INVESTIGATION OF DURALUMIN CHANNEL SECTION STRUT UNDER COMPRESSION

Thesis by

Warren S. Parr
and
Wallace M. Beakley

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I. TABLE OF CONTENTS:

II. ACKNOWLEDGMENTS:

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III. SUMMARY OF RESULTS:

This investigation has clearly established the fact that, for sections built up of straight line elements, theoretical calculations can be made with a degree of accuracy sufficient for practical application and will be on the conservative side.

IV. STATEMENT OF PROBLEM:

Sections built up of straight line elements are extensively used as stiffeners in aeronautical construction and consequently knowledge of their strength under compressive load is of importance to the industry. All shapes of stiffener sections made up of thin flát sheet of reasonable length, are subject to three classes of failure, namely:

1) Local wrinkling or buckling of the sheet in the component parts,

2) Euler failure as long slender columns due to their use in lengths which are ordinarily large compared to the cross-section, and

3) Torsion failure due to torsional instability.

The question at once arises as to the points of transition from one type of failure to another, i.e. the limits within which each type of failure may be expected to occur.

It is apparent from consideration of the problem that the stiffener will fail at whichever critical value of the three types of failure is the minimum for a given size of stiffener. An investigation of angle section stiffeners of equal leg lengths has previously been made by A. B. Vosseller and C. C. Jerome[1], consequently, as an extension of their research, the extensively used channel section was chosen for this investigation.

V. THEORETICAL CALCULATIONS:

In order to present the results of the investigation in a clear and concise manner it is necessary to express the formulas for the three different types of failure in terms of the same parameters. In addition, it is desirable to have these parameters dimensionless. The critical stress to cause buckling of a flat plate under compression was selected as the type equation. This equation and the edge conditions affecting the constant, are summarized in a recent paper by Donnell(2) in which he gives the equation in the form

$$\sigma_{cr} = KE \left[\frac{\text{thickness}}{\text{width}}\right]^2$$

This formula satisfies the conditions of dimensionless parameters in that K is a dimensionless coefficient depending upon the ratio of the length to the width of the plate and upon the edge conditions. The edge conditions in the problem under investigation result in

K becoming a function of b, w, L, t and μ

where b = length of leg (depth of channel)

w = width of back

L = length of channel

t = thickness

μ = Poissons ratio (Taken as 0.3 for material tested)

Since the variables which were to be considered in this investigation were b, w and t, it was thought advisable to obtain K as a function of the dimensionless ratio b/w since by holding L, t and μ constant K can be expressed as a function of (b/w). E, (Young's Modulus), of course, is a constant for any given material.

(a) Euler Failure:

The theory for the failure of struts as Euler columns is well known and has been checked closely by Dr. von Karman[3)] who originated the general method of varying the end conditions to counteract initial eccentricities, which was used in this investigation. The Euler long column formula for critical stress is expressed as

$$\sigma_{cr} = C\pi^2\left(\frac{r}{L}\right)^2 E$$

where: C = coefficient of end fixity

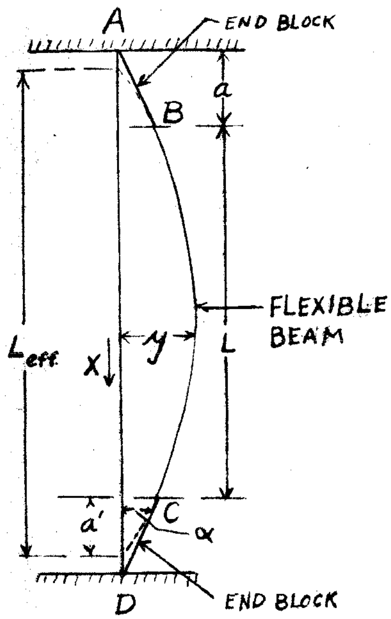
r = least radius of gyration of cross-section

E = Young's Modulus

L = length of column

The end conditions under which the stiffeners were tested were made as nearly pin ended as possible by mounting the channels in grooves on steel plates which rested on a hemisphere of hardened steel (Fig. I), hence the end fixity was taken as unity for all calculations. The effective length of the channel (L) to be used in the Euler column formula was deduced by Dr. von Kármán as follows:

The equation for the bending of a beam is



$$EI\,\frac{d^2y}{dx^2} = -Py \qquad (1)$$

the solution of which is

$$y = A\cos\sqrt{\frac{P}{EI}}\;x \qquad (2)$$

Boundary conditions:

AB and CD are supposed to be of infinite rigidity and hence are straight lines, therefore

$$\tan\alpha = -\left(\frac{dy}{dx}\right)_{x=\frac{L}{2}} = \alpha \qquad \text{for small angles}$$

$$Y_{x=\frac{L}{2}} = a\alpha = -a\left(\frac{dy}{dx}\right)_{x=\frac{L}{2}}$$

or $A\cos\sqrt{\frac{P}{EI}}\,\frac{L}{2} = -a\left[-A\sqrt{\frac{P}{EI}}\,\sin\sqrt{\frac{P}{EI}}\,\frac{L}{2}\right]$

or $\cot\sqrt{\frac{P}{EI}}\,\frac{L}{2} = a\sqrt{\frac{P}{EI}}$ (3)

and equation 3 gives the exact value of the P_{Euler} for the case considered.

For a = 0 we obtain $\cot\sqrt{\frac{P}{EI}}\,\frac{L}{2} = 0$

i.e. $\sqrt{P/EI}\ L/2 = \frac{\pi}{2}$ or $P = \frac{\pi^2 EI}{L^2}$ (4)

Putting $\sqrt{P/EI}\ \frac{L}{2} = z$ equation (3) may be rewritten

$$z \tan z = \frac{L}{2a} \tag{5}$$

and $\sqrt{P/EI} = \frac{2z}{L}$

or $P = \frac{EI \cdot 4z^2}{L^2}$ (6)

and comparing equation (6) with equation (4)

it is seen that the effective length

$$L_{eff} = \frac{L\pi}{2z} \tag{7}$$

therefore $a' = \frac{1}{2}(L_{eff} - L) = \frac{L}{2}\left(\frac{\pi}{2z} - 1\right)$ or substituting from equation (5):

$$a' = a\left(\frac{\pi}{2} - z\right)\tan z \tag{I}$$

and from (5)

$$a/L = \frac{1}{2z \tan z} \tag{II}$$

and so if we take different values of Z we can calculate corresponding values of a' and a/L

z	$\tan z$	a/L (II)	a'/a (I)
$\pi/2 = 90°$	∞	0	1
$17\pi/36 = 85°$	11.43	0.029	0.999
$5\pi/12 = 75°$	3.732	0.102	0.982
$\pi/3 = 60°$	1.732	0.275	0.907

It may be seen from the table that even in such extreme cases where the rigid block equals 27.5% of the length of the beam proper, 90,7% of the length of the rigid portion is to be added to the length of the beam to give the effective long column length. A standard channel length of 22 inches was used in this investigation, and the combined depth of the two end blocks from the bottom of the grooves to the top of the hemispheres was determined as 2.3" giving an effective Euler column length of 24.3" to be used in the Euler formula.

In order to determine the value of K for the Euler column formula, the Euler critical stress is equated to the critical stress of the type equation which gives

$$E\pi^2\left(\frac{r^2}{L^2}\right) = E\frac{t^2}{b^2}K \qquad (8)$$

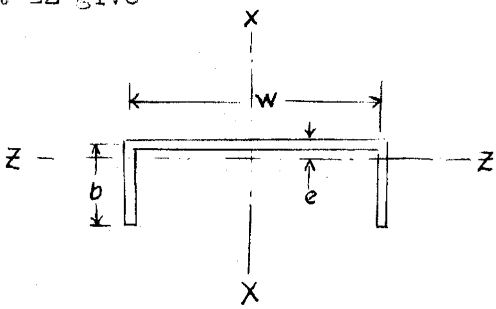
from which

$$K_{EULER} = \frac{\pi^2 r^2 b^2}{L^2 t^2} \qquad (9)$$

Considering that the channels are of thin sheet and the area is approximately (2b+w)t; calculation of the C.G. gives

$$e = \frac{b^2}{2b+w} \quad \text{when } t^2 \text{ terms are neglected.} \qquad (10)$$

