Chapter 7

LCS and the Global Ocean

The Intergovernmental Panel on Climate Change (IPCC) has declaratively assessed that "warming of the climate system is unequivocal" and that "many natural systems are being affected by regional climate changes" [Solomon 2007]. Addressing climate change through accurate global climate prediction and sustainable engineering is arguably the most important scientific challenge of the twenty-first century.

Mesoscale eddies account for the majority of kinetic energy in the global ocean, and are largely responsible for the dispersion of heat and mass [Li 2008]. Chelton has similarly observed that more than fifty percent of ocean variability in satellite data is due to mesoscale eddies [Chelton 2007]. Consequently, accurate modeling of eddy activity, and the effect it has on heat budgets in the ocean, is imperative for faithful predictions of global climate. Yet, in the recent *Fourth Assessment Report of the IPCC* [Solomon 2007], only one of the twenty-four global climate models explicitly simulated mesoscale eddy activity [Bryan 2008]. Constraints on computing power, and the need for ensemble averages over long time scales, necessitate model simulations at large enough gridscales that only the large-scale laminar flow is captured, while the effects of unresolved eddies are included entirely by modeling, or *parameterization*. In a recent *Nature* article, Scheiermeier has observed that the wealth of ocean data now available from satellites and globally deployed drifters has confirmed the important role of mesoscale eddies in transport and stirring, and also revealed that widely-used model parameterizations of mesoscale eddies (variations of the Gent-McWilliams scheme introduced in 1990 are most commonly used [Gent 1990]) fail to capture eddy mixing accurately [Schiermeier 2007]. Most notably, parameterizations of eddy stirring are typically employed uniformly under the assumption that stirring is homogeneous throughout the ocean; an assumption that the data now reveals is a significant weakness of ocean models. Although parameterizations have led to many modeling improvements, discrepancies between ocean observations and the parametrized models are not unexpected since the parameterization schemes were developed primarily using theoretical considerations of the primitive equations without access to large sets of ocean data or eddy-resolving models. For this reason, parameterizations fail to capture the influence of unresolved topography, bottom drag, and regionally specific sensitivities to variations in parameters [Fox-Kemper 2008].

Similarly, the energy exchange between the ocean and atmosphere reveals a rich interplay between rising ocean temperatures and the frequency and intensity of tropical storms, and motivates the need to more fully understand parameterization of transport and flux rates in the atmosphere as well. Emanuel, for instance, has proposed (not without controversy) that the rise in ocean temperatures has led to increased intensity of tropical storms, which in turn drives mixing in the ocean [Emanuel 2001, Emanuel 2007].

Increased computing power has recently permitted the advent of *eddy-resolving* global simulations; however, ensemble runs of climate-length simulations at eddy-resolving resolutions will not be feasible in the foreseeable future [Hecht 2008a]. Nevertheless, the availability of high resolution models, at least on short time scales, and the stream of new observational data is a boon not previously enjoyed by ocean modelers. To this end, the UK Meteorological Office hosted a "Workshop on Mesoscale Eddies" in April, 2009, with the stated purpose of "educating the research community regarding the importance of mesoscale eddies" and to "identify best practices for parameterizing ocean mesoscale eddies in coarse resolution climate models, and to discuss various research avenues for improved parameterizations" [1].

Certainly the use of LCS has a role to play in providing insight in to the regional

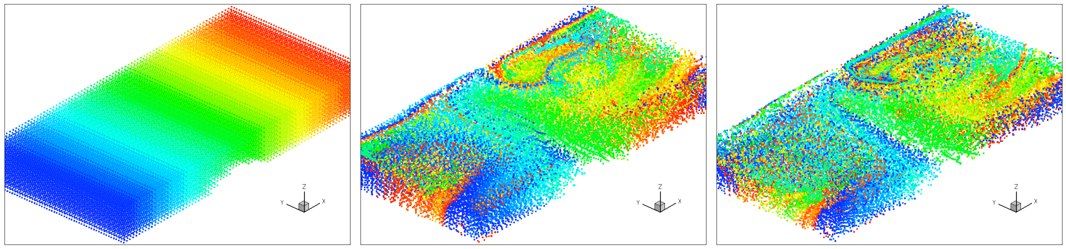
[1] See the Workshop webpage online at `http://www.metoffice.gov.uk/conference/mesoscale_workshop/`

variation of eddy activity and mixing, and revealing the mechanisms by which mixing occurs. Initial results in this regard are promising. The LCS are barriers to transport that define the boundaries to eddies and reveal the pathways for transport. Moreover, the LCS analysis provides a visualization of the transport mechanisms induced by mesoscale eddies, thus providing a clearer understanding of the mixing process. The transport mechanisms revealed in this way are turbulent, but their underlying structure is remarkably low-dimensional.

