RANGE CALCULATIONS FOR AIRPLANES WITH CONTINUOUSLY CONTROLLABLE PITCH PROPELLERS, AND THE EFFECT OF CERTAIN GEOMETRIC PARAMETERS OF THE AIRPLANE ON RANGE

Thesis

by

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I. SUMMARY

This investigation was undertaken to develop a satisfactorily exact method for the calculation of range of airplanes; and with such method, to determine the effect of certain geometrical parameters of the airplane on range, namely, span, aspect ratio, and wing loading.

With satisfactory aerodynamic data from wind tunnel reports, and with satisfactory engine performance data from test stand reports, the method developed permits determination of range, optimum speeds to use, and optimum propellor rpm to use for each speed. It further permits determination of endurance.

In general, increasing aspect ratio for constant span will increase maximum range if reasonable upper limits are placed on wing loading. The gain is but slight for aspect ratios in excess of 11. Similarly, decreasing span for constant aspect ratio will increase maximum range. Both of these variations result in increased wing loading, which can therefore be considered as the fundamental factor giving the increase. There will in general be an optimum combination of aspect ratio and span for a given

wing loading which will give maximum range. Satisfactory takeoff and sea-level rate of climb will limit the wing loading; however, for wing loadings in excess of 50, the maximum gain obtainable is of the order of 5%.

II. INTRODUCTION

It was felt that with the advent of continuously controllable pitch propellors, more efficient engines with devices for automatically controlling fuel consumption, and general aerodynamic improvement in the trend toward larger airplanes, some method was needed which would permit calculation of range, endurance, optimum speeds and propellor rpm, using aerodynamic and engine test stand data, with greater accuracy than methods in use at present. An excellent recent treatment of the problem, in which several of the parameters are replaced by constant, mean values is given in Ref. 1, while the procedure which has become classical in this country is outlined in Ref. 3. The phrase "more exact" is sometimes used in this report to indicate that the variation of all the terms of the range formula (weight, L/D, propulsive efficiency, specific fuel consumption) has been taken into account.

Inasmuch as it appeared advisable to investigate the effect of change of propellor rpm and thereby propulsive efficiency, parameters were developed which were independent of this term.

This led in turn to successive variations of the remaining parameters.

When it became evident that the detailed calculations were too laborious for general use, two approximate methods were developed whereby the designer can obtain maximum range and average cruising speed with a minimum of calculation.

These approximate methods were used in determining the effect of certain geometric parameters on the range and cruising speed of airplanes.

III. DESCRIPTION OF THE METHOD

This method consists of a more exact evaluation of the ultimate range parameter, namely, miles per pound of fuel consumed, which, when integrated over the weight range of the fuel capacity, will give the range. This parameter is designated as C_R, and is defined by the basic equation

$$C_R = \frac{V}{\text{bhp} \times c} = \frac{V}{\text{thp}} \times \frac{\eta}{c}$$

where V is in miles per hour (true airspeed),

bhp is in brake horsepower,

thp is the thrust horsepower,

c is in pounds per brake horsepower per hour.

For convenience in evaluating this parameter, it is rearranged algebraically as follows:

$$C_R = \frac{V}{\text{bhp} \times c} = \frac{V\,\eta}{\text{thp} \times c} = \frac{1}{\text{thp}/V} \times \frac{\eta}{c}$$

$$\eta = \text{propulsive efficiency} = \frac{\text{thp}}{\text{bhp}}.$$

But Drag x V = thp; therefore, thp/V is proportional to Drag and C_R is proportional to $\frac{1}{\text{Drag}} \times \frac{\eta}{c}$.

This will be recognized as a portion of the familiar Breguet formula for range:

$$dR = \frac{\text{Lift}}{\text{Drag}} \times \frac{\eta}{c} \times \frac{dW}{W}$$

Here Lift is equal to Weight; hence they may be cancelled, and the formula becomes

$$R = \int \frac{1}{\text{Drag}} \times \frac{\eta}{c}\, dW = \int C_R\, dW$$

which must be evaluated between the limits of initial and final Gross Weights.

Maximum range will correspond to maximum C_R, and inasmuch as this parameter is a non-analytical function of Drag, η and c, all of which vary with altitude, it must be evaluated by a step-by-step process for a specific altitude. The general procedure is as follows:

The limits of the integral being initial and final gross weights, the maximum values of C_R are evaluated for these weights, and for as many intermediate weights as may be necessary to define a curve. For the altitude and weight chosen, a series of true air speeds is selected. This fixes for each speed, the term 1/Drag of the integral. The remaining terms (η, c) of the integral, being dependent upon the engine-propellor combination, cannot be separated, and must therefore be evaluated as the ratio η/c. The maximum value of this ratio is determined for each speed, and used in the basic equation to obtain the value of C_R for each speed. A plot of these values of C_R vs. speed will determine for the fixed weight and altitude the maximum value of C_R and the speed at which it will be obtained. The maximum value of C_R for each weight is plotted against the weight, and the area under the curve will be the evaluation of the integral, or the maximum range.

As an example to show the detailed calculations, an airplane was selected of a size and power to meet the performance specifications outlined by Pan-American Airways for trans-oceanic service. The data on this airplane are given in Table I, and the detailed calculations for mean weight are given in Table II. Necessary formulae are also listed in Table I.

Table II is prepared for the series of true airspeeds as shown. The total thrust horsepower may be computed by Oswald's method (Ref. 2) or from the full scale polar of the airplane. To account for the propulsive efficiency of each engine-propellor combination, C_R must be computed for the individual engine-propellor combination, and then divided by the number of engines to obtain the value for the airplane. Accordingly, the thrust horsepower per engine is obtained.

Inasmuch as both η and c are dependent upon rpm, use is made of a parameter which is independent of rpm and which is defined as follows:

$$C_{SJ} = \frac{C_S^{5}}{\eta J^2} = \frac{\rho V^5}{N^2 \text{ bhp}} \frac{1}{\eta} \frac{N^2 D^2}{V^2} = \frac{\rho V^3 D^2}{\text{bhp} \times \eta} = \frac{\rho V^3 D^2}{\text{thp}}$$

where C_S is the propellor speed-power coefficient,

J is the propellor advance ratio.

Note that consistent units must be used to make C_{SJ} dimensionless.

