THESIS

The Cause of the Production of Air Bubbles In a Supersound Beam in Water

by

George C. Henny

Presented in partial fulfillment of the requirements
for the degree of
Master of Science in Physics

CALIFORNIA INSTITUTE OF TECHNOLOGY

1922

When standing waves of a supersound beam are produced in water containing dissolved air, small bubbles of the air are liberated from the water. The exact cause for the liberation of these bubbles was unknown and it is this problem which was undertaken and is discussed here.

The supersound is produced by making use of the piezoelectric property (see appendix) of quartz. A thin sheet of quartz is cut from a large clear crystal in the proper manner and cemented between between two blocks of steel, as indicated in the figure. When a high frequency voltage is applied across the two steel blocks the quartz expands and contracts in resonance with the current and the steel blocks are set into high frequency vibration. If the bottom block is in contact with water a supersound beam is projected downward.

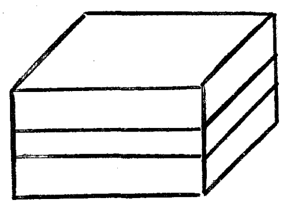


Fig. 1

The apparatus to produce the high frequency current to excite the projectors was a standard 20 K W Poulsen arc with inductance and large variable sheet metal and air condenser. The arc was run by a 500 volt D. C. generator, the apparatus being connected up as in Fig. 2. Although the high frequency current across the condenser plates is not of

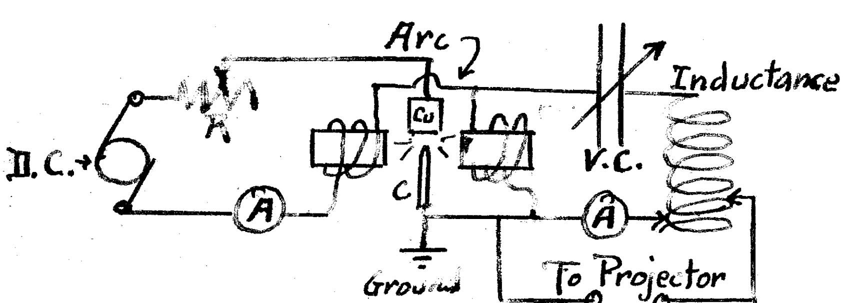


Fig. 2

the sine wave form, that taken from a small portion of the inductance is very nearly so, and was considered as such.

In order to measure the frequency, a frequency meter consisting of an inductance, variable capacity, and a neon tube, was set up, as in Fig. 3, and calibrated. At resonance the induced voltage across the inductance becomes a maximum and the neon tube glows a bright reddish color.

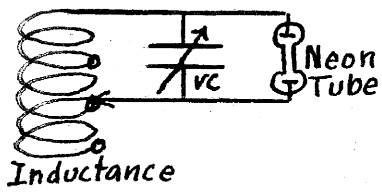


Fig. 3

The apparatus used in the research was devised after some preliminary experiments. In the first place it was found that bubbles would not form in boiled water, no matter how intense the sound beam. This immediately suggests that the degree of saturation has a direct connection with the ease with which the bubbles form and shows that the bubbles form from the dissolved air and not in any way by the decomposition of the water. For a given intensity of the sound beam more bubbles will form for highly saturated water than for water with little dissolved air. This leads to the method of using water of different known saturations and finding the threshold value of the intensity of the sound beam that will just cause the bubbles to start to appear. Now, since the saturation point of a gas in a liquid depends upon the temperature, the above method may be more readily accomplished by simply varying the temperature instead of changing the whole solution in the apparatus. But the saturation point of a gas in a liquid also depends on the pressure. Thus, instead of doing either of the above things, we may vary the pressure instead. As a matter of fact, in the experiments all three methods were used in connection with each other. The apparatus was set up as in Fig. 4. Here P is a quartz steel projector cemented so as to make a water tight joint onto the top of the beaker B, which is filled with water saturated at a known temperature and pressure. C is a copper container through which the water is circulated by a small pump P. C is either heated by a burner or cooled by ice and serves for the temperature regulation of the water. This temperature is read constantly from the thermometer T, whose bulb is right inside the beaker. The whole circulation system is pressure tight and a hydrostatic head may be applied by raising the glass container G full of water up or down. This pressure is read directly from the calibrated water manometer tube M.

