THE CALCULATED FLIGHT PATH OF THE U.S.S. MACON

Thesis by

T. M. Whelan

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Introduction

These calculations are concerned with the flight path of the Macon after the loss of the upper fin and the deflation of the three after gas cells. Taking into account the actual flight conditions, the question was raised whether the ship would rise to an altitude of 4800 feet. For at this altitude, which the record shows was actually reached, the ship had exceeded pressure height by about 2000'. In going over pressure height, the ship loses approximately 3% of her lift for every 1000 feet. Hence 4800 feet corresponds to a heaviness of nearly 11 tons. It was felt that if this additional heaviness could have been avoided, the ship might have been flown safely in spite of the loss of some 20 tons of lift from the three after cells. The calculations were undertaken to determine whether the aerodynamic and aerostatic forces alone were sufficient to force the ship up to 4800 feet or whether there were external forces acting, i.e. gust forces.

Assumptions

General

Because of the scarcity of exact information, many of the assumptions are somewhat arbitrary. It was desired, however, aside from the control maneouvers, to assume those conditions leading to the highest rise.

Zero time for the calculations was taken when the gust struck the after part of the ship. Previous to this time, it was assumed that the ship was flying at 1250 feet in equilibrium, and in horizontal flight with neutral elevator. The last assumption required a negative or nosing-down moment of the ballast of 468 ft.-tons.

The sharp inclination of the ship caused by the gust was simulated in the calculations by delaying the elevator action. Full down elevator was not applied until 1 minute and 50 seconds after the casualty. It was assumed that the elevator control was unaffected by the casualty. The changes in superheat were small (5°) and were neglected in the calculations.

Gas Loss

From the record we have the following sequence of events in regard to the loss of gas:

0':05''	- No. 1 cell damaged
0:10	- Davis driven from frame 17.5 by escaping gas
0:14	- No. 2 cell was damaged
0:25	- No. 0 cell was damaged
0:40	- Deflation of cells proceeding No. 0 cell deflating slowly.
1:01	- Message received in control car No. 1 cell deflated
2:30	- Message received in control car No. 2 cell deflated

It was assumed from this that most of the lift of the three after cells had left the ship 2 minutes and 30 seconds after the casualty.

The sea-level volume of the three after cells at the time of the casualty, was estimated by the ship's First Lieutenant as follows:

Cell No. 0	109,200 c.f.
" " 1	245,700 "
" " 2	386,750 "
Total	741,650 c.f.

The gas was stated to be 95% pure; this gave a lift of 62.7 lbs. per 1000 c.f. of gas. The estimated total lift in the three after cells was then 23.3 short tons. If we leave sufficient gas in the cells to support their own weight, we have a total loss of lift of about 20.6 tons. At the start of the calculations, it appeared that the static moment caused by this loss of lift was more important than the static heaviness. For this reason the total lift of 20.6 tons was assumed to be lost in the first 2-1/2 minutes.

Because a reduction in static heaviness means a reduction also of the static moment, it is difficult to estimate the effect of a more moderate rate of gas loss. However, it seems improbable that such high pitch angles (25°) could be attained a few minutes after the casualty with a rate gas loss greatly less than that assumed. For the purpose of calculation, a linear rate of gas loss was used somewhat retarded in the last minute of the 2-1/2 minute total period.

Ballast

The distribution of the fuel and water ballast just before the casualty is shown in Table 1. An arbitrary schedule for dropping ballast was assumed and is shown in Figure 2. In the calculations only the ballast up to and including frame 102.5 was dropped. To this amount was added the weight of the upper fin which was about 2000 lbs. This left 9 tons of ballast in the fore part of the ship which was dropped before the ship hit the water. Ordering the crew into the bow three minutes after the casualty was assumed to give a change of moment of 450 ft.-tons.

Speed and Thrust

The initial speed was taken as 68.2 knots (100 f/s) which was about 8 knots higher than the observed speed. There is no accurate record of how the engines were operated during the period of the calculations. Although standard speed was registered on the engine telegraph most of the period, there is evidence that some of the engines cut out and others ran irregularly at high pitch angles. The engines were also idled from the control car but the time and duration of this order is not known. Since the moment due to the engine thrust was generally small in comparison with the other terms in the moment equation, it was decided to vary the thrust in accordance with the observed air speed, which was important in all the equations. The resulting speed curve used is shown in Figure 3. It is initially and throughout the calculations somewhat higher than observed.

Force and Moment Coefficients

The normal force and moment coefficients due to translation and angle of attack were based on N.A.C.A. wind tunnel tests on a model of the Akron (Figs. 8 and 9). The force and moment coefficients due to rotation were obtained analytically and are shown in Figure 10. They are in fair agreement with the British experimental results for the R-101. The derivation is contained in the Appendix.

The Calculations

The notation, equations, and assumed dimensions are shown in Figure 1. Two-second intervals were used for the first half of the calculations; later it was found that four-second intervals were satisfactory, since small errors in assuming mean values, were quickly damped and had a negligible effect on the resultant motion. A summary of the calculations with sample calculating page is given in the Appendix.

Results

Using the above assumptions, the calculated flight path is shown in Figure 4. The best available record of the observed path is contained in Dr. Hovgaard's report on the loss of the Macon. For comparison his results are also plotted in Figure 4. The agreement here is closer than was anticipated. It will be noted that Dr. Hovgaard's plot shows a decided tendency of the ship to level off at about 4 minutes. This was followed a minute later by another rapid rise of the ship. At this point in the calculations, it seemed that this second rise might be due to the increased engine thrust following the order for standard speed which occurred about this time. To test this effect an increasing engine thrust up to 9 tons was used (Fig. 3). The pitch angle increased slightly as can be seen in Figure 5 but was soon reduced by the increase in speed and consequent increase in effectiveness of the elevators. It does not appear then that the sudden rise occurring around 5 minutes was due to increased engine thrust. It is difficult to estimate the effect of a change in the original assumptions on the behavior of the ship at this point, but it seems improbable that a reduction in the rate of gas loss would lead to this second rise of the ship. In order to examine the effect of the engine thrust, the valving of No. 9 cell was delayed. This occurred actually around 4 minutes, but was taken up in the calculations at 5 minutes. The valving of 4 tons of lift from No. 9 cell had an immediate effect. The resulting reduction in the static moment led to complete control of the ship in the calculations. Control was attained with the ship 14 tons heavy, flying at an angle of attack of 14 degrees and at a speed of 30.5 knots. Here it seems that if any lift still remained in the after cells, control of the ship would have been facilitated. As noted before, the ship was not actually brought under control but rose rapidly about 1000 feet after beginning to level off. The ship was observed to level off at about 3800 feet, while the calculated maximum altitude attained was 4200 feet. That the calculated path levels off some 400 feet higher than was observed, may be due to the rather optimistic lift coefficient and higher speed used, and the delaying of the valving of No. 9 cell.

Hence it appears that probably not only the difference of 700' but the total rise of 1100' after the actual path was leveled off has to be considered as unexplained by static and dynamic forces.

Conclusions

In the present work the maximum probable rate of gas loss was assumed. With this assumption the calculation leaves unexplained a rise of 700 to 1100 feet in altitude. With a more moderate rate of gas loss, the steepness of the initial flight path cannot be accounted for. In either case it appears that in addition to the structural failure the ship encountered rising air currents before control of the ship had been attained. In these calculations this meant an increased heaviness of 4 to 6 tons, which, combined with additional heaviness due to subsequent valving from #9 and #10 cells, was apparently more than the ship could support in steady flight. The arbitrary character of some of the assumptions is fully realized. However, it is believed that the calculations brought out the relative influence of the different factors such as gas loss, dropping of ballast, thrust, valving from the forward cells and especially the importance of correcting the unbalanced static moment as early as possible for regaining control of the ship.