This parameter has been evaluated from existing CALCIT* three-bladed propellor-efficiency charts, and

*Guggenheim Aeronautics Laboratory, California Institute of Technology.

is plotted against propulsive efficiency, for values of constant J (Fig. I). Now, for the given W, V, and computed thp the value of C_{SJ} is computed for entering the above curves. A series of values of J is selected, and the N's corresponding to the assumed J are computed. Then, for each C_{SJ} and J, the corresponding η is determined from the chart. With these η's and the computed thp, values of bhp are calculated from which, together with the corresponding N (converted to engine rpm), the specific fuel consumption, c, is determined. In this connection, it is found convenient to plot fuel consumption curves with the ratio of N/N_{rated} against specific fuel consumption, for constant values of bhp/bhp_{rated}, as in Fig. II, although for a specific airplane it may be advisable to dispense with ratios. The data upon which Fig. II is based are discussed later in the paper. It will be noted that propellor rpm can be used in the ratio, thus saving a step in the calculations. With the values of η and c thus determined for each N, the ratio of η/c is plotted against N, and, in general there will be a maximum in this curve which will give the maximum η/c obtainable and the corresponding optimum N for the specified weight and velocity. Limiting manifold pressure or full throttle for the engine may prohibit the use of values of N which would give a true maximum to the η/c curve. In such cases, the curve of η/c vs. N stops at the value of N corresponding to full throttle, and this will give the maximum attainable η/c. It is important to note, in this connection, that, in general, the maximum η/c will not occur at either maximum η or at minimum c. This note is made because of the fact that in many cases the lowest possible specific fuel consumption has been assumed as the criterion for maximum range.

The above process is repeated for each true airspeed. Then, from the values of V, c (for maximum η/c), and bhp (determined from thp and the η for maximum η/c), the value of C_R per engine is computed and divided by the number of engines to obtain the value for the airplane. A plot of this value against V for the specified weight will, in general, reveal a maximum which will represent the maximum miles obtainable per pound of fuel consumed for the given weight of airplane (e.g. Fig. IIIa). The maximum generally occurs at 1.1 to 1.4 x V for L/D maximum. This curve will also give the miles per pound of fuel consumed for any speed, provided the optimum rpm is used.

Results for initial and final gross weights are tabulated at the bottom of Table II.

If values of $C_{R_{max}}$ for the given weights be plotted against weight, the area under the curve, between the initial and final gross weights, will be seen to represent the maximum range obtainable (e.g. Fig. IIIb).

Endurance, which is of more importance to the military than to the commercial operator, is also obtainable directly by this method. If C_R be divided by the velocity in miles per hour for which it was determined, the result is hours per pound. Therefore, the endurance parameter is defined as $C_E = \frac{1}{bhp \times c}$ See Table II. This, too, is plotted against V as before, and, in general, there will be a maximum on this curve which will represent the maximum hours obtainable per pound of fuel consumed for the given weight of airplane (e.g. Fig. IVa). Then, if values of $C_{E_{max}}$ be plotted against weight, as before, the area under the curve will be seen to represent the maximum endurance obtainable (e.g. Fig. IVb).

If values of C_E be taken from Fig. IVa for the V corresponding to $C_{R_{max}}$, and these values be integrated over the weight range, the result will be the time required to fly the maximum range (e.g. Fig. IIIb).

IV. LIMITATIONS OF THE METHOD

In the preliminary calculations, standard charts were used for determining propulsive efficiency. There is considerable evidence at present that there must be some sort of a multiplicative correction applied to make performance calculations consistent with flight test data. Correction curves have been proposed, as well as the use of arbitrary percentage corrections. However, recent torque indicator tests made to determine accurately the brake horsepower of engines in flight, suggest that the correction might possibly be more properly applied to the test stand brake horsepower, in which case chart efficiencies will apply. The authors feel that further flight tests with the torque indicator will give the correct answer to this question. If, however, a multiplicative factor appears necessary, note that it can be applied directly to the range parameter C_R.

As for the variation of brake horsepower in flight and at altitude, it is felt that the torque indicator will again provide the answer. This in turn will permit charting of true specific fuel consumptions.

The specific fuel consumption curves shown in Fig. II represent the average of test stand runs and flight test reduction for several modern commercial and military aircraft engines. It may be noted that a close agreement exists with curves in Ref. 1 in the region of high horsepowers, but there is a consistent deviation in the low horsepowers. The authors have noted particularly that there is a great lack of data, either from test stand or from flight test, on fuel consumption at low power and low engine speeds, where maximum C_R and C_E are likely to occur.

V. APPROXIMATIONS TO THE METHOD

First Approximate Method:

In computing ranges for several airplanes of gross weights from 20,000 lbs. to 400,000 lbs., it became apparent that the curves of $C_{R_{max}}$ vs. Weight (Fig. IIIb), were very nearly straight lines. Further, the peaks of the C_R vs. V curves (Fig. IIIa) lay approximately along straight lines.

Therefore, if detailed calculations be made for a weight corresponding to the initial gross weight less half fuel (Table II), the value of $C_{R_{max}}$ will be average for the entire range. Hence, this value multiplied by the total fuel weight will give the maximum range. The optimum speed obtained is the average cruising speed over the range.

Second Approximate Method:

If it is desired to use the Breguet formula, the calculation is carried out as above for half fuel weight. Then, using the η/c corresponding to $C_{R_{max}}$, and computing the L/D for the velocity corresponding to $C_{R_{max}}$, the Breguet formula will give the maximum range. The optimum speed is, as before, the average cruising speed.

Sample calculations justifying these approximations are given in Table III for the airplane of Tables I and II.

These approximate methods give only maximum range, and average cruising speed, and are suitable only for preliminary design and comparative calculations. For a specific airplane, it will be necessary to carry out complete calculations, in order to determine optimum propellor rpm, and optimum cruising speeds for various weights, and at various altitudes. The calculations, though laborious, are quicker and more economical than the extensive flight testing now necessary.

VI. EFFECT OF CERTAIN GEOMETRICAL PARAMETERS ON RANGE

In investigating the effect of geometrical parameters upon range, it is necessary to hold as many factors constant as possible, permitting variation in only those which logically must vary. The gross weight was first fixed at 300,000 lbs. A power plant was chosen which preliminary estimates indicated would produce maximum range. Later in the paper, the effect of increasing the power will be discussed.

The general data on this airplane are as follows:

Gross weight . . . . 300,000 lbs.

Payload . . . . 25,000 lbs.

Power plant weight . . . . 30,500 lbs.
6 engines . . 2500 hp @ 1200 rpm (takeoff)
2000 hp @ 1000 rpm (rated)
1500 hp @ 955 rpm (cruising)

Basic weight . . . . 141,000 lbs.
Structure, furnishings, payload, passengers, power plant, less wing weight and fuel weight.

Remaining weight . . . . 159,000 lbs.
for fuel weight and wing weight

From the curve of Fig. V the wing weight is determined, using a design load factor of 5. The balance is then the weight available for fuel, oil and tanks; and the actual fuel weight is assumed as 6/6.25 times the weight available for fuel, oil and tanks.

