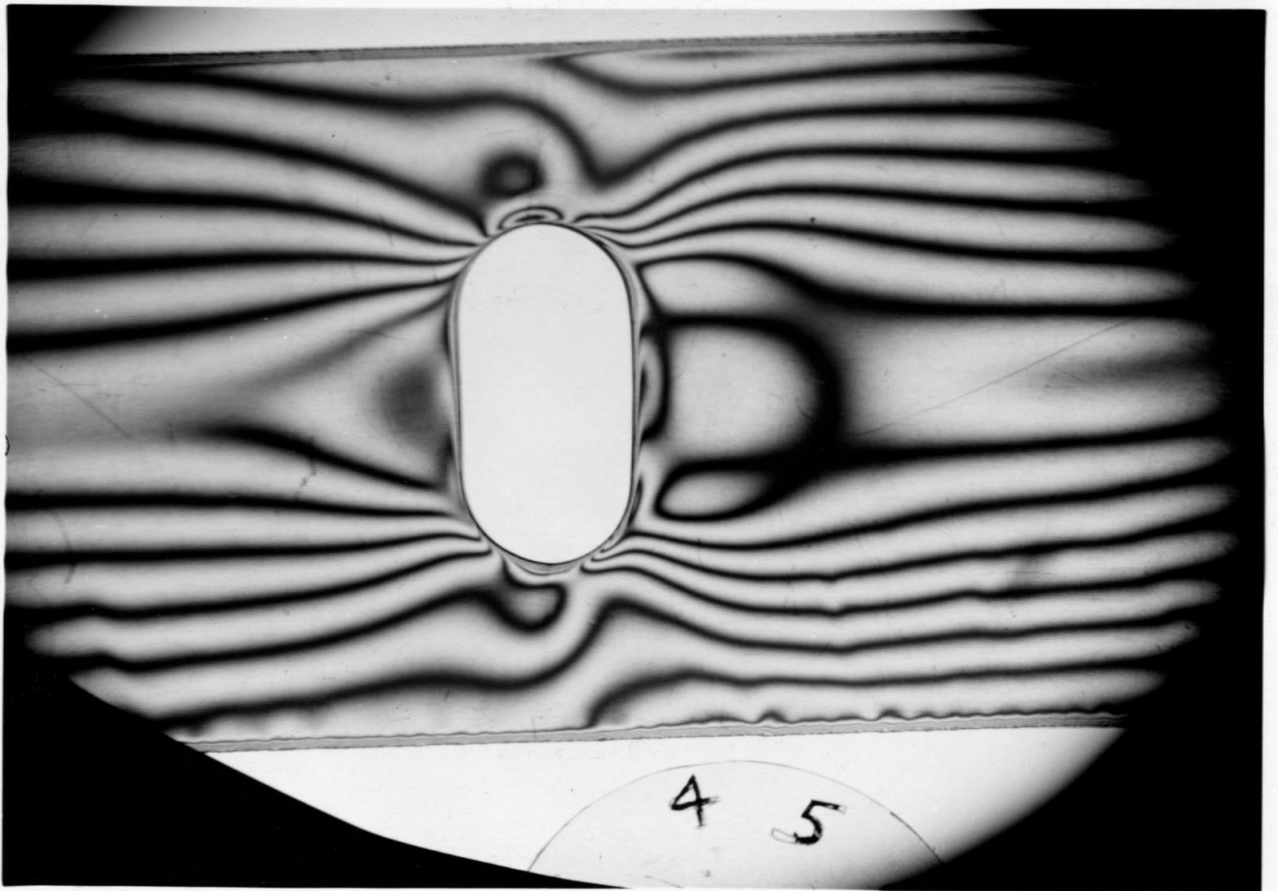
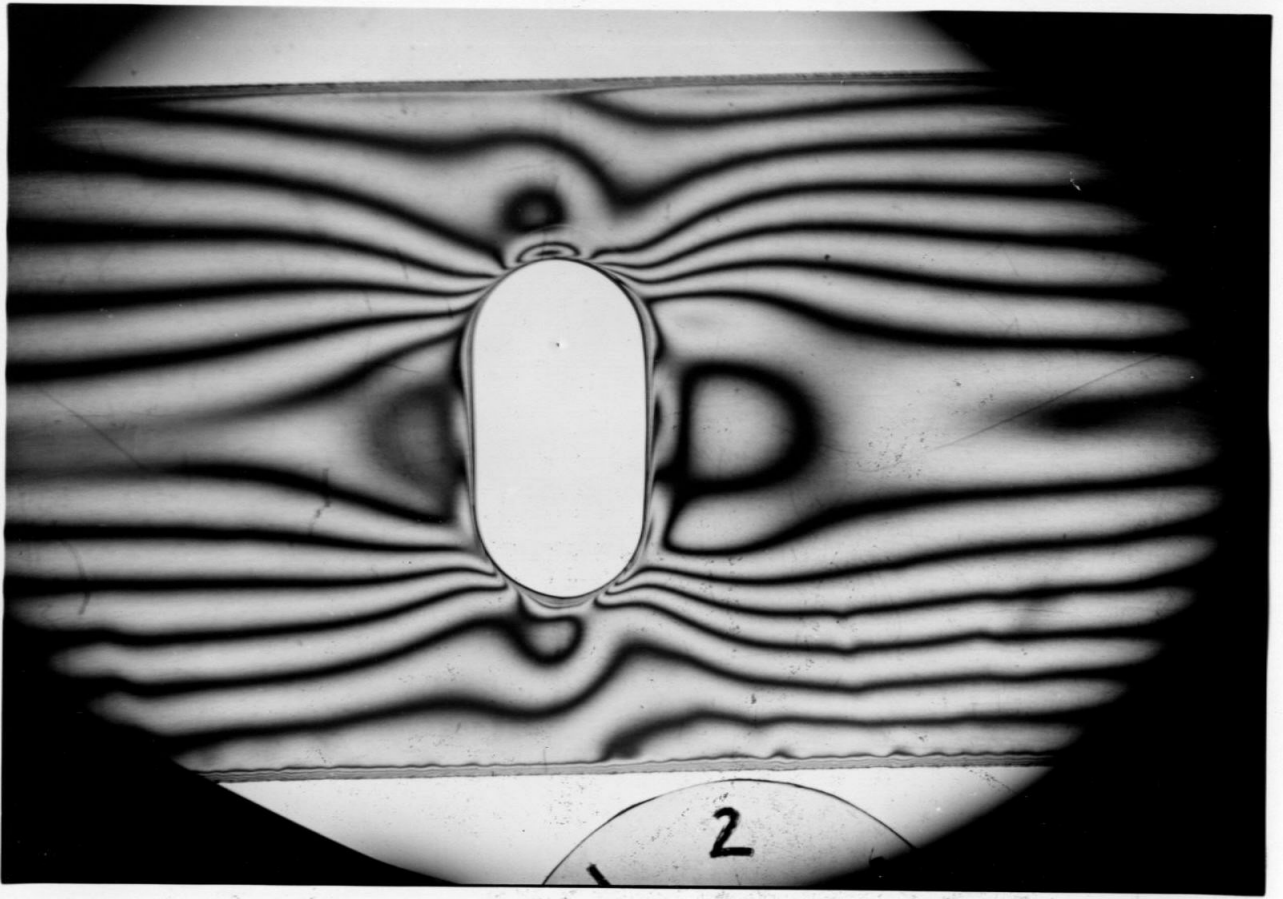


