A DYNAMIC LONGITUDINAL STABILITY ANALYSIS FOR A CANARD TYPE AIRPLANE IN SUPERSONIC FLIGHT

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I INTRODUCTION AND SUMMARY

A dynamic longitudinal stability analysis is made for a Canard (tail forward) type airplane in steady horizontal flight at Mach numbers of 1.7 and 1.3. Four different wing configurations (Fig. 1) are investigated:

Case I. Delta wing with the Mach wave ahead of the leading edge. The planform of the delta wing is characterized by one-half the apex angle, w_o. In this case it has been taken to be 18°.

Case II. Delta wing with the Mach wave ahead of the leading edge ($w_o = 25^\circ$).

Case III. Delta wing with the Mach wave behind the leading edge ($w_o = 54^\circ$).

Case IV. Rectangular, bi-convex, wing with an aspect ratio of 2.

The shell or fuselage of the airplane consists of a conical nose and cylindrical afterbody with no boat tailing at the aft end. The stabilizing surface is bi-convex and rectangular in plan form with an aspect ratio of 2. Power is assumed to be supplied by a constant thrust jet motor. Other characteristics may be found in Table I.

The design of the airplane is based on the Mach number of 1.7 at an altitude of 30,000 ft. and a gross weight of 10,000 lbs. Static stability is assumed to be the major design variable. The dynamic stability is first investigated for a static stability just sufficient to allow a four-g maneuver without exceeding a 20 degree angle of attack on the fin. Then the static stability is increased in multiples of 2, 3, and 4, to establish a trend.

It is found that the effects of compressibility have a powerful influence on some of the coefficients of the stability quartic and hence on the dynamic stability, and that dynamic instability will result in certain cases regardless of the amount of static stability provided.

II THEORETICAL CONSIDERATIONS

A. Equations of Motion and Condition for Stability.

The Eulerian axes ("jumping" or moving axes) are used in describing the motion of the airplane. These axes are fixed in space at any instant but change their position from instant to instant, coinciding at each instant with a definite set of axes fixed in the airplane. The axes fixed in the airplane are called the "wind axes" in which the x-axis is in the direction of motion. In other words, the x and z-axes assume different positions relative to the airplane for different altitudes of flight.

Considering the simple case of small disturbances from steady rectilinear flight, it can be shown (Ref. 1, 2, 3) that the equations of motion for an airplane split up into two independent sets of three equations. One set which completely describes the longitudinal motion, and the other set which completely describes the lateral motion. Assuming small disturbances from the steady state and neglecting squares and products of small quantities, the equations describing the longitudinal motion of an airplane are:

$$\frac{du}{dt} = uX_u + wX_w - g\theta \cos \theta_o + X_q \frac{d\theta}{dt}$$

$$\frac{dw}{dt} - U\frac{d\theta}{dt} = uZ_u + wZ_w + Z_q \frac{d\theta}{dt} - g\theta \sin \theta_o \qquad (1)$$

$$K_y^2 \frac{d^2\theta}{dt^2} = uM_u + wM_w + M_q \frac{d\theta}{dt}$$

or, in terms of the operator $D = \frac{d}{dt}$,

$$(D-X_u)u - X_w w - (-g \cos \theta_o + X_q D)\theta = 0$$

$$-Z_u u + (D-Z_w)w - (-g \sin \theta_o + Z_q D + UD)\theta = 0 \qquad (2)$$

$$-\frac{M_u}{K_y^2} u - \frac{M_w}{K_y^2} w + (D^2 - \frac{M_q}{K_y^2} D)\theta = 0$$

where $U + u$ and w are the components of the rectilinear velocity in the x and z-directions; $\theta + \theta_o$ and q are the pitching displacement from the horizontal and the pitching velocity, respectively; and X_u, Z_w, M_q, etc., are the resistance and rotary derivatives. For example,

$$X_u = \frac{\partial X}{\partial u}, \; Z_w = \frac{\partial Z}{\partial w}, \; M_q = \frac{\partial M}{\partial q}, \text{ etc.} \qquad (3)$$

where X_u is the variation of drag due to a change in velocity in the x-direction, Z_w is the variation of lift due to a velocity disturbance in the z-direction, M_q is the variation of pitching moment due to a velocity of pitch, etc.

Expressing equation (2) in determinate form we have

$$\begin{vmatrix} D - X_u & -X_w & -(-g \cos \theta_o + X_q D) \\ -Z_u & D - Z_w & -(-g \sin \theta_o + Z_q D + UD) \\ -\frac{M_u}{K_y^2} & -\frac{M_w}{K_y^2} & (D^2 - M_q D) \end{vmatrix} (u,w,\theta) = 0 \qquad (4)$$

where $F(D)\left[u,w,\theta\right] = 0$ is a fourth order, linear, homogeneous differential equation in (u,w,θ) and t. Hence,

$$(A_1 D^4 + B_1 D^3 + C_1 D^2 + D_1 D + E_1)(u,w,\theta) = 0 , \qquad (5)$$

for which the solution is

$$u, w, \theta = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t} , \qquad (6)$$

where the λ_i are the roots of the "stability quartic"

$$F(\lambda) = A_1\lambda^4 + B_1\lambda^3 + C_1\lambda^2 + D_1\lambda + E_1 = 0. \tag{7}$$

The condition for stability is that all the roots of the stability quartic shall be pseudo-negative. According to Routh (Ref. 4), the rules for the roots of this quartic to be pseudo-negative are

$$A_1\ ,\ B_1\ ,\ C_1\ ,\ D_1\ ,\ E_1\ ,\ R_1\ ,\ \text{all} > 0, \tag{8}$$

where

$$R_1 = B_1\, C_1\, D_1 - B_1^{\,2}\, E_1 - A_1\, D_1^{\,2}. \tag{9}$$

B. Equations of Motion in Dimensionless Form.

The dimensionless form of the stability equations has been used in this analysis. Referring lengths to the shell diameter and areas to the square of the diameter, equation (4) may be put into the dimensionless form by letting

$$X_u = -\frac{x_u}{\tau}, \qquad Z_u = -\frac{z_u}{\tau}, \qquad \frac{M_u}{K_y^{\,2}} = -\frac{m_u}{d\tau},$$

$$X_w = -\frac{x_w}{\tau}, \qquad Z_w = -\frac{z_w}{\tau}, \qquad \frac{M_w}{K_y^{\,2}} = -\frac{m_w}{d\tau}, \tag{10}$$

$$X_q = -\frac{x_q d}{\tau}, \qquad Z_q = -\frac{z_q d}{\tau}, \qquad \frac{M_q}{K_y^{\,2}} = -\frac{m_q}{\tau},$$

and

$$g \cos \theta_0 = \frac{1}{2}\frac{C_L U}{\tau},$$

$$g \sin \theta_0 = \frac{1}{2}\frac{C_L U}{\tau} \tan \theta_0, \tag{11}$$

where $\tau = mU/2q'd^2$, C_L is the total lift coefficient referred to the square of the shell diameter, d is the shell diameter, m is the mass of the airplane, U is the steady state velocity, and q' is the dynamic pressure. Using this notation equation (4) becomes

$$\begin{vmatrix} \left(\lambda + \frac{x_u}{\tau}\right) & \frac{x_w}{\tau} & \left(\frac{1}{2}\frac{C_L U}{\tau} + \frac{d}{\tau} x_q \lambda\right) \\ \frac{z_u}{\tau} & \left(\lambda + \frac{z_w}{\tau}\right) & \left(\frac{1}{2}\frac{C_L U}{\tau}\tan\theta_0 + \frac{d}{\tau} z_q \lambda - U\lambda\right) \\ \frac{m_u}{\tau d} & \frac{m_w}{\tau d} & \left(\lambda^2 + m_q \frac{\lambda}{\tau}\right) \end{vmatrix} = 0 \qquad (12)$$

Equation (12) may be put into neater form

$$\begin{vmatrix} (\lambda' + x_u) & x_w & \left(\frac{1}{2}C_L + \frac{x_q}{\mu}\lambda'\right) \\ z_u & (\lambda' + z_w) & \left(\frac{1}{2}C_L \tan\theta_0 + \frac{z_q}{\mu}\lambda' - \lambda'\right) \\ \mu m_u & \mu m_w & \left(\lambda'^2 + m_q \lambda'\right) \end{vmatrix} = 0, \qquad (13)$$

where

$$\mu = \frac{U\tau}{d} = \frac{m}{\rho d^3} \qquad (14)$$

$$\lambda' = \tau\lambda = \frac{mU}{2q'd^2}\lambda . \qquad (15)$$

The stability quartic for the dimensionless case then becomes

$$F(\lambda') = A\lambda'^4 + B\lambda'^3 + C\lambda'^2 + D\lambda' + E = 0. \qquad (16)$$

The coefficients of the new stability quartic are given in terms of the dimensionless derivatives x_u, x_w, x_q, etc. in Appendix IV.

