EFFECT OF SMALL VARIATIONS OF PARAMETERS IN THE TURBOPROP CYCLE

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LCdr. Carl O. Holmquist, USN

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TABLE OF CONTENTS

PART	TITLE	PAGE
	Summary	
	Introduction	1
I	Analysis of Basic Turboprop Cycle	3
II	Analysis of Cycle with Regeneration	16
III	Discussion of Results	23
IV	Conclusions	29
V	Schematic Diagrams of Turboprop Cycle	31
VI	Appendix A. Symbols and Definitions	32
VII	Graphs of Coefficients of Equations (76), (77) and (78)	34
	References	60

SUMMARY

A cycle analysis of the turbine-propeller engine is given in terms of the parameters of the cycle and the component efficiencies of the engine with and without regeneration. By means of a Taylor expansion about the point of ideal efficiencies, total power output, specific fuel consumption, and optimum jet pressure ratio are given in terms of the ideal values plus corrections for small variations in the component efficiencies from 100%. In this way, the relative importance of component efficiencies in affecting the performance of the ideal turboprop cycle is demonstrated by means of simple analytical expressions involving the basic cycle parameters and the component efficiencies.

The analysis of the ideal turboprop cycle is given in terms of three basic parameters which are functions of the forward flight speed, the compressor pressure ratio, and the limiting combustion chamber temperature. For the ideal cycle, the jet pressure ratio for optimum division of power between propeller and jet results in the total work of the cycle being done by the propeller.

Of the component efficiencies, the turbine efficiency was found to be the most important in affecting the performance criteria of the ideal turboprop cycle. Since all work is done by the propeller in the ideal cycle, the propeller efficiency was also found to be important in affecting performance. Regeneration,

as considered in this analysis, decreased the total power output and decreased the specific fuel consumption of the ideal turbine-propeller engine.

INTRODUCTION

In the analysis and design of an aircraft gas-turbine propeller-driving power plant, it is important not only to have a simplified method of analyzing the cycle with a convenient notation, but also to find the relative effect on performance of small variations of the component efficiencies in the ideal cycle. In the actual design work of component parts of a turboprop engine, the engineer must work with complex performance and characteristics curves in order to obtain quantitative data for the construction of the machine. However, it would seem that a simple analytical expression, containing the known parameters of the cycle and giving the relative effect of a small change in component efficiencies on the performance of the ideal cycle, would be helpful in overall design considerations.

The problem of analyzing the flow in the gas-turbine power plant has been treated extensively (see References). A method for determining the optimum division of power between jet and propeller for maximum thrust has been graphically analyzed in Reference 3. This method involves the use of complex curves in order to find the optimum jet pressure ratio for maximum thrust with a given set of engine operating conditions. It would seem less cumbersome if a simple analytical expression could be derived for the ideal cycle optimum jet pressure ratio and the variation therefrom for a small change in component efficiencies.

An analysis of the basic turboprop cycle, with and without regeneration, was made in terms of the parameters of the system. In this analysis, the weight of fuel injected in the combustion chamber and the variation of specific heats were neglected. Combustion efficiency was assumed to be 100%, and the momentum pressure loss in the combustion chamber was also neglected. Isentropic, frictionless compression in the inlet diffuser was assumed. From the cycle analysis, performance expressions were derived for total power output and specific fuel consumption. A relation defining the optimum jet pressure ratio was obtained from the total power output of the cycle.

The relative effect on performance criteria of small changes in component efficiencies was then investigated to show, individually, the influence of losses in each component. Using the relations derived for total power output, specific fuel consumption, and optimum jet pressure ratio, a Taylor expansion was used from the ideal operating point (all component efficiencies 100%). The coefficients of these derived expansions were then used to show the relative effect of component efficiencies on the ideal performance of the turboprop cycle.

I. ANALYSIS OF THE BASIC TURBOPROP CYCLE

A schematic diagram of the basic turbine-propeller engine without regeneration is presented as Figure 1. The various points in the cycle through which the working fluid passes are numbered and are referred to in the derivation. Symbols used in the analysis are defined in Appendix A. Expressions for work output of component parts, temperature ratios at various points in the ideal and non-ideal cycle, and all performance criteria have been derived in terms of four basic parameters of the cycle:

$$\mu = \frac{T_1}{T_0} = 1 + \frac{\gamma - 1}{2} M_0^2$$

The factor μ, therefore, is related to the aircraft flight speed.

$$\delta = \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

The factor δ is related to the pressure ratio across the compressor.

$$K = \frac{T_3}{T_0}$$

The factor K is related to the temperature rise in the combustion chamber and, hence, to the quantity of fuel injected and burned.

$$X = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_5}{T_6}$$

When the factors μ, δ, and K have been prescribed in any given turboprop cycle, there is one additional parameter needed to define completely the performance. This parameter has been chosen as the jet pressure ratio X, and defines the pressure and temperature ratio across the jet exhaust nozzle. The value of the jet pressure ratio in any given cycle determines the proportion of the total work of the engine that goes to driving the propeller and the proportion that goes to jet thrust. If the optimum jet pressure ratio X^*, for optimum division of power between propeller and jet, is introduced in terms of μ, σ, and K, then the performance criteria of the complete ideal cycle can be expressed in terms of these three parameters only.

The Basic Turbopropeller Cycle. In the basic cycle, air passes from free stream at station 0 and is compressed isentropically and without friction in the inlet diffuser to station 1. Stagnation state conditions are considered at the compressor inlet:

$$C_p (T_1 - T_0) = \frac{V_0^2}{2gJ}$$

$$\frac{T_1}{T_0} = 1 + \frac{\gamma-1}{2} M_0^2 \qquad (1)$$

$$\frac{p_1}{p_0} = \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}} \qquad (2)$$

Compression in the compressor is carried out between stations 1 and 2. Introducing the notation described above, the temperature ratio across the compressor is derived as follows:

$$\delta = \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \mu = \frac{T_1}{T_0}$$

$$\frac{T_2'}{T_0} = \delta\mu \quad (3) \qquad \eta_c = \frac{T_2' - T_1}{T_2 - T_1}$$

$$\frac{T_2}{T_1} = \left[1 + \frac{\delta - 1}{\eta_c}\right] \qquad (4)$$

$$\frac{T_2}{T_0} = \mu\left[1 + \frac{\delta - 1}{\eta_c}\right] \qquad (5)$$

The work of the compressor for ideal and non-ideal compression is:

$$W_c(\text{ideal}) = C_p(T_2' - T_1) = C_p T_0 \left(\frac{T_2'}{T_0} - \frac{T_1}{T_0}\right)$$

$$W_c(\text{ideal}) = C_p T_0 \mu(\delta - 1) \quad \left(\frac{BTU}{lb}\right) \qquad (6)$$

$$W_c = C_p(T_2 - T_1) = C_p T_0 \left(\frac{T_2}{T_0} - \frac{T_1}{T_0}\right)$$

$$W_c = C_p T_0 \frac{\mu(\delta - 1)}{\eta_c} \quad \left(\frac{BTU}{lb}\right) \qquad (7)$$

The heat added at station 3 in the combustion chamber per pound of air is given below. Combustion efficiency is assumed to be 100% and the momentum pressure loss in the combustion chamber is neglected.

$$Q_c = C_p(T_3 - T_2) = C_p T_0\left(\frac{T_3}{T_0} - \frac{T_2}{T_0}\right) \qquad K = \frac{T_3}{T_0}$$

$$Q_c = C_p T_0\left[K - \mu\left(1 + \frac{\delta - 1}{\eta_c}\right)\right] \quad \left(\frac{BTU}{lb}\right) \qquad (8)$$

The turbine in the turboprop cycle not only supplies the power to drive the compressor, but also drives the propeller. A hypothetical point 4 of the turbine has been chosen to theoretically divide the turbine work between compressor and propeller. By equating the work of the compressor to the work of the turbine to station 4, the following temperature ratios are derived:

$$\frac{T_6'}{T_0} = \frac{T_3}{T_2'} = \frac{T_3}{T_0} \cdot \frac{T_0}{T_2'} = \frac{K}{\delta\mu} \qquad (9)$$

$$C_p(T_2 - T_1) = C_p(T_3 - T_4) = C_p\,\eta_t\,(T_3 - T_4')$$

$$T_3 - T_4' = \frac{T_0\,\mu(\delta - 1)}{\eta_c\,\eta_t}$$

$$\frac{T_4'}{T_0} = K - \frac{\mu(\delta - 1)}{\eta_c\,\eta_t} \qquad (10)$$

$$\frac{T_4}{T_0} = K - \frac{\mu(\delta - 1)}{\eta_c} \qquad (11)$$

Having obtained an expression for the temperature ratio at point 4, the work to drive the propeller can be derived by introducing the jet pressure ratio X:

$$W_p = \eta_p C_p (T_4 - T_5) \qquad X = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_5}{T_6}$$

$$(T_3 - T_5) = \eta_t (T_3 - T_5')$$

$$T_5' = T_6' \left(\frac{T_5}{T_6}\right)$$

$$\frac{T_5'}{T_0} = \frac{KX}{\delta\mu} \qquad (12)$$

$$(T_3 - T_5) = \eta_t T_0 K \left(1 - \frac{x}{\delta\mu}\right)$$

$$W_p = \eta_p C_p \left[(T_3 - T_5) - (T_3 - T_4)\right]$$

$$W_p = \eta_p C_p T_0 \left[\eta_t K \left(1 - \frac{x}{\delta\mu}\right) - \frac{\mu(\delta - 1)}{\eta_c}\right] \left(\frac{BTU}{lb}\right) \tag{13}$$

$$\frac{T_5}{T_0} = K\left[1 - \eta_t \left(1 - \frac{x}{\delta\mu}\right)\right] \tag{14}$$

$$\frac{T_6}{T_0} = \frac{K}{X}\left[1 - \eta_t \left(1 - \frac{x}{\delta\mu}\right)\right] \tag{15}$$

Knowing these temperature ratios in terms of the cycle parameters, the jet velocity V_6 can be determined:

$$C_p (T_5 - T_6) = \frac{1}{C_v^2} \frac{V_6^2}{2gJ}$$

$$V_6 = C_v \sqrt{2gJC_pT_0 \left(\frac{T_5}{T_0} - \frac{T_6}{T_0}\right)}$$

$$V_6 = C_v \sqrt{2gJC_pT_0K\left[1 - \eta_t\left(1 - \frac{x}{\delta\mu}\right)\right]\left(1 - \frac{1}{X}\right)} \left(\frac{ft}{sec}\right) \tag{16}$$

The output of the turboprop cycle can be expressed either in pounds thrust or in power dimensions. Since it is believed that the output of the gas-turbine propeller-driving engine can best be given in terms of power, these dimensions are used throughout the analysis. Therefore, the power to thrust is expressed as:

$$W_j = \frac{V_0^2}{2gJ}\left(\frac{V_6}{V_0} - 1\right) \qquad V_0^2 = 2gJC_pT_0(\mu - 1) \tag{17}$$

$$W_j = 2C_pT_0(\mu-1)\left[C_v\sqrt{\frac{K}{\mu-1}\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right]\left(1-\frac{1}{X}\right)}-1\right] \quad (18)$$

The total output of the cycle without regeneration including propeller work and work to thrust is:

$$W_{tot} = \eta_p C_p T_0\left[\eta_t K\left(1-\frac{X}{\delta\mu}\right)-\frac{\mu(\delta-1)}{\eta_c}\right]$$

$$+2C_pT_0(\mu-1)\left[C_v\sqrt{\frac{K}{\mu-1}\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right]\left(1-\frac{1}{X}\right)}-1\right] \quad (19)$$

Optimum Jet Pressure Ratio. Equation (19) gives the total power output of the turboprop cycle in terms of four basic parameters μ, δ, K, and X and the component efficiencies. By introducing the optimum jet pressure ratio X* for ideal division of power between jet and propeller, the basic parameters for the ideal cycle can be reduced to three, namely, δ, μ, and K. In order to obtain an expression defining the optimum jet pressure ratio X*, the total work relation is differentiated with respect to X and set equal to zero. The relation defining the optimum jet pressure ratio X* is then found to be:

$$\sqrt{\frac{K}{\mu-1}\left[1-\eta_t\left(1-\frac{X^*}{\delta\mu}\right)\right]\left(1-\frac{1}{X^*}\right)} = \frac{C_v\,\delta\mu(1-\eta_t)}{\eta_p\,\eta_t\,X^{*2}}+\frac{C_v}{\eta_p} \quad (20)$$

For ideal efficiencies $(\eta_p = \eta_t = C_v = 1)$, this reduces to the following simple expression:

$$X^{*} = \frac{\delta u(u-1)}{K} + 1 \tag{21}$$

It is interesting to note that the total work expression for the turboprop cycle, with all component efficiencies 100% and with optimum jet pressure ratio, reduces to:

$$W_{tot}(\text{ideal}) = C_p T_0 \left(\delta u - 1\right)\left(\frac{K}{\delta u} - 1\right) \tag{22}$$

and is obtained with such a jet pressure ratio X* that the jet velocity V_6 is equal to the free stream velocity V_0. Thus for the ideal turboprop cycle, the total work output would go to the propeller if the condition of optimum jet pressure ratio is imposed. This follows from the definition of overall efficiency of the turbojet and propeller cycles. From kinetic energy considerations, the following expression for overall efficiency of jet and propeller driven systems can be easily derived:

$$\eta = \frac{2}{1 + \frac{V}{V_0}} \tag{23}$$

Here V represents the jet velocity or the velocity behind the propeller disc, and V_0 represents free stream velocity. Thus, maximum efficiency is obtained for a jet cycle when the velocity of the jet is equal to the free stream velocity. The condition imposed on the ideal turboprop cycle that propeller efficiency η_P is 100%, implies a propeller disc of infinite radius and an infinitesimal change in velocity across the disc. Since the propeller disc is greater than the jet exhaust area, it follows that for minimum kinetic energy loss in the slipstream, ideally all work from the turboprop cycle will be done by the propeller.

Effects of Small Changes in Basic Cycle Parameters.

In order to obtain an expression for the change in total work output due to a small change ϵ in the optimum jet pressure ratio X^*, the third term of the following Taylor expansion was calculated:

$$W_{tot}(X^*+\epsilon) = W_{tot}(X^*) + \left.\frac{\partial W}{\partial X}\right|_{X^*}\epsilon + \frac{1}{2}\left.\frac{\partial^2 W}{\partial X^2}\right|_{X^*}\epsilon^2 + \cdots \qquad (24)$$

where $\left.\frac{\partial W}{\partial X}\right|_{X^*}$ has been set equal to zero and defines the optimum jet pressure ratio X^*.

$$\Delta W = W_{tot}(X^*+\epsilon) - W_{tot}(X^*) = \frac{1}{2}\left.\frac{\partial^2 W}{\partial X^2}\right|_{X^*}\epsilon^2 \qquad (25)$$

$$\Delta W = \frac{-\epsilon^2 C_p T_0 C_v \left[K(\mu-1)\right]^{\frac{1}{2}}\left\{(1-\eta_t)^2\left(4-\frac{3}{X^*}\right)+(1-\eta_t)\left(\frac{\eta_t}{\delta\mu}\right)(6X^*-4)+\left(\frac{\eta_t}{\delta\mu}\right)^2 X^{*3}\right\}}{4X^{*3}\left(\left[1-\eta_t+\frac{\eta_t X^*}{\delta\mu}\right]\left(1-\frac{1}{X^*}\right)\right)^{3/2}} \qquad (26)$$

For ideal conditions $\left(\eta_t = C_v = 1,\ X^* = \frac{\delta\mu(\mu-1)}{K}+1\right)$, this expression reduces to:

$$\Delta W = \frac{-\epsilon^2 C_p T_0 K^2}{4\delta^2\mu^2(\mu-1)} \qquad (27)$$

The effect on total work output of the cycle of a small change ϵ in δ (related to the pressure ratio across the compressor) was found to be:

$$\Delta W = \epsilon C_p T_o \left[\eta_p \left(\frac{\eta_t K X}{\delta^2 \mu} - \frac{\mu}{\eta_c} \right) - \frac{C_v K \eta_t (X-1)}{\delta^2 \mu \sqrt{\frac{K}{\mu-1}\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right]\left(1-\frac{1}{X}\right)}} \right] \quad (28)$$

For $\eta_p = \eta_t = \eta_c = C_v = 1,\ X = X^* = \frac{\delta\mu(\mu-1)}{K} + 1$, this reduces to

$$\Delta W = \epsilon C_p T_o \left[\frac{K}{\delta^2 \mu} - \mu \right] \quad (29)$$

The effect on total work of a small change ϵ in K (related to temperature rise in the combustion chamber) is:

$$\Delta W = \epsilon C_p T_o \left[\eta_p \eta_t \left(1 - \frac{X}{\delta\mu}\right) + C_v \sqrt{\frac{\mu-1}{K}\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right]\left(1-\frac{1}{X}\right)} \right] \quad (30)$$

For $\eta_p = \eta_t = C_v = 1,\ X = X^* = \frac{\delta\mu(\mu-1)}{K} + 1$, this reduces to:

$$\Delta W = \epsilon C_p T_o \left(\frac{\delta\mu - 1}{\delta\mu} \right) \quad (31)$$

The effect on total work of a small change ϵ in μ (related to the forward flight velocity) is:

$$\Delta W = \epsilon C_p T_o \left\{ \frac{\eta_p \eta_t K X}{\delta\mu^2} - \frac{\eta_p(\delta-1)}{\eta_c} - 2 + C_v \left[\frac{K\left(1-\frac{1}{X}\right)\left(\delta\mu^2(1-\eta_t) + \eta_t X\right)}{\delta\mu^2(\mu-1)\sqrt{\frac{K}{\mu-1}\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right]\left(1-\frac{1}{X}\right)}} \right] \right\} \quad (32)$$

For $\eta_p = \eta_t = \eta_c = C_v = 1$, $X = X^*$:

$$\Delta W = \epsilon C_p T_0 \left[\frac{K - \delta^2 \mu^2}{\delta \mu^2} \right] \tag{33}$$

Effect of Small Changes in Component Efficiencies on the Power Coefficient. The effect of small changes in component efficiencies on the optimum jet pressure ratio X* for the idealized cycle was found by determining the coefficients in the following Taylor expansion:

$$X^*(\eta_t, \eta_p, \eta_c, C_v) = X^*(1,1,1,1) + \left.\frac{\partial X^*}{\partial(1-\eta_t)}\right|_{1111}(1-\eta_t) + \left.\frac{\partial X^*}{\partial(1-\eta_p)}\right|_{1111}(1-\eta_p)$$
$$+ \left.\frac{\partial X^*}{\partial(1-\eta_c)}\right|_{1111}(1-\eta_c) + \left.\frac{\partial X^*}{\partial(1-C_v)}\right|_{1111}(1-C_v) + \cdots\cdots \tag{34}$$

To obtain these coefficients, the relation defining the optimum jet pressure ratio X* was differentiated in respect to the various efficiencies and evaluated at the point of ideal efficiency. The following derivatives were obtained:

$$\frac{\partial X^*}{\partial(1-\eta_t)} = \delta\mu(\mu-1)\left[\frac{1}{K} + \frac{\delta\mu[K-\delta\mu(\mu-1)]}{[K+\delta\mu(\mu-1)]^2}\right] \tag{35}$$

$$\frac{\partial X^*}{\partial(1-\eta_p)} = \frac{2\delta\mu(\mu-1)}{K} \tag{36}$$

$$\frac{\partial X^*}{\partial(1-\eta_c)} = 0 \tag{37}$$

$$\frac{\partial X^*}{\partial(1-C_v)} = -\frac{2\delta\mu(\mu-1)}{K} \tag{38}$$

and the resulting Taylor expansion is:

$$X^*(\eta_t,\eta_p,\eta_c,C_v) = \left[\frac{\delta\mu(\mu-1)}{K}+1\right] + \delta\mu(\mu-1)\left[\frac{1}{K}+\frac{\delta\mu[K-\delta\mu(\mu-1)]}{[K+\delta\mu(\mu-1)]^2}\right](1-\eta_t)$$

$$+\left[2\frac{\delta\mu(\mu-1)}{K}\right](1-\eta_p) + o(1-\eta_c) - \left[\frac{2\delta\mu(\mu-1)}{K}\right](1-C_v) \tag{39}$$

The coefficients of this expansion give the changes in the optimum jet pressure ratio X* from the ideal value when the component efficiencies are varied slightly from 100%. The coefficients also show the relative effects on X* of the various component efficiencies of the cycle.

