Brane Models and the Hierarchy Problem

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Abstract

It has been recently proposed that higher-dimensional field theory models in the presence of extended defects ("branes") may play a role in addressing the gauge hierarchy problem. In this thesis we consider several aspects of such field theories. First we perform the Kaluza-Klein reduction of a bulk scalar field propagating in the scenario of Randall and Sundrum, which consists of a region of five-dimensional anti-deSitter space bounded by two three-branes. We then propose a simple mechanism, based on the dynamics of a bulk scalar field, for stabilizing the modulus field (the "radion") corresponding to the size of the compact dimension in the Randall-Sundrum scenario. Some implications of this stabilization mechanism for low-energy phenomenology are described. Next, we investigate the one-loop quantum corrections to the radion effective potential. We show that for large brane separation, the quantum effects are power suppressed and therefore have a negligible effect on the bulk dynamics once a classical stabilization mechanism is in place. Finally, we study the ultraviolet divergence structure of field theory in the presence of branes and find that brane-localized divergences arise both at the classical and quantum level. We show how to interpret the classical divergences by the usual regularization and renormalization procedure of quantum field theory.

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Chapter 1 Introduction

1.1 The gauge hierarchy problem

The Standard Model (SM) of particle physics [1], a renormalizable quantum field theory based on the gauge group $SU(3) \times SU(2) \times U(1)$, is a highly successful description of nature at small scales. As a scientific theory it has passed an impressive number of experimental tests, including precision measurements of electroweak and low energy QED observables that probe the predictions of the theory at the level of radiative corrections. It is only recently that experiments have begun to see evidence of new physics beyond the framework of the SM, in the form of neutrino oscillations [2], and most recently perhaps also in the form of deviations of the value of the muon g-factor from the predictions of the SM [3].

However, besides these experimental hints, we know that the SM must be an effective field theory, giving way to a more fundamental description of particle interactions at high energies. For one, the SM does not incorporate gravitational interactions. While this is irrelevant for phenomenology at experimentally accessible energies, it is an indication that the SM ceases to be valid description at energies scales at most as large as the Planck scale, $M_{Pl} \sim 10^{19}$ GeV, where gravity becomes strongly coupled and its quantum mechanical nature must be taken into account.

In fact, we can understand some features of the SM if we make the minimal assumption that there is no new physics (i.e., no new particle content) between the scale of electroweak symmetry breaking, $m_W \sim 250$ GeV, and the Planck scale (or perhaps the scale of gauge coupling unification, $M_{GUT} \sim 10^{16}$ GeV). Then starting with a Lagrangian that includes arbitrarily complicated non-renormalizable interactions at scales just below the cutoff $\Lambda \sim M_{Pl}$, and integrating out high momentum modes until reaching energies of order m_W, the theory flows to a model with the original non-renormalizable operators suppressed by powers of m_W/M_{Pl}. All that survives are the renormalizable interactions: the gauge and Yukawa couplings probed by high-energy accelerator experiments. Although this Wilsonian picture cannot explain why, for example, the SM is a chiral gauge theory, or predict the specific value of the renormalizable parameters, we can at least understand why nature at low energies is described by renormalizable quantum field theory. We can also understand other facts, such as the suppression of neutrino masses and proton decay rates, based solely on the gauge symmetries of the SM and the existence of an energy "desert" between the weak scale and the Planck scale.

This picture is minimal and attractive, but it leads to a problem. In the minimal SM, electroweak symmetry breaking is achieved through the Higgs mechanism, implemented by a weakly coupled scalar field whose mass determines the weak scale m_W. However, it is unnatural to have fundamental scalars with mass much lighter than the cutoff scale of the theory, since there is generally no symmetry that protects a scalar mass term from acquiring quadratically divergent radiative corrections[1]. In order to obtain a Higgs scalar mass $m_H \simeq m_W$ in the SM, the bare parameters at the cutoff scale Λ must be sensitively tuned. The necessity to adjust parameters in such a way that $m_H \ll M_{Pl}$ is known as the gauge hierarchy problem [4, 5].

To understand this fine-tuning, consider the radiative corrections to the Higgs mass due to its quartic self-couplings. Schematically, this is given by

$$-i\delta m_H^2 =$$

$$= -i\lambda \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2}, \tag{1.1}$$

where λ is the Higgs self-coupling. Imposing a physical cutoff $\Lambda \sim M_{Pl}$ on the loop integral, the physical mass of the Higgs has the form

$$m_H^2 \simeq m_0^2 + \frac{\lambda}{16\pi^2}\Lambda^2, \tag{1.2}$$

[1]We say that it is not "technically natural," in the senss introduced by 't Hooft [6], to have light scalars. This should be contrasted with radiative corrections to fermion masses, which due to chiral symmetry must vanish in the limit of zero bare mass and therefore can only depend logarithmically on the cutoff scale.

where m_0 is the bare mass. If λ is of order unity, this equation implies that for $\Lambda \simeq M_{Pl}$, the $(m_0/\Lambda)^2$ must be tuned to one part in 10^{-32} to generate a physical Higgs mass $m_H \simeq 1$ TeV. Otherwise $m_H \simeq M_{Pl}$. While it is certainly possible that nature is fine-tuned in this sense, the adjustment necessary to maintain $m_W \ll M_{Pl}$ seems rather artificial. It indicates that the SM is only valid up to energies of order the TeV scale. Past these energies, it must be replaced by a more fundamental theory. Then the corrections to the Higgs mass are cut off not at $\Lambda = M_{Pl}$, but instead at $\Lambda \simeq$ TeV, and consequently $m_H \sim$ TeV is natural[2].

The hierarchy problem motivates most proposals for new physics at the TeV scale. One way around the hierarchy problem is to avoid fundamental scalars altogether. This is the idea behind technicolor models [5, 7]. In such models, electroweak symmetry is broken dynamically, through strong gauge theory dynamics at the TeV scale. While technicolor is a beautiful idea, a large number of constraints (for instance, suppression of potentially large flavor-changing neutral currents) make its implementation difficult, and no realistic models have yet been built.

Perhaps the most popular extension of the SM is the existence of supersymmetry (SUSY) broken at the TeV scale [9, 10]. SUSY transforms bosonic fields into fermions, and therefore implies a doubling of the particle spectrum[3]. If SUSY were an exact symmetry, loop corrections to the Higgs mass from SM fields would cancel against the Feynman diagrams involving their supersymmetric partners. Because SUSY is broken at the TeV scale, this exact cancellation does not occur in the supersymmetric Standard Model. However, if SUSY is broken at the TeV scale by "soft" terms (operators with mass dimension less than four), then a Higgs mass of order the TeV scale is technically natural, in much the same way that chiral symmetry makes small

[2]This still leaves us with another hierarchy problem, involving quantum corrections to the vacuum energy. No symmetry protects this cosmological constant term from receiving radiative corrections of order Λ^4. Even if the cutoff is of order the TeV scale, the natural value of $(\mathrm{TeV})^4$ leads to a cosmological constant which is many orders of magnitude larger than cosmological observations require. Although it appears here only in a footnote, the cosmological constant problem is one of the great unsolved problems in particle physics. We will have nothing further to say about it, but see [8] for a review.

[3]Actually, besides doubling the particle spectrum, in the supersymmetric extension of the SM one must also introduce a super multiplet involving an additional Higgs scalar to cancel the anomalies generated by the chiral fermion superpartners of the SM bosons.

fermion masses natural.

A more recent class of ideas for addressing the hierarchy problem involves bringing the gravitational scale down to the TeV range [11, 12]. In order to accommodate this, one also needs to introduce additional compactified dimensions beyond the $3+1$ that are observed in nature. The phenomenology of these models is radically different from that of more conventional extensions of the SM. If these ideas are relevant to nature, they imply that we may see signatures of quantum gravity at collider experiments at the TeV scale. We will describe the basic features of these models in the following section.

1.2 Extra dimensions and the hierarchy problem

The hierarchy problem arises because there are two widely separated fundamental scales in nature, the electroweak scale m_W and the Planck scale M_{Pl}. While our understanding of weak scale physics comes from direct experimental observation at distances of order $1/m_W$, gravitational interactions have only been probed down to millimeter scale distances (see [13] for recent experimental results). The Planck scale represents an extrapolation of this observation across 33 orders of magnitude. This simple observation lead Arkani-Hamed, Dimopolous and Dvali (ADD) [11] to entertain the possibility that perhaps M_{Pl} is not fundamental, but instead strong gravitational dynamics sets in at a new scale M. If this scale is $M \sim 1$ TeV, then a Higgs with $m_H \sim$ TeV is natural, since M is the ultraviolet cutoff on the SM.

How can a gravitational scale M in the TeV range be compatible with the observed strength of gravity $G_N \sim M_{Pl}^{-2}$? Suppose that spacetime is not four dimensional, but rather it is the product of a compact n-dimensional manifold with four-dimensional Minkowski space. At energies below the compactification scale of this internal manifold, the higher-dimensional gravitational action[4]

$$S = 2M^{n+2} \int d^{4+n}X\sqrt{G}R + \ldots, \tag{1.3}$$

behaves like a four-dimensional theory described by

$$S = 2M^{n+2}V_n \int d^4x\sqrt{g}R_4 + \ldots, \tag{1.4}$$

where M is the $4+n$ dimensional Planck scale, and V_n is the volume of the internal manifold. Also, g is the determinant of the four-dimensional part of the metric, and R_4 is the four-dimensional Ricci scalar. From this we can identify the four-dimensional Planck mass as a derived quantity, given by

$$M_{Pl}^2 = M^{n+2}V_n. \tag{1.5}$$

For example, take V_n to be a torus with radii $\sim R$. Then if $M \sim 1$ TeV (although for $n = 2$, astrophysical bounds [14] already exclude $M < 30$ TeV), we obtain the correct strength four-dimensional gravitational interactions provided that we set

$$R^n \sim \left(\frac{M_{Pl}}{M}\right)^2 M^{-n}. \tag{1.6}$$

If $n = 1$, $R \sim 10^{23}$ cm, which is clearly ruled out, while for $n = 2$, $R \sim 1$ mm. The problem of understanding the disparity between the weak scale and the Planck scale has been transformed into the dynamical problem of stabilizing an internal manifold with radius R hierarchically larger than the fundamental scale of the theory. See [15] for some early proposals.

While it seems that gravity can be accommodated by this large extra dimensions picture, the idea as presented so far is not yet consistent with collider experiments. Clearly the SM fields cannot propagate in a bulk space with millimeter size extra dimensions. If this were so, at distance scales shorter than the millimeter range, non-

[4] G_{MN} is the higher-dimensional metric, and G is its determinant. R is the $n+4$-dimensional Ricci scalar.

gravitational physics would appear higher dimensional: in addition to the ordinary SM particles, experiments would also see towers of states with the same quantum numbers as the SM fields and with masses separated by a gap of order 1 $\text{mm}^{-1} \sim 10^{-4}\text{eV}$. To avoid this phenomenological disaster it is necessary to assume that fields charged under the SM gauge group are confined on a three-dimensional surface embedded in the higher-dimensional bulk.

It is highly nontrivial to localize a chiral gauge theory like the SM on a three-dimensional wall, or "brane". Although no specific mechanism for confining the SM fields on a 3-brane has been proposed, it is possible that this may be done within the context of field theoretic models on topological defects [16] or with D-branes [17] in string theory. However, despite this difficulty, a scenario like ADD, where gravity propagates in $4+n$-dimensions while SM fields are trapped on a submanifold makes sense as a low energy effective field theory (with respect to the fundamental scale M) and we are free to consider its implications without detailed knowledge of a specific localization mechanism.

A picture such as the one we just described is a radical departure from the usual ways of thinking about physics beyond the SM, and clearly it has implications not only for millimeter tests of the gravitational force, but also for collider experiments near the TeV scale [18, 19, 20, 21], in astrophysical settings [14, 18, 22], as well as cosmology [18, 23]. For example, in accelerator experiments, a model-independent signature of the extra dimensions is a large cross section for production of Kaluza-Klein graviton states as energies approach the TeV scale. Individual Kaluza-Klein modes are coupled to Standard Model fields with strength M_{Pl}, and are therefore not detectable. However, there is a large multiplicity of graviton states available in the production processes, so the total emission rate is observable in the form of missing energy, for instance, in collider processes such as $e^+e^- \to \gamma + \not{E}_T$ or $q\bar{q} \to \text{jet} + \not{E}_T$. See [19] for a detailed analysis of these processes. Graviton Kaluza-Klein modes may also manifest themselves through virtual exchange, modifying reactions involving external SM states at energies of order M, see [20]. But certainly the most striking consequence for collider physics is the possibility of probing the quantum theory of gravity itself. A specific analysis of such effects is obviously model dependent, however studies of the effect of TeV scale string theory on collider phenomenology can be found in [24].

So far we have assumed that the spacetime metric factorizes into a product of the four-dimensional Minkowski metric and the metric for an internal compact space. We have also neglected the the backreaction of our 3-brane universe on the bulk metric, as well as the possibility of a nontrivial spacetime geometry for the bulk $4+n$ dimensional spacetime. Randall and Sundrum [12] considered a five-dimensional brane scenario where the spacetime metric is no longer factorizable. Their model consists of a spacetime with a single S^1/Z_2 orbifold extra dimension. Three-branes with opposite tensions reside at the orbifold fixed points, and together with a finely tuned negative bulk cosmological constant serve as sources for five-dimensional gravity[5]. The resulting spacetime metric contains a "warp factor" which depends exponentially on the radius of the compactified dimension:

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \tag{1.7}$$

where x^μ are Lorentz coordinates on the four-dimensional surfaces of constant ϕ, and $-\pi \leq \phi \leq \pi$ (with (x,ϕ) and $(x,-\phi)$ identified, and the 3-branes located at $\phi = 0, \pi$). Here, r_c sets the size of the extra dimension, and k is taken to be on the order of the Planck scale.

As we will discuss in the next chapter, the four-dimensional Planck scale is given by

$$M_{Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}], \tag{1.8}$$

so that M_{Pl} is of order M. Also, a field confined to the 3-brane at $\phi = \pi$ with mass parameter m_0 will have a physical mass given by $m = m_0 e^{-kr_c\pi}$. Thus, if kr_c is around 12, the weak scale is dynamically generated while all fundamental mass scales are on

[5]This setup is analogous to the Horava-Witten scenario [26] which arises in M-theory. For a discussion of how the scenario described above Eq. (1.7) may arise from Type IIB string theory compactifications, see [27]. Supersymmetric extensions can be found in [28, 29]. For connections with the AdS/CFT correspondence [30], see [27, 31, 32, 33, 34, 35, 36].

the order of the Planck scale, i.e., there is no very large hierarchy between M and the compactification scale $1/r_c$.

Besides this, it is also found that on account of the exponential warp factor in Eq. (1.7), Kaluza-Klein gravitational modes in this spacetime have TeV scale mass splittings and couplings [12, 25]. Although in this scenario the extra dimension will not be measurable in any foreseeable test of the gravitational force, the strongly coupled graviton Kaluza-Klein modes will manifest themselves as spin-two resonances in TeV scale collider experiments. Some of the relevant phenomenology can be found in [37]. This is in sharp contrast to the Kaluza-Klein decomposition in ADD scenarios, which as we discussed earlier, gives rise to a high number of light modes (with splittings of the order of the compactification scale) that are coupled only with gravitational strength.

In this thesis, we will explore aspects of field theory relevant to the brane models described in this chapter for addressing the hierarchy problem. In Chapter 2, we will give a more detailed discussion of the RS scenario. We will also develop the four-dimensional effective field theory for the massless modes which arise in their minimal setup, with only gravity propagating in the bulk. It is found that this effective field theory contains the ordinary four-dimensional graviton, as well as a massless scalar field, the radion, which is a modulus for the compactification radius r_c of the fifth dimension. The results on the radion effective field theory presented here are based on [38] (which appeared also in [39]).

In Chapter 3, we will continue the discussion of the RS scenario, by performing the Kaluza-Klein decomposition of a massive bulk scalar in the nonfactorizable metric of Eq. (1.7). We will find that even if the bulk field mass and self-interactions are characterized by parameters of order the Planck scale, its Kaluza-Klein modes have masses and couplings of order the TeV scale. A similar pattern arises for bulk fields with spin. This result has important phenomenological consequences, since it implies that Planck scale bulk physics may be accessible to experiments at the TeV scale. The results presented in this chapter are based on work that appeared in [40].

As discussed in Chapter 2, in the minimal RS setup the radion potential is flat.