Moments of Inertia: Calculations about ZZ give



$$I_{zz} = \frac{2tb^3}{12} + 2tb\left(\frac{b}{2} - e\right)^2 + wte^2$$

$$= t\left[\frac{b^3}{6} + \frac{b^3 w}{2(2b+w)^2} + \frac{w b^4}{(2b+w)^2}\right]$$

$$= t\left[\frac{b^4 + 2b^3 w}{3(2b+w)}\right]$$

Note: $\frac{b}{2} - e = \frac{b}{2} - \frac{b^2}{2b+w} = \frac{wb}{2(2b+w)}$

then $$r_{zz}^2 = \frac{I_{zz}}{A} = \frac{b^2(b^2+2bw)}{3(2b+w)^2} = \frac{b^2\left((b/w)^2 + 2(b/w)\right)}{3(2\,b/w+1)^2}$$

and substituting in (9)

$$K_{EULER_{zz}} = \pi^2 \frac{b^4}{L^2 t^2}\left[\frac{(b/w)^2 + 2(b/w)}{3(2\,b/w+1)^2}\right]$$

and similarly for the Euler failure in the other direction, which is outside the region investigated since it occurs only in channels which have very narrow backs and only then provided they are kept from torsion or leg plate failure,

$$K_{xx} = \pi^2 r_{xx}^2 \frac{b^2}{L^2 t^2}$$

$$= \frac{\pi^2 b^4}{L^2 t^2}\left[\frac{1+6\,b/w}{12[(b/w)^2+3(b/w)^3]}\right] = \frac{\pi^2 b^4}{L^2 t^2}\left[\frac{1+6\,b/w}{12(b/w)^2[2\,b/w+1]}\right]$$

(b) Plate Failure:

If the critical stresses for the flat plate failure of the legs and back are considered separately, the formula for the legs is given by the type equation

$$\sigma_{cr} = KE(t/b)^2$$

in which b = length of leg and the critical stress for flat plate failure of the back is expressed as

$$\sigma_{cr} = KE(t/w)^2$$

where w = width of back. If the leg is considered as having simple support along the edge where it joins the back, the value of Donnell's K[2] for this condition is 0.4 and considering the same edge conditions for the back, which would result in simple support on all four sides, gives a K of 3.6. When the edge joining the leg and back is considered fixed, K becomes 1.2 for leg plate failure and 6.3 for back plate failure. (For the purpose of comparison, the value of K used by Donnell in Ref. 2 is related to the K found in Timoshenko[4] and other books on elasticity by the equation $K_D = K_T \times \frac{\pi^2}{12(1-\mu^2)}$ when μ is taken as 0.3). It is apparent that the exact value of K for the specimens tested is somewhere between these two extremes of simple support and fixed edge conditions and must be determined by combining the effect of the legs and back on each other.

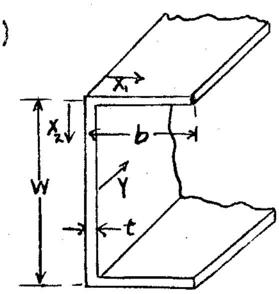
The failure of the legs as plates buckling under compressive load is essentially a problem of the elastic support given the legs by the back as the width of the back varies. It is, however, still a problem of plate buckling and may be analyzed in one of two or three methods all quite complex. Of these the Energy Method was taken as the least involved, and the following theoretical expressions were developed by Dr. von Kármán.

The internal energy stored up in the plate during deformation is first set up using a sine curve approximation to the true deflection curve in the length direction and in the direction of the width of the back considering that the elements of the leg length b remain straight. The work done by the external force is then set up in terms of the deflection, and considering that the energy change is zero, the work done is put equal to the energy stored up in the plate. The critical value of the compressive stress for buckling is so determined and the value of the coefficient K for this type of failure found in terms of the parameters of the channel.

Considering that the channel is made up of three thin plates simply supported or hinged at their top and bottom edges and are deformed by the compression load, the energy of bending and twisting for the deformed sheet is:

$$U = \frac{Et^3}{12(1-\mu^2)} \int_0^L \int_0^b \frac{1}{2} \left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2 + 2(1-\mu)\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + 2\mu \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} \right] dx\, dy$$

and, if it is assumed that the deflection of the leg may be represented by $Z_1 = AX_1 \sin \frac{N\pi Y}{L}$ (2)



and that the deflection of the back is

$$Z_2 = B \sin \frac{\pi X_2}{W} \sin \frac{N\pi Y}{L} \qquad (3)$$

the internal energy for both legs is

$$U_1 = \frac{Et^3}{12(1-\mu^2)} \int_0^L \int_0^b \left[A^2 X_1^2 \left(\frac{N\pi}{L}\right)^4 \sin^2 \frac{N\pi Y}{L} + 2(1-\mu) A^2 \left(\frac{N\pi}{L}\right)^2 \cos^2 \frac{N\pi Y}{L} \right] dx\, dy \qquad (4)$$

the internal energy of the back is

$$U_2 = \frac{Et^3}{12(1-\mu^2)} \int_0^L \int_0^W \frac{1}{2} \Big[B^2 \frac{\pi^4}{W^4} \sin^2 \frac{\pi X_2}{W} \sin^2 \frac{N\pi Y}{L} + B^2 \left(\frac{N\pi}{L}\right)^4 \sin^2 \frac{\pi X_2}{W} \sin^2 \frac{N\pi Y}{L}$$
$$+ 2(1-\mu) B^2 \left(\frac{\pi}{W}\right)^2 \left(\frac{N\pi}{L}\right)^2 \cos^2 \frac{\pi X_2}{W} \cos^2 \frac{N\pi Y}{L}$$
$$+ 2\mu B^2 \left(\frac{\pi}{W}\right)^2 \left(\frac{N\pi}{L}\right)^2 \sin^2 \frac{\pi X_2}{W} \sin^2 \frac{N\pi Y}{L} \Big] dx\, dy$$

and the total internal elastic energy is:

$$U = U_1 + U_2 = \frac{Et^3}{12(1-\mu^2)} \left\{ A^2 \left[\left(\frac{N\pi}{L}\right)^4 \frac{b^3 L}{6} + 2(1-\mu)\left(\frac{N\pi}{L}\right)^2 \frac{bL}{2} \right] + \frac{B^2 wL}{8} \left[\left(\frac{\pi}{w}\right)^4 + \left(\frac{N\pi}{L}\right)^4 + 2\left(\frac{\pi}{w}\right)^2 \left(\frac{N\pi}{L}\right)^2 \right] \right\}$$

But if the angle between the back and leg is preserved, we have at the joint:

$$X_1 = X_2 = 0 \text{ and } \frac{\partial Z_1}{\partial X_1} = \frac{\partial Z_2}{\partial X_2}$$

Then

$$A \sin \frac{N\pi Y}{L} = B\frac{\pi}{W}\cos\frac{\pi X_2}{2}\sin\frac{N\pi Y}{L}$$

or $A = \frac{B\pi}{W}$

and $U = \frac{Et^3}{12(1-\mu^2)} B^2 \left\{ \left(\frac{\pi}{W}\right)^2 \left[\left(\frac{N\pi}{L}\right)^4 \frac{b^3 L}{6} + 2(1-\mu)\left(\frac{N\pi}{L}\right)^2 \right] + \frac{WL}{8}\left[\left(\frac{\pi}{W}\right)^2 + \left(\frac{N\pi}{L}\right)^2 \right]^2 \right\}$

The external work done during buckling is given by:

$$W = \tfrac{1}{2}\sigma t \int_0^L \int_0^b \left(\frac{\partial z}{\partial y}\right)^2 dx\,dy$$

then for the two legs:

$$W_1 = \sigma t \int_0^L \int_0^b A X_1^2 \left(\frac{N\pi}{L}\right)^2 \cos^2\frac{N\pi Y}{L}\,dx\,dy$$

and for the back:

$$W_2 = \tfrac{1}{2}\sigma t \int_0^L \int_0^W B^2 \left(\frac{N\pi}{L}\right)^2 \sin^2\frac{\pi X_2}{W}\cos^2\frac{N\pi Y}{L}\,dx\,dy$$

and the total external work done is: substituting for A:

$$W = W_1 + W_2 = \sigma t B^2 \left(\frac{N\pi}{L}\right)^2 \left\{ \frac{b^3 L}{6}\left(\frac{\pi}{W}\right)^2 + \frac{WL}{8} \right\}$$

Putting E = W

$$\sigma = \frac{Et^3}{12(1-\mu^2)} \frac{\left\{ \left(\frac{\pi}{W}\right)^2 \left[\left(\frac{N\pi}{L}\right)^4 \frac{b^3 L}{6} + 2(1-\mu)\left(\frac{N\pi}{L}\right)^2 \frac{bL}{2} \right] + \frac{WL}{8}\left[\left(\frac{\pi}{W}\right)^2 + \left(\frac{N\pi}{L}\right)^2 \right]^2 \right\}}{t\left(\frac{N\pi}{L}\right)^2 \left[\frac{b^3 L}{6}\left(\frac{\pi}{W}\right)^2 + \frac{WL}{8} \right]}$$

and putting this in the form $\sigma = \frac{Et^2}{b^2} K$

$$K = \left[\frac{1}{12(1-\mu^2)}\right] \frac{\frac{N^2\pi^2}{3}\left(\frac{b}{W}\right)^3\left(\frac{b}{L}\right)^2 + 2(1-\mu)\left(\frac{b}{W}\right)^3 + \frac{1}{4N^2}\left(\frac{b}{W}\right)^4\left(\frac{L}{b}\right)^2 + 1\left(\frac{b}{W}\right)^2 + \frac{N^2}{4}\left(\frac{b}{L}\right)^2}{\frac{1}{3}\left(\frac{b}{W}\right)^3 + \frac{1}{4\pi^2}}$$

To find the minimum σ, N must take a value such that

K is a minimum or $\frac{\partial K}{\partial N} = 0 = \frac{2\pi^2 N}{3}\left(\frac{b}{W}\right)^3\left(\frac{b}{L}\right)^2 - \frac{1}{2N^3}\left(\frac{b}{W}\right)^4\left(\frac{L}{b}\right)^2 + \frac{N}{2}\left(\frac{b}{L}\right)^2$

and $N = \frac{L}{W}\sqrt[4]{\frac{3}{4\pi^2\left(\frac{b}{W}\right)^3 + 3}}$

Let $\frac{b}{W} = \alpha$ and $V = \sqrt{\frac{3}{4\pi^2\left(\frac{b}{W}\right)^3 + 3}}$

Simplifying

$$K = \frac{2\pi^2}{12(1-\mu^2)} V\alpha^2\left[(V+1) + 4V\alpha(1-\mu)\right] \qquad (7)$$

It is realized that the assumption made in approximating the deflected shape of the leg is somewhat in error when it is assumed that the leg element in the B direction remains straight. To approach the actual condition any closer, however, an analytical expression must be chosen to get the actual deformation, and the calculation becomes too involved to be presented here.

