EXPERIMENTAL INVESTIGATION OF ULTIMATE LOADS
CARRIED BY FLAT, UNSTIFFENED PANELS
UNDER COMBINED SHEAR AND COMPRESSION

Thesis

by

George Yoshio Tsubota

In Partial Fulfillment of the Requirements for the Degree of

Master of Science in Aeronautical Engineering

California Institute of Technology

Pasadena, California

1939

TABLE OF CONTENTS

INDEX OF FIGURES

TABLE OF NOTATION

Symbol	Definition	Units
a	Length of panel	inches
b	Width of panel	inches
t	Thickness of panel	inches
P	Applied compression load	lbs.
S	Applied shear load	lbs.
σ_c	Applied compression stress	#/sq.in.
σ_{c_o}	Ultimate compression stress in pure compression	#/sq.in.
τ	Applied shear stress	#/sq.in.
τ_o	Ultimate shear stress in pure shear	#/sq.in.
δ_c	Total deflection parallel to compression axis	inches
ε	Unit deflection parallel to compression axis	in./in.
δ_s	Total deflection perpendicular to compression axis	inches
γ	Unit deflection perpendicular to compression axis	in./in.

I. ACKNOWLEDGMENT

The author wishes to take this opportunity to express his appreciation to Dr. E. E. Sechler of the Guggenheim Aeronautical Laboratory, California Institute of Technology for his useful suggestions on the design of equipment and careful supervision of the research program, Mr. W. L. Howland for general consultation on the problems concerning the test, and Messrs. B. Antonenko and H. Worcester for their valuable assistance during the performance of experiments and the reduction of data.

II. SUMMARY

The problem of the behavior of flat, unstiffened panels under combined shear and compression load was investigated for one length over width ratio and three thicknesses. Due to the lack of data obtained in a limited amount of time, no definite conclusion was reached. Until further investigation is carried out, the ultimate failure stress relationship of

$$\frac{\sigma_c}{\sigma_{c_0}} + \frac{\tau}{\tau_0} = 1$$

is recommended. The effect of shear load on the modulus of elasticity and compression load on the shear modulus was found to be nonexistent.

III. INTRODUCTION

The problem of ultimate failing load of panels subjected to combined shear and compression is of considerable interest to the designers of modern aircrafts. They are continually confronted with the analysis of such structures as semi-monocoque wings and fuselages, in which this type of loading is a routine occurrence. So far, experimental research along this line and workable formulae for such analyses have been lacking.

With these facts in mind the investigation in the flat, unstiffened panel series was begun last year by Mr. W. T. Butterworth at the Guggenheim Aeronautical Laboratory, California Institute of Technology and continued by the author this year.

Considerable time was spent by the author in the modification of the loading mechanism(described in detail in section IV a). The present loading mechanism was developed for the purpose of applying combined shear and compression load at a constant ratio throughout a given test run. The previous setup (reference 1) was found difficult to handle; furthermore it was thought that the loading of the panels in a constant shear over compression ratio would be preferable to failing the specimen in shear after an application of a given compression load.

As a representative material, 17ST duralumin sheet was used throughout the tests. The nominal thicknesses chosen were the ones most frequently encountered in airplane structures. Due to the limited time which was available for experimental work, only one panel size was investigated in three thicknesses. It is hoped that a further study will subsequently be conducted to determine the problem of ultimate load under combined shear and compression over a more complete field with thickness, width, and length over width ratio as parameters.

The tests were conducted for the following loads:-- pure compression, shear over compression ratios of tan. 15°, tan. 30°, tan. 45°, tan. 60°, tan. 75°, and pure shear. The above loading ratios were chosen so that the curves of τ/τ_0 vs. σ_c/σ_{c_0} may be determined from sufficient number of points and that the points represent an equal angular spacing between the shear and the compression axes. Adequate amount of data was gathered to provide a means of computing not only the ultimate load but also the effects of shear load on the modulus of elasticity and compression load on the shear modulus.

IV. EXPERIMENTAL PROCEDURE

(a) Apparatus

The apparatus used for this test is basicly that employed by Mr. Butterworth (reference 1). It consists essentially of a fixed upper head of ten-inch I-beam (a, fig. 1) supported by two upright ten-inch channels (b), a floating lower head (c) directly below and parallel to the upper head, also a ten-inch I-beam, and the loading mechanism (d).

The upper head can be bolted at various positions along the channels to accomodate the testing of different sizes of panels. The lower head is restrained to a parallel motion on a vertical arc of seven-foot radius whose axis is located at e on the same horizontal plane with the lower head, all other motions being restricted by two steel tubes (f) and a vertical bar (g) guided by ball bearings mounted at the upper head and at the base I-beam (h).

The panels are mounted at the upper end between a pair of two-inch angle irons (i) which are bolted to the upper head and between another such pair (j) bolted to a flat bar (k) at the lower end. In order to allow shear deflection, the bar k is separated from the lower head by steel rollers mounted on another flat bar (l) and is restricted to a motion in the plane of the panel and parallel to the bar l by a pair of guides and by roller bearings (m). The bar l is allowed to rest freely on the lower head or is clamped tightly to the latter depending on the type of shear deflection desired for the test (to be explained later). The vertical edges of the specimen are supported by means of a pair of steel tubes (n) slotted along an element and clamped on to the edges sufficiently loosely to allow motion between the tubes and the panel in order to eliminate as much possible the tendency for the tubes to take compression load through the action of friction. Due to the deflection of the panel in the direction of the compression load, the vertical edges cannot be supported along the entire length by means of the tubes alone. Those portions unsupported by the tubes, both at the upper and the lower fractions of the edges, are provided with loosely fitting clamps (o) so constructed that they produce a line support at the edges just inside of the tubes. Hence the only load which the tubes can take is through friction. The clamps are bolted on the upper and the lower angle irons.

The loading device (fig. 2) is a specially designed turnbuckle (a) mounted on a trunnion to the tension gage (b). The turnbuckle is so constructed that during its tightening process the flexible shear cable (c) remains untwisted. The shear cable passes over a ball bearing pulley (d) and a leveling pulley (e), which is adjustable vertically, and terminates at the lower pair of angle irons where it is anchored by means of a pin joint. The tension gage consists of two steel straps clamped at two ends and initially bent away from each other at the center. The contraction of the gap thus formed, measured by a sensitive dial gage, determines the applied load. The lower end of the tension gage is attached by another trunnion, at right angle to the upper trunnion, to a female knife edge bracket (f) which loads the lever (g). The load is transferred to a beam lever (h) by a compression ball joint and thence by another compression ball joint to a screw jack (i) fastened to the lower head. The beam lever h is provided with five knife edges corresponding to the five loading ratios discussed in the introduction. The mating part for the above knife edges is mounted under the desired knife edge, in a slot cut into the legs of an H-beam (j) which is mounted on the base of the machine. All knife edges and the acting center of the ball joints are on the same level. Thus it is possible to load a panel in shear and compression simultaneously and at five different ratios. For a pure compression load the shear cable is anchored to a bracket (k) bolted to the upright channel, and similarly for a pure shear load the lower end of the tension gage is disengaged and pinned to a bracket (l). A lead weight (m) is placed on the lever g to counterweigh the dead weight in the beam system and the weight of the lower head assembly.

A dial gage (p, fig. 1) supported from the upper head and applied at the end of the flat bar k is used to measure the shear deflection of the panel as a whole. The deflection in compression is measured at two places (q and r) by dial gages, the average of the readings being taken as the resultant deflection, the difference of the two showing the angular motion at the lower edge of the specimen.

Two types of shear loading are possible in the above described machine. First, by leaving the lower bar l unclamped, unrestricted deflection of the lower edge in the plane of the panel can take place. In this case a shear load introduces bending in the plane of the panel thus inducing tensile stress on one side and compressive stress on the other tending to rotate the lower edge of the specimen. A superposition of a compression load will be taken up by the side in tension until the deflection in the direction of compression is equalized throughout the lower edge. When such condition is realized, the first type of loading becomes equivalent to the second type.