The curve of Fig. V is taken from a report on wing weights of contemporary airplanes by Dr. J. E. Lipp, of the Douglas Aircraft Company, soon to be published. The particular curve used is based upon an allowable stress of 45,000 lb./in.^{2}, which is in accord with modern practice in the construction of large airplanes. This linear variation of wing weight versus span checks very closely with a similar analysis made by A. E. Lombard, of the Curtiss Wright Corporation.

The procedure followed was to assume a constant aspect ratio and for different wing spans, to compute the range and average cruising speed. This was carried out covering aspect ratios from 9 to 17, and wing spans from 210 ft. to 330 ft. Results are plotted on Fig. VI. Noting that

$$AR = b^2/S \qquad \text{or } S = b^2/AR$$

$$\text{and} \quad \ell_w = W/S = W \times AR/b^2$$

where AR = Aspect ratio
b = wing span
S = wing area
ℓ_w = wing loading
W = gross weight

we see that when we have investigated range variation with aspect ratio and span, we have also investigated its variation with wing loading, and we may therefore draw in lines of constant wing loading, which has been done on Fig. VI. The investigation was not carried beyond aspect ratio 17 because it was felt that wing weight data might no longer hold for the type of structure which would be necessary. Furthermore, the percentage increase in maximum range with higher aspect ratios is close to the limit of the accuracy of the calculations. Similarly, constant wing loading lines are not carried beyond 70 because takeoff and sea-level rate of climb requirements eliminate higher values from practical consideration, unless additional power is provided, and this in itself reduces maximum range. Fig. VI also shows the variation of average cruising speed with the several parameters.

If the figure were plotted on a normal scale (i.e., from zero on the range ordinate) the variation would be difficult to pick out. Therefore Fig. VI is plotted to a larger scale. Note that a half inch on the range scale corresponds to about a 3% variation in maximum range, while the same division corresponds to about a 6% variation in average cruising speed.

A study of the figure indicates that in the range of values considered there is a gain in maximum range to be expected from increasing aspect ratio, and from decreasing wing span, the combination producing optimum range. It follows, as further indicated on the figure, that we may expect increases in maximum range with increased wing loading, and, in general, there will be an optimum combination of aspect ratio and span for a given wing loading. However, the gain in range caused by increase in wing loading is relatively small for wing loadings above 50.

Takeoff calculations were carried out for this airplane as a landplane following the method of Ref. 3, to determine limiting wing loading for practical takeoff. Inasmuch as present regulations require that landplanes climb over a fifty-foot obstacle after takeoff, with the air temperature at 110° F., the additional distance required by these specifications is included in takeoff distance. Calculations indicated that takeoff distance is approximately independent of aspect ratio, and depends principally on wing loading. Calculated takeoff distances are as follows:

Wing Loading	Takeoff Distance
30	4745 ft.
40	5535 ft.
50	7075 ft.
60	8880 ft.
70	10230 ft.

Takeoff distances in excess of 7000 ft. are considered as impractical for a landplane, and therefore further landplane considerations were restricted to airplanes with wing loadings of 50 or less.

Seaplane (flying boat) takeoff calculations were made for comparison with landplane takeoffs. The method of Ref. 4 was used, noting that seaplanes are not subjected to the same temperature and climb-over-obstacle penalties as landplanes. Further, the effect of increased power on takeoff of both seaplanes and landplanes, and on range and average cruising speed was determined. To compensate for increased power plant weight, fuel weight was deducted, keeping the gross weight constant.

All of the final takeoff results are collected in the following table, where range and cruising speed results are also included. Landplanes are assumed to be aerodynamically equivalent to the corresponding seaplanes.

Power-6 Engines
Power Loading 20

Wing Loading	Type	Takeoff Time	Takeoff Distance	Range	Speed
30	Sea	62	4360	5580	145
	Land	51	4745		
40	Sea	93	7430	6030	155
	Land	54	5535		
50	Sea	145	15400	6320	165
	Land	64	7075		

Power-8 Engines
Power Loading 16.7

Wing Loading	Type	Takeoff Time	Takeoff Distance	Range	Speed
30	Sea	31	2200	4870	145
	Land	38	3525		
40	Sea	43	3430	5280	155
	Land	39	4285		
50	Sea	56	5460	5350	165
	Land	43	4695		

This tabulation indicates quite simply that increasing power to gain a better takeoff invariably results in a considerable decrease in maximum range, with no gain in average cruising speed. Remembering that increased maximum range corresponds directly to increased payload for a fixed shorter range, it is apparent that with unassisted takeoff the relatively low-wing-loading, low-powered airplane with satisfactory takeoff, and with suitable cruising speed, is the more practical commercially. If increased speed is needed for express service, the penalty must be taken in payload. Further, the takeoff differential between seaplanes (flying boats) and landplanes of large size for long range, is not as great as might be expected from analyses on smaller planes, and in some cases the seaplanes will have better takeoff.

Fig. VI indicates that the range of a large airplane with a wing loading of 20-30 and marginal takeoff characteristics may be substantially increased by overloading with fuel to a wing loading of 40-50 and employing assisted takeoff. However, the increase in range will not be directly proportional to the amount of fuel added, because the new average C_R over the range will be considerably lowered. This can be seen from Fig. IIIb, where the increased range will be represented approximately by the area of a trapezoid produced by extending the C_R curve to the overload weight. This area will be further reduced by an amount roughly proportional to the increased structural weight incident to stressing for the overload weight, and for assisted takeoff.

VII. CONCLUSIONS

1. Range calculations, including the effect of variation in all important parameters, are practicable.

2. For maximum range at a given altitude, there will be an optimum true air speed for any given weight condition of the airplane, and for this optimum true air speed, there will be an optimum propellor rpm. These optima are obtainable from the method outlined in this paper.

3. For comparative work, and for preliminary design work, it is possible to determine maximum range and average cruising speed by an approximate method. Results will be within 2% of those obtainable by complete calculations. For the preparation of operating charts, complete calculations are necessary.

4. In the range of values considered, maximum range will increase with increased wing loading. In general, for a given wing loading, there will be an optimum combination of span and aspect ratio. Increases due to wing loadings in excess of 50 are small.

5. Takeoffs are generally satisfactory for either a landplane or a seaplane, for wing loadings below 50.

VIII. ACKNOWLEDGMENT

The authors desire to express their appreciation for the invaluable assistance rendered by Dr. Clark B. Millikan, and Mr. W. C. Rockefeller, in the preparation of this paper.

