

A STUDY OF VORTEX SHEDDING  
AS RELATED TO  
TORSIONAL OSCILLATIONS OF A THIN AIRFOIL

Thesis by  
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## Summary

This report covers an experimental investigation of the relationship between the vortex shedding frequency and self excited torsional oscillation frequency for a thin airfoil. The work consisted of measurements of velocity fluctuations in the airstream in the vicinity of a wing model mounted in a wind tunnel so that it could oscillate about the wing axis. The velocity fluctuation measurements were made with the wing restrained and with the wing oscillating at various angles of attack and wind velocities.

Two distinct types of oscillations were found. One type was self sustaining and increased in amplitude with increasing wind velocity while the other type stopped for velocities beyond some critical value.

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## I. Introduction

An investigation of the causes of failure of the Tacoma Narrows Bridge showed that its destruction was in part a result of large torsional oscillations of the bridge floor under action of wind. To determine the nature of these oscillations a research program was begun at GALCIT under the sponsorship of the National Advisory Committee for Aeronautics. Two reports on previous work, References 1 and 2, cover the amplitude response of NACA 0006 and 0012 airfoils in both torsional and bending oscillations for various angles of attack and wind velocities.

This report is an extension of the work reported on in Reference 2. Similar equipment and techniques were used; the only new work being done was the measurement of velocity fluctuations in the airstream in the vicinity of the airfoil by means of hot wire anemometers to determine the relationship between the vortex shedding frequency and the torsional oscillation.

## II. Apparatus

Since most of the apparatus used was the same as was used in the previous investigations of this series, only a short outline of it will be given. A more detailed account may be found in References 1 and 2.

The tests were carried out in the open return type wind tunnel with 36 by 42 inch test section which is shown in Figure 1. Three turbulence screens and a large contraction ratio were used so that the airflow in the test section was quite smooth and uniform throughout. Wind velocities of from 0 to 48 miles per hour could be attained.

The model used was a rectangular planform NACA 0006 airfoil of 41 inch span and 9 inch chord. In most of the tests it was elastically suspended so that it could oscillate about an axis through its 38% chord point. With the wind velocity available the maximum Reynolds' Number reached was about 330,000.

The model was suspended from aluminum alloy rods along the wing axis. The rods were rigidly clamped on their ends so that torsional oscillation of the airfoil caused the rods to twist as torsion springs. The suspension system is shown in Figure 2. Two different diameter torsion rod sets were used in order to have tests at two airfoil oscillation frequencies. Rods of 0.188 inch diameter and 3 inch length gave a frequency of about 12 cycles per second while rods of 0.250 inch diameter and 3 inch length gave a frequency of about 21 cycles per second. The use of torsion rods had several advantages; the system possessed small mechanical damping; it produced a fairly linear spring force; and it restricted the oscillation to one degree of freedom.

To measure the amplitude of the torsional oscillations of the airfoil a strain gage was wrapped spirally around one of the torsion rods at an angle of  $45^\circ$  to its axis. In order to have the strain gage pick up only torsional deflections of the rod it was necessary to remove bending loads from the rod between the airfoil and the part of the rod on which the strain gage was wrapped. A ball bearing was installed for this purpose as is shown in Figure 2. It also served to cut down the bending stress in the rod by shortening the cantilever beam length.

Changes in strain gage resistance caused by airfoil oscillations were amplified and fed into a recording oscillograph, Heiland type A400 R6. The deflections were recorded on 2 inch wide photographic tape along with timing marks at  $1/100$  second intervals. Frequency of the oscillation could easily be determined by counting the number of cycles in any convenient time interval. The amplitude of the oscillations was also determined from the waves on the photographic record tape. In order to do this the resistance change of the strain gage for various angular deflections of the airfoil was measured using a Wheatstone bridge circuit. Standard resistance changes of the strain gage circuit were produced by throwing combinations of precision resistors in parallel with the strain gage. The transient response of the circuit, when these resistors were shunted across the strain gage by a microswitch, appeared as a stepped trace on the photographic record tape. The step heights on the tape corresponded to known angular deflections of the torsion rod. Oscillation amplitudes were determined by comparing oscillation wave heights on the record to the calibration mark heights on the record. Changes in amplifier gain did not affect

the accuracy of the calibration marks on the tape.

To measure the variations in the airstream velocity and the turbulence of the flow, hot wire anemometers were used. Survey of the flow downstream of the airfoil was accomplished by mounting the hot wires on traversing mechanisms which could be moved vertically and axially in the tunnel. To investigate the flow in the vicinity of the oscillating airfoil, the hot wires were attached to light frameworks and secured to the airfoil with "Scotch tape". The position of the hot wires then could be easily changed by bending the framework or moving the tape. This method was found to be quite convenient and did not damage the model. Since only the frequency of the velocity changes was of particular interest, no attempt was made to calibrate the wires for velocity magnitude measurements. A typical hot wire as used in the tests was a  $1/4$  mil platinum wire suspended rather tightly between supports  $1/4$  inch apart and carrying about 100 milliamperes of current.

The frequency of velocity variations caused by vortex shedding was found by counting waves on the photographic record tape and rough checks were made by matching the frequency with that from an audio oscillator. A cathode ray oscillograph was found to be very convenient for checking wave forms, amplitudes, and frequencies before recording them on the photographic tape. While the hot wires were being moved about with the traversing mechanism the oscilloscope was referred to in order to choose points with regular velocity fluctuations.



## III. Tests

A 30 point velocity survey of the test section was made at a tunnel velocity of 30 feet per second. The flow was found to be uniform within 2%; results of the survey are shown in Figure 4.

The model was mounted in the tunnel and a series of runs were made with tufts on the model in order to determine the general character of the flow about the model. These tests clearly indicated that the flow was not two dimensional. There was considerable lateral flow in from the tips on the upper surface of the model in spite of the small end plates and the closeness of the tips to the tunnel walls. No attempt was made to correct the angle of attack for induction effects.

Survey of the wake downstream of the airfoil was carried out in order to determine the height of the wake at different angles of attack.

A qualitative survey of the airflow in the vicinity of the airfoil was made using hot wire anemometers. The flow behind the stalled airfoil was found to be turbulent without any apparent regularity. The flow for a short distance outside the turbulent wake was found to have well defined waves of velocity change which could easily be seen on the oscilloscope and easily recorded. Outside the region of well defined waves the flow was very smooth. Apparently there was no transmission of pressure impulses upstream from the propeller. If the airfoil was allowed to oscillate, the velocity fluctuations over a large region of the test section were found to have the natural frequency of the airfoil. The measureable influence of the oscillating airfoil extended to a considerable distance, possibly 1 chord length upstream

and 2 chords above and below the airfoil. Again the wake immediately behind the airfoil contained only random turbulence. Attempts were made to determine whether the vortex shedding occurred at a different frequency than that at which the airfoil was oscillating. The waves picked up by a hot wire downstream from the oscillating airfoil were fed into a sound analyzer and the analyzer showed response at the natural frequency and multiples of the natural frequency. The response at the natural frequency multiples was attributed to the characteristics of the sound analyzer rather than the existence of these multiples of the natural frequency in the wake.

In another attempt to find other frequencies in the wake behind the oscillating airfoil the hot wires were mounted on the oscillating airfoil. It was hoped that hot wires mounted in this manner might pick up other frequencies than the natural frequency but again only the natural frequency could be detected.

The principal tests were a series of runs to determine frequency of vortex shedding as a function of angle of attack and velocity. The model was suspended on an axis at its 38% chord point and tested at angles of attack of from  $7^{\circ}$  to  $29^{\circ}$  and wind velocities of from 15 to 50 feet per second. In a run the angle of attack was set and the velocity increased in increments of about 1 foot per second. At each velocity the vortex shedding was picked up by a hot wire mounted downstream from the airfoil and near the lower edge of the wake. A record of the vortex shedding was made on the photographic record tape. This procedure was continued until the airfoil began to oscillate.

After the beginning of oscillation the airfoil was temporarily restrained so that a record could be made of the vortex shedding of the stationary wing. Then a calibration mark was inserted in the photographic record tape and the airfoil released. When the oscillations had built up to their highest amplitude a record of the oscillations was made. Thus for each point where oscillation occurred three records were made; vortex frequency of the stationary airfoil, an amplitude calibration for checking oscillation amplitude and a record of airfoil oscillation.

It is important to note that in this part of the investigation the airfoil was held stopped while the velocity was being increased. In this way the conditions under which the oscillations would start from rest were obtained.

Next, the suspension location on the airfoil chord was changed to the 65% chord point. In this manner the natural frequency of the airfoil was lowered. It was hoped that a series of runs could be made using this different natural frequency, but the angular deflections of the airfoil were so large that the tests had to be stopped to prevent damage to the torsion rods. The change in incidence due to the steady aerodynamic pitching moment for an airfoil suspended in this manner also proved to be quite large.

To get some data on the effect of an increased natural frequency a new set of torsion rods was installed and tests were run at angles of attack of  $8^\circ$ ,  $12^\circ$  and  $25^\circ$  with wind velocities of from 35 to 55 feet per second. Higher wind velocities were required to start oscillation because of the increased stiffness of the torsion rods. The original

torsion rods gave a system natural frequency of approximately 12 cycles per second and data obtained while using these rods is marked in the results with the words "Suspension # 1". The stiffer rods gave a natural frequency of 21 cycles per second and the data obtained using them is marked "Suspension # 2".

The vortex frequencies were determined by counting the number of waves in any convenient time interval on the photographic record tape. In the tests at low angles of attack the records were quite irregular. Decisions had to be made on such questions as whether the rough undulation of the trace was a wave or just a phase shift in the shedding and, when a record contained successive groups of waves of different frequencies, which group was representative of the vortex frequency. In some cases the frequency had to be determined from two or three well defined waves occurring in less than  $1/10$  of a second. In view of this, the vortex frequency accuracy for runs at angles of attack of  $7^\circ$  to  $12^\circ$  should be considered as in the vicinity of 15%. At higher angles of attack the vortices were much more defined and regular so the accuracy in the range from  $15^\circ$  to  $29^\circ$  could be considered as about 5%.

Velocities were measured with a micro-manometer and were probably accurate within 1%.

Angles of attack were set within  $1/2^\circ$  on a metal protractor.

The amplitude of the torsional oscillation was obtained from the photographic record tapes. Considering the method used to obtain these amplitudes, their accuracy was probably within  $0.2^\circ$ .

## IV. Results

The results of the tests using suspension system # 1 with natural frequency of 12 cycles per second at angles of attack of from 7° to 29° are given in Figures 9 to 20. The curves show vortex frequency, ratio of vortex frequency to natural frequency, and amplitude of oscillation all plotted against wind velocity. The vortex frequency curves are, of course, for a stationary airfoil.

Considering the vortex frequency, the curves are not smooth, but all show vortex frequency increasing approximately linearly with velocity.

The ratio of vortex frequency to oscillation frequency follows the same trend as the vortex frequency itself since the oscillation frequency is essentially constant with one suspension system.

Vortex frequencies are usually computed from the following equation:

$$N = \frac{V K}{b \sin \alpha} .$$

K is usually 0.15 to 0.21 for airfoils. The vortex frequencies for different velocities were plotted as functions of angle of attack in Figures 34 and 35. The experimental values were substituted in the above equation and values of K determined. These were found to range between 0.112 and 0.178. For the higher angles of attack, Figure 35 shows that the vortex frequency does vary approximately in the manner which was stated by the above equation. However, in the angle of attack range below 15° the vortex frequency seems to decrease somewhat with an angle of attack decrease.

The amplitude of oscillation increased with velocity, apparently

linearly for angles of attack of from  $7^\circ$  to  $12^\circ$ . At these angles of attack the oscillation was capable of starting itself for all velocities from a minimum depending on angle of attack up to the maximum tunnel velocity. For the angles of attack of from  $14^\circ$  to  $29^\circ$  the velocity at which the oscillations were self excited had an upper limit. Increasing the angle of attack narrowed the velocity range over which the oscillation was self starting. The range of velocities for which the oscillation was self excited is plotted against angle of attack in Figure 21.

It should be noted that for low angles of attack the oscillations, once started, would persist and grow with increasing wind velocity. Figure 25 shows the oscillation amplitude as a function of angle of attack for various wind velocities. That increasing velocity would increase vibration amplitude to dangerous magnitude is indicated by Figure 24, showing the maximum oscillation amplitude obtained at various angles of attack. Also shown in the same figure are the vortex frequency of the stationary airfoil and the wind velocity at which the maximum amplitude occurred. Below  $14^\circ$  the maximum amplitude seems to go up rapidly. It would not be quite correct to say the amplitude grows without limit since the suspension system would be destroyed by large oscillations.

To attempt explanation of the starting of oscillation, the range of vortex frequencies at which oscillation began was plotted in Figure 22 and the ratios of vortex frequency to natural frequency were plotted in Figure 23.

The lift curve for the model used is shown in Figure 3. Wake surveys taken downstream of the model are shown in Figure 5 through 8.

Curves showing the data obtained using Suspension # 2 are shown in Figures 26 to 28. Since very little data was taken using this suspension, the results could not be suitably coordinated with those using Suspension # 1.

Figures 29 to 33 show the vortex shedding frequencies from stationary airfoils at various angles of attack. Note that at low angles of attack the frequency is quite erratic while at high angles of attack the frequency varies almost linearly with the wind velocity.

Figure 37a shows a record taken by a hot wire fastened approximately 1 inch back from the leading edge and  $\frac{3}{8}$  inch above the upper surface of the airfoil while it was oscillating about  $8^\circ$  angle of attack with amplitude of  $6.6^\circ$ . The direction of increasing time and the angular position of the airfoil are indicated on the record. The direction of high velocity on the record is downward and that of the low velocity is, of course, upward. The irregular trace is a record of the velocity past the hot wire while the sine shaped wave is the angular position of the airfoil. The highest velocity during a cycle, the flat portion of the velocity trace between 1 and 2, corresponds to the angle of attack decreasing from  $2.0^\circ$  to its lowest value  $1.5^\circ$  and then increasing to  $13.5^\circ$ . From 2 the velocity drops to a lower irregular value signifying a region of turbulence. While this is occurring, the angle of attack has reached its highest value,  $14.6^\circ$  and then dropped again to  $2.0^\circ$  at which time the flow past the hot wire again becomes smooth.

Figure 37b shows a record taken by a hot wire approximately  $\frac{1}{2}$  of an inch behind and  $\frac{1}{2}$  inch below the trailing edge of the airfoil

under the same conditions as for the record given in Figure 37a.

Figure 37c shows the trace taken by a hot wire located as in Figure 37a but with angle of attack of  $10^\circ$ . The hot wire seems to be in the smooth part of the flow for less time during each cycle than does the one shown in Figure 37a.

Figure 37d shows a record taken by a hot wire located as in Figure 37a but with the airfoil oscillating about  $18^\circ$  angle of attack with amplitude of  $7^\circ$ . The irregularity of the trace indicates that the hot wire was in a region of random turbulence during the entire oscillation cycle.



## V. Discussion

The discussion of the starting and growing of oscillations shall regard three distinct behavior regions of angle of attack.

First, consider a region of angles of attack of from  $23^\circ$  to  $29^\circ$ . Reference to Figure 23 shows that self excited vibrations occur for a narrow range of  $N/n$  about  $N/n = 1.0$ . Here the airfoil begins vibration when the vortex shedding frequency approximates the natural frequency of the airfoil. Vibrations started in this manner would not grow to amplitudes larger than about  $2^\circ$ , and if the velocity were increased about 5 feet per second, they would die out. In the investigation by Levy, Reference 2, the tests were stopped before the velocity at which these oscillations die out was reached. However, in this angle of attack range, another, larger amplitude, sustained vibration could be started by giving the airfoil an initial torsional oscillation of sufficient amplitude. Oscillations of this type are mentioned by Levy, Reference 2, and have been studied as to the conditions for their initiation by Bratt, Wight, and Chinneck, Reference 3. One or two tests at angle of attack of  $29^\circ$  with these larger amplitude oscillations showed that they would grow with increasing wind velocity to an amplitude of about  $20^\circ$ , after which they held constant amplitude while the velocity was increased further, and finally, they halted quite abruptly.

Next, consider the angle of attack region of from  $6^\circ$  to  $13^\circ$ . In this region the airfoil would not experience self excited oscillations for ratios of vortex frequency to natural frequency of less than one. In fact, at  $7^\circ$  the vibration would not start until  $N/n$  had reached 3.3. However, once these oscillations did start they would sustain themselves

and continue to grow with increasing wind velocity. Of course, the tests were stopped when the amplitude had reached a dangerous value, but the indications of Figures 24 and 25 are that they would have continued to grow much more. This amplitude growth behavior would seem to put these vibrations in the same class as the self sustaining type found in the  $21^\circ$  to  $29^\circ$  angle of attack range.

The angle of attack region of from  $14^\circ$  to  $18^\circ$  shows a pattern of oscillation behavior between that observed in the two regions already considered. The range of  $N/n$  values for which oscillation would start was limited. Vibrations started in this angle of attack region also seemed to be of the self sustaining type. Once started, they increased in amplitude with increasing wind velocity until the tests had to be stopped to prevent damage to the suspension.

The large amplitude oscillations are thought to be a product of stalled flow over the oscillating airfoil at least during part of the oscillation cycle.

## VI. Conclusions

Two distinct types of oscillations were found. One type occurred only at high angles of attack and only when the vortex shedding frequency from the stationary airfoil was very near the natural frequency of the airfoil. This type of vibrations could not maintain itself if the wind velocity was increased beyond some critical value. The other type of oscillation was able to increase to large amplitudes if the wind velocity was increased.

An important finding not covered in the previous reports of this series is that the range of velocities for which torsional oscillations were self excited had an upper limit. The range of velocities for which self excited oscillations would start narrowed as the angle of attack was increased.

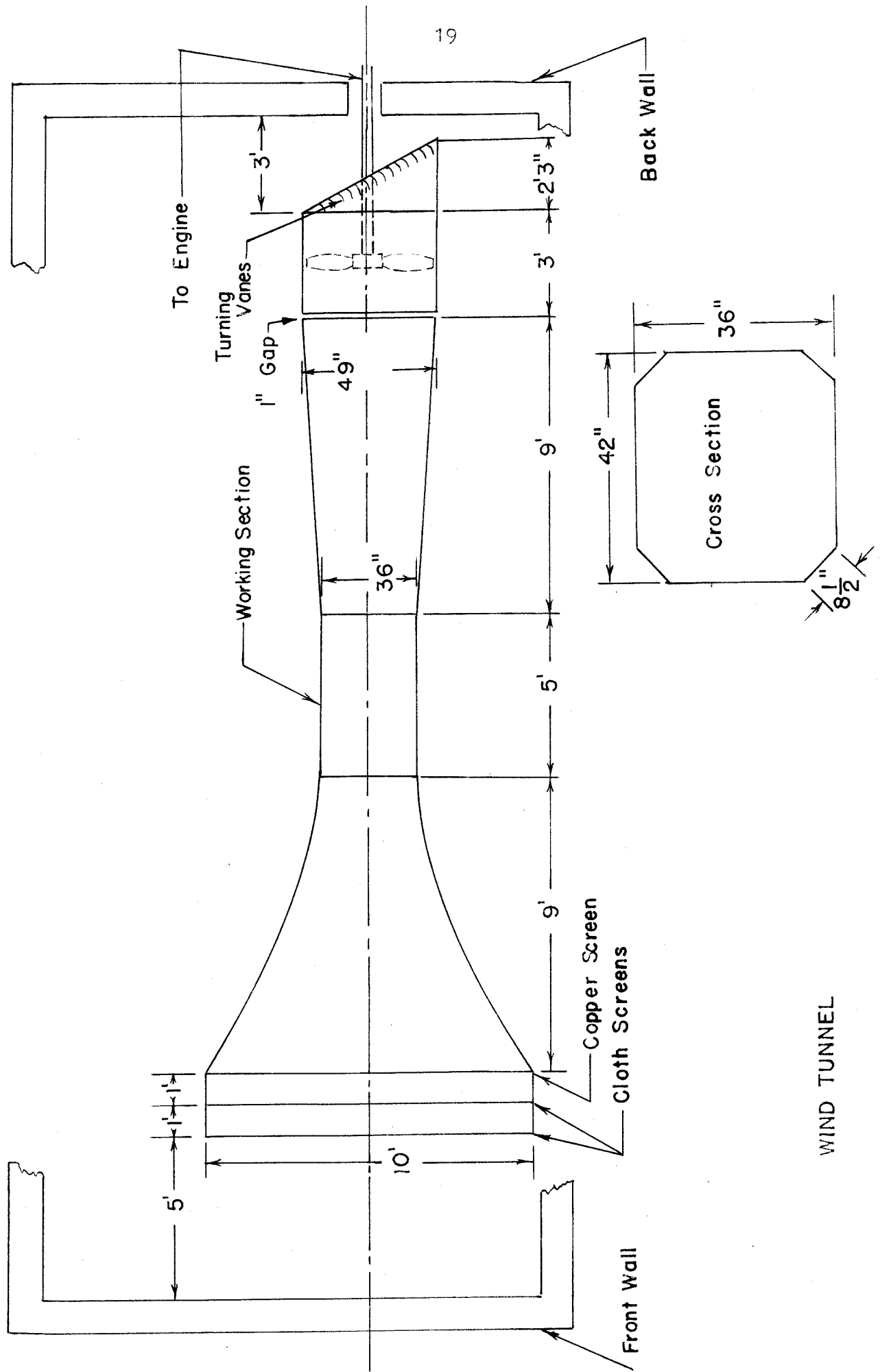
## VII. Notation

- $\alpha$  Geometric angle of attack of airfoil when stationary, in degrees.
- $V$  Wind velocity in feet per second.
- $V_1$  Lower critical velocity at which oscillation starts.
- $V_2$  Upper critical velocity at which oscillation no longer starts.
- $V_3$  Velocity at which maximum amplitude occurs.
- $\theta$  Amplitude of oscillation in degrees.
- $\theta_{\max}$  Maximum amplitude of oscillation at a given angle of attack.
- $n$  Frequency of oscillation in cycles per second.
- $N$  Frequency of vortex shedding from the stationary airfoil.
- $N_1$   $N$  corresponding to  $V_1$ .
- $N_2$   $N$  corresponding to  $V_2$ .
- $N_3$   $N$  corresponding to  $V_3$ .
- $b$  Airfoil chord in feet.
- $K$  Dimensionless Karman number.
- $\Delta H$  Difference in total head between free stream and wake.