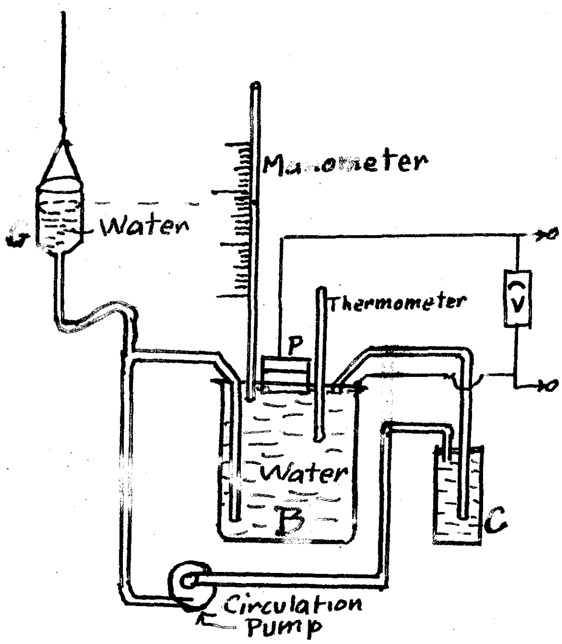
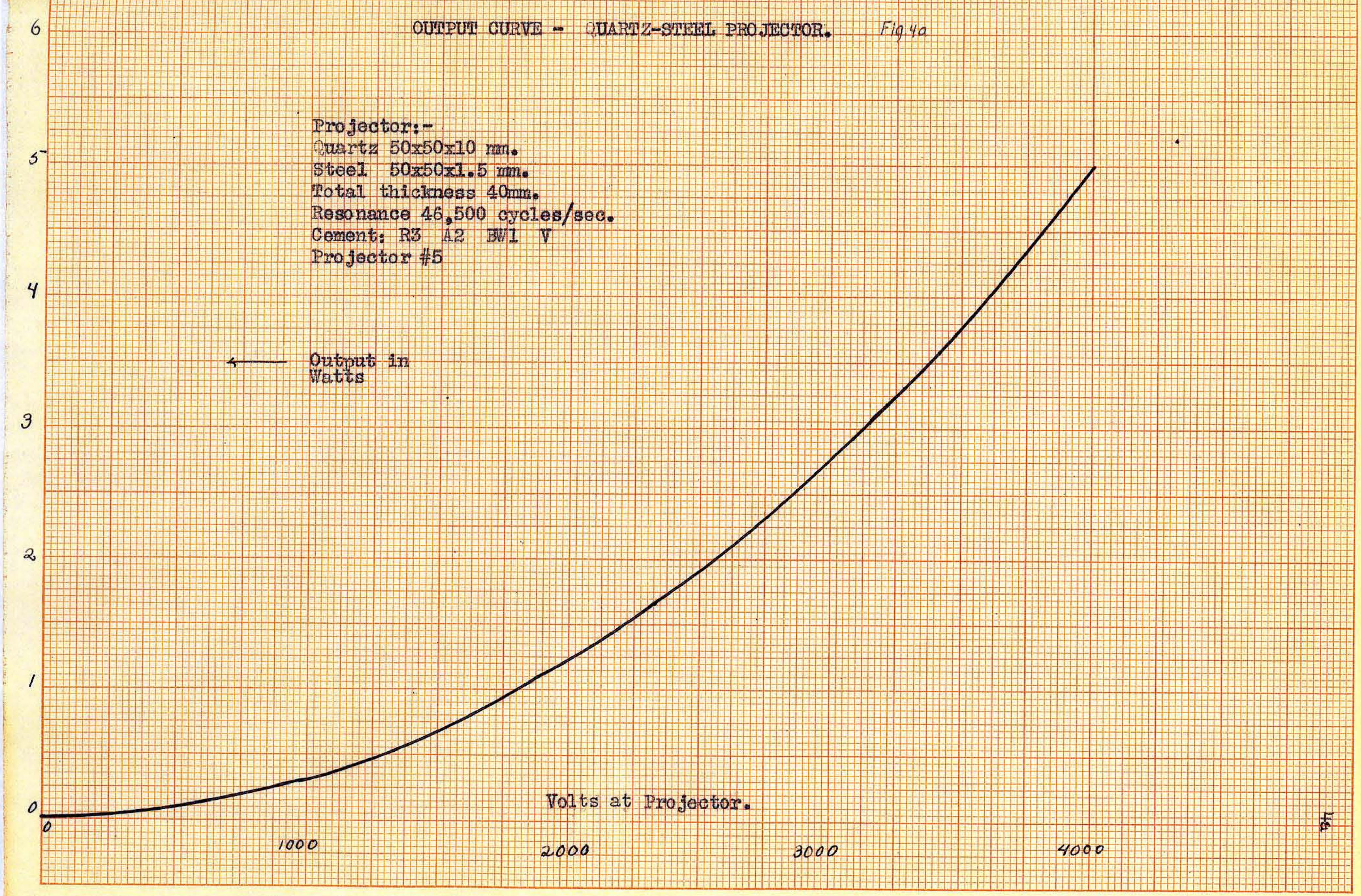


Fig. 4

It was found that for a given sound beam from the projector P, the bubbles would start to form at a particular pressure as indicated by M. At a different temperature or intensity of the sound beam, this pressure was found to be different. Readings were taken using water saturated at various temperatures, using different temperatures and using different intensities of the sound beam.

It is evident that to do any quantitative calculations with the above mentioned kind of readings it is necessary to know the output of the sound projector under different conditions. To get the best efficiency the projector was always run at resonance, the intensity being controlled by varying the voltage delivered to the projector as measured directly by the electrostatic voltmeter V. Perhaps the best way to find the total output is to absorb the sound in a calorimeter and measure the rate of rise in temperature. Since only one projector was used throughout the experiments this could be carefully done and a curve showing relations between output in water and the voltage at the projector was drawn, Fig. 4a. Knowing the voltage (at resonance) as read by V, delivered to the projector, we can, by referring to the curve, find the total output, and this divided by the area of the projector gives the intensity of the beam.

The water investigated should be saturated at a known temperature and pressure. This was done by placing a large jar full of distilled water in a carefully regulated water thermostat, and gently bubbling several streams of air through it for at least 36 hours. While handling, the water was always kept below its saturation temperature so as to prevent supersaturation and possible evolution of part of the dissolved gases. The apparatus was filled with this water, the projector brought to resonance at some voltage, and the hydrostatic head applied. This pressure was slowly diminished until tiny air bubbles just began to form under the projector. This value of the pressure is herein called the "threshold value". All readings were taken at this point, the temperature of the whole apparatus changed and the performance


OUTPUT CURVE - QUARTZ-STEEL PROJECTOR.
Fig 4a
Projector:-
Quartz 50x50x10 mm.
Steel 50x50x1.5 mm.
Total thickness 40mm.
Resonance 46,500 cycles/sec.
Cement: R3 A2 BW1 V
Projector #5
Output in Watts
Volts at Projector.
0
1
2
3
4
5
6
1000
2000
3000
4000

repeated. Many sets of data were obtained in this way, although only a few are given as the others are similar and lead to the same conclusion. A typical short run with the apparatus is indicated in the following tables.

Table 1.

Saturation temperature of water, 19.5° C.
Projector: Quartz-steel cemented with R cement.
Steel reactors each 50 x 50 x 10 mm.
Quartz driver, 50 x 50 x 10 mm.
Run at resonance, 46,500 cycles per second.

Temperature	Total output	Output per sq. cm.	Threshold pressure for bubbles
22°	2.9 watts	.116 watts	185 g. per sq. cm.
25.5	2.9 "	.116 "	240 g. " " "
30	3.1 "	.124 "	243 g. " " "
35.4	3.2 "	.128 "	270 g. " " "
25	2.75 "	.110 "	155 g. " " "
20.5	3.1 "	.124 "	95 g. " " "
19.5	2.37 "	.095 "	0 g. " " "

Another run taken with the same projector using water saturated at 35° C gave the following results:

Table 2.