(c) Torsion Failure:

The theory for failure of this type of stiffener due to torsional instability has been treated recently by H. Wagner and W. Pretschner[5]. This method was chosen by the authors as being most suitable for their use and expresses the value for the critical load of a section under compression as

$$P = \frac{1}{i_{sp}^2}\left(G I_e + \frac{E\pi^2}{L^2} C_{BT}\right)$$

where i_{sp} = polar radius of gyration of the section with respect to the shear point or elastic centrum.

G = Modulus of elasticity in shear

E = Young's Modulus of elasticity

I_e = Effective moment of inertia for thin sheet open sections

C_{BT} = Constant determined by the bending rigidity of the flanges

This formula is made up of two parts which will be treated separately and combined later.

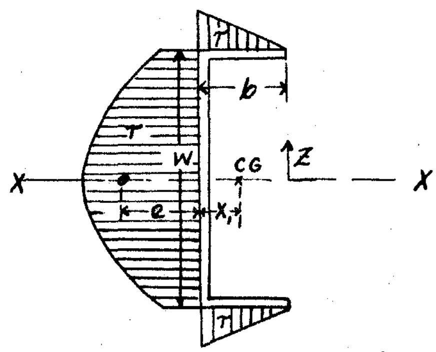
For Part I

$$P_1 = \frac{GI_e}{i_{sp}^2} = \frac{EI_e}{2(1+\mu)\, i_{sp}^2} \qquad \text{or} \qquad \sigma = P/A = \frac{Et^2}{b^2}\,\frac{b^2}{6(1+\mu)\, i_{sp}^2}$$

From which $K = \frac{b^2}{6(1+\mu)\, i_{sp}^2}$

Calculation of shear center

to determine i_{sp}^2



SHEAR DISTRIBUTION

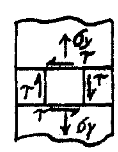
In each leg $\frac{dM}{dy} = Q$

and $\frac{d\sigma_y}{dy} + \frac{d\tau}{dx} = 0$

but $\sigma_y = \frac{Mz}{I_{xx}}$

then differentiating and substituting

$$\frac{d\sigma_y}{dy} = \frac{dM}{dy}\frac{z}{I} = \frac{Qz}{I} = -\frac{d\tau}{dx}$$



and integrating

$$\tau = -\frac{Qz}{I}x + CONSTANT$$

for each leg $z = W/2$

at the edge of each leg $x = b$, $\tau = 0$ then by substitution

$$-\frac{Qwb}{2I} + C = 0 \quad or \quad C = \frac{Qwb}{2I}$$

the solution becomes

$$\tau = \frac{Qw}{2I}(b-x)$$

and the shear over each leg is given by $t \cdot \tau = \frac{Qwt}{2I}\int_0^b (b-x)\,dx$

from which $t \cdot \tau = \frac{Qwtb^2}{4I}$

Then the moment of shear force in both legs about shear center e is

$$M_e = \frac{Qw^2b^2t}{4I}$$

This is balanced by the direct shear force in the back (Q) acting at distance e.

Hence $M_e = Q \cdot e = \frac{Qw^2b^2t}{4I}$

and $e = \frac{w^2b^2t}{4I_{xx}}$

but $I_{xx} = \frac{w^3t}{12}\left(6\,b/w + 1\right)$

and, substituting for I_{xx}

$$e = \frac{w^2b^2t \cdot 12}{4w^2t(6b+w)} = \frac{3b^2}{6b+w} = \frac{3b\,(b/w)}{(6\,b/w + 1)}$$

Calculation of i_{sp}^2

$$i_{sp}^2 = \frac{I_{P(SP)}}{A}$$

$$= \frac{I_{xx} + I_{zz}}{A} + (X_1 + e)^2$$

$$= \frac{w^3 + 6w^2b}{12(2b+w)} + \frac{b^4 + 2b^3w}{3(2b+w)^2} + \left[\frac{b^2}{(2b+w)} + \frac{3b^2}{(6b+w)}\right]^2$$

$$\frac{i_{sp}^2}{b^2} = \frac{1 + 6\frac{b}{w}}{12(\frac{b}{w})^2(2\frac{b}{w}+1)} + \frac{\frac{b}{w}(2+\frac{b}{w})}{3(2\frac{b}{w}+1)^2} + \left[\frac{\frac{b}{w}}{2\frac{b}{w}+1} + \frac{3\frac{b}{w}}{(6\frac{b}{w}+1)}\right]^2$$

from which for Part I of Torsion Formula:

$$K = \frac{b^2}{6(1+\mu)\, i_{sp}^2}$$

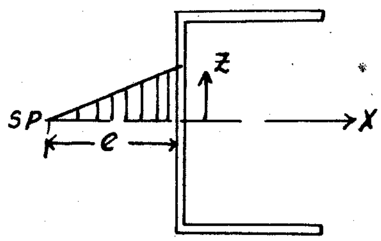
for Part II $\quad P_z = \frac{E\pi^2}{L^2}\,\frac{C_{BT}}{i_{sp}^2}$

$$C_{BT} = \int \omega^2 dA$$

where ω = twice the area swept through by a radius vector from the shear center to outline of section. ω = warping = 0 at line of symmetry.

For the back $\quad \omega = 2F = \frac{2ez}{2}$

For half the back then



$$\int \omega^2 dA = t\int_0^{\frac{w}{2}} z^2 e^2 dz = \frac{te^2w^3}{24}$$

For one leg

$$\int_0^b \omega^2 dA = t\int_0^b \left(\frac{ew}{2} - \frac{wx}{2}\right)^2 dx = \frac{tw^2}{4}\left[e^2 b - eb^2 + \frac{b^3}{3}\right]$$

Then for total section

$$C_{BT} = \frac{2tw^2}{4}\left[\frac{e^2 w}{6} + e^2 b - eb^2 + \frac{b^3}{3}\right]$$

substituting for C_{BT}

$$P_2 = \frac{E\pi^2}{L^2}\frac{tw^2}{2}\frac{1}{i_{sp}^2}\left[\frac{e^2 w}{6} + e^2 b - eb^2 + \frac{b^3}{3}\right]$$

Then $\sigma = \dfrac{P}{A} = \dfrac{Et^2\pi^2}{b^2\, 2L^2}\dfrac{w^2}{t^2}\dfrac{b^2}{i_{sp}^2}\dfrac{1}{(2b+w)}\left[\dfrac{e^2 w}{6} + e^2 b - eb^2 + \dfrac{b^3}{3}\right]$

and substituting the value of e above gives

$$K_2 = \frac{\pi^2}{12}\frac{b^4}{L^2 t^2}\frac{b^2}{i_{sp}^2}\frac{w^2}{b^2}\left[\frac{3b^2 + 2bw}{12b^2 + 8bw + w^2}\right]$$

or

$$K_2 = \frac{\pi^2}{12}\frac{b^4}{L^2 t^2}\frac{b^2}{i_{sp}^2}\frac{w}{b}\left[\frac{3\left(\frac{b}{w}\right) + 2}{12\left(\frac{b}{w}\right)^2 + 8\left(\frac{b}{w}\right) + 1}\right]$$

and total K for torsion $= K_1 + K_2$.

Having determined K in relation to b/w for the three different types of failure, the theoretical curves for each type can now be plotted on the same coordinate system as will be seen in Figs. 3 to 9.

VI. EXPERIMENTAL INVESTIGATION:

The determination of the compressive strength of channel sections involves three variables, namely thickness of material, width of back, and length of leg. In order to determine the effect of each variable, it becomes necessary to vary only one at a time and this was done by using two thicknesses of material, .025" and .051", variation of width of back from 0.4" to 2.0", and variation of length of leg from 0.4" to 1.5". The tests were made with equal length channel section struts 22" long which, with the supporting blocks, etc., gave an effective length of 24.3" for Euler failure.

