THE STUDY OF A LIFTING AIR BREATHING BOOST FOR SATELLITE LAUNCH

Thesis by

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ABSTRACT

The effect on performance of variations in power plant, aircraft, and rocket parameters was calculated for a lifting air breathing boost system for launching satellites. A limited variation in aircraft flight plan was considered also. In addition, comparisons were made between the air breathing boost system and a three stage all rocket system. For the air breathing boost computations were made for launch Mach numbers ranging from two to five.

The air breathing boost was assumed to be a turbojet or dual cycle engine powered aircraft. The rocket used in conjunction with the boost had two stages. In computing aircraft performance thrust and engine specific fuel consumption were taken as constants. The lift to drag ratio was also considered constant for each portion of the flight profile which consisted of a take off and acceleration to climb speed, a two step climb, and a pull up to the maximum angle attainable for rocket launch.

In computing rocket performance burning times, effective exhaust velocities, payload weight ratios, and structural weight ratios were assumed to be the same for each stage. Drag was neglected in rocket calculations, and the acceleration of gravity was assumed constant. The calculations were made by computing the kinetic and potential energies for a sounding rocket and equating them to the energy required for orbit.

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TABLE OF SYMBOLS

A) Aircraft

Symbol	Definition	Units
c	Engine specific fuel consumption	$\frac{\text{lbs(M)}}{\text{sec lb(F)}}$
c_L	Lift coefficient	
D	Drag	lbs.
F	Thrust	lbs.
g	Acceleration of gravity	ft./sec.^{2}
H	Altitude	ft.
m	System mass	lbs. (M)
n	Additional "g" loading	
q	Dynamic pressure	lbs./ft.^{2}
R	Radius of circular arc pull up	ft.
t	Time	sec.
v	Velocity	ft./sec.
θ	Angle of climb	
λ	Lift to drag ratio	
ρ	Atmospheric density	$\frac{\text{lb. sec.}^2}{\text{ft.}^4}$

Subscripts and Superscripts

o	Initial conditions or conditions at start of climb	
$'$	End of first leg of climb	
$''$	End of second leg of climb	
2	End of pull up	

B) Rocket

C	Characteristic velocity, $\frac{V_e}{\sqrt{R_o g_o}}$	

TABLE OF SYMBOLS (Cont'd)

E_1	Energy per unit mass	$\frac{ft.^2}{sec.^2}$
E	Specific energy, $\frac{E_1}{\sqrt{g_0 R_0 / 2}}$	
F_R	Thrust	lbs.
R_o	Radius of earth	ft.
r	Radius of orbit	ft.
t_b	Burning time	sec.
T	Characteristic time, $\frac{t_b}{\sqrt{R_0/g_0}}$	
V_e	Effective exhaust velocity	ft./sec.
$V_\bullet$	Tangential satellite velocity	ft./sec.
W_p	Payload weight	lbs.
W_R	Rocket weight	lbs.
W_{Rf}	Rocket fuel weight	lbs.
W_{RS}	Rocket structural weight	lbs.
α	Payload weight ratio, $\frac{W_p}{W_R}$	
α_1	Payload weight ratio per stage, $\frac{W_{R2}}{W_R}$	
β	Structural weight ratio, $\frac{W_{RS}}{W_{RS} + W_{Rf}}$	
γ	$\frac{\text{First stage weight at burnout}}{\text{Initial weight of first stage}}$, $\beta + \alpha_1(1-\beta)$	

Subscripts - unless otherwise noted above

0	Initial rocket conditions
1	Burnout of first stage
2	Burnout of second stage

I. INTRODUCTION

The launching of large satellites into orbit by means of rocket propulsion requires multi-stage rocket systems of large thrust and weight. It has appeared possible that by replacing the first stage of an all rocket system by a lifting air breathing boost, the rocket thrust and weight requirements could be reduced. The air breathing boost could offer other obvious advantages over the all rocket system. The fact that the manned boost would be recoverable should result in much lower system cost for a sufficiently large number of launchings. Furthermore, there would be no requirement for the construction of launching platforms which are required for an all rocket system. Launching of various size rockets by the air breathing boost could be accomplished without modification of the boost system. The air breathing boost also has obvious disadvantages when compared with the all rocket system. In particular, the cost of an air breathing boost would be high, and the development of an aircraft and power plant for launching rockets at high Mach numbers would take a number of years.

The use of an air breathing boost for launching satellites has been considered by a variety of authors. Sandorff (1) made a comparison between a three stage rocket system and a two stage rocket boosted by a conventional airplane, both designed to place 500 pounds in permanent orbit. It was assumed the air breathing boost would launch the two stage rocket from an altitude of 50,000 ft. at a speed of 1000 ft./sec. Comparisons of cost, rocket thrusts and weights, and system weights were made for the two launching schemes. Rocket costs were based on the average cost of the Viking rocket per pound. It was assumed in the analysis that an existing airplane such as the B-52 could be modified to launch the two stage rocket, and only modification costs were estimated for the airplane.

Kappus (2) discussed existing air breathing power plants and their possible use in an air breathing boost system for launching satellites or ballistic missiles. High Mach number engines were analyzed, and design problems were enumerated for such engines. The advantages of the air breathing boost system were discussed briefly.

A more detailed analysis of the air boost system for launching satellites was made by Ferri, Nucci, Daskin, and Feldman (3). The problem considered was the placing of 10,000 pounds in orbit. The air breathing boost was a conventional airplane powered by afterburning turbojets and ramjets. Comparisons were made between an all rocket system and air boost systems which launched their payload over a range of Mach numbers from 2.2 to 6.0.

These previous analyses of the air breathing boost launch were concerned with very particular systems for which a detailed result was computed based upon a given flight plan. The aim of the present work was to undertake a somewhat more crude analysis but to investigate systematic variations in power plant, aircraft and rocket parameters as well as a limited variation in aircraft flight plan.

In the present analysis the air breathing boost was considered to be a turbojet or dual cycle engine powered airplane. The rocket system analyzed in conjunction with the boost had two stages. The analysis consisted of computing the performance of the air boost system for different values of airplane and rocket parameters affecting system performance. The performance of a three stage all rocket system was computed also for comparison purposes.

The results of the analysis appear in the form of graphs showing the effect of varying certain parameters. The air boost system was analyzed for launch Mach numbers from 2.0 to 5.0, and, generally, the results are plotted versus the Mach number of launch.

II. ANALYSIS

Type of Analysis and Basic Assumptions. -- The flight profile envisioned for the air breathing boost is shown in Fig. 1. Following take off and acceleration to climb speed, a two step climb was made to the pull up point followed by a pull up to the maximum attitude attainable for launching the rocket system. In computing the airplane performance, it was assumed that the engine thrust, engine specific fuel consumption, and the lift to drag ratio were constants. Warm up, take off and acceleration to climb speed was assumed to result in a fixed amount of fuel consumed independent of the performance characteristics chosen for the airplane. In addition, it was assumed that the airplane characteristics did not depend on the Mach number of launch.

In computing the rocket performance burning times, effective exhaust velocities, payload weight ratios, and structural weight ratios were taken arbitrarily to be the same for each stage. Also, the structural weight ratio for each stage was assumed equal to the overall structural weight ratio. The energy possessed by the rocket system at the end of burnout was computed as if there were no coasting between stages. The calculations were made by computing the kinetic and potential energies for a vertical sounding rocket. Drag was neglected, and the acceleration of gravity was taken as a constant equal to the sea level value.

Air Breathing Boost Performance. --The balance of forces on the system for the climb portion of the flight is shown in Fig. 2. The equation of motion along the flight path is, neglecting any small deviation of the thrust axis from the flight path,

$$F - D - mg\sin\theta = m\frac{dv}{dt} \tag{1}$$

F and D are the thrust and drag, respectively, in pounds, m is the mass in slugs, θ is the angle of climb, v is the velocity in ft./sec., and t is the time in seconds. Normal to the flight path

$$L = mg\cos\theta \tag{2}$$

where L is the lift in pounds.

Also,

$$\frac{dm}{dt} = -\frac{cF}{g_o} \tag{3}$$

so that

$$\frac{m}{m_o} = 1 - \frac{cF}{m_o g_o}t$$

if the acceleration of gravity and the engine specific fuel consumption, $\frac{\text{lbs}}{\text{sec lb}}$, are taken as constants. Dividing equation 1 by equation 2 and substituting equation 3 into the result, there is obtained the differential equation

$$\frac{1}{g_o}\frac{dv}{dt} = \frac{F}{m_o g_o}\left(\frac{1}{1 - \frac{Fc}{m_o g_o}t}\right) - \frac{1}{\lambda}\cos\theta - \sin\theta \tag{4}$$

where λ is the lift to drag ratio, and the subscript o indicates initial conditions. Since F, λ, and θ are taken as constants in the analysis, equation 4 readily integrates to

$$v_1 = -\frac{g_o}{c}\ln\left(1 - \frac{Fc}{m_o g_o}t_1\right) - \left(\frac{1}{\lambda_1}\cos\theta_1 + \sin\theta_1\right)g_o t_1 + v_o \tag{5}$$

for the velocity at the end of the first leg of the climb. For the velocity at the end of the second leg of the climb

$$v_1' = \frac{-g_0}{c} \ln\left[1 - \frac{Fc(t_1'-t_1)}{m_0 g_0\left(1-\frac{Fc}{m_0 g_0}t_1\right)}\right] - \left[\frac{1}{\lambda_1'}\cos\theta_1' + \sin\theta_1'\right] g_0(t_1'-t_1) + v_1 \tag{6}$$

Integrating the equation for velocity the altitudes in feet at the end of the first and second portions of the climb are found to be

$$H_1 = \sin\theta_1 \left\{ \left(\frac{g_0}{c}\right)\left(\frac{m_0 g_0}{Fc}\right)\left\{\left(1 - \frac{Fc}{m_0 g_0}t_1\right)\left[\ln\left(1-\frac{Fc}{m_0 g_0}t_1\right) - 1\right] + 1\right\} - \left(\frac{1}{\lambda_1}\cos\theta_1 + \sin\theta_1\right)\frac{g_0}{2}t_1^2 + v_0 t_1 \right\} \tag{7}$$

and

$$H_1' = \sin\theta_1' \left\{ \frac{\left(\frac{g_0}{c}\right)\left(\frac{m_0 g_0}{Fc}\right)}{1 - \frac{Fc}{m_0 g_0}t_1} \left\{\left[1 - \frac{Fc(t_1'-t_1)}{m_0 g_0\left(1-\frac{Fc}{m_0 g_0}t_1\right)}\right]\left[\ln\left(1 - \frac{Fc(t_1'-t_1)}{m_0 g_0\left(1-\frac{Fc}{m_0 g_0}t_1\right)}\right) - 1\right] + 1\right\} - \left(\frac{1}{\lambda_1'}\cos\theta_1' + \sin\theta_1'\right)\frac{g_0}{2}(t_1'-t_1)^2 + v_1(t_1'-t_1) \right\} + H_1 \tag{8}$$

A circular arc pull up was considered for the pull up to launch attitude from the climb. The balance of forces on the system is shown in Fig. 3. The equations of motion are

$$F - D - mg_0 \sin\theta = m\frac{dv}{dt} \tag{9}$$

along the flight path, and

$$L - \frac{mv^2}{R} - mg_0\cos\theta = 0 \tag{10}$$

normal to the flight path. R is the radius of the circular arc in feet. Dividing equation 10 into equation 9 and making use of equation 3 results in the differential equation

$$\frac{1}{g_0}\left(\frac{dv}{dt}+\frac{v^2}{R\lambda}\right) = -\left(\frac{1}{\lambda}\cos\theta+\sin\theta\right)+\frac{F}{m_0 g_0}\frac{1}{1-\frac{FC}{m_0 g_0}t} \tag{11}$$

But

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \frac{dv}{d\theta}\frac{v}{R}$$

Thus,

$$v\lambda\frac{dv}{d\lambda}+v^2 = -g_0 R\lambda\left(\frac{1}{\lambda}\cos\theta+\sin\theta\right)+g_0 R\lambda\left(\frac{F}{m_0 g_0}\frac{1}{1-\frac{FC}{m_0 g_0}t}\right) \tag{12}$$

Since the time of pull up is short, from twenty-four to thirty seconds, compared to the time of climb, the solution of equation 12 can be approximated closely by replacing the variable t in the last term of equation 12 by a constant $\bar{t}$ approximately equal to $\frac{(t_1'+t_2)}{2}$. If this is done, the velocity at the end of pull up is found to be

$$v_2 = (R)^{1/2}\left\{Ae^{-\frac{2}{\lambda_2}\theta_2}+\frac{2g_0}{\lambda_2^2+4}\left[(\lambda_2^2-2)\cos\theta_2-3\lambda_2\sin\theta_2\right]\right.$$
$$\left.+g_0\lambda_2\left[\frac{F}{m_0 g_0}\frac{1}{1-\frac{FC}{m_0 g_0}\bar{t}}\right]\right\}^{1/2}+v_1' \tag{13}$$

where

$$A = -e^{\frac{2}{\lambda_2}\theta_1'}\left\{\frac{2g_0}{\lambda_2^2+4}\left[(\lambda_2^2-2)\cos\theta_1'-3\lambda_2\sin\theta_1'\right]\right.$$
$$\left.+g_0\lambda_2\left[\frac{F}{m_0 g_0}\frac{1}{1-\frac{FC}{m_0 g_0}t_1'}\right]\right\}$$

and the subscript 2 denotes the end of pull up.

The altitude at the end of pull up is

$$H_2 = R(\cos\theta_1'-\cos\theta_2)+H_1' \tag{14}$$

In the solution of the performance problem for the air breathing boost, additional equations were needed relating atmospheric density to altitude and the radius of the circular arc pull up to the velocity at the start of pull up. For the altitudes of interest in this analysis, the atmospheric density at the end of climb can be represented very closely by the equation

$$\rho_1' = 0.003955 \, e^{-\frac{1.685 H_1'}{35,332}} \tag{15}$$

where the density is in slugs/ft^3. Since $\rho = \frac{2q}{v^2}$, where q is the dynamic pressure in lbs./ft.^{2}, equation 15 can be rewritten as

$$H_1' = 41,800 \ln \left[\frac{v_1'}{\left(\frac{2q_1'}{0.003955} \right)^{1/2}} \right] \tag{16}$$

Also, R = $\frac{(V_1')^2}{n_1' g_0}$ where n_1' is the additional "g" loading imposed on the aircraft at the start of pull up. Substituting equation 16 into equation 14 and making use of the relation for the radius of pull up, another equation is obtained for launch altitude:

$$H_2 = 41,800 \left[\frac{v_1'}{\left(\frac{2q_1'}{0.003955} \right)^{1/2}} \right] + \frac{(v_1')^2}{n_1' g_0} (\cos \theta_1' - \cos \theta_2) \tag{17}$$

In the solution of the equations for the air breathing boost performance, the dynamic pressure at the end of climb was taken as 2,000 lbs./ft.^{2} for all cases. Depending on the value of velocity desired at the end of climb, the launch altitude could be estimated by choosing the lift coefficient at the end of pull up. This lift coefficient was taken as 0.5 for all cases. The angle of launch was chosen compatible with the choice of launch altitude. Selecting a value of launch altitude according to the launch Mach number desired, equation 17 was solved for the velocity at the start of pull up, V_1'. Equation 16 was solved for the altitude at the end of climb, H_2, in the solution of equation 17. Having velocity and altitude at the end of climb, equations 6 and 8 were solved simultaneously for the time at the end of each leg of the climb. In solving equations 6 and 8, the velocity and altitude at the end of the first leg of the climb were determined also. Equation 13 was solved next for the velocity at the end of pull up, V_2. Then the lift coefficient at the end of pull up was computed from

$$C_{L2} = \left(\frac{W}{S}\right)_2 \frac{1}{q_2}\left(\cos\theta_2 + n_2\right) \tag{18}$$

Any variation of terminal lift coefficient, C_{L2}, from 0.5 was corrected by changing the value of launch angle, and thus launch altitude, launch velocity, and the additional "g" loading at the launch point.

Rocket Performance.--The energy necessary to bring a satellite into a circular orbit around the earth consists of its kinetic and potential energy. Thus,

$$E_1 = \frac{V_\theta^2}{2} + \int_{R_0}^{r} g\,dr \tag{19}$$

where R_o is the radius of the earth, r is the radius of orbit, V_θ is the tangential velocity of the satellite and E_1 is the energy per unit mass. For a circular orbit the centrifugal force of the satellite must balance the earth's gravitational pull. Therefore,

$$\frac{V_\theta^2}{r} = g. \text{ Since } g = g_o\left(\frac{R_o}{r}\right)^2, \tag{20}$$

$$V_\theta^2 = \frac{R_o^2}{r} g_o \tag{21}$$

Substituting equations 20 and 21 into equation 19, the required energy is found to be

$$E_1 = g_o R_o - \frac{g_o R_o^2}{2r} \tag{22}$$

Normalizing equation 22 by dividing by the energy necessary for a circular orbit at the surface of the earth, the specific energy, E, becomes

$$E = 2 - \frac{R_o}{r} \tag{23}$$

The problem considered was the placing of a satellite in a 200 mile circular orbit. Thus, E = 1.047.

The equation of motion for a sounding rocket is

$$m\ddot{y} = -mg - V_e \frac{dm}{dt} \tag{24}$$

where m is the rocket mass in slugs, and V_e is the effective exhaust velocity in ft./sec.. Drag has been neglected. Taking $g = g_o$, equation 24 integrates to

$$\dot{y} - \dot{y}_o = -g_o t + V_e \ln \frac{m_R}{m} \tag{25}$$

where m_R is the initial rocket mass. But

$$m = m_R - (m_R - m_b)\frac{t}{t_b}$$

or

$$\frac{m}{m_R} = 1 - \left(1 - \frac{m_b}{m_R}\right)\frac{t}{t_b} \tag{26}$$

where the subscript b indicates burnout conditions. The total rocket weight is

$$W_R = W_{RS} + W_{Rf} + W_P \tag{27}$$

where W_{RS} is the structural weight, W_{Rf} is the fuel weight, and W_P is the payload weight which includes structure for payload. Introducing the structural weight ratio, $\beta = \frac{W_{RS}}{W_{RS} + W_{Rf}}$, and payload ratio, $\alpha = \frac{W_P}{W_R}$, equation 26 can be rewritten, making use of equation 27, as

$$\frac{m}{m_R} = 1 - (1-\alpha)(1-\beta)\frac{t}{t_b} \tag{28}$$

Substituting into equation 25 and integrating, the velocity at the end of burnout of the first stage is

$$\dot{y}_{b1} = \dot{y}_0 - g_0 t_{b1} - V_e \ln[\beta_1(1-\alpha_1) + \alpha_1] \tag{29}$$

Since it was assumed that $\beta = \beta_1 = \beta_2$, that $\alpha_1 = \alpha_2 = \alpha^{1/2}$, and that $t_{b1} = t_{b2} = \frac{1}{2} t_b$, the velocity at burnout of the second stage becomes

$$\dot{y}_{b2} = \dot{y}_{b1} - \frac{g_0 t_b}{2} - V_e \ln[\beta(1-\alpha_1) + \alpha_1] \tag{30}$$

Integration of equation 25 twice and substitution of the appropriate constants of integration yields for the altitude at burnout of the second stage:

$$y_{b2} = y_0 + \dot{y}_0 t_b - \frac{g_0 t_b^2}{2} - V_e \ln \xi + \frac{V_e t_b \xi}{(1-\xi)} \ln \xi + V_e t_b \tag{31}$$

where $\zeta = \beta(1-\alpha_1) + \alpha_1$ (32)

By using equations 30 and 31 the specific energy of the system is found to be

$$E = E_0 + 2CT\left[1 - \frac{2\dot{y}_{b2}}{g_0 t_b}\ln\zeta + \frac{3-\zeta}{2(1-\zeta)}\ln\zeta + \frac{2V_e}{g_0 t_b}\ln^2\zeta\right] \quad (33)$$

where $E_0 = \dfrac{y_0 g_0 + \frac{(\dot{y}_0)^2}{2}}{g_0 R_0/2}$, the specific energy imparted to the system by the air breathing boost; $C = \dfrac{V_e}{\sqrt{R_0 g_0}}$, and $T = \dfrac{t_b}{\sqrt{R_0/g_0}}$.

