Wireless Network Design and Control

Thesis by

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To the Memory of My Late Grandfather, Fuhua Chen.

To My Wife Liyun and Daughter Xiaozhi.

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Abstract

Optimization theory and game theory provide a suite of tools that are flexible in modelling various network systems, and a rich series of equilibrium solution concepts and convergent algorithms. In this thesis, we view network protocols as distributed algorithms achieving the corresponding network equilibria, and study wireless network design and control in optimization and game-theoretic frameworks.

Specifically, we first take a holistic approach and design an overall framework for the protocol architecture in ad hoc wireless networks. The goal is to integrate various protocol layers into a unified framework, by regarding them as distributed computations over the network to solve some optimization problem. Our current theory integrates three functions—congestion control, routing and scheduling—in transport, network and link layers into a coherent framework. These three functions interact through and are regulated by congestion price so as to achieve a global optimality, even in a time-varying environment. This framework is promising to be extended to provide a mathematical theory for network architecture, and to allow us to systematically carry out cross-layer design.

We then develop a general game-theoretic framework for contention control. We define a general game-theoretic model, called random access game, to study the contention/interaction among wireless nodes, and propose a novel medium access method derived from carrier sensing multiple access with collision avoidance in which each node estimates its conditional collision probability and adjusts its persistence probability or contention window, according to a distributed strategy update mechanism achieving the Nash equilibrium of random access game. This results in simple dynamics, controllable performance objectives, good short-term fairness, low collision, and

high throughput. As wireless nodes can estimate conditional collision probabilities by observing consecutive idle slots between transmissions, we can decouple contention control from handling failed transmissions. This also opens up other opportunities such as rate adaptation to channel variations. In addition to providing a general and systematic design methodology for medium access control, the random access game model also provides an analytical framework to understand the equilibrium properties such as throughput, loss and fairness, and dynamic properties of different medium access protocols and their interactions.

Finally, we conclude this work with some suggestions for future research.

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Chapter 1

Introduction

Networked computer systems such as the Internet and wireless networks have become an inseparable and vital part of daily life of human being. They are continuing to grow in several dimensions – the scale of these networks, the heterogeneity at various layers, emerging network technologies and new business models. Accompanying this increasing complexity are the existing network architecture and protocols being stressed and even working adversely in face of new environments. Significantly and even radically new designs are hence necessary to efficiently utilize network resources, to support new applications and to meet current and future demands on these networks.

Many network designs are based on intuition and heuristics, and then validated by simulations and experiments. If problems are found or the performance is not satisfactory, the same design cycle is repeated. Design based on intuition and heuristics, we argue, is not going to meet emerging requirements for network architecture and protocols, as it is usually limited in the scope of the system features that will be taken into account, can easily underestimate the importance of certain factors and lead to suboptimal performance or even worse, cannot foresee potentially adversarial interactions and lead to disastrous implementations. Instead, network design and control should be based on mathematically rigorous and systematic methodologies.

This thesis examines two analytical frameworks, optimization theory and game theory, that hold promise for providing such systematic methodologies for network design. Optimization theory and game theory provide a suite of tools that are flexible in modelling various network systems, and a rich series of equilibrium solution concepts and convergent algorithms. In this thesis, we view network protocols as distributed algorithms achieving the corresponding network equilibria, and study wireless network design and control in optimization and game-theoretic frameworks.

1.1 Computational and Informational Constraints in Network Design and Control

Several features of networks make their design and control very challenging – of predominance are the large scale and the distributed nature of the system. These two features impose strong constraints on the system designer, as individual agents (e.g., end users such as transmission control protocol (TCP) sources and network components such as links or autonomous systems) must make local decisions based on local information. These constraints roughly fall into two tangled categories: computational and informational.

Computational constraint refers to the necessity of distributed computing with low complexity in a network setting. It also refers to bounded computing capability, or bounded rationality in the language of game theory, of individual agents. Informational constraint is concerned with the communication complexity of network algorithms and protocols. Due to the large scale of the network and the scarcity of the network resources, low communication complexity is required. Informational constraint also refers to the informational structure of the system, which specifies what kind and what amount of information is available at each time. For example, in the utility maximization framework of congestion control [48] [60], the utility of each source is not available to other sources and the network, either because it is impractical to communicate this information to other sources and the network or because the source wants to keep it as private information. Taken together, computational and informational constraints determine what kind of equilibrium, or stable operating point, the system can achieve, and what kind of convergent algorithm to the equilibrium is feasible.

There is actually another kind of constraint – incentive. Rather than following specifications of the system designer, the end users or the network components are self-interested and may strategize to manipulate the system to their advantage. For example, a TCP source may choose not to respond to congestion signal, in order to obtain higher throughput. Though it is not a direct concern of this thesis, to include incentive constraint and design systems that are robust to the self-interest of agents will be a future research direction.

1.2 Design Desiderata, Equilibria and Dynamics

In network design we usually associate a utility function, a cost function or any other objective function to each agent. The goal of network design and control is to design a system in which individual agents interact in a way that achieves an equilibrium (or stable operating point) with some desired systemwide properties. The most widely adopted property or design desiderata is probably the social optimality, where the system optimizes some global objective function such as the aggregate user utility or the aggregate cost. Indeed, optimization theory permeates all engineering disciplines. From the classical network flow problems that are often formulated as linear programs, to the recent and more general network utility maximization problems that are nonlinear programs, the optimization theory has played a major role both in designing new systems and protocols, and in analyzing and understanding the existing systems and protocols.

Two reasons contribute to the prevalence of optimization framework in network design. First, many performance metrics are "natural" objectives to be optimized. For example, in maximum flow problem [11], it is natural to ask what is the maximum throughput that the network can support; in optimal routing problem [10], we would like to route the traffic in a way that minimizes the aggregate cost such as the aggregate delay. The other reason is that many network equilibrium models can be reverse engineered as optimality conditions for some appropriately defined optimization problems. A classic example is the Wardrop equilibrium [87], which can be mathematically expressed as the Karush-Kuhn-Tucker (KKT) optimality condition for some optimization problem, known as the Beckmann transform [8]. Another outstanding example is the duality model of TCP congestion control, where again the network equilibrium can be seen as the KKT optimality condition for some network utility maximization problem [62], which we will discuss in detail in chapter 2. Based on the insights obtained from reverse engineering, we can conversely achieve desired network equilibrium and systematically improve the protocol through carefully specifying the objective functions or designing better algorithms to achieve the equilibrium (forward engineering).

The underlying mathematical structure responsible for the majority of these results is convexity and strong duality, i.e., the optimal value of an optimization problem equals that of its Lagrangian dual, see, e.g., [12] [17]. When convexity and strong duality hold, many desirable properties follow. First, the problem is computationally tractable and efficient algorithms such as gradient-based, primal-dual algorithms exist. Second, the additivity structure in the primal and the linearity of the constraints translate into decomposition structure in the dual, which leads to distributed algorithms across distributed agents and across different network layers. Third, the dual variables have a physical interpretation and especially, the Lagrangian multipliers can be viewed as the "equilibrium prices." Thus, these simple iterative (e.g., gradient-based) algorithms applied to the primal or the dual are implementable in the network, which describe or specify the network dynamics achieving the network equilibrium that is specified by the optimality condition. In words, in optimization based network design and control, we view the network as an optimization solver and design network protocols according to distributed algorithms solving the corresponding optimization problems and their duals.

However, optimization framework is not universal and has its limitations. First, the optimization problems may be very difficult nonlinear, nonconvex optimization with integral constraints, for which in general no efficient, distributed algorithms exist. Second, informational constraint may prevent the system from achieving global optimality, as extensive message passing is usually required to align the behaviors of individual agents to achieve the global optimality. For example, in wireless ad hoc networks, extensive message passing may be impractical, due to the scarcity of wireless spectrum and the interference incurred in simultaneous transmissions. Third, some design problems intrinsically do not fit in an optimization paradigm, because of informational structure or incentive issue of the problems. For example, network equilibria such as Wardrop equilibrium that involve several classes of users with different cost functions cannot be described in an optimization framework; the stable path problem in interdomain routing [36] cannot be cast into an optimization problem. So, we need to look for other modelling frameworks that involve seeking an equilibrium of the system.

A more general modelling framework is (noncooperative) game theory. Game-theoretic models are inherently distributed, since the agents are independent decision makers. So, they are flexible in modelling various situations. Game theory provides a series of equilibrium solution concepts, such as the Nash equilibrium and the dominant strategy equilibrium, that differ in assumptions about agents' computational (rationality) and informational constraints and thus are suitable for different situations. For example, in interdomain routing, complete autonomy is a desired property of autonomous systems. Thus, the dominant strategy equilibrium is the right solution concept, since it makes no assumptions about the rationality and information available to agents about each other. Indeed, the stable path problem for interdomain routing can be viewed as seeking a dominant strategy equilibrium. Game-theoretic analysis also provides a basis for designing systems to achieve the given desired goals (such as to maximize the aggregate utility), which is the scope of mechanism design [64] [31]. For example, TCP congestion control algorithms can also be interpreted from this perspective – each TCP source maximizes its own payoff, i.e., the utility gain minus the congestion cost, and active queue management (AQM) provides a pricing scheme that induces the maximization of the aggregate utility. Game theory is also closely related to optimization theory. On the one hand, as we just saw, social optimality can be the goal of a game design. On the other hand, optimization theory provides valuable tools to characterize equilibria and derive distributed convergent algorithms of games. For example, for potential games [66], distributed convergent algorithms follow naturally from the greedy or gradient-based algorithms solving the corresponding optimization problems.

In game-theoretic framework, we design network protocols according to distributed algorithms (strategy update mechanisms) achieving various kinds of equilibria. Different equilibrium solution concepts impose different computational and informational requirements for the convergent algorithms, as they have different assumptions about agents' computational and informational constraints. For some solution concepts, these computational and informational requirements are usually satisfied and general distributed convergent algorithms exist. For example, iterated elimination of dominated strategies is a distributed algorithm to achieve the dominant strategy equilibrium, and indeed, in the example of interdomain routing, routing protocols such as border gateway protocol (BGP) implement such an algorithm. For other solution concepts such as the Nash equilibrium, while in general distributed convergent algorithms do not exist, there are convergent algorithms for several general classes of games. For example, greedy or gradient-based algorithms are general convergent algorithms for the potential games.

Traditionally, game theory has been predominantly focusing on equilibrium solution concepts, but often neglected the dynamic aspect of game, i.e., how interacting agents could converge to an equilibrium. The dynamics of game is important for network design in game-theoretic framework. To identify those key mathematical structures (e.g., the properties of the objective functions and the information structure of the system) that guarantee distributed convergence of different equilibria will be a future research direction.

1.3 Outline and Contributions

While the techniques are general, this thesis applies optimization theory and game theory to two important design problems in wireless networks, respectively. Besides their practical importance, each design problem is intended to demonstrate the rationale, merit and power of the corresponding model and design framework. The outline and contributions are as follows.

In chapter 2, we provide some background material that are required by this thesis. We review the utility maximization framework of congestion control, especially the duality model of TCP/AQM. We also introduce basic equilibrium solution concepts and dynamics in game theory.

In chapter 3, we take a holistic approach and design an overall framework for the protocol architecture in ad hoc wireless networks. The goal is to integrate various protocol layers into a unified framework, by regarding them as distributed computations over the network to solve some optimization problem. Our current theory integrates three functions—congestion control, routing and scheduling—in transport, network and link layers into a coherent framework. These three functions interact through and are regulated by congestion price so as to achieve a global optimality, even in a time-varying environment. This framework is promising to be extended to provide a mathematical theory for network architecture, and to allow us to systematically carry out cross-layer design. Also, we present a general technique and results regarding the stability and optimality of dual algorithms in face of time-varying parameters. As the dynamics of many systems can be modelled as a dual algorithm and dual decomposition has motivated many cross-layer design schemes, these results provide an avenue to establish the stability and study the performance of these systems in a time-varying environment.

In chapter 4, we develop a game-theoretic framework for contention control. We define a general game-theoretic model, called random access game, to capture the contention/interaction among wireless nodes in wireless networks with contention-based medium access, and propose a novel medium access method derived from carrier sensing multiple access with collision avoidance (CSMA/CA) in which each node estimates its conditional collision probability and adjusts its persistence probability or contention window, according to distributed strategy update mechanism achieving the Nash equilibrium of random access game. This results in simple dynamics, controllable performance objectives, good short-term fairness, low collision and high throughput. As wireless nodes can estimate conditional collision probabilities by observing consecutive idle slots between transmissions, we can decouple contention control from handling failed transmissions. This also opens up other opportunities such as rate adaptation to channel variations. As a case study of medium access control design in game-theoretic framework, we present a concrete medium access method and show that it achieves superior performance over the standard 802.11 distributed coordination function (DCF), and can provide flexible service differentiations among wireless nodes. In addition to providing a general and systematic design methodology for medium access control, the random access game model also provides an analytical framework to understand equilibrium properties such as throughput, loss and fairness, and dynamics property of different medium access protocols and their interactions.

We conclude the thesis in chapter 5 with some discussions and suggestions for future research.

Chapter 2

Background Material

In this chapter, we provide an overview of the network utility maximization framework of TCP congestion control and basic game-theoretic concepts required by this thesis. This chapter is not intended to be a complete introduction or summary of the topics. For more details, see, e.g., [48] [60] [62] [53] for the utility maximization framework of TCP congestion control, and see, e.g., [70] [34] for game theory.

2.1 The Network Utility Maximization Framework of Congestion Control

2.1.1 Congestion Control

Congestion control is a distributed adaptive algorithm to share network resources (e.g., link bandwidth) among competing users so as to avoid congestion while ensuring high utilization and fair allocation of available resources. There are two components in a congestion control algorithm: a source algorithm that dynamically adjusts the sending rate based on congestion along its path, and a link algorithm that updates a congestion measure and sends it back to the sources. On the current Internet, the source algorithm is implemented in TCP such as TCP Reno [46], and the link algorithm is carried out by AQM scheme such as DropTail or RED [33] at the routers. The source controls its sending rate usually through controlling its congestion window size. The congestion measure can be loss probability, as used in, e.g., TCP Reno and its variants, or queueing delay, as used in TCP Vegas [18] and FAST TCP [89]. They are fed back to the sources in the form of packet loss, marking, or delay. In the next subsection, we will review a duality model of TCP/AQM [62] in the network utility maximization framework.

2.1.2 Duality Model of TCP/AQM

A network is modelled as a set L of links with finite capacities $c = \{c_l, l \in L\}$. These links are shared by a set S of sources indexed by s. Each source s uses a subset $L_s \subseteq L$ of links. The sets L_s define an $|L| \times |S|$ routing matrix with $R_{ls} = 1$ if $l \in L_s$ and $R_{ls} = 0$ otherwise. Each source attains a utility $U_s(x_s)$ when it transmits at rate x_s bits per second. We assume $U_s(\cdot)$ is continuously differentiable, increasing, and strictly concave. It has been shown that, if bandwidth is allocated in such a way that maximizes the aggregate source utility, then there is a natural decomposition that allows distributed allocation algorithms [48] [60] [53].

Specifically, consider the following network utility maximization problem [48] [60],

$$\max_x \quad \sum_s U_s(x_s) \tag{2.1}$$

$$\text{subject to} \quad Rx \leq c, \tag{2.2}$$

where the constraint says that the aggregate flow rate through a link should not exceed its capacity. To obtain a distributed algorithm, we consider the equivalent Lagrangian dual problem

$$\min_{p \geq 0} \quad D(p) := \sum_s \max_{x_s} (U_s(x_s) - x_s \sum_l R_{ls} p_l) - p^T c. \tag{2.3}$$

The Lagrangian dual function decomposes into subproblems that can be optimized separately by individual sources based on p_l along their paths

$$x_s = U_s'^{-1}(\sum_l R_{ls} x_s). \tag{2.4}$$

There exist other algorithms to solve the problems, and as we shall see, different congestion control algorithms can be interpreted as distributed primal-dual algorithms to solve the utility maximization problem and its dual [62].

Let $y_l(t) = \sum_s R_{ls} x_s(t)$ be the aggregate source rate through link l at time t. Each link l is associated with a congestion price $p_l(t)$ at time t, and let $q_s(t) = \sum_l R_{ls} p_l(t)$ be the end-to-end congestion price for source s. Source s can observe only its own rate $x_s(t)$ and the end-to-end congestion price $q_s(t)$ along its path, and link l can observe only local congestion $p_l(t)$ and aggregate flow rate $y_l(t)$. Since sources adjust their rates and links update their congestion prices based on local information, we can model the dynamics of TCP and AQM by a general model

$$x_s(t+1) = F_s(x_s(t), q_s(t)), \quad (2.5)$$

$$p_l(t+1) = G_l(p_l(t), y_l(t)), \quad (2.6)$$

where the exact forms of the functions $F_s(\cdot)$ and $G_l(\cdot)$ are determined by specific TCP/AQM protocols we consider. Take, for example, TCP Vegas/DropTail [18]. It is shown in [61] that Vegas uses queueing delay as congestion measure, and the update rules for source rate and congestion price are given by

$$x_s(t+1) = x_s(t) + \frac{1}{D_s^2(t)} \mathbf{1}\left(\frac{\alpha_s d_s}{q_s(t)} - x_s(t)\right), \quad (2.7)$$

$$p_l(t+1) = \left[p_l(t) + \frac{y_l(t)}{c_l} - 1\right]^+, \quad (2.8)$$

where α_s is a parameter of Vegas, d_s is the round trip propagation delay and $D_s(t) = d_s + q_s(t)$ is the round trip delay of source s, and the step function $\mathbf{1}(z) = 1$ if $z > 0$, -1 if $z < 0$ and 0 if $z = 0$.

Suppose that the system (2.5)–(2.6) has an equilibrium (x^*, p^*). Under some mild conditions, the fixed point equation $x_s^* = F_s(x_s^*, q_s^*)$ implicitly defines a relation between equilibrium congestion prices and source rates

$$q_s^* = f_s(x_s^*) > 0. \quad (2.9)$$

Define a utility function for each source s as

$$U_s(x_s) = \int f_s(x_s)dx_s, \quad x_s \geq 0. \tag{2.10}$$

Since the source decreases its sending rate with increasing congestion, it is reasonable to assume that $f_s(\cdot)$ is a strictly decreasing function. So, $U_s(\cdot)$ is a continuous, increasing and strictly concave function. For example, derived from equation (2.7), the utility functions for TCP Vegas are given by $U_s(x_s) = \alpha_s d_s \log x_s$.

With the above defined utility functions, we can formulate a utility maximization problem as (2.1)–(2.2). Now we interpret the source rate x as primal variables of the primal problem (2.1)–(2.2), and the congestion price p as dual variables of the corresponding dual (2.3). Note that, given the equilibrium prices p_l^*, the equilibrium source rates x_s^* solve

$$\max_{x_s} U_s(x_s) - x_s q_s^*. \tag{2.11}$$

By strong duality of convex optimization [12] [17], if the equilibrium congestion price p^* is a dual optimum, the corresponding equilibrium source rate x^* is the primal optimum, i.e., the solution to the primal problem (2.1)–(2.2). Indeed, it has been shown in [62] that, under some reasonable assumptions, the equilibrium (x^*, p^*) of the system (2.5)–(2.6) satisfies the KKT condition

$$y_l^* \leq c_l, \tag{2.12}$$

$$p_l^*(y_l^* - c_l) = 0, \tag{2.13}$$

$$p_l^* \geq 0, \tag{2.14}$$

$$U_s'(x_s^*) - q_s^* = 0, \tag{2.15}$$

and thus solves the primal and dual problems. The condition (2.15) follows from the way we define the utility function. The complementary slackness condition (2.12)–(2.14) is satisfied by any AQM that stabilizes the queues [62]. Hence, various TCP/AQM protocols can be interpreted as different distributed primal-dual algorithms (F, G) to solve the global optimization problem (2.1)–(2.2) and its dual (2.3), with different utility functions U_s.

Besides as an analytical tool of reverse engineering TCP congestion control, the network utility maximization and the approach of protocol as distributed solution to some global optimization problem through dual decomposition has recently been used to guide the systematic design of new congestion control algorithms such as FAST TCP [89]. It can also be extended to provide a mathematical theory for network architecture and a general approach to cross-layer design, as we will discuss in chapter 3.

2.2 Basic Game Theory Concepts

Game theory is the study of mathematical models of strategic interaction between rational agents. It provides general techniques for analyzing situations where agents' decisions will influence one another's decisions and payoffs. Game theory has been primarily studied in economics, and is applied to other areas such as politics and recently to networking.

A game is a strategic interaction between multiple independent agents or players. A game in strategic form has three elements: the set N of agents (players), the (pure) strategy space S_i which specifies the set of permissible actions for each agent $i \in N$, and payoff function $u_i : S_1 \times S_2 \times \cdots \times S_{|N|} \longmapsto \mathcal{R}$ which assigns a payoff to agent i for each combined choice (profile) $s = (s_1, s_2, \ldots, s_{|N|})$ of strategies. The payoff function, $u_i(\cdot)$, of agent i describes its preference over its own strategy and the strategies of other agents. The basic model of agent rationality in game theory is that of a payoff maximizer. An agent will select a strategy that maximizes its (expected) payoff, given its beliefs about the strategies of other agents and the structure of the game.

Consider, for example, the classic example of the prisoner's dilemma game. In this game, we have two players P1 and P2. The strategy of each player is either cooperate (C) or defect (D). The players move simultaneously, and based on their strategies,

Table 2.1: Payoffs for the prisoner's dilemma game

P1\P2	C	D
C	(1,1)	(-1,2)
D	(2,-1)	(0,0)

each player receives a payoff as given by the matrix in Table 2.2. In this matrix, each cell represents the payoff of a particular strategy profile. For example, if the strategies of the players are C and D respectively, player P1 receives payoff $u_1(C, D) = -1$ and player P2 receives payoff $u_2(C, D) = 2$.

2.2.1 Equilibrium Solution Concepts

Game theory provides a number of equilibrium solution concepts to compute the outcome of a game, given assumptions about agent rationality (computational constraint) and information available to agents about each other.

The predominant solution concept is the Nash equilibrium, where each agent will select a payoff-maximizing strategy given the strategies of other agents. Denote by $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_{|N|})$ the strategy profile of all agents other than i. A strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_{|N|}^*)$ is a Nash equilibrium if, for all agent $i \in N$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \tag{2.16}$$

for all $s_i \in S_i$. We see that at the Nash equilibrium, no agent has unilateral incentive to change. For example, for the prisoner's dilemma game, the strategy profile (D, D) is the Nash equilibrium.

A stronger solution concept is the dominant strategy equilibrium, where each agent has the same payoff-maximizing strategy no matter what strategies of other agents. A strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_{|N|}^*)$ is a dominant strategy equilibrium if, for all agent $i \in N$,

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}), \tag{2.17}$$

for all s_i and s_{-i}. In the example of the prisoner's dilemma game, the strategy profile (D, D) is also a dominant strategy equilibrium. Indeed, the dominant strategy equilibrium is a subset of the Nash equilibrium.

Nash equilibrium makes very strong assumptions about agents' rationality and knowledge. To play a equilibrium strategy in a one-shot game, each agent must have perfect information about all agents' strategy spaces and the corresponding payoffs, and agent rationality must also be common knowledge. In contrast, the dominant strategy equilibrium is a very robust solution concept. It makes no assumptions about other agents' rationality and the information available to agents about each other.

There exist other solution concepts such as correlated equilibrium. Most of them are the refinements or extensions of the Nash equilibrium, under different assumptions about agent rationality and information. Since they are not essential to this thesis, we will not further elaborate on them.

2.2.2 Dynamics

These equilibrium solution concepts are usually very strong concepts. For example, as mentioned above, the Nash equilibrium assumes that agent strategies, payoffs and rationality are common knowledge. This assumption usually does not hold in reality, and as pointed out in [34], game theory lacks a general and convincing argument that a Nash outcome will occur. One justification is that equilibria arise as a result of adaptation or learning. In this context, we consider repeated play of the game, and assume that the agents are myopic and adjust their strategies based on the strategies of other players in previous rounds.

This justification of equilibrium is particularly legitimate in network design and control, as the network consists of distributed entities having limit information and almost all network algorithms and protocols are repeated (or iterative) and adaptive. We can design network algorithms and protocols according to distributed strategy update mechanisms achieving various equilibria. In chapter 4, we apply this general design methodology to contention control, and design medium access protocols according to distributed strategy update algorithms achieving the Nash equilibria of random access games.

