

# An Experimental and Numerical Study of Normal Particle Collisions in a Viscous Liquid

Thesis by

Xiaobai Li

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*To my grandfather, Chao Hong.*

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# Abstract

When two solid bodies collide in a liquid environment, the collision process is influenced by viscous effects and the increased pressure in the interstitial liquid layer between the two solid boundaries. A normal collision process is investigated for a range of impact Stokes numbers using both experimental and numerical methods. Experiments of a steel sphere falling under gravity and colliding with a Zerodur wall with Stokes number ranging from 5 to 100 are performed, which complement previous investigations of immersed particle-wall collision processes. The incompressible Navier-Stokes equations are solved numerically to predict the coupled motion of the falling particle and the surrounding fluid as the particle impacts and rebounds from the planar wall. The numerical method is validated by comparing the numerical simulations of a settling sphere with experimental measurements of the sphere trajectory and the accompanying flow-field. A contact model of the liquid-solid and solid-solid interaction is developed that incorporates the elasticity of the solids to permit the rebound trajectory to be simulated accurately. The contact model is applied when the particle is sufficiently close to the wall that it becomes difficult to resolve the thin lubrication layer. The model is calibrated with measured particle trajectories and is found to represent well the observed coefficient of restitution over a range of impact Stokes numbers from 1 to 1000. In addition, the model is modified to simulate the normal collision of two spheres. The effective coefficient of restitution obtained from the simulation shows a strong dependence on the binary Stokes number accordant with other researcher's experimental results. The unique behaviors of the two spheres at low binary Stokes number including target motion prior to contact and group motion after collision are simulated by the current work.

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# Chapter 1

## Introduction

### 1.1 Background and Motivation

Liquid-solid flows are involved in a wide variety of geophysical and industrial processes, including nearshore sediment transport, debris surges and landslides, the handling of dredging slurries and hydraulic fracture technologies (see [Crowe \*et al.\*, 1998a](#); [Lorenzini & Mazza, 2004](#)). These flows are heterogeneous and the macroscopic behavior of the mixture is not adequately described by continuum theories. The rheology of liquid-solid mixtures depends on many factors, including the hydrodynamic forces, inter-particle interactions, the volume fraction, gravity, and the size and shape of the particles.

Particle-particle and particle-wall collisions play an important role in a liquid-solid flow because they affect particle accumulation and dispersion, and interphase transport and mixing. Compared to a dry collision in which the fluid resistance is negligible and the particle inertia is dominant, the kinetic energy of a particle in a liquid environment is dissipated by viscous stresses in the liquid and by inelasticity during collision. The ratio of particle inertia to viscous forces is quantified through the Stokes number,  $St = \frac{1}{9}(\rho_p/\rho_l)Re$ , where  $Re = \rho_l DU/\mu$  is the particle Reynolds number based on impact velocity  $U$ , the particle diameter  $D$ , the liquid viscosity  $\mu$ , and  $\rho_p$  and  $\rho_l$  are the density of the particle and the liquid, respectively.

By simultaneously accounting for elastic deformation and viscous forces, [Davis, Serayssol & Hinch](#)

(1986) established the range of conditions for deformation and rebound of colliding spheres. They found that elasticity of the particle and the impacting surface influences the value of coefficient of restitution,  $e$ , defined as the ratio of the rebound to impact velocity. However, the Stokes number determines whether there is a rebound and the trend of  $e$ . By measuring the approach and rebound of a particle colliding with a wall in a viscous fluid, Joseph *et al.* (2001) presented the coefficient of restitution as a function of the Stokes number. Below a Stokes number of approximately 10, no rebound of the particle occurs. At a Stokes number greater than 1000, the particle rebound speed is not affected by the surrounding fluid. Hence, the coefficient of restitution,  $e$ , increases from 0 at  $St \approx 10$  to a dry value,  $e_d$ , which occurs for a collision with negligible fluid resistance. In their pendulum-wall collision experiments, they observed a velocity decrease prior to contact as the result of the presence of the wall. The dependence of the coefficient of restitution on the Stokes number was also presented in the work by Gondret *et al.* (2002); in this study, the authors measured the normal trajectory of the particle over multiple bounces. However, they did not observe the velocity decrease as the particle approaches to a horizontal wall under gravity. The work by Joseph & Hunt (2004) examined the oblique collisions between a particle and a wall. Later work by Yang & Hunt (2006) measured the coefficient of particle-particle collisions in a liquid and found a similar dependence on particle Stokes number. However, for Stokes numbers less than approximately 20 the authors observed that the target particle moved prior to impact due to the increase in hydrodynamic pressure as the impacting particle approached within a half a particle diameter.

Computational studies have also considered the problem of particle collisions in a liquid. By fixing the particle velocity at a constant value as it approaches a wall, Leweke, Thompson & Hourigan (2004) computed the flow generated by a particle colliding normally to a surface without rebound; these simulations showed the development of the vortex rings around the particle over a range of Reynolds numbers. TenCate *et al.* (2002) simulated a sphere settling toward a solid wall without rebound and compared with experiment results. Both the spatial structure and the temporal behavior of the flow field were obtained. Ardekani & Rangel (2008) proposed a collision strategy that assumed no liquid is present between the two solid surfaces at collision and used the dry coefficient of

restitution  $e_d$  to calculate the instant rebound velocity directly when the distance between particles becomes equal to particle surface roughness height,  $h_{\min}$ , of order  $1\mu m$ . To resolve the flow at the length scale of  $h_{\min}$ , a finer mesh is used near the contact region; the simulation results depend on the mesh size and  $h_{\min}$ . Their model neglects the elastic deformation of the particles, which occurs over lengths that are comparable with the roughness height according to the elastohydrodynamics analysis in [Davis \*et al.\* \(1986\)](#).

While extensive work has been done independently, a complete description of an immersed collision process is not readily available. Such a description would include accurate solution of the flow field, the interaction between the solid particle and the surrounding liquid, and the rebound trajectory of the impact particle. A better understanding of the bulk behavior of a liquid-solid flow would depend critically on such an exact description on a single collision scale. Moreover, a contact model is required to reveal the contact mechanism so that a flow with many particles can be simulated with a simplified contact strategy. The work presented in this thesis provides a first step toward that goal.

## 1.2 Thesis outline

The primary goal of this thesis is to investigate the collision between two solid surfaces when the effect of the surrounding liquid is non-negligible. Chapter [2](#) describes the experiments performed for a solid sphere falling under gravity and colliding with a solid wall. The detailed velocity profile before the sphere contacts the wall is presented by using a higher frame rate for the recording camera than used in some previous studies. The coefficient of restitution measured from the experiments are presented as a function of the Stokes number. Chapter [3](#) introduces the numerical method employed in the current simulations. Chapter [4](#) presents a contact model for a particle-wall collision based on the hydrodynamic forces and the elastic force on an impact sphere. In Chapter [5](#), the simulations for an individual particle-wall collision in a viscous liquid are examined with the measured results from the current experiments. The coefficient of restitution calculated from the simulations are compared with the experimental data and the other researchers' result. In Chapter [6](#), the proposed contact



model is modified to simulate an immersed collision between two solid spheres. The unique sphere behavior before and after a collision at low Stokes numbers are presented. The effective coefficient of restitution and the binary Stokes number are correlated, and the dependance between them is compared with the other researchers' results. Conclusions are summarized in Chapter [7](#), and the future work is discussed in brief.