The effect on total work was obtained by a similar procedure. The expression for the optimum jet pressure ratio X* was inserted in the total work equation, and the derivatives of total work in respect to the component efficiencies was obtained. In this case, X* is a function of the efficiencies also, and the derivatives obtained above were inserted appropriately. The following derivatives were obtained:

$$\frac{\partial W_{tot}}{\partial(1-\eta_t)} = -C_pT_o\left[\frac{K^2}{K+\delta\mu(\mu-1)} - \frac{K}{\delta\mu}\right] \tag{40}$$

$$\frac{\partial W_{tot}}{\partial(1-\eta_p)} = -C_pT_o\left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right] \tag{41}$$

$$\frac{\partial W_{tot}}{\partial(1-\eta_c)} = -C_p T_0 \mu(\delta-1) \tag{42}$$

$$\frac{\partial W_{tot}}{\partial(1-C_v)} = -2C_p T_0(\mu-1) \tag{43}$$

And a similar Taylor expansion for C_ℓ, the power coefficient, is:

$$C_\ell = \frac{W_{tot}(\eta_t,\eta_p,\eta_c,C_v)}{C_p T_0} = \left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right] - \left[\frac{K^2}{K+\delta\mu(\mu-1)} - \frac{K}{\delta\mu}\right](1-\eta_t)$$

$$-\left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right](1-\eta_p) - \left[\mu(\delta-1)\right](1-\eta_c) - \left[2(\mu-1)\right](1-C_v) \tag{44}$$

Effect of Small Changes in Component Efficiencies on the Specific Fuel Consumption. The effect of component efficiencies on the specific fuel consumption S was obtained as follows:

$$S = \frac{3600\ f}{W_{tot}\ (h.p.)} \quad \left(\frac{\text{lb fuel}}{hp - hr}\right) \tag{45}$$

With the proper expressions for fuel consumption f and total work in terms of the power coefficient C_ℓ, this becomes:

$$S = \frac{2545\left[K-\mu\left(1+\frac{\delta-1}{\eta_c}\right)\right]}{H\ C_\ell} \tag{46}$$

If the power coefficient C_ℓ in the preceding Taylor expansion is considered to be equal to $C_{\ell_0} - \alpha$, where C_{ℓ_0} is the power coefficient with all efficiencies 100% and with optimum jet pressure ratio X*, the above relation for S can be expanded into a similar form by neglecting squares of small quantities. The expansion thus obtained for the specific fuel consumption is:

$$S = \frac{2545}{H}\left(\frac{\delta\mu}{\delta\mu-1}\right) + \frac{2545}{H}\left\{\left[\frac{K\delta\mu}{(\delta\mu-1)^2(K-\delta\mu)}\left(\frac{K\delta\mu}{K+\delta\mu(\mu-1)}-1\right)\right](1-\eta_t) + \left(\frac{\delta\mu}{\delta\mu-1}\right)(1-\eta_p) + \left[\frac{\delta\mu^2(\delta-1)}{(\delta\mu-1)^2(K-\delta\mu)}\right](1-\eta_c) + \left[\frac{2(\mu-1)\delta^2\mu^2}{(\delta\mu-1)^2(K-\delta\mu)}\right](1-C_v)\right\} \tag{47}$$

Here $S_0 = \frac{2545}{H}\left(\frac{\delta\mu}{\delta\mu-1}\right)$ and is the value of the specific fuel consumption under ideal conditions. By evaluating the coefficients of this expansion for any given set of cycle parameters, the relative effect of the component efficiencies on the specific fuel consumption can be calculated.

II. ANALYSIS OF CYCLE WITH REGENERATION

A schematic diagram of the turbine-propeller engine with regeneration is given as Figure 2. It is assumed that an ideal heat exchanger is installed after the compressor and after the turbine so that a small amount of heat can be taken from the exhaust gases to heat the air leaving the compressor. The same assumptions and symbols as for the basic cycle are used in the development of the equations.

Compression in the diffuser:

$$\frac{T_1}{T_0} = 1 + \frac{\gamma-1}{2} M_0^2 \tag{48}$$

$$\frac{p_1}{p_0} = \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}} \tag{49}$$

Compression in the compressor:

$$\delta = \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \frac{T_2}{T_0} = \mu\left(1 + \frac{\delta-1}{\eta_c}\right) \tag{50}$$

$$\mu = \frac{T_1}{T_0} \qquad \frac{T_2}{T_1} = \left(1 + \frac{\delta-1}{\eta_c}\right) \tag{51}$$

Work of non-ideal compression:

$$W_c = C_p T_0 \frac{\mu(\delta-1)}{\eta_c} \quad \left(\frac{BTU}{lb}\right) \tag{52}$$

Total heat added in the combustion chamber (including heat from regeneration:

$$K = \frac{T_3}{T_0} \qquad Q_c = C_p T_0 \left[K - \mu\left(1 + \frac{\delta-1}{\eta_c}\right)\right] \quad \left(\frac{BTU}{lb}\right) \tag{53}$$

Setting the work of the turbine (to station 4) equal to the work of the compressor, the following relations are obtained:

$$T_3 - T_4 = T_2 - T_1 = \eta_t (T_3 - T_4')$$

$$\frac{T_4'}{T_0} = K - \frac{\mu(\delta - 1)}{\eta_c \eta_t} \quad (54)$$

$$\frac{T_6'}{T_0} = \frac{K}{\delta\mu} \quad (56)$$

$$\frac{T_4}{T_0} = K - \frac{\mu(\delta - 1)}{\eta_c} \quad (55)$$

Regenerator heating effectiveness is defined as:

$$\eta_x = \frac{T_2'' - T_2}{T_5 - T_2} \quad (57)$$

$$C_p (T_2'' - T_2) = C_p (T_5 - T_5'')$$

$$\frac{T_2'' - T_2}{T_5 - T_2} = \frac{T_5 - T_5''}{T_5 - T_2} = \eta_x \quad (58)$$

$$T_5'' = T_5 - \eta_x (T_5 - T_2)$$

Power to drive the propeller:

Assuming no pressure loss across the regenerator, the jet pressure ratios for the cycle with and without regeneration are the same:

$$X = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_5''}{T_6''} = \frac{T_5}{T_6} \quad (59)$$

$$W_p = \eta_p C_p T_0 \left[\eta_t K \left(1 - \frac{X}{\delta\mu}\right) - \frac{\mu(\delta - 1)}{\eta_c}\right] \left(\frac{BTU}{lb}\right) \quad (60)$$

$$\frac{T_5}{T_0} = K\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right] \tag{61}$$

The temperature ratios $\frac{T_5''}{T_0}$ and $\frac{T_6''}{T_0}$ are found to be:

$$\frac{T_6''}{T_5''} = \frac{1}{X} \qquad \frac{T_6''}{T_0} = \frac{T_5''}{T_0}\frac{1}{X}$$

$$\frac{T_5''}{T_0} = \frac{T_5}{T_0} - \eta_x\left(\frac{T_5}{T_0} - \frac{T_2}{T_0}\right)$$

$$\frac{T_5''}{T_0} = K\left[1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right](1-\eta_x) + \eta_x\mu\left(1+\frac{\delta-1}{\eta_c}\right) \tag{62}$$

$$\frac{T_6''}{T_0} = \frac{1}{X}\left[K(1-\eta_x)\left(1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right) + \eta_x\mu\left(1+\frac{\delta-1}{\eta_c}\right)\right] \tag{63}$$

With these temperature ratios the thrust velocity V_6'' can be obtained:

$$C_p\left(T_5''-T_6''\right) = \frac{V_6''^2}{2gJ}$$

$$V_6'' = C_v\sqrt{2gJC_pT_0\left\{K(1-\eta_x)\left(1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right)+\eta_x\mu\left(1+\frac{\delta-1}{\eta_c}\right)\right\}\left(1-\frac{1}{X}\right)} \tag{64}$$

Power to thrust:

$$W_j = \frac{V_0^2}{2gJ}\left(\frac{V_6''}{V_0}-1\right) \tag{65}$$

$$W_j = 2C_pT_0(\mu-1)\left[C_v\sqrt{\frac{\left\{K(1-\eta_x)\left(1-\eta_t\left(1-\frac{X}{\delta\mu}\right)\right)+\eta_x\mu\left(1+\frac{\delta-1}{\eta_c}\right)\right\}\left(1-\frac{1}{X}\right)}{(\mu-1)}}-1\right] \tag{66}$$

The total work output of the turboprop cycle with regeneration is:

$$W_{tot} = \eta_p C_p T_0 \left[\eta_t K \left(1 - \frac{X}{\delta \mu} \right) - \frac{\mu(\delta - 1)}{\eta_c} \right]$$

$$+ 2 C_p T_0 (\mu - 1) \left[C_v \sqrt{\frac{\left\{ K(1-\eta_x)\left(1 - \eta_t\left(1 - \frac{X}{\delta\mu}\right)\right) + \eta_x \mu \left(1 + \frac{\delta - 1}{\eta_c}\right) \right\} \left(1 - \frac{1}{X}\right)}{(\mu - 1)}} - 1 \right] \quad (67)$$

<u>Fuel Consumption Decrease due to Regeneration.</u> For the basic cycle without regeneration, the actual fuel consumption in $\frac{\text{lb fuel}}{\text{sec}}$ was found to be:

$$f = \frac{m C_p T_0}{H} \left[K - \mu \left(1 + \frac{\delta - 1}{\eta_c} \right) \right] \left(\frac{\text{lb fuel}}{\text{sec}} \right) \quad (68)$$

Assuming the same temperature rise in the combustion chamber for the cycle with and without regeneration, the actual fuel consumption with regeneration is found as follows:

$$H f'' + m C_p \eta_x (T_5 - T_2) = m C_p T_0 \left[K - \mu \left(1 + \frac{\delta - 1}{\eta_c} \right) \right] \quad (69)$$

from which f'' can be determined as:

$$f'' = \frac{m C_p T_0}{H} \left\{ \left[K - \mu \left(1 + \frac{\delta - 1}{\eta_c} \right) \right] (1 - \eta_x) + \eta_x \eta_t K \left(1 - \frac{X}{\delta \mu} \right) \right\} \quad (70)$$

Thus, the actual fuel consumption decrease due to regeneration:

$$f - f'' = \frac{m C_p T_0}{H} \eta_x \left\{ \left[K - \mu \left(1 + \frac{\delta - 1}{\eta_c} \right) \right] - \eta_t K \left(1 - \frac{X}{\delta \mu} \right) \right\} \quad (71)$$

<u>Power Output Decrease due to Regeneration.</u> The corresponding work output decrease for the regenerative cycle (assuming

the same temperature rise in the combustion chamber) becomes:

$$W - W'' = \eta_t c_p T_0 C_v \left[K\left(1 - \eta_t\left(1 - \frac{X}{\delta\mu}\right)\right) - \mu\left(1 + \frac{\delta - 1}{\eta_c}\right) \right] \sqrt{\frac{(\mu - 1)\left(1 - \frac{1}{X}\right)}{K\left[1 - \eta_t\left(1 - \frac{X}{\delta\mu}\right)\right]}} \qquad (72)$$

Optimum Jet Pressure Ratio with Regeneration. By differentiating the total work expression for the regenerative cycle with respect to X and setting the derivative equal to zero, the expression determining the optimum jet pressure ratio is determined to be:

$$\sqrt{\frac{\left\{K(1-\eta_x)\left(1 - \eta_t\left(1 - \frac{X^*}{\delta\mu}\right)\right) + \eta_x \mu\left(1 + \frac{\delta - 1}{\eta_c}\right)\right\}\left(1 - \frac{1}{X^*}\right)}{(\mu - 1)}} =$$

$$(1 - \eta_x)\left[\frac{C_v \delta\mu(1 - \eta_t)}{\eta_p \eta_t X^{*2}} + \frac{C_v}{\eta_p}\right] + \frac{\eta_x C_v \delta \mu^2}{K \eta_p \eta_t X^{*2}}\left(1 + \frac{\delta - 1}{\eta_c}\right) \qquad (73)$$

Effect of Small Changes in the Component Efficiencies of the Regenerative Turboprop Cycle on Optimum Jet Pressure Ratio, Power Coefficient and Specific Fuel Consumption. Similar Taylor expansions were carried out for the turboprop cycle with regeneration to obtain expressions for optimum jet pressure ratio, total work, and specific fuel consumption for small variations in η_t, η_p, η_c, C_v and η_x. Because of the fact that the coefficients are evaluated at the point of ideal efficiency, the derivatives of X* and W_{tot} in respect to η_t, η_p, η_c and C_v remain unchanged. The derivatives in respect to η_x were derived as:

$$\frac{\partial X^*}{\partial \eta_x} = \frac{\delta\mu(\mu - 1)}{K}\left[\frac{\delta^2\mu^2(K - \delta\mu(\mu - 1))}{[K + \delta\mu(\mu - 1)]^2} - 1\right] \qquad (74)$$

$$\frac{\partial W_{tot}}{\partial \eta_x} = C_p T_0 (\mu - 1)\left[\frac{\delta^2 \mu^2}{K + \delta\mu(\mu-1)} - 1\right] \tag{75}$$

and the complete expansions for X* and W_{tot} about the point of ideal efficiencies are:

$$X^* = \left[\frac{\delta\mu(\mu-1)}{K} + 1\right] + \delta\mu(\mu-1)\left[\frac{1}{K} + \frac{\delta\mu[K-\delta\mu(\mu-1)]}{[K+\delta\mu(\mu-1)]^2}\right](1-\eta_t)$$
$$+\left[\frac{2\delta\mu(\mu-1)}{K}\right](1-\eta_p) - \left[\frac{2\delta\mu(\mu-1)}{K}\right](1-C_v) + 0(1-\eta_c) + \frac{\delta\mu(\mu-1)}{K}\left[\frac{\delta^2\mu^2(K-\delta\mu(\mu-1))}{[K+\delta\mu(\mu-1)]^2} - 1\right]\eta_x \tag{76}$$

$$C_p = \left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right] - \left[\frac{K^2}{K+\delta\mu(\mu-1)} - \frac{K}{\delta\mu}\right](1-\eta_t) - \left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right](1-\eta_p)$$
$$-\left[2(\mu-1)\right](1-C_v) - \left[\mu(\delta-1)\right](1-\eta_c) - \left[(\mu-1)\left[1-\frac{\delta^2\mu^2}{K+\delta\mu(\mu-1)}\right]\right]\eta_x$$

(77)

The coefficients of these expansions are plotted in Figures 3 - 9 for various values of μ, δ, and K, and show the relative effects on optimum jet pressure ratio and total work output of small changes in the component efficiencies from the ideal value.

An expansion for specific fuel consumption S of the regenerative cycle was obtained by a method similar to that described previously. By dropping terms such as $\eta_x(1-\eta_t)$ and $\eta_x(1-\eta_c)$ as being of higher order, the following result was obtained:

$$S = \frac{2545}{H}\left(\frac{\delta\mu}{\delta\mu-1}\right) + \frac{2545}{H}\left\{\left(\frac{\delta\mu}{\delta\mu-1}\right)(1-\eta_p) + \frac{K\delta\mu}{(\delta\mu-1)^2(K-\delta\mu)}\left(\frac{K\delta\mu}{K+\delta\mu(\mu-1)}-1\right)(1-\eta_t)\right.$$

$$+\left[\frac{\delta\mu^2(\delta-1)}{(\delta\mu-1)^2(K-\delta\mu)}\right](1-\eta_c) + \left[\frac{2(\mu-1)\delta^2\mu^2}{(\delta\mu-1)^2(K-\delta\mu)}\right](1-C_v)$$

$$\left.-\left[\frac{K+\delta\mu(\mu-1)-\delta^2\mu^2}{(\delta\mu-1)(K-\delta\mu)} + \frac{\delta^2\mu^2(\mu-1)}{(\delta\mu-1)^2(K-\delta\mu)}\left(\frac{\delta^2\mu^2}{K+\delta\mu(\mu-1)}-1\right)\right]\eta_x\right\} \quad (78)$$

Relative values of the coefficients of this expansion are plotted on Figure 10 for $M_0 = .50$, $\frac{p_2}{p_1} = 3$ and various values of K to show the effects of component efficiencies on the specific fuel consumption of the turboprop cycle with regeneration.

III. DISCUSSION OF RESULTS

By the technique described in the foregoing sections, it is possible to determine the relative effects on performance criteria of a small change in any of the component efficiencies of the cycle from their ideal values. The expansions submitted as Equations (76), (77), and (78) give these variations as functions of only three of the cycle parameters δ, μ, and K. This simplification was made possible by the introduction of the optimum jet pressure ratio X*. The value of X* for the ideal cycle (all component efficiencies 100%) was found to be an explicit function of δ, μ, and K, and had a value such that all of the work output of the cycle went to drive the propeller, none going to jet thrust.

The expansion (Equation 76) for the optimum jet pressure ratio X* as a function of the cycle parameters and efficiencies was found to be:

$$X^* = \left[\frac{\delta\mu(\mu-1)}{K} + 1\right] + \delta\mu(\mu-1)\left[\frac{1}{K} + \frac{\delta\mu(K-\delta\mu(\mu-1))}{[K+\delta\mu(\mu-1)]^2}\right](1-\eta_t)$$

$$+\left[\frac{2\delta\mu(\mu-1)}{K}\right](1-\eta_p) - \left[\frac{2\delta\mu(\mu-1)}{K}\right](1-C_v) + 0(1-\eta_c)$$

$$+ \frac{\delta\mu(\mu-1)}{K}\left[\frac{\delta^2\mu^2(K-\delta\mu(\mu-1))}{[K+\delta\mu(\mu-1)]^2} - 1\right]\eta_x \qquad (76)$$

As the efficiencies of the turbine and propeller decrease from the ideal value, the value of the optimum jet pressure ratio is found to increase, showing an increase in power to jet thrust. The effect of a change in η_t or η_p on X* is of the same order, the turbine efficiency having the greater value. The effect of a decrease in the jet exhaust nozzle coefficient C_v or an increase in regenerator effectiveness η_x is shown to lower the value of X*. Compressor efficiency η_c is found to have no effect. The relative importance of changes in component efficiencies on the value of the optimum jet pressure ratio X* is shown in Figures 3-6, Part VII, for various values of δ, μ, and K.