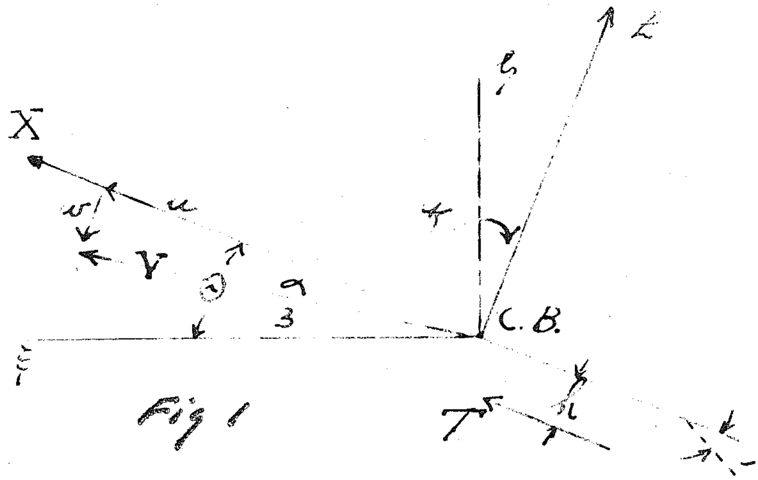


Fig 1

Equations and Assumptions

$$m_1\dot{u} = -X_\alpha V^2 + T - (W + \delta W)\sin\theta + (X_q V_q - m w q)$$

$$m_2\dot{w} = Z_\alpha V^2 - (W + \delta W)\cos\theta + Z_q V_q - m_1 u q$$

$$B\dot{q} = M_\alpha V^2 - M\sin\beta + (k\,\delta W - M_B)\cos\theta - M_q V_q + Th$$

$\tan\alpha = -w/u \quad (X_q V_q - m w q)$ neglected (R & M 748)

$V = u\sec\alpha$

$\xi = V\cos\beta$

$\zeta = V\sin\beta$

$X_\alpha V^2,\ Z_\alpha V^2,\ M_\alpha V^2$ = Normal lift, Drag and Moment due to translation and angle of attack

T = Thrust

W = Heaviness

δW = Change in heaviness, loss of gas or ballast

$X_q V_q,\ Z_q V_q,\ M_q V_q$ = Drag, Lift, Moment due to rotation

M = Static Moment (4000 ft.-tons)

M_B = Initial Moment for horizontal flight (488 ft.-tons)

h = Distance from C.B. to thrust line 45 ft.

α = Angle of attack

β = Inclination of flight path

θ = Inclination of ship's axis

ϵ = Elevator angle

$k\,\delta W$ = Moment of gas lost or ballast dropped

m = Mass of displaced air = 9.07 tons mass

m_1 = Longitudinal apparent mass = 8.65 tons mass

m_2 = Transverse apparent mass = 13.14 tons mass

B = Apparent moment of inertia = 475,000 tons-ft.^{2}

x_1 = Nose to C.B. = 362 ft.

x_2 = Stern to C.B. = 423 ft.

x_3 = C.P. of fins to C.B. = 317 ft.

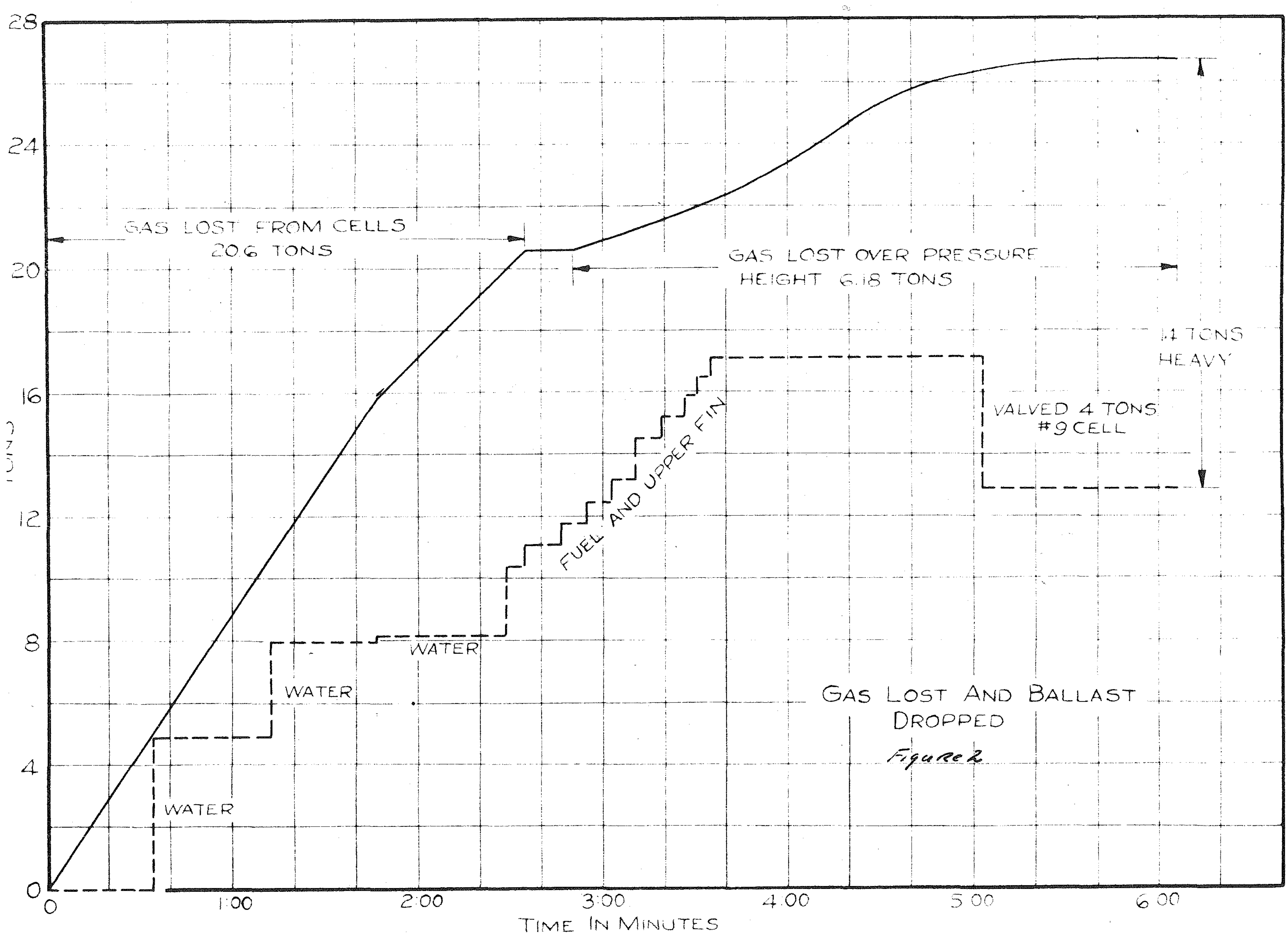
Volume = 7.4 x 10^6 : (Vol.)$^{2/3}$ = 38100 : (Vol.)$^{1/3}$ = 195