## VIII. References

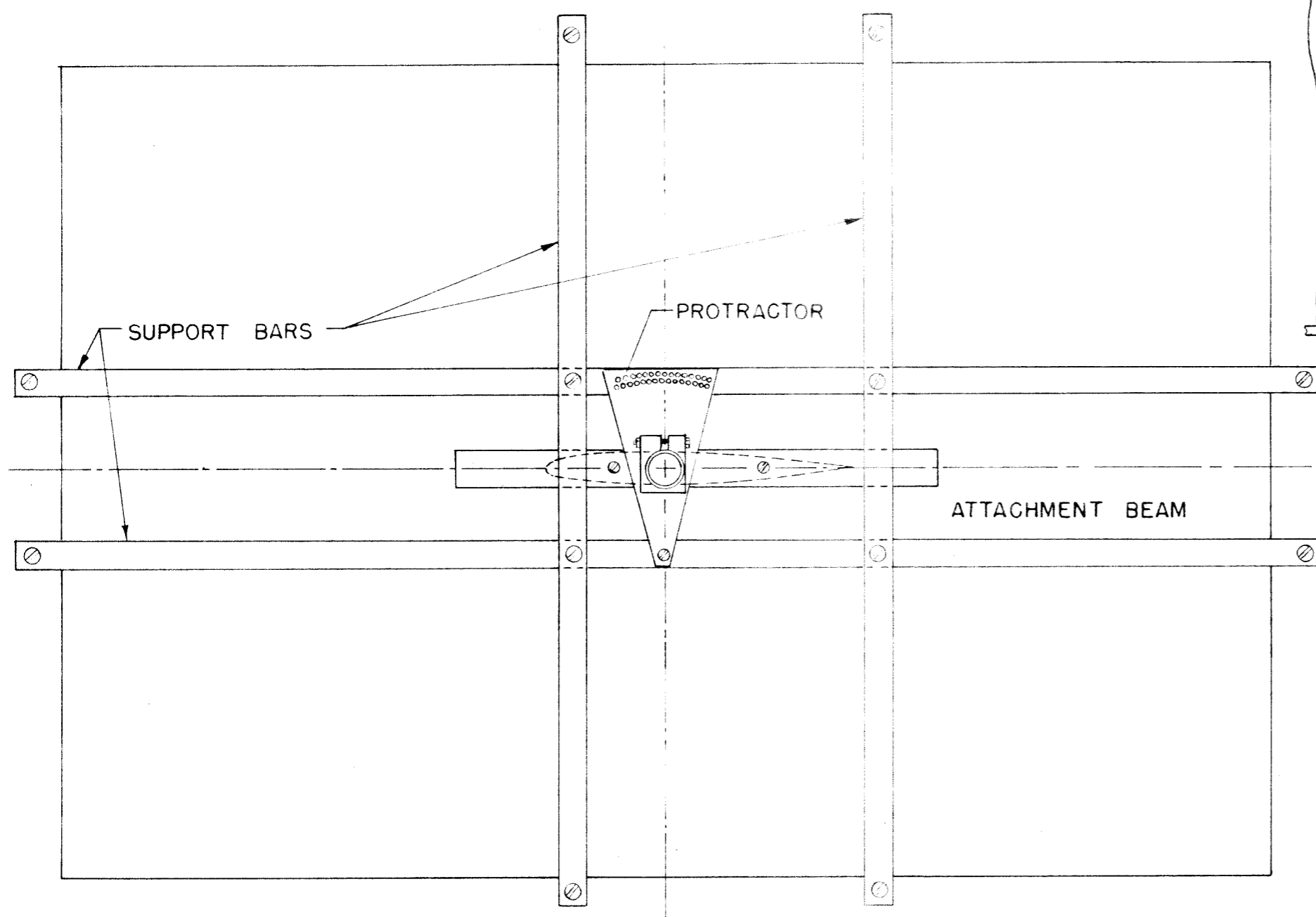
1. Dunn, L. G., and Finston, M., "Self Excited Oscillations of Airfoils", Report in Final Fulfillment of Contract NAW 2329, April, 1945, California Institute of Technology. (Confidential).
2. Levy, C. N., "Self-excited Torsional Oscillations of an Airfoil", Thesis, California Institute of Technology, June, 1945. (Confidential).
3. Bratt, J. B., Wight, K. C., and Chinneck, A., "Free Oscillations of an Aerofoil about the Half-chord Axis at High Incidences and Pitching Moment Derivatives for Decaying Oscillations", R. & M. No. 2214, London, 1940.

IX. Figures

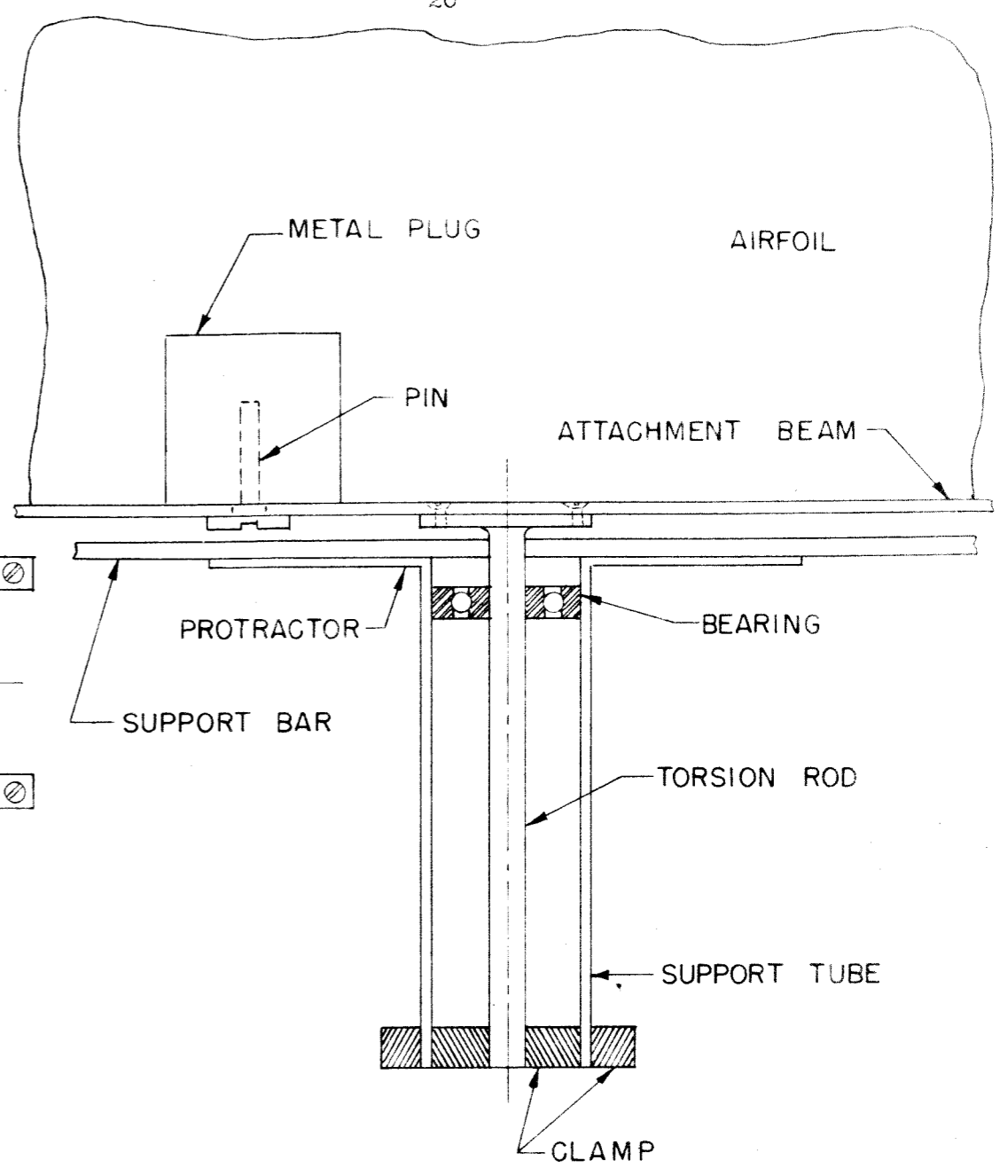


WIND TUNNEL

FIG. 1'