A simple expression can be derived from Equations (76) and (77) to show how the ratio of jet work to propeller work varies as the value of jet pressure ratio varies from the optimum. Assuming, as a first approximation, that $(X - X^*) \ll 1$ the following ratio is obtained:

$$\frac{\text{Jet Work}}{\text{Propeller Work}} = \left[\frac{\frac{K}{\delta\mu}}{(\delta\mu - 1)\left(\frac{K}{\delta\mu} - 1\right)}\right](X - X^*)$$

The coefficient of $(X - X^*)$ gives the slope of the straight line showing the above ratio. For $M_o = .50$, $\frac{p_2}{p_1} = 3$, and K = 4, this relation is plotted as Figure 10.

The power coefficient expansion C_P (Equation (77)) as a function of the cycle parameters and efficiencies was found to be:

$$C_P = \left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right] - \left[\frac{K^2}{K+\delta\mu(\mu-1)} - \frac{K}{\delta\mu}\right](1-\eta_t)$$

$$-\left[(\delta\mu-1)\left(\frac{K}{\delta\mu}-1\right)\right](1-\eta_p) - \left[2(\mu-1)\right](1-C_v) - \left[\mu(\delta-1)\right](1-\eta_c)$$

$$-(\mu-1)\left[1-\frac{\delta^2\mu^2}{K+\delta\mu(\mu-1)}\right]\eta_x \qquad (77)$$

A decrease in the component efficiencies η_t, η_p, C_v, and η_c from their ideal values is shown to result in a corresponding decrease in C_P and hence in the power output of the cycle. The effect of the turbine efficiency is predominant in decreasing the power output, followed by propeller efficiency, compressor efficiency, and jet exhaust nozzle coefficient in relative importance. Regeneration was found to decrease the power output of the ideal turboprop cycle. Figures 7-9, Part VII, show the relative importance of these coefficients for various values of the cycle parameters. For typical cycle parameter values of $M_o = .50$, $\frac{p_2}{p_1} = 3$ and K = 4, a 1% change in η_t results in a change in C_P of -.01146, while a 1% change in η_c results in a change of -.00510 in the value of C_P. The relative effects are apparent in this example. It is interesting to note that the change in power coefficient due to a change in compressor efficiency is independent of the amount of heat added in the combustion chamber and depends on flight speed and compressor ratio only.

To show the variation of C_{ℓ} with change in turbine and compressor efficiency, values of power coefficient are plotted against compressor pressure ratio on Figure 11 for $\eta_x = 0$, $\eta_p = 1$, $C_v = 1$, $M_o = .50$, and K = 4. The expansion given in Equation (77) was used for the plot which shows how the compressor pressure ratio for maximum power coefficient changes with the efficiencies of compressor and turbine.

Specific fuel consumption S as a function of the cycle parameters and component efficiencies is given as Equation (78). The coefficients of this expansion are plotted on Figure 12 for a typical set of values of δ, μ, and K. The turbine and propeller efficiencies are seen to be predominant in increasing the fuel consumption. Lesser effects are shown for compressor efficiency and exhaust nozzle coefficient. For cycle parameter values of $M_o = .50$, $\frac{p_2}{p_1} = 3$, and K = 4, a 1% change in turbine efficiency η_t results in an increase of the specific fuel consumption of .0061 (lb fuel/hp.hr.) while a 1% change in compressor efficiency η_c results in an increase of .0015 (lb.fuel/hp.hr.). These typical values show the relative effect of turbine efficiency as compared with compressor efficiency on the specific fuel consumption of the ideal turboprop cycle.

Regeneration, as considered in this analysis, decreases the power coefficient and decreases the specific fuel consumption. For $\eta_x = 10\%$, and for the parameter values given above, it is found that C_P is decreased by -.00246 while S is decreased by .178 (lb fuel/hp.hr.).

The results of these calculations can be used analytically to immediately show the effect and importance of the efficiencies in a turbine-propeller engine. The relative importance of the effects of these efficiencies on performance criteria can be easily deduced from simple mathematical expressions without the use of complex charts and graphs. As an example, by the use of Equation (77) it is possible to show how the percentage work done by the propeller changes with variations in component efficiencies. For the ideal cycle with optimum jet pressure ratio X*, it was found that the total power output of the cycle went to propeller work. As the component efficiencies change, the power division between jet and propeller likewise changes.

This type of cycle analysis can be used to solve various types of problems. As an example, Equation (29) can be used to determine the value of compressor pressure ratio that will give the maximum power output of the cycle for any given values of cycle parameters and efficiencies. Assuming $\eta_p = \eta_t = C_V = 1$

in Equation (28) and introducing the value of the optimum jet pressure ratio, the following expression is obtained:

$$\frac{\Delta C_{\ell}}{\epsilon} = \frac{\partial (C_{\ell})}{\partial (\delta)} = \left[\frac{K}{\delta^2 \mu} - \frac{\mu}{\eta_c}\right]$$

This derivative is set equal to zero to find the value of δ which gives maximum C_{ℓ} for the specified parameter values:

$$\frac{K}{\delta^2 \mu} - \frac{\mu}{\eta_c} = 0$$

$$\delta = \frac{\sqrt{\eta_c K}}{\mu} \qquad \text{(for max. } C_{\ell}\text{)}$$

This simple equation gives the value of compressor pressure ratio for maximum power output and can be plotted for various values of compressor efficiency to show the trend with change in efficiency.

Similar problems can be likewise easily solved using Equations (30) and (31) to give the values of μ and K for optimum power output for a given set of cycle parameters.

IV. CONCLUSIONS

From the results of this cycle analysis of the gas turbine propeller-driving engine, it is concluded that:

1. A complete analysis of the performance of the ideal turboprop cycle can be made in terms of three basic parameters δ, μ, and K by introducing the optimum jet pressure ratio X* as a means of dividing the power output between propeller and jet thrust.

2. An analysis of the non ideal turboprop cycle, with or without regeneration, can be made in terms of the four parameters, δ, μ, K and X and the component efficiencies of the system. The relative effect of these efficiencies on performance criteria can be shown by a Taylor expansion about the point of ideal efficiencies. In this method, the optimum jet pressure ratio X* can again be introduced, and the resulting performance expressions can be given in terms of three basic parameters δ, μ, and K.

3. Of the component efficiencies, the turbine efficiency η_t was found to be predominant in affecting the performance criteria of the ideal turboprop cycle.

4. Propeller efficiency η_p was also found to have an important effect on the performance of the ideal cycle since, for ideal conditions, all work is done by the propeller.

5. Regeneration, as considered in the analysis, has the effect of decreasing the total power output and decreasing the specific fuel consumption of the ideal turboprop cycle.

V SCHEMATIC DIAGRAMS OF TURBOPROP CYCLES

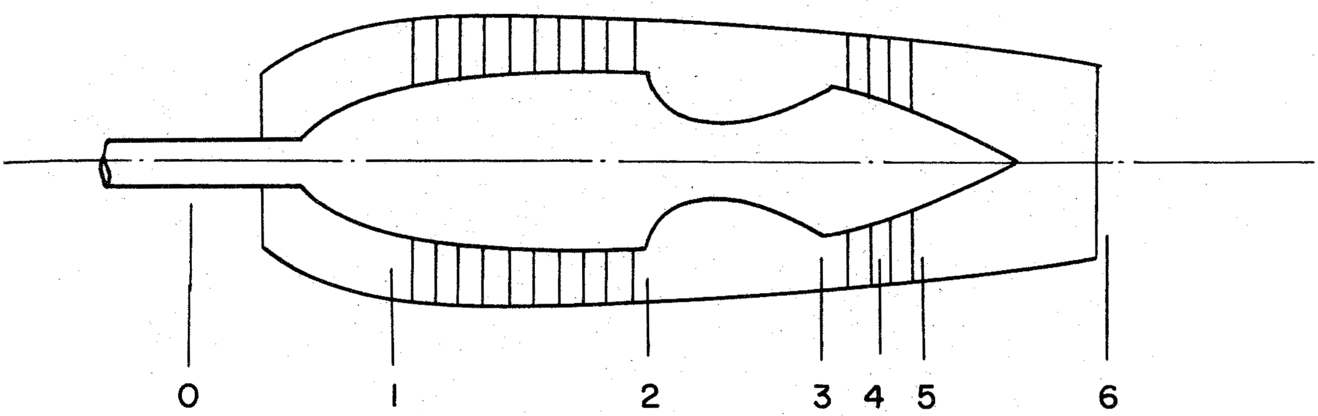


FIG. I

BASIC TURBOPROP CYCLE

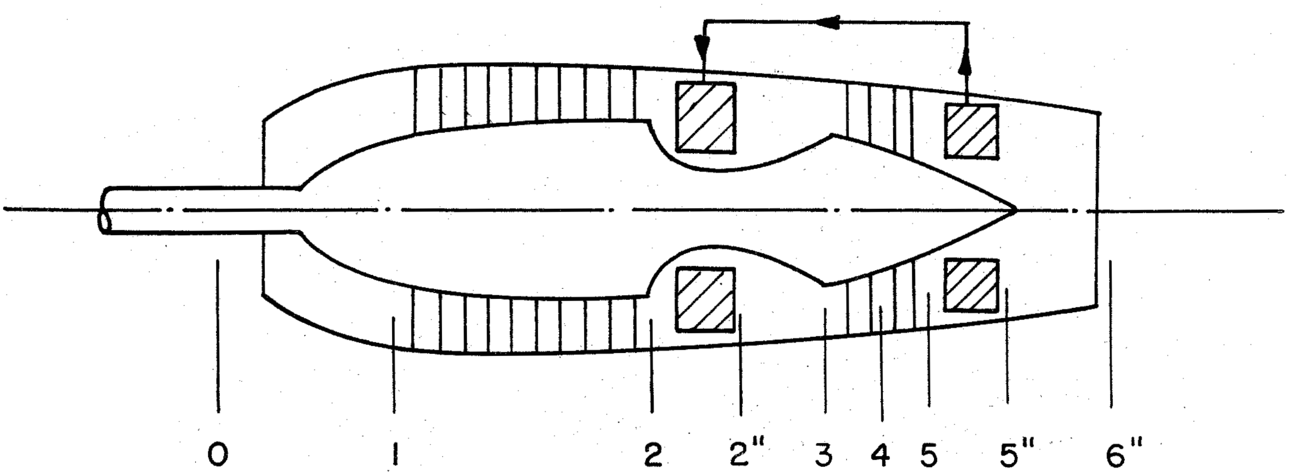


FIG. 2

TURBOPROP CYCLE WITH REGENERATION

VI. APPENDIX A

SYMBOLS AND DEFINITIONS

C_p - Specific heat at constant pressure $\left(\frac{BTU}{lb\ ^\circ F}\right)$.

C_P - Power coefficient, defined as $\frac{W_{tot}}{C_p T_o}$.

C_{P_o} - Power coefficient with all efficiencies 100% and optimum jet pressure ratio.

C_v - exhaust nozzle velocity coefficient.

$\delta = \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ (related to pressure rise across the compressor).

ϵ - A small change in a given parameter.

η_c - Efficiency of compressor.

η_p - Propeller efficiency, including gear box losses and other power transmission losses.

η_t - Turbine efficiency.

η_x - Regenerator heating effectiveness.

f - Actual fuel consumption $\left(\frac{\text{lb fuel}}{\text{sec}}\right)$

g - Acceleration due to gravity (32.17 ft/sec^2)

γ - Ratio of specific heats

H - Heating value of the fuel $\left(\frac{\text{BTU}}{\text{lb fuel}}\right)$

J - Mechanical equivalent of heat $\left(778.3\frac{\text{ft lb}}{\text{BTU}}\right)$

K $= \frac{T_3}{T_o}$ (related to the temperature rise in the combustion chamber

M - Local Mach number

m - Mass air flow $\left(\frac{\text{lb air}}{\text{sec}}\right)$

μ $= \frac{T_1}{T_0}$ (related to forward flight speed)

p - Static pressure $(\frac{lb}{in^2})$

Q - Heat added per pound of air $(\frac{BTU}{lb})$

S - Specific fuel consumption $(\frac{lb\ fuel}{Hp\text{-}hour})$

S_o - Specific fuel consumption with all efficiencies equal to 100% and with optimum jet pressure ratio

T_o - Free stream static temperature (^oR)

T - Total temperature (^oR)

T' - Total temperature for an ideal isentropic process

T'' - Total temperature at certain points in the regenerative cycle

V - Velocity (ft/sec)

W - Work per pound of air $(\frac{BTU}{lb})$

X $= \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_5}{T_6}$ (related to the pressure ratio across the exhaust nozzle)

X* - Optimum jet pressure ratio for ideal division of power between propeller and jet

PART VII

On the following Figures 3-6 are plotted the coefficients of the expansion of optimum jet pressure ratio X* as a function of the cycle parameters δ, μ, and K, and the component efficiencies η_t, η_p, C_v, η_c and η_x.

Equation (76):

$$X^* = \left[\frac{\delta\mu(\mu-1)}{K}+1\right] + \delta\mu(\mu-1)\left[\frac{1}{K}+\frac{\delta\mu\left(K-\delta\mu(\mu-1)\right)}{[K+\delta\mu(\mu-1)]^2}\right](1-\eta_t)$$

$$+\left[\frac{2\delta\mu(\mu-1)}{K}\right](1-\eta_p) - \left[\frac{2\delta\mu(\mu-1)}{K}\right](1-C_v) + 0(1-\eta_c)$$

$$+ \frac{\delta\mu(\mu-1)}{K}\left[\frac{\delta^2\mu^2\left(K-\delta\mu(\mu-1)\right)}{[K+\delta\mu(\mu-1)]^2} - 1\right]\eta_x$$

The following notation has been used in plotting:

$Z_1 = \left[\frac{\delta\mu(\mu-1)}{K}+1\right]$ (ideal optimum jet pressure ratio)

$Z_2 = \delta\mu(\mu-1)\left[\frac{1}{K}+\frac{\delta\mu\left(K-\delta\mu(\mu-1)\right)}{[K+\delta\mu(\mu-1)]^2}\right]$ (coef. of $(1-\eta_t)$ term)

$Z_3 = \left[\frac{2\delta\mu(\mu-1)}{K}\right]$ (coef. of $(1-\eta_p)$ term)

$Z_4 = -Z_3$ (coef. of $(1-C_v)$ term)

$Z_5 = 0$ (coef. of $(1-\eta_c)$ term)

$Z_6 = \frac{\delta\mu(\mu-1)}{K}\left[\frac{\delta^2\mu^2[K-\delta\mu(\mu-1)]}{[K+\delta\mu(\mu-1)]^2} - 1\right]$ (coefficient of η_x term)