	Tank No.	Capa-city	Fuel on Board	Fuel Dropped	Water Ballast (lbs)	Tank No.	Capa-city	Fuel on Board	Fuel Dropped	Water Ballast (lbs)
F	1(D)	700#				2(D)	700#	700	700	
R	3(S)	700	700	700	E-900*	4(S)	700	700	700	E-900*
A	5(D)	700			E-900*	6(D)	700	700	700	
	7(D)	700	700	700		8(D)	700	700	700	
E	9(S)	700	700	700		10(S)	700	700	700	
26.5	11	700				12	700			
FR.	15	2200			2000*	16	2200			2000*
35					1900*					1900*
	23	700				24	700			
	25(S)	700	700	700		26(S)	700	700	700	
	27	700				28	700			
FR.	29	700				30	700			
57.5	31(S)	700	700	700		32(S)	700	700	700	
	33	700				34	700			
	I-A	2400	2000		1500*	II-A	2400	2000		1500*
	I-F	2400	500		1200*	II-F	2400	500		1200*
	35	700				36	700			
	37(S)	700	700	700		38(S)	700	700	700	
	39	700				40	700			
FR.	41	700				42	700			
80	43(S)	700	700	700		44(S)	700	700	700	
	45	700				46	700			
	III-A	2400	2000			IV-A	2400	2000		
	III-F	2400	1700			IV-F	2400	1700		
	47	700	700			48	700			
	49(S)	700	700	700		50(S)	700	700	700	
	51	700				52	700			
FR.	53	700				54	700			
02.5	55(S)	700	700	700		56(S)	700	700	700	
	57	700				58	700			
	V-A	2400	2000		1200	VI-A	2400	2000		1200
	V-F	2400	600			VI-F	2400	700		
	63	700				64	700			
FR.	VII-A	2400	1300			VIII-A	2400	2000		
125	VII-F	2400				VIII-F	2400	600		
						74	2200			
	79	2200	1700		1000	80	2200	1600		1000
FR.	81	2200				82	2200			
47.5	83	700				84	700			
	85(S)	700	700			86(S)	700	700		
	87	700				88	700			
	89	700				90	700			
	91(S)	700	700			92(S)	700	700		
FR.	93	700				94	700			
70	95	2200			1900	96	2200			1900
	97	2200			1900	98	2200			1900
	99	700				100	700			
	101(S)	700	700			102(S)	700	700		
	103(D)	700	700			104(D)	700	700		
	105(D)	700	700		E-900	106(D)	700	700		
R.	107(S)	700	700		E-900	108(S)	700	700		E-900
78.5	109(D)	700			E-900	110(D)	700	700		E-900

(D) indicates fuel <u>dump</u> tank
(S) indicates fuel <u>slip</u> tank
E indicates emergency water ballast bag

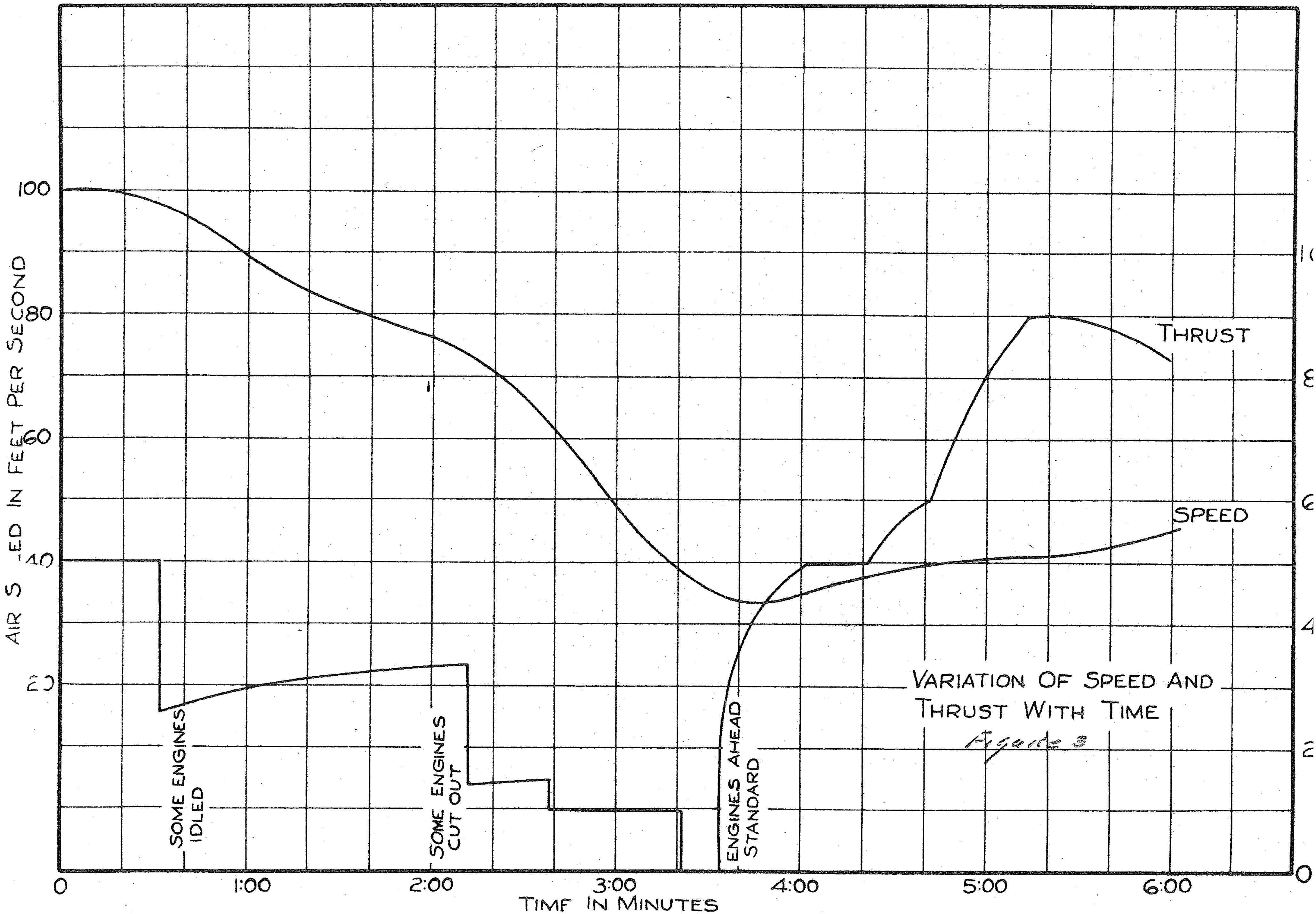
Fuel was dropped almost instantly
Ballast marked "*" dropped instantly
Remainder of ballast dropped before ship hit the water