By a similar development the specific energy of a three stage all rocket system is

$$E = 2CT\left[1 + \frac{15-8\zeta}{7(1-\zeta)}\ln\zeta + 9\frac{C}{T}\ln\zeta\right] \quad (34)$$

In the performance calculations ζ was determined from equation 33. Thus, α_1 could be determined from equation 32. W_R and W_{R2} were then computed from the expressions

$$W_R = \frac{W_P}{\alpha} \quad (35)$$

and

$$W_{R2} = W_R\alpha_1 \quad (36)$$

The rocket thrusts were calculated from

$$F_R = \frac{W_R(1-\alpha_1)(1-\beta)}{g_0}\frac{V_e}{t_{b1}} \quad (37)$$

and

$$F_{R2} = \frac{W_{R2}(1-\alpha_1)(1-\beta)}{g_0}\frac{V_e}{t_{b1}} \quad (38)$$

which result from solving the differential equation $F = -\frac{V_e}{g_o}\frac{dm}{dt}$ and making the appropriate substitutions.

The weight characteristics of the system were computed by subdividing the system into five components; namely, rocket, airplane fuel, airplane fuel tanks, aircraft structure, and engines. The structural weight, which included crew and equipment weight, was taken as a constant. Aircraft fuel was broken down into three categories: fuel for warm up, take off, and acceleration to climb speed; fuel for climb and pull up; and fuel reserve for landing. Fuel tank weights were taken as a constant percentage of the fuel weight. Finally, engine weights were chosen which were compatible with engine specific fuel consumptions.

III. CALCULATION METHODS

Choice of Parameters. -- For the air breathing boost system the following parameters were varied in the analysis to show their effect on system performance: engine weight to thrust ratio, engine specific fuel consumption, airplane thrust to weight ratio, the angles of climb, airplane structural weight ratio, and the lift to drag ratio. Rocket parameters varied were the effective exhaust velocity and the burning time. The other parameters having an effect on system performance were considered fixed throughout the analysis.

In computing the rocket performance it was assumed that fuel for warm up, take off, and acceleration to climb speed was two per cent of the take off weight. The fuel for landing and reserve was taken as five per cent of the airplane structural weight plus engine weight. This would be approximately four per cent of landing weight. The fuel tanks were considered to weigh five per cent of the fuel weight. This is a standard estimate for gasoline and JP type fuels.

The structural weight was taken as twenty-five per cent of the total weight, a percentage which appears realistic when compared with some of the modern day bomber aircraft. The crew and equipment weight was included in the structural weight. For the size airplane required to launch a large satellite, the crew and equipment weight would be a minor consideration. However, for smaller airplanes the crew and equipment weight would be a greater percentage of system weight. This is so because the crew and equipment weight would be essentially independent of system weight.

The wing loading was set at 100 lbs./ft.^{2}, a value common to modern operation aircraft.

The lift to drag ratio was taken as five for both legs of the climb. Experimental evidence indicates that high supersonic aircraft can be designed for maximum lift to drag ratios in excess of five. However, in the analysis the lift coefficient during a major portion of the second leg of the climb is slightly less than 0.05. Thus, it would appear that the value of five for lift to drag ratio during climb is somewhat optimistic. For the pull up the lift to drag ratio was taken as three.

The angles of climb were chosen so that the air breathing boost accelerated continually along the flight path during climb. Thus, the choice of climb angles was dictated largely by the thrust to weight ratio. The climb angle for the second leg of the climb was chosen less than that for the first leg in all cases. A two step climb was chosen so that the same angles of climb could be used over the Mach number range considered for each case.

It was assumed that a lift coefficient of 0.4 could be attained at the launch Mach numbers. To obtain the maximum launch angle, the radius of pull up would be increased as maximum lift coefficient was approached in an actual pull up. Since a circular arc pull up was chosen for convenience in this analysis, the maximum lift coefficient was chosen as 0.5, and the additional "g" loading was taken as two at the start of pull up. The same launch angles could be attained in the actual case with a maximum lift coefficient of 0.4 by increasing the additional "g" loading at the start of pull up to 2.5 and decreasing it as necessary as the pull up progressed. The pull up was included in the analysis since launching of the rocket at the low angles of climb could impair the rocket performance.

The rocket structural weight ratio has a large effect on the amount of payload that can be placed in orbit for a given system weight. A value of 0.05 was chosen for the rocket structural weight ratio. This is attainable for a liquid propellant rocket with low initial accelerations. This acceleration varies inversely with burning time. For the standard case burning time was taken as 120 second, sixty seconds per stage. For the three stage all rocket system, total burning time was taken as 180 seconds, 60 seconds per stage.

Table I shows how the other parameters were chosen in the analysis and how these parameters were varied. Case I was taken as the standard against which all other cases were compared. In cases II through V the ratio of engine thrust to system weight was varied for comparison purposes. The lift to drag ratio was reduced for two separate values of engine thrust to system weight ratio in cases VI and VII. The effect of reduced thrust during pull up was investigated in case VIII. Cases IX and X involved the change in angles of climb and in airplane structural weight ratio, respectively. The effect of engine thrust to engine weight ratio and engine specific fuel consumption was evaluated in cases XI through XV. Rocket effective exhaust velocities were changed in cases XVI through XVIII. Finally, rocket burning time was varied in case XIX.

For the standard case the thrust to system weight was set at 0.75. This is very high, but it was chosen since preliminary calculations seemed to indicate that values much lower would result in poor system performance. The engine thrust to weight ratio was taken as 0.12. A value of 2.5, $\frac{\text{lbs(M)}}{\text{lb(F)hr}}$, was chosen for the engine specific fuel consumption. These values, although somewhat optimistic, correspond fairly well to those for afterburning turbojet engines now in use. An effective exhaust velocity of 9,000 ft./sec. was selected for the rocket corresponding to an actual specific impulse of 280 seconds. Since in practice the actual specific impulse is about ninety per cent of the ideal, the value chosen is realistic today only for liquid propellant rockets.

Calculations for Standard Case.--It was found that the altitude and angle of launch was fixed within very narrow limits for any launch Mach number. For example, choosing the launch altitude as 85,000 ft. and the angle of launch as thirty degrees re ulted in a launch Mach number of approximately four.

By choosing the altitude of launch as 85,000 ft. and the angle of launch as thirty degrees for the standard case, it was found that the velocity at the end of climb was 3800 ft./sec. from equation 17. The first term of equation 17, the altitude at the end of climb, was found to be 55,600 ft. Next, equations 6 and 8 were solved simultaneously for the times at the end of the first and second legs of the climb. These were seventy and two hundred and ten seconds. The velocity and altitude at the end of the first leg of the climb were respectively 1360 ft./sec. and 26,000 ft., both values being obtained in the solution of equations 6 and 8. From equation 13 the velocity at the end of pull up was found to be 3965 ft./sec. which corresponds to a launch Mach number of 4.09. Using equation

18 it was found that the lift coefficient at the end of pull up was 0.501. Thus, the altitude of launch and the angle of launch were chosen correctly. Fuel consumed during the climb and pull up was computed from equation 3 and was found to be 14.58 per cent of system weight.

Having the velocity at launch and the launch altitude, the rocket payload weight ratio per stage was calculated from equation 32 after solving equation 33. The payload ratio per stage was 0.218. Therefore, the overall payload ratio was .0474. The ratio of rocket weight to payload weight was the reciprocal of .0474 or 21.1. From equation 36 the weight of the second stage of the rocket per pound of payload was found to be 4.6. Using equations 37 and 38 the rocket thrusts per pound of payload were computed as 89.0 and 17.7 for the first and second stages, respectively.

The fuel for return and landing was computed as five per cent of the engine weight plus structure weight or 1.7 per cent of take off weight. Thus, fuel for the mission was 16.28 per cent of system weight at take off. Subtracting the ratio of the sum of engine, structure, fuel, and fuel tank weight to system weight from one, the ratio of rocket to system weight was found to be 0.4891. Thus, system weight at take off per pound of payload was 43.1.

The same procedure was followed in solving for system performance at launch Mach numbers of approximately 2, 3 and 5. The results were then plotted versus Mach number.

IV. RESULTS AND DISCUSSION

For the standard case the velocity, altitude, lift coefficient, and total pressure were computed as a function of time for the launch Mach number of 4.09. These are plotted in Fig. 4. The rise in stagnation pressure with velocity along the flight path, as shown in Fig. 4, would result in an increased air weight flow through the engine at the higher speeds even though diffuser efficiency would decrease with speed. This could conceivably counteract the decrease in heat addition per pound of air flow with increased speed. Thus, with a properly designed engine with variable diffuser inlet and nozzle exhaust, the thrust could be maintained nearly constant up to the pull up point. Thrust would definitely decrease during pull up as indicated by the sharp drop in total pressure. It is noted also that system speed actually increases during pull up. This is because of the high thrust to weight loading. Because of the very sharp rise in lift coefficient near the end of pull up, the launch altitude and launch angle vary very little with a change in the climb angle for the second leg of the climb.

In Fig. 5 some of the results of the solution of the performance problem are shown for the standard case as a function of the launch Mach number. Although plotted up to a launch Mach number of five, the results are not particularly significant past a launch Mach number of four. Turbojet engines are limited by temperature to operation at Mach numbers not much in excess of four. Thus, for launch Mach numbers of five and greater, ramjets or other type engines would have to be used in conjunction with turbojet engines. Neither was structural weight increased in the analysis for the higher Mach numbers of launch. At a Mach number of five the stagnation temperature is approximately 1800° F for altitudes above the tropopause. An aircraft structure designed to withstand the temperatures encountered at such a Mach number would necessarily be heavier than one designed for operation at lower speeds.

It is seen in Fig. 5 that the ratio of rocket weight to payload weight is reduced by approximately twenty-four per cent if the rocket is launched at a Mach number of four instead of a Mach number of two. It was found in the analysis that the reduction of rocket weight with launch Mach number depends almost entirely on launch Mach number. The altitude of launch affects the rocket weight slightly, contributing only to the specific energy at launch.

The ratio of system weight to payload weight also decreases with Mach number of launch even though the airplane weight goes up. Although the decrease might not be as great as shown, sixteen per cent for launch at a Mach number of four compared to a Mach number two launch, it is evident that the higher launch Mach number yields more favorable results.

The fuel to system weight ratio increases from about 10.8 per cent for a Mach two launch to approximately 15.8 per cent for a Mach four launch. These values are not conservative in that the fuel reserve for return and landing was taken as approximately four per cent of landing weight leaving very little for reserve.

Adding the structural weight to the engine, fuel and fuel tank weight, it is seen in Fig. 5 that the airplane weight is approximately forty-five per cent of system weight at take off for a Mach two launch, while for a Mach number four launch, it is slightly in excess of fifty per cent.

In the design of a turbojet engine the engine specific fuel consumption can be improved at the expense of engine weight to thrust ratio. Cases XIII, XIV, and XV are combinations of engine weight to thrust ratios and engine specific fuel consumptions which represent the optimum in turbojet engine design today. Cases XI and XII represent engines of poorer performance and are characteristic of engines now operational.

The effects of varying these two parameters, engine weight to thrust ratio and engine specific fuel consumption, are shown in Figs. 6, 7, and 8. In Fig. 6 the obvious result is shown that the systems having engines with the best specific fuel consumption use the least fuel. The difference becomes greater at the higher launch Mach numbers. The difference in fuel requirements for systems having engines with the same specific fuel consumption is due to the amount of fuel required for return and landing. The fuel requirements for the system having the engine of best specific fuel consumption, 1.1 $\frac{\text{lbs}}{\text{lb hr}}$, are about thirty-nine per cent less than those for the system with the worst specific fuel consumption, 2.5 $\frac{\text{lbs}}{\text{lb hr}}$, for a launch Mach number of two. At Mach number four the difference is about forty per cent.

The effect on engine weight, fuel, and fuel tank weight is shown in Fig. 7. Here it is seen that the engines of lower weight to thrust ratio and higher specific fuel consumption yield better system performance. This result is especially true for low launch Mach numbers. For lower system thrust to weight ratios, engines with lower specific fuel consumption and higher weight to thrust ratios would compare more favorably.

The effect of engine weight to thrust ratio and specific fuel consumption on system weight to payload weight ratio is shown in Fig. 8 as a function of launch Mach number. The effect is quite large. A reduction of about sixteen per cent in system weight to payload weight ratio is obtained by use of the best engine, weight to thrust ratio of 0.08 and specific fuel consumption of 2.0 lbs./lb.hr., compared to the worst engine, weight to thrust ratio of 0.20 and specific fuel consumption of 2.0 $\frac{\text{lbs}}{\text{lbs hr}}$. This is for a launch Mach number of two. At a launch Mach number of four the reduction is closer to fifteen per cent.

The effects of rocket effective exhaust velocity and burning time on system performance are shown in Figs. 9 through 16. These two effects are by far the greatest of any considered in the analysis. In Fig. 9 system weight to payload weight ratio is plotted versus launch Mach number for effective exhaust velocities varying from 8,000 to 12,000 ft./sec. and for the one case where burning time was taken as 200 seconds. It is noted that the higher Mach numbers of launch are especially desirable when rocket performance is poor. For a rocket effective exhaust velocity of 8,000 ft./sec., a system weight of 500,000 pounds would be required to launch a 5,920 pound payload into a 200 mile circular orbit under the assumptions of this analysis. For a Mach number of launch of four system weight would have to be only 394,000 pounds to place the same weight payload in orbit. For an effective exhaust velocity of 12,000 ft./sec.

a payload of 23,400 pounds could be placed in orbit with a system weight of 500,000 pounds, rocket launch being at a Mach number of two.

In Fig. 10 the system to payload weight ratio is plotted versus effective exhaust velocity for different values of launch Mach number. The three stage all rocket system is plotted on the graph also. It is seen that the boosted rocket system becomes more desirable when rocket performance is poor. At the higher effective exhaust velocities the air breathing boost system compares less favorably with the all rocket system. For a three stage all rocket system with a burning time of 300 seconds and an effective exhaust velocity of 9,000 ft./sec., the system to payload weight ratio is 51.6 under the assumptions of this analysis. For the two-stage boosted rocket with burning time of 200 seconds and an effective exhaust velocity of 9,000 ft./sec., the system to payload ratio is 53.0 for a launch Mach number of four. Thus, the boosted rocket system also compares more favorably with the all rocket system when initial accelerations of the first stage rocket are low. This is the case for the first stage of an all rocket system because of the control problems after launch, and because low accelerations are required initially if the rocket structural weight ratio is low. Also, the first stage of an all rocket system is less efficient because the rocket is exhausting against the back pressure of the atmosphere, and rocket drag is at its highest in the denser air of the lower atmosphere.

In Fig. 11 the rocket to payload weight ratios are plotted versus launch Mach number for the different effective exhaust velocities and burning times. In Fig. 12 a cross plot is made of rocket to payload weight ratios versus effective exhaust velocities for the different launch Mach numbers. For an effective exhaust velocity of 8,000 ft./sec., the rocket to payload weight ratio for the rocket boosted to Mach number four before launch is forty-eight per cent less than the ratio for a three stage all rocket system. For an effective exhaust velocity of 10,000 ft./sec., the reduction is forty-two and one half per cent. Thus, it is demonstrated again that the boost system compares more favorably with the all rocket system when rocket performance is poor. It is noted also that the launch Mach number has a great effect on the system to payload weight ratio for the boosted rocket system. Again, the effect is more pronounced when rocket performance is poor.

Figs. 13 and 14 show the effect of effective exhaust velocities and burning times on the second stage rocket to payload weight ratio. Comparing the weight ratios for the boosted and all rocket system, it is seen that the reduction in weight of the second stage rocket for the boosted system is greater than the reduction for the first stage. This is because the payload weight ratio per stage is greater for the all rocket system.

The rocket thrusts to payload weight ratios are compared in Figs. 15 and 16. The ratios are not the same as the rocket to payload weight ratios because of the difference in payload ratios per stage between the boosted system and the all rocket system. For the first stage initial accelerations of the all rocket system are approximately two times the gravitational acceleration. For the boosted rocket systems the initial rocket accelerations are approximately two and one half to three times that of gravity. Thus, in practice the thrust requirements for the rocket launched from the air breathing boost would go down, and the comparison with the all rocket system, as shown in Fig. 16, would be more favorable. However, reducing the initial rocket accelerations of the boosted rocket would require an increase in rocket and system weight and the overall performance of the boosted rocket system would deteriorate. As shown in Fig. 16 the thrust requirements for the first stage of the air boosted rocket are less by a very small margin if the boosted rocket is launched at a Mach number of two. The reduction in thrust requirements is appreciable only at the higher launch Mach numbers. Again, the air boost system compares more favorably when rocket performance is poor.

The effects of system thrust to weight ratio on the performance of the air breathing boost system are shown in Figs. 17 through 20. In Fig. 17 it is seen that the increase in airplane fuel requirements is appreciably only for the case where the airplane thrust to system weight ratio is reduced to 0.25. At a launch Mach number of four, the fuel required is twenty-nine per cent of system weight if airplane thrust to system weight ratio is 0.25, while the fuel required is only sixteen per cent of system weight if airplane thrust to system weight ratio is 0.75.

The sum of fuel, fuel tanks, and engine to system weight ratios is plotted against launch Mach number in Fig. 18 for the range of airplane thrust to system weight ratios considered. Here it is seen that the effect is large only for the case where engine thrust to weight ratio is 0.25. This is because the excess thrust for acceleration of the airplane is so small at the low thrust loading.

Figs. 19 and 20 show the effect of airplane thrust to system weight ratios on system to payload weight ratios. In Fig. 20 it is seen that best performance is obtained for power loadings greater than 0.55 and less than 0.64. As seen in Fig. 20, the optimum power loading increases with launch Mach number. For optimum conditions system weight to payload weight ratio is reduced sixteen per cent if launch Mach number is four instead of two.

The variation in the altitude at the end of the first leg of the climb as a function of Mach number is shown in Fig. 21 for the five different values of thrust to system weight ratios chosen in the analysis. It is noted that as the Mach number of launch is increased past three, the major portion of the flight is along the second leg of the climb.

In Fig. 22 it is seen that the gain in altitude during pull up is approximately 30,000 ft. and is essentially independent of launch Mach number. As the launch Mach number is increased, the launch altitude increases but at a steadily decreasing rate.

Although the gain in altitude is essentially independent of launch Mach number, the angle of launch is not as is shown in Fig. 23. In this analysis it was found that the angle of launch depended on launch Mach number only. In the actual case it would be a function of airplane thrust to system weight ratio also. Since a circular arc pull up was considered in the analysis, the additional "g" loading on the airplane varied as speed changed, but the lift coefficient was almost independent of this speed change. Thus, for the lower airplane thrust to system weight ratios the additional "g" loading necessary for flight along the circular arc was less than two at the end of pull up, as low as one and one half, while for the higher airplane thrust to system weight ratios, the value was in excess of two in some cases. By increasing the radius of pull up, the angle of launch could be increased very slightly in the latter case.

Since the maximum launch angle for a launch at Mach number five is of the order of twenty-five degrees, it would probably be necessary to increase the attitude of the rocket system after launch at Mach numbers of five and higher in order to obtain optimum rocket performance. At the low Mach numbers of launch this would not be necessary.

The change in speed during pull up depended only on the launch angle and the airplane thrust to system weight ratio. This change is shown in Fig. 24 where the ratio of Mach number at the end of climb to Mach number at the end of pull up is plotted versus the angle of launch. Remembering that the higher angles correspond to lower launch Mach numbers, it is seen that the loss in speed during pull up is great only at the lower launch Mach numbers.

The time for climb and pull up is very short for the systems with high power loadings as is seen in Fig. 25. Only for the power loading of 0.25 does it become appreciable.