Since the vacuum expectation value of the radion determines the compactification scale (and therefore the weak/Planck hierarchy), additional dynamics must be specified to yield the weak scale dynamically from the spacetime geometry. Furthermore, for the scenario to be phenomenologically relevant, the radion must acquire a mass. In Chapter 4, we describe a proposal, first presented in [41], for generating a radion potential whose minimum yields $kr_c \sim 12$ without fine-tuning of parameters. This potential arises classically, due to the presence of a bulk scalar field with interaction terms localized on the two branes (see [42] for a similar mechanism in the context of ADD models with two extra dimensions). We also discuss some of the phenomenological features [38, 39] of the radion stabilized by the mechanism of [41]. It is found that the radion mass is generically in the TeV range, although it is somewhat lighter if the large separation between branes arises from a small bulk scalar mass. Consequently, it may be the first signal of the RS scenario. Four-dimensional general covariance completely determines the couplings of the modulus to SM fields. The strength of these couplings is determined by a single parameter which is set by the TeV rather than the Planck scale.

The radion stabilization picture described in Chapter 4 is purely classical. One expects quantum corrections to generate a nontrivial potential for the radius modulus. In Chapter 5 we calculate the one-loop effective radion potential induced by the quantum fluctuations of fields that propagate in the bulk and on the two branes (this calculation is based on [43]). We find that the resulting potential cannot naturally stabilize the brane separation at the distance needed to generate the hierarchy. Consequently, a classical stabilization mechanism, such as that presented in Chapter 4, is required. However, for large brane separation, the quantum effects are power suppressed and therefore have a negligible impact on the bulk dynamics once such a mechanism is in place. Our result provides some evidence that, at least at the one-loop level, the RS scenario is quantum mechanically stable even in the absence of bulk supersymmetry.

In Chapter 6, we study the structure of ultraviolet divergences for field theory in the presence of branes. Such field theories encounter localized divergences that renormalize brane couplings. The sources of these brane-localized divergences are understood as arising either from broken translation invariance or from short distance singularities as the brane thickness vanishes. While the former are generated only by quantum corrections, the latter can appear even at the classical level. Using as an example six-dimensional scalar field theory in the background of a 3-brane, we show how to interpret such classical divergences by the usual regularization and renormalization procedure of quantum field theory. In our example, the zero thickness divergences are logarithmic, and lead classically to nontrivial renormalization group flows for the brane couplings. We construct the tree level renormalization group equations for these couplings, as well as the one-loop corrections to these flows from bulk-to-brane renormalization effects. The classical logarithms considered here may play a role in the mechanism of [42] for stabilizing ADD with two extra dimensions, as well as the proposal of [44] for obtaining gauge coupling unification. These results closely follow the presentation in [45].

Finally, in Chapter 7 we present concluding remarks.

Chapter 2 The Randall-Sundrum scenario

In this chapter we describe the proposal due to Randall and Sundrum [12] to address the hierarchy problem via exponential warp factors in the spacetime metric. In Section 2.1 we write down a five-dimensional low energy effective theory (with respect to the five dimensional gravitational scale), including five-dimensional Einstein-Hilbert gravity with a bulk cosmological constant, as well as world-volume actions for three-branes. We also solve for the five-dimensional metric generated by the brane tensions and the cosmological constant, and show how the resulting spacetime geometry can generate hierarchies between the scale of physics for field theories localized at different points in the bulk space. In Section 2.2, we describe a low energy four-dimensional effective action for the massless fields, which in the minimal setup of Section 2.1, are identified with the four-dimensional graviton and with a scalar modulus, the radion, that describes fluctuations in the separation between the branes.

2.1 The five-dimensional theory

We assume that spacetime is five-dimensional, with the fifth dimension compactified on an interval. It will be convenient in what follows to represent this interval as an S^1/Z_2 orbifold. Choose coordinates (x^μ, ϕ), where x^μ, with $\mu = 0 \ldots 3$, parametrize the noncompact directions, and $-\pi < \phi \leq \pi$ are coordinates on the circle. Z_2 acts on our coordinates as $(x^\mu, \phi) \to (x^\mu, -\phi)$, so that after taking into account the periodicity $\phi \to \phi + 2\pi$ on the circle, the points $(x^\mu, 0)$ and (x^μ, π) are fixed points of the orbifold action. On these fixed points reside three-branes, whose tensions serve as sources for bulk gravity. An action describing this system at energies low compared to the five-dimensional Planck scale (where we loose predictability due to poorly understood quantum gravity effects) is given by

$$S = S_b + S_v + S_h, \tag{2.1}$$

where

$$S_b = \int d^5x\sqrt{G}\left[2M^3 R - \Lambda\right] + \cdots \tag{2.2}$$

is the gravitational action for the bulk spacetime, where M denotes the five dimensional Planck scale, and Λ is a bulk cosmological constant. ... denotes terms in the action involving additional bulk fields not present in the minimal setup of [12]. Such fields will eventually play an important role in the dynamics of the model and will be discussed in the following chapters. The terms $S_{v,h}$ denote world-volume actions for fields trapped on the branes

$$\begin{aligned} S_v &= \int d^4x\sqrt{-g_v}[-V_v + \mathcal{L}_v], && (2.3)\\ S_h &= \int d^4x\sqrt{-g_h}[-V_h + \mathcal{L}_h], && (2.4) \end{aligned}$$

where the subscripts v, h label actions on a visible and and hidden sector branes respectively[1]. In these two equations, g_v and g_h denote the determinants of the induced metric on each brane, which in our coordinate system are given simply by

$$g^h_{\mu\nu}(x) = G_{\mu\nu}(x, \phi = 0), \quad g^v_{\mu\nu}(x) = G_{\mu\nu}(x, \phi = \pi). \tag{2.5}$$

Each 3-brane can support four-dimensional field theories with Lagrangians $\mathcal{L}_{v,h}$, and in particular the SM fields are assumed to be confined to the visible brane at $\phi = \pi$. Finally, the constants $V_{v,h}$ represent the tensions of the branes, which together with the bulk cosmological constant act as sources for five-dimensional gravity.

[1]We will also refer to these as the TeV and Planck brane, since, as we shall see, these are the characteristic energy scales for the dynamics on these two sectors.

2.1.1 Classical configuration

We now solve for the vacuum configuration that corresponds to the above action. This is obtained by solving the classical equations of motion for the five-dimensional metric G_{MN},

$$R_{MN} - \frac{1}{2}G_{MN}R = -\frac{1}{4M^3\sqrt{G}}\left[\Lambda\sqrt{G}G_{MN} + V_v\sqrt{-g_v}G_{\mu\nu}\delta^\mu_N\delta^\nu_M\delta(\phi-\pi) + V_h\sqrt{-g_h}G_{\mu\nu}\delta^\mu_N\delta^\nu_M\delta(\phi)\right], \quad (2.6)$$

where G is the determinant of G_{MN}, and R_{MN} is the five-dimensional Ricci tensor. We assume that the low energy dynamics respects four-dimensional Poincare invariance. An ansatz for the metric which is consistent with this assumption is given by

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \quad (2.7)$$

where in this coordinate system, r_c is the radius of the fifth-dimension compactified on a circle, and $r_c\pi$ is the length of the orbifold. The curvature tensor for this metric ansatz is (a prime denotes a derivative with respect to ϕ)

$$R_{\mu\nu} = \frac{\eta_{\mu\nu}}{r_c^2}e^{-2\sigma}(\sigma'' - 4\sigma'^2), \quad (2.8)$$

$$R_{\phi\phi} = 4(\sigma'' - \sigma'^2), \quad (2.9)$$

with $R_{\mu\phi} = 0$. Then the $\phi\phi$ component of Eq. (2.6) is then simply

$$\frac{6\sigma'^2}{r_c^2} = \frac{-\Lambda}{4M^3}, \quad (2.10)$$

while the $\mu\nu$ components are given by

$$\frac{3\sigma''}{r_c^2} = \frac{V_h}{4M^3 r_c}\delta(\phi) + \frac{V_v}{4M^3 r_c}\delta(\phi-\pi). \quad (2.11)$$

Eq. (2.10) is trivially solved

$$\sigma(\phi) = \sqrt{\frac{-\Lambda}{24M^3}} r_c|\phi| \equiv kr_c|\phi|, \tag{2.12}$$

where we have imposed $\phi \to -\phi$ symmetry and absorbed an arbitrary constant of integration into the definition of the four-dimensional coordinates x^μ. Note that in order to have a solution we must have $\Lambda < 0$, meaning that away from the branes, the spacetime geometry is locally five-dimensional anti-deSitter space (AdS_5) with curvature radius $1/k$. Taking into account the periodicity in ϕ,

$$\sigma'' = 2kr_c[\delta(\phi) - \delta(\phi - \pi)], \tag{2.13}$$

so that comparison with Eq. (2.11) gives the constraints

$$V_v = -V_h = 24M^3 k. \tag{2.14}$$

These conditions impose two fine-tunings on the parameters of the theory. Recall that the original motivation for introducing this setup was to address the hierarchy problem. In the minimal SM, only one fine-tuning of parameters is required in order to achieve $m_W \ll M_{Pl}$. It does not seem that progress is being made if we then have to trade a single constraint among the parameters of the SM for the two conditions of Eq. (2.14) which are necessary to generate our AdS_5 solution. In the next section we will discuss the physical origin of the conditions Eq. (2.14) and we will see that additional dynamics beyond what we presented here is necessary to get rid of one of these relations. However, one relation among parameters will remain. We will be able to match this onto the usual fine-tuning required in the four-dimensional field theory that is necessary to achieve a suitably small cosmological constant.

The solution for the metric is

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \tag{2.15}$$

with the radius r_c a free parameter, not determined by the physics of Eq. (2.1). Note that in order for this solution to be valid, we must take $k < M$. In this case, terms with higher powers of the curvature in the bulk action are negligible, and Eq. (2.1) captures the relevant physics.

2.1.2 Generating the TeV/Planck hierarchy

Imagine that all scales appearing in Eq. (2.1) are comparable, and of the same order of magnitude[2] as the fundamental scale M. To see how a hierarchy of scales can be generated by the solution of Eq. (2.15), consider as an example scalar field h (for instance, the SM Higgs scalar) confined to the visible brane at $\phi = \pi$. Keeping only the free action for h,

$$S_v = \frac{1}{2} \int d^4x \sqrt{-g_v} \left[g_v^{\mu\nu} \partial_\mu h \partial_\nu h - \mu_0^2 h^2 \right] + \cdots, \tag{2.16}$$

where $g^v_{\mu\nu}$ is the induced metric on the surface $\phi = \pi$, and μ_0 is a mass parameter of order M. Substituting the solution for $g^v_{\mu\nu}$ from Eq. (2.15) into this equation, this becomes

$$S_v = \frac{1}{2} \int d^4x e^{-4kr_c\pi} \left[e^{2kr_c\pi} \eta^{\mu\nu} \partial_\mu h \partial_\nu h - \mu_0^2 h^2 \right] + \cdots. \tag{2.17}$$

Now rescale $h \to e^{-kr_c\pi} h$ to obtain a canonically normalized field

$$S_v = \frac{1}{2} \int d^4x \left[\eta^{\mu\nu} \partial_\mu h \partial_\nu h - (\mu_0 e^{-kr_c\pi})^2 h^2 \right] + \cdots. \tag{2.18}$$

From this we can determine the physical mass of the canonically normalized scalar to be

$$\mu = \mu_0 e^{-kr_c\pi}. \tag{2.19}$$

Because this mass depends on the exponential of kr_c, it is possible to generate a strong hierarchy between the fundamental scale of the theory and the scale of physics on the visible brane without having to introduce unnatural values for parameters of the

[2] Perhaps somewhat smaller, however, so that quantum gravity/string effects are negligible, and the effective five-dimensional theory is a valid description of the physics.

theory. Indeed, if we take the fundamental scale M to be of order the four-dimensional Planck mass, one only needs $kr_c \sim 12$ to get TeV scale mass parameters on the visible brane. We see that in this scenario, the large hierarchy between the weak scale and the gravitational scale is a consequence of five-dimensional gravitational dynamics. The hierarchy problem of the minimal SM then gets translated into the problem of finding a mechanism that generates the small exponential $\exp(-kr_c\pi)$ from a theory with parameters of order one (in units of the fundamental scale). We will suggest such a mechanism in Chapter 4.

2.2 The four-dimensional effective theory

In this section we begin to explore some of the physical consequences of the RS model. At low energies, the physics is best described in terms of a four-dimensional effective field theory that includes the massless modes about the background solution, Eq. (2.15). Although not intuitive at this stage, we will see in the next chapter that this four-dimensional description is valid only up to energies of order the TeV scale. Past these energies, the Kaluza-Klein modes of bulk fields must be included in the effective field theory, and a five-dimensional description becomes more appropriate.

Below the TeV scale, the only light degrees of freedom in the minimal setup of the previous section are massless fields. They arise as gravitational fluctuations about the background AdS_5 solution. A convenient ansatz [12] that describes these modes is

$$ds^2 = e^{-2k|\phi|T(x)} g_{\mu\nu}(x) dx^\mu dx^\nu - T^2(x) d\phi^2. \tag{2.20}$$

In this equation, $g_{\mu\nu}(x)$ is identified with the four-dimensional massless graviton, which for our AdS_5 solution with four-dimensional Poincare invariance satisfies $\langle g_{\mu\nu}\rangle = \eta_{\mu\nu}$. The scalar $T(x)$ is a modulus field (the "radion") whose vacuum expectation value (VEV) determines the radius of the compact dimension, $r_c = \langle T\rangle$. This field describes fluctuations in the separation between the branes, and must be included in the theory in order to give a complete description of the gravitational dynamics.

Note that the Kaluza-Klein gauge field $ds^2 = A_\mu(x)dx^\mu d\phi + \ldots$ corresponding to the gauge transformation $\phi \to \phi + \chi(x)$ is projected out of the spectrum due to the Z_2 symmetry of the theory, since it must be Z_2 odd in order to keep the line element a Z_2 invariant. Likewise, our branes cannot be dynamical, since the massless scalar fields that describe individual transverse fluctuations of the branes are Z_2 odd and therefore cannot appear in the setup of [12] with the branes placed at the orbifold fixed points. This is fortunate, since from Eq. (2.14) we see that the TeV brane tension must be negative, in which case the field corresponding to its fluctuations is tachyonic, giving rise to an instability. Although it is difficult to realize a negative tension object from a more fundamental theory which describes the structure of the brane, at least the scenario with a negative tension brane is consistent as a low energy effective theory.

To derive a low energy theory for the fields $g_{\mu\nu}(x)$ and $T(x)$, we substitute the ansatz Eq. (2.20) into Eq. (2.1) and integrate out the compact dimension. First, we concentrate on the low energy dynamics of $g_{\mu\nu}$ alone. Assuming that the radion T is frozen at its VEV r_c, we obtain

$$S = 2M^3 \int d^4x \sqrt{-g} \int_{-\pi}^{\pi} r_c d\phi e^{-4kr_c|\phi|} \left[e^{2kr_c|\phi|} g^{\mu\nu} R[g]_{\mu\nu} + \cdots \right], \tag{2.21}$$

where $R[g]_{\mu\nu}$ is the four-dimensional Ricci tensor constructed from the metric $g_{\mu\nu}$, and $\cdots$ denotes terms not relevant to constructing the effective action for $g_{\mu\nu}$. Performing the ϕ integration,

$$S = \frac{2M^3}{k}(1 - e^{-2kr_c\pi}) \int d^4x \sqrt{g} R[g] + \ldots \tag{2.22}$$

this becomes a four-dimensional Einstein-Hilbert action for the graviton. From this we can read the four-dimensional Planck scale

$$M_{Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}]. \tag{2.23}$$

Since to address the hierarchy problem we assumed that M and k were of order the observed four-dimensional Planck scale, we end up with (for $kr_c \sim 12$) $M_{Pl}^2 \simeq M^3/k$

also of order the Planck scale. Thus, the model recovers general relativity with the correct strength of gravitational interactions at low energies. It is interesting to note that M_{Pl} is finite even for $r_c \to \infty$, suggesting that one can obtain a sensible four-dimensional theory of gravity even if the bulk spacetime is noncompact. As Randall and Sundrum pointed out [25], and as we will briefly discuss in the next chapter, this can be attributed to a localization of the bulk graviton zero mode on the Planck brane ($\phi = 0$).