Chapter 3

Cross-Layer Design in Ad Hoc Wireless Networks

In this chapter, we take a holistic approach and design an overall framework for the protocol architecture in ad hoc wireless networks. Our goal is to integrate various protocol layers into a unified framework, by regarding them as distributed computations over the network to solve some optimization problem. Different layers carry out distributed computation on different subsets of the decision variables using local information to achieve individual optimality. Taken together, these local algorithms (with respect to different layers) achieve a global optimality. Our current theory integrates three functions—congestion control, routing and scheduling—in transport, network and link layers into a coherent framework. These three functions interact through and are regulated by congestion price so as to achieve a global optimality, even in a time-varying environment. This framework is promising to be extended to provide a mathematical theory for network architecture, and to allow us to systematically carry out cross-layer design.

3.1 Introduction

The success of communication network has largely been a result of adopting a layered architecture. With this architecture, its design and implementation is divided into simpler modules that are separately designed and implemented and then interconnected. As a result, protocol stack for the network has five layers, application, transport (TCP), network (IP), data link (include MAC) and physical layer. Each layer controls a subset of the decision variables, hides the complexity of the layer below and provides well-defined services to the layer above. Together, they control and allocate networked resources to provide a reliable and usually best-effort communication service to a large pool of competing users.

However, the layered structure addresses only abstract and high-level aspects of the whole network protocol design. Various layers of the network are put together often in an ad hoc manner, and the add-up of these layers might not be optimal as a whole. In order to improve the performance and achieve efficient resource allocation, we need to understand interactions across layers and carry out cross-layer design. Moreover, in wireless networks, as there does not exist a good interface between the physical and network layers, cross-layer design seems a must. Wireless links are unreliable and wireless nodes usually rely on random access mechanism to access wireless channel. Thus, the performance of link layer is not guaranteed, which will result in performance problems for the whole network such as degraded TCP performance. So, we need cross-layer design, i.e., to exchange information between physical/link layer with higher layers in order to provide better performance.

Motivated by the duality model of TCP congestion control [48] [60] [62], one approach to understand interactions across layers and carry out cross-layer design is to view the network as an optimization solver and various protocol layers as distributed algorithms solving an optimization problem. This approach and the associated utility maximization problem, as reviewed in chapter 2, were originally proposed as an analytical tool for reverse engineering TCP congestion control where a network with fixed link capacities and prespecified routes is implicitly assumed. A natural framework for cross-layer design is then to extend the basic utility maximization problem to include decision variables of other layers, and seek a decomposition such that different layers carry out distributed computation on different subsets of decision variables using local information to achieve individual optimality, and taken together, these local algorithms achieve the global optimality.

We apply this approach to design an overall framework for the protocol architecture in ad hoc wireless networks, with the goal of achieving efficient resource allocation through cross-layer design. We first consider the network with fixed channel or single-rate devices, and formulate network resource allocation as a utility maximization problem with rate constraints at the network layer and schedulability constraints at the link layer. We then apply duality theory to decompose the system problem vertically into congestion control, routing and scheduling subproblems that interact through congestion prices. Based on this decomposition, a distributed subgradient algorithm for joint congestion control, routing and scheduling is obtained, and proved to approach arbitrarily close to the optimum of the system problem. We next extend the dual subgradient algorithm to wireless ad hoc networks with time-varying channels and adaptive multi-rate devices. The stability of the resulting system is proved, and its performance is characterized with respect to an ideal reference system. We finally apply the general algorithm to the joint congestion control and medium access control design over the network with single-path routing and to the cross-layer congestion control, routing and scheduling design in the network without prespecified paths.

Our current theory integrates three functions— congestion control, routing and scheduling—in transport, network and link layers into a coherent framework. This framework is promising to be extended to provide a mathematical theory for network architecture, and allow us to systematically carry out cross-layer design. We also present a general technique and results regarding the stability and optimality of dual algorithm in face of time-varying parameters. As the flow contention graph that will be used to characterize feasible rate regions of the networks is a rather general construction and can be used to capture the interdependence or contention among parallel servers of any queueing networks, these results are applicable to any systems that can be modelled by a general model of queueing network that is served by a set of interdependent parallel servers with time-varying service capabilities.

The remainder of this chapter is organized as follows. The next section briefly discusses related work. Section 3.3 presents details of the system model for the network with fixed channel or single-rate devices, and section 3.4 presents a distributed algorithm for joint congestion control, routing and scheduling via dual decomposition. Section 3.5 extends the dual algorithm to handle the network with time-varying channel and adaptive multi-rate devices. Section 3.6 discusses joint congestion control and medium access control design in ad hoc wireless networks with single-path routing. Section 3.7 discusses cross-layer congestion control, routing and scheduling design in the network without prespecified paths. Section 3.8 concludes the chapter with a brief discussion of "layering as dual decomposition" as a general theory for network architecture and a systematic approach to cross-layer design.

3.2 Related Work

The utility maximization framework [48] [60] on TCP congestion control has been extensively applied and extended to study resource allocations, especially congestion control (see, e.g., [95] [96]), fair channel access (see, e.g., [67] [84] [54] [30] [79] [29] [25]), and cross-layer design (see, e.g., [91] [26] [58] [21] [22]), in wireless networks. Xue et al. [95] and Yi et al. [96] are among the first to formulate schedulability constraints at link layer for congestion control over ad hoc wireless networks. Xiao et al. [91] study joint routing and resource allocation, and are among the first to apply dual decomposition to cross-layer design in ad hoc wireless networks. Chiang [26] is among the first to study joint congestion and power control. Lin et al. [58] and Chen et al. [21] are among the first to study joint congestion control and scheduling. Chen et al. [21] and Wang at al. [86] are among the first to study cross-layer design in the network with contention-based medium access.

The work presented in section 3.6 (see also [21]) is originally motivated to solve TCP unfairness problem over ad hoc wireless networks, see, e.g., [35], [81], [92], [93], [94]. The model used in section 3.7 (see also [22]) is motivated by Neely et al. [68] that studies dynamic power control and routing for time-varying wireless networks and by Hajeck et al. [37] and Kodialam at al. [49] that study the problem of jointly routing the flows and scheduling the transmissions to determine the achievable rates in multi-hop wireless networks; and similar decomposition for the network with deterministic wireless channel has also been revealed in [58] and the journal version of [68].

The utility maximization in time-varying networks is first studied in the context of fair scheduling. It has been shown that a family of primal scheduling algorithms maximize the sum of the utilities of the long-run average data rates provided to the users, see, e.g., [84] [54] [79]. In contrast, the result presented in section 3.5 (see also [22]) is for the dual algorithms. An earlier result for the dual scheduling algorithm is by Eryilmaz et al. [29] that studies fair resource allocation using queue-length based scheduling and congestion control. Another similar result is by Neely et al. [69] that studies fairness and optimal stochastic control for heterogeneous networks. All these three work use stochastic Lyapunov method to establish the stability, but the technical details are somewhat different. Especially, the stability and optimality result presented in section 3.5 is based only on general properties of convexity and the definition of subgradients, and can be directly applied to a variety of time-varying systems that can be solved or modelled by the dual algorithms. Another comparable result is by Stolyar [80] that proposes greedy primal-dual algorithm to maximize network utility. It uses a very different technique to establish the optimality result.

We have been focused on dual decomposition, which leads to a natural "vertical" decomposition into separate designs of different layers that interact through congestion price. There are many different ways to decompose a given problem, each of which corresponds to a different layering scheme. See the survey article [27] and the references therein for various recent work on cross-layer design or layering as optimization decomposition.

3.3 System Model

Consider an ad hoc wireless network with a set N of nodes and a set L of directed logical links. We assume a static topology and each link $l \in L$ has a fixed finite capacity c_l bits per second when active, i.e., we implicitly assume that the wireless channel is fixed or some underlying mechanism is used to mask the channel variation so that the wireless channel appears to have a fixed rate. This assumption will be relaxed in section 3.5. Wireless channel is a shared medium and interference limited, where links contend with each other for exclusive access to the channel. We will use the flow contention graph to capture the contention relations among links. The feasible rate region at link layer is then a convex hull of the corresponding rate vectors of independent sets of the flow contention graph. We will further describe rate constraints at the network layer by linear inequalities in terms of user service requirements and allocated link capacities. The resource allocation of the network is then formulated as a utility maximization problem with the schedulability and rate constraints.

3.3.1 Flow Contention Graph and Schedulability Constraint

The interference among wireless links is usually specified by some interference model that describes physical constraints regarding wireless transmissions and successful receptions. For example, in a network with primary interference, links that share a common node cannot transmit or received simultaneously but links that do not share nodes can do so. It models a wireless network with multiple channels where simultaneous communications in a neighborhood are enabled by using orthogonal CDMA or FDMA channels. In a network with secondary interference, links mutually interfere with each other whenever either the sender or the receiver of one is within the interference range of the sender or receiver of the other. Given an interference model, we can construct a flow contention graph that captures the contention relations among the links, see, e.g., [67]. In the contention graph, each vertex represents a link, and an edge between two vertices denotes the contention between the corresponding links: two links interfere with each other and cannot transmit at the same time. Figure 3.1 shows an example of a simple ad hoc wireless network with primary interference and the corresponding flow contention graph.

Given a flow contention graph, we can identify all its independent sets of vertices. An independent set is a set of vertices that have no edges between each other [28]. The links in an independent set can transmit simultaneously. Let E denote the set

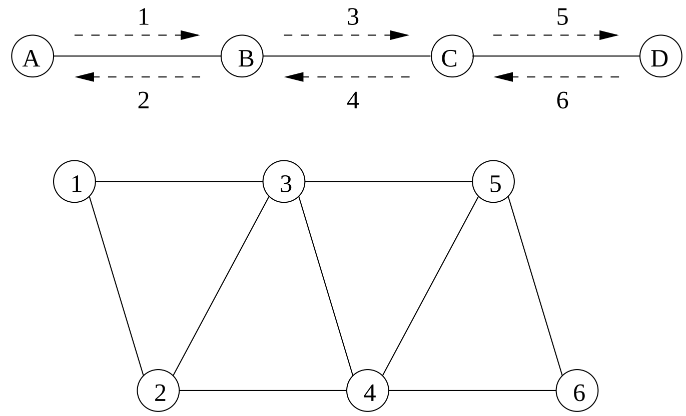

Figure 3.1: Example of an ad hoc wireless network with 4 nodes and 6 logical links and the corresponding flow contention graph.

of all independent sets with each independent set indexed by e. We represent an independent set e as a $|L|$-dimensional rate vector r^e, where the lth entry is

$$r_l^e := \begin{cases} c_l & \text{if } l \in e, \\ 0 & \text{otherwise.} \end{cases}$$

The feasible rate region Π at the link layer is then defined as the convex hull of these rate vectors

$$\Pi := \left\{ r : r = \sum_e a_e r^e,\ a_e \geq 0,\ \sum_e a_e = 1 \right\}. \tag{3.1}$$

Thus, given a link flow vector y, the schedulability constraint says that y should satisfy $y \in \Pi$.

The contention graph is a rather general construction, and can be used to capture the interdependence or contention among parallel servers of any queueing networks. For example, it includes wireline networks as a special case where the contention graph is just a set of vertices with no edges, since there is no interference among the links. It can be used to characterize the interference relations among wireless and wired links in hybrid wireline-wireless networks. It can also be modified to characterize the contention relations in the network where wireless nodes are equipped with multiple radios or communicate through multiple channels. Also, note that in some references the conflict graph is used to capture the contention relations among the links, see, e.g., [43]. The conflict graph is almost the same as the flow contention graph. We do not distinguish between them in this thesis.

3.3.2 Rate Constraint

Let $f_l \geq 0$ denote the amount of capacity allocated to link l. From the schedulability constraint, f should satisfy

$$f \in \Pi. \tag{3.2}$$

Assume that the network is shared by a set S of sources, with each source $s \in S$ transmitting at rate x_s bits per second. In the following, we will formulate rate constraints for networks with different kinds of routing respectively.

The Network with Single-Path Routing

Each source s uses a path consisting of a set $L_s \subset L$ of links. The sets L_s define an $|L| \times |S|$ routing matrix

$$R_{ls} = \begin{cases} 1 & \text{if } l \in L_s, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the aggregate rate over link l is $\sum_{s\in S} R_{ls}x_s$. The rate constraint is written as

$$Rx \leq f, \tag{3.3}$$

i.e., the aggregate link rate should not exceed the link capacity.

The Network with Multipath Routing

Each source s can send traffic along a set T_s of given paths. Each path $r \in T_s$ contains a set of $L_r^s \subset L$ of links, which defines a $|L| \times |T_s|$ routing matrix H^s whose (l, r)th entry is given by

$$H^s_{lr} = \begin{cases} 1 & \text{if } l \in L^s_r, \\ 0 & \text{otherwise.} \end{cases}$$

Denote by x^r_s the rate at which source s sends along path r. Thus, the source rate $x_s = \sum_{r \in T_s} x^r_s$. The rate constraint is written as

$$\sum_{s,\ r \in T_s} H^s_{lr} x^r_s \leq f_l,\ l \in L. \tag{3.4}$$

The Network without Prespecified Paths

Since no end-to-end path is given, we will use multicommodity flow model for routing. Let D denote the set of destination nodes of network layer flows. Let $f^k_{i,j} \geq 0$ denote the amount of capacity of link (i, j) allocated to the flows to destination k. Then the aggregate capacity on link (i, j) is $f_{i,j} := \sum_{k \in D} f^k_{i,j}$. Let $x^k_i \geq 0$ denote the flow generated at node i towards destination k. Then the aggregate capacity for its incoming flows and generated flow to the destination k should not exceed the summation of the capacities for its outgoing flows to k

$$x^k_i \leq \sum_{j:(i,j) \in L} f^k_{i,j} - \sum_{j:(j,i) \in L} f^k_{j,i},\ i \in N,\ k \in D,\ i \neq k. \tag{3.5}$$

Equation (3.5) is the rate constraint for resource allocation. For simplicity of presentation, we assume that there is at most one flow between any node and destination pair $[i, k] \in S \times D$. Thus, $x^k_i = x_s$ if i is the source node of flow $s = [i, k]$, and $x^k_i = 0$ otherwise.

3.3.3 Problem Formulation

We see from the last subsection that all three kinds of rate constraints are expressed as linear inequalities. If we represent the "routing" of the user (source) service requirement by a linear function $H(x)$ of the source rates x, and represent the "allocation" of the service capacity by a linear function $A(f)$ of the link capacities f, since the service requirement should not exceed the allocated service capacity, we have the following inequality constraint

$$H(x) \leq A(f). \tag{3.6}$$

The linear constraint (3.6) is a very general relation. The rate constraints (3.3), (3.4) and (3.5) are just its different concrete representations.

Following [48] [60], assume each source s attains a utility $U_s(x_s)$ when it transmits at a rate x_s. We assume $U_s(\cdot)$ is continuously differentiable, increasing, and strictly concave. Our objective is to choose source rates x and allocated capacities f so as to solve the following global problem

$$\max_{x,f} \quad \sum_s U_s(x_s) \tag{3.7}$$

$$\text{subject to} \quad H(x) \leq A(f), \tag{3.8}$$

$$f \in \Pi. \tag{3.9}$$

The system problem (3.7)–(3.9) is convex optimization problem, and is polynomially solvable if all the utility and constraint information is provided, but this is impractical in real networks. Since it is convex optimization problem with strong duality, distributed algorithm can be derived by formulating and solving corresponding Lagrange dual problem, as we will show in the next section.

3.4 Distributed Algorithm via Dual Decomposition

3.4.1 Distributed Algorithm

Consider the Lagrangian of the problem (3.7)–(3.9) with respect to the rate constraint

$$L(p, x, f) = \sum_s U_s(x_s) - p^T(H(x) - A(f)).$$

Given p, the above Lagrangian has a nice decomposition structure: it is the summation of two independent terms, in terms of source rates and link capacities respectively. Interpreting p as the "congestion price" and maximizing the Lagrangian over x and f for fixed p, we obtain the following joint congestion control and scheduling algorithm:

Congestion control: At time t, given congestion price $p(t)$, the sources adjust flow rates x according to the congestion price

$$x(t) = x(p(t)) = \arg\max_x \sum_s U_s(x_s) - p^T(t)H(x). \tag{3.10}$$

Scheduling: Over link l, send an amount of data for each flow according to the rates f such that

$$f(t) = f(p(t)) \in \arg\max_{f \in \Pi} p^T(t)A(f). \tag{3.11}$$

Note that there does not exist an explicit routing component in the dual decomposition. Instead, the routing is implicitly solved in (3.10) if the set of paths from which a source can choose is given, and solved in (3.11) if no path is prespecified for the source. We see that, by dual decomposition, the flow optimization problem decomposes into separate "local" optimization problems of transport, network and link layers respectively, and they interact through congestion prices.

Defining dual function $D(p) = \max_{x, f\in\Pi} L(p, x, f)$, by duality we have (see, e.g., Chapter 5 in [12])

$$\max_{x,f} \sum_s U_s(x_s) = \min_{p \geq 0} D(p) = \min_{p \geq 0} \max_{x,f \in \Pi} L(p,x,f).$$

The dual problem $\min_p D(p)$ can be solved by using the subgradient method [76] [12], where the Lagrangian multipliers are adjusted in the opposite direction to the subgradient of the dual function

$$g(p) = A(f(p)) - H(x(p)). \tag{3.12}$$

Congestion price update: The network (links or nodes) updates the congestion price, according to

$$p(t+1) = [p(t) + \gamma_t(H(x(p(t))) - A(f(p(t))))]^+, \tag{3.13}$$

where γ_t is a positive scalar stepsize, and "+" denotes the projection onto the set $\Re^+$ of nonnegative real numbers. The algorithm has a nice interpretation in terms of law of supply and demand and their regulation through pricing. Equation (3.13) says that, if the demand $H(x(p(t)))$ for service capacity exceeds the supply $A(f(p(t)))$, the price p will rise, which will in turn decrease the demand (see equation (3.10)) and increase the supply (see equation (3.11)).

Before proceeding, we explain the notation used in this chapter. We denote a link either by a single index l or by the directed pair (i, j) of nodes it connects. We use s or alternatively node pair $[i, k]$ to denote a network layer flow. We overload the use of the source rate x, the link capacity f and the congestion price p throughout this chapter, depending on different kinds of routing involved. For example, x refers to the source rate x_s or the source rate x_i^k at node i towards destination k, depending on the specific contexts. Similarly, f refers to both the link capacity $\{f_{i,j}\}$ and the capacity $\{f_{i,j}^k\}$ over link (i, j) that is allocated to destination k.

3.4.2 Convergence Analysis

Subgradient may not be a direction of descent, but makes an angle less than 90 degrees with all descent directions. Thus, the new iteration may not improve the dual cost for all values of the stepsize. Using results on the convergence of the subgradient method [76] [12], we show that, for constant stepsize, the algorithm is guaranteed to converge to within a neighborhood of the optimal value. For diminishing stepsize, the algorithm is guaranteed to converge to the optimal value. We would like a distributed implementation of the subgradient algorithm, and thus a constant stepsize $\gamma_t = \gamma$ is more convenient. Note that the dual cost usually will not monotonically approach the optimal value, but wander around it under the subgradient algorithm. The usual criterion for stability and convergence is not applicable. Here we define convergence in a statistical sense [21]. Let $\overline{p}(t) := \frac{1}{t}\sum_{\tau=1}^{t} p(\tau)$ be the average price by time t.

Definition 3.1 *Let p^* denote an optimal value of the dual variable. Algorithm (3.10)–(3.13) with constant stepsize is said to converge* statistically *to p^*, if for any $\delta > 0$ there exists a stepsize γ such that* $\limsup_{t\to\infty} D(\overline{p}(t)) - D(p^*) \leq \delta$.

Clearly, an optimal value p^* exists. The following theorem guarantees the statistical convergence of the subgradient method.

Theorem 3.2 *Let p^* be an optimal price. If the norm of the subgradients is uniformly bounded, i.e., there exists G such that $||g(p)||_2 \leq G$ for all p, then*

$$D(p^*) \leq \limsup_{t\to\infty} D(\overline{p}(t)) \leq D(p^*) + \gamma G^2/2, \tag{3.14}$$

i.e., the algorithm (3.10)–(3.13) converges statistically to p^*.

Proof. The first inequality $D(p^*) \leq \limsup_{t\to\infty} D(\overline{p}(t))$ always holds, since $D(p^*)$ is the minimum of the dual function $D(p)$. Now we prove the second inequality. By equation (3.13), we have

$$\begin{aligned} ||p(t+1)-p^*||_2^2 &= ||[p(t)-\gamma g(p(t))]^+ - p^*||_2^2 \\ &\leq ||p(t)-\gamma g(p(t)) - p^*||_2^2 \\ &= ||p(t)-p^*||_2^2 - 2\gamma g(p(t))^T(p(t)-p^*) + \gamma^2||g(p(t))||_2^2 \\ &\leq ||p(t)-p^*||_2^2 - 2\gamma(D(p(t))-D(p^*)) + \gamma^2||g(p(t))||_2^2, \end{aligned}$$

where the last inequality follows from the definition of subgradient. Applying the inequalities recursively, we obtain

$$||p(t+1)-p^*||_2^2 \leq ||p(1)-p^*||_2^2 - 2\gamma\sum_{\tau=1}^{t}(D(p(\tau))-D(p^*)) + \gamma^2\sum_{\tau=1}^{t}||g(p(\tau))||_2^2.$$

Since $||p(t+1)-p^*||_2^2 \geq 0$, we have

$$\begin{aligned} 2\gamma\sum_{\tau=1}^{t}(D(p(\tau))-D(p^*)) &\leq ||p(1)-p^*||_2^2 + \gamma^2\sum_{\tau=1}^{t}||g(p(\tau))||_2^2 \\ &\leq ||p(1)-p^*||_2^2 + t\gamma^2G^2. \end{aligned}$$

From this inequality we obtain

$$\frac{1}{t}\sum_{\tau=1}^{t}D(p(\tau)) - D(p^*) \leq \frac{||p(1)-p^*||_2^2}{2t\gamma} + \frac{\gamma G^2}{2}.$$

Since D is a convex function, by Jensen's inequality,

$$D(\overline{p}(t)) - D(p^*) \leq \frac{||p(1)-p^*||_2^2}{2t\gamma} + \frac{\gamma G^2}{2}.$$

Thus, $\limsup_{t\to\infty} D(\overline{p}(t)) \leq D(p^*) + \frac{\gamma G^2}{2}$, i.e., the algorithm converges converges statistically to p^*. ■

The assumption of bounded norm for subgradient $g(p)$ is reasonable, since f is finite and we always have an upper bound on x in practice. Theorem 3.2 implies that the congestion price p approaches p^* statistically when the stepsize γ is small enough.

Let the primal function be $P(x) := \sum_s U_s(x_s)$ and achieve its optimum at x^*. Define $\overline{x}(t) := \frac{1}{t}\sum_{\tau=1}^{t} x(\tau)$, the average data rate up to time t. As time goes to infinity, $\overline{x}(t)$ must be in the feasible rate region (determined by equations (3.8)–(3.9)), otherwise $\overline{p}(t)$ will go unbounded as time goes to infinity, which contradicts Theorem 3.2.

Theorem 3.3 *Let x^* be the optimal source rates. Under the same assumption of Theorem 3.2, the algorithm (3.10)–(3.13) converges statistically to within a small neighborhood of the optimal values $P(x^*)$, i.e.,*

$$P(x^*) \geq \liminf_{t\to\infty} P(\overline{x}(t)) \geq P(x^*) - \frac{\gamma G^2}{2}. \tag{3.15}$$

Proof. The first inequality $P(x^*) \geq \liminf_{t\to\infty} P(\overline{x}(t))$ holds, since $\overline{x}(t)$ is in the feasible rate region as t goes to infinity. Now we prove the second inequality. By equation (3.13), we have

$$\begin{aligned}
||p(t+1)||_2^2 &\leq ||p(t) - \gamma g(p(t))||_2^2 \\
&= ||p(t)||_2^2 - 2\gamma g(p(t))^T p(t) + \gamma^2 ||g(p(t))||_2^2 \\
&= ||p(t)||_2^2 + 2\gamma \sum_s U_s(x_s(t)) - 2\gamma(\sum_s U_s(x_s(t)) - p^T(t)H(x(t))) \\
&\quad -2\gamma p^T(t)A(f(t)) + \gamma^2||g(p(t))||_2^2 \\
&\leq ||p(t)||_2^2 + 2\gamma \sum_s U_s(x_s(t)) - 2\gamma(\sum_s U_s(x_s^*) - p^T(t)H(x^*)) \\
&\quad -2\gamma p^T(t)A(f(t)) + \gamma^2||g(p(t))||_2^2 \\
&= ||p(t)||_2^2 + 2\gamma P(x(t)) - 2\gamma P(x^*) + \gamma^2||g(p(t))||_2^2 \\
&\quad -2\gamma p^T(t)(A(f(t)) - H(x^*)) \\
&\leq ||p(t)||_2^2 + 2\gamma P(x(t)) - 2\gamma P(x^*) + \gamma^2||g(p(t))||_2^2,
\end{aligned}$$

where the second inequality follows from the fact that $x(t)$ is the maximizer in the problem (3.10), and the third inequality follows from the fact that $f(t)$ is the maximizer in problem (3.11). Applying the inequalities recursively, we obtain

$$||p(t+1)||_2^2 \leq ||p(1)||_2^2 + 2\gamma \sum_{\tau=1}^{t} (P(x(\tau)) - P(x^*)) + \gamma^2 \sum_{\tau=1}^{t} ||g(p(\tau))||_2^2.$$

Since $||p(t+1)||_2^2 \geq 0$, we have

$$\begin{aligned} 2\gamma \sum_{\tau=1}^{t} (P(x(\tau)) - P(x^*)) &\geq -||p(1)||_2^2 - \gamma^2 \sum_{\tau=1}^{t} ||g(p(\tau))||_2^2 \\ &\geq -||p(1)||_2^2 - t\gamma^2 G^2. \end{aligned}$$

From this inequality we obtain

$$\frac{1}{t} \sum_{\tau=1}^{t} P(x(\tau)) - P(x^*) \geq \frac{-||p(1)||_2^2 - t\gamma^2 G^2}{2t\gamma}.$$

Since P is a concave function, by Jensen's inequality,

$$P(\overline{x}(t)) - P(x^*) \geq \frac{-||p(1)||_2^2 - t\gamma^2 G^2}{2t\gamma}.$$

Thus, $\liminf_{t\to\infty} P(\overline{x}(t)) \geq P(x^*) - \frac{\gamma G^2}{2}$, i.e., the algorithm (3.10)–(3.13) converges statistically to within a small neighborhood of the optimal values $P(x^*)$. ■

Since $P(x)$ is continuous, Theorem 3.3 implies that the average source rate approaches the optimal x^* when γ is small enough.

