OPERATIONAL FLIGHT TESTING

Thesis by

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I also wish to express my appreciation to members of the Engineering Flight Test Group of the Lockheed Aircraft Corp. for their help in the development of the ideas presented.

Mr. Max Williams has been of great assistance in evaluating the importance of various portions of the subject matter, and in deciding the final form in which the data is presented.

Last, but not least, my very sincere appreciation is due my wife, Virginia Seager, for so diligently typing this thesis.

INDEX

SUMMARY

Operational data are those which are used in routine flights to indicate the weight of payload and fuel that can safely be carried. Therefore, operational data consist of information on take-off, climb, cruise, descent, and landing.

Airplanes which make the most use of this type of information at present are long range, propeller driven transports and bombers; the material presented on the following pages is specifically for use with this general type of aircraft.

Methods of reducing take-off tests, climb tests, and of obtaining high speed cruise control are presented in the body of the thesis.

PART I

TAKE-OFF GROUND RUN

TAKE-OFF GROUND RUN

Introduction

Flight testing to determine the take-off ground run distance to reach some airspeed involves making measurements, usually elapsed time and distance, under a somewhat arbitrary set of test conditions. These measurements are corrected, by means of readings of free air temperature, BHP, wind velocity, etc., made during the run, and the final result is the distance that would be measured under a desired different set of conditions.

If it were possible to determine the correct airplane acceleration during the run, the problem would be relatively simple. However, instrumentation to measure acceleration is, as yet unperfected, and taking second derivatives of a time-distance curve is laborious and of doubtful accuracy, even though a velocity-distance curve is usually so obtained, and is considered sufficiently accurate for use in reduction.

The method proposed here assumes that the airplane excess thrust decreases with velocity squared, or

$$\text{(A)} \quad T_{EX} = T_{EX_S} - B_1 V^2$$

where T_{EX} = excess thrust - lbs.

T_{EX_S} = excess thrust at $V = 0$ - lbs.

V = airplane speed - mph.

B_1 = a constant indicating the rate of thrust decrease

As will be shown later this type of equation is a good approximation for any normal zero-wind test conditions.

As a check of the validity of (A), the following airplane was considered -

$(W/BHP) = 10$

$(W/S) = 45$

$(T_o/BHP) = 4$

$(TS)_{TO} = 950$

$C_{D_{RUN}} = .060$

$C_{L_{RUN}} = .4$

$\mu_{TO} = .05$

where W = airplane weight - lbs.

BHP = total airplane brake horsepower

S = wing area - ft.^{2}

T_o = total static thrust

$(TS)_{TO}$ = propeller tip speed during T.O. - fps

$C_{D_{RUN}}$ = drag coefficient during ground run, gear down flaps extended

$C_{L_{RUN}}$ = lift coefficient during ground run, considered constant for modern tricycle-gear planes

μ_{TO} = coefficient of rolling friction

The above values are taken as giving a good average for the type of airplane to be considered.

During the ground run

$$\text{(B)} \quad T_R = \mu_{TO} W + (C_D - \mu_{TO} C_L)\left\{\frac{S\sigma V^2}{391}\right\}$$

where T_R is the thrust required to maintain the airplane, unaccelerated, at velocity V - lbs.

σ is the density ratio

$$\left\{\frac{T_R}{T_o}\frac{T_o}{BHP}\right\} = \mu_{TO}\left\{\frac{W}{BHP}\right\}_{TO} + (C_D - \mu_{TO} C_L)\left\{\frac{S}{W}\right\}_{TO}\left\{\frac{W}{BHP}\right\}_{TO}\frac{\sigma V^2}{391}$$

The value of $(TS)_{TO} = 950$ is used to determine the airplane speed in terms of the propeller advance ratio J,

$$J = V/206$$

Inserting the numerical values in (B), the following relationship results,

$$\text{(C)} \quad T_R/T_o = .125 + .241(J)^2$$

Most transport airplanes take-off at a J of about 0.6. The results of combining equation (C) with thrust available curves from references 1 and 2, from $J = 0$ to $J = .6$ are shown on page 104. The linear relationship between T_{EX} and J^2 is valid between $J = .3$ and $J = .6$, which corresponds to about 80% of the average ground run.

As a further check on the accuracy of the thrust variation considered, the results of zero-wind tests made on two widely different transports in current use are shown together with the distance-velocity relationships which would exist were the thrust variation in accordance with equation (A). Pages 105 and 106 indicate that any errors resulting from that approximation are negligible.

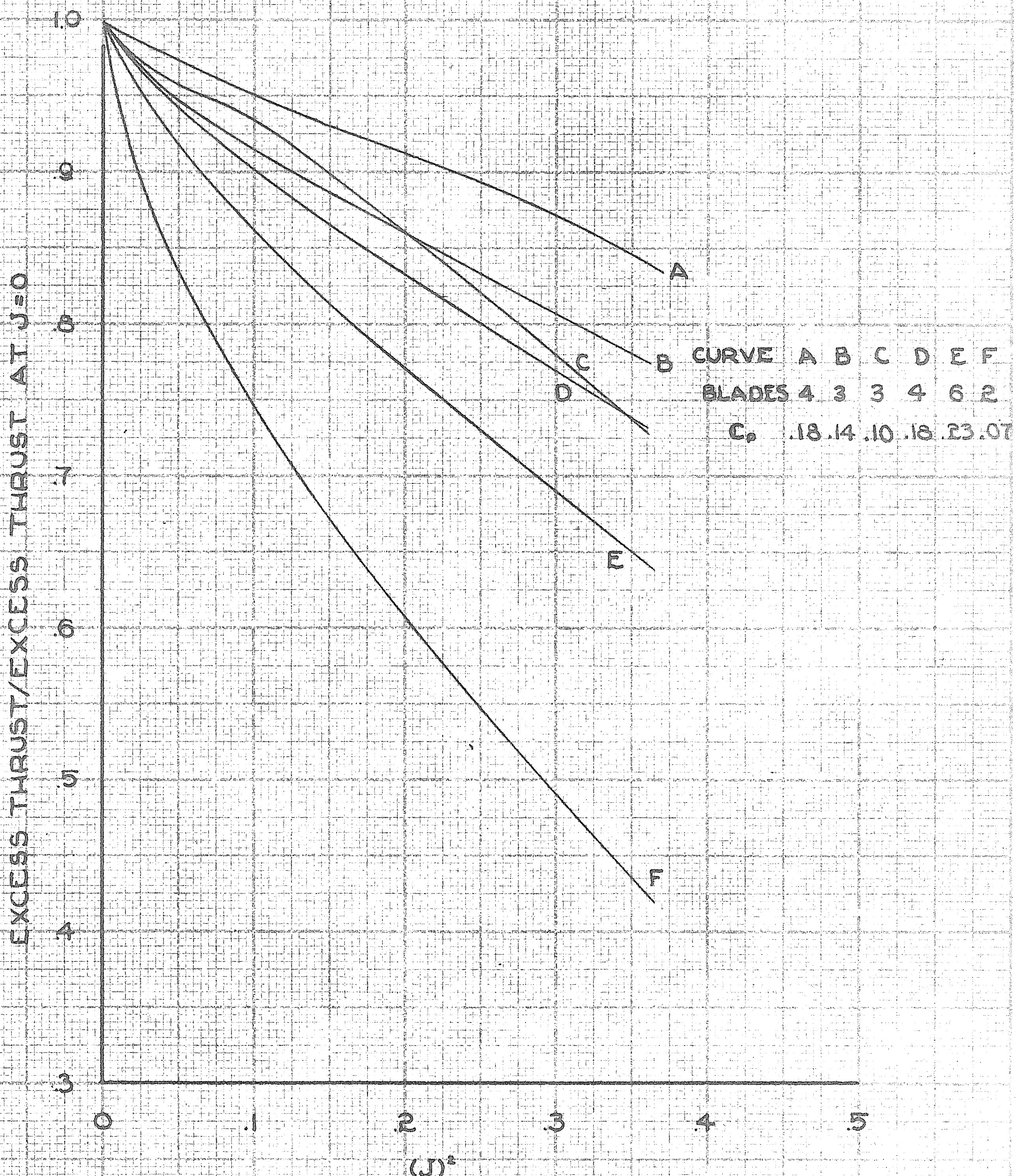
EXCESS THRUST vs. $(J)^2$

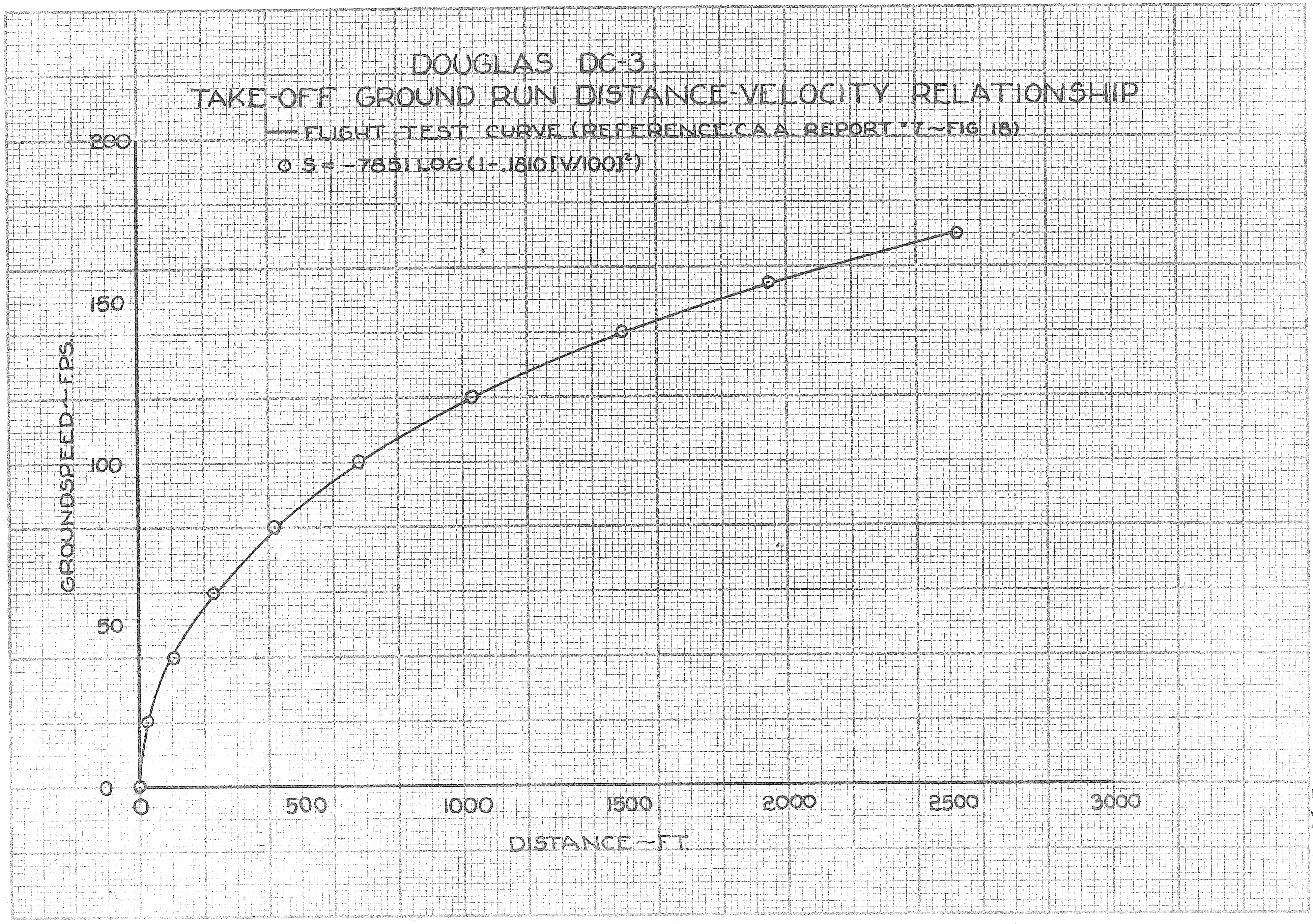
FOR A TYPICAL TRANSPORT AIRPLANE

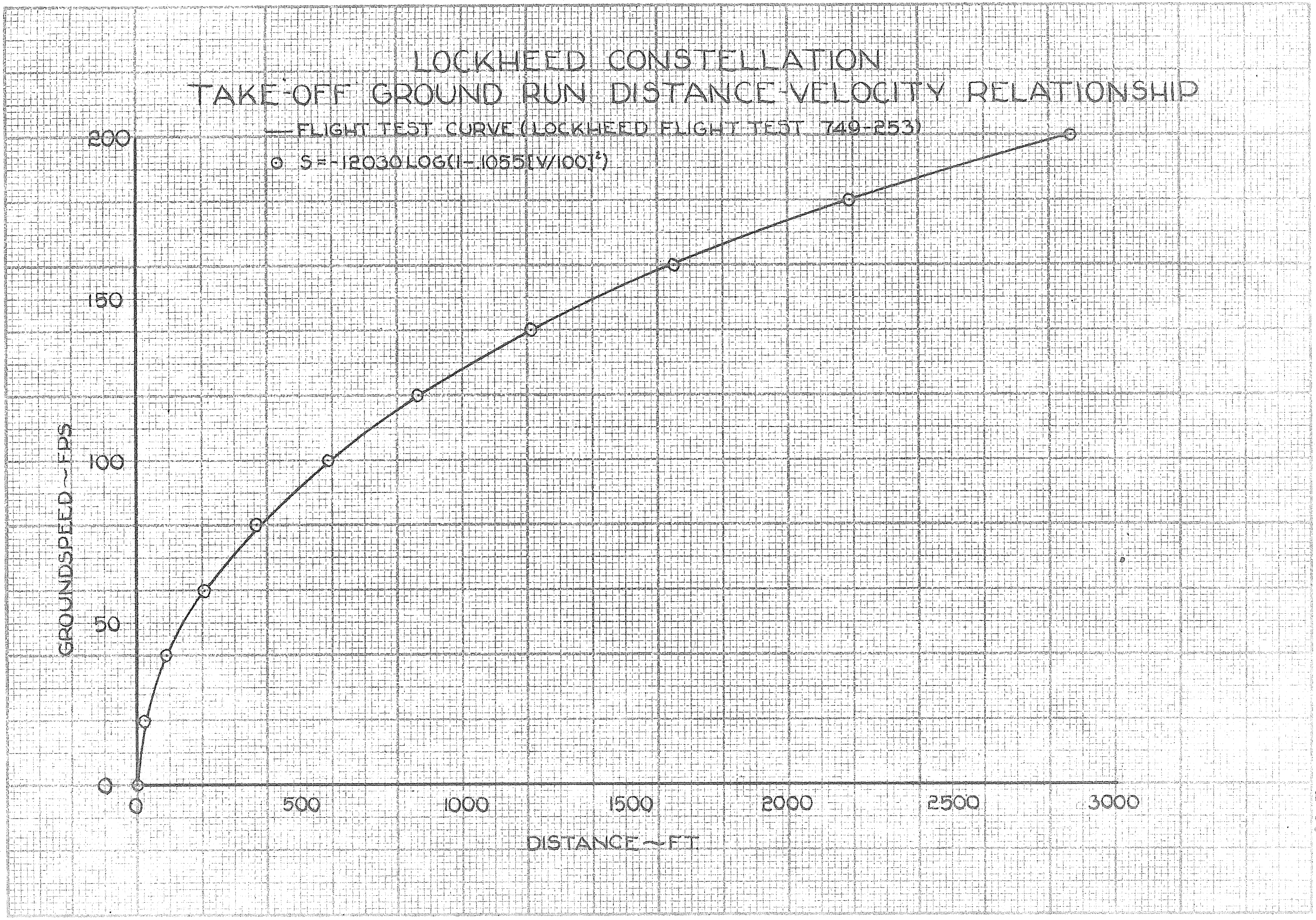
BASED ON DATA ISSUED BY -

(1) CURTISS PROPELLER CO. ~ CURVES A, C, & F

(2) HAM. STD. PROPELLER CO. ~ CURVES B, D, & E




DOUGLAS DC-3
TAKE-OFF GROUND RUN DISTANCE-VELOCITY RELATIONSHIP
— FLIGHT TEST CURVE (REFERENCE: C.A.A. REPORT #7 ~ FIG 18)
⊙ S = -7851 LOG (1 - .1810 [V/100]²)
GROUNDSPEED ~ F.P.S.
200
150
100
50
0
0
500
1000
1500
2000
2500
3000
DISTANCE ~ FT.


LOCKHEED CONSTELLATION
TAKE-OFF GROUND RUN DISTANCE-VELOCITY RELATIONSHIP
— FLIGHT TEST CURVE (LOCKHEED FLIGHT TEST 749-253)
⊙ S=-12030 LOG(1-.1055[V/100]²)
GROUNDSPEED ~ FPS
0
50
100
150
200
DISTANCE ~ FT
0
500
1000
1500
2000
2500
3000

Derivation

The approximation used states -

V = airplane speed - fps.

