Chapter 11

Conclusion

In his study of planetary motion and celestial mechanics, Poincaré was the first to discover the geometric structure of the homoclinic tangle. Others, such as Cartwright, Littlewood, and Levinson, saw the effects of the tangle exhibited in the bizarre behavior of trajectories for periodically forced electric circuits. Later, Smale would abstract the action of the tangle and fully come to terms with the implications of chaotic trajectories.

This thesis adds infinitesimally to these works, by providing evidence for the persistence of the homoclinic tangle in *aperiodic* flows – a realm where Poincaré's theory is not directly applicable. Indeed, the action of horseshoes has been shown to arise not just in an abstract manner on the beaches of Rio [Smale 1998], but in a very real way in the waters a few miles offshore, and in the swirling atmosphere above. The approach that allows for visualization of the homoclinic tangle in aperiodic flows is the FTLE-LCS method.

In hurricanes, we have seen that the FTLE-LCS recovers the relevant Lagrangian boundary to a storm – a boundary that cannot be perceived from plots of velocity or vorticity alone. Moreover, the mechanism that governs transport of airmass across this boundary is lobe dynamics, the process of tranporting regions of the flow that necessarily attends the action of a homoclinic tangle.

In the turbulent flows of the global ocean, the FTLE-LCS approach provides a mental picture of how mesoscale eddies are responsible for inducing mixing and stirring. The LCS reveals the boundaries to eddies, and their mutual interaction, in a time-dependent manner that underscores the importance of eddies and lobe structures for lateral mixing in the ocean.

The method has been applied further in a model for bio-molecules. Computation of the LCS for aperiodic flow in a reduced order model that faithfully captures the statistics of the full system reveals that the transport mechanism that governs global conformation change is again the homoclinic tangle.

Further applications of the method include coastal ocean flows, the atmosphere of Titan, storms on Netpune, laboratory flows, persistent eddies in the North Atlantic, and celestial mechanics. In each case, the LCS provides interesting insight into the the important separatrices that dictate transport, and encode Lagrangian information that cannot be discerned from Eulerian fields.

Beyond the applications, we have addressed some important theoretical underpinnings of the LCS method. We have discovered that the FTLE-LCS are neither invariant, nor are they generalizations of stable and unstable manifolds, yet they optimally shed as little invariance as needed in order to track the most important features in the flow. An algorithm has been presented which allows for the derivation of explicit evolution equations for the LCS, which the author hopes will spur further progress in the theory of the FTLE-LCS method.

Future applications of FTLE-LCS appear to be as wide and varied as there are flows, and there are many avenues in which to proceed. Immediate research goals that follow from this thesis, include:

- A continued study of the theoretical underpinnings of FTLE-LCS using the initial results provided here as a starting point. What are the key properties that can be shown for LCS defined as ridges in the FTLE?

- Development of regionally specific mesoscale eddy parameterizations for use in global ocean models based on the insights learned from LCS. Can LCS provide a parametrization that improves upon the diffusion and random-walk parametrizations currently employed?

- Incorporating FTLE-LCS calculations to guide real-time deployment of sensory drifters in ocean and hurricane experiments. An experiment to this effect is planned for Prince William Sound, Alaska in July and August, 2009. An ideal situation will be to compute the FTLE-LCS in realtime using ocean model velocity data, identify homoclinic dynamics in the flow, and seed the corresponding lobes in the Sound with drifters in order to verify the presence of horseshoe dynamics.