3.5 Extension to the Networks with Time-Varying Channels

In the last section, we consider wireless ad hoc networks with fixed channels or single-rate devices, i.e., the capacity c_l is a constant when link l is active. However, recent years have seen the growing popularity and demand of multi-rate wireless network devices (e.g., 802.11a cards) that can adjust transmission rate according to the time-varying channel state and improve overall network utilization. Here, we consider the networks with time-varying channels and adaptive multi-rate devices.

3.5.1 Distributed Algorithm

We assume that time is slotted, and the channel is fixed within a time slot but independently changes between different slots.[1] Let $h(t)$ denote the channel state in time slot t. Corresponding to the channel state h, the capacity of link l is $c_l(h)$ when active and the feasible rate region at the link layer is $\Pi(h)$, which is defined in a similar way as in (3.1). We further assume that the channel state is a finite state process with identical distribution $q(h)$ in each time slot,[2] and define the mean feasible rate region as

$$\overline{\Pi} := \{\overline{r} : \overline{r} = \sum_{h} q(h)r(h), r(h) \in \Pi(h)\}. \tag{3.16}$$

Ideally, we would like to have a distributed algorithm that solves the following utility maximization problem

$$\max_{x,f} \quad \sum_{s} U_s(x_s) \tag{3.17}$$

$$\text{subject to} \quad H(x) \le A(f), \tag{3.18}$$

$$f \in \overline{\Pi}. \tag{3.19}$$

However, if we solve the above problem via dual decomposition, we may get a link rate assignment which is infeasible for the channel state at a given time slot. Instead we directly extend the algorithm (3.10)–(3.13) with a modification to handle time-varying channel. For convenience, we describe the algorithm in detail in the following:

[1] It is straightforward to extend our results to the network where the channel state process is modulated by a hidden Markov chain.

[2] Even if the channel state is a continuous process, we only have finite choices of modulation schemes. The corresponding capacities take discrete values.

Congestion control: At time t, given congestion price $p(t)$, the sources adjust flow rates x according to the congestion price

$$x(t) = x(p(t)) = \arg\max_{x} \sum_{s} U_s(x_s) - p^T(t)H(x). \tag{3.20}$$

Scheduling: In the beginning of period t, each node monitors the channel state $h(t)$, and over link l send an amount of data for each flow according to the rates f such that

$$f(t) = f(p(t)) \in \arg \max_{f \in \Pi(h(t))} p^T(t)A(f). \tag{3.21}$$

Congestion price update: The network (links or nodes) updates the congestion price, according to

$$p(t+1) = \lfloor [p(t) + \gamma(H(x(p(t))) - A(f(p(t))))]^+ \rfloor. \tag{3.22}$$

Here "$\lfloor\ \rfloor$" denotes the integer function floor, and for the simplicity of the presentation we let congestion price take integer values with appropriate unit.

The above algorithm cannot be derived from the dual decomposition of the problem (3.17)–(3.19). However, we will use the problem (3.17)–(3.19) as a reference system, and characterize the performance of the above algorithm with respect to it.

3.5.2 Stochastic Stability

Note that congestion price $p(t)$ is proportional to the queue lengths in the networks (at links or nodes). It takes discrete values, i.e., the queue lengths scaled by γ. Thus, congestion price $p(t)$ evolves according to a discrete-time, discrete-space Markov chain. We need to show that this markov chain is stable, i.e., the congestion price process reaches a steady state and does not become unbounded. It is easy to check that the Markov chain has a countable state space, but is not necessarily irreducible. In such a general case, the state space is partitioned in transient set T and different recurrent classes R_i. We define the system to be *stable* if all recurrent states are positive recurrent and the Markov process hits the recurrent states with probability one [83]. This will guarantee that the Markov chain will be absorbed/reduced into some recurrent class, and the positive recurrence ensures the ergodicity of the Markov chain over this class. We have the following result.

Theorem 3.4 *The Markov chain described by equation (3.22) is stable.*

Proof. Denote the dual function of the problem (3.17)–(3.19) by $\overline{D}(p)$ with an optimal price p^* and subgradient $\overline{g}(p)$, i.e., $\overline{g}(p) = A(\overline{f}(p)) - H(x(p))$ with $\overline{f}(p) \in \arg\max_{f \in \overline{\Pi}} p^T A(f)$. Consider the the Lyapunov function $V(p) = \|p - p^*\|_2^2$, we have

$$
\begin{aligned}
E[\Delta V_t(p)|p] &= E[V(p(t+1)) - V(p(t)) \mid p(t) = p] \\
&= E[V(\lfloor [p(t) - \gamma g(p(t))]^+ \rfloor) - V(p(t)) \mid p(t) = p] \\
&\leq E[V(p(t) - \gamma g(p(t))) - V(p(t)) \mid p(t) = p] \\
&= E[-\gamma g(p(t))^T (2(p(t) - p^*) - \gamma g(p(t))) \mid p(t) = p] \\
&= 2\gamma \overline{g}(p)^T (p^* - p) + \gamma^2 E[\|g(p(t))\|_2^2 \mid p(t) = p] \\
&\leq 2\gamma \overline{g}(p)^T (p^* - p) + \gamma^2 G^2,
\end{aligned}
$$

where we again use the assumption that the norm of $g(p(t))$ is bounded above by G. By the definition of subgradient, we further get

$$
E[\Delta V_t(p)|p] \leq 2\gamma(\overline{D}(p^*) - \overline{D}(p)) + \gamma^2 G^2.
$$

Let

$$
\delta = \max_{\overline{D}(p) - \overline{D}(p^*) \leq \gamma G^2} \|p - p^*\|_2
$$

and define $\mathcal{A} = \{p : \ \|p - p^*\|_2 \leq \delta\}$. We obtain

$$
E[\Delta V_t(p)|p] \leq -\gamma^2 G^2 \mathcal{I}_{p \in \mathcal{A}^c} + \gamma^2 G^2 \mathcal{I}_{p \in \mathcal{A}},
$$

where $\mathcal{I}$ is the index function. Thus, by Theorem 3.1 in [83], which is an extension of Foster's criterion [7], the Markov chain $p(t)$ is stable. ■

The above proof shows that the distance to the optimal p^* has negative conditional mean drift for all prices that have sufficiently large distance to p^*, and implies that the congestion price will stay near p^* when γ is small enough.

3.5.3 Performance Evaluation

We now characterize the performance of the algorithm (3.20)–(3.22) in terms of the dual and primal objective functions of the reference system problem (3.17)–(3.19).

Theorem 3.5 *The algorithm (3.20)–(3.22) converges statistically to within a small neighborhood of the optimal value* $\overline{D}(p^*)$*, i.e.,*

$$\overline{D}(p^*) \leq \overline{D}(E[p(\infty)]) \leq \overline{D}(p^*) + \gamma G^2/2, \tag{3.23}$$

where $p(\infty)$ *denotes the state of the Markov chain* $p(t)$ *in the steady state.*

Proof. The first inequality $\overline{D}(p^*) \leq \overline{D}(E[p(\infty)])$ always holds, since $\overline{D}(p^*)$ is the minimum of the dual function $\overline{D}(p)$. Now we prove the second inequality. From the proof of Theorem 3.4, we have

$$\begin{aligned} E[\Delta V_t(p)|p] &= E[V(p(t+1)) - V(p(t)) \mid p(t) = p] \\ &\leq 2\gamma(\overline{D}(p^*) - \overline{D}(p)) + \gamma^2 G^2. \end{aligned}$$

Taking expectation over p, we get

$$\begin{aligned} E[\Delta V_t(p)] &= E[V(p(t+1)) - V(p(t))] \\ &\leq 2\gamma(\overline{D}(p^*) - E[\overline{D}(p)]) + \gamma^2 G^2. \end{aligned}$$

Taking summation from $\tau = 0$ to $\tau = t - 1$, we obtain

$$E[V(p(t))] \quad \leq \quad E[V(p(0))] - 2\gamma \sum_{\tau=0}^{t-1} E[\overline{D}(p(\tau))] + 2\gamma t \overline{D}(p^*) + t\gamma^2 G^2.$$

Since $E[V(p(t))] \geq 0$, we have

$$2\gamma \sum_{\tau=0}^{t-1} E[\overline{D}(p(\tau))] - 2\gamma t \overline{D}(p^*) \leq E[V(p(0))] + t\gamma^2 G^2.$$

From this inequality we obtain

$$\frac{1}{t} \sum_{\tau=0}^{t-1} E[\overline{D}(p(\tau))] - \overline{D}(p^*) \leq \frac{E[V(p(0))] + t\gamma^2 G^2}{2t\gamma}.$$

Note that $p(t)$ is stationary and ergodic in some steady state by Theorem 3.4, and so is $\overline{D}(p(t))$. Thus,

$$\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[\overline{D}(p(\tau))] = E[\overline{D}(p(\infty))].$$

So,

$$E[\overline{D}(p(\infty))] - \overline{D}(p^*) \leq \gamma G^2/2.$$

Since $\overline{D}(p)$ is a convex function, by Jensen's inequality,

$$\overline{D}(E[p(\infty)]) - \overline{D}(p^*) \leq \gamma G^2/2,$$

i.e., the algorithm converges statistically to within $\gamma G^2/2$ of the optimal value $\overline{D}(p^*)$. ■

Since $\overline{D}(p)$ is a continuous function, Theorem 3.5 implies that the congestion price p approaches p^* statistically when γ is small enough.

Corollary 3.6 *$x(t)$ is a stable Markov chain. Moreover, the average arrival rates*

$E[x(\infty)] \in \overline{\Pi}$, where $x(\infty)$ denotes the state of the process $x(t)$ in the steady state.

Proof. $x(t)$ is a deterministic, finite-value function of $p(t)$. $x(t)$ is a stable Markov chain, since $p(t)$ is. $E[x(\infty)] \in \overline{\Pi}$, otherwise the average congetsion price $E[p(\infty)]$ will go unbounded, which contradicts Theorem 3.4. ∎

Theorem 3.7 *Let $\overline{P}(x)$ be the primal function and x^* be the optimal source rates of the reference system problem (3.17)–(3.19). The algorithm (3.20)–(3.22) converges statistically to within a small neighborhood of the optimal value $\overline{P}(x^*)$, i.e.,*

$$\overline{P}(x^*) \geq \overline{P}(E[x(\infty)]) \geq \overline{P}(x^*) - \frac{\gamma G^2}{2}. \tag{3.24}$$

Proof. The proof for the theorem is a straightforward extension of the proof of Theorem 3.3, following similar procedure as in the proof of Theorem 3.5. We skip the detail here. ∎

Since $\overline{P}(x)$ is a continuous function, Theorem 3.7 implies that the average source rate approaches the optimal of the ideal reference system (3.17)–(3.19) when stepsize γ is small enough. Theorems 3.5 and 3.7 show that, surprisingly, the algorithm (3.20)–(3.22) can be seen as a distributed algorithm to approximately solve the ideal reference system problem that is not readily solvable due to stochastic channel variations.

Our proofs for stability and performance bounds are rather general. They only use general properties of convexity and Markovity and the definition of subgradients. We thus have presented a general technique and results regarding the stability and optimality of dual algorithm for convex optimization in face of time-varying parameters. As the flow contention graph is a rather general construction and can be used to capture the interdependence or contention among parallel servers of any queueing networks, the aforementioned results are applicable to any systems that can be modelled by a general model of queueing network that is served by a set of interdependent parallel servers with time-varying service capabilities. In the next two sections, we will discuss two such applications. Other examples include fair scheduling in a generalized switch [78] [23], and TCP [60] with time-varying capacity as in last-hop wireless networks. It can include power control as well [26], as power does not change convexity of the feasible rate region.

3.6 Joint Congestion Control and Media Access Control Design

TCP was originally designed for wireline networks, where links are assumed to have fixed capacities. However, as wireless channel is a shared medium and interference-limited, wireless links are "elastic" and the capacities they obtain depend on the bandwidth sharing mechanism used at the link layer. This may result in various TCP performance problems in wireless networks.

One such problem is TCP unfairness over ad hoc wireless networks. Many existing wireless MAC protocols, such as DCF specified in IEEE 802.11 standard [41], are traffic independent and do not consider the actual requirements of the flows competing for the channel. These MAC protocols suffer from the unfairness problem, caused by the location dependency of the contentions, and exacerbated by the contention resolution mechanisms such as the binary exponential backoff algorithm adopted in DCF. When they interact with TCP, TCP will further penalize these flows with more contention. This will result in significant TCP unfairness in ad hoc wireless networks [35] [81] [92] [93] [94]. To illustrate this, consider the example in Figure 3.2, and assume there are four network-layer flows $A{\rightarrow}B$, $C{\rightarrow}D$, $E{\rightarrow}F$ and $G{\rightarrow}H$. The flow $C{\rightarrow}D$ experiences more contention and will build up queue faster than the other three flows. TCP will further penalize it by reducing the congestion window more aggressively, and the resulting throughput of flow $C{\rightarrow}D$ will be much less than that of other flows.

In addition to the location dependency of contentions, correlation among links is also the key to understand the interaction between transport and MAC layers. In wireline networks, link bandwidth is well defined and links are disjoint resources. But in wireless networks, as we mentioned above, links are correlated due to the

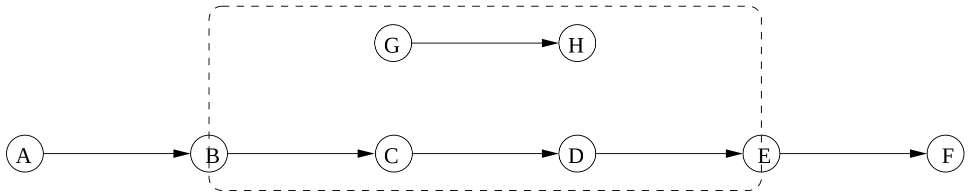

Figure 3.2: An example of a simple ad hoc wireless network.

interference with each other, and network-layer flows, which do not transverse a common link, may still compete with each other. Thus, congestion is located at some spatial contention region [94]. Consider again the example in Figure 3.2, and assume there are two network-layer flows $A{\rightarrow}F$ and $G{\rightarrow}H$. Link-layer flows BC, CD, DE and GH contend with each other, and congestion is located in the spatial contention region denoted by the rectangle. So, unlike wireline networks where link capacities provide constraints for resource allocation, in ad hoc wireless networks the contention relations between link-layer flows provide fundamental constraints for resource allocation. We need to exploit the interaction between transport and link (MAC) layers, in order to improve the performance.

The equations (3.2)–(3.3) capture the constraints that arise from channel contention among wireless links. We model the resource allocation for ad hoc wireless networks as a utility maximization problem with these constraints,

$$\max_{x,f} \quad \sum_s U_s(x_s) \tag{3.25}$$

$$\text{subject to} \quad Rx \leq f, \tag{3.26}$$

$$f \in \Pi. \tag{3.27}$$

With this formulation, we can explicitly exploit the interaction between transport and MAC layers, and systematically carry out joint design of congestion and media access control. In the next subsection, a dual algorithm is derived by solving the dual problem of the system problem (3.25)–(3.27). The solution to the dual problem motivates a scheme for media access control in which link-layer flows are scheduled according to congestion prices.

3.6.1 Distributed Algorithm

Consider the Lagrangian of the problem (3.25)–(3.27) with respect to the rate constraint

$$L(p, x, f) = \sum_s U_s(x_s) - p^T (Rx - f). \tag{3.28}$$

Interpret p_l as the congestion price at link l, we can use the algorithm (3.10)–(3.13) to solve the problem (3.25)–(3.27) and its dual.

Rate control: At time t, given congestion price $p(t)$, source s adjusts its sending rate x_s according to the aggregate congestion price $\sum_l R_{ls}p_l$ along its path

$$x_s(t) = U_s'^{-1}(\sum_l R_{ls}p_l(t)). \tag{3.29}$$

Scheduling: Over link l, send an amount of data for each flow according to the rate f such that

$$f(t) = f(p(t)) \in \arg\max_{f \in \Pi} p^T f. \tag{3.30}$$

If the network with time-varying channel is considered, each node monitors channel state $h(t)$ and over link l sends an amount of data for each flow according to the rate f such that

$$f(t) = f(p(t)) \in \arg\max_{f \in \Pi(h(t))} p^T f. \tag{3.31}$$

Congestion price update: Each link l updates its price, according to

$$p_l(t+1) = [p_l(t) + \gamma_t(\sum_s Rlsx_s(p(t)) - f_l(p(t)))]^+. \tag{3.32}$$

The above algorithm motivates a joint design scheme where the link layer flows are scheduled according to congestion prices of the links. Also, note that equations (3.29) and (3.32) are completely distributed and can be implemented at individual sources and links using only local information. We will discuss the distributed solution to scheduling problem (3.30) in the next subsection.

3.6.2 Scheduling over Ad Hoc Networks

We now come to the scheduling problem (3.30), which will also show out in the next section. Scheduling over ad hoc networks is a difficult problem and in general NP-hard. To see this, note that problem (3.30) is equivalent to a maximum weight independent set problem over the flow contention graph, which is NP-hard for general graphs. It is easy to design some heuristic algorithm but is hard to bound its performance.

With the primary interference model, the scheduling problem (3.30) is equivalent to the maximum weighted matching problem[3] over the connectivity graph $\{N, L\}$ of the network. Maximum weighted matching problem can be computed in polynomial time (see, e.g., [71]), but this requires centralized implementation. If implemented over an ad hoc network, each node needs to notify the central node of its weight and local connectivity information such that the central node can reconstruct the network topology as a weighted graph. This will lead to an immense communication overhead which is expensive in time and resources. There also exist simpler greedy sequential algorithms to compute a weighted matching at most a factor of 2 away from the maximum, see, e.g., [73]. But they also require centralized implementation. We seek a distributed algorithm where each node participates in the computation itself using only local information.

A few distributed approximation algorithms exist for maximum weighted matching problem, see, e.g., [85] [88] [39]. In [39], the author presents a simple distributed algorithm to compute a weighted matching at most a factor of 2 away from the maximum in linear running time $O(|L|)$. This algorithm is a distributed variant of the sequential greedy algorithm presented in [73]. We have utilized this algorithm to solve

[3] A matching in a graph is a subset of links, no two of which share a common node. The weight of a matching is the total weight of all its links. A maximum weighted matching in a graph is a matching whose weight is maximized over all matchings of the graph.

the scheduling problem (3.30) distributedly, see [22] for details. The resulting scheduling algorithm for ad hoc wireless networks is one of the best distributed algorithms in terms of computational complexity and performance bound. It has a linear complexity $O(|L|)$. Such a low complexity is important for the scalability and efficiency of ad hoc wireless networks. It achieves a performance of 1/2 of the maximum weight in the worst case, and in practice, numerical simulations show it typically achieves a performance within about 4/5 of the maximum weight. There also exist few other distributed approximation algorithms, see, e.g., [59] [90] [65]. Especially, in [65] the authors present a distributed randomized algorithm with comparable complexity that achieves nearly 100% throughput.

As for the overall performance of our cross-layer design with our approximate scheduling, we can extend the result in [59] to show that the performance is no worse than that achieved by an exact design with a feasible rate region $\frac{1}{2}\Pi$ (and in practice, $\frac{4}{5}\Pi$) at the link layer. Moreover, in [22] we also see that this distributed scheduling algorithm only results in a very small degradation in the performance of the cross-layer design for the network with time-varying channel, since in this situation the exact solution of the scheduling is not as important and reasonable approximations work well.

3.6.3 Numerical Examples

In this subsection, we provide numerical examples to complement the analysis in previous subsections. We consider a simple network with secondary interference as shown in Figure 3.2, and assume that there are three network layer flows $G \rightarrow H$, $A \rightarrow F$ and $D \rightarrow F$ with the same utility function $U_s(x_s) = \log x_s$. We have chosen such a small, simple topology to facilitate the detailed discussion of the results.

The Networks with Fixed Channel and Single-Rate Devices

We first consider the network with fixed link capacities. For simplicity, we assume that all the links have one unit of capacity when active. Figure 3.3 shows the evolution of source rates and their averages with the joint algorithm (3.29), (3.30) and (3.32) with stepsize $\gamma = 0.2$. We see that the source rates converge quickly to a neighborhood of the optimal and oscillate around the optimal. This oscillating behavior mathematically results from the non-differentiability of the dual function and physically can be interpreted as due to the scheduling process. However, the average source rates are smooth and approach the optimum monotonically. Figure 3.4 shows the evolution of the corresponding end-to-end congestion prices and their averages of the three flows. Similarly, the congestion prices approach the optimal quickly. We also note that the performance of the algorithm is much better than the bound of $\gamma G^2/2$ specified in Theorems 3.2 and 3.3.

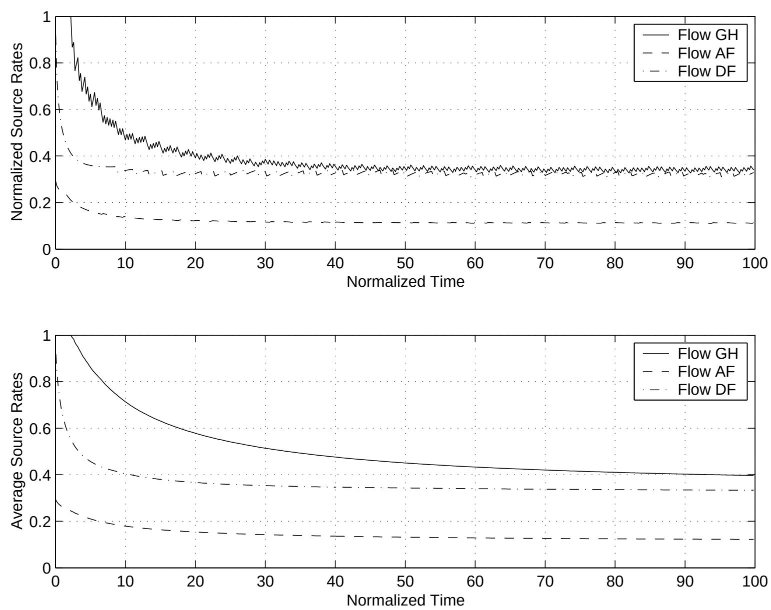

Figure 3.3: The evolution of source rates in the network with fixed link capacities.