Let us now include the radion field in the low energy dynamics. Promoting r_c to the four-dimensional field $T(x)$, the five-dimensional Ricci scalar becomes

$$\begin{aligned} R &= e^{2kT(x)|\phi|}\left[-\frac{2\Box T}{T} + \frac{4k|\phi|}{T}\partial_\alpha T\partial^\alpha T + 6k|\phi|\Box T - 6k^2\phi^2\partial_\alpha T\partial^\alpha T\right] \\ &+ \frac{8\sigma''}{r_c T(x)} - \frac{20\sigma'^2}{r_c^2} + e^{2kT(x)|\phi|}R[g]. \end{aligned} \tag{2.24}$$

Following the same procedure as we did for the graviton, insert this expression into Eq. (2.1) and integrate over ϕ. In deriving the effective action, it will be instructive to keep the brane tensions general, not setting them to the values which they must take in order to obtain the solution of Eq. (2.15). Then,

$$\begin{aligned} S &= 2M^3\int d^4x\sqrt{-g}\int_{-\pi}^{\pi} d\phi e^{-2k|\phi|T}[6k|\phi|\partial_\mu T\partial^\mu T - 6k^2\phi^2 T\partial_\mu T\partial^\mu T] \\ &+2M^3\int d^4x\sqrt{-g}\int_{-\pi}^{\pi} d\phi T e^{-4k|\phi|T}\left[e^{2k|\phi|T}R - 20k^2 + \frac{16k}{T}(\delta(\phi) - \delta(\phi-\pi))\right] \\ &-\int d^4x\sqrt{-g}\int_{-\pi}^{\pi} d\phi T e^{-4k|\phi|T}\Lambda - V_v\int d^4x\sqrt{-g}e^{-4k\pi T} - V_h\int d^4x\sqrt{-g}, \end{aligned} \tag{2.25}$$

where we integrated by parts to obtain the first line of this equation. Now substitute $\Lambda = -24M^3k^2$ and perform the integral over the fifth dimension. After the ϕ integration there is a cancellation between the first two terms in Eq. (2.25), and only the part that depends on the exponential of T remains:

$$S = \frac{2M^3}{k}\int d^4x\sqrt{-g}\left(1 - e^{-2k\pi T}\right)R + \frac{12M^3}{k}\int d^4x\sqrt{-g}\partial_\mu\left(e^{-k\pi T}\right)\partial^\mu\left(e^{-k\pi T}\right)$$

$$-\int d^4x\sqrt{-g}(V_h - 24M^3k) - \int d^4x\sqrt{-g}e^{-4k\pi T}(V_v + 24M^3k) \qquad (2.26)$$

The first term of this equation includes the Einstein-Hilbert term already derived in Eq. (2.22), and a dilaton-like coupling of the radion to four-dimensional curvature. The second term is a kinetic term for the radion, while the terms on the second line give a radion potential. From this action we can understand why the solution of Eq. (2.15) requires two fine-tunings. For generic values of the brane tensions, the modulus T is not a flat direction, but rather develops a potential. Depending on the sign of $V_v + 24M^3k$, this potential forces the brane separation to run away to $r_c \to \infty$ or collapse to zero. Thus for our static ansatz to be consistent, the TeV brane tension must be tuned exactly to the critical value $V_v = -24M^3k$. Once this adjustment has been made, there is a constant term remaining. This serves as an effective cosmological constant for the four-dimensional theory. Since our vacuum ansatz had $g_{\mu\nu} = \eta_{\mu\nu}$, the only consistent choice of four-dimensional cosmological constant is zero, hence $V_h = 24M^3k$.

After tuning the brane tensions to their critical values, the radion-graviton system is described by [38, 39]

$$S = \frac{2M^3}{k}\int d^4x\sqrt{-g}\left(1-(\varphi/f)^2\right)R + \frac{1}{2}\int d^4x\sqrt{-g}\partial_\mu\varphi\partial^\mu\varphi, \qquad (2.27)$$

where we have introduced the canonically normalized radion, with $\varphi = f\exp(-k\pi T)$ and $f = \sqrt{24M^3/k}$. Unfortunately, this equation has no dynamics built into it that could generate the radion VEV $kr_c \sim 12$ necessary to account for the TeV/Planck hierarchy. Furthermore, the physics described by this equation as it stands is phenomenologically unacceptable. It describes a massless scalar which, as we will see in Chapter 4, couples universally to TeV brane matter. For a brane separation roughly ten times the curvature scale $1/k$, the strength of this coupling is set by the TeV scale. Clearly, such a long range force is at odds with observation. Thus we see that the minimal RS setup cannot be considered a satisfactory resolution of the hierarchy puzzle unless additional dynamics which stabilizes the radion is specified. As was proposed in [41], and as we will see in Chapter 4, the presence of an additional bulk scalar propagating in the RS background can generate a potential $V(\varphi)$ that stabilizes the modulus (see also [29] for a proposal based on supersymmetric gauge dynamics). The minimum of $V(\varphi)$ can be arranged to yield the desired value of kr_c without extreme fine-tuning of parameters. Once this mechanism is in place, the TeV brane tension no longer needs to be tuned to the value given by Eq. (2.14), since $V(\varphi)$ counteracts the effects of a noncritical brane tension on the radion dynamics.

Chapter 3 Bulk fields in a nonfactorizable geometry

In the previous chapter, we introduced the RS scenario and described some of its physical consequences. In particular, we showed how a five-dimensional metric with a warp factor can generate a hierarchy of scales between separate locations in the bulk spacetime. We also wrote down a four-dimensional action for the massless modes that appear in the minimal setup. However, our derivation of this four-dimensional action is not yet complete. In Chapter 2, we stated that the four-dimensional theory is valid up to energies of order the TeV scale, but it was not clear in the discussion of the previous chapter why this had to be so. In order to answer this question, one must know the characteristic mass scale at which Kaluza-Klein excitations of bulk fields (for instance, the graviton) must be included. At energies below this scale, the theory can be treated as four-dimensional. However, as one starts to probe distances shorter than the Compton wavelength of the low-lying Kaluza-Klein modes, the four-dimensional description is no longer practical, since one must keep a growing number of modes in the effective action. In this case, a five-dimensional description is more adequate.

In order to understand the transition from four to five dimensions, in this chapter we will carry out the Kaluza-Klein decomposition of a bulk field propagating in the spacetime corresponding to Eq. (2.15). In Section 3.1, we will consider, for purposes of illustration, a nongravitational scalar bulk field. Naively, one would expect even the lightest Kaluza-Klein modes to have masses comparable to the mass of the bulk scalar and to have self-interactions set by the Planck scale. Instead, we find that the mass spectrum of the four-dimensional Kaluza-Klein modes is suppressed by a factor of $e^{-kr_c\pi}$ relative to the five-dimensional scalar mass. We also find, in Section 3.2, that the same exponential factor suppresses the scale that sets the non-renormalizable self-couplings of the light modes.

The Kaluza-Klein modes of higher-spin bulk fields have a similar pattern of masses and interactions as those explicitly discussed here for a scalar (see [25, 37, 46, 47]). Thus, if the masses and couplings of bulk fields are set by the Planck scale and if we take kr_c to be around 12, the low-lying Kaluza-Klein modes would be characterized by a scale which is on the order of a TeV and could therefore have significant phenomenological implications, which we will briefly comment on in Section 3.3.

Throughout this chapter, we will assume that the radius r_c is fixed by some yet unspecified dynamics in such a way that $kr_c \sim 12$. Also, we will neglect the backreaction of the bulk matter on the spacetime metric, taking the geometry to be fixed by Eq. (2.15).

3.1 Kaluza-Klein decomposition of bulk scalars

First consider a free scalar field in the bulk. The action is

$$S = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G}\left(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2\right), \tag{3.1}$$

where G_{AB} with $A, B = \mu, \phi$ is given by Eq. (2.15), and m is of order M. After an integration by parts, this can be written as

$$S = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} r_c d\phi \left(e^{-2\sigma(\phi)}\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{r_c^2}\Phi\partial_\phi\left(e^{-4\sigma(\phi)}\partial_\phi\Phi\right) - m^2 e^{-4\sigma(\phi)}\Phi^2\right), \tag{3.2}$$

with $\sigma(\phi) = kr_c|\phi|$. To perform the Kaluza-Klein decomposition, write $\Phi(x,\phi)$ as a sum over modes:

$$\Phi(x,\phi) = \sum_n \psi_n(x)\frac{y_n(\phi)}{\sqrt{r_c}}. \tag{3.3}$$

If the $y_n(\phi)$ are chosen to satisfy

$$\int_{-\pi}^{\pi} d\phi e^{-2\sigma(\phi)} y_n(\phi) y_m(\phi) = \delta_{nm} \tag{3.4}$$

and

$$-\frac{1}{r_c^2}\frac{d}{d\phi}\left(e^{-4\sigma(\phi)}\frac{dy_n}{d\phi}\right)+m^2e^{-4\sigma(\phi)}y_n=m_n^2e^{-2\sigma(\phi)}y_n, \tag{3.5}$$

then Eq. (3.2) simplifies to

$$S=\frac{1}{2}\sum_n\int d^4x\left[\eta^{\mu\nu}\partial_\mu\psi_n\partial_\nu\psi_n-m_n^2\psi_n^2\right]. \tag{3.6}$$

As in usual Kaluza-Klein compactifications, the bulk field $\Phi(x,\phi)$ manifests itself to a four-dimensional observer as an infinite "tower" of scalars $\psi_n(x)$ with masses m_n which we find by solving the above eigenvalue problem. After changing variables to $z_n=m_ne^{\sigma(\phi)}/k$ and $f_n=e^{-2\sigma(\phi)}y_n$, Eq. (3.5) can be written as (for $\phi\neq 0,\pm\pi$)

$$z_n^2\frac{d^2f_n}{dz_n^2}+z_n\frac{df_n}{dz_n}+\left[z_n^2-\left(4+\frac{m^2}{k^2}\right)\right]f_n=0. \tag{3.7}$$

The solutions of this equation are Bessel functions of order $\nu=\sqrt{4+\frac{m^2}{k^2}}$. We thus find

$$y_n(\phi)=\frac{e^{2\sigma(\phi)}}{N_n}\left[J_\nu\left(\frac{m_n}{k}e^{\sigma(\phi)}\right)+b_{n\nu}Y_\nu\left(\frac{m_n}{k}e^{\sigma(\phi)}\right)\right], \tag{3.8}$$

where N_n is a normalization factor. The condition that the differential operator on the left-hand side of Eq. (3.5) be self-adjoint forces the derivative of $y_n(\phi)$ to be continuous at the orbifold fixed points. This gives two relations that can be used to solve for m_n and $b_{n\nu}$, yielding

$$b_{n\nu}=-\frac{2J_\nu\left(\frac{m_n}{k}\right)+\frac{m_n}{k}J'_\nu\left(\frac{m_n}{k}\right)}{2Y_\nu\left(\frac{m_n}{k}\right)+\frac{m_n}{k}Y'_\nu\left(\frac{m_n}{k}\right)}, \tag{3.9}$$

and

$$\begin{aligned}0 \;=\; & [2J_\nu(x_{n\nu})+x_{n\nu}J'_\nu(x_{n\nu})][2Y_\nu(x_{n\nu}e^{-kr_c\pi})+x_{n\nu}e^{-kr_c\pi}Y'_\nu(x_{n\nu}e^{-kr_c\pi})] \\ & -[2Y_\nu(x_{n\nu})+x_{n\nu}Y'_\nu(x_{n\nu})][2J_\nu(x_{n\nu}e^{-kr_c\pi})+x_{n\nu}e^{-kr_c\pi}J'_\nu(x_{n\nu}e^{-kr_c\pi})],\end{aligned} \tag{3.10}$$

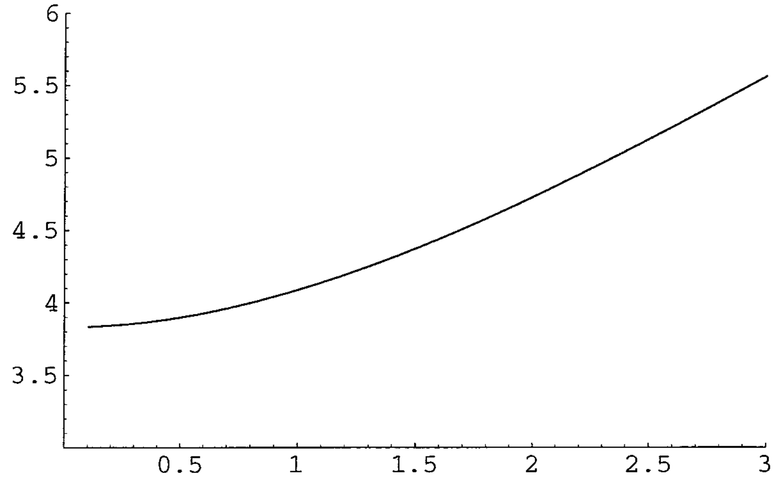


Figure 3.1: Plot of $x_{1\nu}$ versus m/k in the region where m/k is order unity.

where $x_{n\nu} = m_n e^{kr_c\pi}/k$. For $e^{kr_c\pi} \gg 1$, this last condition simplifies to

$$2J_\nu(x_{n\nu}) + x_{n\nu}J'_\nu(x_{n\nu}) = 0. \tag{3.11}$$

Figure 3.1 shows the lowest root of Eq. (3.10), $x_{1\nu}$, as a function of m/k. Because the lightest modes have $x_{n\nu}$ of order unity, we see that their masses are suppressed exponentially with respect to the scale m appearing in Eq. (3.2). Since we take m of order the Planck scale and kr_c around 12, these light modes have masses in the TeV range. The exponential suppression can be understood from Eq. (3.8): the modes $y_n(\phi)$ are larger near the 3-brane at $\phi = \pi$, and consequently it is more likely to find the Kaluza-Klein excitations in that region. Therefore their masses behave in the same way as masses of fields confined to the brane at $\phi = \pi$, which are characterized by the TeV scale.

For the massless case, $m = 0$, there is a mode with y_1 constant and $x_{12} = 0$. It can be obtained from Eq. (3.8) and Eq. (3.9) by a limiting procedure. When $x_{1\nu}$ is small, Eq. (3.10) yields

$$x_{1\nu} \simeq \frac{1}{\sqrt{2}}\left(\frac{m}{k}\right) e^{kr_c\pi}. \tag{3.12}$$

Consequently, $x_{1\nu}$ increases to near the minimum value shown in Fig. 3.1 over an exponentially small region of m/k.

For the low-lying modes, the coefficient $b_{n\nu}$ is of order $e^{-2\nu k r_c \pi}$ and we can safely ignore the $Y_\nu(z_n)$ term with respect to $J_\nu(z_n)$ when performing integrals involving the $y_n(\phi)$. Thus, to a good approximation

$$N_n \simeq \frac{e^{kr_c\pi}}{\sqrt{kr_c}} A_n, \tag{3.13}$$

where

$$A_n = J_\nu(x_{n\nu})\sqrt{1 + \frac{4-\nu^2}{x_{n\nu}^2}}. \tag{3.14}$$

3.2 Scalar self-interactions

We now turn our attention to possible self-interactions of the bulk scalar. From the four-dimensional point of view, these induce couplings between the Kaluza-Klein modes. Here, we concentrate on the self-couplings of the light modes. Just as in the case of the mass spectrum, we find that the exponential factor in Eq. (2.15) plays a crucial role in determining the effective scale of the couplings. If the Planck scale sets the scale of the five-dimensional couplings, the low-lying Kaluza-Klein modes have TeV range self-interactions. First consider a term in the action which is of the form:

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G} \frac{\lambda}{M^{3m-5}} \Phi^{2m}, \tag{3.15}$$

where λ is of order unity. Expanding in modes, the self-interactions of the light Kaluza-Klein states are given by

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} r_c d\phi e^{-4\sigma(\phi)} \frac{\lambda}{M^{3m-5}} \psi_n^{2m} \left(\frac{y_n}{\sqrt{r_c}}\right)^{2m}. \tag{3.16}$$

Thus, the effective four-dimensional coupling constants for the ψ_n^{2m} interactions are

$$\lambda_{eff} = \frac{2\lambda}{(Mr_c)^{m-1} M^{2m-4}(kr_c)} \int_0^{kr_c\pi} d\sigma e^{-4\sigma} y_n^{2m}, \tag{3.17}$$

which in the large kr_c limit become

$$\lambda_{eff} \simeq 2\lambda \left(\frac{k}{M}\right)^{m-1} (Me^{-kr_c\pi})^{4-2m} \int_0^1 r^{4m-5} dr \left[\frac{J_\nu(x_{n\nu}r)}{A_n}\right]^{2m}. \tag{3.18}$$

Such couplings can also be induced by derivative self-interactions of the bulk field. For example, the term

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G} \frac{\lambda}{M^{5m-5}} \left(-G^{AB}\partial_A\Phi\partial_B\Phi\right)^m \tag{3.19}$$

has a piece which contains only derivatives with respect to ϕ :

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} r_c d\phi e^{-4\sigma(\phi)} \frac{\lambda}{M^{5m-5}} \psi_n^{2m} \left(\frac{(\partial_\phi y_n)^2}{r_c^3}\right)^m. \tag{3.20}$$

From the point of view of four-dimensional observers, this yields a ψ_n^{2m} interaction with a coupling constant

$$\lambda_{eff} = \frac{2\lambda(kr_c)^{2m-1}}{(Mr_c)^{3m-1}M^{2m-4}} \int_0^{kr_c\pi} d\sigma e^{-4\sigma} \left(\frac{dy_n}{d\sigma}\right)^{2m}. \tag{3.21}$$

For large kr_c, this becomes

$$\lambda_{eff} \simeq 2\lambda \left(\frac{k}{M}\right)^{3m-1} (Me^{-kr_c\pi})^{4-2m} \int_0^1 r^{2m-5} dr \left[\frac{d}{dr}\left(r^2 \frac{J_\nu(x_{n\nu}r)}{A_n}\right)\right]^{2m}. \tag{3.22}$$

In either case, we see that the scale relevant to four-dimensional physics is not M, but rather $v = Me^{-kr_c\pi}$. The Kaluza-Klein reduction has lead to an exponential enhancement of irrelevant couplings from Planck scale to only TeV scale suppression.