We begin our study of global ocean currents by investigating an idealized model for flow in the Antarctic Circumpolar Current. We then proceed to an investigation of reanalysis data for the global ocean.

7.1 A model for the Antarctic Circumpolar Current

The Antarctic Circumpolar Current is the only major ocean current that flows continuously without impinging on a continental shelf, and is an important component in the ocean "conveyor belt" that transfers heat energy and momentum between the major ocean basins. The current carries both warm water at its surface, as well as cool waters in its deep bottom currents. In [Hecht 2008a, Hecht 2008b], Hecht presented a Lagrangian-Averaged Navier Stokes (LANS-α) model for ocean flow, and applied the model to flow in a three-dimensional channel that idealizes the Antarctic Circumpolar Current. The channel used in the model has solid rectangular sides in the zonal directions, and the boundary conditions at the open ends are determined by reentrant flow which flows predominantly from West to East. The model includes wind and thermal forcing at the surface. The topography along the bottom of the channel includes a deep sea ridge, the effect of which is to induce northward flow immediately before the ridge, and southward flow immediately after the ridge. During the course of this meandering flow, eddies are produced that induce stirring and mixing. The intent of [Hecht 2008b] was to confirm that the LANS-α model captures



(a) (b) (c)

Figure 7.1: Snapshots of drifter trajectories in the idealized channel model of the Antarctic Circumpolar Current. The drifters are colored by their initial longitude, and flow is predominantly from West to East. A North-South ridge in the topography can be seen near the center of the channel. After a short time (less than the time for the majority of particles to travel the length of the channel), the drifters appear well-mixed.

the energy cascades to smaller scales correctly so that eddies are produced (a major problem with many coarse-grid ocean models is that they capture insufficient eddy kinetic energy.)

Our purpose, will be to study the transport structures that are responsible for mediating mixing in this channel flow. Simulation of passive drifters reveals that the turbulent flow due to the deep sea ridge quickly induces mixing of particle trajectories. Figure 7.1 shows three frames in the simulation of trajectories, in which drifters are colored by their initial zonal location. The final figure indicates that the particles are well-mixed, and the hypothesis that the mixing is fully turbulent and well-modeled by diffusion or a random walk appears valid. The computation of the LCS, however, reveals a different picture.

Figures 7.2(a) and 7.2(b) depict the FTLE scalar field for the channel flow model, colored so that the LCS appear as white curves. Considering the complexity of the flow, the low dimensionality, coherence, and stability of the resulting flow structures are quite surprising. The LCS reveal the boundaries of several major eddy structures that in concert provide the stirring mechanisms that induce the chaotic drifter trajectories. Some of the eddies pair into doublets that are reminiscent of familiar flows in dynamical systems texts such as the Duffing oscillator. The overall eddy structure is remarkably stable to perturbations in the flow. In fact, the perturbations allow for exchange of drifters between the eddies through lobe dynamics and enhance mixing in the same manner we have observed previously in the perturbed pendulum. Here we observe that the LCS provide the "skeleton" of turbulence, as Haller has demonstrated previously for laboratory flows [Mathur 2007].

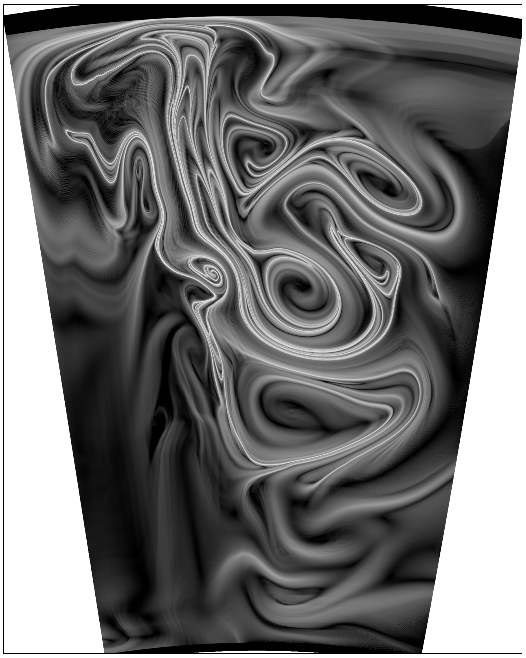
The insight gained by the LCS analysis of flow in the idealized channel, and the identification of eddy structures, bode well for the use of FTLE-LCS for understanding the role of mesoscale eddies in global ocean data, which we turn to next.