Table 1



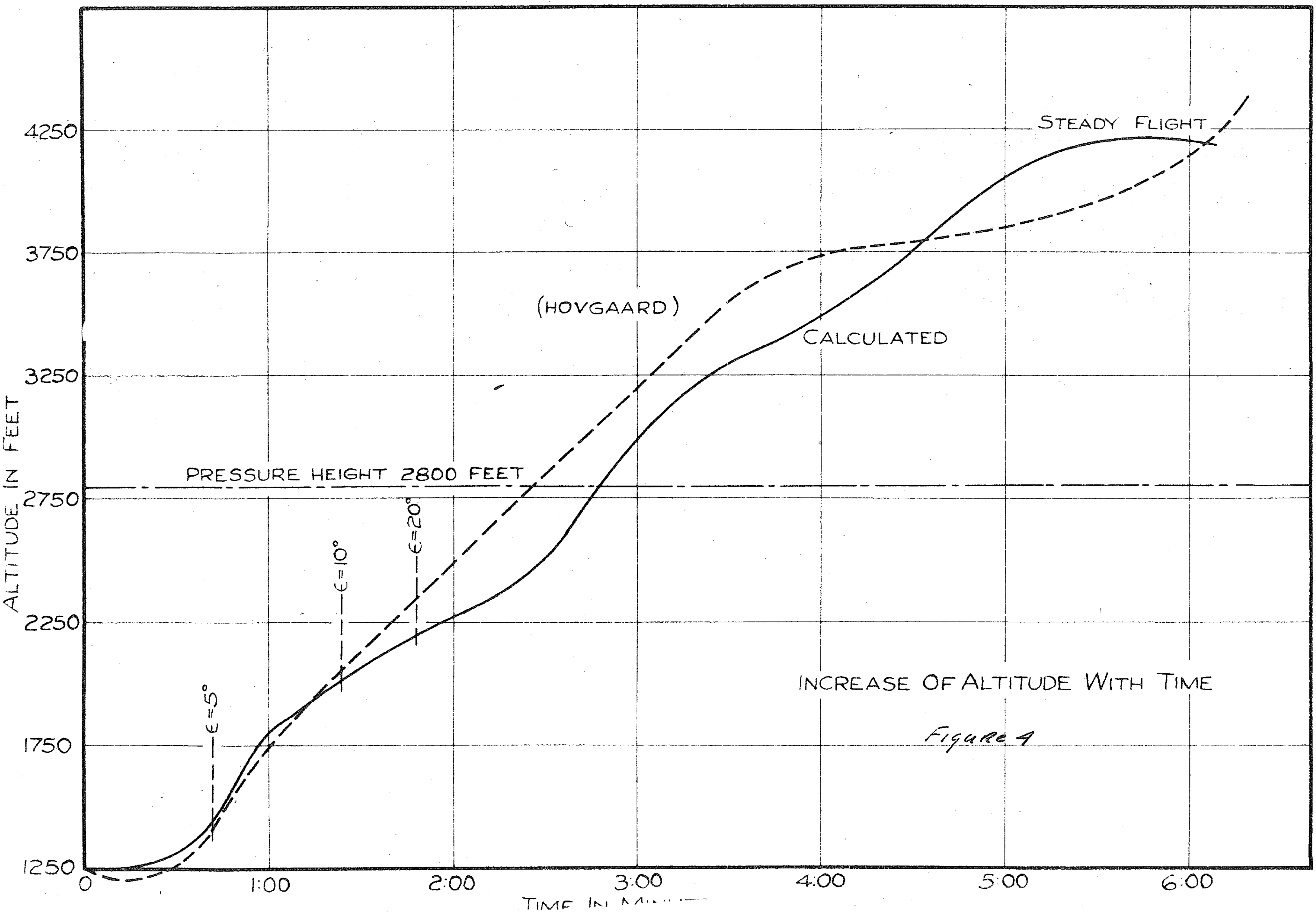
GAS LOST AND BALLAST DROPPED

Figure 2



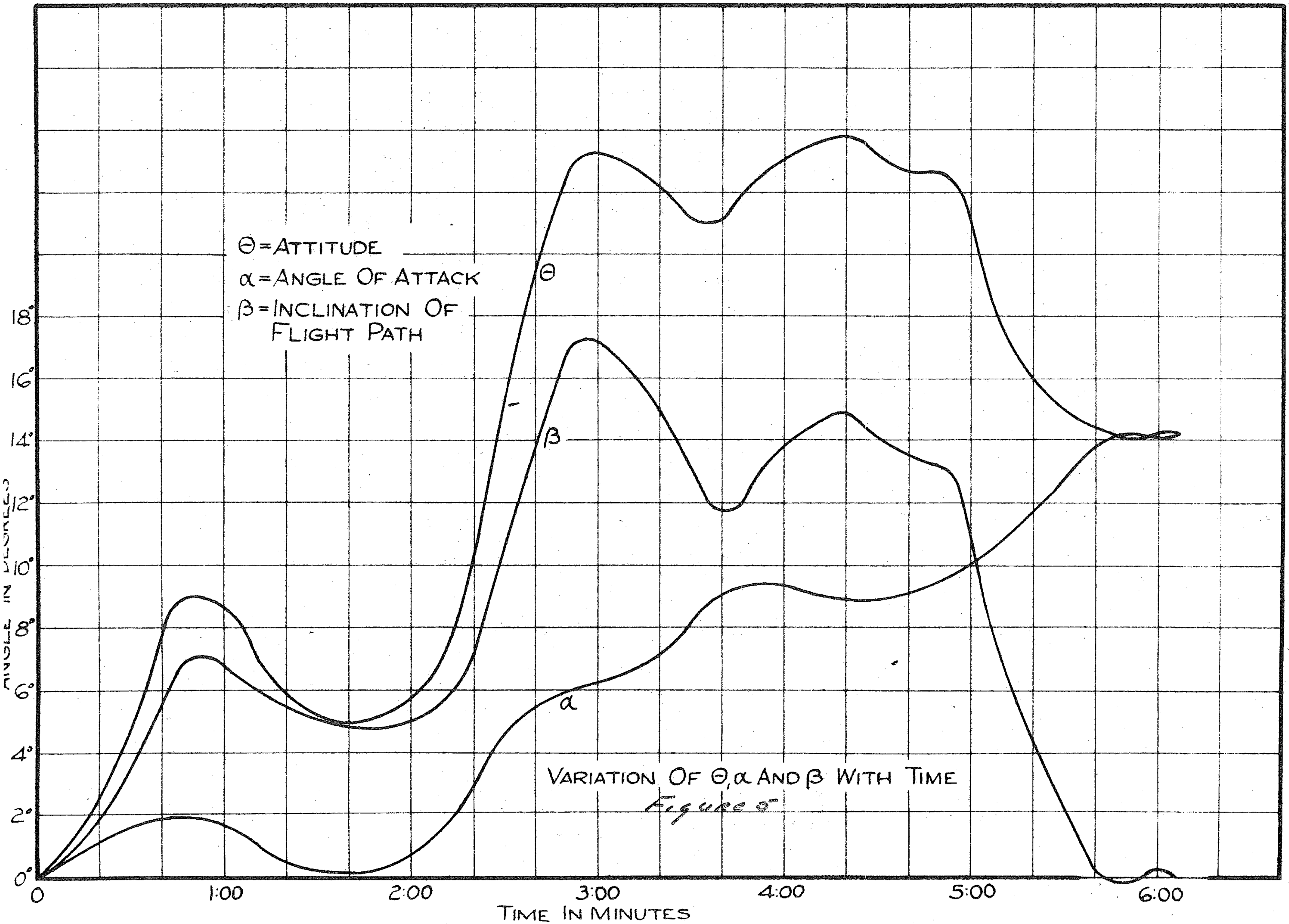
VARIATION OF SPEED AND THRUST WITH TIME

Figure 3



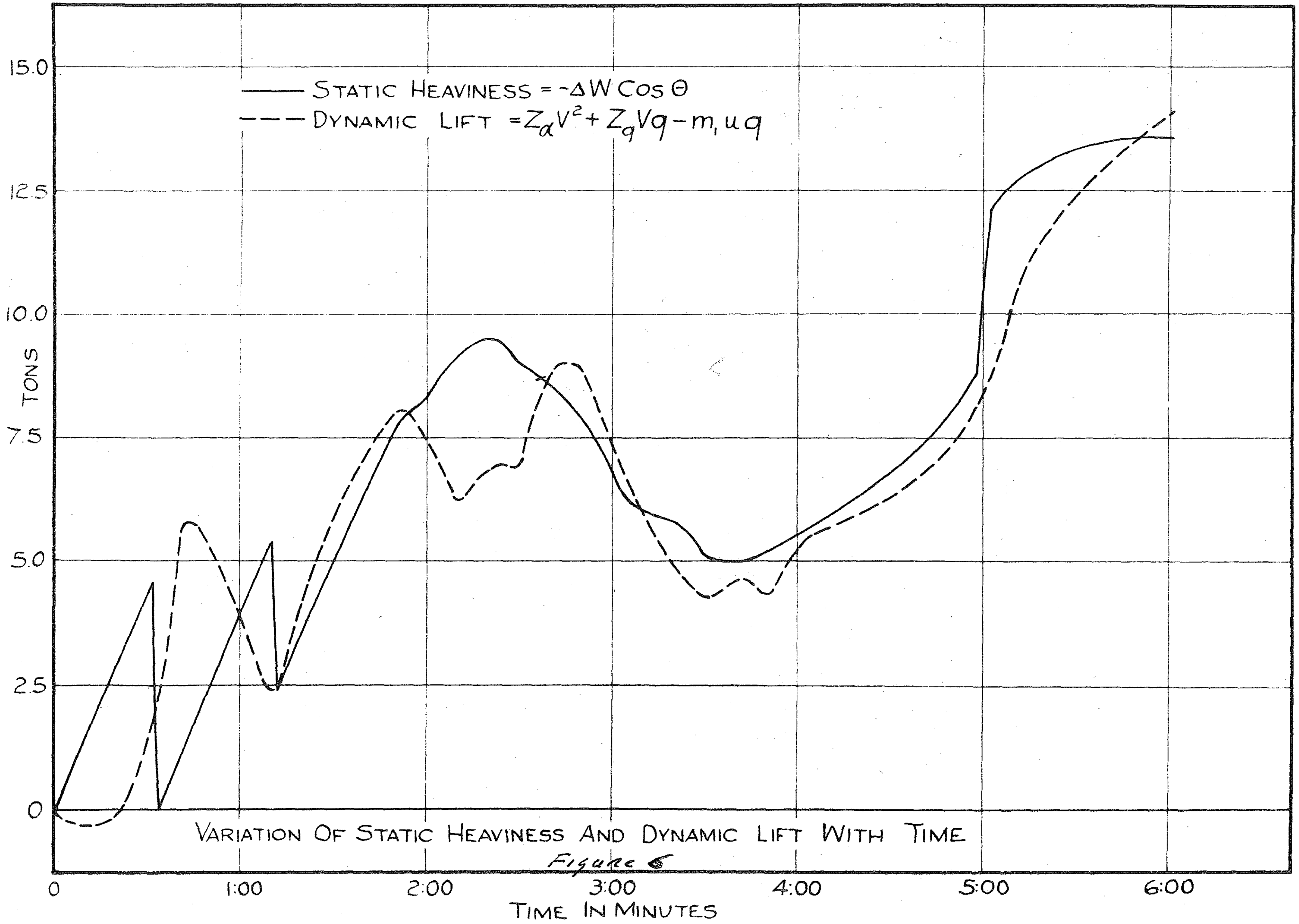