The effect of reducing lift to drag ratio from five to four during climb was investigated for two cases; namely, for system power loadings of 0.75 and 0.50. The increase in fuel consumption for the former case is 1.4 per cent of system gross weight at a launch Mach number of two. The increase is 2.2 per cent at a launch Mach number of five. This is shown in Fig. 26. For the airplane thrust to system weight ratio of 0.50, the fuel consumption increase is two per cent of system gross weight at a launch Mach number of two and 3.2 per cent at a launch Mach number of five. The effect on system to payload weight ratio is shown in Fig. 27. The reduction in lift to drag ratio has a greater effect on the system with the lower airplane thrust to system weight ratio. For the two cases studied the effect, although significant, is not too large. At a Mach number of launch of five the reduction of system to payload ratio is about five per cent for the case with thrust loading of 0.50. This would become greater for lower thrust loadings when there is less excess power available for acceleration of the air breathing boost. The reduced lift to drag ratio has the added effect of shifting the optimum power loading curve in Fig. 20 to the right.

The effect on fuel consumption of increased structural weight and of different climb angles is shown in Fig. 28. Increasing the airplane structural weight ratio increases the fuel to system gross weight ratio by approximately 0.2 per cent due to the increased amount of fuel needed for landing and reserve. Changing the climb angles to fifteen and seven degrees decreases the fuel consumption, the maximum decrease being approximately one per cent of system weight at a launch Mach number of five.

In Fig. 29 the effect of reducing the thrust by fifty per cent during pull up is shown as a function of launch Mach number.

The effect is small. The increase in system to payload ratio is less than one per cent for the range of launch Mach numbers considered. The effect is greatest at the lower Mach numbers of launch where the launch angle is greatest. This observation justifies the assumption of constant thrust for most of the investigation.

The system to payload weight ratios for changes in airplane structural weight ratio and angles of climb are plotted in Fig. 29 as functions of launch Mach number. It is seen that an increase in the airplane structural weight ratio of five per cent increases the system to payload ratio approximately nine per cent at a launch Mach number of two and twelve per cent at a launch number of five. The effect of changing the angles of climb is small, the maximum decrease in system to payload weight ratio being about two per cent at a Mach number of launch of five.

V. CONCLUSIONS

The analysis of the air breathing boost system for satellite launch reveals that the ratio of rocket weight to payload weight depends strongly on the Mach number of launch. For a two stage rocket launched at Mach number two the rocket to payload weight ratio is 28.3 for the standard case. By launching at Mach number four the rocket to payload weight ratio is 21.4, a reduction of about twenty-four per cent. For the three stage all rocket system the ratio is 38.0.

The airplane weight percentage goes up as the launch Mach number is increased. However, because of the strong decrease in rocket weight, the system to payload ratio goes down as the Mach number of launch is increased up to four. For launch at Mach numbers in excess of four, the analysis is not strictly accurate since no increase was made in airplane or power plant component weights. For launch at Mach number two, the system to payload weight ratio is 51.6 while for launch at Mach number four the ratio is 43.6, a reduction of about sixteen per cent. These ratios are for the standard case. The all rocket system weighs less as is seen from the rocket to payload weight ratio mentioned above.

The airplane to rocket weight ratio varies from just over forty per cent to just under sixty per cent, depending on the Mach number of launch and the performance parameters chosen for the boost system. For launch Mach numbers of four it appears that the system to rocket weight ratio cannot be reduced much below fifty per cent.

The comparison of the all rocket system with the air boost system reveals also that the comparison depends on rocket performance. The air boost system compares much more favorably when rocket performance is poor. With effective exhaust velocities of 8,000 ft./sec., the analysis shows that the system to payload weight ratio for the all rocket system is 63.0. For the air boost system the ratio is 66.5 at a launch Mach number of four. For an effective exhaust velocity of 11,000 ft./sec., corresponding to an actual specific impulse of 342 seconds, the ratio of system weight to payload weight is 19.0 for the all rocket system and 25.0 for the air boost system when the rocket is launched at Mach number four. Thus, as better rocket propellant combinations are developed, the advantages of reduced weight and thrust for the air breathing boost system become less.

The choice of engines for the air breathing boost has a large effect on system performance. Because the time of flight is short, it is desirable that the engines designed for the air breathing boost have low weight at the expense of engine specific fuel consumption.

The results of the analysis show also that the airplane thrust to system weight ratio should be greater than 0.50 and less than 0.70. Although the effect of power loading is very small for power loadings between 0.40 and 0.70, the effect becomes greater outside of the indicated range. For low power loadings fuel consumption is excessive. For power loadings in excess of 0.70 the added engine weight causes deterioration in overall system performance.

The reduction of lift to drag ratio has less effect on overall system performance if the power loading is high. For a power loading of 0.75 the increase in system weight to payload weight ratio is about two per cent for a launch Mach number of four if lift to drag ratio is decreased from five to four during climb. For a power loading of 0.50 the increase is about four per cent.

Changing the climb angles for the air breathing boost has only a small effect on system performance. The system to payload weight ratios varied up to a maximum of two per cent for the two cases studied. The choice of climb angles would have some effect on engine performance.

Increasing the airplane structural weight ratio by five per cent results in an increase in system to payload weight ratio of from nine to twelve per cent depending on the launch Mach numbers.

The decrease in speed during pull up due to a decrease in engine thrust has small effect on overall system performance for the case considered. Reducing power loading from 0.75 to 0.375 during pull up results in a maximum increase in system to payload weight ratio of less than one per cent. The increase is greatest at the lower Mach numbers of launch where the pull up angle is greater.

The one big advantage of an air breathing boost for satellite launch appears to be recoverability. To reduce rocket thrusts and weights appreciably launch Mach numbers must be high, four or greater. The problems involved in airplane design for this mission become an item of major concern. The need for the air breathing boost system decreases as higher performance propellants are developed.

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TABLE I

SYSTEM PERFORMANCE PARAMETERS

Case	F/W_o	C- $\frac{lbs}{hr\ lb}$	$\frac{W_e}{F}$	λ_1	λ_2	$\frac{W_s}{W_o}$	θ-deg.	θ_1'-deg.	v - ft/sec	t_b - sec
I	0.75	2.5	0.12	5	3	0.25	20	5	9,000	120
II	0.625	X	X	X	X	X	12	4	X	X
III	0.50	X	X	X	X	X	10	3	X	X
IV	0.375	X	X	X	X	X	5	2	X	X
V	0.25	X	X	X	X	X	2	1	X	X
VI	X	X	X	4	X	X	X	X	X	X
VII	0.50	X	X	4	X	X	10	3	X	X
VIII	0.75 P.U.=0.375	X	X	X	X	X	X	X	X	X
IX	X	X	X	X	X	X	15	7	X	X
X	X	X	X	X	X	0.30	X	X	X	X
XI	X	2.0	0.20	X	X	X	X	X	X	X
XII	X	1.5	0.20	X	X	X	X	X	X	X
XIII	X	2.0	0.08	X	X	X	X	X	X	X
XIV	X	1.5	0.12	X	X	X	X	X	X	X
XV	X	1.1	0.20	X	X	X	X	X	X	X
XVI	X	X	X	X	X	X	X	X	8,000	X
XVII	X	X	X	X	X	X	X	X	10,500	X
XVIII	X	X	X	X	X	X	X	X	12,000	X
XIX	X	X	X	X	X	X	X	X	X	200

Case I - standard
X - indicates parameter same as for Case I

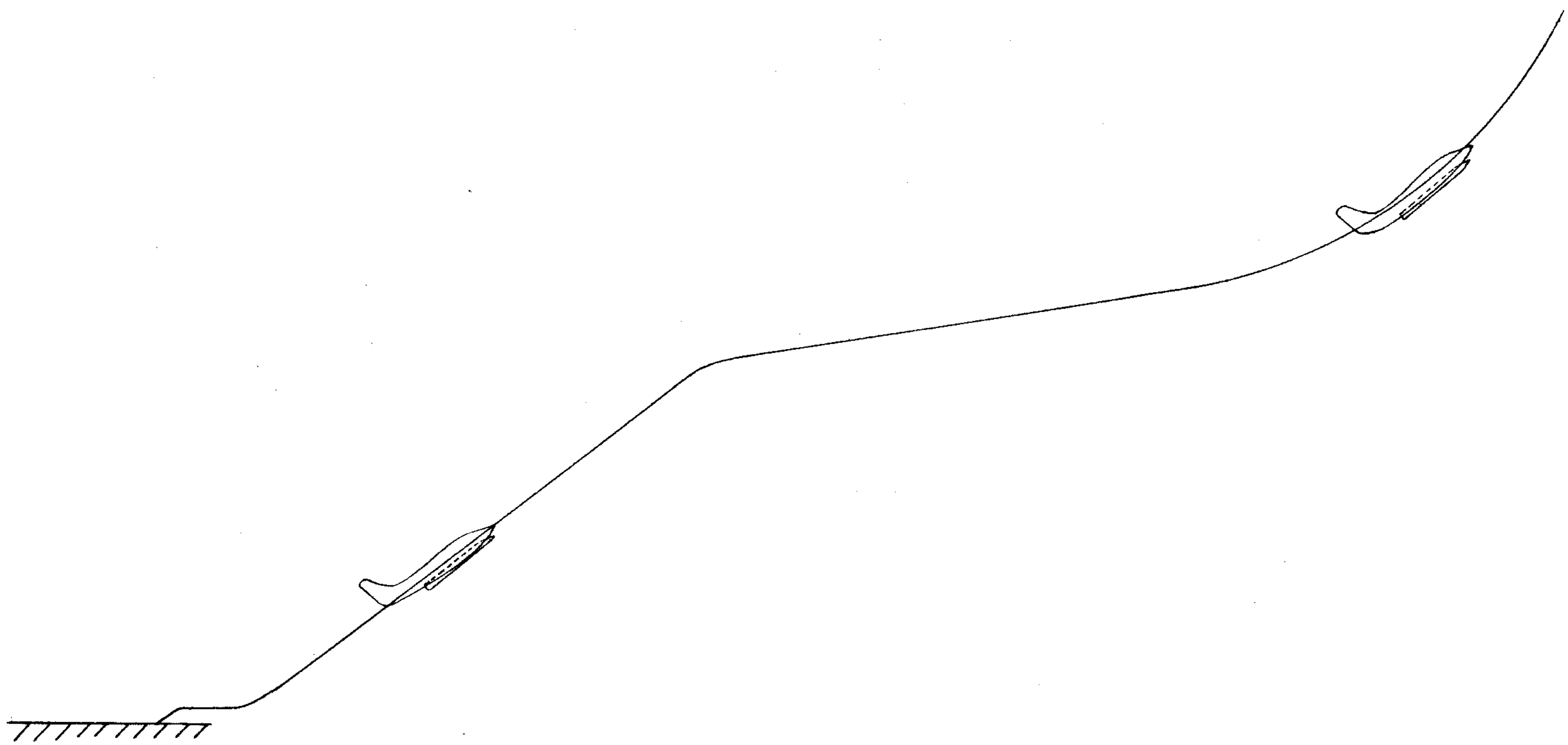

FIG. I — AIR BREATHING BOOST FLIGHT PROFILE FOR SATELLITE LAUNCHING

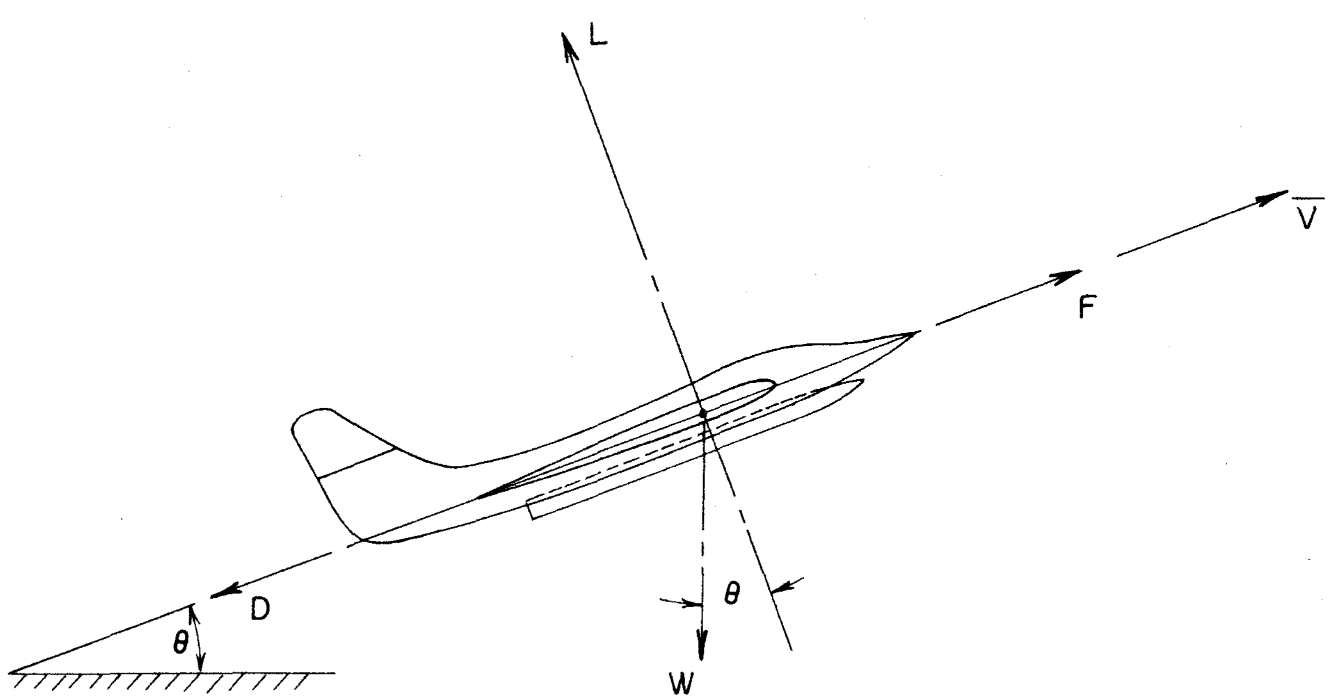

FIG. 2 - FORCES ON AIR BREATHING BOOST DURING CLIMB

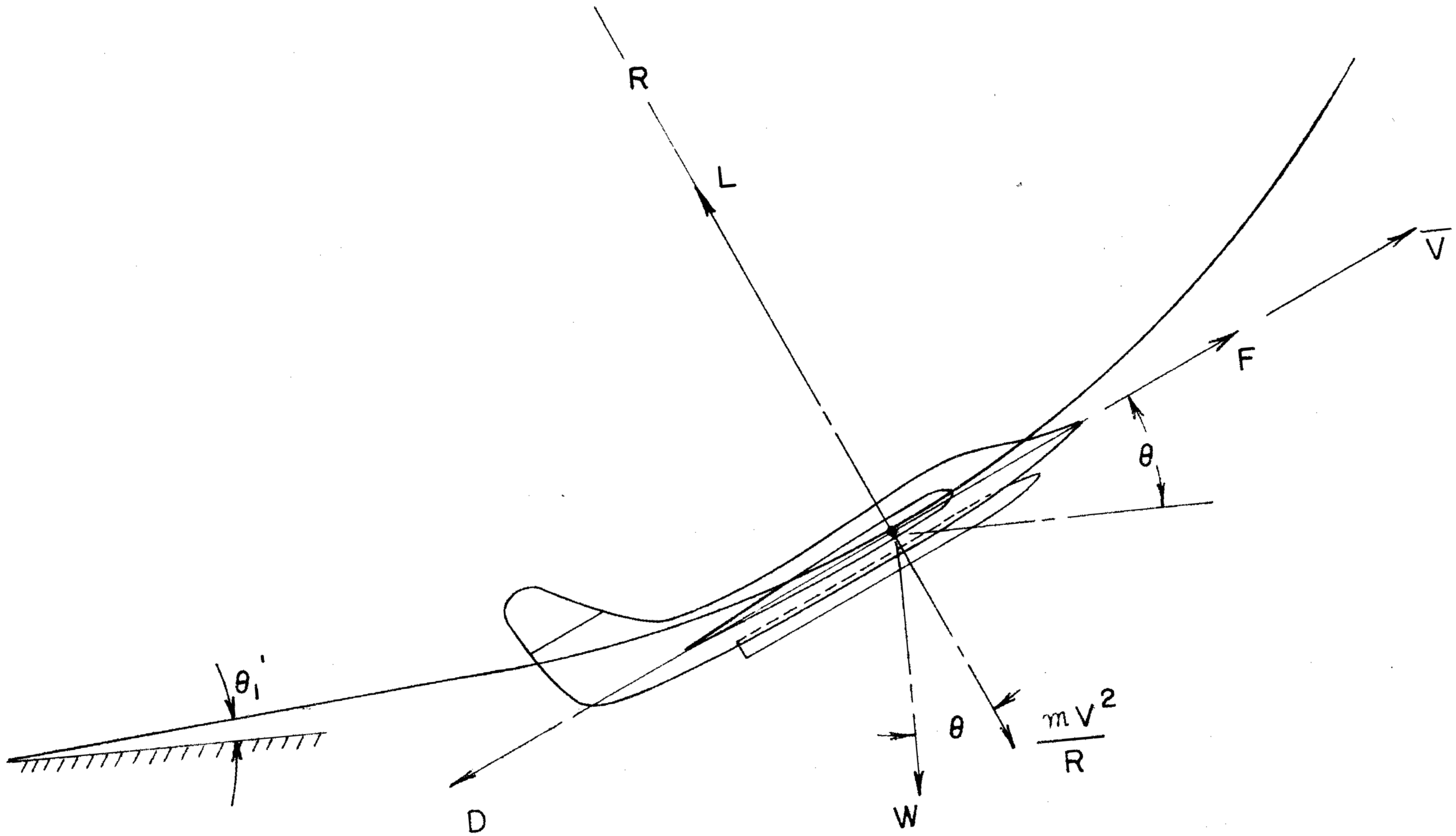

FIG. 3 - FORCES ON AIR BREATHING BOOST DURING PULL UP

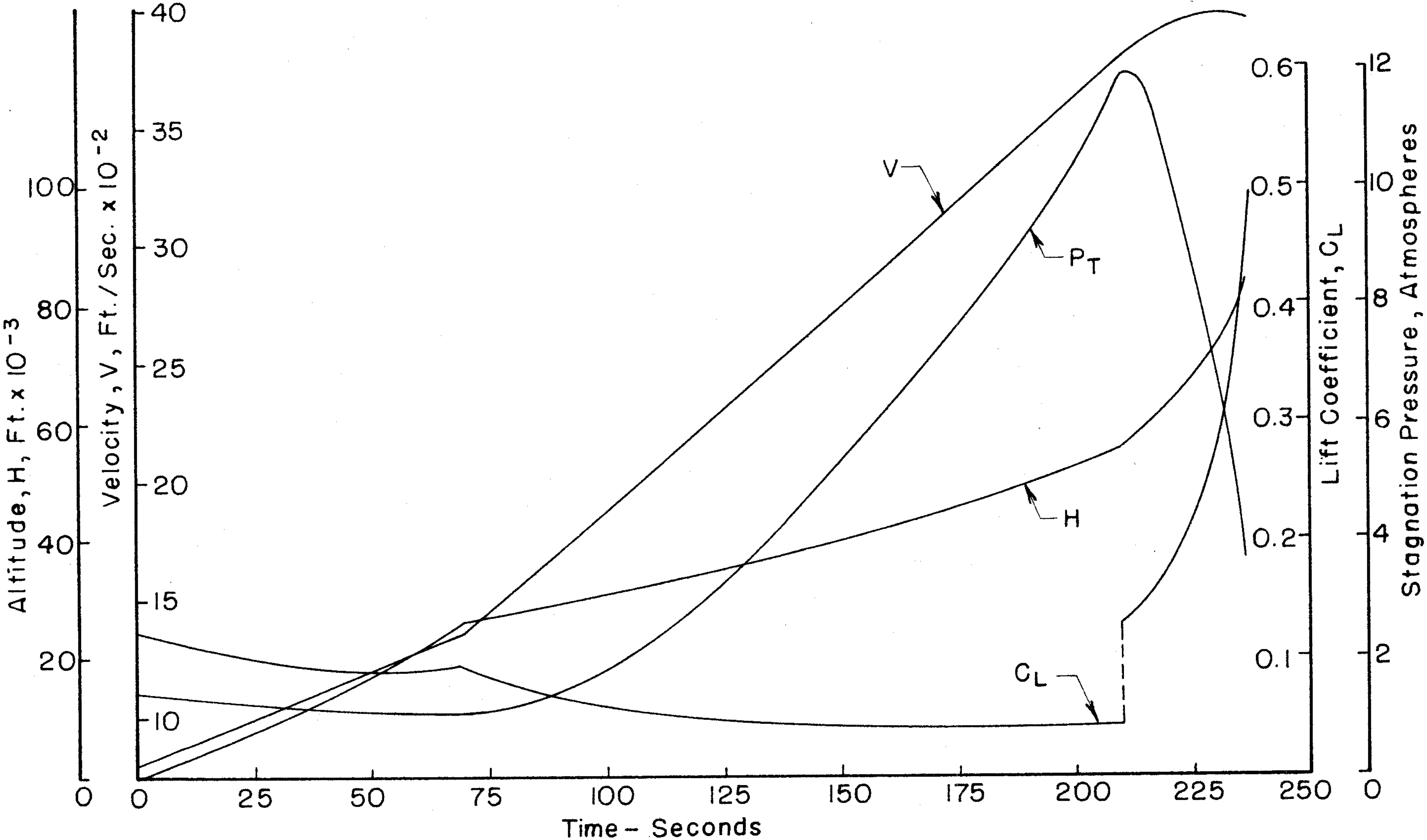

Fig. 4 Variation in Velocity, Altitude, Lift Coefficient and Stagnation Pressure with Time for Launch at Mach Number of Four, Standard Case.