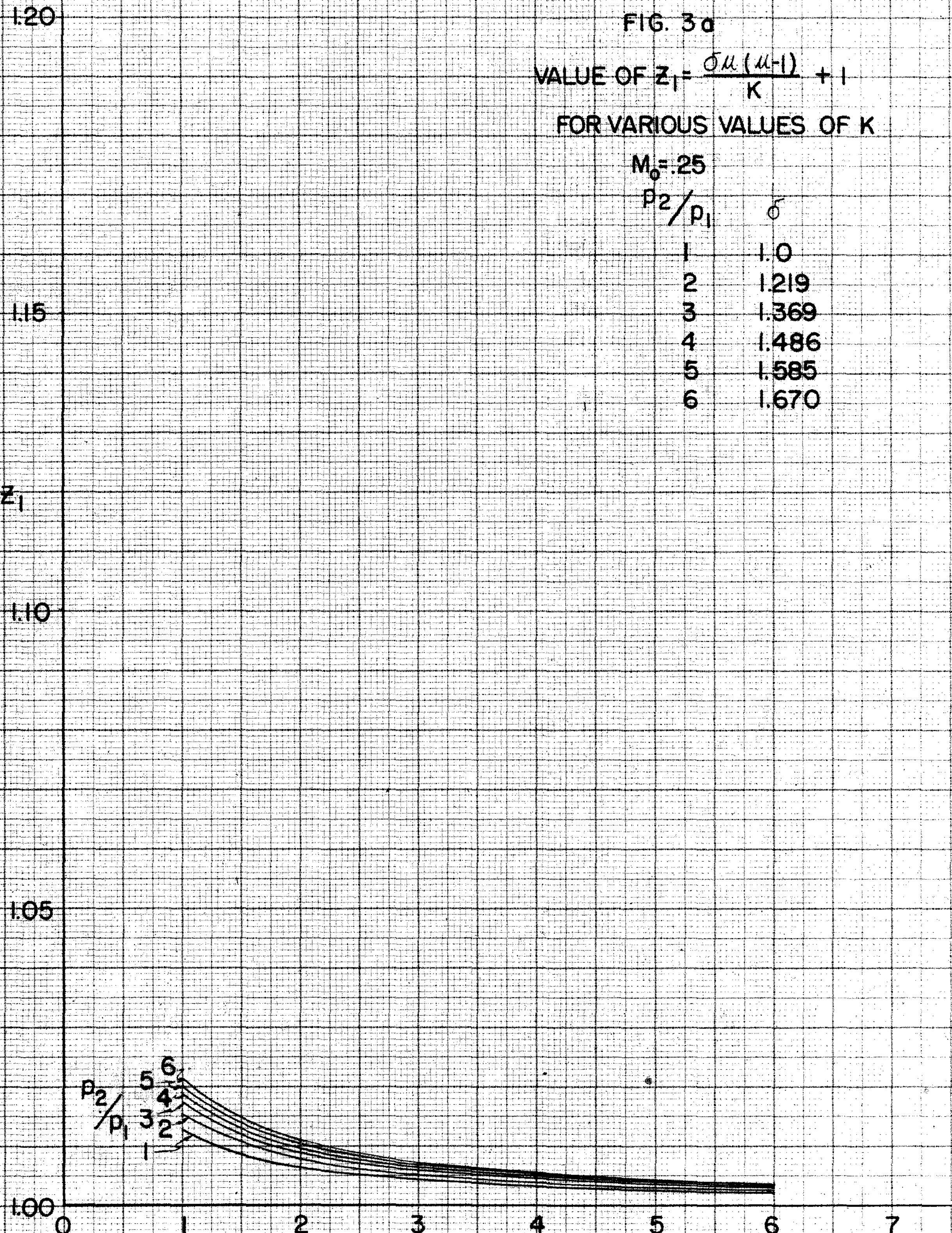


FIG. 3a

VALUE OF $Z_1 = \frac{\sigma \mu (\mu - 1)}{K} + 1$

FOR VARIOUS VALUES OF K

$M_0 = .25$

p_2/p_1	σ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

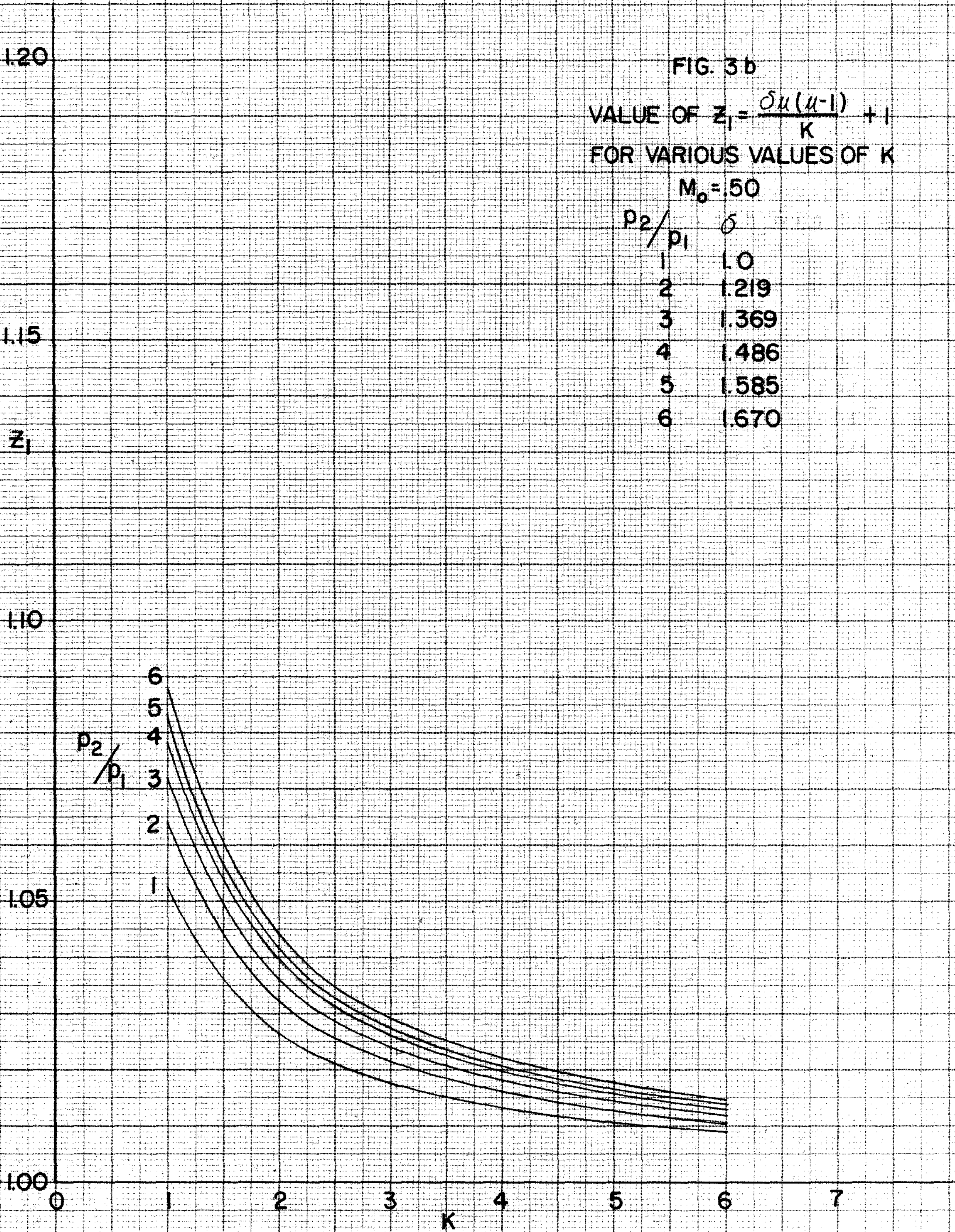
FIG. 3 b

VALUE OF $Z_1 = \frac{\delta u(u-1)}{K} + 1$

FOR VARIOUS VALUES OF K

$M_o = .50$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670



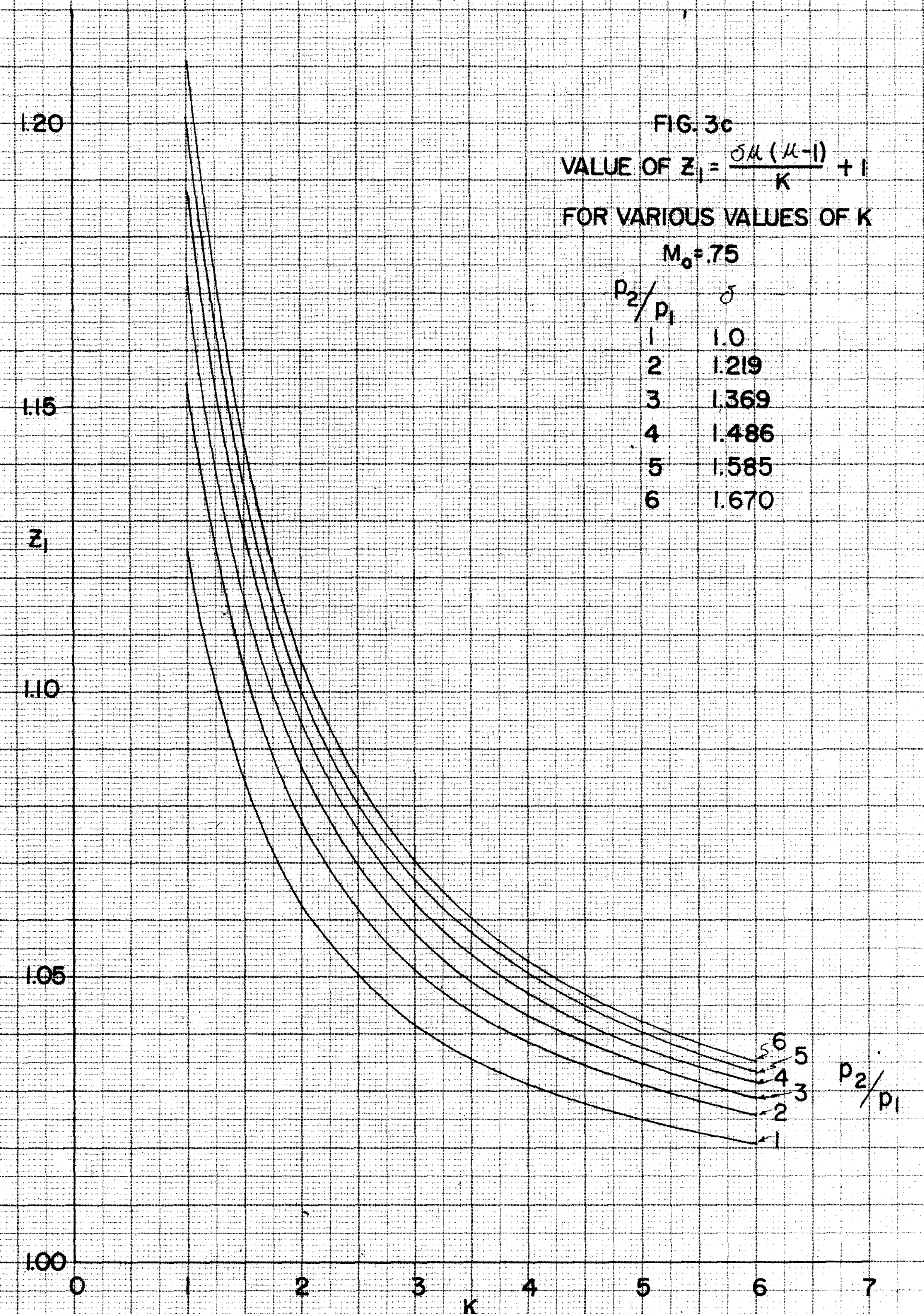


FIG. 3c
VALUE OF $Z_1 = \frac{\sigma \mu (\mu - 1)}{K} + 1$
FOR VARIOUS VALUES OF K
$M_o = .75$
p_2/p_1 σ
1 1.0
2 1.219
3 1.369
4 1.486
5 1.585
6 1.670
1.20
1.15
Z_1
1.10
1.05
1.00
0
1
2
3
4
5
6
7
K
6
5
4
3
2
1
p_2/p_1

FIG. 4a

VALUE OF $Z_2 = \delta\mu(\mu-1)\left[\frac{1}{K} + \frac{\delta\mu\left[K-\delta\mu(\mu-1)\right]}{\left[K+\delta\mu(\mu-1)\right]^2}\right]$

FOR VARIOUS VALUES OF K

$M_0 = 25$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

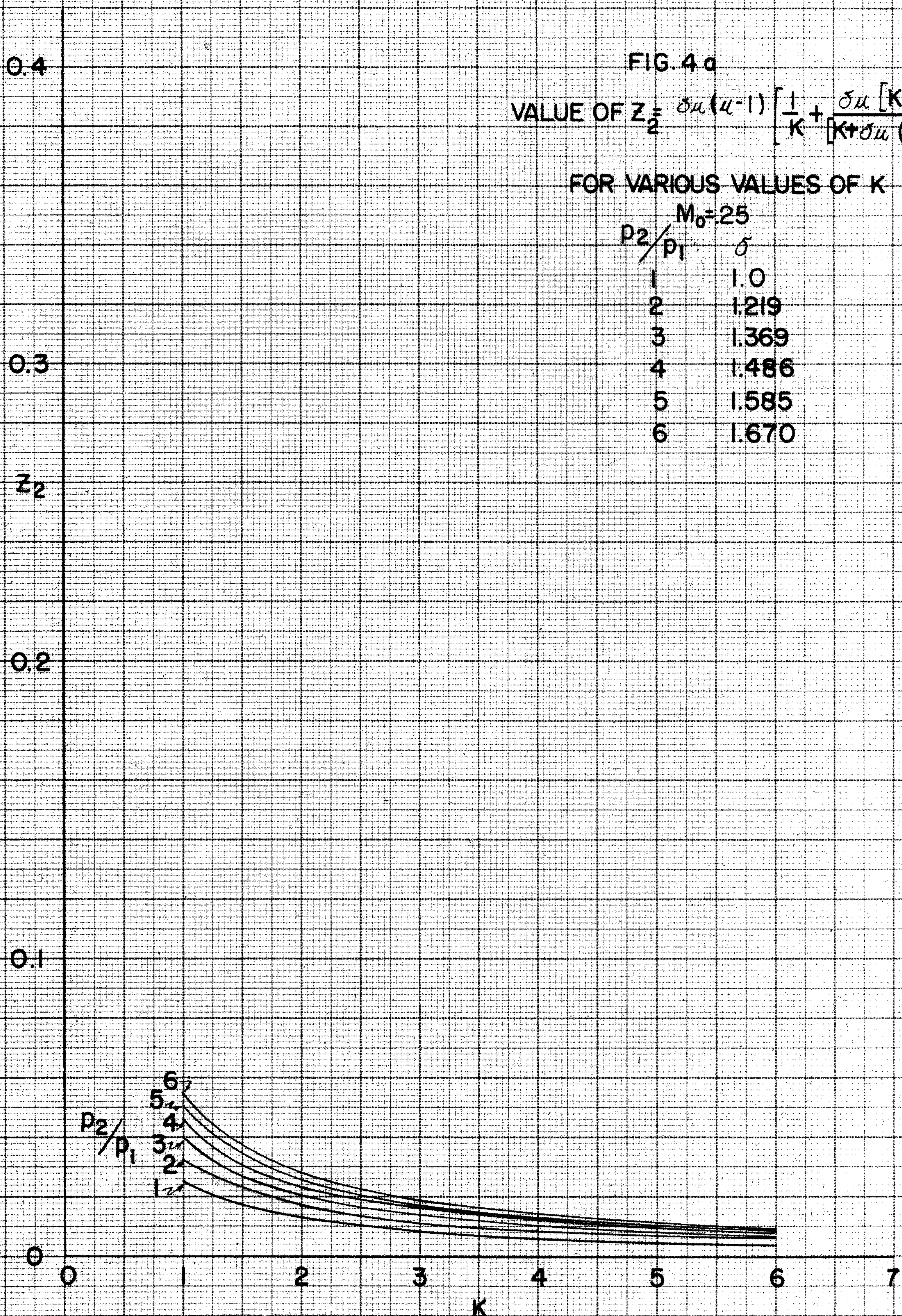
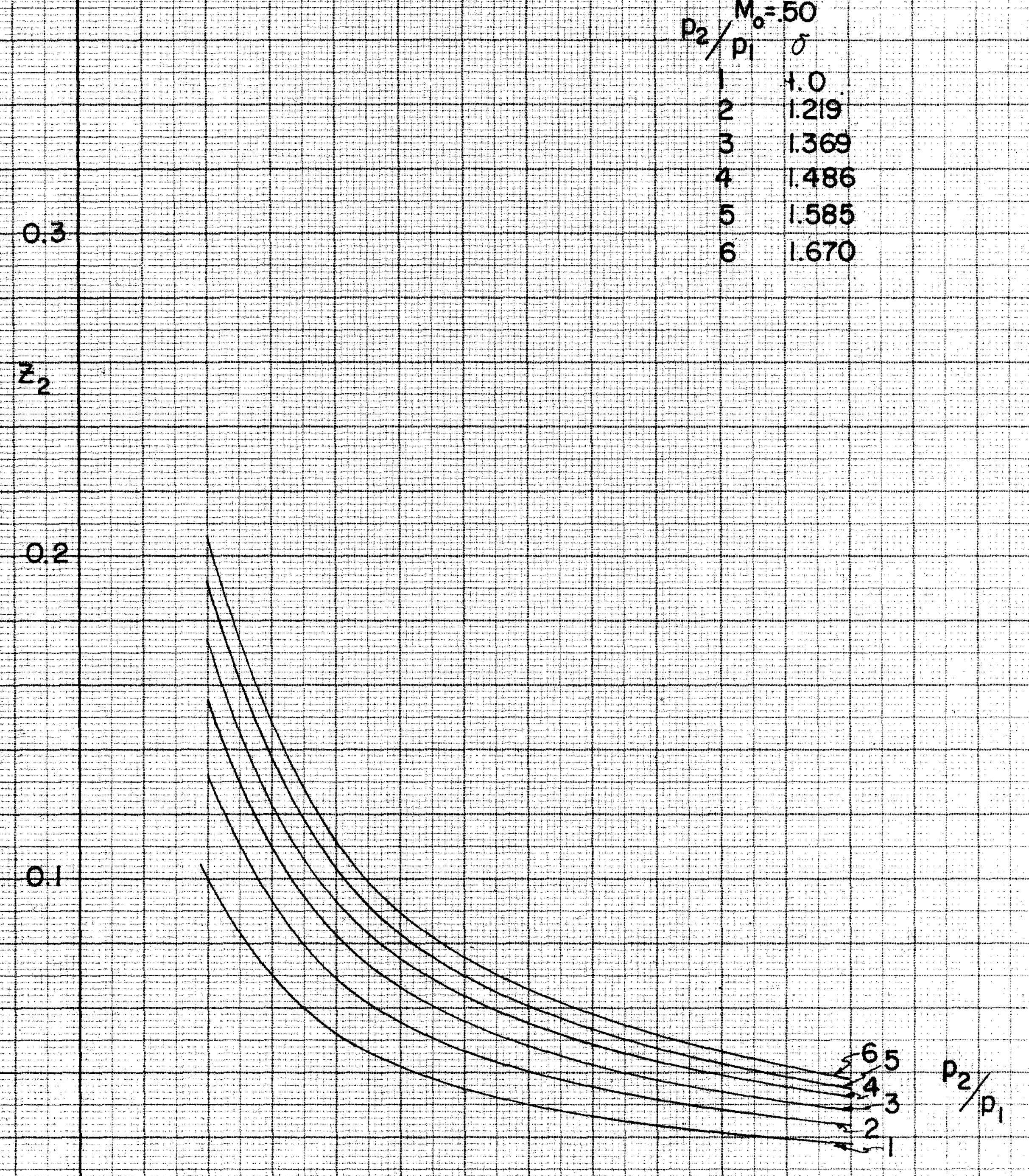


FIG. 4b

VALUE OF $Z_2 = \delta\mu\,(\mu-1)\left[\frac{1}{K} + \frac{\delta\mu\,[K-\delta\mu\,(\mu-1)]}{[K+\delta\mu(\mu-1)]^2}\right]$

FOR VARIOUS VALUES OF K



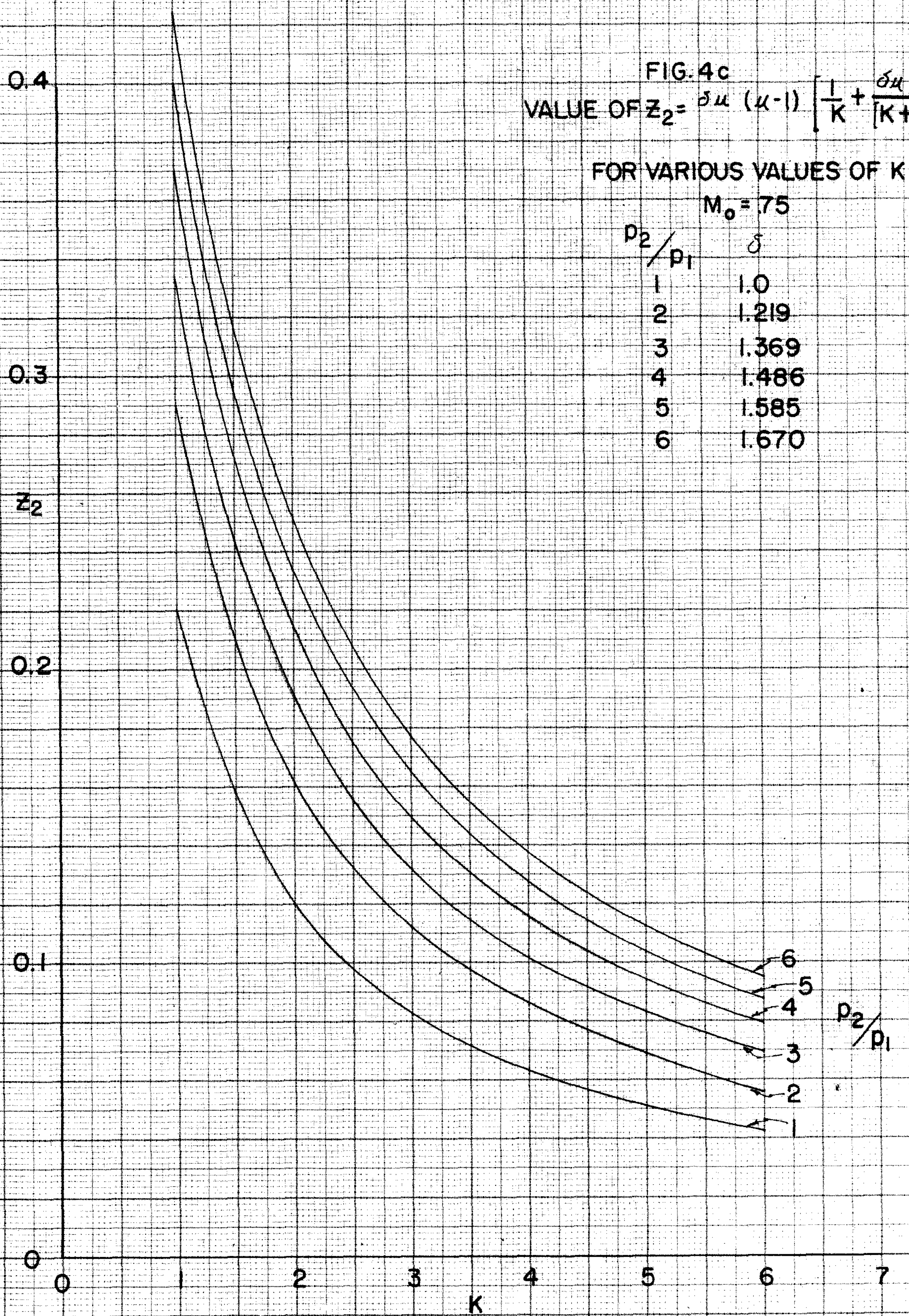


FIG. 4c

VALUE OF $Z_2 = \delta\mu\,(\mu-1)\left[\frac{1}{K} + \frac{\delta\mu\,[K-\delta\mu(\mu-1)]}{[K+\delta\mu\,(\mu-1)]^2}\right]$