The choice of the stepsize γ is important. It characterizes the "optimality" of the algorithm, as shown in Theorems 3.2 and 3.3 (and also in Theorems 3.5 and 3.7). It also affects the convergence speed. In oder to study the impact of different choices of the stepsize on the performance of the algorithm, we have run simulations with different stepsizes. We found that the smaller the stepsize, the slower the convergence and the closer to the optimal, which is a general characteristic of any gradient based algorithm. So, there is a tradeoff between convergence speed and optimality. In

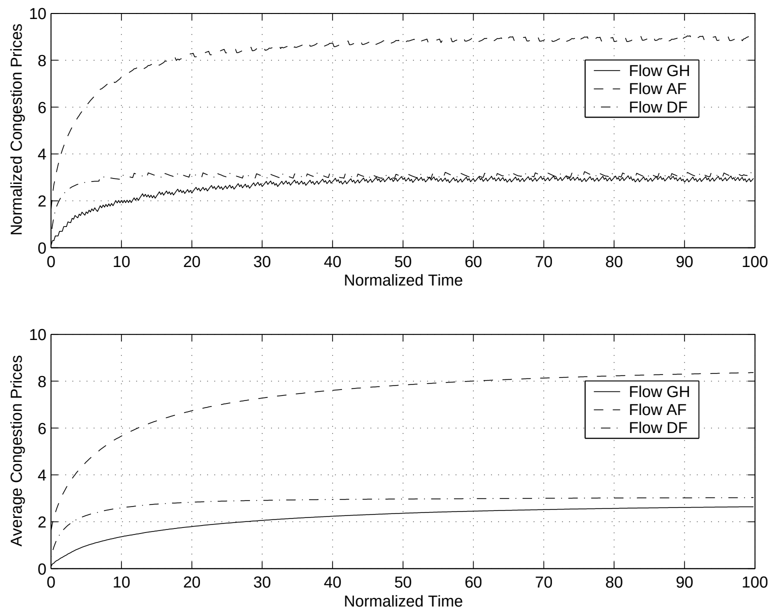

Figure 3.4: The evolution of congestion prices in the network with fixed link capacities.

practice, the end user can first choose large stepsizes to ensure fast convergence, and subsequently, the stepsizes can be reduced once the source rate starts oscillating around some mean value.

The Networks with Time-Varying Channel and Multirate Devices

We now consider the network with time-varying link capacities. For simplicity, we assume that the capacities of all links are identically, uniformly distributed over 0.5, 1 and 1.5 units. Thus, the average capacity for each link when active is the same as that in the example with fixed link capacities.

Figures 3.5 and 3.6 show the evolution of source rates, congestion prices and their averages with the same stepsize $\gamma = 0.2$. The source rates and congestion prices have much larger variations than those with fixed channel, due to the channel variations. But the average source rates and congestion prices are still smooth, and converge quickly and monotonically to optimal values. Our simulation results have confirmed the conclusions from Theorems 3.5 and 3.7, which say that the average source rates and congestion prices approach the optimum of an ideal system with the best feasible rate region at the link layer, and that algorithm (3.29), (3.31) and (3.32) can been seen as a distributed algorithm to solve this ideal system problem. Also note that, although the average link capacities when active are the same as those in fixed channel, each flow achieves larger sending rate. This is due to the multi-user diversity that we exploit when doing scheduling. Our "optimal" scheduling (3.31) has implicitly considered multi-user diversity.

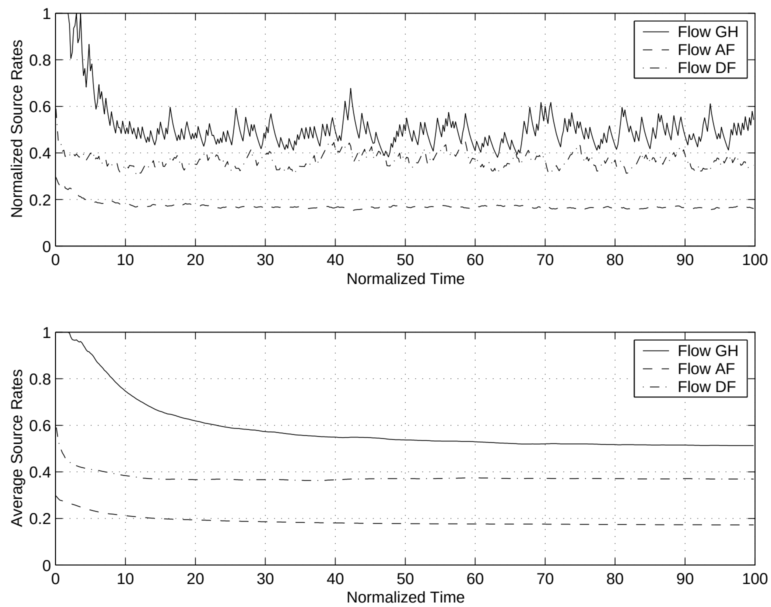

Figure 3.5: The evolution of source rates in the network with time-varying link capacities.

3.6.4 Summary

We have presented a model for the joint design of congestion control and media access control for ad hoc wireless networks, where the resulting dual algorithm is to solve a utility maximization problem with constraints that arise from contention for the wireless channel. This algorithm motivates a joint design where link-layer flows are scheduled according to the congestion prices of the links.

There exist other ways to solve the resource allocation problem (3.25)–(3.27). In [21], we also derive a primal algorithm by solving the relaxation of the system problem (3.25)–(3.27). Based on the algorithm, we propose a traffic-dependent scheme for contention-based medium access control and generate congestion price directly from

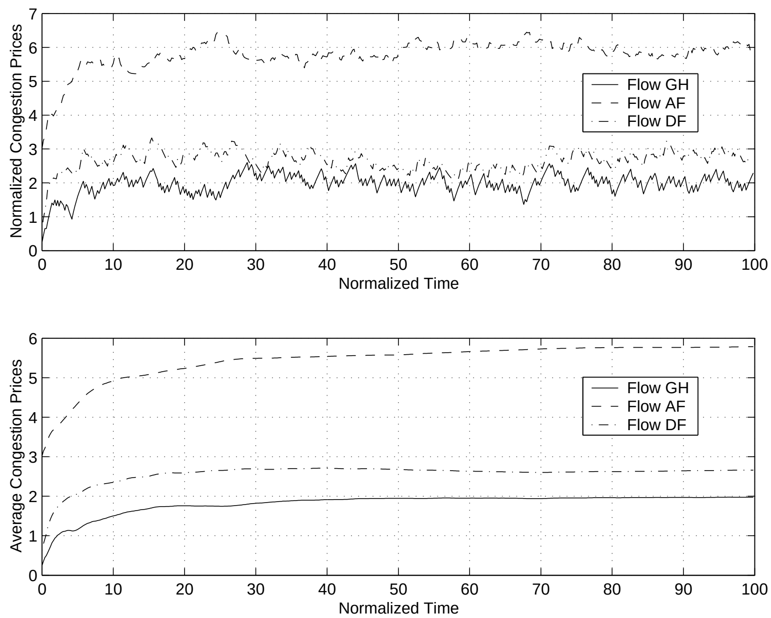

Figure 3.6: The evolution of congestion prices in the network with time-varying link capacities.

the MAC layer. As scheduling in ad hoc wireless networks is an intrinsically hard problem, contention-based medium access seems a must. To further integrate congestion control and contention-based medium access in utility maximization framework will be a future research step.

3.7 Joint Congestion Control, Routing and Scheduling Design

In the last section, we have discussed the resource allocation in ad hoc wireless networks where the path for each network layer flow is given. However, as wireless spectrum is a scarce resource, it may be costly to maintain end-to-end paths, and congestion control based on end-to-end feedback may consume too much bandwidth in signalling. Moreover, most routing schemes for ad hoc networks select paths that minimize hop count, see, e.g., [47] [74]. This implicitly predefines a path for any source-destination pair, independent of the pattern of traffic demand and interference/contention among links. This may result in congestion at some region while other regions are underutilized. In order to achieve high end-to-end throughput and efficient resource allocation, the paths should not be decided exogenously but jointly optimized with congestion control and scheduling.

Since the actual paths that will be used are not specified *a priori*, we will use multicommodity flow model for routing and model the resource allocation as a utility maximization problem with the constraints (3.2) and (3.5),

$$\max_{x,f} \quad \sum_s U_s(x_s) \tag{3.33}$$

$$\text{subject to} \quad x_i^k \leq \sum_{j:(i,j)\in L} f_{i,j}^k - \sum_{j:(j,i)\in L} f_{j,i}^k, \tag{3.34}$$

$$f \in \Pi, \tag{3.35}$$

where $i \in N, \ \ k \in D$, $i \neq k$, and $x_i^k = 0$ if $[i,k] \notin S \times D$. In the next subsection, we apply duality theory to obtain a distributed subgradient algorithm for joint congestion control, routing and scheduling. This algorithm motivates a joint design where the source adjusts its sending rate according to the congestion price generated locally at the source node, and backpressure from the differential price of neighboring nodes is used for optimal scheduling and routing.

3.7.1 Distributed Algorithm

Consider the Lagrangian of the problem (3.33)–(3.35) with respect to the rate constraint

$$L(p,x,f) = \sum_s U_s(x_s) - \sum_{i\in N, k\in D, i\neq k} p_i^k (x_i^k - \sum_{j:(i,j)\in L} f_{i,j}^k + \sum_{j:(j,i)\in L} f_{j,i}^k). \tag{3.36}$$

Interpret p_l^k as the congestion price at node i for the flows to destination k, we can use the algorithm (3.10)–(3.13) to solve the problem (3.33)–(3.35) and its dual.

Rate control: At time t, given congestion price $p(t)$, the source s adjusts its sending rate x_s according to the local congestion price at the source node

$$x_s(p) = U_s'^{-1}(p_s), \tag{3.37}$$

where $p_s = p_k^i$ for $s = [i,k] \in S \times D$. In contrast to traditional TCP congestion control where the source adjusts its sending rate according to the aggregate price along its path, in this algorithm the congestion price is generated locally at the source node.

Note that, since

$$\sum_{i,k} p_i^k \left(\sum_j f_{i,j}^k - \sum_j f_{j,i}^k \right) = \sum_{i,j,k} f_{i,j}^k \left(p_i^k - p_j^k \right),$$

the scheduling problem is equivalent to the following problem

$$\max_{f \in \Pi} \sum_{i,j} f_{i,j} \max_k \left(p_i^k - p_j^k \right). \tag{3.38}$$

This motivates the following joint scheduling and routing algorithm:

Scheduling: Each node i collects congestion price information from its neighbor j, finds destination $k(t)$ such that $k(t) \in \arg\max_k (p_i^k(t) - p_j^k(t))$, and calculates differential price $w_{i,j}(t) = p_i^{k(t)}(t) - p_j^{k(t)}(t)$ and passes this information to its neighbors. Allocate capacities $\widetilde{f}_{i,j}(t)$ over links (i,j) such that

$$\widetilde{f}(t) \in \arg\max_{f \in \Pi} \sum_{(i,j) \in L} w_{i,j}(t) f_{i,j}. \tag{3.39}$$

If the network with time-varying channel is considered, each node monitors the channel state $h(t)$ and allocates capacities $\widetilde{f}_{i,j}(t)$ over links (i,j) such that

$$\widetilde{f}(t) \in \arg\max_{f \in \Pi(h(t))} \sum_{(i,j) \in L} w_{i,j}(t) f_{i,j}. \tag{3.40}$$

Routing: Over link (i,j), send a number of bits for destination $k(t)$ according to the rate determined by the scheduling.

The $w_{i,j}$ values represent the maximum differential congestion price of destination k flows between nodes i and j. The above algorithm uses backpressure to do optimal scheduling and find optimal routing. Also note that the scheduling problem is solved by the following assignment,

$$f_{i,j}^k(t) = \begin{cases} \widetilde{f}_{i,j}(t) & \text{if } k = k(t), \\ 0 & \text{if } k \neq k(t). \end{cases}$$

Congestion price update: Each node i updates its price with respect to destination k, according to

$$p_i^k(t+1) = [p_i^k(t) + \gamma_t(\ x_i^k(p(t)) - (\sum_{j:(i,j)\in L} f_{i,j}^k(p(t)) - \sum_{j:(j,i)\in L} f_{j,i}^k(p(t))\ )\ )]^+, \quad (3.41)$$

and passes the price p_i^k to all its neighbors. Note that $p_i^k(t)$ is interpreted as the congestion price at the beginning of time slot t.

The above dual algorithm motivates a joint congestion control, routing and scheduling design where at the transport layer sources s individually adjust their rates according to the local congestion price at the source nodes, and nodes i individually update their prices according to (3.41), and at the network/link layer nodes i solve the scheduling (3.39) and route data flows accordingly. Also, note that the congestion control is not an end-to-end scheme. There is no need to maintain end-to-end paths and no communication overhead for congestion control.

3.7.2 Numerical Examples

In this subsection, we provide numerical examples to complement the analysis in the previous subsections. We consider a simple ad hoc network with primary interference as shown in Figure 3.7, and assume that there are two network layer flows $A \rightarrow F$ and $B \rightarrow E$ with the same utility $U_s(x_s) = \log x_s$.

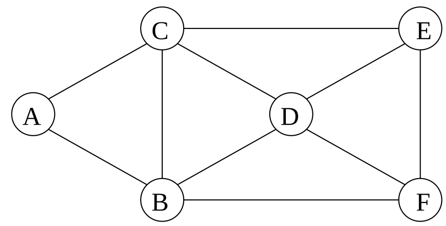

Figure 3.7: A simple network with two network layer flows. All links are bidirectional.

The Network with Fixed Channel and Single-Rate Devices

We consider first the network with fixed link capacities. For simplicity, we assume that links CE, EC, BF and FB have one unit of capacity and all other links have 2 units of capacity when active. Figure 3.8 shows the evolution of source rate and congestion price of each flow with the joint algorithm (3.37), (3.39) and (3.41) with stepsize $\gamma = 0.2$. We see that they converge quickly to a neighborhood of the optimal and oscillate around the optimal. However, Figure 3.9 shows that the average source rates and congestion prices are smooth and approach the optimum monotonically. We again note that the performance of the algorithm is much better than the bound of $\gamma G^2/2$ specified in Theorems 3.2 and 3.3.

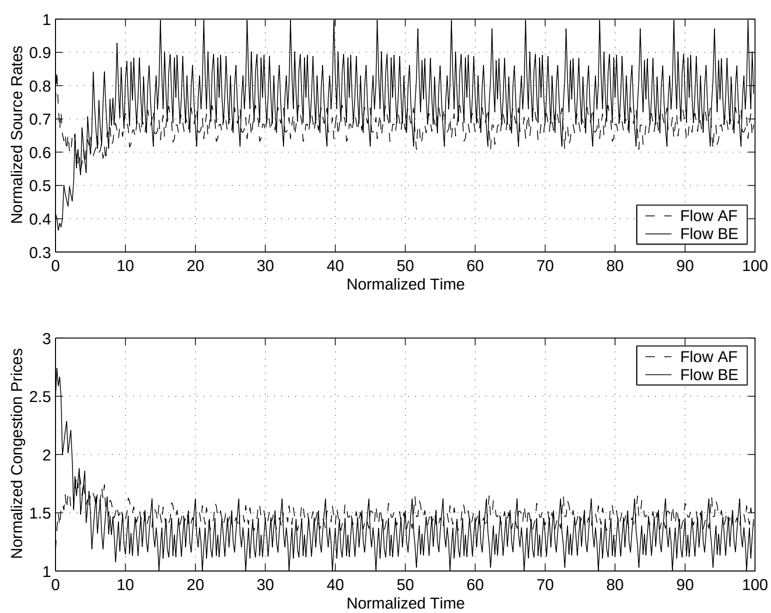

Figure 3.8: Source rates and congestion prices in the network with fixed link capacities.

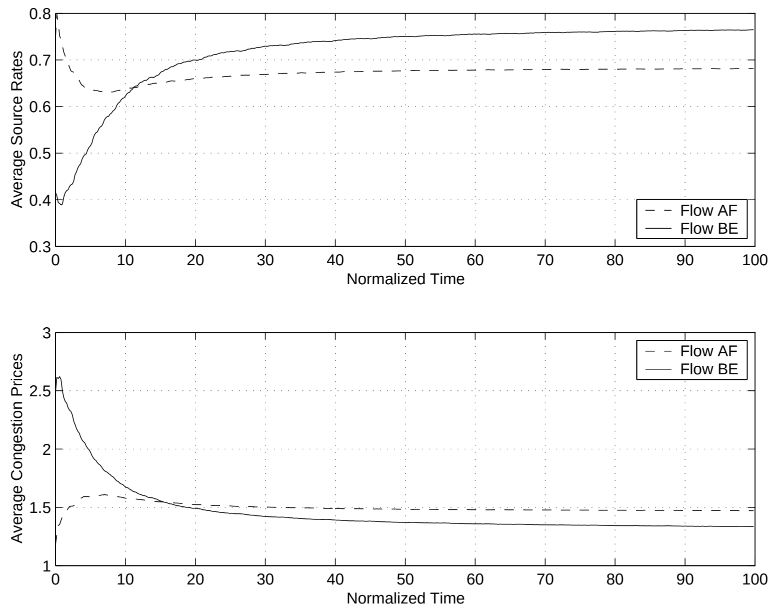

Figure 3.9: The average source rates and congestion prices in the network with fixed link capacities.

Table 3.1 shows the average link rates allocated to each flow.[4] In this table (and another table in this subsection), the first column contains the sending nodes and the first row contains the receiving nodes of each directed link. From this table, we can tell which paths each flow has used. Note that links BC and CB are not used. This is due to the fact that BC and CB are near the sources and is the link with most contention. So, an optimal routing and scheduling will not activate it.

The Networks with Time-Varying Channel and Multirate Devices

We now consider the network with time-varying link capacities. For simplicity, we assume that links CE, EC, BF and FB's capacities are identically, uniformly distributed over 0.5, 1 and 1.5 units, while other links' capacities are identically, uniformly distributed over 1, 2 and 3 units. Thus, the average capacity for each link when active is the same as that in the example with fixed link capacities.

Figures 3.10 and 3.11 show the evolution of source rates, congestion prices and their averages with the same stepsize $\gamma = 0.2$. The source rates and congestion

[4] In this and another tables, flows are slightly not conserved at some nodes. This is because we run numerical simulations for finite time and some residual effect of the initial condition remains.

Table 3.1: Average rates of flows AF (upper table) and BE (lower table) through different links in the network with fixed link capacities

Rates	A	B	C	D	E	F
A	0	0.265	0.404	0	0	0
B	0	0	0	0	0	0.262
C	0	0	0	0.222	0.182	0
D	0	0	0	0	0	0.222
E	0	0	0	0	0	0.182
F	0	0	0	0	0	0

Rates	A	B	C	D	E	F
A	0	0.000	0.000	0	0	0
B	0	0	0	0.510	0	0.225
C	0	0	0	0	0	0
D	0	0	0	0	0.510	0
E	0	0	0	0	0	0
F	0	0	0	0	0.225	0

prices have much larger variations than those with fixed channel, due to the channel variations. But the average source rates and congestion prices are still smooth, and converge quickly and monotonically to optimal values. Note that, although the average link capacity when active is the same as that in fixed channel, each flow achieves larger sending rates. This is again due to the multi-user diversity that we exploit when doing scheduling. Also note that the increase in sending rate of flow BE is much more notable. This is because node B has more neighbors and thus a much larger multi-user diversity.

Table 3.2 summarizes the average link rates allocated to each flow. We see that the routing pattern has changed for flow BE, while almost all the data for flow AF are routed along the same paths as those for the network with fixed link capacities. This change is due to the time-varying capacities, which makes every link have a chance to be a globally heavy link for some channel state and thus affects the paths each flow takes.

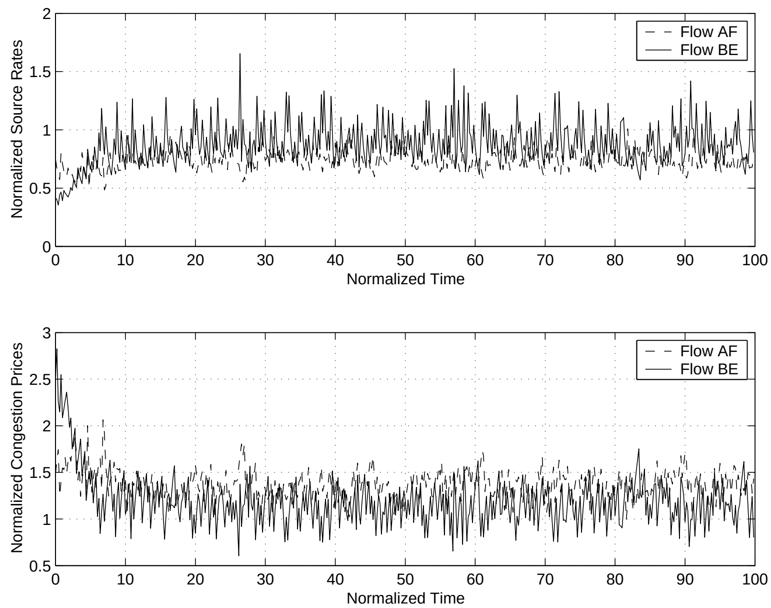

Figure 3.10: Source rates and congestion prices in the network with time-varying link capacities.

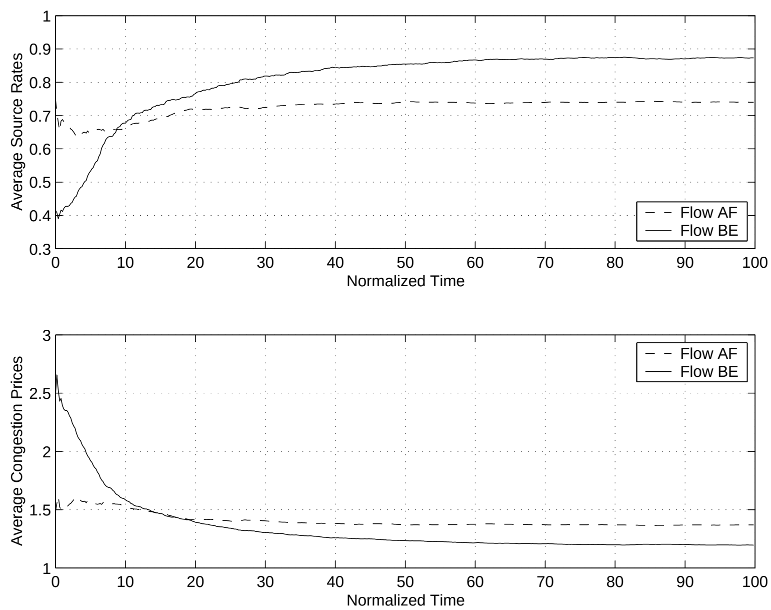

Figure 3.11: The average source rates and congestion prices in the network with time-varying link capacities.

3.7.3 Summary

We have presented a model for the joint design of congestion control, routing and scheduling for ad hoc wireless networks. The resulting dual algorithm motivates a

Table 3.2: Average rates of flows AF (upper table) and BE (lower table) through different links in the network with time-varying link capacities

Rates	A	B	C	D	E	F
A	0	0.328	0.390	0	0	0
B	0	0	0	0.074	0	0.253
C	0	0.001	0	0.243	0.147	0
D	0	0	0	0	0.022	0.295
E	0	0	0	0	0	0.169
F	0	0	0	0	0	0

Rates	A	B	C	D	E	F
A	0	0	0.104	0	0	0
B	0.104	0	0.032	0.504	0	0.211
C	0	0	0	0.012	0.124	0
D	0	0	0	0	0.443	0.072
E	0	0	0	0	0	0
F	0	0	0	0	0.283	0

joint design where at the transport layer, sources s adjust their rates according to the local congestion price at the source nodes, and at the network/link layer nodes solve the scheduling and route data flows according to backpressure in congestion between neighboring nodes. As our design only requires nodes exchanging local information with their neighbors and does not need to maintain end-to-end paths, it has a very low communication overhead and can adapt to changing topologies such as those in mobile ad hoc networks.

3.8 Conclusions

We have seen in previous sections that, by formulating a general utility maximization problem for the network design, duality theory leads to a natural "vertical" decomposition into functional modules of various layers of the protocol stack and "horizontal" decomposition into distributed computation across various network nodes or links. As shown in Figure 3.12, our current theory integrates three functions—congestion control, routing and scheduling—in transport, network and link layers into a coherent framework. With this layering scheme, the dual variables of the utility maximization problem capture the network state information and are the information that is passed across the interfaces among different layers. These layers are interacting through and coordinated by the dual variables, i.e., congestion prices, so as to achieve a global optimality. Even though this framework does not provide all the design and implementation details (such as the implementation of congestion prices and signalling mechanism), it helps us understand issues, clarify ideas, and suggests directions, leading to better and more robust designs for ad hoc wireless networks.

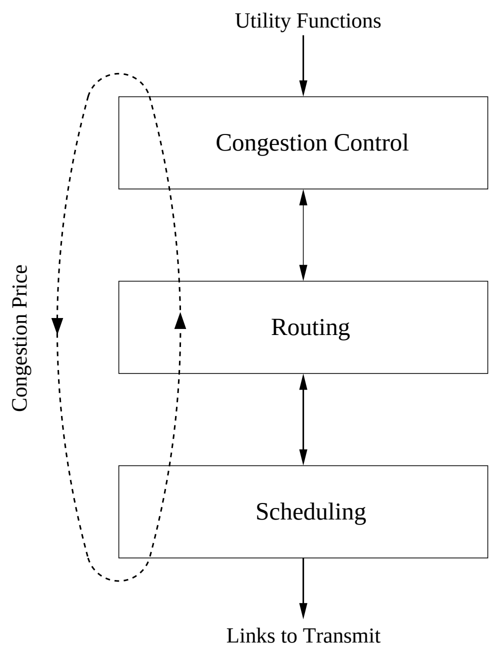

Figure 3.12: Layering as dual decomposition.

