THE EFFECT OF FLOATING RINGS ON THE STABILITY OF LONGITUDINALLY STIFFENED CYLINDERS IN AXIAL COMPRESSION

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ABSTRACT

An experimental investigation was conducted to determine the effect of floating rings on the stability of longitudinally stiffened, thin-walled cylinders in axial compression. Mylar and Plexiglas were used as the skin and stiffening materials, respectively. The primary objectives of the investigation were: (1) to determine the increase in buckling stress that could be obtained by the addition of floating rings; (2) to determine the ring stiffness required for the maximum cylinder strength; (3) to determine what effect the number of longitudinal stiffeners had on the ability of the rings to increase the buckling stress; and (4) to examine the mechanism of buckling in the various ring-stringer combinations. A strength-weight analysis was also made.

The addition of rings was found to be very effective in increasing the cylinder buckling load when 24 longitudinal stiffeners were used. The addition of three rings to a cylinder with 24 longitudinal stiffeners increased the buckling load 123 per cent with only a 20 per cent increase in weight. The ring stiffness beyond which there was no more increase in cylinder buckling strength was determined for the 24 stringer, two and three ring configurations. The ability of a ring to increase the buckling stress was markedly decreased when the number of longitudinal stiffeners was decreased from 24 to 12. The mechanism of buckling was also affected by the number of rings and longitudinal stiffeners.

In the light of the experimental results, two analytical methods are suggested for predicting the buckling stress of floating ring stiffened cylinders.

LIST OF SYMBOLS

L	length of cylinder, in
c	distance between rings, in
R	radius of cylinder, in
t	thickness of cylinder wall, in
E_m	Mylar modulus of elasticity, psi
E_p	Plexiglas modulus of elasticity, psi
W	average total weight of 24 stringer cylinders, lb
σ	compressive stress in cylinder, psi
σ_{cr}	compressive buckling stress in cylinder, psi
P_{cr}	compressive load at cylinder buckling, psi
δ	cylinder axial shortening, in
I_{r_1}	ring moment of inertia resisting in-plane bending, in^4
I_{r_2}	ring moment of inertia resisting out-of-plane bending, in^4
I_s	least stringer moment of inertia, in^4

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I. INTRODUCTION

The increasing use of stiffened cylindrical shells, as exemplified in the field of missiles and spacecraft, has led to a need for strength data to aid in the design of structures of this type. A shell configuration of interest is one stiffened longitudinally with stringers and circumferentially with internal, "floating" rings. A floating ring is one which is supported at a fixed axial position but is not attached to either the skin or the stringers so that outward radial motion is permitted between the rings and stringers. When unpressurized, the rings are just in contact with the longitudinal stiffeners. This structure has the advantage of having a sufficiently high strength when unpressurized, and, when pressurized, it does not induce the bending stresses in the skin and stringers that a rigidly attached ring would cause. Since strength data for a shell of this type is of interest, an experimental investigation was conducted to determine the effect of the number of rings and the ring bending stiffness on the buckling load of a circular cylinder in axial compression.

The experimental program consisted of testing cylinders longitudinally stiffened with 12 and 24 equally spaced stringers and with 0 to 3 rings. All the cylinders were 12.5 inches long and had a radius of 4.0 inches. The in-plane bending stiffness of the rings as measured by their moment of inertia ranged from 625×10^{-6} in^4 to 1.08×10^{-6} in^4. The cylinder skins were constructed from DuPont's polyester film, Mylar, and the rings and stringers from Rohm and Haas' acrylic plastic, Plexiglas. The use of these materials has several advantages over their metallic counterparts. A cylinder can be put together with relative ease and at a relatively low cost. Secondly, since the modulus of elasticity of both these materials is quite low, the deformations encountered in shell buckling cause stresses low enough to be within their elastic limits. Hence, one cylinder may be buckled several times before permanent set is encountered.

The tests were conducted in the Structures Laboratory of the Guggenheim Aeronautical Laboratory at the California Institute of Technology (GALCIT).

II. EQUIPMENT AND PROCEDURE

A. Cylinder Components and Construction

The test cylinders were constructed from 4 mil Mylar sheet for the skin material and 0.060 inch (nominal thickness) Plexiglas for the stiffening material. Tensile tests conducted at GALCIT in the summer of 1960 indicated that the average Young's modulus in tension for 4 mil Mylar was 725,000 psi, and tensile tests on the Plexiglas showed an average Young's modulus of 500,000 psi. These values were used in the calculations reported herein.

Strips 0.118 inches wide (± 0.001 inch) and 15 inches long were machined from the 0.060 inch Plexiglas sheet and used for longitudinal stiffening. In one case these strips were formed into rings, but this will be reported later. Rings were machined from the 0.060 inch Plexiglas with an outside diameter of 7.88 inches and inside diameters of 6.88, 7.28, 7.48, and 7.68 inches. This gave a radial thickness of 0.5, 0.3, 0.2, and 0.1 inches, respectively. Hereafter these rings will be referred to as the 0.5 inch rings, 0.3 inch rings, etc. Because of the difficulty in machining a ring with a radial thickness of less than 0.1 inch, the remaining rings used in this program were formed from Plexiglas strips. These were again machined from 0.060 inch thick Plexiglas sheet into strips 27 inches long and widths of 0.080, 0.070, and 0.060 inches. These strips were made into rings by wrapping them around an 8 inch diameter mandrel and heating them in an oven to 200°F for approximately one hour. After cooling, the strips maintained their circular shape. They were then cut down to a size to give an outside diameter of 7.88 inches. The ends were bonded together with either "Weld-on", an acrylic solvent, or Eastman 910 Adhesive. In some cases, it was necessary to reinforce the point of bonding with small pieces of Mylar which were bonded to the Plexiglas with Eastman 910 Adhesive. This then supplied rings of 7.88 inch outside diameter and 7.72, 7.74, and 7.76 inside diameter. These rings are hereafter referred to as the 0.08 inch, the 0.07 inch, and the 0.06 inch rings, respectively. Rings with 0.06 inch radial thickness and 0.118 inch axial depth were made in the manner described above from the strips used for the stringers. Since these strips had been made only 15 inches long, two were required to make one ring, thus necessitating bonding in two places. These rings are referred to as the 0.118 x 0.06 inch rings.

Small clips to support the rings in the cylinder were fashioned from 0.005 inch thick sheet Phosphor bronze. The clips were made so that they would grip the stringers tightly, and a small tab extending from the clip towards the axis of the cylinder provided support for the rings.

The cylinders were constructed in the following manner. Grooves 0.125 inch wide and 0.120 inch deep were machined longitudinally into an 8 inch diameter wooden mandrel every 15° for a total of 24 grooves. 24 of the 0.118 x 0.060 inch Plexiglas strips were cut to a length of 13 inches. A sheet of 4 mil Mylar was cut to 25.5 x 13.0 inches. 24 strips of 8 mil Mylar, 0.118 inches wide and 15 inches long, were placed in the mandrel grooves. Double edged Scotch brand tape was applied to one side of the 13 inch Plexiglas strips, and they were placed in the grooves, sticky side out. This left a gap of about

0.06 inches between the tape and the outer surface of the mandrel. The Mylar sheet was wrapped around the mandrel and the one-half inch seam bonded together with double edged tape. Then the 15 inch long 0.118 x 0.060 inch Plexiglas strips were wedged between the 8 mil Mylar strip and the bottom of the mandrel groove, forcing the sticky side of the 13 inch strips against the Mylar sheet. A good bond between the strips and the sheet was obtained by applying pressure to the sheet where the strips were touching. The cylinder with 12 stringers was constructed in a similar manner.

B. Cylinder Mounting

The cylinders were given a clamped end fixity by placing the ends of the cylinder in melted Cerrobend, a low melting point alloy, and allowing the Cerrobend to harden. Two 10 inch diameter end plates were machined from aluminum. Each end plate had a circular groove machined into it, 1/2 inch wide and 3/8 inch deep, to hold the Cerrobend. Small holes were provided in the end plates to allow equalization of the pressure inside and outside the cylinder. A cylinder was mounted in the first end plate by centering it on the end plate while it was still on the mandrel and then sliding 1/4 inch of the cylinder into the melted Cerrobend. The solidified Cerrobend clamped the cylinder securely to the end plate. The cylinder was removed from the mandrel and its unbonded end placed 1/4 inch deep in the Cerrobend contained in the second end plate. It was supported and leveled by three equally spaced, adjustable rods around the outside of the end plates. This end plate was removed for access to the interior of the cylinder when ring configurations were changed.

Equally spaced rings were located in the cylinder by visual reference to a 12 inch scale. The rings were supported from both above and below by the clips described above. One set of the clips supporting each ring was bonded permanently to the stringers with radio cement.

C. Testing Apparatus and Experimental Procedure

The cylinders were tested in the Riehle 3,000 lb testing machine in the Structures Laboratory of GALCIT. This apparatus is shown in Figure 1. A jig was placed in the machine to allow the testing of specimens in compression. The load was applied through a calibrated load ring consisting of a piece of 6 inch outside diameter Shelby tubing, 1/2 inch wide and 1/8 inch thick, a dial gage, and a restraining device. The restraining device prevented the load ring from deflecting the test cylinder an excessive amount when buckling occurred.

The cylinders were tested in the following manner. The test cylinder was centered in the testing machine. The load disk was placed on the top end plate, a ball bearing placed in the load disk, and the bottom of the load ring placed over the ball bearing. The top of the load ring was placed in a small hole in the top of the loading jig. The cylinder was loaded at a uniform rate until it collapsed, and the position where complete buckling started was noted. The cylinder was unloaded, the load disk moved a small distance away from the point of initial buckling, and the cylinder reloaded. This procedure was continued until the maximum buckling load was obtained, and this value recorded.