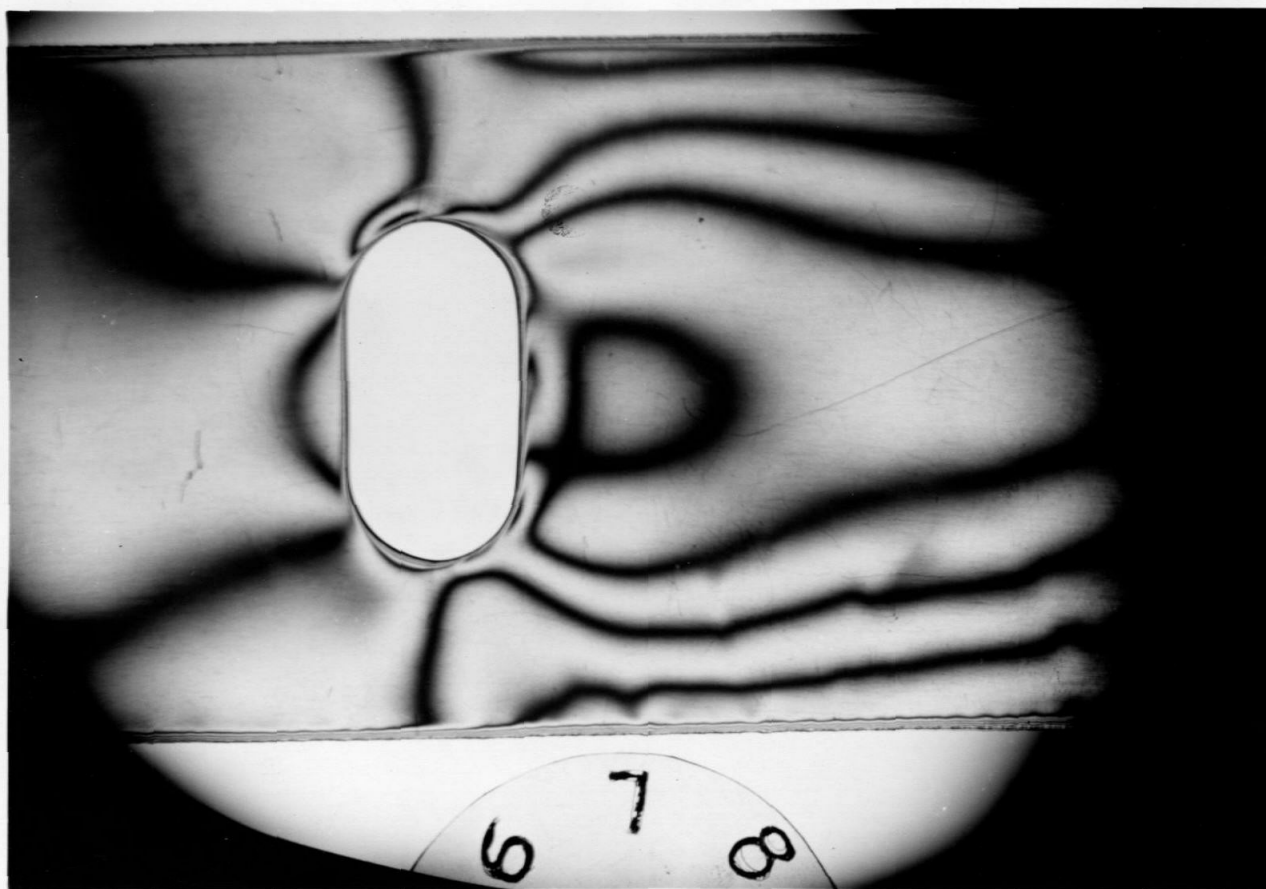
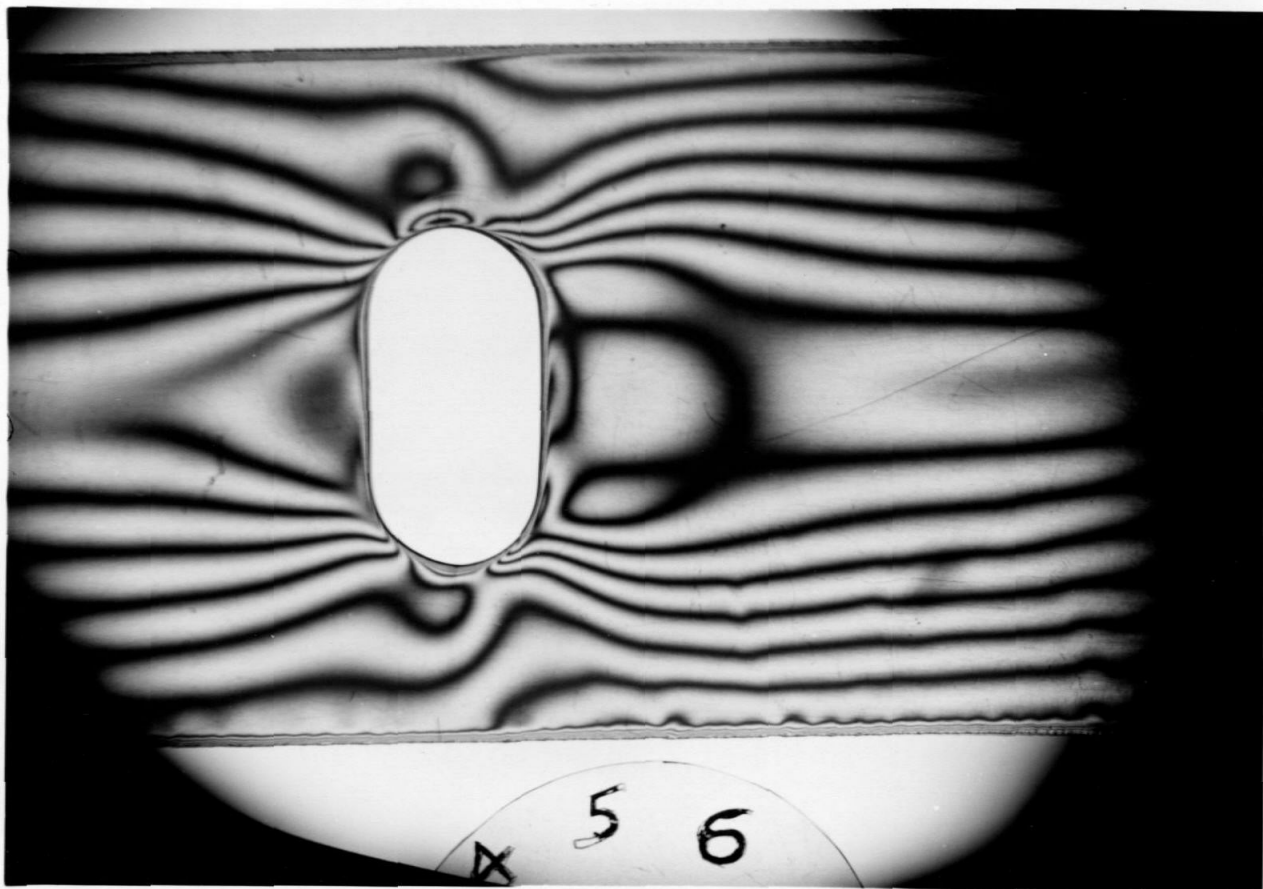