T_{EX} = excess thrust corresponding to V - lbs.

T_{ST} = excess thrust at $V = 0$ - lbs.

B_1 = a constant relating T_{EX} and V - lbs·sec^2/ft^2.

m = airplane mass - slugs

t = time - seconds

S = distance traversed when V is reached - ft.

V_2 = take-off speed - fps.

S_0 = distance required to reach speed V_2

$$(1) \quad T_{EX} = T_{ST} - B_1 V^2$$

$$T_{EX} = m \frac{dV}{dt}$$

In general

$$\frac{dV}{dt} = V \frac{dV}{dS}$$

$$mV \frac{dV}{dS} = T_{ST} - B_1 V^2$$

$$dS = \frac{mVdV}{T_{ST} - B_1 V^2}$$

$$S_0 = \frac{m}{2B_1} \int_0^{V_2} \frac{(2B_1/T_{ST})VdV}{1 - (B_1/T_{ST})V^2}$$

$$S_0 = -(m/2B_1) \mathrm{Log}_e (1 - [B_1/T_{ST}]V_2^2)$$

Letting $-m/2B_1 = A'$ $\quad B_1/T_{ST} = F_a$

$$S_0 = A' \mathrm{Log}_e (1 - F_a V_2^2) \qquad M^{-1} = 2.3026 = \mathrm{Log}_e 10$$

$$= A' M^{-1} \mathrm{Log}_{10} (1 - F_a V_2^2)$$

$$(2) \quad S_0 = A \,\mathrm{Log}_{10} (1 - F_a V_2^2)$$

The above expression will be assumed to be valid for standard conditions, i.e., zero wind, zero runway slope, standard weight, density and power. Of the constants appearing in equation (2), "A" will not be introduced into any of the correction formulae, and need not be discussed here. The factor F_a is a measure of the decrease in excess thrust of the airplane with increasing velocity during the take-off run, and will be called the acceleration factor. This term will always be used in the form $F_a V_2^2$ which is dimensionless, and allows V_2 to be measured in any convenient units. The logarithm to the base 10 is used in calculations to facilitate the work.

If V is taken as the ground speed during the take-off run, and w is the headwind encountered, the acceleration of the airplane will be

$$(3) \qquad \frac{dV}{dt} = -\frac{1 - F_a(V+w)^2}{2AF_aM}$$

This is true because thrust available is a function only of airspeed at a given power and density, and since thrust required can be expressed as

$$T_R = \mu W + (C_D - \mu C_L)(S\sigma V_A^2/391)$$

As shown in the preliminary discussion, excess thrust is a function only of airspeed.

$$dS_w = -\frac{2AMF_a V dV}{1 - F_a(V+w)^2}$$

where S_w is the ground-run distance to reach airspeed V_2 with a headwind w.

$$S_w = A\int_0^{V_2-w} \frac{-2MF_a V dV}{1 - F_a(V+w)^2}$$

$$S_w = AM\int_0^{V_2-w}\left[\frac{-2F_a(V+w)dV}{1-F_a(V+w)^2}+\frac{2F_a w dV}{1-F_a(V+w)^2}\right]$$

$$= A \operatorname{Log}_{10}\left[1-F_a(V+w)^2\right]\Big|_0^{V_2-w} + A(F_a w^2)^{\frac{1}{2}} \operatorname{Log}_{10}\frac{1+[F_a(V+w)^2]^{\frac{1}{2}}}{1-[F_a(V+w)^2]^{\frac{1}{2}}}\Bigg|_0^{V_2-w}$$

$$= A \operatorname{Log}_{10}\left\{\frac{1-F_a V_2^2}{1-F_a w^2}\right\} + A(F_a w^2)^{\frac{1}{2}} \operatorname{Log}_{10}\frac{1+(F_a V_2^2)^{\frac{1}{2}}}{1-(F_a V_2^2)^{\frac{1}{2}}}\,\frac{1-(F_a w^2)^{\frac{1}{2}}}{1+(F_a w^2)^{\frac{1}{2}}}$$

$$(4)\quad S_w = A \operatorname{Log}_{10}\left[\frac{1-(F_a V_2^2)^{\frac{1}{2}}}{1-(F_a w^2)^{\frac{1}{2}}}\right]^{1-(F_a w^2)^{\frac{1}{2}}}\left[\frac{1+(F_a V_2^2)^{\frac{1}{2}}}{1+(F_a w^2)^{\frac{1}{2}}}\right]^{1+(F_a w^2)^{\frac{1}{2}}}$$

Evaluation of the constants in the equation for S_w is accomplished by means of an airspeed V_1 at $\frac{1}{2}S_w$, defined so -

$$\frac{1}{2}S_w = A \operatorname{Log}_{10}\left[\frac{1-(F_a V_1^2)^{\frac{1}{2}}}{1-(F_a w^2)^{\frac{1}{2}}}\right]^{1-(F_a w^2)^{\frac{1}{2}}}\left[\frac{1+(F_a V_1^2)^{\frac{1}{2}}}{1+(F_a w^2)^{\frac{1}{2}}}\right]^{1+(F_a w^2)^{\frac{1}{2}}}$$

Finally,

$$(5)\quad \frac{1}{2} = \frac{\operatorname{Log}_{10}\left[\frac{1-(F_a V_1^2)^{\frac{1}{2}}}{1-(F_a w^2)^{\frac{1}{2}}}\right]^{1-(F_a w^2)^{\frac{1}{2}}}\left[\frac{1+(F_a V_1^2)^{\frac{1}{2}}}{1+(F_a w^2)^{\frac{1}{2}}}\right]^{1+(F_a w^2)^{\frac{1}{2}}}}{\operatorname{Log}_{10}\left[\frac{1-(F_a V_2^2)^{\frac{1}{2}}}{1-(F_a w^2)^{\frac{1}{2}}}\right]^{1-(F_a w^2)^{\frac{1}{2}}}\left[\frac{1+(F_a V_2^2)^{\frac{1}{2}}}{1+(F_a w^2)^{\frac{1}{2}}}\right]^{1+(F_a w^2)^{\frac{1}{2}}}}$$

In order to facilitate the solution of these equations, two limiting cases will be considered. First, when $w = 0$

$$(5a)\quad \frac{1}{2} = \operatorname{Log}_{10}\frac{1-F_a V_1'^2}{1-F_a V_2'^2}$$

Rewriting equation (5),

$$\left[1-(F_a W^2)^{1/2}\right]\mathrm{LOG}_{10}\left\{\frac{\left[1-(F_a V_1^2)^{1/2}\right]^2}{\left[1-(F_a W^2)^{1/2}\right]\left[1-(F_a V_2^2)^{1/2}\right]}\right\}$$

$$+\left[1+(F_a W^2)^{1/2}\right]\mathrm{LOG}_{10}\left\{\frac{\left[1+(F_a V_1^2)^{1/2}\right]^2}{\left[1+(F_a W^2)^{1/2}\right]\left[1+(F_a V_2^2)^{1/2}\right]}\right\}=0$$

$$2\,\mathrm{LOG}_{10}(1-F_a V_1^2)-\mathrm{LOG}_{10}(1-F_a W^2)-\mathrm{LOG}_{10}(1-F_a V_2^2)$$

$$-(F_a W^2)^{1/2}\,\mathrm{LOG}_{10}\left\{\frac{\left[1-(F_a V_1^2)^{1/2}\right]^2}{\left[1-(F_a W^2)^{1/2}\right]\left[1-(F_a V_2^2)^{1/2}\right]}\right\}$$

$$+(F_a W^2)^{1/2}\,\mathrm{LOG}_{10}\left\{\frac{\left[1+(F_a V_1^2)^{1/2}\right]^2}{\left[1+(F_a W^2)^{1/2}\right]\left[1+(F_a V_2^2)^{1/2}\right]}\right\}$$

Considering the case where $F_a \rightarrow 0$

$$-2F_a V_1''^2+F_a W^2+F_a V_2''^2-(F_a W^2)^{1/2}\left[-2(F_a V_1''^2)^{1/2}+(F_a W^2)^{1/2}+(F_a V_2''^2)^{1/2}\right]$$

$$+(F_a W^2)^{1/2}\left[2(F_a V_1''^2)^{1/2}-(F_a W^2)^{1/2}-(F_a V_2''^2)^{1/2}\right]$$

$$-2V_1''^2+W^2+V_2''^2+2WV_1''-W^2-WV_2''+2WV_1''-W^2-WV_2''=0$$

$$\left(\frac{V_1''}{V_2''}\right)^2+\frac{1}{2}\left(\frac{W}{V_2''}\right)^2-2\left(\frac{V_1''}{V_2''}\right)\left(\frac{W}{V_2''}\right)+\left(\frac{W}{V_2''}\right)-\frac{1}{2}=0$$

(5b) $$\frac{V_2''}{V_1''}=\frac{\sqrt{2}}{(W/V_2'')(\sqrt{2}-1)+1}$$

Solutions of equations (5), (5a), and (5b) will give the term $F_a V_2^2$ as a function of the dimensionless ratios V_2/V_1 and w/V_2. A plot of that relationship is given on page 112.

The limits given on page 112 are chosen so as to include the range of conditions met in ordinary flight test work. The value of $w/V = .20$ corresponds to a wind of about 25 mph for most modern transports. Suitable flight test conditions (i.e., lack of gusts) would seldom be found with winds in excess of this value. A value of $F_a V_2^2 = .7$ corresponds to an excess thrust at the take-off point equal to 30% of the static value, while $F_a V_2^2 = 0$ indicates constant excess thrust. All modern conventional airplanes fall somewhere between these extremes.

Letting P_w be the ratio between take-off distance with a headwind w to zero wind take-off distance,

$$(6) \qquad P_w = \frac{\text{Log}_{10}\left[\dfrac{1-(F_a V_2^2)^{1/2}}{1-(F_a W^2)^{1/2}}\right]^{1-(F_a W^2)^{1/2}}\left[\dfrac{1+(F_a V_2^2)^{1/2}}{1+(F_a W^2)^{1/2}}\right]^{1+(F_a W^2)^{1/2}}}{\text{Log}_{10}\,(1-F_a V_2^2)}$$

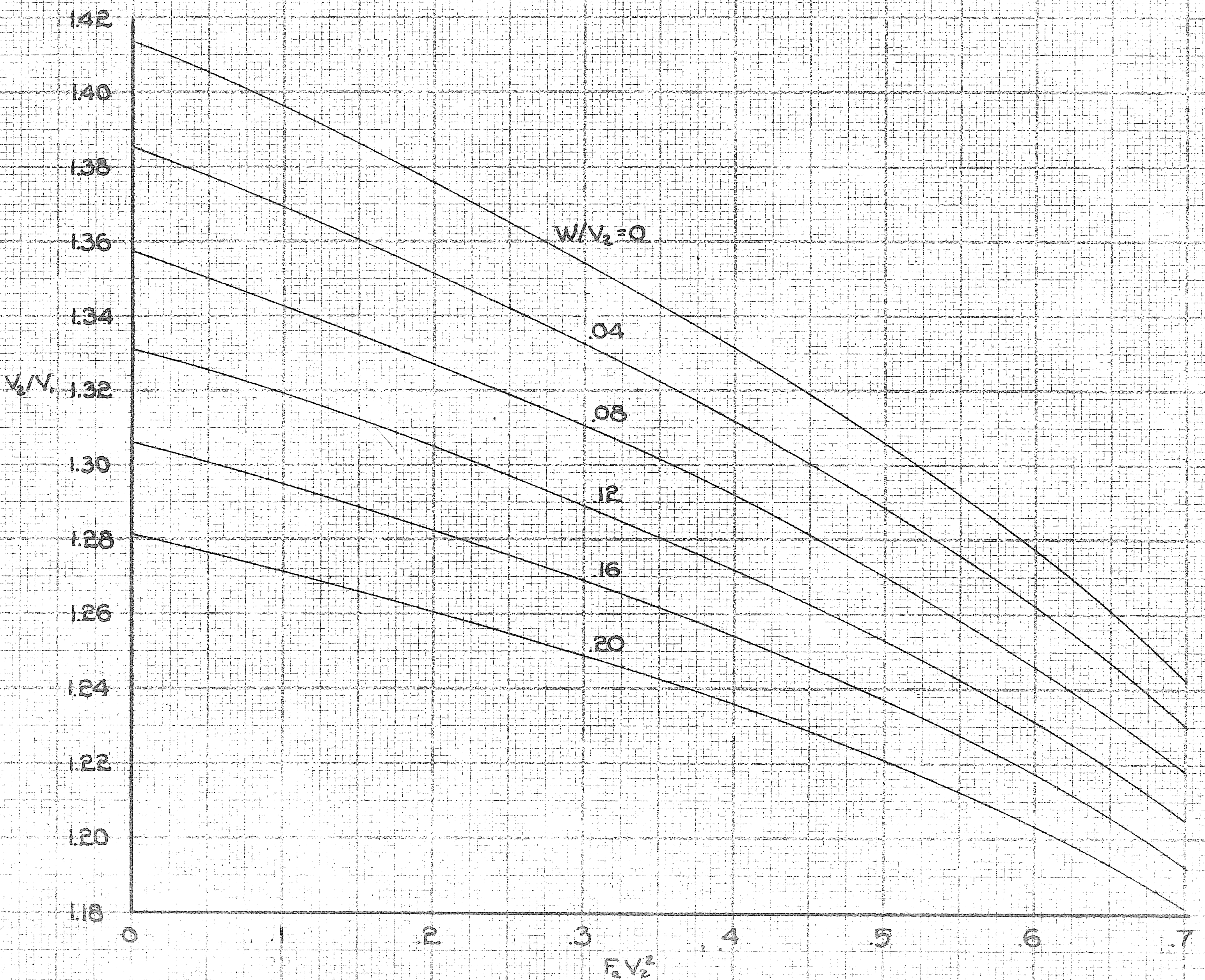
A plot of this equation is given on page 113. Page 112 is superimposed on this plot so as to reduce the steps required in the reduction of a take-off test.

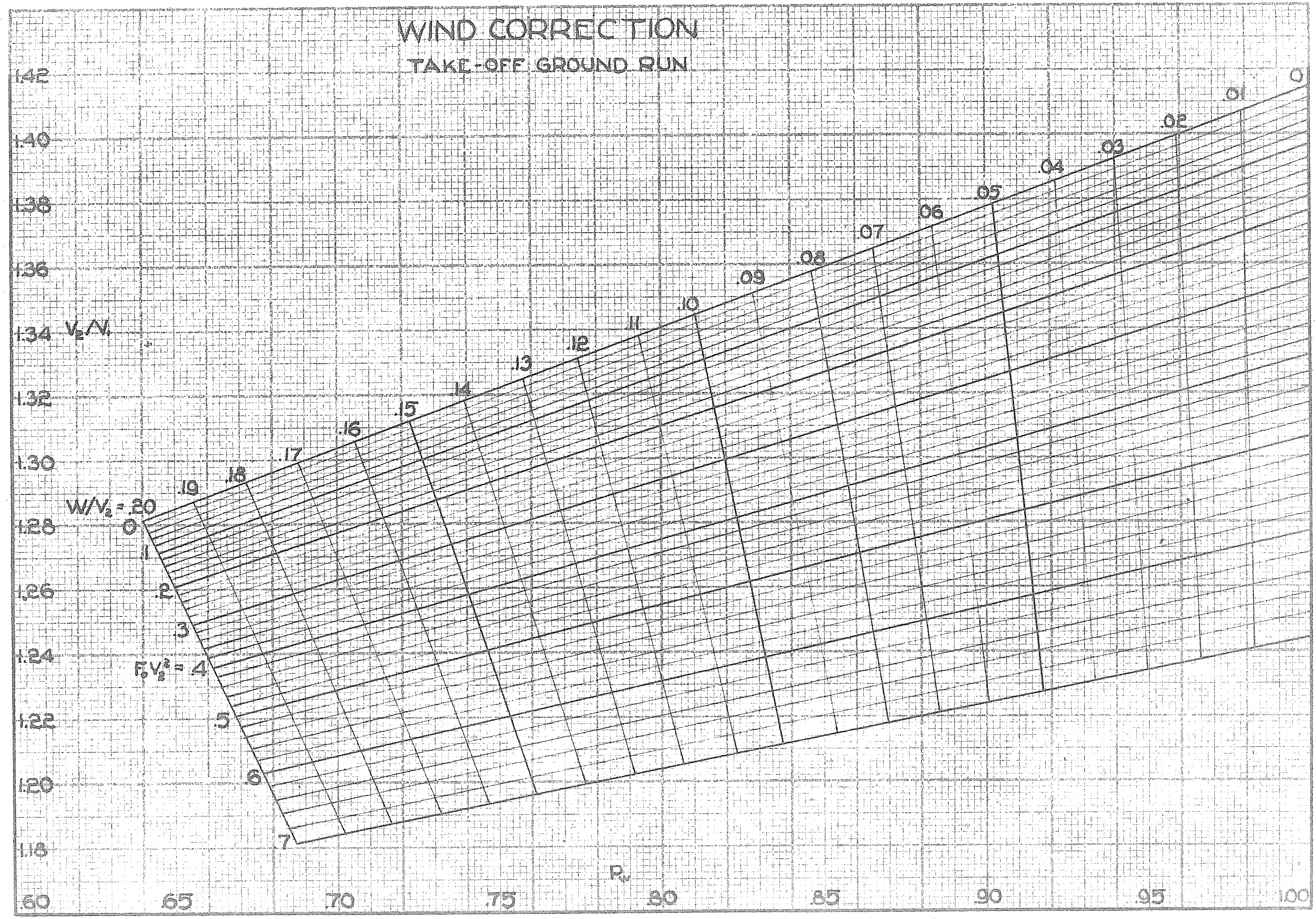
A test is corrected to zero wind in the following manner: -

- V_2 (take-off airspeed) and V_1 (airspeed at one-half observed take-off distance) are noted.
- w (component of headwind parallel to take-off direction) is noted.
- w/V_2 and V_1/V_2 are used to obtain P_w and $F_a V_2^2$. $S_w = P_w S_o$.