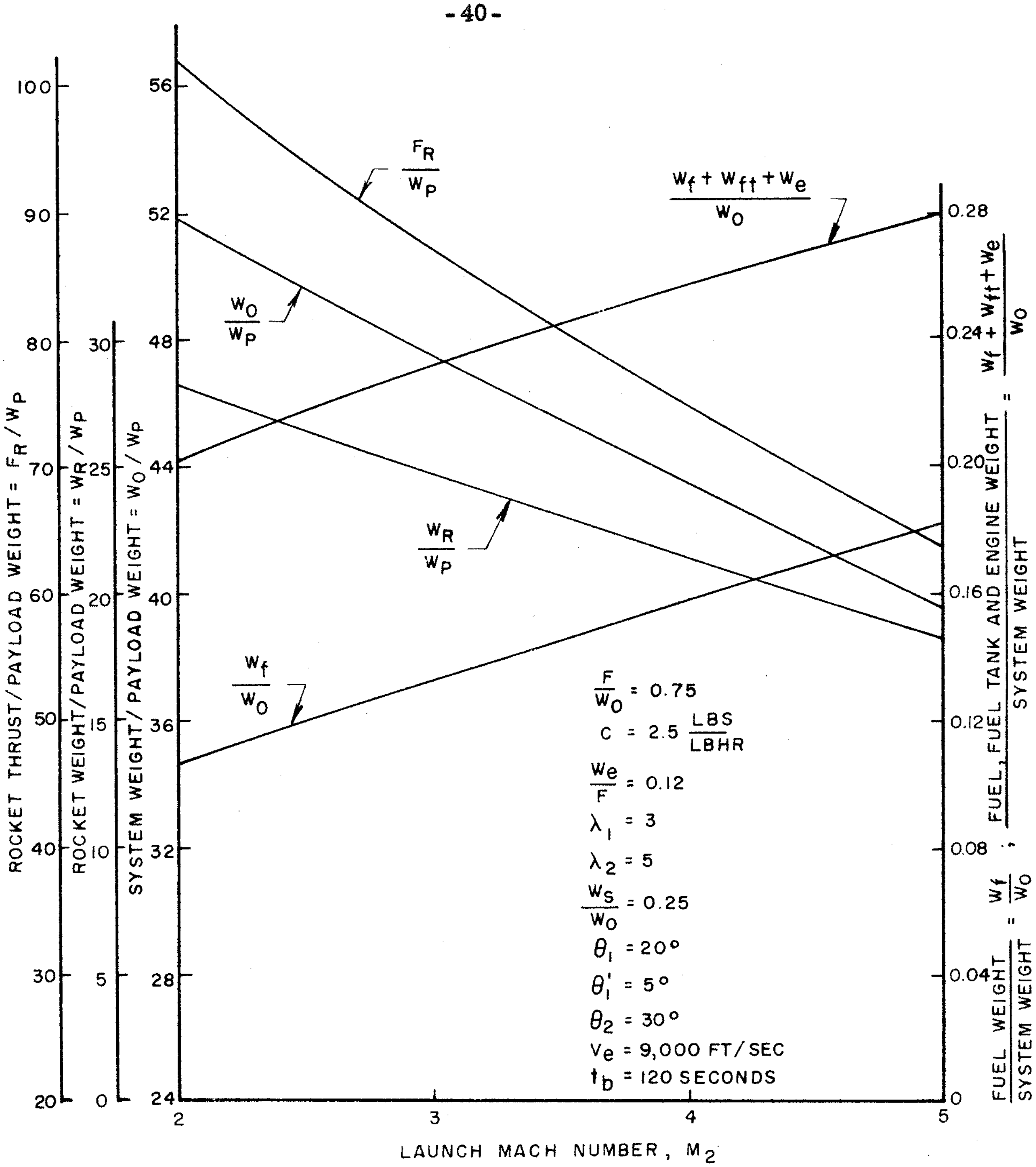

Fig. 5. Rocket Thrust/Payload Weight; System Weight/Payload Weight; Rocket Weight/Payload Weight; Fuel Weight/System Weight; and Fuel, Fuel Tanks and Engine Weight/System Weight Versus Launch Mach Number for Standard Case.

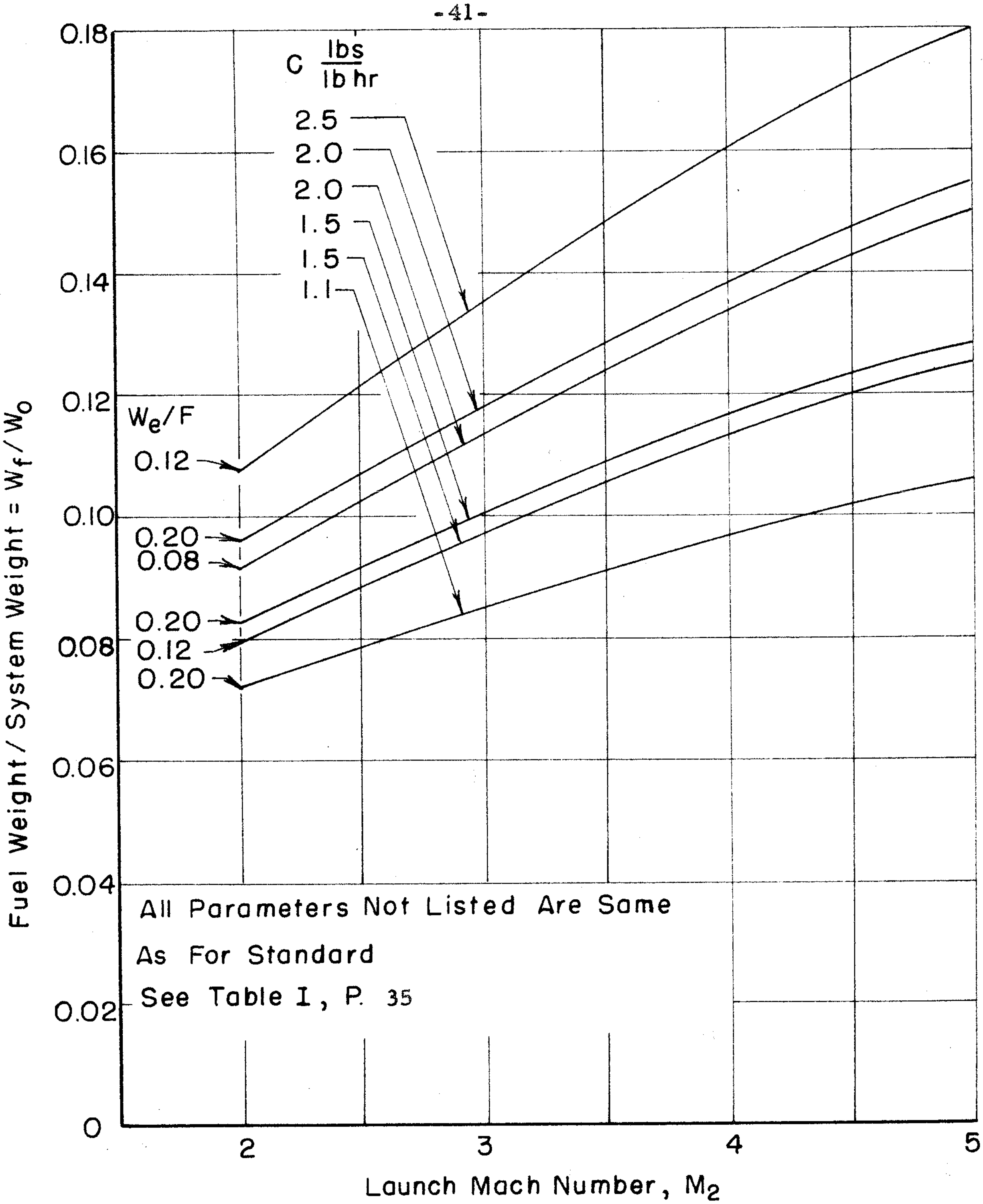

Fig. 6. Ratio of Fuel Weight to System Weight Versus Launch Mach Number for Different Values of Thrust Loading and Engine Specific Fuel Consumption.

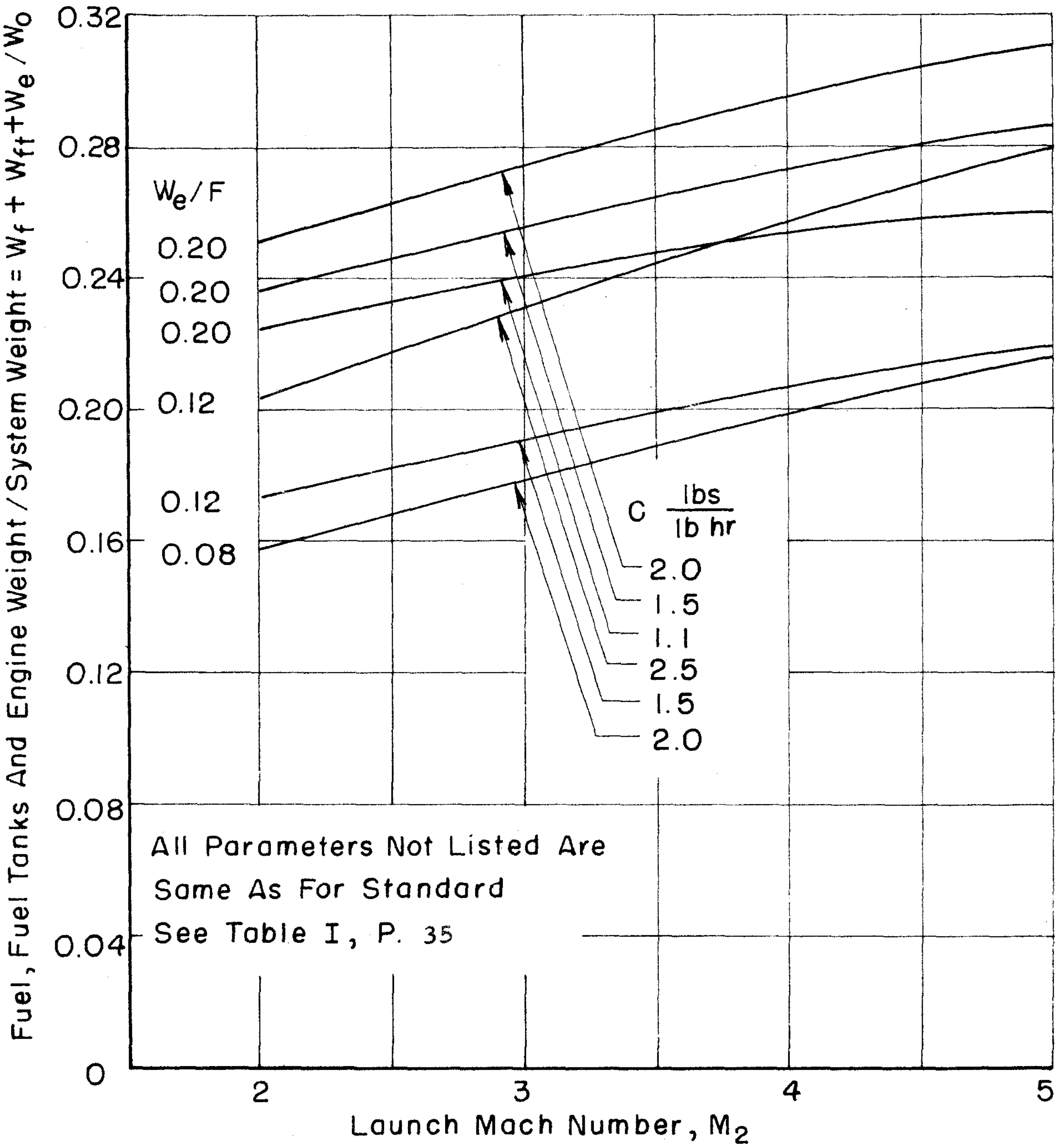

Fig. 7. Ratio of Fuel, Fuel Tanks, and Engine Weight to System Weight Versus Launch Mach Number for Different Values of Thrust Loading and Engine Specific Fuel Consumption.

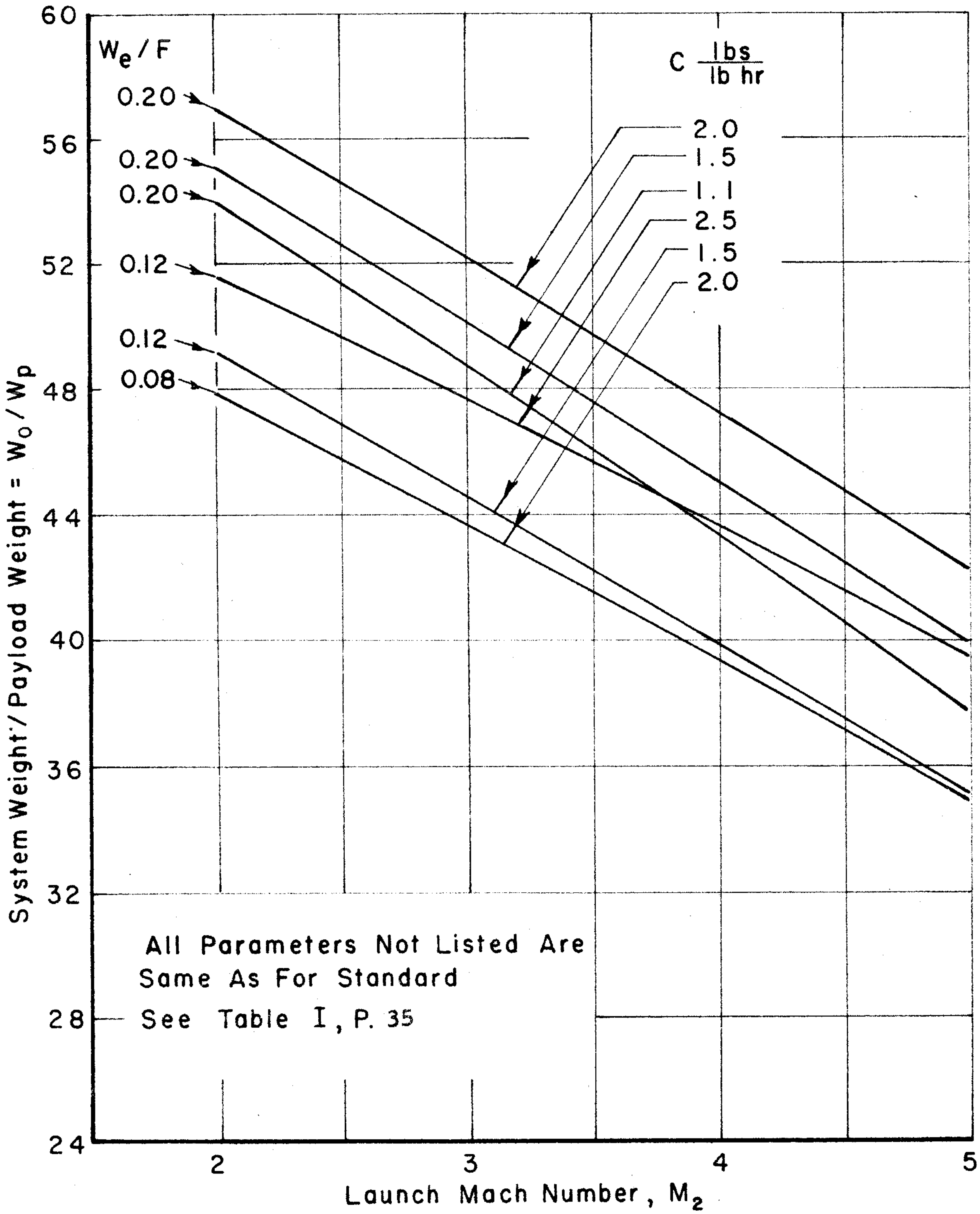

Fig. 8. Ratio of System Weight to Payload Weight Versus Launch Mach Number for Different Values of Thrust Loading and Engine Specific Fuel Consumption.

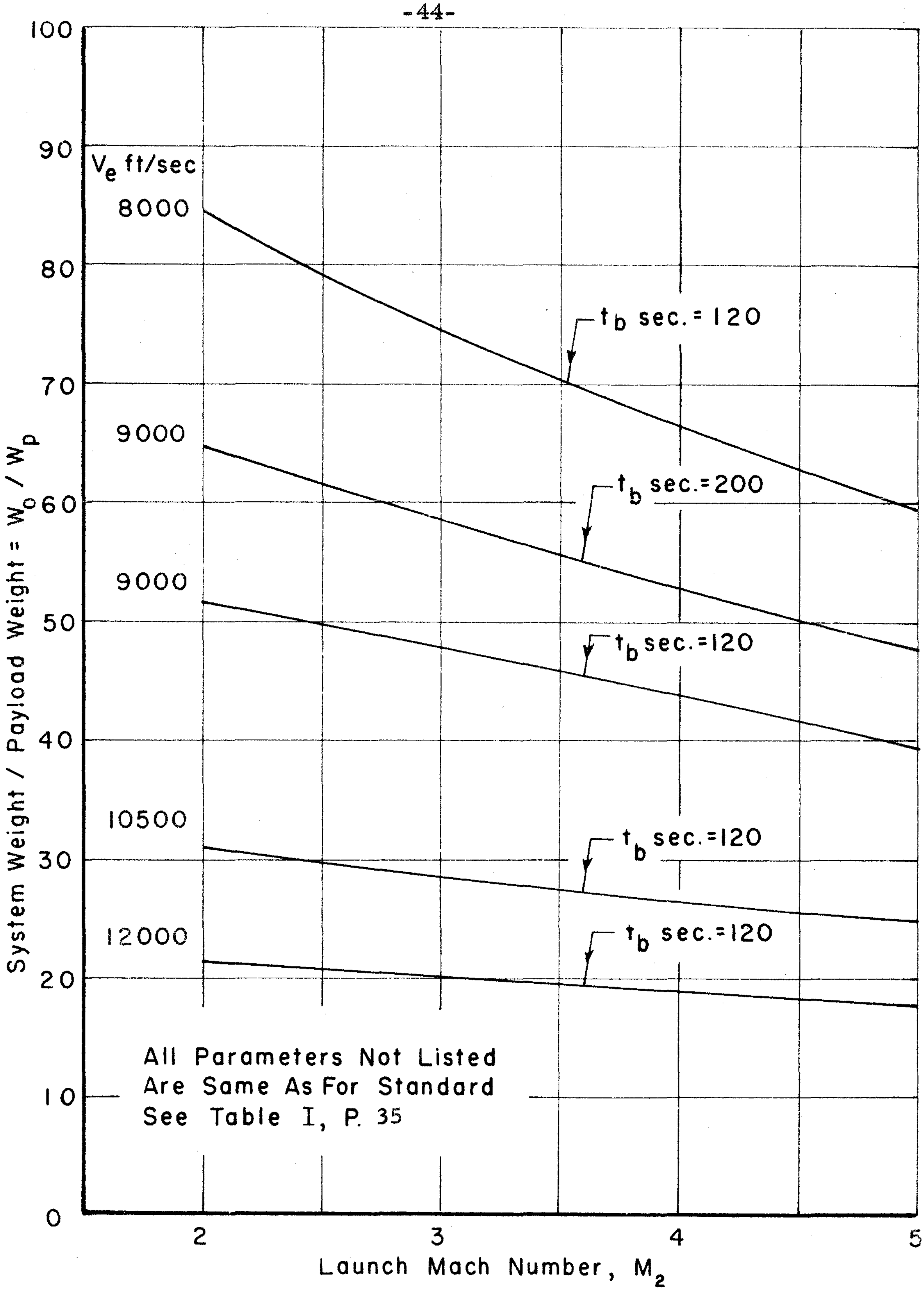

Fig. 9. Ratio of System Weight to Payload Weight Versus Launch Mach Number for Different Values of Rocket Burning Time and Effective Exhaust Velocity.