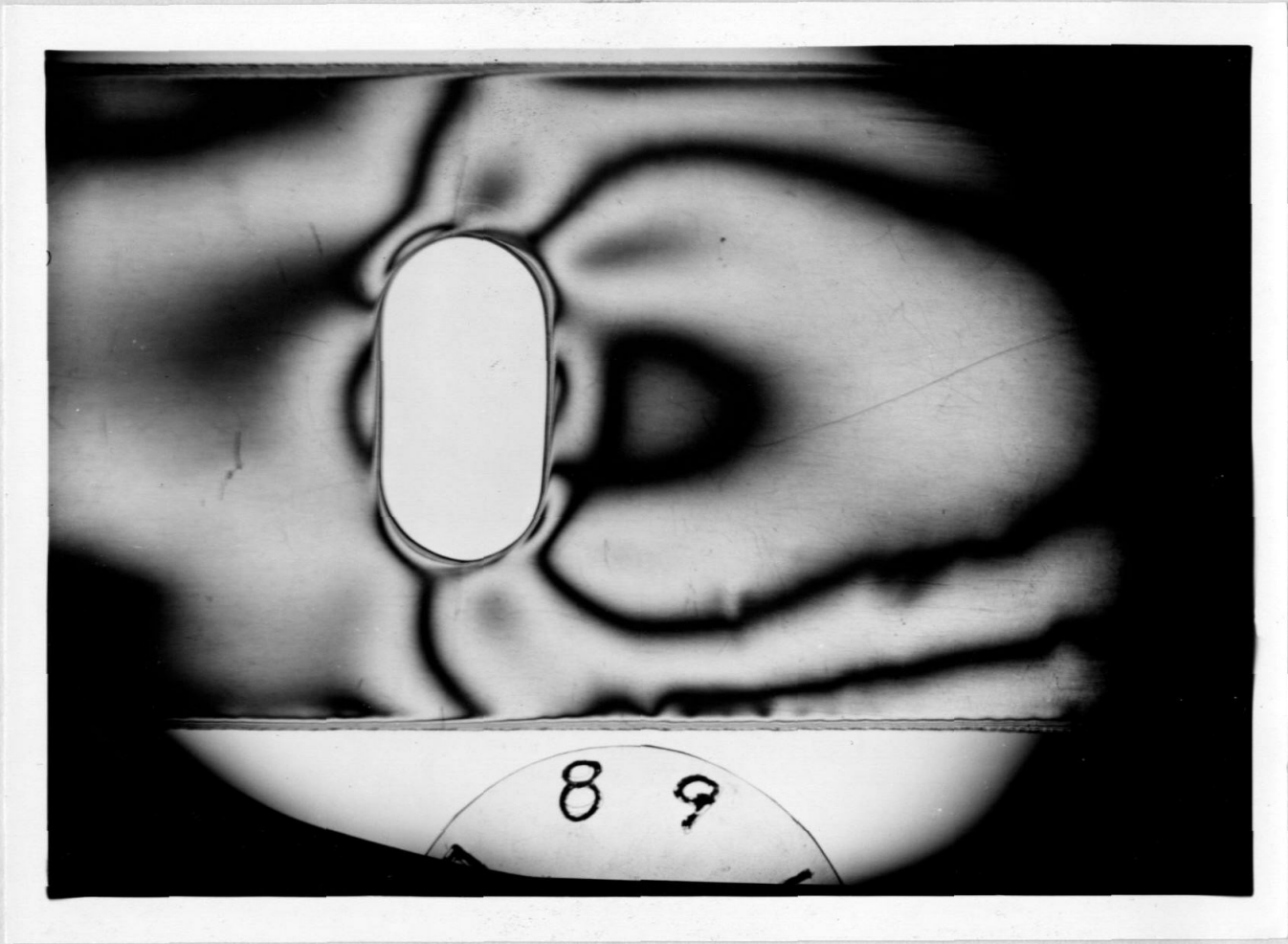


GROUP 3

Rectangular Cutout with $\frac{1}{2}$ " Fillets

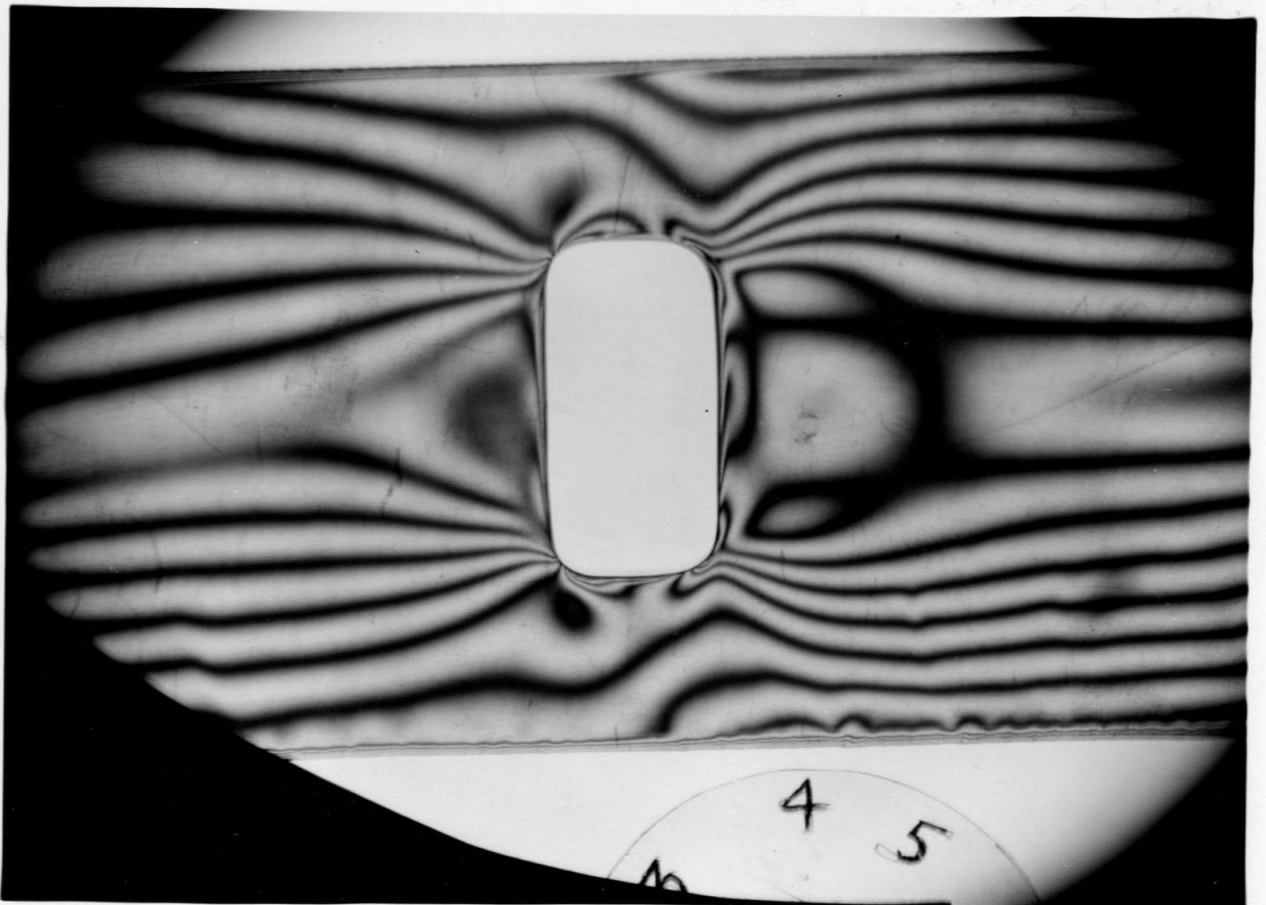
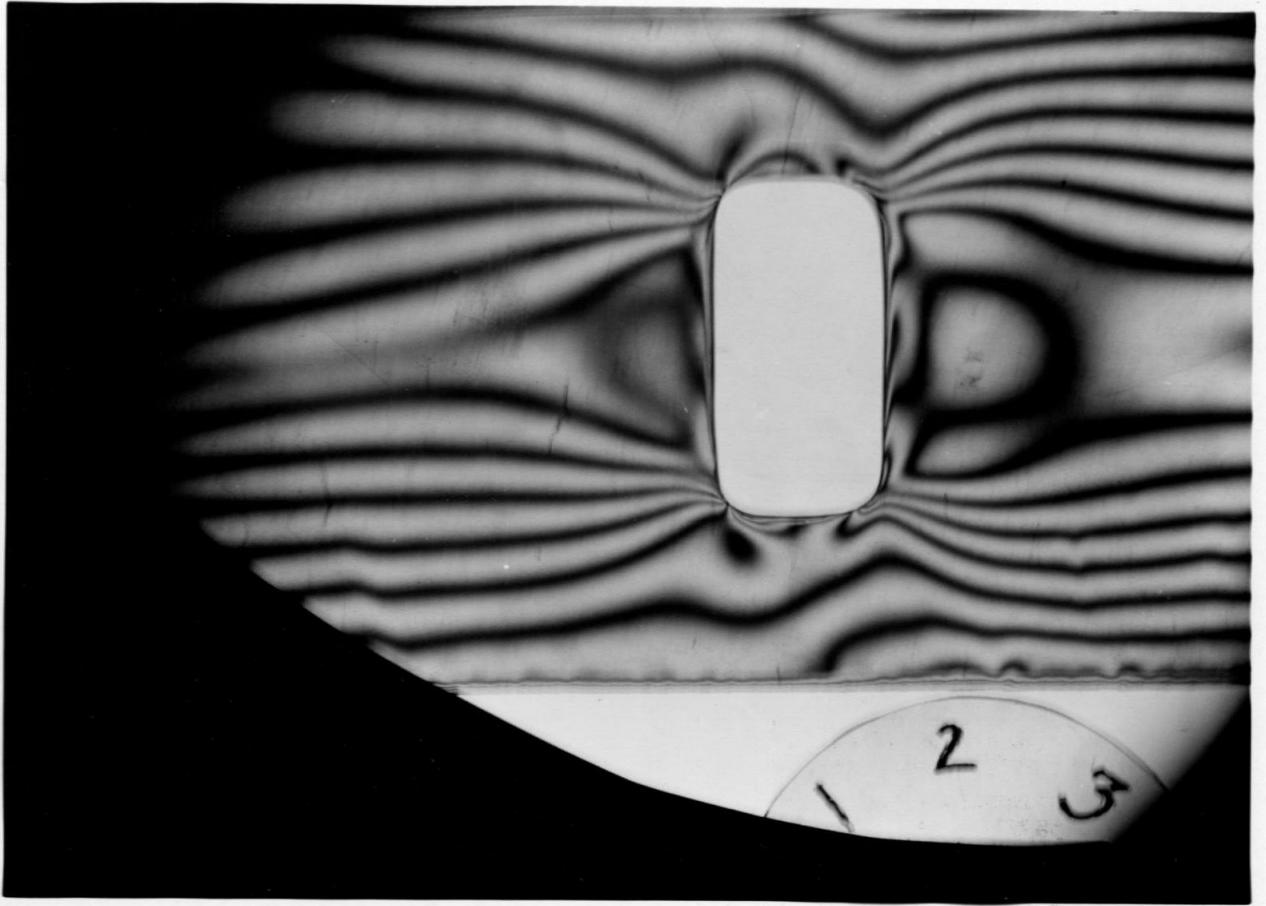


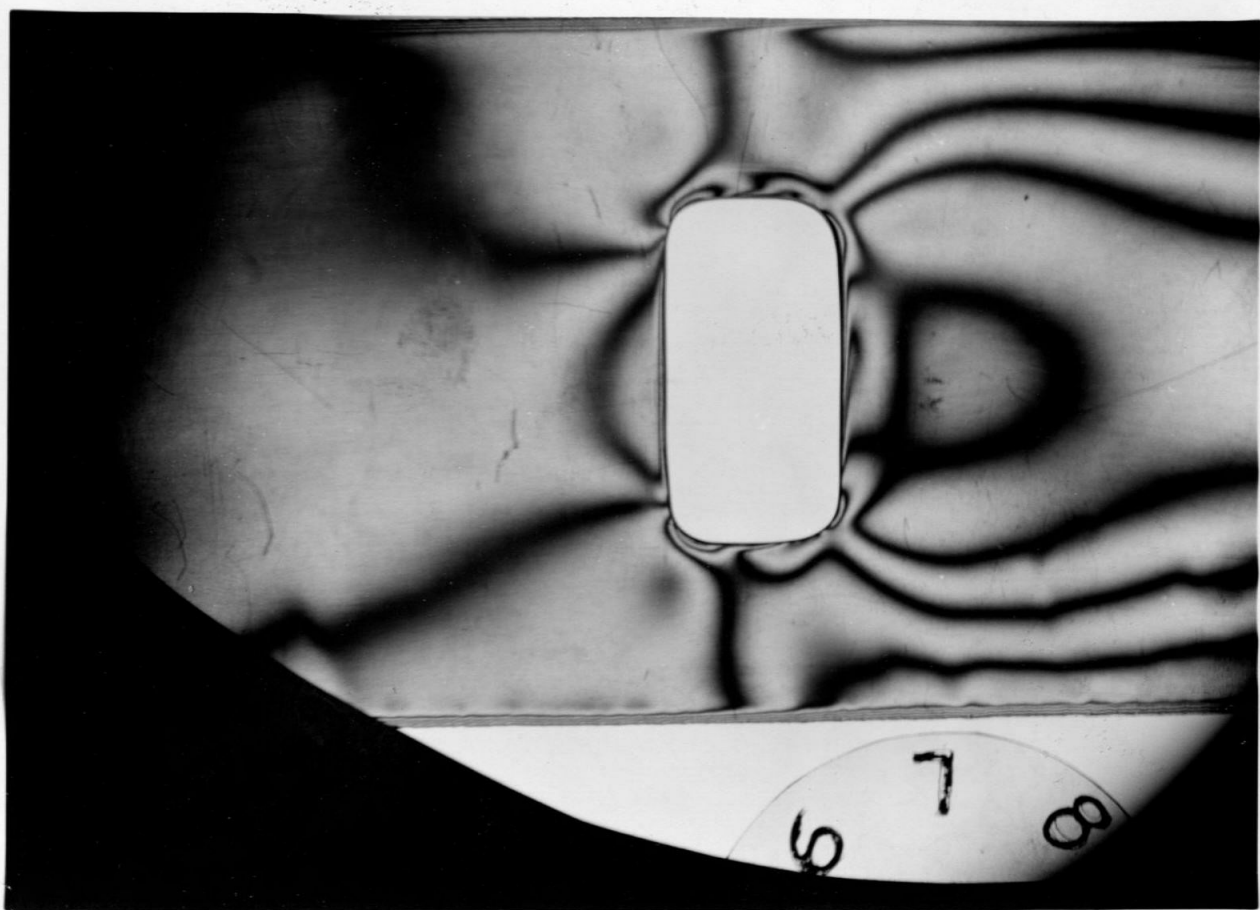
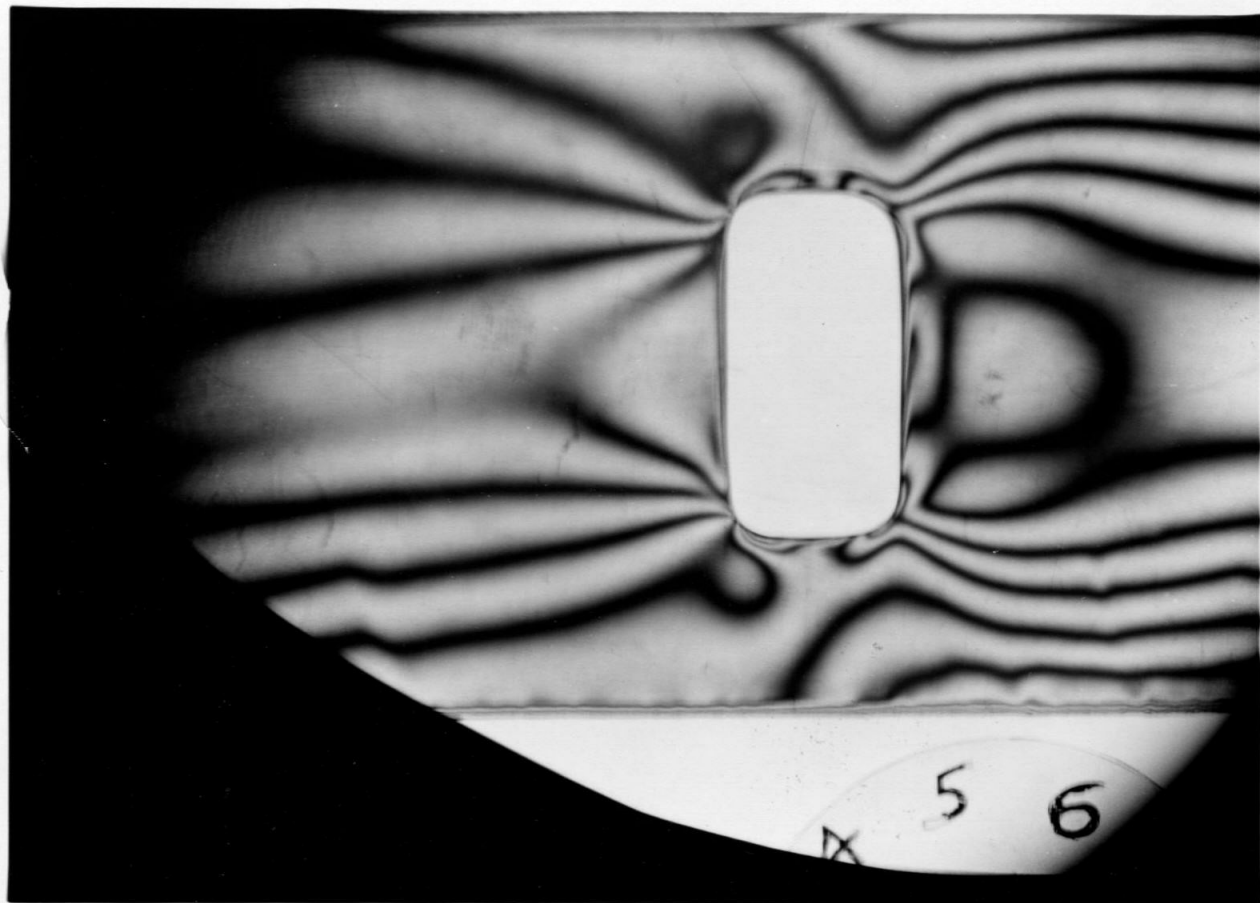


