Appendix E A discriminant for P- and S-waves

In the application of the Virtual Seismologist method, it is important to be able to distinguish between P- and S-waves. In Chapter 4, we discussed how the likelihood function is defined in terms of envelope attenuation relationships and linear discriminant analysis methods. There are different envelope attenuation relationships for P- and S-waves (as presented in Chapters 2 and 3), and it is thus important to be able to distinguish between the two phases. The linear discriminant analysis based magnitude relationships are less sensitive to whether the observed amplitudes are P- or S-waves (see Appendix F). However, better source estimates can obtained if we can identify phases. If we could detect an S-wave arrival, the epicentral distance can be estimated from the $S - P$ time, and the envelope attenuation relationships can be used to determine magnitude.

In this Appendix, we use linear discriminant analysis to address the following question: how can we use ratios of vertical to horizontal ground motions to distinguish between P- and S-waves?

Recall that linear discriminant analysis involves solving the following eigenvalue problem:

$$\sum\nolimits_w^{-1} \cdot \sum\nolimits_a u = \lambda \cdot u \tag{E.1}$$

where $\sum_a$ and $\sum_w$ are the *among group* and *within group* covariance matrices, which are in turn, defined as a function of the data (or observation) matrix X. (See Chapter 4: A short note on linear discriminant analysis, Eqns 4.20 through 4.26).

We will present 4 different P/S discriminants using vertical and horizontal channels of ground motion

- acceleration

- velocity
- acceleration and velocity
- acceleration, velocity, and filtered displacement

These 4 analyses differ only in the definition of the data or observation matrix X.

E.1 P/S discriminant using vertical and horizontal acceleration

The data or observation matrix X has two columns, corresponding to vertical and horizontal acceleration. We use the P- and S-wave envelope amplitudes obtained from fitting Eqn. 2.2 to observed ground motion envelopes. There are 3336 records in our database that have both vertical and horizontal acceleration envelopes. Thus, X is a 3336×2 matrix. We have 2 groups: group 1 consists of the P-wave envelope amplitudes, group 2 of the S-waves. The eigenvalues and eigenvectors of $\sum_w^{-1} \cdot \sum_a$ are:

$$\begin{aligned} \lambda_{1,a} = 7906.5 \quad &, \qquad u_{1,a}^T = \begin{bmatrix} 0.69 & -0.73 \end{bmatrix} \\ \lambda_{2,a} = 0 \quad &, \qquad u_{2,a}^T = \begin{bmatrix} -0.97 & 0.25 \end{bmatrix} \end{aligned} \tag{E.2}$$

The decision boundary between P- and S-waves is $Z_a = 0.01$. Given a new set of observed vertical and horizontal accelerations, $X_{new} = \begin{bmatrix} log(acc_Z) & log(acc_H) \end{bmatrix}$, we calculate $Z_{a,new} = X_{new} \cdot u_{1,a} = 0.69log(acc_Z) - 0.73log(acc_H)$, where acc_Z denotes vertical ground motion acceleration amplitude, and acc_H horizontal ground motion acceleration amplitude. If $Z_{a,new} > 0.01$, the observed amplitudes are from a P-wave. When $Z_{a,new} < 0.01$, the observed amplitudes are from an S-wave. The distance between the means the P- and S-wave groups is 0.29 (in units of Z). The standard deviations within each group are $\sigma_{a,P} = 0.15$ and $\sigma_{a,S} = 0.13$ for the P- and S-waves respectively. Figure E.1 and Table E.1 summarize the performance of this discriminant. This discriminant can be used on the output data streams of accelerometers or strong motion instruments.

