

Abstract

This thesis is comprised primarily of work from three independent papers, [1], [2], and [3], written in collaboration with Sean Carroll, Tim Dulaney, and Heywood Tam. The original motivation for the projects undertaken came from revisiting the standard assumption of spatial isotropy during inflation. Each project relates to the spontaneous breaking of Lorentz symmetry—in early Universe cosmology or in the context of effective field theory, in general. Chapter 1 is an introductory chapter that provides context for the thesis. Chapter 2 is an investigation of the stability of theories in which Lorentz invariance is spontaneously broken by fixed-norm vector “æther” fields. It is shown that models with generic kinetic terms are plagued either by ghosts or by tachyons, and are therefore physically unacceptable. Chapter 3 is an investigation of the phenomenological properties of the one low-energy effective theory of spontaneous Lorentz symmetry breaking found in the previous chapter to have a globally bounded Hamiltonian and a perturbatively stable vacuum—the theory in which the Lagrangian takes the form of a sigma model. In chapter 4 cosmological perturbations in a dynamical theory of inflation in which an Abelian gauge field couples directly to the inflaton are examined. The dominant effects of a small, persistent anisotropy on the primordial gravitational wave and curvature perturbation power spectra are found using the “in-in” formalism of perturbation theory. It is found that the primordial power spectra of cosmological perturbations gain significant direction dependence and that the fractional direction dependence of the tensor power spectrum is suppressed in comparison to that of the scalar power spectrum.

Contents

Acknowledgments	iii
Abstract	v
1 Introduction	1
1.1 The Big Picture	1
1.2 How This Thesis Emerged	6
1.3 Synopsis	7
1.4 CMB Temperature Correlations	8
1.5 From Primordial Perturbations to CMB Temperature Correlations . .	11
1.6 The Cosmic No-Hair Theorem	15
1.7 Æther	16
1.8 Hairy Inflation	17
1.9 Standard Slow-Roll Inflation	18
1.9.1 Background Equations	18
1.9.2 Perturbations from Single-Field Slow-Roll Inflation	21
2 Instabilities in the Æther	29
2.1 Introduction	29
2.2 Models	35
2.2.1 Validity of Effective Field Theory	38
2.3 Boundedness of the Hamiltonian	40
2.3.1 Timelike Vector Field	44
2.3.2 Spacelike Vector Field	46

2.3.3	Smooth Potential	47
2.3.4	Discussion	49
2.4	Linear Instabilities	50
2.4.1	Timelike Vector Field	51
2.4.2	Spacelike Vector Field	53
2.4.3	Stability is Not Frame Dependent	55
2.5	Negative Energy Modes	55
2.5.1	Spin-1 Energies	57
2.5.2	Spin-0 Energies	59
2.6	Maxwell and Scalar Theories	60
2.6.1	Maxwell Action	62
2.6.2	Scalar Action	65
2.7	Conclusions	67
2.A	Appendix: Solutions to the Linearized Equations of Motion	69
3	Sigma-Model Æther	74
3.1	Introduction	75
3.2	Excitations in the Presence of Gravity	77
3.3	Experimental Constraints	82
3.4	Cosmological Evolution	83
3.5	Extra Dimensions	85
3.6	Conclusions	87
4	Primordial Perturbations from Anisotropic Inflation	89
4.1	Introduction	89
4.2	Model and Background Solution	94
4.3	Perturbations: Setup and Strategy	101
4.3.1	Physical Scenario	102
4.3.2	Correlations Using “in-in” Formalism	103
4.3.3	Decomposition of Perturbations	104
4.3.4	Canonically Normalized Variables	105

4.3.5	Comparison with Data	106
4.4	Perturbations: Odd Sector	107
4.4.1	Preliminary Look at Stability	108
4.4.2	Diagonalized Action	110
4.4.3	Correlations Using Perturbation Theory	111
4.4.4	Discussion	116
4.5	Perturbations: Even Sector	118
4.5.1	Diagonalizing the Action	122
4.5.2	Correlations Using Perturbation Theory	123
4.6	Conclusions	128
4.A	Appendix: Parametrization of Perturbations	129
4.B	Appendix: Quadratic Action and Einstein's Equations	133
4.C	Appendix: Diagonalizing a Kinetic Term	135
4.D	Appendix: Estimates of Integrals	136
	Bibliography	139

List of Figures

1.1	Cosmological evolution timeline.	2
1.2	WMAP 7-year full-sky map of the cosmic microwave background.	8
1.3	Spectrum of CMB multipole coefficients from 7-year WMAP data.	9
2.1	Hamiltonian density as a function of Goldstone field values	45
3.1	Æther rest frame mode phase velocities	79
3.2	Sigma-model æther Parameter space allowed by constraints from Čerenkov radiation and PPN	83
4.1	Log plot of anisotropy parameter as a function of e -foldings	100
4.2	Plot of primordial perturbation correlation integrand as a function of e -foldings	138