If the take-off distance at headwind w' is desired, P_w' is obtained at $F_a V_2^2$ and w'/V_2. $S'(\text{AT } W') = S_w(P_w'/P_w)$

CHART FOR DETERMINATION OF $F_a V_2^2$




WIND CORRECTION
TAKE-OFF GROUND RUN
V_2/V_1
W/V_2 = .20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
0
1
2
3
$F_0V_2^2$ = 4
5
6
7
1.42
1.40
1.38
1.36
1.34
1.32
1.30
1.28
1.26
1.24
1.22
1.20
1.18
.60
.65
.70
.75
.80
.85
.90
.95
1.00
P_w

Effect of Runway Slope

If the equation for the take-off ground run corrected for wind only is

(2) $$S_0 = A \log_{10}(1 - F_a V_2^2)$$

The acceleration is

$$\frac{dV}{dt} = -\frac{1 - F_a V^2}{2AF_a M}$$

where airspeed = ground speed = V.

If the test was made on a runway sloping downward at an angle γ, the acceleration on a level runway would be -

(7) $$\left(\frac{dV}{dt}\right)_{\gamma=0} = -\frac{1 - F_a V^2}{2AF_a M} - g\gamma$$

$$= -\frac{1 + 2AF_a Mg\gamma - F_a V^2}{2AF_a M}$$

$$= \frac{1 - \dfrac{F_a}{1 + 2F_a AMg\gamma} V^2}{\dfrac{2AF_a M}{1 + 2F_a Mg\gamma}}$$

Letting

$$F_a / (1 + 2AF_a Mg\gamma) = F_a'$$

the acceleration factor for a level runway

$$\left(\frac{dV}{dt}\right)_{\gamma=0} = -\frac{1 - F_a' V^2}{2AF_a' M}$$

and,

(2a) $$S_1 = A \log_{10}(1 - F_a' V_2^2)$$

where S_1 is the distance to reach V_2 on a level runway.

$$F_a' = \frac{F_a}{1 + 2AF_a M g Y}$$

where $A = S_o / \mathrm{LOG}_{10}(1 - F_a V_z^2)$

$$F_a' = \frac{F_a}{1 + \dfrac{2 S_o F_a M g Y}{\mathrm{LOG}_{10}(1 - F_a V_z^2)}}$$

$$= \frac{F_a}{1 + \left(\dfrac{S_o Y}{V_z^2}\right) \dfrac{F_a V_z^2}{\mathrm{LOG}_{10}(1 - F_a V_z^2)} (2Mg)}$$

Defining

$$C_s = S_o Y / V_z^2$$

where S_o = take-off ground run corrected for wind - ft.

V_z = zero wind take-off speed - mph

Y = runway slope during test (down taken as positive)-deg.

(8) Finally

$$F_a' = \frac{F_a}{1 + C_s \dfrac{.2267 F_a V_z^2}{\mathrm{LOG}_{10}(1 - F_a V_z^2)}}$$

(9) And

$$P_s = \frac{\mathrm{LOG}_{10}(1 - F_a V_z^2)}{\mathrm{LOG}_{10}(1 - F_a' V_z^2)} = \frac{S_o}{S_1}$$

where S_1 is the ground distance for zero slope.

If the test data are taken at zero slope, and the ground run distance at an angle Y_1 is desired, the acceleration at Y_1 will be

$$\left(\frac{dV}{dt}\right)_{Y_1} = -\frac{1 - F_a' V^2}{2AF_a' M} + g Y_1$$

where F_a' is the value corresponding to zero slope.

If F_a is the acceleration factor at slope Y_1 ,

$$F_a = \frac{F_a'}{1 - C_s \frac{.2267 F_a' V_2^2}{\mathrm{Log}_{10}(1 - F_a' V_2^2)}}$$

Therefore, for correcting from a slope Y to a zero slope,

$$C_s = S_o Y / V_2^2$$

And, for correcting from a zero slope to a slope Y_1 ,

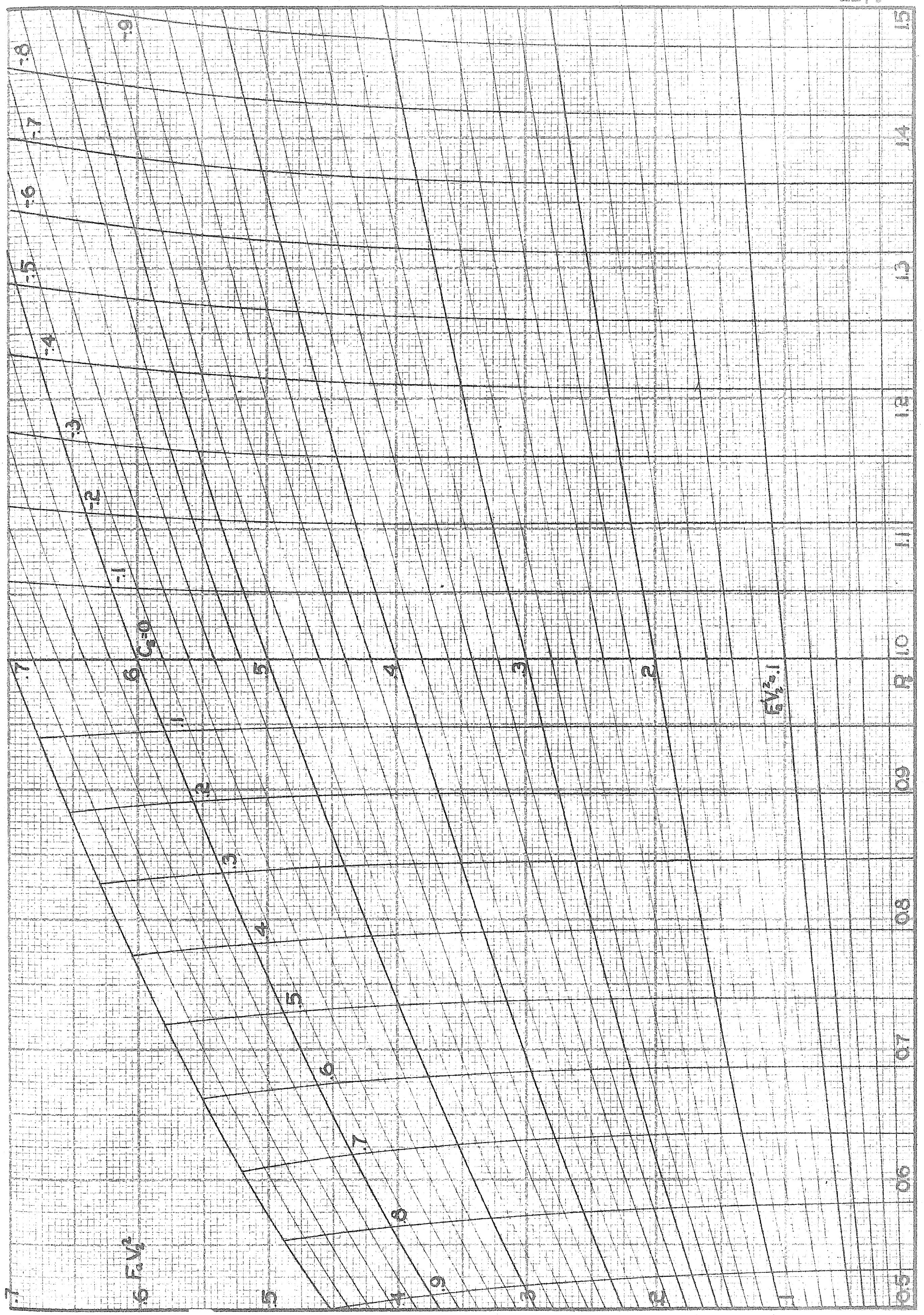
$$C_s' = -S_o' Y_1 / V_2^2$$

If a take-off is made at an angle Y , and the distance for an angle Y_1 is desired -

(10) $$S_1 = (P_Y / P_{Y_1}) S_1'$$

where S_1 is the distance for the final slope Y_1 .

A chart for obtaining values of P_Y and $F_a' V_2^2$ is included on page 117.


R
.05
.06
.07
.08
.09
1.0
1.1
1.2
1.3
1.4
1.5
C_s=0
F_1V_1^2
F_2V_2^2=.1

Effect of Weight Variations

If the test is made at an airplane weight W_1, the zero wind, zero runway slope ground run acceleration is given by

$$\frac{dV}{dt} = -\frac{1-F_a'V^2}{2AF_a'M}$$

If the test is to be corrected to a gross weight W_0,

$$\left(\frac{dV}{dt}\right)_{W_0} = \left(\frac{W_1}{W_0}\right)\left(-\frac{1-F_a'V^2}{2AF_a'M}\right) + g\left(\mu_1\frac{W_1}{W_0} - \mu_0\right) - \frac{qC_LS}{W_0/g}(\mu_1 - \mu_2)$$

where μ is the coefficient of rolling friction, and qC_LS is the airplane lift. For usual reduction purposes, $\mu_1 = \mu_2 = \mu$, and,

$$(11)\quad \left(\frac{dV}{dt}\right)_{W_0} = \left(\frac{W_1}{W_0}\right)\left(-\frac{1-F_a'V^2}{2AF_a'M}\right) + g\mu\left(\frac{W_1}{W_0} - 1\right)$$

$$= \left(\frac{W_1}{W_0}\right)\left\{-\frac{1-F_a'V^2}{2AF_a'M} + g\mu\left(1 - \frac{W_0}{W_1}\right)\right\}$$

$$= \left(\frac{W_1}{W_0}\right)\left\{-\frac{1-2AF_a'Mg\mu(1-W_0/W_1) - F_a'V^2}{2AF_a'M}\right\}$$

setting

$$F_a''V^2 = F_a'V^2/[1-2AF_a'Mg\mu(1-W_0/W_1)]$$

$$\left(\frac{dV}{dt}\right)_{W_0} = \frac{W_1}{W_0}\left\{-\frac{1-F_a''V^2}{2AF_a''M}\right\}$$

setting

$$dS = \frac{VdV}{dV/dt}$$

$$(dS)_{W_0} = -\frac{W_0}{W_1}\frac{2AF_a''MVdV}{1-F_a''V^2}$$

If the airplane is to be airborne at the same C_L at the two weights, the take-off speed at W_0 will be $V_2\sqrt{W_0/W_1} = V_2'$

$$S_{W_0} = -\frac{W_0}{W_1}M\int_0^{V_2\sqrt{\frac{W_0}{W_1}}}\frac{2AF_a''VdV}{1-F_a''V^2}$$

$$= \frac{W_0}{W_1}A\,\mathrm{Log}_{10}\left(1-\frac{W_0}{W_1}F_a''V_2^2\right)$$

$$(12)\quad S_{W_0} = \frac{W_0}{W_1}A\,\mathrm{Log}_{10}\left(1-F_a''V_2'^2\right)$$

In order to make these results more readily usable

$$(13)\qquad \left(\frac{\mu S_{W_1}}{V_2^2}\right)\left(1-\frac{W_0}{W_1}\right) = C_W'$$

This factor is to be calculated from values known and observed during the test. The magnitude of μ is not measured directly, but can be taken as .03 for hard surface runways.

And

$$F_a''V^2 = \frac{W_0}{W_1}\frac{F_a'V^2}{1-2AF_a'Mg(1-W_0/W_1)}$$

$$= \frac{W_0}{W_1}\frac{F_a'V^2}{1-\frac{2S_{W_1}F_a'V_2^2Mg}{V_2^2\mathrm{Log}_{10}(1-F_a'V_2^2)}\left(1-\frac{W_0}{W_1}\right)}$$

$$F_a''V^2 = \frac{\mathbb{W}_0}{\mathbb{W}_1}\,\frac{F_a'V^2}{1 - C'_{\mathbb{W}}\dfrac{F_a'V^2}{\log_{10}(1-F_a'V^2)}2Mg}$$

Letting

$$\frac{-C'_{\mathbb{W}}F_a'V_2^2}{\log_{10}(1-F_a'V_2^2)}2Mg = C_{\mathbb{W}}$$

(14) $$F_a''V^2 = \frac{\mathbb{W}_0}{\mathbb{W}_1}\,\frac{F_a'V^2}{1+C_{\mathbb{W}}}$$

A chart for obtaining $C_{\mathbb{W}}$ from $C'_{\mathbb{W}}$ is included on page 121.

One other parameter $K_{\mathbb{W}}$ is defined as

(15) $$K_{\mathbb{W}} = (\mathbb{W}_0/\mathbb{W}_1)/(1+C_{\mathbb{W}})$$

This value is calculated, knowing $C_{\mathbb{W}}$ and $\mathbb{W}_0/\mathbb{W}_1$.

(16) $$\frac{S_{\mathbb{W}_0}}{S_{\mathbb{W}_1}} = \frac{(\mathbb{W}_0/\mathbb{W}_1)A\log_{10}(1-F_a''V_2'^2)}{A\log_{10}(1-F_a'V_2^2)}$$

$$= \frac{\mathbb{W}_0}{\mathbb{W}_1}\,\frac{\log_{10}(1-F_a''V_2'^2)}{\log_{10}(1-F_a'V_2^2)}$$

$$= P_{\mathbb{W}}$$

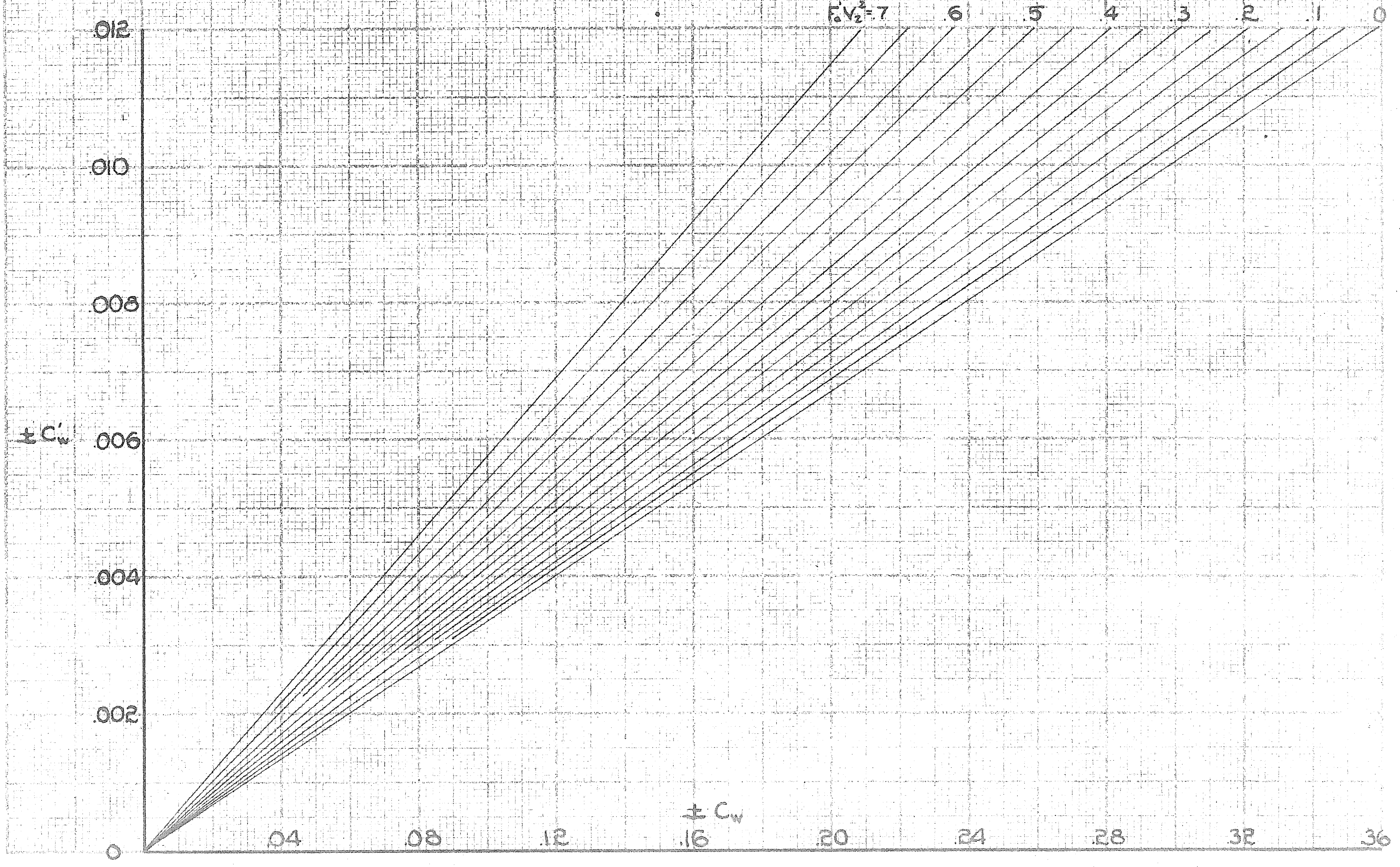
Knowing $K_{\mathbb{W}}$ and $F_a'V_2^2$, $(\mathbb{W}_1/\mathbb{W}_2)P_{\mathbb{W}}$ can be read from the chart on page 122, and $P_{\mathbb{W}}$ is thereby obtained.