C. Approximate Factorization of the Stability Quartic.

Bairstow has shown (Ref. 5) that in certain cases the roots of a quartic may be approximated with sufficient accuracy by the following factorization:

$$F(\lambda') = \left[\lambda'^2 + B\lambda' + C \right] \left[\lambda'^2 + \frac{DC - BE}{C^2}\lambda' + \frac{E}{C} \right] = 0. \tag{17}$$

The sufficient (but not necessary) conditions for this to be valid are,

$$C \geq B, \; C^2 > 20\,E, \;\; BC > 20\,D. \tag{18}$$

The first term of the approximate factoriaation gives what is called the "short oscillation", and the second term the "phugoid oscillation". The period and time to damp to half amplitude for the short oscillation are given by

$$T_s = \frac{2\pi\tau}{\sqrt{C - \left(\frac{B}{2}\right)^2}} \quad \text{sec.}, \tag{19}$$

$$t_{\frac{1}{2}s} = \frac{1.386}{B}\tau \quad \text{sec.}; \tag{20}$$

and for the phugoid oscillation by

$$T_p = \frac{2\pi\tau}{\sqrt{\frac{E}{C} - \frac{CD - BE}{2C^2}}} \quad \text{sec.}, \tag{21}$$

$$t_{\frac{1}{2}p} = \frac{1.386\,C^2}{CD - BE}\tau \quad \text{sec.} \tag{22}$$

D. Resistance and Rotary Derivatives. *

The resistance and rotary derivatives are summarized in Appendix III in both the dimensional and dimensionless form. The derivatives are derived assuming small disturbances from the steady state and small angles of attack. They are of the conventional form except for those which are referred to u, the perturbation velocity in the x-direction. Here, the variation of Mach number must be considered. For example, the actual force in the x-direction is given by

$$m X = -\sum \frac{\rho d^2}{2} C_D (U + u)^2 . \tag{23}$$

The summation sign represents the total of the contributions from the shell, fin, and wing. Neglecting the products and squares of small quantities,

$$X_u = \frac{\partial X}{\partial u} = -\sum \frac{q' d^2}{mU} \left[\frac{\partial C_D}{\partial u} U + 2 C_D \right] . \tag{24}$$

The coefficient of drag is assumed to be of the form

$$C_D = C_{D_f} + C_{D_w} + \frac{\partial C_L}{\partial \alpha} \left[1 + f(\beta) \right] \alpha^2 \tag{25}$$

where $\beta = \sqrt{M^2 - 1}$, C_{D_f} is the friction drag coefficient, C_{D_W} is the wave drag coefficient at zero lift, and the remaining term is the drag increment due to lift which, in general, includes a correction factor for second order effects, i.e., $f(\beta)$. In many instances $f(\beta)$ can be taken to be zero. It is assumed for convenience that the friction drag coefficient is independent of Mach number. The wave drag coefficient at zero lift and the wave drag increment due to lift are both functions of

* Jet effects on the derivative m_u and x_u are neglected.

the Mach number, hence,

$$\frac{\partial C_D}{\partial u} = \left[\frac{\partial C_{D_w}}{\partial \beta} + \alpha^2 \frac{\partial}{\partial \beta} \frac{\partial C_L}{\partial \alpha}\left(1 + f(\beta)\right)\right]\frac{\partial \beta}{\partial u}$$
$$= \frac{M}{a\beta}\left[\frac{\partial C_{D_w}}{\partial \beta} + \alpha^2 \frac{\partial}{\partial \beta} \frac{\partial C_L}{\partial \alpha}\left(1 + f(\beta)\right)\right], \qquad (26)$$

where a = free stream speed of sound. Therefore

$$X_u = -\sum \frac{2q'd^2}{mU}\left[C_D + \frac{M^2}{2\beta}\left\{\frac{\partial C_{D_w}}{\partial \beta} + \alpha^2 \frac{\partial}{\partial \beta} \frac{\partial C_L}{\partial \alpha}\left(1 + f(\beta)\right)\right\}\right]. \qquad (27)$$

Similarly it can be shown that

$$Z_u = -\sum \frac{2q'd^2}{mU}\left[C_L + \frac{M^2\alpha}{2\beta} \frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right], \qquad (28)$$

and

$$M_u = \sum \frac{2q'd^3}{mU} x'\left[C_L + \frac{M^2\alpha}{2\beta} \frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right]. \qquad (29)$$

III CALCULATIONS

A. Aerodynamic Coefficients.

The basic aerodynamic coefficients for the component parts of the airplane are given in Table II for Mach numbers of 1.7 and 1.3. These coefficients are derived in the manner indicated in Appendix II and are based on the characteristic area of the individual component. In applying these to the complete airplane, all coefficients are referred to the square of the shell diameter, and the following assumptions are made:

1. the principal of superposition applies;
2. the influence of body upwash on the fin, neglecting the area of the fin occupied by the body, cancel the tip losses;
3. the influence of body upwash on the wing can be approximated by including the area occupied by the body;
4. downwash, or upwash, effects of the fin on the wing are negligible;
5. base pressure drag on the shell is zero since the exhaust jet of the motor occupies the full cross sectional area of the body.

B. Static Stability and Areas.

The static stability is determined in the conventional manner. Summing moments about the center of gravity,

$$C_M = \left(\frac{\partial C_L}{\partial \alpha}\right)_s (x')_s \alpha + (S')_f \left(\frac{\partial C_L}{\partial \sigma}\right)_f (x')_f \sigma + (S')_w \left(\frac{\partial C_L}{\partial \alpha}\right)_w (x')_w \alpha, \qquad (30)$$

where $(S')_f = (S)_f/d^2$, $(x')_f = (x)_f/d$, etc., $\sigma = \sigma_0 + \alpha$.

Differentiating with respect to the angle of attack

$$\frac{dC_M}{d\alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_s (x')_s + (S')_f\left(\frac{\partial C_L}{\partial \sigma}\right)_f (x')_f + (S')_w\left(\frac{\partial C_L}{\partial \alpha}\right)_w (x')_w . \tag{31}$$

The static stability is then given by

$$-\frac{dC_M}{dC_L} = -\frac{\frac{dC_M}{d\alpha}}{\frac{dC_L}{d\alpha}} . \tag{32}$$

The wing and fin areas are determined from conditions at the Mach number of 1.7. The angle of incidence of the wing is taken to be zero, and, hence, the entire airplane flys at an angle of attack to obtain lift. The angle of incidence of the fin (σ_0) is chosen to give a reasonable degree of static stability and yet provide sufficient margin to obtain a four-g maneuver without exceeding an angle of attack of 20 degrees. Eighty per cent of the lift is assumed to be carried by the wing, and the remainder by the shell and fin. With these considerations in mind, α and σ_0 are chosen to be 3 and 2 degrees, respectively. The areas for the fin and wings, given in Table I, are then obtained from the equation for the total lift:

$$(S)_w = \frac{.80\,W}{q'\left(\frac{\partial C_L}{\partial \alpha}\right)_w \alpha} , \tag{33}$$

$$(S)_f = \frac{.20\,W - q'd^2\left(\frac{\partial C_L}{\partial \alpha}\right)_s \alpha}{q'\left(\frac{\partial C_L}{\partial \sigma}\right)_f \sigma} . \tag{34}$$

Given the areas, the angle of attack, the center of pressure of the shell and assuming the wing center of pressure location to be $(x')_W = -1.50$, the location of the center of pressure of the fin is determined from the equation of equilibrium,

$$C_M = \left(\frac{\partial C_L}{\partial \alpha}\right)_s (x')_s \alpha + (S')_f \left(\frac{\partial C_L}{\partial \sigma}\right)_f (x')_f \sigma + (S')_w \left(\frac{\partial C_L}{\partial \alpha}\right)_w (x')_w \alpha = 0. \qquad (35)$$

Under the above conditions the static stability at the Mach number of 1.7 becomes $-\frac{dC_M}{dC_L} = .3169$, based on the shell diameter.

The lift coefficient slopes are different at a Mach number of 1.3 than at a Mach number of 1.7, while the areas and wing and fin locations remain fixed. The airplane is thus required to fly at different angles of attack, with different fin incidences, to obtain the required lift and to maintain equilibrium. The new angles of attack and fin incidences are obtained using the equation of equilibrium and the equation for the total lift, as before.

The static stability at the design Mach number is varied in multiples of n = 2, 3, 4, to determine the effect on the dynamic stability, and to establish a trend, i.e.,

$$-\frac{dC_M}{dC_L} = n(.3169). \qquad (36)$$

This is accomplished by moving the wing aft from the center of gravity. Wing and fin areas and fin location remain fixed, but the angle of attack and fin incidence change for each condition.

The center of pressure locations, angles of attack, and static stability, as determined by the procedure outlined above, are given in

Table III.

C. Dynamic Stability Derivatives and Coefficients of the Stability Quartic.

The resistance and rotary derivatives, as presented in Appendix III, are calculated in Tables IV-A through D. These are determined for the various degrees of static stability (n) at Mach numbers of 1.7 and 1.3 for each wing configuration. The aerodynamic coefficients are referred to the square of the shell diameter and are based on the angles of attack given in Table III.

The coefficients of the stability quartic as defined in Appendix IV, are then determined in Table V.

D. Roots of the Stability Quartic.

The roots of the stability quartic are determined for the dynamically stable cases by means of Bairstow's approximate factorization. These are presented in Table VI. The period and time to damp to half amplitude are given where applicable.

IV RESULTS AND DISCUSSION

A. General Discussion of the Nature of Instability.

The conditions for stability and the nature of instability which can arise are examined first in the general case and then in the particular case of supersonic motion. It was shown that the conditions for stability are

$$B,\ C.\ D.\ E.\ \text{and}\ R > 0,$$

where

$$R = B\,C\,D - D^2 - B^2\,E.$$

Considering a stable airplane it is apparent that if B, D, and C decrease and pass through zero independently then R will decrease and reach zero first. If E decreases and passes through zero then R increases. Hence, the only way in which stability can change continuously over into instability is for R or E to pass through zero first.