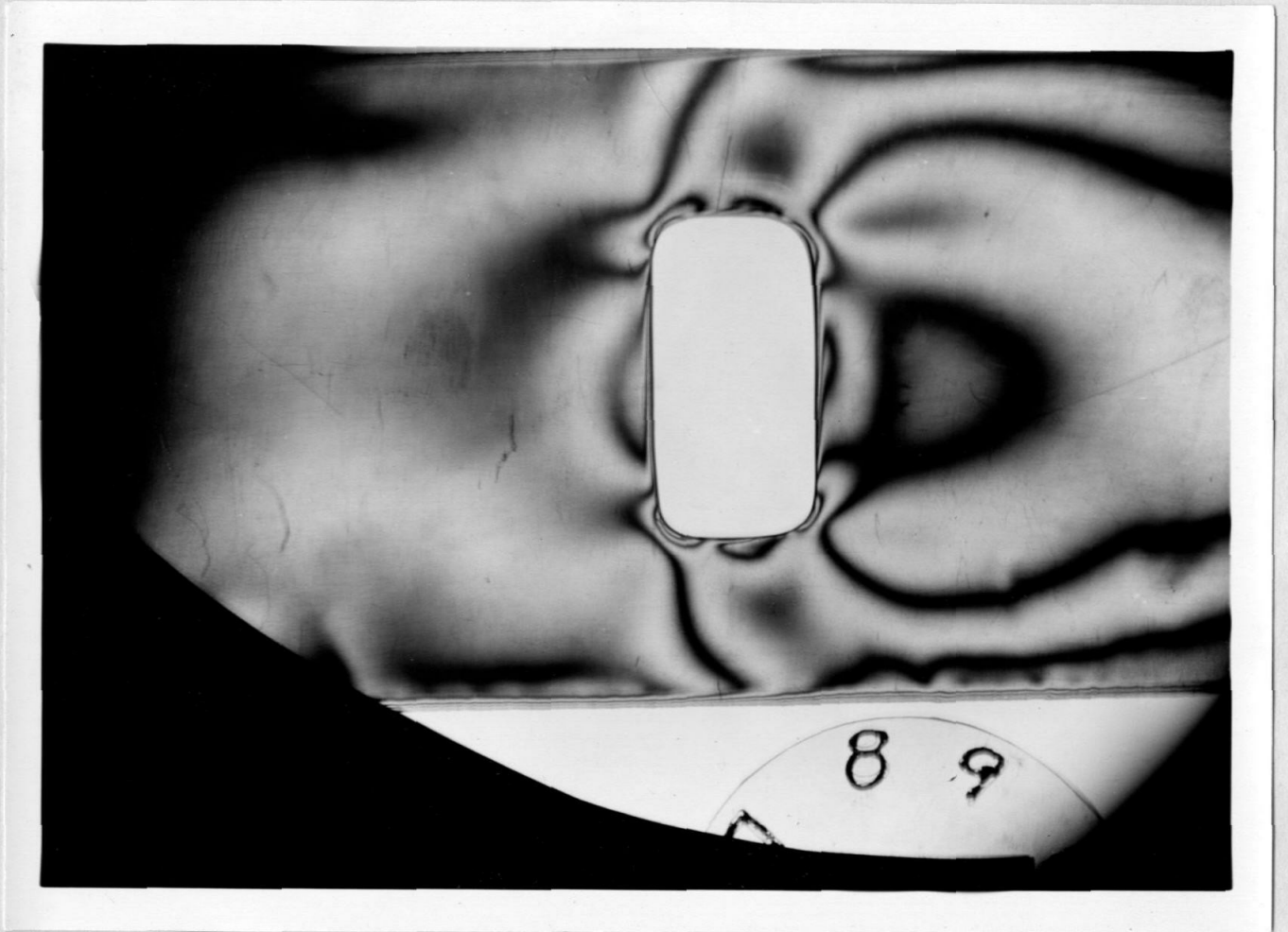


GROUP 4

Rectangular Cutout with 1/4" Fillets

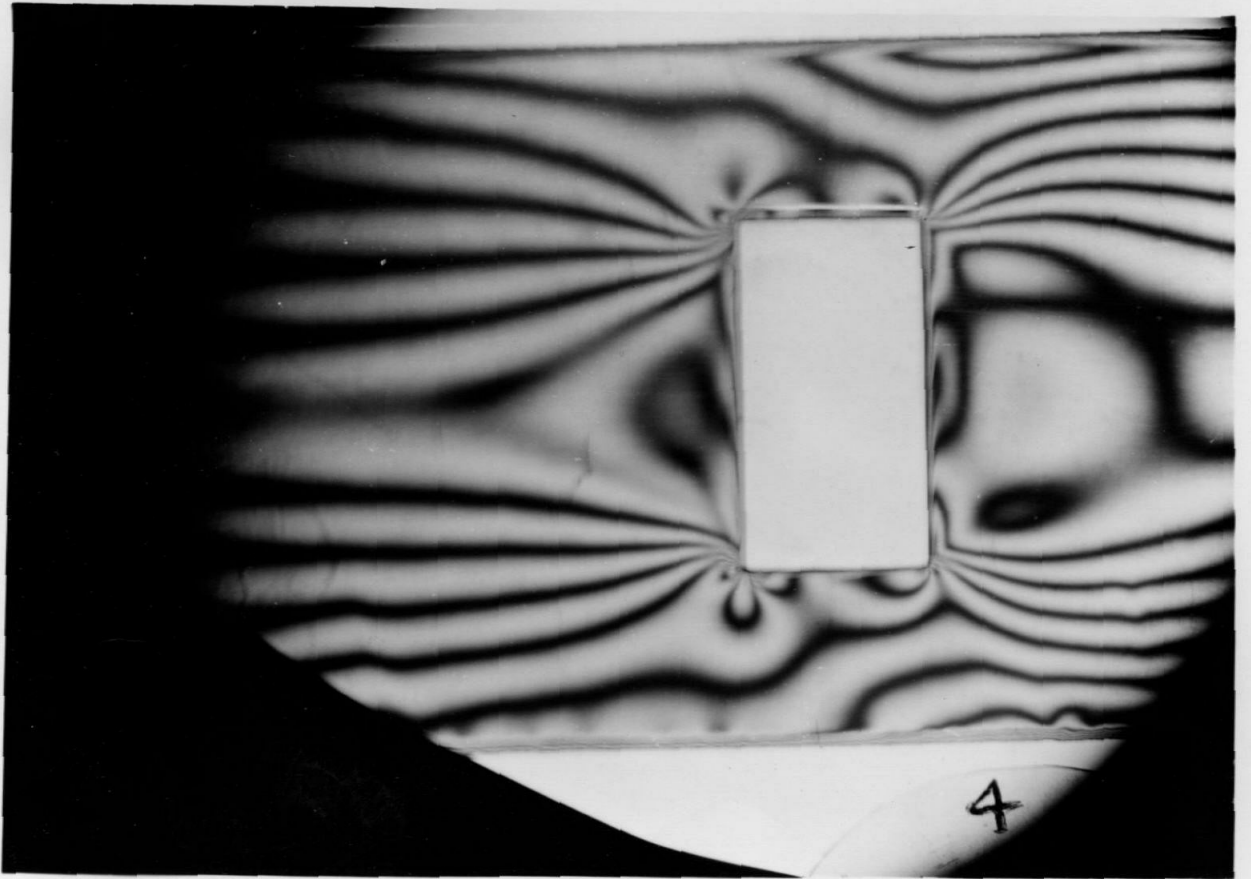
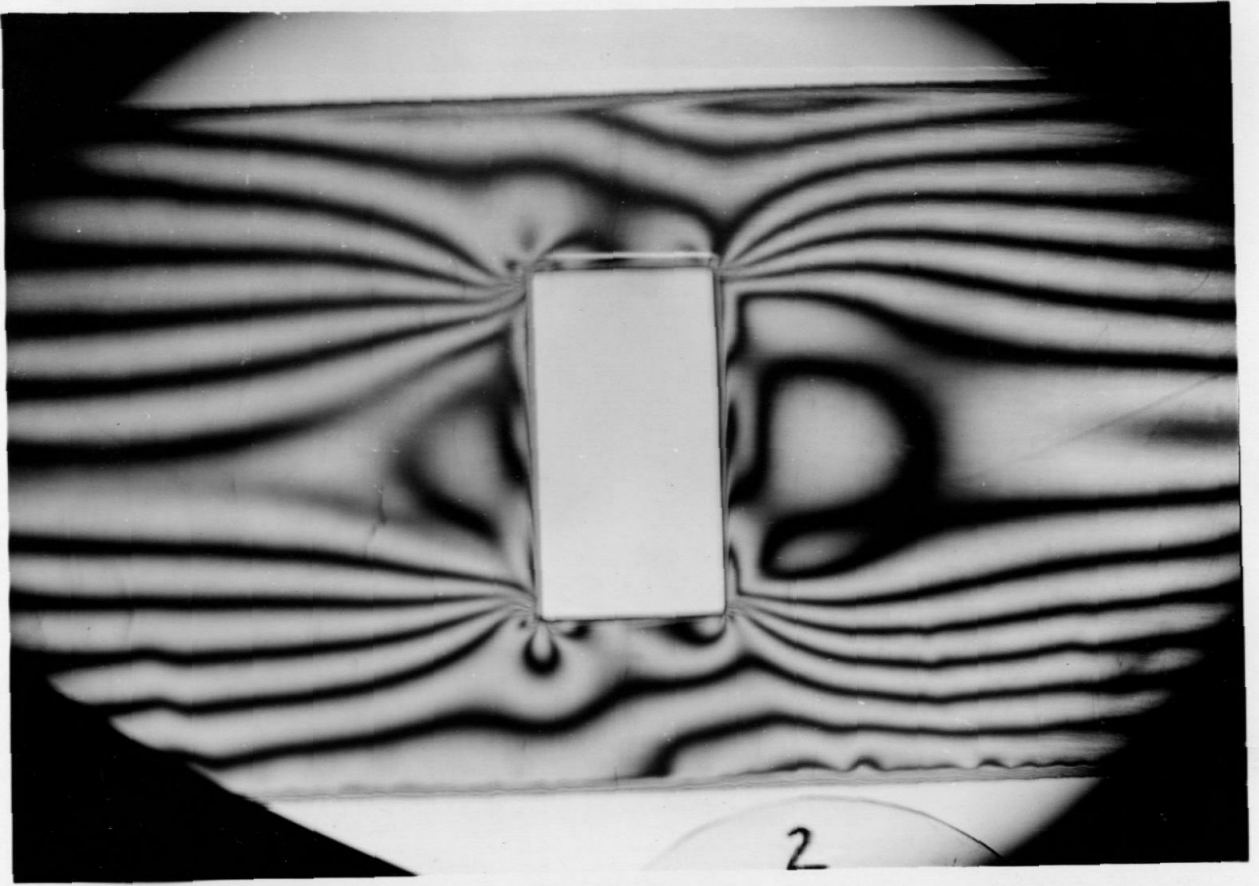