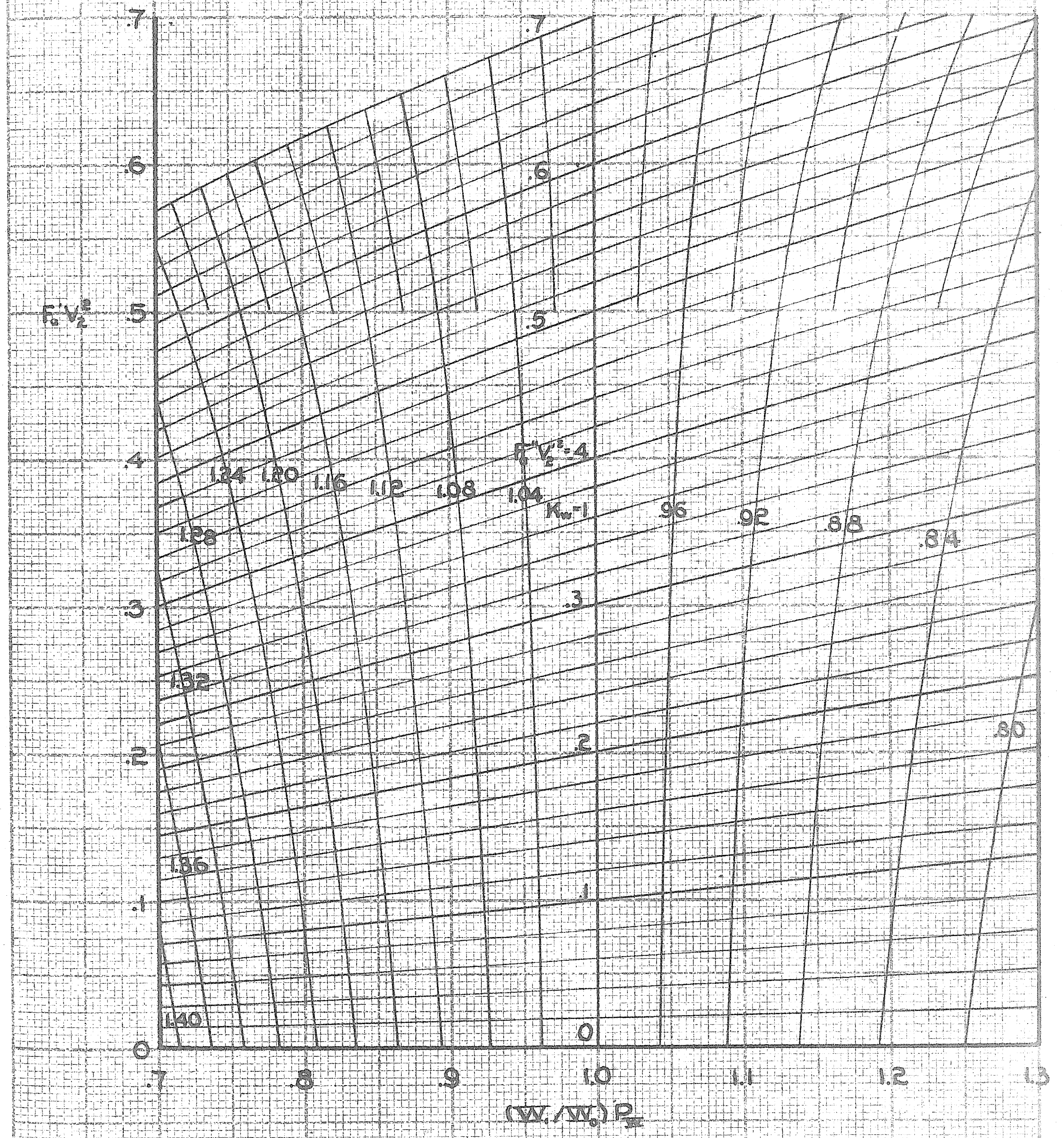
And

$$S_{\mathbb{W}_0} = P_{\mathbb{W}}S_{\mathbb{W}_1}$$

where $S_{\mathbb{W}_0}$ is the ground distance at $\mathbb{W}_0$.

CHART FOR DETERMINING C_w





.7
.6
.5
.4
.3
.2
.1
0
$F_0' V_2^2$
$F_0' V_2^2 = .4$
1.24
1.20
1.16
1.12
1.08
1.04
$K_w = 1$
.96
.92
.88
.84
.80
1.28
1.32
1.36
1.40
.7
.8
.9
1.0
1.1
1.2
1.3
$(W_1/W_0) P_{II}$

Effect of Density Variations

A change in density causes variation both in the airplane lift and drag, and, in general, in the thrust available. The first variation will be considered here; the second under "Effects of Power".

$$\frac{dV}{dt} = -\frac{1 - F_a'' V'^2}{2AF_a''M}$$

Since both airplane lift and drag depend upon $V_i = V'\sigma^{1/2}$, and if the test is made at a density ratio σ_1 and correction to σ_0 is desired.

$$\left(\frac{dV}{dt}\right)_{\sigma_0} = -\frac{1 - (F_a''/\sigma_1)V_i^2}{2AF_a''M}$$

The acceleration at σ_0 will be

$$(17) \quad \left(\frac{dV}{dt}\right)_{\sigma_0} = -\frac{1 - (F_a''/\sigma_1)V'^2\sigma_0}{2AF_a''M}$$

If the take-off is made at $V_i = V_2'\sigma_1^{1/2}$, acceleration to the same indicated airspeed at σ_0 will be required in the final corrected distance, or

$$V_2''\sigma_0^{1/2} = V_2'\sigma_1^{1/2}$$

$$S_{\sigma_0} = \frac{\sigma_1}{\sigma_0} A \operatorname{Log}_{10}\left(1 - F_a'' V_2''^2 \frac{\sigma_0}{\sigma_1}\right)$$

$$= \frac{\sigma_1}{\sigma_0} A \operatorname{Log}_{10}\left(1 - F_a'' V_2'^2\right)$$

And

$$P_\sigma = (S_{\mathrm{II}_0} / S_2) = \sigma_0/\sigma_1$$

Where S_2 is corrected for density.

It is to be noted that while the density correction does not alter $F_a'' V_2'^2$, the actual take-off velocity if $V_2'' = V_2'\sqrt{\sigma_1/\sigma_0}$ at density σ_0.

Effect of Thrust Variations

It has been shown that the relationship

$$\frac{dV}{dt} = -\frac{1 - F_o V^2}{2AF_o M}$$

holds to a very good approximation for various test conditions, and it is therefore a valid assumption that any variations in thrust caused by density changes, lowered free air temperatures, etc., would result is an equally good assumption for acceleration.

If ΔT_{ST} is the increase in static thrust to be used in correcting the test data, and

ΔT_2 is the increase at V_2', both in lbs., and,

$$\frac{dV}{dt} = \frac{1 - F_a'' V'^2}{2AF_a'' M}$$ is the acceleration

corrected except for thrust, then

$$(19)\quad \left(\frac{dV}{dt}\right)_T = \frac{\left(1 - 2AF_a''M \frac{\Delta T_{ST}}{W} g\right) - \left[F_a'' - \frac{2AF_a''Mg}{W V_2'^2}(\Delta T_2 - \Delta T_{ST})\right]V^2}{2AF_a''M}$$

is the acceleration corrected for thrust differences ΔT_{ST} and ΔT_2.

Letting

$$(20)\quad F_a''' V_2'^2 = \frac{F_a'' - \frac{2AF_a''Mg}{W V_2'^2}\left[\Delta T_{ST} - \Delta T_2\right]}{1 - 2AF_a''M(\Delta T_{ST}/W)g} V_2'^2$$

$$(21)\quad S_3 = \frac{A}{1 - \frac{2AMg}{W V_2'^2}(\Delta T_{ST} - \Delta T_2)} \operatorname{Log}_{10}(1 - F_o''' V_2'^2)$$

where S_3 is corrected for thrust.

$$P_T = \frac{S_2}{S_3} = \left[1 - \frac{2AMg}{W V_2'^2}(\Delta T_{ST} - \Delta T_2)\right] \frac{\mathrm{Log}_{10}(1 - F_a'' V_2'^2)}{\mathrm{Log}_{10}(1 - F_a''' V_2'^2)}$$

$$A = \frac{S}{\mathrm{Log}_{10}(1 - F_a'' V_2'^2)}$$

(22) $$P_T = \frac{\mathrm{Log}_{10}(1 - F_a'' V_2'^2)}{\mathrm{Log}_{10}(1 - F_a''' V_2'^2)} - \frac{2MgS}{W V_2'^2} \frac{(\Delta T_{ST} - \Delta T_2)}{\mathrm{Log}_{10}(1 - F_a''' V_2'^2)}$$

(23) If $$\frac{S(\Delta T_{ST} - \Delta T_2)}{V_2'^2 W} = C_T'$$

And $$\frac{2F_a'' V_2'^2 Mg}{\mathrm{Log}_{10}(1 - F_a'' V_2'^2)} \frac{S(\Delta T_{ST} - \Delta T_2)}{V_2'^2 W} = -C_T$$

$$F_a''' V_2'^2 = \frac{F_a'' V_2'^2 + C_{T_1}}{1 - 2AF_a'' M \frac{\Delta T_{ST}}{W} g} = \frac{F_a'' V_2'^2 + C_{T_1}}{1 - \frac{F_a'' V_2'^2}{\mathrm{Log}_{10}(1 - F_a'' V_2'^2)} \frac{S \Delta T_{ST}}{V_2'^2 W} 2Mg}$$

(24) Letting $$\frac{S \Delta T_{ST}}{V_2'^2 W} = C_{T_2}'$$

$$\frac{2F_a'' V_2'^2 Mg}{\mathrm{Log}_{10}(1 - F_a'' V_2'^2)} \frac{S \Delta T_{ST}}{V_2'^2 W} = -C_{T_2}$$

(25) $$F_a''' V_2'^2 = \frac{F_a'' V_2'^2 + C_{T_1}}{1 + C_{T_2}}$$

C_{T_1} and C_{T_2} can be obtained from the chart on page 128. Once these two factors are obtained, $F_a''' V_2'^2$ is calculated.

If $$P_T' = \frac{\log_{10}(1 - F_a'' V_2'^2)}{\log_{10}(1 - F_a''' V_2'^2)}$$

And $$P_T'' = \frac{-2Mg}{W}\left[\Delta T_{ST} - \Delta T_2\right]\frac{1}{\log_{10}(1 - F_a''' V_2'^2)}$$

(26) And $$P_T = P_T' + P_T''$$

P_T' is obtained from the values of $F_a'' V_2'^2$ and $F_a''' V_2'^2$ using page 129.

If $$K_T = \frac{(\Delta T_{ST} - \Delta T_2)}{W}$$

is calculated, the same page can be used to obtain P_T''.

Knowing P_T' and P_T'',

(27) $$S_3/S_2 = P_T' + P_T'' = P_T$$

And S_3 is the distance corrected for thrust.

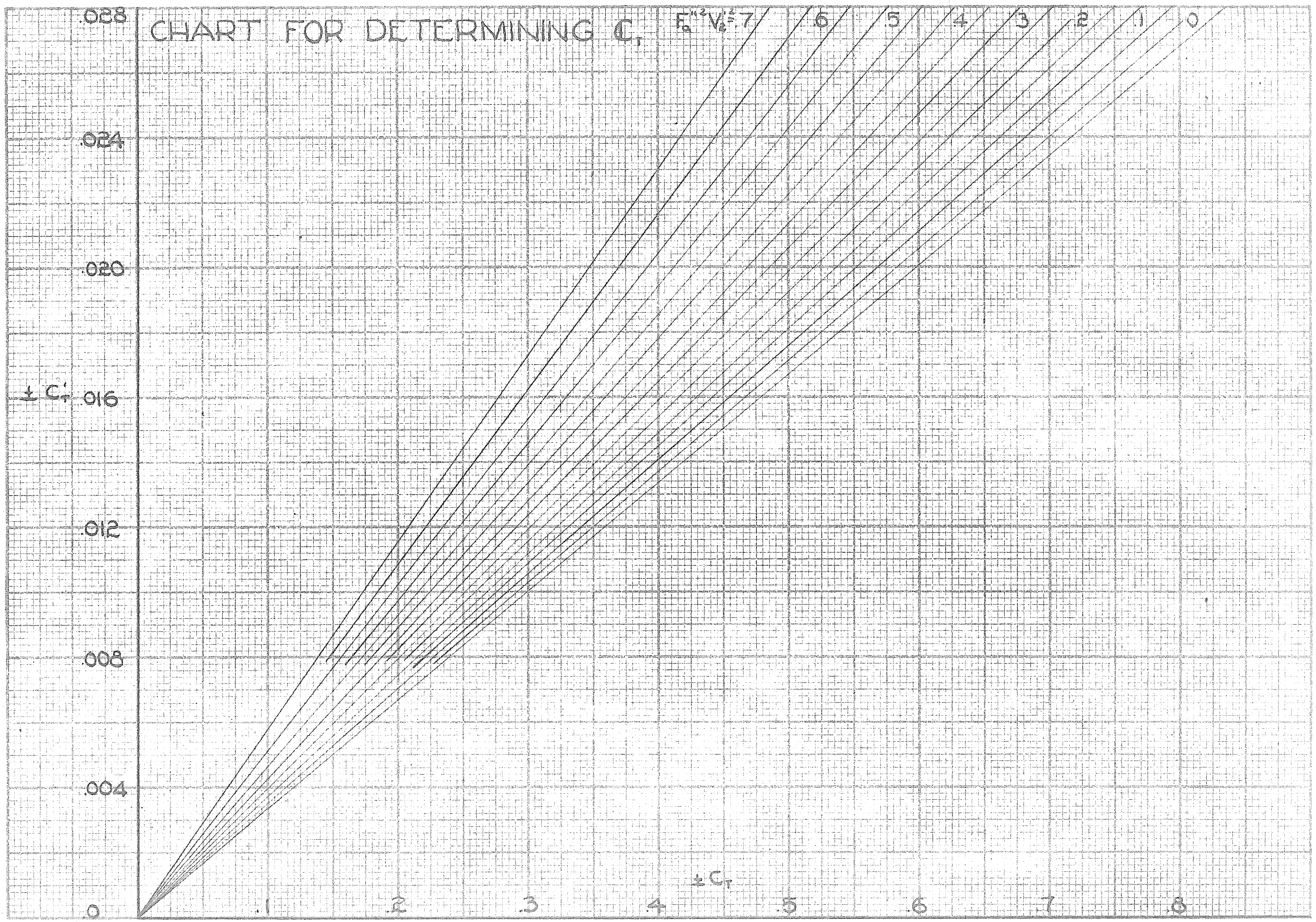
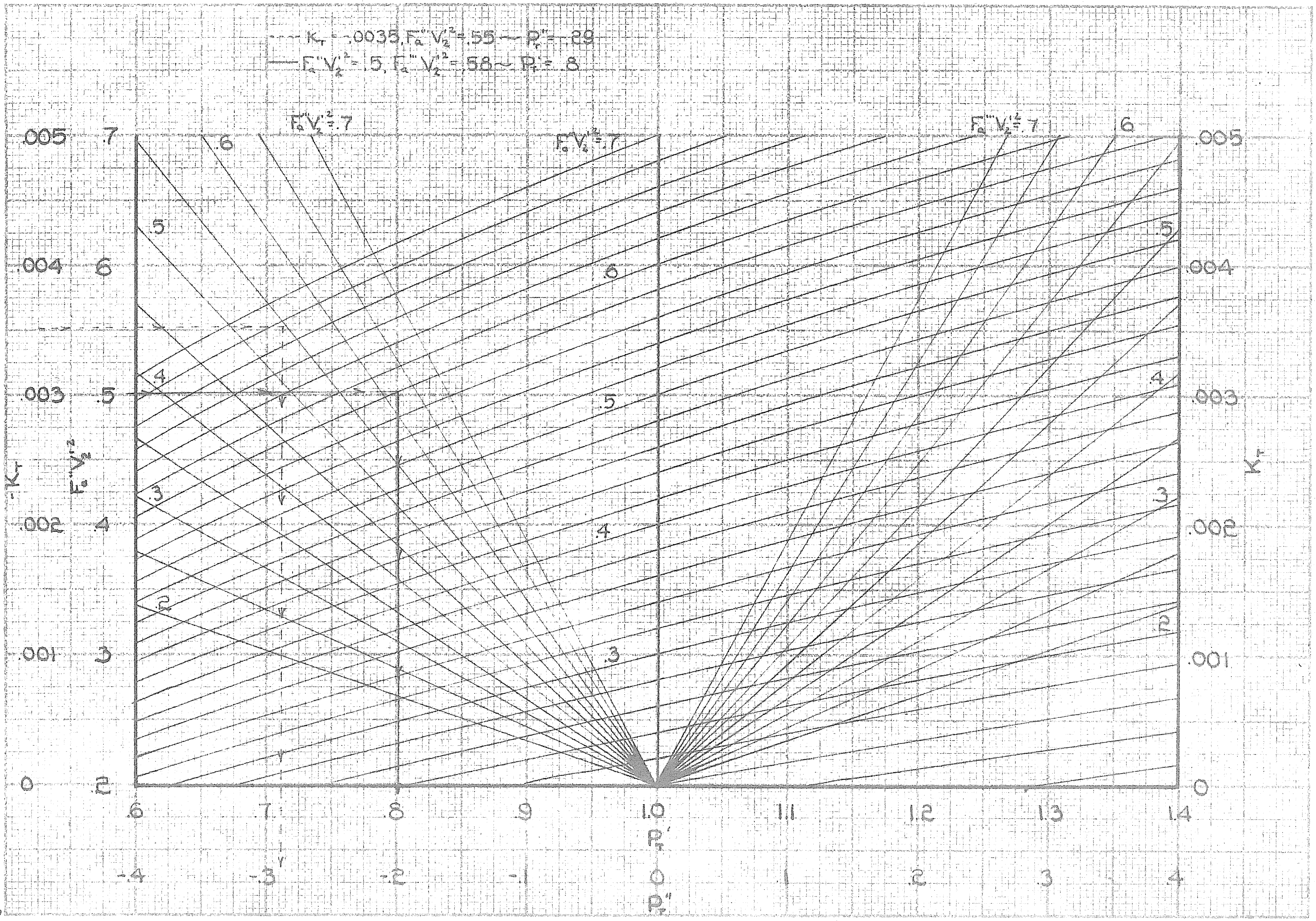