By clamping the lower bar to the lower head, the second type of loading, in which the angular motion of the lower edge is restrained, can be obtained. The restriction placed on the lower head introduces a moment, in the plane of the panel, which reacts the bending moment induced by the shear load. A superposition of compression load, in this case, acts along the total width of the panel at all times.

The first type of loading is often encountered in Wagner beams while the second type prevails in the stressed-skin coverings of wings, tail surfaces and fuselages. Both types of loading are of primary interest to the designer. The first type was tested only in the a/b ratio of 1.5 in this investigation for the purpose of comparison with the second type.

(b) <u>Specimen</u>

The following series of panels were tested for this thesis:

Nominal thickness, $t = 0.020, 0.032, 0.040$ inches

$$a/b = 1.5$$

Width, $b = 6$ inches.

The actual thicknesses varied somewhat from the above figures.

The length of the panel, a, was measured between the edges of the upper and the lower pair of angle irons. The width, b, was taken as the actual width of the panel.

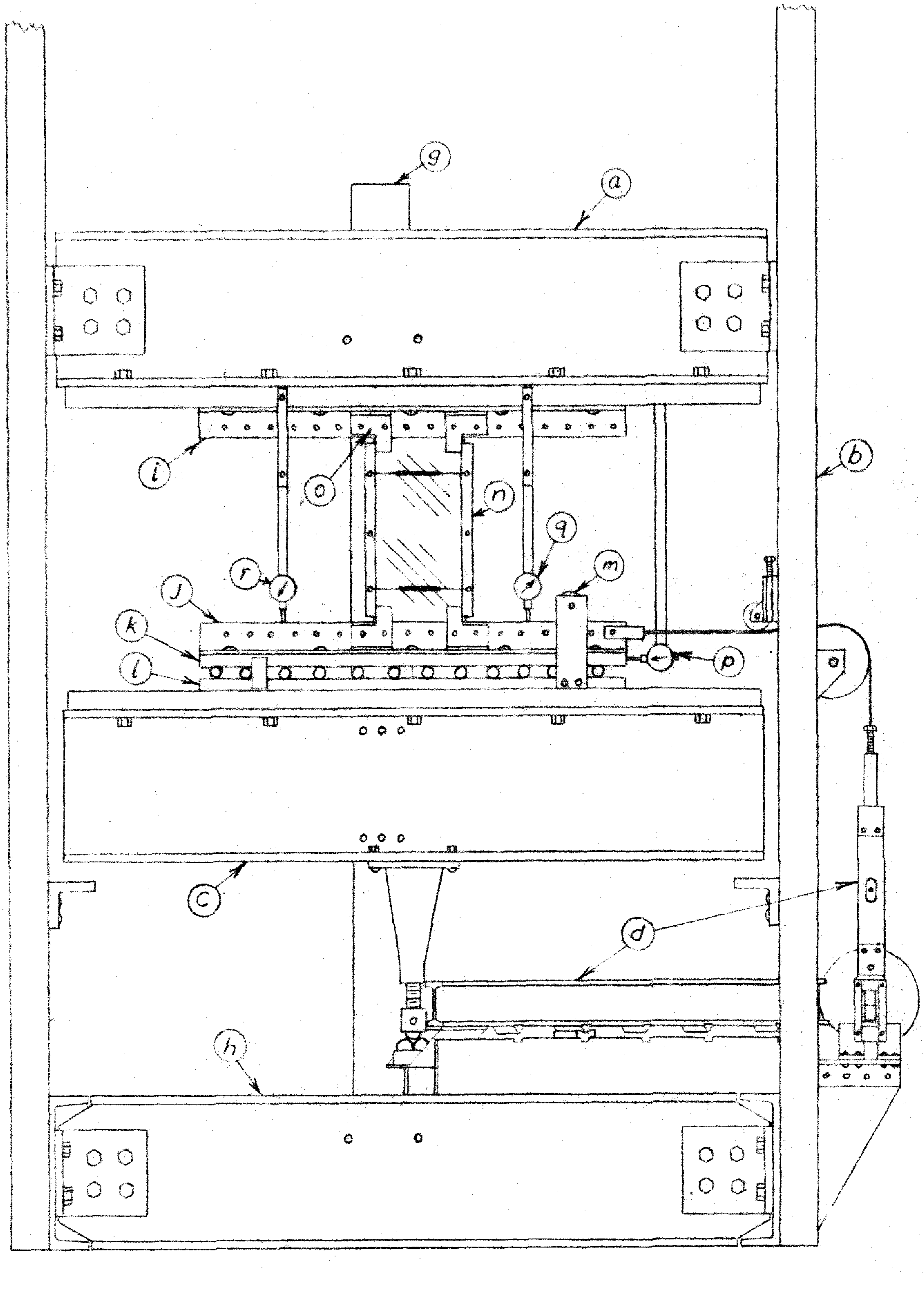
(c) Test Procedure

The tension gage was calibrated in a standard tension machine several times. The calibration curve is given in the appendix.

After setting the upper head in the proper position for the given panel size and aligning both the upper and the lower heads, the counterweight setting was determined for the given knife edge setting with the whole of the lower head assembly in place before mounting the panel. The panel was gaged in several places, the average being taken as its thickness, and was then bolted between the angles with its axis parallel to the direction of the compression load. The clamps were mounted near each corner, leaving sufficient space at the edges of the panel to mount the slotted tubes. The tubes were then clamped to the edges until a sliding fit was obtained, leaving clearance at the upper and lower corners, and were drawn against the clamps by means of four turnbuckle bracings bridged between the tubes on each side of the panel as shown in figure 1. These bracings were required in order to prevent the tubes from springing off the edges when the waves in the panel, resulting from the load, had become large.

After setting the lower bar of the lower head assembly to the type of shear loading desired, and engaging the loading mechanism at the proper place for the given run, i. e. the shear cable, the tension gage, and the knife edge of the beam lever in their respective position, the panel was loaded, in such increments as were found necessary to produce a load-deflection curve, by tightening the turnbuckle (a, fig. 2). Care was taken not to twist the tension gage during the tightening process. The lever system and that portion of the shear cable between the leveling pulley and the lower angle irons were kept on a horizontal plane by adjusting the jack screw and the leveling pulley respectively. It was found that near the ultimate load the jack screw provided a better loading device than the turnbuckle due to the fact that the jack screw did not tend to twist the tension gage during the loading process. Thus it was possible to obtain by this means an accurate ultimate load value. Such precaution was unnecessary for smaller loads because for each load there was a definite and stable deflection. The loading mechanism was tapped slightly to remove friction forces in the system.

The ultimate load was taken as that final load beyond which the deflection increased indefinitely without an increase in load. This precaution eleminated any point analogous to the yield point of mild steel from being considered as an ultimate load.