To examine the resulting motion which occurs under these circumstances, Bairstows approximate factorization of the stability quartic may be utilized.

$$F(\lambda') = \left[\lambda'^2 + B\lambda' + C\right]\left[\lambda'^2 + \frac{DC - BE}{C^2}\lambda' + \frac{E}{C}\right] = 0.$$

The resulting motion for the "short oscillation" is given by

$$x = C_1 e^{\left(-\frac{1}{2}B + \sqrt{\left(\frac{B}{2}\right)^2 - C}\right)t} + C_2 e^{\left(-\frac{1}{2}B - \sqrt{\left(\frac{B}{2}\right)^2 - C}\right)t}. \qquad (37)$$

Generally speaking $(B/2)^2 < C$ and B is always positive, hence the resulting motion is damped oscillation.

The resulting motion for the "phugoid oscillation" is given by

$$x = C_3 e^{\left(-D.F. + \sqrt{(D.F.)^2 - \frac{E}{C}}\right)} + C_4 e^{\left(-D.F. - \sqrt{(D.F.)^2 - \frac{E}{C}}\right)}, \tag{38}$$

where $$D.F.\text{(damping factor)} = \frac{1}{2} \frac{DC - BE}{C^2} \tag{39}$$

Considering the case when D.F. $>$ 0 and $E/C > 0$, the following generalizations may be made:

damped oscillation will occur if $E/C > (D.F.)^2$,

subsidence will occur if $E/C < (D.F.)^2$.

Suppose $E/C > (D.F.)^2$, then as D. F. goes from (+) to (-), the motion goes from damped oscillation to divergent oscillation. Hence, D.F. = 0 is the boundary between damped and divergent oscillation. Or, explicitly,

$$\frac{1}{2} \frac{DC - BE}{C^2} = 0. \tag{40}$$

This can be written as

$$R - D^2 = 0, \tag{41}$$

where in general $D^2 << R$ and we can say D.F. = 0 corresponds to R = 0.* Hence if R goes through zero from (+) to (-) the motion changes from a damped oscillation to an undamped oscillation.

If we take D.F., B, C, D, and R $>$ 0 and let E go from (+) to (-), then if $E/C < (D.F.)^2$ we go from subsidence to divergence.

In general then (1) if R goes through zero the motion goes from damped to divergent oscillation, and (2) if E goes through zero the motion goes from subsidence to divergence.

To study the particular case of dynamic stability at supersonic speeds, the coefficients of the stability quartic must be examined in detail. It was previously pointed out that the effects of compressibility enter directly only into the stability derivatives x_u, z_u, and m_u. The signs

* R = 0 is actually the rigorous boundary between divergent and damped oscillations.

of these terms may be affected by the relative magnitude and signs of the compressibility correction factors. Hence, to examine the motion at supersonic speeds, it is necessary to determine in particular the effects of these terms on the coefficients of the stability quartic. The coefficients given in Appendix IV may be further simplified when m_q appears in the expressions for C and D in such a manner as to be small compared to the other terms. The coefficients then become:

$$\begin{aligned} B &= x_u + z_w + m_q \\ C &= \mu m_w \\ D &= \mu m_w x_u - \mu m_u (x_w + \tfrac{1}{2} C_L) \\ E &= \tfrac{1}{2} C_L \mu (m_w z_u - m_u z_w). \end{aligned} \qquad (42)$$

Considering the coefficients separately, one can arrive at the following conclusions:

1. The second and third terms in coefficient B are always positive and of the same order of magnitude, while the first term may be either positive or negative but is of a lower order of magnitude than the other terms. Therefore B will always be positive.

2. The derivative m_w represents the static stability and is always positive, therefore, the coefficient C will always be positive if there is a reasonable margin of static stability.

3. The sign of coefficient D will depend on the magnitude of the respective terms and on the signs of x_u and m_u only, since m_w and $(x_w - \frac{1}{2}C_L)$ are always positive.

4. Similarly, the sign of coefficient E will depend on the magnitude of the respective terms and on the signs of z_u and m_u only, since m_w and

z_w are always positive.

Hence, it is seen that the dynamic stability and resultant motion of an airplane, provided with a reasonable margin of static stability and flying at supersonic speeds, depend only upon the signs of coefficients D and E, and that these coefficients are positive or negative depending on the signs and magnitude of the terms which account for compressibility.

B. Specific Results of this Analysis.

The stability quartic coefficients obtained for the various conditions investigated in this analysis are given in Table V. It is seen that B and C are always positive as previously indicated, and that D and E are negative in certain cases, giving rise to instability. The coefficients D and E are plotted in Figs. 2 and 3 against static stability (dC_M/dC_L), and for all practical purposes they vary linearly. It is seen that D is always positive for the cases investigated at $M = 1.7$ and increases with dC_M/dC_L. At $M = 1.3$, D is negative for Cases I and II, but become positive with increasing dC_M/dC_L. It is positive for Case III but decreases and becomes negative. For Case IV, it is negative and nearly constant throughout the range investigated. Hence, D can be made positive by varying the static stability in all cases except IV.

The coefficient E increases with increasing static stability at $M = 1.7$. For Cases I and II, it is negative for low values of dC_M/dC_L but becomes positive. At $M = 1.3$, E decreases with increasing static stability. For Cases I, II, and IV, it is negative and never becomes positive for positive values of dC_M/dC_L. For Case III, it is positive for the practical range of dC_M/dC_L.

It is evident that Cases I, II, and IV, are always unstable at $M = 1.3$. This is entirely due to the effects of compressibility which enter into the coefficient E in a predominating manner through the stability derivative m_u. The predominating effect of this term may be clearly seen by examining columns 14 and 15 of Table V. It is interesting to note that the derivative,

$$m_u = -\sum \frac{d^2 x'}{K_y^2}\left[C_L + \frac{M^2 \alpha}{2\beta}\ \frac{\partial}{\partial\beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right], \qquad (43)$$

appears in the analysis only because of the compressibility effects, since the summation of the C_L part of the expression is identically equal to zero. The resulting motion for the unstable cases cited above will be divergence.

The physical interpretation of this may be seen by examining the terms m_u and m_w. Neglecting the effect of the shell, for discussional purposes, we need only consider the moments due to the fin and wing. In general, except for Case III, if we consider a small increase in forward velocity, the moment increment due to the change in Mach number, i.e.,

$$K_1 x' \frac{\partial}{\partial\beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\alpha , \qquad (44)$$

decreases more rapidly for the fin than for the wing. This gives rise to a negative pitching moment, and hence the airplane goes into a dive. If we consider a small down gust, the moment increment due to the gust, i.e.,

$$K_2 x' \left(\frac{\partial C_L}{\partial \alpha}\right)\alpha \quad , \qquad (45)$$

decreases more rapidly for the wing than for the fin, since static stability is positive. This gives rise to a positive pitching moment tending to restore the airplane to equilibrium. Actually these effects are simultaneous. For instance, when the airplane pitches due to an increase in forward velocity and goes into a dive, the angle of attack is effectively reduced. This is essentially the same as a small down gust, hence, there is a tendency to restore equilibrium if the moment due to the down gust overpowers that due to the increase in forward velocity. Such a condition leads to stability. On the other hand instability will result if the diving moment due to the increase in forward velocity is predominate.

As might be expected, a further deduction of this analysis is that the direct effects of compressibility on dynamic stability at supersonic speeds become less important as the Mach number increases.

The roots of the stability quartic, the time to damp to half-amplitude, and the period, are given for the stable cases in Table VI. The periods for the short and phugoid oscillation show the essential difference between the two. Both types of motion appear to be unobjectionable since the short oscillation has a small period which damps out rapidly, and the phugoid oscillation has a very long period which damps out slowly. The latter motion is unobjectionable because the long period allows plenty of time for corrections to be applied.

V CONCLUSIONS

The effects of compressibility have a powerful influence on the dynamic stability quartic coefficient E for supersonic speeds in the range of Mach numbers where the variation of $\partial C_L/\partial\alpha$ is relatively large. This is due primarily to the derivative m_u which appears only because the effects of compressibility have been considered. This influence is felt even at $M = 1.7$ for ordinarily reasonable values of static stability.

It is found that the delta wing, with the Mach wave ahead of the leading edge, and the rectangular wing (Cases I, II, and IV) for a Canard type airplane are unstable under the assumptions made in this analysis. Cases I and II may be made stable by choosing a fin with the same planform as the wing. Under these circumstances, the variation of $\partial C_L/\partial\alpha$ with Mach number will be the same for both fin and wing assuming upwash or downwash effects are constant. The derivative m_u, then, is effectively reduced to zero, since, for the shell the variation of $\partial C_L/\partial\alpha$ with Mach number is small. With the assumption made in this analysis $\partial C_L/\partial\alpha$ varies with Mach number in a different manner for the rectangular fin than for the rectangular wing, therefore this reasoning does not apply to Case IV.