CHART FOR DETERMINING C_1
$F_0''^2 V_2^2 = 7$
6
5
4
3
2
1
0
.028
.024
.020
$\pm C_1'$ .016
.012
.008
.004
0
$\pm C_1$
1
2
3
4
5
6
7
8


$K_T = .0035$, $F_a''V_2'^2 = .55$ — $P_T'' = .29$
— $F_a''V_2'^2 = .5$, $F_a'''V_2'^2 = .58$ — $P_T' = .8$
$F_a''V_2'^2 = .7$
$F_a''V_2'^2 = .7$
$F_a'''V_2'^2 = .7$
K_T
$F_a''V_2'^2$
K_T
P_T'
P_T''
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
-.4
-.3
-.2
-.1
0
.1
.2
.3
.4
0
.001
.002
.003
.004
.005

Discussion

A reduction method of the type outlined should be (1) accurate, (2) simple, and (3) have a physically apparent continuity. An attempt will be made here to evaluate this method in light of these factors.

First, the method depends on actual test curves of velocity-distance and calculated curves $S = A\log(1 - F_a V^2)$ agreeing closely. This has been shown to be true for two representative airplanes. The weight correction depends upon a knowledge of μ, which will have to be estimated. However, μ is known for modern runways to lie between .02 and .05, and the difference in the correction obtained using each extreme is very small. And, it is assumed that if the thrust is altered in some manner from the test values, the final acceleration will be of the same form as the uncorrected variation. This is true for normal thrust corrections, if the original variation is accurate, which it has been shown to be. These are the primary assumptions made using this method.

The use of correction charts to reduce calculations has simplified this method to a point where it is comparable to other simple reduction procedures in labor required. It is so set up that if some correction is not desired, a series of correction steps is eliminated without destroying the continuity. The fact that the distance-velocity equation has the same form for any zero wind condition makes this possible. Were this not so, an approximation for some set of flight conditions would have to be assumed, and the corrections made in a manner so this type of relation would be obtained, each correction causing some necessary change in the initial relationship.

The term $F_a \; V_2^2$ has an easily understood physical meaning, and makes the effect of each correction readily understood. Two parameters S and $F_a \; V_2^2$ completely define a ground run, to a good approximation, for a given V_2 , and give a good insight, for example, into the effect of increasing the weight of two engine-propeller combinations of the same airplane. If S is the same for the original weight, the configuration with the least $F_a \; V_2^2$ would require the lesser ground run for the increased weight. Certain other of the parameters calculated have a not readily apparent physical significance to the casual user of this method, but this condition is found, as a rule to a greater degree, in similar methods. And, these parameters indicate clearly what is important in any correction.

In view of these facts, it is felt that the method described is superior to others in common use.

Sample Calculation

In order that this method can be more readily understood, a calculation form, as would be used in flight test reduction, is included here. The ground run on page 106 is corrected so as to give the corresponding distance for $W = 90{,}000$ lbs., $\sigma = 1$, $\gamma = 0$, ΔT_{ST}/engine = 100 lbs., ΔT_2/engine = 50 lbs., w = 20 mph..

(1)	$h_p = 495$ ft.		
(2)	F.A.T. = 58° F.		
(3)	$w = 0$		Observed data
(4)	$V_2 = 200$ fps = 136.5 mph		
(5)	$V_1 = 151$ fps		
(6)	$S_w = 2860$		
(7)	$W = 84680$ lbs.		
(8)	$\gamma = .63^\circ$		
(9)	$V_2/V_1 = 1.324$	(4)/(5)	
(10)	$w/V_2 = 0$	(3)/(4)	Correction to zero
(11)	$F_a V_2^2 = .470$	Page 113	wind
(12)	$P_w = 1.000$	Page 113	
(13)	$S_o = 2860$	Page 113	
(14)	$C_s = .0967$	[(13)(8)]/(4)	Correction to final
(15)	$P_s = .950$	Page 117	slope
(16)	$F_a' V_2^2$ .488	Page 117	
(17)	$S_1' = 3010$	(13)/(15)	
(18)	$\gamma' = 0$		
(19)	$C_s' = 0$	$(18)(17)/(4)^2$	
(20)	$P_s' = 1.000$	Page 117	

(21)	$F_a' V_2^2 =$ .488	Page 117	
(22)	$S_1 =$ 3010	(17)/(20)	
(23)	$\mathbf{W}_0 =$ 90,000 lbs.		
(24)	$\mathbf{W}_0/\mathbf{W}_1 =$ 1.062	(23)/(7)	
(25)	$1 - \mathbf{W}_0/\mathbf{W}_1 =$ −.062	−[(24) − 1]	
(26)	$\mu =$ .04		
(27)	$C_W' =$ −.00040	$[(26)(22)/(4)^2](25)$	Correction
(28)	$C_W =$ −.0090	Page 121	to final
(29)	$K_W =$ 1.071	(24)/[1+(28)]	weight
(30)	$(\mathbf{W}_1/\mathbf{W}_0)P_W =$ .907	Page 122	
(31)	$F_a'' V_2'^2 =$ .520	Page 122	
(32)	$S_W =$ 3520	(22)(24)/(30)	
(33)	$V_2' =$ 141 mph	(4)(24)	
(34)	$h_d =$ 600 ft.	From (1) and (2)	
(35)	$\sigma =$ .980	From (34)	Correction
(36)	$\sigma_0 =$ 1.000		to final
(37)	$\sigma_0/\sigma_1 =$ 1.020	(36)/(35)	density
(38)	$S_2 =$ 3450	(32)/(37)	
(39)	$\Delta T_{ST} =$ 400		
(40)	$\Delta T_2 =$ 200		
(41)	$C_{T_1}' =$ .000393	$[(39)-(40)](32)/(23)(33)^2$	
(42)	$C_{T_2}' =$ .000786	$(39)(32)/(23)(33)^2$	Correction
(43)	$C_{T_1} =$ .008	Page 128	to final
(44)	$C_{T_2} =$ .016	Page 128	thrust
(45)	$F_a''' V_2'^2 =$ .520	[(31)+(43)] / [1+(44)]	
(46)	$K_T =$ .00222	[(39)−(40)] / (23)	
(47)	$P_T' =$ 1.000	Page 129	
(48)	$P_T'' =$ .200	Page 129	

(49)	$P_T = 1.200$	(47)+(48)	
(50)	$S_3 = 2880$	(38)/(49)	
(51)	$V_2'' = 139.5$	$(33)/\sqrt{(37)}$	
(52)	$w = 20$		Correction to
(53)	$w/V_2'' = .1435$	(52)/(51)	final wind
(54)	$P_w' = .759$	Page 113	
(55)	$S = 2190$ ft.	(50)(55)	

2190 ft. represents the ground run distance desired.

PART II

INITIAL CLIMB AFTER TAKE-OFF

INITIAL CLIMB AFTER TAKE-OFF

Discussion

In order to complete the take-off reduction procedure, it is necessary to prescribe some means of analysis for the distance from the unstick point to the place where the airplane clears an obstacle of prescribed height above the runway, usually taken as 50 ft.. Unlike the ground run, the initial climb does not lend itself readily to rigorous reduction methods, the reasons being these: First, pilot technique, which is a small factor in the ground run as a rule, is a major factor once the airplane has left the ground; and second, the problem is two dimensional.

Reduction methods used for the take-off flare are almost altogether empirical or semi-empirical. It has been a common practice to approximate the distance by that measured in a climb away from the ground corrected with an empirical safety factor.

The method presented on the following pages is based on the assumption that the airplane lift coefficient will be the same at any height above the runway regardless of airplane weight, air density, etc.. It is assumed that the airplane flight path distance is approximately equal to the horizontal distance to the obstacle. Since the ratio of horizontal distance to obstacle height is of the order of 15:1, the error introduced can have a maximum magnitude of 1/16 or 6%, and is much less for any normal take-off.

The excess thrust during the test is represented by a mean thrust, and corrections are made assuming wind and runway slope have no effect on the mean thrust; and appropriate factors account for the change in thrust with power, air density, and weight. Also, mean values of airplane and wind velocity are used in making the wind correction.*

It is felt that greater "rigor" would be unwarranted, as the additional complexity of analysis necessary to obtain it would be very great. In fact, rigor is hard to define in a matter such as this, as the action of a pilot under a different set of circumstances is difficult to predict. In operational data where a safety factor is included in all results, it is felt that the reduction methods presented are sufficiently accurate.

As further justification for the semi-rigorous approach used, the flight distance to cover a fifty foot obstacle is about one-fourth or one-third of the total take-off distance. An error in the flare distance of 10% would introduce about a 3% error in the total take-off distance.

* The airplane velocity along the flight path and the horizontal component are used inter-changeably in the following discussion; this is possible because the cosine of the flight path angle is very close to unity. Most accurate methods of measuring take-off data give the horizontal component directly.

The method assumes, for correction purposes, the gear has not started to retract at the 50 ft. point. This will always be a conservative assumption.

Derivation

(1) Correction For Runway Slope

During the initial climb after take-off -

$$\int_0^S F\,dS = W\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + h\right\}$$

where

F = the airplane excess thrust - lbs.

H = a predetermined height above the runway - ft.

h = the height above the take-off point where the airplane is a distance H above the runway - ft.

S = the distance along the flight path corresponding to the height h - ft.

W = the airplane weight - lbs.

V_2 = take-off speed (with respect to the ground) - mph.

V_H = airplane speed at height h (with respect to the ground) - mph.

If the take-off is made on a runway sloping downward at an angle γ,

$$(1)\quad \int_0^{S_\gamma} F\,dS = W\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + H\cos\gamma - X_\gamma \sin\gamma\cos\gamma\right\}$$

where X_γ is the horizontal distance corresponding to S_γ.

If the runway is level,

$$(2)\quad \int_0^{S_L} F\,dS = W\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + H\right\}$$

The approximation is made that

$$(3)\quad \int_0^{S_Y} F\,dS = \bar{F}X_Y \qquad \int_0^{S_L} F\,dS = \bar{F}X_L$$

Dividing (1) by (2), and, since for all normal airports Y is very small, -

$$\frac{X_Y}{X_0} = 1 - \frac{X_Y(Y/57.3)}{\frac{2.151}{2g}(V_H^2 - V_2^2) + H}$$

Taking $g = 32.17$

$$\frac{X_Y}{X_0} = 1 - \frac{X_Y Y}{1.916(V_H^2 - V_2^2) + 57.3H}$$

For $H = 50$ ft.

$$(4)\quad \frac{X_Y}{X_0} = 1 - \frac{X_Y Y}{3.831(\Delta V)V_{AV} + 2865}$$

$$\Delta V = V_H - V_2$$

$$V_{AV} = \frac{1}{2}(V_H + V_2)$$

and,

$$(4a)\quad \frac{X_0}{X_Y} = 1 + \frac{X_0 Y}{3.831(\Delta V)V_{AV} + 2865}$$

For usual take-off procedures ΔV is positive and of the order of 10 mph for zero wind and somewhat less as the headwind increases. Headwinds of the magnitude required to make $\Delta V \leq 0$ are larger than those usually existing during test take-offs. If extremely high headwinds are encountered, the flight path distance and the horizontal distance cannot be equated, the method breaks down.

(2) Correction For Wind

At any height h above the runway, the airspeed, and consequently the excess thrust, is independent of wind conditions. This means that the rate at which the sum of airplane potential and kinetic energy increases is constant, or

$$(5)\quad 2.151\frac{W}{g}V\frac{dV}{dt}+W\left(\frac{dh}{dt}\right)_o=2.151\frac{W}{g}V_A\frac{d(V_A-W)}{dt}+W\left(\frac{dh}{dt}\right)_w$$

where

V_A	is the airplane velocity with respect to the air - mph
w	is the headwind velocity at height h - mph
$d(\;)/dt$	is a time rate of change, t in seconds

The subscripts o and w refer to zero wind and wind condition respectively.

$$\left\{2.151\frac{V}{g}\frac{dV}{dh}+1\right\}\left(\frac{dh}{dt}\right)_o=\left\{2.151\frac{V_A}{g}\left[\frac{dV_A}{dh}-\frac{dw}{dh}\right]+1\right\}\left(\frac{dh}{dt}\right)_w$$

$$\left(\frac{dt}{dh}\right)_w=\left(\frac{dt}{dh}\right)_o\left\{1-\frac{(dw/dh)(2.151)}{2.151(dV/dh)-g/V}\right\}$$

Since $V_A = V$.

$$1.467\int_0^H\left(\frac{dt}{dh}\right)_w(V-W)\,dh=S_w\doteq X_w$$

$$1.467\int_0^H\left(\frac{dt}{dh}\right)_o V\,dh\doteq X_o$$

$$(6) \quad X_w = X_o - 1.467\left\{\int_0^H w_o\left(\frac{dt}{dh}\right)_o dh + \int_0^H \Delta w\left(\frac{dt}{dh}\right)_o dh + \int_0^H \frac{(V-w)(dt/dh)_o(dw/dh)}{dV/dh + g/2.151V}dh\right\}$$

Where w_o is the headwind at $h = 0$, and $\Delta w = w - w_o$ at height h.

The three terms on the left have readily apparent physical significance. The first is the decrease in distance were there no gradient. The second is the further decrease in distance because of the airplane airspeed being less above the ground due to the wind gradient. The third term appears because the increase in airplane kinetic energy required to maintain a given airspeed is less than with no wind gradient, and more energy is available to lift the airplane, increasing the rate of climb.