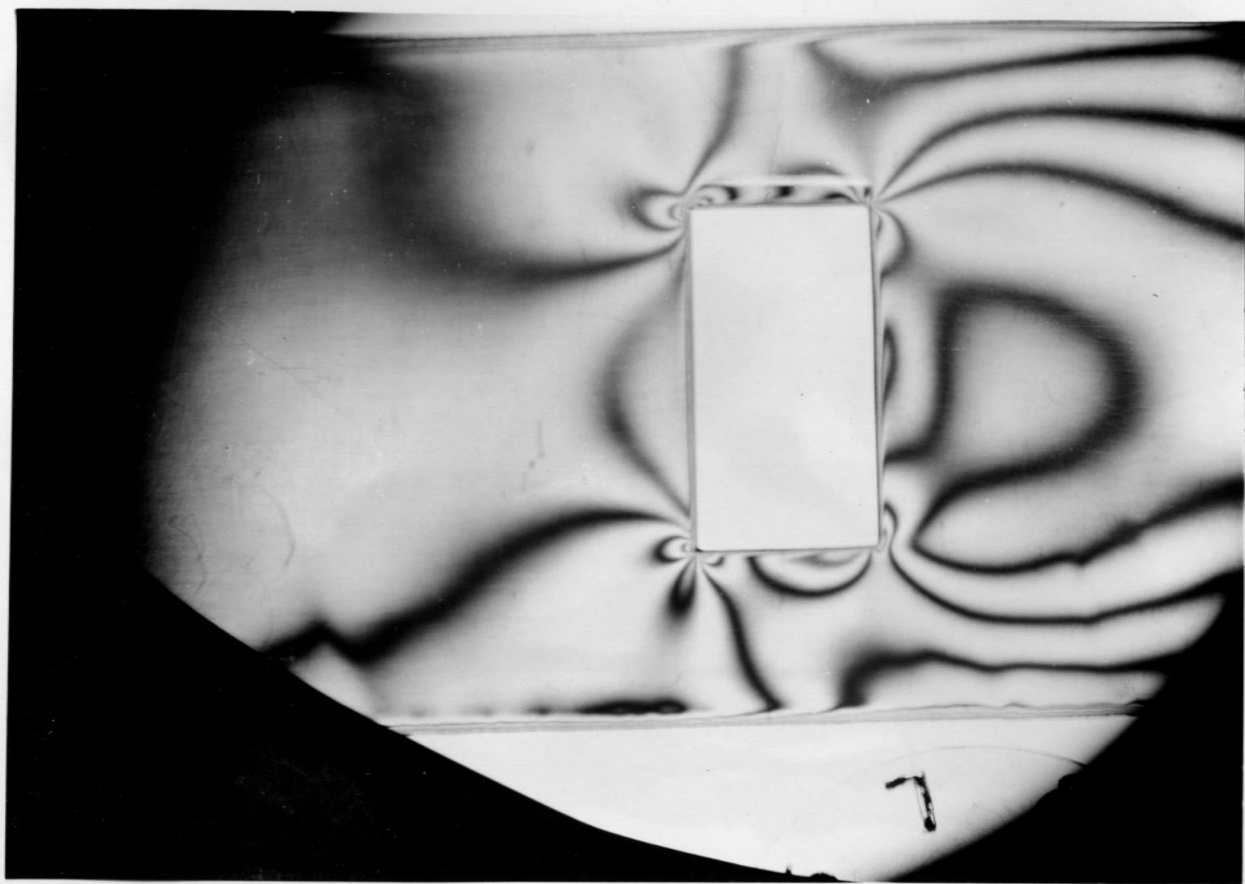
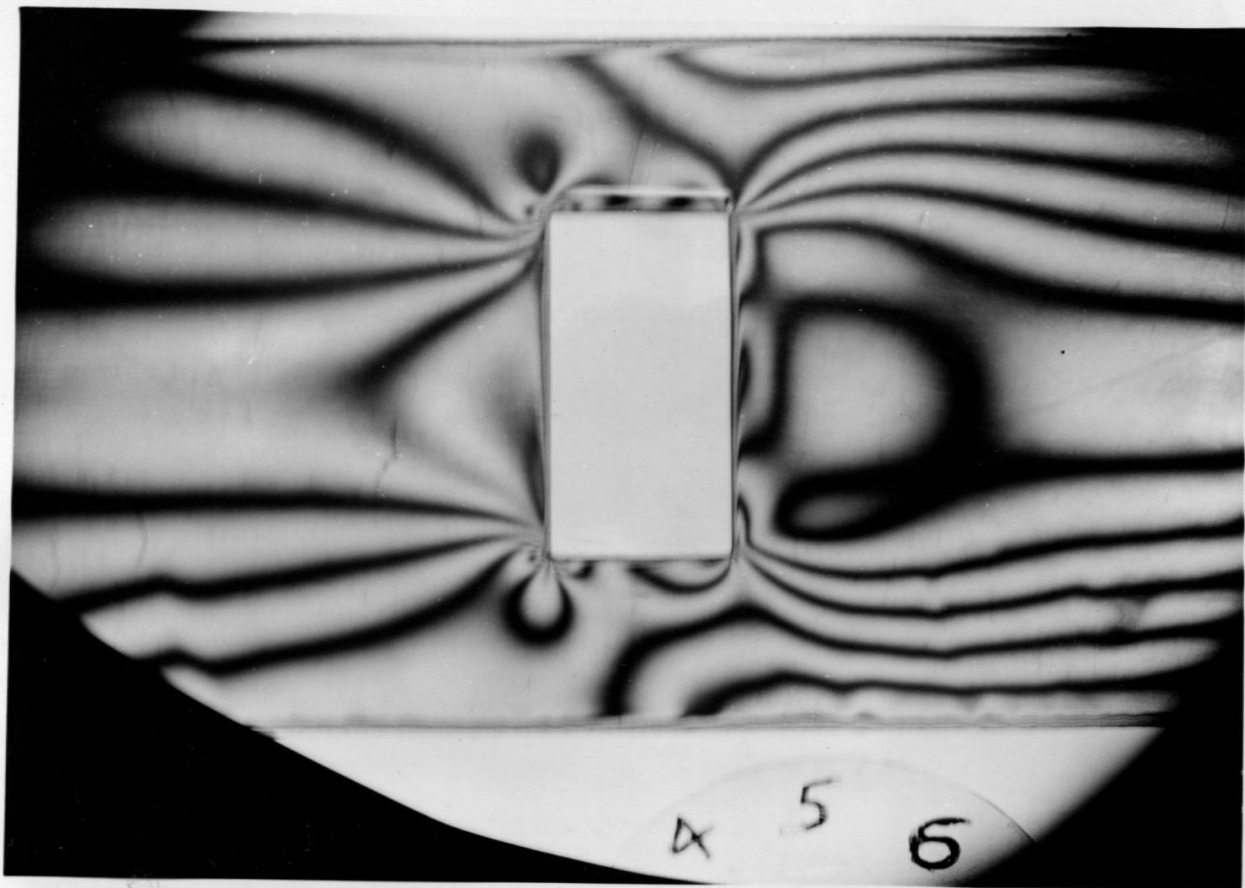