Late in the test program, a dial gage was placed in the center of the cylinder in order to determine its load-shortening characteristics.

III. RESULTS AND DISCUSSION

The effect of the number of floating rings is shown in Figure 10. These curves were plotted from the data tabulated in Table I. The buckling stresses plotted are the highest values obtained for a given number of rings. For the 24-stringer cylinders they correspond to the 0.5 inch rings ($I_{r_1} = 625 \times 10^{-6}$ psi), the stiffest rings tested, and for the 12-stringer cylinder they correspond to the 0.3 inch rings ($I_{r_1} = 103.6 \times 10^{-6}$ psi)*. In all except two cases, the skin would develop small buckles between the stringers before complete buckling took place. The two exceptions to this were the 12-stringer no-ring and one-ring configurations. In these two cases, the buckling of the skin and complete buckling took place simultaneously. With no rings, large diamond shaped buckles were formed between every second stringer. This pattern is illustrated in Figure 2.

Three cylinders with 24 stringers were built and tested, and the scatter in their buckling stresses was quite reasonable, as Figure 10 shows. Each cylinder was buckled over and over again with no appreciable change in the buckling load, even though a small amount of permanent set could be detected in the skin. When the buckling load did drop, it was due to the stringers pulling away from the skin. When this occurred, a new cylinder was built using the same stringers. The pre-buckling and post-buckling states for the 24-stringer cylinders are shown in Figures 3 to 8.

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* The 24-stringer curve and the 12-stringer curve can be compared on the same basis, even though the rings used had different stiffnesses, because the 0.5 inch rings and the 0.3 inch rings gave identical results when tested in the 12-stringer cylinder.

In testing the 24-stringer cylinders with the 0.5 inch, 0.3 inch, and 0.2 inch rings it was sufficient to support the rings at every fourth stringer. However, when the 0.1 inch rings were tested with an L/c * of 4, the rings would bend out of their planes and buckling would occur. This is illustrated in Figure 9. Thereafter, the rings were supported at each stringer. This illustrates an important point when trying to determine the optimum ring in the sense of minimizing weight. While the ability to resist in-plane bending is the main factor in a ring design, the ability to resist out-of-plane bending must also be taken into account.

The effect of varying the ring bending stiffness is illustrated in Figure 11 where the buckling stress is plotted against the moment of inertia resisting in-plane bending. These data are from the third 24-stringer cylinder tested. The range of ring stiffnesses was wide enough so that the upper bound on the rings ability to stiffen the cylinder was determined. It would be expected that a ring of infinite stiffness would give no higher a buckling load than the asymptotic values shown in Figure 11. The no-ring configuration was considered the limiting case for each L/c ratio.

It was felt that by supporting the 0.1 inch rings at all 24 stringers an increase in the buckling stress would be obtained. However, an increase was not found in this particular case. It is felt that this increase would be found with rings of less out-of-plane bending stiffness, and, therefore, for their most efficient use the rings should be supported at each stringer.

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* Ratio of the cylinder length to the distance between the rings.

The ability of a floating ring to strengthen a cylinder is affected to a large degree by the number of stringers. This point is illustrated in Figure 12 where the buckling stress for the stiffest rings tested is plotted against the number of stringers for each value of L/c. McCoy (Ref. 1) found that floating rings had no effect on otherwise unstiffened cylinders. Therefore, the buckling stress for no stringers is that for a pure monococque thin-walled cylinder, as calculated from the empirical equation of Kanemitsu and Nojima:

$$\sigma_{cr} = E_m \left[9 \left(\frac{t}{R}\right)^{1.6} + 0.16 \left(\frac{t}{L}\right)^{1.3} \right]$$

In most applications of cylindrical shells, it is desirable to keep their weight to a minimum. It naturally follows that it is most desirable to have a highly efficient ring. Figure 13 illustrates the efficiency of the rings used in these tests in a plot of the strength-to-weight ratio against the ring weight, and ring-to-cylinder weight ratio. The strength-to-weight ratios, average ring weights, and the ring-to-cylinder weight ratios tabulated in Table II are for the third cylinder tested. The total weight is the cylinder weight (weight of skin plus stringers) plus the total ring weight. The weight of the support clips was neglected. It can be seen that, for an L/c of 3 and 4, the most efficient rings with an axial thickness of 0.060 inch lie between the 0.08 inch and the 0.1 inch rings. The symbols with the tails are for the 0.06 x 0.118 inch rings. These rings were stiffer in resisting out-of-plane bending than in-plane bending, and their lack of efficiency is obvious. This illustrates again the desirability of distributing the ring material so that it is highly resistant to in-plane bending compared to out-of-plane resistance. The ratio of both the in-plane and out-of-plane bending stiffness to the stringer bending stiffness is tabulated in Table III.

A fifth cylinder was built to investigate the load-shortening characteristics of a 24-stringer cylinder. The results of these tests are tabulated in Table IV and presented in Figures 14 and 15. The curves start out with a slightly varying slope until a certain point, after which the slope remains constant until complete buckling occurs. This point of change from a varying slope to a constant slope coincided with the buckling of the stringers in all four cases tested. In the first part of the curve, the cylinder exhibits the stress-strain properties of the cylinder materials. Once the stringers buckle, the cylinder exhibits the linear load-deflection property of a centrally compressed column on an elastic foundation.

The behavior of the $L/c = 1$ case and the $L/c = 2$ case was very much the same for this cylinder, unlike the results obtained earlier. In all the tests with an $L/c = 1$, the stringers assumed the shape of the second mode of a compressed column with fixed ends, which is the shape that one ring forces the stringers to assume. Since as the load is increased it is harder and harder for the stringers to stay in this mode while unsupported, it is natural that the $L/c = 1$ case gave a lower complete buckling load than the $L/c = 2$ case.

The experimental results obtained in these tests suggest two methods for predicting the effect floating rings can produce. The first method is to calculate the load carried by the stringer plus the effective width of the cylinder skin acting with the stringer. The load carried by the remainder of the skin must be added to this to obtain the total buckling load. An average buckling stress can be obtained by dividing the total load by the total area. This method gives very conservative results since it is unable to predict buckling loads in the large deflection regime encountered once the stringers buckle initially. A method for calculating the buckling load of the stringer plus effective skin width is outlined in Sechler and Dunn (Ref. 2). This method is shown in detail in Appendix A.

In several of the configurations tested in this investigation, the buckling stress of the stringers alone was less than for the skin alone. The method described above was not designed to operate under these conditions. Hence, a more comprehensive method was considered. This second method considers the stringer as a centrally compressed column on an elastic foundation with the cylinder skin acting as the elastic foundation. The rings add to the strength of the cylinder by forcing the stringers into higher modes. The critical buckling load for one stringer on an elastic foundation can be found by using the energy method, i.e., equating the strain energy and the work done by the external forces, and then multiplying by the total number of stringers to obtain the buckling load of the complete cylinder. This method is shown in detail in Appendix B.

IV. CONCLUSIONS

From the results of this investigation the following conclusions were reached:

1. Floating rings are effective devices for increasing the buckling stress of thin-walled, longitudinally stiffened cylinders loaded in axial compression.
2. The degree to which floating rings are effective is dependent upon both the number of longitudinal stiffeners and the stiffness of the rings.
3. The in-plane bending stiffness of the floating rings was the important parameter for optimum stiffening effectiveness.
4. Because of the large deflections encountered after initial buckling of the longitudinal stiffeners, the analytical methods described are incapable of predicting the ultimate buckling load.

Since it was impossible to vary all the important parameters in this investigation, it is recommended that further studies be conducted to determine the effect of stringer stiffness, the thickness-to-radius ratio, and the length-to-radius ratio on the ability of a floating ring to increase the buckling load of a cylinder. It is further recommended that subsequent tests include metal cylinders to determine if the same conclusions carry over.

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APPENDIX A

Calculation of the Cylinder Buckling Stress Considering the Effective Sheet Width

If a stiffened flat sheet loaded in compression in the direction of the stiffeners is considered, the stress is uniformly distributed over the sheet and stiffeners until the sheet buckles. Once buckled, the central portion of the sheet can carry little or no additional stress, while that near the stiffeners can carry additional stress since it remains straight. The total load which the stiffener-sheet panel carries can be obtained by integrating the stress distribution over the panel width and multiplying by the sheet thickness. To simplify the calculations the concept of an "effective width" was introduced. This method assumes that there is a uniform compressive stress σ_{se}, the stress at the supported edges of the sheet, acting on a width of the plate, w_e, directly adjacent to the supported edges. The value of w_e is adjusted so that $2\sigma_{se} w_e$ times the sheet thickness is equal to the total load carried by the panel. The buckling stress of the effective column, i.e., the stiffener plus the effective width, can be obtained by well known methods.

The obvious difficulty in this method is in accurately predicting the stress distribution in the buckled sheet. As the panel approaches its ultimate load, large deflections are encountered and more and more of the sheet goes into the plastic regime. Since a theoretical analysis is not available, the effective width can be obtained from a set of curves based on conservative experimental information such as those given by Sechler and Dunn (Ref. 2). These curves give values of w_e/b for various values of σ_{se}/σ_{cr} and σ_{se}/σ_{yp} where b is the panel width, σ_{cr} the sheet buckling stress, and σ_{yp} the sheet yield point stress.

The effective column gives a new value of the stress at the supported edge of the sheet, and this value must be used to determine a new effective width. This iterative method converges rapidly to the solution. The load carried by the remainder of the sheet is based on the buckling stress of the unstiffened cylinder. To illustrate this method the calculations made for predicting the buckling stress of the cylinders tested in this program will be shown.