Reference 7 indicates that the wind velocity w at a height h' from the ground is approximately related to a windspeed w_λ measured at a height λ above the ground by the relationship

$$w/w_\lambda = (h'/\lambda)^{1/7}$$

which is valid in the range covered during the climb to 50 ft.. A height Δ is defined as the distance from the ground to a point on the airplane resting on the ground; this point is defined so that the drag of the airplane would be the same with a wind gradient as it would with no gradient were the wind velocity at the point Δ the same. Obviously such a dimension will vary with wind gradient, but for purposes of analysis it will be assumed to be constant. With gradients as described above, the variation will be small.

For modern low wing transports in take-off configuration, Δ can be taken as the distance to the wing fuselage junction.

$$h' = h + \Delta \qquad W/W_\lambda = [(h+\Delta)/\lambda]^{1/7} \qquad W_0 = W_0$$

Rewriting equation (6) in terms of mean values -

$$(7) \qquad X_w = X_0 - 1.467 T_0 \left[\frac{1}{2} \left\{ \left(\frac{\Delta}{\lambda}\right)^{1/7} + \left(\frac{50+\Delta}{\lambda}\right)^{1/7} \right\} W_\lambda \right.$$

$$\left. + \frac{1}{2} \left\{ \frac{[(V-W)_{50} + (V-W)_0](W_{50} - W_0)}{(V_{50} - V_0) + 1496/(V_{50} + V_0)} \right\} \right]$$

where T_0 is the time required to reach the obstacle with no wind. Since the time T_w, corresponding to T_0 with wind, is measured in most take-off tests, this value will be used to determine T_0.

For any normal take-off flare the average ground speed lies between the take-off speed and the speed at the obstacle.

$$\frac{X_w}{T_w} = A[(V-W)_{50} + (V-W)_0]^{*} \qquad A \doteq \frac{1}{2}$$

$$\frac{X_0}{T_0} = A'[V_{50} - V_0]$$

It will be assumed $A = A'$.

$$(8) \quad \frac{T_0}{X_0} = \frac{T_w}{X_w} \left[\frac{V_{50} + V_0}{(V-W)_{50} + (V-W)_0} \right]$$

Finally,

$$(9) \quad \frac{X_w}{X_0} = 1 - .733 \left(\frac{T_0}{X_0}\right) \left[\left\{ \left(\frac{\Delta}{\lambda}\right)^{1/7} + \left(\frac{50+\Delta}{\lambda}\right)^{1/7} \right\} W_\lambda + \frac{[(V-W)_{50} - (V-W)_0](W_{50} - W_0)}{(V_{50} - V_0) + 1496/(V_{50} + V_0)} \right]$$

* X_w and T_w are not corrected for runway slope.

(3) Correction For Density

The density correction made here accounts only for the fact that for a lower value of the density ratio, σ, the airplane takes off at a higher speed, and for the same C_L, reaches a higher speed at the 50 ft. point. Any variation in thrust due to density (and/or temperature) at the same indicated airspeed is considered under power.

If the test is made at σ_0, and a correction to σ_s is desired, using the relationships $V_{2_0}\sigma_0 = V_{2_S}\sigma_S$ and $V_{H_0}\sigma_0 = V_{2_S}\sigma_S$,

$$\frac{X_{\sigma_S}}{X_{\sigma_0}} = \frac{\frac{2.151}{2g}(V_H^2 - V_2^2)\frac{\sigma_0}{\sigma_S} + H}{\frac{2.151}{2g}(V_H^2 - V_2^2) + H}$$

for $H = 50$

$$(10) \quad \frac{X_{\sigma_S}}{X_{\sigma_0}} = 1 - \frac{(1 - \sigma_0/\sigma_S)}{1 + 748/V_{AV}\Delta V}$$

V_{AV} and ΔV being zero wind values.

(4) Correction For Weight

If, as assumed, the C_L at various points during the initial climb does not vary with weight, a change in weight produces two principle effects. First, the airplane increase in kinetic and potential energy is changed, and second, the drag and thrust available are different. The first effect is considered here, the second is considered under power.

Letting W_T and W_0 be the test and corrected weights respectively

$$\frac{X_T}{X_0} = \frac{W_T\{(2.151/2g)(V_H^2 - V_2^2) + H\}}{W_0\{(2.151/2g)(V_H^2 - V_2^2)(W_0/W_T) + H\}}$$

where V_H and V_2 are measured at W_T

$$\frac{X_T}{X_0} = \frac{W_T}{W_0}\left[1 - \frac{[(W_T - W_0)/W_T](2.151/2g)(V_H^2 - V_2^2)}{(2.151/2g)(V_H^2 - V_2^2) + H}\right]^{-1}$$

For $H = 50$ ft.

(11) $$\frac{X_T}{X_0} = \frac{W_T}{W_0}\left[1 - \frac{1 - W_0/W_T}{1 + 748/V_{AV}\Delta V}\right]^{-1}$$

(11a) $$\frac{X_T}{X_0} = \frac{W_T}{W_0}\left[1 + \frac{1 - W_0/W_T}{1 + 748/V_{AV}\Delta V}\right]$$

when $$\frac{1 - W_0/W_T}{1 + 748/V_{AV}\Delta V} \ll 1$$

(5) Correction For Power

The correction noted here accounts for the following factors:

(1) In general the corrected take-off is at a different airspeed, density and temperature than the test take-off. Also, if engine performance is faulty during take-off, the final results must be given in terms of what the airplane would do in normal operation.

(2) Any changes in weight cause a change in drag if the C_L is held constant.

In order to account for these factors, mean values will again be considered.

$$\overline{F_u} = \overline{T_u} - \overline{\tfrac{1}{2} C_D \rho V_u'^2 S}$$

$$\overline{F_o} = \overline{T_o} - \overline{\tfrac{1}{2} C_D \rho V_o'^2 S}$$

Where the subscripts u and o define values uncorrected and corrected for power respectively.

S = wing area - ft^2.

C_D = drag coefficient

ρ = air density - $slugs/ft^3$.

V = air speed - fps.

T = total airplane thrust - lbs. This can be determined using power charts and propeller curves with sufficient accuracy for this correction.

At any height, h, -

$$V_o'^2 = (W_o/W_u) V_u'^2$$

and

$$\overline{\tfrac{1}{2} C_D \rho V_o'^2 S} \doteq \overline{\tfrac{1}{2} C_D \rho V_u'^2 S}\,(W_o/W_u)$$

$$(12)\qquad \overline{F}_u X_u = \mathbb{W}_0\left[(2.151/2g)(V_H^2 - V_2^2) + H\right] = \overline{F}_0 X_0$$

$$= \left(\overline{T}_u - \overline{\tfrac{1}{2} C_D \rho V_u^2 S}\right) X_u$$

$$\overline{\tfrac{1}{2} C_D \rho V_u'^2 S} = \overline{T}_u - \frac{\mathbb{W}_0}{X_u}\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + H\right\}$$

$$\left[\overline{T}_0 - \left(\overline{T}_u - \frac{\mathbb{W}_0}{X_u}\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + H\right\}\right)\frac{\mathbb{W}_0}{\mathbb{W}_u}\right] X_0$$

$$= \mathbb{W}_0\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + H\right\}$$

$$\left(\overline{T}_0 - \frac{\mathbb{W}_0}{\mathbb{W}_u}\overline{T}_u\right) X_0 = \mathbb{W}_0\left\{\frac{2.151}{2g}(V_H^2 - V_2^2) + H\right\}\left[1 - \frac{X_0}{X_u}\frac{\mathbb{W}_0}{\mathbb{W}_u}\right]$$

$$\frac{\overline{T}_0 - (\mathbb{W}_0/\mathbb{W}_u)\overline{T}_u}{\mathbb{W}_0\left[(2.151/2g)(V_H^2 - V_2^2) + H\right]} = \left[\frac{1}{X_0} - \frac{1}{X_u}\frac{\mathbb{W}_0}{\mathbb{W}_u}\right]$$

$$\frac{X_u}{X_0} = \frac{\mathbb{W}_0}{\mathbb{W}_u} + \frac{X_u\left(\overline{T}_0 + [\mathbb{W}_0/\mathbb{W}_u]\overline{T}_u\right)}{\mathbb{W}_0\left[(2.151/2g)(V_H^2 - V_2^2) + H\right]}$$

For $H = 50$ ft.

$$(13)\qquad \frac{X_u}{X_0} = \frac{\mathbb{W}_0}{\mathbb{W}_u} + X_u\,\frac{\overline{T}_0/\mathbb{W}_0 - \overline{T}_u/\mathbb{W}_u}{(.0167\,V_{AV}\,\Delta V + 50)}$$

Where X_u is corrected for everything but power, and $\overline{T}$ can be taken as the average of T at V_2 and at V_H .

Sample Calculation

As an example of the use of this method, the following case will be considered. *

(1)	$X = 800$ ft.	Observed	
(2)	$V_2 = 120$ mph *	Observed	$(V - w)_2$ } ground
(3)	$V_H = 125$ mph	Observed	$(V - w)_H$ } speeds
(4)	$\gamma = 1^\circ$	Observed	
(5)	$w_\lambda = 10$ mph	Observed	
(6)	$\lambda = 25$ ft.	Observed	
(7)	$\Delta = 8$ ft.	Observed	
(8)	$\sigma = .95$	Observed	
(9)	$T_w = 4.45$ sec.	Observed	
(10)	$\overline{T_J / W_J} = .21$	Calculated	
(11)	$\Delta V = 5$ mph	(3) - (2)	
(12)	$V_{AV} = 122.5$ mph	$\frac{1}{2}$ [(3) + (2)]	
(13)	$3.831(\Delta V)V_{AV} = 2345$	3.831(11)(12)	
(14)	$X_\gamma / X_o = 1.154$	(1)(4)/[(13) + 2865]	
(15)	$w_{50} = 11.3$ mph	(5) $[50 \quad (7)/(6)]^{1/7}$	
(16)	$w_o = 8.5$ mph	(5) $[(7)/(6)]^{1/7}$	
(17)	$V_{50} = 136.3$ mph	(15) + (3)	
(18)	$V_o = 128.5$ mph	(16) + (2)	
(19)	$T_w / X_w = .00556$	(9)/(1)	
(20)	$V_{50} + V_o = 264.8$	(17) + (18)	
(21)	$(V - w)_{50} + (V - w)_o = 245$	(2) + (3)	
(22)	$T_o / X_o = .00601$	(19)(20)/(21)	
(23)	$[(\Delta/\lambda)^{1/7} + (\{50 + \Delta\}/\lambda)^{1/7}] w_\lambda = 19.8$	(15) + (16)	

* Data will be corrected to zero runway slope, zero wind, $\sigma = 1.00$, and T/W and W_T/W_o as indicated.

(24)	$V_{50} - V_0 = 7.8$	(17) - (18)
(25)	$w_{50} - w_0 = 2.8$	(15) - (16)
(26)	$\dfrac{[(V-w)_{50} + (V-w)_0](w_{50}-w_0)}{(V_{50}-V_0) + 1496/(V_{50}+V_0)} = 51.0$	$\dfrac{(21)(25)}{(24) + 1496/(20)}$
(27)	$1 - X_w/X_0 = .312$	$.733(22)[(23) + (26)]$
(28)	$X_w/X_0 = .688$	1 - (27)
(29)	$\sigma_0 = 1.00$	given
(30)	$1 - \sigma_0/\sigma_s = -.052$	1 - (29)/(8)
(31)	$1 - (X_{\sigma_S}/X_{\sigma_0}) = -.0256$	$(30)/\{1+748/[(17)-(18)]\frac{1}{2}[(17)+(18)]\}$
(32)	$X_{\sigma_S}/X_{\sigma_0} = 1.026$	1 - (31)
(33)	$\mathbf{W}_T/\mathbf{W}_0 = .95$	given
(34)	$1 - \mathbf{W}_0/\mathbf{W}_T = -.052$	1 - 1/(33)
(35)	$(\mathbf{W}_0/\mathbf{W}_T)(X_T/X_0) - 1 = -.0256$	(30)(34)/(31)
(36)	$X_T/X_0 = .905$	$[1 + (35)](33)$
(37)	$X_U = 1083$	(1)/(14)(28)(32)(36)
(38)	$T_0/\mathbf{W}_0 = .200$	Calculated from observed
(39)	$T_U/\mathbf{W}_U = .210$	and corrected data
(40)	$\dfrac{X_U(T_0/\mathbf{W}_0 - T_U/\mathbf{W}_U)}{.0167 V_{AV}\Delta V + 50} = -.133$	$\dfrac{(37)[(38) - (39)]}{.0167[(17)-(18)]\frac{1}{2}[(17)+(18)] + 50}$
(41)	$X_U/X_0 = .919$	(40) + 1/(33)
(42)	$X_0 = 1180$ ft.	(37)/(41)

It is noted that some duplication in nomenclature is present in the derivation, -

$$V_2 \equiv V_0$$
$$\mathbf{W}_U \equiv \mathbf{W}_T$$

This was done for additional clarification in the separate sections of the derivation, and will cause no confusion if the above calculation form is used.

PART III

CLIMB ANALYSIS

CLIMB ANALYSIS

Discussion

The method of reducing climb data to standard conditions presented here differs from commonly used reduction procedures (See references 3 and 4.) in that it does not require a graphical differentiation of the observed pressure altitude-time curve. Such a difference is important for several reasons: First, a smoothly faired pressure altitude-time curve is considered necessary to obtain satisfactory results graphically, but such a fairing does not give proper weight to short time variations such as the power fluctuations often encountered. The method presented here is set up to account for those factors. Second, obtaining data by graphical methods is a job handled satisfactorily only by trained personnel; this method can be handled by anyone capable of making standard calculations. Furthermore, it is often desirable to make in-flight reductions of flight test data, which would not be feasible if extensive graphical work were required, but which can be handled using the numerical analysis outlined here.

The analysis made here is possible only because the pressure altitude-time variation occuring during any satisfactory climb test will have a fairly smooth and continuous derivative. It is therefore possible to represent segments of the variation by simple mathematical expressions, and base reduction methods upon these.

The following method will, in general, require more test data than other methods. Additional accuracy in the reduction of test climbs in the neighborhood of rapid changes or variations of rate of change of such factors as velocity or power naturally requires more data in that region. In spite of this fact, this method will require approximately the same amount of total time as the graphical method, since satisfactory fairing and graphical differentiation require a considerable amount of time.

An attempt is made to indicate the importance of various corrections so that the reduction procedure used will conform to the accuracy desired.

The final presentation of the climb data consists of a curve of rate of climb vs. Standard Altitude or some specified non-Standard pressure altitude. This curve will permit fairing out wild points due to instrument lag, unusual atmospheric conditions, and other factors not accounted for with ordinary flight test methods. Since the slope of this final curve is usually unimportant, the fairing does not require excessive care.

It is assumed that the reader is familiar with ordinary flight test procedures and the presentation does not discuss elementary problems of taking and reducing data.

This method does not consider the effect of wind gradients on climb. It is felt that any satisfactorily conducted climb will be made cross wind due to the inadequate winds-aloft data usually available.

Outline Of Method

The unreduced climb data consist of pressure altitude readings vs. time, and corresponding values of engine RPM, manifold pressure, outside air temperature, etc.. These values, corrected for instrument errors and, when necessary, pitot errors, are used in the following manner.

First, density altitudes are calculated at each test point and the desired I.A.S. at that altitude is taken from the previously determined flight plan. These values are used to correct the pressure altitude readings for velocity variations according to the following equation -

$$(1) \qquad \Delta h_p = -\frac{1}{.469}\left(\frac{V_i}{\sigma g}\right)\Delta V_i^{*} = -\frac{1}{15.1}\left(\frac{V_i}{\sigma}\right)\Delta V_i$$

Δh_p = difference in pressure altitude reading were $\Delta V_i = 0$ - Ft.

V_i = flight plan I.A.S. corresponding to the density altitude at the test point - mph.

$\Delta V_i = V_i - V_{i_{OBS}}$ - mph.

The above relationship assumes that any difference between observed and flight plan airplane kinetic energy is manifested in an equal potential energy difference. It assumes that $\Delta h_p = \Delta h = \Delta h_d$; since corrections are usually small, the error in these corrections is less than the minimum readable altimeter differences, - five feet or so. Also, the assumption is made that the planned true airspeed at the observed altitude is not appreciably different from that at the corrected altitude, which, since the corrections are small, is justified.

*(See reference 5.)

At $h_d = 10{,}000$ ft., $\sigma = .738$; for $V_i = 180$ mph, $\Delta V_i = 5$ mph, and

$$\Delta h_p = -\frac{1}{15.1}\left(\frac{180}{.738}\right)(5) = -80 \text{ FT.}$$

which will indicate the order of magnitude to be expected.

The approximate relationship between pressure altitude and time used to determine dh_p/dt was chosen because it was accurate, did not require an extremely large amount of test data to obtain this accuracy, and did not require complicated mathematics for its use.