SIDE VIEW OF SUPPORT SYSTEM



TOP VIEW OF TORSION ROD SUPPORT SYSTEM

TORSIONAL OSCILLATION SUSPENSION

FIG. 2



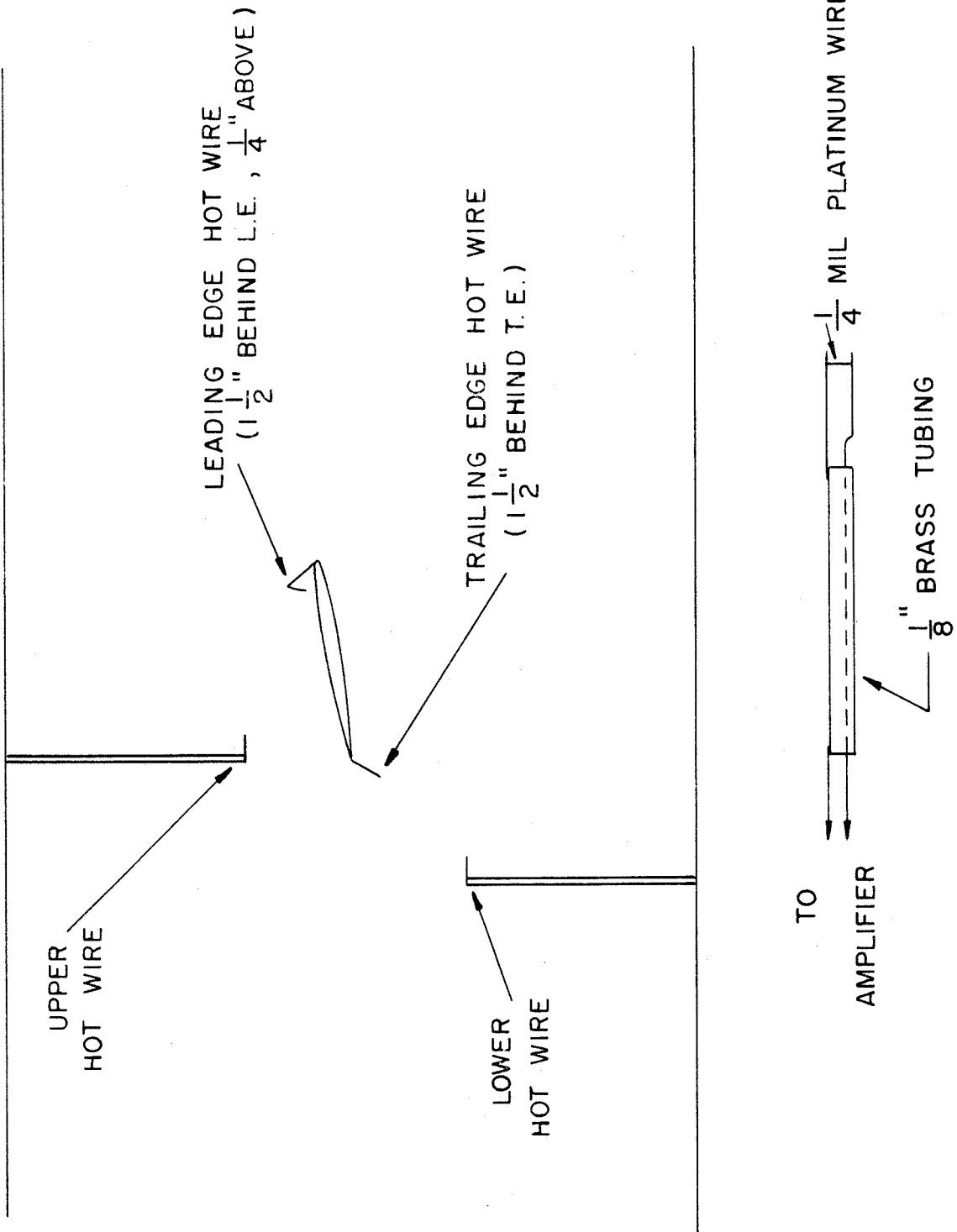


FIG 2 a

LIFT CURVE  
NACA 0006 AIRFOIL  
SPAN = 41" CHORD = 9"  
V = 30.6 fps

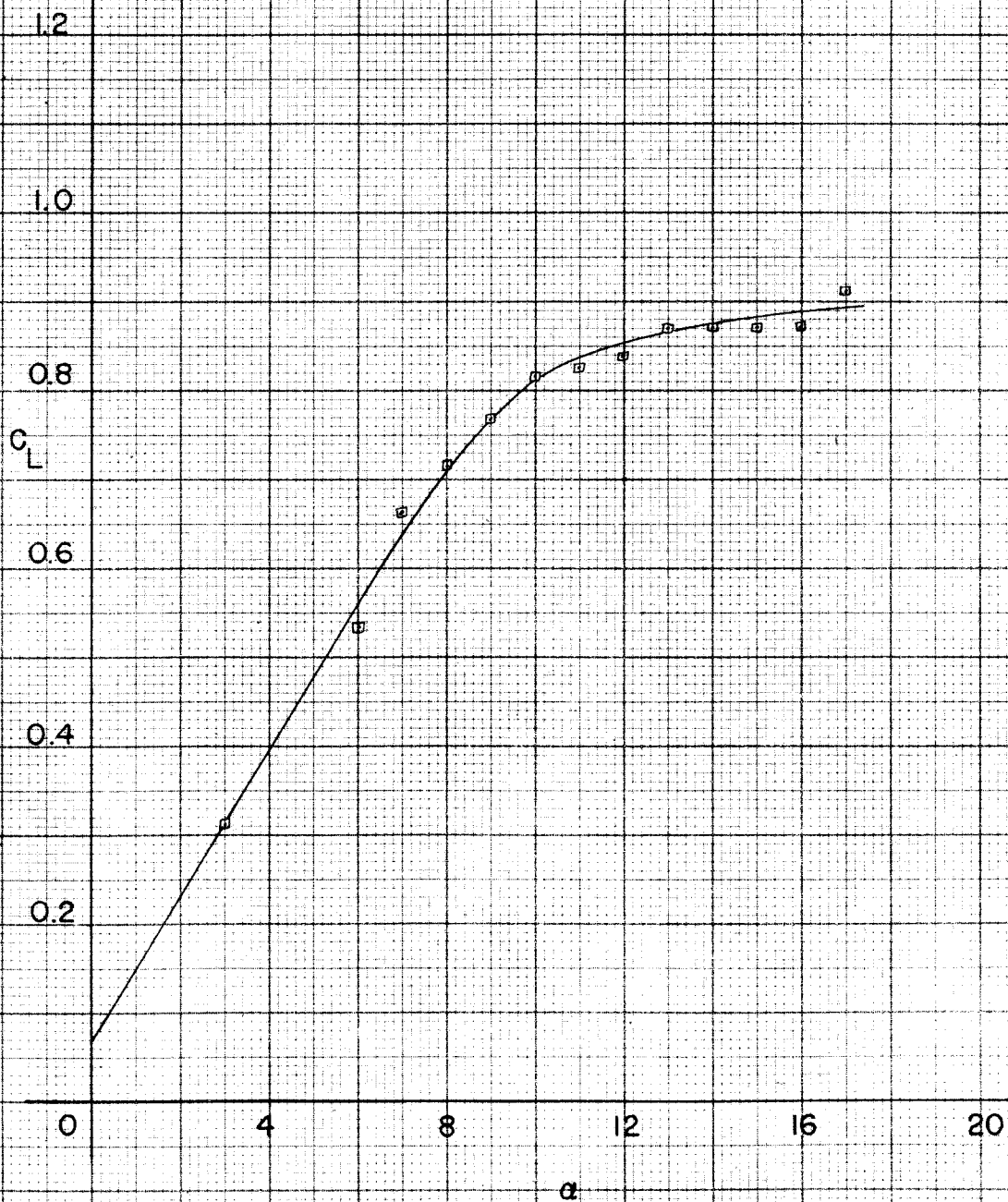


FIG. 3

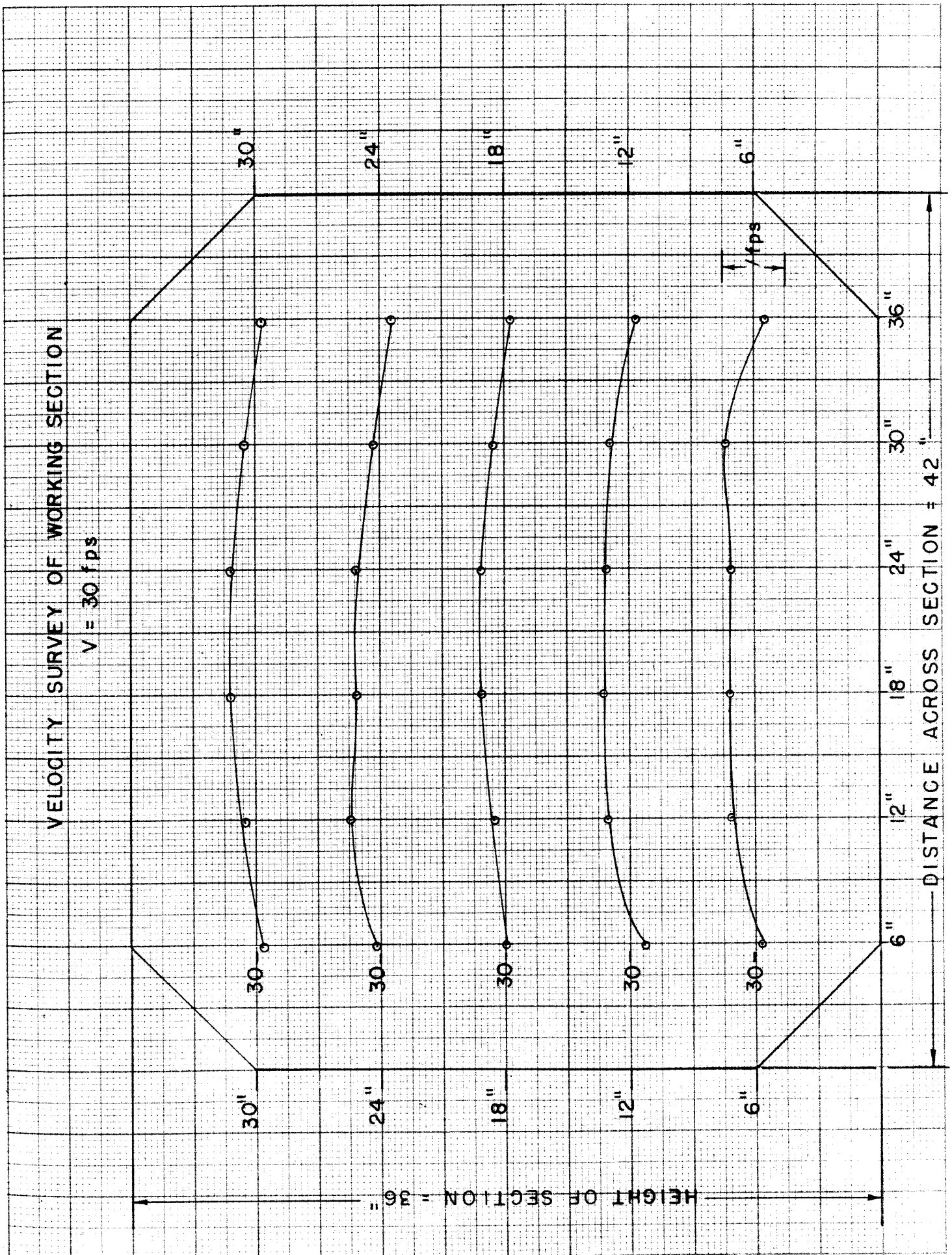


FIG. 4

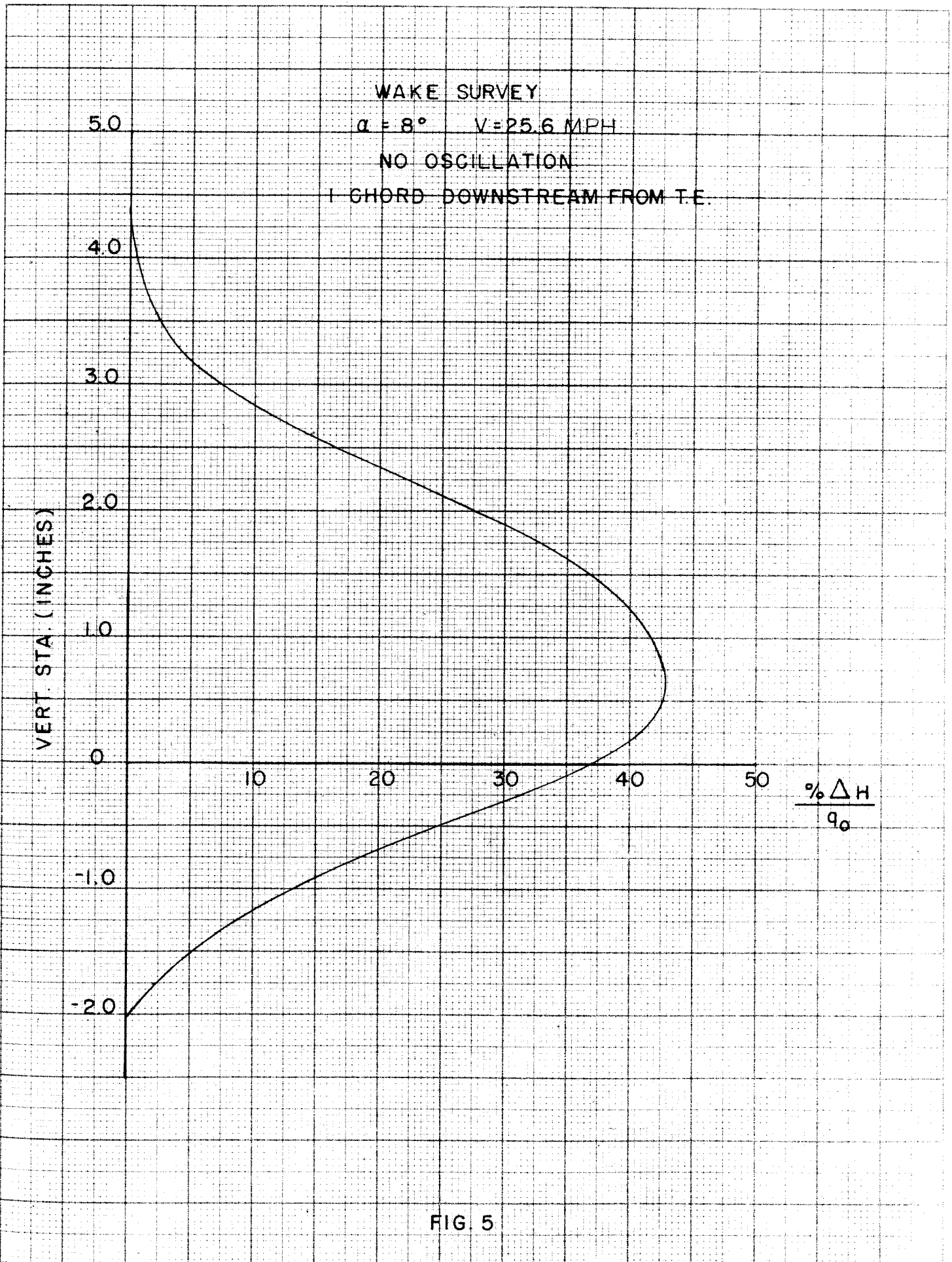


FIG. 5

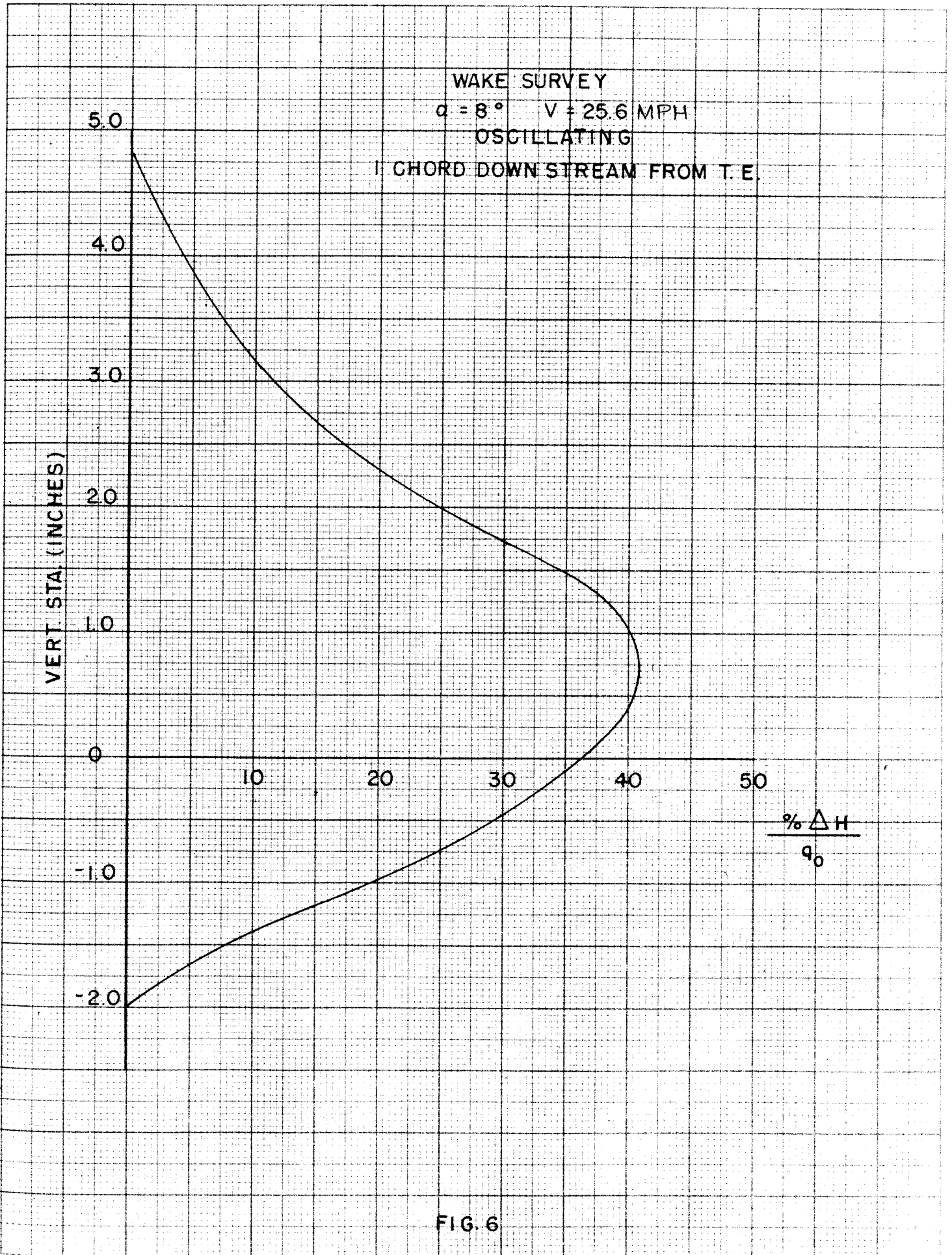


FIG. 6

## WAKE SURVEY

 $\alpha = 12^\circ$   $V = 24.8$  MPH

OSCILLATING

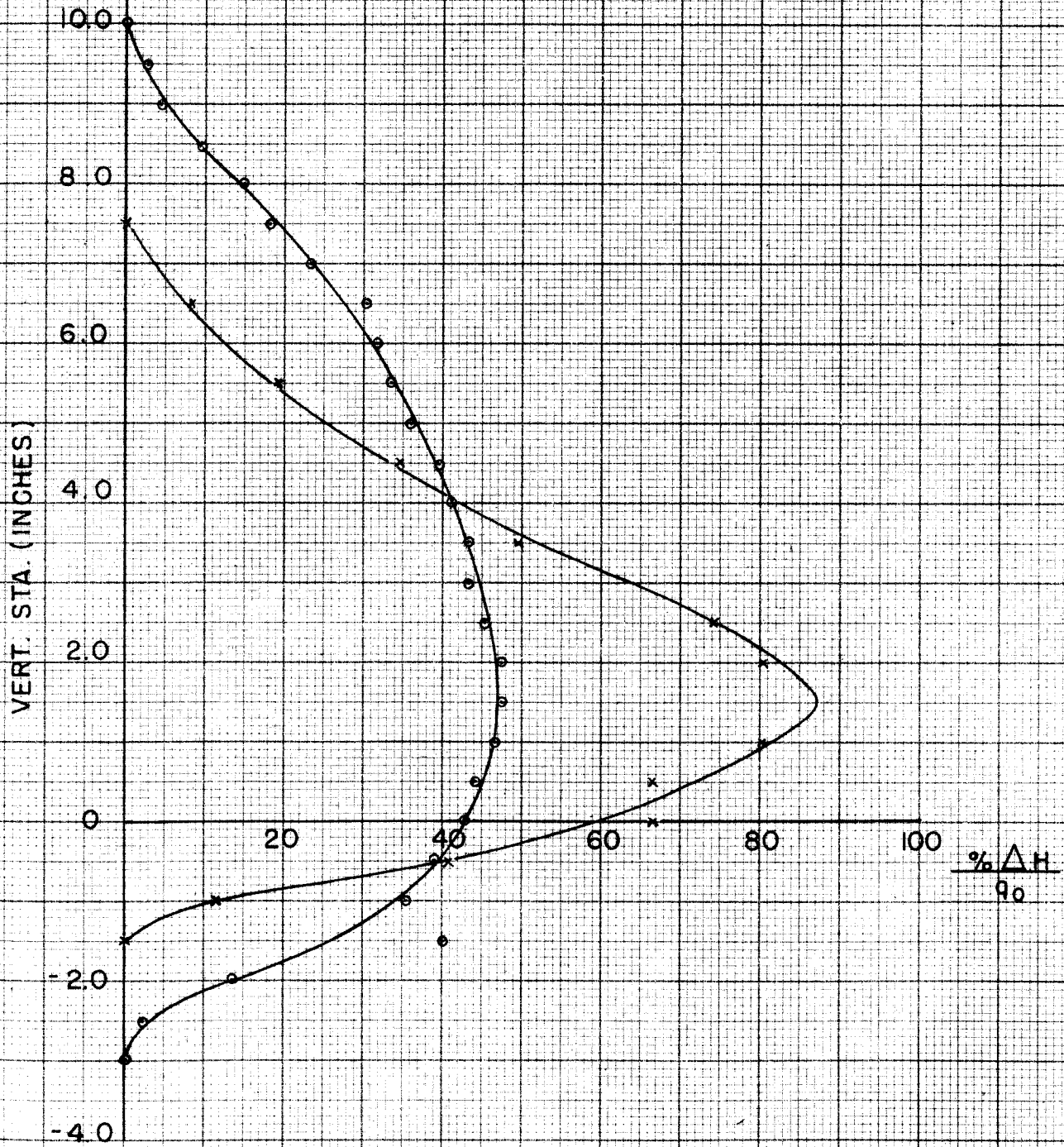
 $\circ = 1$  CHORD DOWNSTREAM FROM T.E. $+ = \frac{1}{4}$  " " " "

FIG. 7

WAKE SURVEY  
 $\alpha = 15^\circ$   $V = 25.0$  MPH  
 OSCILLATING  
 1 CHORD DOWNSTREAM FROM T.E.

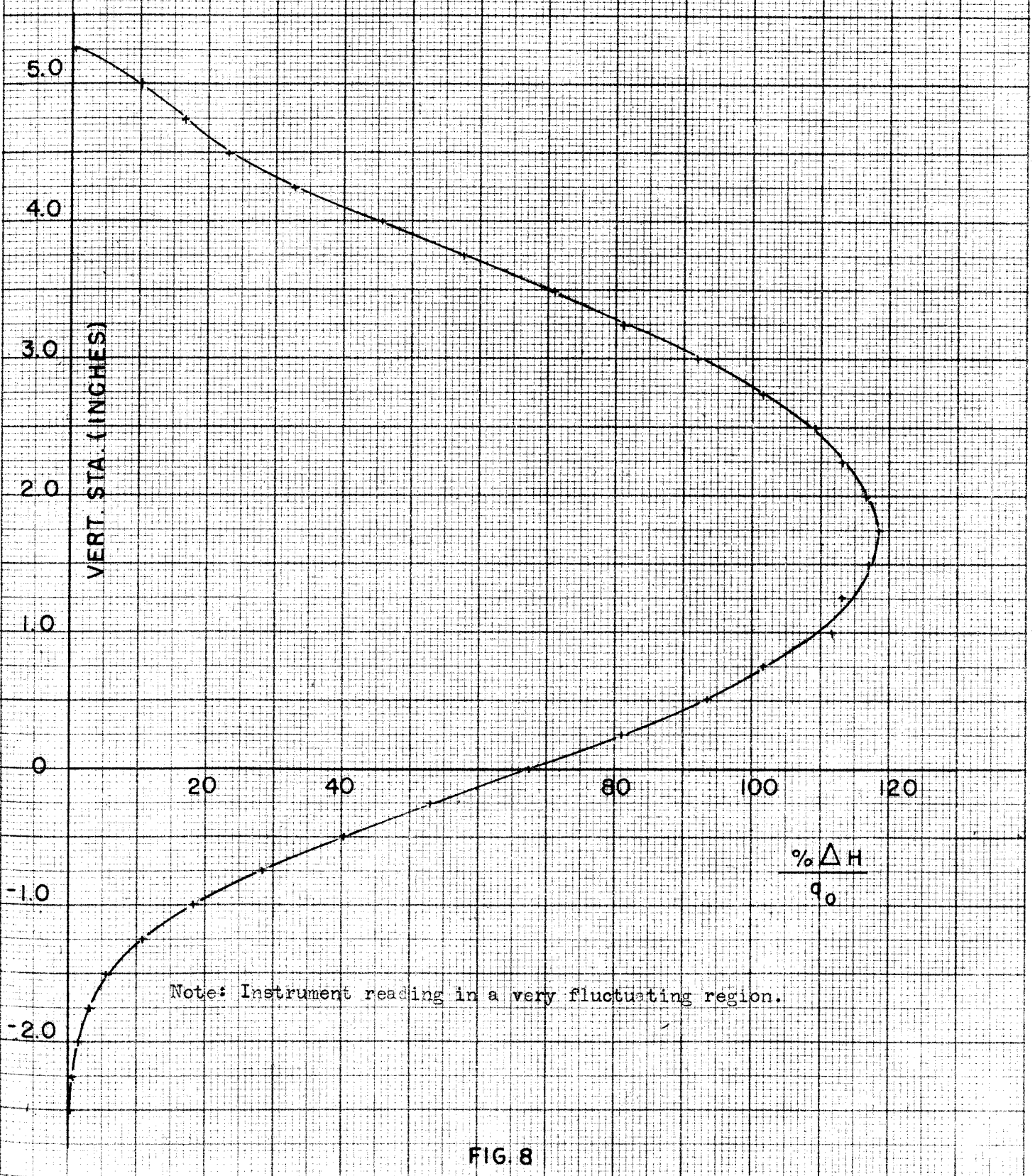
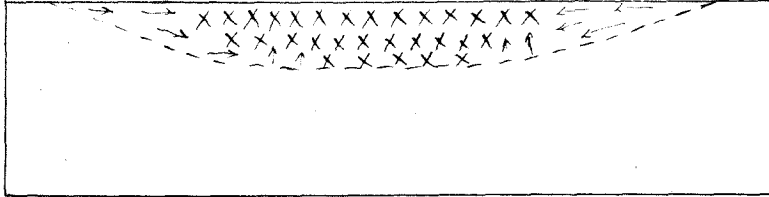
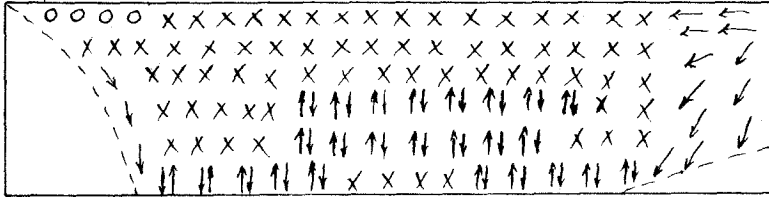