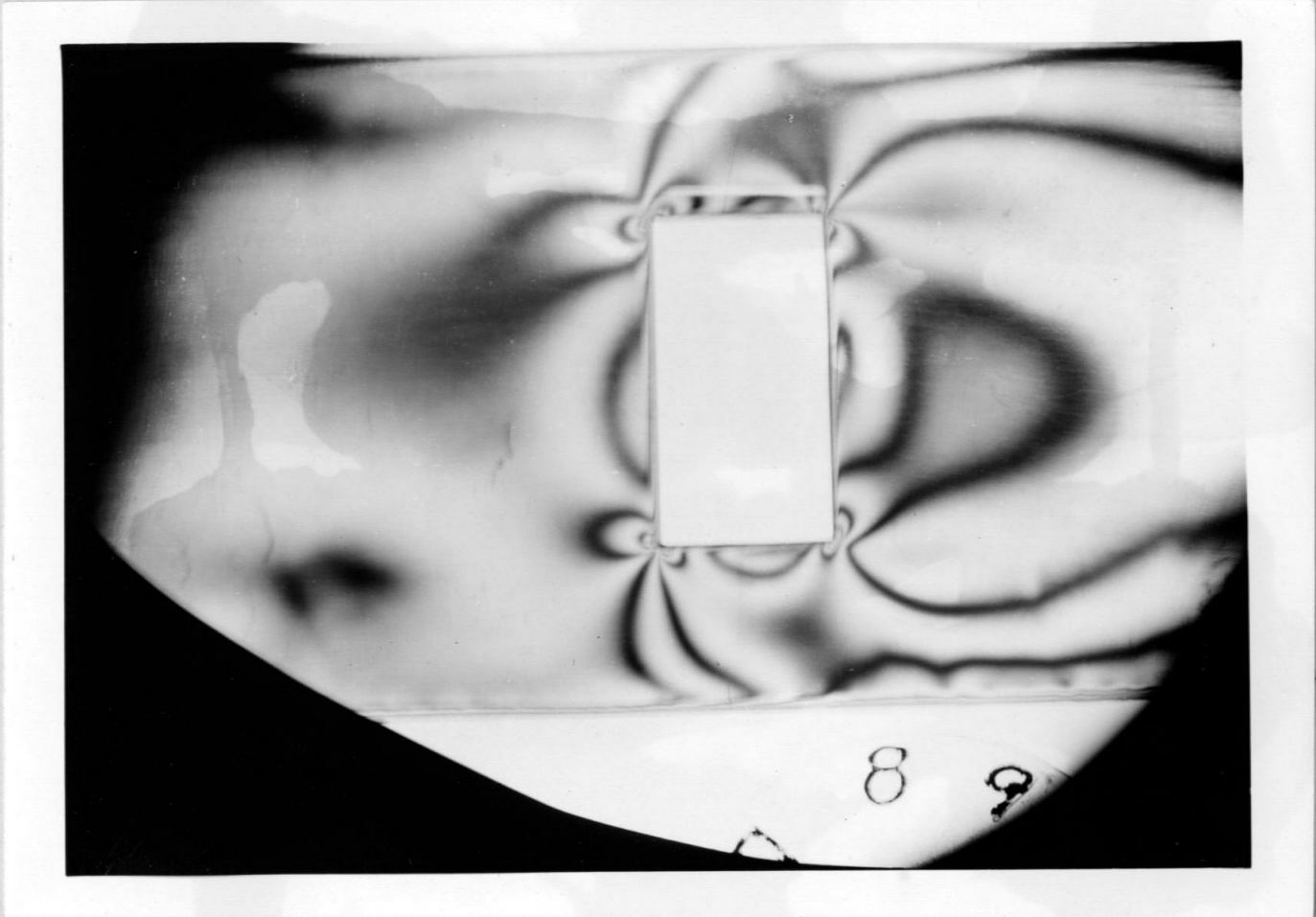


GROUP 5

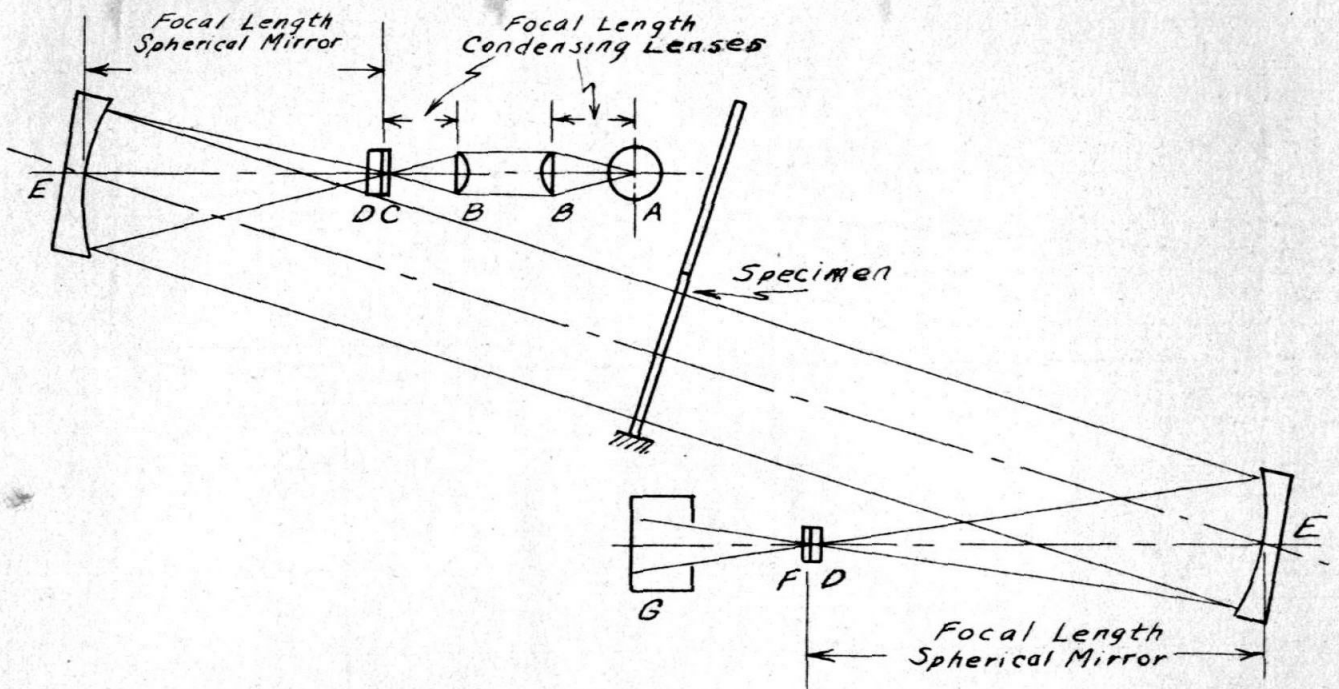
Rectangular Cutout





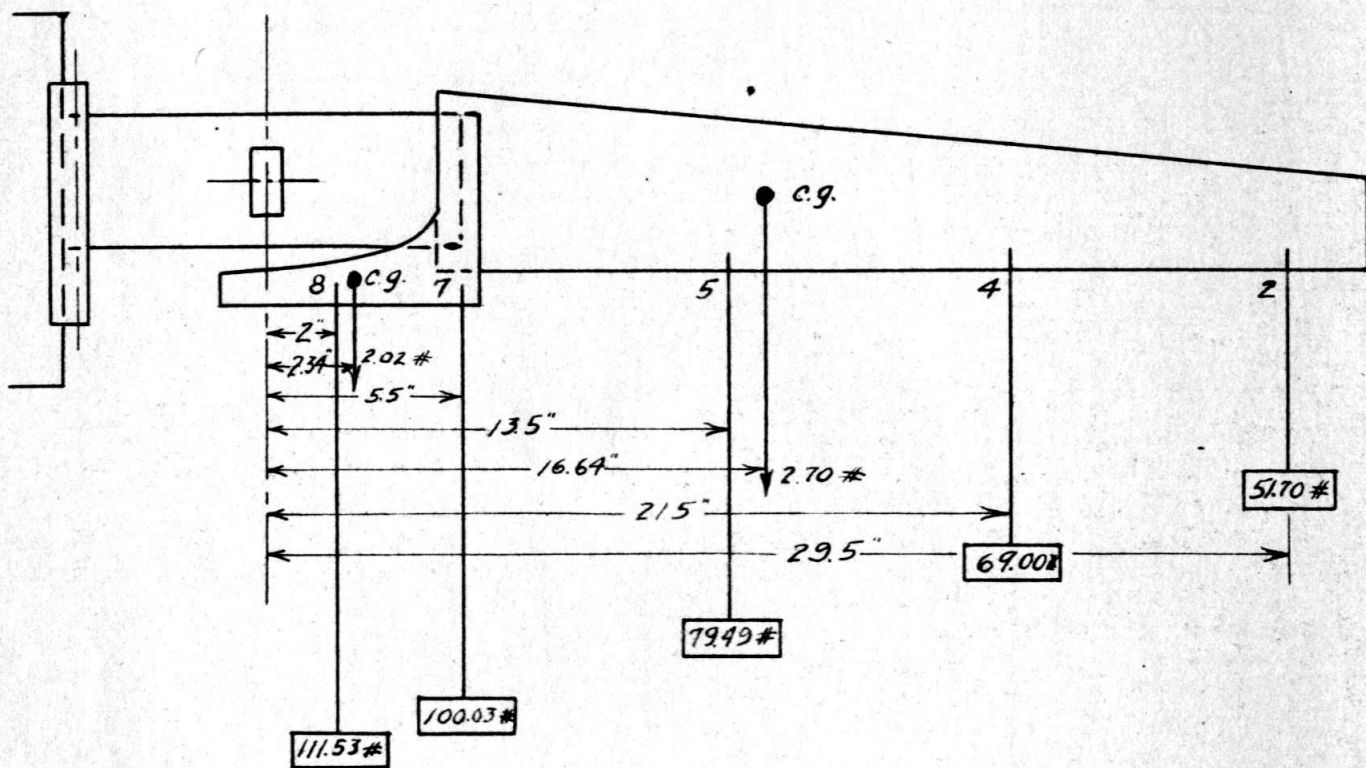


Figures and Curves



- DIAGRAMATIC PLAN OF POLARASCOPE
- A - Light Source - 60 Watt Sodium Vapor Lamp
 - B - Condensing Lenses
 - C - Polarizer
 - D - Quarter Wave Plates
 - E - Spherical Mirrors
 - F - Analyser
 - G - Box Camera

Fig. 1



MODEL
&
METHOD OF LOADING

Note: Loads applied separately.