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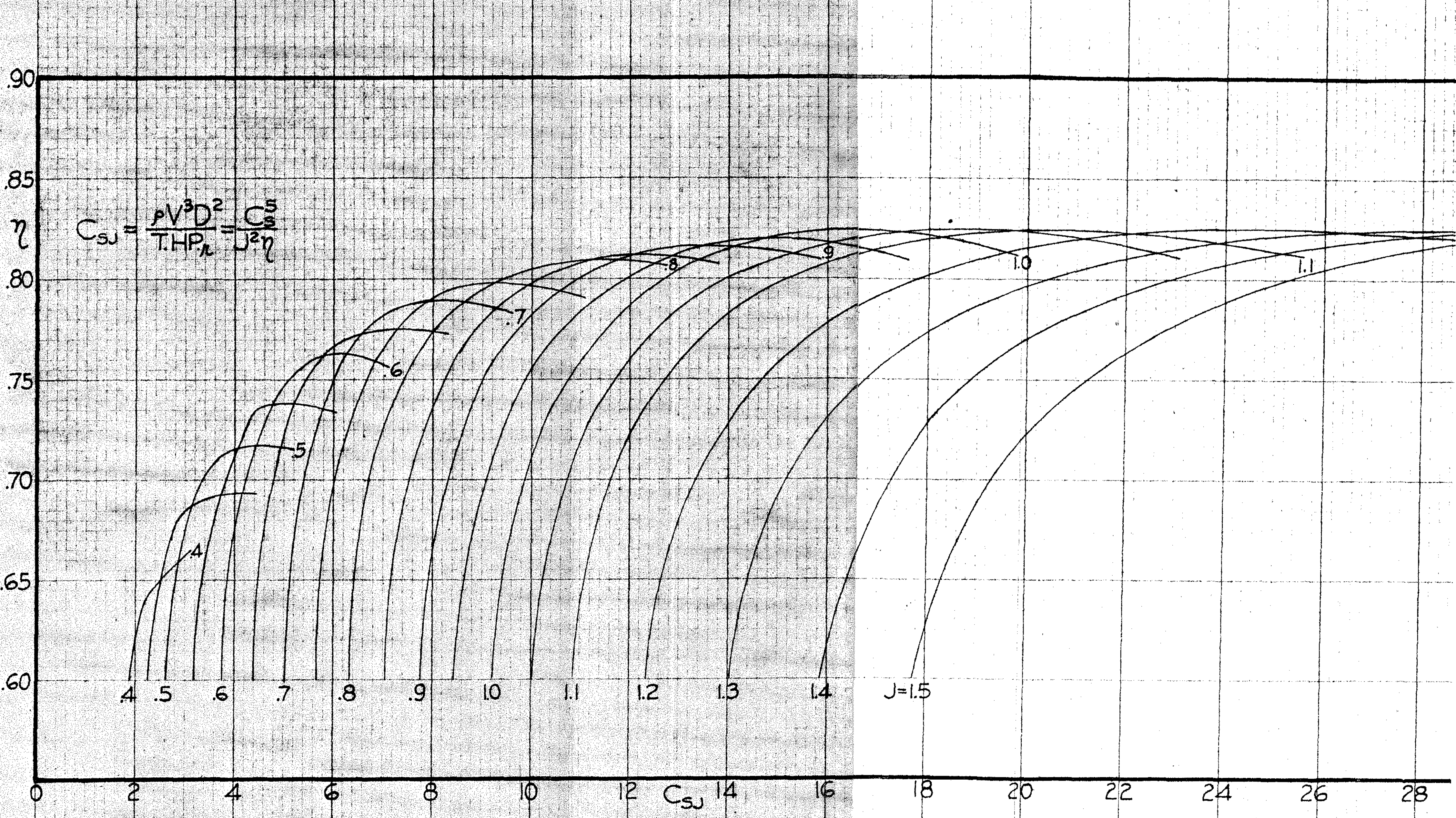
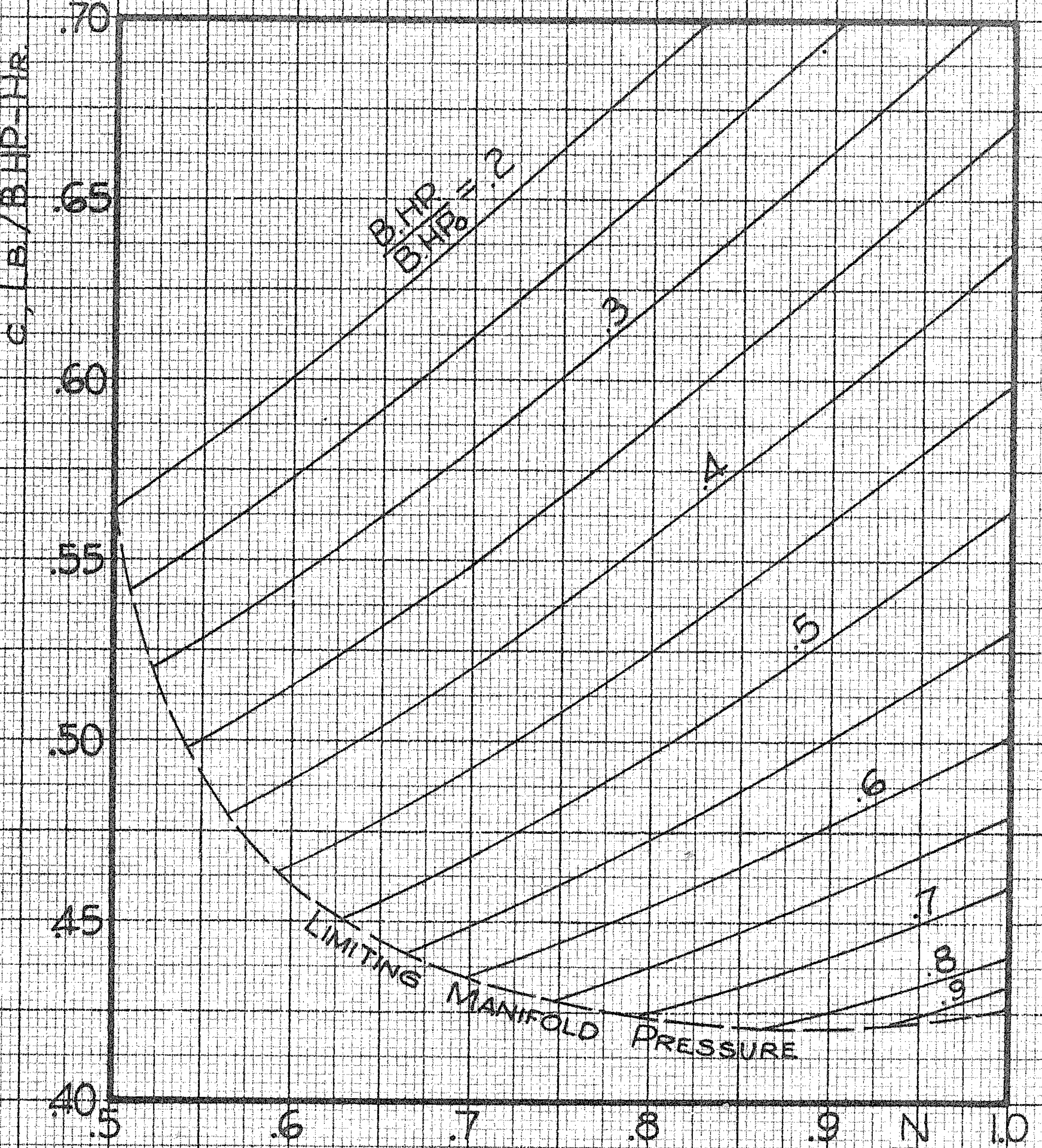