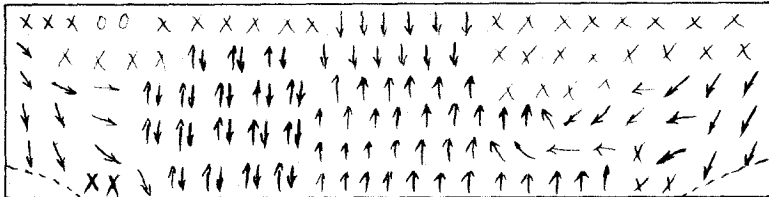
FIG. 8



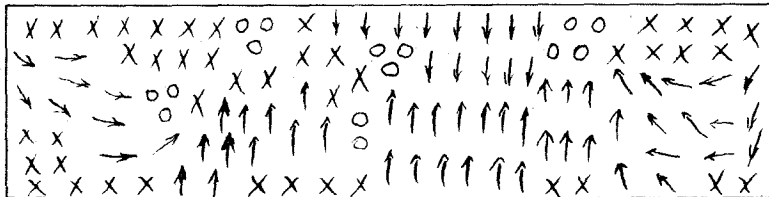
$\alpha = 6^\circ$



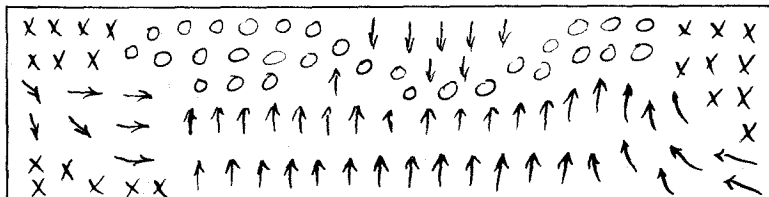
$\alpha = 8^\circ$



$\alpha = 12^\circ$



$\alpha = 15^\circ$



$\alpha = 19^\circ$

LEGEND

- STEADY FLOW
- ⇌ OSCILLATING FLOW
- x TURBULENT FLOW
- o STAGNANT

FIG. 8 a



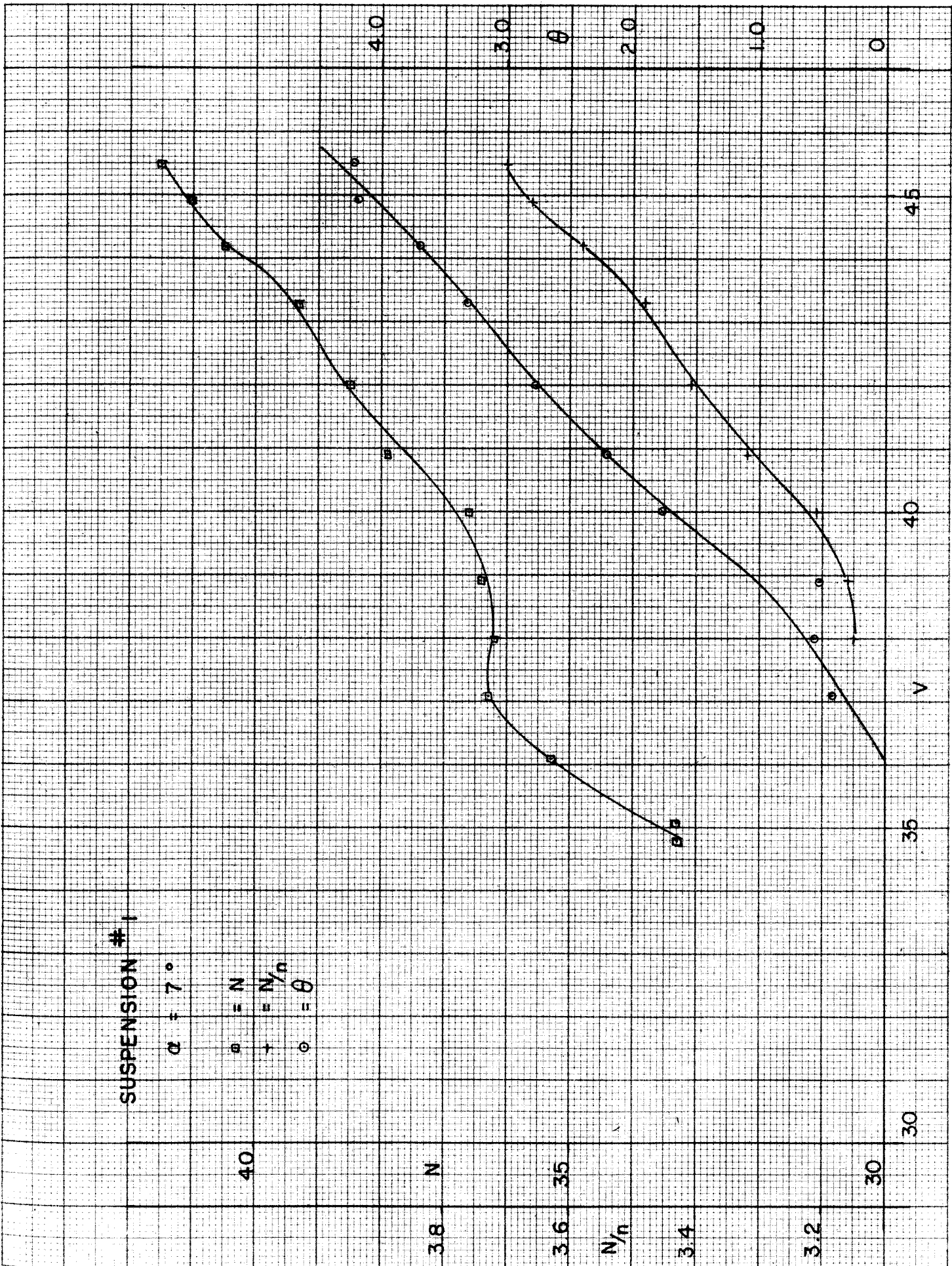


FIG. 9

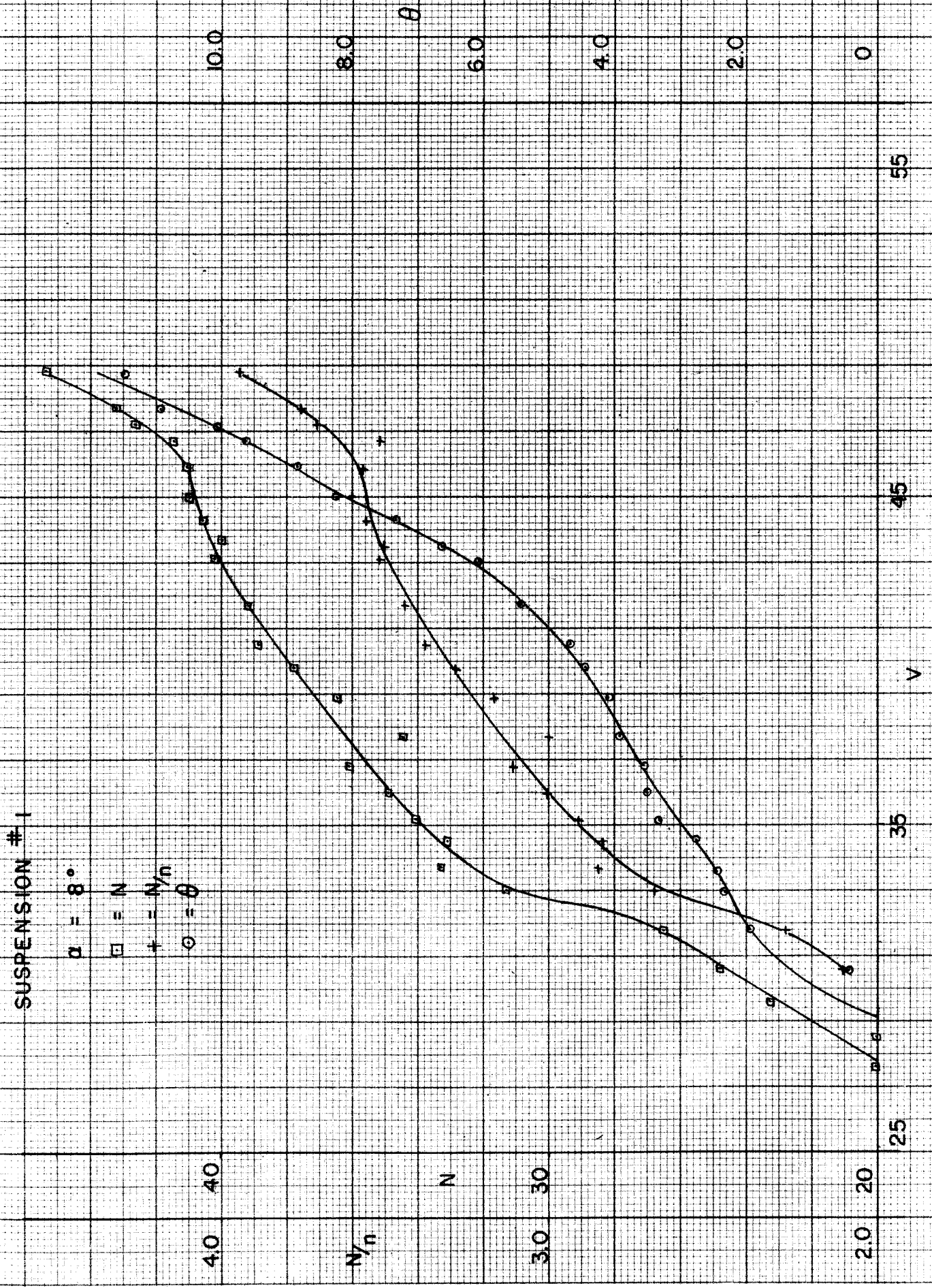


FIG. 10

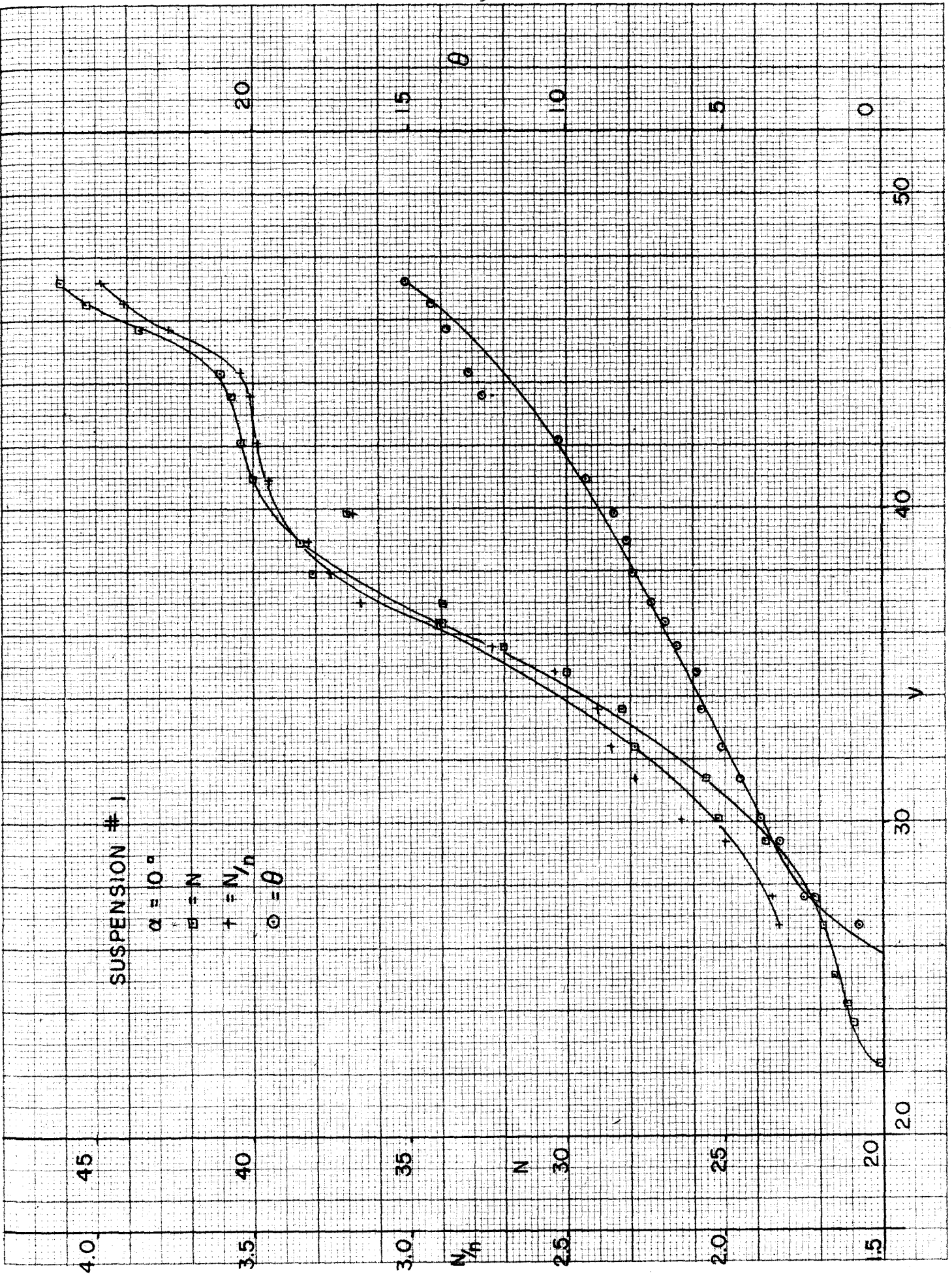


FIG. II

SUSPENSION # 1

$$\alpha = 12^\circ$$

$$\square = N$$

$$+ = N/n$$

$$\circ = \theta$$