3.3 Physical consequences

Because the results for bulk fields with higher spin are qualitatively similar to those for scalars [37, 47], we can use the conclusions of the previous sections to understand some important phenomenological consequences of the RS scenario. First, since bulk fields have excitations that are characterized by the TeV scale, it is natural to ask [40] whether it is possible that the SM itself is a bulk field theory. Much work has been dedicated to this issue [48, 49]. Unfortunately, the actual implementation of the SM as five-dimensional theory has not led to promising results. For example, it has been noticed [49] that unless the Higgs boson is confined to the TeV brane, the theory must be sensitively fine-tuned in order to yield masses for the W and Z bosons which are below the TeV range (this can be understood essentially from Figure 3.1). A scenario in which some of the SM fields reside in the bulk, while others must be confined to the TeV brane does not seem well motivated, however.

As a second application of the dimensional reduction of bulk fields, consider the bulk graviton. Our presentation follows that of [25]. To study massive spin-2 excitations of the bulk metric, we expand the metric about the background AdS solution. We can choose a gauge in which only the $\mu\nu$ components of the metric perturbation are nonvanishing. Writing

$$G_{\mu\nu}(x,\phi) = e^{-2kr_c|\phi|}\left(\eta_{\mu\nu} + h_{\mu\nu}(x,\phi)\right), \tag{3.23}$$

and

$$h_{\mu\nu}(x,\phi) = \sum_n h^{(n)}_{\mu\nu}(x)\frac{y_n(\phi)}{\sqrt{r_c}}, \tag{3.24}$$

one finds, after imposing the further gauge-fixing conditions $\eta^{\alpha\beta}\partial_\alpha h_{\beta\mu} = 0$ and $\eta^{\mu\nu}h_{\mu\nu} = 0$

$$(\eta^{\alpha\beta}\partial_\alpha\partial_\beta + m_n^2)h^{(n)}_{\mu\nu} = 0. \tag{3.25}$$

This is true provided that the $y_n(\phi)$ satisfy Eq. (3.5) with the bulk mass set to zero, and the m_n are given by Eq. (3.10) with $\nu = 2$. These eigenfunctions are identical to those of a bulk scalar, so one finds the same pattern of TeV scale Kaluza-Klein masses and self-couplings for the graviton as we did for a bulk scalar. We can also compute the coupling of these linearized modes to the SM fields. Assuming that the SM resides on the TeV brane, this is given by

$$S_{int} \sim \frac{1}{M^{3/2}}\int d^4x\tilde{h}_{\mu\nu}(x,\pi)T^{\mu\nu}(x) + \cdots, \tag{3.26}$$

where $T_{\mu\nu}$ is the stress tensor for the SM, written in terms of canonical fields and rescaled masses, $\tilde{h}$ is the canonically normalized graviton, and tensor indices are contracted with the Minkowski metric $\eta_{\mu\nu}$. Using Eq. (3.13) and Eq. (3.14), as well as the relation $M_{Pl}^2 \simeq M^3/k$ from the previous chapter, the coupling of the SM to the massive modes is

$$S_{int} \sim \frac{1}{M_{Pl} e^{-kr_c\pi}} \sum_{n\neq 0} \frac{1}{J_2(x_n)} \int d^4x \tilde{h}_{\mu\nu}^{(n)}(x) T^{\mu\nu}(x) + \cdots . \tag{3.27}$$

Since $J_2(x_n)$ is order one, this equation implies that massive Kaluza-Klein graviton states couple to ordinary matter with TeV rather than Planck suppression. Since their masses are also of order the TeV scale, they may therefore be accessible to collider experiments. The observation of massive spin-2 resonances which decay into SM modes would be a strong signal[1] of the scenario of [12]. See [37] for more details on the graviton phenomenology of the RS model.

So far in this discussion, we have ignored the zero mode component of the bulk graviton. While the massive mode eigenfunctions of bulk fields are peaked towards the vicinity of the TeV brane, leading to behavior similar to that of fields confined to $\phi = \pi$, the zero mode eigenfunction y_0 is a constant (if the bulk mass is zero). Because the zero-mode graviton has a profile which from Eq. (3.23) is given by $\exp(-2kr_c|\phi|)y_0$, it is actually dominated by the region near the Planck brane, at $\phi = 0$. This leads to another understanding of why, from a four-dimensional perspective, gravitational interactions are still suppressed by the Planck scale.

The fact that the zero mode graviton is localized on the Planck brane led [25] to consider a variant of the two-brane scenario in which the TeV brane is taken to infinity, and SM matter resides on the Planck brane. In this case, the Kaluza-Klein spectrum for gravitons becomes a continuum without a mass gap above the zero mode. However, as the analysis of [25] shows, for observers located on the Planck brane the continuum modes decouple in such a way that at long distances,

[1]Another distinctive signature is the appearance of a TeV scale mass scalar with universal couplings, the stabilized radion. We will discuss this in the next chapter.

gravity appears four-dimensional despite the fact that the bulk space is noncompact. Because the bulk space has infinite volume, this effective four-dimensional theory cannot be derived by the usual dimensional reduction techniques (such as the ones we used in the last chapter). Perhaps one way to interpret the low-energy effective field theory is holographically, via the AdS/CFT correspondence [30]. In the version of this correspondence that is pertinent to the RS model [31], one replaces the bulk spacetime with a dual four-dimensional conformal field theory (CFT), which couples to the matter fields on the Planck brane purely through four-dimensional gravity. Several checks of this equivalence [33, 34, 35, 36, 50] give agreement between the two pictures. The most convincing result so far [33, 34, 35] is the agreement between the corrections to the gravitational potential due to Kaluza-Klein graviton exchange in the five-dimensional theory with the corrections to the zero-mode graviton propagator due to CFT loops in the dual four-dimensional theory.

Chapter 4 Radion stabilization

We have seen that in the scenario presented in [12], r_c is associated with the vacuum expectation value of a massless four-dimensional scalar field, the radion. If the brane tensions are properly tuned, the radion has zero potential and, consequently, r_c is not determined by the dynamics of the model. For this scenario to be relevant, it is necessary to find a mechanism for generating a potential to stabilize the value of r_c. In Section 4.1 we show that such a potential can arise classically from the presence of a bulk scalar with interaction terms that are localized to the two branes [41]. The minimum of this potential can be arranged to yield a value of $kr_c \sim 12$ without fine-tuning of parameters. In Section 4.2, we explore some of the phenomenological features of the radion coupled to a stabilizing potential, such as that generated by the scalar of Section 4.1.

4.1 Modulus stabilization with bulk fields

To stabilize the radion, we add to the model a scalar field Φ. We will consider the free bulk action

$$S_b = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G}\left(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2\right), \tag{4.1}$$

where G_{AB} with $A, B = \mu, \phi$ is given by Eq. (2.15). We also include interaction terms on the TeV and Planck branes given by

$$S_h = -\int d^4x\sqrt{-g_h}\lambda_h\left(\Phi^2 - v_h^2\right)^2, \tag{4.2}$$

and

$$S_v = -\int d^4x\sqrt{-g_v}\lambda_v\left(\Phi^2 - v_v^2\right)^2, \tag{4.3}$$

where g_h and g_v are the determinants of the induced metric on the hidden and visible branes respectively. Note that Φ and $v_{v,h}$ have mass dimension 3/2, while $\lambda_{v,h}$ have mass dimension -2. Kinetic terms for the scalar field can be added to the brane actions without changing our results. The terms on the branes cause Φ to develop a ϕ-dependent vacuum expectation value $\Phi(\phi)$ which is determined classically by solving the differential equation

$$\begin{aligned} 0 &= -\frac{1}{r_c^2}\partial_\phi\left(e^{-4\sigma}\partial_\phi\Phi\right) + m^2 e^{-4\sigma}\Phi + 4e^{-4\sigma}\lambda_v\Phi\left(\Phi^2 - v_v^2\right)\frac{\delta(\phi-\pi)}{r_c} \\ &\quad + 4e^{-4\sigma}\lambda_h\Phi\left(\Phi^2 - v_h^2\right)\frac{\delta(\phi)}{r_c}, \end{aligned} \tag{4.4}$$

where $\sigma(\phi) = kr_c|\phi|$. Away from the boundaries at $\phi = 0, \pi$, this equation has the general solution

$$\Phi(\phi) = e^{2\sigma}[Ae^{\nu\sigma} + Be^{-\nu\sigma}], \tag{4.5}$$

with $\nu = \sqrt{4 + m^2/k^2}$. Putting this solution back into the scalar field action and integrating over ϕ yields an effective four-dimensional potential for r_c which has the form

$$\begin{aligned} V_\Phi(r_c) &= k(\nu+2)A^2(e^{2\nu kr_c\pi} - 1) + k(\nu-2)B^2(1 - e^{-2\nu kr_c\pi}) \\ &\quad + \lambda_v e^{-4kr_c\pi}\left(\Phi(\pi)^2 - v_v^2\right)^2 + \lambda_h\left(\Phi(0)^2 - v_h^2\right)^2. \end{aligned} \tag{4.6}$$

The unknown coefficients A and B are determined by imposing appropriate boundary conditions on the 3-branes. We obtain these boundary conditions by inserting Eq. (4.5) into the equations of motion and matching the delta functions:

$$k\left[(2+\nu)A + (2-\nu)B\right] - 2\lambda_h\Phi(0)\left[\Phi(0)^2 - v_h^2\right] = 0, \tag{4.7}$$

$$ke^{2kr_c\pi}\left[(2+\nu)e^{\nu kr_c\pi}A + (2-\nu)e^{-\nu kr_c\pi}B\right] + 2\lambda_v\Phi(\pi)\left[\Phi(\pi)^2 - v_v^2\right] = 0. \tag{4.8}$$

Rather than solve these equations in general, we consider the simplified case in which the parameters λ_h and λ_v are large. It is evident from Eq. (4.6) that in this limit, it is energetically favorable[1] to have $\Phi(0) = v_h$ and $\Phi(\pi) = v_v$. Thus, from Eq. (4.5) we get for large kr_c

$$A = v_v e^{-(2+\nu)kr_c\pi} - v_h e^{-2\nu kr_c\pi}, \tag{4.9}$$

$$B = v_h(1 + e^{-2\nu kr_c\pi}) - v_v e^{-(2+\nu)kr_c\pi}, \tag{4.10}$$

where subleading powers of $\exp(-kr_c\pi)$ have been neglected. Now suppose that $m/k \ll 1$ so that $\nu = 2 + \epsilon$, with $\epsilon \simeq m^2/4k^2$ a small quantity. In the large kr_c limit, the potential becomes

$$V_\Phi(r_c) = 4ke^{-4kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi})^2, \tag{4.11}$$

where terms of order ϵ and higher are dropped (but ϵkr_c is not treated as small), since such terms are of the same order as the effects of the backreaction of the scalar field on the background geometry (see below). Because we are not solving for this backreaction, we have no right to keep such terms in $V_\Phi(r_c)$. A formalism for finding exact solutions of the gravity plus bulk scalar system including the effects of the gravitational backreaction can be found in [51].

This potential has a minimum at

$$kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right]. \tag{4.12}$$

With $\ln(v_h/v_v)$ of order unity, we only need m^2/k^2 of order $1/10$ to get $kr_c \sim 12$. Clearly, no extreme fine-tuning of parameters is required to get the right magnitude for kr_c. For instance, taking $v_h/v_v = 1.5$ and $m/k = 0.2$ yields $kr_c \simeq 12$.

The stress tensor for the scalar field can be written as $T_s^{AB} = T_k^{AB} + T_m^{AB}$, where for large kr_c:

$$T_k^{\phi\phi} \simeq -\frac{k^2}{2r_c^2}\left[(4+\epsilon)(v_v - v_h e^{-\epsilon kr_c\pi})e^{-(4+\epsilon)(kr_c\pi-\sigma)} - \epsilon v_h e^{-\epsilon\sigma}\right]^2, \tag{4.13}$$

[1]The configuration that has both VEVs of the same sign has lower energy than the one with alternating signs and therefore corresponds to the ground state. Clearly, the overall sign is irrelevant.

$$T_k^{\mu\nu} \simeq \frac{k^2}{2} e^{2\sigma} \eta^{\mu\nu} \left[(4+\epsilon)(v_v - v_h e^{-\epsilon k r_c \pi}) e^{-(4+\epsilon)(k r_c \pi - \sigma)} - \epsilon v_h e^{-\epsilon\sigma} \right]^2, \quad (4.14)$$

and

$$T_m^{\phi\phi} \simeq -\frac{2k^2}{r_c^2} \epsilon \left[(v_v - v_h e^{-\epsilon k r_c \pi}) e^{-(4+\epsilon)(k r_c \pi - \sigma)} + v_h e^{-\epsilon\sigma} \right]^2, \quad (4.15)$$

$$T_m^{\mu\nu} \simeq -2k^2 e^{2\sigma} \eta^{\mu\nu} \epsilon \left[(v_v - v_h e^{-\epsilon k r_c \pi}) e^{-(4+\epsilon)(k r_c \pi - \sigma)} + v_h e^{-\epsilon\sigma} \right]^2. \quad (4.16)$$

As long as v_h^2/M^3, v_v^2/M^3, and ϵ are small, T_s^{AB} can be neglected in comparison to the stress tensor induced by the bulk cosmological constant. It is therefore safe to ignore the influence of the scalar field on the background geometry for the computation of $V(r_c)$. A similar criterion ensures that the stress tensor induced by the bulk cosmological constant is dominant for $kr_c \sim 1$.

One might worry that the validity of Eq. (4.11) and Eq. (4.12) requires unnaturally large values of λ_h and λ_v. We will check that this is not the case by computing the leading $1/\lambda$ correction to the potential. To obtain this correction, we linearize Eq. (4.7) and Eq. (4.8) about the large λ solution. Neglecting terms of order ϵ, the VEVs are then shifted by

$$\delta\Phi(0) = \frac{k}{\lambda_h v_h^2} e^{-(4+\epsilon) k r_c \pi} (v_v - v_h e^{-\epsilon k r_c \pi}), \quad (4.17)$$

$$\delta\Phi(\pi) = -\frac{k}{\lambda_v v_v^2} (v_v - v_h e^{-\epsilon k r_c \pi}), \quad (4.18)$$

and thus (neglecting subleading exponentials of $kr_c\pi$)

$$\delta A = -\frac{k}{\lambda_v v_v^2} e^{-(4+\epsilon) k r_c \pi} (v_v - v_h e^{-\epsilon k r_c \pi}), \quad (4.19)$$

$$\delta B = e^{-(4+\epsilon) k r_c \pi} (v_v - v_h e^{-\epsilon k r_c \pi}) \left[\frac{k}{\lambda_v v_v^2} + \frac{k}{\lambda_h v_h^2} \right]. \quad (4.20)$$

Hence, the correction to the potential is

$$\delta V_\Phi(r_c) = -\frac{4k^2}{\lambda_v v_v^2} e^{-4 k r_c \pi} (v_v - v_h e^{-\epsilon k r_c \pi})^2. \quad (4.21)$$

This has the same form as the leading $\epsilon \to 0$ behavior of Eq. (4.11) and therefore does not significantly affect the location of the minimum. See [52] for a variant of the model of [41] that leads to a stabilized radion, but that has boundary potentials for which the boundary conditions can be treated exactly. Also, see [51] for exact solutions for a wide class of bulk and boundary potentials for the bulk scalar Φ.

Note that the forms of the potentials in Eq. (4.11) and Eq. (4.21) are only valid for large kr_c. For small kr_c, the potential becomes

$$V_\Phi(r_c) = \frac{(v_v - v_h)^2}{\pi r_c}, \tag{4.22}$$

when terms of order ϵ and $1/\lambda$ are neglected. The singularity as $r_c \to 0$ is removed by finite λ corrections which become large for small r_c, and yield

$$V_\Phi(0) = \frac{\lambda_h \lambda_v}{\lambda_h + \lambda_v} \left(v_v^2 - v_h^2\right)^2. \tag{4.23}$$

We saw in Chapter 2 that for Eq. (2.15) to be a solution of the field equations that follow from Eq. (2.1), one must arrange $V_h = -V_v = 24M^3k$. This amounts to having a vanishing four-dimensional cosmological constant plus an additional fine tuning which causes the r_c potential to vanish. However, imagine perturbing the 3-brane tensions by small amounts [2]:

$$V_h \to V_h + \delta V_h, \tag{4.24}$$

$$V_v \to V_v + \delta V_v. \tag{4.25}$$

As long as $|\delta V_h|$ and $|\delta V_v|$ are small compared to $-\Lambda/k$, these shifts in the brane tensions induce the following potential for r_c

$$V_\Lambda(r_c) = \delta V_h + \delta V_v e^{-4kr_c\pi}. \tag{4.26}$$

[2]It has been noted that given the action in Eq. (2.1), changes in the relation between the brane tensions and the bulk cosmological constant result in bent brane solutions [53]. It is possible that there are higher dimension induced curvature terms in the brane actions that make it energetically favorable for them to stay flat. For $V_h = -V_v = 24M^3k$, Eq. (2.15) remains a solution to the field equations in the presence of such terms.