This framework—layering as dual decomposition—is promising to be extended to provide a mathematical theory for network architecture, and to allow us to systematically carry out cross-layer design. In this general framework, application needs (possibly, plus other performance metrics such as network cost) form the objective function (i.e., network utility to be maximized) and the restrictions in resource provisioning are translated into the constraints of the generalized network utility maximization problem. By choosing different objective functions and having different sets of decision variables involved, we can explicitly characterize and trade off different design objectives such as performance, scalability and robustness.

There exist, however, some challenging issues with this framework. First, utility design, i.e., how to model the user or application needs, is not an easy task, especially for real-time applications. Second, the general utility maximization problems may be very difficult nonlinear, nonconvex optimization with integer constraints. Third, this framework only involves the functionalities of the data plane of the network, but leaves out the issues related to the control plane such as the implementation and management complexity. To address these issues will be a future research step.

Chapter 4

Contention Control: A Game-Theoretic Approach

In this chapter, we develop a general game-theoretic framework for contention control [24]. We define a general game-theoretic model, called random access game, to capture the contention/interaction among wireless nodes in wireless networks with contention-based medium access, and design a novel medium access method derived from CSMA/CA according to distributed strategy update mechanism achieving the Nash equilibrium of random access game. Our access method uses a continuous contention measure – conditional collision probability, and could stabilize the network into a steady state with a target fairness (or service differentiation) and high efficiency. As wireless nodes can estimate conditional collision probabilities by observing consecutive idle slots between transmissions, our access method can decouple contention control from handling failed transmissions. This also opens up other opportunities such as rate adaptation to channel variations. In addition to providing a general and systematic design methodology for medium access control, the random access game model also provides an analytical framework to understand equilibrium properties such as throughput, loss and fairness, and dynamic properties of different medium access protocols and their interactions.

4.1 Introduction

Wireless channel is a shared medium and interference limited. Contention-based medium access control (contention control) is a distributed strategy to access and share wireless channel among contending wireless nodes. From a control-theoretic point of view, it consists of two components: a contention resolution algorithm that dynamically adjusts persistence probability or contention window in response to contention in the network, and a feedback mechanism that updates a contention measure and sends it back to wireless nodes. Contention resolution is usually achieved through two mechanisms: persistence and backoff [67]. In the persistence mechanism, each wireless node maintains a persistence probability and accesses the channel with this probability when it perceives an idle channel. In the backoff mechanism, each wireless node maintains a contention window and waits for a random amount of time bounded by the contention window before a transmission. When simultaneous accesses to the channel by different nodes cause contention, the persistence probability or contention window is adjusted appropriately so that contention is reduced. Different medium access control methods differ in terms of how they adjust these parameters in response to contention and what contention measure they use. For example, the standard IEEE 802.11 DCF uses a backoff mechanism and a binary contention signal – packet collision or successful transmission, in which each wireless node doubles its contention window upon a collision (binary exponential backoff) and sets it to the base value upon a successful transmission [41].

The choice of contention measure and contention resolution algorithm is key to the performance of medium access methods. "Inappropriate" choice of these two components will result in poor performance. For example, when the number of contending nodes is large, 802.11 DCF results in too many collisions and hence low throughput, because setting to the base contention window upon successful transmission is too drastic and each new transmission starts with the base contention window independent of the contention level in the network. It also has short-term unfairness problem, due to oscillation in contention window. The binary exponential backoff directly causes short-term unfairness. However, this oscillation in contention window is unavoidable because DCF uses a binary contention signal. In order to achieve high efficiency (high throughput and low collision) and better fairness, we need to stabilize the network into a steady state which sustains an appropriate contention window size (or equivalently, persistence probability) for each node. Furthermore, how we can estimate and implement the contention measure is important. Almost all medium access methods, including 802.11 DCF, adapt to packet collisions. However, they cannot distinguish collisions from corrupted frames that are common in wireless networks. This leads to lower throughput and increased unfairness. To ensure good performance, we need to use a contention measure whose estimation is not based on packet collisions, and decouple contention control from handling failed transmissions.

The main motivation of this work is to provide an analytical framework to systematically study the contention/interation among wireless nodes and design medium access methods that could stabilize the network around a steady state with a target fairness (or service differentiation) and high efficiency. To this end, we define a general game-theoretic model, called random access game, to capture the contention/interaction among wireless nodes in wireless networks with contention-based medium access. Here the game-theoretic model is not intended to model selfish behaviors of wireless nodes, but rather to capture the constraints encountered in real networks. In real networks, we prefer distributed algorithms with no or minimal explicit message passing, and each wireless node does not know how many nodes are present, is not aware of the actions (such as transmission or channel access probability) of others *a priori*, and can only sense limited information about the channel state (such as packets encountering collisions, or channel being idle or busy). In such a situation, the best a node can do is to optimize some local or private objective and adjust its action based on limited information about the network state. Noncooperative game is best to model such a situation, and we design random access game to guide individual nodes to seek an equilibrium that achieves some systemwide performance objective.

In random access games, a player (wireless node) strategy is its persistence probability or equivalently its contention window size, and its payoff function includes both utility gain from channel access and cost from packet collision. Through the specification of per-node utility function, we can model a large class of systemwide quality of service models, similar to that in utility framework for network flows [48] [60]. We characterize the Nash equilibrium of random access games, study their dynamics and propose algorithms (strategy evolutions) to achieve the Nash equilibrium. We show that systemwide fairness or service differentiation can be achieved in a distributed manner as long as each node executes a contention resolution algorithm that is designed to achieve the Nash equilibrium.

Based on the understanding of the equilibrium and dynamics of random access games, we propose a novel medium access method derived from CSMA/CA in which each node estimates its conditional collision probability and adjusts its persistence probability or equivalently contention window accordingly. Unlike other medium access methods, our method adapts to continuous feedback signal (conditional collision probability) rather than binary contention signal (packet collision or successful transmission), and each node tries to keep a fixed persistence probability or equivalently contention window specified by the Nash equilibrium of random access game. In addition to simpler dynamics resulting from responding to continuous feedback and controllable performance objectives via the specification of per-node utility functions, as the conditional collision probability is a more accurate measure of contention in the network, our medium access method achieves better contention control (collision reduction) and hence higher throughput. Moreover, as wireless nodes can estimate conditional collision probabilities by observing consecutive idle slots between transmissions, we can decouple contention control from handling failed transmissions. This also opens up other opportunities such as rate adaptation to channel variations. As a case study of medium access control design in game theory framework, we present a concrete medium access method and show that it achieves higher throughput, lower collision and better short-term fairness than the standard 802.11 DCF, and can provide flexible service differentiations among wireless nodes.

The remainder of this chapter is organized as follows. The next section briefly discusses some related work. Section 4.3 presents details of the random access game model and medium access control design in general for a single-cell wireless LAN, and section 4.4 presents a concrete medium access control design and evaluates its performance. Section 4.5 extends the game-theoretic framework to multicell wireless LANs. Section 4.6 discusses utility function and reverse engineering of medium access control protocols in our framework. Section 4.7 concludes the chapter with some discussions on further research.

4.2 Related Work

Game-theoretic approach has been applied extensively to study random access. Jin et al. [45] study noncooperative equilibrium of Aloha networks and their local convergence. MacKenzie et al. [63] study the stability of multipacket slotted Aloha. Altman et al. [4] and Borkar et al. [16] study distributed scheme for adapting random access. Altman et al. [3] [5] study distributed choice of retransmission probability in slotted Aloha with partial information, and with priorities and random power. Tang et al. [82] reverse engineer binary exponential backoff algorithm in game theory framework. Čagalj et al. [19] study selfish behavior in CSMA/CA networks using game-theoretic approach and propose a distributed protocol to guide multiple selfish nodes to a Pareto-optimal Nash equilibrium. We do not consider such selfish behaviors of wireless users that tamper with wireless interfaces to increase their share of channel access as in [19]. In contrast, we use game-theoretic model to capture the information and implementation constraints encountered in real networks and design games to guide distributed users to achieve systemwide performance objectives. Another major difference of our work from most other game-theoretic works is that we take a control-theoretic viewpoint and regard channel access probabilities as dynamic variables. As such, we define a general utility for each user directly in terms of its channel access probability, and specify a special structure for random access game that respects the distributed and adaptive nature of contention-based medium access, see the first paragraph in the introduction. Optimization-theoretic framework has also been used to design medium access control, see, e.g., [67] [57]. However, there is serious limitation for the optimization-based design: extensive message passing among wireless nodes is needed to align the behaviors of individual nodes to achieve some global optimality; otherwise, the convergence to the optimality is actually not guaranteed.

There are many papers on various enhancements and improvements to 802.11 DCF. We will only briefly discuss some designs that propose better contention resolution algorithms and that improve throughput by tuning contention window according to the number of contending nodes. Aad et al. [2] introduce slow decrease method to improve efficiency and fairness. Kwon et al. [55] propose fast collision resolution algorithm for throughput improvement. Our design is different in terms of both contention measure and contention resolution algorithm. Bianchi et al. [13] and Cali et al. [20] propose to choose and compute an optimal contention window to maximize the throughput. They need sophisticated methods to estimate the number of contending nodes in the system, while in our access method wireless nodes do not need that information but still are able to choose optimal contention windows. There also exists extensive work on 802.11 QoS provisioning, see, e.g., [1] [42]. Our access method can provide general and more flexible service differentiations through the specification of per-node utility functions, except for manipulating the length of inter-frame space.

Related work also includes [38], which proposes idle sense access method for a single-cell wireless LAN that compares the mean number of idle slots between transmission attempts with the optimal value and adopts an additive increase and multiplicative decrease algorithm to dynamically control the contention window in order to improve throughput and short-term fairness. In our access method, wireless nodes estimate conditional collision probabilities by observing consecutive idle slots between transmissions. So, like idle sense access method, our access method can decouple contention control from handling failed transmissions (that could be due to packet collisions or corrupted frames) and be able to estimate frame error rate and based on that to do rate adaptation. However, idle sense method intends to make contention windows equal for all wireless nodes and requires the calculation of optimal average number of idle slots between transmissions. It is not clear how to achieve this for a single-cell wireless LAN of heterogenous users with different system parameters or different service requirements and how to extend to the multicell networks. In contrast, our access method is fully distributed and adaptive, and works for single-cell wireless LANs of heterogeneous users and multicell wireless LANs.

Finally, several works, see, e.g., [14] [52] [75], have proposed analytical models for 802.11 DCF and studied its performance by fixed point analysis. Our analysis of nontrivial Nash equilibrium has some similarity with those fixed point analysis.

4.3 Random Access Game

Consider a set N of wireless nodes in a wireless LAN with contention-based medium access. We first focus on single-cell wireless LANs,[1] and will consider multicell wireless LANs in section 4.5. We consider the case of greedy nodes, i.e., they always have a frame to transmit. We will mainly present our theory and analysis in terms of "channel access probability." If a persistence mechanism is implemented, the channel access probability is just the persistence probability. If a backoff mechanism is implemented, channel access probability p is related to a constant contention window cw according to

$$p = \frac{2}{cw + 1}. \tag{4.1}$$

Relation (4.1) can be derived under the decoupling approximation for a set of wireless nodes with constant contention windows, see, e.g., [14] [52]. The decoupling approximation is an extremely accurate approximation, as validated by extensive simulations reported in, e.g., [14] [52].

Assume that each node $i \in N$ attains a utility $U_i(p_i)$ when it accesses the channel with probability p_i. We assume that $U_i(\cdot)$ is continuously differentiable, strictly concave, and with finite curvatures that are bounded away from zero, i.e.,

[1] Single-cell means that every wireless node can hear every other node in the network. For example, a single-cell ad hoc wireless network or a single-cell wireless access network.

there exist some constants μ and λ such that $1/\mu \geq -1/U_i''(p_i) \geq 1/\lambda > 0$. Let $q_i(\mathbf{p}) := 1 - \prod_{j \in N/\{i\}}(1 - p_j)$ denote the conditional collision probability of node i.[2] Our objective is to choose $\mathbf{p} := (p_1, p_2, \ldots, p_{|N|})$ such that each node maximizes its payoff $U_i(p_i) - p_i q_i$. Since wireless nodes are not aware of channel access probabilities of others *a priori*, we model their interaction as a noncooperative game. Formally, we define a random access game as follows.

Definition 4.1 *A random access game $\mathcal{G}$ is defined as a triple $\mathcal{G} := \{N, (S_i)_{i \in N}, (u_i)_{i \in N}\}$, where N is a set of players (wireless nodes), player $i \in N$ strategy $S_i := \{p_i | p_i \in [\nu_i, \omega_i]\}$ with $0 \leq \nu_i < \omega_i \leq 1$, and payoff function $u_i(\mathbf{p}) := U_i(p_i) - p_i q_i(\mathbf{p})$ with $q_i(\mathbf{p}) := 1 - \prod_{j \in N/\{i\}}(1 - p_j)$.*

We may constrain the strategy space S_i of each node to a strict subset of $[0, 1]$, in order to prevent a node from exclusively occupying wireless channel or being completely excluded from the channel. We further assume that utility $U_i(p_i)$ is an increasing function in the strategy space S_i. Wireless nodes (players) interact through collisions. Note that the throughput of node i is proportional to p_i if there is no collision, and $p_i q_i$ is the collision probability experienced by node i and can be seen as collision cost. Thus, the payoff function $u_i(\cdot)$ has a nice interpretation: the net gain of utility from channel access, discounted by collision cost.[3] Correspondingly, the payoff function has a simple structure, which makes the analytical study of equilibrium and dynamic properties of random access game manageable. This will in turn facilitate the design of medium access control.

Random access game $\mathcal{G}$ is defined in a rather general manner. Each node i can choose any utility function $U_i(\cdot)$ it thinks appropriate. We classify different choices into two categories as follows. If all nodes have the same utility functions, the system is said to have homogeneous users. If the nodes have different utility functions, the system is said to have heterogeneous users. The motivation for studying systems of heterogeneous users is to provide differentiated services to different wireless nodes.

[2] For any two sets A and B, define $A/B := \{i | i \in A \text{ and } i \notin B\}$.

[3] In most references on game theory, utility function and payoff function mean the same thing. In this thesis we distinguish between them.

4.3.1 Nash Equilibrium

We now analyze the equilibrium of random access game. The solution concept we use is the Nash equilibrium [34]. Denote the strategy (channel access probability) selection for all nodes but i by $\mathbf{p}_{-i} := (p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_{|N|})$, and write $(p_i, \mathbf{p}_{-i})$ for the strategy profile $(p_1, p_2, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_{|N|})$. A vector of access probability $\mathbf{p}^*$ is a Nash equilibrium if, for all nodes $i \in N$, $u_i(p_i^*, \mathbf{p}_{-i}^*) \geq u_i(p_i, \mathbf{p}_{-i}^*)$ for all $p_i \in S_i$. We see that the Nash equilibrium is a set of strategies for which no player has an incentive to change unilaterally. The following result is immediate.

Theorem 4.2 *There exists a Nash equilibrium for any random access game $\mathcal{G}$.*

Proof. Since the strategy spaces S_i are compact convex sets, and the payoff functions u_i are continuous and concave in p_i, there exists a Nash equilibrium [34]. ■

Since utility function $U_i(\cdot)$ is concave, at the Nash equilibrium, $p_i^\star$ either takes value at the boundaries of the strategy space S_i or satisfies

$$U_i'(p_i^*) = q_i(\mathbf{p}^*), \tag{4.2}$$

where $U_i'(p_i^*) = \frac{dU_i(p_i^*)}{dp_i}$, the marginal utility at p_i^*. We call a Nash equilibrium $\mathbf{p}^*$ a *nontrivial* equilibrium if, for all nodes i, p_i^* satisfies equation (4.2), and *trivial* equilibrium otherwise. In the remainder of this section, we will mainly focus on nontrivial Nash equilibria.

Theorem 4.3 *Random access game $\mathcal{G}$ has a nontrivial Nash equilibrium if, for each node $i \in N$, inverse function $(U_i')^{-1}(q_i)$ maps any $q_i \in [0,1]$ into a point $p_i \in S_i$.*

Proof. From equation (4.2), we get

$$p_i^* = G_i(\mathbf{p}^*) := (U_i')^{-1}(q_i(\mathbf{p}^*)). \tag{4.3}$$

Since $q_i(\mathbf{p})$ maps any $\mathbf{p} \in S_1 \times S_2 \times \cdots \times S_{|N|}$ into a point in $[0, 1]$ and by assumption $(U_i')^{-1}(q_i)$ maps any $q_i \in [0, 1]$ into a point in S_i, vector function

$$G(\mathbf{p}) := (G_1(\mathbf{p}), G_2(\mathbf{p}), \ldots, G_{|N|}(\mathbf{p}))$$

maps compact set $S_1 \times S_2 \times \cdots \times S_{|N|}$ into itself. Hence, by Brouwer's fixed point theorem [15], there exists at least one fixed point of G in $S_1 \times S_2 \times \cdots \times S_{|N|}$, i.e., random access game $\mathcal{G}$ has a nontrivial Nash equilibrium. ■

The assumption that $(U_i')^{-1}(q_i)$ maps any $q_i \in [0, 1]$ into a point in S_i is a mild assumption. For convenience, we call it assumption **A1**. It gives a sufficient condition for the existence of nontrivial Nash equilibrium. For some utility functions that do not satisfy this condition, nontrivial Nash equilibrium may exist too. For example, take $U_i(p_i) := a \ln(a + p_i)$ with $a > 0$, we have $(U_i')^{-1}(q_i) = a(1 - q_i)/q_i$, which does not satisfy the assumption A1. However, there exists at least one nontrivial Nash equilibrium $\mathbf{p}^*$ that satisfies $\frac{a}{a+p_i^*} = 1 - (1 - p_i^*)^{|N|-1}$, $i \in N$.

Since $U_i(p_i)$ is a continuously differentiable concave function, $U_i'(p_i)$ is a continuous, decreasing function and so is $(U_i')^{-1}(q_i)$. Note that $[0, 1]$ is a connected and compact set. Hence the range of $(U_i')^{-1}(q_i)$ is a connected and compact set. Without loss of generality, with the assumption A1 we constrain the strategy space S_i to this set, i.e., $(U_i')^{-1}(0) = \omega_i$ and $(U_i')^{-1}(1) = \nu_i$, in the following discussion.

Define idle probability $\gamma(\mathbf{p}) := \prod_{i \in N}(1 - p_i)$, and $\Gamma_i(p_i) := (1 - p_i)(1 - U_i'(p_i))$. It follows from equation (4.2) that, at nontrivial Nash equilibrium,

$$\Gamma_i(p_i^*) = \gamma(\mathbf{p}^*). \tag{4.4}$$

Note that the right-hand side of the above equation is independent of i. Thus, $\Gamma_i(p_i^*) = \Gamma_j(p_j^*)$ for any $i, j \in N$.

Theorem 4.4 *Suppose A1 holds. If additionally $\Gamma_i(p_i)$ is a monotone function in S_i for all $i \in N$, then random access game $\mathcal{G}$ has a unique nontrivial Nash equilibrium.*

Proof. The assumption A1 guarantees the existence of nontrivial Nash equilibrium. Note that $\Gamma_i(p_i)$ is an increasing function, since it is monotone by assumption, nonnegative and $\Gamma_i(\nu_i) = (1-\nu_i)(1-1) = 0$. Suppose that there are two nontrivial Nash equilibria $\bar{\mathbf{p}}$ and $\hat{\mathbf{p}}$. From equation (4.4) we require that there exist $\gamma_1, \gamma_2 > 0$ such that, for all i,

$$\Gamma_i(\bar{p}_i) = \gamma_1,$$
$$\Gamma_i(\hat{p}_i) = \gamma_2.$$

Since $\Gamma_i(p_i)$ is monotone, $\gamma_1 \neq \gamma_2$. Without loss of generality, assume $\gamma_1 > \gamma_2$. Thus $\bar{p}_i > \hat{p}_i$ for all i. By equation (4.2), $U_i'(\bar{p}_i) = q_i(\bar{\mathbf{p}}) > q_i(\hat{\mathbf{p}}) = U_i'(\hat{p}_i)$, which contradicts the fact that $U_i'(p_i)$ is a decreasing function. Thus, random access game $\mathcal{G}$ has a unique nontrivial Nash equilibrium. ■

The assumption that $\Gamma_i(p_i)$ is a monotone function in S_i is also a mild assumption. Again, for convenience we call it assumption **A2**. When this assumption is not satisfied, multiple nontrivial Nash equilibria are possible. Consider the same example with utilities $U_i(p_i) = a\ln(a+p_i)$, we have $\Gamma_i(p_i) = (1-p_i)p_i/(a+p_i)$, which is not monotone in $[0,1]$. When the number of wireless nodes $|N| > 2$, in addition to the Nash equilibrium mentioned above, there exists a family of nontrivial Nash equilibria $\mathbf{p}^*$ that satisfy $p_i^* = 1 - (\frac{1}{a+1})^{\frac{1}{|N|-2}}$, $p_j^* = \frac{a(1-p_i^*)}{a+p_i^*}$ for all $j \in N$ and $j \neq i$.

In order to study quality of service differentiation among wireless nodes, we further differentiate among symmetric and asymmetric equilibria as follows.

Definition 4.5 *A Nash equilibrium* $\mathbf{p}^*$ *is said to be a symmetric equilibrium if* $p_i^* = p_j^*$ *for all* $i, j \in N$*, and an asymmetric equilibrium otherwise.*

From the former examples, for a system of homogeneous users, both symmetric and asymmetric Nash equilibria are possible. By symmetry, if a system of homogeneous users has an asymmetric Nash equilibrium, all its permutations are Nash equilibria. However, for symmetric nontrivial equilibrium, it must be unique.

Theorem 4.6 *For a system of homogeneous users, if random access game $\mathcal{G}$ has symmetric nontrivial Nash equilibrium, it must be unique. More generally, for a system with several classes of homogeneous users, if $\mathcal{G}$ has symmetric nontrivial Nash equilibrium,*[4] *it must be unique.*

Proof. For a system of homogeneous users, suppose that there are two symmetric Nash equilibria $\bar{\mathbf{p}}$ and $\hat{\mathbf{p}}$. Without loss of generality, assume $\bar{\mathbf{p}} > \hat{\mathbf{p}}$. It follows from equation (4.2) that, for all i,

$$\begin{aligned} U_i'(\bar{p}_i) &= q_i(\bar{\mathbf{p}}), \\ U_i'(\hat{p}_i) &= q_i(\hat{\mathbf{p}}). \end{aligned}$$

Note that q_i is an increasing function of $\mathbf{p}$ and U_i' is a decreasing function of p_i, which contradicts the above equations. Thus, $\bar{\mathbf{p}} = \hat{\mathbf{p}}$, i.e., the symmetric Nash equilibrium must be unique. Following the same argument, we can prove the second part of the theorem. ■

Since by symmetry there must be multiple asymmetric Nash equilibria if there exists any, the following result follows directly from Theorems 4.4 and 4.6.

Corollary 4.7 *For a system of homogeneous users, suppose A1 and A2 hold, then random access game $\mathcal{G}$ has a unique nontrivial Nash equilibrium which is a symmetric equilibrium. More generally, for a system with several classes of homogeneous users, under the same assumptions, $\mathcal{G}$ has a unique nontrivial Nash equilibrium which is symmetric among each class of users.*

Corollary 4.7 is a powerful result. It guarantees the uniqueness of nontrivial Nash equilibrium, and moreover, it guarantees fair sharing of wireless channel among the same class of wireless nodes and provides service differentiation among different classes of wireless nodes. This will facilitate the analysis of dynamic property of random access games and the design of medium access control.

[4] For a system with several classes of users, a Nash equilibrium is symmetric if at equilibrium the users of the same class choose the same strategy.

Note that we establish the uniqueness of nontrivial Nash equilibrium by exploiting special symmetry in the single-cell wireless LAN, see equation (4.4). It is not clear how to extend this to the multicell wireless LAN. In section 4.5, we will use contraction mapping theorem to specify conditions that guarantee the uniqueness of and convergence to the nontrivial Nash equilibrium for any networks.

Remark 1: Since at trivial Nash equilibrium some player takes a strategy (channel access probability) at the boundary of the strategy space, a trivial Nash equilibrium usually has great unfairness or low payoff. So, nontrivial Nash equilibrium is desired. If for a random access game there does not exist any nontrivial Nash equilibrium, we may need to look for alternative solution other than the Nash equilibrium. For example, we may use Nash bargaining framework in cooperative game theory to derive a desired equilibrium solution, as in, e.g., [19].