FOR VARIOUS VALUES OF K

$M_o = .75$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

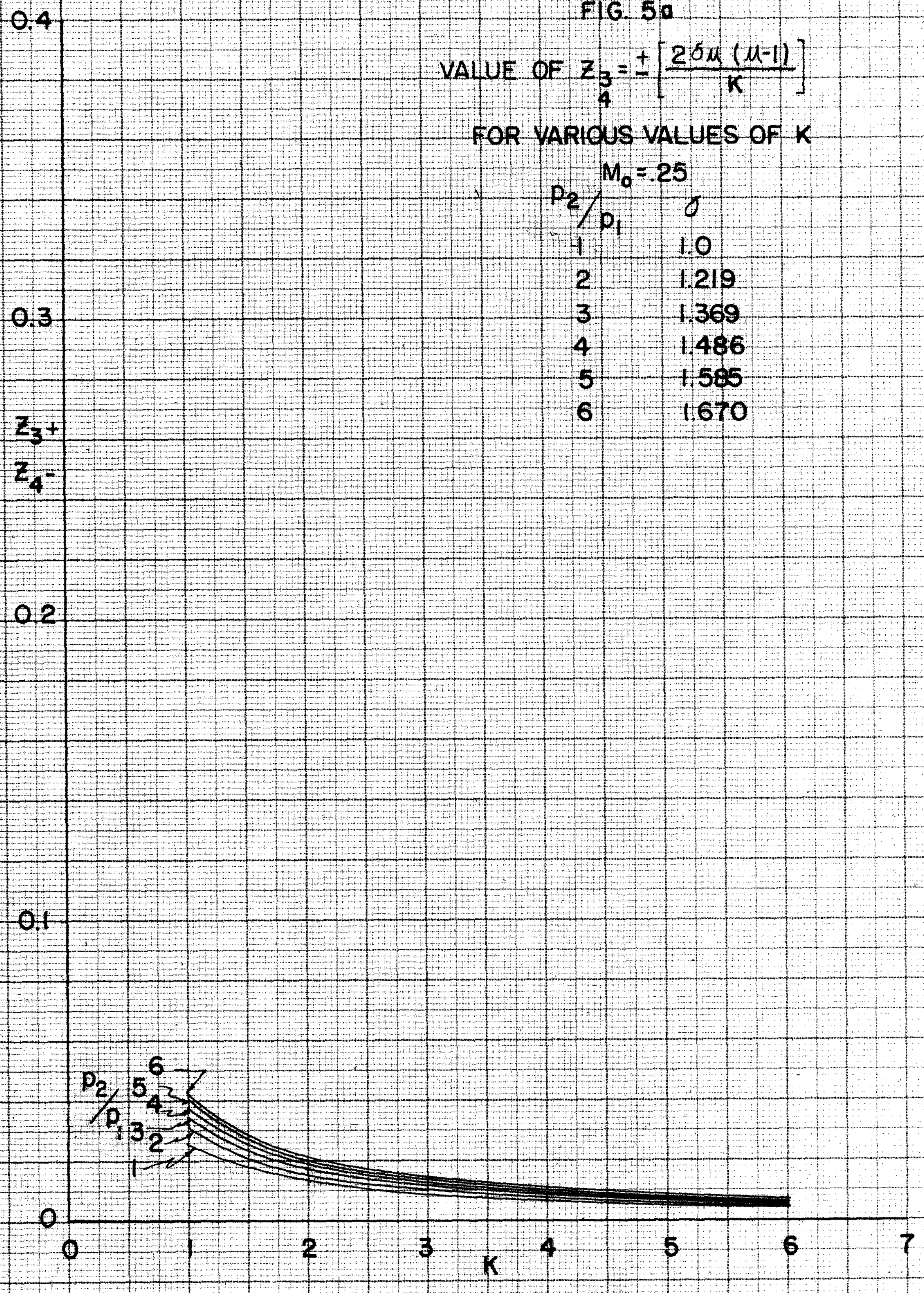


FIG. 5a

VALUE OF $Z_{\frac{3}{4}} = \pm \left[\frac{2\sigma\mu(\mu-1)}{K} \right]$

FOR VARIOUS VALUES OF K

$M_o = .25$

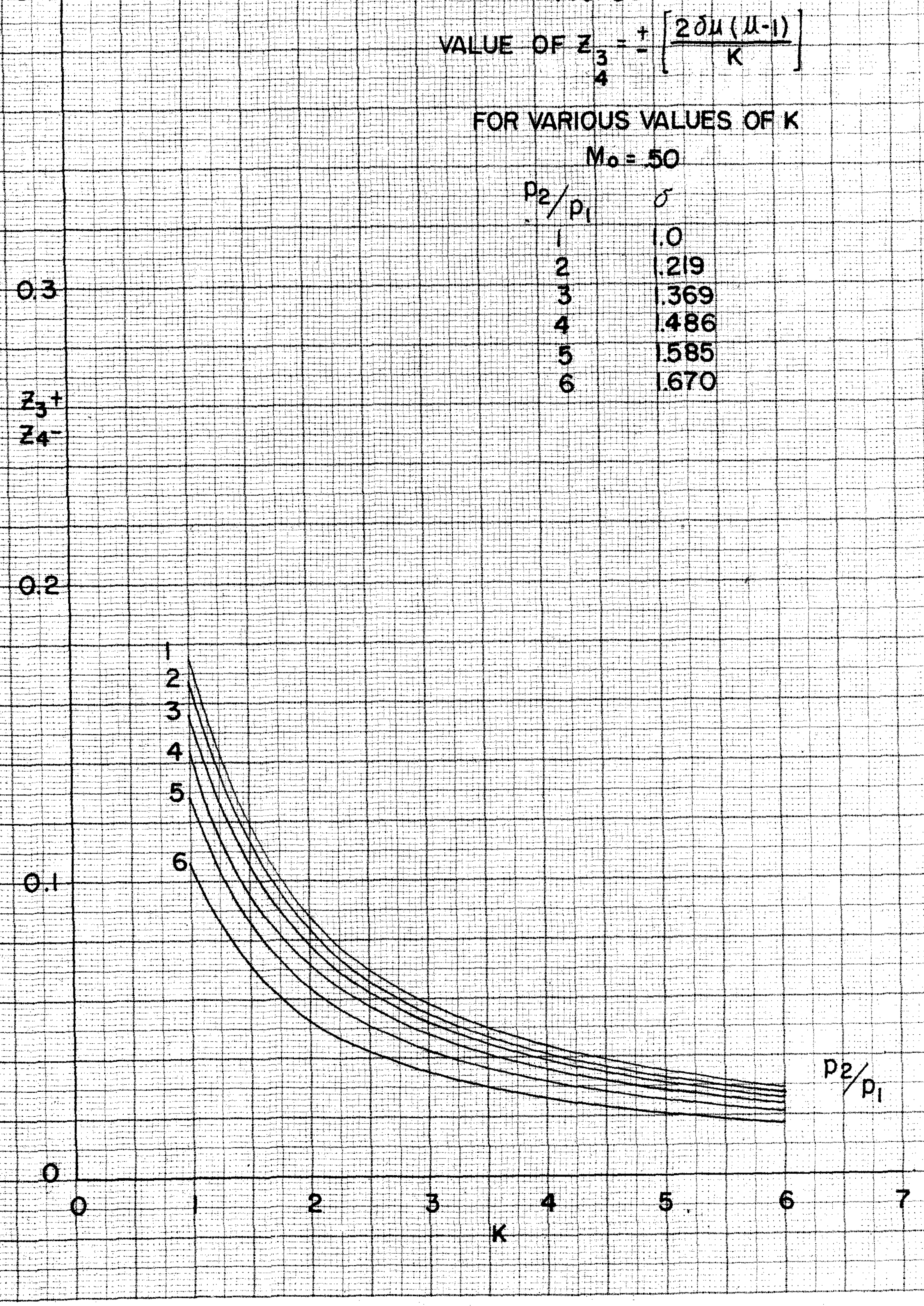
p_2/p_1	σ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

FIG. 5b

VALUE OF $Z_{\substack{3\\4}} = \pm\left[\dfrac{2\delta\mu(\mu-1)}{K}\right]$

FOR VARIOUS VALUES OF K

$M_o = .50$



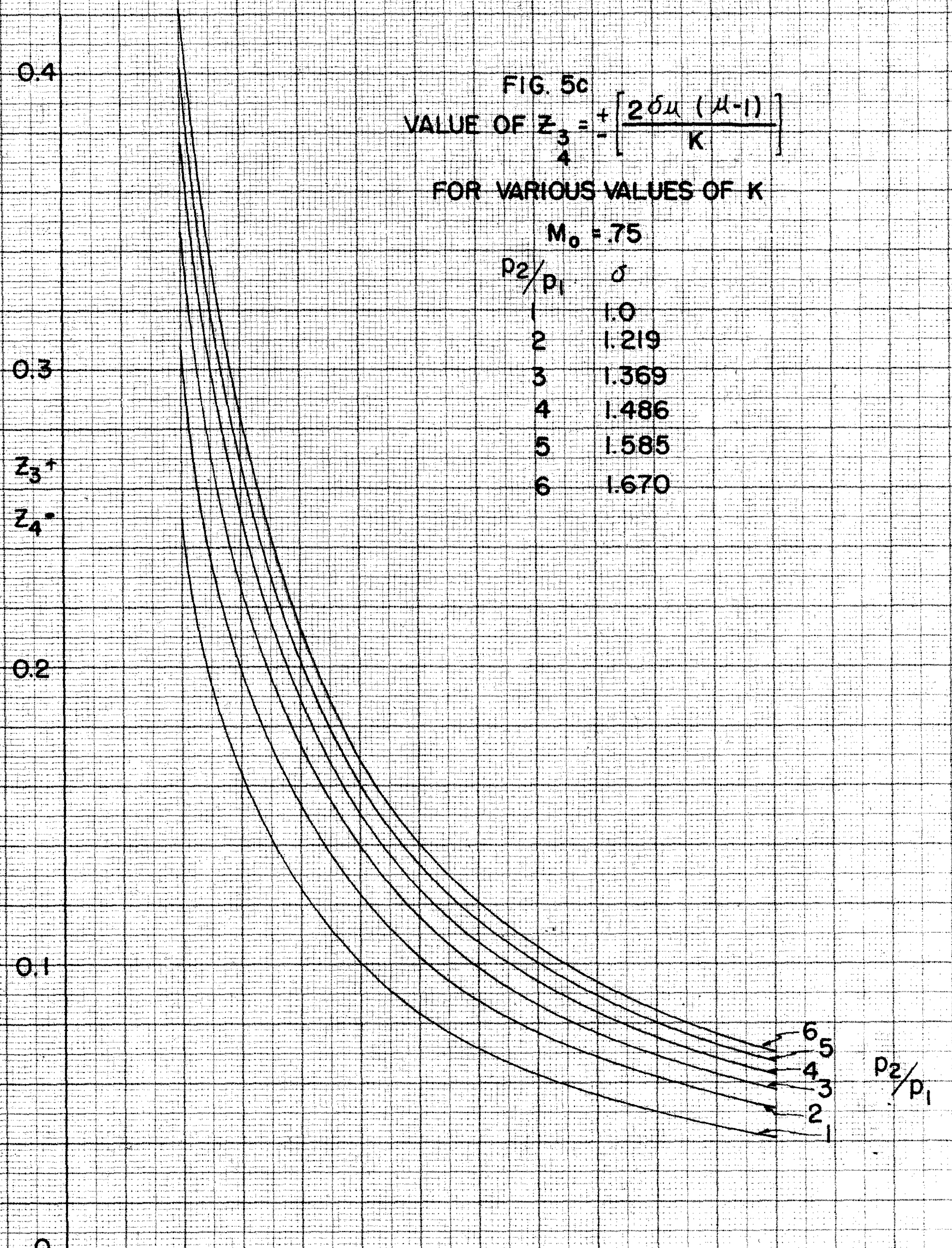


FIG. 5c
VALUE OF $Z_{3/4} = \pm\left[\frac{2\sigma\mu(\mu-1)}{K}\right]$
FOR VARIOUS VALUES OF K
$M_0 = .75$
p_2/p_1 σ
1 1.0
2 1.219
3 1.369
4 1.486
5 1.585
6 1.670
0.4
0.3
Z_3 +
Z_4 -
0.2
0.1
0
0
1
2
3
4
5
6
7
K
6
5
4
3
2
1
p_2/p_1

FIG. 6a

VALUE OF $Z_6 = \frac{\delta u\,(u-1)}{K}\left[\frac{\delta^2 u^2\left[K-\delta u\,(u-1)\right]}{\left[K+\delta u\,(u-1)\right]^2}-1\right]$

FOR VARIOUS VALUES OF K

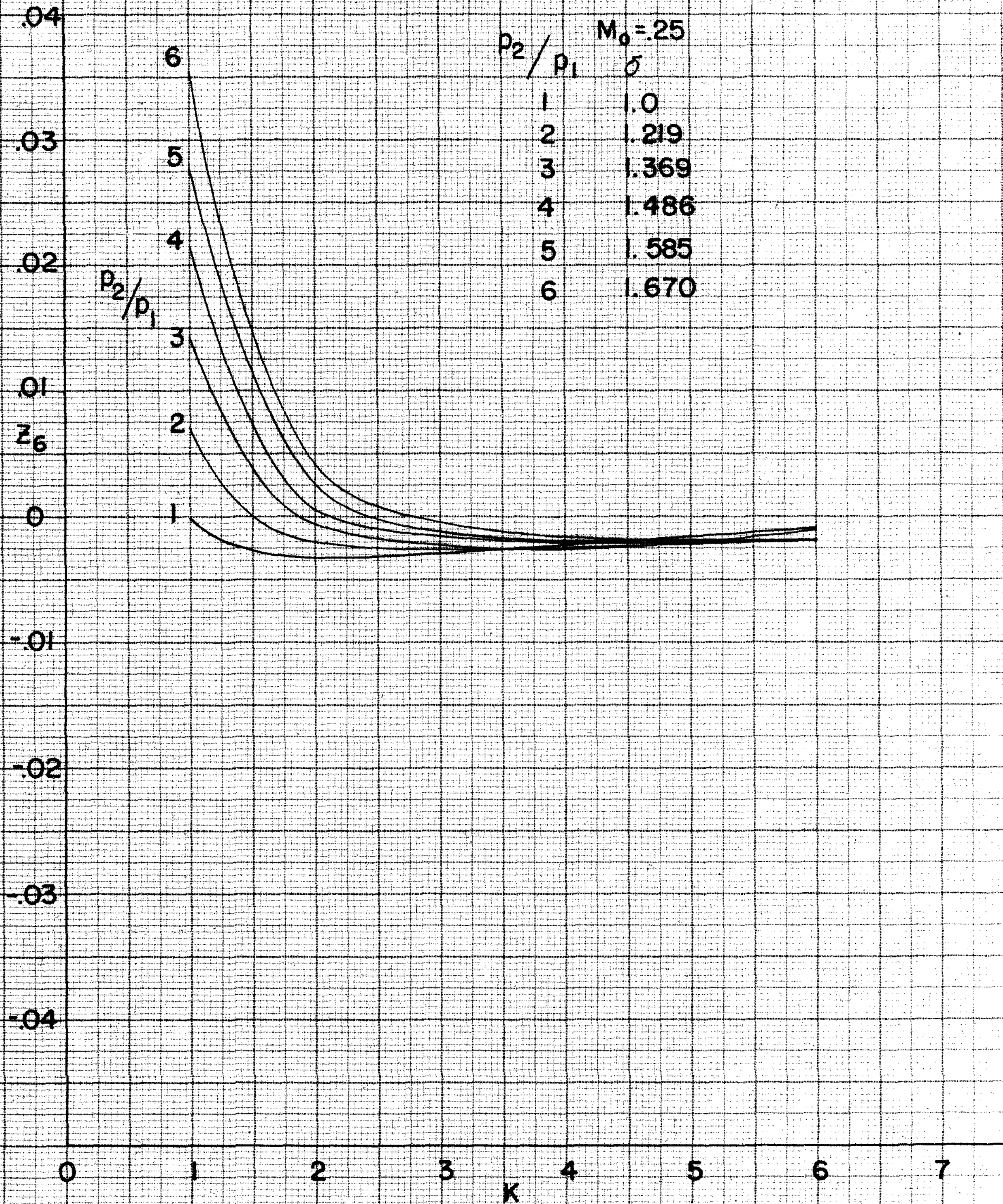


FIG. 6 b

VALUE OF $Z_6 = \frac{\delta\mu(\mu-1)}{K}\left[\frac{\delta^2\mu^2[K-\delta\mu(\mu-1)]}{[K+\delta\mu(\mu-1)]^2}-1\right]$