Temperature	Total Output	Output per sq. cm.	Threshold pressure of bubbles
35.5°	3.1 watts	.124 watts	100 g. per sq. cm.
35	2.9 "	.116 "	76 g. " " "
32	2.8 "	.112 "	25 g. " " "
28	3.1 "	.124 "	35 g. " " "
27	2.9 "	.116 "	0 g. " " "
34.6	2.9 "	.116 "	43 g. " " "
37	2.9 "	.116 "	78 g. " " "
40	2.9 "	.116 "	147 g. " " "
35	2.39 "	.095 "	0 g. " " "

From these tables it is evident that the more saturated the water is the more easily the bubbles are driven out. The theory is that due to rarefactions in the sound waves or standing waves, the pressure is momentarily released and a tiny bubble set free. When the accompanying condensation comes along the tendency is for the bubbles to redissolve and perhaps most of them do. So, even with the best measurements we can make, we cannot expect an exact check. But, judging from the accuracy of our readings and the accuracy of the check we do get, it is possible to say there is good evidence for the theory.

The method of attack will be to derive theoretically the pressure amplitude of the sound waves. It is found that even at saturation a certain energy in a sound beam is necessary before bubbles form. The pressure amplitude of this wave plus the threshold pressure necessary for the bubbles to form (i. e., the hydrostatic head which will just prevent the bubbles from forming) when a more powerful beam is employed, should check with the pressure amplitude of the more powerful beam.

The method of deriving the expression for the pressure amplitude is the classical one. I am indebted to Dr. H. A. Lorentz for his kind help in this derivation. The work was later checked by him.

The following letters will denote the same things throughout.

Let x, y, and z = coordinates of any point considered
u, v, and w = the components in the x, y, and z, directions of the velocities of the medium under consideration. These are functions of x, y, or z, and t.
ρ = density of the medium at any instant.
ρ_0 = " " " " under normal conditions.
p = pressure of the medium at any instant.
p_0 = " " " " under normal conditions.
A = amplitude of vibration of the particles of the medium.

Then ρu = mass of the medium passing unit cross sectional area perpendicular to x, per second.

$\frac{\partial \rho u}{\partial x}$ = change of mass of the medium with respect to x. But an increase in density $\frac{\partial \rho}{\partial t}$, is due to an increase of mass in unit volume.

Considering the propagation along the x axis we may write:

$(\rho u)_{\text{leaving the unit cube}} - (\rho u)_{\text{entering}} = (-\frac{\partial \rho}{\partial t})$ for unit volume.

This equation indicates that the change in mass per unit volume, or the change in density, equals the mass leaving the section minus that entering. Instead of considering a unit volume or unit cube as above, we may take a section of thickness dx. We may then write

$$(1) \quad \frac{\partial \rho u}{\partial x} dx = -\frac{\partial \rho}{\partial t} dx \text{ or}$$

$$(1) \quad \frac{\partial \rho u}{\partial x} + \frac{\partial \rho}{\partial t} = 0.$$

We have considered propagation only along the axis of x. Considering the general case, we have, considering the matter entering and leaving the sides dx, dy, and dz

$$(2) \frac{\partial \rho u}{\partial x} dx(dydz) + \frac{\partial \rho v}{\partial y} dy(dxdz) + \frac{\partial \rho w}{\partial z} dz(dxdy) = -\frac{\partial \rho}{\partial t} dxdydz.$$

dydz, dxdz, and dxdy are the areas of the faces of the cube. The quantity dxdydz occurs in each expression and may be cancelled out. We then have

$$(3) \quad \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \frac{\partial \rho}{\partial t} = 0.$$

This is called the equation of continuity and states that the matter is continuous and is neither created nor destroyed.

The derivation of the acceleration of each particle in the sound beam is as follows:

The definition of acceleration is the change in velocity in unit time. Hence to find the change we multiply the rate of change by the interval.

If dt = change in t, or the small interval,

and u = velocity in any direction,

then the general change in velocity is time dt is

$$\frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}udt + \frac{\partial u}{\partial y}vdt + \frac{\partial u}{\partial z}wdt = Gdt$$

$$G = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}U + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}.$$

But if the x axis is taken along u, then in the case of sound $v = 0$ and $w = 0$, so we get as the expression for the acceleration of each particle

(d) $G = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$.

Since the force must equal the mass times the acceleration, we have

(4) $(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\rho dx = -\frac{\partial p}{\partial x}dx$ on unit area, where $\frac{\partial p}{\partial x}dx$ = the force or pressure at point dx.

(5) Now $p = f(\rho)$ due to characteristics of medium

and $p_o = f(\rho_o)$

Considering u as infinitely small and $\rho - \rho_o$ as infinitely small

If s = condensation (a variable)

(6) $\rho = \rho_o(1 + s)$

An expression for the pressure is given by

(7) $p = p_o + \frac{dp}{d\rho}(\rho - \rho_o) = p_o + \rho_o s\frac{dp}{d\rho}$ since from (6)
rate x interval

(8) Now we can put $\frac{dp}{d\rho} = \underline{a}^2$ = (velocity of propagation squared, as later found)

(9) With (7) and (8) $p = \quad + p_o + \rho_o sa^2$

From (1) and (6) $\frac{\partial \rho}{\partial t} = \rho_o\frac{\partial s}{\partial t}$ we get

(10) $\frac{\partial(\rho_o u)}{\partial x} + \rho_o\frac{\partial s}{\partial t} = 0$ or $\frac{\partial u}{\partial x} + \frac{\partial s}{\partial t} - = 0$

(11) $\rho_o\frac{\partial u}{\partial t} = -\rho_o a^2\frac{\partial s}{\partial x}$ from (4) and (7). From (7)

(12) $\frac{\partial p}{\partial x} = \rho_o\frac{\partial p}{\partial \rho}\frac{\partial s}{\partial x} = \rho_o a^2\frac{\partial s}{\partial x}$