g
a
b
i
o
n
q
r
m
j
k
p
l
c
d
h

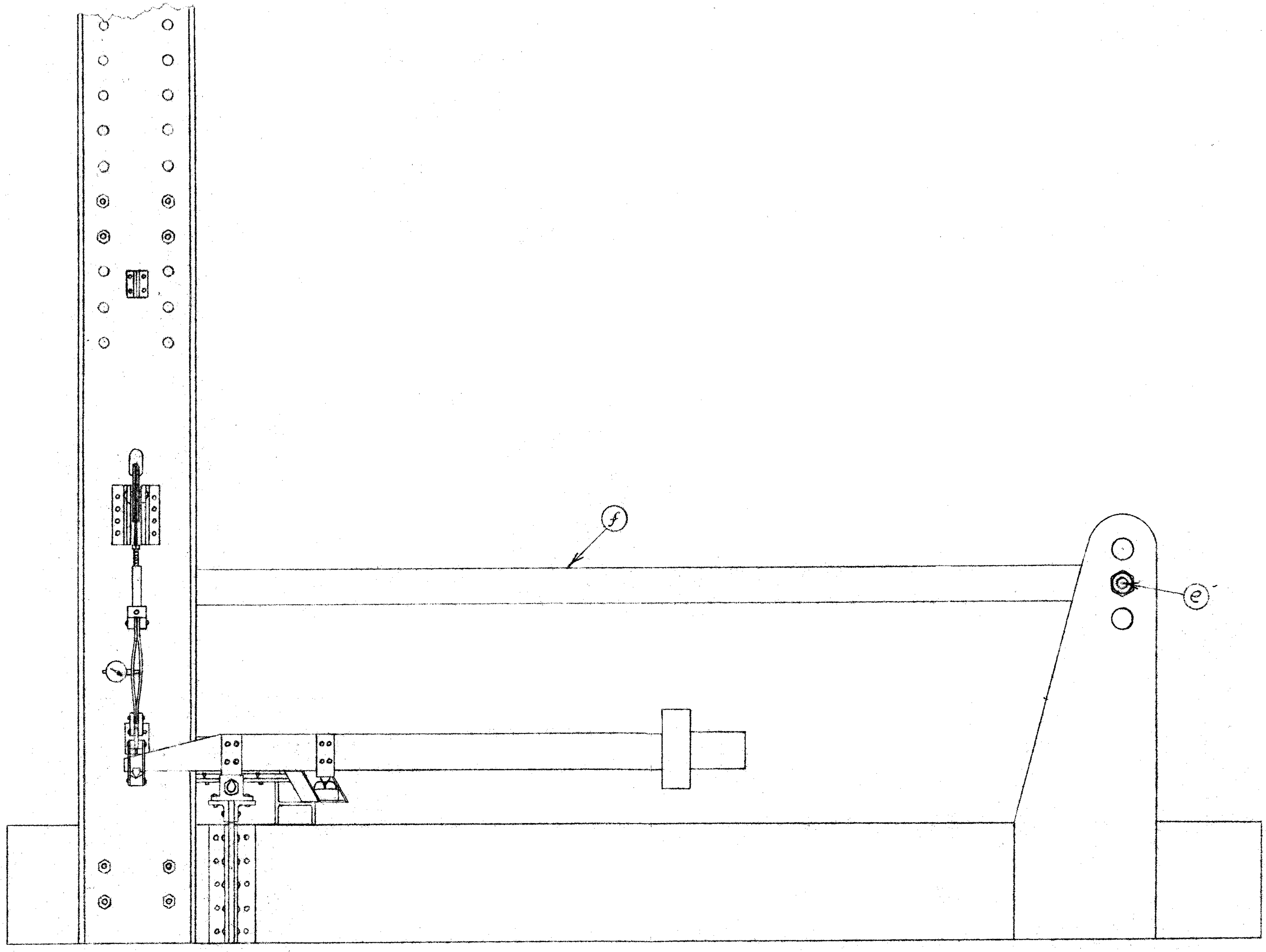
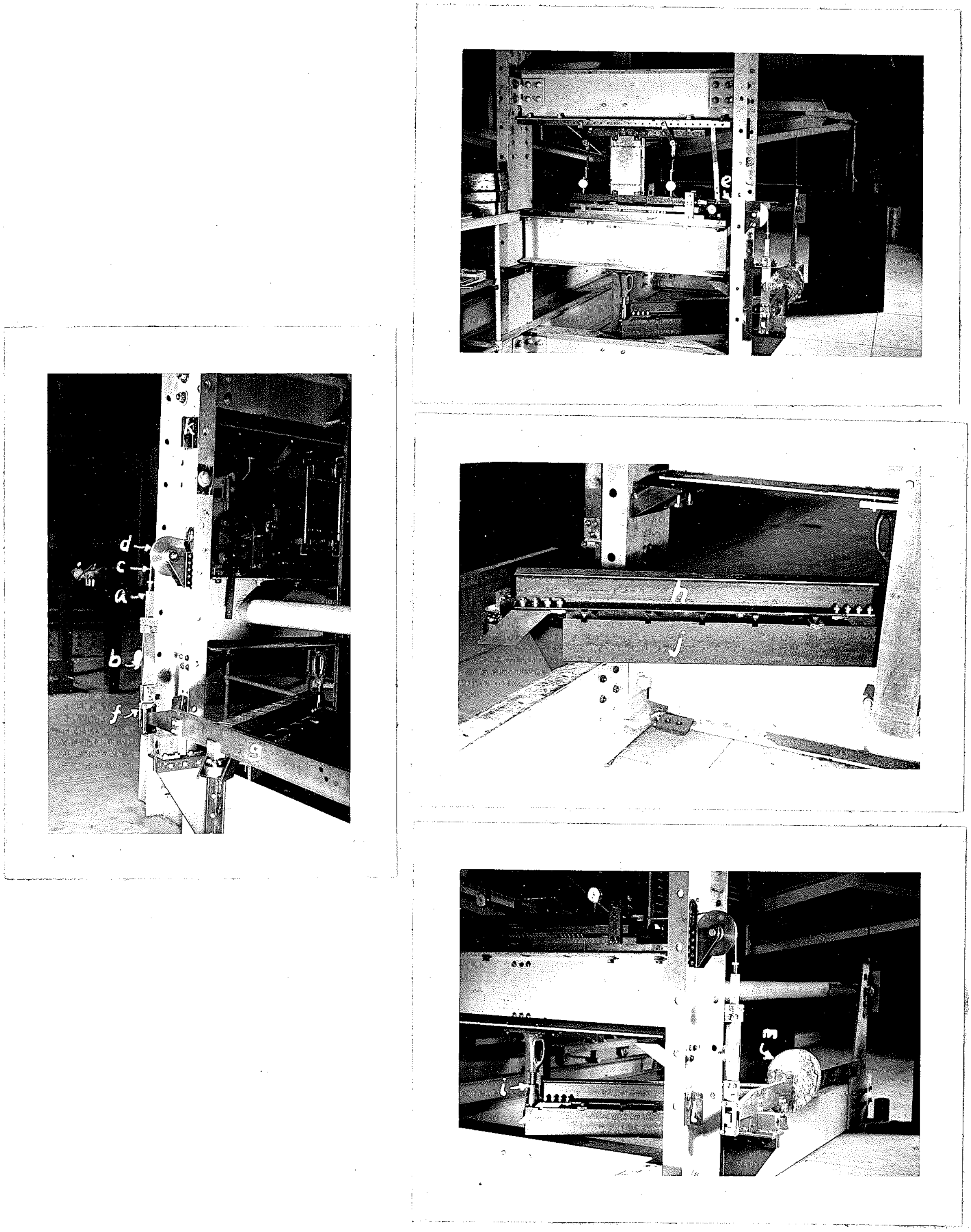


FIGURE 1

Figure 2 - Loading Mechanism



V. DISCUSSION OF RESULTS

(a) Type of Failure

Observation during the test indicated clearly that the failure is due to wave formation in a manner generally assumed by all investigators, i.e. by formation of waves whose length is equal to the width of the panel. The above law was complied with throughout the test by a formation of one complete wave and a half wave or one complete and two quarter waves.

When the waves had become very deep, the induced force in the vertical edges acting perpendicular to the plane of the panel separated the slot in the edge tubes causing immediate failure of the edges as an Euler column.

(b) Load-Deflection Curves

Figures 3, 4, and 5 show the load-deflection relationship for shear for nominal thicknesses of .020, .032, and .040 inches respectively. The ordinate τ is defined by

$$\tau = \frac{S}{bt}$$

where S = applied shear load,

b = width of specimen,

t = thickness of specimen,

and the abscissa by

$$\gamma = \frac{\delta_s}{a}$$

where δ_s = total deflection in the direction of applied shear load,

a = length of specimen.

Figures 6, 7, and 8 show the corresponding load-deflection curves for compression. The ordinate σ_c is defined by

$$\sigma_c = \frac{P}{b\,t}$$

where P = applied compression load

and the abscissa ε by

$$\varepsilon = \frac{\delta_c}{a}$$

where δ_c = total compression deflection in the direction of applied compression load.

The curves indicated "unclamped" are those obtained for the case of unrestricted shear deflection of the lower edge in the plane of the panel as described in section IVa, the remainder being the clamped series. Actually, the tests were performed unclamped for the loading cases of $S/P = \tan. 45^\circ$, $\tan. 30^\circ$, $\tan. 15^\circ$, and pure compression; but there was no tendency for the lower edge of the specimen to rotate as indicated by the compressive deflection readings at the two ends (r and q, fig. 1) so that they were considered equivalent to the clamped series. Inspection of the curves show that such condition is nearly realized for the loading case of S/P = tan. 60°.