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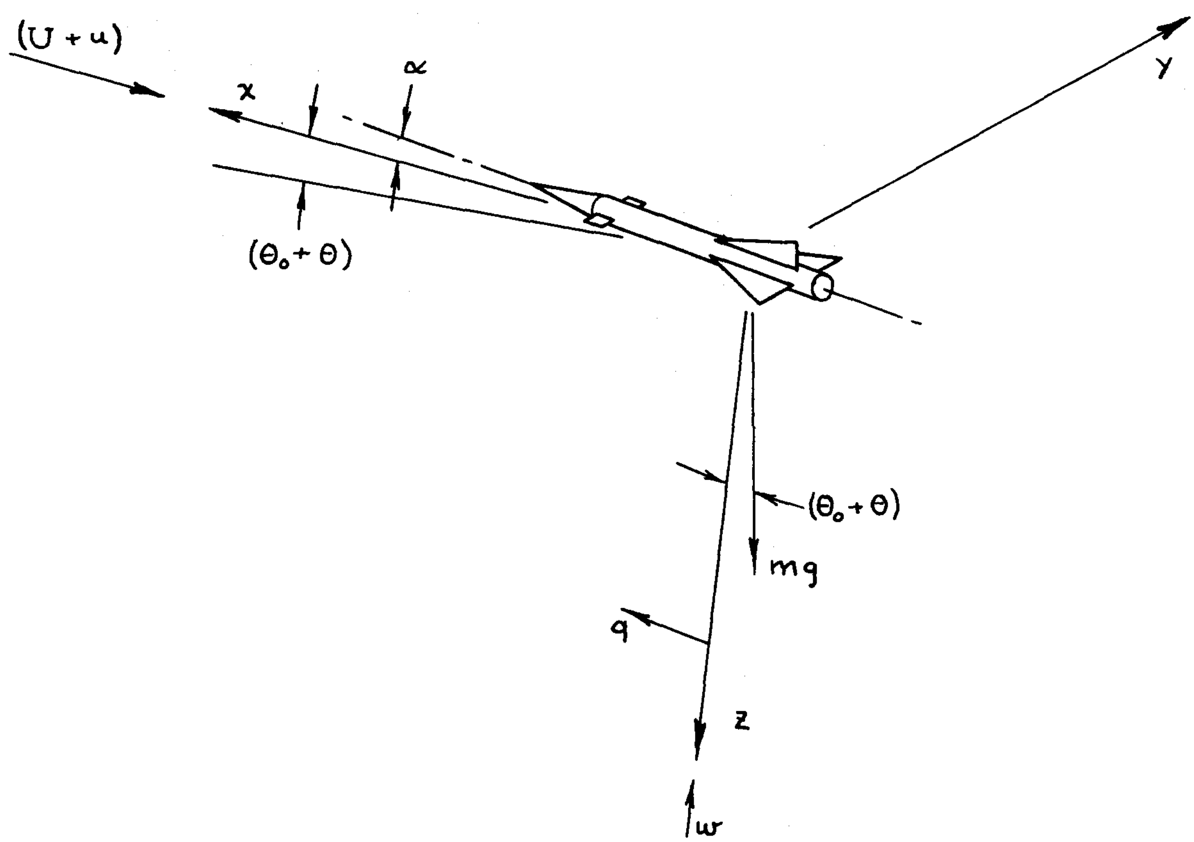
APPENDIX I

SIGN CONVENTION AND NOMENCLATURE

A. Sign Convention.



B. Nomenclature.

U	Steady state velocity of flight (ft./sec.).
ρ	Free stream density (slugs/cu.ft.).
q'	Dynamic pressure (lb./sq.ft.).
M	Mach number.
β	$\sqrt{M^2 - 1}$
g	Acceleration due to gravity (ft./sec.^{2}).
u	Perturbation velocity in x-direction (ft./sec.).
w	Perturbation velocity in z-direction (ft./sec.).
q	Perturbation velocity in pitch (rad/sec.).
θ_o	Angle of flight path with the horizontal (rad.).
θ	Deviation in angular displacement in pitch from the steady state (rad.).
mX	Actual force in the x-direction (lb.).
mZ	Actual force in the z-direction (lb.).
mM	Actual pitching moment (ft.-lb.).
m	Mass of the airplane (slugs).
X_u, Z_u, M_u	Derivatives representing change of drag, lift, and moment due to u.
x_u, z_u, m_u	Dimensionless form of the above.
X_w, Z_w, M_w	Derivatives representing change of drag, lift, and moment due to w.
x_w, z_w, m_w	Dimensionless form of the above.
X_q, Z_q, M_q	Derivatives representing change of drag, lift, and moment due to velocity of pitch.
x_q, z_q, m_q	Dimensionless form of the above.

τ	Dimensionless dynamic stability parameter (= $mU/2q'd^2$).
μ	Dimensionless dynamic stability parameter (= $m/\rho d^3$).
λ_i	Actual roots of the dynamic stability quartic.
λ'_i	Roots of the dimensionless form of the stability quartic.
A_1, B_1, C_1, D_1, E_1	Actual coefficients of the dynamic stability quartic.
A, B, C, D, E	Coefficients of the dimensionless form of the stability quartic.
T_s, T_p	Period for the short and phugoid oscillation.
$t_{\frac{1}{2}s}$, $t_{\frac{1}{2}p}$	Time to damp to half amplitude for the short and phugoid oscillation.
C_L	Lift coefficient.
C_M	Moment coefficient.
C_D	Total drag coefficient.
C_{D_f}	Skin friction drag coefficient.
C_{D_w}	Wave drag coefficient.
n	Static stability parameter (= 1, 2, 3, 4).
α	Airplane angle of attack (rad.).
σ_o	Fin angle of incidence (rad.).
σ	Fin angle of attack (rad.).
$(x')_s$, $(x')_f$, $(x')_w$	Distance of the center of pressure of the shell, fin, and wing to the c.g. in terms of diameters.
d	Diameter of the fuselage or shell (ft.).

ℓ	Total length of the airplane (ft.).
b	Span (ft.).
c	Chord (ft.).
t	Maximum thickness of the surface (ft.).
η	Maximum thickness in percent of maximum chord.
w_o	One-half the apex angle of the delta wing (degrees).
$(A')_s$, $(A')_f$, $(A')_w$	Wetted surface areas of the shell, fin, and wing in terms of the square of the shell diameter.
$(S')_f$, $(S')_w$	Surface areas of the fin and wing in terms of square of the shell diameter.
W	Gross weight of the airplane (lb.).
x_o	Center of gravity position from the aft end of the airplane (ft.).
I_y	Moment of inertia about the y-axis (lb.-ft.-sec.^{2}).
K_y	Radius of gyration (ft.).

Subscripts

$(\ )_s$	For the shell or fuselage.
$(\ )_f$	For the fin.
$(\ )_w$	For the wing.

APPENDIX II

AERODYNAMIC COEFFICIENTS USED IN THE STABILITY ANALYSIS

A. Shell or Fuselage.

1. The lift coefficient slope and center of pressure is estimated by Tsien's method (Ref. 6) based on the linearized equations of motion for an axial body of symmetry.

2. Variations of the lift coefficient slope with Mach number, $\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$, is estimated by assuming a linear variation of $\frac{\partial C_L}{\partial \alpha}$ with Mach between M = 1.3 and 1.7. This assumption was substantiated by a study of numerous calculations in which the variation of $\frac{\partial C_L}{\partial \alpha}$ with Mach number was nearly linear for shell lengths greater than 10d.

3. The skin friction drag coefficient, C_{D_f}, is assumed to be constant at 0.003 based on the wetted surface area. This is assumed to be the case for each of the airplane's components.

4. The wave drag coefficient at zero angle of attack, C_{D_W}, is estimated from figure 2, Ref. 7, which is an interim report on Kopal's calculations for drag of cones by the Taylor Maccoll method.

5. Variation of the wave drag coefficient with Mach number, $\frac{\partial}{\partial \beta} C_{D_W}$, is estimated from figure 2, Ref. 7.

6. The drag coefficient due to lift is assumed to be directly proportional to the angle of attack and the lift coefficient, i.e.,

$$\alpha^2 \left(\frac{\partial C_L}{\partial \alpha}\right).$$

7. Variation of the drag coefficient due to lift with Mach number is assumed to be

$$\alpha^2 \frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right),$$

where $\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$ is the same as in paragraph 2, above.

B. Fin.

1. For the lift coefficient slope it is assumed that the two dimensional thin airfoil theory is applicable since it is believed that the influence of the body upwash will approximately cancel the tip losses, hence,

$$\frac{\partial C_L}{\partial \alpha} = \frac{4}{\beta} .$$

In computing the total lift coefficient the body area is neglected. The center of pressure is assumed to be constant at the mid-chord.

2. Variation of the lift coefficient slope with Mach number is simply

$$\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right) = -\frac{1}{\beta}\frac{\partial C_L}{\partial \alpha} .$$

3. The wave drag coefficient at zero angle of attack is based on the two dimensional airfoil theory for bi-convex airfoils, i.e.,

$$C_{D_w} = \frac{16}{3}\frac{\eta^2}{\beta} .$$

4. Variation of the wave drag coefficient with Mach number is simply

$$\frac{\partial}{\partial \beta} C_{D_w} = -\frac{1}{\beta} C_{D_w} .$$

5. The drag coefficient due to lift and its variation with respect to Mach number is found in the same manner as indicated for the shell.

C. Wing - Case I and II.

1. The lift coefficient slope for the delta wing with the Mach cone ahead of the leading edge was determined by Stewart, (Ref. 8) to be

$$\frac{\partial C_L}{\partial \alpha} = \frac{2\pi \tan \omega_0}{E(k')} ,$$

where w_0 is one-half the apex angle of the delta and $E(k')$ is the complete elliptic integral of the second kind having a modulus

$$k' = \sqrt{1 - \beta^2 \tan^2 \omega_0}$$

The center of pressure is constant at two-thirds the maximum chord aft of the wing apex.