Differentiating (10) in respect to x we get

(13) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 s}{\partial x \partial t} = 0$ or $a^2 \frac{\partial^2 u}{\partial x^2} + a^2\frac{\partial^2 s}{\partial x\partial t} = 0.$

and from (11), differentiating in respect to t

(14) $\frac{\partial^2 u}{\partial x^2} + a^2\frac{\partial^2 s}{\partial x\partial t} = 0.$

Subtracting (13) from (14) we get

(15) $\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = 0$

In the same way differentiating (1) in respect to t and (11) in respect to x we get

(16) $\frac{\partial^2 u}{\partial t\partial x} + \frac{\partial^2 s}{\partial t^2} = 0$ and

(17) $\frac{\partial^2 u}{\partial t\partial x} + a^2 \frac{\partial^2 s}{\partial x^2} = 0$ and subtracting (17) from (16)

(18) $\frac{\partial^2 s}{\partial t^2} - a^2 \frac{\partial^2 s}{\partial x^2} = 0$

The solution of (15) is

(19) $u = F(x - at) + f(x + at)$ (One wave travelling in one direction and the other in the opposite direction
a = velocity of propagation
U = velocity of particle inself at instant, t)

(20) Then $\frac{\partial u}{\partial t} = - aF'(x - at) + af''(x + at)$ and

(21) $\frac{\partial^2 u}{\partial t^2} = + a^2 F''(a - at) + a^2 f''(x + at)$

In the same way from (19)

(22) $\frac{\partial u}{\partial x} = F'(x - at) + f'(x + at)$ and

(23) $\frac{\partial^2 u}{\partial x^2} = F''(x - at) + f''(x + at)$

Substituting (21) and(23) into (18) satisfies the equation showing (19) is a solution.

(24) If $u = F(x - at)$, then the value of u at time t and place x is also equal to the value of u at time $t + \Delta t$ and place $x + a\Delta t$ for from (24)

$$x + a\Delta t - a(t + \Delta t) = x - at$$

Thus a = velocity of propagation. From (8) page 8 $a^2 = \frac{\partial p}{\partial \rho}$. Thus velocity of propagation, a, is given by

(25) $a = \sqrt{\frac{dp}{d\rho}} =$

Let K = coefficient of compressibility =

$$\frac{\text{fraction of diminution of vol.}}{\text{increase in pressure}}$$

(26) or $K = \frac{\text{fraction increase of density}}{\text{increase of pressure}}$ (for very small changes)

(27) $K = \frac{\frac{d\rho}{\rho}}{dp} = \frac{1}{\rho}\frac{d\rho}{dp}$

(28) $\frac{dp}{d\rho} = \frac{1}{K\rho}$ and from (25)

(29) Velocity of propagation $= \sqrt{\frac{1}{K\rho}} = a$. Example, for water

$K = 49 \times 10^{-6}$ per atmosphere $= 49 + 10^{-12}$ per dyne.

Then for water, since $\rho = 1$ $a = \sqrt{\frac{10^{12}}{49}}$ = 1430. meters per second.

Experimental value is 1437 meters at 15° C.

Applying (29) to steel, we must use for K

Young's Modulus $= E = \frac{\text{pressure per unit area}}{\text{fraction of change of length}}$

Then $\frac{dp}{d\rho} = \frac{E}{\rho}$ and

(30) $a = \sqrt{\frac{E}{\rho}} = V$

For steel E = approximately 19.5×10^{11} $\rho = 7.8$

and $a = \frac{19.5 \times 10^{11}}{7.8}$ $= 5 \times 10^5$ cm./second
= 5000 meters per second.

Experimental value is approximately 5000 for soft steel.

Thus this derivation for the velocity of the propagation of sound in any medium agrees with experiment. But we are concerned with finding the <u>pressure</u> amplitude. This can be done as follows:

$p = p_0 + \rho_0 sa^2$ from (9) page 8

where p = the pressure at any instant

p_0 = " " under normal conditions.

s = the condensation.

$a = \sqrt{\frac{dp}{d\rho}}$ = velocity of propagation of sound.

from (10) $\frac{\partial \rho_0 u}{\partial x} + \rho_0 \frac{\partial s}{\partial t} = 0$ or $\frac{\partial u}{\partial x} + \frac{\partial s}{\partial t} = 0$

(31) $-\frac{\partial u}{\partial x} = \frac{\partial s}{\partial t}$. But the velocity of a particle u is given by

(32) $u = B \cos n\,(t - \frac{x}{a} - q)$ where B is the maximum velocity of the particle

(33) $B = A \times 2\pi f$ where A = amplitude of vibration

$= A \times n$ and f = frequency

q = some arbitrary phase angle.

From (31) and (32)

(34) $\frac{\partial s}{\partial t} = -B\frac{n}{a} \sin n\,(t - \frac{x}{a} - q)$

or

(35) $S = +\frac{B}{a} \cos n\,(t - \frac{x}{a} - q) - \phi(x)$

But from (9) page 8

$p = p_0 + \rho_0 a^2 s$. Thus

(36) $p = p_0 + \rho_0 a^2 \frac{B}{a} \cos n\,(t + q)$ at $x = o$.

From this we get the expression for the pressure

(37) amplitude as $P_m = \rho_0 a B$

But the problem is not solved yet because to find the pressure amplitude the amplitude of vibration, A, must be known. The relation between A and B is given in (33) i. e., $B = A \times 2\pi f = A n$

To find A we must make use of the fact that the energy of the beam depends on A. Thus we must deal with the equations of energy.

The kinetic energy of a system of plane waves moving in the direction of x, having volume $\iiint dx\,dy\,dz$, density ρ_0 and velocity of each element of volume = u at point (x, y, z) and at time t is

$T = \frac{1}{2}\rho_0 \iiint u^2\, dxdydz$ where u = velocity at the point (x, y, z) and at time t. Now the potential energy from unit mass in expanding through a small range from volume v to standard volume v_0 is to the second order terms

(38) $\frac{1}{2}(p + p_0)\ (v_0 - v)$

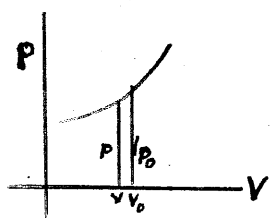


Fig. 5

Now $p = p_0 + Es$ since $E = \rho\left(\frac{dp}{d\rho}\right)_{\rho=\rho_0}$, by definition, and s is by (6) page 8, $\rho = \rho_0(1 + s)$.