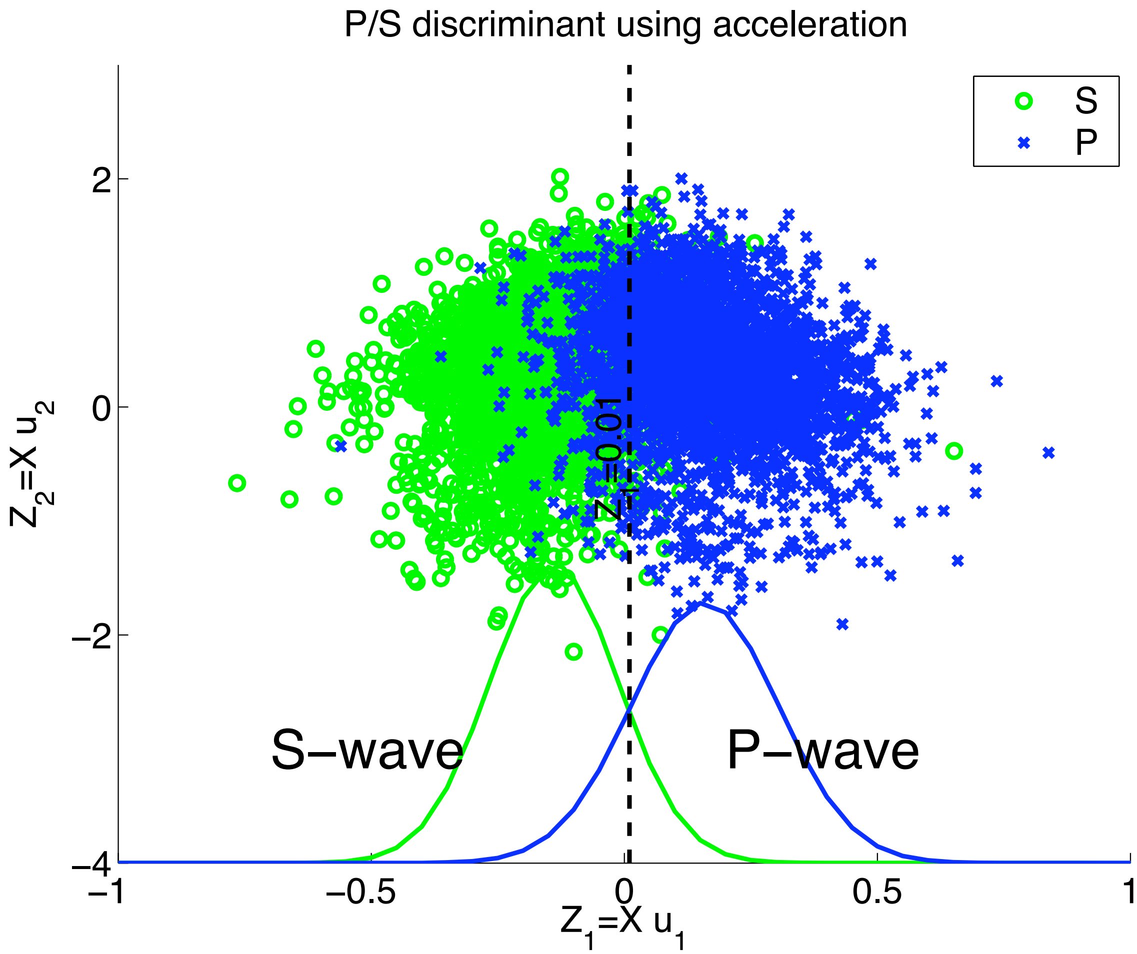

Figure E.1: P/S wave discriminant using vertical and horizontal ground motion acceleration.

E.2 P/S discriminant using vertical and horizontal velocity

The data or observation matrix X again has two columns, corresponding to vertical and horizontal ground motion velocity, vel_Z, vel_H. X is a 3336×2 matrix. Group 1

Confusion matrix for P/S discriminant using vertical and horizontal ground motion acceleration

	P-wave	S-wave	Total obs.
P-wave	86% (2865)	14% (471)	=3336
S-wave	11% (356)	89% (2979)	=3335