2. Variation of the lift coefficient slope with Mach number is

$$\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right) = \frac{2\pi \beta \tan^3 \omega_0}{(k')^2}\left[\frac{E(k') - K(k')}{[E(k')]^2}\right],$$

where $K(k')$ is the complete elliptic integral of the first kind.

3. The wave drag coefficient at zero angle of attack was found by Puckett (Ref. 9) to be

$$C_{D_w} = \frac{2\eta^2}{\pi \beta}\left\{\frac{G_2'(n,r)}{r(1-r)^2} - \frac{F'(n,r)}{(1-r)^2} + \frac{1}{r(1-r)}\left[\frac{\log nr}{\sqrt{r^2 n^2 - 1}} - \frac{\log n}{\sqrt{n^2 - 1}} + \sin^{-1}\frac{1}{rn} - \sin^{-1}\frac{1}{n}\right]\right\},$$

where r is the distance from the trailing edge of the delta wing to the maximum thickness point and $n = \cot w_0/\beta$. $G'_2(n,r)$ and $F'(n,r)$ are functions of n and r as defined by Puckett in the given reference.

4. Variation of the wave drag coefficient with Mach number is given by

$$\frac{\partial}{\partial \beta}(C_{D_w}) = \frac{2\eta^2}{\pi \beta^2}\left\{\frac{\beta \frac{\partial}{\partial \beta}(G_2')}{r(1-r)^2} - \frac{\beta \frac{\partial}{\partial \beta}(F')}{(1-r)^2} + \frac{1}{r(1-r)}\left[\frac{r^2 n^2 \log nr}{(r^2 n^2 - 1)^{3/2}} - \frac{n^2 \log n}{(n^2-1)^{3/2}}\right]\right\}.$$

5. The drag coefficient due to lift was found by Stewart and Puckett (Ref. 10) to be

$$\frac{\partial C_L}{\partial \alpha}\left[1 - \frac{k'}{2E(k')}\right] .$$

6. Variation of the drag coefficient due to lift with Mach number is given by

$$\frac{\partial}{\partial \beta}\left[\frac{\partial C_L}{\partial \alpha}\left(1 - \frac{k'}{2E(k')}\right)\right] = \frac{2\pi\beta \tan\omega_0}{(k')^2}\left\{\frac{E(k')-K(k')}{[E(k')]^2} - \frac{k'}{2}\left[\frac{E(k')-2K(k')}{[E(k')]^3}\right]\right\} .$$

D. Wing - Case III.

1. The lift coefficient slope for the delta wing with the Mach cone behind the leading edge was found by Puckett (Ref. 9) to be identical to that for the two dimensional thin airfoil, i.e.,

$$\frac{\partial C_L}{\partial \alpha} = \frac{4}{\beta} .$$

The center of pressure is again at two-thirds the maximum chord aft of the wing apex.

2. Variation of the lift coefficient slope with Mach number is simply

$$\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right) = -\frac{1}{\beta}\frac{\partial C_L}{\partial \alpha} .$$

3. The wave drag coefficient at zero angle of attack was found by Puckett (Ref. 9) to be

$$C_{D_w} = \frac{2\eta^2}{\pi\beta(1-r^2)}\left\{\frac{\cos^{-1} n}{\sqrt{1-n^2}} + \frac{\frac{\pi}{2} + \sin^{-1} rn}{r\sqrt{1-r^2n^2}}\right\} ,$$

where the symbols are as defined under part C, paragraph 3.

4. Variation of the wave drag coefficient with Mach number is given by

$$\frac{\partial}{\partial\beta}(C_{D_w}) = \frac{2\eta^2}{\pi\beta^2(1-r^2)}\left\{\frac{n}{1-n^2} - \frac{n}{1-r^2n^2} - \frac{\pi}{2r(1-r^2n^2)^{3/2}} - \frac{\cos^{-1}n}{(1-n^2)^{3/2}} - \frac{\sin^{-1}rn}{r(1-r^2n^2)^{3/2}}\right\}.$$

5. The drag coefficient due to lift and its variation with respect to Mach number is found in the same manner as indicated for the shell.

E. Wing - Case IV.

1. The lift coefficient slope for the rectangular wing is given by the two dimensional thin airfoil theory and includes a correction for tip losses, i.e.,

$$\frac{\partial C_L}{\partial\alpha} = \frac{4}{\beta}\left(1 - \frac{1}{2A\!\!R\beta}\right).$$

The center of pressure in per cent of chord from mid-chord is given by

$$c.p. = \frac{1}{12A\!\!R\beta - 6}.$$

2. Variation of the lift coefficient slope with Mach number is given by

$$\frac{\partial}{\partial\beta}\left(\frac{\partial C_L}{\partial\alpha}\right) = -\frac{4}{\beta^2}\left(1 - \frac{1}{A\!\!R\beta}\right).$$

3. The wave drag coefficient at zero angle of attack is given by

$$C_{D_w} = \frac{16}{3}\frac{\eta^2}{\beta}.$$

4. Variation of the wave drag with Mach number is given by

$$\frac{\partial}{\partial \beta}(C_{D_w}) = -\frac{1}{\beta}(C_{D_w}).$$

5. The drag coefficient due to lift and its variation with respect to Mach number is found in the same manner as indicated for the shell.

APPENDIX III

SUMMARY OF LONGITUDINAL STABILITY DERIVATIVES

Dimensional Form / **Dimensionless Form**

$$X_u = -\sum \frac{2q'd^2}{mU}\left[C_D + \frac{M^2}{2\beta}\left\{\frac{\partial C_{Dw}}{\partial \beta} + \alpha^2 \frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right\}\right] \qquad x_u = \sum \left[C_D + \frac{M^2}{2\beta}\left\{\frac{\partial C_{Dw}}{\partial \beta} + \alpha^2 \frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right\}\right]$$

$$Z_u = -\sum \frac{2q'd^2}{mU}\left[C_L + \frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right] \qquad z_u = \sum \left[C_L + \frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right]$$

$$\frac{M_u}{K_y^2} = \sum \frac{2q'd^3}{mU}\frac{x'}{K_y^2}\left[C_L + \frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right] \qquad m_u = -\sum \frac{d^2x'}{K_y^2}\left[C_L + \frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)\right]$$

$$X_w = \sum \frac{q'd^2}{mU}\left[C_L - \frac{\partial C_D}{\partial \alpha}\right] \qquad x_w = -\sum \frac{1}{2}\left[C_L - \frac{\partial C_D}{\partial \alpha}\right]$$

$$Z_w = -\sum \frac{q'd^2}{mU}\left[C_D + \frac{\partial C_L}{\partial \alpha}\right] \qquad z_w = \sum \frac{1}{2}\left[C_D + \frac{\partial C_L}{\partial \alpha}\right]$$

$$\frac{M_w}{K_y^2} = \sum \frac{q'd^3}{mU}\frac{x'}{K_y^2}\left[C_D + \frac{\partial C_L}{\partial \alpha}\right] \qquad m_w = -\sum \frac{1}{2}\frac{d^2x'}{K_y^2}\left[C_D + \frac{\partial C_L}{\partial \alpha}\right]$$

$$X_q = -\sum \frac{q'd^3}{mU}x'\left[C_L - \frac{\partial C_D}{\partial \alpha}\right] \qquad x_q = \sum \frac{1}{2}x'\left[C_L - \frac{\partial C_D}{\partial \alpha}\right]$$

$$Z_q = \sum \frac{q'd^3}{mU}x'\left[C_D + \frac{\partial C_L}{\partial \alpha}\right] \qquad z_q = -\sum \frac{1}{2}x'\left[C_D + \frac{\partial C_L}{\partial \alpha}\right]$$

$$\frac{M_q}{K_y^2} = -\sum \frac{q'd^4}{mU}\frac{x'^2}{K_y^2}\left[C_D + \frac{\partial C_L}{\partial \alpha}\right] \qquad m_q = \sum \frac{1}{2}\frac{d^2x'^2}{K_y^2}\left[C_D + \frac{\partial C_L}{\partial \alpha}\right]$$

Note

Aerodynamic Coefficients are referred to d^2

APPENDIX IV

COEFFICIENTS OF THE STABILITY QUARTIC

In the general case the coefficients of the stability quartic are as follows:

$$A = 1$$

$$B = x_u + z_w + m_q$$

$$C = m_q(z_w + x_u) + m_w(\mu - z_q) + x_u z_w - x_w z_u - x_q m_u$$

$$D = m_q(x_u z_w - x_w z_u) + m_w(\mu x_u - \tfrac{1}{2} C_L \mu \tan \theta_o + x_q z_u - x_u z_q)$$
$$+ m_u(-\mu x_w - \tfrac{1}{2} C_L \mu + x_w z_q - x_q z_w)$$

$$E = \tfrac{1}{2} C_L \mu \left[m_w(z_u - x_u \tan \theta_o) - m_u(z_w - x_w \tan \theta_o) \right] .$$