It is apparent from the curves that the rigidity as well as the ultimate failure load of the unclamped series is considerably less than those of the clamped series. Generally an application of compression load to the unclamped series tended to increase the ultimate shear load and shear rigidity, due to the fact that the compression introduced a counteracting bending moment to that induced by the shear load. The curves show no indication of the variation of modulii of rigidity in compression and in shear in the elastic regime of the clamped series with an application of shear and compression loads respectively.

(c) Effect of Thickness on Failure Load

In figure 9 the failure stress is plotted against the thickness of the specimens. The failure stress in shear for various τ/σ_c ratios can be easily calculated from the ratio given for each curve. In accordance with expectation, the failure stress increases with the thickness; however, the exact nature of the relationship between the ultimate stress and thickness could not be determined from the amount of data obtained.

For the purpose of comparison a calculated curve of the ultimate compression stress for compression alone based on the work of Sechler (reference 2) is included. In calculating

$$\lambda = \frac{t}{b}\sqrt{\frac{E}{\sigma_{yield}}}$$

the yield stress in compression was assumed to be 35,000 lbs./sq. in. and the value of E to be 10,300,000 lbs./sq. in. Sechler's tests were conducte d with simple supports of V-grooves at all four edges. In the present investigation the horizontal edges were built in and the vertical edges were supported by slotted tubes. Thus the ratio of the ultimate compression stress for the present case to that of Sechler gives an indication of the extent of the fixity of the edges. The ratio varied from 1.45 to 1.60.

(d) Ultimate Combined Stress

In order to eliminate the thickness effect on the ultimate load for a given series, the experimental points in figure 10 were taken from the faired curves of figure 9 at the specified thicknesses. The ultimate shear stress in pure shear of the clamped series was used for the value of τ_o.

No definite law governing the ultimate combined load was found. Butterworth gives

$$\frac{\sigma_c}{\sigma_{c_0}} + \left(\frac{\tau}{\tau_0}\right)^4 = 1.$$

His experimental points lie fairly close to the curve given by the above equation. The present test indicates that it is not safe to design beyond the value given by the equation

$$\frac{\sigma_c}{\sigma_{c_0}} + \frac{\tau}{\tau_0} = 1.$$

A s was discussed previously, the failure occurred as an Euler column at the vertical edges. If this type of failure can be eliminated by the use of stiffer tubes for edge support or in other manner, there is a possibility that the relationship as given by Butterworth may be reached.

It is of interest to note that in the clamped series the ultimate shear load decreases sharply with an application of compression load. With the exception of t = .020 the reverse situation holds for the unclamped series.

VI. MISCELLANEOUS DISCUSSION OF THE TEST

The test completed thus far is altogether inadequate to draw any definite conclusion. Furthermore, no time was available to check some of the points which appeared questionable.

The accuracy of the loading mechanism is believed to be good, although no check was made in this respect. The slotted tubes, which were used to support the vertical edges, acted as an elastic support of unknown loading and hence proved somewhat unsatisfactory. With the formation of deep waves in the panel, the friction force between the tubes and the panel was found to be very high. The author recommends the use of a continuous ball or roller support, preferably the former. The balls should be of the smallest diameter practicable and mounted in a groove milled into a sufficiently heavy piece of steel so that the support may be considered infinitely rigid. The friction forces will then be rolling friction instead of sliding.

VII. REFERENCES AND BIBLIOGRAPHY

1. Butterworth, W. T.: "Experimental Investigation of Ultimate Loads Carried by Flat, Unstiffened Panels under Combined Shear and Compression." Thesis for M. S. in Aeronautical Engineering, California Institute of Technology, 1938.
2. Sechler, E. E.: "Stress Distribution in Stiffened Panels under Compression." Journ. Aero. Sciences, June, 1937, pp. 320-323.
3. Wagner, H.: "Structures of Thin Sheet Metal." TM 490, N.A.C.A.
4. Schapitz, E.: "Contribution to the Theory of Incomplete Tension Bay." TM 831, N.A.C.A.
5. Timoshenko, S.: "Theory of Elastic Stability." McGraw-Hill, 1936.

VIII. GRAPHICAL RESULTS

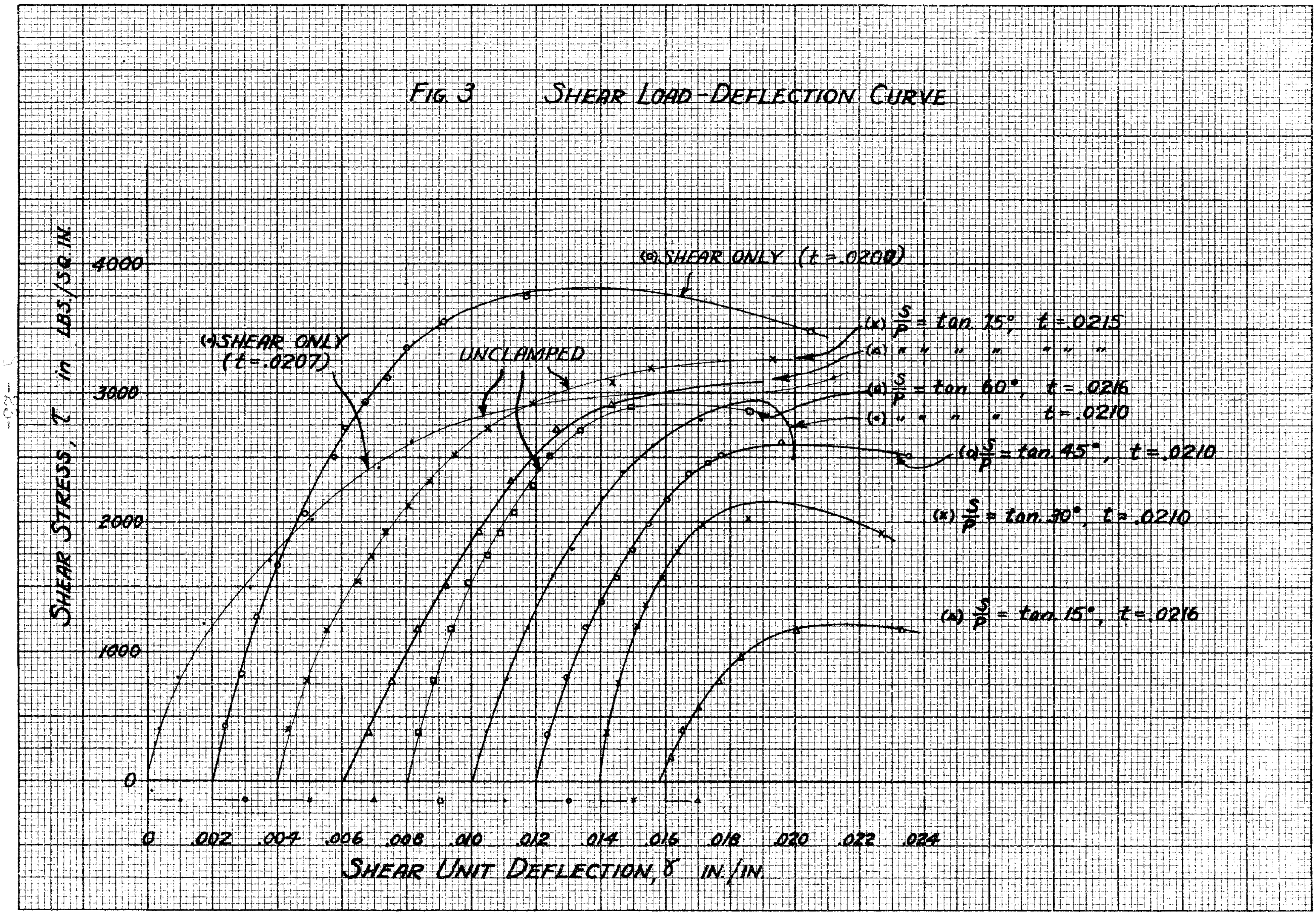


FIG. 3 SHEAR LOAD-DEFLECTION CURVE
(⊙) SHEAR ONLY (t = .0208)
(•) SHEAR ONLY (t = .0207)
UNCLAMPED
(x) S/P = tan 75°, t = .0215
(o) " " " " " " "
(o) S/P = tan 60°, t = .0216
(•) " " " " t = .0210
(o) S/P = tan 45°, t = .0210
(x) S/P = tan 30°, t = .0210
(x) S/P = tan 15°, t = .0216
4000
3000
2000
1000
0
0 .002 .004 .006 .008 .010 .012 .014 .016 .018 .020 .022 .024
SHEAR UNIT DEFLECTION, γ IN./IN.
SHEAR STRESS, τ in LBS./SQ. IN.