Table E.1: Confusion matrix for P/S wave discriminant using vertical and horizontal ground motion acceleration

consists of P-wave envelope amplitudes, group 2 of S-wave envelope amplitudes. The eigenvalues and eigenvectors of $\sum_w^{-1} \cdot \sum_a$ are:

$$\begin{aligned} \lambda_{1,v} &= 8218.6 \quad , \qquad u_{1,v}^T = \begin{bmatrix} 0.69 & -0.72 \end{bmatrix} \\ \lambda_{2,v} &= 3.64 \times 10^{-12} \quad , \qquad u_{2,v}^T = \begin{bmatrix} -0.94 & 0.35 \end{bmatrix} \end{aligned} \tag{E.3}$$

The decision boundary between P- and S-waves is $Z_v = 0.0063$. Given a new set of observed vertical and horizontal velocity amplitudes, $X_{new} = \begin{bmatrix} log(vel_Z) & log(vel_H) \end{bmatrix}$, we calculate $Z_{v,new} = X_{new} \cdot u_{1,v} = 0.69log(vel_Z) - 0.72log(vel_H)$. If $Z_{v,new} > 0.0063$, it is most likely that the observed amplitudes are from a P-wave. (From the confusion matrix in Table **??**, this criteria correctly identified P-waves in our database 88% of the time.) If $Z_{v,new} < 0.0063$, it is most likely that the observed amplitudes are from an S-wave. The distance between the means of the P- and S-wave groups is 0.304 (in units of Z). The standard deviations within each group are $\sigma_{v,P} = 0.14$ and $\sigma_{v,S} = 0.13$ for the P- and S-wave groups, respectively. Figure E.2 and Table E.2 summarize the performance of this discriminant.