Fig. 2

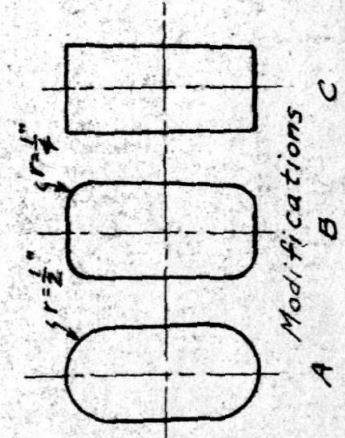
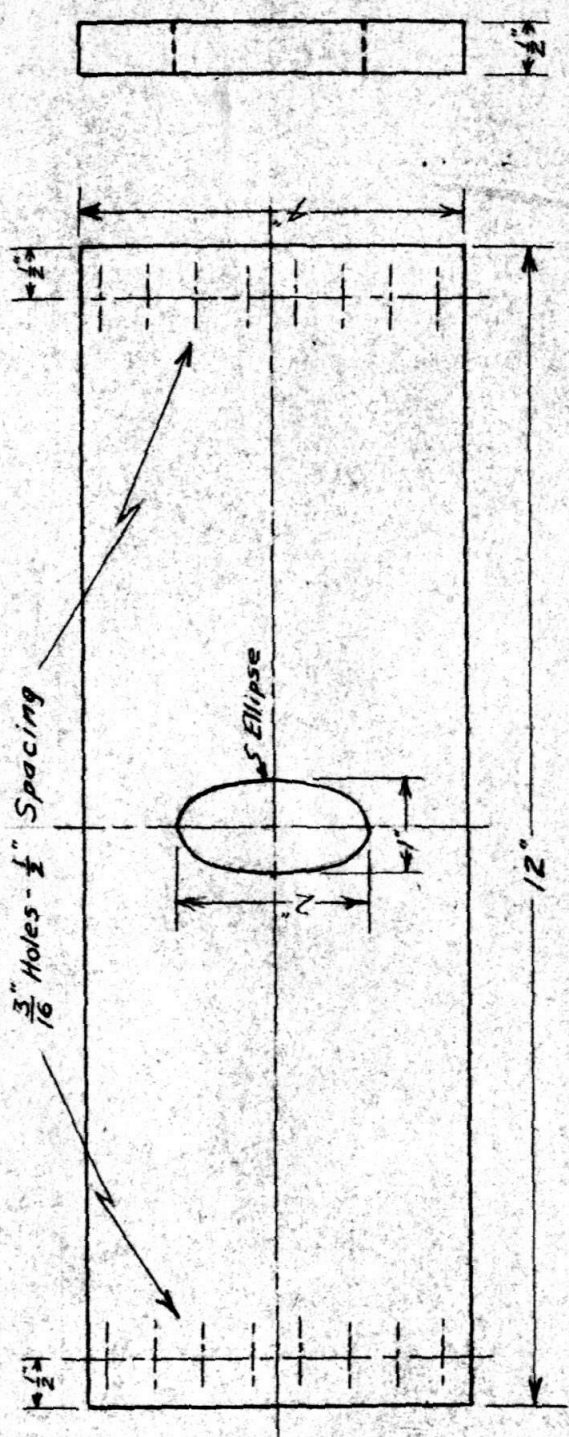
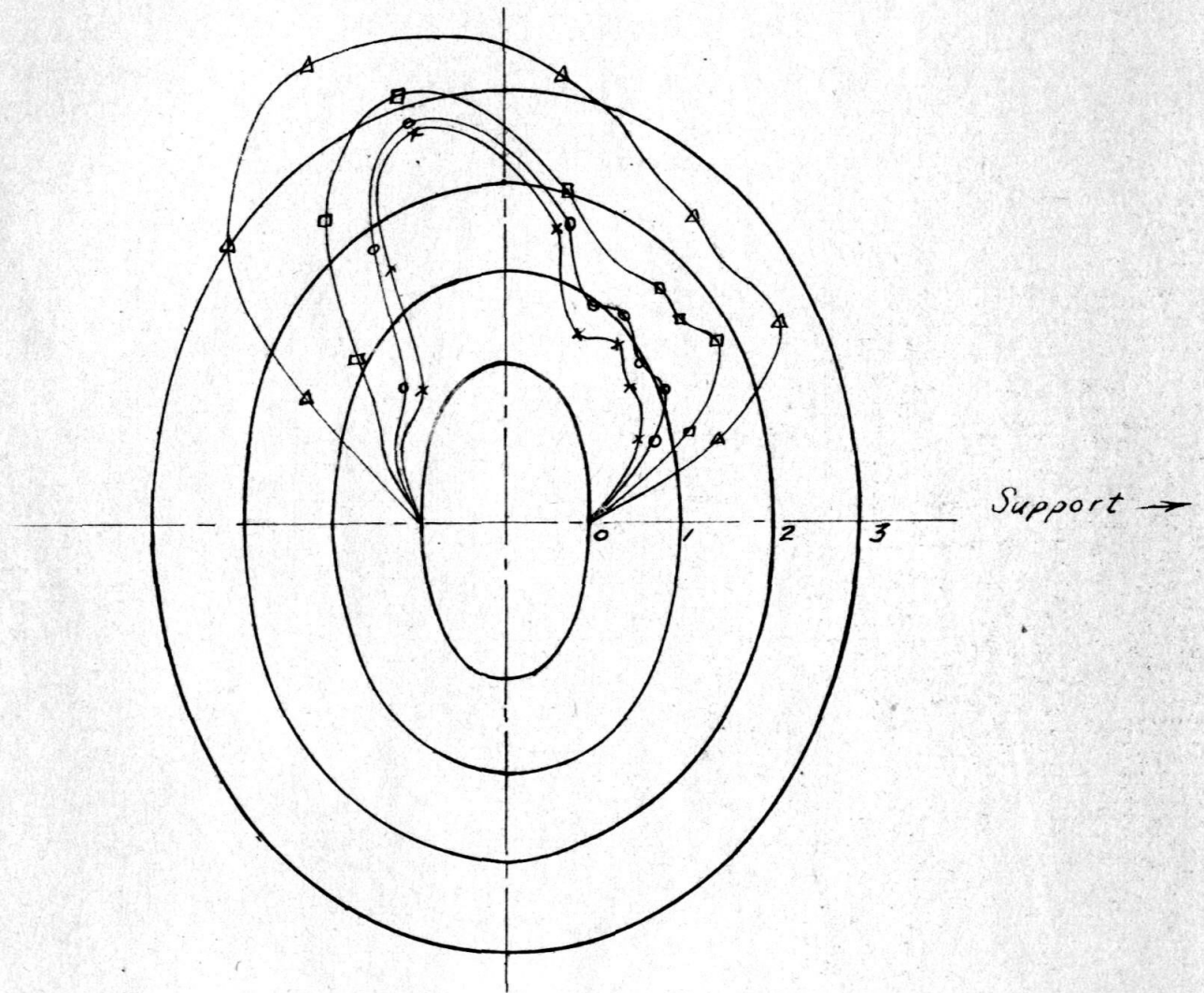


Fig. 3
56

| | | | |
|---|--|-----------------|--|
| TOLERANCES ± .010 OR 1/64 UNLESS OTHERWISE NOTED | | Scale 1/2 Size | |
| MATERIAL | | FINISH | |
| GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY | | HEAT TREAT | |
| R.M. MOORE | | DRAFTSMAN | |
| NAME | | ENGINEER | |
| MOORE | | APPROVED | |
| DRAWING NO. | | PHOTOELASTICITY | |

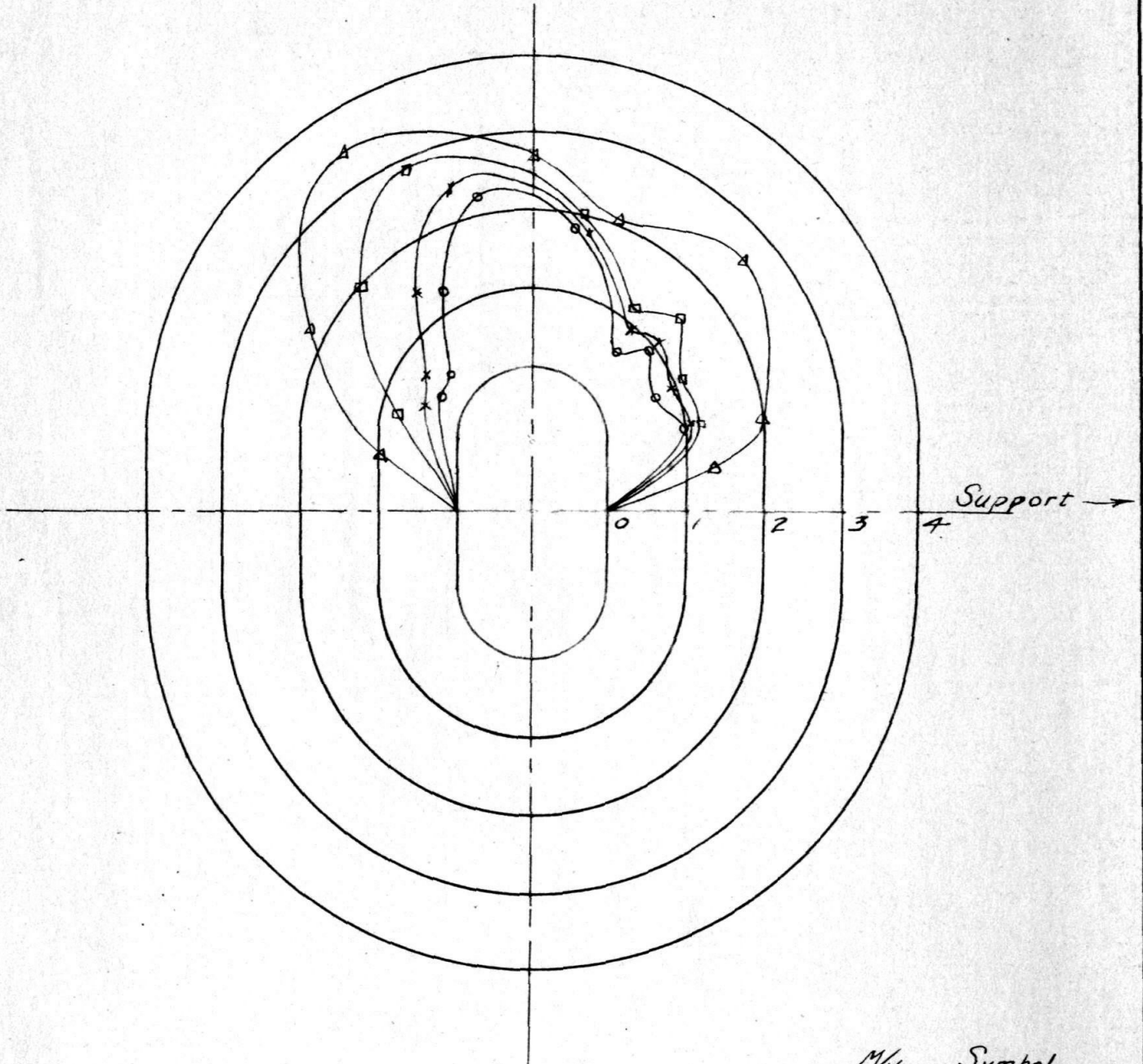
Top



| <u>M/V</u> | <u>Symbol</u> |
|------------|---------------|
| 27.9 | o |
| 13.3 | x |
| 5.74 | □ |
| 2.34 | Δ |