FIG. 4

SHEAR LOAD-DEFLECTION CURVE

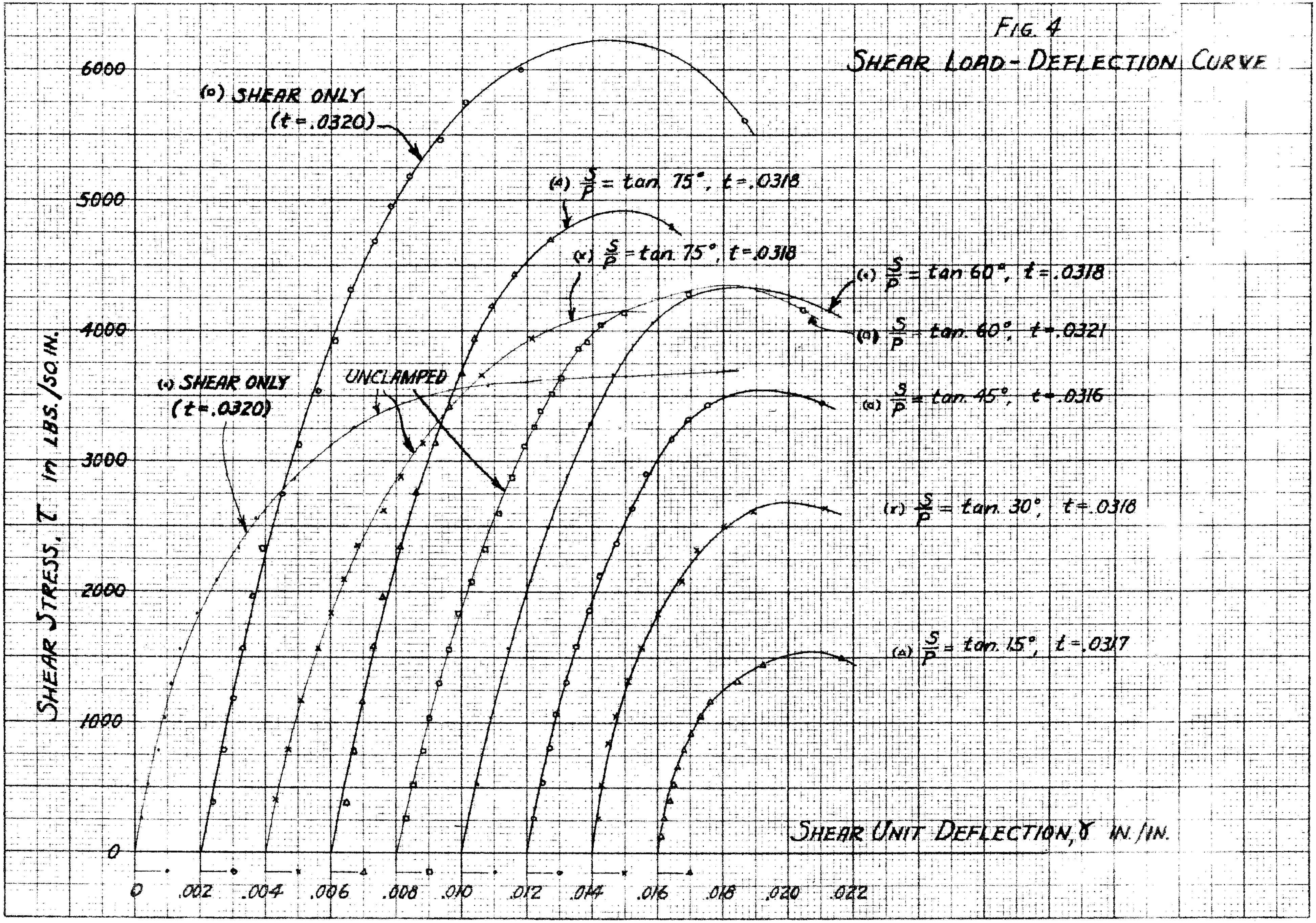
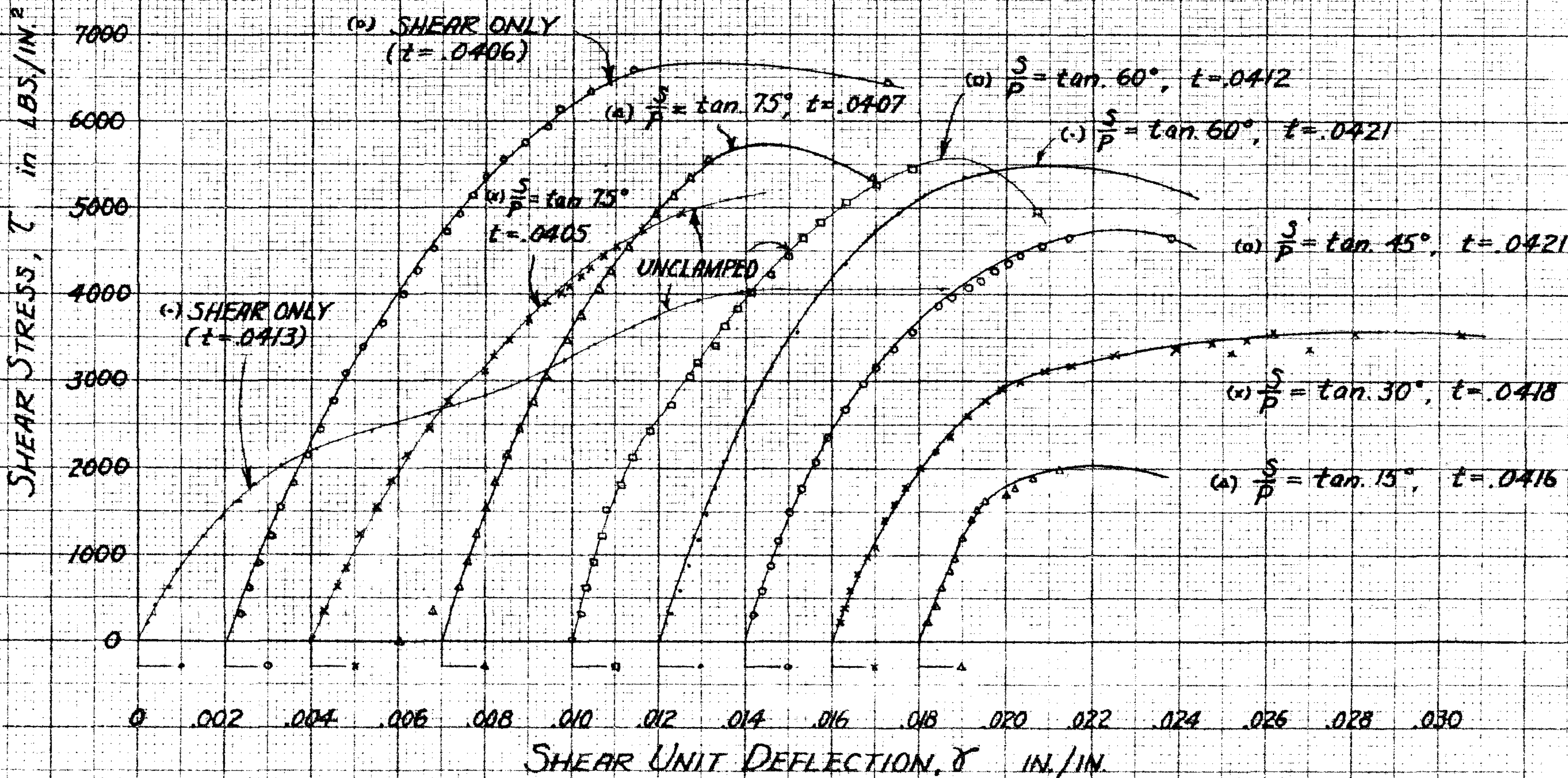