(39) Then $p = p_0 + \rho s \frac{dp}{d\rho}$ where s is increase in density and $\frac{dp}{d\rho}$ is rate of change of p with the denisty. Also

(40) $v_0 - v = sv_0$ where v_0 and v are volumes.

(41) $\frac{1}{2}(p + p_0)(v_0 - v) = \frac{1}{2}pv_0 - \frac{1}{2}pv + \frac{1}{2}p_0 v_0 - \frac{1}{2}p_0 v$ or

(42) $(p_0 v_0 - p_0 v) + \frac{1}{2}pv_0 - \frac{1}{2}pv - \frac{1}{2}p_0 v_0 + \frac{1}{2}p_0 v = p_0(v_0 - v)$

$+ \frac{1}{2}(p - p_0)(v_0 - v)$

(43) or $= (v_0 - v) + \frac{1}{2}Es^2 v_0$ since $p = p_0 + Es$ and $v_0 - v = sv_0$

The first term $p_0(v_0 - v)$ will disappear when there is no total change of volume. Since this is assumed, we have for the

<u>potential energy</u>

(44) $w = \frac{1}{2}E\iiint s^2 dxdydz$

where $E = a^2\rho_0$ from (30) page 10.

From (19) page 9 $u = F(x - at) + f(x + at)$

and from (10) page 5 $\frac{\partial u}{\partial x} = -\frac{\partial s}{\partial t}$ $\frac{\partial u}{\partial x} = F'(x - at) + f'(x + at)$

(45) $\frac{ds}{dt} = -f'(x + at) - F'(x - at)$ $S = -\frac{1}{a}f(x + at) + \frac{1}{a}F(x - at)$

(46) or $aS = -f(x + at) + F(x - at)$ This is for a double wave or two systems of waves, travelling in opposite directions with constant velocity a. Taking just one part of the wave, i. e., a pro-

gressive wave, since $u = F(x + at)$

(47) $\pm as = u$

Thus we see, since $E = \rho_0 a^2$ that

(48) $T = W$ for the progressive wave. Consider the harmonic motion of any particle.

(49) $s = A \cos n(t - \frac{x}{a} + q)$ A = amplitude of vibration.

(50) $\frac{ds}{dt} = - An \sin \quad (t - \frac{x}{a} + q)$ = velocity. Now Kinetic energy $= \frac{1}{2}m\, u^2$ and $m = \rho$ per unit volume

(51) Then $KE = \frac{1}{2}\rho A^2 n^2 \sin^2 n(t - \frac{x}{a} + q) = T$

(52) But $W + T$ = constant. Hence maximum T gives

(53) total energy, or $T\ max = \frac{1}{2}\rho\ A^2 n^2$ per unit length where A = amplitude of vibration and $n = 2\pi \times$ frequency. The energy current or rate of flow of energy is given by

(54) $\frac{1}{2}\ A^2 n^2 (\frac{\lambda}{T})$ where T = period

λ = wave length in cm.

$\frac{\lambda}{T}$ = a in cm. per second

(55) Hence we get energy current $= \frac{1}{2}\rho a A^2 n^2$ per second.

From (29) page 10 $a = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{1}{K\rho}}$

So we can write Energy current $= \frac{1}{2}\rho\sqrt{\frac{1}{K\rho}}\ A^2 n^2 = \frac{1}{2} A^2 n^2 \sqrt{\frac{\rho}{K}}$

Since in our experiments we had stationary waves, we should derive the equation for these.

(56) Let $y = A \sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$ be the equation for the displacement given to the particles of the medium by a progressive wave. The reversed wave must be one for which at some instant, say $t = o$, the displacements given to the same series of particles will be equal and opposite. The equation

(57) $y' = A \sin 2\pi(\frac{t}{T} + \frac{x}{\lambda})$ satisfies this condition and represents

the reverse wave. The resultant for the standing wave is then the sum of y and y'. Thus

(58) $Y = A \sin 2\pi(\frac{t}{T} - \frac{x}{\lambda}) + A \sin 2\pi(\frac{t}{T} + \frac{x}{\lambda})$

Now from trigonometry

(59) $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$ and

(60) $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$.

Using these we get

(61) $Y = (2 A \cos 2\pi\frac{x}{\lambda})\sin nt$

Since $n = \frac{2\pi}{T}$

(62) $Y = 2 an \cos(\frac{2\pi x}{\lambda}) \cos nt$ and for K. E. $= \frac{1}{2} mu^2$

(63) K.E. $= 2\rho A^2 n^2 \cos^2(\frac{2\pi x}{\lambda}) \cos^2 nt$

We may write (63) as

(64) K.E. $= \rho A^2 n^2 (1 - \cos\frac{4\pi x}{\lambda})(\cos^2 nt)$

to get the total energy per wave length we may integrate in respect to x from 0 to λ and multiply by 2 since T = W. See (48), page 13. This gives

(65) $E = \rho A^2 n^2 \lambda$ since the average value of $\cos^2 nt$ is $\frac{1}{2}$ and the energy density $D_s = \rho A^2 n^2$

From (55) the energy density is $\frac{1}{2} \frac{\rho a A^2 n^2}{a}$ or

(66) $D_p = \frac{1}{2} \rho A^2 n^2$

This we see is just double the energy of the progressive wave, as might have been suspected. Using (54) and knowing all the values but A, this amplitude may be calculated. The energy current from the projector was measured calorimetrically and is given in the curve Fig. 4a page 4a. This amplitude may be substituted in equation (37) and the pressure amplitude calculated. We may then compare this with the experimental values and either sustain or disprove the theory for the formation of the bubbles.