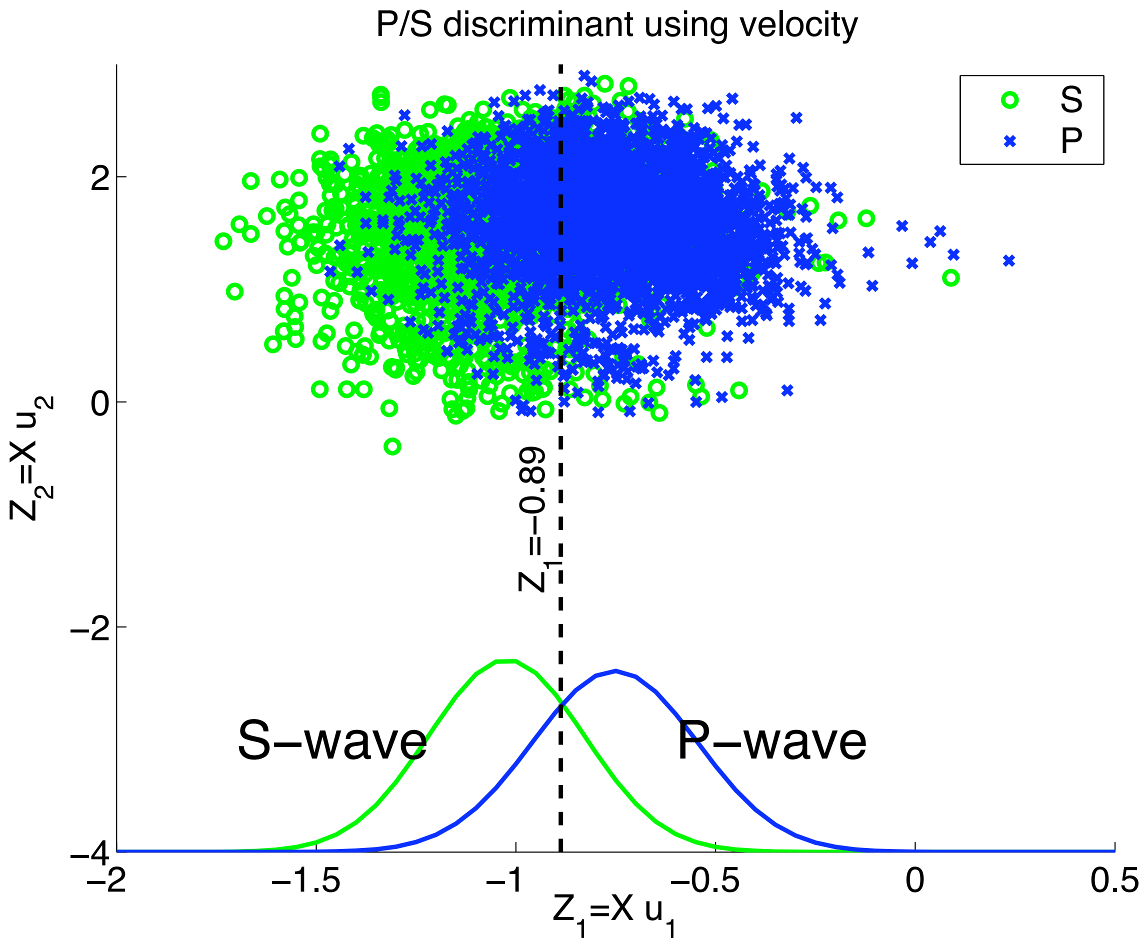

Figure E.2: P/S wave discriminant using vertical and horizontal ground motion velocity

Confusion matrix for P/S discriminant using vertical and horizontal ground motion velocity

	P-wave	S-wave	Total obs.
P-wave	74% (2478)	26% (856)	=3336
S-wave	23% (783)	77% (2553)	=3336

Table E.2: Confusion matrix for P/S wave discriminant using vertical and horizontal ground motion velocity

E.3 P/S discriminant using vertical and horizontal acceleration and velocity

The data or observation matrix has 4 columns, corresponding to vertical acceleration, vertical velocity, horizontal acceleration, and horizontal velocity. X is a 3336×4 matrix. Again, group 1 consists of P-wave envelope amplitudes, and group 2 of S-wave envelope amplitudes. The eigenvalues and eigenvectors of $\sum_w^{-1} \cdot \sum_a$ are:

$$\begin{aligned} \lambda_{1,av} &= 10061 \quad , & u_{1,av}^T &= \begin{bmatrix} 0.44 & 0.55 & -0.46 & -0.55 \end{bmatrix} \\ \lambda_{2,av} &= 3.24 \times 10^{-12} \quad , & u_{2,av}^T &= \begin{bmatrix} -0.35 & 0.89 & 0.21 & 0.23 \end{bmatrix} \\ \lambda_{3,av} &= -5.11 \times 10^{-13} \quad , & u_{3,av}^T &= \begin{bmatrix} 0.96 & 0.16 & -0.08 & -0.21 \end{bmatrix} \end{aligned} \quad \text{(E.4)}$$

The decision boundary between P- and S-waves is $Z_{av} = -0.01$. Given a new set of observed vertical and horizontal acceleration and velocity amplitudes, $X_{new} = \begin{bmatrix} log(acc_Z) & log(vel_Z) & log(acc_H) & log(vel_H) \end{bmatrix}$, we calculate $Z_{av,new} = X_{new} \cdot u_{1,av} = 0.43 log(acc_Z) + 0.55 log(vel_Z) - 0.46 log(acc_H) - 0.55 log(vel_H)$. If $Z_{av,new} > -0.01$, it is most likely that the observed amplitudes are from a P-wave. If $Z_{av,new} < -0.01$, it is most likely they are from an S-wave. The distance between the means of the P- and S-wave groups in units of Z_{av} is 0.424. The standard deviations within the P- and S-wave groups are $\sigma_{av,P} = 0.183$ and $\sigma_{av,S} = 0.162$, respectively. Figure E.3 and Table **??** summarize the performance of this discriminant.