FIG. 5

SHEAR LOAD-DEFLECTION CURVE



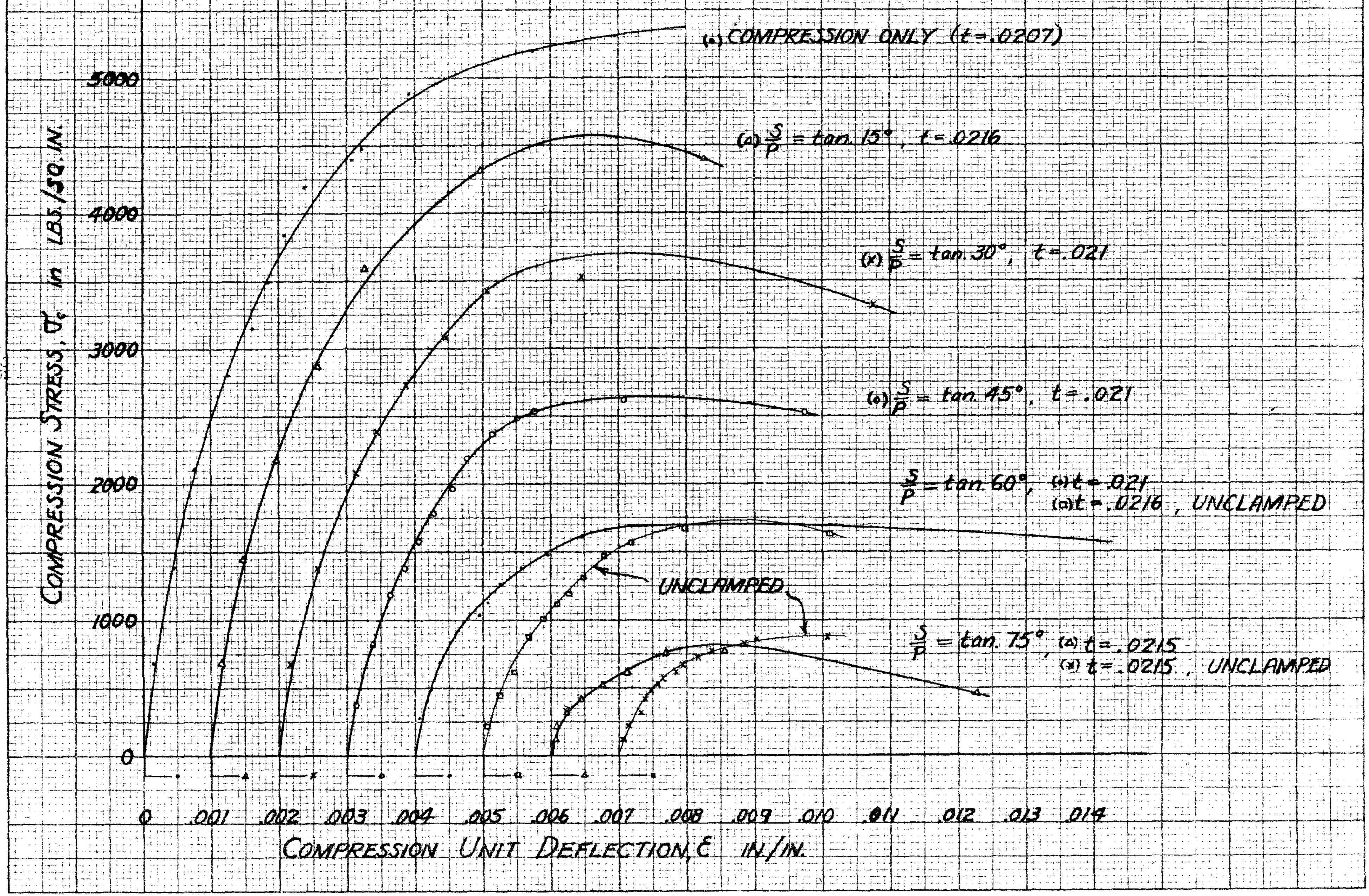


FIG. 6 COMPRESSION LOAD-DEFLECTION CURVE

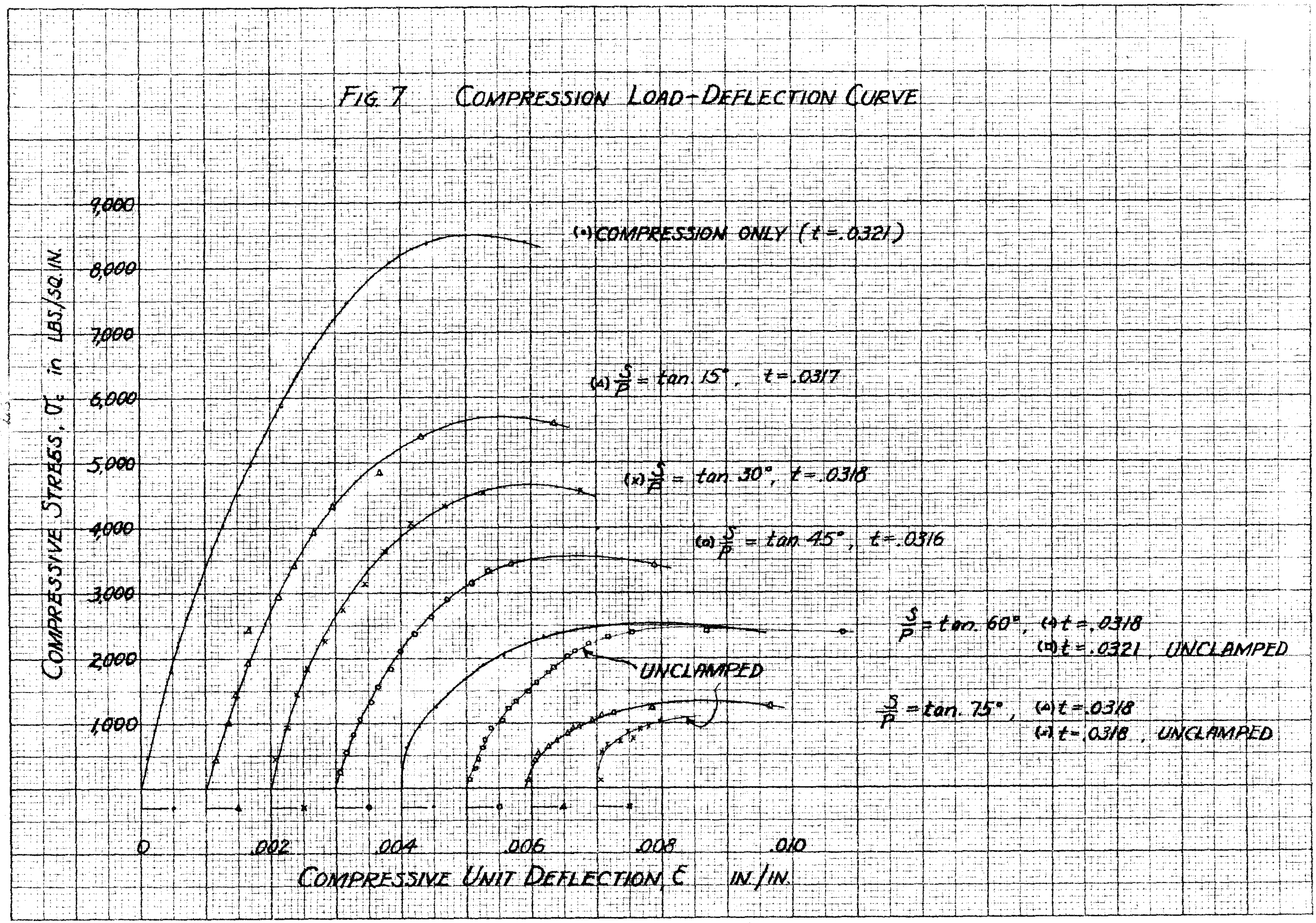


FIG. 7 COMPRESSION LOAD-DEFLECTION CURVE