To show that the maximum pressure of two waves progressing in opposite directions is the sum of the two, we have the following:

To find the intensity of the reflected wave we may consider the energy current which the transmitted wave will have. The bottom of the beaker of the apparatus was practically all in contact with air during the experiments. Since the density of the glass bottom is less than 3 times that of water the amplitude of vibration is nearly the same. The density of the glass is, however, about 2400 times that of the air. Now the energy current was, by equation (54) seen to be directly proportional to the density. Hence the energy of the reflected wave is, well within experimental error, equal to that of the incident wave and will be considered as such in the calculations.

Equation (57), representing two waves of the <u>same</u> amplitude progressing in opposite directions may be written

(67) $Y = A \sin n\left(t - \frac{x}{u}\right) + A \sin n\left(t + \frac{x}{u}\right)$ because $n = 2\pi f = \frac{2\pi}{T}$ and $f = \frac{a}{\lambda}$ = frequency.

(68) Then $y = A \sin nt \cos \frac{nx}{a} - A \cos nt \sin \frac{nx}{a} + A \sin nt \cos \frac{nx}{a} + A \cos nt \sin \frac{nx}{a}$

(69) or $Y = 2A \sin nt \cos \frac{nx}{a}$

(70) Then $\frac{\partial y}{\partial x} = \frac{-2An}{a} \sin nt \sin \frac{nx}{a}$

Now the change in pressure is given (since $s = \frac{\partial y}{\partial x}$) by

(71) $dp = -\rho_0 a^2 \frac{\partial y}{\partial x}$. Therefore

(72) $dp = +\rho_0 a^{(+2An)} \sin nt \sin \frac{nx}{a}$ and the maximum change in pressure is

(73) $P_m = 2\rho_0 anA$ or twice that of the progressive wave. Thus the maximum variation in pressure for the two waves progressing in

opposite directions is additive.

From (54), page 13, we have

(74) $A^2 = \frac{2E}{\rho_o a n^2}$ and from (33) and (37) we have

(75) $Pm = \rho_o Aan$ and substituting

(76) $Pm = \sqrt{2aE\rho_o}$. In the case of water this gives

(77) $Pm = 535\sqrt{E}$ where E = energy in ergs per second and Pm is in dynes per square cm.

Putting E in terms of watts and Pm in grams per square centimeter, we have

(78) $Pm = \frac{535 \times 1000}{980} \sqrt{10\ E} = 1724\sqrt{E}$. In the case of the standing wave we find this to be twice as great or

(79) Total $Pm = 3448\sqrt{E}$

This formula will now be used with the data in the tables already given and also those below.

Table 3

Saturation Temperature of water 20° C.

Temperature	Total Output	Output per sq. cm.	Threshold pressure for bubbles
20°	2.0 watts	.080 watts	0 g. per sq. cm.
20°	3.12 "	.124 "	205 " " " "
11.75°	3.12 "	.124 "	0 " " " "
17°	3.12 "	.124 "	165 " " " "
25°	3.12 "	.124 "	210 " " " "
30°	3.0 "	.120 "	270 " " " "

Using water saturated at 30° C. the following results were obtained.

Table 4

Temperature	Total Output	Output per sq. cm.	Threshold pressure for bubbles
30°	2.10 watts	.084 watts	0 g. per sq. cm.
30°	2.75 "	.11 "	110 " " " "
22°	2.75 "	.11 "	0 " " " "
26°	2.75 "	.11 "	108 " " " "
34°	3.00 "	.12 "	130 " " " "
35°	2.75 "	.11 "	200 " " " "

Using water saturated at 40° C.

Table 5

Temperature	Total output	Output per sq. cm.	Threshold pressure for bubbles
40°	2.65 watts	.106 watts	0 g. per sq. cm.
40°	3.05 "	.122 "	48 " " " "
37°	3.05 "	.122 "	0 " " " "
38°	4.00 "	.160 "	12 " " " "

From Table 3, page 16, using the readings at 20° the results are

Total Pm = $3448\sqrt{.123}$ = 1210 grams per sq. cm.

Total Pm for threshold value = $344\sqrt{.08}$ = 976 grams per sq. cm.

Difference = 234 " " " "

Experimental value = 205 " " " "

Error 29 " " " "

From Table 4, using the readings at 30°

Total Pm = $3448\sqrt{.11}$ = 1144 grams per sq. cm.

Total Pm for threshold value = $3448\sqrt{.084}$ 1000 " " " "

Difference = 144 " " " "

Experimental value = 110 " " " "

Error 34 " " " "

From Table 2, page 5, using the readings at 35° the results are:

Total Pm = $3448\sqrt{.116}$ = 1173 grams per sq. cm.

Total Pm for threshold value = 1064 " " " "

Difference = 109 " " " "

Experimental value = 76 " " " "

Error 33 " " " "

From Table 5, page 17, using the readings at 40° the results are:

Total Pm = $3448\sqrt{.122}$ =	1205	grams	per	sq.	cm.
Total Pm for threshold value = $3448\sqrt{.106}$ =	1124	"	"	"	"
Difference =	81	"	"	"	"
Experimental value =	48	"	"	"	"
Error	33	"	"	"	"

The results do not check accurately, but the nearly constant error suggests some experimental error common to all the readings. For instance, the output curve of the projector might be slightly in error. Taking the results of Table 2 as an example, we see if instead of .116 watts per sq. cm. output we had .114 and if instead of .095 watts per sq. cm. output for the threshold value we had .0965, the results would check exactly. This means an error in the curve of about 1.7 per cent, which is as close as might be expected taking all things into consideration.

Using the results of Table 1, page 5, we get in the same manner, for the reading at 20.5° and 19.5° respectively:

Total Pm = $3448\sqrt{.124}$ =	1214	grams	per	sq.	cm.
Total Pm for threshold value = $3448\sqrt{.095}$ =	1064	"	"	"	"
Difference =	150	"	"	"	"
Experimental value =	95	"	"	"	"
Discrepancy =	55	"	"	"	"

This latter difference is seen to be larger than any of the others, but according to the theory it should be so because the temperature at which the hydrostatic head (of 95 g. per sq. cm.) was applied was above the saturation temperature of the water.