Confusion matrix for P/S discriminant
using vertical and horizontal
ground motion acceleration and velocity

	P-wave	S-wave	Total obs.
P-wave	88% (2928)	12% (408)	=3336
S-wave	8% (270)	92% (3065)	=3335

Table E.3: Confusion matrix for P/S discriminant using vertical and horizontal ground motion acceleration and velocity

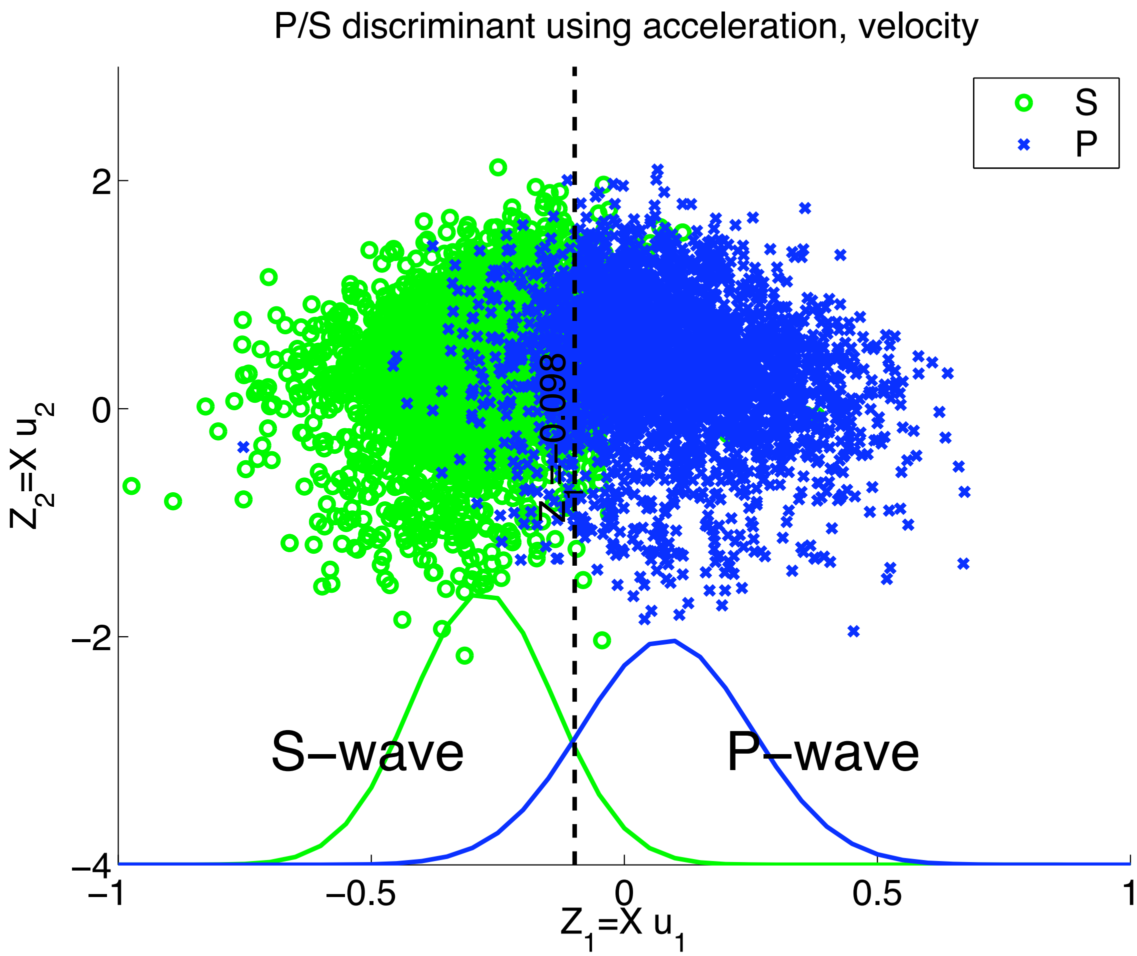

Figure E.3: P/S wave discriminant using vertical and horizontal ground motion acceleration and velocity

E.4 P/S discriminant using vertical and horizontal acceleration, velocity, and displacement