The relationship

$$h_p = h_o + At$$

where h_p the pressure altitude at time t - Ft.

h_o the pressure altitude at time $t = 0$ - Ft.

t elapsed time - min.

would result in simple reduction methods, but would give satisfactory indicated rates of climb between points h_p and $h_p + \Delta h_p$ only if Δh_p were very small.

The relationship

$$\frac{dh_p}{dt} = B - Ch_p$$

would require, in general, less test points, since a linear or near linear rate of climb variation with altitude exists for most airplanes. However, the mathematics involved in the use of such a relationship are cumbersome and do not provide a good reduction procedure.

The variation

$$h_p = h_0 + at + bt^2$$

was chosen for the approximate relationship to be used because it provided accuracy of results with a minimum of calculation.

The constants in this relationship are determined by considering three test points at successive pressure altitudes, h_0 , h_1 and h_2 where $t=0$, t_1 and $t_1 + t_2$. If the approximate relationship is exact at these points.

$$a = \frac{(h_1 - h_0)(t_1 + t_2)^2 - t_1^2(h_2 - h_0)}{t_1 t_2 (t_1 + t_2)}$$

$$b = -\frac{(h_1 - h_0)(t_1 + t_2) - t_1(h_2 - h_0)}{t_1 t_2 (t_1 + t_2)}$$

At $h_p = h_1$,

$$\left(\frac{dh_p}{dt}\right)_1 = a + 2bt_1 = \frac{(h_1 - h_0)(t_2^2 - t_1^2) + (h_2 - h_0)t_1^2}{t_1 t_2 (t_1 + t_2)}$$

If $\Delta h_1 = h_1 - h_0$

$\Delta h_2 = h_2 - h_1$

$$(2) \quad \left(\frac{dh_p}{dt}\right)_1 = \frac{\Delta h_1 (t_2^2 - t_1^2) + (\Delta h_1 + \Delta h_2) t_1^2}{t_1 t_2 (t_1 + t_2)}$$

$$= \frac{\Delta h_1 (t_2/t_1) + \Delta h_2/(t_2/t_1)}{(t_1 + t_2)}$$

If the test program is set up so that the time intervals at which data are read are held constant except in critical regions where they can be halved, the above equation simplifies, and a large reduction in calculation time is realized. If data are taken from airplane instruments, an approximate knowledge of the airplane rates of climb will permit determination of a suitable constant time interval. If a photographic "automatic observer" is used to record data, suitable time intervals can be determined upon completion of the test.

It will be necessary to plan the test so that knowledge of the apparent rate of climb at the first test point is not required, as this method does not provide for obtaining climbs at the end points.

Once the rate of change of pressure altitude is known, the true rate of climb can be obtained from the relationship

$$(3) \qquad R/C = \frac{dh_p}{dt}\frac{T}{T_o}$$

where R/C is the true rate of climb at h_p - ft./min.

T is the absolute free air temperature at h_p - °F.

T_o is the absolute free air temperature at h_p were the atmosphere Standard - °F.

Derivation of the above relationship is found in reference 4 and many others and will not be included here.

The following equation is the equilibrium equation for an airplane, and will be used in the subsequent discussion.

$$(4) \qquad \eta BHP_A = \frac{C_D q S V}{375} + \frac{W R/C}{33000} + \frac{1}{375}\frac{W}{g}V\frac{dV'}{dt}$$

where, for the airplane at the point considered,

$\eta =$ propeller efficiency

$BHP_A =$ total brake horsepower available

$C_D =$ drag coefficient

$q =$ dynamic pressure - lb/ft.^{2}

$S =$ wing area - ft.^{2}

$V =$ airspeed - mph

$W =$ weight - lbs.

$dV'/dt =$ acceleration with respect to air - ft/sec.

Letting the subscript "T" refer to test conditions, and "O" refer to the conditions to which the test is to be corrected,

$$(4a) \qquad \eta_0 BHP_{A_0} - \frac{W_0 R/C_0}{33000} - \frac{1}{375}\frac{W_0}{g}V\frac{dV'}{dt} - \frac{C_{Di_0} q SV}{375} = \frac{C_{DP} q SV}{375}$$

$$= \eta_T BHP_{A_T} - \frac{W_T R/C_T}{33000} - \frac{1}{375}\frac{W_T}{g}V\frac{dV'}{dt_T} - \frac{C_{Di_T} q SV}{375}$$

$$(5) \qquad R/C_0 - R/C_T = \left\{\frac{W_T}{W_0} - 1\right\} R/C_T + \frac{33000}{W_0}\left\{\eta_0 BHP_{A_0} - \eta_T BHP_{A_T}\right\}$$

$$+ 2.74V\left\{\frac{W_T}{W_0}\frac{dV'}{dt_T} - \frac{dV'}{dt_0}\right\} + \frac{88qSV}{W_0}\left\{C_{Di_T} - C_{Di_0}\right\}$$

LETTING $C_{Di} = \frac{C_L^2}{\pi A\!\!R e} = \frac{W^2}{q^2 S^2}\frac{1}{\pi A\!\!R e}$

$$(6) \qquad 88qSV\left\{C_{Di_T} - C_{Di_0}\right\} = \frac{88V}{qS}\frac{1}{\pi A\!\!R e}\left\{W_T^2 - W_0^2\right\}$$

$$\frac{88qSV}{W_0}\left\{C_{Di_T} - C_{Di_0}\right\} = \frac{34400}{V_i \sigma^{1/2} \pi A\!\!R e}\left\{\frac{W_T}{W_0} - 1\right\}\left\{\frac{W_T + W_0}{S}\right\}$$

The above form is acceptable for flight test calculations even though it contains the airplane efficiency factor e which cannot be determined satisfactorily from a test of this type. For airplanes with a laminar flow wing $e \doteq 1.0$; for airplanes with other type airfoils $e \doteq .85$; these values will be acceptable if more accurate knowledge of e is not available.

One of the reasons that an approximate value of e can be used is that the above correction term is small -

As an illustration,

$$V_i = 180$$

$$\sigma^{-1/2} = .8$$

$$W_T/W_0 - 1 = .02$$

$$(W_T + W_0)/S = 100$$

$$AR = 10$$

$$e = .8$$

$$\frac{34400}{V_i \sigma^{1/2} \pi AR e}\left\{\frac{W_T}{W_0} - 1\right\}\left\{\frac{W_T + W_0}{S}\right\}$$

$$= \frac{34400}{(180)(.8)} \frac{1}{\pi(10)(.8)}(.02)(100) = 19 \text{ FT/MIN.}$$

or, if e were actually 1.0 rather than 0.8, the error in the correction would be about 4 ft/min..

Another reason that the accuracy with which e is determined is not too important is that for most test work W_T/W_0 will be very close to one. If the power during a climb varies within a few percent, as a rule the resulting fuel required to reach a given height does not change appreciably. An increase in power will increase the ratio of excess thrust to thrust required, but will increase the s.f.c., and these two factors tend to compensate for each other.

Also, as a first approximation, the term

$$(6a) \qquad 2.74V\left\{\frac{W_T}{W_o}\frac{dV'}{dt_T} - \frac{dV'}{dt_o}\right\} \doteq 4.02\left\{\frac{dV}{dt_T} - \frac{dV}{dt_o}\right\} \doteq 0$$

for a climbing speed of 180 mph indicated,

$dV/dt_o =$ .100 ft/sec.^{2} at 10,000 ft.

at a rate of climb of 1250 ft/min..

If atmospheric conditions are such that the test height climbed between two density altitudes is .95 the actual height,

$$dV/dt_T = .105 \text{ FT./SEC.}^2$$

and,

$$2.74V\left\{\frac{dV'}{dt_T} - \frac{dV'}{dt_o}\right\} = 4.02\,\frac{180}{.859}(.005)$$

$$= 3 \text{ ft/min.}$$

So, for a good first approximation,

$$(7) \quad R/C_o - R/C_T = \frac{33000}{W_o}\left\{\eta_o BHP_{A_o} - \eta_T BHP_{A_T}\right\}$$

and weight corrections can be neglected.

Once the approximate rates of climb, neglecting the effect of weight and acceleration variations, are determined, a more accurate determination of the climb picture is immediately possible. Weight effects are important if the density altitudes at take-off for test and corrected conditions are appreciably different, and if much variation between actual climb height and the change in density altitude exists.

If times to climb are determined using the correction of equation (7), fuel consumptions for climb power are taken either from flight test or engine manufacturers data, and a good approximation of the fuel consumed and the height attained in reaching a stabilized climb is available, the following calculation procedure is recommended which gives readily a first and second approximation of the correct rates of climb.

1 $h_{p_{obs}}$	Observed test data corrected for instrument and pitot error
2 F.A.T.	Observed test data corrected for instrument error and adiabatic rise
3 V_i	Observed test data corrected for instrument and pitot error
4 t	Observed data
5 h_d	From 1 and 2
6 σ	From 5
7 Δh	From 3, 5, and the flight plan using equation (1)
8 $h_{p_{cor}}$	1 + 7
9 $h_{D_{cor}}$	5 + 7
10 Δt	From 4 – the time difference between two successive altitude readings
11 Δh_p	From 8 – the altitude difference between two successive readings
12 t_2/t_1	From 10 using the form of equation (2)
13 t_1/t_2	1/ 12

14 t_1+t_2	From 10 using the form of equation (2)
15 $\left(\frac{dh_p}{dt}\right)_1$	From 11, 12, 13, and 14 using equation (2)
16 T_s	Standard F.A.T. at 9
17 R/C	From 2, 15 and 16 using equation(3)
18 BHP_T	Observed data
19 η_T	Calculated using test conditions
20 $\eta_o BHP_o$	Standard values from corrected test data or power charts
21 W_T	Observed data from displacement tank readings, flowmeter, etc., knowing take-off weight
22 R/C_o	First approximation for corrected rate of climb, using 17, 18, 19, 20, 21, and equation (7)

The reduction outlined on the previous pages can be used to obtain a final result, or as a first step in obtaining a more accurate result, depending upon the accuracy desired. For most test conditions the first result will be sufficient.

For additional accuracy, the fuel used to reach a stabilized climb must be calculated. The point of stability is generally a few hundred feet above the take-off height, and the fuel used includes taxi, warm-up, take-off, and initial climb. Above this point a stabilized climb is assumed to exist, and using the rate of climb data calculated on the previous pages, extrapolated, if necessary, to reach the density altitude for initial stability, and the fuel flow characteristics of engine, a second approximation of weight vs. altitude will be obtained. A similar consideration will result in a relationship between altitude and time to climb used in getting Δt.

The correction of (6a) is approximated by assuming that test and Standard day accelerations at the same density altitude are equal to the mean accelerations, based on the test points immediately above and below the point considered. Or, if the three test points considered are h_0, h_1, h_2; the corresponding true velocities are V_0, V_1, V_2; and the test time between h_0 and h_2 is T_T, the approximate climb time is T_0, at point (1) $dV/dt_T = (V_2 - V_0)/T_T$, and $dV/dt_0 = (V_2 - V_0)/T_0$. As long as the distance climbed between readings is not too great, (say, less than 1000 ft.) this is a satisfactory approximation.

23 W_0	Calculated using 22 and fuel flow data
24 $\frac{W_T}{W_0}$	21 / 23
25 $\frac{W_T}{W_0}-1$	24 - 1
26 $\Delta R/C_1$	25 17
27 $\Delta R/C_2$	24 22
28 σ_0	From 9 (usually sufficiently accurate to say 28 = 6)
29 $\Delta R/C_3$	Induced drag correction using 21, 23, 25, 26, 28, and correct I.A.S. in equation (6)
30 Δt_0	From altitude vs. time to climb curve
31 V_T	3 28
32 $\frac{dV}{dt_T}$	From 9 and 10
33 $\frac{dV}{dt_0}$	From 9 and 30
34 $\Delta R/C_4$	From 24, 31, 32, and 33 using equation (6a)
35 R/C	17 + 26 + 27 + 29 + 34 - final corrected rate of climb

$$\Delta R/C_1 = \left\{\frac{W_T}{W_0} - 1\right\} R/C_T$$

$$\Delta R/C_2 = \frac{33000}{W_0}\left\{\eta_0 BHP_{A_0} - \eta_T BHP_{A_T}\right\}$$

$$\Delta R/C_3 = \frac{34400}{V_i \sigma^{1/2}} \frac{1}{\pi A R e}\left\{\frac{W_T}{W_0} - 1\right\}\left\{\frac{W_T + W_0}{S}\right\}$$

$$\Delta R/C_4 = 4.02\left\{\frac{W_T}{W_0}\frac{dV}{dt_T} - \frac{dV}{dt_0}\right\}$$

No proof of the fact that this method converges is given, but it is obvious that it will do so. As a rule, any correction beyond the second step indicated on the preceding page will give results more accurate than the airplane instruments.

The term flight-plan I.A.S., spoken of in the preceding analysis, is that speed variation which is most suitable for operational work. The speed is faster than the speed for best rate of climb because the airplane flies better at faster speeds, cooling is better, and more horizontal distance can be covered in reaching altitude, an important point in operational flying. Most operational climb plans specify a constant I.A.S. up to some altitude, (say 9500 ft.) , a decrease in speed to a higher point, (say 10500 ft.), and constant I.A.S. above that, repeating the process if necessary at higher altitudes. This flight plan, though not as "rigorous" as a continuously varying climb speed, is much easier for the pilot to fly.

This climb speed is somewhat arbitrary and not the result of extensive sawtooths, etc., as would be used in engineering flight test. Consequently, the question might arise as to why accurate reduction procedures are necessary. The answer is that when using such a plan the pilot must know accurately the fuel required for climb if he is to carry a maximum payload, even though the flight plan is not rigorously determined.

Engine critical altitudes and best altitudes for blower change can be approximated from power chart data, and checked against the torquemeter readings made during the flight (corrected for non Standard conditions) and final corrections made accordingly.

PART IV

HIGH SPEED CRUISE CONTROL

HIGH SPEED CRUISE CONTROL

Introduction

The term "cruise control" refers to data which, for a particular airplane, indicates the power settings giving a maximum value of ground miles traveled per pound of fuel consumed for any flight plan. Methods of determining cruise control data for airplanes which fly under conditions where the effects of compressibility are unimportant are covered in standard references; (See, for example, references 4 and 5.) the methods indicated here are specifically for use with airplanes where compressibility effects are important.

The method presented is based on the principal of correcting test data to non-test conditions holding temperature constant and correcting for pressure variations; (See reference 6.) this type of data reduction requires no Mach Number corrections. The basic equations used are

$$\text{(a)} \quad \frac{BHP_s}{BHP_t} = \frac{P_{a_s}}{P_{a_t}}$$

$$\text{(b)} \quad \frac{W_s}{W_t} = \frac{P_{a_s}}{P_{a_t}}$$

$$\text{(c)} \quad \frac{P_{m_s}}{P_{m_t}} = \frac{P_{a_s}}{P_{a_t}}$$

where P_a = Absolute Atmospheric Pressure - in. Hg.

BHP = Brake Horsepower

W = Gross Weight - lbs.

P_m = Manifold Pressure - in. Hg.

and the subscripts t and s refer to test and corrected conditions respectively. The above relationships hold if test and corrected free air temperature are the same.

The temperature altitude method has two drawbacks. First, according to equation (b), the reduced weight may differ appreciably from the test weight; however, careful flight planning should reduce this difficulty to a minimum. Second, the method permits data to be reduced to standard only if the free air temperature lies between 59° and -67° F; the fact that most operational flying is done below 30,000 ft. tends to make this restriction unimportant, and the test data can be extrapolated to give results not directly obtainable.

In addition, use is made of the fact that altitude has a negligible effect on the specific fuel consumption of an engine at constant RPM, power, and blower ratio.

The test program outlined requires an accurate knowledge of free air temperature; it is recommended that a shielded thermocouple giving very close to a full adiabatic temperature rise be used. In addition a positive displacement tank to measure fuel flow is recommended, and a knowledge of the altimeter pitot error is desirable in addition to the more common instrument corrections. Torquemeters are also required for accurate testing.

The method presented does not reduce airplane performance directly to such basic data as speed power curves and specific fuel consumptions, but gives cruise data directly. The importance of this fact depends upon the overall airplane test program, of which cruise data may or may not be a major factor, and modifications of the proprosed program may be desirable to permit two purpose testing and consequent reduced overall flight time.

The problem of <u>high speed</u> cruise control, at present, is not too important. However, future airplanes will cruise at higher Mach Numbers, compressibility effects will have to be considered, and methods of data reduction similar to that outlined here will have to be used. In general, the consideration of an additional parameter (Mach Number) will require increased test time, and greater care in setting up a test program; this fact is evident in the following discussion. A test program such as this should be used only if preliminary data indicate the effect of compressibility cannot be neglected.