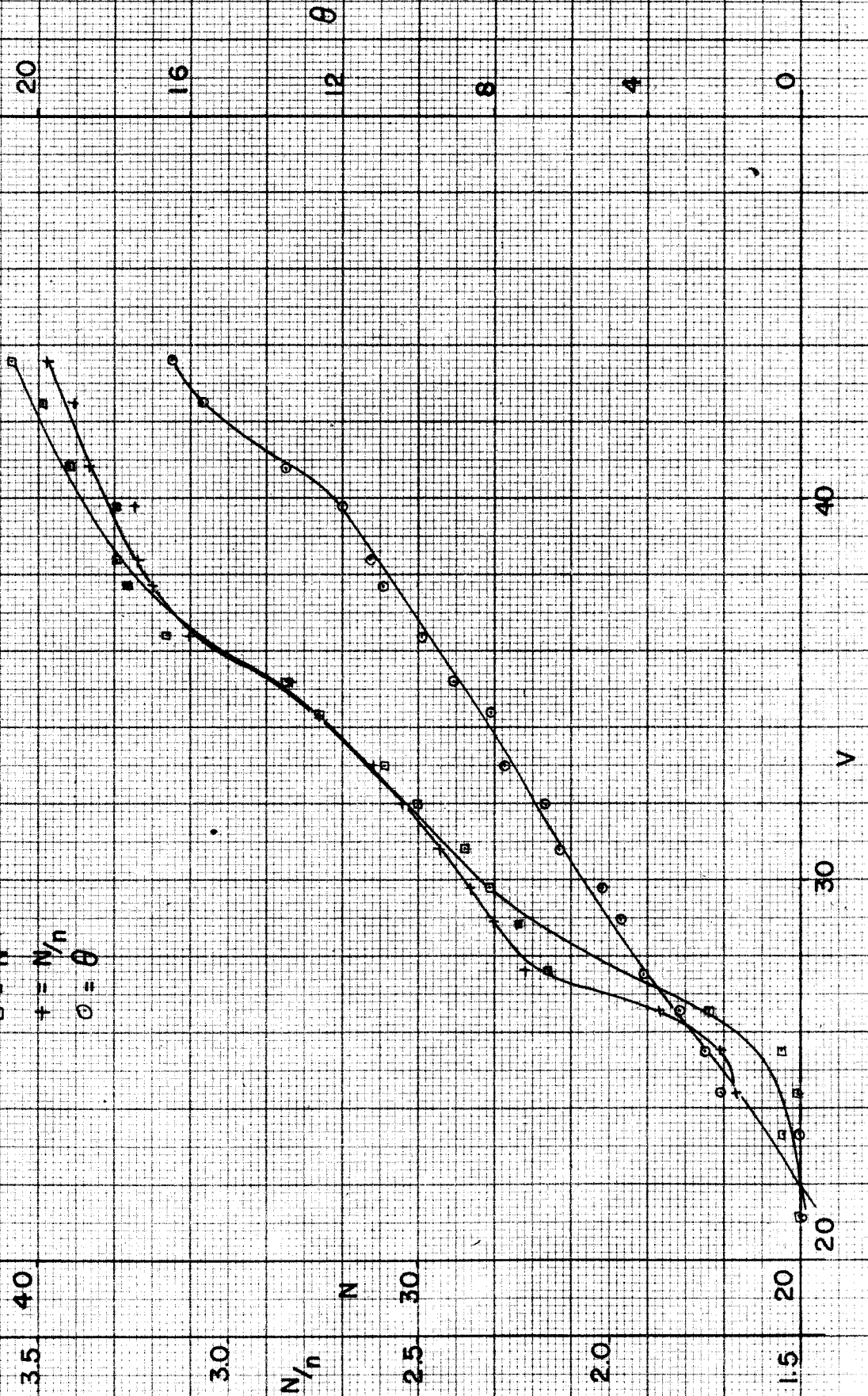


FIG. 12

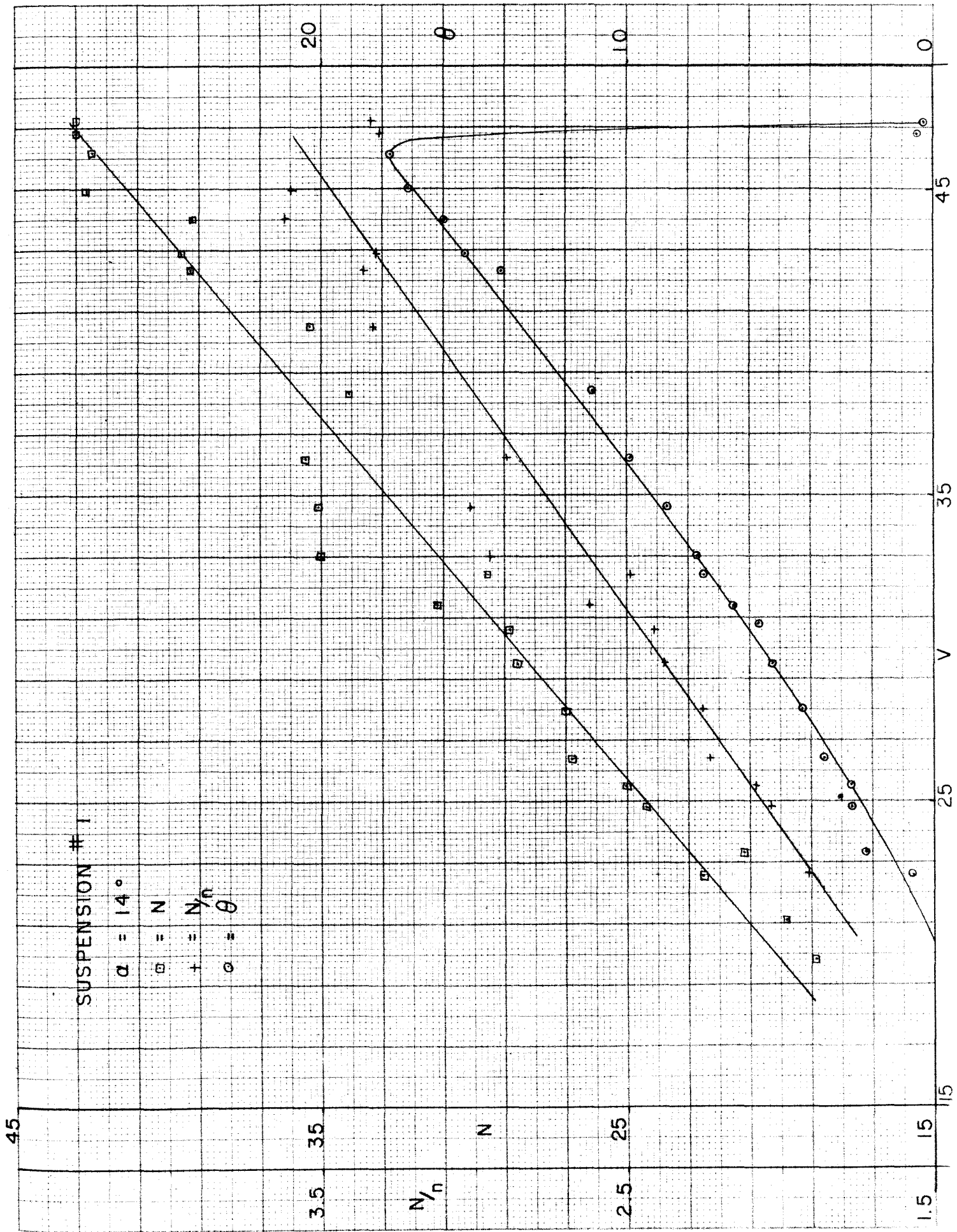


FIG. 13

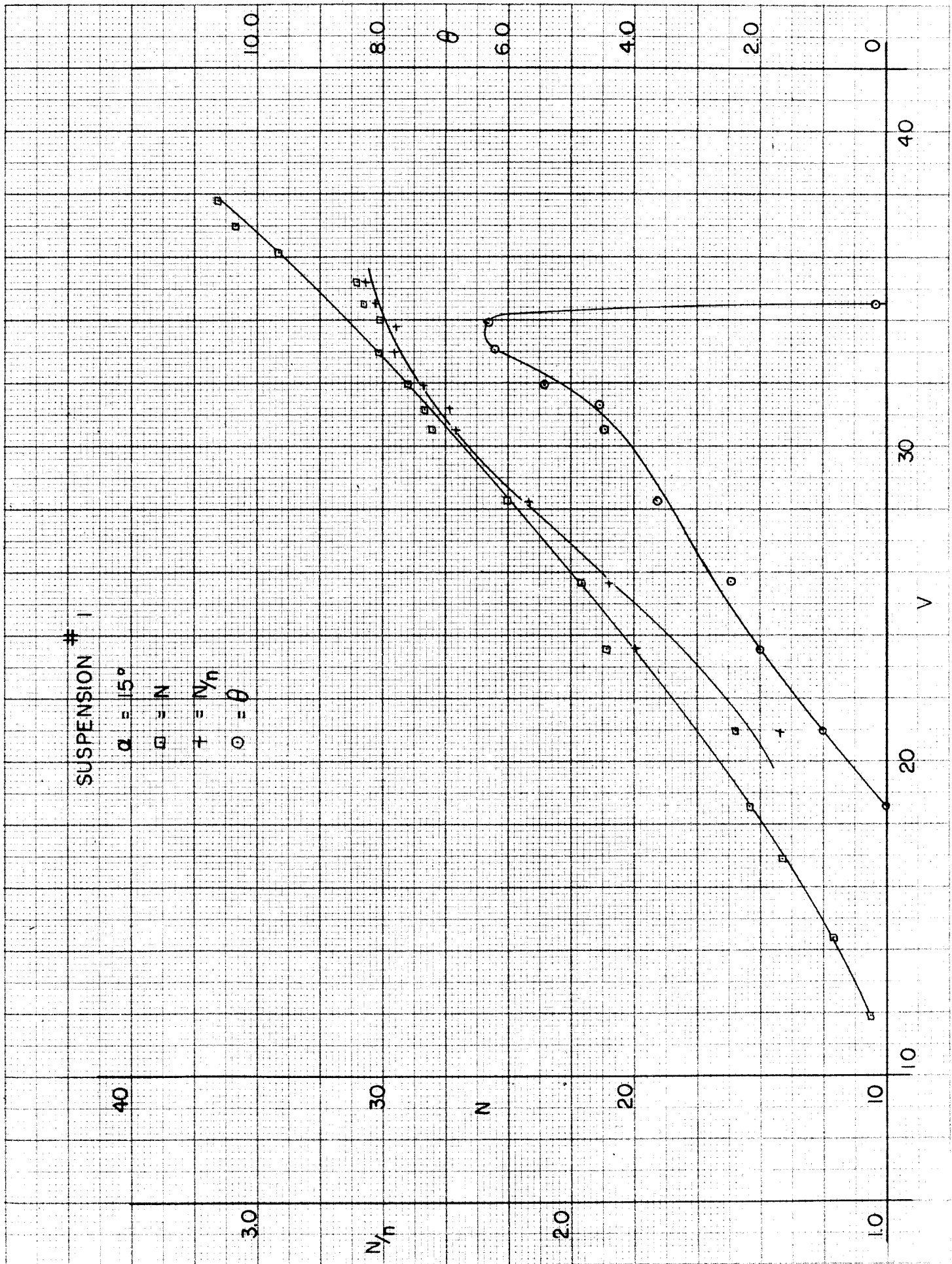


FIG. 14

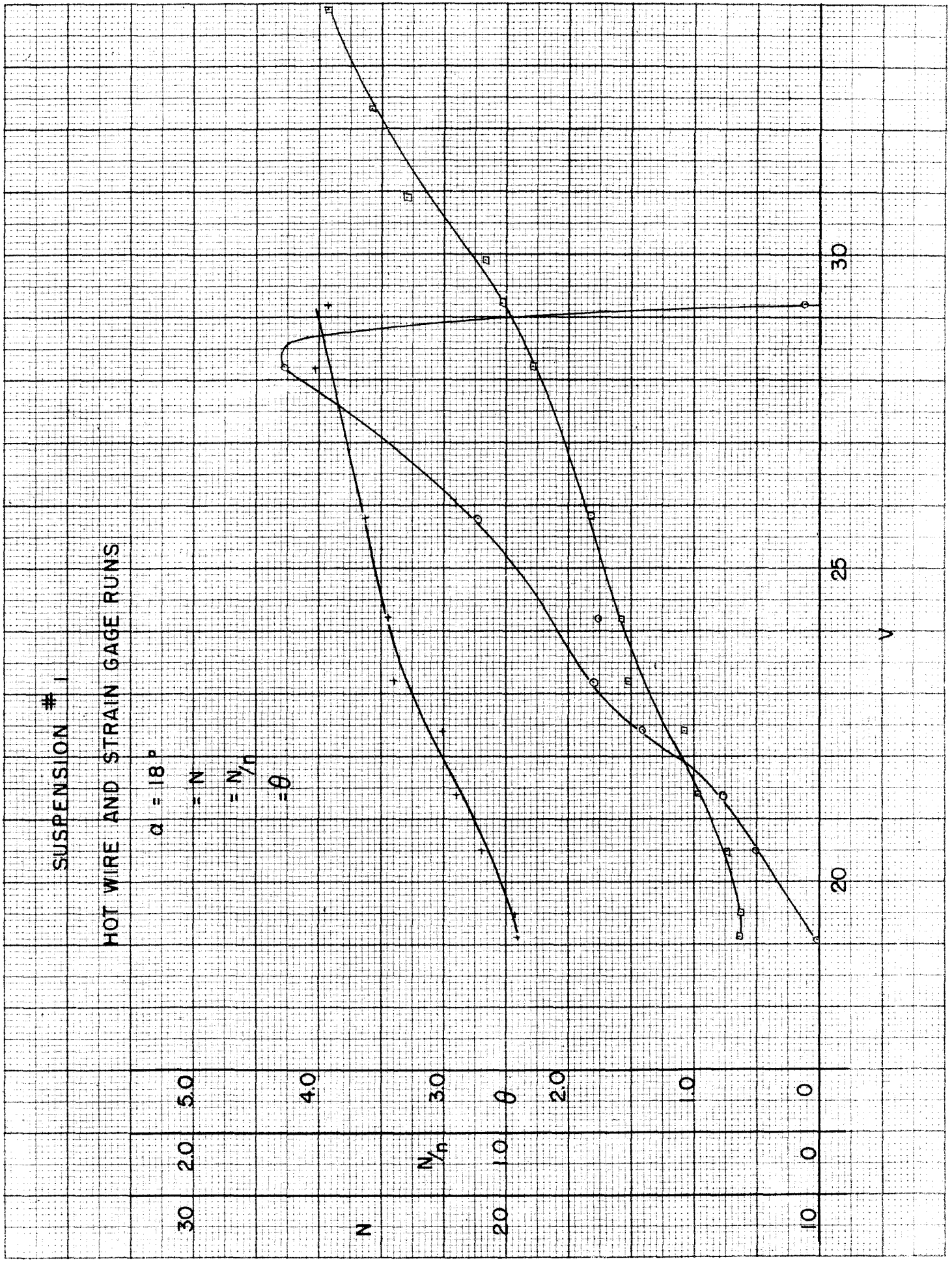


FIG. 15

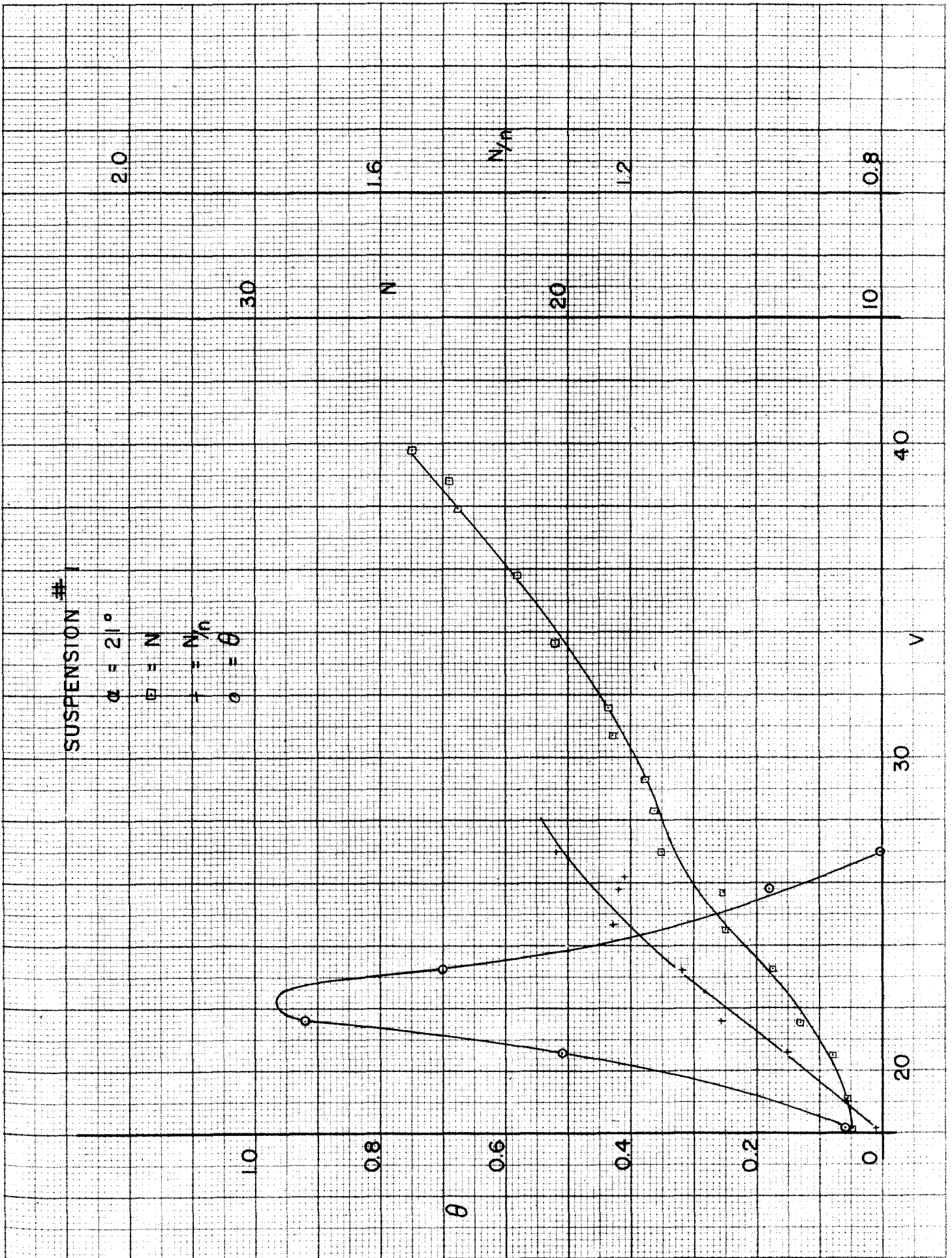


FIG. 16



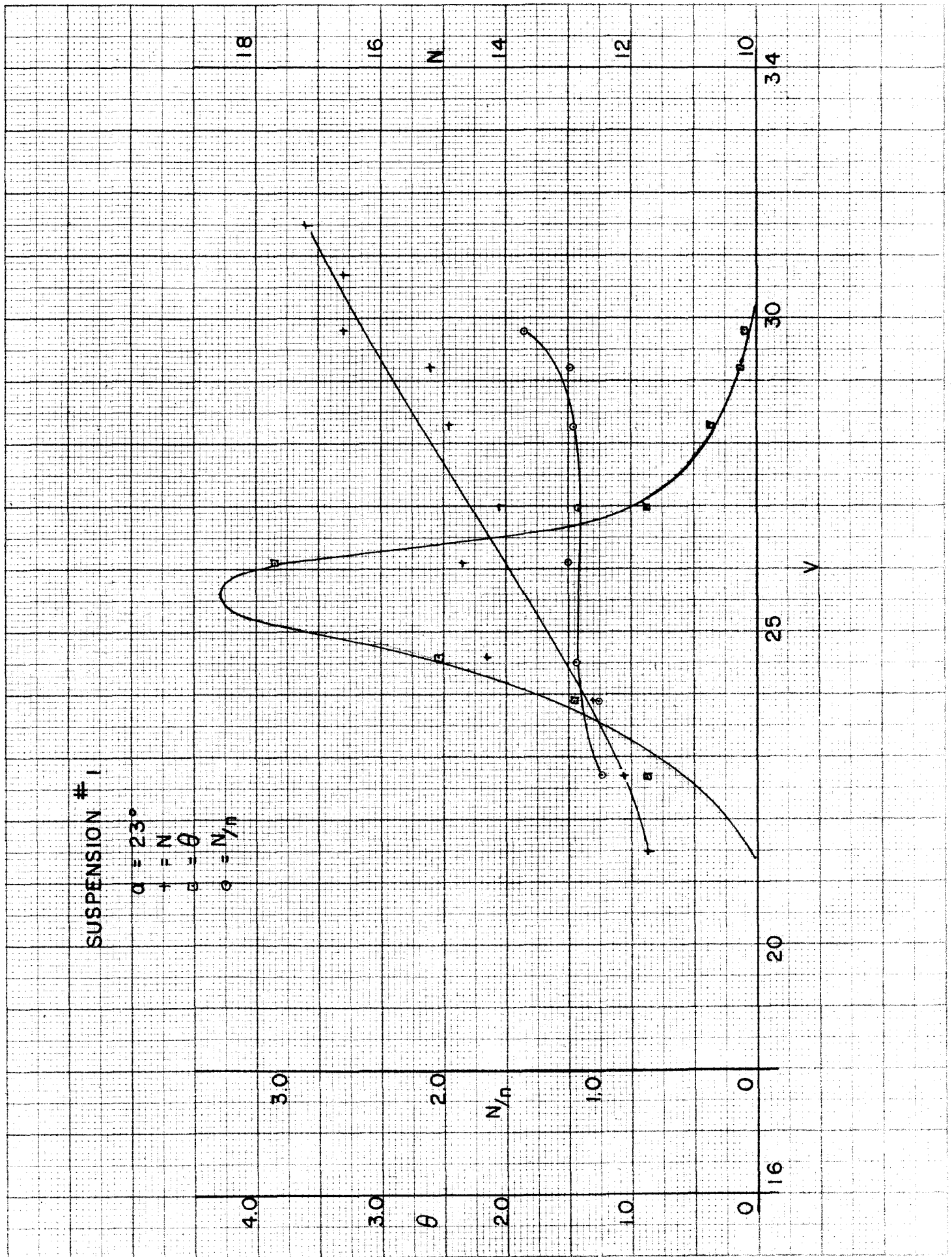


FIG. 17

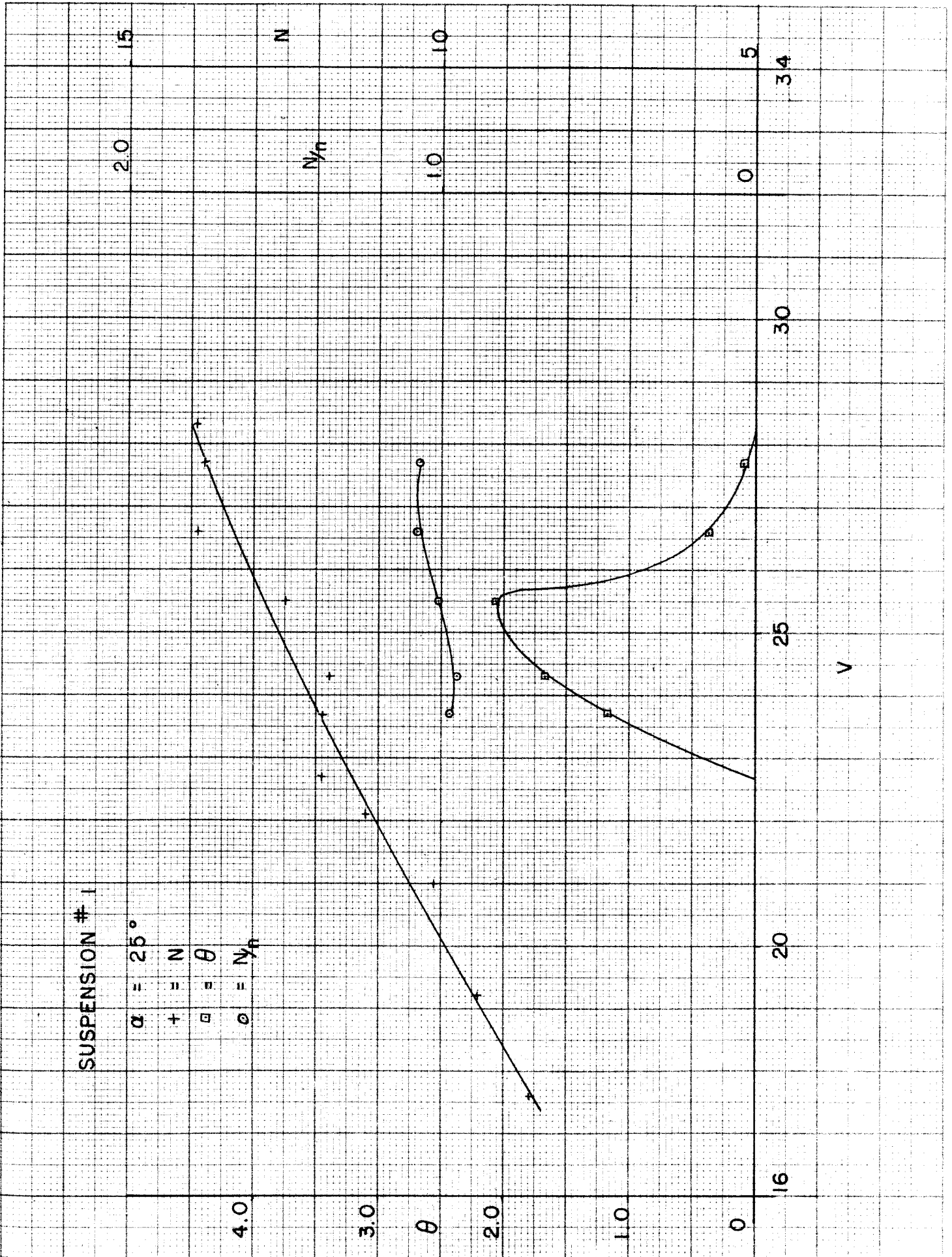


FIG. 18

SUSPENSION # 1

$\alpha = 27^\circ$

$\mu = \theta$

$t = N$

$\phi = N/n$

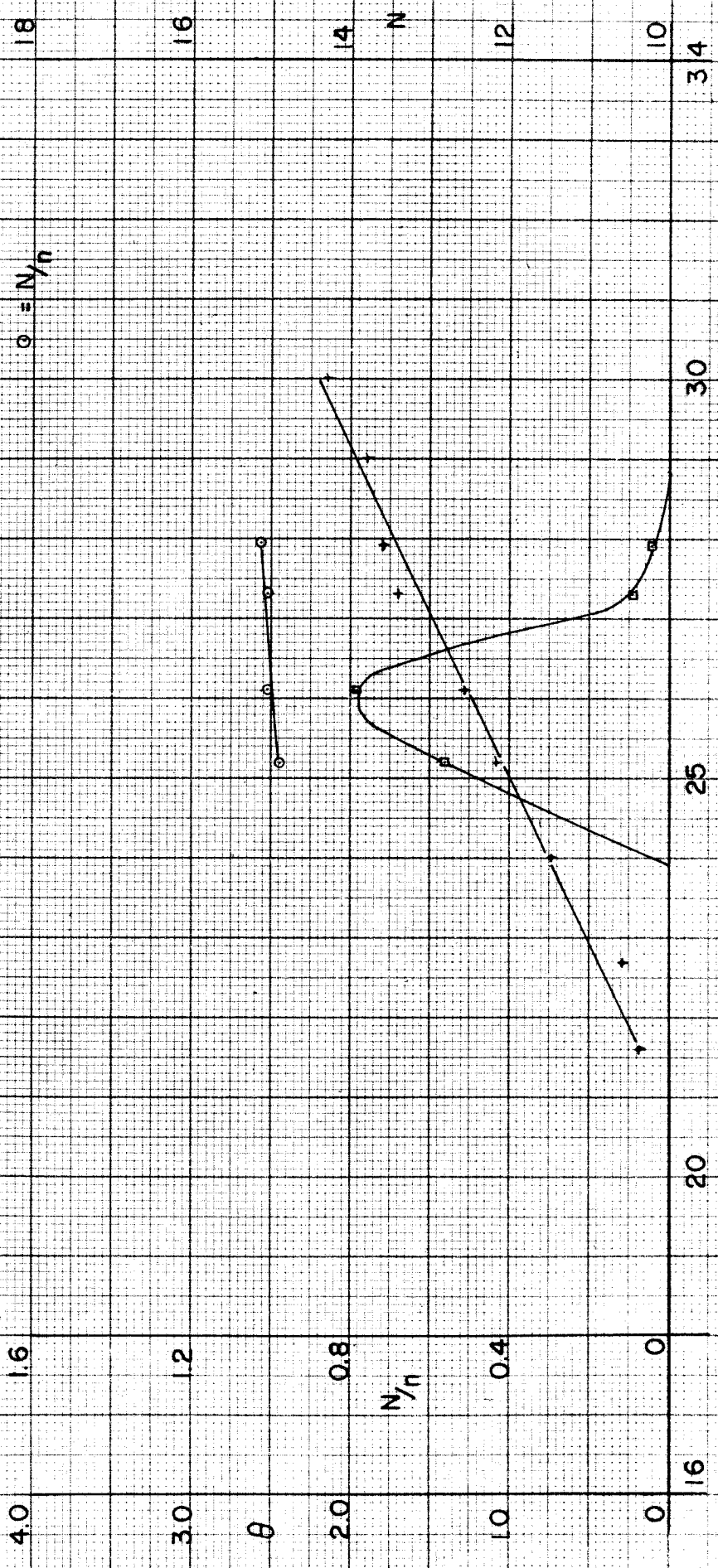


FIG. 19

SUSPENSION # 1

$\alpha = 29^\circ$

$+ = N$

$\square = \theta$