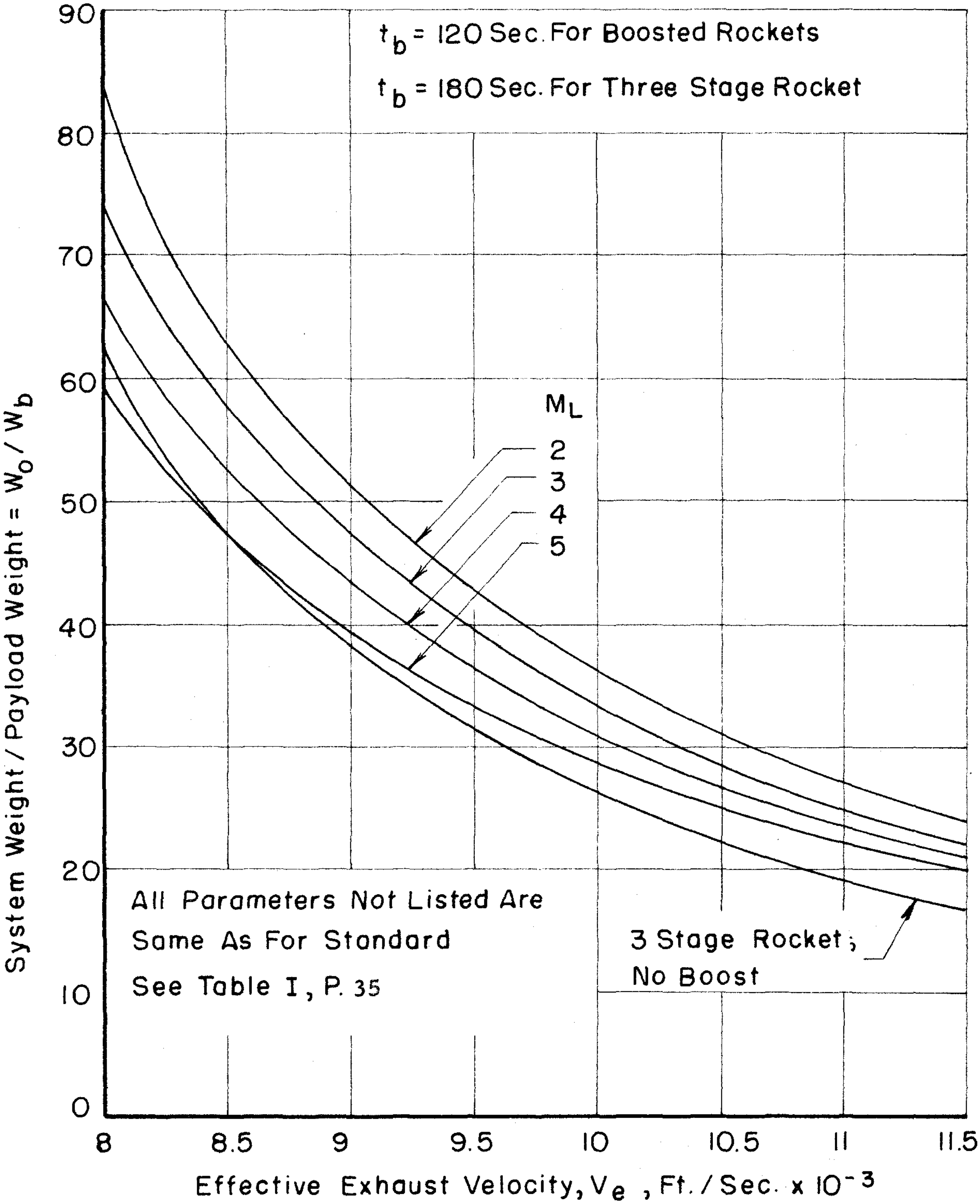

Fig. 10. Ratio of System Weight to Payload Weight Versus Effective Exhaust Velocity for Different Launch Mach Numbers.

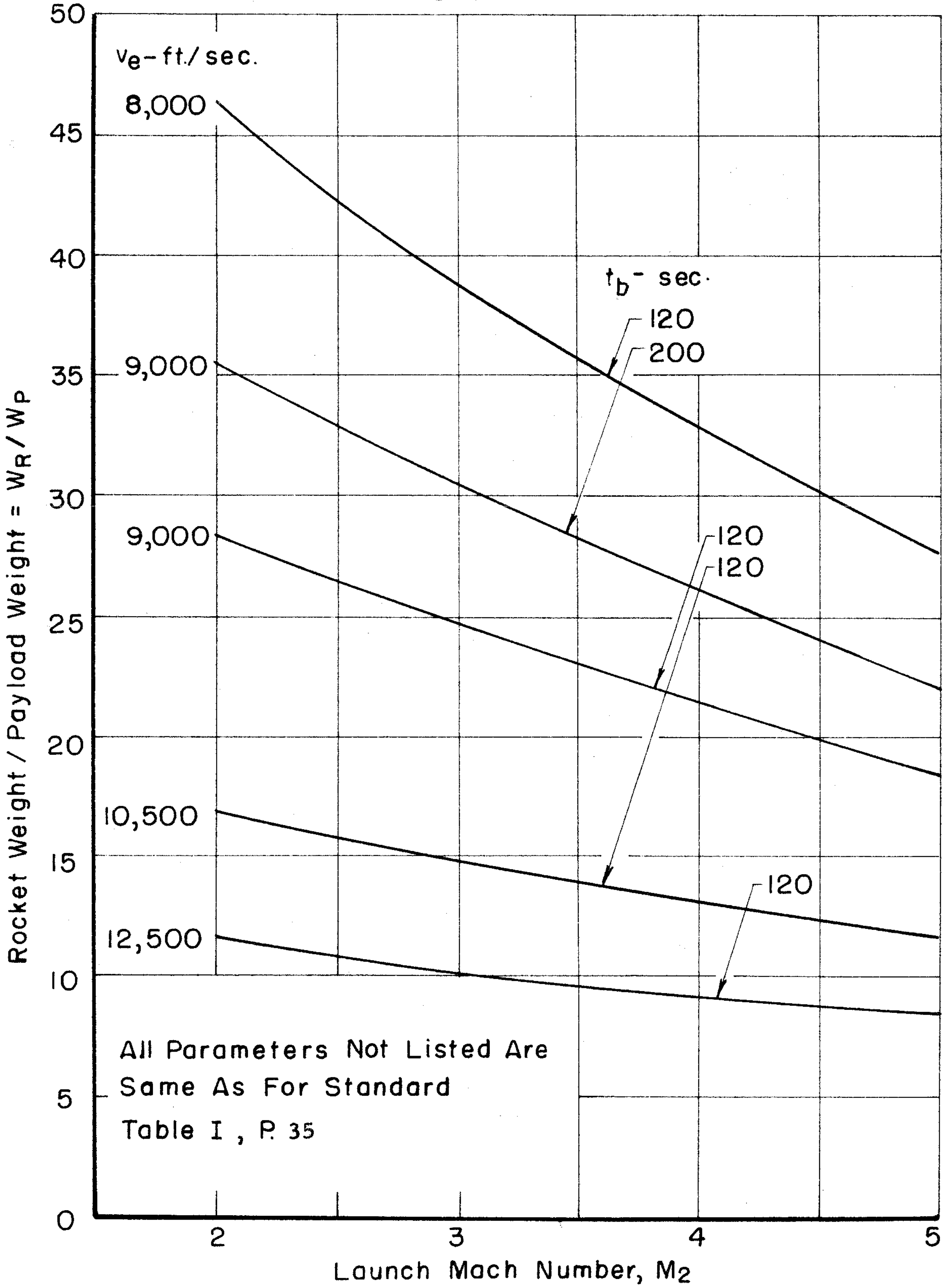

Fig. 11. Ratio of Rocket Weight to Payload Weight Versus Launch Mach Number for Different Rocket Burning Times and Effective Exhaust Velocities.

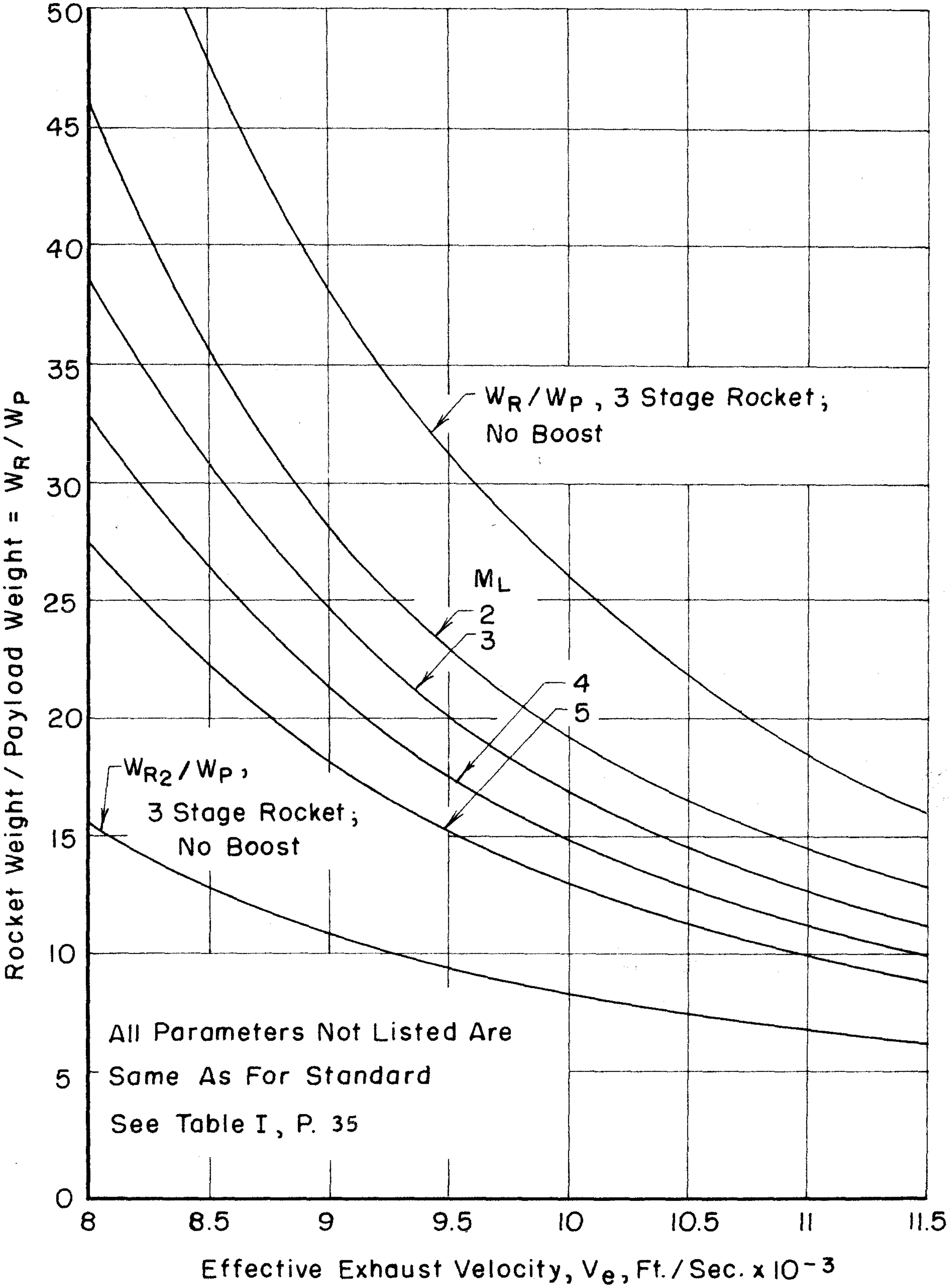

Fig. 12. Ratio of Rocket Weight to Payload Weight Versus Effective Exhaust Velocity for Different Launch Mach Numbers.

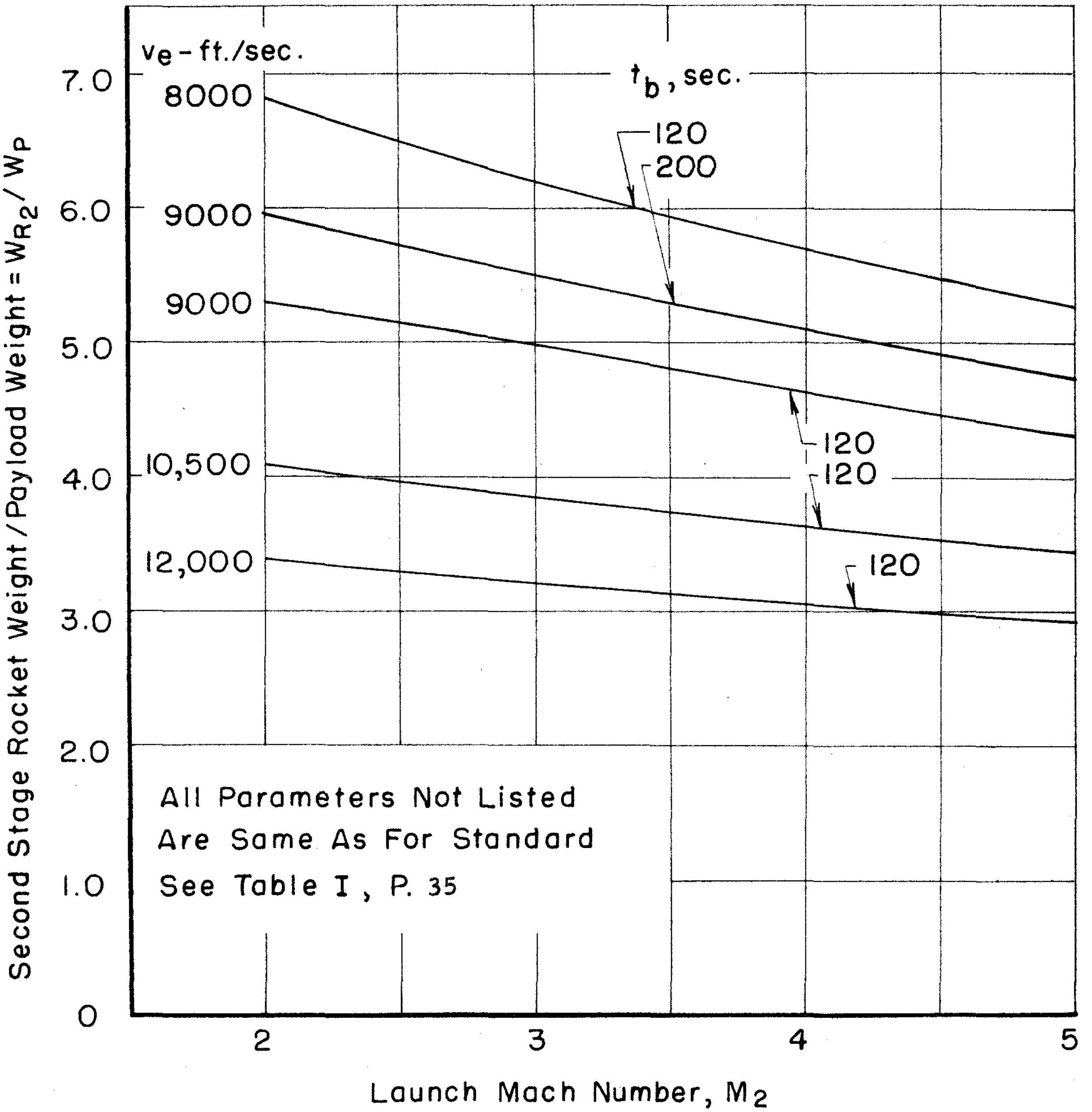

Fig. 13. Ratio of Second Stage Rocket Weight to Payload Weight Versus Launch Mach Number for Different Burning Times and Effective Exhaust Velocities.

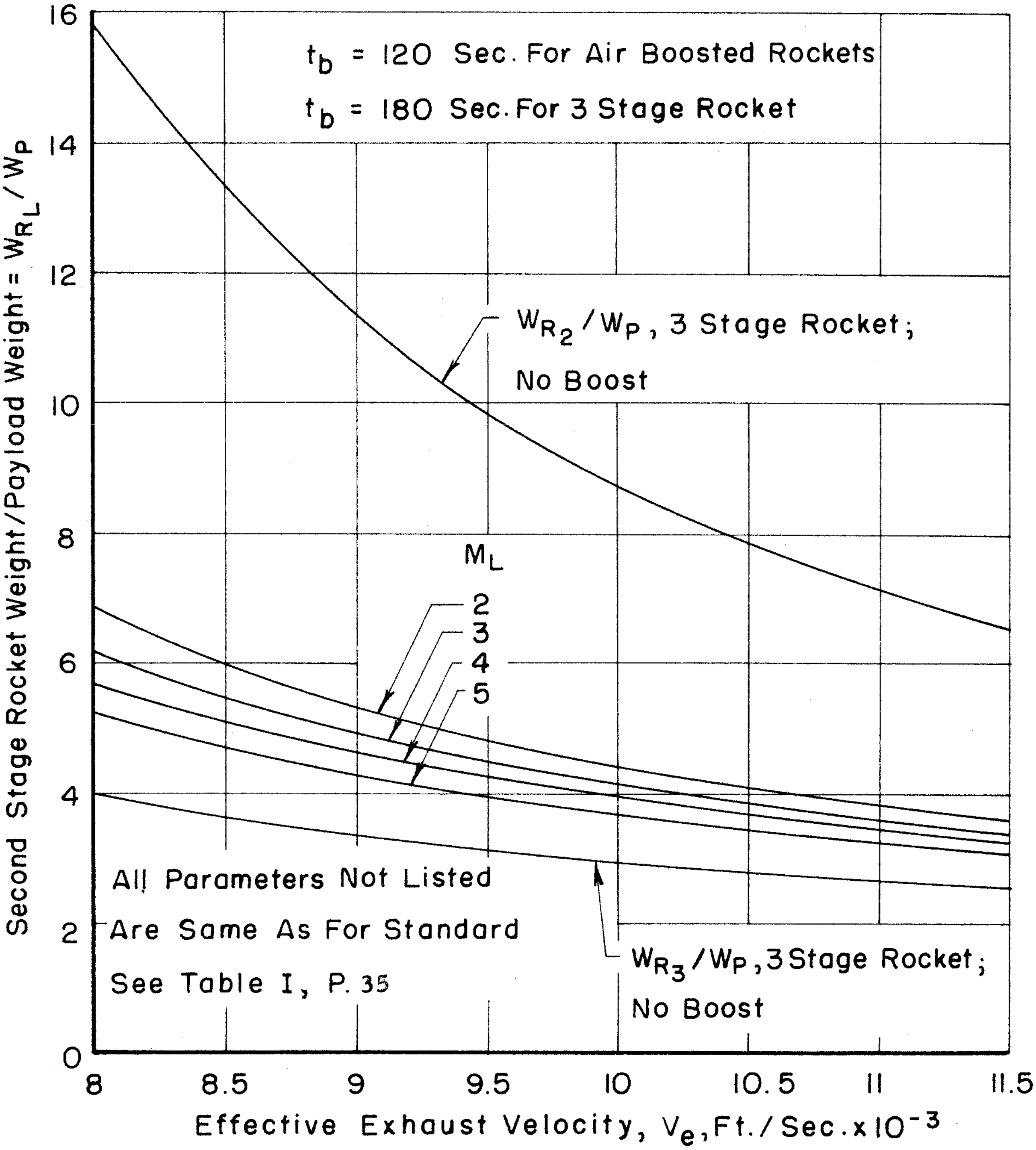

Fig. 14. Ratio of Second Stage Rocket Weight to Payload Weight Versus Effective Exhaust Velocity for Different Launch Mach Numbers.