FIG. 8 COMPRESSION LOAD-DEFLECTION CURVE

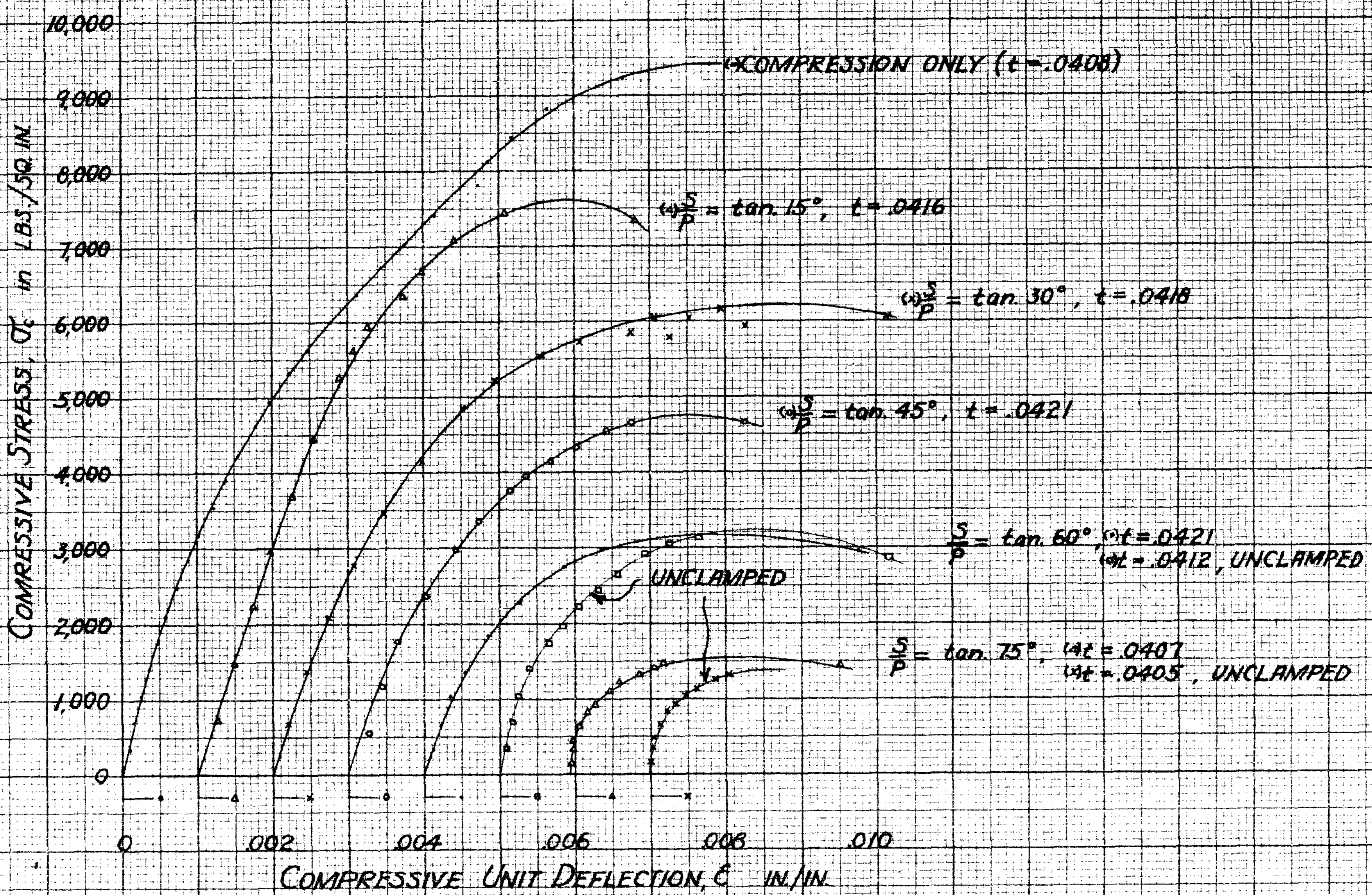


FIG. 9

ULTIMATE STRESS VS. THICKNESS

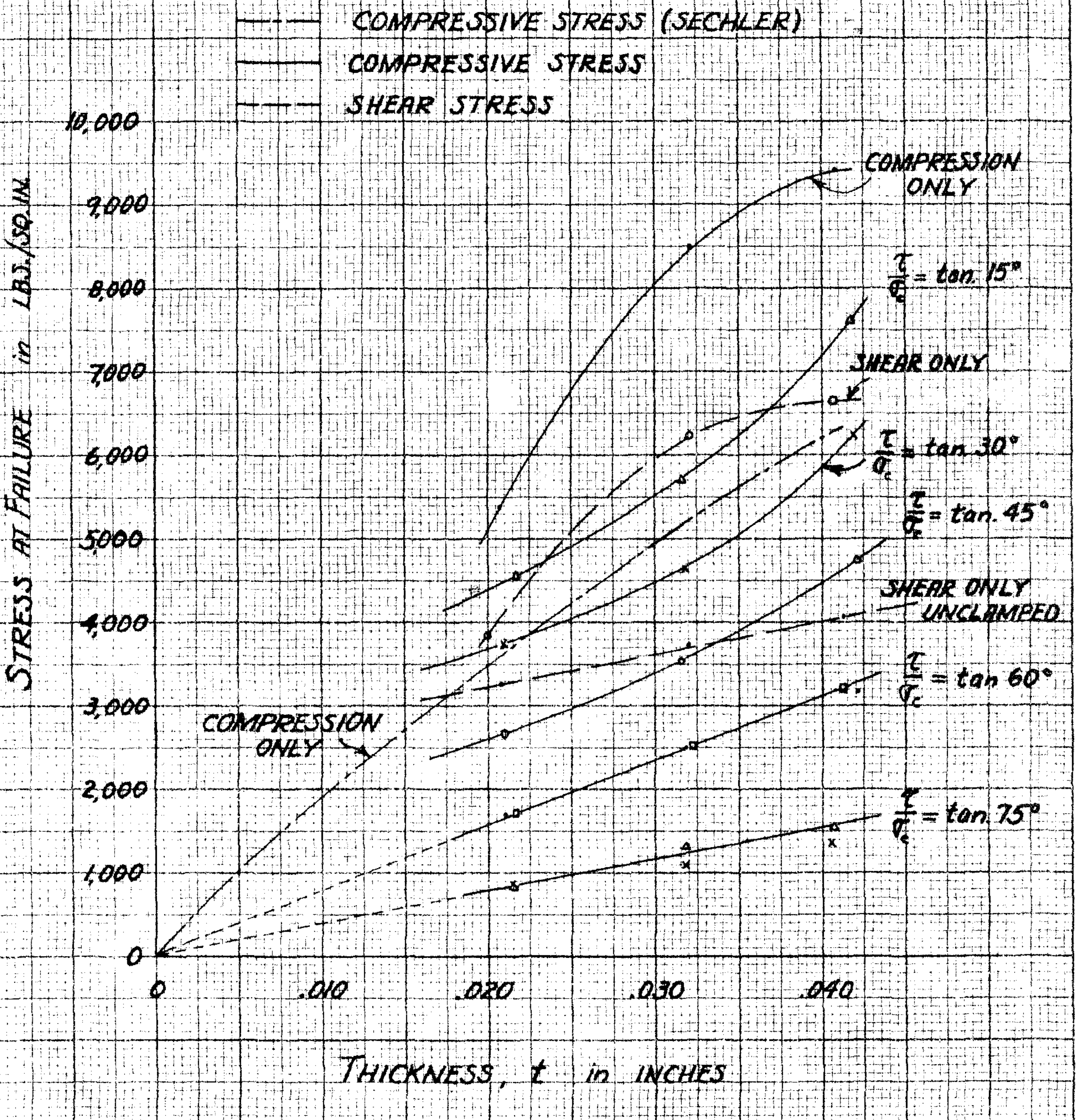
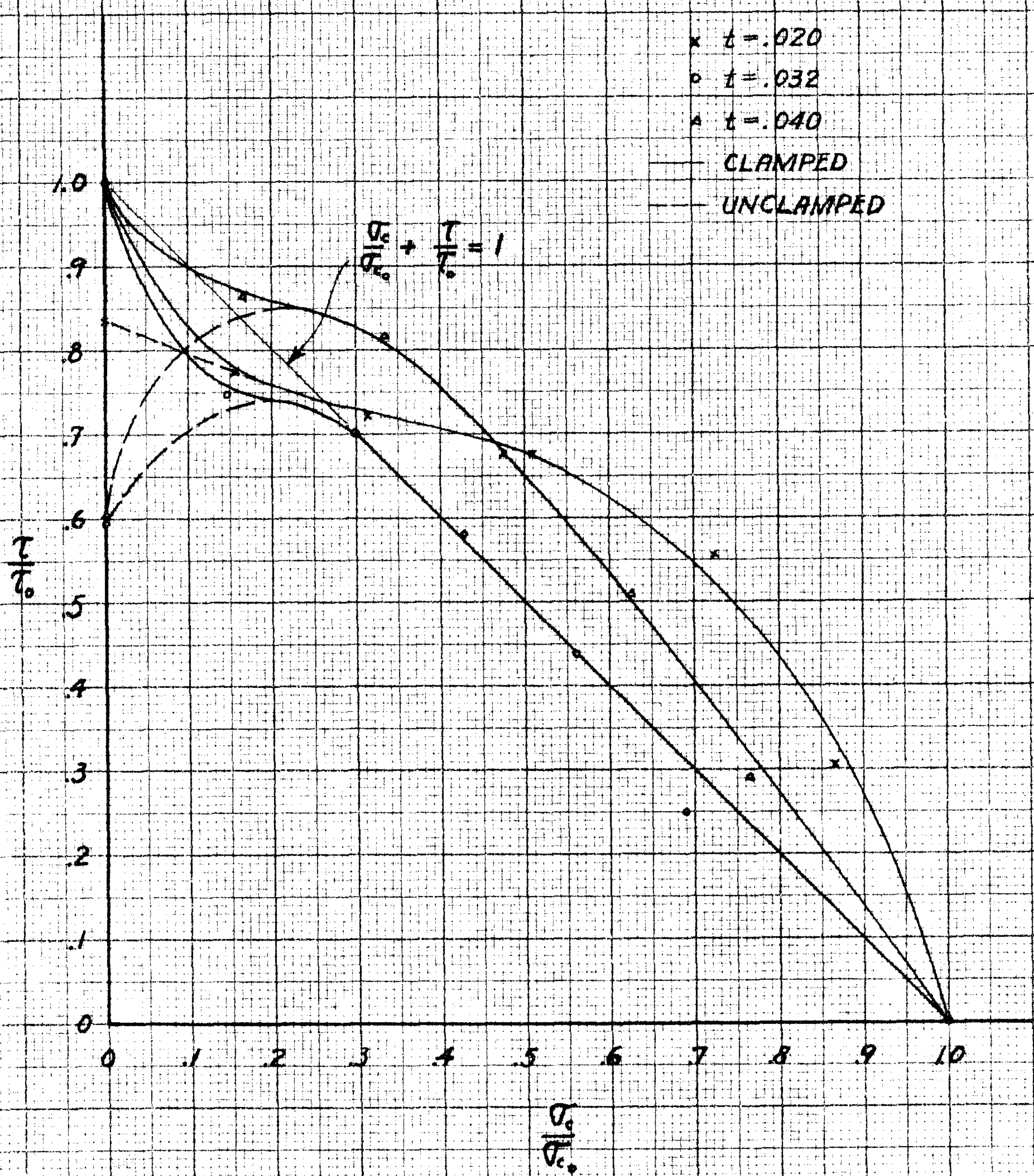


FIG. 10