$\circ = N/A$

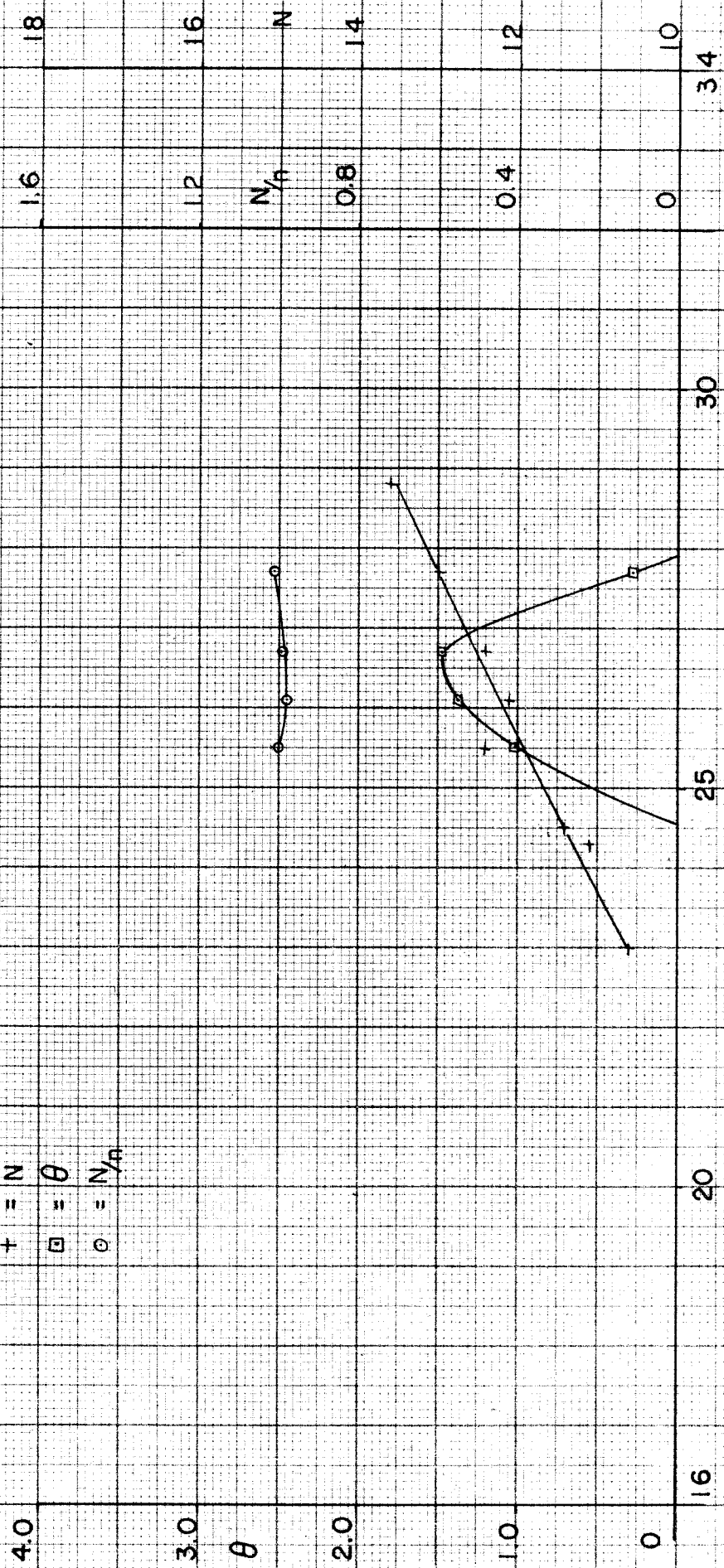


FIG. 20

SUSPENSION # 1

□ = V1

+ = V2

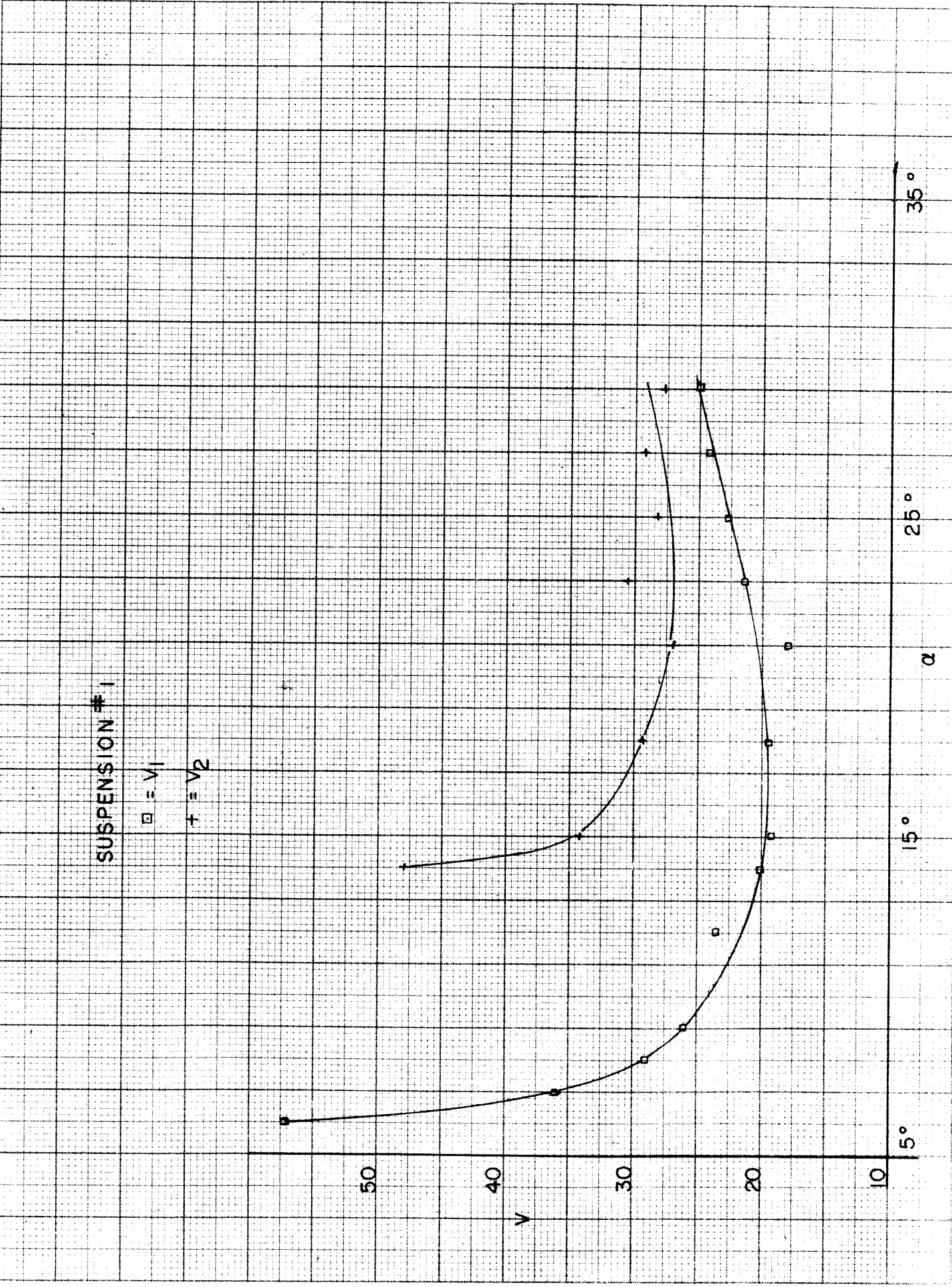


FIG. 21

SUSPENSION #1

+ = N1

□ = N2

50

40

N

30

20

10

n = 12

5°

15°

$\alpha$

25°

35°

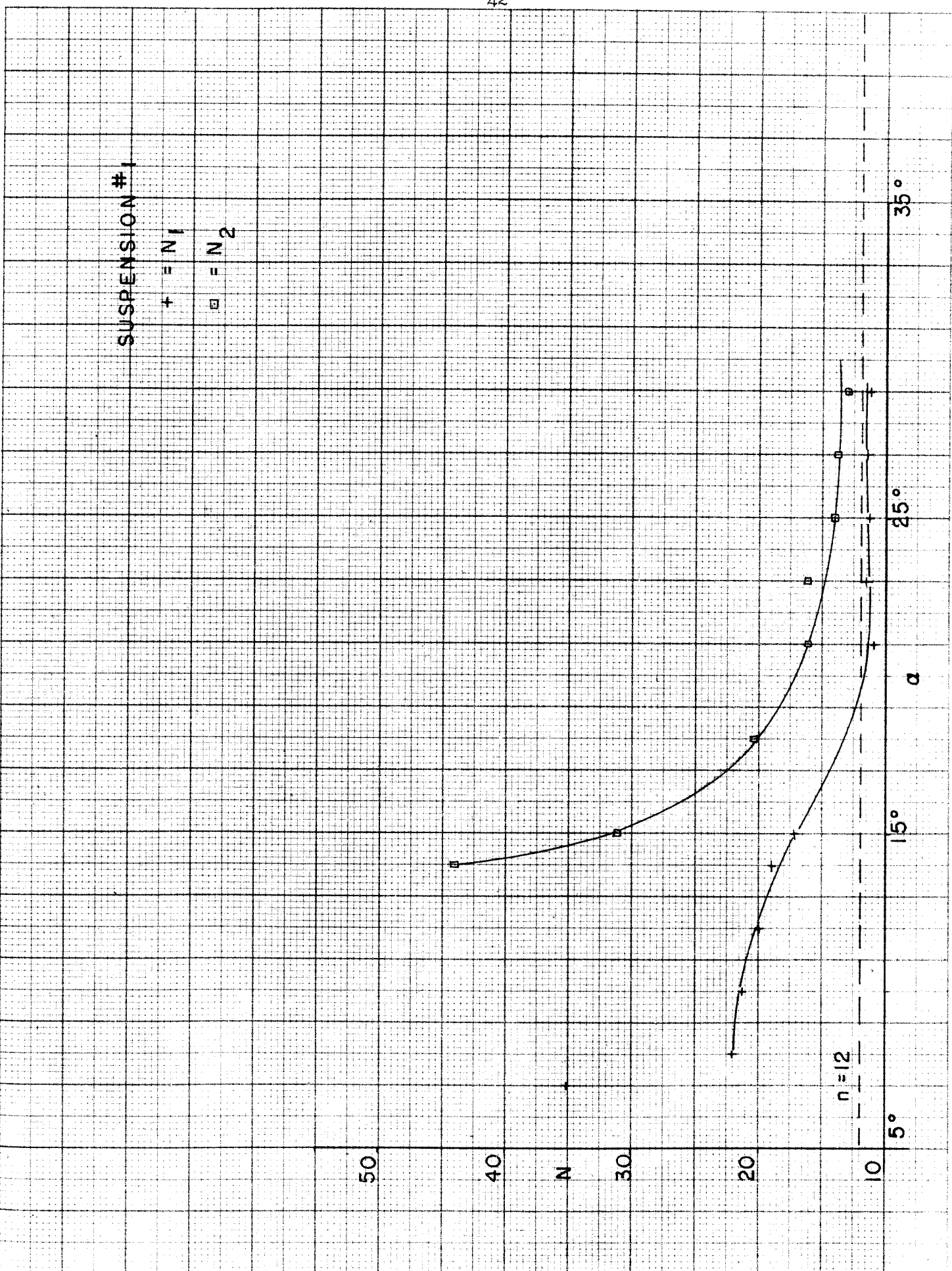


FIG. 22

SUSPENSION # 1

$$\sigma = \frac{N_1}{n}$$

$$+ = \frac{N_2}{n}$$

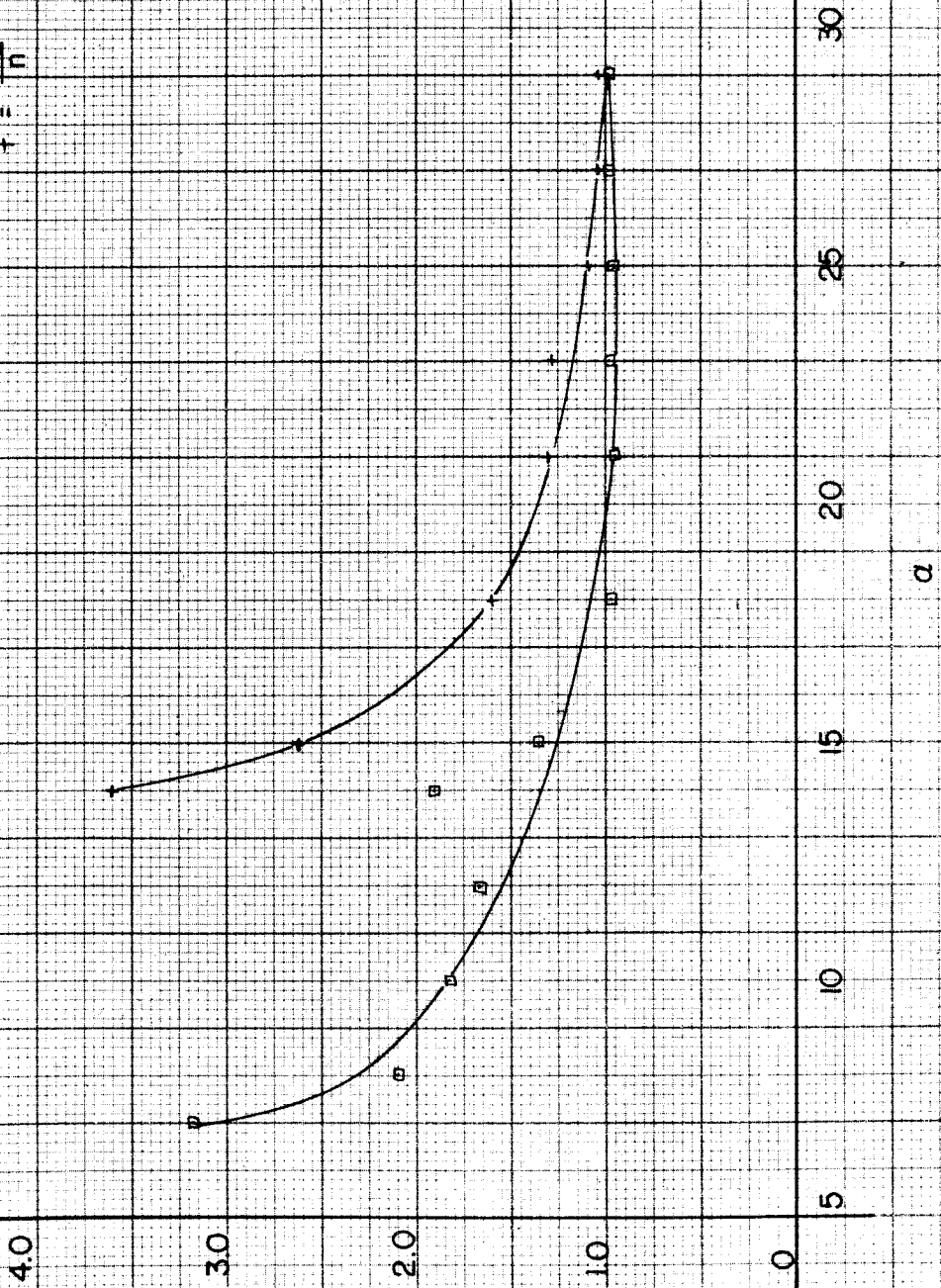


FIG. 23

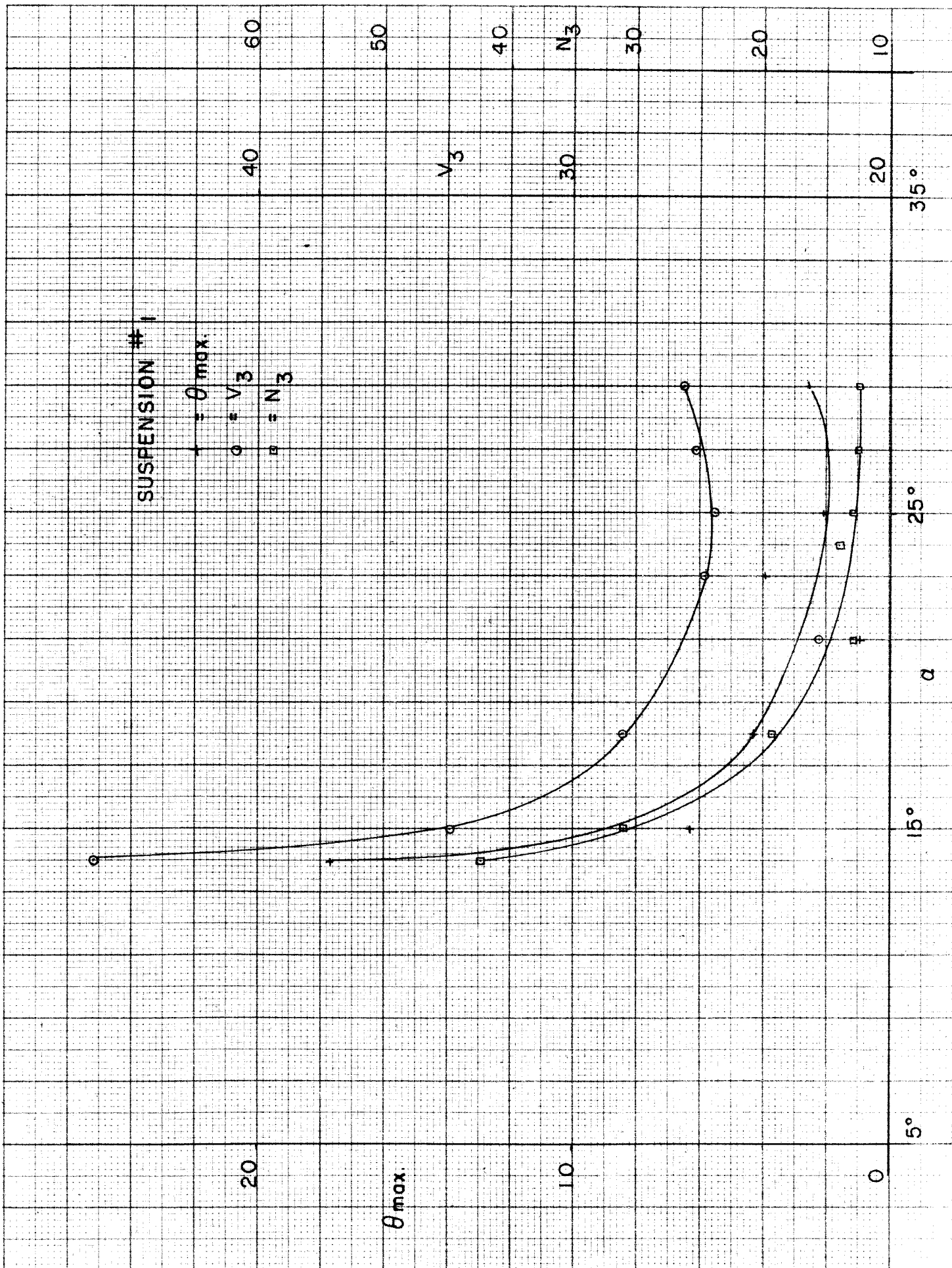


FIG. 24



SUSPENSION # |  
 □ V = 20 fps  
 + V = 25 " "  
 ○ V = 30 " "  
 △ V = 35 " "  
 × V = 40 " "  
 ◇ V = 45 " "