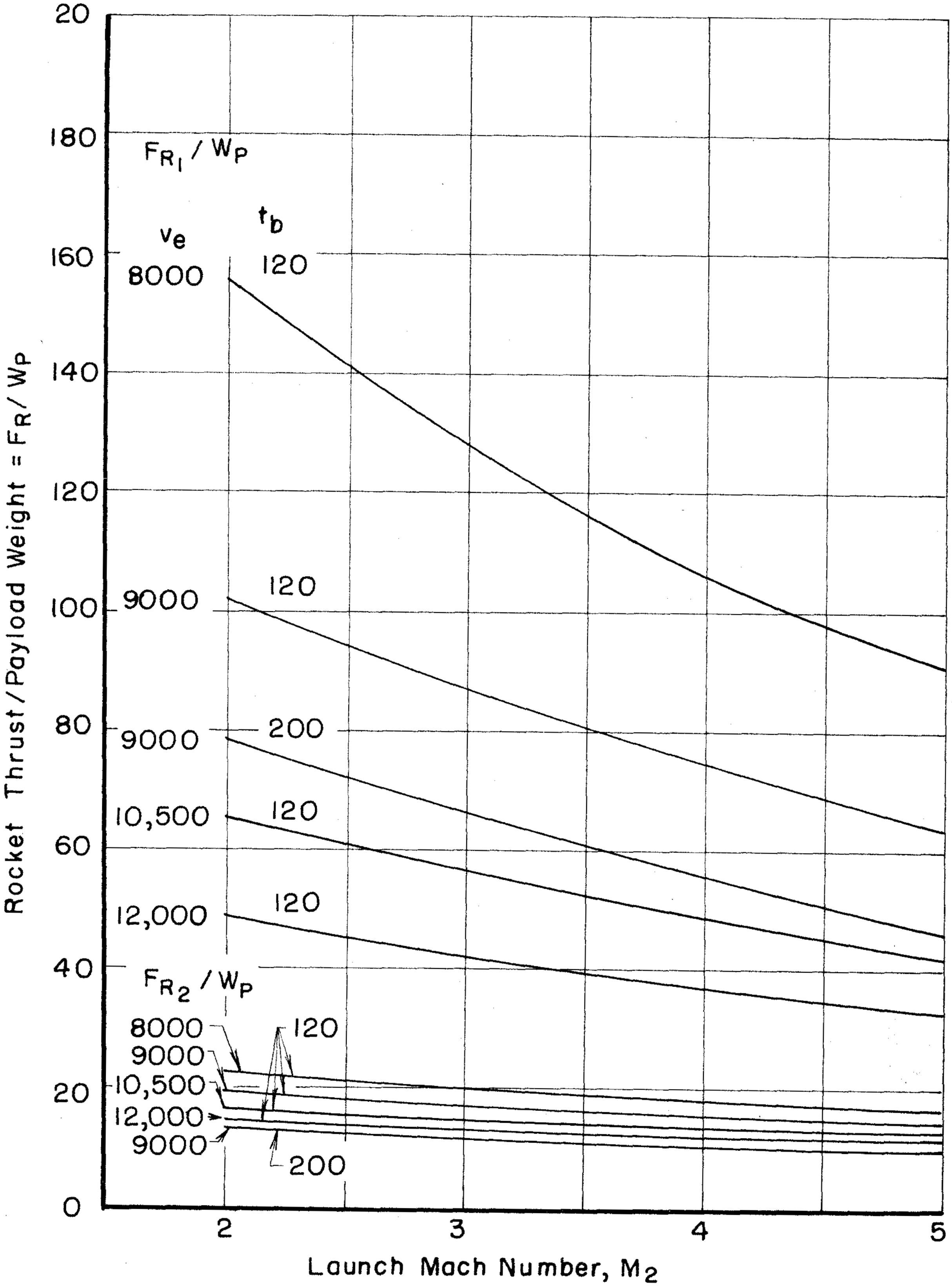

Fig. 15. Ratio of Rocket Thrusts to Payload Weight Versus Launch Mach Number for Different Burning Times and Effective Exhaust Velocities.

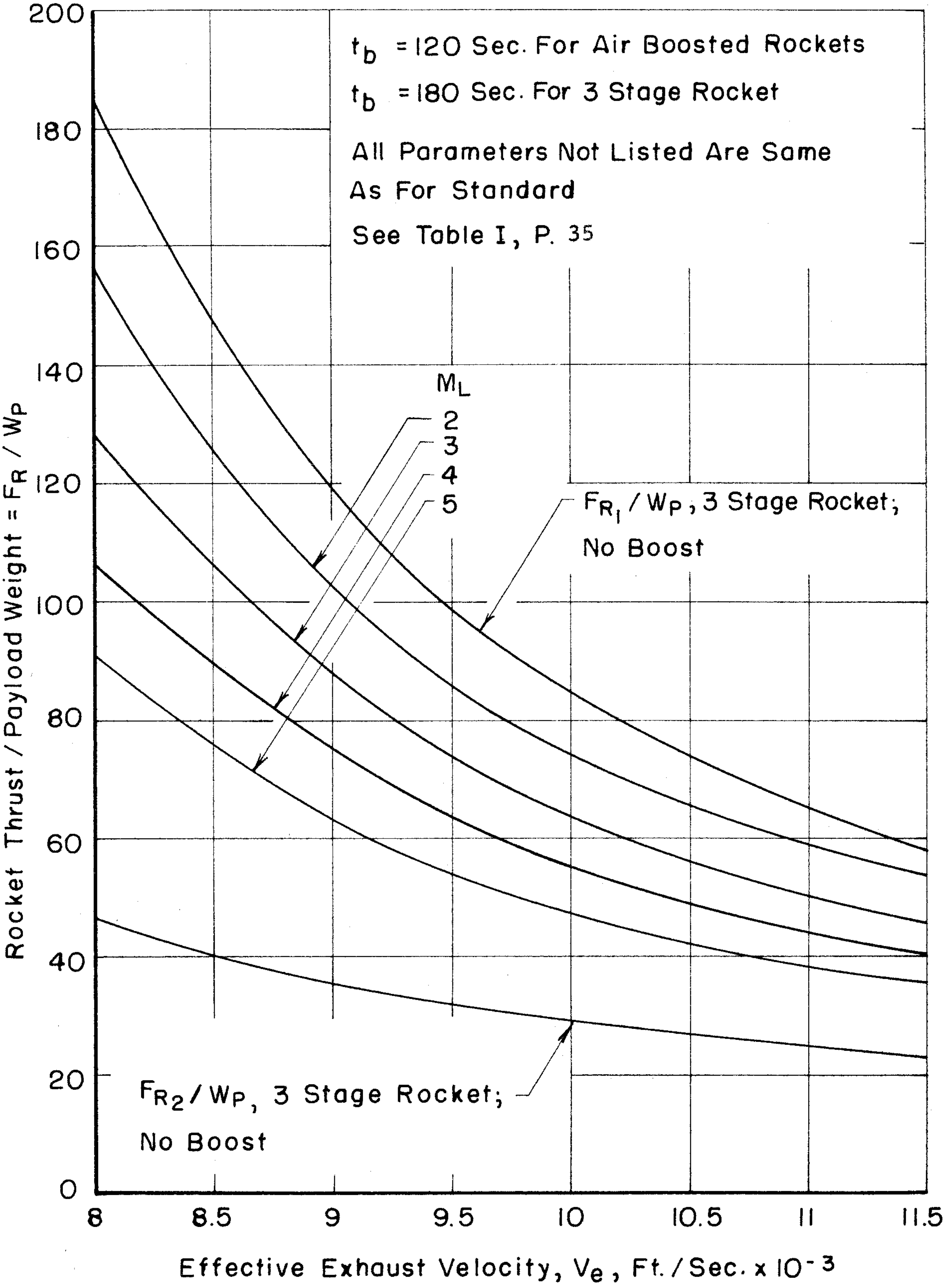

Fig. 16. Ratio of Rocket Thrust to Payload Weight Versus Effective Exhaust Velocity for Different Launch Mach Numbers.

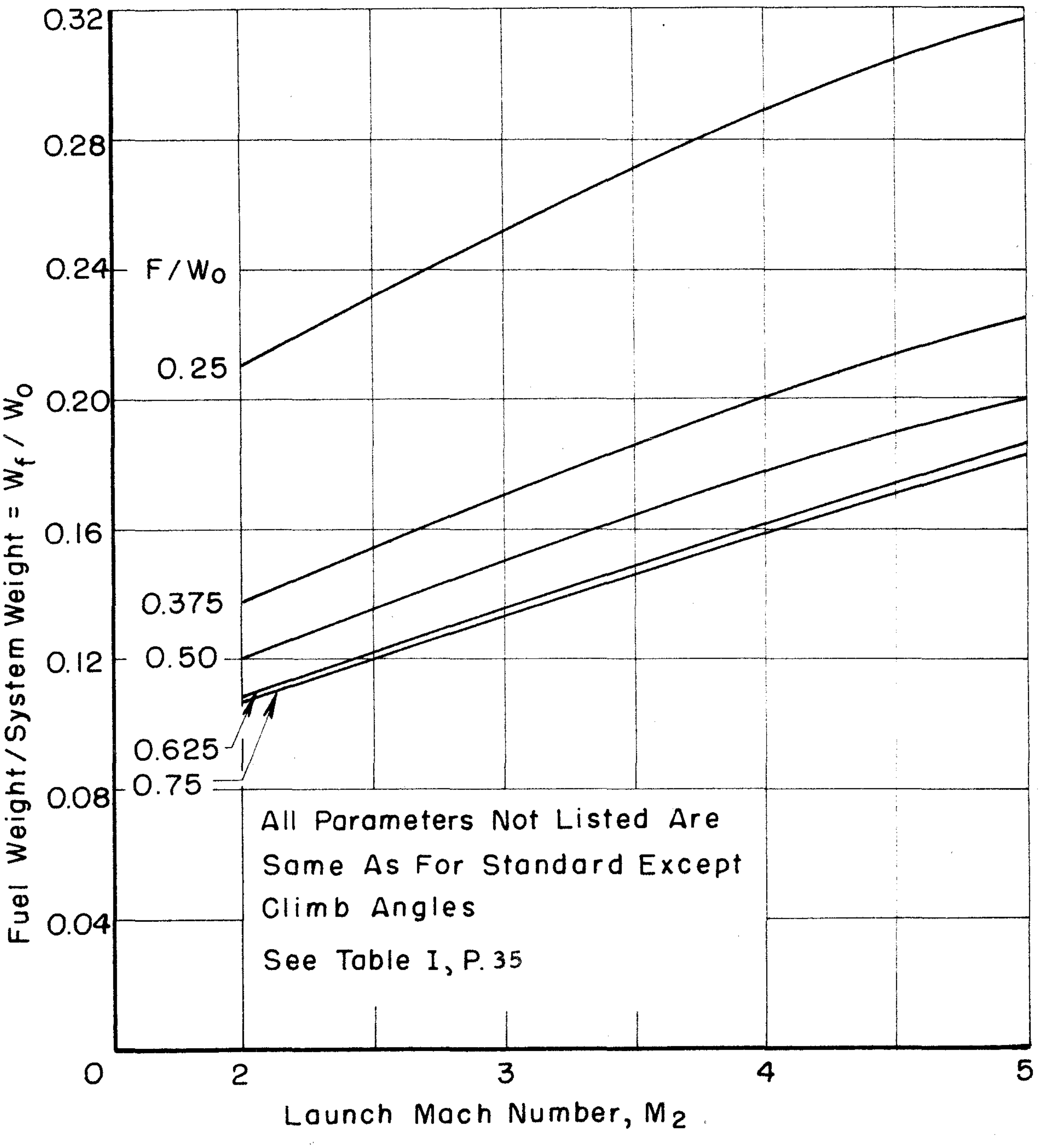

Fig. 17. Ratio of Fuel Weight to System Weight Versus Launch Mach Number for Different Thrust Loadings.

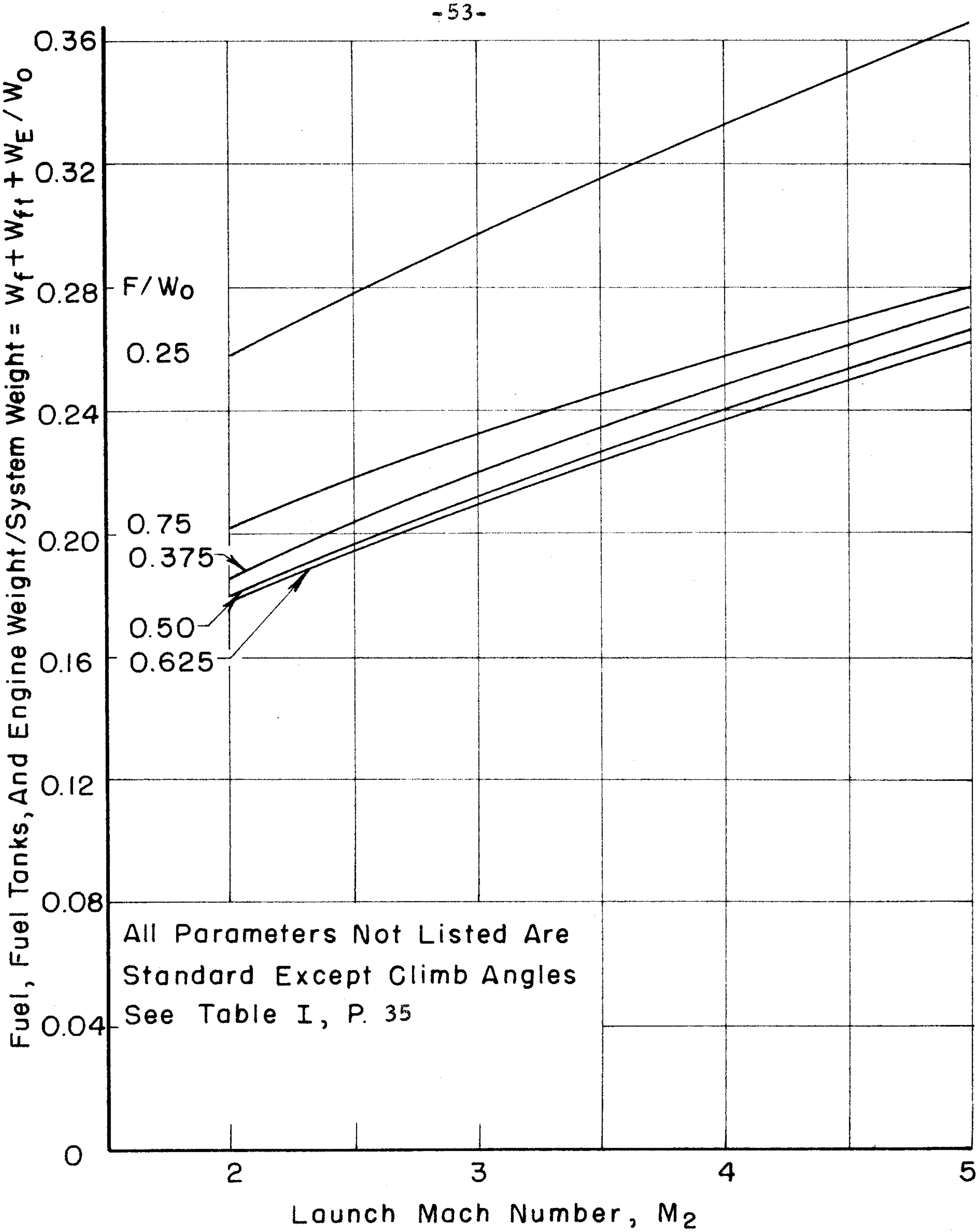

Fig. 18. Ratio of Fuel, Fuel Tanks and Engine Weight to System Weight Versus Launch Mach Number for Different Thrust Loadings.

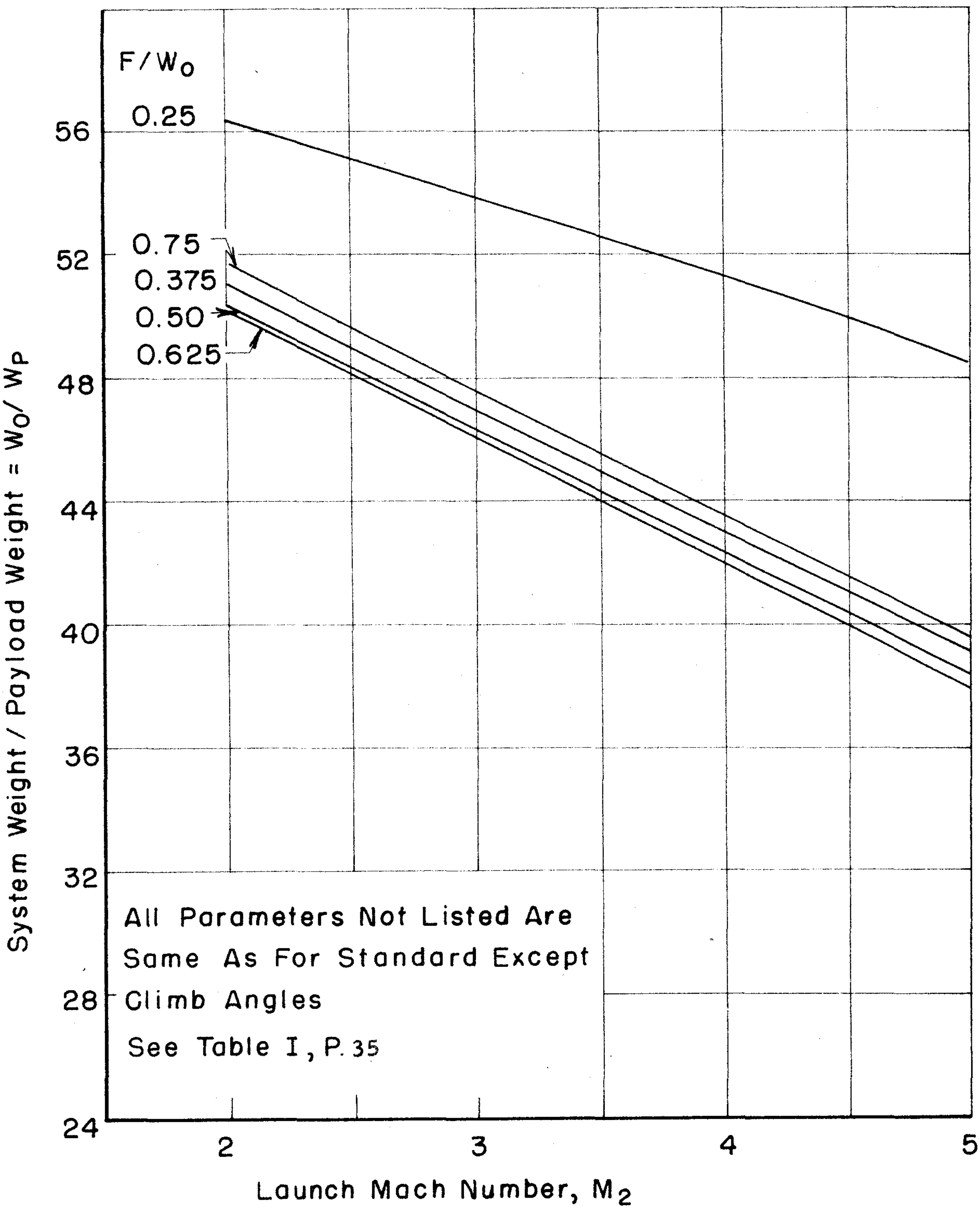

Fig. 19. Ratio of System Weight to Payload Weight Versus Launch Mach Number for Different Thrust Loadings.

