Abstract

The method of using Finite Time Liapunov Exponents (FTLE) to extract Lagrangian Coherent Structures (LCS) in aperiodic flows, as originally developed by Haller, is applied to geophysical flows, and flows in the phase space of second order dynamical systems. In this approach, the LCS are identified as surfaces of greatest separation that parse the flow into regions with different dynamical behavior. In this way, the LCS reveal the underlying skeleton of turbulence. The time-dependence of the LCS provides insight into the mechanisms by which fluid is transported from one region to another. Of especial interest in this study, is the utility with which the FTLE-LCS method can be used to reveal homoclinic and horseshoe dynamics in aperiodic flows.

The FTLE-LCS method is applied to turbulent flow in hurricanes and reveals LCS that delineate sharp boundaries to a storm. Moreover, intersections of the LCS define lobes that mediate transport into and out of a storm through the action of homoclinic lobe dynamics. Using FTLE-LCS, the same homoclinic structures are seen to be a dominant transport mechanism in the Global Ocean, and provide insights into the role of mesoscale eddies in enhancing lateral mixing.

Beyond geophysical flows, we also study transport in the phase space of a coupled oscillator model for biomolecules. Before we can analyze transport in this model, we first introduce an appropriate model reduction that captures the relevant statistics of the full system. In the reduced model, we see that transport is again mediated by the process of horseshoe dynamics in a perturbed homoclinic tangle.

We also consider some theoretical aspects of FTLE-LCS, including the relationship between LCS and stable/unstable manifolds, the invariance of LCS, and the possibility of an evolution equation describing the motion of the LCS. A parallelized

software for computing FTLE is also introduced.

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