STRESS RATIO DIAGRAM
ELLIPTICAL CUTOUT
Fig. 4

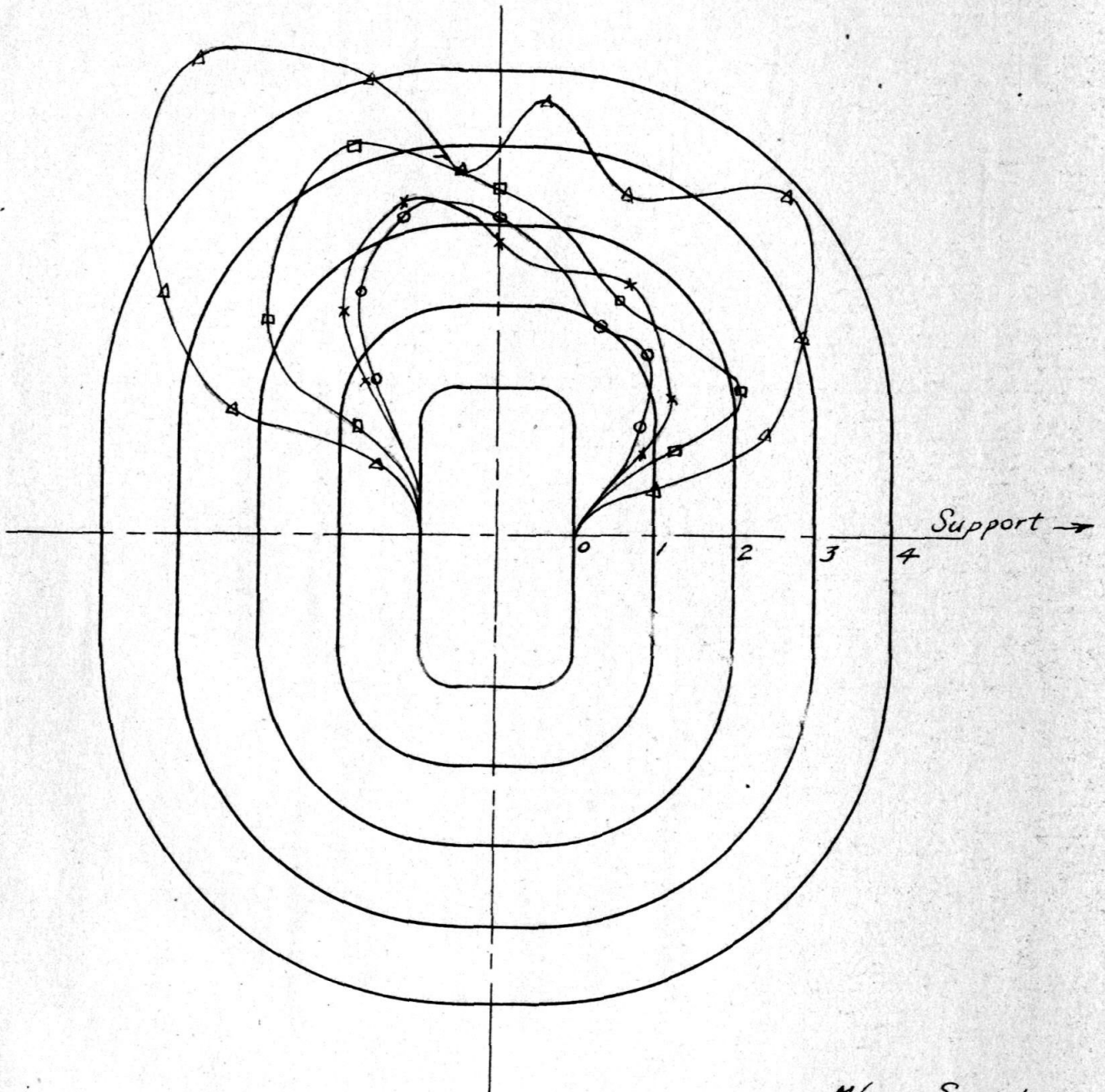
TOP



| <u>M/V</u> | <u>Symbol</u> |
|------------|---------------|
| 27.9 | o |
| 13.3 | x |
| 5.74 | □ |
| 2.34 | △ |

STRESS RATIO DIAGRAM
1/2" FILLETS
FIG. 5

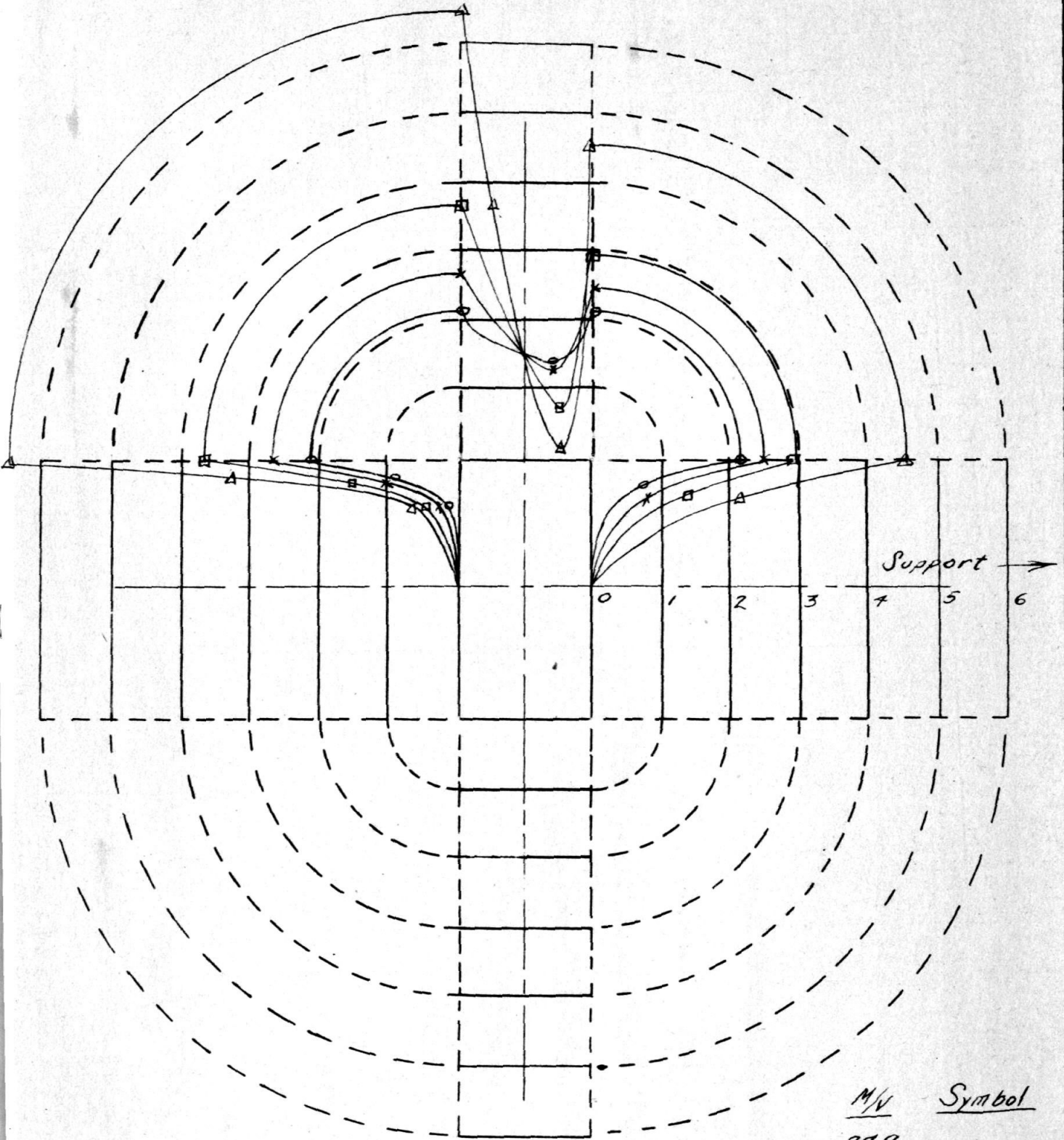
Top



| M/V | Symbol |
|-------|--------|
| 27.9 | o |
| 13.3 | x |
| 5.74 | □ |
| 2.34 | Δ |

STRESS RATIO DIAGRAM
 $\frac{1}{4}$ " FILLETS
FIG. 6

Top



| M/V | Symbol |
|-------|--------|
| 27.9 | o |
| 13.3 | x |
| 5.74 | □ |
| 2.34 | Δ |

STRESS RATIO DIAGRAM
NO FILLETS
FIG. 7

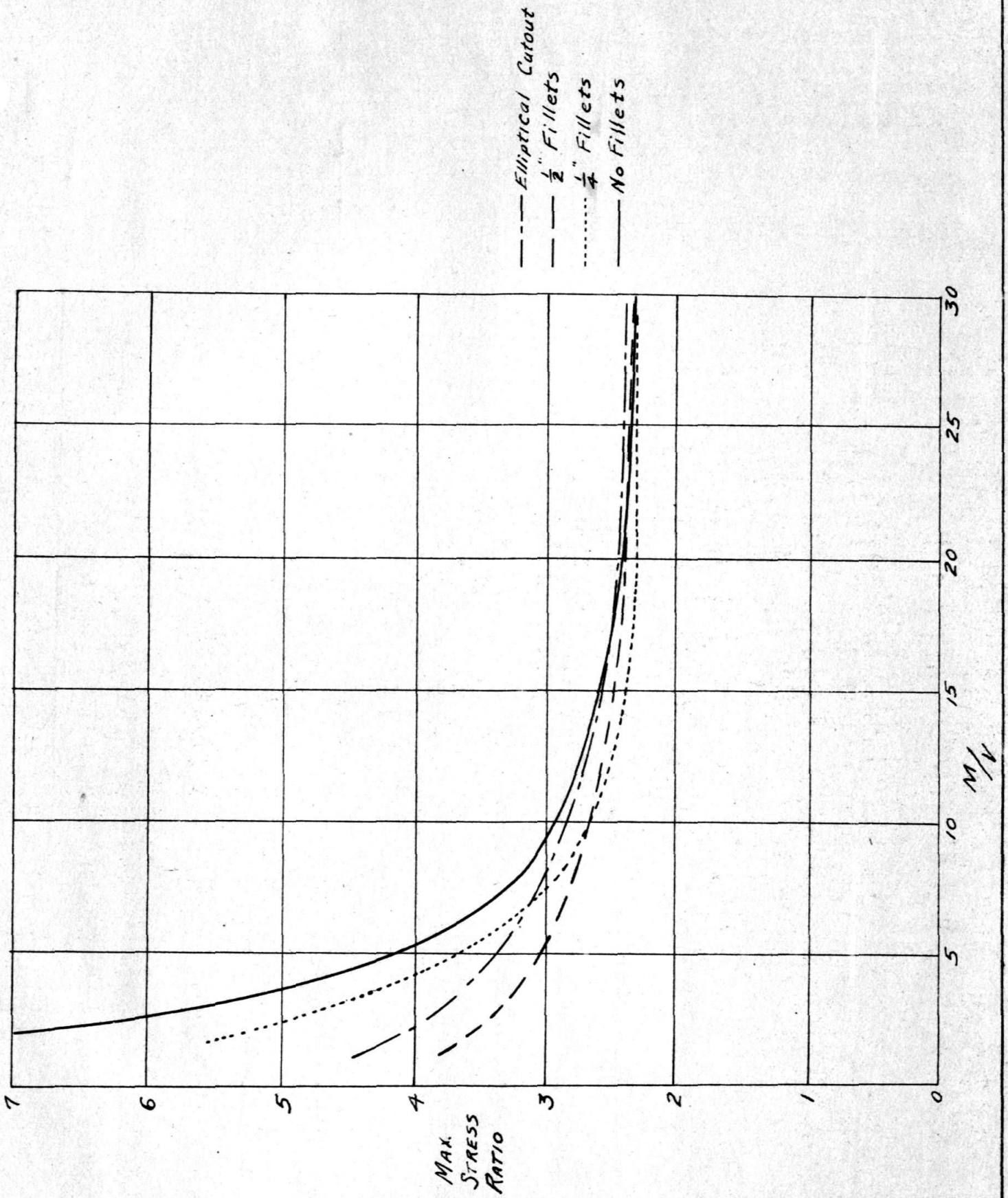


FIG. 8