Above the saturation temperature the water is supersaturated, the air is more easily freed, and hence the pressure should be greater to suppress the formation of bubbles. Considering the unsteadiness of the Poulsen arc at the time of the experiment, due partly to the fact that one person ran it and took all readings at the same time, the experimental results may, with a fair degree of certainty, be said to agree with the theoretical ones.

The whole theory is, perhaps, not yet clear. From a glance at the tables it is easily seen that the more saturated the water is, the more readily the bubbles form. But another thing that may be noticed is that for a saturated solution a certain intensity of the sound beam is necessary to free the air in bubbles. At saturation, any reduction in pressure should free some air. However, the reduction in pressure is always followed, one-half period later, by an increase in pressure. This tends to redissolve any air that may have been set free. This is what is done, up to the threshold value. The larger the bubbles that form, the less readily they will dissolve because of the smaller ratio of surface to volume. Thus, when the pressure passes a certain value (herein called the "threshold" value) the bubbles become large enough during the half period so as not to dissolve when the compression passes. Since the maximum pressures have been shown to be additive (equation 73), it is legitimate to subtract the threshold value from the total maximum value and expect to get the small additional "experimental" hydrostatic pressure which had to be applied to repress the bubbles. This is what was done and a check (within experimental error, of course) obtained.

APPENDIX

PIEZO-ELECTRIC EFFECT AND THE SUPERSOUND PROJECTOR

In 1880 the brothers J. and P. Curie in France discovered that when certain crystals, - as, for example, quartz, tourmaline, boracite, and Rochelle salts, cut in the proper way, - are subjected to compression or tension, opposite and equal charges of electricity appear at the ends of the crystals. They found that the amount of electricity appearing at the end faces of the crystal is proportional to the force applied. But the reverse action, within certain limits, was also found to take place. That is, if at the proper places on the piece of quartz electric charges of opposite signs are applied, the crystal will lengthen. If the charges are reversed, the crystal will shorten. This change in length may correspond to that produced by a force of several kilograms. This action will be discussed later. Let us further investigate piezo-electricity.

We find the piezo-electricity property to be associated only with crystals. Crystals are distinguished by an orderly and practically invariable space distribution of planes of cleavage. These planes, together with the planes of growth, are in specific arrangements in which planes and axes of symmetry are found. These arrangements possess different degrees of symmetry; so crystallized matter may be divided on this basis into about thirty-two classes, differentiated by definite symmetry relationships, which are perhaps due to definite arrangement of the molecules or atoms. These classes are further grouped in several systems according to the defined axes to which these geometrical arrangements are naturally referred.

In about twenty of these crystallographic classes no center of symmetry is found. These classes are technically designated as hemi-morphic, hemi-hedral and tetarto-hedral. In these classes in general, opposite ends of a line or direction, through the crystalline medium are not characterized by the same grouping of planes or association of physical properties. In other words, these directions are polar. So all matter whose crystalline structure is lacking in a center of symmetry has at least one direction, and usually many, with two or more principal directions, along which the application of a force, as pressure or traction, will produce electric polarization and the accumulation of electric charges of opposite sign at opposite ends of the axes. In most cases this effect is exceedingly small. In a few substances, such as Rochelle salt, the effect produced by the application of force is very much greater for some directions than for others.

When there are two or more principal directions through a substance which are principal axes of piezo-electricity, it happens that the application of force along one direction may produce perceptible polarization, not only at the extremities of that line, but at the extremities of another, also. This is the case with quartz, and plays a very important part in the action of the quartz projector.

Evenly developed quartz crystals look like hexagonal prisms terminated by hexagonal pyramids. The axis running longitudinally from the top to the bottom pyramid is called the optical or principal axis of the crystal and is not an axis of piezo-electricity, so that force exerted in its direction produces no piezo-electric effect.

But the three horizontal directions which bisect the angles between the horizontal crystal axes are the chief axes of piezo-electricity (see fig. 7). These axes emerge at the angles between the prism faces. Force applied as pressure or tension upon planes cut perpendicular to any one of these intermediate directions develops electric polarization and the accumulation of charges whose quantities are given by the formula

$$q = D\ F$$

where

q = electrical quantity of electrostatic units,

F = applied force measured in dynes,

D = the piezo-electric constant, an empirical coefficient of the piezo-electric effect reduced from observation and experiment.

(for quartz the Curies found for the chief directions

$$D = 6.32 \times 10^{-8} \text{ e. s. u. of quantity per dyne.)}$$

But, now, if we compress or dilate a crystal plate out from quartz in a direction perpendicular to both the principal