GALCIT

3 Bladed Propeller Characteristics

η vs. C_{SJ} for Various J



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AVERAGE SPECIFIC FUEL CONSUMPTION CURVES
FOR RADIAL ENGINES OF 600 TO 1000 HP.

FIG II

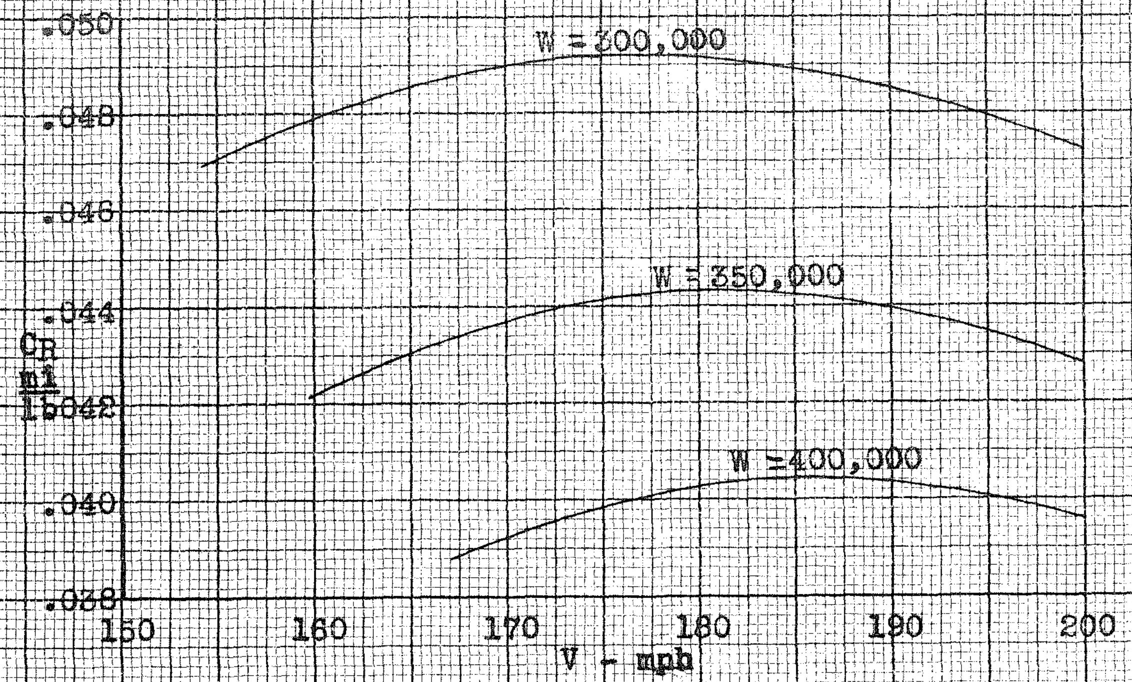


Fig. IIIa

C_R vs. V for constant gross weight.

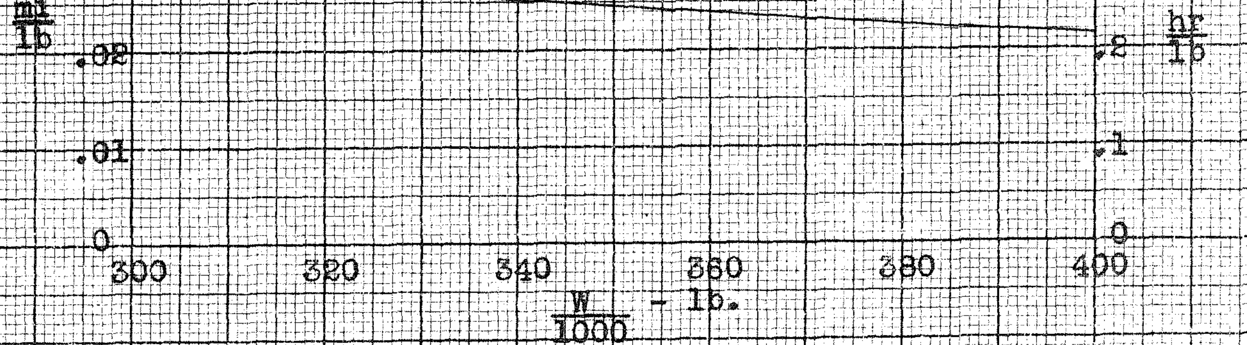


Fig. IIIb.

C_R max and C_E $C_R$max vs. Weight.