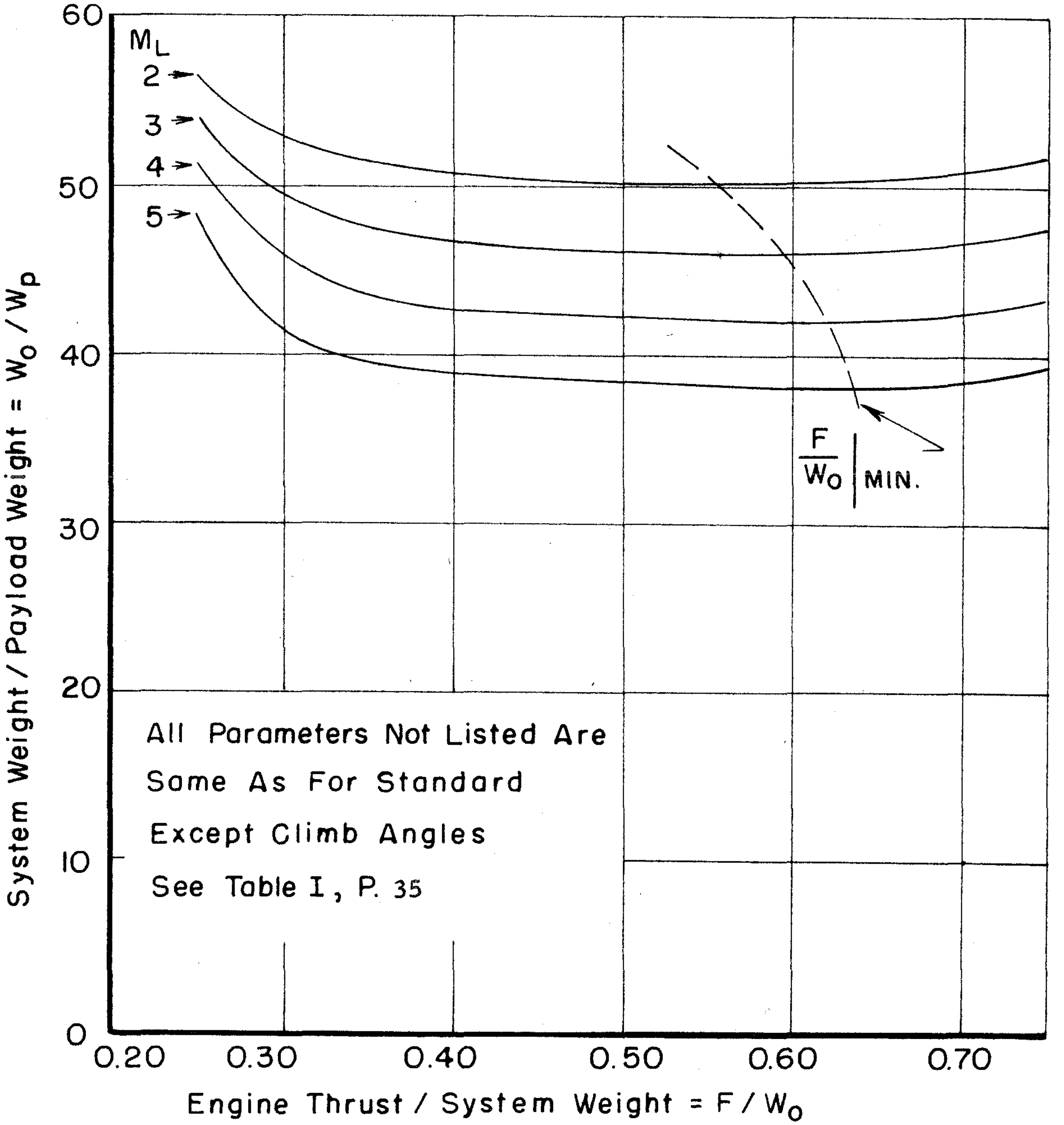

Fig. 20. Ratio of System Weight to Payload Weight Versus Engine Thrust to System Weight Ratio for Different Launch Mach Numbers.

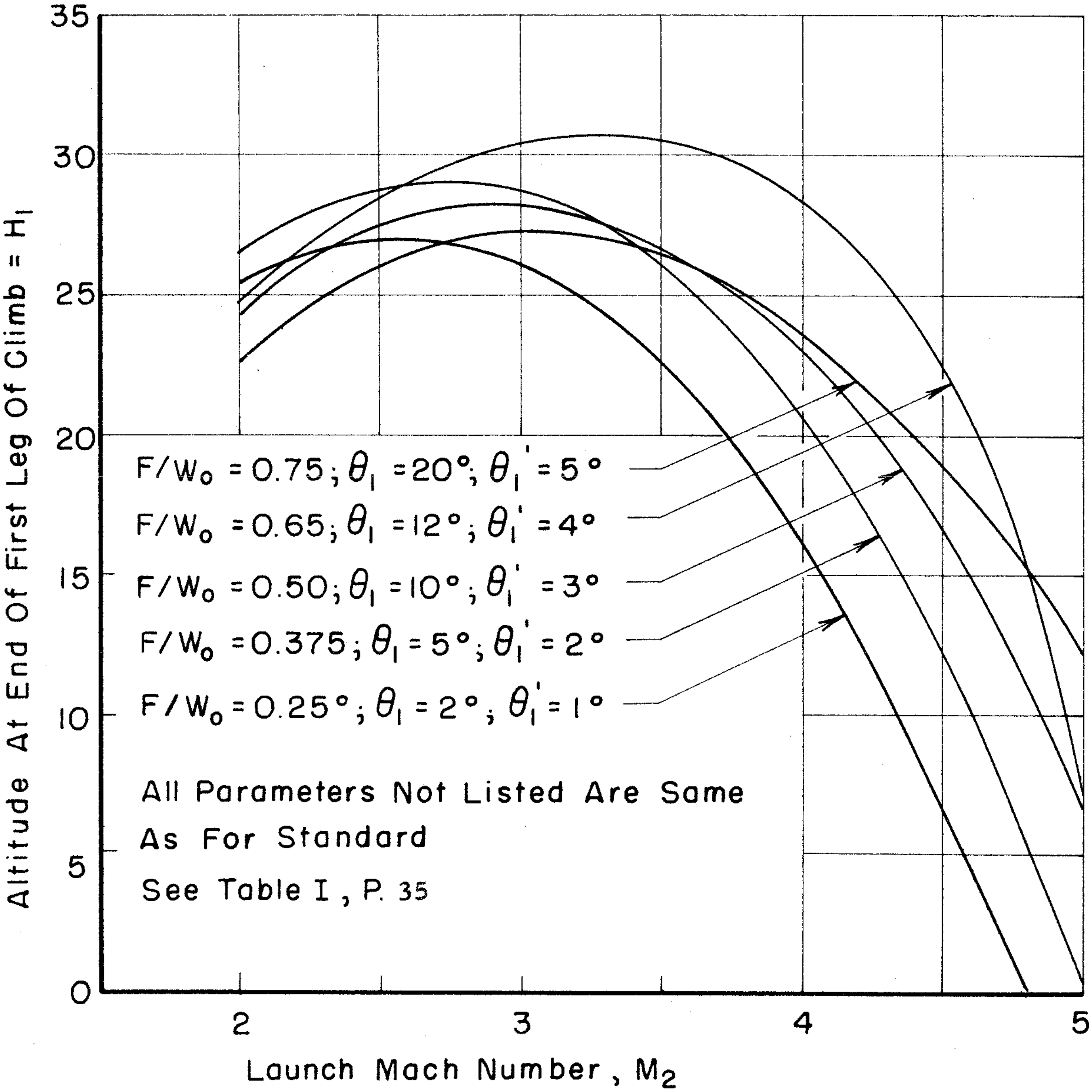

Fig. 21. Altitude at End of First Leg of Climb Versus Launch Mach Number for Different Thrust Loadings.

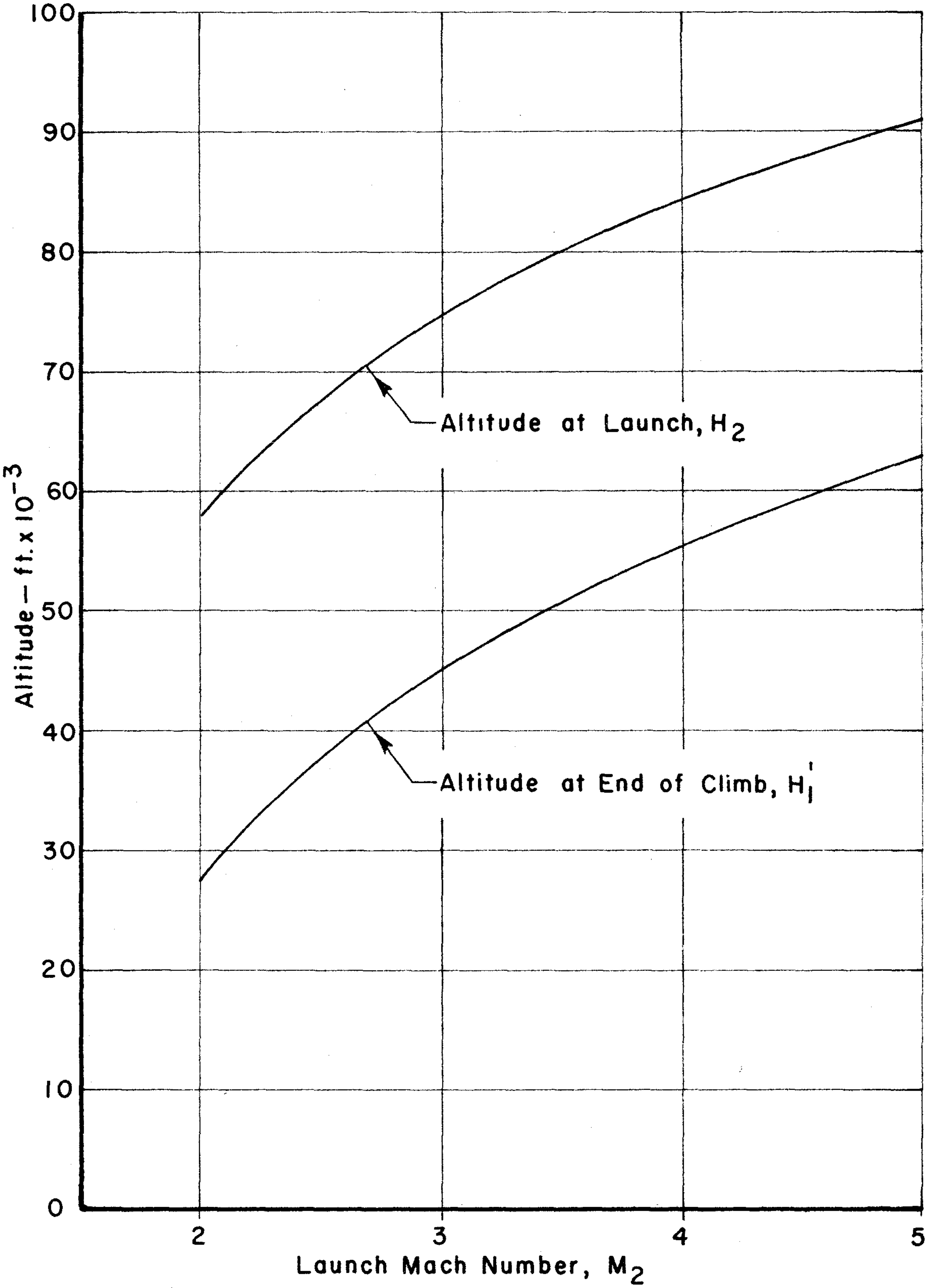

Fig. 22. Variation of Altitudes at End of Climb and Pull Up with Launch Mach Number for All Cases.

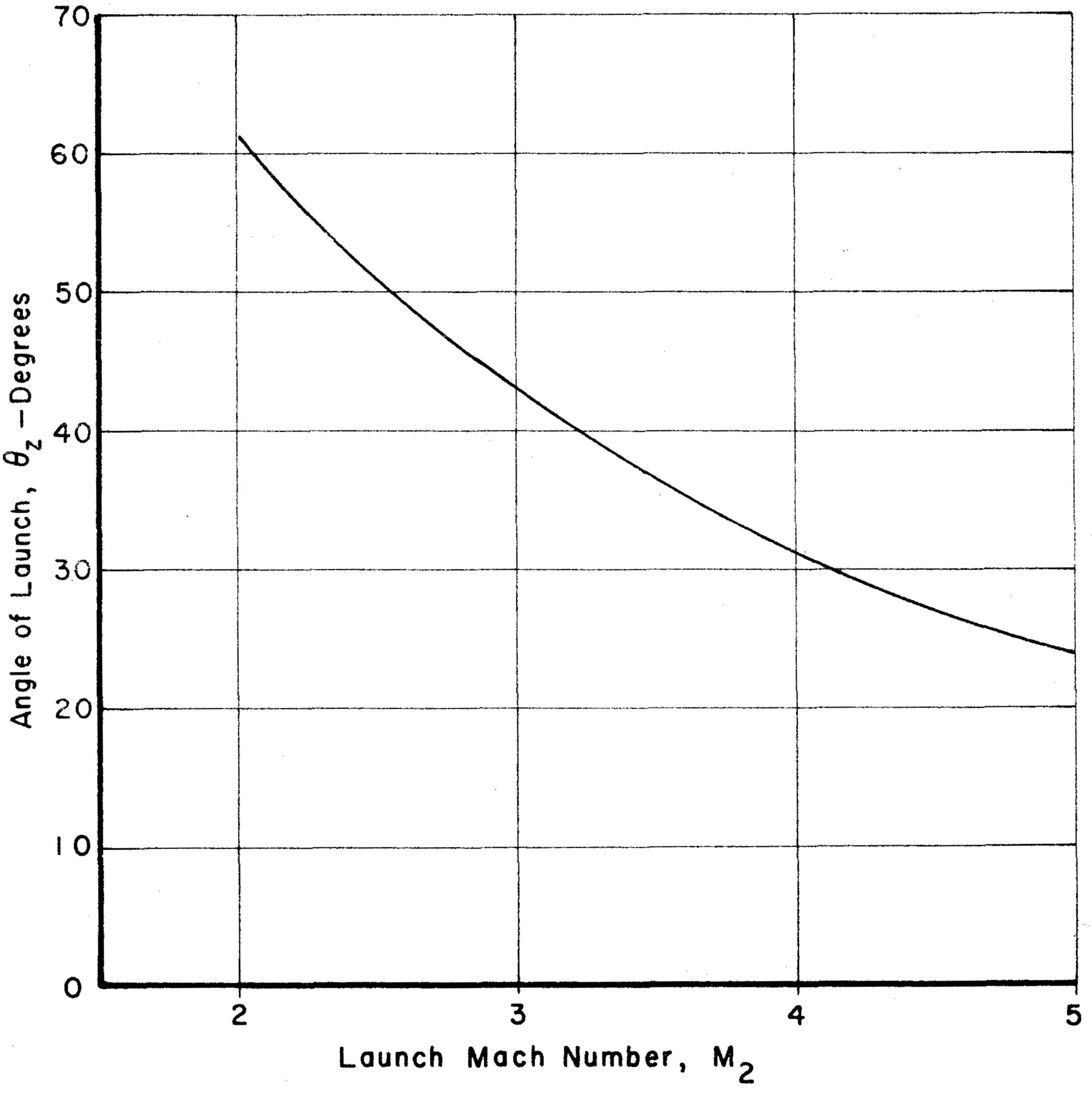

Fig. 23. Variation of Angle of Launch with Launch Mach Number for All Cases.

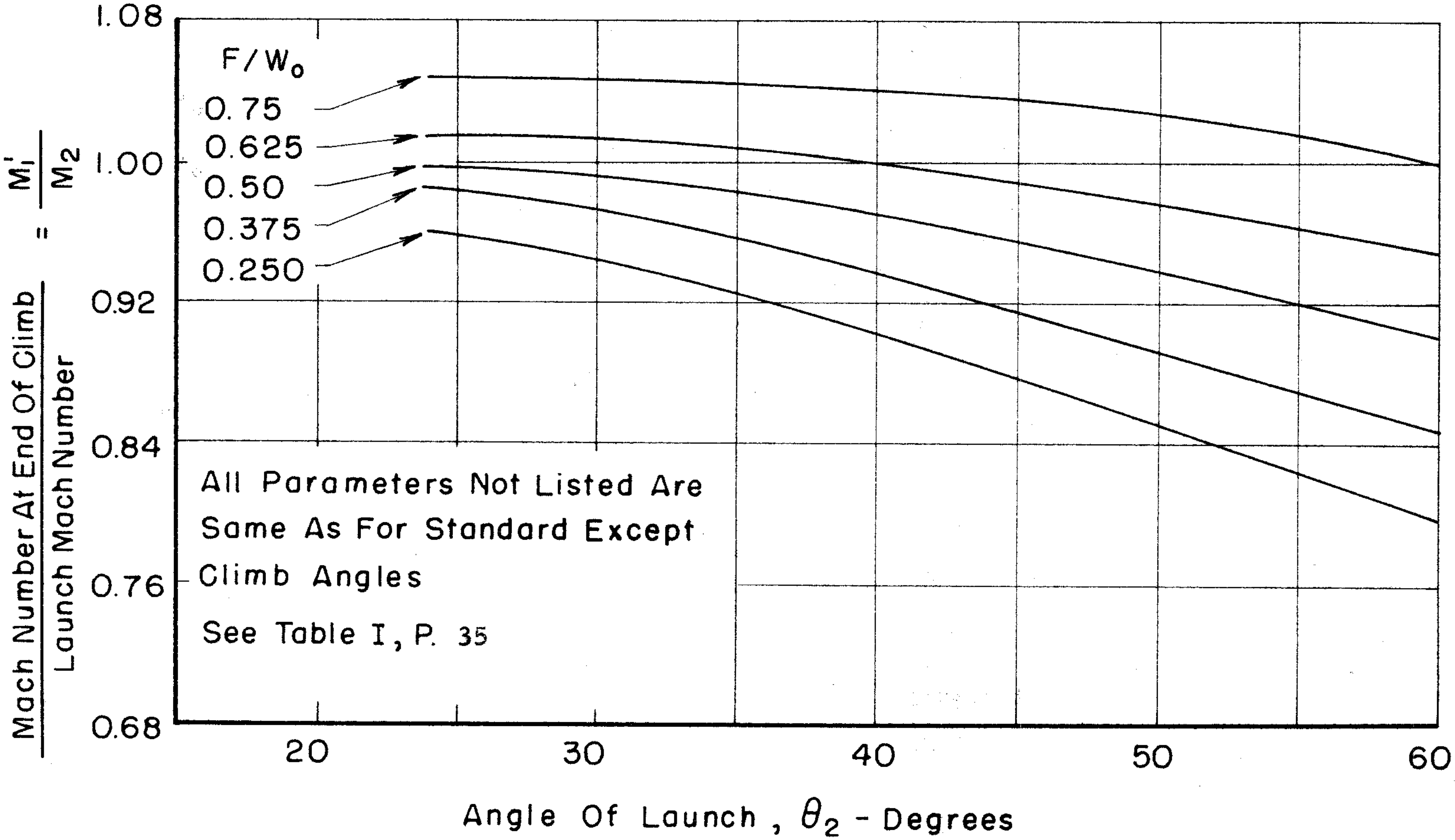

Fig. 24. Ratio of Mach Number at End of Climb to Launch Mach Number Versus Angle of Launch for Different Thrust Loadings.