The value of σ_{cr} is calculated from the equation of Kanemitsu and Nojima, and σ_{se} is assumed to be the stringer's buckling stress calculated from the Euler Theory for the modes which the rings force the stringers to take. If the case for three rings is considered,

σ_{se} = 229 psi, σ_{cr} = 115 psi, σ_{yp} = 5,000 psi, and b = 1.047 in.

$$\frac{\sigma_{se}}{\sigma_{cr}} = 0.0458, \quad \frac{\sigma_{se}}{\sigma_{yp}} = 1.99$$

From Figure 6.2, page 205, Reference 2,

$$\frac{w_e}{b} = 0.37$$

Hence, $$w_e = 0.387 \text{ in.}$$

To obtain the radius of gyration of the effective sheet width plus the stringer, the following are defined:

S = distance between the centroidal axis of the sheet and the centroidal axis of the stringer

ρ_0 = radius of gyration of the stringer alone

ρ_1 = radius of gyration of the stringer plus the effective sheet width

t = sheet thickness

A_0 = cross sectional area of the stringer

$b_e = 2 w_e$

For this case, $S = 0.032$ in, $\rho_0 = 0.01735$ in, $t = 0.004$ in, $A_0 = 0.0075$ in^2, $b_e = 0.774$ in,

$$S/\rho_0 = 1.845 \quad t/A_0 = 0.533$$

From Figure 6.10, page 214, Reference 2,

$$(\rho_1/\rho_0)^2 = 1.4$$

The new stringer-effective sheet buckling stress, σ_{cr_1}, is $1.4 \times \sigma_{se}$ = 320 psi.

Using σ_{cr_1} as the second trial stress, the new effective width is 0.335 in, but the same value of $(\rho_1/\rho_0)^2$ is obtained indicating convergence to the solution. Therefore, the effective column buckling stress is 320 psi. The total buckling load is then

$$P_{cr} = 320(0.18 + 24 \times 0.004 \times 0.67) + 115\left[24 \times 0.004 \times (1.047 - 0.67)\right]$$
$$= 82.2 \text{ lb.}$$

The average stress, σ_{cr}', is $\frac{82.2}{0.0281} = 293$ psi.

For two rings σ_{se} was assumed to be 151 psi which gave a value of 212 psi for σ_{cr_1} and 0.398 in for w_e. Hence, $P_{cr} = 41.6$ lb and $\sigma_{cr}' = 148$ psi. Since for no rings and one ring the stringer buckling stress was less than the sheet buckling stress, it was assumed that the average buckling stress was 115 psi, the critical stress of the unstiffened cylinder. Comparison of these values with the experimental values for the 24-stringer cylinders plotted in Figure 10 shows the conservative nature of this method of analysis and indicates the need for a theory which can take into account the large stringer deflections encountered in this type of stiffened cylinder when subjected to axial compression.

APPENDIX B

Calculation of the Cylinder Buckling Stress Considering the Stringers as Beams on an Elastic Foundation

The longitudinal stiffeners in a stiffened cylinder can be thought of as centrally compressed beams on an elastic foundation, the cylinder skin providing the elastic foundation. The elastic foundation makes the column more stable, thus increasing the load at which it buckles. The insertion of floating rings into the cylinder adds to its strength by forcing the longitudinal stiffeners into higher buckling modes. An analysis of a stiffened cylinder made on this basis can predict the stress at which the stringers will initially buckle. Since it cannot predict when complete buckling of the cylinder will occur, its application is limited to cases where stringer buckling and complete buckling occur simultaneously, or when it is desirable to know when severe deformation of the cylinder will first occur.

The energy method is used to find the buckling load of a beam on an elastic foundation, and the spring constant of the cylinder skin is calculated from knowledge of the deflection of the skin due to unit point loadings. The energy method will be considered first.

If y is the displacement of the stringer (see Figure 16), the strain energy of bending is

$$\Delta V_1 = \frac{EI}{2}\int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx$$

The strain energy of the elastic medium is given by Timoshenko (Ref. 3) to be

$$\Delta V_2 = \frac{\beta}{2} \int_0^L y^2 \, dx$$

where β is called the modulus of foundation.

$$\beta = \frac{\alpha}{a}$$

where α is the spring constant of an individual elastic support and a is the distance between the supports. The work done by the compressive force P is

$$\Delta T = \frac{P}{2} \int_0^L \left(\frac{dy}{dx}\right)^2 dx$$

Then,

$$P_{cr} = \frac{EI \int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx + \beta \int_0^L y^2 \, dx}{\int_0^L \left(\frac{dy}{dx}\right)^2 dx}$$

The spring constant of the cylinder skin can be determined by considering a unit strip of the skin. The deformation due to opposing unit radial loads (see Figure 17) is given by Timoshenko (Ref. 4) to be

$$u = \frac{R^3}{EI} \left(\frac{1}{\pi} - \frac{\phi \sin \phi}{4} - \frac{\cos \phi}{4}\right)$$

By superimposing the deflections due to other unit radial loads, the deflection at the point of application of each unit load can be found. In the cylinder buckling tests, it was noted that the stringers deflected alternately inward and outward. For a loading of this type the absolute value of the deflection at each load point is a constant, and the spring constant of the cylinder skin is

$$\alpha = \frac{1}{u}$$

Once the displacement equation is known, the problem is completely specified and P_{cr} can be calculated.

The effect of different ring stiffnesses can be taken into account in the displacement equations. To determine the maximum effectiveness of a given number of rings, the support given by the ring can be considered infinitely rigid when determining the displacement equation. For rings giving elastic support, the displacement equation can be calculated for a column with rigidly fixed ends and elastic supports where the rings add support. The spring constant for the elastic supports would be that of the ring and could be calculated in the same manner as the cylinder skin's spring constant is calculated. However, in this case the loading instead of alternating in and out would all be towards the center of the ring.

To demonstrate this method, calculations will be made for a cylinder with the same properties as those tested. First the spring constant of the skin must be calculated. Recalling that $u = \frac{Pr^3}{EI}\left(\frac{1}{\pi} - \frac{\phi \sin \phi}{4} - \frac{\cos \phi}{4}\right)$, consider a one inch strip of the cylinder skin.

P = unit load = 1 lb, r = 4 in, E = 725,000 psi

$$I = \frac{bd^3}{12} = \frac{(1)(0.004)^3}{12} = 5.33 \times 10^{-9} \text{ in}^4$$

then, $\frac{Pr^3}{EI}$ = 16,580 in.

Defining $\overline{u} = \frac{uEI}{Pr^3} = \frac{1}{\pi} - \frac{\phi \sin\phi}{4} - \frac{\cos\phi}{4}$ as the dimensionless deflection, and using the notation $\overline{u}_{ij}$ as the dimensionless deflection at 1 on the circular strip due to a unit load $(j - 1) \times 15^\circ$ from 1 (j = 1, . . . , 12), the following are obtained: $\overline{u}_{11}$ = 0.07439, $\overline{u}_{12} = \overline{u}_{1,12}$ = 0.06250, $\overline{u}_{13} = \overline{u}_{1,11}$ = 0.03342, $\overline{u}_{14} = \overline{u}_{1,10}$ = - 0.00269, $\overline{u}_{15} = \overline{u}_{19}$ = - 0.03635, $\overline{u}_{16} = \overline{u}_{18}$ = - 0.05989, and $\overline{u}_{17}$ = - 0.06831.

Summing these up for the alternating load assumed,

$$\overline{u} = \overline{u}_{11} - \overline{u}_{12} + \ldots + (-1)^{1+j}\overline{u}_{1j} + \ldots - \overline{u}_{1,12}$$

$$= 0.00028$$

$$u = \frac{Pr^3}{EI}\,\overline{u} = 4.64 \text{ in}$$

$$\alpha = \frac{1}{u} = 0.2155 \text{ lb/in}$$

Since one inch strips were taken, a, the distance between them, will be one inch. Hence,

$$\beta = \frac{\alpha}{a} = 0.2155 \text{ lb/in}$$

If the stringer deflection is assumed to be that of the first mode of a fixed end column, then

$$y = \frac{A}{2}\left(1 - \cos\frac{2\pi x}{L}\right)$$

$$\Delta V_1 = \frac{EI}{2} \int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx = EI \frac{A^2 \pi^2}{L^3}$$

$$\Delta V_2 = \frac{\beta}{2} \int_0^L y^2 \, dx = \frac{3\beta A^2 L}{16}$$

$$\Delta T = \frac{P}{2} \int_0^L \left(\frac{dy}{dx}\right)^2 dx = \frac{PA^2 \pi^2}{4L}$$

Hence,
$$P_{cr} = \frac{4\pi^2 EI}{L^2} + \frac{3\beta L^2}{4\pi^2}$$

The first term is the critical buckling load for the stringer alone, while the second term is the additional load due to the elastic foundation. For this case,

$$P_{cr} = \frac{(4)(5 \times 10^5)(2.12 \times 10^{-6})\pi^2}{(12.5)^2} + \frac{(3)(0.2155)(12.5)^2}{4\pi^2}$$

$$= 0.267 + 2.56 = 2.827 \text{ lb/stringer}$$

Hence, $\sigma_{cr} = 400$ psi.

For the second mode of a fixed end column, which is the mode one ring forces the stringers to take, the deflection is given by

$$y = C\left(-\frac{1}{k} \sin ky - \frac{\eta}{k} \cos ky + y + \frac{\eta}{k}\right)$$

where $\eta = \dfrac{\cos kL - 1}{\sin kL}$, $kL = 2.87\,\pi$

In this case,

$$P_{cr} = EIk^2 + 1.6 \frac{\beta}{k^2}$$

$$= 0.552 + 0.661 = 1.213 \text{ lb/stringer}$$

$$\sigma_{cr} = 172 \text{ psi}$$

The fact that the second mode buckles at a lower stress than the first mode explains why the cylinders with no rings buckled into the second mode.