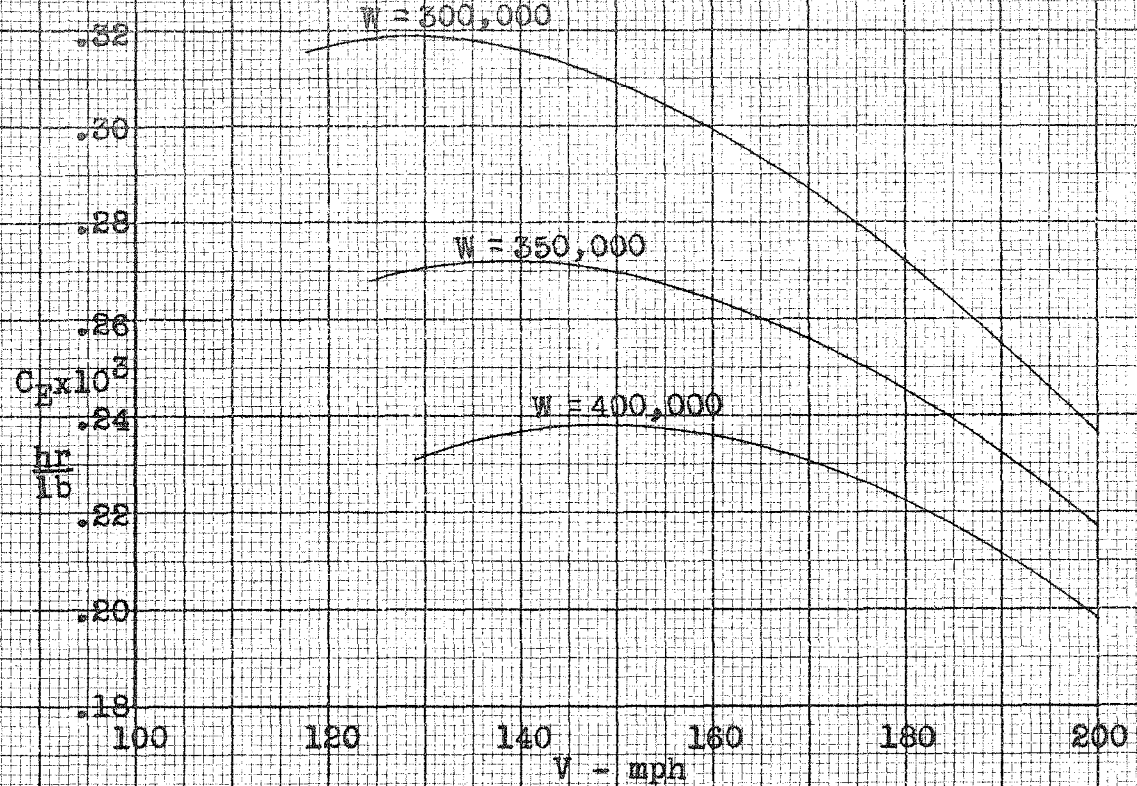


Fig. IV a

C_E vs. V for constant gross weight.

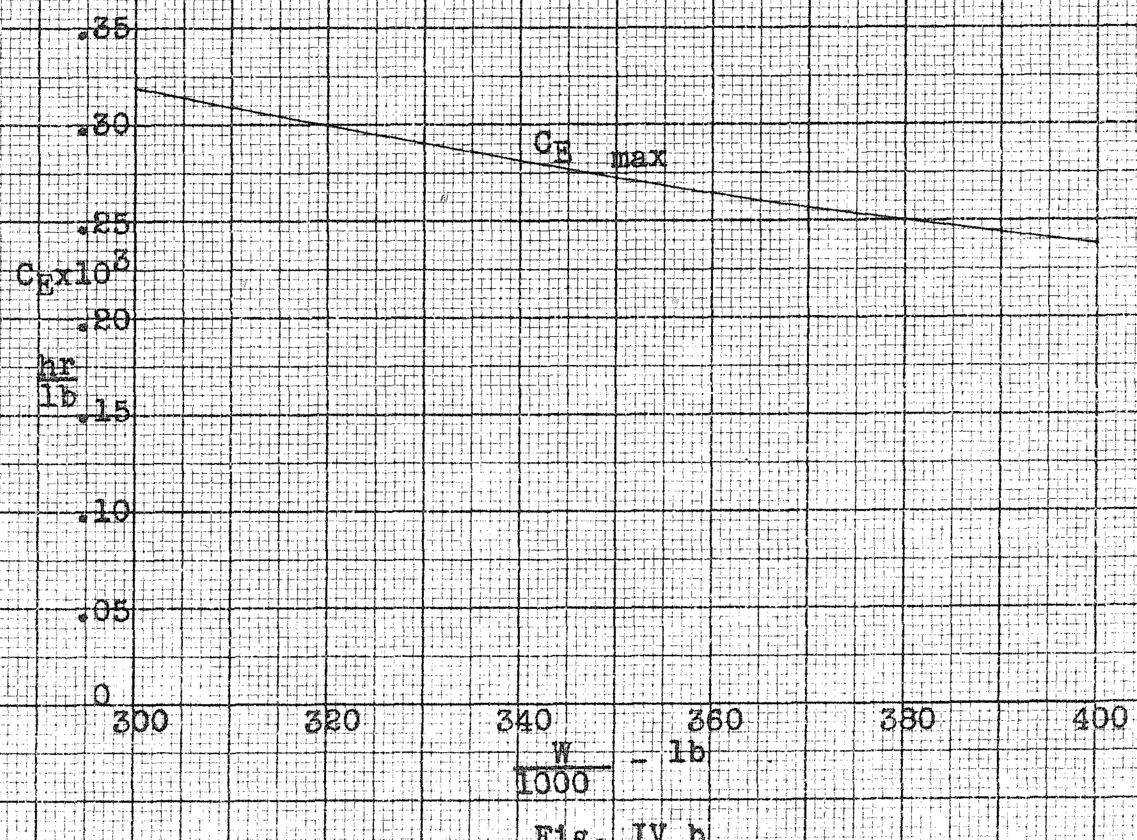
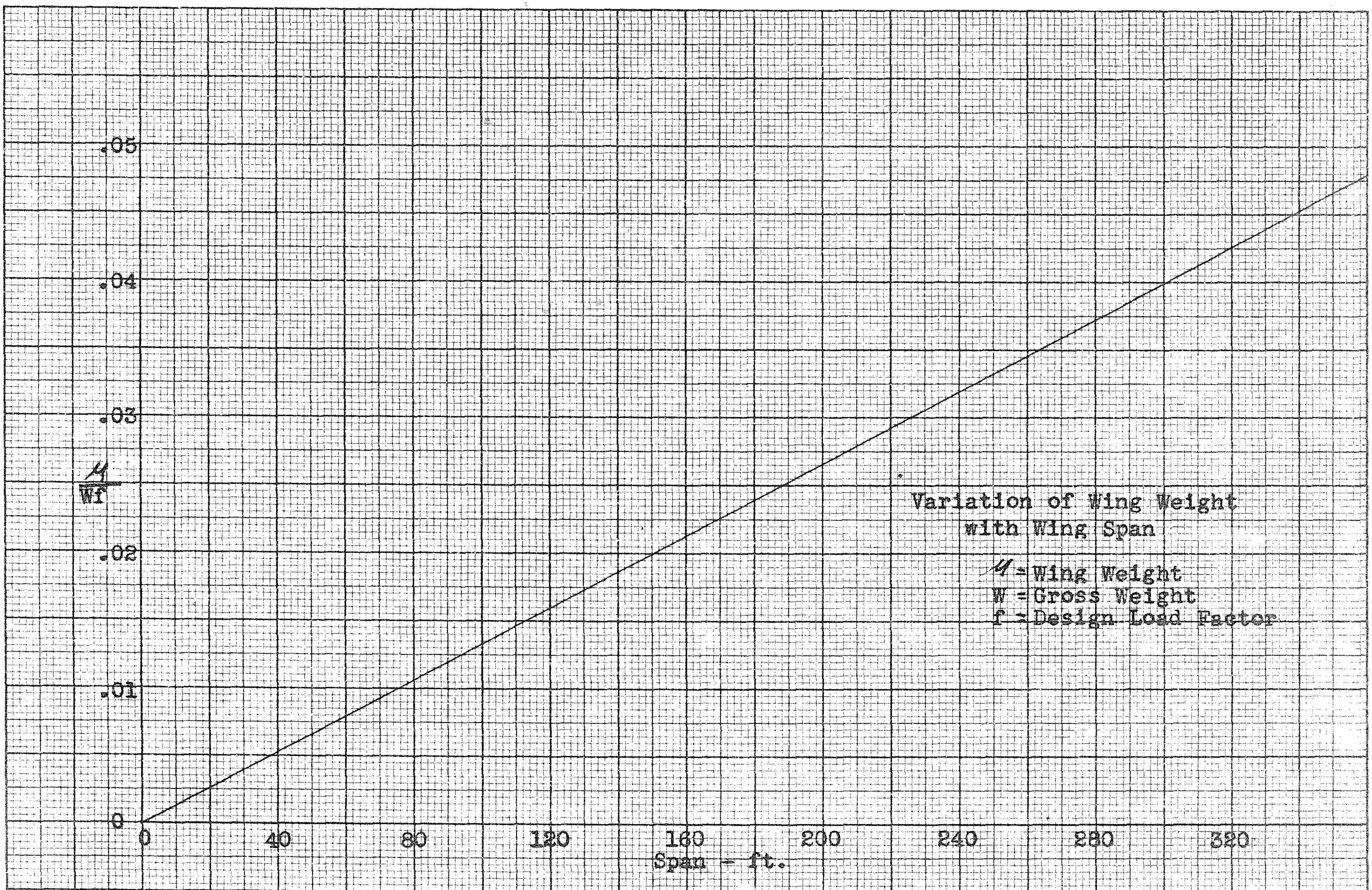