Usually x_q and z_q appear in the equations in such a manner as to be negligible compared to other terms. If $x_u z_w$ and $x_w z_u$ are small also, and, if we consider the special case of horizontal flight, then the coefficients become:

$$A = 1$$

$$B = x_u + z_w + m_q$$

$$C = m_q(z_w + x_u) + \mu m_w$$

$$D = m_q(x_u z_w - x_w z_u) + \mu m_w x_u - \mu m_u(x_w - \tfrac{1}{2} C_L)$$

$$E = \tfrac{1}{2} C_L \mu (m_w z_u - m_u z_w) .$$

TABLE I DIMENSIONS AND AREAS

	Dimensional Symbol	Dimensional	Dimensionless Symbol	Dimensionless
General				
Overall Length	ℓ	37.5 ft.	ℓ'	15
Center of Gravity (2/5 from aft end)	x_o	15 ft.	x_o'	6
Moment of Inertia	I_y	19000 lb ft. sec.^{2}		
Radius of Gyration	K_y	7.822 ft.	K_y'	3.129
Body Density (Approx.)	ρ_b	60 lb/cu ft.		
Shell or Body				
Length	ℓ	37.5 ft.	ℓ'	15
Diameter	d	2.5 ft.		1
Center of Gravity (from aft end of shell)	x_o	15 ft.	x_o'	6
Length of Nose	ℓ_n	7.5 ft.	ℓ_n'	3
Nose half angle	ϕ	10 degrees		
Wetted Area	A_s	265.5 ft.^{2}	A_s'	42.48
Fin (Rectangular) [4]				
Span (excluding shell)	b_f	2.944 ft.	b_f'	1.1776
Chord	c_f	1.472 ft.	c_f'	.5888
Surface Area (excluding shell)	S_f	4.333 ft.^{2}	S_f'	.6933
Wetted Area	A_f	8.666 ft.^{2}	A_f'	1.3866
Aspect Ratio (excluding shell)			AR [2]	2.00
Max Thickness at 50% c_f	t	0.1472 ft.	η [3]	0.10
Wing - Case I (w_o = 18° 1')				
Span (including shell)	b_w	9.496 ft.	b_w'	3.798
Max Chord	$(c_w)_{max}$	14.598 ft.	$(c_w)'_{max}$	5.839
Average Chord	$\bar{c}_w$	7.299 ft.	$\bar{c}_w'$	2.920
Surface Area (including shell)	S_w	69.31 ft.^{2}	S_w'	11.090
Wetted Area	A_w	75.24 ft.^{2}	A_w'	12.038
Aspect Ratio			AR [2]	1.301
Max Thickness at 25% $(c_w)_{max}$	t	1.4598 ft.	η [3]	0.10
Wing - Case II (w_o = 25° 0')				
Span (including shell)	b_w	9.951 ft.	b_w'	3.980
Max Chord	$(c_w)_{max}$	10.670 ft.	$(c_w)'_{max}$	4.268
Average Chord	$\bar{c}_w$	5.335 ft.	$\bar{c}_w'$	2.134
Surface Area (including shell)	S_w	53.09 ft.^{2}	S_w'	8.494
Wetted Area	A_w	59.53 ft.^{2}	A_w'	9.525
Aspect Ratio (including shell)			AR [2]	1.865
Max Thickness at 25% $(c_w)_{max}$	t	1.0670 ft.	η [3]	0.10
Wing - Case III (w_o = 54° 3')				
Span (including shell)	b_w	15.093 ft.	b_w'	6.037
Max Chord	$(c_w)_{max}$	5.473 ft.	$(c_w)'_{max}$	2.189
Average Chord	$\bar{c}_w$	2.737 ft.	$\bar{c}_w'$	1.095
Surface Area (including shell)	S_w	41.30 ft.^{2}	S_w'	6.608
Wetted Area	A_w	57.51 ft.^{2}	A_w'	9.202
Aspect Ratio (including shell)			AR [2]	5.516
Max Thickness at 50% $(c_w)_{max}$	t	0.5473 ft.	η [3]	0.10
Wing - Case IV (Rectangular) [4]				
Span (including shell)	b_w	10.047 ft.	b_w'	4.019
Chord	c_w	5.024 ft.	c_w'	2.010
Surface Area (including shell)	S_w	50.48 ft.^{2}	S_w'	8.077
Wetted Area	A_w	75.73 ft.^{2}	A_w'	12.117
Aspect Ratio (including shell)			AR [2]	2.00
Max Thickness at 50% c_w	t	0.5024 ft.	η [3]	0.10

1 Dimensions are referred to d and areas to d^2 except as noted.

2 $AR = \frac{b^2}{S}$

3 Given in percent chord.

4 Fin and rectangular wing are bi-convex.

Note: Free stream conditions -

ρ = .000889 slugs / cu. ft. (h = 30,000 ft.)
a = 995 ft. / sec.

M =	1.7	1.3	
U =	1691	1294	ft. / sec.
q' =	1271	743.7	lb. / sq. in.
τ =	33.05	43.21	

TABLE II BASIC AERODYNAMIC COEFFICIENTS

Mach No.	Component	Ref. Area	c.p. Location	$\frac{\partial C_L}{\partial \alpha}$	C_{D_f} [1]	C_{D_w}	$\frac{\partial}{\partial \beta} C_{D_w}$	$\frac{M^2}{2\beta}\frac{\partial}{\partial \beta} C_{D_w}$	$\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	$\frac{M^2}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	$\frac{M^2}{\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$
	Shell	d^2	1.474d aft of tip	1.445	0.003	0.0895	-0.01890	-0.01986	0.06672	-0.07013	0.14025
	Fin	Fin	.50c aft l.e.	2.910	"	0.03879	-0.02822	-0.02966	-2.117	-2.225	-4.450
	Wing Case I	Wing	.67c " "	1.734	"	0.01333	0.01339	0.01407	-0.28855	0.05266 [2]	-0.60657
1.7	Wing Case II	"	.67c " "	2.264	"	0.02334	0.04056	0.04263	-0.55352	0.11295 [2]	-1.16357
	Wing Case III	"	.67c " "	2.910	"	0.03090	-0.02509	-0.02637	-2.117	-2.225	-4.450
	Wing Case IV	"	.4630c " "	2.3806	"	0.03879	-0.02822	-0.02966	-1.347	-1.416	-2.832
	Shell	d^2	1.63d aft of tip	1.412	0.003	0.1095	-0.03380	-0.03438	0.05272	0.05363	0.10725
	Fin	Fin	.50c aft l.e.	4.815	"	0.06420	-0.07728	-0.07862	-5.798	-5.898	-11.793
	Wing Case I	Wing	.67c " "	1.889	"	0.01355	0.01906	0.01939	-0.27229	0.05059 [2]	-0.55400
1.3	Wing Case II	"	.67c " "	2.562	"	0.01891	0.02876	0.02926	-0.59311	0.10643 [2]	-1.20674
	Wing Case III	"	.67c " "	4.815	"	0.05661	-0.09048	-0.09205	-5.798	-5.898	-11.793
	Wing Case IV	"	.4282c " "	3.3657	"	0.06420	-0.07728	-0.07862	-2.308	-2.348	-4.695

1 C_{D_f} is based on wetted area

2 A drag correction factor has been included in this term.
The term is:

$$\frac{M^2}{2\beta}\frac{\partial}{\partial \beta}\left\{\frac{\partial C_L}{\partial \alpha}\left[1 - \frac{k'}{2E(k')}\right]\right\}$$

where

$$\left.\begin{array}{ll} M = 1.3 & \text{Case I} \\ \text{"} & \text{Case II} \\ M = 1.7 & \text{Case I} \\ \text{"} & \text{Case II} \end{array}\right\} \frac{\partial}{\partial \beta}\left\{\frac{\partial C_L}{\partial \alpha}\left[1 - \frac{k'}{2E(k')}\right]\right\} \left\{\begin{array}{l} = 0.04973 \\ = 0.10462 \\ = 0.05009 \\ = 0.10746 \end{array}\right.$$

TABLE III STATIC STABILITY

Mach No.	n	Case	$(x')_s$	$(x')_f$	$(-x')_w$	α	σ	$-\frac{dC_M}{dC_L}$
1.7	1	All	7.526	5.344	1.500	.05235	.08725	.3169
	2	"	"	"	1.874	.04958	.11568	.6338
	3	"	"	"	2.248	.04798	.14124	.9507
	4	"	"	"	2.622	.04483	.16438	1.2676
1.3	1	I	7.369	5.344	1.500	.08182	.09641	.1237
		II	"	"	1.500	.07870	.09812	.1660
		III	"	"	1.500	.05350	.11194	.5328
		IV	"	"	1.465	.06309	.10405	.3627
	2	I	7.369	5.344	1.874	.07750	.12536	.4286
		II	"	"	1.874	.07455	.12696	.4730
		III	"	"	1.874	.05069	.13987	.8583
		IV	"	"	1.839	.05975	.13261	.6810
	3	I	7.369	5.344	2.248	.07361	.15140	.7335
		II	"	"	2.248	.07081	.15290	.7799
		III	"	"	2.248	.04817	.16504	1.1837
		IV	"	"	2.213	.05675	.15830	.9994
	4	I	7.369	5.344	2.622	.07009	.17495	1.0384
		II	"	"	2.622	.06743	.17637	1.0869
		III	"	"	2.622	.04588	.18781	1.5091
		IV	"	"	2.587	.05404	.18154	1.3177