FOR VARIOUS VALUES OF K

$M_0 = .50$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

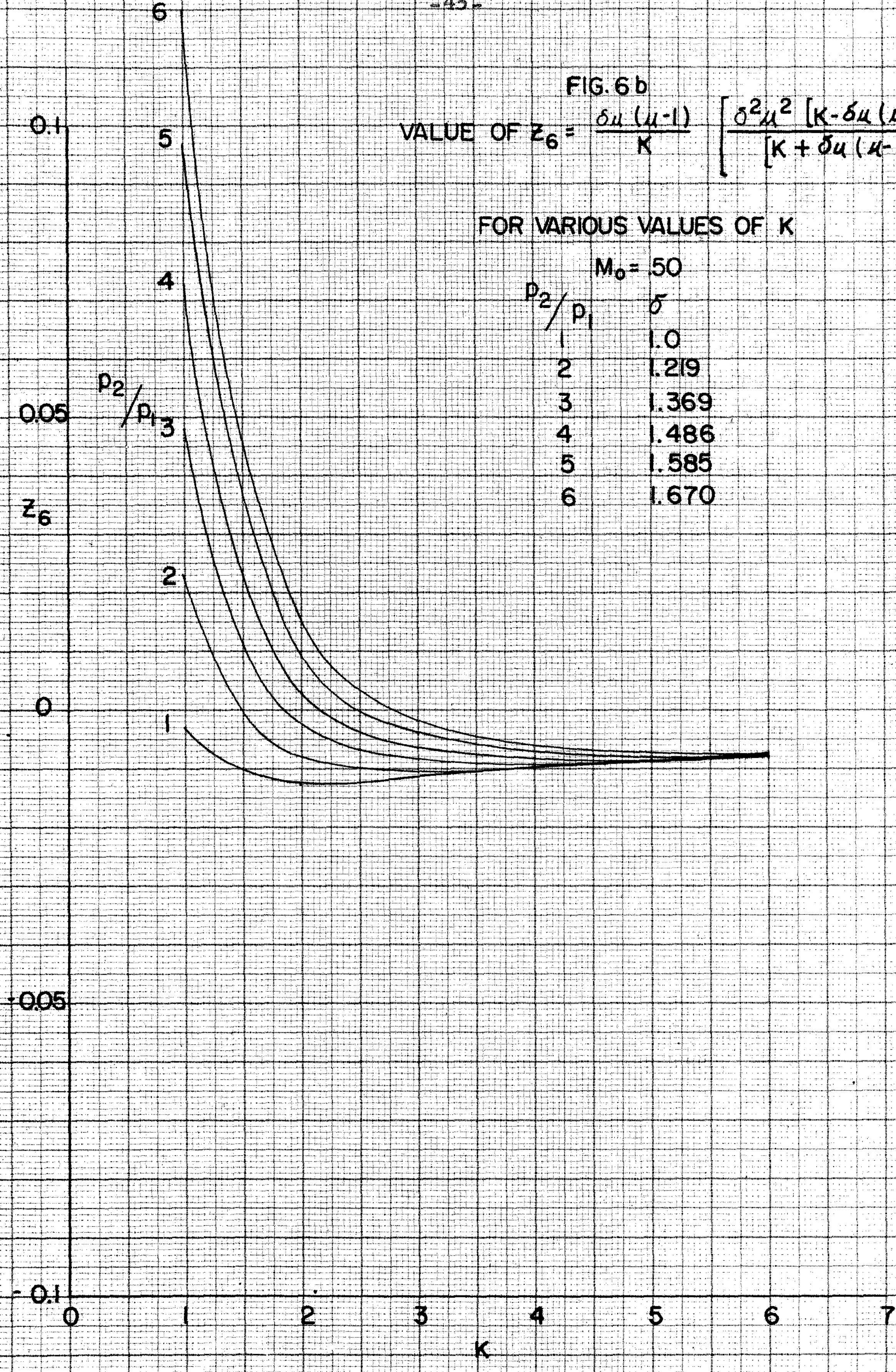


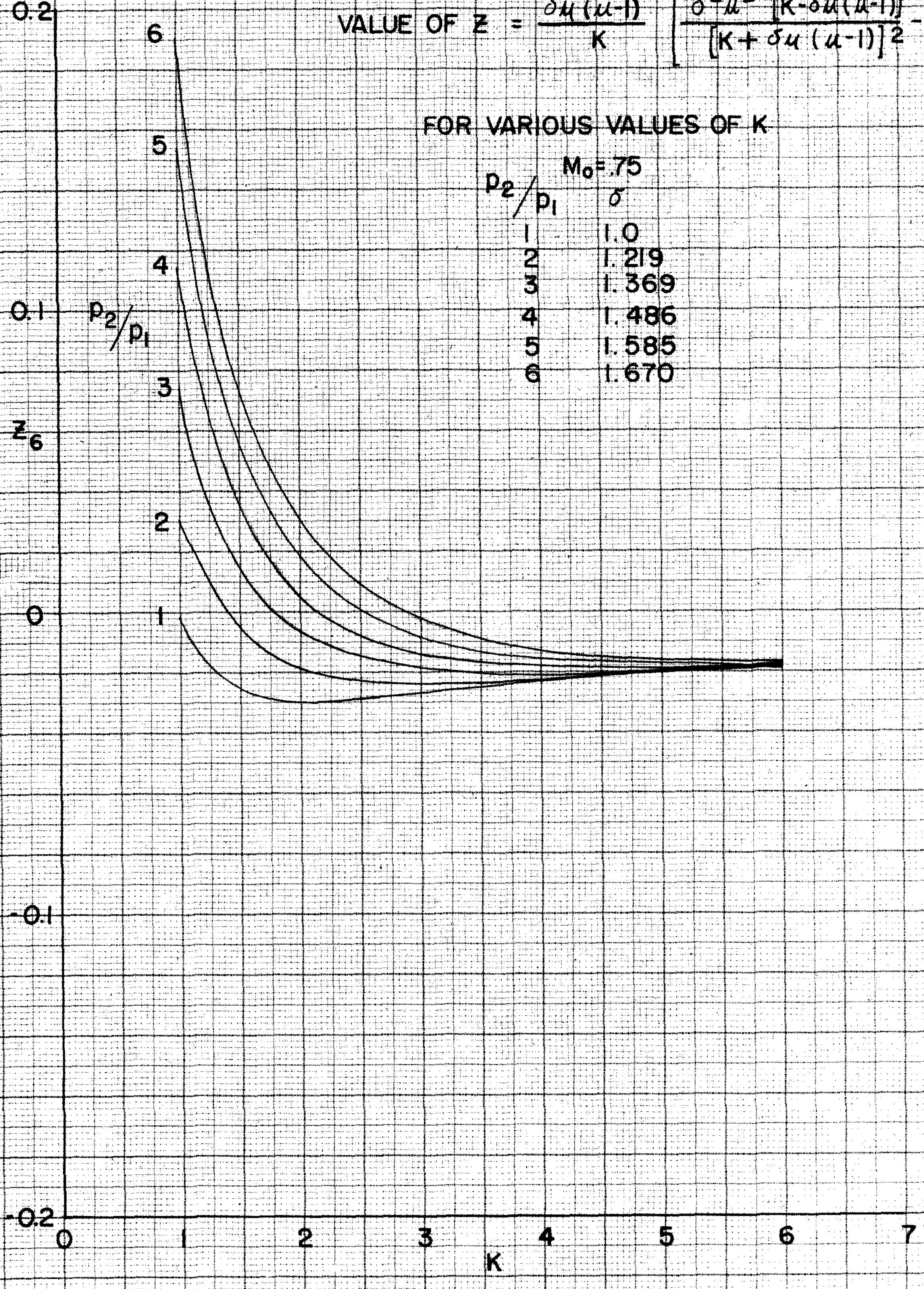
FIG. 6c

VALUE OF $Z = \frac{\delta\mu(\mu-1)}{K}\left[\frac{\delta^2\mu^2\left[K-\delta\mu(\mu-1)\right]}{\left[K+\delta\mu(\mu-1)\right]^2}-1\right]$

FOR VARIOUS VALUES OF K

$M_0 = .75$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670



On the following Figures 7-9 are plotted the coefficients of the expansion of the power coefficient C_p as a function of the cycle parameters δ, μ, and K and the component efficiencies η_t, η_p, C_v, η_c and η_x.

Equation (77)

$$C_p = (\delta\mu - 1)\left(\frac{K}{\delta\mu} - 1\right) - \left[\frac{K^2}{K + \delta\mu(\mu - 1)} - \frac{K}{\delta\mu}\right](1 - \eta_t) - \left[(\delta\mu - 1)\left(\frac{K}{\delta\mu} - 1\right)\right](1 - \eta_p)$$

$$- \left[2(\mu - 1)\right](1 - C_v) - \left[\mu(\delta - 1)\right](1 - \eta_c) - \left\{(\mu - 1)\left[1 - \frac{\delta^2\mu^2}{K + \delta\mu(\mu - 1)}\right]\right\}\eta_x$$

The following notation has been used in plotting:

$Z_7 = (\delta\mu - 1)\left(\frac{K}{\delta\mu} - 1\right)$ (ideal power coefficient)

$Z_8 = -\left[\frac{K^2}{K + \delta\mu(\mu - 1)} - \frac{K}{\delta\mu}\right]$ (coef. of $(1 - \eta_t)$ term)

$Z_9 = -\left[(\delta\mu - 1)\left(\frac{K}{\delta\mu} - 1\right)\right] = -Z_7$

$Z_{12} = -(\mu - 1)\left[1 - \frac{\delta^2\mu^2}{K + \delta\mu(\mu - 1)}\right]$ (coef. of η_x term)

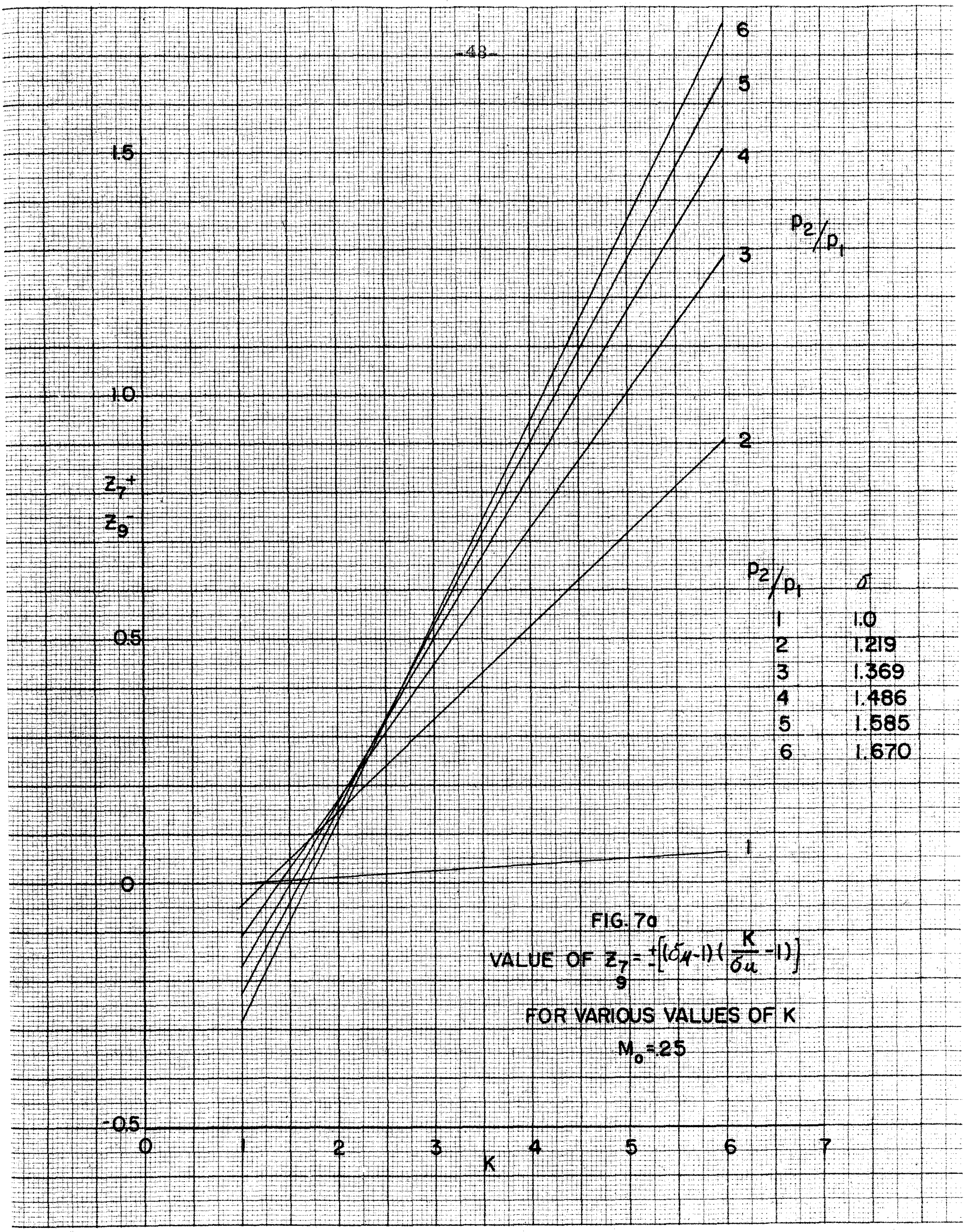


FIG. 7a

VALUE OF $Z_{\frac{7}{9}} = \pm\left[(\sigma_u - 1)\left(\frac{K}{\sigma_u} - 1\right)\right]$

FOR VARIOUS VALUES OF K

$M_o = 25$

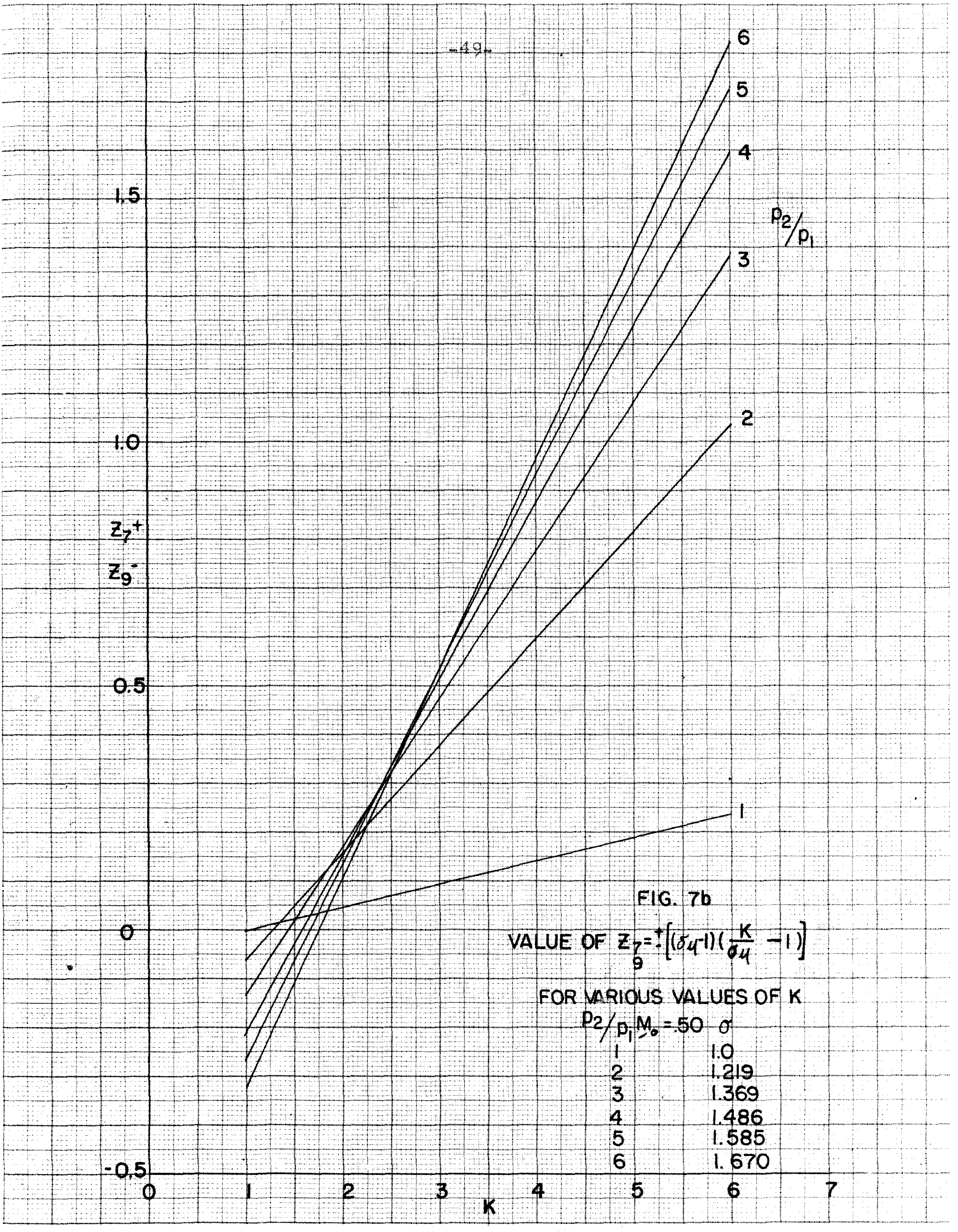


FIG. 7b

VALUE OF $Z_{\frac{7}{9}} = \pm\left[(\sigma_4 - 1)\left(\frac{K}{\sigma_4} - 1\right)\right]$

FOR VARIOUS VALUES OF K

p_2/p_1, $M_0 = .50$	σ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

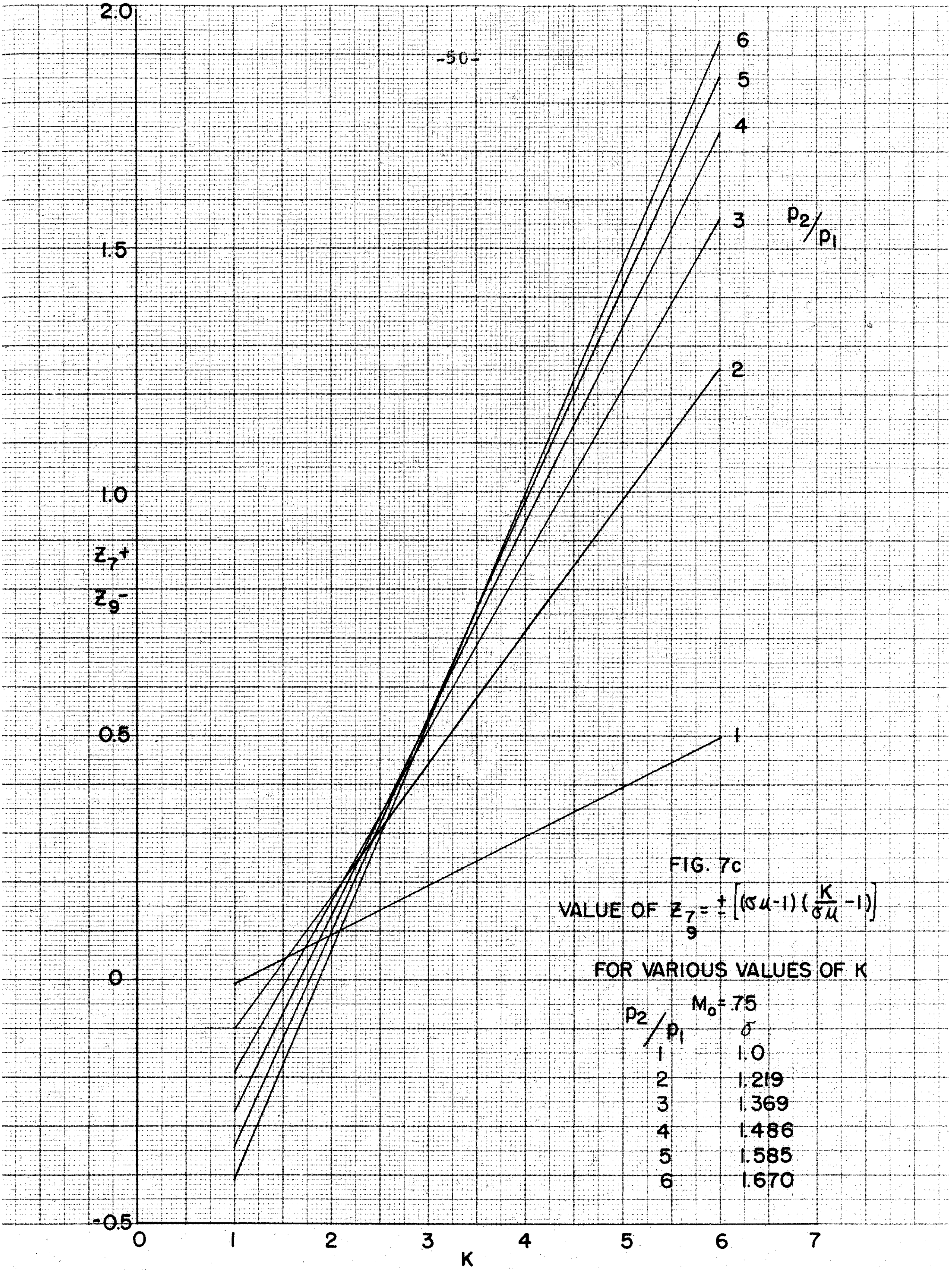


FIG. 7c

VALUE OF $Z_{7,9} = \pm\left[(\sigma\mu - 1)\left(\frac{K}{\sigma\mu} - 1\right)\right]$