Since accurate knowledge of the end fixity is essential for the determination of column buckling loads, a great deal of care was taken to approach as nearly as possible the hinged ends condition for the channels tested. A sketch of an end block and plate used is shown in Fig. 1. The plate A was machined on the upper side with V grooves and these grooves received the end of the specimen, which had been filed to an angle of 60 degrees, as shown in sketch. In filing the ends, great care was taken not to file away any of the center line and to keep the ends square. The key and keyway connecting A and B allow a movement of A, (and thus the end of the specimen) by means of the set screws C, relative to E (the point of application of the load). The motion of A and B is such that E always moves along the axis of symmetry of the specimen, and in this manner the point of application could be placed exactly on the center of gravity of the section. In order to obtain definite end conditions for Euler buckling, a 5/8" hardened steel ball inserted in the opposite side of the end blocks transmitted the compressive load from the test machine to the end block, plates and channels. The struts thus had pure hinged ends and could buckle in any direction. It was found, however, that a slight dent made by the steel ball in the cage plate by one or two tests in the same spot would give erratic results for Euler failure, due to the change in fixity of the ends if the ball was placed in the dent for subsequent tests. To eliminate this source of error, hardened steel discs were made for the ball to rest on and the discs shifted after each test. It was also found that the slight support given a channel by the Ames dial gauge used for lining up the specimen would prevent Euler failure at the theoretical point and carry the load, in many cases, far above the Euler value. For this reason, after the channel was lined up (initial eccentricities removed), the dial gauge was removed before continuing the test.

The tests were made on a Riehle Bros. testing machine of 3000 lbs. capacity. The machine was a tension type but was converted for compression tests by means of the cage shown in the illustrations, Fig. 2.

The specimens tested were of 17ST aluminum alloy and center line dimensions were used for all calculations. The specimens of .025" thickness were made in an ordinary hand brake with radius of curvature at the corners of approximately 1/16", but the specimens of .051" thickness were manufactured by the Douglas Aircraft Company of Santa Monica, California, since this material was too heavy to bend on a hand brake without cracking. Extreme care was necessary in squaring the ends, due to the tendency to have torsional failure from uneven stress distribution if the entire end of the channel was not flush in the bottom of the grooves. This was accomplished by milling off approximately .001" on each end after the ends had been chamfered. The channels were made with the grain (i.e. direction of rolling) of the material parallel to the angles. This was done due to the marked difference in the stress strain curve taken normal or parallel to the direction of rolling found by tests of the Bureau of Standards for N.A.C.A. on duralumin sheet,[6)] and the common practice of manufacturing stiffeners with the grain in this direction.

In order to properly line up the specimen in the cage, wooden jigs were constructed which would place the two points of contact (E) directly over one another when the end blocks were fitted in the jigs. An Ames dial gauge was then placed at the center of the specimen and the end plates adjusted equally, as load was applied, until all the eccentricity of the channel was removed as indicated by a constant reading on the gauge for considerable part of the expected load. The removal of all initial eccentricity must be very exacting for satisfactory results due to its influence both in Euler and torsion failure. The gauge was then removed and the load increased by small increments until the specimen failed.

VII. DISCUSSION OF EXPERIMENTAL RESULTS:

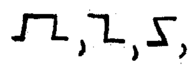
The first tests made were of the t = .025" b = 0.4" channels varying the back from 0.4" to 2.0" for which quite consistent Euler failures were obtained and which followed the Euler curve very closely (Fig. 3) showing that the end conditions were as desired (approximately unity). The next set of channels tested was t = .025" b = 0.6" which gave three torsion failures with the shorter backs and the remainder Euler failure as may be seen on Fig. 4. It will be noted that the torsion failures obtained during the investigation were all consistently higher than their theoretical torsion curves. The theoretical curves are based on an assumption of free ends, i.e. the ends are allowed to warp when the channel twists. This condition naturally is practically impossible to obtain and resulted in the channels resisting the torsional moments until a higher load than the theoretical value was obtained. The channel would then fail in torsion and in failing would cause a local buckling of one corner of a leg indicating the tendency for the legs to warp in torsion failure. If, however, the stress reached the Euler value before this occurred, Euler failure would result, as clearly shown by this set of points following the Euler curve after three torsional failures.

It being desirable to obtain plate failure also, channels were chosen t = .025" b = 0.8"; t = .025" b = 1.0"; t = .025" b = 1.3"; t = .051" b = 1.5"; to insure plate failure of the legs. The test values obtained for these channels, as may be seen in Fig. 9, followed the general curvature of the theoretical plate failure curve except for those points where $\frac{b}{w} \rightarrow 1$. It is suggested for design that the dotted curve faired through the experimental points shown in this figure be used.

In this region, considerable scatter is apparent due to the tendency toward torsional failure, five of which were definitely so. It is felt that several of the other points were probably a combination of torsion and plate failure, since a small additional load after plate failure in this region would result in a twisting of the channel, but the initial failure could not be definitely designated as other than plate failure. For all the points shown, not marked as torsional, in this group of channels plate failure of the legs occurred similar to Fig. 2. The low values obtained for this set of channels as compared to the theoretical plate failure curve is explainable by the fact that in the theoretical analysis for this case, the assumption was made that the energy of the deformed section could be obtained by assuming that the legs deform as a straight line. This assumption will hold for very large values of w, or small b/w ratios, but as w becomes small, or b large (large b/w ratios), then the assumption of a straight line deflection for the legs breaks down. This must be true since at the limit ($w \rightarrow 0$, $b/w \rightarrow \infty$), the legs of the channel will act as plates fixed along one side, and will obviously not have a straight line deflection curve. Thus for large values of b/w the treatment previously given leads to a higher value of K than would be expected from experimental tests. Probable discrepancies in the end conditions may also be held partially accountable for the low value.

VIII. CONCLUSIONS:

It is fully appreciated that the method of securing a channel strut in a structure will vary from no support to rigid support for the back, and that the degree of this support will greatly influence the amount of fixity given the edge of the legs. It is hoped, however, that this investigation will at least give some basis for making a first approximation of the strength of channel section struts.

The authors believe that this investigation has demonstrated the feasibility of analyzing all sections built up of flat sheet elements such as  etc., theoretically with some degree of accuracy, since fundamentally they all reduce to the same question of determination of K at the corners. It will be noted that the experimental points check the theoretical values for the plate failure curve quite closely in the usual range of channel sections, i.e. $0.4 < \frac{b}{w} < 0.7$ and beyond this range the torsion failure region is entered and failure occurs in this manner.

It is recommended that further investigation of this type of channel be made for different methods of securing in a structure. Investigation of thin sheet stiffener sections made up of curved sections would also be of considerable value to the aeronautical industry.

TEST DATA

Channel Dimensions				Test Values		
Thickness	Back	Leg	B/W	Load	Stress	K
.025	1.0 x 1.0		1.000	388	5220	.835
	1.2 x 1.0		.833	329	4150	.665
	1.4 x 1.0		.714	400	4730	.76
	1.6 x 1.0		.625	395	4420	.708
	1.8 x 1.0		.500	383	3860	.618
.025	.6 x	.8	1.330	330	6140	.628
	.8 x	.8	1.000	388	6500	.663
	1.0 x	.8	.800	581	9020	.915
	1.2 x	.8	.667	503	7220	.735
	1.4 x	.8	.571	493	6620	.673
	1.6 x	.8	.500	481	6040	.615
	1.8 x	.8	.444	474	5610	.574
	2.0 x	.8	.400	499	5570	.574
.025	.4 x	.6	1.500	136	3510	.202
	.6 x	.6	1.000	200	4575	.263
	.8 x	.6	.750	293	6000	.347
	1.0 x	.6	.600	348	6480	.375
	1.2 x	.6	.500	385	6550	.382
	1.4 x	.6	.428	396	6210	.360
	1.6 x	.6	.375	400	5850	.339
	1.8 x	.6	.333	397	5390	.313
	2.0 x	.6	.300	420	5340	.310
.025	.4 x	.4	1.000	84	2900	.074
	.6 x	.4	.670	103.5	3040	.078
	.8 x	.4	.500	125	3200	.082
	1.0 x	.4	.400	117	2670	.068
	1.2 x	.4	.330	142	2900	.074
	1.4 x	.4	.286	147	2720	.070
	1.6 x	.4	.250	153	2600	.067
	1.8 x	.4	.222	155	2470	.063
	2.0 x	.4	.200	148	2150	.056

TEST DATA (cont'd)

Channel Dimensions			Test Values		
Thickness	Back Leg	B/W	Load	Stress	K
.051	1.2 x 1.3	1.08	2197	11400	.747
	1.2 x 1.3	1.08	1978	10350	.672
	1.4 x 1.3	.930	2100	10420	.678
	1.4 x 1.3	.930	2330	11550	.752
	1.6 x 1.3	.814	2469	11650	.760
	1.6 x 1.3	.814	2427	11450	.746
	1.8 x 1.3	.722	2545	11450	.747
	1.8 x 1.3	.722	2610	11750	.765
	2.0 x 1.3	.650	2200	9500	.617
	2.0 x 1.3	.650	2588	11150	.726
.051	1.4 x 1.5	1.020	1920	8650	.750
	1.4 x 1.5	1.020	2200	9900	.858
	1.6 x 1.5	.938	2200	9480	.820
	1.6 x 1.5	.938	2175	9380	.811
	1.8 x 1.5	.834	2106	11500	.752
	1.8 x 1.5	.834	2205	10975	.789
	2.0 x 1.5	.750	2044	12340	.700
	2.0 x 1.5	.750	2185	11550	.748

References:

1) A. B. Vosseller and C. C. Jerome "Investigation of Relation Between Euler and Flat Plate Buckling of "L" Section Struts", Trans. A.S.M.E. Applied Mechanics Division, August 1934, pp. 569-578.