As the number of rings increases, the elastic foundation has less and less effect. If the deflection for the three ring cylinder is assumed to be approximated by the fourth mode of a fixed end column, and the effect of the elastic foundation neglected, then

$$P_{cr} = \frac{24.2\pi^2 EI}{L^2} = 1.616 \text{ lb/stringer}$$

$$\sigma_{cr} = 228 \text{ psi}$$

The values calculated for the case of one ring and the case of three rings compare favorably to the stress values for initial stringer buckling obtained experimentally and indicated in Figures14 and 15.

TABLE I

RESULTS OF AXIAL COMPRESSION TESTS ON

FLOATING RING STIFFENED CYLINDERS AND RING MOMENT OF INERTIA VALUES

Cylinder No.*	No. of Rings	L/c (in/in)	Nominal Ring Size (in)	Ring Moment of Inertia** x 10^6		Load, P_{cr} (lb)	Nominal Stress***, σ_{cr} (psi)
				In-Plane (in^4)	Out-of-Plane (in^4)		
1	0	1				49	180
↓	1	2	0.5	625.00	9.00	68	250
↓	2	3	0.5	625.00	9.00	81	297
↓	3	4	0.5	625.00	9.00	105	385
2	0	1				53	195
↓	1	2	0.5	625.00	9.00	74	272
↓	2	3	0.5	625.00	9.00	88	323
↓	3	4	0.5	625.00	9.00	119	436
3	0	1				56	206
↓	1	2	0.5	625.00	9.00	74	272

TABLE I (cont'd.)

Cylinder No.*	No. of Rings	L/c (in/in)	Nominal Ring Size (in)	Ring Moment of Inertia** $\times 10^6$ In-Plane (in^4)	Ring Moment of Inertia** $\times 10^6$ Out-of-Plane (in^4)	Load, P_{cr} (lb)	Nominal Stress***, σ_{cr} (psi)
3	2	3	0.5	625.00	9.00	89	326
↓	3	4	0.5	625.00	9.00	125	459
↓	1	2	0.3	103.60	5.40	74	272
↓	2	3	0.3	103.60	5.40	88	323
↓	3	4	0.3	103.60	5.40	124	455
↓	3	4	0.2	40.00	3.60	124	455
↓	1	2	0.1	5.01	1.85	72	264
↓	2	3	0.1	5.01	1.85	88	323
↓	3	4	0.1	5.01	1.85	122	448
↓	2	3	0.08	2.47	1.30	85	312
↓	3	4	0.08	2.47	1.30	119	436
↓	2	3	0.06 x 0.118	2.12	8.21	83	304
↓	3	4	0.06 x 0.118	2.12	8.21	117	429

TABLE I (cont'd.)

Cylinder No.*	No. of Rings	L/c (in/in)	Nominal Ring Size (in)	Ring Moment of Inertia** x 10^6 In-Plane (in^4)	Ring Moment of Inertia** x 10^6 Out-of-Plane (in^4)	Load, P_{cr} (lb)	Nominal Stress***, σ_{cr} (psi)
3	2	3	0.07	1.70	1.05	85	312
↓	3	4	0.07	1.70	1.05	117	429
↓	1	2	0.06	1.02	0.97	73	268
↓	2	3	0.06	1.02	0.97	84	308
↓	3	4	0.06	1.02	0.97	101	371
4	0	1	0.3	103.60	5.40	29	155
↓	1	2	0.3	103.60	5.40	27	144
↓	2	3	0.3	103.60	5.40	29	155
↓	3	4	0.3	103.60	5.40	37	197

* Cylinders 1, 2, and 3 had 24 stringers; cylinder 4, 12 stringers.

** The ring moment of inertia is based on the average dimensions of the rings.

*** The nominal stress is based on the total cross sectional area of the cylinder.

Total area = area of stringers + area of skin

$= 0.170\ in^2 + 0.1025\ in^2 = 0.2725\ in^2$ (24 stringers)

$= 0.085\ in^2 + 0.1025\ in^2 = 0.1875\ in^2$ (12 stringers)

TABLE II

STRENGTH TO WEIGHT RATIOS FOR CYLINDER NO. 3

L/c (in/in)	Nominal Ring Size (in)	Ring Weight (lb)	Total Weight (lb)	Ring-to-Cylinder Weight Ratio* (lb/lb)	Strength-to-Total Weight Ratio** (lb/lb)
1			0.176		319
2	0.3	0.01853	0.195	0.1052	379
	0.2	0.01188	0.188	0.0675	
	0.1	0.00625	0.182	0.0355	396
	0.08	0.00476	0.181	0.0270	
	0.06 x 0.118	0.00723	0.183	0.0410	
	0.07	0.00401	0.180	0.0228	
	0.06	0.00346	0.179	0.0196	408
3	0.3		0.213		413
	0.2		0.200		440
	0.1		0.187		466
	0.08		0.186		457
	0.06 x 0.118		0.190		436
	0.07		0.184		461
	0.06		0.183		459
4	0.3		0.232		534
	0.2		0.212		585
	0.1		0.195		625
	0.08		0.190		625

TABLE II (cont'd.)

L/c (in/in)	Nominal Ring Size (in)	Ring Weight (lb)	Total Weight (lb)	Ring-to-Cylinder Weight Ratio* (lb/lb)	Strength-to-Total Weight Ratio** (lb/lb)
4	0.06 x 0.118		0.198		588
↓	0.07		0.188		622
	0.06		0.186		543

* Cylinder Weight = Weight of skin + 24 stringers
= 0.0633 lb + 0.1128 lb = 0.1761 lb

** Strength = P_{cr} as tabulated in Table I.

TABLE III

RING TO STRINGER BENDING STIFFNESS RATIO

Ring Size (in)	In-Plane Ratio $E_p I_{r_1}/E_p I_s$* (lb in^2/lb in^2)	Out-of-Plane Ratio $E_p I_{r_2}/E_p I_s$* (lb in^2/lb in^2)
0.5	295.8	4.24
0.3	48.8	2.54
0.2	18.9	1.70
0.1	2.36	0.87
0.08	1.17	0.61
0.07	0.80	0.50
0.06	0.48	0.36
0.06 x 0.118	1.00	3.87

*$E_p I_s = 1.06$ lb in^2.

TABLE IV

CYLINDER LOAD-SHORTENING CHARACTERISTICS

Load, P_{cr} (lb)	Nominal Stress, σ (psi)	Deflection x 10^3, δ (in)			
		L/c = 1	L/c = 2	L/c = 3	L/c = 4
12	42	0.1			
18	66	0.2	0.1	0.1	
24	88	0.5	0.5	0.2	
31	112	0.8	1.2	0.6	0.7
37	136	1.7	2.2	1.5	1.5
43	158	4.5	4.7	2.8	2.6
50	182	7.7	7.2	6.2	3.7
56	205	10.3	10.2	11.7	4.7
62	227	13.4	12.2	16.5	5.8
66	240	17.7	14.7	21.0	9.5
69	251			26.0	13.7
75	275		18.9	31.0	22.5
82	299			37.5	28.5
88	323				34.5
94	345				41.5
101	368				49.5
107	392				56.0
113	415				62.5
121	442				70.5
123	451				75.5
126	462				77.5