INCREASE OF ALTITUDE WITH TIME

Figure 4



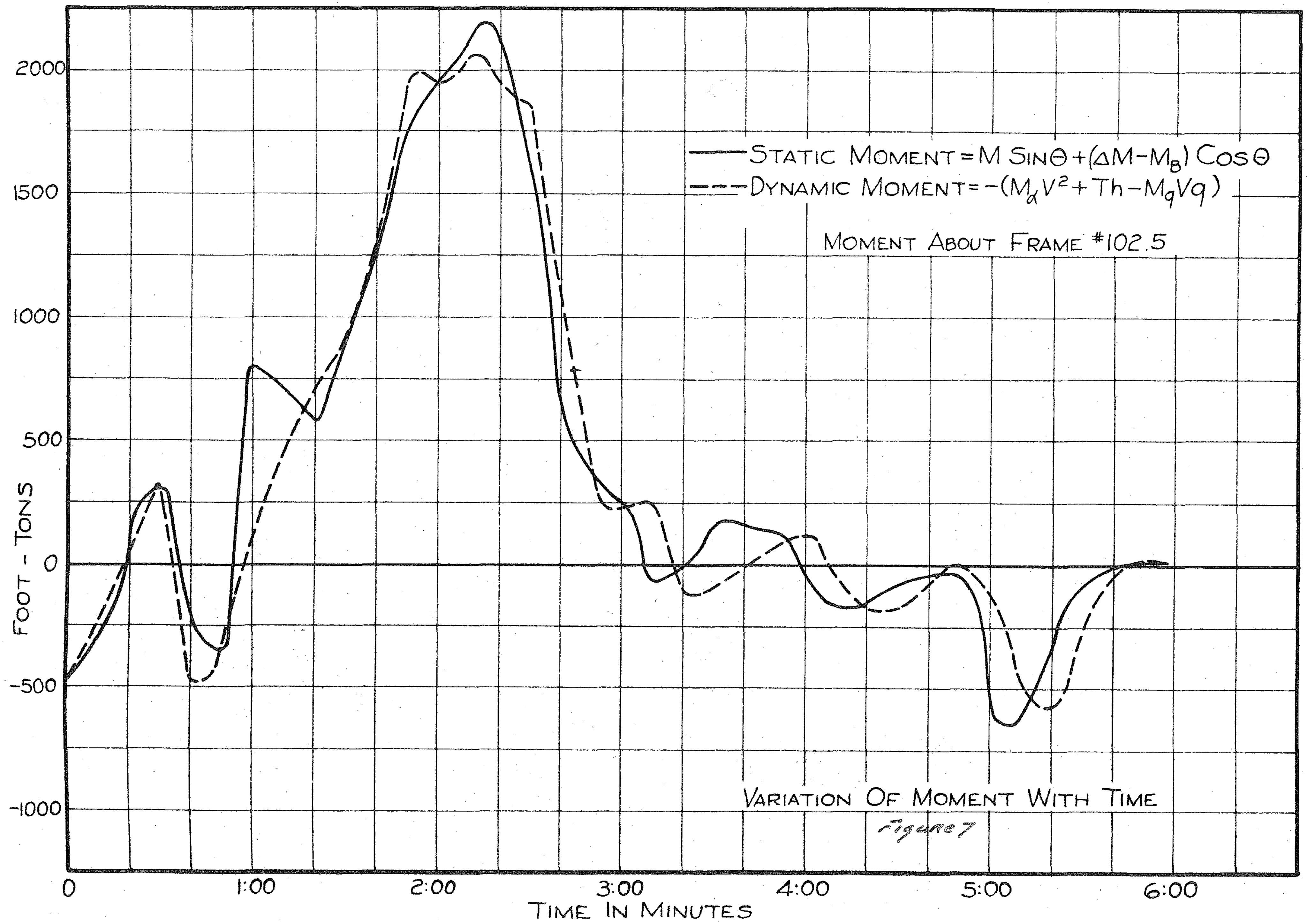
Variation Of Θ, α And β With Time

Figure 5



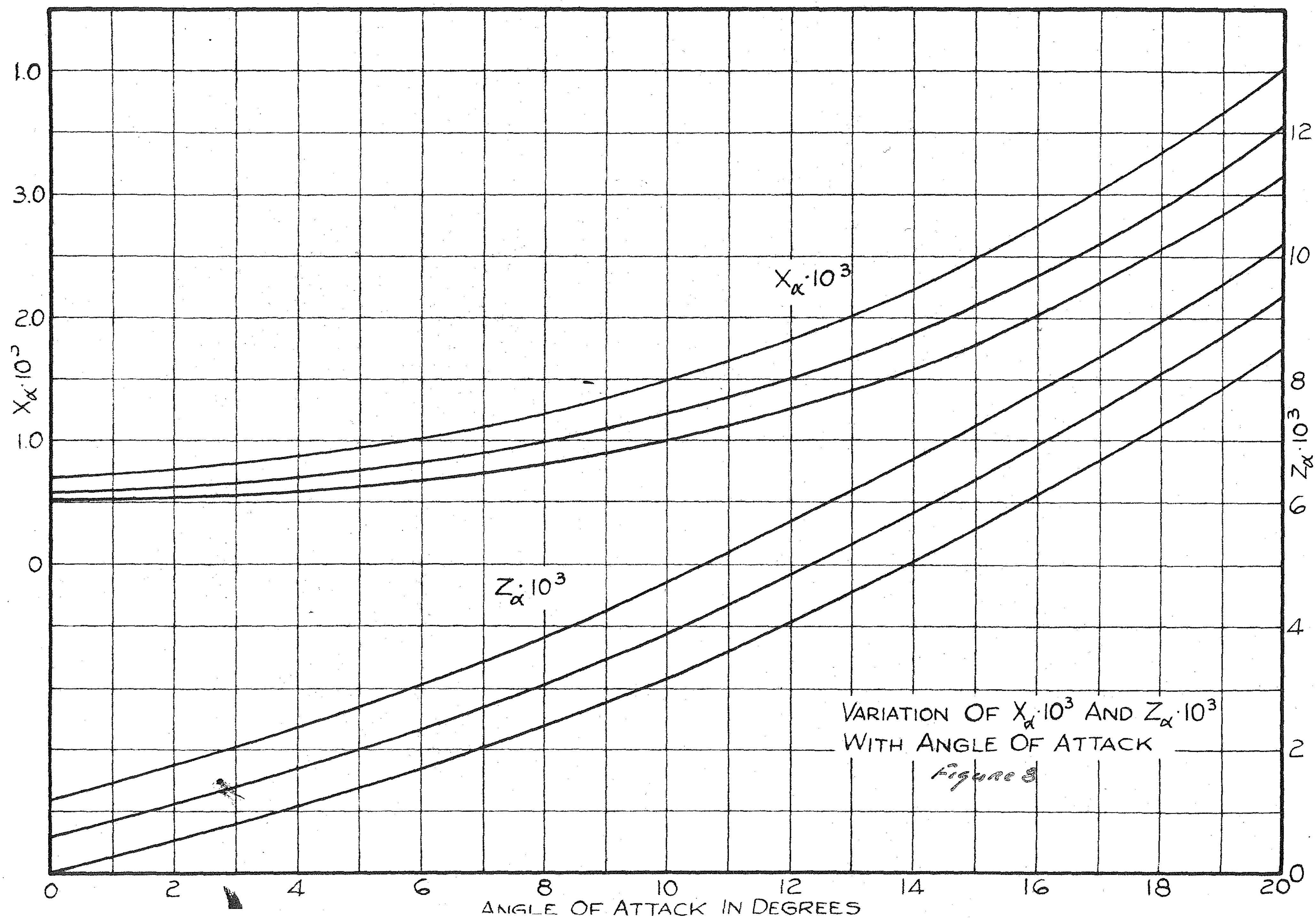
Variation Of Static Heaviness And Dynamic Lift With Time