For δV_v small, the sum of potentials $V_\Phi(r_c) + V_\Lambda(r_c)$ has a minimum for large kr_c. The effective four-dimensional cosmological constant can be tuned to zero by adjusting the value of δV_h.

Finally, we note that in the minimal RS setup without stabilization mechanism, an analysis of the cosmology below the TeV scale with matter on the branes [54], leading to time evolution of the Hubble constant in contradiction with Friedmann cosmology. This arises because no dynamics is present to set the radion VEV to a fixed value. Since the brane matter acts as a source for the radion potential (in a similar manner as the brane tensions did in Chapter 2), it is not surprising that it is necessary to impose unnatural constraints on the relation between the expansion rate and the densities of the brane localized fluids. Indeed, once r_c is stabilized, this constraint is not required and the cosmology associated with the model has been shown to be standard for temperatures below the weak scale [39] (see also [55] for related work). However, for temperatures above this scale, it will be different from the usual Friedmann cosmology, since bulk degrees of freedom, as well as the motion of the radion, must be taken into account.

4.2 Phenomenology of the stabilized radion

In this section, we point out a few of the basic phenomenological features [38, 39] of a radion that is stabilized by a bulk scalar such as that of the previous section[3]. The radion potential generated in [41] is nearly flat near its minimum for values of the modulus VEV that solve the hierarchy problem in the manner of [12]. Consequently, the radion is likely to be lighter than the Kaluza-Klein modes of any bulk field, and may be the first experimental signal for a scenario such as the Randall-Sundrum model. In addition, its couplings to fields confined to the visible brane are suppressed by the TeV scale and are completely fixed by four-dimensional general covariance on the brane. This leads to a well-defined set of predictions that can be compared with experiment.

[3]Other aspects of radion phenomenology are found in [56, 57, 58]

In terms of the canonical radion φ that we introduced in Chapter 2,

$$V(\varphi) = \frac{k^3}{144M^6}\varphi^4 \left(v_v - v_h(\varphi/f)^\epsilon\right)^2, \tag{4.27}$$

where terms of order ϵ are dropped, since they are of the same order as uncomputed backreaction effects. The mass of φ excitations about the minimum of this potential, $\langle\varphi\rangle/f = (v_v/v_h)^{1/\epsilon}$, is given by

$$m_\varphi^2 = \frac{\partial^2 V}{\partial\varphi^2}(\langle\varphi\rangle) = \frac{k^2 v_v^2}{3M^3}\epsilon^2 e^{-2kr_c\pi}. \tag{4.28}$$

Note that the exponential factor rescales m_φ from a quantity of order the Planck scale down to the TeV scale. As we discussed in the previous chapter, low-lying Kaluza-Klein excitations of bulk fields in the RS model have masses which are typically slightly larger than the TeV scale. (This also includes the lowest excitation of the scalar Φ. Although it has a bulk mass which is smaller than the Planck mass, its lowest Kaluza-Klein mode still has a mass which is on the order of a few TeV, see Figure 1.) However, if the large value of kr_c (i.e, $kr_c \sim 12$) arises from a small bulk scalar mass, then in addition to the factor $\exp(-2kr_c\pi)$ in Eq. (4.28) there is suppression by the small quantity ϵ. Consequently, m_φ is somewhat smaller than the TeV scale, and therefore lighter than the Kaluza-Klein excitations of bulk fields. It would be the first clear signal of the scenario of [12].

Because the modulus arises as a gravitational degree of freedom, its couplings to brane matter are constrained by four-dimensional general covariance. These couplings arise from the induced metric on the brane. On the $\phi = 0$ brane, the induced metric obtained from Eq. (2.20) is simply $g_{\mu\nu}$: the modulus does not couple directly to hidden brane matter. It can, however, couple to the Planck brane through the mixing term with the graviton in Eq. (2.27). The induced metric on the visible brane is given by $(\varphi/f)^2 g_{\mu\nu}$ and consequently, φ interacts directly with SM fields. For example, consider a scalar $h(x)$ confined to the visible brane:

$$S = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}(\varphi/f)^4[(\varphi/f)^{-2}\bar{g}^{\mu\nu}\partial_\mu h\partial_\nu h - \mu_0^2 h^2]. \tag{4.29}$$

Rescaling $h \to (f/\langle\varphi\rangle)h$ to obtain a canonically normalized field, this becomes

$$S = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}[(\varphi/\langle\varphi\rangle)^2\bar{g}^{\mu\nu}\partial_\mu h\partial_\nu h - \mu^2(\varphi/\langle\varphi\rangle)^4 h^2], \tag{4.30}$$

where

$$\mu = \mu_0\frac{\langle\varphi\rangle}{f} = \mu_0 e^{-kr_c\pi}. \tag{4.31}$$

For μ_0 of order the Planck scale and $kr_c \sim 10$, the physical mass μ is of order the weak scale. This result can be generalized to any operator appearing in the visible brane Lagrangian: a parameter with mass dimension d is rescaled by d powers of $\exp(-kr_c\pi)$. Also, any operator with n powers of the inverse metric is multiplied by $4-2n$ powers of $\varphi/\langle\varphi\rangle$ (for fermions, a power of the inverse vierbein counts as $n = 1/2$). Note that the non-renormalizable couplings of the modulus φ to visible brane fields are characterized by the scale $\langle\varphi\rangle$, which is in the TeV range. Expanding φ about its VEV, $\varphi = \langle\varphi\rangle + \delta\varphi$, we see that $\delta\varphi$ couples to ordinary matter through the trace of the SM energy-momentum tensor $T_{\mu\nu}$,

$$\mathcal{L}_{int} = \frac{\delta\varphi}{\langle\varphi\rangle}T^\mu{}_\mu. \tag{4.32}$$

Neglecting the quark masses, the energy momentum tensor for QCD is traceless at tree level. This suppresses some production mechanisms for $\delta\varphi$ at high energy hadron colliders.

If the almost complete cancellation of the ϕ integral of the first two terms in the square brackets of Eq. (2.25) did not occur, the canonically normalized modulus field would have couplings suppressed by the Planck scale instead of the weak scale, as well as a much lighter mass, of order $(\text{TeV})^2/M_{Pl}$. It would be interesting to use the methods of [51] to examine precisely how deviations from the pure anti-deSitter metric of Eq. (2.15) which arise due to classical Φ configuration influence the kinetic term for T. When the backreaction of the classical Φ configuration is included, the induced metric will have a more complicated dependence on φ than $(\varphi/f)^2 g_{\mu\nu}$. Consequently, Eq. (4.30) is not general. Nevertheless, expanding the induced metric to linear order in $\delta\varphi$ gives Eq. (4.32) multiplied by the additional coupling constant

$$\kappa = \frac{1}{2}\left[\varphi\frac{\partial}{\partial\varphi}\ln F(\varphi)\right]\Big|_{\langle\varphi\rangle}, \tag{4.33}$$

where the induced metric on the visible brane is now $F(\varphi)g_{\mu\nu}$.

There are several factors that could affect our result for the mass of the radion. For example, the radion φ mixes with the Kaluza-Klein modes of Φ. However, the part of this mixing that is not Planck suppressed is generated by the same mechanism as the potential $V(\varphi)$, so it seems reasonable that it will also be suppressed by ϵ and not drive m_φ up to the TeV scale. In [51] other regions of parameter space which generate a large value of kr_c were explored. For example, $kr_c \sim 10$ can be obtained if the bulk scalar has negative mass squared and its VEV on the visible brane is large compared with that on the hidden brane. It seems plausible that even in these cases, φ will be light for large kr_c since a natural way to get a large VEV for T (i.e., a large value of kr_c) is to have its potential be broad. There are also potential corrections to Eq. (4.28) due to gravitational backreaction effects. Finally, there is the issue of whether the lightness of φ will survive quantum corrections. We will consider a restricted class of quantum effects in the next chapter and show that these are under control.

Given the number of effects that are missing in our calculation of the radion mass, it is remarkable that Eq. (4.28) in fact agrees *exactly* (in the limit of small ϵ) with the more thorough treatment of [56]. Ref. [56] preformed the calculation of the radion mass not in the four-dimensional effective field theory framework that we presented here, but rather considered the radion from a five-dimensional perspective. They computed the mass spectrum of the coupled radion and bulk scalar system by solving the linearized equations of motion, using the exact solution of [51] for the metric and bulk scalar profile as a background, and found that the radion mass is indeed of order the TeV scale, but lighter in the limit of small backreaction. Furthermore, after taking into account the mixing between the bulk scalar modes and the radion, they found that the bulk scalar can couple to the SM fields, providing an experimental probe of the stabilization mechanism of [41]. It would be interesting to check explicitly why our four-dimensional calculation of the radion mass agrees with this more detailed analysis. In order to do this, one would need to be able to compute in the effective theory the backreaction and mixing effects as a power series in the small parameter ϵ. This computation is beyond the scope of this thesis.

Chapter 5 Quantum corrections to the radion potential

So far, our analysis of the radion dynamics has been purely classical. However, quantum fluctuations of fields which propagate in the bulk or on the TeV brane will also generate contributions to the effective radion potential[1]. In this chapter, we explore the possibility that it is these quantum corrections which stabilize the radion [43]. We calculate the effective potential arising from bulk fields as well as fields confined to the TeV brane. For the confined fields we calculate using three different regulators and show clearly that the effective cutoff on the brane is indeed of order TeV. After proper regularization, the sole effect of the brane field fluctuations is the renormalization of the brane tension. The physical contribution to the effective potential from integrating out bulk fields is suppressed by large powers of $\exp(-kr_c\pi)$ in the large r_c limit. As a consequence, the resulting vacuum energy cannot be used to stabilize the branes at the separation required to generate the TeV/Planck hierarchy. Furthermore, the quantum effects do not spoil the classical mechanism of [40]. Our results resolve a discrepancy between two previous results in the literature [59, 60].

5.1 Vacuum energy from brane localized fields

First, we consider the contribution to the radion potential coming from a field on the TeV brane. We will show that the fluctuations of fields on the brane serve only to renormalize the brane tension. Given the subtlety in the regularization procedure we will calculate using three different regulators.

For concreteness, we will take a scalar field theory confined to the TeV brane.

[1] Fields on the Planck brane, at $\phi = 0$, do not couple directly to the radion. We expect that their contribution to the one-loop effective potential is suppressed relative to the sources mentioned above.

First we shall compute the effective potential using dimensional regularization. In $n = 4 - \epsilon$ dimensions, the action is given by

$$S = \frac{1}{2} \int d^n x a^n \left(\frac{1}{a^2} (\partial h)^2 - m_0^2 h^2 \right), \tag{5.1}$$

where m_0 is of order the Planck scale, and the powers of a multiplying the kinetic and mass terms come from the induced metric on the brane. To compute the radion potential, take a constant and rescale $h \to ah$. The rescaled field has a canonically normalized kinetic term and an effective mass $m = m_0 a$. The effective potential obtained from integrating out h can be trivially expressed as the zero point energy in the presence of a constant φ field configuration:

$$V = \frac{1}{2} \mu^{4-n} \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \sqrt{k^2 + a^2 m_0^2}, \tag{5.2}$$

with μ an arbitrary mass scale which has been introduced to keep V a four-dimensional energy density. The resulting expression,

$$V = -\frac{1}{2} \frac{\mu^{4-n}}{(4\pi)^{n/2}} \Gamma\left(-\frac{n}{2}\right) (m_0 a)^n, \tag{5.3}$$

contains a divergent piece that must be absorbed into a local counterterm. Such a counterterm is provided by the brane tension on the TeV brane

$$S_{ct} = - \int d^n x a^n \delta V \mu^{n-4}, \tag{5.4}$$

which is generally covariant in n dimensions. Comparing this with our result, we see that V is in fact pure counterterm: the effect of the scalar h is simply to renormalize the brane tension. Given a bare mass of order the Planck scale (this is the appropriate choice for our set of coordinates), there are no large logs for $\mu \simeq M_{Pl}$.

An alternative way of understanding this result is to regulate the divergent integral using a physical (coordinate invariant) cutoff Λ. The vacuum energy for h is then, for $\Lambda \gg m_0$

$$V = \frac{1}{2}\int^{\Lambda a} \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + a^2 m_0^2}$$
$$= \frac{a^4}{32\pi^2}\left[2\Lambda^4 + \Lambda^2 m_0^2 + m_0^4 \ln\left(\frac{m_0}{2\Lambda}\right)\right], \tag{5.5}$$

which simply induces a shift in the brane tension. Note that the coordinate cutoff on the momentum integral is rescaled by a factor of a with respect to the physical cutoff. Had we used an a-independent cutoff on the momentum integral, we would have generated terms in the effective action proportional to $(\Lambda m_0 a)^2$. On the other hand, the rescaled cutoff yields results that are consistent with four-dimensional general covariance on the brane, and which are in agreement with dimensional regularization.

The same conclusion can be reached by using a Pauli-Villars regulator. To get a consistent result, the regulator fields must couple to the induced metric on the TeV brane in the same way as our scalar field. Performing the calculations in two dimensions for simplicity, we make the subtraction

$$V \rightarrow V - \frac{1}{2(M_1^2 - M_2^2)}\int \frac{d^2k}{(2\pi)^2}\left[(m^2 - M_2^2)\sqrt{k^2 + M_1^2}\right.$$
$$\left. + (M_1^2 - m^2)\sqrt{k^2 + M_2}\right]. \tag{5.6}$$

Where *all* the masses, including the regulator masses get rescaled by the warp factor. Performing the momentum integral, it is easily seen that all logarithmic dependence on a cancels from the regulated expression. The remaining dependence on a is a pure counterterm.

5.2 Vacuum energy from bulk fields

The quantum fluctuations of bulk fields also contribute to the radion effective potential. Decomposing the bulk field into four-dimensional Kaluza-Klein modes, the potential can again be expressed as a sum over zero point energies

$$V = (-1)^F \frac{g}{2} \mu^\epsilon \sum_n \int \frac{d^{3-\epsilon}k}{(2\pi)^{3-\epsilon}} \sqrt{k^2 + m_n^2}, \tag{5.7}$$

where $F = 0, 1$ for bosons and fermions respectively, and g is the number of physical polarizations of the Kaluza-Klein modes. In this equation, the dependence on a enters through the Kaluza-Klein masses m_n. Defining $m_n = kx_n a$, the above expression becomes

$$V = (-1)^{F+1} g \frac{k^4 a^4}{32\pi^2} \left(\frac{k^2 a^2}{4\pi\mu^2}\right)^{-\epsilon/2} \Gamma(-2 + \epsilon/2) \sum_n x_n^{4-\epsilon}. \tag{5.8}$$

We now evaluate Eq. (5.8) for a bulk scalar field with action

$$S_b = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G} \left(G^{AB} \partial_A \Phi \partial_B \Phi - \left(m^2 + \alpha \frac{\sigma''}{r_c^2} \right) \Phi^2 \right), \tag{5.9}$$

where G_{AB} with $A, B = \mu, \phi$ is given by Eq. (2.15). Because $\sigma'' = 2kr_c[\delta(\phi) - \delta(\phi - \pi)]$, the parameter α controls a possible mass term on the boundaries of the space. Such mass terms arise if the field Φ is a component of a supermultiplet on AdS_5 with one dimension compactified on an S^1/Z_2 orbifold (see [46]). It is found in [46] that the roots x_n satisfy

$$j_\nu(x_n) y_\nu(a x_n) - j_\nu(a x_n) y_\nu(x_n) = 0, \tag{5.10}$$

where $\nu = \sqrt{4 + m^2/k^2}$, $j_\nu(z) = (2 - \alpha) J_\nu(z) + z J_\nu'(z)$, and y_ν is given by the same expression with Y_ν replacing J_ν. The a dependence from the sum over x_n in Eq. (5.8) can be calculated by zeta function regularization techniques [61], which we now review.