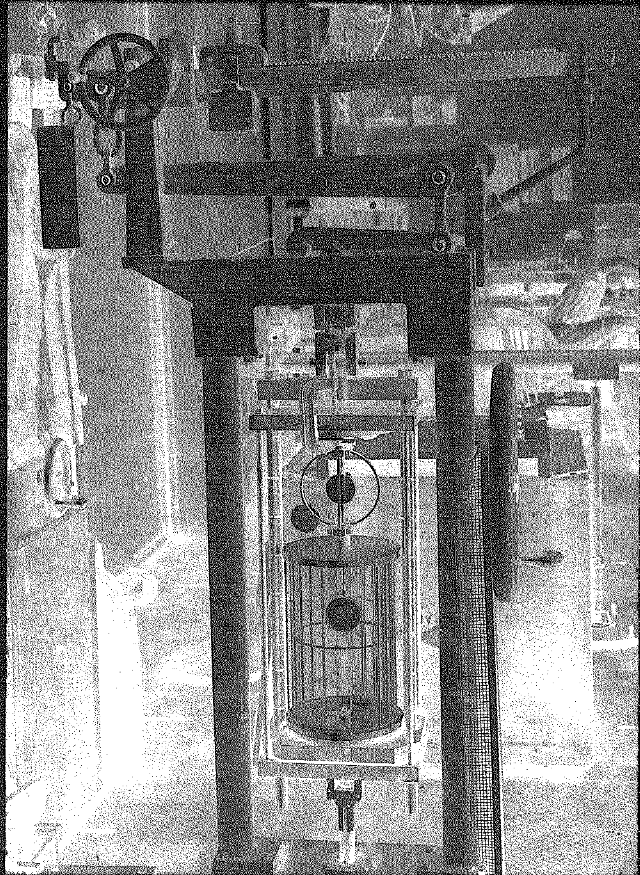

Fig. 1 Cylinder Loading Apparatus with Unbuckled 24-Stringer, One Ring Cylinder

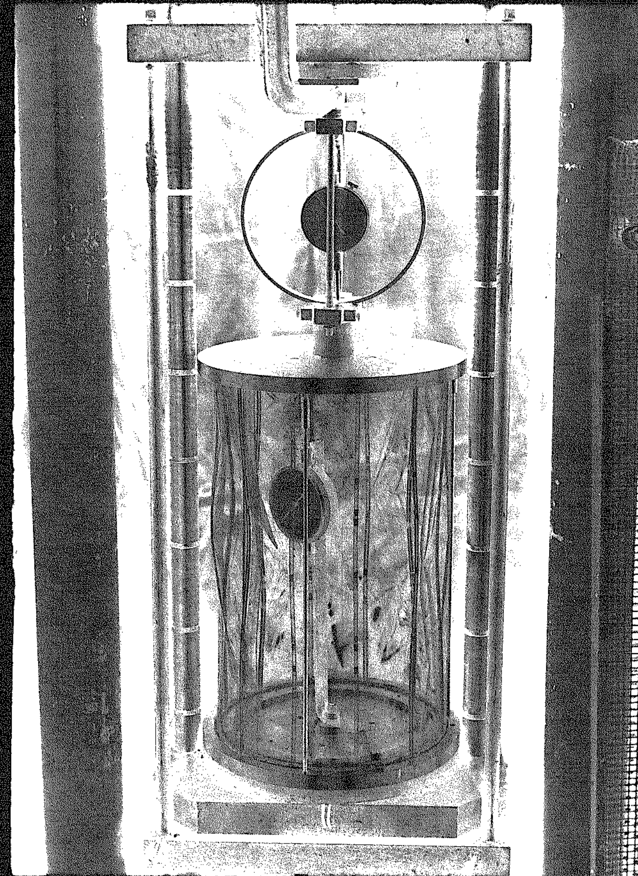

Fig. 2 Buckle Pattern - 12-Stringer, No Ring Cylinder

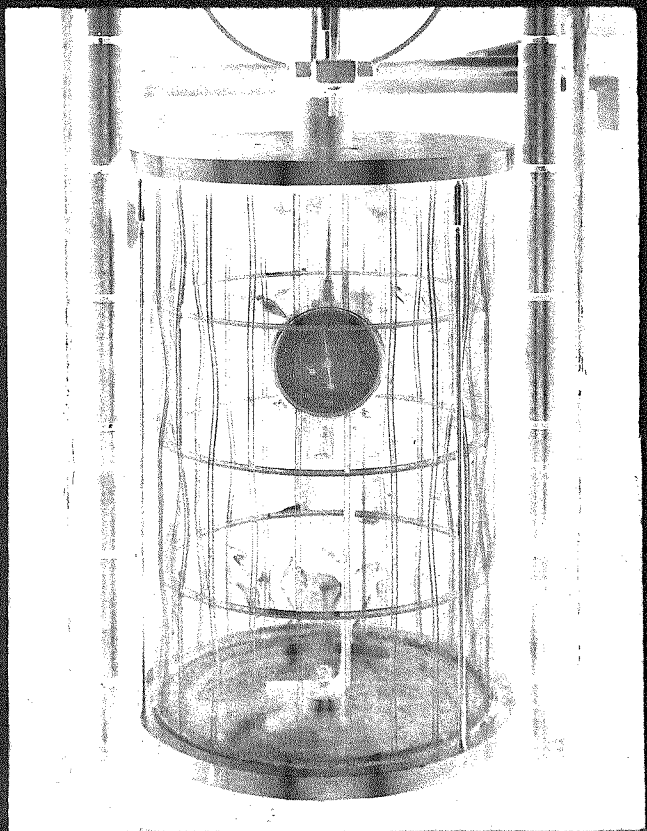

Fig. 3 Buckle Pattern, Pre-Collapse - 24-Stringer, Three Ring Cylinder

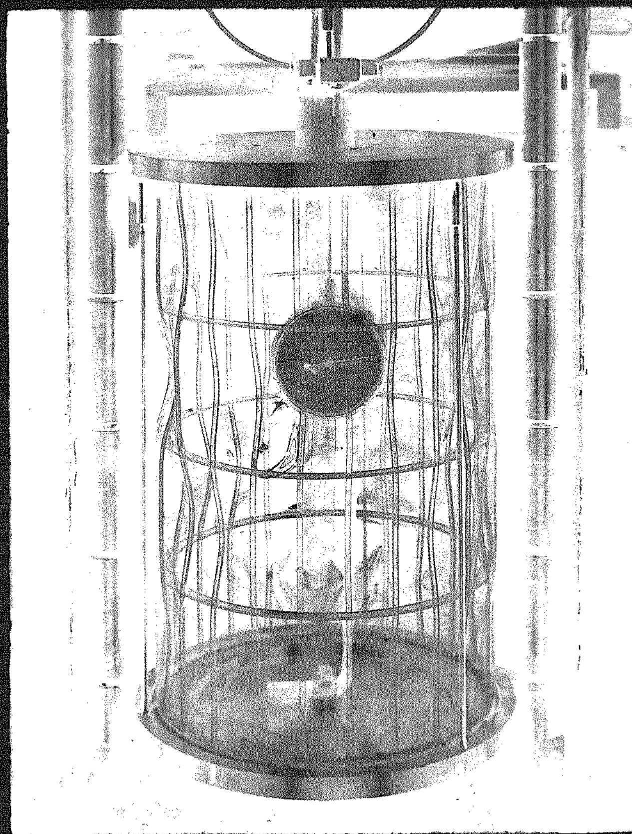

Fig. 4 Buckle Pattern, Post-Collapse - 24-Stringer, Three Ring Cylinder