2) L. H. Donnell "The Problem of Elastic Stability", Trans. A.S.M.E. 1933, Vol. 5, No. 4, p. 4.

3) Th. von Kármán "Mitteilungen über Forschungsarbeiten" Ver. deuts. Ing., Berlin 1909.

4) S. Timoshenko "Strength of Materials", Part II, p. 605.

5) Von H. Wagner and W. Pretschner "Verdrehung und Knickung von Profilen", Luftfahrtforschung, Band 11, Nr. 6.

6) E. E. Lundquist "The Compressive Strength of Duralumin Columns of Equal Angle Section", N.A.C.A. Technical Note 413, 1932.

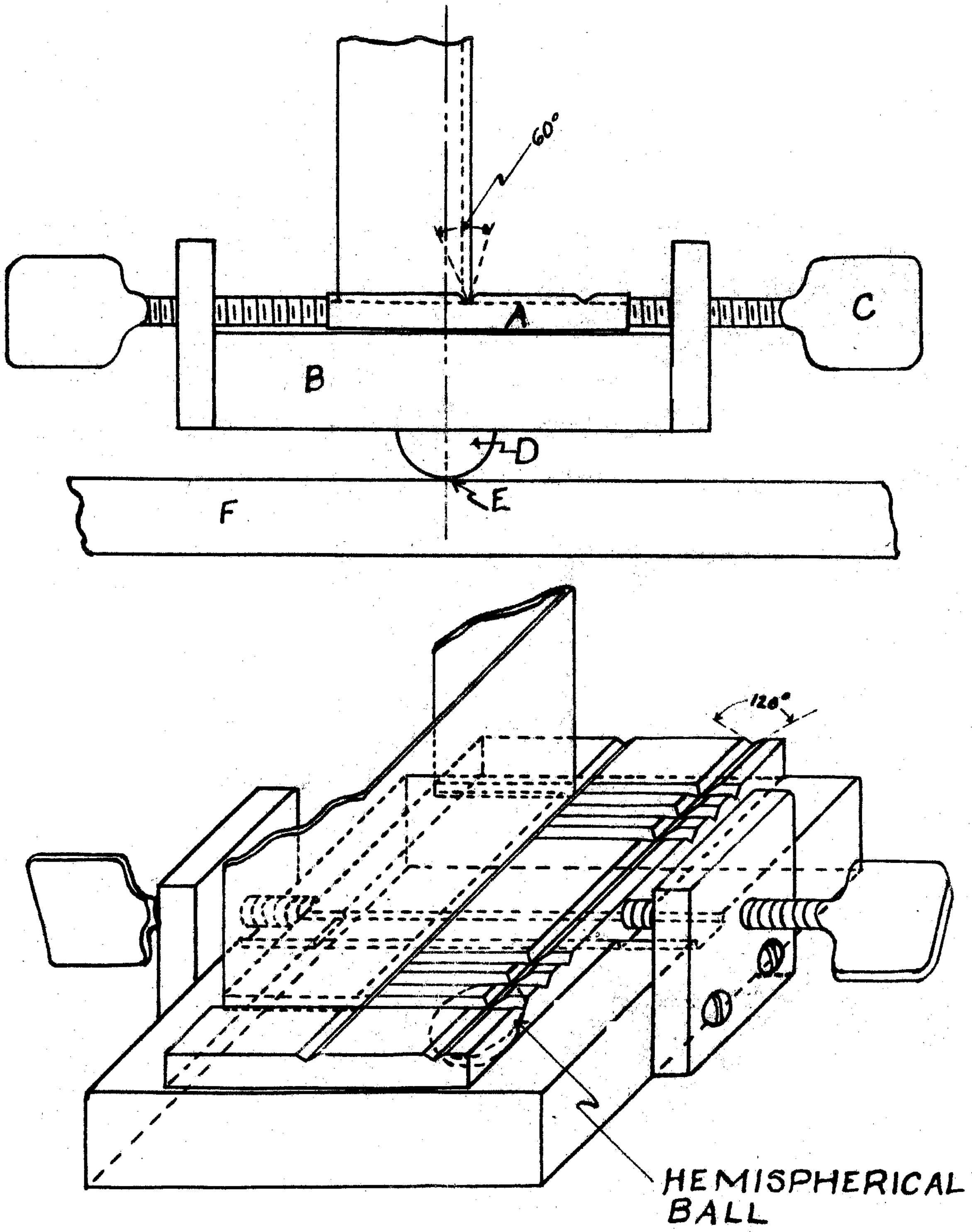
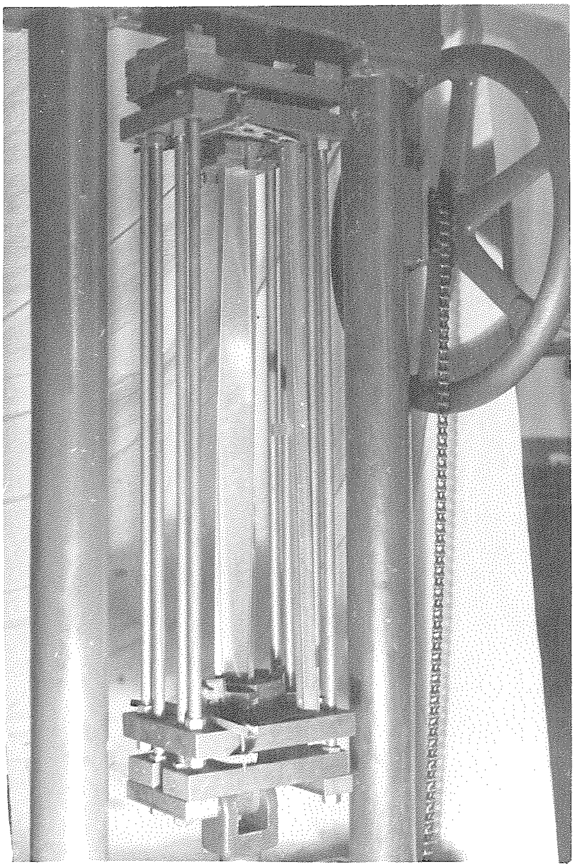
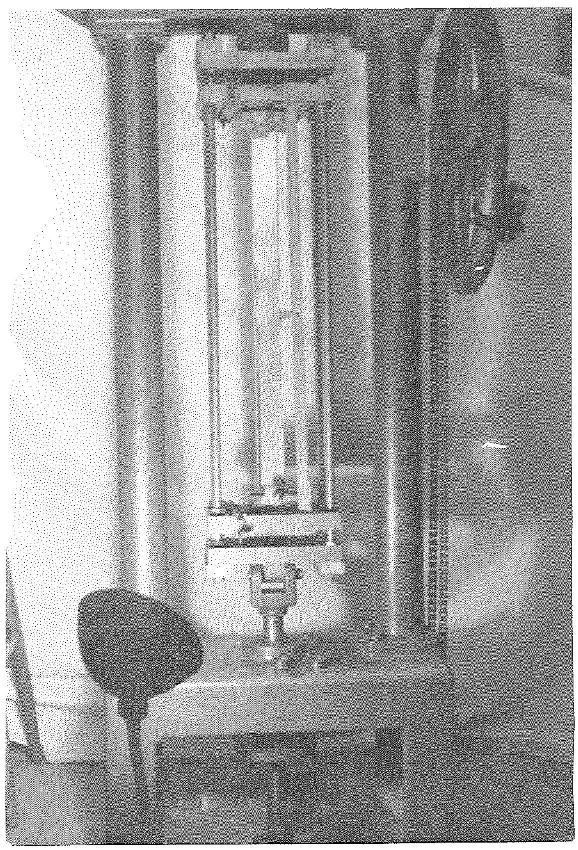


FIGURE 1



View of Testing Machine Showing Compression Cage, Method of Mounting, and Typical Torsion Failure of the Specimen



View of Testing Machine Showing the Apparatus Used and Typical Plate Failure of the Legs of the Specimen