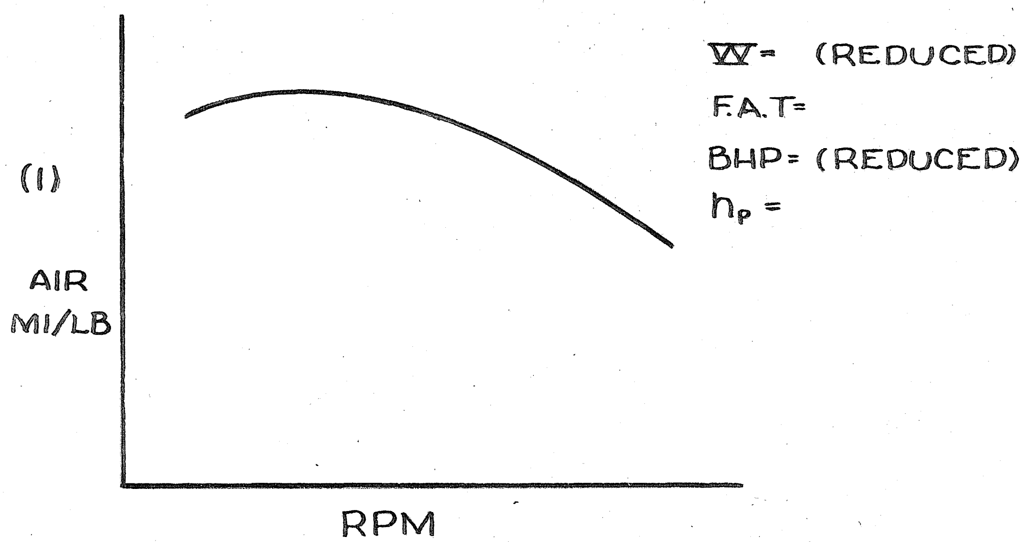
Calculations and Testing Procedure

The engine power chart and an approximate knowledge of the airplane speed-power relationship for the test weight range are desirable in setting up the test program. Several test temperature altitudes are selected in accordance with the engine installation used, test time available, accuracy desired, etc.. At each altitude several powers are selected in constant increments; at least four or five between maximum BMEP or open throttle and the maximum RPM condition, and a similar range of RPM's is selected. The powers chosen are reduced powers; test powers are chosen using equation (a) to give the desired reduced power at standard conditions.

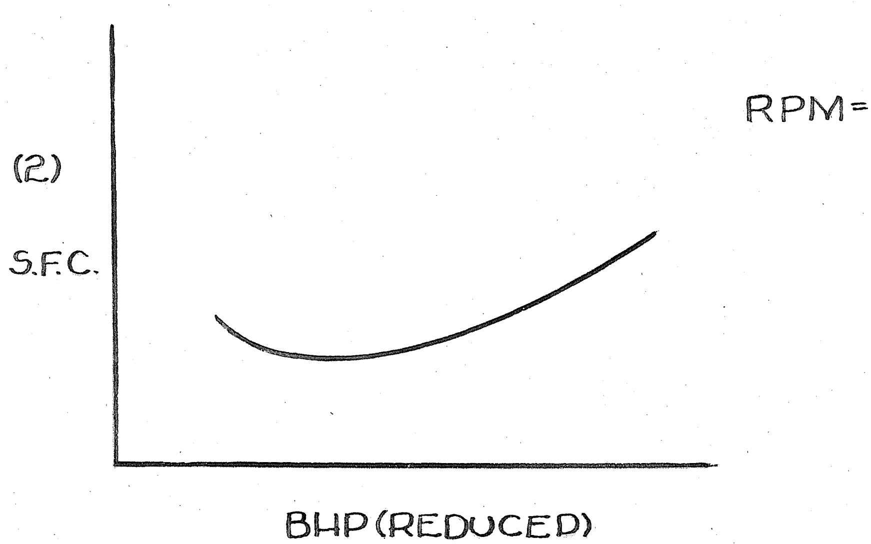
The tests are conducted in the following manner: First, at a given free air temperature, a power is set at the lowest obtainable (or permissable) RPM, fuel flow and airspeed are read, and air miles per pound of fuel calculated. This procedure is repeated at the same power for at least two more RPM's. The purpose is to find a maximum value of miles per pound for that power; this maximum will occur at or near the minimum RPM. If that maximum occurs at full throttle, it is desirable to take test points at higher RPM's since some operators prefer not to fly at full throttle even though some economy is sacrificed. This desire arises because momentary increases in power are desirable, for example, in rough weather, and such increases can be obtained most easily if some throttle play is available. And, variations in engine characteristics make some leeway desirable.

The variation of weight during the testing at each power is neglected; it is usually very small. Since the fuel flow can be determined during the test, it is possible to keep a running record of the weight, which is therefore known at each test point. A thermometer in the displacement tank can be used to determine the fuel density; one relationship between temperature and weight will usually be adequate for a series of tests if occasional checks are made.

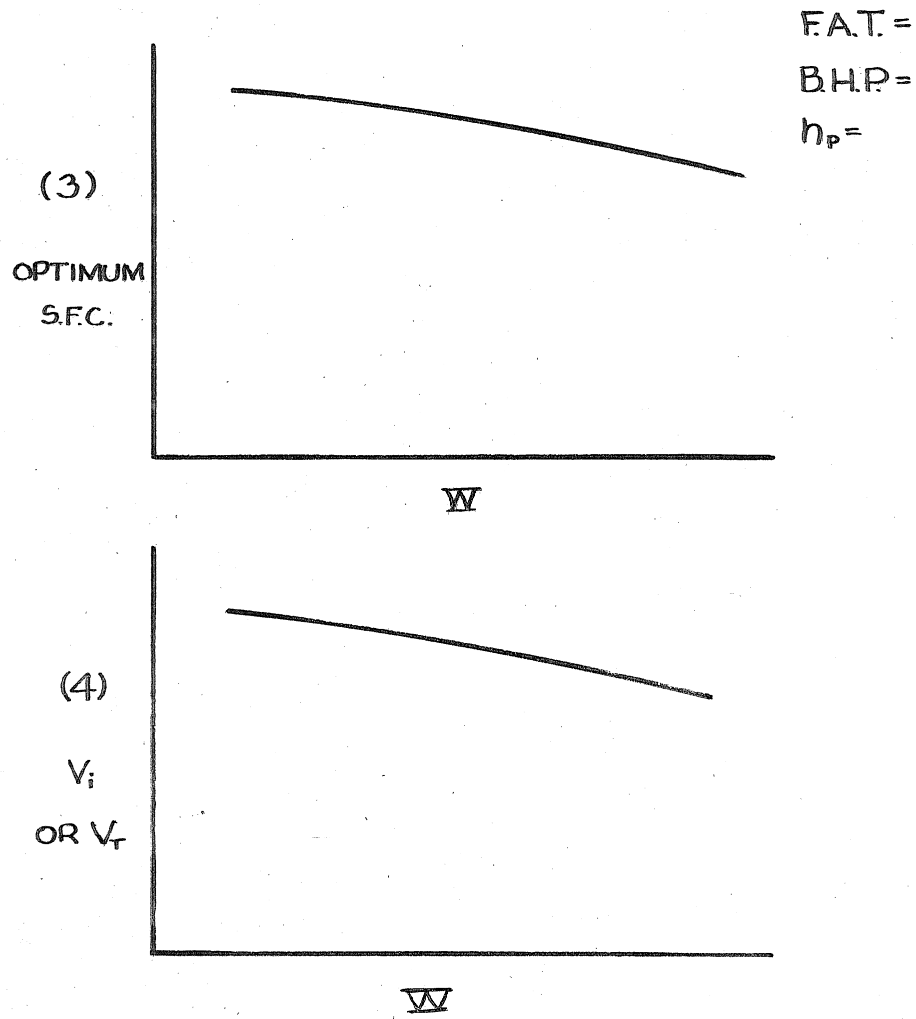
The resulting curves will look like the sketch -



Also, curves of specific fuel consumption vs. power are drawn from the same data, and allow determination of curves (1) for various temperatures.

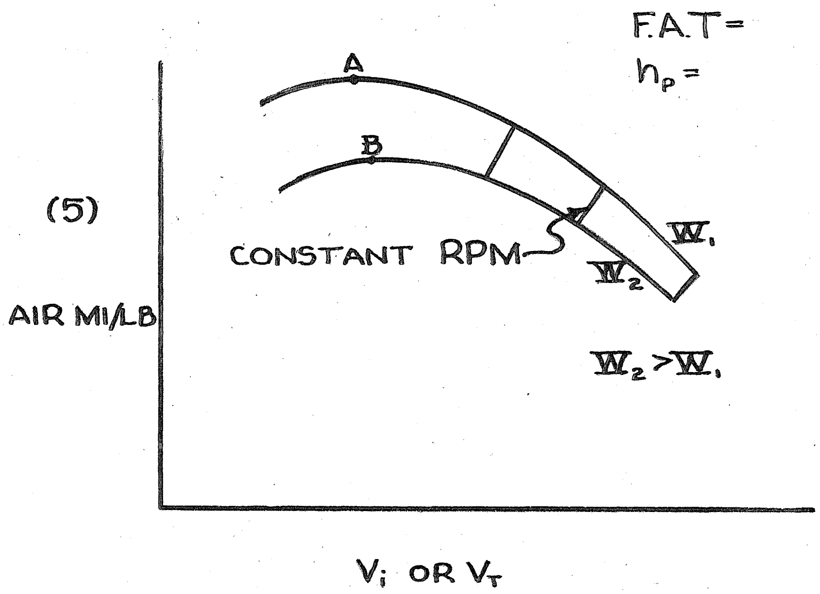


Similar tests are made at several weights, and the following cross plots are obtained.



In addition, if RPM varies with weight, a plot of RPM vs. weight will be desired; this is usually not necessary.

Curves (3) and (4) for various powers are used to obtain the following type of curve for desired weights.



Points A and B are obviously flight conditions for maximum range, i.e.

$$\frac{d(MI/LB)}{dV_T} = 0 = \frac{d[(MI/HR)/(LB/HR)]}{dV_T}$$

The maximum mi/lb for various headwinds can be obtained using curves (5) in the following manner. For a headwind w mph -

$$(MI/LB)_w = (MI/LB)_{w=0}\,\frac{V_T - W}{V_T} = (MI/LB)_{w=0}\left\{1 - \frac{W}{V_T}\right\}$$

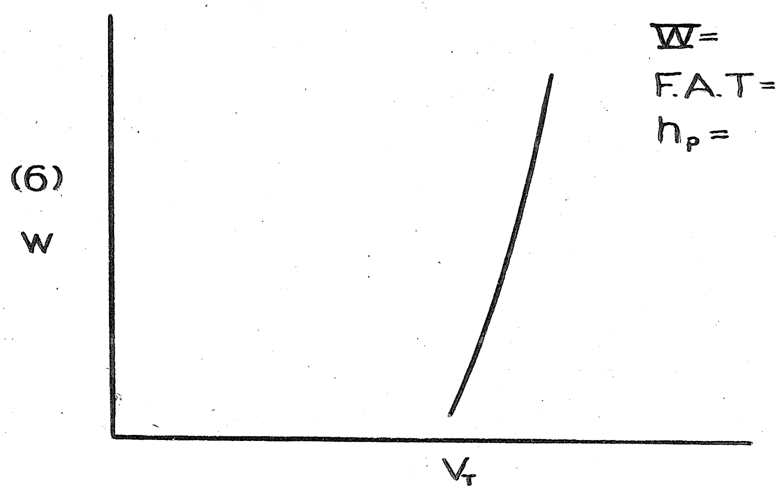
For a maximum $(MI/LB)_w$ $\quad \dfrac{d(MI/LB)_w}{dV_T} = 0$

$$= \left[\frac{d}{dV_T}(MI/LB)_{w=0}\right]\left\{1 - \frac{W}{V_T}\right\} - (MI/LB)_{w=0}\,\frac{d}{dV_T}\left\{\frac{W}{V_T}\right\}$$

(d) $\quad W/V_T = \left[1 - \{(MI/LB)_{w=0}/V_T\}\left\{1/\frac{d(MI/LB)_{w=0}}{dV_T}\right\}\right]^{-1}$

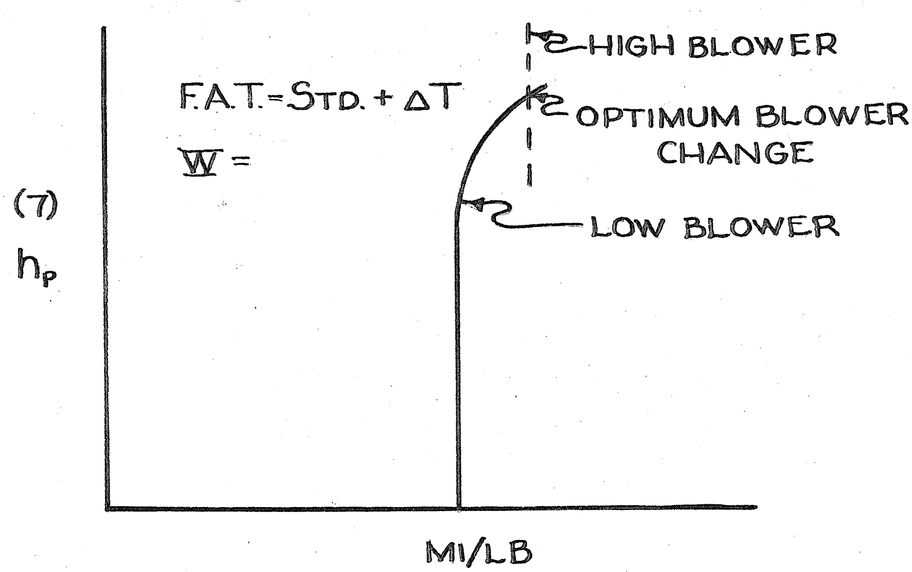
Slopes taken from curves (5) will give values of wind vs. V_T , and values of V_T at the desired headwinds can be obtained.

(6)

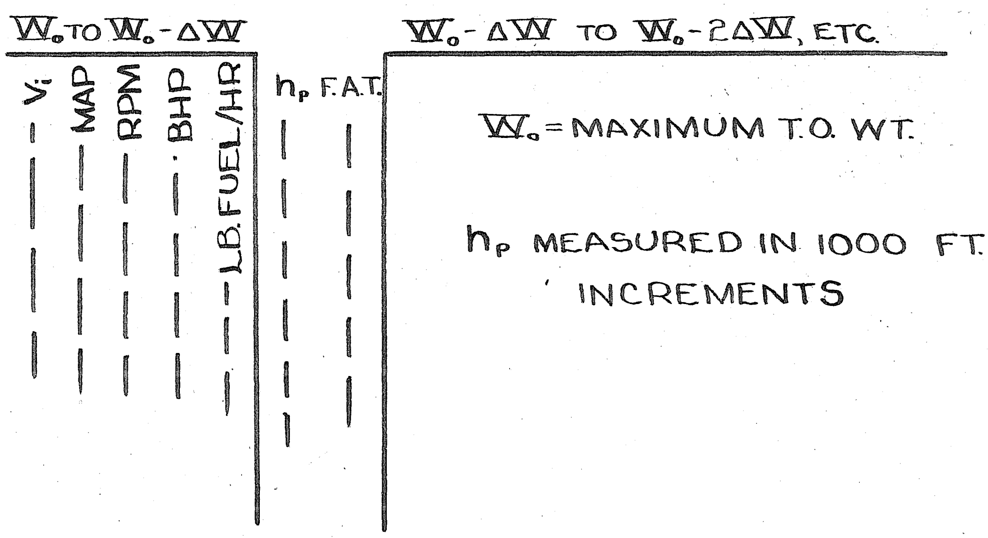


The aforementioned type of testing will be completed at several altitudes, and the following type of curve is drawn for each temperature variation with altitude desired. (i.e. Standard F.A.T.+ 20° F.)

(7)



Once the best altitude for blower change is determined, curves of optimum engine RPM, manifold pressure, power, fuel flow, and I.A.S. vs. altitude are obtained for each weight. These data are presented in a manner similar to this -



One of the above charts is drawn up for each of a range of temperature variations from Standard and for each of a range of headwinds. ΔW is usually taken as about 5% of the take-off weight. Data for each weight range is taken for the middle of the range, consequently V_i will vary slightly through the range. The use of these charts is familiar to anyone with flight test experience, and will not be discussed here.

Another useful set of curves which can be obtained from the data taken are power charts for the various temperature conditions considered; they are obtained using corrections (a) and (b), and standard power chart methods. These charts are particularly useful because they are valid for the engine installation tested, and are helpful in making up cruise charts.

Differences in airplanes, time from last engine overhaul, etc.

will cause the operating characteristics of airplanes to vary. However, cruise control will still indicate best flying conditions, and give a good indication of the fuel consumed.

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