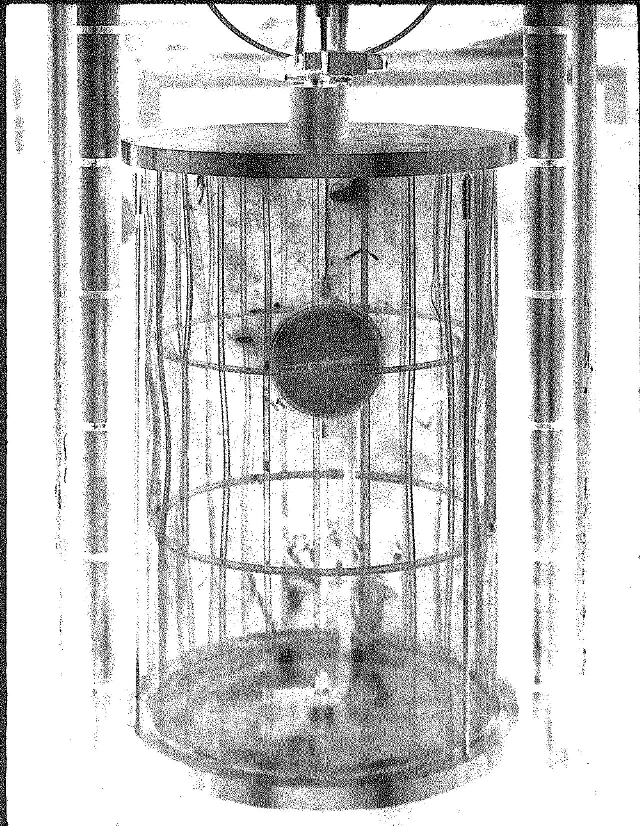

Fig. 5 Buckle Pattern, Pre-Collapse - 24-Stringer, Two Ring Cylinder

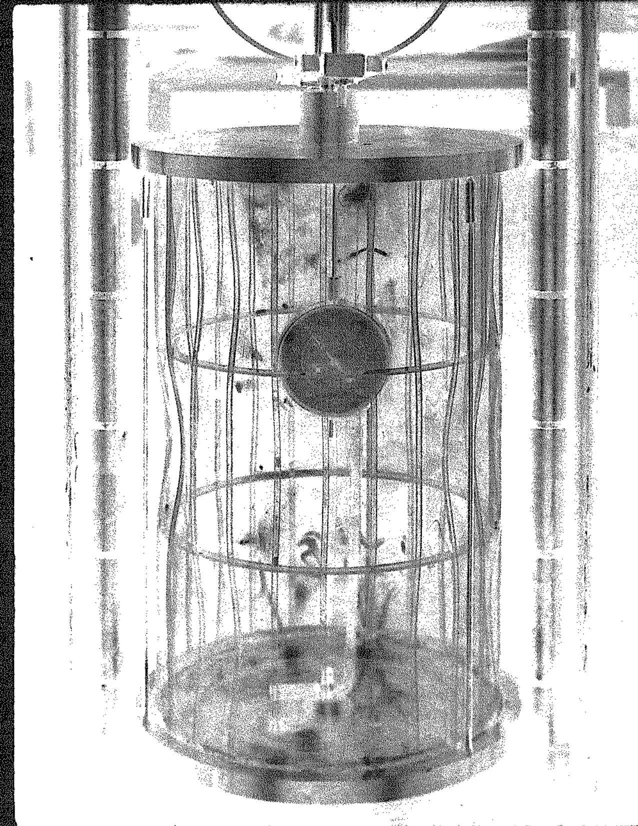

Fig. 6 Buckle Pattern, Post-Collapse - 24-Stringer, Two Ring Cylinder

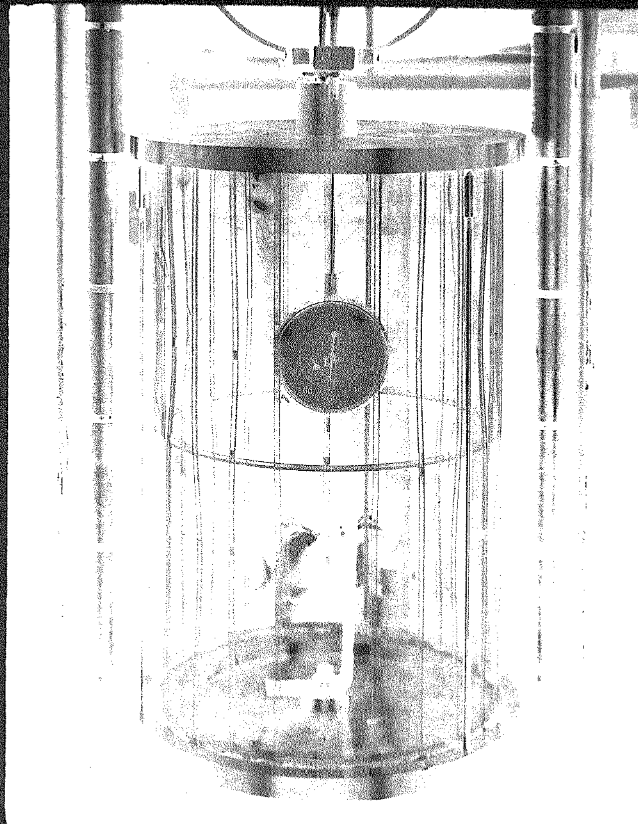

Fig. 7 Buckle Pattern, Pre-Collapse - 24-Stringer, One Ring Cylinder

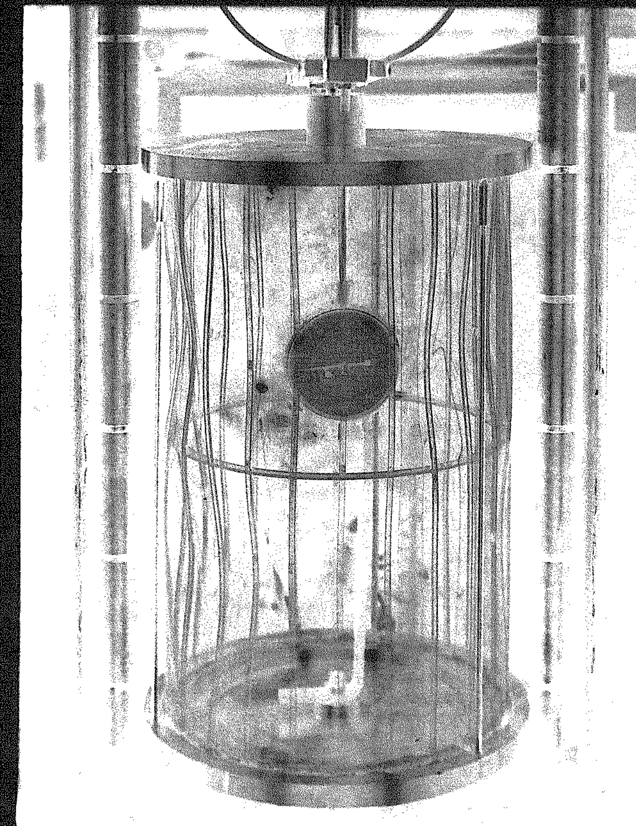

Fig. 8 Buckle Pattern, Post-Collapse - 24-Stringer, One Ring Cylinder