FIG. 2

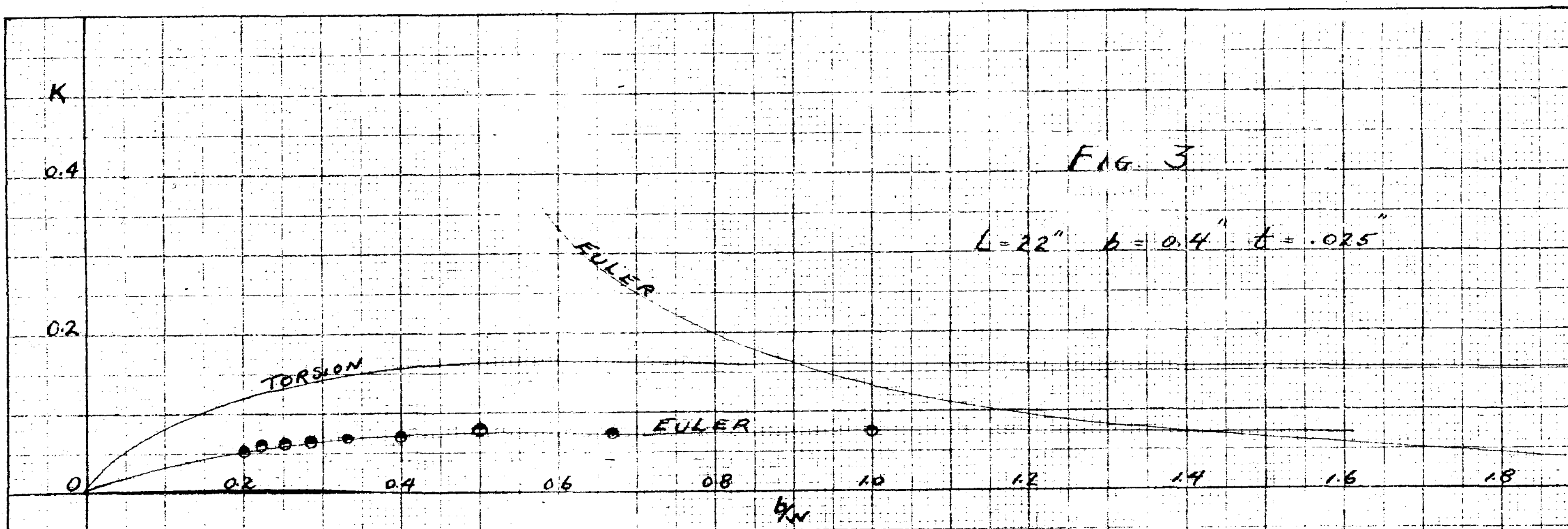


FIG. 3
L = 22"   b = 0.4"   t = .025"
K
0.4
0.2
0
TORSION
EULER
EULER
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
b/w

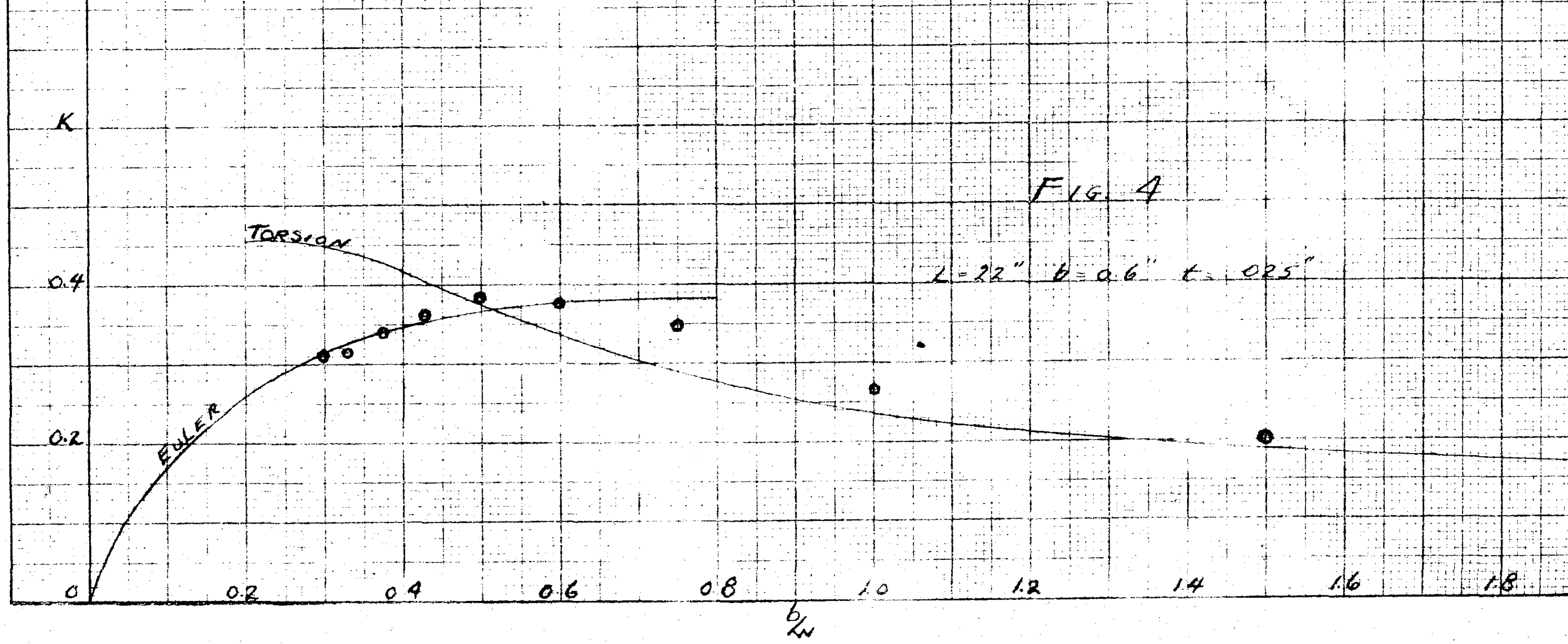
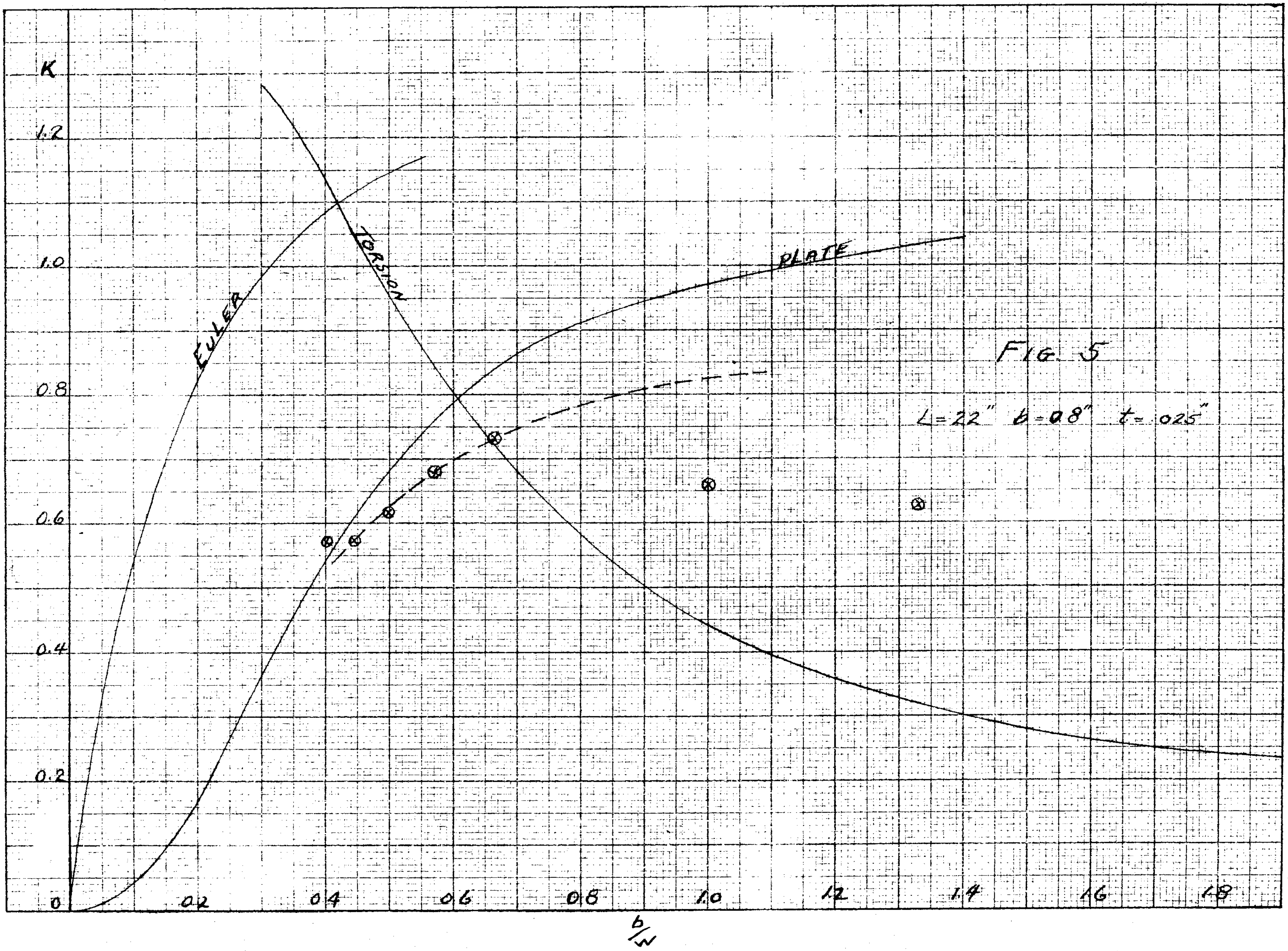