TABLE IV-A DYNAMIC STABILITY DERIVATIVES - CASE I

Mach. No.	n	Component	$\frac{\partial C_L}{\partial \alpha}$	C_L	C_{D_f}	C_{D_w}	$\frac{\partial C_L}{\partial \alpha}\alpha^2$ [1]	C_D	$\frac{\partial C_D}{\partial \alpha}$ [2]	$\frac{M^2}{2\beta}\frac{\partial}{\partial \beta}C_{D_w}$	$\frac{M^2\alpha^2}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	$\frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	x_u	z_u	m_u	x_w	z_w	m_w	x_q	z_q	m_q
1.7	1	Shell	1.445	.07566	.12743	.08950	.00396	.22089	.15132	-.01986	.00019	.00367	.20122	.07933	-.06099	.03783	.8329	-.64033	-.28471	-6.2684	4.8191
		Fin	2.017	.17607	.00416	.02689	.01537	.04642	.35214	-.02056	-.01175	-.13462	.01411	.04145	-.02263	.08804	1.0319	-.56331	-.47049	-5.5145	3.0104
		Wing	19.230	1.00686	.03615	.14783	.03269	.21667	1.24928	.15603	.00155	-.17610	.37425	.83076	.12730	.12121	9.7233	1.48990	.18182	14.5850	2.2349
		Total	22.692	1.25859	.16774	.26422	.05202	.48398	1.75274	.11561	-.01001	-.30705	.58958	.95154	.04368	.24708	11.5881	.28626	-.57338	2.8021	10.0644
	2	Shell	do	.07164	do	do	.00355	.22048	.14328	do	.00017	.00348	.20079	.07512	-.05775	.03582	.8327	-.64020	-.26958	-6.2672	4.8182
		Fin		.23339			.02701	.05806	.46678		-.02065	-.17844	.01685	.05495	-.03000	.11670	1.0377	-.56649	-.62364	-5.5456	3.0273
		Wing		.95340			.02931	.21329	1.18318		.00139	-.16675	.37071	.78665	.15059	.11489	9.7216	1.86101	.21530	18.2183	3.4875
		Total		1.25843			.05987	.49183	1.79324		-.01909	-.34171	.58835	.91672	.06284	.26741	11.5920	.65432	-.67792	6.4055	11.3330
	3	Shell	do	.06803	do	do	.00320	.22013	.13606	do	.00015	.00330	.20042	.07133	-.05484	.03402	.8326	-.64007	-.25603	-6.2659	4.8172
		Fin		.28496			.04026	.07131	.56992		-.03078	-.21787	.01997	.06709	-.03662	.14248	1.0441	-.57011	-.76141	-5.5811	3.0467
		Wing		.90533			.02643	.21041	1.12352		.00125	-.15834	.36769	.74699	.17154	.10910	9.7202	2.23211	.24526	21.8510	5.0178
		Total		1.25832			.06989	.50185	1.82950		-.02938	-.37291	.58808	.88541	.08008	.28560	11.5969	1.02193	-.77218	10.0040	12.8817
	4	Shell	do	.06478	do	do	.00290	.21983	.12956	do	.00014	.00314	.20011	.06792	-.05222	.03239	.8324	-.63996	-.24377	-6.2648	4.8163
		Fin		.33164			.05453	.08558	.66328		-.04169	-.25357	.02333	.07807	-.04262	.16582	1.0512	-.57401	-.88614	-5.6192	3.0675
		Wing		.86206			.02396	.20794	1.06982		.00114	-.15077	.36511	.71129	.19051	.10388	9.7190	2.60315	.27237	25.4832	6.8255
		Total		1.25848			.08139	.51335	1.86266		-.04041	-.40120	.58855	.85728	.09567	.30209	11.6026	1.38918	-.85754	13.5992	14.7093
1.3	1	Shell	1.412	.11553	.12743	.10950	.00945	.24639	.23106	-.03438	.00036	.00439	.21237	.11992	-.09027	.05777	.8292	-.62417	-.42571	-6.1103	4.5995
		Fin	3.338	.32184	.00416	.04451	.03103	.07970	.64368	-.05451	-.03801	-.39413	-.01282	-.07229	.03946	.16092	1.7088	-.93292	-.85996	-9.1326	4.9855
		Wing	20.949	1.71402	.03611	.15027	.07783	.26421	1.90259	.21504	.00376	-.25134	.48301	1.46268	.22413	.09429	10.6066	1.62525	.14144	15.9099	2.4379
		Total	25.699	2.15139	.16771	.30428	.11831	.59030	2.77733	.12615	-.03389	-.64108	.68256	1.51031	.17332	.31298	13.1446	.06816	-1.14423	.6670	12.0229
	2	Shell	do	.10944	do	do	.00848	.24541	.21888	do	.00032	.00416	.21135	.11360	-.08551	.05472	.8287	-.62381	-.40323	-6.1068	4.5969
		Fin		.41848			.05246	.10113	.83696		-.06426	-.51248	-.01764	-.09400	.05131	.20924	1.7197	-.93876	-1.11818	-9.1899	5.0167
		Wing		1.62360			.06984	.25622	1.80214		.00337	-.23808	.47463	1.38552	.26523	.08927	10.6026	2.02968	.16729	19.8693	3.8036
		Total		2.15152			.13078	.60276	2.85798		-.06057	-.74640	.66834	1.40512	.23103	.35323	13.1510	.46711	-1.35412	4.5726	13.4172
	3	Shell	do	.10394	do	do	.00765	.24458	.20788	do	.00029	.00395	.21049	.10789	-.08121	.05197	.8283	-.62350	-.38297	-6.1037	4.5946
		Fin		.50541			.07652	.12519	1.01082		-.09373	-.61893	-.02305	-.11352	.06197	.25271	1.7317	-.94533	-1.35048	-9.2542	5.0518
		Wing		1.54209			.06300	.24938	1.71168		.00304	-.22613	.46746	1.31596	.30219	.08480	10.5992	2.43396	.19063	23.8270	5.4715
		Total		2.15144			.14717	.61915	2.93038		-.09040	-.84111	.65490	1.31033	.28295	.38948	13.1592	.86513	-1.54282	8.4691	15.1179
	4	Shell	do	.09897	do	do	.00694	.24387	.19794	do	.00026	.00376	.20976	.10273	-.07733	.04949	.8279	-.62323	-.36469	-6.1011	4.5926
		Fin		.58403			.10217	.15084	1.16806		-.12515	-.71521	-.02882	-.13118	.07161	.29201	1.7445	-.95233	-1.56050	-9.3227	5.0893
		Wing		1.46839			.05713	.24351	1.62983		.00276	-.21532	.46131	1.25307	.33562	.08072	10.5962	2.83810	.21165	27.7832	7.4412
		Total		2.15139			.16624	.63822	2.99583		-.12213	-.92677	.64224	1.22462	.32990	.42222	13.1686	1.26254	-1.71354	12.3594	17.1231

1 For wings of cases I and II the drag due to lift is given by:

$$\frac{d C_L}{d \alpha}\left[1 - \frac{k'}{2E(k')}\right]\alpha^2$$

2 $\frac{d C_D}{d \alpha} = 2 C_L$ except for wings of cases I and II.

TABLE IV-B DYNAMIC STABILITY DERIVATIVES - CASE II

Mach No,	n	Component	$\frac{\partial C_L}{\partial \alpha}$	C_L	C_{D_f}	C_{D_W}	$\frac{\partial C_L}{\partial \alpha}\alpha^2$ [1]	C_D	$\frac{\partial C_D}{\partial \alpha}$ [2]	$\frac{M^2}{2\beta}\frac{\partial}{\partial \beta}C_{D_w}$
1.7		Shell	1.445	.07566	.12743	.08950	.00396	.22089	.15132	-.01986
	1	Fin	2.017	.17607	.00416	.02689	.01537	.04642	.36214	-.02056
		Wing	19.230	1.00686	.02854	.19824	.03706	.26383	1.41573	.36209
		Total	22.692	1.25859	.16013	.31463	.05633	.53114	1.91919	.32167
		Shell		.07164			.00355	.22048	.14328	
	2	Fin	do	.23339	do	do	.02701	.05806	.46678	do
		Wing		.95340			.03323	.25990	1.34056	
		Total		1.25843			.06379	.53844	1.95062	
		Shell		.06803			.00320	.22013	.13606	
	3	Fin	do	.28496	do	do	.04026	.07131	.56992	do
		Wing		.90533			.02996	.25663	1.27297	
		Total		1.25832			.07342	.54807	1.97895	
		Shell		.06478			.00290	.21983	.12956	
	4	Fin	do	.33164	do	do	.05453	.08558	.66328	do
		Wing		.86206			.02717	.25384	1.21213	
		Total		1.25848			.08460	.55925	2.00497	
1.3		Shell	1.412	.11113	.12744	.10950	.00875	.24569	.22226	-.03438
	1	Fin	3.338	.32754	.00416	.04451	.03214	.08081	.65508	-.05451
		Wing	21.762	1.71270	.02857	.16062	.08041	.26961	2.04353	.24853
		Total	26.512	2.15137	.16017	.31463	.12130	.59611	2.92087	.15964
		Shell		.10527			.00785	.24479	.21054	
	2	Fin	do	.42383	do	do	.05381	.10248	.84766	do
		Wing		1.62239			.07216	.26136	1.93577	
		Total		2.15149			.13382	.60863	2.99397	
		Shell		.09999			.00708	.24402	.19998	
	3	Fin	do	.51042	do	do	.07804	.12671	1.02084	do
		Wing		1.54100			.06510	.25430	1.83866	
		Total		2.15141			.15022	.62503	3.05948	
		Shell		.09522			.00642	.24336	.19044	
	4	Fin	do	.58877	do	do	.10384	.15251	1.17754	do
		Wing		1.46745			.05903	.24823	1.75090	
		Total		2.15144			.16929	.64410	3.11888	

