PHASE NOISE OF NANOELECTROMECHANICAL SYSTEMS

Thesis by

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In Partial Fulfillment of the Requirements for the

Degree of

Doctor of Philosophy

CALIFORNIA INSTITUTE OF TECHNOLOGY

Pasadena, California

2007

(Defended August 9, 2004)

Dedicated to

my family

ACKNOWLEDGEMENTS

Through the course of my study in Caltech, I have gained much more than I originally anticipated. During the process of working on graduate research and writing this thesis, many things have happened that were better and worse than I expect. The period of time I spent at Caltech certainly brought major transformations to my entire life as a result of friendship, team work, persistence, intellectual education, sorrow, and merriment.

First, I would like to thank the Lord for His bountiful supply through these years and I would certainly acknowledge the reigning of His throne over all things through my toughest time in my personal life and school years. He is active and living God and my ultimate protector. I sincerely hope this thesis will in some way glorify Him for His purpose on earth.

I would like to acknowledge the research opportunity provided by my graduate advisor, Prof. Michael Roukes. Through the training in his research group, I have acquired incredibly many technical and personal skills that are indispensable in my career development. I am also privileged to work with so many talented people and benefit from them. Special thanks go to Prof. Carlo Callegari for his support both on the professional and the personal level. Carlo has made many breakthroughs in our research sometimes through our "constructive" confrontation and brainstorming. I very much enjoyed his company for the time we spent chatting about many small but important things about life in the cafe and the lab. I would also like to thank Philip Xiao-li Feng for his friendship, hands-on help in the lab, and efforts to get our results published. I have collaborated on various projects with Prof. Kamil Ekinci, Larry Schiviaone, Dr. Warren Fon, Dr. Darrel Harrington, Dr. Henry Xue-Ming Huang, Dr. Hong Tang, Dr. Ali Husain, Prof. Chris Zorman, Prof. Jim Hone, Dr. Henk Postma, Prof. Keith Schwab, Meher Prakash, Inna Kozinsky, and many others, whose help has been invaluable for making my graduate research productive. I also thank our honorary group member, Prof. Philps Wigen from Ohio State University, for his personal support and encouragement.

I am grateful to my mother Li-chu Chu for her unconditioned love and support and dedicate my doctorate degree to her. During my stay at Caltech, major tragedy has happened to my family. Ever since, the prayer, fellowship, support, and practical advice from the brothers and sisters of the church in Monterey Park and Santa Clara have eased my pain and anxiety in this prolonged academic process of obtaining the diploma. Last but not least, I would like thank my fiancée, Jessie Yung-chieh Yu, for her thoughtfulness and prayer for me and my family. She has brought me joy and peace beyond words and knowledge and dissolved my confusion about life in many cases. As we begin the new phase in our life, I am indebted to her for her love through this period and for many years to come.

ABSTRACT

Nanoelectromechanical systems (NEMS) are microelectromechanical devices (MEMS) scaled down to nanometer range. NEMS resonators can be fabricated to achieve high natural resonance frequencies, exceeding 1 GHz with quality factors in excess of 10^4. These resonators are candidates for ultrasensitive mass sensors and frequency determining elements of precision on-chip clocks. As the size of the NEMS resonators is scaled downward, some fundamental and nonfundamental noise processes will impose sensitivity limits to their performance. In this work, we examine both fundamental and nonfundamental noise processes to obtain the corresponding expressions for phase noise density, Allan deviation, and mass sensitivity. Fundamental noise processes considered here include thermomechanical noise, momentum-exchange noise, adsorption-desorption noise, diffusion noise, and temperature-fluctuation noise. For nonfundamental noise processes, we develop a formalism to consider the Nyquist-Johnson noise from transducer-amplifier implementations.

As an initial step to experimental exploration of these noise processes, we demonstrate the phase noise measurement of NEMS using the phase-locked loop scheme. We analyze control servo behavior of the phase-locked loop and describe several implementation schemes at very high frequency and ultra high frequency bands. By incorporating the ~190 MHz NEMS resonator into the frequency modulation phase-locked loop, we investigate the diffusion noise arising from xenon atoms adsorbed on the device surface. Our experimental results can be explained with the diffusion noise theory. The measured spectra of fractional frequency noise confirm the predicted functional form from the diffusion noise theory and are fitted to extract the diffusion coefficients of adsorbed xenon atoms. Moreover, the observed Allan deviation is consistent with the theoretical

estimates from diffusion noise theory, using the total number of adsorbed atoms and extracted diffusion times.

Finally, very high frequency NEMS devices provide unprecedented potential for mass sensing into the zeptogram level due to their minuscule mass and high quality factor. We demonstrate *in situ* measurements in real time with mass noise floor ~20 zeptogram. Our best mass sensitivity corresponds to ~7 zeptograms, equivalent to ~30 xenon atoms or the mass of an individual 4 kDa molecule. Detailed analysis of the ultimate sensitivity of such devices based on these experimental results indicates that NEMS can ultimately provide inertial mass sensing of individual intact, electrically neutral macromolecules with single-Dalton (1 amu) sensitivity. This is an exciting prospect—when realized it will blur the traditional distinction between inertial mass sensing and mass spectrometry. We anticipate that it will also open intriguing possibilities in atomic physics and life science.

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