4.3.2 Dynamics of Random Access Game

The dynamics of game studies how interacting players could converge to a Nash equilibrium. As discussed in chapter 2, it is a difficult problem in general, and game theory lacks a general and convincing argument that a Nash outcome will occur. In the setting of random access, players (wireless nodes) can observe the outcome (packet collision or successful transmission) of the actions of others, but do not have direct knowledge of other player actions and payoffs. We consider repeated play of random access game, and look for update mechanism in which players repeatedly adjust strategies in response to observations of other player actions so as to achieve the Nash equilibrium. In the following discussion, we will suppose that assumptions A1 and A2 hold, i.e., a unique and nontrivial Nash equilibrium exists. Note that the assumption A2 also implies $\omega_i < 1$. In practice, this can be used to prevent a node from exclusively occupying wireless channel.

The simplest strategy update mechanism is of best response sort: at each stage, every node chooses the best response to the actions of all the other nodes in the previous round. Mathematically, at stage $t + 1$, node $i \in N$ chooses a channel access probability

$$p_i(t+1) = G_i(\mathbf{p}(t)) := \arg\max_{p \in S_i} \left(U_i(p) - p q_i(\mathbf{p}(t)) \right). \tag{4.5}$$

Clearly, if the above dynamics reaches a steady state, then this state is a Nash equilibrium. Nonetheless, there are no convergence results for general games using this dynamics.

Theorem 4.8 *Suppose A1 and A2 hold. If function $G^{(2)}(\mathbf{p})$ has a unique fixed point in the strategy space,*[5] *then the best response strategy (4.5) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$.*

Proof. First, note that the unique nontrivial Nash equilibrium $\mathbf{p}^*$, which is the fixed point of $G(\mathbf{p})$, is also a fixed point of $G^{(2)}(\mathbf{p})$. Thus, by assumption $\mathbf{p}^*$ is the unique fixed point of $G^{(2)}(\mathbf{p})$. Let $\mathbf{p}_{min} := (\nu_1, \nu_2, \ldots, \nu_{|N|})$ and $\mathbf{p}_{max} := (\omega_1, \omega_2, \ldots, \omega_{|N|})$, we have $\mathbf{0} < \mathbf{p}_{min} \le \mathbf{p}^* \le \mathbf{p}_{max} < \mathbf{1}$. Since $G(\mathbf{p})$ is a decreasing function with respect to relation $<$ over vectors $\mathbf{p}$, we have

$$\mathbf{0} < \mathbf{p}_{min} < G(\mathbf{p}_{max}) \le \mathbf{p}^* \le G(\mathbf{p}_{min}) < \mathbf{p}_{max} < \mathbf{1},$$

and further,

$$\begin{aligned} \mathbf{0} < \mathbf{p}_{min} < G(\mathbf{p}_{max}) < G^{(2)}(\mathbf{p}_{min}) \le \mathbf{p}^* \\ \le G^{(2)}(\mathbf{p}_{max}) < G(\mathbf{p}_{min}) < \mathbf{p}_{max} < \mathbf{1}. \end{aligned}$$

Recursively applying the best response strategy (4.5), we can obtain, for any $m \in \mathcal{N}$,

$$\begin{aligned} &\mathbf{0} < \mathbf{p}_{min} < G(\mathbf{p}_{max}) < G^{(2)}(\mathbf{p}_{min}) < G^{(3)}(\mathbf{p}_{max}) \\ &< G^{(4)}(\mathbf{p}_{min}) < \cdots < G^{(2m-1)}(\mathbf{p}_{max}) < G^{(2m)}(\mathbf{p}_{min}) \end{aligned}$$

[5] We denote m recursive operations of function $G(\mathbf{p})$ by $G^{(m)}(\mathbf{p})$.

$$\leq \mathbf{p}^* \leq G^{(2m)}(\mathbf{p}_{max}) < G^{(2m-1)}(\mathbf{p}_{min}) < \cdots$$
$$< G^{(4)}(\mathbf{p}_{max}) < G^{(3)}(\mathbf{p}_{min}) < G^{(2)}(\mathbf{p}_{max})$$
$$< G(\mathbf{p}_{min}) < \mathbf{p}_{max} < \mathbf{1}.$$

We see that $\{\ldots, G^{(2m-1)}(\mathbf{p}_{max}), G^{(2m)}(\mathbf{p}_{min}), \ldots\}$ is an increasing sequence bounded above by $\mathbf{p}^*$, and thus converges to a point $\bar{\mathbf{p}}$. $\{\ldots, G^{(2m-1)}(\mathbf{p}_{min}), G^{(2m)}(\mathbf{p}_{max}), \ldots\}$ is a decreasing sequence bounded below by $\mathbf{p}^*$, and thus converges to a point $\hat{\mathbf{p}}$. Furthermore, $\bar{\mathbf{p}}$ and $\hat{\mathbf{p}}$ are fixed points of $G^{(2)}(\mathbf{p})$. So, we must have $\bar{\mathbf{p}} = \hat{\mathbf{p}} = \mathbf{p}^*$, i.e., starting with $\mathbf{p}_{min}$ or $\mathbf{p}_{max}$ the best response strategy converges to $\mathbf{p}^*$. Since $G(\mathbf{p})$ is a continuous function and for any $\mathbf{p}$ in the strategy space $\mathbf{p}_{min} \leq \mathbf{p} \leq \mathbf{p}_{max}$, sequence $\{\ldots, G^{(m)}(\mathbf{p}), \ldots\}$ converges to $\mathbf{p}^*$. We conclude that, if $G^{(2)}(\mathbf{p})$ has a unique fixed point in the strategy space, repeated play of the best response strategy (4.5) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$. ■

The condition specified in the last theorem is not easy to verify in general. Moreover, best response is expected to have large fluctuations. We thus consider an alternative strategy update mechanism called gradient play [32]. Compared to "best response" strategy, gradient play can be viewed as a "better response." In gradient play, every player adjusts a current channel access probability gradually in a gradient direction suggested by observations of other player actions. Mathematically, each node $i \in N$ updates its strategy according to

$$p_i(t+1) = [p_i(t) + f_i(p_i(t))(U_i'(p_i(t)) - q_i(\mathbf{p}(t)))]^{S_i}, \tag{4.6}$$

where the stepsize $f_i(\cdot) > 0$ can be a function of the strategy of player i, and "S_i" denotes the projection onto the player i strategy space. We can interpret the conditional collision probability q_i as contention price for node i. If the marginal utility $\frac{dU_i(p_i)}{dp_i}$ is greater than contention price, we increase the access probability, and if the marginal utility is less than contention price, we decrease the access probability. Since at each stage, players update channel access probabilities by a small amount, gradient play is expected to have a smooth dynamics. The following result is immediate.

Lemma 4.9 *By the definition of nontrivial Nash equilibrium, nontrivial Nash equilibria of random access game $\mathcal{G}$ are fixed points of the gradient play (4.6) and vice versa.*

Theorem 4.10 *Suppose A1 and A2 hold, the gradient play (4.6) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$ if for any $i \in N$, the stepsize $f_i(p_i) < \frac{1}{\lambda+|N|-1}$.*

Proof. Consider Lyapunov function $V(\mathbf{p}) := \sum_{i\in N}(U_i(p_i) - p_i) - \prod_{i\in N}(1-p_i)$. Define a matrix $B(\mathbf{p}) := -\nabla^2 V(\mathbf{p})$, we have (see, e.g., pp. 635 in [9])

$$||B(\mathbf{p})||_2^2 \leq ||B(\mathbf{p})||_\infty \cdot ||B(\mathbf{p})||_1.$$

Since $B(\mathbf{p})$ is symmetric, $||B(\mathbf{p})||_\infty = ||B(\mathbf{p})||_1$ and hence

$$\begin{aligned}
||B(\mathbf{p})||_2 &\leq ||B(\mathbf{p})||_\infty \\
&= \max_i \sum_j [B(\mathbf{p})]_{ij} \\
&= \max_i \sum_j (-U_i''(p_i)\delta_{i,j} + (1-\delta_{i,j}) \prod_{k\neq i,j}(1-p_k)) \\
&\leq \max_i \{-U_i''(p_i)\} + |N| - 1 \\
&\leq \lambda + |N| - 1.
\end{aligned}$$

By Taylor expansion, we have

$$\begin{aligned}
& V(\mathbf{p}(t+1)) - V(\mathbf{p}(t)) \\
= \ & \nabla V(\mathbf{p}(t)) \cdot (\mathbf{p}(t+1) - \mathbf{p}(t)) \\
& + \frac{1}{2}(\mathbf{p}(t+1) - \mathbf{p}(t))^T \cdot \nabla^2 V(\bar{\mathbf{p}}) \cdot (\mathbf{p}(t+1) - \mathbf{p}(t)) \\
= \ & \nabla V(\mathbf{p}(t)) \cdot (\mathbf{p}(t+1) - \mathbf{p}(t)) \\
& - \frac{1}{2}(\mathbf{p}(t+1) - \mathbf{p}(t))^T \cdot B(\bar{\mathbf{p}}) \cdot (\mathbf{p}(t+1) - \mathbf{p}(t)) \\
\geq \ & \nabla V(\mathbf{p}(t)) \cdot (\mathbf{p}(t+1) - \mathbf{p}(t)) \\
& - \frac{\lambda + |N| - 1}{2} ||\mathbf{p}(t+1) - \mathbf{p}(t)||_2^2,
\end{aligned}$$

where $\bar{\mathbf{p}} \in \{\mathbf{p}|\mathbf{p} = a\mathbf{p}(t) + (1-a)\mathbf{p}(t+1),\ a \in [0,1]\}$. Now, note that there exists some nonnegative number $\Delta_i \leq |U_i'(p_i(t)) - q_i(\mathbf{p}(t))|$ such that equation (4.6) can be written as

$$\begin{aligned} p_i(t+1) &= p_i(t) + f_i(p_i(t))(U_i'(p_i(t)) - q_i(\mathbf{p}(t))) \\ &\quad -f_i(p_i(t))\Delta_i \text{sign}(U_i'(p_i(t)) - q_i(\mathbf{p}(t))), \end{aligned} \tag{4.7}$$

where we define $\text{sign}(a) = 1$ if $a \geq 0$ and $\text{sign}(a) = -1$ if $a < 0$. Plug equation (4.7) into the above inequality, we have

$$\begin{aligned} & V(\mathbf{p}(t+1)) - V(\mathbf{p}(t)) \\ \geq & \sum_i (f_i(p_i(t)) - \frac{(\lambda + |N| - 1)f_i^2(p_i(t))}{2})(U_i'(p_i(t)) - q_i(\mathbf{p}(t)))^2 \\ & -\sum_i \frac{(\lambda + |N| - 1)f_i^2(p_i(t))}{2}\Delta_i^2 \\ & +\sum_i ((\lambda + |N| - 1)f_i^2(p_i(t)) - f_i(p_i(t)))\Delta_i |U_i'(p_i(t)) - q_i(\mathbf{p}(t))| \\ \geq & \sum_i (f_i(p_i(t)) - (\lambda + |N| - 1)f_i^2(p_i(t)))(U_i'(p_i(t)) - q_i(\mathbf{p}(t)))^2 \\ & +\sum_i ((\lambda + |N| - 1)f_i^2(p_i(t)) - f_i(p_i(t)))\Delta_i |U_i'(p_i(t)) - q_i(\mathbf{p}(t))| \\ = & \sum_i (f_i(p_i(t)) - (\lambda + |N| - 1)f_i^2(p_i(t)))\{(U_i'(p_i(t)) - q_i(\mathbf{p}(t)))^2 \\ & -\Delta_i |U_i'(p_i(t)) - q_i(\mathbf{p}(t))|\}. \end{aligned}$$

Thus, if $f_i(p_i) < \frac{1}{\lambda + |N| - 1}$, $V(\mathbf{p}(t+1)) - V(\mathbf{p}(t)) \geq 0$. Note that $\Delta_i < |U_i'(p_i(t)) - q_i(\mathbf{p}(t))|$ if $U_i'(p_i(t)) - q_i(\mathbf{p}(t)) \neq 0$. We see that $V(\mathbf{p})$ will keep increasing until the system reaches a fixed point of equation (4.6). Since by Theorem 4.4 equation (4.6) has a unique fixed point in strategy space, gradient play (4.6) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$. ∎

Theorem 4.10 guarantees the convergence of distributed gradient play to the desired Nash equilibrium. If a backoff mechanism is implemented, by equation (4.1) each node $i \in N$ updates its contention window cw_i as follows:

$$cw_i(t) = \frac{2 - p_i(t)}{p_i(t)}. \tag{4.8}$$

Remark 2: For a general game model, there may exist multiple nontrivial Nash equilibria. In this situation, a “naive” strategy such as best response or gradient play may not converge to the desired equilibrium, and concepts from incentive design or control theory may come in to work, see, e.g., [77].

4.3.3 Medium Access Control Design

Our ultimate purpose for studying random access games is to design medium access method with better performance and simpler dynamics. Corollary 4.7 and Theorem 4.10 (and Theorem 4.8) suggest that random access games provide a general analytical framework to model a large class of systemwide quality of service models (mainly in terms of throughput) via the specification of per-node utility functions, and systemwide fairness or service differentiation can be achieved in a distributed manner as long as each node executes a contention resolution algorithm that is designed to achieve the Nash equilibrium.

Based on this understanding of the equilibrium and dynamics of random access games, we propose a novel medium access method derived from CSMA/CA: instead of executing exponential backoff upon collisions, each node estimates its conditional collision probability and adjusts its channel access probability and contention window accordingly. As gradient play guarantees the convergence to the Nash equilibrium and results in smoother dynamics, we will design medium access method according to equations (4.6) and (4.8), see Table 4.1 for a formal description. Unlike other medium access methods, our method adapts to continuous feedback signal (conditional collision probability) rather than binary feedback (packet collision), and stabilizes the network around a steady state specified by the Nash equilibrium of random access game. Our access method is an equation-based control, and its performance (such as throughput, collision and fairness) is determined by the Nash equilibrium. Note

Table 4.1: Medium access method via gradient play

```
After each transmission
{
 /*wireless node observes n idle
   slots before a transmission*/
 isum ← isum + n
 ntrans ← ntrans + 1
 if(ntrans >= maxtrans){
   /*compute the estimator*/
   n̄ ← isum / ntrans
   q_i ← (1-(n̄+1)p_i) / ((n̄+1)(1-p_i))
   /*update access probability*/
   p_i ⟵ p_i + f_i(p_i)(U'_i(p_i) - q_i)
   /*update contention window*/
   cw_i ⟵ (2-p_i) / p_i
   /*reset variables*/
   isum ← 0
   ntrans ← 0
  }
}
```

that $U_i'(p_i(t)) - q_i(\mathbf{p}(t))$ specifies how far the current state is from the equilibrium. The contention window adjustment is small when the current state is close to the equilibrium and large otherwise, independent of where the equilibrium is. This is in sharp contrast to the approach taken by 802.11 DCF, where window adjustment depends on just the current window size and is independent of where the current state is with respect to the target equilibrium. So, our access method can achieve better contention control (collision reduction) and better short-term fairness. In addition to simple dynamics resulting from using continuous feedback and controllable performance objectives via the specification of per-node utility functions, as conditional collision probability is a more accurate measure of the contention among wireless nodes than instantaneous packet collision, our access method can achieve better balance/tradeoff between channel access and collision avoidance, and hence a higher throughput.

Furthermore, wireless nodes can estimate conditional collision probabilities by observing idle period of the channel. Let n denote the number of consecutive idle slots between two transmissions. Here "a transmission" corresponds to a busy period in the channel when only a node transmits (i.e., a successful transmission) or multiple nodes transmit simultaneously (i.e., a collision). Since n has the geometric distribution with parameter $\gamma(\mathbf{p})$, its mean $\bar{n}$ is given by

$$\bar{n} = \frac{\gamma(\mathbf{p})}{1-\gamma(\mathbf{p})}.$$

Thus, each node can estimate its conditional collision probability by observing the average number of consecutive idle slots, according to

$$q_i = 1 - \frac{\gamma(\mathbf{p})}{1-p_i} = \frac{1-(\bar{n}+1)p_i}{(\bar{n}+1)(1-p_i)}. \tag{4.9}$$

So, our access method can decouple contention control from handling packet losses, and is immune to all the problems incurred in methods that infer channel contention from packet collisions.

Moreover, our access method provides a way for adapting transmission rates to channel variations. Rate adaptation is not easy, since it depends on the frame error rate perceived at the receiver and it is difficult for the sender to obtain or estimate the frame error rate. In our access method, with the conditional collision probability at hand, wireless nodes i can estimate frame error rates e_i. This can be achieved by first estimating conditional packet loss probabilities l_i. Each wireless node i keeps a counter tx_sum_i for its total transmission and another counter l_sum_i for its total packet losses over some period. The conditional loss probability over that period is given by $l_i = \frac{l\_sum_i}{tx\_sum_i}$, and the frame error rate can be estimated by $e_i = \frac{l_i - q_i}{1-q_i}$. Denote its current rate by r_i, wireless node i can switch to a lower rate $\hat{r}_i$ roughly when

$$\hat{r}_i(1-\hat{e}_i) \geq r(1-e_i),$$

where $\hat{e}_i$ is the error rate if operating at the lower rate. Note that $\hat{e}_i$ is expected to be small. As a first approximation, node i can switch to the lower rate when $e_i = 1 - \hat{r}_i / r_i$.

In the next section, we will study a concrete random access game and the corresponding medium access control design, as a case study for the proposed design methodology in game theory framework. We will describe there in detail the design of our medium access method.

4.4 A Case Study

Consider the following utility

$$U_i(p_i) := \frac{1}{a_i}(\frac{(a_i - 1)\omega_i}{a_i} \ln(a_i p_i - \omega_i) - p_i), \tag{4.10}$$

where $0 < \omega_i < 1$, $a_i > 1$ and $p_i \in [2\omega_i/(1+a_i),\ \omega_i]$. Define a random access game $\mathcal{G}_1 := \{N, (S_i)_{i \in N}, (u_i)_{i \in N}\}$, where N is a set of players (wireless nodes), player i strategy $S_i := \{p_i | p_i \in [2\omega_i/(1+a_i),\ \omega_i]\}$ and payoff function $u_i(p) := U_i(p_i) - p_i q_i(\mathbf{p})$ with $q_i(\mathbf{p}) := 1 - \prod_{j \in N/\{i\}}(1 - p_j)$. In the following, we will study the equilibrium and dynamic aspects of random access game $\mathcal{G}_1$, and design a medium access method accordingly.

4.4.1 Nash Equilibrium

The marginal utility

$$U_i'(p_i) = \frac{\omega_i - p_i}{a_i p_i - \omega_i},$$

thus the inverse function

$$(U_i')^{-1}(q_i) = \frac{\omega_i(1 + q_i)}{1 + a_i q_i}.$$

It is easy to check that $(U_i')^{-1}(q_i)$ is a decreasing function in $[0,1]$ and satisfies the assumption A1. Also,

$$\Gamma_i(p_i) = (1-p_i)(1-U_i'(p_i)) = \frac{(1-p_i)((1+a_i)p_i - 2\omega_i)}{a_i p_i - \omega_i}.$$

Lemma 4.11 *$\Gamma_i(p_i)$ is monotone in $[2\omega_i/(1+a_i),\ \omega_i]$, if $a_i\omega_i < 1$.*

Proof. The derivative of Γ_i is

$$\Gamma_i'(p_i) = -1 + \frac{\omega_i - p_i}{a_i p_i - \omega_i} - \frac{(1-a_i)\omega_i}{(a_i p_i - \omega)^2}(1-p_i). \tag{4.11}$$

Note that $\frac{\omega_i - p_i}{a_i p_i - \omega_i} \geq 0$, and $\frac{(1-a_i)\omega_i}{(a_i p_i - \omega)^2} \leq \frac{1}{(1-a_i)\omega_i}$. Thus, $\Gamma_i'(p_i) \geq -1 + \frac{1-w}{(a_i-1)\omega_i}$. So, if $a_i\omega_i < 1$, we have $\Gamma_i'(p_i) > 0$, which means $\Gamma_i(p_i)$ is monotone in $[2\omega_i/(1+a_i),\ \omega_i]$. ■

Lemma 4.11 shows that, If $a_i\omega_i < 1$, the assumption A2 is satisfied. Thus, the following result follows directly from Corallary 4.7.

Theorem 4.12 *If $a_i\omega_i < 1$, random access game $\mathcal{G}_1$ has a unique nontrivial Nash equilibrium. Moreover, for a system of homogeneous users the unique nontrivial Nash equilibrium of $\mathcal{G}_1$ is a symmetric equilibrium, and for a system with several classes of homogeneous users the unique nontrivial Nash equilibrium of $\mathcal{G}_1$ is symmetric among each class of users.*

4.4.2 Dynamics

Assume that each node $i \in N$ adjusts its strategy according to gradient play

$$p_i(t+1) = [p_i(t) + f_i(p_i(t))(\frac{\omega_i - p_i(t)}{a_i p_i(t) - \omega_i} - q_i(\mathbf{p}(t)))]^{S_i}, \tag{4.12}$$

$$cw_i(t) = \frac{2 - p_i(t)}{p_i(t)}, \tag{4.13}$$

where $f_i(p_i) > 0$ is the stepsize. Note that nontrivial Nash equilibria are the fixed points of equation (4.12) and vice versa. The following result is immediate.

Theorem 4.13 *Suppose $a_i \omega_i < 1$, the system described by equation (4.12) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}_1$ if for any $i \in N$, the stepsize $f_i(p_i) < \frac{1}{\lambda + |N| - 1}$.*

Proof. Note that, if $a_i \omega_i < 1$, assumptions A1 and A2 hold. The result follows directly from Theorem 4.10. ■

The condition $a_i \omega_i < 1$ is a mild assumption and admits a very large region in parameter space. The Nash equilibrium can be easily calculated numerically with equation (4.2). Note that Γ_i is a decreasing function of ω_i and an increasing function of a_i. Since Γ_i is an increasing function of p_i, larger value of ω_i or smaller value of a_i will results in larger channel access probability p_i^* at equilibrium. Thus, in order to provide differentiated services, we can choose larger value of ω_i or smaller value of a_i for the users of a higher priority class. For example, in wireless access network, we can assign a large ω_i value or small a_i value to the access point, because usually downlink traffic is greater than the traffic of mobile nodes.

4.4.3 Medium Access Control Design

We design a medium access method according to channel access probability and contention window update mechanism (4.12)–(4.13), by modifying a CSMA/CA access method such as 802.11 DCF [41]. The basic access mechanism in DCF works as follows: a node wishing to transmit senses the channel for a period of time equal to the distributed interframe space (DIFS) to check if it is idle. If the channel is determined to be idle, the node starts to transmit a DATA frame. If the channel is considered to be busy, the node waits for a random backoff time b, an integer uniformly distributed in the window $[0,\ cw - 1]$ before attempting to transmit. Upon successful reception of the DATA frame, the receiving node waits for a short interframe space (SIFS) interval and then sends an ACK frame. When the node detects a failed transmission, it doubles the contention window cw (exponential backoff). In order to avoid channel capture, a node must wait for a random backoff time, in the same way as if the channel is sensed busy, between two consecutive new packet transmissions. Note that DCF employs a discrete-time backoff scale. The time immediately following an idle DIFS is slotted with slot time size σ. The backoff time counter is decremented as long as the channel is sensed idle during a time slot, and frozen when the channel is sensed busy, and reactivated when the channel is sensed idle again for a DIFS. The node transmits when the backoff time counter reaches zero. See the reference [41] for more details.

In our medium access method, we make two key modifications to 802.11 DCF. Instead of adjusting contention window cw_i to a binary feedback signal (packet loss or successful transmission) and using exponential backoff algorithm, each node i estimates its conditional collision probability q_i, which is a continuous feedback, and adjusts cw_i according to algorithm (4.12)–(4.13). The detailed medium access method is described as follows:

(1) Initially, each node i is in state *INITIAL* and sets its channel access probability p_i to be ω_i, and contention window cw_i to be $(2 - \omega_i)/\omega_i$.

(2) in state *INITIAL* or *NEW*, node i monitors the channel. If the channel is idle for a period of time equal to DIFS, it will generate a backoff time b_i, an integer uniformly distributed in the interval $[0, cw_i - 1]$, before transmitting. We say that the node enters state *BACKOFF*.

(3) In state *BACKOFF*, a node will decrease its backoff time counter by one as long as the channel is sensed idle during a time slot, freeze the backoff time counter when the channel is sensed busy, and reactivate it when the channel is idle again for a DIFS interval. When the backoff time counter reaches zero, the node will transmit a DATA frame.

(4) Upon receiving the ACK frame, the node will enter state *NEW*.

(5) If the node does not receive the ACK frame within a DIFS interval, it decides that a packet collision has happened, and enters state *BACKOFF*.