FIG. 4
L = 22"   b = 0.6"   t = .025"
K
0.4
0.2
0
TORSION
EULER
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
b/w



K
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
b/W
EULER
TORSION
PLATE
FIG. 5
L = 22" b = 0.8" t = .025"

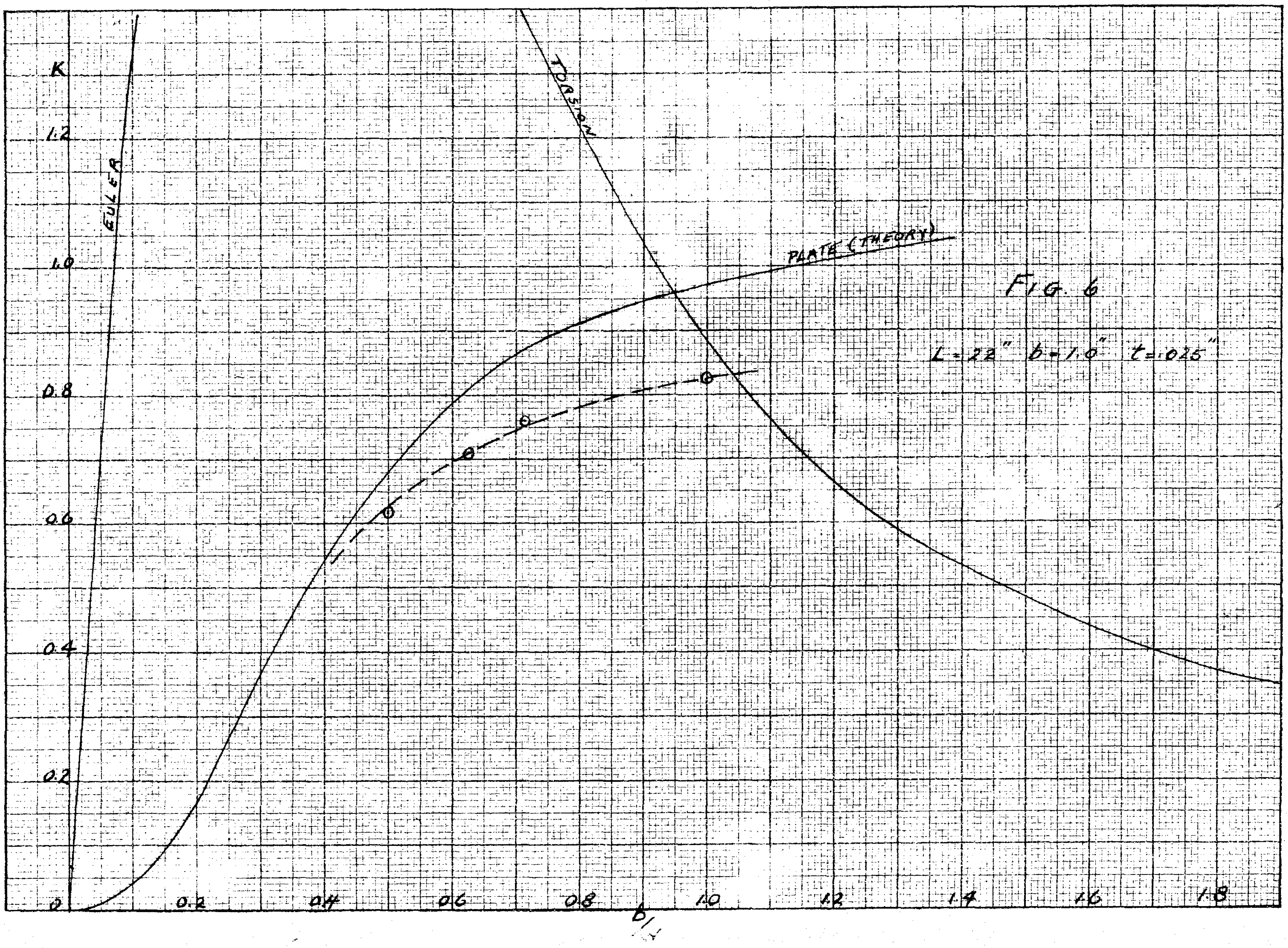

K
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
EULER
TORSION
PLATE (THEORY)
L = 22" b = 1.0" t = .025"