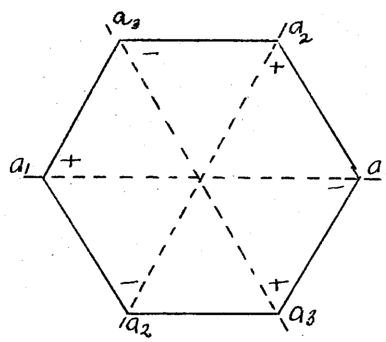


Fig. 7

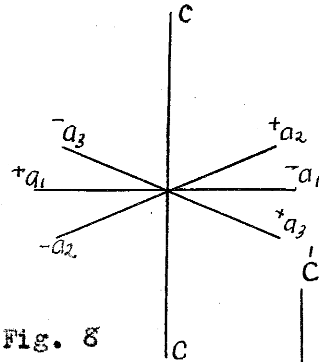


Fig. 8

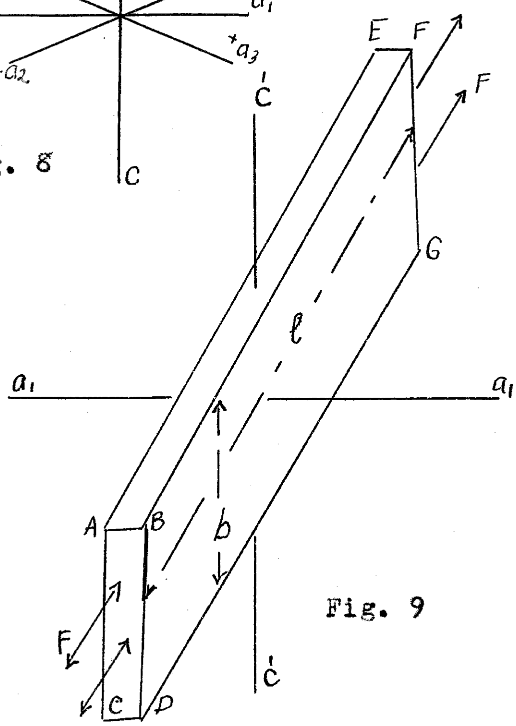


Fig. 9

crystallographic axis $\acute{c}$ and to any one of the piezo-electric axes, as a, Fig. 8, thus applying the force upon the planes A,B,C,D, and E,F,G, Fig. 9, again electric polarization is produced with reference to the piezo-electric axis, a_1, and a charge is set free and accumulated on the faces ACE and BDFG, which stand perpendicular to the electric axis a_1. (The electricity set free and thus accumulated is opposite in sign to that which would be produced by force similarly applied directly upon these faces.) In this case, however, the quantity of electricity thus set free is given by the formula

$$q = D \frac{l}{d} F$$

(when, as before $D = 6.32 \times 10^{-8}$ = the constant piezo-electric coefficient found by the Curies for this direction also) where l and I are measured in centimeters.

As a result of extended investigations with quartz and other piezo-electric substances, the Curies were able to state the following laws:

1. The quantities of electricity set free at the end of an axis due to a deformation are equal and of opposite sign.

2. The quantities of electricity set free at one of the ends of an axis are equal and of opposite sign for two deformations inverse in sense to each other.

3. The quantities of electricity set free on each end are proportional to the variation in force applied.

4. For one and the same variation in force applied, exerted in the direction at whose extremities the electricity is collected, the quantities of electricity set free are independent on the dimensions of the crystal.

5. For one and the same variation in pressure, exerted normal to the direction along which we collect the electricity, the quantities of electricity set free are proportional to the ratio of the length to the thickness, they are independent of the third dimension, and these lead, together with the phenomena of pyro-electricity, to the supplementary generalization.

6. For one and the same crystal the quantities of electricity set free are proportional to the intensity of the deformation at each point.

7. For one and the same intensity of deformation at each point, the quantities of electricity set free are proportional to the surface upon which the electricity is collected and are independent of the third dimension of the crystal.

These laws plainly teach that when we are limited to a given total quantity of force and to a given volume of the crystallized substance also, when we collect the charge upon planes perpendicular to a piezo-electric axis which is parallel to the direction of this applied force, we will obtain a given <u>quantity</u> of liberated electricity; dependent <u>only</u> upon the <u>total</u> amount of the <u>force</u> applied; but in this case we may alter the dimensions of the crystal plate but in so doing, if we increase the collecting area, we decrease the force per unit of area applied, and so the intensity of deformation at each point in equivalent ratio, and vice versa.

Since the piezo-electric action is reciprocal, we see we may obtain a greater force <u>per unit area</u> for a given charge of electricity by reducing the size of the plane perpendicular to a piezo-electric axis upon which are the metallic electrodes. However, if we are interested in producing the largest force with the least quartz, regardless of the charge to produce it, the best way is to cut the crystal perpendicular to a piezo-electric axis into thin (2 or 3 mm.) sheets.

We have now seen something of how the piezo-electric action works and may proceed to see how it is applied in supersound projectors. Since a crystal of quartz, for example, when cut in the right way, will lengthen or shorten when opposite charges are brought to the opposite ends of a piezo-electric axis, it will lengthen and shorten alternately if the charges alternate in sign. Thus, if an alternating current is applied to the crystal, a vibration in the crystal of the same frequency as that of the current will take place. This is found to be the case even when very high frequencies are employed. It is found that as the frequency is gradually increased a point is reached where this frequency and the natural mechanical frequency of vibration of the crystal come into resonance. At this point the amplitude of vibration is very much greater than at any other point. In fact, the amplitude may build up to such an extent that the vibration breaks the crystal. The end of the vibrating crystal, being in contact with the air, causes successive rarefactions and condensations to be propagated outwards. These constitute supersound in air. However, since the amplitude of vibration is so small and the air is so easily compressible, very little energy is dissipated in the form of sound waves. If one face of the vibrating crystal is placed in contact with a large body of water, supersound waves are sent outward from the projector face. A great deal of energy may be communicated to the water in this way, although the amplitude of vibration is very small. This amplitude may be only one-fourth that of the wavelength of sodium light, and, partly due to the high frequency (say 50,000 per second) the energy current of the sound beam would be about 13×10^7 ergs.

In nearly all of our work the projectors were made of quartz crystals which were used as the drivers, although we are now trying tourmaline. Quartz sheets were cut from the crystals so that a piezo-electric axis was perpendicular to the surface. Since the sheets were cut thin (due to the scarcity of clear quartz) their resonant vibration frequency is very high. In order to bring this down to more desirable limits it was found possible to cement a piece of steel to each surface of the quartz slab in such a way that the steel formed part of the vibrating system. These pieces of steel are called reactors, and their effect is to lower the natural frequency of the projector. Equal thicknesses of steel are always cemented to the faces of the quartz, as shown in Fig. 6. This is done so as to have the quartz at the center of the vibrating system. This gives the maximum amplitude of vibration at resonance and puts the least strain on the cement.

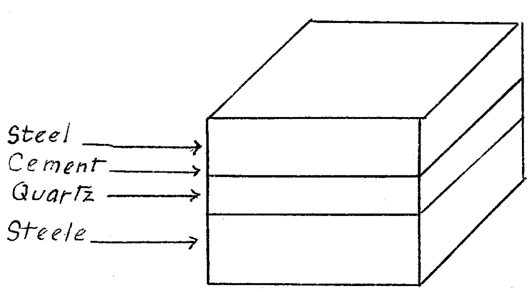


Fig. 6