First, convert the sum into a contour integral

$$\sum_n x_n^{-s} = \frac{s}{2\pi i} \int_C dz z^{-s-1} \ln\left[j_\nu(z) y_\nu(az) - j_\nu(az) y_\nu(z) \right], \tag{5.11}$$

which is valid for Re$s > 1$. In this equation, C is a closed contour bounded by arcs of radius δ (chosen to avoid a possible pole at $z = 0$) and $R \to \infty$ which circles the roots

x_n in a counterclockwise manner. Our goal is to perform the analytic continuation of the RHS of Eq. (5.11) to a neighborhood of $s = -4$. To do this, split the contour into C_+ and C_-, its portions above and below the real axis respectively. On each contour, the asymptotic expansion of the argument of the logarithm is

$$Z_\nu(z,a) \equiv j_\nu(z)y_\nu(az) - j_\nu(az)y_\nu(z) \sim \mp\frac{i}{\pi}z\sqrt{a}e^{\mp iz(1-a)}\left[1+\mathcal{O}(1/z)\right]. \qquad (5.12)$$

We now add and subtract the logarithm of the RHS of this expression to the contour integral above, which yields

$$\begin{aligned}\sum_n x_n^{-s} &= \frac{s}{2\pi i}\sum_{C_\pm}\int_{C_\pm} dz z^{-s-1}\ln\left[\pm\frac{i\pi}{z\sqrt{a}}e^{\pm iz(1-a)}Z_\nu(z,a)\right] \\ &\quad - \frac{s}{2\pi i}\sum_{C_\pm}\int_{C_\pm} dz z^{-s-1}\ln\left[\pm\frac{i\pi}{z\sqrt{a}}e^{\pm iz(1-a)}\right]. \qquad (5.13)\end{aligned}$$

The first line is now defined for Re$s > -1$, while the second is still only defined for Re$s > 1$. However, for the second term in Eq. (5.13), we are free to deform the contour C into a straight line running parallel to the imaginary axis from $z = i\infty + \delta$ to $z = -i\infty + \delta$. The result is

$$\begin{aligned}\sum_n x_n^{-s} &= \frac{s}{2\pi i}\sum_{C_\pm}\int_{C_\pm} dz z^{-s-1}\ln\left[\pm\frac{i\pi}{z\sqrt{a}}e^{\pm iz(1-a)}Z_\nu(z,a)\right] \\ &\quad - \frac{s}{\pi}\left[\frac{2(1-a)}{(1-s)}\delta^{1-s} + \frac{\pi}{2s}\delta^{-s}\right]. \qquad (5.14)\end{aligned}$$

Since the second line of this equation provides its own analytic continuation, we can now extend the definition of the sum on the LHS to $-1 < \text{Re}s < 0$. In this region, it is safe to take the limit $\delta \to 0$. Then the second term above vanishes. To evaluate the piece left over, we can take the straight line contour along the imaginary axis. The result, valid for $-1 < \text{Re}s < 0$, is

$$\sum_n x_n^{-s} = \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\int_0^\infty dt\, t^{-s-1}\ln\left[\frac{2}{t\sqrt{a}}e^{-t(1-a)}\left\{k_\nu(t)i_\nu(at) - k_\nu(at)i_\nu(t)\right\}\right], \qquad (5.15)$$

where $i_\nu(t) = (2-\alpha)I_\nu(t) + tI'_\nu(t)$, and $k_\nu(t)$ is defined in the same way with $K_\nu(t)$ instead of $I_\nu(t)$.

Eq. (5.15) still needs to be extended to a neighborhood of $s=-4$. For $s=-4+\epsilon$, it can be written as

$$\sum_n x_n^{-s} = -2\epsilon\Bigg\{\int_0^\infty dt\, t^{3+\epsilon}\ln\left[1-\frac{k_\nu(t)i_\nu(at)}{k_\nu(at)i_\nu(t)}\right] + \int_0^\infty dt\, t^{3+\epsilon}\ln\left[\sqrt{\frac{2\pi}{t}}e^{-t}i_\nu(t)\right] + \frac{1}{a^{4-\epsilon}}\int_0^\infty dt\, t^{3+\epsilon}\ln\left[-\sqrt{\frac{2}{\pi t}}e^{t}k_\nu(t)\right]\Bigg\}. \quad (5.16)$$

Because of the overall factor of $a^{4-\epsilon}$ in Eq. (5.8), the second two terms in this expression yield contributions that go as $a^{4-\epsilon}$ or independent of a respectively. The term that is independent of a can be absorbed into the renormalization of the Planck brane tension. As we discussed in the case of TeV brane fields, $a^{4-\epsilon}$ can also be cancelled by a local counterterm. The first term in the brackets is well defined at $s=-4$. Taking the limit $\epsilon \to 0$, we end up with

$$V = V_h + V_v a^4 + \frac{k^4a^4}{16\pi^2}\int_0^\infty dt\, t^3 \ln\left[1-\frac{k_\nu(t)i_\nu(at)}{k_\nu(at)i_\nu(t)}\right], \quad (5.17)$$

where $V_{h,v}$ are shifts in the brane tensions. For $a \ll 1$, the a dependence in the above equation is

$$\int_0^\infty dt\, t^3 \ln\left[1-\frac{k_\nu(t)i_\nu(at)}{k_\nu(at)i_\nu(t)}\right] = \frac{2}{\nu\Gamma(\nu)^2}\left(\frac{\nu-\alpha+2}{\alpha+\nu-2}\right)\left(\frac{a}{2}\right)^{2\nu}\int_0^\infty dt\, t^{2\nu+3}\frac{k_\nu(t)}{i_\nu(t)} + \cdots, \quad (5.18)$$

if $\alpha+\nu \neq 2$ (terms of order $a^{2\nu+2}$ and higher are neglected). For $\alpha+\nu=2$

$$\int_0^\infty dt\, t^3 \ln\left[1-\frac{k_\nu(t)i_\nu(at)}{k_\nu(at)i_\nu(t)}\right] = \frac{2(\nu-1)}{\Gamma(\nu)^2}\left(\frac{a}{2}\right)^{2\nu-2}\int_0^\infty dt\, t^{2\nu+1}\frac{k_\nu(t)}{i_\nu(t)} + \ldots, \quad (5.19)$$

with terms of order $a^{2\nu}$ for $\nu \neq 2$ and $a^4 \ln a$ for $\nu = 2$ not shown. Incidentally, the eigenvalues x_n for bulk fields of higher integer spin satisfy equations that are identical to that of the bulk scalar except for the values of ν and α [46]. It follows immediately that in those cases the a dependence is similar to that in Eq. (5.17). Furthermore, we can use the scalar result to calculate the effective potential in these cases as well. For instance, the contribution from a bulk $U(1)$ gauge field can be obtained by taking $\nu = 1$ and $\alpha = 1$. To calculate the effective potential due to transverse traceless metric fluctuations (corresponding to Kaluza-Klein modes of the graviton), use Eq. (5.17) with $\nu = 2$ and $\alpha = 0$ for each of the five graviton polarization states.

5.3 One-loop radion effects

In addition to contributions from fields in the bulk and on the TeV brane, the vacuum energy receives corrections from loops of the radion itself. First, we compute this for the following radion effective Lagrangian,

$$\mathcal{L} = \frac{f^2}{2}(\partial a)^2 - \delta V_v a^4, \tag{5.20}$$

where δV_v is a small classical shift in the TeV brane tension relative to the value which generates the background metric. The one-loop effective potential generated by the radion is

$$\begin{aligned} V &= \frac{1}{2}\int^{\Lambda a} \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + \hat{m}^2 a^2} \\ &= \frac{a^4}{32\pi^2}\left[2\Lambda^4 + \Lambda^2\hat{m}^2 + \hat{m}^4 \ln\left(\frac{\hat{m}}{2\Lambda}\right)\right], \end{aligned} \tag{5.21}$$

where $\hat{m}^2 = 12\delta V_v/f^2$. Note that as in the case of TeV brane fields, we have used an a-dependent cutoff on the momentum integral. It is not immediately clear that this is the correct cutoff to use in the dimensionally reduced theory, which provides an effective description of the physics at energy scales for which the fifth dimension cannot be resolved[2]. However, had we not used the rescaled cutoff, we would have obtained cutoff-dependent terms that are proportional to a^2 in the effective potential.

[2]Although perhaps this is not so surprising, since as we saw in the previous chapter, the radion has mass and couplings in the TeV range and therefore behaves similarly to a TeV brane field. It makes sense that we have to regulate loops involving the radion in the same way that we regulate loop effects of fields on the TeV brane.

No counterterm exists to absorb such terms. On the other hand, the rescaled cutoff yields $V \propto a^4$, which is a pure counterterm that can be absorbed into the TeV brane tension.

We can also see this result using dimensional regularization. The dimensionally reduced theory becomes

$$\mathcal{L} = \frac{f_n^{n-2}}{2}\left(\partial a^{\frac{n-2}{2}}\right)^2 - \delta V_v \mu^{n-4} a^n, \tag{5.22}$$

where f_n has dimensions of mass. Introducing a canonically normalized radion field $\varphi = (f_n a)^{(n-2)/2}$, the vacuum energy scales as

$$V \sim \left(\frac{\delta V_v \mu^{n-4}}{f_n^n}\right)^{n/2} \varphi^{\frac{2n}{n-2}} = \left(\frac{\delta V_v \mu^{n-4}}{f_n^{n-2}}\right)^{n/2} a^n, \tag{5.23}$$

which simply renormalizes the TeV brane tension. One could also use a Pauli-Villars regulator. To avoid cutoff-dependent terms that cannot be absorbed into counterterms, the regulator masses should scale with a in the same way as masses on the TeV brane. In this case, the resulting effective potential is proportional to a^4 in agreement with the two other methods described here. Finally, if we repeat the calculation of Eq. (5.23) using the stabilizing potential of the previous chapter as the classical background, we generate terms at the one-loop level that either renormalize the parameters of the classical potential or that do not affect its qualitative behavior.

Chapter 6 Renormalization group flows for brane couplings

In this chapter, we consider the structure of ultraviolet divergences for field theories in the presence of branes, such as the brane scenarios motivated by the hierarchy problem that we discussed in Chapter 1. The computation of loop corrections to the effective action for such theories has been studied in [62, 63], where it has been noted that quantum effects generate localized ultraviolet divergences that must be renormalized by field theory operators on the branes (other work on renormalization of field theory on singular spaces can be found in [64]). These ultraviolet divergences arise because, in the limit of large tension, the branes break translation invariance and, therefore, lead to nonconservation of transverse momentum.

We now consider another source of brane localized short distance divergences which come up in the renormalization of brane models. These divergences, which arise in the limit of zero brane thickness, require renormalization even at the classical level. They signify a breakdown of the field theory at scales at which the finite thickness of the brane cannot be neglected, and are analogous to the singularities of classical field theory, such as the ones that are found in classical electrodynamics with point sources. While these singularities appear on brane backgrounds of codimension greater than one, they lead to particularly interesting classical effects for codimension two, since in this case the divergences are logarithmic, and therefore give rise to nontrivial renormalization group (RG) flows.

To illustrate these effects, we consider a specific toy model in six dimensions in the vicinity of a 3-brane. Within the context of this model, we show how to systematically account for the zero thickness classical divergences by using the usual regularization and renormalization procedure of quantum field theory (the necessity for renormalization of classical field theories with singular sources has been pointed out in [65]). We also construct the tree level RG equations for the brane localized couplings, as well as the one-loop corrections induced by the same type of bulk-to-brane renormalization effects considered in [62, 63, 64].

6.1 The Model

We consider Euclidean scalar field theory in a six-dimensional flat space with a 3-brane. The metric is taken to have a conical singularity:

$$ds^2 = \delta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 d\theta^2, \tag{6.1}$$

where the brane is located at $r = 0$, $0 \leq \theta < 2\pi\alpha$, with $\alpha \leq 1$, and x^μ, with $\mu = 0, \ldots 3$ are flat space coordinates parallel to the brane. If gravity is included, then α is related to the brane tension [66]. Our scalar field theory is given by

$$S = \int d^6x \sqrt{g} \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{g_4}{4!}\phi^4 + \cdots \right] + \int d^4x \sum_{n=0}^{\infty} \frac{\lambda_{2n}}{(2n)!} \phi^{2n}, \tag{6.2}$$

where $\cdots$ denotes a series of ϕ^{2n} bulk couplings and the second term includes brane localized interactions, such as a brane tension λ_0 and a brane mass λ_2. As discussed in [63], such terms must be included as counterterms for bulk-to-brane ultraviolet divergences that arise in the computation of loops with insertion of bulk interactions.

If the brane is dynamical, the scalar ϕ will also couple to a set of Goldstone fields localized at $r = 0$ that arise due to the breaking of translation invariance by the presence of the brane. For simplicity, we will consider only the limit of large brane tension. In this limit, the brane is rigid, so the backreaction on the fluctuations of the brane can be neglected, and the couplings of our scalar to the Goldstone modes are suppressed. Note that for a cone deficit angle of π, ($\alpha = 1/2$), the conical singularity can be interpreted as a Z_2 orbifold fixed point. On the orbifold, the fluctuations of the brane are projected out due to the Z_2 symmetry.

We will treat the bulk mass as well as the brane localized coupling λ_2 as small

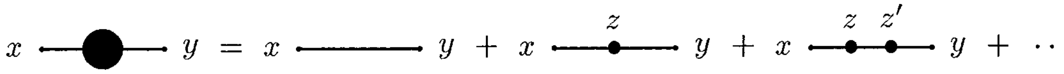


Figure 6.1: Brane mass corrections to the scalar propagator. A • denotes an insertion of the coupling λ_2.

perturbations. Then the scalar propagator is given by the solution of

$$\Box_x D(x,x') = -\delta^4(x^\mu - x'^\mu)\frac{\delta(r-r')\delta(\theta-\theta')}{r}. \qquad (6.3)$$

Using standard techniques, one finds

$$D(x,x') = \sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\int_0^\infty\frac{qdq}{2\pi\alpha}\frac{e^{ik_\mu(x^\mu-x'^\mu)}e^{in(\theta-\theta')/\alpha}}{k_4^2+q^2}J_{|n/\alpha|}(qr)J_{|n/\alpha|}(qr'), \qquad (6.4)$$

where k_4^2 is shorthand for $\delta_{\mu\nu}k^\mu k^\nu$, and J_ν is a Bessel function of the first kind. It is easy to check that for vanishing deficit angle ($\alpha = 1$), this formula recovers the usual scalar propagator in six dimensions (see Appendix A).

6.2 Classical RG equations

Consider now the renormalization of the brane-localized couplings appearing in Eq. (6.2). Besides the loop bulk-to-brane ultraviolet divergences of [63, 64], there are also divergences at the classical level. This can be seen by computing the tree level Green's functions for this theory. For example, let us compute the corrections to the scalar propagator from inclusion of the brane mass term. Summing the diagrams of Fig. (1) yields

$$\begin{aligned} G^{(2)}(x,y) &= D(x,y) - \lambda_2\int d^6z D(x,z)\delta^2(\vec{z})D(z,y) \\ &\quad + \lambda_2^2\int d^6z d^6z' D(x,z)\delta^2(\vec{z})D(z,z')\delta^2(\vec{z}')D(z',y) + \cdots, \end{aligned} \qquad (6.5)$$

where we denote coordinates transverse to the brane by a two-dimensional vector. We will find it convenient to work in four-dimensional momentum space. Introducing the Fourier transform of Eq. (6.4) along four-dimensional momentum,

$$D_k(\vec{x},\vec{x}') = \sum_{n=-\infty}^{\infty} \int_0^{\infty} \frac{qdq}{2\pi\alpha} \frac{e^{in(\theta-\theta')/\alpha}}{q^2+k_4^2} J_{|n/\alpha|}(qr) J_{|n/\alpha|}(qr'), \tag{6.6}$$

Eq. (6.5) becomes

$$\begin{aligned} G_k^{(2)}(\vec{x},\vec{y}) &= D_k(\vec{x},\vec{y}) - \lambda_2 D_k(\vec{x},0) D_k(0,\vec{y}) + \lambda_2^2 D_k(\vec{x},0) D_k(0,0) D_k(0,\vec{y}) + \cdots \\ &= D_k(\vec{x},\vec{y}) - \frac{\lambda_2}{1+\lambda_2 D_k(0,0)} D_k(\vec{x},0) D_k(0,\vec{y}). \end{aligned} \tag{6.7}$$

In this representation for the Green's functions, momentum parallel to the brane is conserved at each vertex for a brane localized interaction. However, due to the delta function at $r = 0$, momentum transverse to the brane is not conserved and must be integrated over each internal line for the graphs in Fig. (1). In Eq. (6.7), this integration over two-dimensional momentum appears first at $\mathcal{O}(\lambda_2^2)$ and leads to the factor of $D_k(0,0)$, which is ultraviolet divergent. We emphasize that this tree level divergence is not an artifact of our large tension limit, in which momentum appears not to be conserved due to the resistance of the brane to changes in its configuration. Rather, it arises because we have also taken the limit in which our brane is represented by a delta function, i.e., it is infinitely thin. In reality, the brane has internal structure at short distance, and the divergence we encounter simply reflects the fact that the field theory we wrote down in Eq. (6.2) is not a valid description of the physics at these scales.