7.2 Reanalysis data for the global ocean

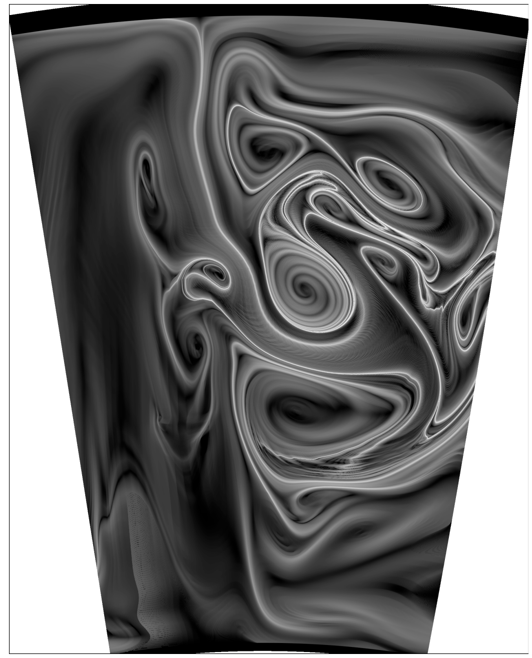
For our study we have used reanalysis data for global ocean currents provided by the Mercator Ocean Modeling group [Drevillon 2008]. The Mercator data product is produced by running a global ocean model at high resolution while assimilating satellite observations of sea surface height data – the primary driver of mesoscale eddy activity. The model also incorporates data received from merchant ships, as well as data retrieved from thousands of drifters deployed throughout the global ocean that relay current data through satellite uplinks.[2]

Computation of the FTLE for the global ocean reveals a rich labyrinth of coherent mesoscale eddy structures. Figure 7.3 shows the LCS computed for the global ocean for an integration time of three weeks. Perhaps most striking is the inhomogeneity in the distribution of the LCS. The Antarctic Circumpolar Current, the Atlantic Gulf Stream, the Cape Cauldron at the Southern tip of Africa, and the equatorial jets in the Eastern Pacific are all highlighted as regions of robust activity. Images of all these regions are shown in higher resolution in Figures 7.4(a) through 7.4(c). Taken purely from a visual perspective, the FTLE computation conveys intricate details about coherent structures in the ocean. These images are to be contrasted with plots of velocity, vorticity (see Figure 7.5), and sea surface height that are the more traditional plots used by oceanographers. Certainly, these are all important quantities to study, but do not capture the Lagrangian information necessary to understand transport

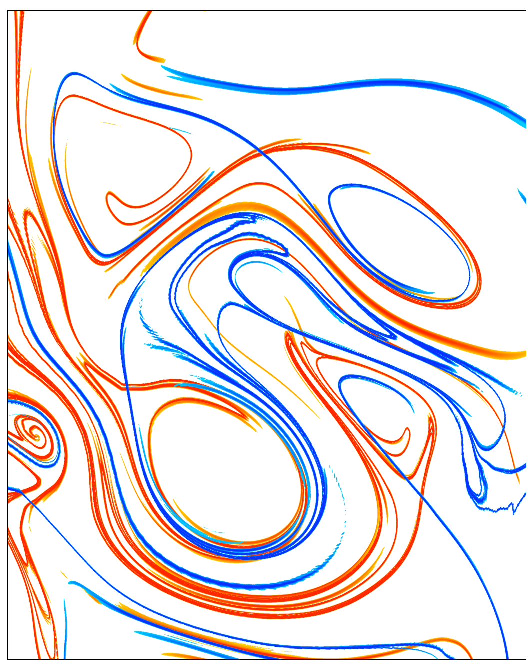
[2]See the Argo project at http://www.argo.ucsd.edu/index.html



(a) Repelling FTLE.



(b) Attracting FTLE.



(c) Superimposing the repelling LCS (red) and the attracting LCS (blue) indicates the presence of familiar objects from low-dimensional dynamical systems, such as the Duffing oscillator.

Figure 7.2: LCS in flow of the idealized channel model of the Antarctic Circumpolar Current for the section of channel immediately behind the deep sea ridge.