20

θ

10

0

5°

15°

25°

35°

α

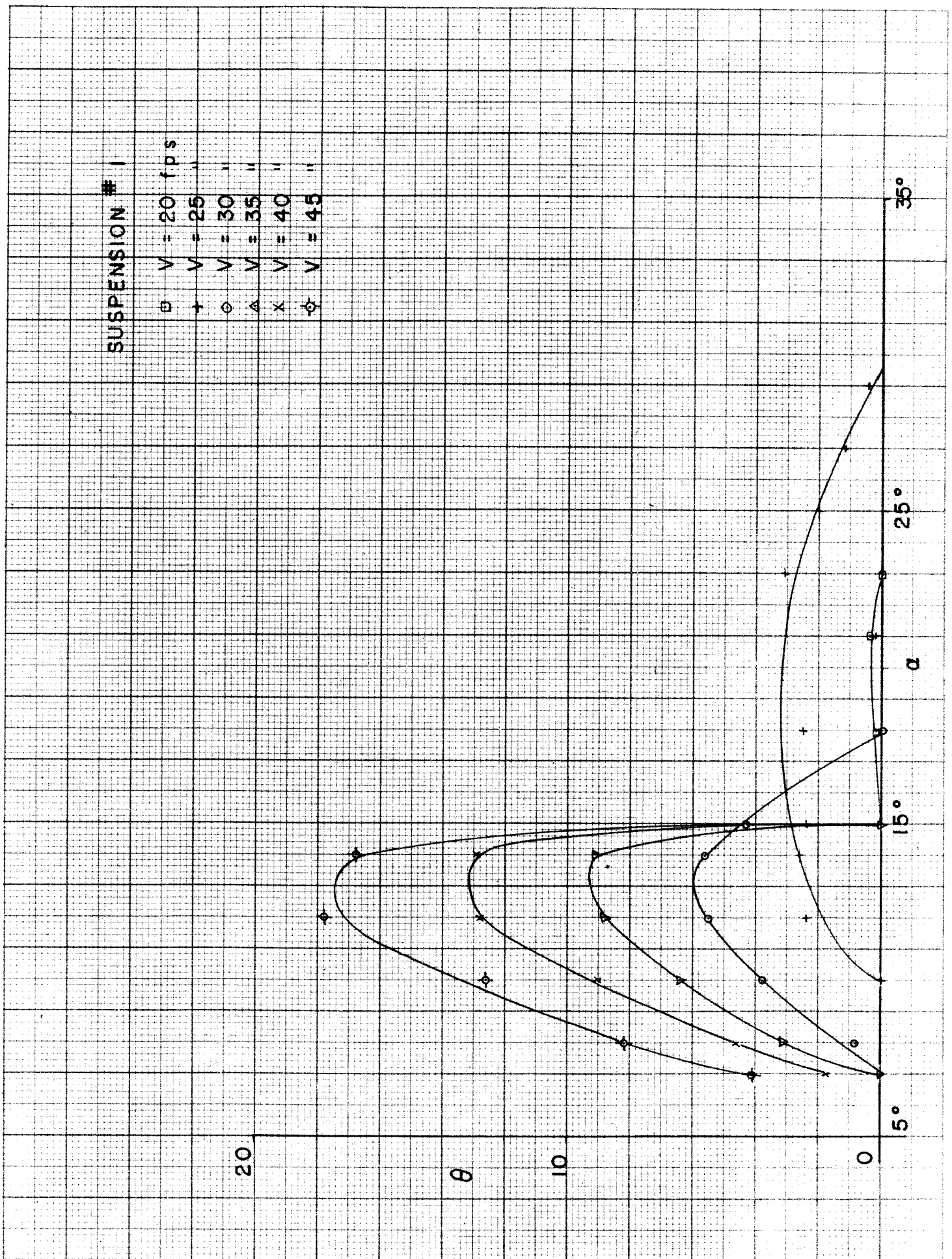


FIG. 25

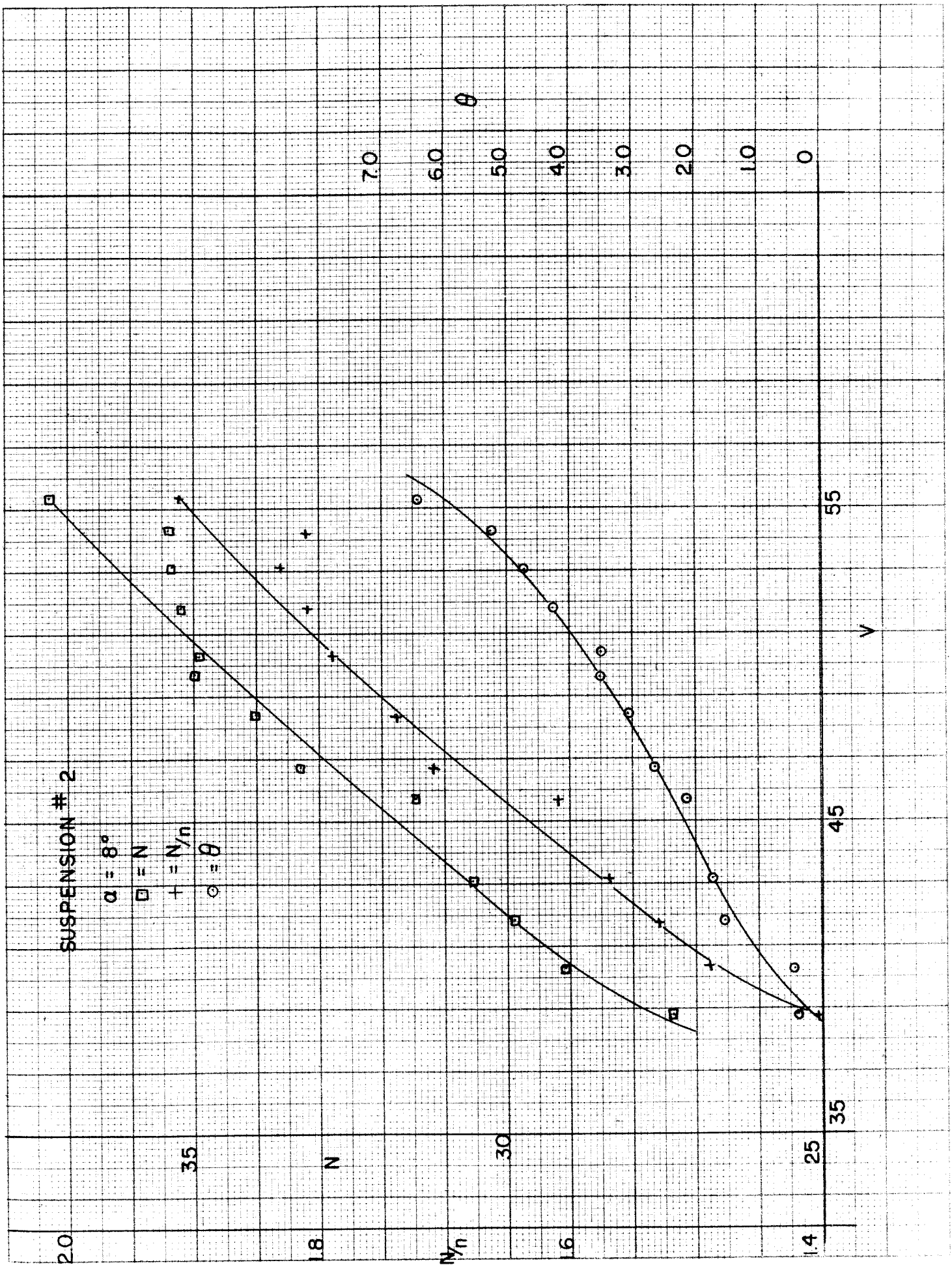


FIG. 26

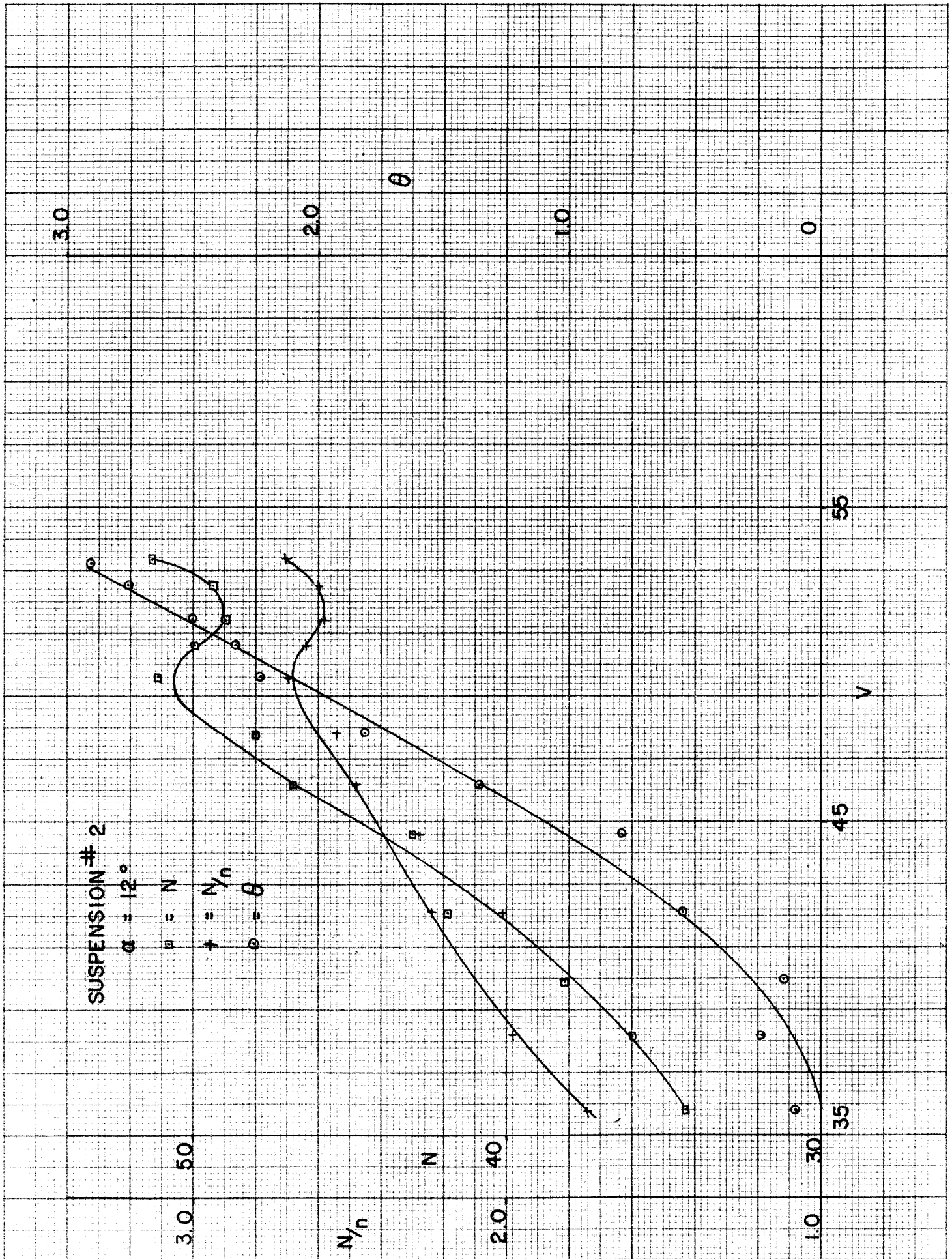


FIG. 27

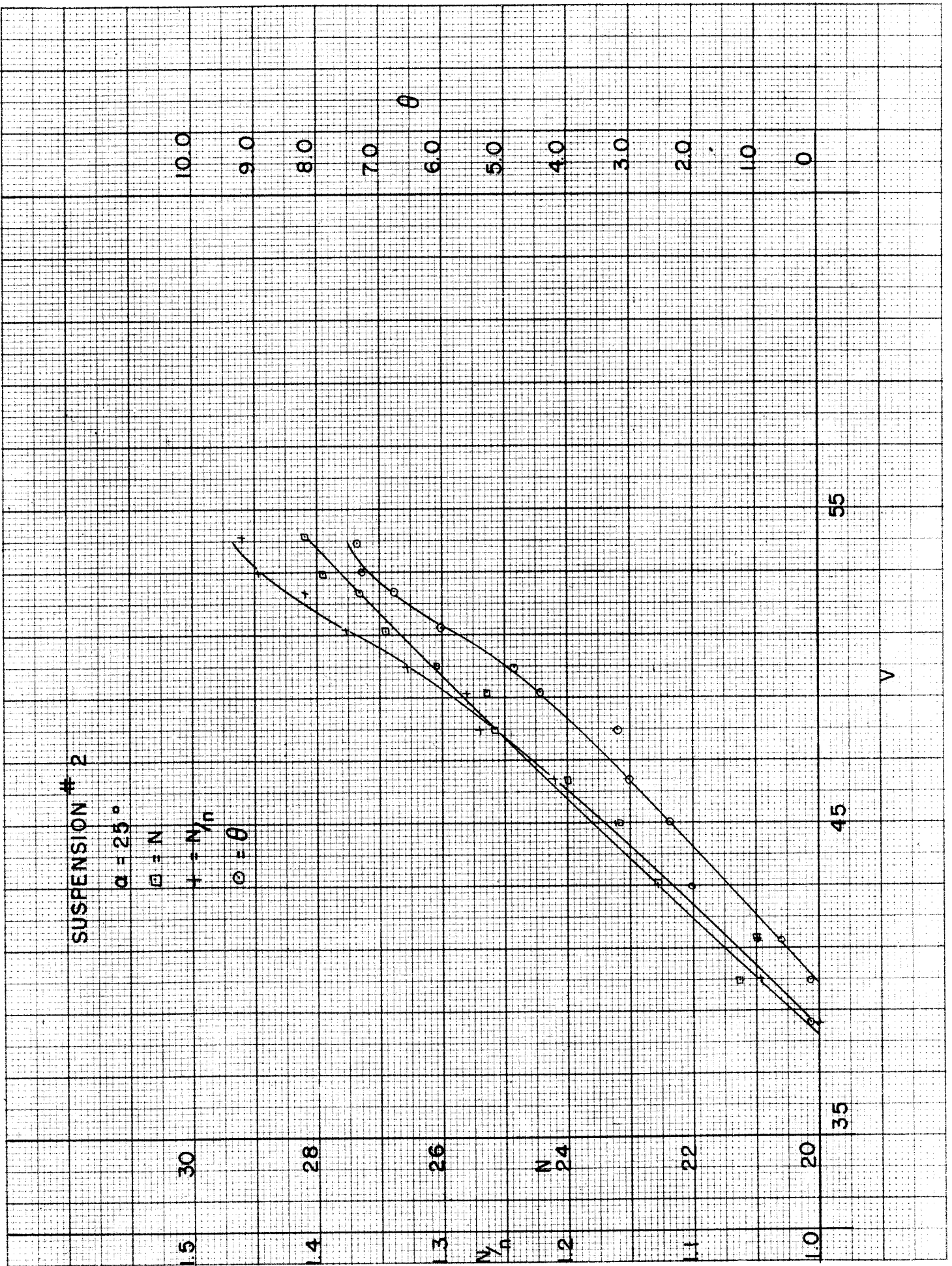


FIG. 28

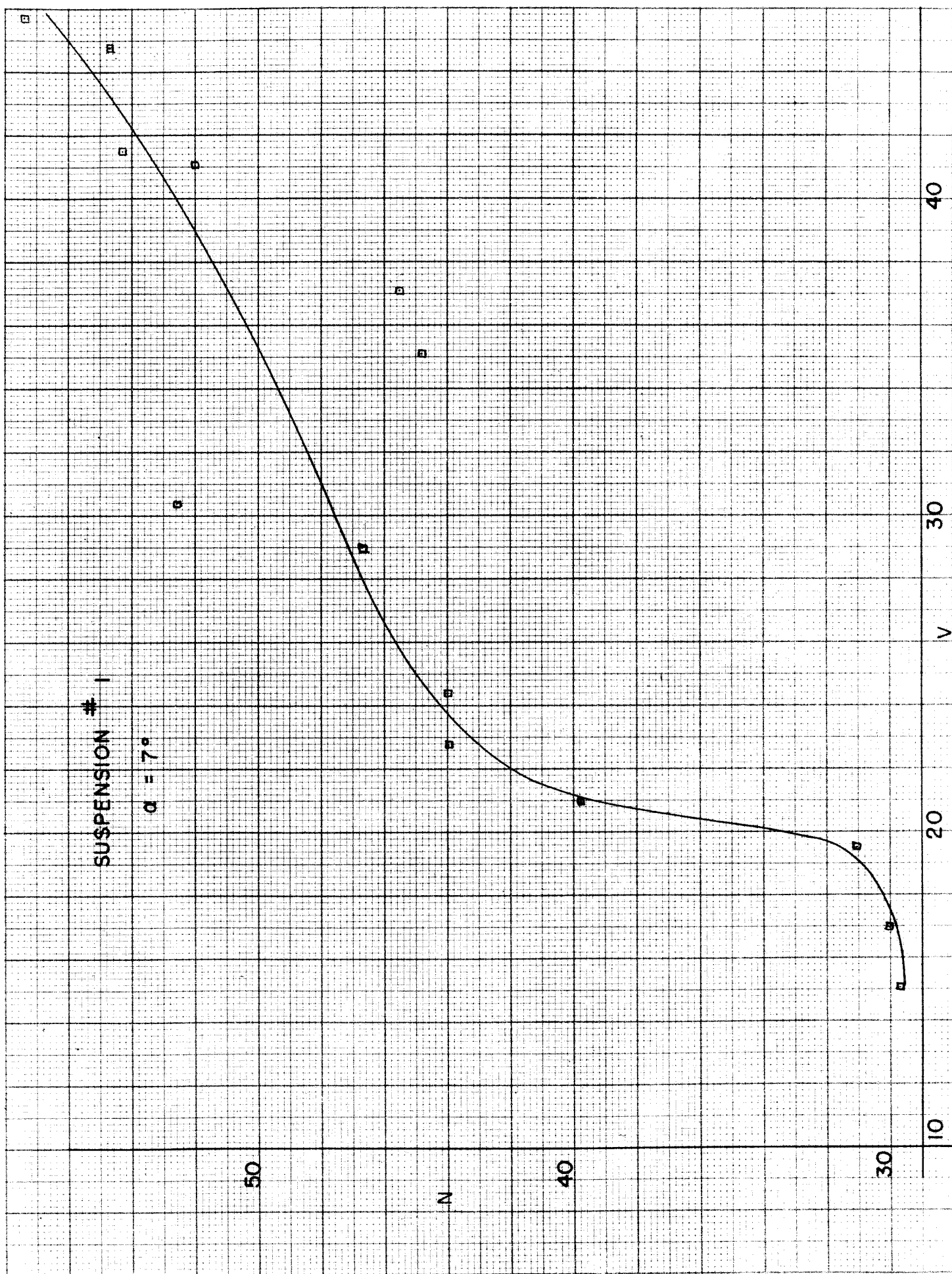


FIG. 29

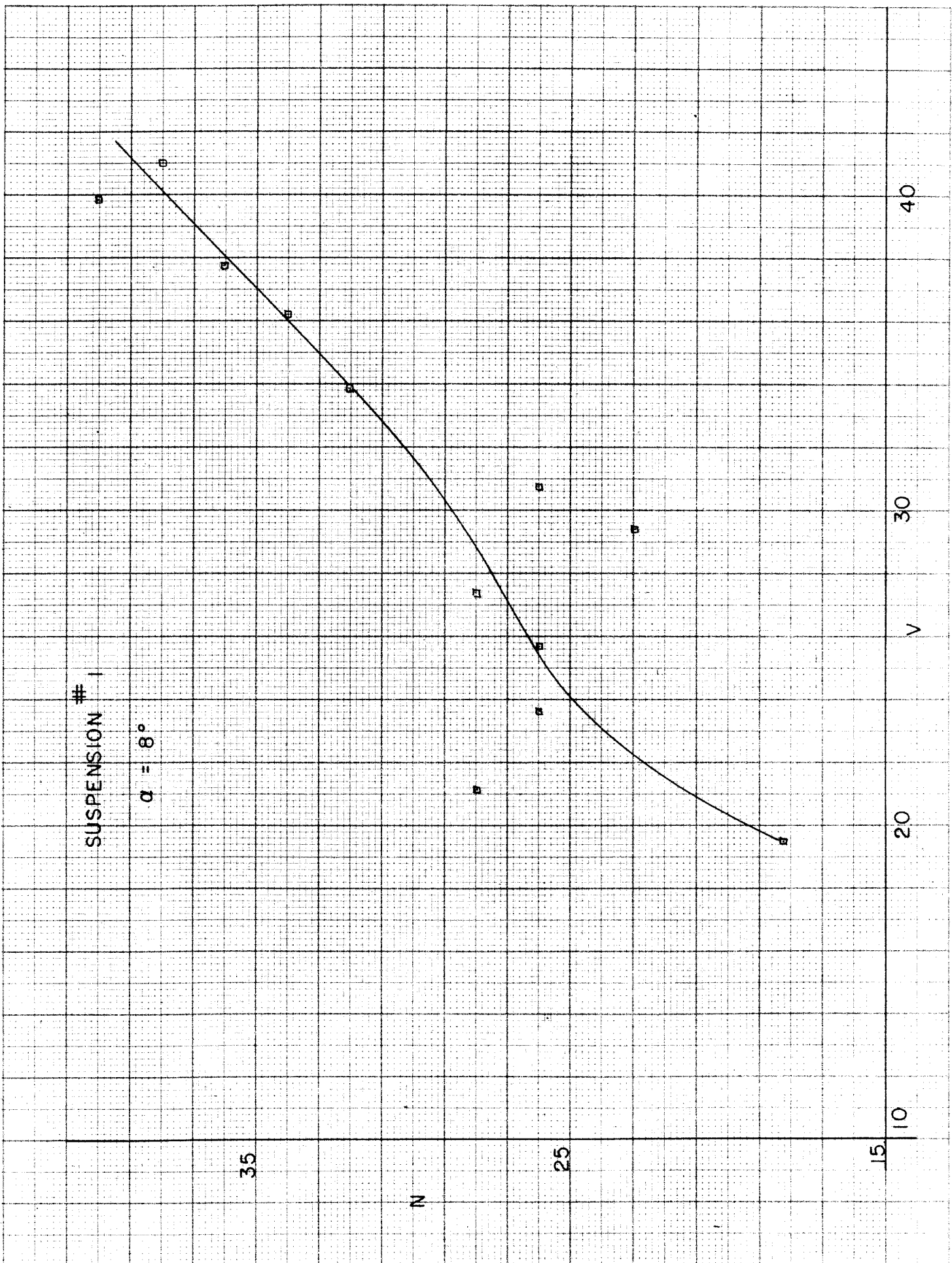


FIG. 30

SUSPENSION # 1

$\alpha = 23^\circ$

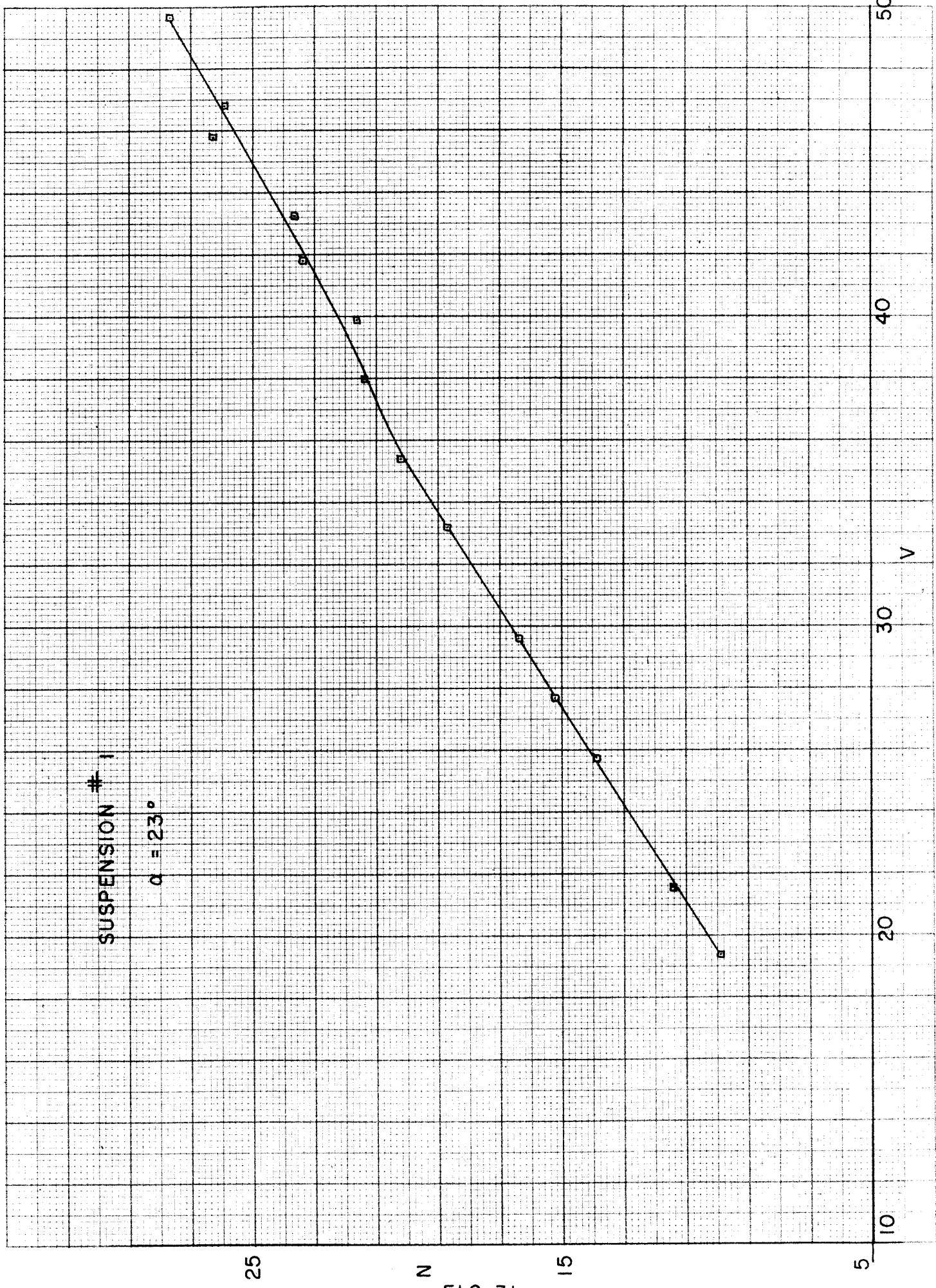


FIG. 31

SUSPENSION # 1  
 $\alpha = 27^\circ$

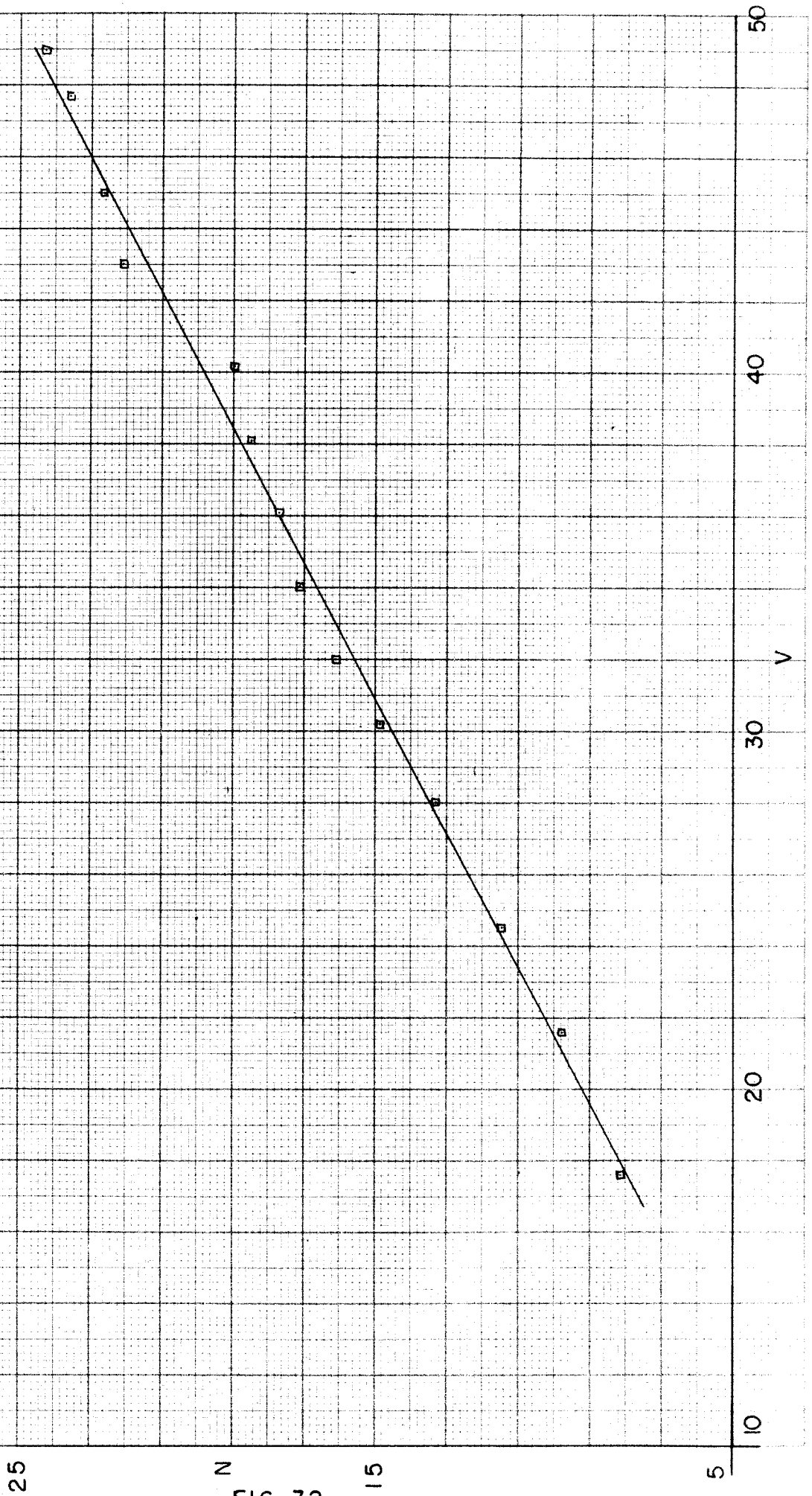


FIG 32



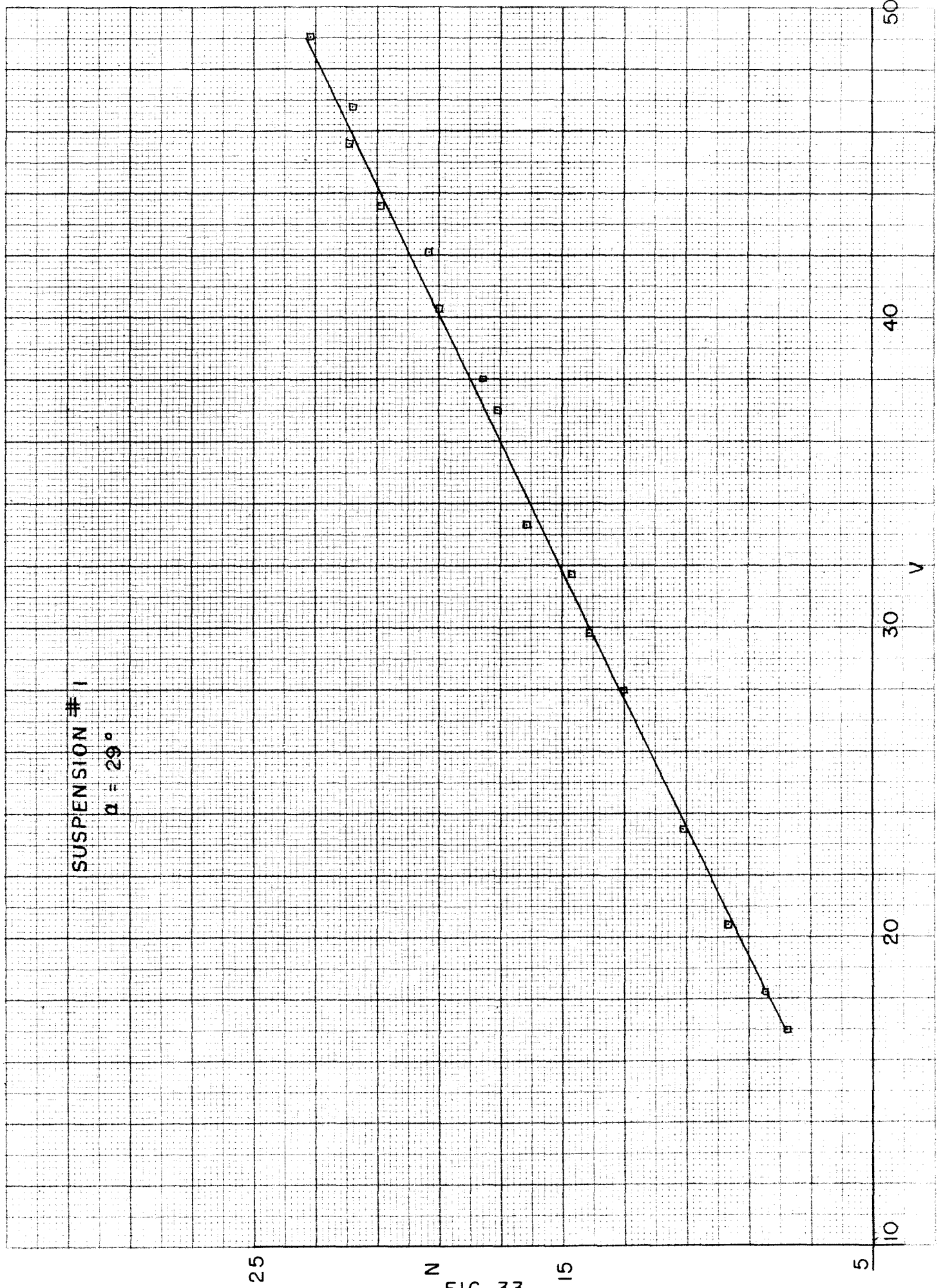