Figure 6



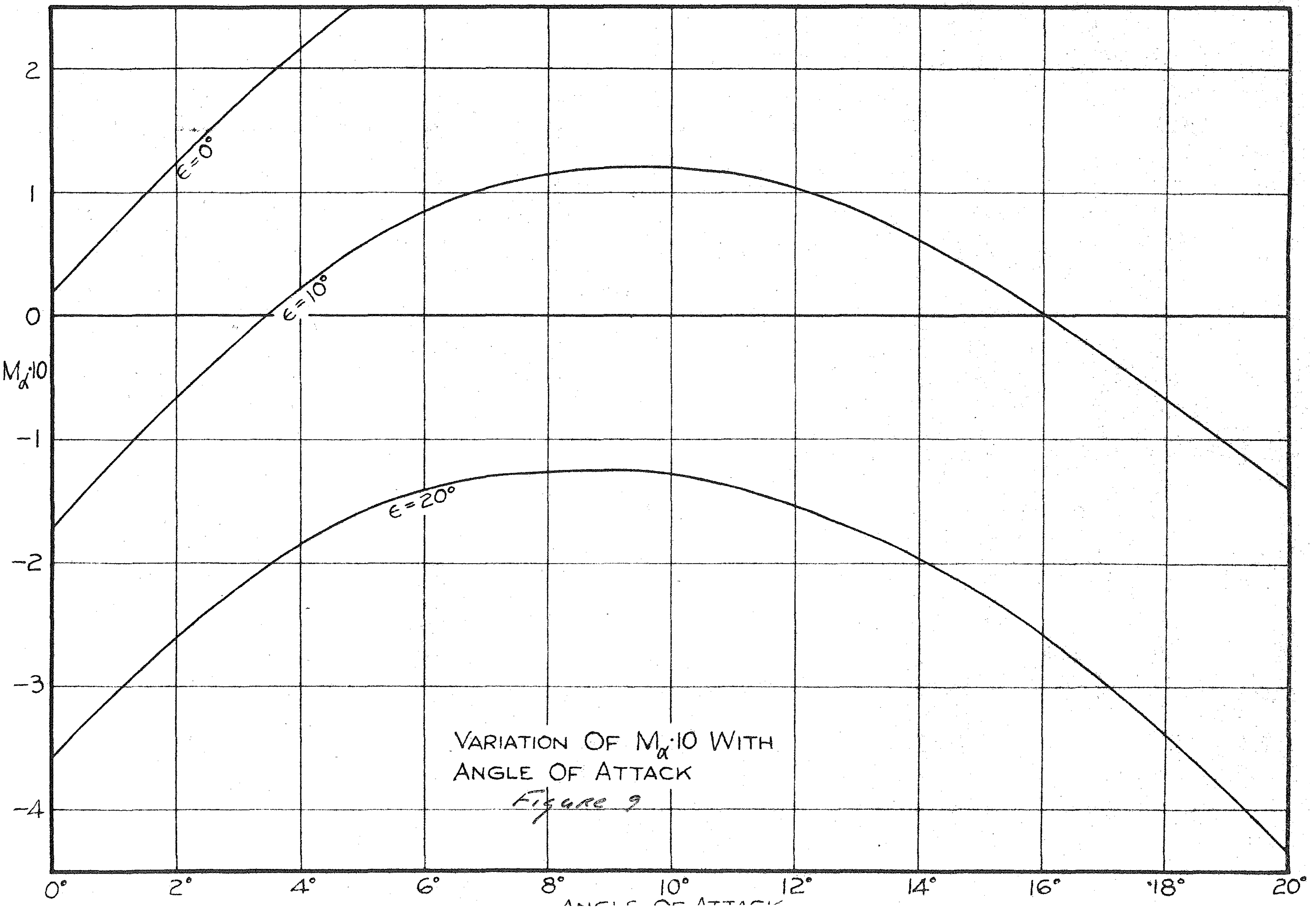
VARIATION OF MOMENT WITH TIME

Figure 7



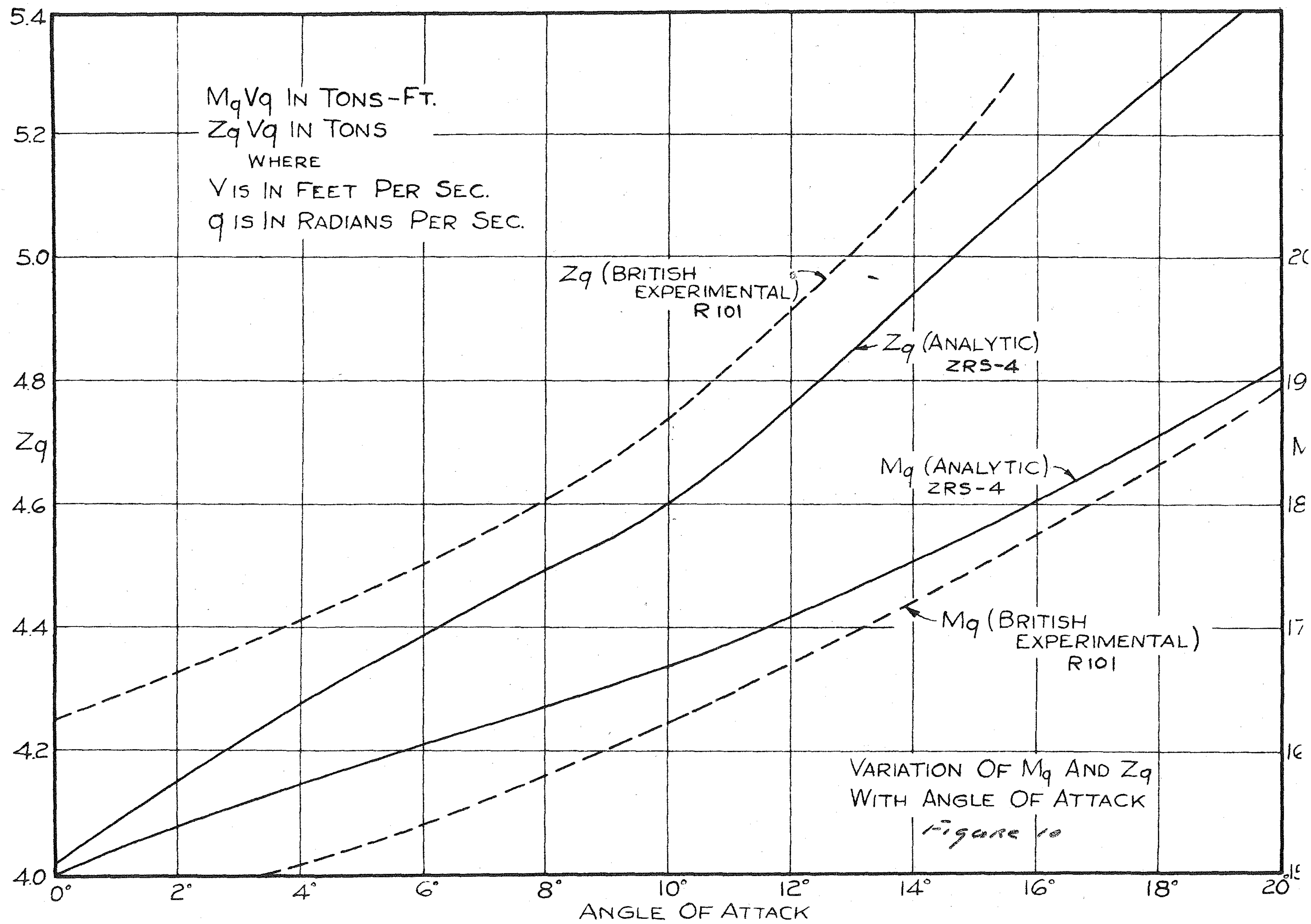
VARIATION OF $X_\alpha \cdot 10^3$ AND $Z_\alpha \cdot 10^3$ WITH ANGLE OF ATTACK