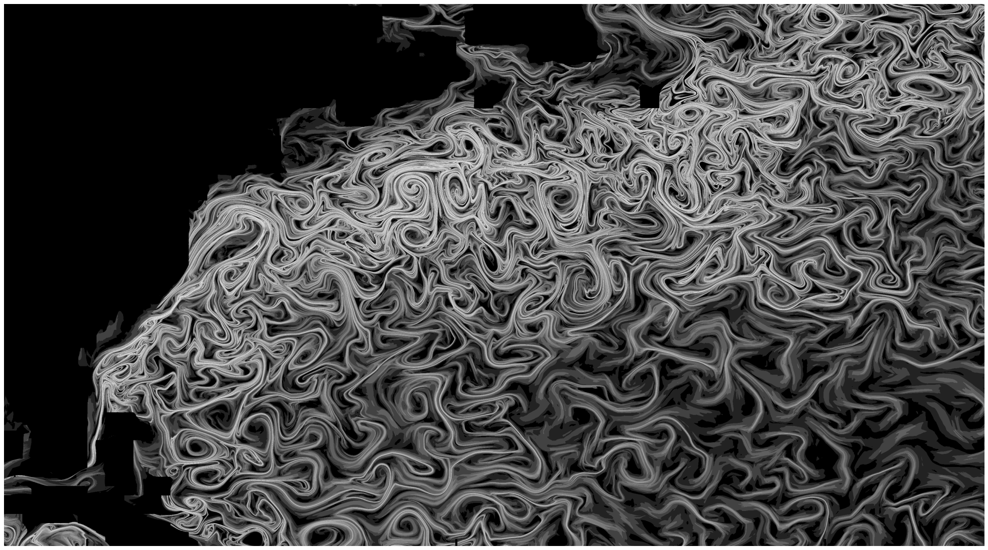
mechanisms in mesoscale eddies. The LCS in the higher resolution images reveal the boundaries of the mesoscale eddies, and the time-dependence of the LCS reveals the manner in which eddies are created, interact with each other, and are finally destroyed, or subsumed by neighboring eddies.

Figure 7.4(c) shows a close-up of the Cape Cauldron off the Southern coast of Africa. The Agulhas retroflection is a well-known current formation that brings warm water from the equatorial ocean along the Eastern coast of Africa to the Southernmost tip of Africa, where the current undergoes a dramatic retroflection and joins the Eastward flow of the Antarctic Circumpolar Current. As a consequence, the Agulhas retroflection plays an important role in mixing warm surface water into the Antarctic Circumpolar Current. In the visualization provided by the FTLE computation, we see that the retroflection is not simply retrograde motion of bulk laminar flow, but rather the current is teeming with eddies that are vigorously inducing and enhancing mixing of tracers. Animations of the LCS reveal that the eddies interact strongly with each other while being advected in the direction of the mean flow.

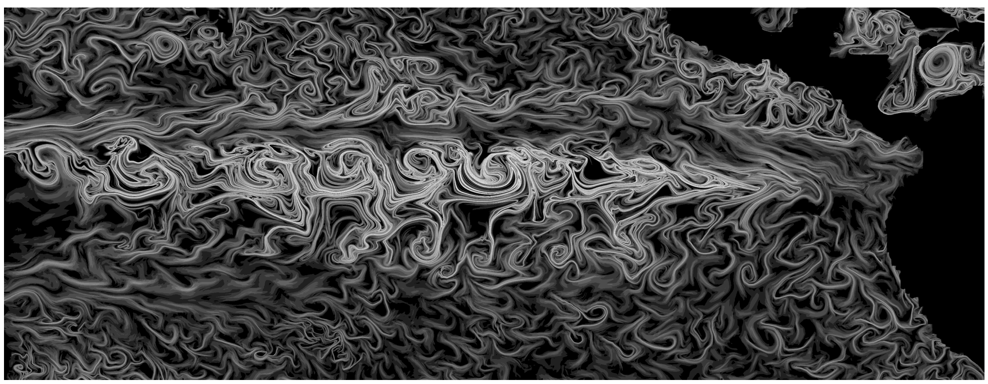
An important feature revealed by the LCS, in relation to the role of mesoscale eddies is that the eddies are indeed the driving "atoms" of transport and mixing. If mixing in the ocean were diffusion-dominated, then we would not expect to see sharp LCS structures – the FTLE would be a diffuse field without strong ridges. To the contrary, strong LCS ridges are evident, and eddies retain their integrity and exchange material through complex networks of lobes. Zooming in on a single eddy in the FTLE field, as in Figure 7.6, reveals that the boundary of the eddy, even for this turbulent ocean flow, has the familiar shape of the perturbed homoclinic tangle observed previously in the pendulum and in hurricanes.