(6) Each node i also keeps a transmission counter *ntrans* and an idle slots counter *isum* that are initially set to zero. The node increases *ntrans* by one after

a transmission. The node also counts the consecutive idle slots n before a transmission and increases $isum$ by n. Note that, as explained in Subsection 4.3.3, "a transmission" here corresponds to a busy period in the channel.

(7) If $ntrans \geq maxtrans$, each node i will estimate the average number of consecutive idle slots $\bar{n} = \frac{isum}{ntrans}$ and its conditional collision probability via $q_i \leftarrow \frac{1-(\bar{n}+1)p_i}{(\bar{n}+1)(1-p_i)}$, and update its channel access probability p_i and contention window cw_i according to the following algorithm

$$\begin{aligned} p_i &\longleftarrow p_i + f_i(p_i)(\frac{\omega_i - p_i}{a_i p_i - \omega_i} - q_i) \\ cw_i &\longleftarrow \frac{2 - p_i}{p_i}, \end{aligned}$$

and reset $ntrans$ and $isum$ to be zero.

(8) Upon successful receiption of the DATA frame, the receiving node waits for a SIFS interval and then sends an ACK frame.

We can see that our medium access method works in similar way as 802.11 DCF, except for the contention window update mechanism. So, it can be easily implemented with existing 802.11 hardware. Note that we do not specify all the implementation details. For example, in (5) we have described a simple time-out mechanism that works for a single-cell system. In practice, we can have other choices such as setting an appropriate ACK_Timeout interval. If the transmitting node does not receive the ACK frame within ACK_Timeout or detects the transmission of a different frame on the channel, it will decide that there is a collision and will reschedule the frame transmission according to the given contention window update mechanism and backoff rules.

There are several parameters in our medium access method. The parameters ω_i and a_i determine the strategy space and the equilibrium properties such as throughput, loss (collision) and fairness. The parameters $f_i(\cdot)$ and $maxtrans$ determine the dynamical properties such as stability and responsiveness. The stepsize $f_i(\cdot)$ affects the convergence speed. In practice, we will choose a constant stepsize for all nodes.

The number of transmissions, *maxtrans*, for each node before updating its channel access probability and contention window, affects the convergence speed and the accuracy of the conditional collision probability estimation. Note that in strategy update algorithm (4.12)–(4.13), t is not real time but represents the stages at which the random access game is played. In our design, each node repeatedly plays game $\mathcal{G}_1$ every *maxtrans* transmissions, and between consecutive plays the channel access probability and contention window are fixed. If *maxtrans* is too large, it will take longer time to reach the Nash equilibrium, but if *maxtrans* is too small, it will result in large estimation error in the average number of consecutive idle slots between transmissions and thus conditional collision probability. Since by gradient play nodes update p_i and cw_i gradually, in order to achieve a good tradeoff between convergence speed and estimation accuracy we will choose a relatively small value for *maxtrans* and estimate average number of consecutive idle slots between transmissions using an exponential weighted running average

$$\bar{n} \longleftarrow \beta\bar{n} + (1-\beta)\frac{isum}{ntrans},$$

where $\beta \in [0, 1)$. If β is small we weight history less, and if β is large we weight history more. By choosing appropriate β value, exponential weighted running average gives better estimate than the "naive" estimator $isum/ntrans$.

By our access method, the system is designed to reach and operate around the Nash equilibrium of random access game $\mathcal{G}_1$. Thus, its performance is determined by the Nash equilibrium of $\mathcal{G}_1$. Consider a system of greedy nodes. Denote the channel access probability of node i at Nash equilibrium by p_i, we can calculate its throughput T_i and conditional collision probability q_i as follows.

$$T_i = \frac{p_i(1-q_i)P}{\gamma(\mathbf{p})\sigma + \sum_i p_i(1-q_i)T_s + (1-\gamma(\mathbf{p}) - \sum_i p_i(1-q_i))T_c}, \tag{4.14}$$

$$q_i = 1 - \prod_{j\neq i}(1-p_j), \tag{4.15}$$

Table 4.2: Parameters used to obtain numerical results

Slot Time (σ)	20 μs
SIFS	10 μs
DIFS	50 μs
Basic Rate (br)	1 Mbps
Data Rate (dr)	11 Mbps
Propagation Delay (δ)	1 μs
PHY Header (ph)	192 bits
MAC Header (mh)	272 bits
ACK	112 bits
Packet Payload (P)	12000 bits

where P is the packet payload, idle probability $\gamma(\mathbf{p}) = \prod_i (1 - p_i)$ and

$$\begin{aligned} T_s &= \frac{ph}{br} + \frac{mh + P}{dr} + SIFS + \frac{ph}{br} + \frac{ACK}{dr} + DIFS + 2\delta, \\ T_c &= \frac{ph}{br} + \frac{mh + P}{dr} + DIFS + \delta, \end{aligned}$$

are the time the channel is sensed busy because of a successful transmission and during a collision, respectively. See Table 4.2 for other notations. Here for simplicity, we have assumed an equal payload size. The throughput for general payload size distribution can calculated in a similar way [14], and the aggregate throughput is the summation of T_i over all nodes $i \in N$.

4.4.4 Performance

To evaluate the performance of our medium access method, we develop a discrete-event simulator that implements our method and the standard 802.11 DCF basic access method (i.e., no RTS/CTS). The values for the parameters used to obtain numerical results are summarized in Table 4.2. The system values are those specified in the 802.11b standard with DSSS PHY layer [41]. In all simulations, we set the following values of the control parameters: $maxtrans = 10$, $f_i = 0.01$ and $\beta = 0.2$.

The Networks with Perfect Channel

We first consider the networks with perfect channel, i.e., there is no corrupted frame.

Throughput and Collision Overhead: We consider a system of homogeneous users, and compare the throughput achieved by our method and 802.11b DCF. In our design each node i is limited to choose a contention window size between $(2-\omega)/\omega$ and $(1+a-\omega)/\omega$, corresponding to channel access probability $p_i \in [2\omega/(a+1), \omega]$. To compare the performance of our design with that of 802.11 DCF on the same ground, we choose values for those related parameters such that $(2-\omega)/\omega = CWmin$ and $(1+a-\omega)/\omega = 2^m CWmax$, corresponding to a maximum backoff stage m. Also note that in our numerical experiments with DCF, we assume that after a packet's $(m+1)$th failed transmission the contention window resets to $CWmin$. This is also equivalent to the packet being discarded after m failed retransmissions.

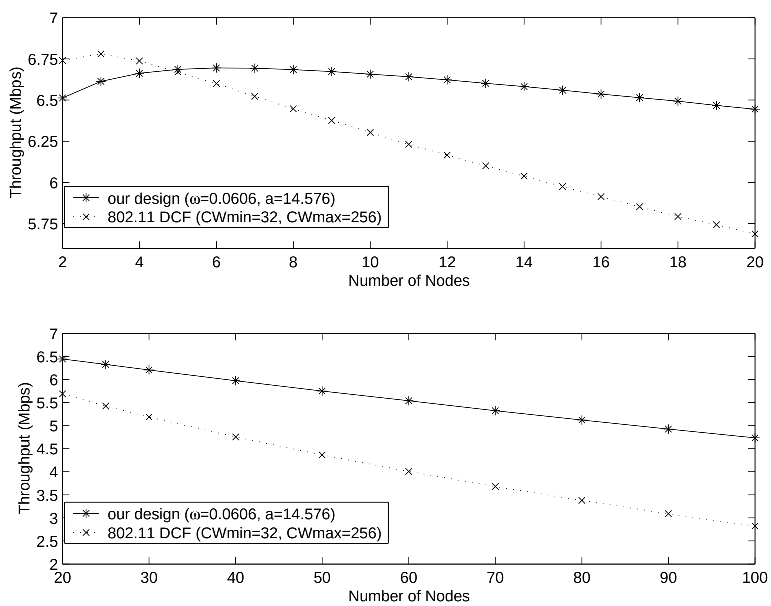

Figure 4.1: Throughput comparison.

Figure 4.1 shows the aggregate throughout achieved by our design with $\omega = 0.0606$ and $a = 14.576$, and DCF with $CWmin = 32$ and $CWmax = 256$. We see that for a small number of wireless nodes, DCF provides a slightly higher throughput. But as the number of nodes increases, our access method achieves much higher throughput. With DCF each new transmission will start with the base contention window and

Table 4.3: Throughput (in Mpbs) for our design with $\omega = 0.0606$ and $a = 14.576$, and 802.11 DCF with $CWmin = 32$ and $CWmax = 256$

N	our design*	our design	DCF
2	6.5193	6.513	6.740
4	6.6658	6.663	6.738
6	6.6961	6.695	6.600
10	6.6574	6.657	6.303
15	6.5553	6.560	5.975
20	6.4380	6.445	5.688
25	6.3193	6.327	5.427
40	5.9677	5.975	4.754
60	5.5224	5.540	4.007
80	5.1255	5.123	3.377
100	4.7318	4.735	2.824

*Analytical value calculated with Nash equilibrium of game $\mathcal{G}_1$

execute binary exponential backoff upon collisions, while with our access method nodes will choose a constant contention window determined by the Nash equilibrium, which is "optimal" for the current contention level in the network. Thus, for a system of many competing nodes where the contention in the network is heavy, DCF will incur much more packet collisions than our access method, which results in much lower throughput, as shown in Figure 4.1. This is further confirmed by the comparison of collision overhead between DCF and our access method, as shown in Figure 4.2. We see that our access method achieves a better tradeoff between channel access and collision avoidance, and hence a higher throughput that is sustainable over a large range of numbers of competing nodes. Mathematically, this behavior results from the structure of the payoff function of random access game, which includes both the gain from channel access and the cost from packet collision. Practically, this means that our access method can achieve higher throughput but with fewer transmissions than DCF, which will benefit the whole system in many aspects such as lower energy usage and less interference to the wireless nodes of neighboring cells.

Tables 4.3 and 4.4 record some of the numerical values used to plot Figures 4.1 and 4.2. They also show the analytical values of throughput and conditional collision

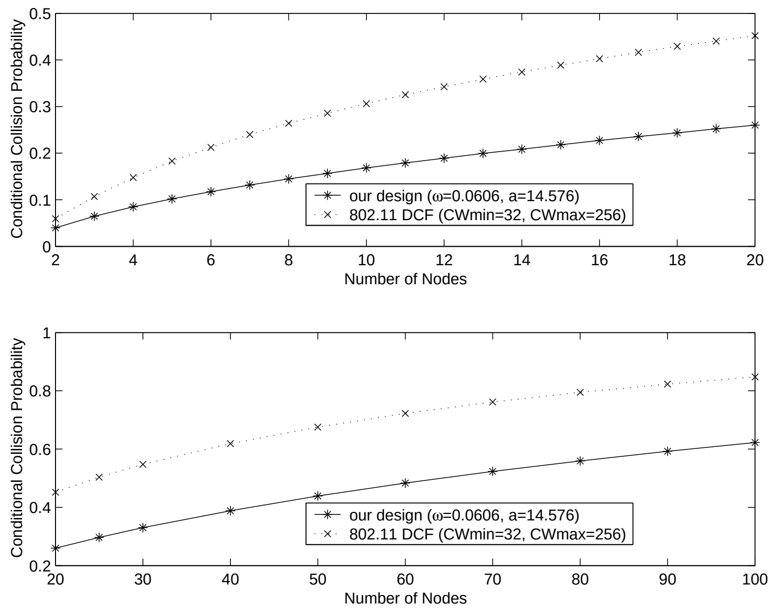

Figure 4.2: Conditional collision probability comparison.

Table 4.4: Conditional collision probability for our design with $\omega = 0.0606$ and $a = 14.576$, and 802.11 DCF with $CWmin = 32$ and $CWmax = 256$

N	our design*	our design	DCF
2	0.0399	0.0396	0.0594
4	0.0853	0.0849	0.1477
6	0.1185	0.1174	0.2125
10	0.1693	0.1683	0.3061
15	0.2201	0.2179	0.3889
20	0.2625	0.2600	0.4518
25	0.2991	0.2967	0.5035
40	0.3901	0.3884	0.6188
60	0.4855	0.4832	0.7224
80	0.5587	0.5592	0.7945
100	0.6228	0.6224	0.8475

*Analytical value calculated with Nash equilibrium of game $\mathcal{G}_1$

probability for our access method, calculated with the Nash equilibrium of random access game $\mathcal{G}_1$ according to equations (4.14)–(4.15). We see that the analytical values and numerical values from simulations match extremely well. This also proves that our medium access method does converge to the desired Nash equilibrium of the random access game. We also track the evolution of contention windows, which

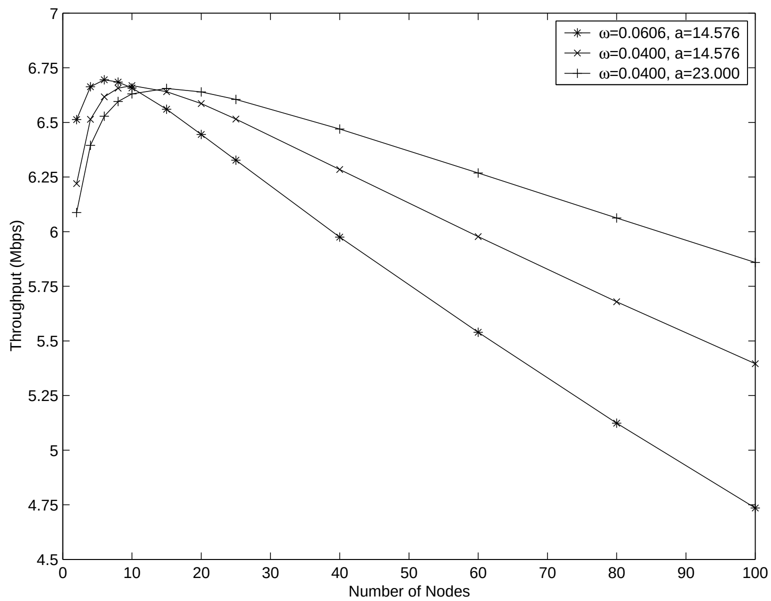

Figure 4.3: Throughput comparison.

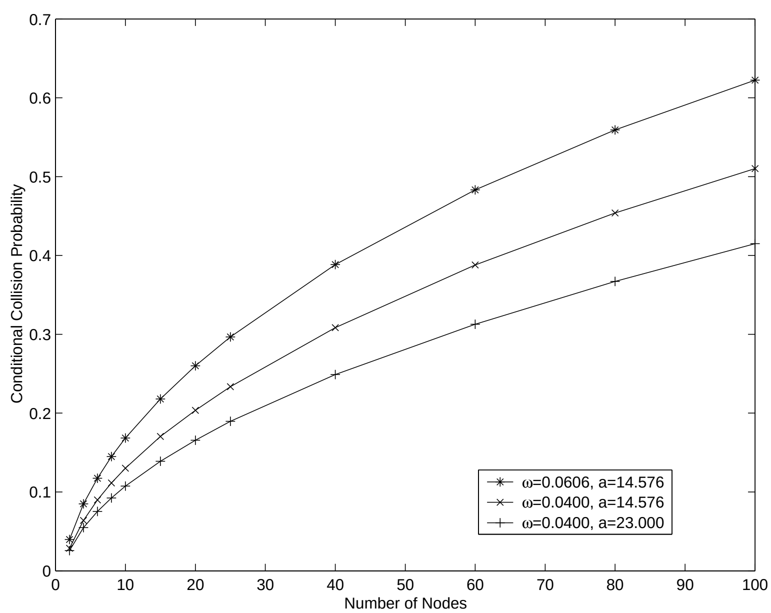

Figure 4.4: Conditional collision probability comparison.

quickly approach and stay around the values specified by the Nash equilibrium.

To investigate the dependency of the throughput and conditional collision probability of our design on parameters ω and a, we report in Figures 4.3 and 4.4 the aggregate throughput and conditional collision probability versus different setting of these parameters. We see that larger ω or smaller a will result in higher throughput

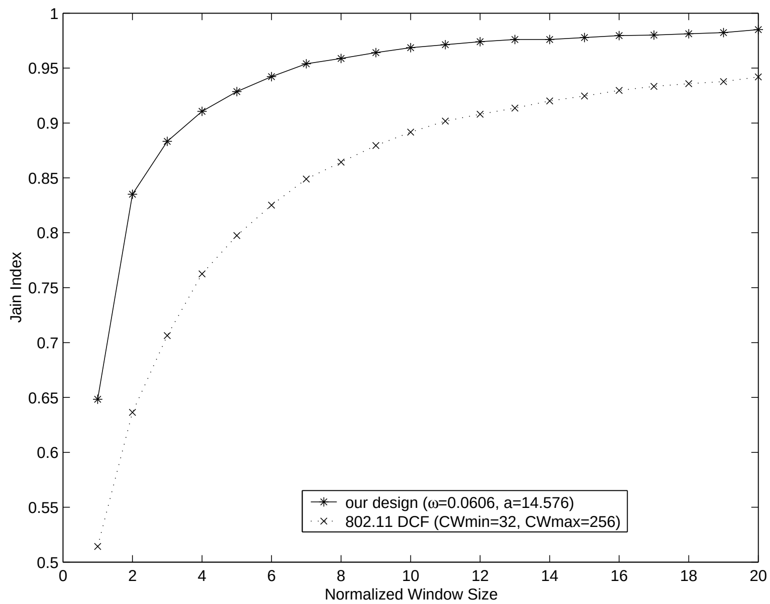

Figure 4.5: Fairness comparison for 40 competing nodes.

for a system of a few competing nodes, since they lead to a higher channel access probability. But smaller ω or larger a will result in much higher throughput for a large system. This is because smaller ω or larger a result in much smaller collision probability. In the practical implementation, we need to choose a set of parameters that could provide sustainable high throughput over a large range of the number of wireless nodes.

Fairness: It is well known that 802.11 DCF has short-term unfairness problem, due to binary exponential backoff process. In our access method for a system of homogeneous users, wireless nodes have the same contention window size, specified by the symmetric Nash equilibrium of random access game $\mathcal{G}_1$. Thus, it is expected to have a better short-term fairness. Figure 4.5 compares short-term fairness of our access method and DCF using Jain fairness index for the window sizes that are multiples of the number of wireless nodes [44] [50], where we normalize window size with respect to the number of nodes. We can see that our method provide better short-term fairness than 802.11 DCF.

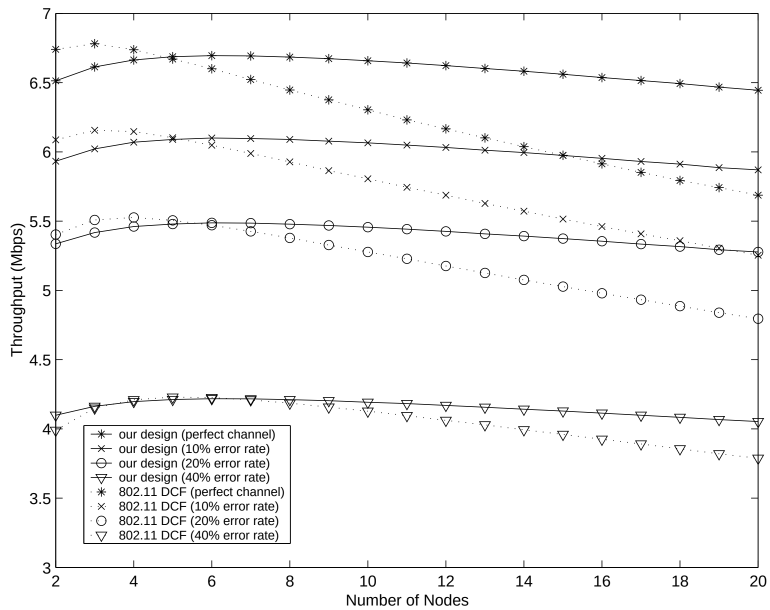

Figure 4.6: Throughput comparison of our design with $\omega = 0.0606$ and $a = 14.576$, and 802.11 DCF with $CWmin = 32$ and $CWmax = 256$, with different frame error rates.

The Networks with Unreliable Channel

We now consider the networks with unreliable channel, i.e., there exist corrupted frames, due to the channel variations. Figures 4.6 and 4.7 show the aggregate throughput achieved by our design with $\omega = 0.0606$ and $a = 14.576$, and 802.11 DCF with $CWmin = 32$ and $CWmax = 256$, with zero, 10%, 20% and 40% frame error rates respectively. We see that the channel error has larger impact on DCF than that on our design, due to the additional backoff in DCF that results from the corrupted frames. With fixed frame error rate, the impact of the channel error on both our design and DCF becomes smaller as the number of contending nodes increases. This is because a lost packet takes a shorter time than a successful transmission. Thus, more packet losses from the channel error will result in more transmissions, which partially compensates the packet losses. For DCF, this is also because the packet losses from the channel error take smaller portion of the total losses when there are more collisions. So, the additional backoff resulting from the corrupted frames is relatively less often as the number of nodes increases.

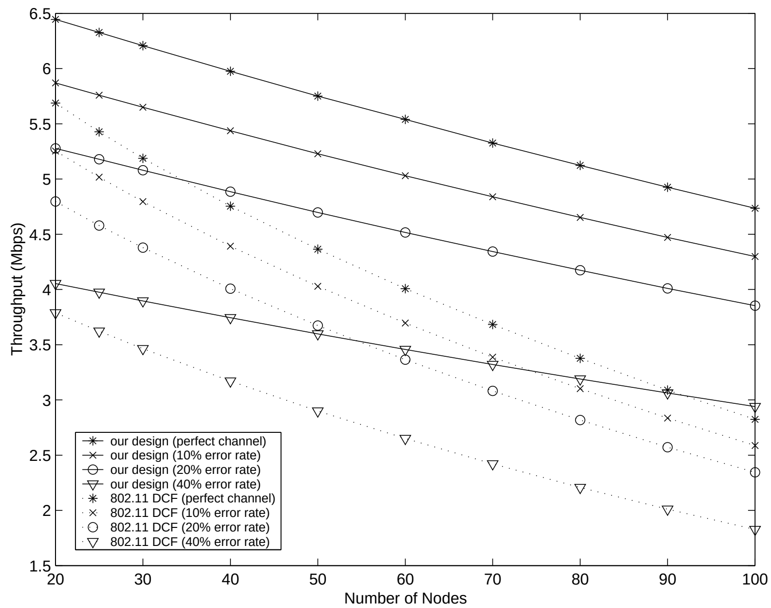

Figure 4.7: Throughput comparison of our design with $\omega = 0.0606$ and $a = 14.576$, and 802.11 DCF with $CWmin = 32$ and $CWmax = 256$, with different frame error rates.

Note that the throughput degradation of DCF resulting from the additional backoff that is only due to the corrupted frames is not very significant for a frame error rate as large as 20%. There are two reasons for this. The first and major reason is that the idle slot has a much shorter duration than a collision or successful transmission. Thus, the additional backoff in DCF due to the corrupted frames does not take much time. The throughput degradation will be more severe for relatively shorter durations of a successful transmission or collision. The second and minor reason is that in the example shown in Figures 4.6 and 4.7, the maximum backoff stage is small. The throughput degradation will be larger if DCF has a larger number of backoff stages.

Service Differentiation

As discussed in section 4.3 and subsection 4.4.2, we can provide service differentiation by choosing different utility functions for different classes of users. Regarding the concrete medium access method we consider, each node i will receive different services by choosing different values for parameters ω_i or a_i (with only constraint $a_i\omega_i < 1$). For the simplicity of presentation, we consider two classes of users. Assume that class

1 has n_1 users with parameters (ω_1, a_1), corresponding to a higher priority of service, and class 2 has n_2 users with parameters (ω_2, a_2), corresponding to a lower priority of service. Let first study the impact of ω_i on the service differentiation by setting the same a_i value. Figure 4.8 shows the throughput ratio of a class 1 node to a class 2 node versus the total number of nodes for two different scenarios: two classes have equal number of users, and class 1 has fixed number of users. We see that, as the total number of nodes increase, the throughput ratio approaches 1.5. When there is a large number of nodes accessing the channel, each user should sense approximately the same environment on average, and we can assume that each user has the same conditional collision probability,[6] denoted by q. Thus, with equal packet payload sizes the throughput ratio between a class 1 user and class 2 user is approximately

$$\frac{(U_1')^{-1}(q)(1-q)}{(U_2')^{-1}(q)(1-q)} = \frac{\omega_1}{\omega_2}.$$

So, when the service differentiation is provided by maximum channel access probability, the throughput ratio between users of different classes is approximately $\frac{\omega_1}{\omega_2}$ for a large number of users.