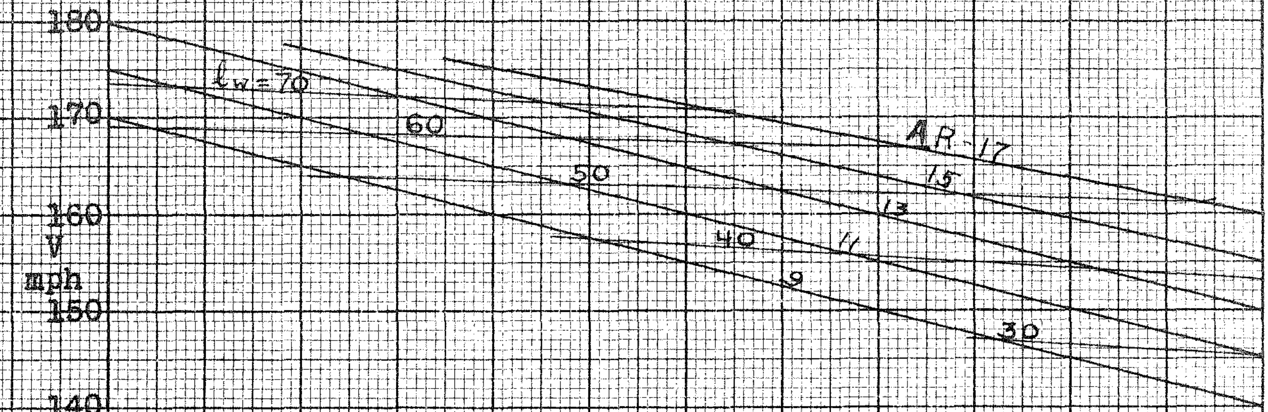
Fig. IV b

$C_E$max vs. Weight

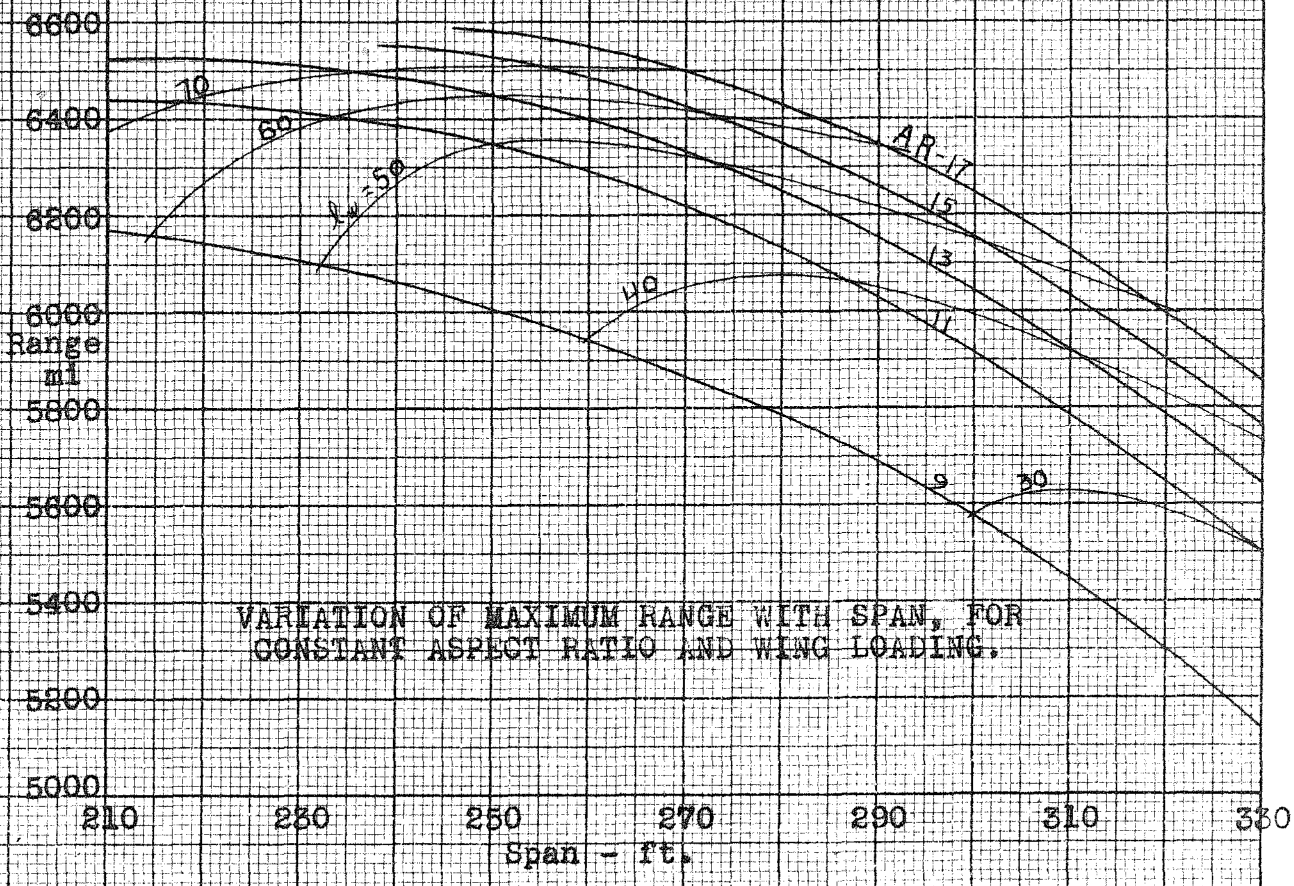


.05
.04
.03
Wf
.02
.01
0
0
40
80
120
160
200
240
280
320
Span - ft.
Variation of Wing Weight
with Wing Span
= Wing Weight
W = Gross Weight
f = Design Load Factor

Fig. V



VARIATION OF AVERAGE CRUISING SPEED FOR MAXIMUM RANGE WITH SPAN, FOR CONSTANT ASPECT RATIO AND WING LOADING.



VARIATION OF MAXIMUM RANGE WITH SPAN, FOR CONSTANT ASPECT RATIO AND WING LOADING.

Fig. VI.

Table I

Specifications on Airplane
For Sample Range Calculations

Gross weight	400,000 lbs.
Power plant	10 engines
2500 hp @ 1200 rpm (takeoff)	
2000 hp @ 1000 rpm (rated)	
1500 hp @ 955 rpm (cruising at 10,000 ft.)	
Fuel weight (assumed)	100,000 lbs.
Aspect ratio	11
Span (b)	330 ft.
Wing loading	40
Propellor diameter (d)	17 ft.
Airplane efficiency factor (e)	0.9
Equivalent parasite area (f)	152 sq. ft.

Formulae involved:

$$\sigma = \rho/\rho_0 = \text{atmospheric density ratio}$$

$$\ell_s = W/eb^2$$

$$\ell_p = W/f$$

$$V_{L/D_{max}} = \frac{14.85}{\sigma^{1/2}} (\ell_s \ell_p)^{\frac{1}{4}}$$

$$\frac{thp}{W} = \frac{6.8 \times \left(\frac{V}{100}\right)^3 \times \sigma}{\ell_p} + \frac{0.322 \times \ell_s}{\sigma \times V}$$

$$C_{SJ} = \frac{\sigma \times 13.67 \times d^2 \times \left(\frac{V}{100}\right)^3}{thp}$$

$$J = \frac{88 \times V}{N \times d}$$

$$C_R = \frac{V}{thp} \times \frac{\eta}{c}$$

$$C_E = \frac{1}{thp} \times \frac{\eta}{c} = \frac{C_R}{V}$$

$$L/D_{V_{opt}} = \frac{1}{\frac{\sigma \times V^2}{391 \times \ell_p} + \frac{124.3 \times \ell_s}{\sigma \times V^2}}$$

Note: V is in mph in all formulae listed above.

Table II.

Sample range calculations.

Mean weight..... 350,000 lb.
Altitude.. 10,000 ft. $\sigma = 0.735$
$\ell_s = 3.61$ $\ell_p = 2300$ $V_{L/D_{max}} = 165$ mph.

V	130	140	150	160	170	180	190
thp	6080	6160	6370	6690	7090	7600	8230
thp/eng	608	616	637	669	709	760	823
C_{SJ}	10.5	12.9	15.4	17.8	20.0	22.2	24.0