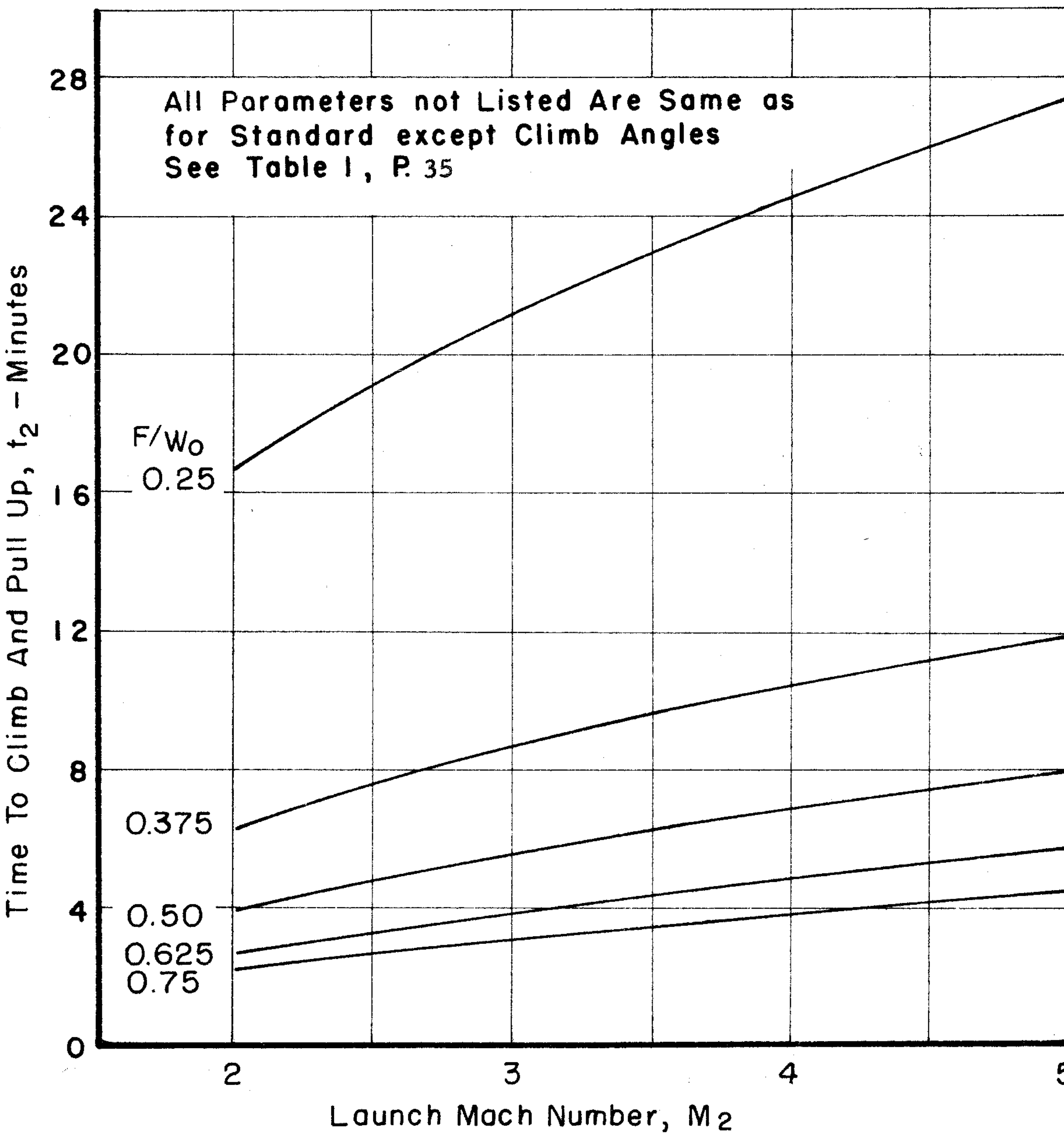

Fig. 25. Variation in Time to Climb and Pull Up to Launch Attitude Versus Launch Mach Number for Different Thrust Loadings.

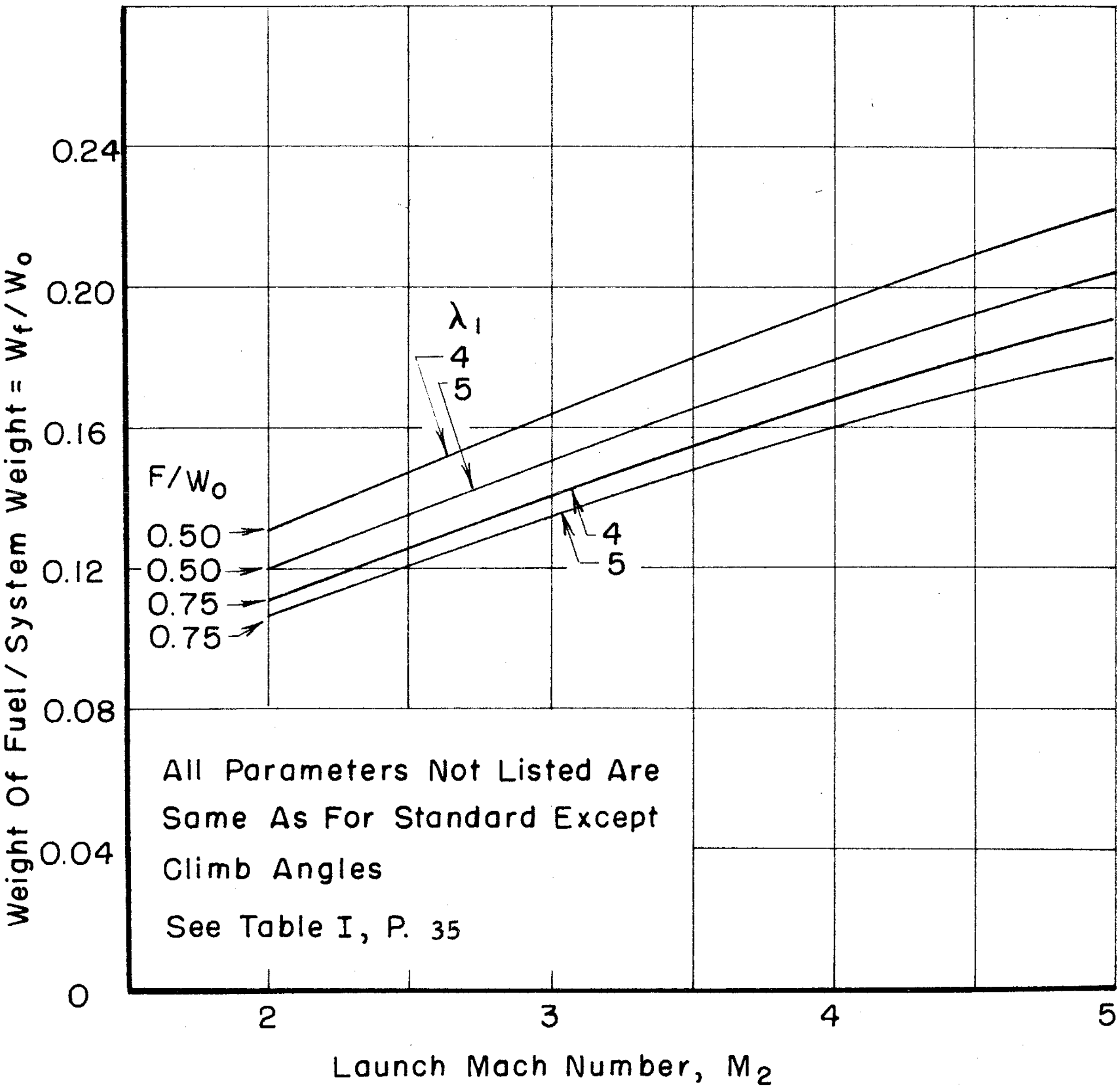

Fig. 26. Ratio of Fuel Weight to System Weight Versus Launch Mach Number for Different Values of Climb Lift to Drag Ratios and Thrust Loadings.

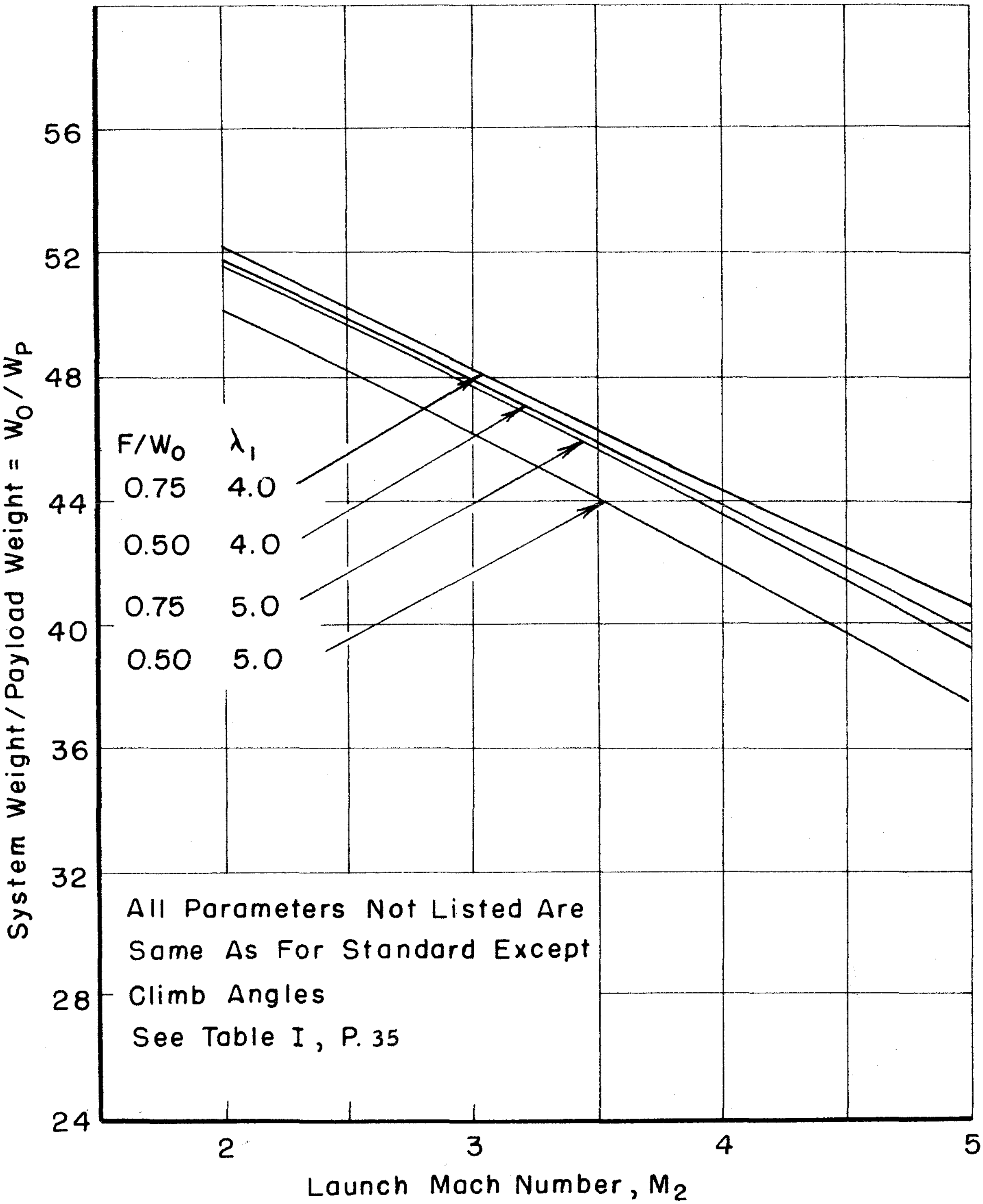

Fig. 27. Ratio of System Weight to Payload Weight Versus Launch Mach Number for Different Values of Climb Lift to Drag Ratios and Thrust Loadings.

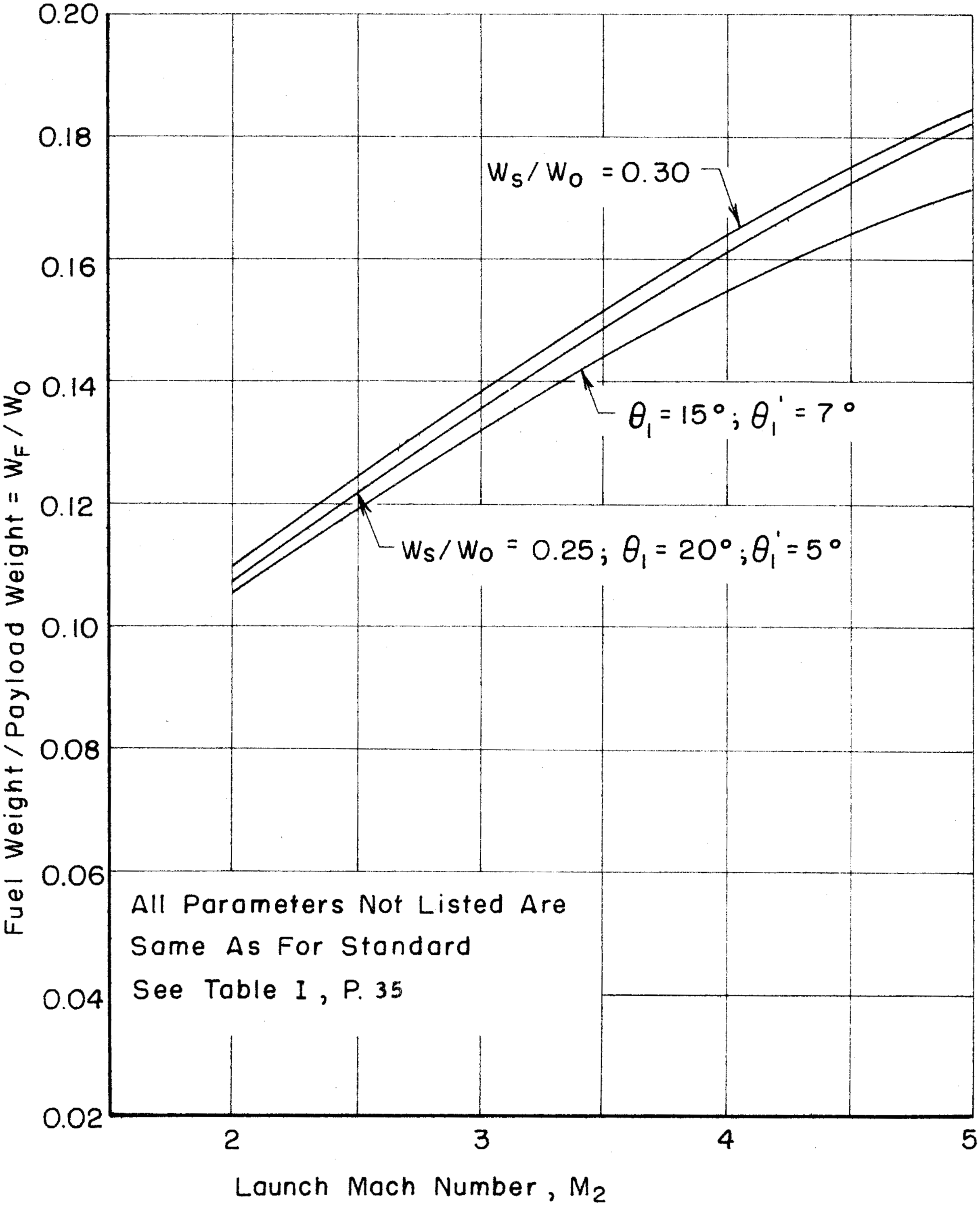

Fig. 28. Effect of Structural Weight and Angles of Climb on System Weight to Payload Weight Ratio.

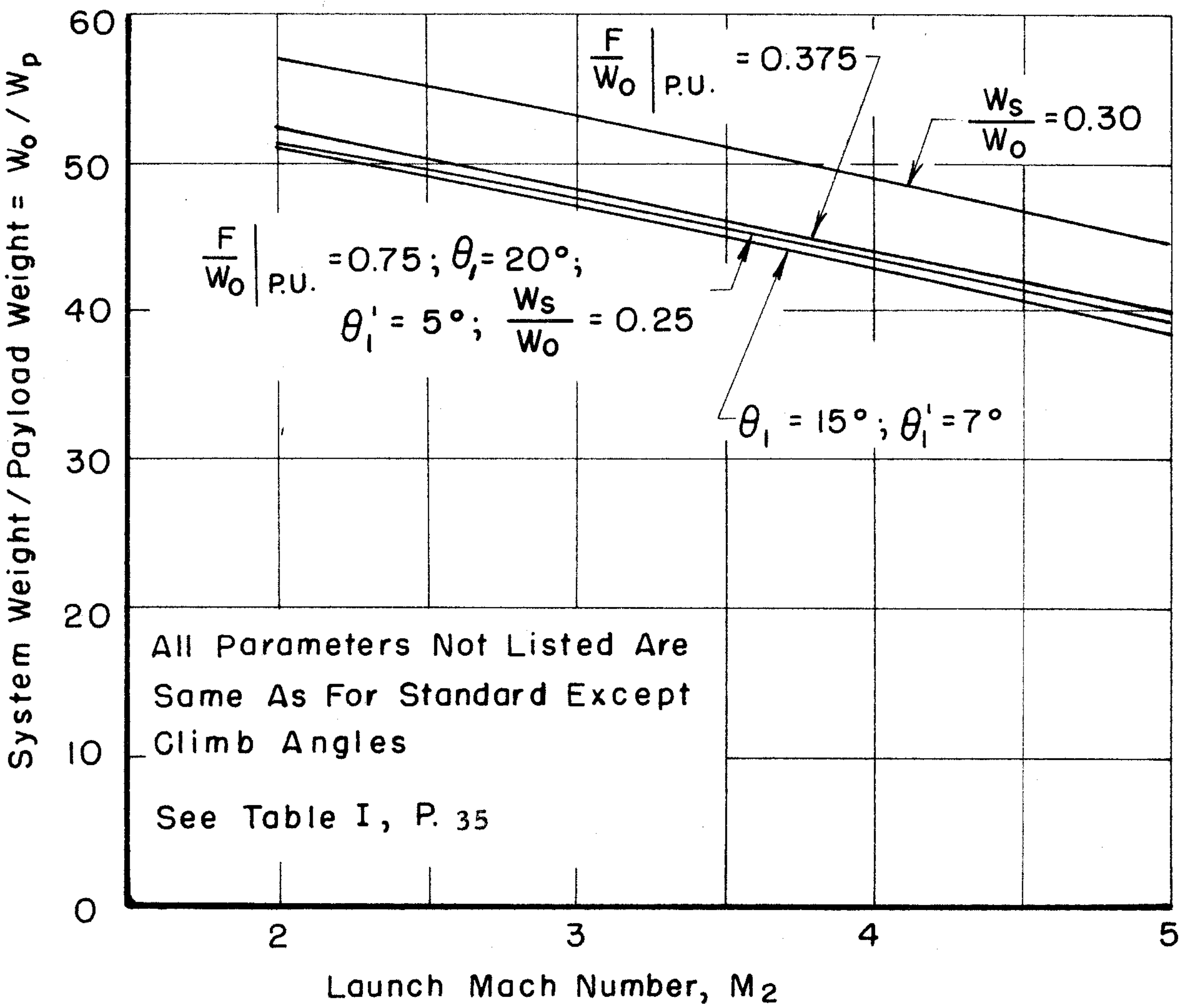

Fig. 29. Effect of Structural Weight, Reduced Thrust Loading During Pull Up, and Angles of Climb on System Weight to Payload Weight Ratio.