FIG. 6

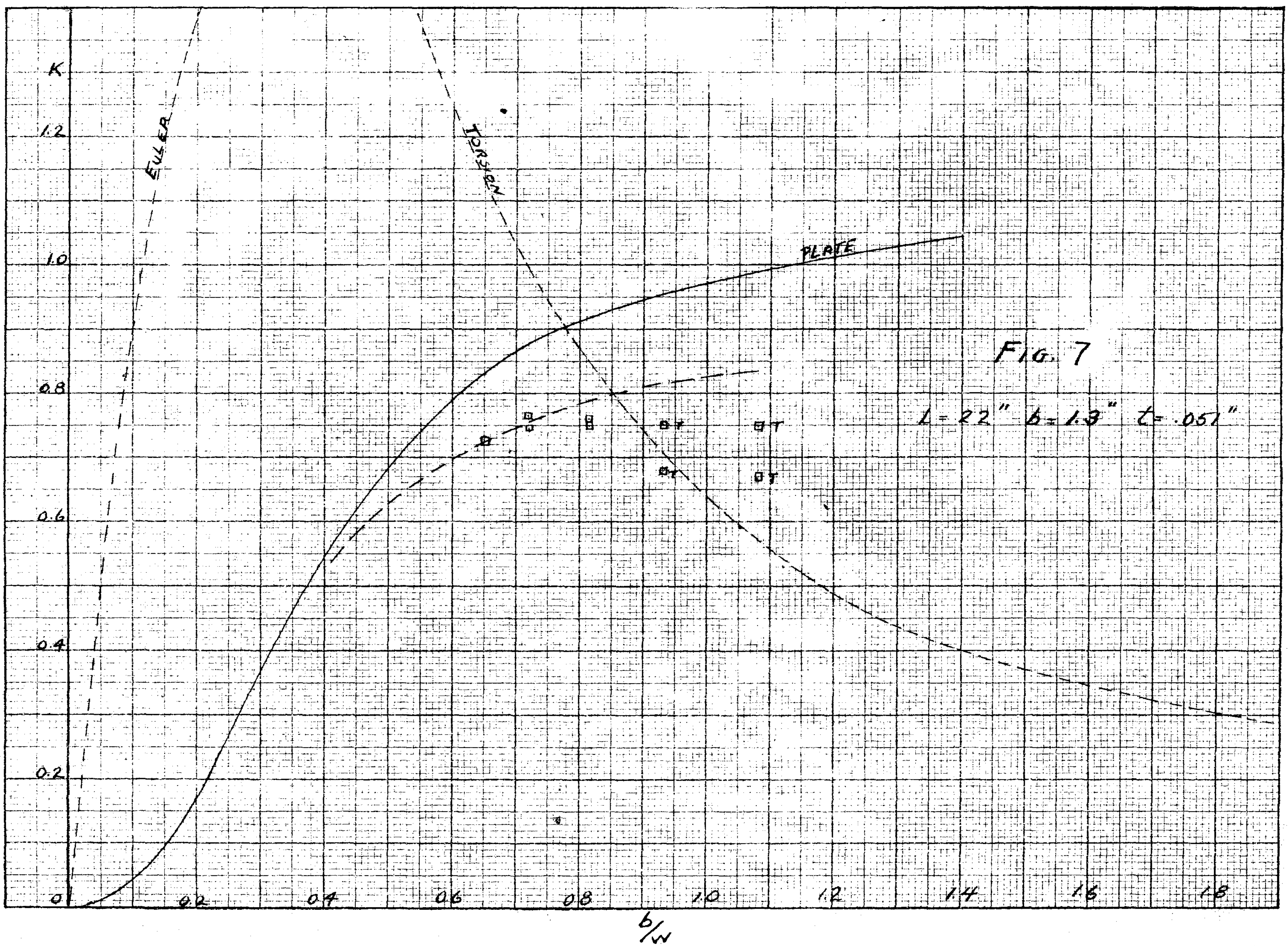
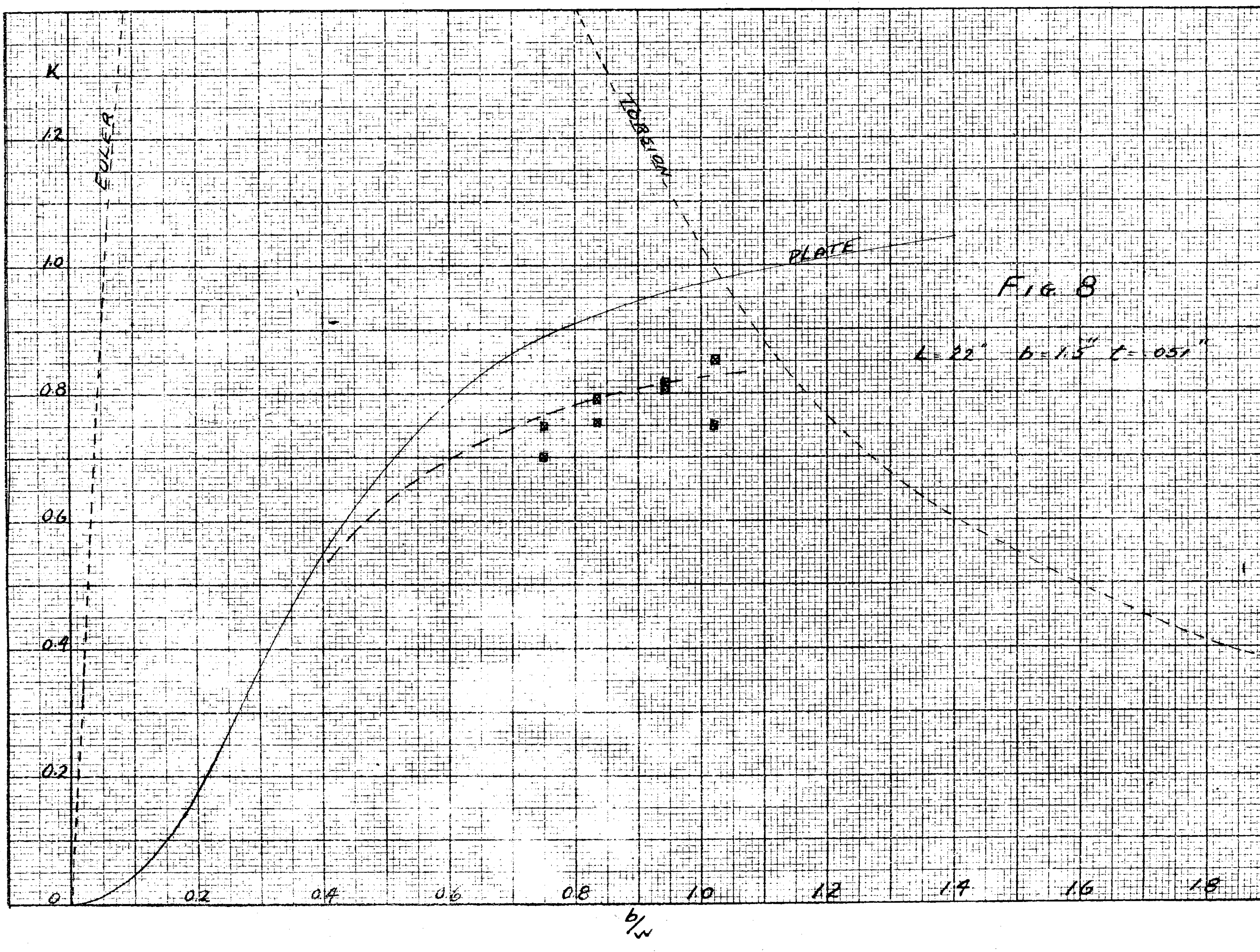


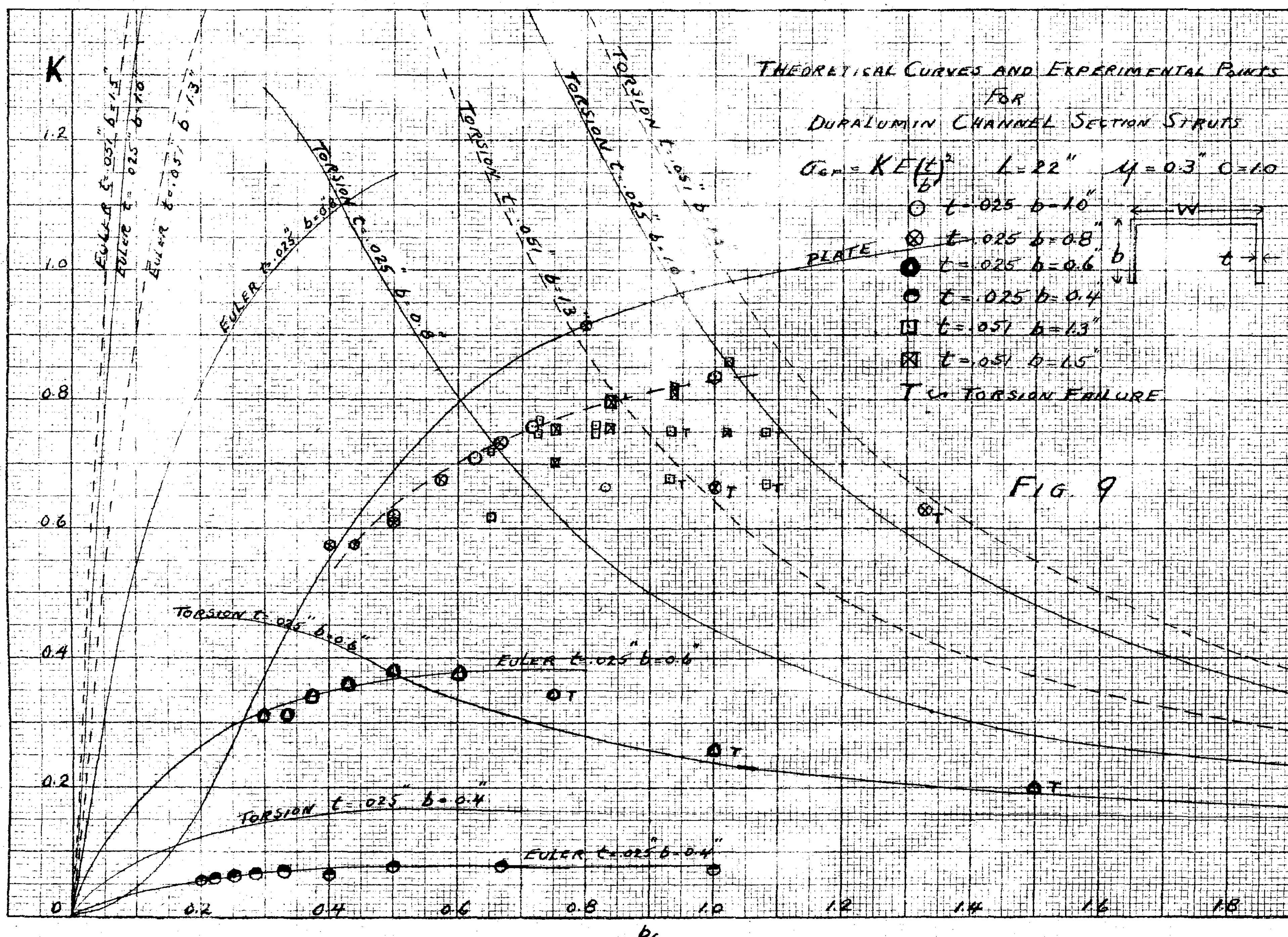
FIG. 7

L = 22" b = 1.3" t = .051"



K
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
b/w
EULER
TORSION
PLATE
L = 22" b = 1.5" t = .051"

FIG. 8


THEORETICAL CURVES AND EXPERIMENTAL POINTS
FOR
DURALUMIN CHANNEL SECTION STRUTS
$\sigma_{cm} = KE\left(\frac{t}{b}\right)^2$ L = 22" y = 0.3" C = 1.0
○ t = .025 b = 1.0"
⊗ t = .025 b = 0.8"
● t = .025 b = 0.6"
● t = .025 b = 0.4"
□ t = .051 b = 1.3"
⊠ t = .051 b = 1.5"
T = TORSION FAILURE
W
b
t
PLATE
FIG. 9
K
b/W
EULER t = .051" b = 1.5
EULER t = .025" b = 1.0
EULER t = .051 b = 1.3
EULER t = .025" b = 0.8"
TORSION t = .025" b = 0.8"
TORSION t = .051" b = 1.3
TORSION t = .025" b = 1.0
TORSION t = .051" b = 1.5
TORSION t = .025" b = 0.6"
EULER t = .025" b = 0.6"
TORSION t = .025" b = 0.4"
EULER t = .025" b = 0.4"
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8