Taking time to view the images of the FTLE provides insight into the nature of turbulent structures in the ocean, and in particular the role that mesoscale eddies play in mediating mixing. The time-dependence of the LCS reveals the mechanisms by which folding, stretching, and transport occur. From a mathematical viewpoint, it is reward enough to see that the structures envisioned by Poincare in the three-body problem, and then reformulated abstractly by Smale, can be found

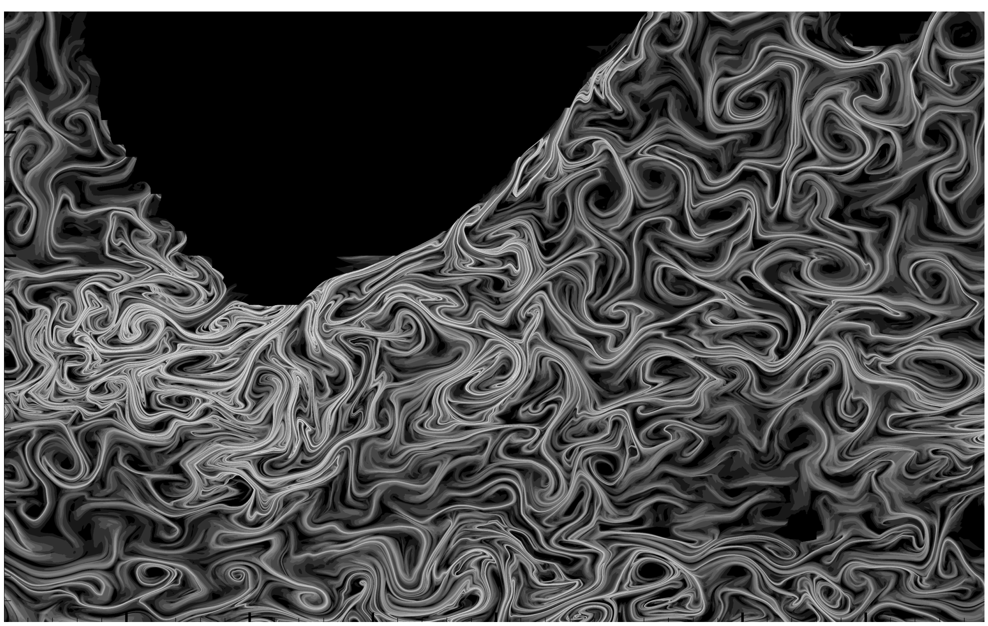
Figure 7.3: Repelling FTLE for the Global Ocean. The LCS reveal boundaries to mesoscale eddies that are responsible for lateral mixing. Regions of intense activity include the Pacific Equatorial jets, the Atlantic Gulf Stream, the Cape Cauldron, and the Antarctic Circumpolar Current.



(a) The Atlantic Gulf Stream.



(b) The Pacific Equatorial Jets.



(c) The Cape Cauldron.

Figure 7.4: Mesoscale eddies visualized by the LCS reveal that the major ocean conveyor belts are teeming with eddies that constantly stir and mix the flow.