The data or observation matrix has 6 columns, corresponding to vertical acceleration, velocity, and displacement, and horizontal acceleration, velocity, and displacement. X is a 3336×6 matrix. Again, group 1 consists of P-wave envelope amplitudes, and group 2 of S-wave envelope amplitudes. The eigenvalues and eigenvectors of $\sum_w^{-1} \cdot \sum_a$ are:

$$\begin{aligned}
\lambda_{1,avd} = 10514 \quad &, \quad u_{1,avd}^T = \begin{bmatrix} -0.64 & 0.26 & -0.33 & 0.35 & -0.12 & 0.53 \end{bmatrix} \\
\lambda_{2,avd} = 8.68 \times 10^{-12} \quad &, \quad u_{2,avd}^T = \begin{bmatrix} -0.88 & 0.09 & -0.31 & -0.11 & 0.02 & 0.32 \end{bmatrix} \\
\lambda_{3,avd} = -4.72 \times 10^{-12} \quad &, \quad u_{3,avd}^T = \begin{bmatrix} 0.21 & -0.75 & 0.49 & -0.05 & 0.16 & 0.35 \end{bmatrix} \\
\lambda_{4,avd} = 3.75 \times 10^{-12} \quad &, \quad u_{4,avd}^T = \begin{bmatrix} -0.95 & -0.14 & 0.24 & -0.12 & -0.1 & -0.02 \end{bmatrix} \\
\lambda_{5,avd} = 3.55 \times 10^{-12} \quad &, \quad u_{5,avd}^T = \begin{bmatrix} 0.95 & -0.14 & 0.24 & -0.12 & -0.1 & -0.02 \end{bmatrix} \\
\lambda_{6,avd} = -1.05 \times 10^{-13} \quad &, \quad u_{6,avd}^T = \begin{bmatrix} 0.50 & -0.03 & -0.81 & 0.09 & 0.29 & -0.02 \end{bmatrix}
\end{aligned} \tag{E.5}$$

The decision boundary between P- and S-waves is $Z_{avd} = 0.04$. Given a new set of observed vertical and horizontal acceleration, velocity, and filtered displacement amplitudes, $X_{new} = \begin{bmatrix} log(acc_Z) & log(acc_H) & log(vel_Z) & log(vel_H) & log(disp_Z) & log(disp_H) \end{bmatrix}$, we calculate $Z_{avd,new} = X_{new} \cdot u_{1,avd} = -0.64 log(acc_Z) + 0.26 log(acc_H) - 0.33 log(vel_Z) + 0.35 log(vel_H) - 0.12 log(disp_Z) + 0.53 log(disp_H)$. If $Z_{avd,new} > 0.04$, the observed amplitudes are most likely from an S-wave. If $Z_{avd,new} < 0.04$, they are most likely from a P-wave. The distance between the means of the P-wave and S-wave groups is 0.47 (in units of Z_{avd}). The standard deviations within each group are $\sigma_{avd,P} = 0.20$ and $\sigma_{avd,S} = 0.17$ for the P- and S-wave groups, respectively. Figure E.4 and Table E.4 summarize the performance of this discriminant.