J	0.90	0.95	1.00	1.05	1.05	1.10	1.20
	0.95	1.00	1.05	1.10	1.10	1.20	1.30
	1.00	1.05	1.10	1.20	1.20	1.30	1.40
	1.05	1.10	1.20	1.30	1.30	1.40	1.50
η	.794	.811	.823	.825	.824	.823	.825
	.773	.803	.815	.819	.825	.823	.820
	.741	.783	.800	.800	.815	.812	.807
	.695	.725	.765	.770	.797	.795	.790
N/N_o	.781	.798	.814	.827	.880	.888	.860
	.740	.757	.775	.790	.839	.813	.794
	.703	.721	.740	.723	.769	.751	.737
	.675	.665	.679	.668	.710	.698	.688
bhp/bhp_o	.510	.506	.516	.540	.574	.615	.665
	.524	.512	.521	.545	.573	.615	.669
	.546	.525	.531	.557	.580	.623	.680
	.570	.550	.555	.579	.593	.638	.695
c	.486	.492	.492	.485	.485	.470	.443
	.468	.478	.480	.472	.474	.450	.430
	.450	.460	.465	.450	.455	.435	x
	.439	.441	.441	x	.439	x	x
η/c	1.63	1.65	1.67	1.70	1.70	1.75	1.86
	1.65	1.68	1.70	1.73	1.74	1.83	1.90
	1.54	1.70	1.72	1.77	1.81	1.86	x
	1.58	1.64	1.73	x	1.82	x	x
C_R/eng	.352	.383	.405	.422	.435	.443	.439
C_R	.0352	.0383	.0405	.0422	.0435	.0443	.0439
$C_E x10^3$	.271	.273	.270	.264	.256	.246	.231

Results for W = 400,000 lb. $V_{L/Dmax}$ = 175 mph.

V	140	150	160	170	180	190	200
C_R	.0332	.0357	.0378	.0392	.0402	.0404	.0397
$C_E x10^3$	.237	.238	.236	.230	.223	.213	.198

Results for W = 300,000 lb. $V_{L/Dmax}$ = 155 mph.

V	130	140	150	160	170	180	190
C_R	.0415	.0442	.0464	.0478	.0489	.0492	.0485
$C_E x10^3$	.319	.316	.309	.299	.287	.272	.255

Table III

Comparison of Results by Complete Method
With Those of Approximate Methods

Complete Method: (by integration under curves)

Range .... 4505 miles

Average speed .... 185 mph.

First Approximate Method:

From Table II, for $W = 350,000$ lbs.,

$C_{R_{max}}$ .... .0443

Range $= .0443 \times 100,000$ .... 4430 miles

Average speed (from Table II) .... 185 mph.

Second Approximate Method:

$\eta / c \backsim V_{opt}$ of 185 mph (by interpolation in Table II) 1.88

$L/D \backsim V_{opt}$ of 185 mph (by formula) 22.0

$\log_{10} \dfrac{400,000}{300,000}$ 0.126

Range $= 863.5 \times 1.88 \times 22 \times 0.126$ .... 4500 miles