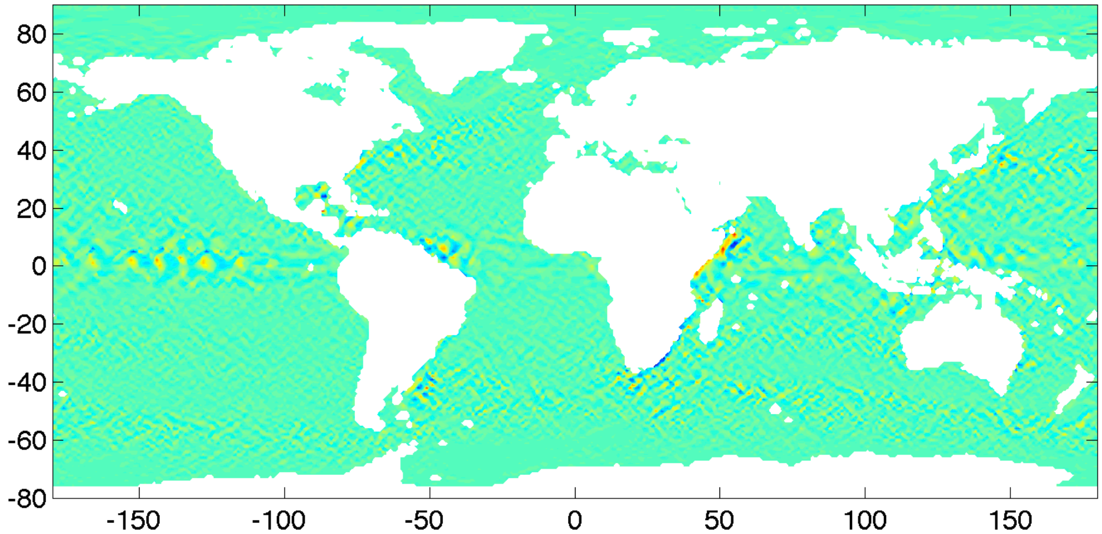
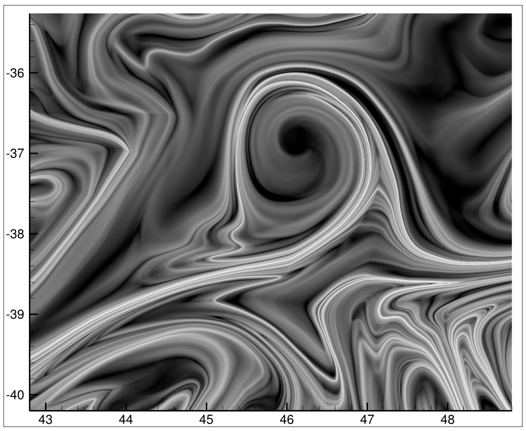
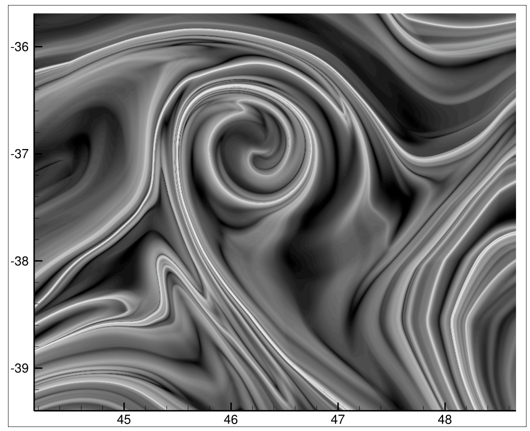


Figure 7.5: A plot of vorticity in the Global Ocean indicates the presence of eddies, but does not provide the same intricate detail as the LCS analysis. Moreover, the time-dependence of the vorticity does not indicate the mechanism responsible for transport and mixing between the eddies.

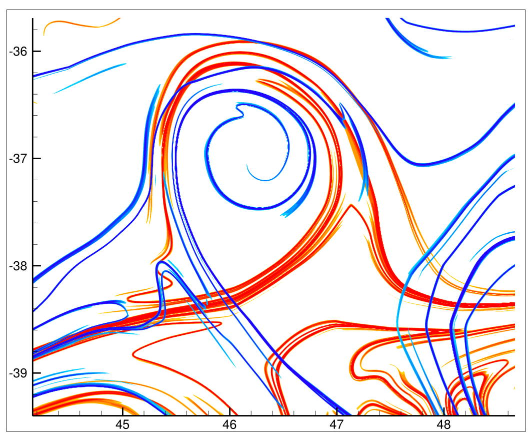
vigorously at work in the global ocean. To the oceanographer, these analyses will hopefully provide insight into the structure of turbulence, its spatial inhomogeneity, and the mechanisms by which mixing occurs, so that future parametrizations of these processes can be better informed. The realization of the importance of lobe dynamics suggests that the dynamical systems community may be able to shed helpful light on the the eddy parametrization problem. The studies of Rom-Kedar [Rom-Kedar 1990, Rom-Kedar 1991], for example, provide parametrized rates of mixing via lobe dynamics in a homoclinic tangle that may find useful application to the eddy parameterization problem in global ocean models.



(a) Repelling FTLE.



(b) Attracting FTLE.



(c) The repelling LCS (red) and the attracting LCS (blue) are superimposed to reveal the boundary of a mesoscale eddy.

Figure 7.6: LCS in the Global Ocean reveals the boundary of a mesoscale eddy. The boundary has the familiar shape of the homoclinic tangle, as well as the boundary of hurricanes computed in Chapter 6. The time-dependence of the LCS reveals the mechanism for transport into and out of the eddy, and demonstrates that the eddy is the 'atom' of mixing.