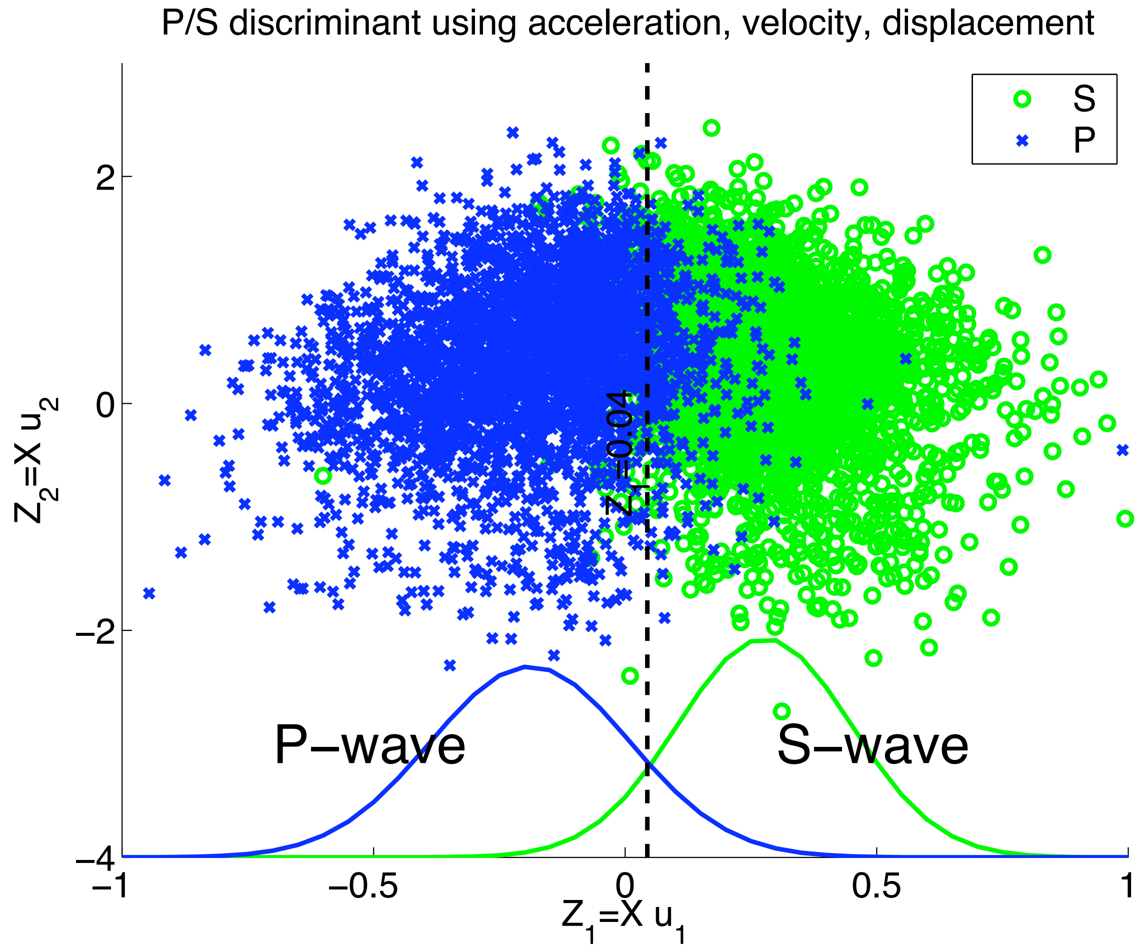

Figure E.4: P/S wave discriminant using vertical and horizontal ground motion acceleration, velocity, and displacement

Confusion matrix for P/S discriminant
using vertical and horizontal
ground motion acceleration, velocity, displacement

	P-wave	S-wave	Total obs.
P-wave	90% (3008)	10% (328)	=3336
S-wave	7% (236)	93% (3100)	=3336

Table E.4: Confusion matrix for P/S discriminant using vertical and horizontal ground motion acceleration, velocity, and displacement.

E.5 Summary

In this Appendix, we discussed how linear discriminant analysis can be used to estimate the phase (P- or S-wave) from which a set of observed ground motion amplitudes originate. Using vertical and horizontal acceleration, velocity, and filtered displacement amplitudes provides the best performance in terms of classification. Of the various confusion matrices presented, that for using acceleration, velocity, and displacement is most peaked along the diagonal. It has the lowest misclassification percentages (off-diagonal values) of the various cases considered, which were using 1) acceleration, velocity, and displacement, 2) acceleration and velocity, 3) acceleration only, and 4) velocity only.

The advantage of using the acceleration, velocity discriminant is that it can be applied to directly to the output of accelerometer and broadband seismometer instruments (after correcting for instrument response) without the additional processing necessary to calculate the filtered displacements. A number of CISN stations have both accelerometer and broadband seismometer instruments. This acceleration, velocity discriminant has a slightly higher misclassification error than that using acceleration, velocity, and displacement. We can see this by comparing Tables E.4 and E.3.

If it is necessary to choose between basing the estimates on accelerometer or seismometer outputs, it is better to use the acceleration discriminant, as it has a smaller percentage of misclassification errors than the velocity-based discriminant (compare Tables E.1 and E.2).