

# Transport and Separatrices in Time-Dependent Flows

Thesis by  
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To my parents,  
Herman and Sandy,  
for following a dream so that  
their children could live theirs.

# Acknowledgments

This thesis is written foremost with the intent to provide a meaningful contribution to the scientific community; but for the author, it also represents an opportunity to create a “scientific scrapbook” of all the beneficial and congenial collaborations I have enjoyed during my years of study at Caltech – and in many ways the writing of it is sweeter because of the friendships found and enjoyed during the development of the scientific results.

The most significant and cherished influence over my studies has been the kind and gentle tutoring of Dr. Jerrold Marsden. I have learned during my travels to conferences worldwide, that when I introduce myself as a student of Dr. Marsden, I am immediately afforded friendship and respect far beyond that merited by my contributions or virtues of character. Indeed, I feel that regardless of my future achievements, my greatest career honor, and the honor that I can reflect upon with the warmest of memories, will be to say: “I was a student of Dr. Jerry Marsden.”

The opportunity to learn from Dr. Marsden (as well as from his amiable collaborators - a veritable School of Athens!) is personally exhilarating. Dr. Marsden’s breadth of insight and contributions, and his willingness to share these with students, are rare in a scientific community where research is evermore narrowed, splintered, and hurried; and, in my opinion, place him in company with the great Fathers of our field. In my mind’s eye, I can imagine in the world of spirits “across the bar,” a salon frequented by Euler, Poincaré, Lie, Hamilton, Whittaker, Jacobi, and Cartan, where they are all seated around a table discussing and comparing, all the while leaving a chair open for Jerry Marsden. We all hope and trust that they will have to wait many more years yet, before he will have the opportunity to join them!

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If there are mistakes in this thesis, they are of my own imagination. From the long list of acknowledgments, it is easy to see that if there is anything of great scientific value contained herein, that these results have come through me, and not because of me.