FOR VARIOUS VALUES OF K

$M_o = .75$

p_2/p_1	σ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

FIG. 8a

VALUE OF $Z_8 = -\left[\frac{K^2}{K+\sigma\mu(\mu-1)} - \frac{K}{\sigma\mu}\right]$

FOR VARIOUS VALUES OF K

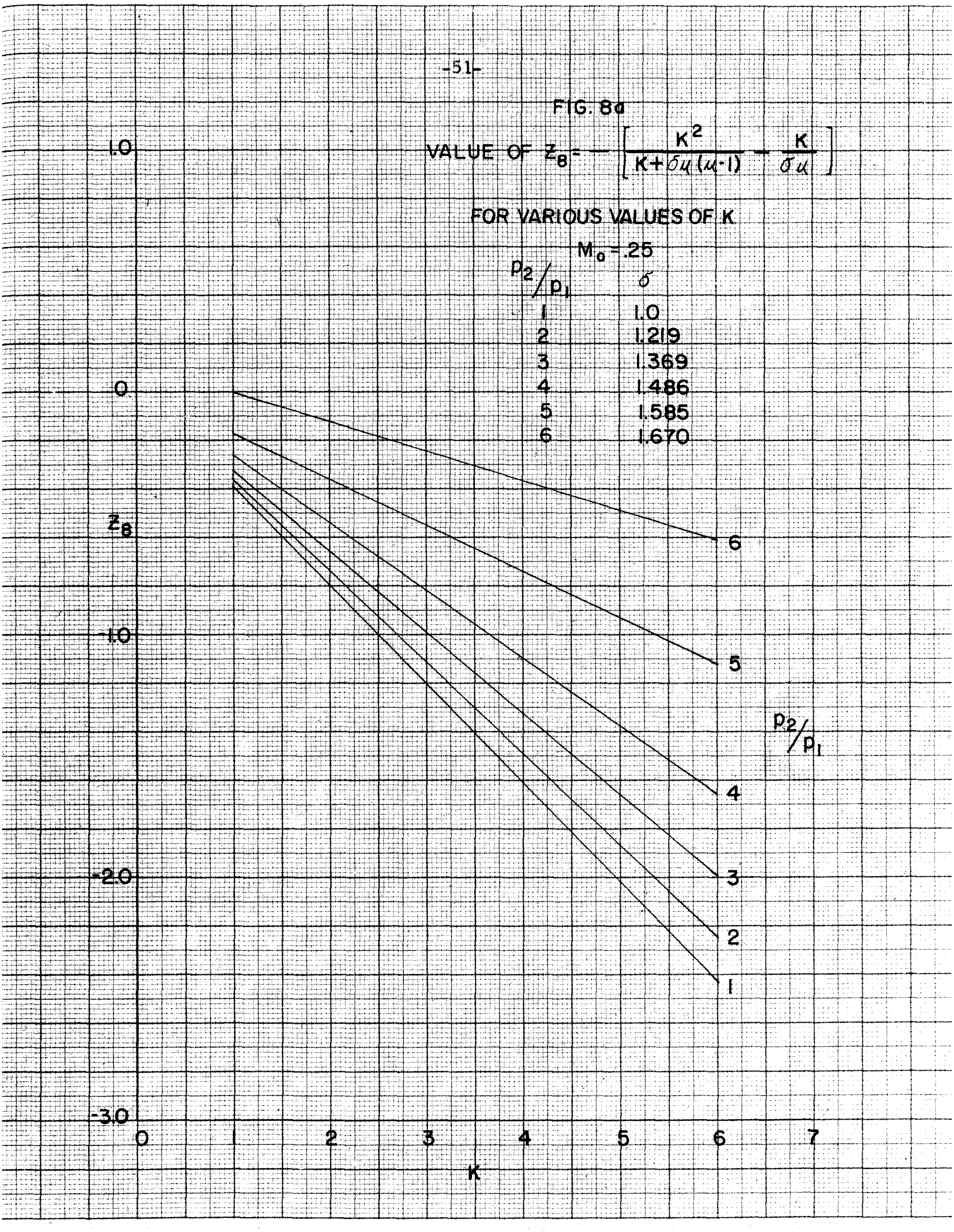


FIG. 8b

VALUE OF $Z_8 = -\left[\frac{K^2}{K+\delta\mu(\mu-1)} - \frac{K}{\sigma\mu}\right]$

FOR VARIOUS VALUES OF K

$M_0 = .50$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

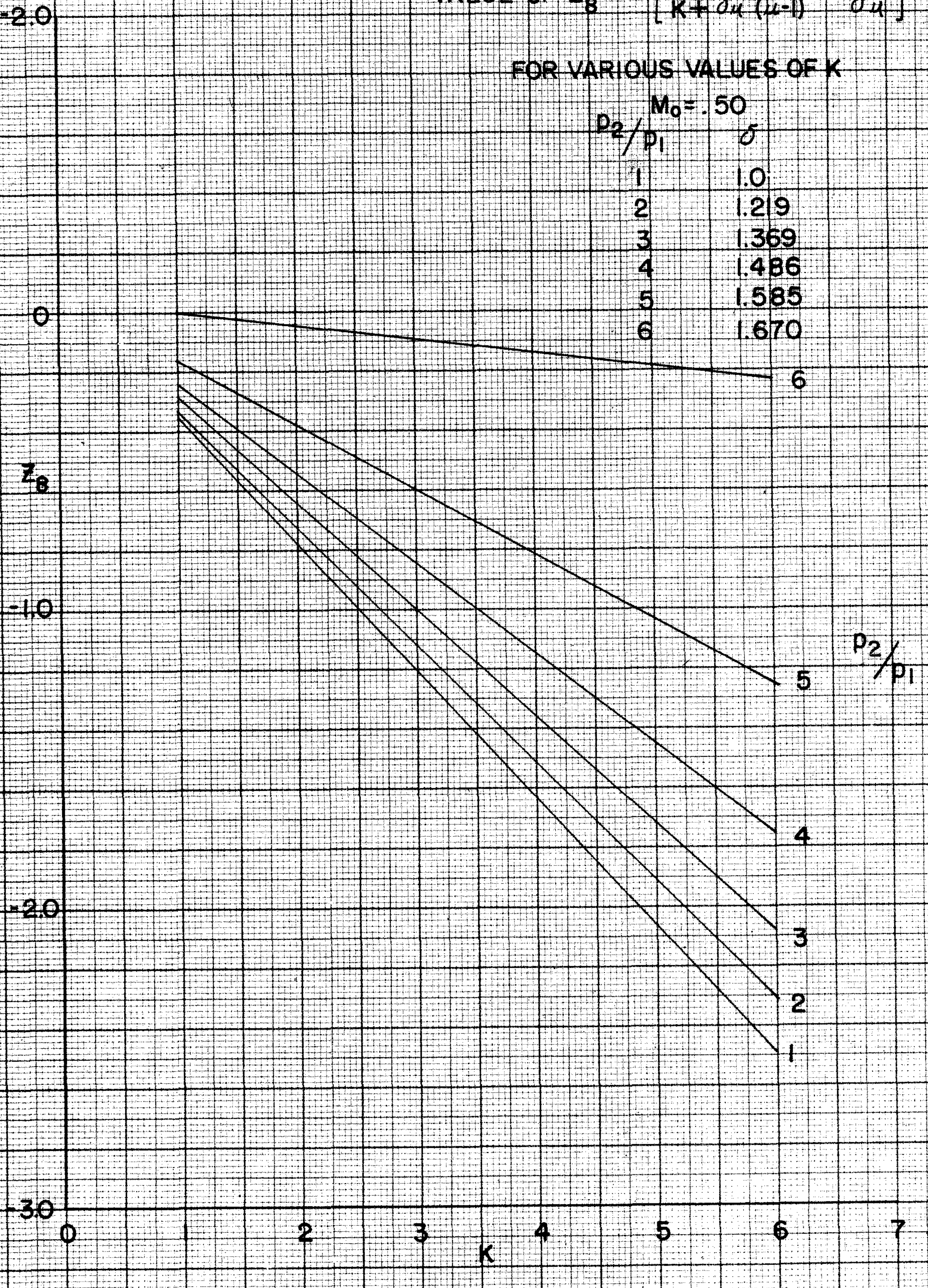


FIG. 8c

VALUE OF $Z_8 = -\left[\frac{K^2}{K+\delta\mu(\mu-1)} - \frac{K}{\delta\mu}\right]$

FOR VARIOUS VALUES OF K

$M_o = .75$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

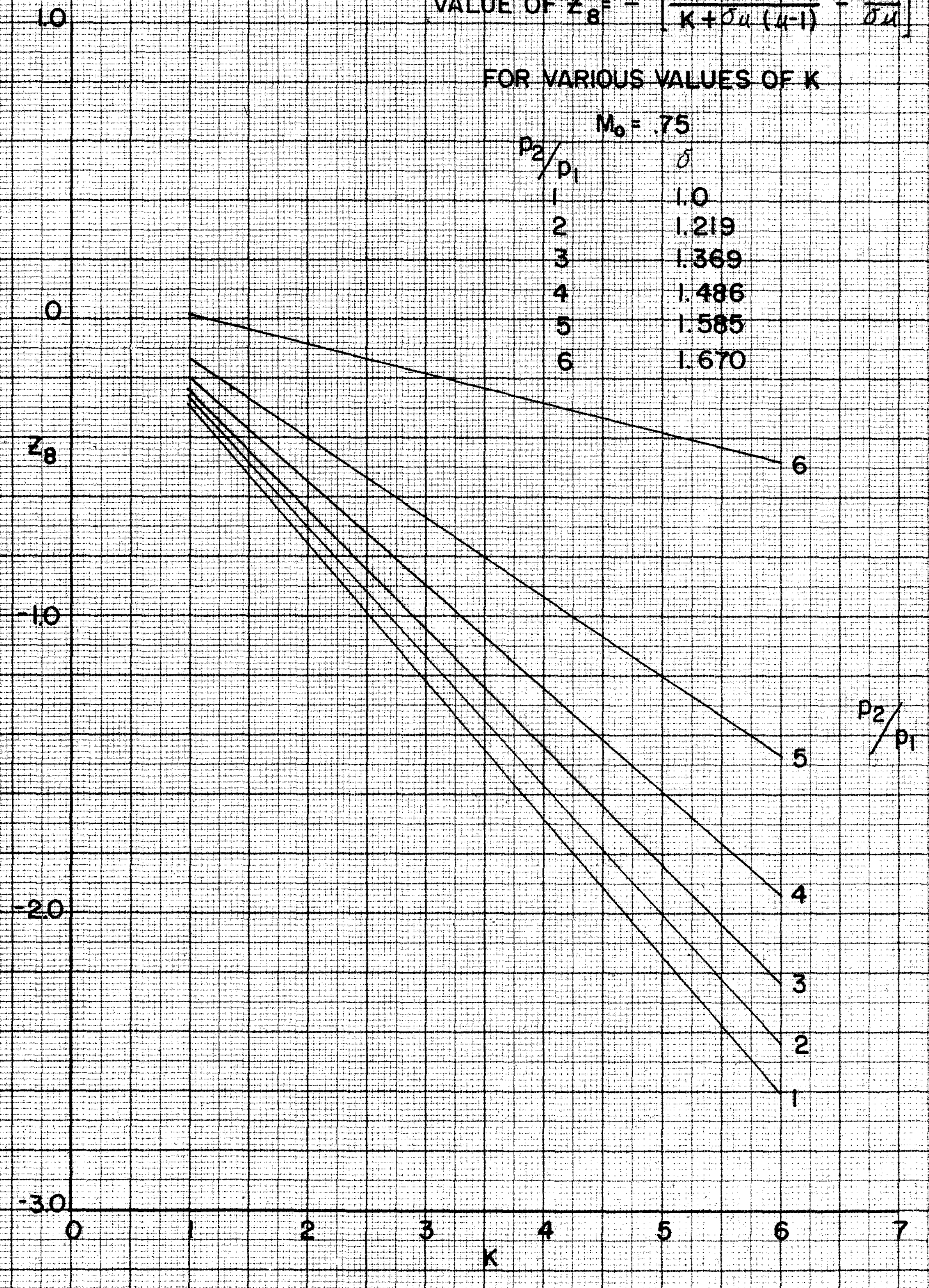


FIG. 9a

VALUE OF $Z_{12} = -(\mu - 1)\left[1 - \frac{\delta^2 \mu^2}{K + \delta\mu\,(\mu - 1)}\right]$

FOR VARIOUS VALUES OF K

$M_0 = .25$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

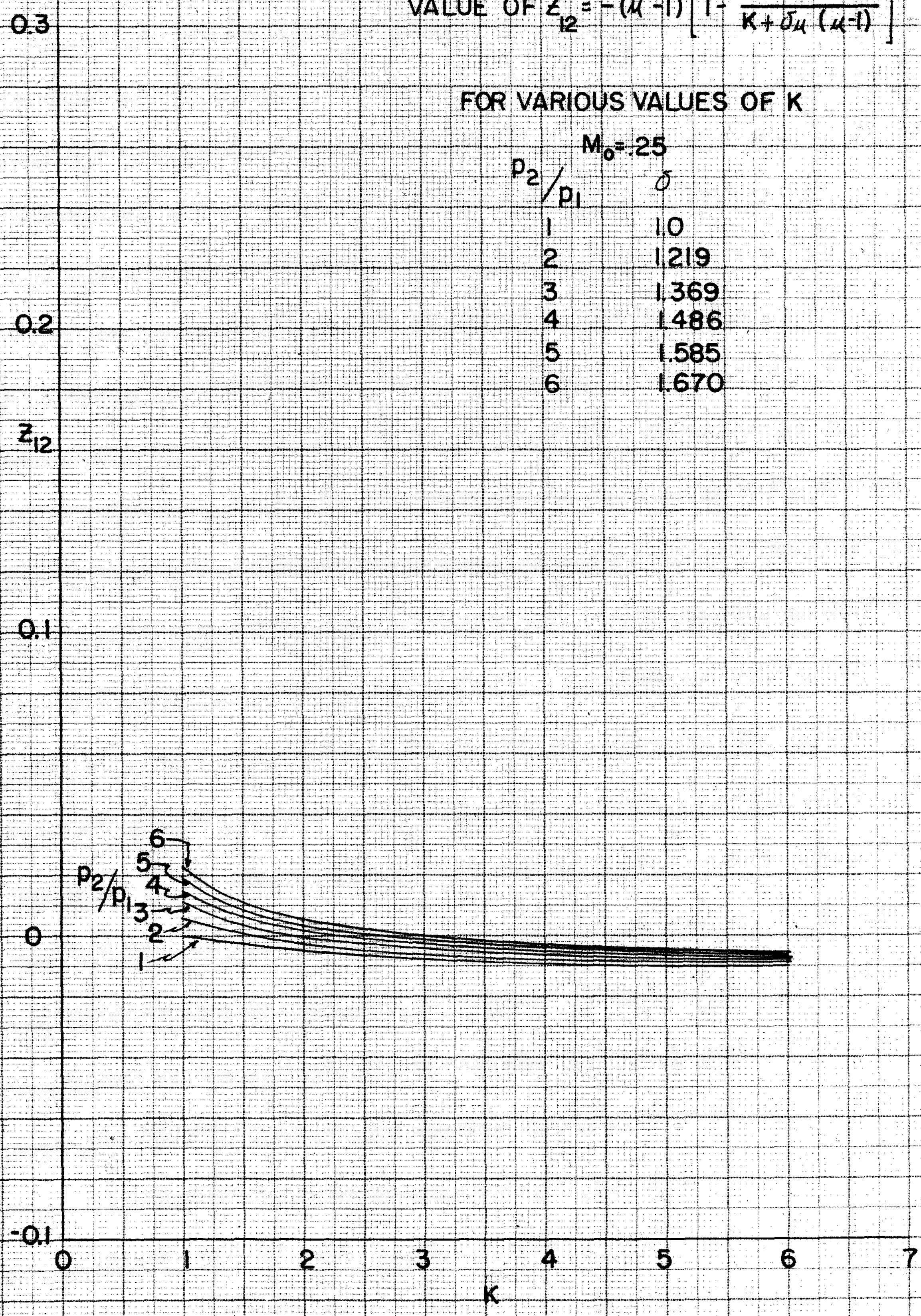


FIG. 9b

VALUE OF $Z_{12} = -(\mu-1)\left[1 - \frac{\delta^2\mu^2}{K+\delta\mu(\mu-1)}\right]$