FIG. 33

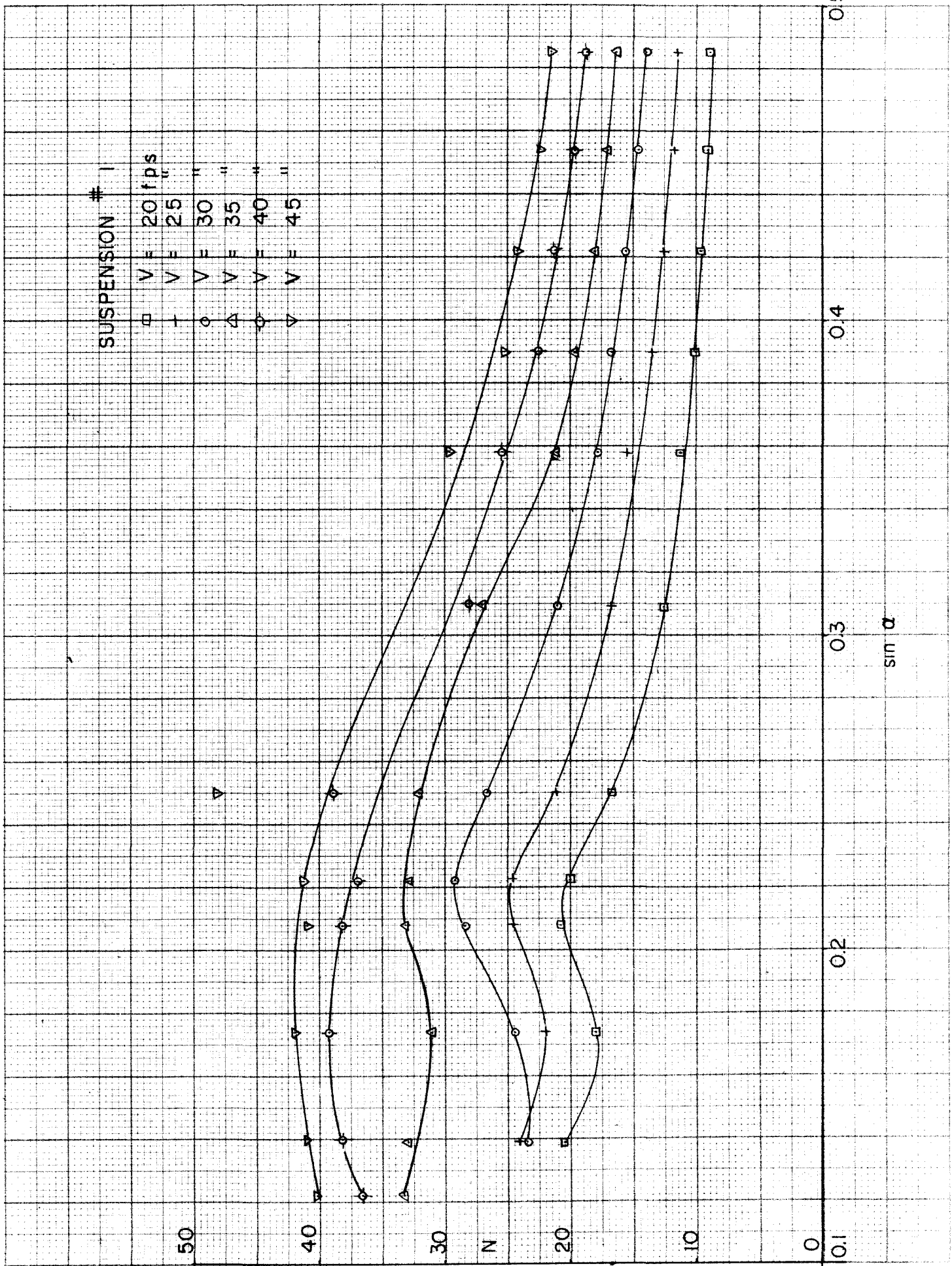


FIG. 34

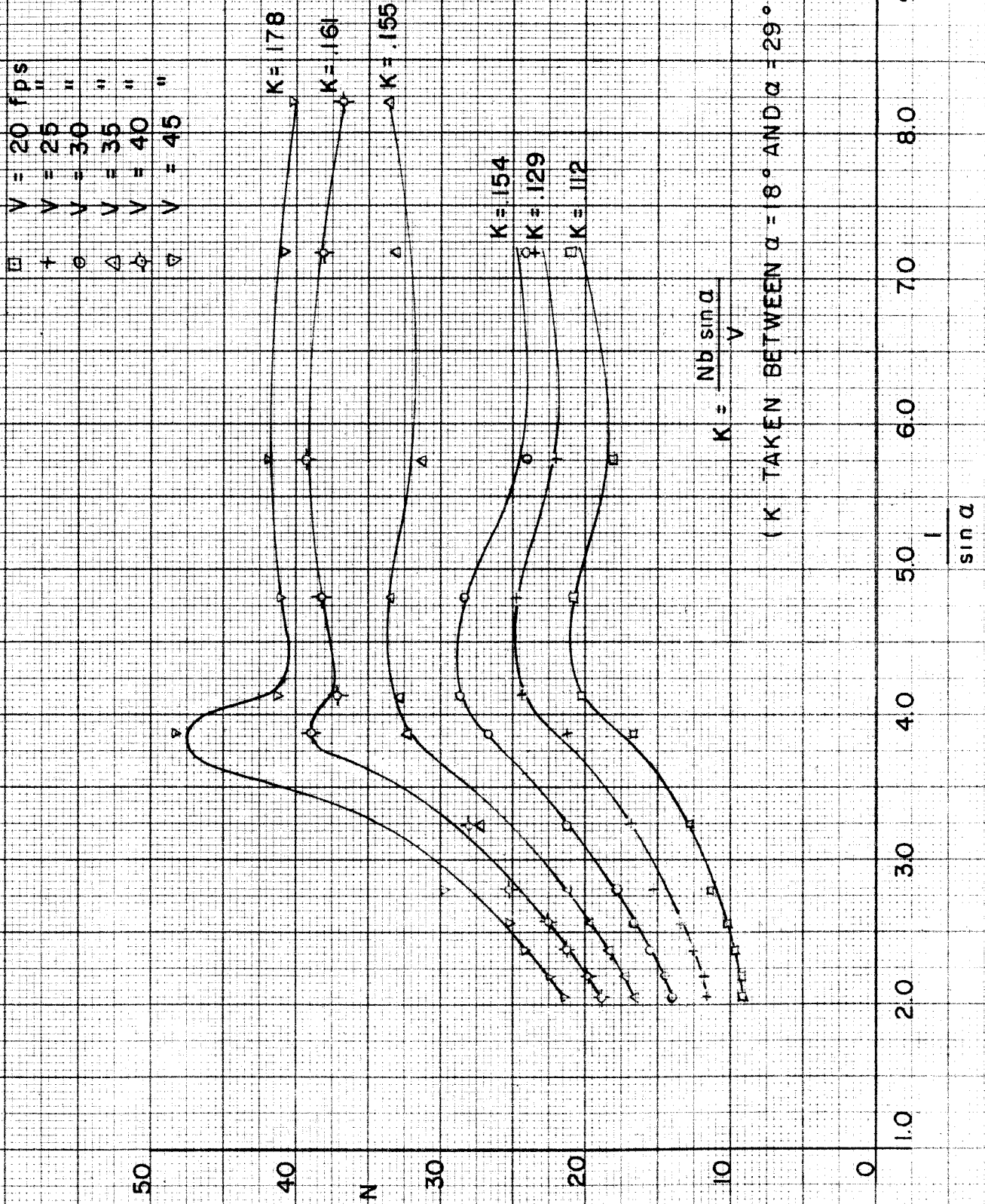


FIG. 35

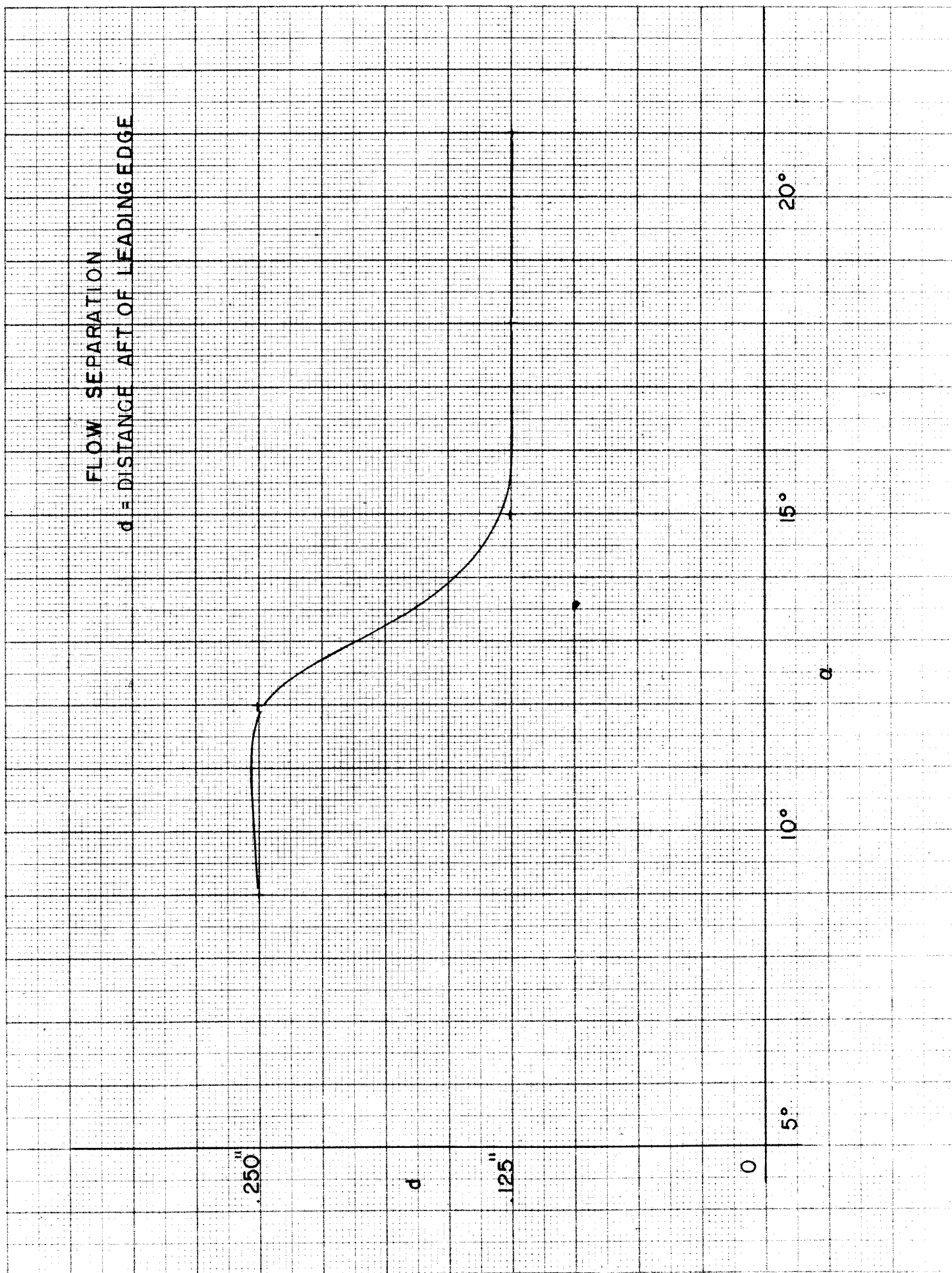
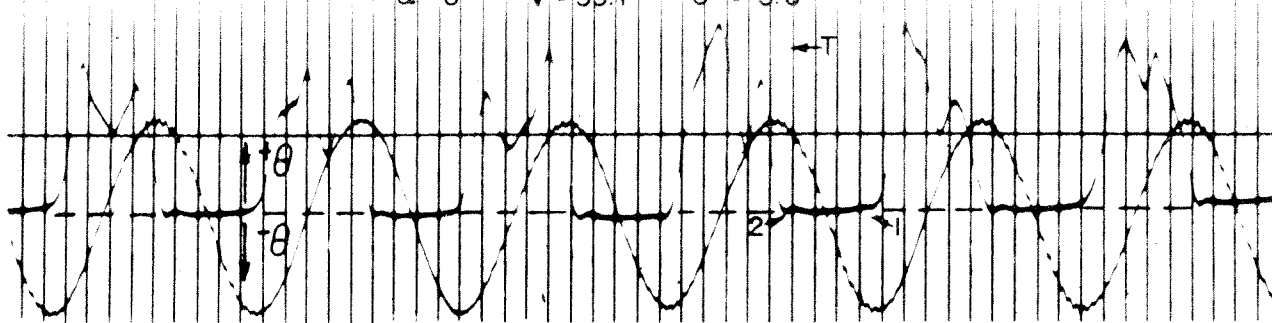
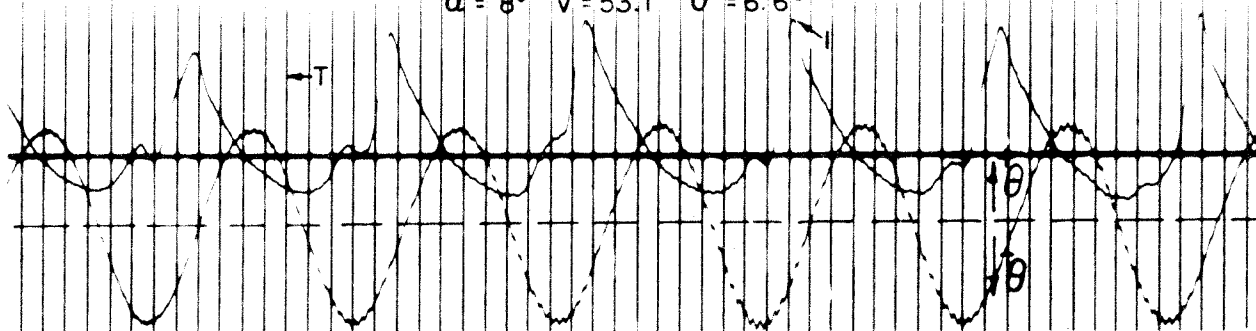


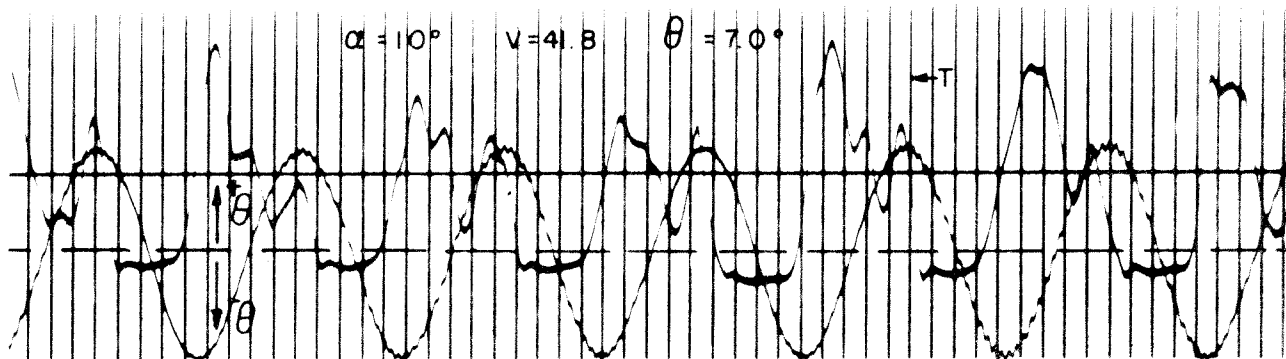
FIG. 36

$\alpha = 8^\circ$   $v = 53.1$  $\theta = 6.6^\circ$ 

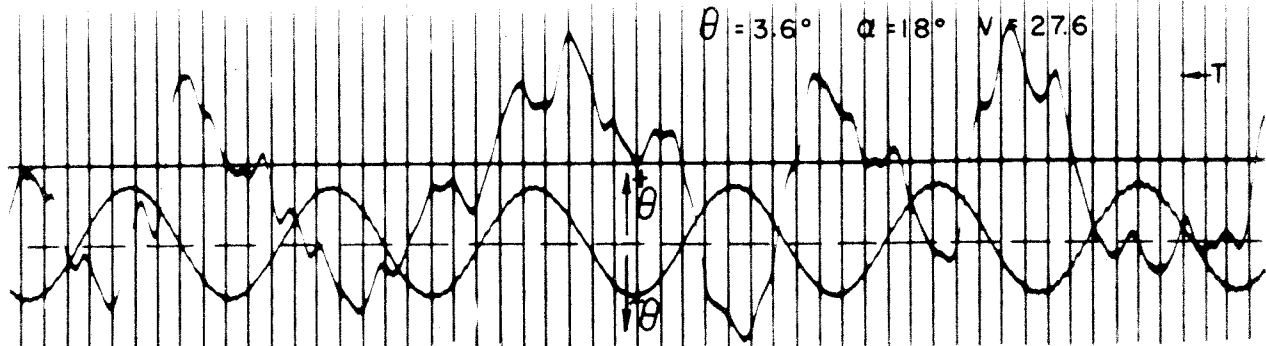
a

 $\alpha = 8^\circ$   $v = 53.1$  $\theta = 6.6^\circ$ 

b

 $\alpha = 10^\circ$   $v = 41.8$  $\theta = 7.0^\circ$ 

c

 $\theta = 3.6^\circ$   $\alpha = 18^\circ$   $v = 27.6$ 

d

FIG. 37