We then study the impact of a_i on the service differentiation by setting the same ω_i value. Figure 4.9 shows the throughput ratio of a class 1 node to a class 2 node versus the total number of nodes for the scenario where two classes have equal number of users and the scenario where class 1 has fixed number of users. We see that, as the total number of nodes increase, the throughput ratio seems to converge to some fixed value. For a large number of users, we can again assume that users of different classes experience the same conditional collision probability q. Thus, with equal packet payload sizes the throughput ratio between a class 1 user and class 2 user is approximately

$$\frac{(U_1')^{-1}(q)(1-q)}{(U_2')^{-1}(q)(1-q)} = \frac{1+a_2q}{1+a_1q}.$$

[6] This assumption is similar to the decoupling approximation made in [14] and other works in performance analysis of 802.11 DCF.

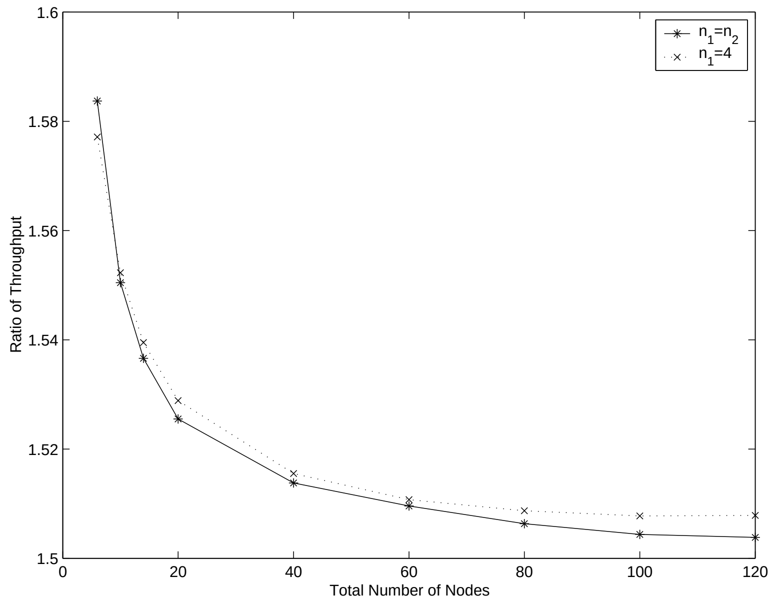

Figure 4.8: The throughput ratio of a class 1 node with $\omega_1 = 0.06$ and $a_1 = 15$ to a class 2 node with $\omega_2 = 0.04$ and $a_2 = 15$.

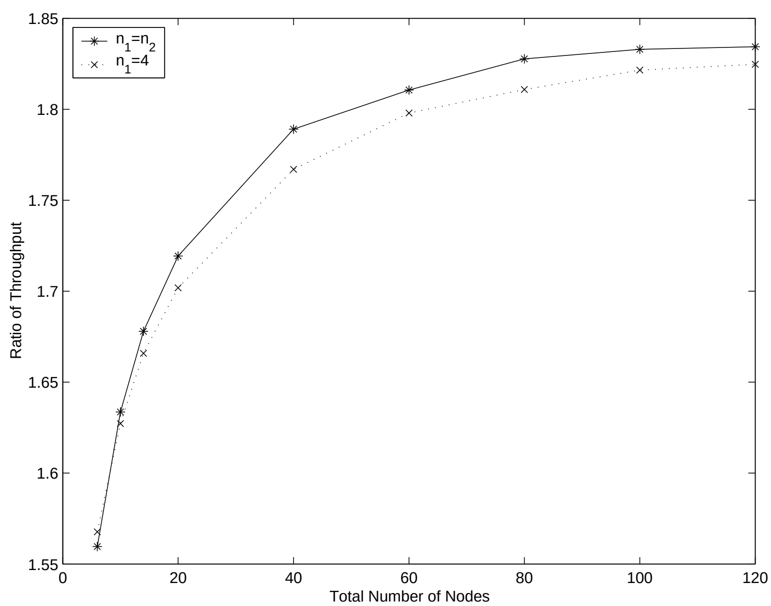

Figure 4.9: The throughput ratio of a class 1 node with $\omega_1 = 0.04$ and $a_1 = 10$ to a class 2 node with $\omega_2 = 0.04$ and $a_2 = 20$.

So, when service differentiation is provided by a_i, as the total number of users increases, the throughput ratio will approach $\frac{1+a_2}{1+a_1}$. Also, note that the scenario with $n_1 = n_2$ has a higher throughput ratio than the scenario with $n_1 = 4$. This is because as the traffic load increase, the throughput ratio is in favor of high priority class, and in the first scenario the traffic load is higher, due to larger number of class 1 users.

4.5 Extension to Multicell Networks

We now extend the above development to multicell networks where wireless nodes could have asymmetric information about contention. For each node i, denote by I_i the set of nodes that interfere with the transmission of node i. Thus, the conditional collision probability is

$$q_i := 1 - \prod_{j \in I_i} (1 - p_j), \ i \in N. \tag{4.16}$$

Random access game $\mathcal{G}$ is defined in the same way as in Definition 4.1, but with conditional collision probabilities (4.16).

As mentioned in section 4.3, we established the uniqueness of nontrivial Nash equilibrium for single-cell wireless LANs by exploiting the special symmetry in the network. It is not clear how to extend that to the multicell networks. In this section, we will establish the uniqueness of and the convergence to the nontrivial Nash equilibrium by general techniques that are applicable to any networks. That said, for single-cell networks, we have two sets of overlapping and complementary conditions for the uniqueness of the equilibrium and the convergence of the strategy update algorithm, which means a larger design space for random access game and medium access control.

4.5.1 Nash Equilibrium

It is straightforward to verify that Theorems 4.2 and 4.3 still hold for the multicell networks, i.e., under the assumption A1 there exists at least one nontrivial Nash equilibrium for random access game $\mathcal{G}$. In the following discussion, we will suppose that the assumption A1 holds. We now establish the uniqueness of the nontrivial Nash equilibrium.

Theorem 4.14 *If* $\sum_{j\in I_i}\frac{-1}{U_i''((U_i')^{-1}(q_i(\mathbf{p})))}\prod_{k\in I_i/\{j\}}(1-p_k) < 1$ *for any node* $i \in N$, *then random access game* $\mathcal{G}$ *has a unique nontrivial Nash equilibrium.*

Proof. Following $U_i' \circ (U_i')^{-1}(q_i) = q_i$, we have for $j \in I_i$,

$$\begin{aligned}\frac{\partial G_i(\mathbf{p})}{\partial p_j} &= \frac{1}{U_i''(G_i(\mathbf{p}))}\prod_{k\in I_i/\{j\}}(1-p_k)\\ &= \frac{1}{U_i''((U_i')^{-1}(q_i(\mathbf{p})))}\prod_{k\in I_i/\{j\}}(1-p_k),\end{aligned} \tag{4.17}$$

and for $j \notin I_i$, $\frac{\partial G_i(\mathbf{p})}{\partial p_j} = 0$.

Denote the Jacobi matrix of the best response function $G(\mathbf{p})$ by J, i.e., $J_{i,j} = \frac{\partial G_i(\mathbf{p})}{\partial p_j}$. Note that $J_{i,i} = 0$. By Geršgorin disc theorem [40], all the eigenvalues λ_i of J are located in the union of $|N|$ discs centered at 0 with radii

$$\sum_{j\neq i}|\frac{\partial G_i(\mathbf{p})}{\partial p_j}| = \sum_{j\in I_i}\frac{-1}{U_i''((U_i')^{-1}(q_i(\mathbf{p})))}\prod_{k\in I_i/\{j\}}(1-p_k),\ i \in N.$$

So, if $\sum_{j\in I_i}\frac{-1}{U_i''((U_i')^{-1}(q_i(\mathbf{p})))}\prod_{k\in I_i/\{j\}}(1-p_k) < 1$ for any node i, then complex norms of all eigenvalues $|\lambda_i| < 1$ and $G(\mathbf{p})$ is a contraction mapping. By contraction mapping theorem [6], $G(\mathbf{p})$ has a unique fixed point, i.e., random access game $\mathcal{G}$ has a unique nontrivial Nash equilibrium. ■

The condition specified in the above theorem is in terms of general utility functions. For convenience, we call it assumption **A3**. It may look restrictive since it requires the summation of $|I_i|$ terms to be less than one, but is not necessarily so. For example, for random access game $\mathcal{G}_1$ considered in section 4.4, the assumption A3 reads $\frac{(a_i-1)\omega_i}{(1+a_iq_i(\mathbf{p}))^2}\sum_{j\in I_i}\prod_{k\in I_i/\{j\}}(1-p_k) < 1$. This requires that

$$\frac{(a_i-1)\omega_i}{(1+a_i-a_i\prod_{j\in I_i}(1-\nu_i))^2}\sum_{j\in I_i}\prod_{k\in I_i/\{j\}}(1-\nu_k) < 1. \tag{4.18}$$

It seems that, as $|I_i|$ becomes large, we need large ν_i values to satisfy this condition, but this is not true. To see this, let us consider a set of parameters that is considered

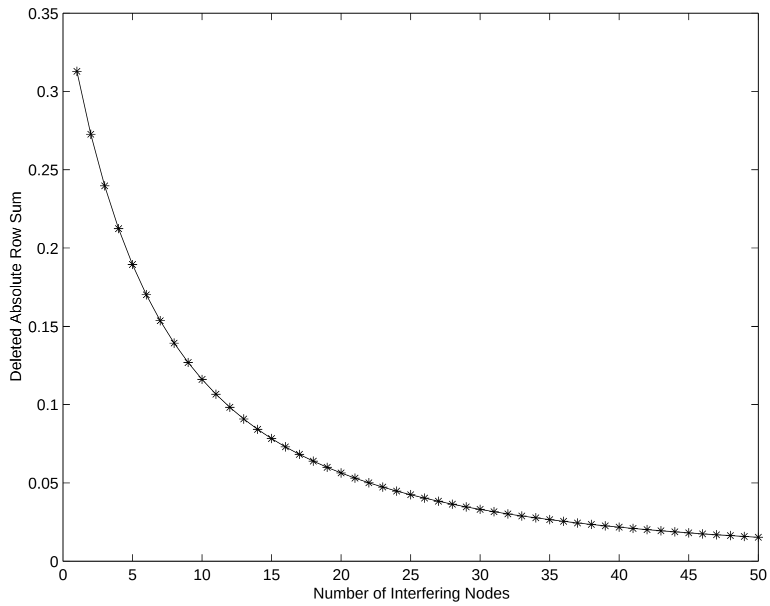

Figure 4.10: The evolution of deleted absolute row sum of the Jacobi matrix versus the number of interfering nodes for random access game $\mathcal{G}_1$ with $\omega_i = 0.04$ and $a_i = 23$.

in section 4.4: $\omega_i = 0.04$, $a_i = 23$ and $\nu_i = 2\omega_i/(a_i + 1) = 0.0033$. Figure 4.10 shows the evolution of the left-hand side of equation (4.18), called deleted absolute row sum [40] of the Jacobi matrix, versus the number of interfering nodes. We see that A3 holds for any number of interfering nodes. Thus, A3 is a mild assumption and allows for a large design space for random access game and medium access control.

4.5.2 Dynamics

Let us first consider the best response strategy. By contraction mapping theorem, the following result is immediate.

Theorem 4.15 *Under the assumption A3, the best response strategy (4.5) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$.*

We next consider the gradient play (4.6). Before proceeding, note that we can reformulate the assumption A3 as $\sum_{j \in I_i} \frac{-1}{U_i''((U_i')^{-1}(q_i(\mathbf{p})))} \prod_{k \in I_i/\{j\}} (1 - p_k) \leq 1 - \delta$ by appropriately choosing a positive constant $\delta > 0$.

Theorem 4.16 *Suppose A3 holds. The gradient play (4.6) converges to the unique nontrivial Nash equilibrium of random access game* $\mathcal{G}$ *if* $1-\delta < \frac{\mu}{\lambda}$ *and for any* $i \in N$*, the constant stepsize* $f_i \leq \frac{1}{\lambda}$.

Proof. First note that the nontrivial Nash equilibrium of random access game $\mathcal{G}$ is the fixed point of the gradient play (4.6) and vice versa. Define a mapping $M(\mathbf{p})$

$$M_i(\mathbf{p}) := p_i + f_i \left(U_i'(p_i) - q_i(\mathbf{p})\right). \tag{4.19}$$

By the assumption A1, for any q_i, there exists a $\bar{p}_i \in S_i$ such that $U_i'(\bar{p}_i) = q_i$. By Taylor expansion, there exists a $\hat{p}_i \in S_i$ such that

$$M_i(\mathbf{p}) = p_i + f_i U_i''(\hat{p}_i)\left(p_i - \bar{p}_i\right).$$

It is easy to verify that if the stepsize $f_i \leq \frac{1}{\lambda}$, $M_i(\mathbf{p}) \in S_i$ for any $\mathbf{p} \in S_1 \times S_2 \times \cdots \times S_{|N|}$. Thus, when $f_i \leq \frac{1}{\lambda}$, the gradient play (4.6) can be written as

$$p_i(t+1) = M_i(\mathbf{p}(t)).$$

Now, assume a constant stepsize f_i and consider the Jacobi matrix J^M of $M(\mathbf{p})$,

$$\begin{aligned} J_{i,j}^M &= \delta_{i,j} + f_i \left(U_i''(p_i)\delta_{i,j} - U_i''((U_i')^{-1}(q_i(\mathbf{p})))J_{i,j}\right) \\ &= (1 + f_i U_i''(p_i))\delta_{i,j} - f_i U_i''((U_i')^{-1}(q_i(\mathbf{p})))J_{i,j}. \end{aligned} \tag{4.20}$$

Since $J_{i,i} = 0$, by Geršgorin disc theorem all the eigenvalues λ_i of J^M are located in the union of $|N|$ discs centered at $1 + f_i U_i''(p_i)$ with radii $-f_i U_i''((U_i')^{-1}(q_i(\mathbf{p}))) \sum_{j \neq i} |J_{i,j}|$. By assumption, $\sum_{j \neq i} |J_{i,j}| \leq 1 - \delta$. We see that the complex norms $|\lambda_i| < 1$ if

$$\begin{aligned} 1 - f_i\mu + f_i\lambda(1-\delta) &< 1, \\ 1 - f_i\lambda - f_i\lambda(1-\delta) &> -1, \end{aligned}$$

i.e., $1-\delta < \frac{\mu}{\lambda}$ and $f_i < \frac{2}{(2-\delta)\lambda}$. Note that, if the complex norms $|\lambda_i| < 1$, then $M(\mathbf{p})$ is a contraction mapping and the gradient play will converge to the unique fixed point of (4.6). Since $\frac{1}{\lambda} < \frac{2}{(2-\delta)\lambda}$, the gradient play (4.6) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$ if $1-\delta < \frac{\mu}{\lambda}$ and for any $i \in N$, the constant stepsize $f_i \leq \frac{1}{\lambda}$. ■

The condition $1-\delta < \frac{\mu}{\lambda}$ in the above theorem may be restrictive. In order to go around it, we consider another alternative strategy update mechanism called Jacobi play.[7] In Jacobi play, every player adjusts current channel access probability gradually towards the best response strategy. Mathematically, each node $i \in N$ updates its strategy according to

$$p_i(t+1) = [p_i(t) + g_i(p_i(t))\left((U_i')^{-1}(q_i(\mathbf{p}(t))) - p_i(t)\right)]^{S_i}, \tag{4.21}$$

where the stepsize $g_i(\cdot) > 0$ can be a function of the current strategy of node i. When $g_i = 1$, we recover the best response strategy. However, we would like a small stepsize, since smoother dynamics are preferred.

Theorem 4.17 *Suppose A3 holds. The Jacobi play (4.21) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$ if for any $i \in N$, the constant stepsize $g_i \leq 1$.*

Proof. First note that the nontrivial Nash equilibrium of random access game $\mathcal{G}$ is the fixed point of Jacobi play (4.21) and vice versa. Define a mapping $\bar{M}(\mathbf{p})$

$$\bar{M}_i(\mathbf{p}) := p_i + g_i\left((U_i')^{-1}(q_i(\mathbf{p})) - p_i\right). \tag{4.22}$$

It is easy to verify that if the stepsize $g_i \leq 1$, $\bar{M}_i(\mathbf{p}) \in S_i$ for any $\mathbf{p} \in S_1 \times S_2 \times \cdots \times S_{|N|}$. Thus, when $g_i \leq 1$, the Jacobi play (4.21) can be written as

$$p_i(t+1) = \bar{M}_i(\mathbf{p}(t)).$$

[7]The name comes from Jacobi update scheme, see, e.g., [56] [45].

Now, assume a constant stepsize g_i and consider the Jacobi matrix $J^{\bar{M}}$ of $\bar{M}(\mathbf{p})$,

$$J^{\bar{M}}_{i,j} = (1 - g_i)\delta_{i,j} + g_i J_{i,j}. \tag{4.23}$$

Similarly, by Geršgorin disc theorem, we see that the complex norms of eigenvalues λ_i of $J^{\bar{M}}$ are less than one, if

$$\begin{aligned} 1 - g_i &\geq 0, \\ 1 - g_i + g_i(1 - \delta) &< 1, \end{aligned}$$

i.e., $g_i \leq 1$. Note that, if the complex norms $|\lambda_i| < 1$, then $\bar{M}(\mathbf{p})$ is a contraction mapping and the Jacobi play will converge to the unique fixed point of (4.21). So, the Jacobi play (4.21) converges to the unique nontrivial Nash equilibrium of random access game $\mathcal{G}$ if for any $i \in N$, the constant stepsize $g_i \leq 1$. ■

4.5.3 Medium Access Control Design

Comparing the gradient play and Jacobi play, we see that the gradient play (4.6) has a nice economic interpretation, i.e., adjusts channel access probability based on marginal utility and contention price, but the Jacobi play (4.21) demands a weaker condition for convergence. However, both of them are equally easy to implement, and can be used to design medium access methods as described in subsections 4.3.3 and 4.4.3.

4.6 Utility Function and Reverse Engineering

As we see from the above discussions, utility functions determine Nash equilibria of random access games and thus the equilibrium (steady) operating points of medium access control protocols. Conversely, utility functions are determined by the equilibrium (steady) operating points of medium access control protocols. Since the medium access control protocol adapts channel access probability p_i according to current access probability and packet collision, the equilibrium operating point defines an implicit relation between equilibrium channel access probability p_i and conditional collision probability q_i,

$$p_i = \mathcal{F}_i(p_i, q_i). \tag{4.24}$$

Assume $\mathcal{F}_i$ is continuously differentiable and $\partial\mathcal{F}_i/\partial q_i \neq 0$ in $[0, 1]$. Then, by implicit function theorem [6], there exists a unique continuously differentiable function F_i such that

$$q_i = F_i(p_i). \tag{4.25}$$

Define the utility function of each node i as

$$U_i(p_i) = \int F_i(p_i)dp_i. \tag{4.26}$$

With the above defined utility functions, we can define a random access game as in section 4.3. Hence, we can reverse engineer medium access control protocols and study them in game theory framework: medium access control can be interpreted as a distributed strategy update algorithm to achieve the Nash equilibrium of the random access game.

For example, if we are first given the medium access method presented in subsection 4.4.3, it can be interpreted as a distributed strategy update algorithm to achieve the Nash equilibrium of random access game $\mathcal{G}_1$ that is defined with the utility functions determined by the equilibrium of equation (4.12). Take another example, 802.11 DCF. It is well established that for a single-cell wireless LAN at steady state, channel access probability p relates to conditional collision probability as follows [14]:

$$p = \frac{2(1-2q)}{(1-2q)(CWmin+1) + qCWmin(1-(2q)^m)},$$

where $CWmin$ is the base contention window and m is the maximum backoff stage.

Following procedures (4.25)–(4.26) to derive a utility function, we can define a random access game and interpret DCF as distributed strategy update algorithm to achieve the corresponding Nash equilibrium.[8]

The random access game model can be used to analyze equilibrium properties such as throughput, collision and fairness of different medium access control protocols. When wireless nodes in a wireless LAN deploy different medium access protocols with different contention measures, we can also study the coexistence and interaction of different protocols in the random access game framework. For example, 802.11 DCF and our design are based on different contention measures. It is interesting to see how they interact.

4.7 Conclusions

We have developed a general game-theoretic model to study the contention/interaction among wireless nodes, and propose a novel medium access method derived from CSMA/CA in which each node estimates its conditional collision probability and adjusts its persistence probability or contention window, according to distributed strategy update mechanism achieving the Nash equilibrium. This results in simple dynamics, controllable performance objectives, good short-term fairness, low collision and high throughput. As wireless nodes can estimate conditional collision probabilities by observing consecutive idle slots between transmissions, we can decouple contention control from handling failed transmissions. This also opens up other opportunities such as rate adaptation to channel variations. As a case study of medium access control design in game-theoretic framework, we present a concrete medium access method and show that it achieves superior performance over the standard 802.11 DCF, and can provide flexible service differentiations among wireless nodes. In addition to guiding medium access control design, the random access game model also provides an analytical framework to understand equilibrium properties such as throughput, loss and fairness, and dynamic property of different medium access pro-

[8]However, the dynamics of 802.11 DCF cannot be described by gradient play.

tocols and their interactions.

There are a number of issues to investigate. We will search for other contention measures and contention resolution algorithms that could achieve Nash equilibria of random access games. We are also studying the issue of ensuring time fairness in a wireless network with rate diversity. We are also investigating the coexistence of our access method and 802.11 DCF that use different contention signals: how the resource is allocated to and shared among wireless nodes using different medium access methods. This issue is important for the deployment of the new access method. We also plan to implement our access method in a testbed to evaluate its performance against realistic physical characteristics of a network, especially to examine the setting of various control parameters that determine the dynamic properties of the access method.

Chapter 5

Discussions and Future Research

In this thesis, we view network protocols as distributed algorithms achieving the corresponding network equilibria, and have explored aspects of this general design methodology in optimization and game-theoretic frameworks through the study of two wireless network design problems, cross-layer design and contention control, respectively. In broader context, the underlying theme of this thesis is that optimization theory and game theory provide a suite of tools that are flexible in modelling various network systems, and a rich series of equilibrium solution concepts and distributed convergent algorithms to guide systematic design of network protocols. Our results have demonstrated the merit and power of such *mathematically rigorous and systematic methodology* for network design.

Before closing, I shall briefly discuss some future research directions.

We have been focusing on the utility maximization that is convex problem in chapter 3. However, nonconvex utility maximization problems often appear. This can be due to nonconvex utility as in modelling inelastic traffic such as voice, or nonconvex constraints such as integral constraints as in some routing scheme or scheduling. Such nonconvex optimization problems are in general difficult and NP-hard. Especially, nonconvex optimization often has non-zero duality gap, and the distributed algorithms based on dual decomposition may lead to suboptimal design and instability in cross-layer interactions. In the framework of "layering as dual decomposition," some key questions to be addressed are to quantify the duality gap as it is a measure of the tradeoff between optimizing the performance (optimality) and adhering to distributed layered structure (decentralization), to seek systematic ways to reduce or even close the duality gap, and to determine conditions under which dual-based subgradient algorithms and their variants still lead to decent designs. These are difficult problems, but fortunately, recent developments such as sum of squares programming [72] seem to provide the right mathematical tool to tackle these issues.

Regarding network design and control in game-theoretic framework, a key issue is the distributed convergence of the equilibria. Various equilibrium solution concepts are somewhat delicate objects, as they usually require complete information of the system. As the network consists of distributed entities having limit information, it is important to understand how the structure of the system affects equilibria and what we can expect in terms of dynamics when the agents are using adaptive algorithms, and to identify those key mathematical structures (e.g., the properties of the objective functions such as monotonicity, and the information structure of the system) that guarantee the distributed convergence of different equilibria. Another issue is concerned with the property of the equilibria: how the system performs under game-theoretic equilibrium, compared with a globally optimal solution. One related and highly studied concept is the price of anarchy [51], which is usually for Nash equilibrium. It would be interesting to characterize other solution concepts such as correlated equilibrium against the globally optimal. Also, one other research direction is simply to study more network design problems in game-theoretic framework.

As discussed in chapter 1, there are computational, informational and incentive constraints in network design and control. These constraints together determine what kind of equilibrium, or stable operating point, the system can achieve, and what kind of convergent algorithm to the equilibrium is feasible. A research agenda in the long run is to seek a general mathematical framework of network design that could handle these constraints in a systematic way, while achieving designs with the best possible performance. Recent developments such as distributed algorithmic mechanism design [31] have taken a thrust at this problem, but usually target to the design for maximal social welfare. However, in many situations an agent's (subjective) preferences, which are encapsulated in utility or cost functions, are not aligned with the (objective) performance of the network. For example, due to the economic incentive, an autonomous system may prefer to transfer traffic through a link with largest delay. So, in view of the network performance, to maximize the social welfare may not be a good design objective. It would be interesting to bring in accountability and study the design for optimal overall performance rather than welfare maximizing.

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