VARIATION OF ULTIMATE COMBINED STRESS



IX. APPENDIX

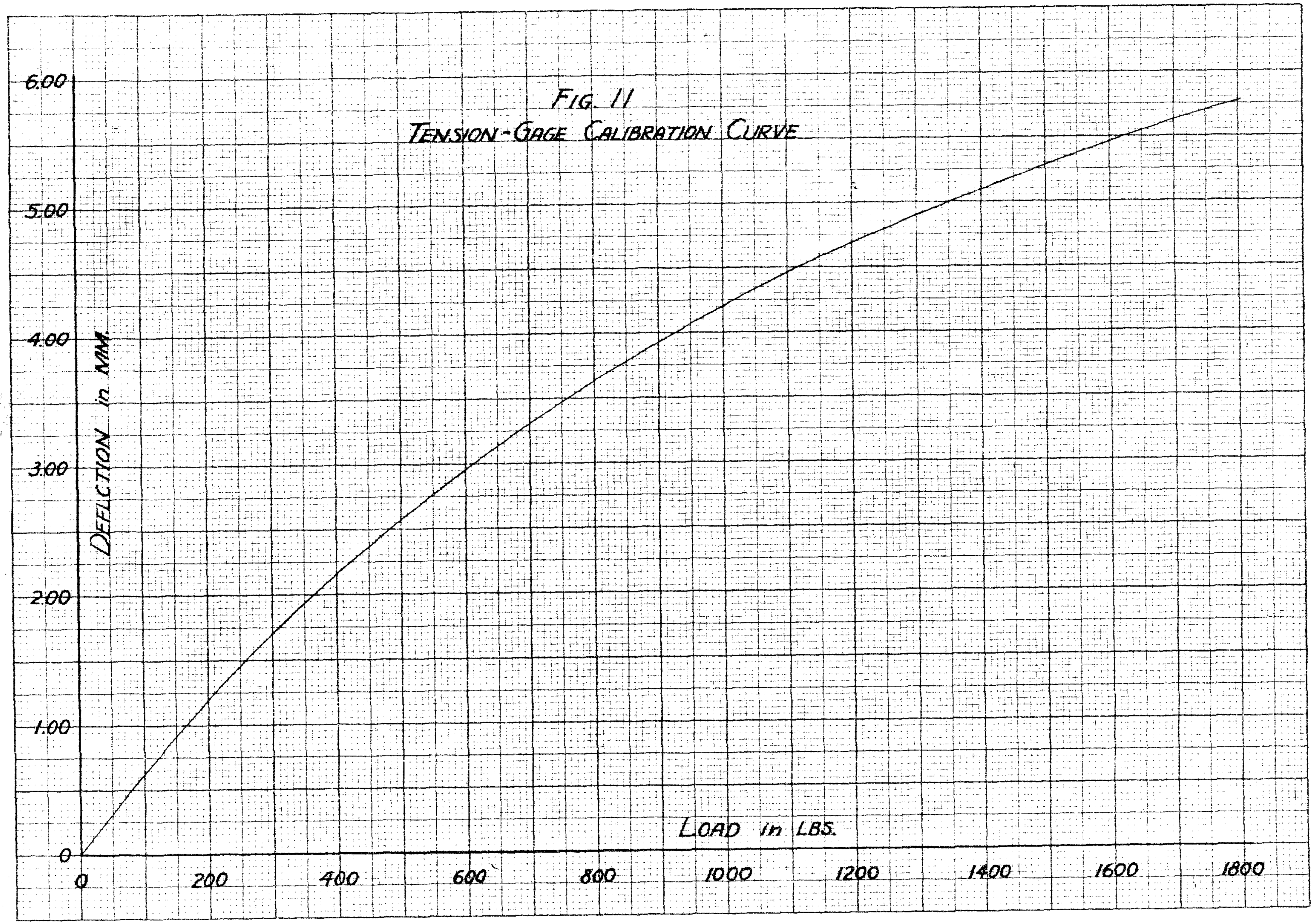


FIG. 11
TENSION-GAGE CALIBRATION CURVE
DEFLECTION in MM.
6.00
5.00
4.00
3.00
2.00
1.00
0
200
400
600
800
1000
1200
1400
1600
1800
LOAD in LBS.