FOR VARIOUS VALUES OF K

$M_o = .50$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

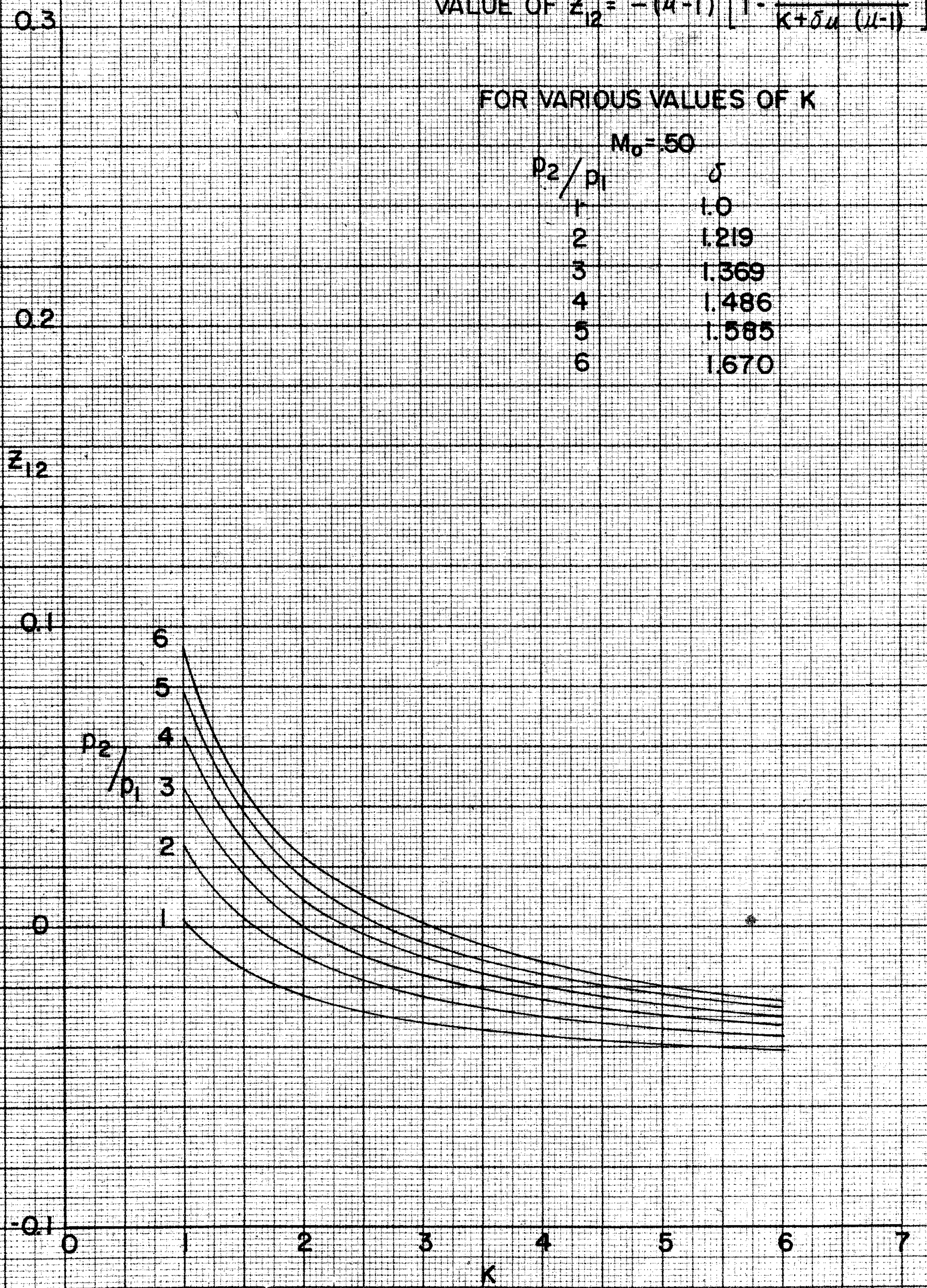


FIG. 9c

VALUE OF $Z_{12} = -(\mu - 1)\left[1 - \frac{\delta^2 \mu^2}{K + \delta\mu\,(\mu - 1)}\right]$

FOR VARIOUS VALUES OF K

$M_0 = .75$

p_2/p_1	δ
1	1.0
2	1.219
3	1.369
4	1.486
5	1.585
6	1.670

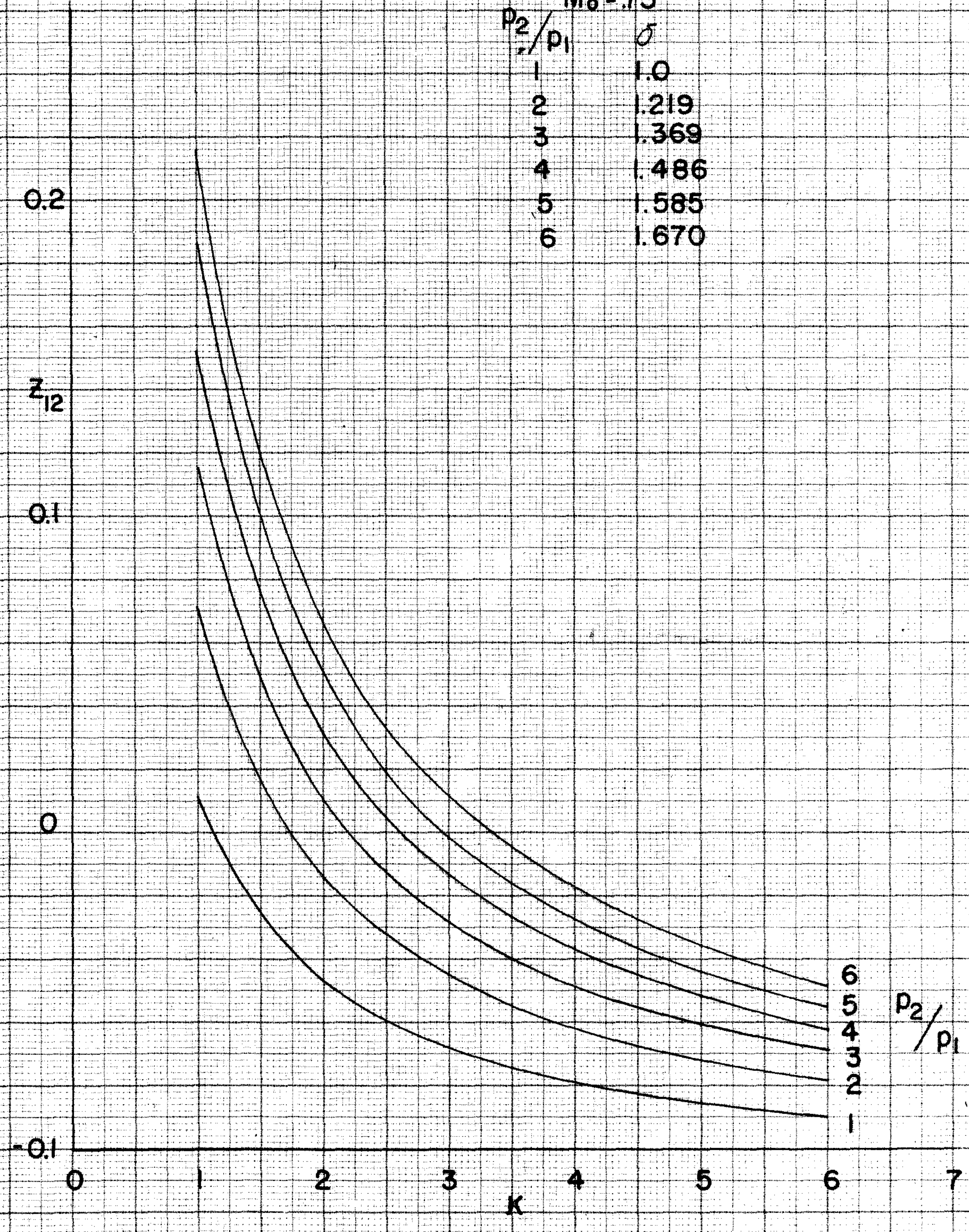


FIG. 10

RATIO OF JET WORK TO PROPELLER WORK VS. $(X-X^*)$ FOR M_o=.50, p_2/p_1 = 3, K = 4

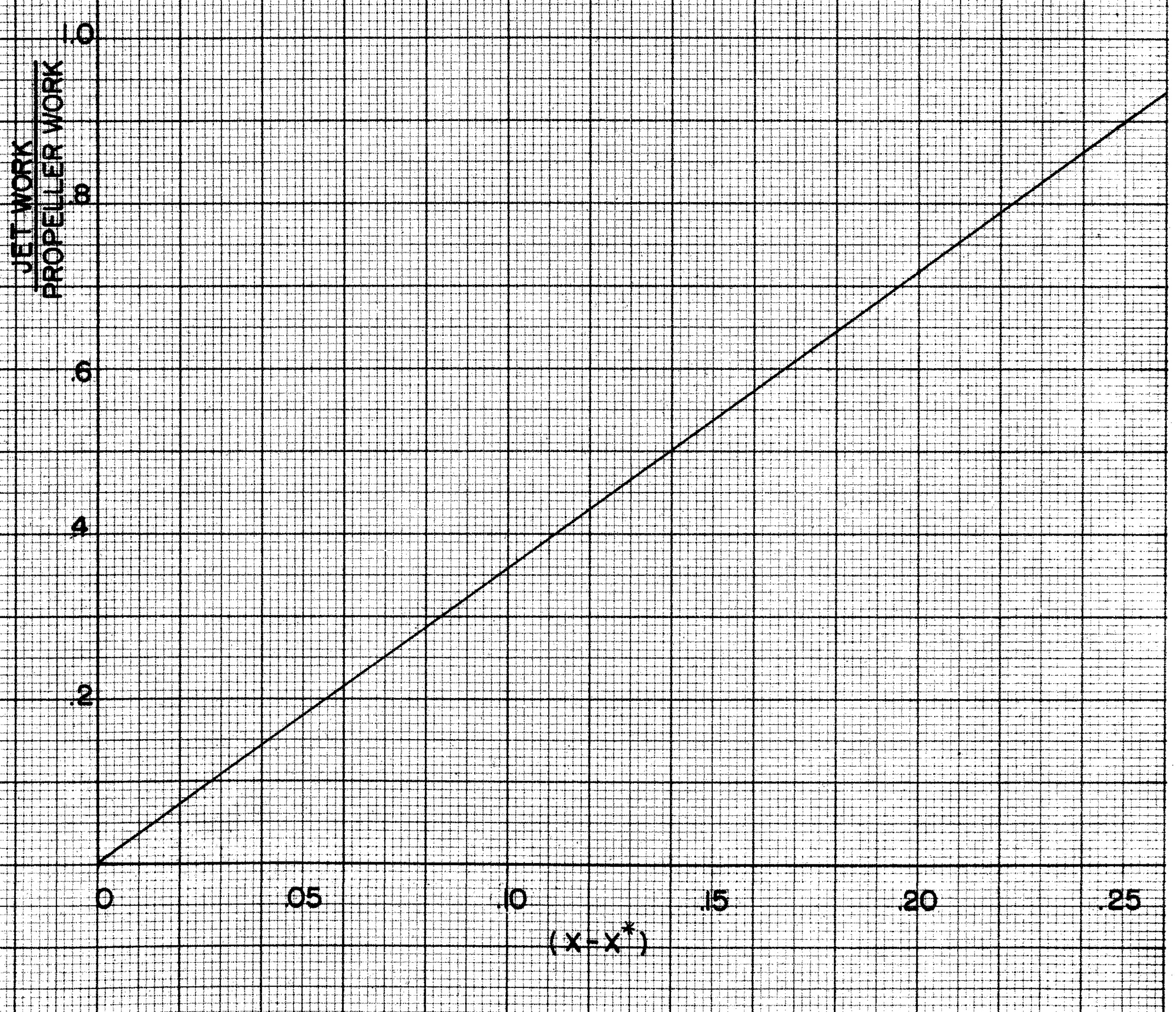


FIG. 11

VARIATION OF POWER COEFFICIENT WITH COMPRESSOR RATIO FOR VARIOUS VALUES OF η_c AND η_t

$M_o = .50, K = 4, \eta_x = 0, C_v = 1, \eta_p = 1$

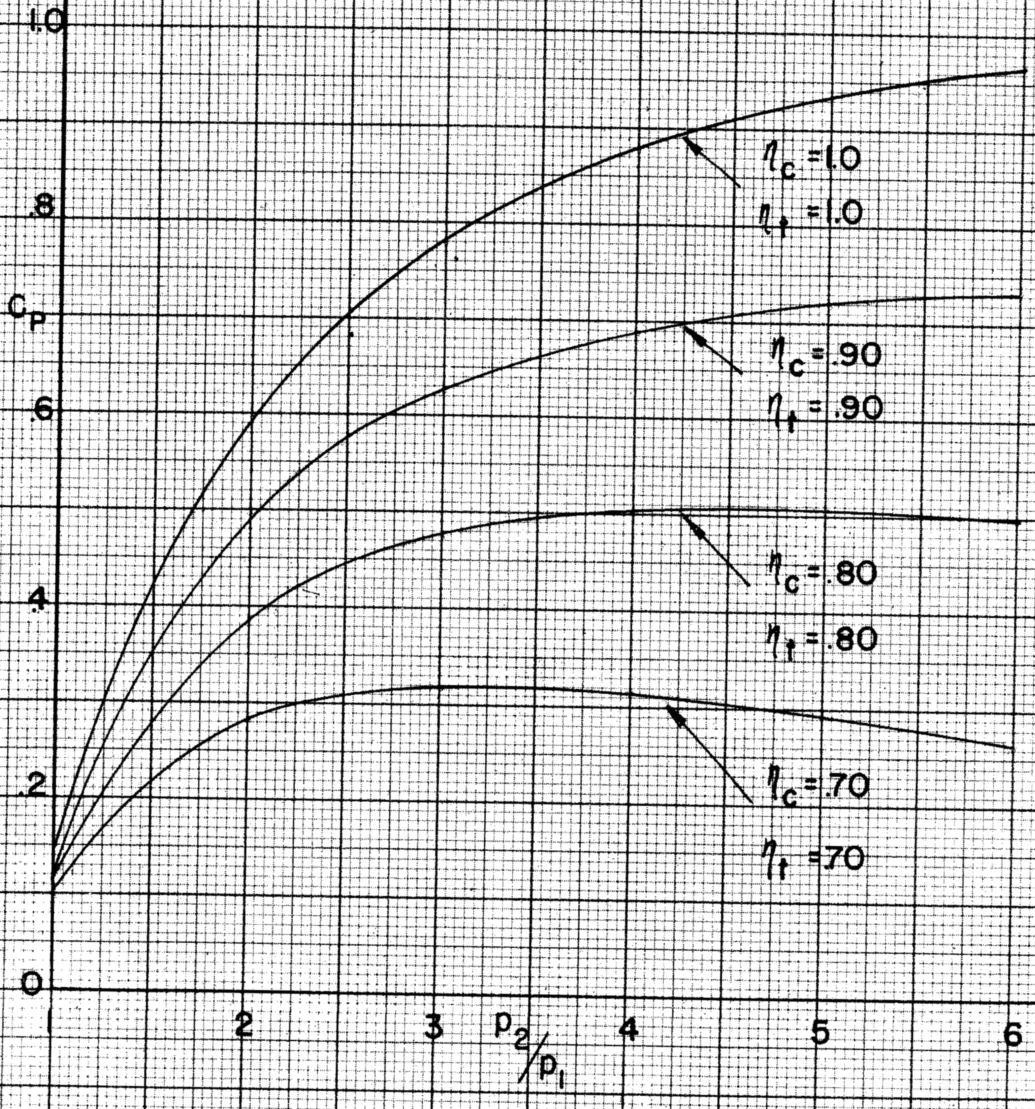
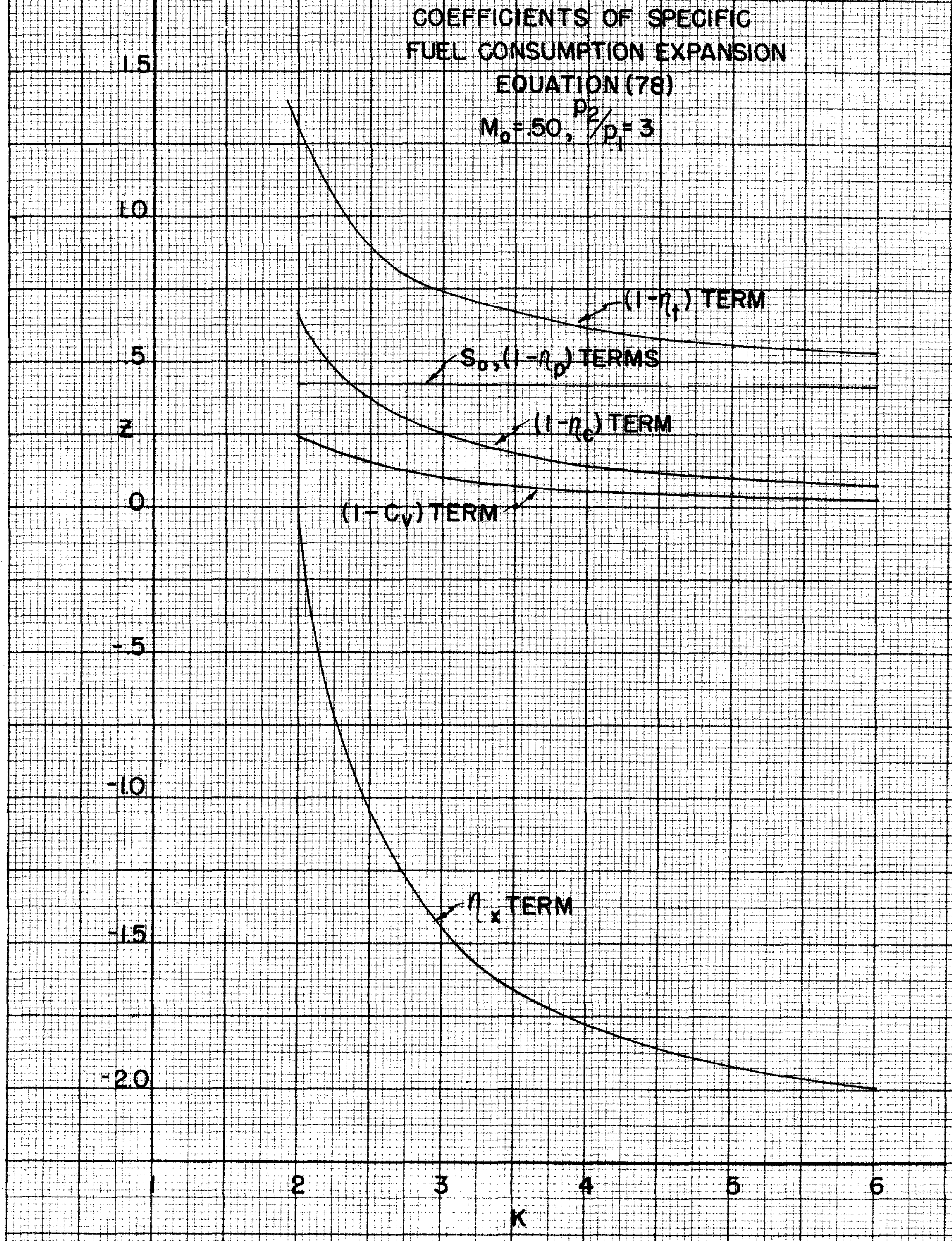


FIG. 12


COEFFICIENTS OF SPECIFIC FUEL CONSUMPTION EXPANSION EQUATION (78)
$M_o = .50,\ p_2/p_1 = 3$
$(1-\eta_t)$ TERM
$S_o, (1-\eta_p)$ TERMS
$(1-\eta_c)$ TERM
$(1-C_v)$ TERM
η_x TERM
1.5
1.0
.5
0
-.5
-1.0
-1.5
-2.0
Z
1
2
3
4
5
6
K

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