These divergences are no different than the types of singularities that arise, for instance, in classical electrodynamics with point sources. They can be dealt with in the same manner as the ultraviolet divergences that appear in quantum field theory, by introducing a regulator and absorbing the regulator dependence into renormalized couplings in such a way that physical quantities are regulator independent. While the divergences described here appear on spaces with branes of codimension greater than one, they are particularly interesting for field theories on codimension two branes, such as the scalar model that we are considering here. For codimension two, the divergences are logarithmic, leading to running couplings and RG flow even at the classical level. To see this, regulate $D_k(0,0)$ with a momentum cutoff[1] Λ and interpret the coupling λ_2 appearing in the above series as a cutoff-dependent bare coupling $\lambda_2(\Lambda)$. Introducing a renormalized coupling $\lambda_2(\mu) = \lambda_2(\Lambda)/Z_2$ that depends on a subtraction point μ, and using[2]

$$D_k(0,0) = \frac{1}{4\pi\alpha} \ln\left(\frac{\Lambda^2}{k_4^2}\right), \tag{6.8}$$

we find

$$G_k^{(2)}(\vec{x},\vec{y}) = D_k(\vec{x},\vec{y}) - \frac{\lambda_2(\mu)}{1 - (\lambda_2(\mu)/4\pi\alpha)\ln(k_4^2/\mu^2)} D_k(\vec{x},0) D_k(0,\vec{y}) \tag{6.9}$$

provided that we adjust

$$Z_2 = \frac{1}{1 - (\lambda_2(\mu)/2\pi\alpha)\ln(\Lambda/\mu)}, \tag{6.10}$$

which corresponds to a scheme in which only powers of $\ln(\Lambda/\mu)$ are subtracted. Therefore, at $\mathcal{O}(\hbar^0)$ we have

$$\mu \frac{d\lambda_2}{d\mu} = \frac{\lambda_2^2}{2\pi\alpha}, \tag{6.11}$$

with solution

$$\lambda_2(\mu) = \frac{\lambda_2(\mu_0)}{1 - (\lambda_2(\mu_0)/2\pi\alpha)\ln(\mu/\mu_0)}. \tag{6.12}$$

In six dimensions $[\phi] = 2$ and, therefore, λ_2 is a dimensionless coupling. Eq. (6.12) indicates that for positive λ_2 this coupling increases in the ultraviolet, reaching a Landau singularity at $\mu = \mu_0 \exp(2\pi\alpha/\lambda_2(\mu_0))$. A derivation of Eq. (6.11) based on regulating the solution of the classical field equations derived from Eq. (6.2) can be

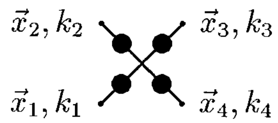
[1] It is straightforward to use dimensional regularization instead of a momentum cutoff. This would be necessary in a more realistic theory in which gauge fields or gravity are included.

[2] Because $J_0(0) = 1$ and $J_\nu(0) = 0$ for $\nu > 0$, only one term in the sum of Eq. (6.6) contributes to $D_k(0,0)$. Terms suppressed by inverse powers of Λ have been neglected.

found in Appendix B. A similar RG equation for a scalar mass term localized on a singular surface has been obtained in [67].

The short distance divergences that arise in the computation of the tree level two-point function also appear at tree level in other Green's functions. For example, the tree-level four-point function, which can be evaluated to all orders in λ_2, is given by

$$G^{(4)}_{k_1\cdots k_4}(\vec{x}_1\cdots\vec{x}_4) =$$



$$+ \cdots \tag{6.13}$$

$$= -\lambda_4(\Lambda)(2\pi)^4\delta^4(\sum_i k_i)\prod_{i=1}^{4} D_{k_i}(\vec{x}_i,0)\left[1-\frac{\lambda_2(\mu)D_{k_i}(0,0)}{1-(\lambda_2(\mu)/4\pi\alpha)\ln(k_{i4}^2/\mu^2)}\right]$$

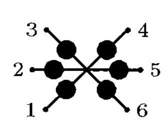
$$= -\lambda_4(\Lambda)Z_2^{-4}(2\pi)^4\delta^4(\sum_i k_i)\prod_{i=1}^{4}\frac{D_{k_i}(\vec{x}_i,0)}{1-(\lambda_2(\mu)/4\pi\alpha)\ln(k_{i4}^2/\mu^2)}.$$

We define the renormalized coupling $\lambda_4(\mu)$ by $\lambda_4(\Lambda) = Z_4\lambda_4(\mu)$ and adjust (in the same scheme used to renormalize the two-point function) $Z_4 = Z_2^4$. Then the four-point function is cutoff independent and

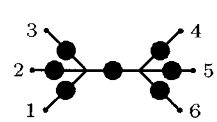
$$\mu\frac{d\lambda_4}{d\mu} = \frac{4\lambda_4\lambda_2}{2\pi\alpha}. \tag{6.14}$$

Similarly, the six-point function is given by

$$G^{(6)}_{k_1\cdots k_6}(\vec{x}_1\cdots\vec{x}_6) =$$



$$+$$



$$+ \text{perms.} + \cdots \tag{6.15}$$

$$= (2\pi)^4 Z_2^{-6}\delta^4(\sum_i k_i)\prod_{i=1}^{6}\frac{D_{k_i}(\vec{x}_i,0)}{1-(\lambda_2(\mu)/4\pi\alpha)\ln(k_{i4}^2/\mu^2)}$$

$$\times\left[-\lambda_6(\Lambda)+\lambda_4(\Lambda)^2\sum_q\frac{Z_2^{-1}D_q(0,0)}{1-(\lambda_2(\mu)/4\pi\alpha)\ln(q_4^2/\mu^2)}\right]$$

$$= (2\pi)^4\delta^4(\sum_i k_i)\prod_{i=1}^{6}\frac{D_{k_i}(\vec{x}_i,0)}{1-(\lambda_2(\mu)/4\pi\alpha)\ln(k_{i4}^2/\mu^2)}$$

$$\times \left[-\lambda_6(\mu) - \lambda_4(\mu)^2 \sum_q \frac{\ln(q_4^2/\mu^2)\,/4\pi\alpha}{1 - (\lambda_2(\mu)/4\pi\alpha)\ln(q_4^2/\mu^2)} \right],$$

where q is the four-momentum going through the internal line in the second graph in the figure, which is fixed in terms of the external momenta. The sum over q is over the momentum in this graph as well as the other nine permutations not shown in the figure. In this expression, the renormalized coupling $\lambda_6(\mu)$ is related to the bare coupling by $\lambda_6(\Lambda) = Z_6\lambda_6(\mu)$, with

$$Z_6 = Z_2^6 + \binom{5}{2} \frac{\lambda_4(\mu)^2}{2\pi\alpha\lambda_6(\mu)} Z_2^7 \ln\left(\frac{\Lambda}{\mu}\right), \tag{6.16}$$

leading to the RG equation for λ_6

$$\mu\frac{d\lambda_6}{d\mu} = \frac{6\lambda_6\lambda_2}{2\pi\alpha} + \binom{5}{2}\frac{\lambda_4^2}{2\pi\alpha}. \tag{6.17}$$

In this and the previous examples, the beta functions, computed to all orders in λ_2, coincide with those that would have been obtained by keeping only terms with two vertex insertions in the expansions for the Green's functions. This is because such graphs are the only sources of tree level divergences that are single powers of $\ln\Lambda$. Knowledge of the exact $\mathcal{O}(\hbar^0)$ coefficient of this log then determines the tree level beta functions to all orders [68]. We can also immediately write the full tree level beta functions for the other brane couplings appearing in Eq. (6.2). Keeping only terms with divergences that are single powers of $\ln\Lambda$, the Green's functions are:

$$G^{(4k)} \sim -\lambda_{4k}(\Lambda) + \sum_{j=1}^{k} \binom{4k}{2j-1} \frac{\lambda_{2j}\lambda_{4k-2j+2}}{2\pi\alpha} \ln\left(\frac{\Lambda}{\mu}\right) + \cdots, \tag{6.18}$$

$$\begin{aligned} G^{(4k+2)} \sim & -\lambda_{4k+2}(\Lambda) + \sum_{j=1}^{k} \binom{4k+2}{2j-1} \frac{\lambda_{2j}\lambda_{4k-2j+4}}{2\pi\alpha} \ln\left(\frac{\Lambda}{\mu}\right) \\ & + \binom{4k+2}{2k+1} \frac{\lambda_{2k+2}^2}{4\pi\alpha} \ln\left(\frac{\Lambda}{\mu}\right) + \cdots, \end{aligned} \tag{6.19}$$

where in the first line k runs over $k = 1, 2, \cdots$ and in the second equation $k = 0, 1, \cdots$.

The combinatoric factors count the number of distinct ways of assigning momentum labels to the external lines in the graphs. Note that for λ_{4k+2}, the combinatorics is slightly different due to the possibility of having a graph with two factors of λ_{2k+2}. Introducing the renormalized couplings $\lambda_{2n}(\Lambda) = Z_{2n}\lambda_{2n}(\mu)$, and choosing Z_{2n} to cancel the logs of Λ, we find

$$\mu\frac{d\lambda_{4k}}{d\mu} = \sum_{j=1}^{k}\binom{4k}{2j-1}\frac{\lambda_{2j}\lambda_{4k-2j+2}}{2\pi\alpha}, \tag{6.20}$$

$$\mu\frac{d\lambda_{4k+2}}{d\mu} = \sum_{j=1}^{k}\binom{4k+2}{2j-1}\frac{\lambda_{2j}\lambda_{4k-2j+4}}{2\pi\alpha} + \binom{4k+2}{2k+1}\frac{\lambda_{2k+2}^2}{4\pi\alpha}. \tag{6.21}$$

Because the equation for λ_{2n} only involves the couplings λ_{2m} with $m \leq n$, it can be easily solved by iteration. Given the solution for $\lambda_2(\mu)$ we construct the RG flow for $\lambda_4(\mu)$ by noting from Eq. (6.14) that $\lambda_4(\mu)\lambda_2(\mu)^{-4}$ is an RG invariant. Then the equation for λ_6 can be written as

$$\frac{d}{d\lambda_2}(\lambda_6\lambda_2^{-6}) = \binom{5}{2}\left(\frac{\lambda_4}{\lambda_2^4}\right)^2 \tag{6.22}$$

so that

$$\lambda_6(\mu)\lambda_2(\mu)^{-6} = \lambda_6(\mu_0)\lambda_2(\mu_0)^{-6} + 10\left(\frac{\lambda_4(\mu)}{\lambda_2(\mu)^4}\right)^2(\lambda_2(\mu) - \lambda_2(\mu_0)), \tag{6.23}$$

and similarly for larger n.

6.3 One-Loop Corrections

Besides the tree level RG flows just considered, there are other corrections to the brane beta functions due to loop effects involving insertions of both the bulk and brane couplings. Loop diagrams with only brane couplings cannot give rise to any further logarithmic divergences than those obtained already at tree level. To see this, note that an L loop diagram contributing to the renormalization of the λ_{2n} vertex with N_{2m} insertions of λ_{2m} vertices ($m = 1\ldots\infty$) is proportional to a product of coupling constants that has mass dimension

$$d = \sum_m N_{2m}[\lambda_{2m}], \tag{6.24}$$

where $[\lambda_{2m}] = 4 - 4m$. Using the relations

$$L = I - \sum_m N_{2m} + 1, \tag{6.25}$$

$$2I = \sum_m 2m N_{2m} - E, \tag{6.26}$$

with I the number of internal lines and $E = 2n$ the number of external lines, we see that $d = 4 - 4n - 4L$. To get a contribution to the beta function, this diagram needs to be logarithmically divergent. This occurs when $d = [\lambda_{2n}] = 4 - 4n$, which happens precisely at tree level. Thus to obtain loop corrections to the RG equations one must include insertions of the bulk couplings in loops. We now turn to the calculation of some of these bulk-to-brane renormalization effects.

For simplicity, we will consider the field theory on a space with deficit angle[3] π, i.e., $\alpha = 1/2$. Then the scalar propagator simplifies to a sum of two terms, the usual scalar propagator and an image charge contribution (see Appendix A)

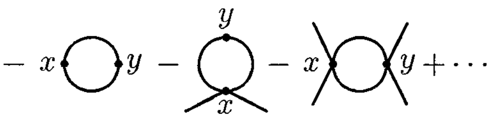
$$D(x, x') = \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2} e^{ik_\mu (x-x')^\mu} \left(e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} + e^{i\vec{k}\cdot(\vec{x}+\vec{x}')} \right). \tag{6.27}$$

In order to see what types of divergences arise from loops with insertions of bulk couplings, we compute one-loop quantum corrections to the effective action. We will consider only the effects of the bulk ϕ^2 and ϕ^4 couplings. Inclusion of the higher powers of ϕ is straightforward. For a term in the effective action to give a logarithmically divergent contribution to the renormalization of λ_{2n}, it must be constructed from insertions of bulk couplings whose product has mass dimension $[\lambda_{2n}]$. Then at one-loop, the relevant terms (i.e., the terms that diverge like a single power of $\ln \Lambda$ and

[3]This space can also be thought of as a Z_2 orbifold.

therefore contribute to the RG equations) are

$$S_{eff} = S_{cl} - \ldots \qquad (6.28)$$



$$= S_{cl} - \frac{m^4}{4}\int d^6x d^6y D(x,y)^2 - \frac{1}{2!}\frac{g_4 m^2}{2}\int d^6x d^6y D(x,y)^2\phi(x)^2$$
$$- \frac{1}{4!}\frac{3g_4^2}{2}\int d^6x d^6y \phi(x)^2 D(x,y)^2\phi(y)^2 + \cdots,$$

where an external line going into a vertex at x denotes an insertion of $\phi(x)$. In this expression, the second, third, and fourth terms contribute to the brane tension λ_0, the brane mass λ_2 and the coupling λ_4, respectively. Since we are only after the counterterms for the brane ϕ^{2n} couplings, we can take $\phi(x) =$ constant. Then all the integrals in Eq. (6.28) are identical

$$\begin{aligned}
\int d^6x d^6y D(x,y)^2 &= \int d^4x \int \frac{d^6k}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{1}{k^2}\frac{1}{q^2}(2\pi)^4\delta^4(k-q) \qquad (6.29)\\
&\times \int d^2\vec{x}d^2\vec{y}\left[e^{i\vec{k}\cdot(\vec{x}-\vec{y})} + e^{i\vec{k}\cdot(\vec{x}+\vec{y})}\right]\left[e^{i\vec{q}\cdot(\vec{x}-\vec{y})} + e^{i\vec{q}\cdot(\vec{x}+\vec{y})}\right]\\
&= \frac{1}{4}\cdot 2\int d^4x \int \frac{d^4k}{(2\pi)^4}\frac{d^2\vec{k}}{(2\pi)^2}\frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{k_4^2+\vec{k}^2}\frac{1}{k_4^2+\vec{q}^2}\\
&\times \left[2(2\pi)^2\delta^2(\vec{k}+\vec{q})\int d^2\vec{x} + (2\pi)^2\delta^2(\vec{k}+\vec{q})(2\pi)^2\delta^2(\vec{k}-\vec{q})\right],
\end{aligned}$$

where the factor of $1/4$ in the second line reflects the fact that both integrals over two-dimensional position run over only the half-plane. Regulating the four-dimensional and two-dimensional momentum integrals with an ultraviolet cutoff[4] Λ,

$$\int d^6x d^6y D(x,y)^2 = \int d^4x \frac{1}{64\pi^2}\ln\left(\frac{\Lambda}{\mu}\right) + \int d^6x \frac{\ln 2}{64\pi^3}\Lambda^2, \qquad (6.30)$$

where we have introduced a subtraction scale μ. Note that the brane-localized ultraviolet divergences encountered here are different in nature than the classical singu-

[4]Here, we choose the same cutoff Λ that we used in the previous section to regulate $D_k(0,0)$. This choice reflects the assumption that both the zero thickness and the large tension divergences are resolved by physics at similar scales, of order the ultraviolet cutoff Λ.

larities discussed in the previous section. In this case, the divergence proportional to four-dimensional volume arises because the brane at $r = 0$ induces a spacetime geometry that breaks translation invariance and leads to nonconservation of momentum transverse to the brane. This is taken into account in the above calculation by the inclusion of the image term in the scalar propagator. It is the cross term between the ordinary scalar propagator and the image term in the second line of Eq. (6.29), which leads to the brane localized logarithm in Eq. (6.30).