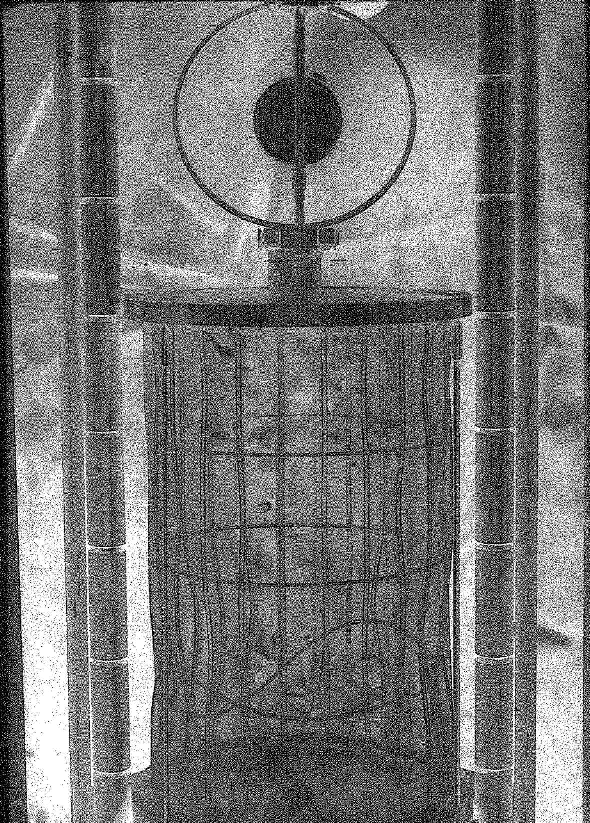

Fig. 9 Out-of-Plane Ring Bending - 24-Stringer, Three Ring Cylinder

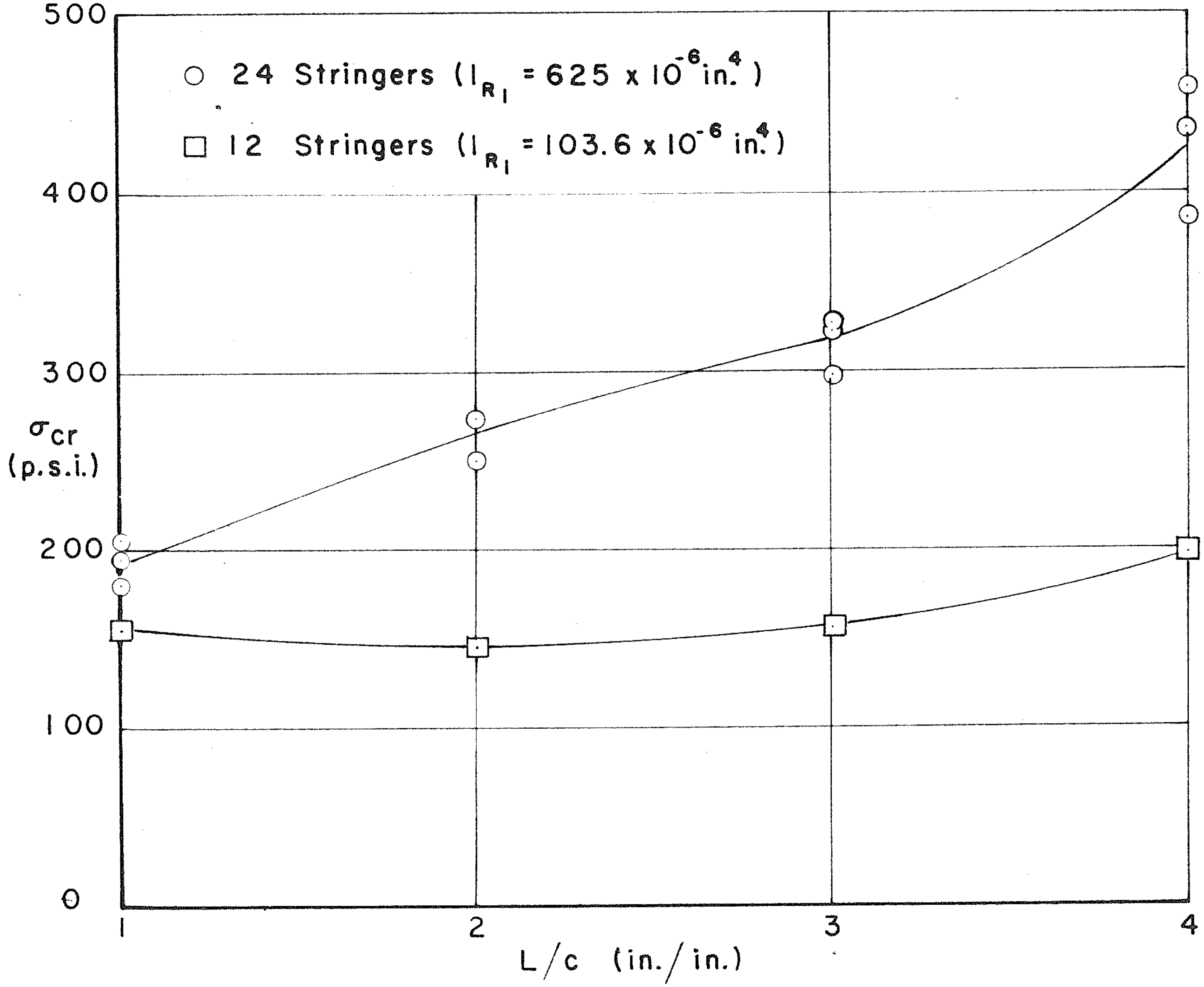

FIG. 10 EFFECT OF NUMBER OF RINGS

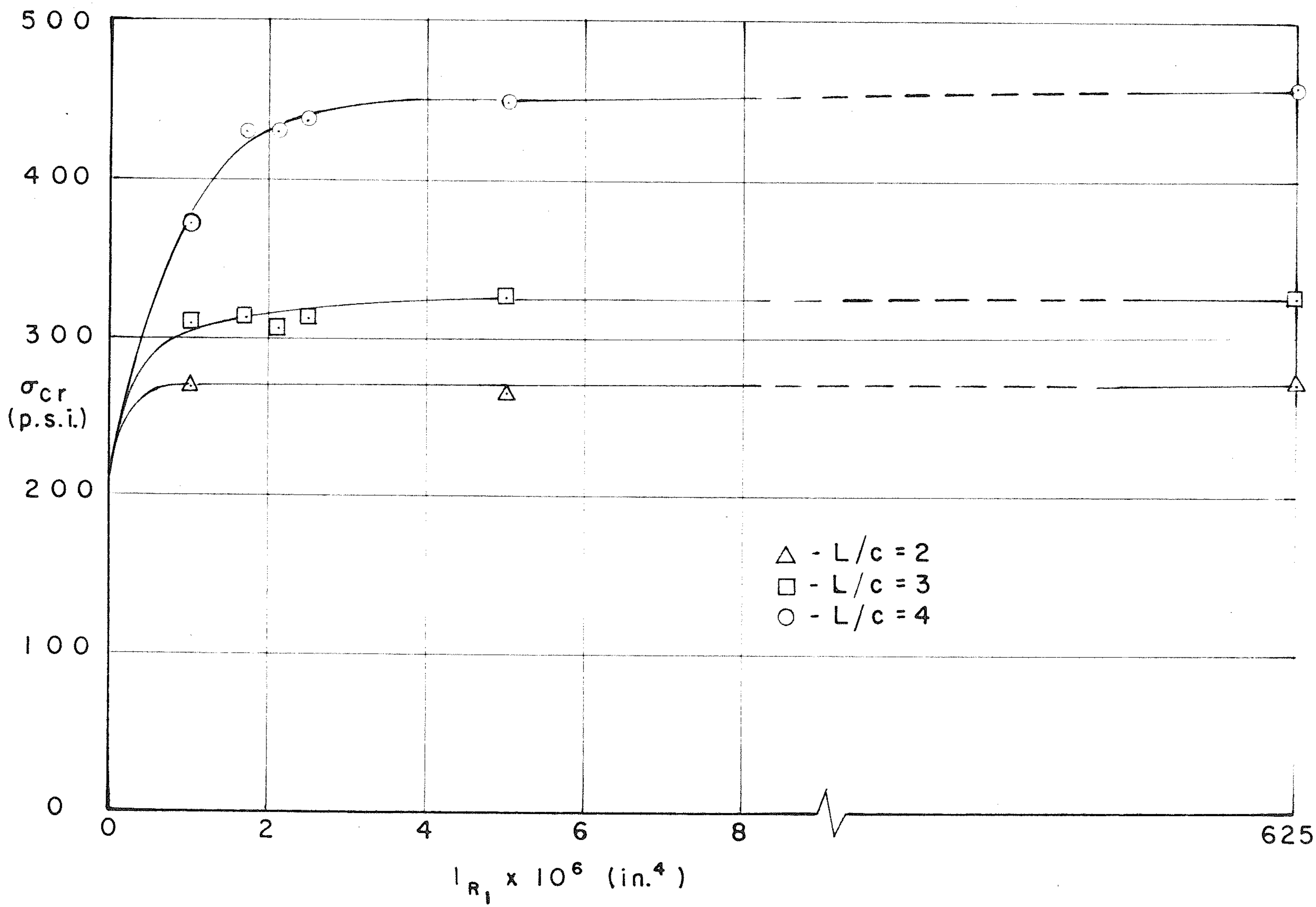

FIG. II EFFECT OF RING BENDING STIFFNESS — 24 STRINGERS

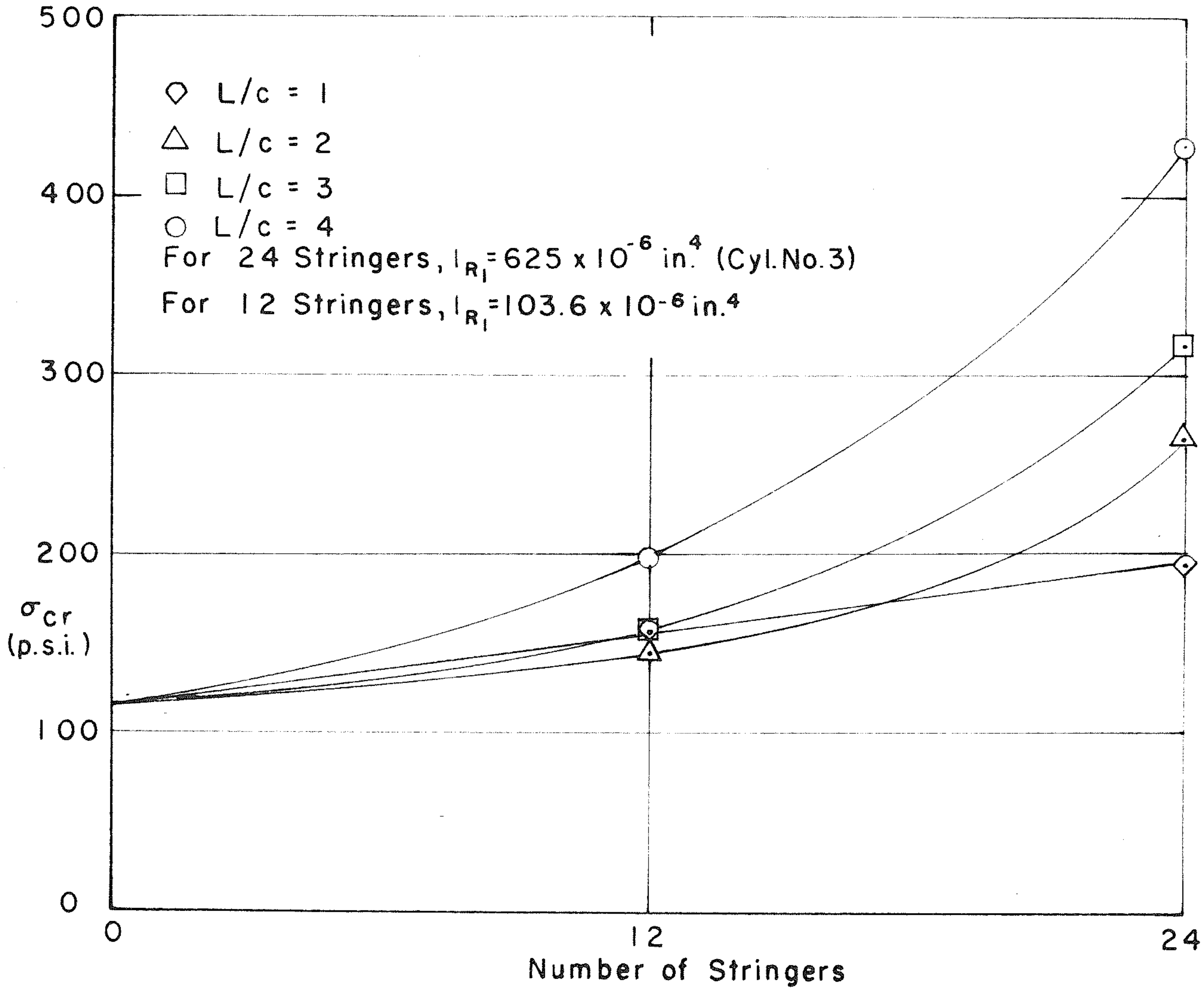

FIG. 12 EFFECT OF THE NUMBER OF STRINGERS

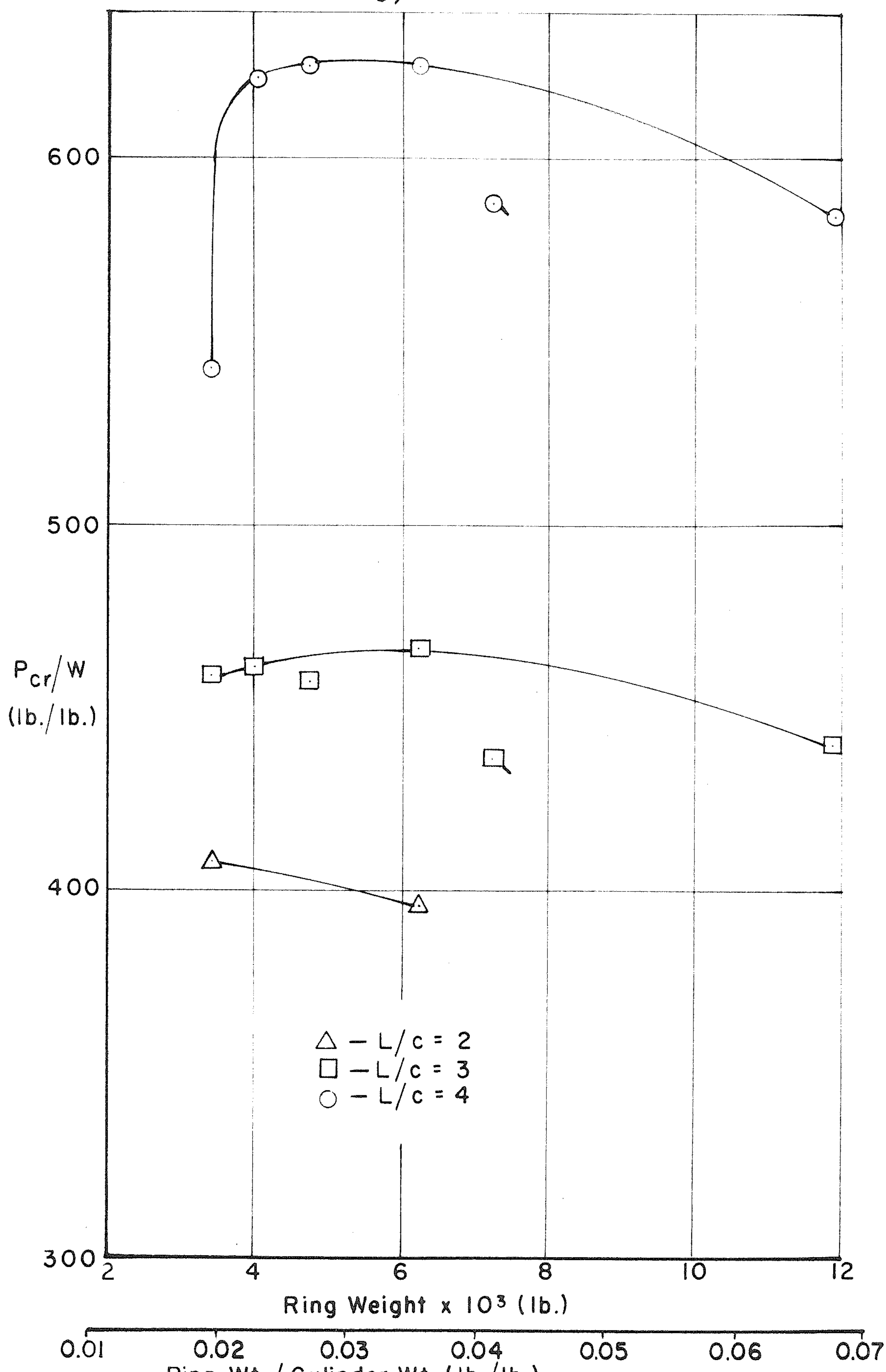

FIG. 13 RING EFFICIENCY — 24 STRINGERS (CYLINDER NO. 3)