Figure 8



VARIATION OF $M_\alpha \cdot 10$ WITH ANGLE OF ATTACK

Figure 9


5.4
5.2
5.0
4.8
Zq
4.6
4.4
4.2
4.0
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
20°
ANGLE OF ATTACK
$M_q V_q$ IN TONS-FT.
$Z_q V_q$ IN TONS
WHERE
V IS IN FEET PER SEC.
q IS IN RADIANS PER SEC.
Zq (BRITISH EXPERIMENTAL) R 101
Zq (ANALYTIC) ZRS-4
Mq (ANALYTIC) ZRS-4
Mq (BRITISH EXPERIMENTAL) R 101
VARIATION OF M_q AND Z_q
WITH ANGLE OF ATTACK
Figure 10

APPENDIX

1. The normal lift, drag and moment due to translation and angle of attack

$$L = C_L q\,(\text{Vol.})^{2/3} = V^2\left(C_L \frac{\rho}{2}\ \text{Vol.}^{2/3}\right) = V^2 L_\alpha$$

$$D = C_D q\,(\text{Vol.})^{2/3} = V^2\left(C_D \frac{\rho}{2}\ \text{Vol.}^{2/3}\right) = V^2 D_\alpha$$

$$M = C_M q\,(\text{Vol.})^{2/3} = V^2\left(C_M \frac{\rho}{2}\ \text{Vol.}^{2/3}\right) = V^2 M_\alpha$$

$$Z = L\cos\alpha + D\sin\alpha = V^2 L_\alpha\cos\alpha + V^2 D_\alpha\sin\alpha$$

$$X = -L\sin\alpha + D\cos\alpha = V^2 L_\alpha\sin\alpha + V^2 D_\alpha\cos\alpha$$

$$Z = V^2(L_\alpha\cos\alpha + D_\alpha\sin\alpha) = V^2 Z_\alpha$$

$$X = V^2(-L_\alpha\sin\alpha + D_\alpha\cos\alpha) = V^2 X_\alpha$$

$$M = V^2 M_\alpha$$

C_L, C_D and C_M taken from NACA Report 443

ρ = density of standard atmosphere at 3000' or .00218 $\frac{\text{lbs. mass}}{\text{c.f.}}$

Vol. = 7.4×10^6 (Vol.) = 38100

The quantities Z_α, X_α, and M_α were thus determined and plotted against angle of attack for use in the equations. (Figs. 8 and 9).

2. The force and moment coefficients due to rotation. On an elemental cross-section, we have

$$\frac{dF}{dx} = \frac{d}{dt}(M'w)$$

$$u\,dt = dx$$

$$w = qx$$

where F = force

M' = apparent mass

u = longitudinal velocity component

w = transverse velocity component

q = angular velocity

x = longitudinal distance from C.B.

If S = cross-sectional area, then the apparent mass is given by $M' = \rho S$.

The force equation then becomes

$$\frac{dF}{dx} = \rho u q \left(S + x \frac{dS}{dx}\right)$$

In order to take care of end effects, we introduce the factor $\cos^2\tau$, where τ is the angle made by a tangent to the hull profile and the longitudinal axis of the ship. In addition we account for the trailing vortex system with a factor suggested by Dr. von Karman, namely $\frac{r_2}{r_0}$, where r_2 is the radius of any rear body cross-section and r_0 is the radius of the largest cross-section.

If we let $\sigma = S \cos^2\tau$ our equation becomes on integrating:

$$F = \rho u q \int_{x_1}^{0} \left(\sigma + x \frac{d\sigma}{dx}\right) \quad \text{for the fore body}$$

$$F = \rho u q \int_{0}^{-x_2} \frac{r_2}{r_0}\left(\sigma + x \frac{d\sigma}{dx}\right) \quad \text{for the rear body}$$

where x_1 is distance to the bow and x_2 is distance to stern measured from the center of buoyancy. $x_1 = 362'$, $x_2 = -423'$.

The moment is given by introducing the factor x under the integral in each case.

A good approximation to the hull profile is given by an expression due to Milton and Frances Clauser.

$$r_1/r_0 = \left(1 - [x/x_1]^3\right)^{1/2} \quad \text{for the fore body}$$

$$r_2/r_0 = \left(1 + [x/x_2]^3\right) \quad \text{for the rear body}$$

APPENDIX (cont'd)

Since $\frac{dr}{dx} = \tan \tau$ it was possible to plot the quantity σ against the ship's length and hence measure $\frac{d\sigma}{dx}$. The quantities under the integrals were then plotted and areas measured by a planimeter (Fig. 11). This method gave the following values for the hull forces due to rotation:

$$F_h = 550.25 \, \rho u q \text{ tons}$$

$$M_h = 302870 \, \rho u q \text{ ft. tons}$$

From the wind tunnel data for the hull alone and the hull with tail surfaces, a normal force coefficient for the tail surfaces was obtained.

The force due to the tail surfaces was then

$$F_T = \rho \frac{V^2}{2} \left(\frac{dC_n}{d\alpha_T} \alpha_T \right) vol^{2/3}$$

$$= \rho \frac{V^2}{2} \left(\frac{dC_n}{d\alpha_T} \right) \frac{\omega_T}{u} \cdot vol^{2/3}$$

$$= \rho \frac{V^2}{2} \left(\frac{dC_n}{d\alpha_T} \right) q \, x_3 / V \cos\alpha \cdot vol^{2/3}$$

where

x_3 = distance from C.B. to C.P. of tail taken as 300'

ρ = .00218 and $Vol.^{2/3}$ = 38100

Then $F_q = F_h + F_T$

$$= V_q \left(1.2 \cos \alpha + \frac{6.23}{\cos \alpha} \frac{dC_n}{d\alpha_T} \right) \text{ tons}$$

and $M'q = V_q \left(660 \cos \alpha + \frac{1865}{\cos \alpha} \frac{dC_n}{d\alpha_T} \right)$ ft. tons

The effect of elevator angle was neglected in evaluating these quantities. From the wind tunnel data we have the following values for $\frac{dC_n}{d\alpha_T}$:

α^o	$dC_n/d\alpha_T$
0	.45
5	.50
10	.54
15	.60
20	.65

For the purpose of calculation the quantities in the parenthesis were evaluated and plotted against angle of attack (Fig. 10). Then

$$F_q = V_q Z_q$$

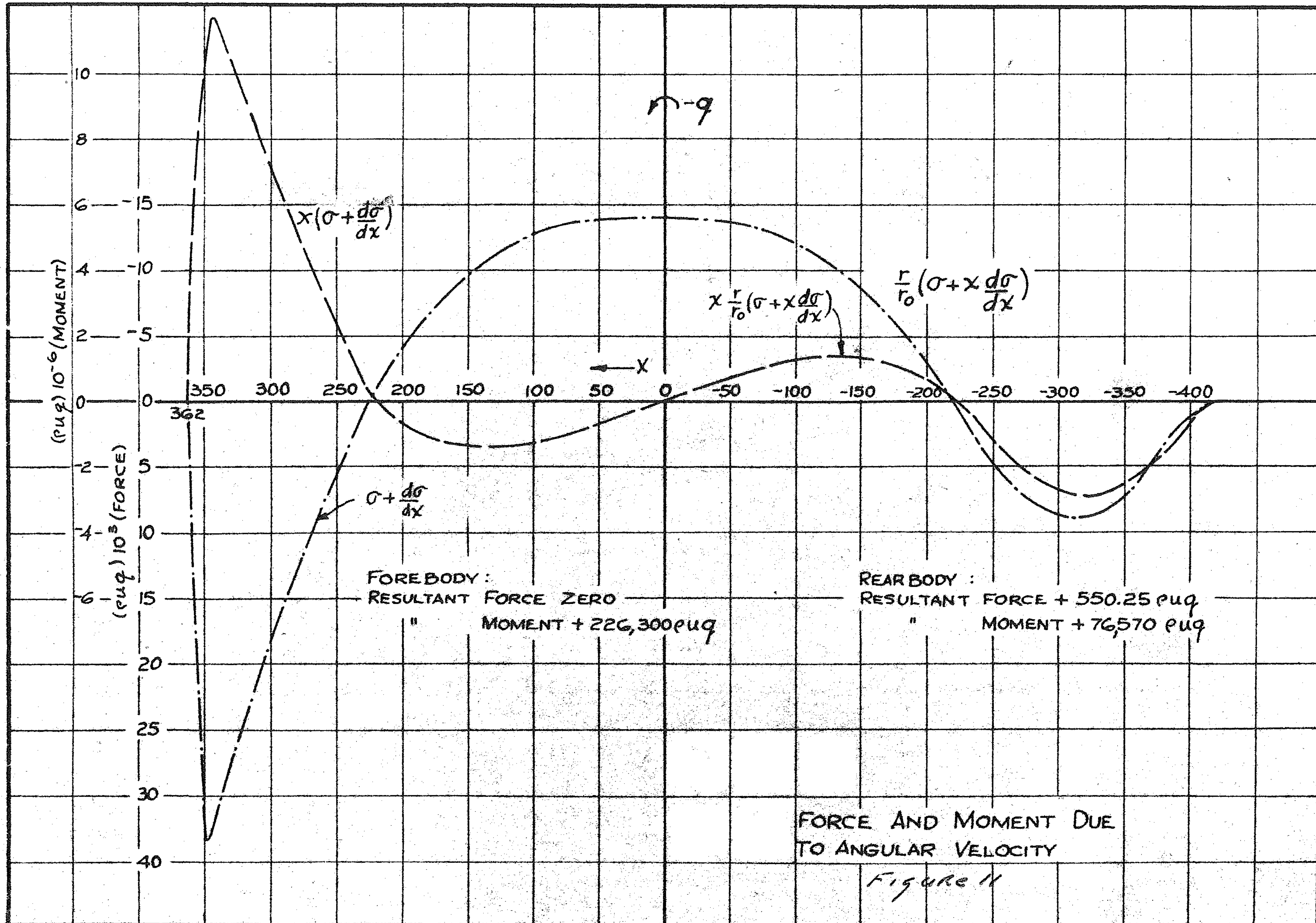
$$M'_q = V_q M_q$$

3. The calculations:

The simultaneous solution of the three equations was facilitated by the use of a ruled form placed under each calculating sheet (Fig. 12).

Items 31 to 34 give the changes in u, w, q and θ for the interval used. These are added to the initial values obtained for the last calculation and entered at the top of the sheet items 5 to 8; these are the initial values for that time heading. A mean value for each is estimated and entered to the right of the initial values. The mean values are used in subsequent calculations. The values of V, α and β are obtained as indicated, and mean values for each estimated by comparison with previous variations. With the mean angle of attack α and elevator angle ϵ, items 35 to 37 are obtained from charts. With the static condition entered in items 11 and 12 and thrust in item 16, the calculations proceed in straightforward fashion using the mean values. The additional heaviness due to going over pressure height is obtained as indicated in item 44.

Errors in assuming mean values are soon apparent, the most difficulty coming from the determination of a proper mean value for q. With sudden changes in static conditions, it was found necessary to calculate all the terms in the moment equation first, except $M_q V_q$, then to determine q by trial and error to give a correct mean value. This procedure is explained also in R. & M. 1401. For simplicity in calculations, a linear rate of ballast dropped is recommended.



FORCE AND MOMENT DUE TO ANGULAR VELOCITY

Figure 11

Sample Calculating sheet   Fig 12

1	t	5.22		5.26		5.30		5.34	
2	δt	04		04		04		04	
3	ϵ	20°		20°		20°		20°	
4	$V = u/\cos\alpha$	40.7	40.7	41.0	41.0	41.9	41.9	42.6	42.6
5	u	40.00	39.89	40.38	40.19	40.88	40.63	41.38	41.13
6	w	-9.90	-9.81	-10.18	-10.04	-10.48	-10.33	-10.70	-10.59
7	q	-00574	-00667	-00361	-00468	-00168	-00265	000504	-00109
8	$\Theta = \alpha + \beta$	17.83	18.73	16.30	17.06	15.24	15.77	14.63	14.93
9	$\alpha = \tan^{-1}(-w/u)$	13.9	13.85	14.10	14.00	14.40	14.25	14.60	14.50
10	β	3.93	4.88	2.20	3.06	.84	1.52	.03	.43
11	$\Delta W + \Delta' W$	13.85		13.92		13.97		14.00	
12	ΔM	1500		1500		1500		1500	
13	Vq	-.212		-.192		-.111		-.0466	
14	uq	-.266		-.188		-.1075		-.0452	
15	$-X_\alpha V^2$	-3.73		-3.84		-4.12		-4.35	
16	T	+4.00		+4.00		+4.00		+4.00	
17	$-\Delta W \sin\Theta$	-4.45		-4.08		-3.80		-3.61	
18	$m_1 u$ (8.63)	+.82		+1.08		+1.08		+1.04	
19	$Z_\alpha V^2$	+11.05		+11.35		+12.10		+12.70	
20	$-\Delta W \cos\Theta$	-13.12		-13.30		-13.40		-13.50	
21	$Z_q Vq$	-1.33		-.94		-.54		-.23	
22	$-m_1 uq$ (8.63)	+2.29		+1.62		+.93		+.34	
23	$m_2 \dot{w}$ (16.14)	-1.11		-1.27		-.91		-.64	
24	$M_\alpha V^2$	-323		-328		-360		-382	
25	Th (45')	+405		+405		+405		+405	
26	$-M \sin\Theta$ (4000)	-1282		-1172		-1090		-1032	
27	$\Delta M \cos\Theta$	+1420		+1435		+1440		+1450	
28	$-M_B \cos\Theta$ (468)	+443		-448		-450		-452	
29	$-M_q Vq$	+476		+337		+195		+82.3	
30	$B\dot{q}$ (475000)	+253		+229		+140		+71	
31	$\dot{u}\,\delta t = \delta u$	+.38		+.50		+.50		+.483	
32	$\dot{w}\,\delta t = \delta w$	-.275		-.315		-.225		-.158	
33	$\dot{q}\,\delta t = \delta q$	+.00213		+.00193		+.00118		+.0006	
34	$57.3\, q\, \delta t = \delta\Theta$	-1.53		-1.06		-.608		-.251	
35	$X_\alpha \times 10^3$	2.25		2.28		2.35		2.40	
36	$Z_\alpha \times 10^3$	6.75		6.75		6.40		7.0	
37	$M_\alpha \times 10$	-1.95		-1.95		-2.05		-2.1	
38	M_q	1755		1755		1760		1765	
39	Z_q	4.89		4.90		4.91		4.94	
40	$\delta\xi = V \cos\beta\, \delta t$	162		163		167		171	
41	$\delta\zeta = V \sin\beta\, \delta t$	13.9		8.8		6.3		3.7	
42	ξ	13645		13808		13975		14146	
43	ζ	2932		2941		2947		2951	
44	$\delta\zeta \times 0054 = \Delta' W$	.075		.0474		.034		.02	