Using Eq. (6.30), the effective action becomes

$$\begin{aligned} S_{eff} &= S_{cl} - \frac{m^4}{256\pi^2} \ln\left(\frac{\Lambda}{\mu}\right) \int d^4x - \frac{1}{2!}\frac{g_4 m^2}{128\pi^2} \ln\left(\frac{\Lambda}{\mu}\right) \int d^4x \phi(x^\mu, 0)^2 \\ &\quad - \frac{1}{4!}\frac{3g_4^2}{128\pi^2} \ln\left(\frac{\Lambda}{\mu}\right) \int d^4x \phi(x^\mu, 0)^4 + \cdots . \end{aligned} \tag{6.31}$$

The logarithmic divergences in this expression can be absorbed into counterterms appearing in S_{cl}. Using our prescription in which only powers of Λ and $\ln(\Lambda/\mu)$ are subtracted, the RG equations for the brane couplings become

$$\mu \frac{d\lambda_0}{d\mu} = \frac{m^4}{256\pi^2}, \tag{6.32}$$

$$\mu \frac{d\lambda_2}{d\mu} = \frac{\lambda_2^2}{\pi} + \frac{m^2 g_4}{128\pi^2}, \tag{6.33}$$

$$\mu \frac{d\lambda_4}{d\mu} = \frac{4\lambda_4\lambda_2}{\pi} + \frac{3g_4^2}{128\pi^2}. \tag{6.34}$$

There are also corrections from one-loop diagrams with insertions of both brane and bulk couplings. First, consider the renormalization of the tension. At linear order in λ_2 this is given by

$$\begin{aligned} G^{(0)} &= \bullet + \bigcirc + \cdots \\ &= -\lambda_0(\Lambda) - \frac{1}{2}\lambda_2(\Lambda) \int \frac{d^4k}{(2\pi)^4} D_k(0,0) + \cdots , \end{aligned} \tag{6.35}$$

where we have included the effects of the bulk mass in the propagator

$$D_k(0,0) = \frac{1}{2\pi} \ln\left(\frac{\Lambda^2}{k_4^2 + m^2}\right). \tag{6.36}$$

To extract the RG equation for the tension, we need the coefficient of $\ln\Lambda$ in the tension counterterm. We shall use

$$\int \frac{d^4k}{(2\pi)^4} D_k(0,0) = \frac{m^4}{64\pi^3} \ln\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots, \tag{6.37}$$

where finite terms, as well as terms proportional to powers of the cutoff or more powers of $\ln\Lambda$ have been suppressed. Hence

$$G^{(0)} = -Z_0\lambda_0(\mu) - \frac{m^4\lambda_2}{128\pi^3} \ln\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots. \tag{6.38}$$

Therefore, the one-loop beta function for the tension at $\mathcal{O}(\lambda_2)$ becomes

$$\mu\frac{d\lambda_0}{d\mu} = \frac{m^4}{256\pi^2} - \frac{m^4\lambda_2}{64\pi^3}. \tag{6.39}$$

We can also include brane couplings in the one-loop renormalization of λ_2 itself. A similar calculation to the one above gives

$$\mu\frac{d\lambda_2}{d\mu} = \frac{\lambda_2^2}{\pi} - \frac{m^4\lambda_4}{64\pi^3} + \frac{m^2 g_4}{128\pi^2}, \tag{6.40}$$

where for the one-loop part, we only included terms linear order in the couplings λ_{2n} and g_4. The pattern is similar for the other λ_{2n} couplings.

Finally, to complete the discussion of the RG flows in this theory, one should also calculate the beta functions for the bulk couplings. Clearly, the brane couplings cannot generate bulk divergences, so these do not contribute. Then the calculation is a standard field theory exercise, which we will not repeat here.

Chapter 7 Conclusions

In this thesis, we have studied some applications of field theory to higher-dimensional brane scenarios motivated by the gauge hierarchy problem. In Chapter 2, we gave a detailed presentation of the proposal of [12] to address the hierarchy problem with a nonfactorizable five-dimensional geometry. We discussed the low energy four-dimensional gravitational dynamics of the model, which includes the ordinary graviton and an additional massless scalar, the radion. The radion plays a crucial role in generating the hierarchy between the Planck scale and the weak scale. However, because in this minimal model the radion has no potential, additional dynamics must be specified in order to stabilize the weak scale.

In Chapter 3, we showed that in the RS model, bulk scalars have low-lying Kaluza-Klein modes with four-dimensional masses of order the weak scale and four-dimensional non-renormalizable interactions suppressed by powers of the weak scale, even though from a five-dimensional perspective their masses and interactions are characterized by the Planck scale. This is similar to what occurs for the fields localized on the 3-brane at $\phi = \pi$. Bulk fields of higher spin exhibit similar patterns. The structure of the Kaluza-Klein reduction for bulk fields leads to interesting phenomenology, including the possibility of observing strongly coupled graviton resonances at collider experiments near the TeV scale.

In Chapter 4, we argued that a bulk scalar with a ϕ-dependent VEV can generate a potential to stabilize the radion without having to fine-tune the parameters of the model (there is still one fine tuning associated with the four-dimensional cosmological constant, however). We also explored some of the physical properties of the modulus field which arises in the Randall-Sundrum scenario. An important feature is that it couples to visible brane matter with TeV rather than Planck scale strength. In the absence of a mechanism that generates a radion mass, this is unacceptable: radion exchange gives rise to a long-range universal attractive force which is 32 orders of magnitude stronger than gravity. On the other hand, this is not a problem if the radion is stabilized by a mechanism such as that of [41]. In addition, the stabilized radion has a very distinctive phenomenology. It has a mass which is likely to be lighter than Kaluza-Klein modes of bulk fields, making it the first direct signal of the extra dimension. Also, the couplings of the radion to the Standard Model fields are fixed by general covariance and depend on the single parameter $\langle\varphi\rangle$. We expect similar phenomenology to arise in other scenarios, such as that of [69].

In Chapter 5, we have calculated the quantum effective potential for the radion in compactified AdS_5. By explicitly performing the computation using three different regulators, we have shown that fields confined to the TeV brane give no nontrivial contributions to the potential. In particular, we find that in dimensional regularization, the disappearance of any contribution that scales as $a^4 \ln a$ is not due to a rescaling of the regulator mass μ by a factor of a. Instead, it can be traced to the fact that the proper generally covariant counterterm in $n = 4 - \epsilon$ dimensions includes this term. Likewise, general covariance requires that within a cutoff regularization procedure, one should use the rescaled cutoff Λa, leading to $V \propto a^4$.

The contribution due to bulk fields yields a nontrivial dependence on the warp factor a. However, as in the case of confined fields, no terms of the form $a^4 \ln a$ are generated. Beyond the pure counterterm a^4, bulk fields generate terms that are suppressed in the large r_c regime. For instance, a massless bulk field yields terms of the form a^6 as well as the finite log term $a^8 \ln a$. This a dependence is too weak to generate an exponentially small value of a without having to choose unnatural values of the brane tensions. Because of this, a classical stabilization mechanism is needed.

Finally, we have also included the quantum effects of the radion field itself. We found that as in the case of TeV brane fields, the correct momentum space cutoff should be rescaled by a factor of a. While this is quite natural for brane fields, it is not obvious that this had to be so for the radion field, since in the dimensionally reduced theory we have integrated over the fifth dimension, and there is no single preferred "scale" $\exp(-kr_c\phi)$.

In Chapter 6, we have analyzed the types of ultraviolet divergences that appear in field theories with branes. Nondynamical branes break translation invariance, leading to localized divergences at the quantum level. Short distance divergences appear also in the limit of vanishing brane thickness. Such divergences signify a failure of the theory to describe finite thickness effects, and manifest themselves already at the classical level. By looking at a toy model with a 3-brane in six dimensions, we showed how to regulate and renormalize these classical singularities into the parameters of the theory, and derived RG equations (which are only generated in backgrounds with codimension-two branes) for the brane localized couplings. We also computed corrections to these flows from one-loop bulk-to-brane effects. Although we have worked with two noncompact extra dimensions, the divergences we encountered are only sensitive to short distance effects, and hence the RG equations we have derived remain valid if the space is compactified.

The brane-localized divergences considered here may have implications in the context of brane-world scenarios. For example, models with two compact extra dimensions may address the hierarchy problem if the compact space is large [11]. A mechanism for naturally generating a volume exponentially larger than the fundamental scale of the theory in such codimension-two models has been proposed by [42]. This mechanism relies on large logarithms of the ratio of the size of the space over brane thickness induced by the bulk profile of a massless scalar that couples to 3-branes. For a scalar field that is massless and noninteracting in the bulk, our classical RG flows could also play a role in this scenario, since they too generate such logarithms in the infrared.

It may be interesting to see what happens when the brane is taken to be dynamical. One should be able to do this by including the contribution of the localized Goldstone modes associated with the brane in loop corrections to the effective action. Also, it may be worthwhile to examine how a more fundamental description of the brane which includes finite thickness effects resolves the singularities and leads, at long wavelengths, to the classical running couplings described here.

Brane scenarios provide an interesting approach to tackling the hierarchy problem. However, they also have features that are less appealing than the SM with minimal particle content. In brane scenarios, higher dimension operators are suppressed by the weak scale and unlike the Standard Model, where the suppression can be by powers of the GUT scale, there is no explanation for the smallness of neutrino masses and the long proton lifetime based simply on dimensional analysis (although extra-dimensional mechanisms for suppressing phenomenologically dangerous operators have been proposed in [70]). Hopefully, experiments at the LHC will soon tell whether nature employs extra dimensions at the TeV scale, low energy supersymmetry, or perhaps something completely unexpected to solve the puzzle of why gravity is so much weaker than the other forces.

Appendix A The scalar propagator on R^6 and $R^4 \times R^2/Z_2$

A.1 The Scalar Propagator on R^6 and $R^4 \times R^2/Z_2$

In this appendix we show the equivalence of the scalar propagator on cones of deficit angle 0 ($\alpha = 1$) and π ($\alpha = 1/2$) from Eq. (6.4) with the scalar propagator in flat six-dimensional space, and on the orbifold (given in Eq. (6.27) as a sum over images), respectively. We will need the Bessel function identities

$$J_0(z) = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{iz\cos\theta}, \tag{A.1}$$

$$J_0(qR) = \sum_{n=-\infty}^{\infty} e^{in\theta} J_n(qr) J_n(qr'), \tag{A.2}$$

with $R = \sqrt{r^2 + r'^2 - 2rr'\cos\theta}$. For $\alpha = 1$, Eq. (6.4) gives

$$\begin{aligned} D(x,x') &= \int \frac{d^4k}{(2\pi)^4} \int_0^\infty \frac{qdq}{2\pi} \frac{e^{ik_\mu(x^\mu - x'^\mu)}}{k_4^2 + q^2} \sum_n e^{in(\theta-\theta')} J_n(qr) J_n(qr') \qquad \text{(A.3)} \\ &= \int \frac{d^4k}{(2\pi)^4} \int_0^\infty \frac{qdq}{2\pi} \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{e^{ik_\mu(x^\mu - x'^\mu)} e^{iq|\vec{x}-\vec{x}'|\cos\theta}}{k_4^2 + q^2}, \end{aligned}$$

or

$$D(x,x') = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^2\vec{q}}{(2\pi)^2} \frac{e^{ik_\mu(x^\mu - x'^\mu) + i\vec{q}\cdot(\vec{x}-\vec{x}')}}{k_4^2 + q^2}, \tag{A.4}$$

which is precisely the six-dimensional scalar propagator. For $\alpha = 1/2$, Eq. (6.4) gives

$$D(x,x') = \sum_{n\,even} \int \frac{d^4k}{(2\pi)^4} \int_0^\infty \frac{qdq}{\pi} \frac{e^{ik_\mu(x^\mu - x'^\mu)} e^{in(\theta-\theta')}}{k_4^2 + q^2} J_n(qr) J_n(qr'). \tag{A.5}$$

We now write

$$\begin{aligned}
\sum_{n even} e^{in(\theta-\theta')} J_n(qr) J_n(qr') &= \frac{1}{2}\left(\sum_{n even} + \sum_{n odd}\right) e^{in(\theta-\theta')} J_n(qr) J_n(qr') \\
&\quad + \frac{1}{2}\left(\sum_{n even} - \sum_{n odd}\right) e^{in(\theta-\theta')} J_n(qr) J_n(qr') \\
&= \frac{1}{2}\sum_n e^{in(\theta-\theta')} J_n(qr) J_n(qr') + \frac{1}{2}\sum_n e^{in(\theta-\theta'+\pi)} J_n(qr) J_n(qr') \\
&= \frac{1}{2} J_0(q|\vec{x}-\vec{x}'|) + \frac{1}{2} J_0(q|\vec{x}+\vec{x}'|).
\end{aligned} \tag{A.6}$$

Then Eq. (A.5) becomes

$$D(x,x') = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^2\vec{q}}{(2\pi)^2} \frac{e^{ik_\mu(x^\mu - x'^\mu)}}{k_4^2 + q^2} \left(e^{i\vec{q}\cdot(\vec{x}-\vec{x}')} + e^{i\vec{q}\cdot(\vec{x}+\vec{x}')}\right), \tag{A.7}$$

which reproduces Eq. (6.27).

Appendix B Brane mass renormalization from classical field equations

In the Chapter 6, we obtained the RG equation for the brane mass λ_2 by renormalizing divergences in the diagrammatic expansion for the two-point function. We now obtain the same beta function by solving the classical field equations for a free, massless bulk scalar with a brane localized mass terms. This classical problem is also singular and must therefore be regularized. The regulator dependence in the classical solution can be absorbed into a renormalized brane mass, which leads to the same result as Eq. (6.11).

The full two-point function in the mixed representation is given by

$$G_k^{(2)}(\vec{x},\vec{x}') = \sum_n \int_0^\infty \frac{qdq}{k_4^2+q^2}\phi^*_{n,q}(\vec{x}')\phi_{n,q}(\vec{x}) \tag{B.1}$$

where $\phi_{n,q}$ are a set of orthonormal functions which satisfy

$$\left(-\nabla^2_{\vec{x}} + \lambda_2\delta^2(\vec{x})\right)\phi_{n,q}(\vec{x}) = q^2\phi_{n,q}(\vec{x}). \tag{B.2}$$

In polar coordinates, away from $r=0$ the solutions are,

$$\phi_{n,q}(\vec{x}) = \frac{N_n(q)}{\sqrt{2\pi\alpha}}e^{in\theta/\alpha}R_{n,q}(r), \tag{B.3}$$

where $N_n(q)$ is determined by normalization and $R_{n,q}(r)$ are linear combinations of Bessel functions

$$R_{n,q}(r) = J_{|n/\alpha|}(qr) + c_n(q)Y_{|n/\alpha|}(qr), \tag{B.4}$$

We obtain c_n by applying the boundary conditions at $r=0$, which follow from integrating Eq. (B.2) over the interior of the surface $r=\epsilon$. For the $n\neq 0$ modes, this integral vanishes due to rotational invariance. Thus λ_2 has no effect on these modes and we conclude that $c_n = 0$ for $n \neq 0$. For the $n = 0$ mode, we get from Eq. (B.2)

$$-\epsilon \left. \frac{dR_{0,q}}{dr} \right|_{r=\epsilon} + \frac{\lambda_2}{2\pi\alpha} \int_0^\epsilon dr \delta(r) R_{0,q}(r) - q^2 \int_0^\epsilon r dr R_{0,q}(r) = 0, \tag{B.5}$$

where we used $\delta^2(\vec{x}) = \delta(r)/2\pi\alpha r$. The second term in this equation is singular as $\epsilon \to 0$, due to the singularity of Y_0 at the origin. We will handle this singularity by regulating the delta function:

$$\delta(r) = \frac{1}{\delta} \left[1 - \theta(r - \delta) \right], \tag{B.6}$$

with $\delta \to 0$. Using the asymptotic form for the Bessel functions near $r = 0$,

$$R_{0,q}(r) \simeq 1 + \frac{2c_0(q)}{\pi} \left[\gamma + \ln \left(\frac{qr}{2} \right) \right], \tag{B.7}$$

we find that the third term in Eq. (B.5) vanishes as $\epsilon \to 0$. On the other hand,

$$\epsilon \left. \frac{dR_0}{dr} \right|_{r=\epsilon} = \frac{2c_0(q)}{\pi} \tag{B.8}$$

and using the regulated form for the delta function (with $\epsilon > \delta$),

$$\begin{aligned} \int_0^\epsilon dr \delta(r) R_0(r) &= \frac{1}{\delta} \int_0^\delta dr R_0(r) \\ &= 1 + \frac{2c_0(q)}{\pi} \left[\gamma - 1 + \ln \left(\frac{q\delta}{2} \right) \right]. \end{aligned} \tag{B.9}$$

Then Eq. (B.5) gives

$$\frac{2c_0(q)}{\pi} = \frac{\lambda_2/(2\pi\alpha)}{1 - (\lambda_2/2\pi\alpha)\ln(q/\Lambda)}, \tag{B.10}$$

where $1/\Lambda = e^{\gamma-1}\delta/2$. As in the text, the dependence of $c_0(q)$ on the regulator can be removed by interpreting λ_2 as a bare coupling $\lambda_2(\Lambda)$ and introducing a renormalized parameter $\lambda_2(\mu) = \lambda_2(\Lambda)/Z_2$. In terms of $\lambda_2(\mu)$,

$$\frac{2c_0(q)}{\pi} = \frac{\lambda_2(\mu)/(2\pi\alpha)}{1 - (\lambda_2(\mu)/2\pi\alpha)\ln(q/\mu)}, \tag{B.11}$$

provided that $1/Z_2 = 1 - (\lambda_2(\mu)/2\pi\alpha)\ln(\Lambda/\mu)$. This gives

$$\mu\frac{d\lambda_2}{d\mu} = \frac{\lambda_2^2}{2\pi\alpha}, \tag{B.12}$$

in agreement with the diagrammatic approach.

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