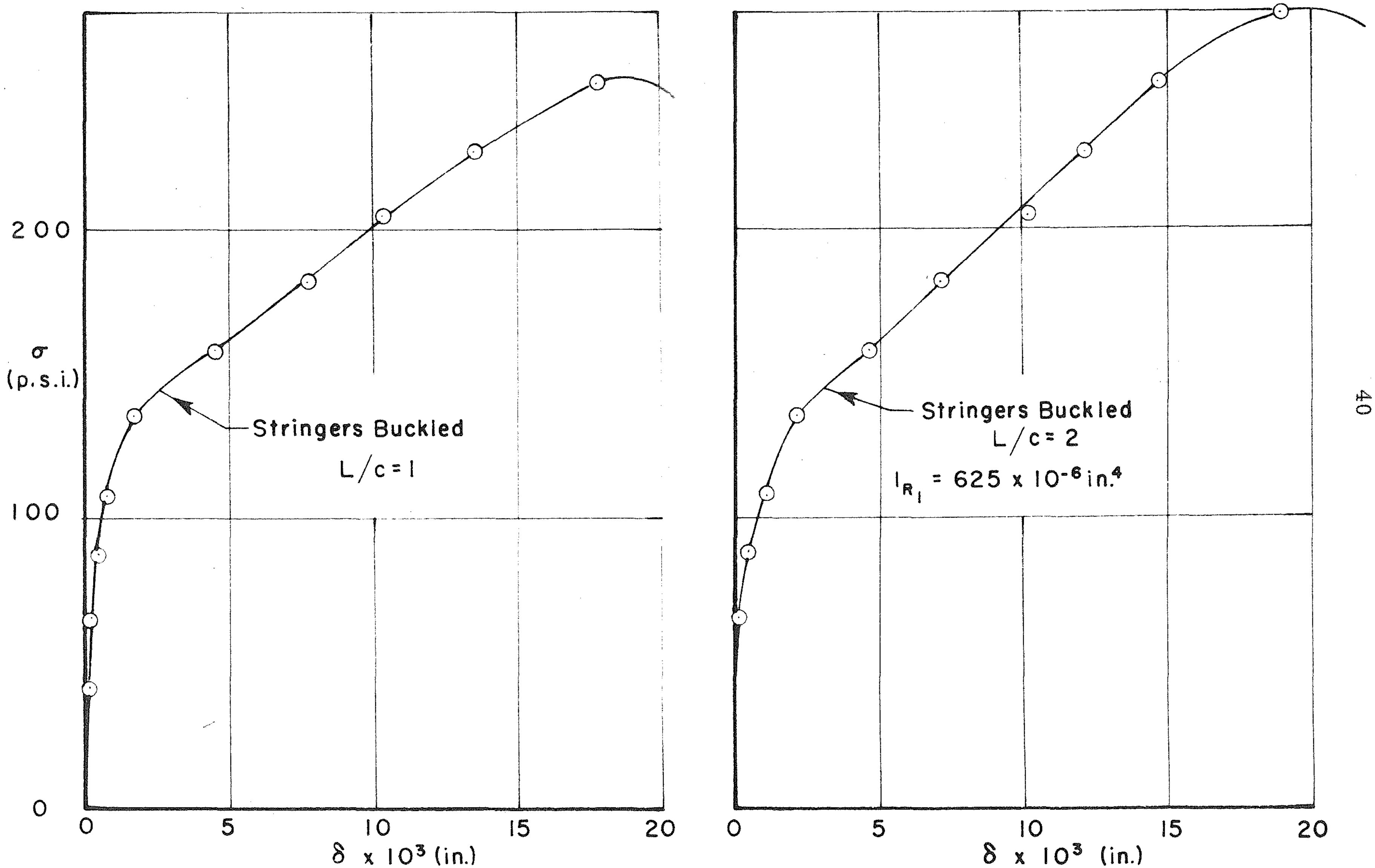

FIG. 14 LOAD—SHORTENING CURVES FOR L/c = 1 AND 2 (24 STRINGERS)

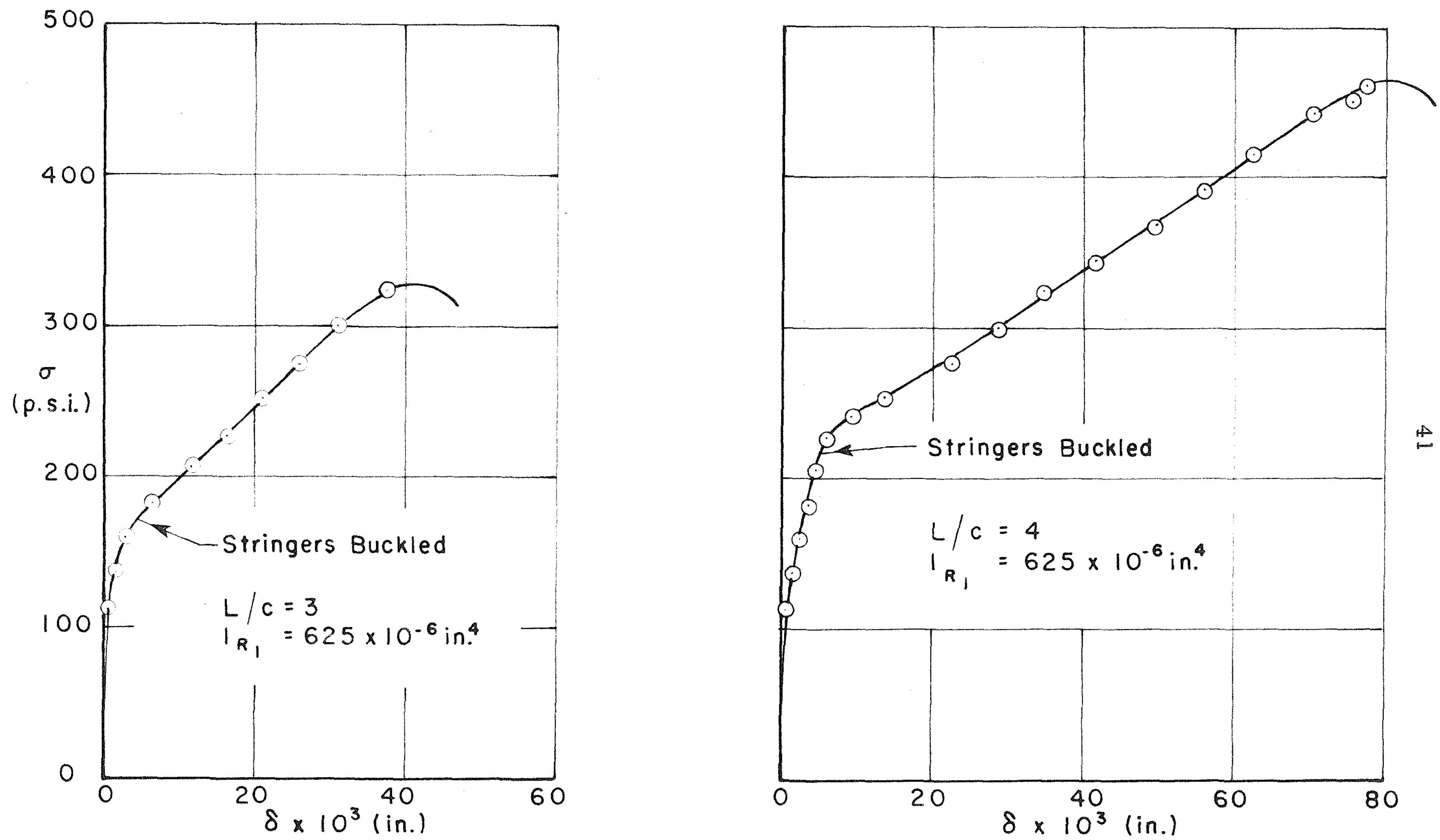

FIG. 15 LOAD-SHORTENING CURVES FOR $L/c = 3$ AND 4 (24 STRINGERS)

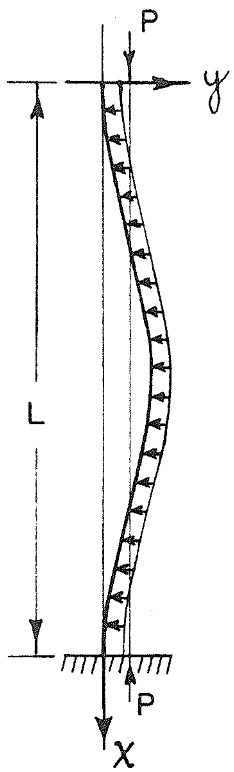

FIG. 16 BEAM ON AN ELASTIC FOUNDATION

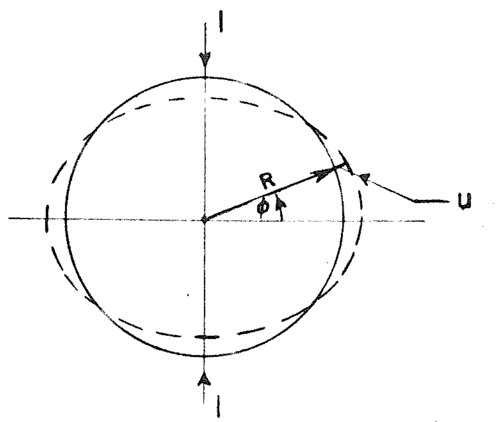

FIG. 17 DEFLECTION OF A RING DUE TO A LOAD