Mach No,	n	Component	$\frac{M^2\alpha^2}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	$\frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	x_u	z_u	m_u	x_w	z_w	m_w	x_q	z_q	m_q
1.7		Shell	.00019	.00367	.20122	.07933	-.06099	.03783	.8329	-.64033	-.28471	-6.2684	4.8191
	1	Fin	-.01175	-.13462	.01411	.04145	-.02263	.08804	1.0319	-.66331	-.47049	-5.5145	3.0104
		Wing	.00263	-.25872	.62855	.74814	.11464	.20444	9.7469	1.49352	.30666	14.6204	2.2403
		Total	-.00893	-.38967	.84388	.86892	.03102	.33031	11.6117	.28988	-.44854	2.8375	10.0698
		Shell	.00017	.00348	.20079	.07512	-.05775	.03582	.8327	-.64020	-.25948	-6.2672	4.8182
	2	Fin	-.02065	-.17844	.01685	.05495	-.03000	.11670	1.0377	-.56649	-.62364	-5.5456	3.0273
		Wing	.00236	-.24498	.62435	.70642	.13561	.19358	9.7450	1.86550	.36277	18.2627	3.4959
		Total	-.01812	-.41994	.84199	.83849	.04786	.34610	11.6154	.65881	-.53045	6.4493	11.3414
		Shell	.00015	.00330	.20042	.07133	-.05484	.03402	.8326	-.64007	-.25603	-6.2659	4.8172
	3	Fin	-.03078	-.21787	.01997	.06709	-.03662	.14248	1.0444	-.57011	-.76141	-5.5811	3.0467
		Wing	.00213	-.23263	.62085	.67270	.15448	.18382	9.7433	2.23741	.41323	21.9029	5.0297
		Total	-.02850	-.44720	.84124	.81112	.06302	.36032	11.6203	1.02723	-.60421	10.0559	12.8936
		Shell	.00014	.00314	.20011	.06792	-.05222	.03239	.8324	-.53996	-.24377	-6.2648	4.8163
	4	Fin	-.04169	-.25357	.02333	.07807	-.04262	.16582	1.0515	-.57401	-.88614	-5.6192	3.0675
		Wing	.00193	-.22151	.61786	.64055	.17157	.17604	9.7419	2.60928	.45895	25.5433	6.8415
		Total	-.03962	-.47194	.84130	.78654	.07673	.37325	11.6258	1.39531	-.67096	13.6593	14.7253
1.3		Shell	.00033	.00422	.21164	.11535	-.08683	.05557	.8288	-.62391	-.40950	-6.1077	4.5976
	1	Fin	-.03936	-.40110	-.01306	-.07356	.04016	.16377	1.7095	-.93322	-.87519	-9.1356	4.9871
		Wing	.00560	-.40336	.52374	1.30934	.20063	.16542	11.0158	1.68795	.24813	16.5237	2.5319
		Total	-.03343	-.80024	.72232	1.35113	.15396	.38476	13.5541	.13082	-1.03656	1.2804	12.1166
		Shell	.00030	.00400	.21071	.10927	-.08225	.05264	.8284	-.62358	-.38790	-6.1045	4.5952
	2	Fin	-.06591	-.51902	-.01794	-.09519	.05196	.21191	1.7203	-.93913	-1.13248	-9.1935	5.0187
		Wing	.00502	-.38209	.51491	1.24030	.23743	.15669	11.0117	2.10799	.29364	20.6359	3.9504
		Total	-.06059	-.89711	.70768	1.25438	.20714	.42124	13.5604	.54528	-1.22674	5.3379	13.5643
		Shell	.00027	.00380	.20991	.10379	-.07813	.05000	.8280	-.62329	-.36845	-6.1016	4.4963
	3	Fin	-.09559	-.62506	-.02339	-.11464	.06258	.25521	1.7325	-.94574	-1.36384	-9.2583	5.0540
		Wing	.00453	-.36292	.50736	1.17808	.27053	.14883	11.0082	2.52788	.33457	24.7464	5.6827
		Total	-.09079	-.98418	.69388	1.16723	.25498	.45404	13.5687	.95885	-1.39772	9.3865	15.2330
		Shell	.00024	.00362	.20922	.09884	-.07440	.04761	.8277	-.62304	-.35084	-6.0992	4.4945
	4	Fin	-.12719	-.72100	-.02919	-.13223	.07218	.29439	1.7454	-.95279	-1.57322	-9.3272	5.0917
		Wing	.00411	-.34560	.50087	1.12186	.30048	.14173	11.0051	2.94762	.37162	28.8554	7.7287
		Total	-.12284	-1.06298	.68090	1.08846	.29826	.48373	13.5782	1.37179	-1.55244	13.4290	17.3149

1 For wings of Cases I and II the drag due to lift is given by:

$$\frac{d C_L}{d \alpha}\left[1 - \frac{k'}{2E(k')}\right]\alpha^2$$

2 $\frac{d C_D}{d \alpha} = 2 C_L$ except for wings of Cases I and II.

TABLE IV-C DYNAMIC STABILITY DERIVATIVES - CASE III

Mach No.	n	Component	$\frac{\partial C_L}{\partial \alpha}$	C_L	C_{D_f}	C_{D_w}	$\frac{\partial C_L}{\partial \alpha}\alpha^2$	C_D	$\frac{M^2}{2\beta}\frac{\partial}{\partial\beta}C_{D_w}$	$\frac{M^2\alpha^2}{2\beta}\frac{\partial}{\partial\beta}\left(\frac{\partial C_L}{\partial\alpha}\right)$	$\frac{M^2\alpha}{2\beta}\frac{\partial}{\partial\beta}\left(\frac{\partial C_L}{\partial\alpha}\right)$	x_u	z_u	m_u	x_w	z_w	m_w	x_q	z_q	m_q
1.7	1	Shell	1.445	.07566	.12743	.08950	.00396	.22089	-.01986	.00019	.00367	.20122	.07933	-.06099	.03783	.8329	-.64033	-.28471	-6.2684	4.8191
		Fin	2.017	.17607	.00416	.02689	.01537	.04642	-.02056	-.01175	-.13462	.01411	.04145	-.02263	.08804	1.0319	-.56331	-.47049	-5.5145	3.0104
		Wing	19.230	1.00686	.02767	.20418	.05268	.28454	-.17425	-.04031	-.76980	.06998	.23706	.03633	.50343	9.7573	1.49511	.75515	14.6360	2.2427
		Total	22.692	1.25859	.15921	.32057	.07201	.55185	-.21467	-.05187	-.90075	.28531	.35784	-.04729	.62930	11.6221	.29147	0	2.8531	10.0722
	2	Shell	do	.07164	do	do	.00355	.22048	do	.00017	.00348	.20079	.07512	-.05775	.03582	.8327	-.64020	-.26958	-6.2672	4.8182
		Fin		.23339			.02701	.05806		-.02065	-.17844	.01685	.05495	-.03000	.11669	1.0377	-.56649	-.62359	-5.5456	3.0273
		Wing		.95340			.04723	.27903		-.03614	-.72893	.06864	.22447	.04297	.47670	9.7545	1.86732	.89334	18.2800	3.4994
		Total		1.25843			.07779	.56757		-.05662	-.90389	.28628	.35454	-.04478	.62921	11.6249	.66063	0	6.4672	11.3449
	3	Shell	do	.06803	do	do	.00320	.22013	do	.00015	.00330	.20042	.07133	-.05484	.03401	.8326	-.64006	-.25596	-6.2659	4.8171
		Fin		.28496			.04026	.07131		-.03078	-.21787	.01997	.06709	-.03662	.14248	1.0444	-.57011	-.76141	-5.5810	3.0467
		Wing		.90533			.04259	.27439		-.03259	-.69217	.06755	.21316	.04895	.45267	9.7522	2.23946	1.01760	21.9230	5.0343
		Total		1.25832			.08605	.56583		-.06322	-.90674	.28794	.35158	-.04251	.62916	11.6292	1.02929	0	10.0761	12.8981
	4	Shell	do	.06478	do	do	.00290	.21983	do	.00014	.00314	.20011	.06792	-.05202	.03239	.8324	-.63996	-.24377	-6.2648	4.8163
		Fin		.33164			.05453	.08558		-.04169	-.25357	.02333	.07807	-.04262	.16582	1.0515	-.57401	-.88614	-5.6192	3.0675
		Wing		.86206			.03862	.27042		-.02955	-.65909	.06662	.20297	.05436	.43103	9.7502	2.61151	1.13016	25.5651	6.8474
		Total		1.25848			.09605	.57583		-.07110	-.90952	.29006	.34896	-.04048	.62924	11.6341	1.39754	0	13.6811	14.7312
1.3	1	Shell	1.412	.07554	.12744	.10950	.00404	.24098	-.03438	.00015	.00287	.20675	.07841	-.05902	.03777	.8265	-.62214	-.27833	-6.0904	4.5845
		Fin	3.338	.37367	.00416	.04451	.04183	.09050	-.05451	-.05124	-.45760	-.01525	-.08393	.04582	.18684	1.7144	-.93589	-.99847	-9.1618	5.0014
		Wing	31.818	1.70220	.02761	.37408	.09107	.49275	-.60827	-.11155	-2.08455	-.22707	-.38235	-.05859	.85110	16.1552	2.47546	1.27665	24.2328	3.7133
		Total	36.568	2.15141	.15921	.52809	.13694	.82423	-.69716	-.16263	-2.53928	-.03557	-.38787	-.07179	1.07571	18.6960	.91743	0	8.9806	13.2992
	2	Shell	do	.07157	do	do	.00363	.24057	do	.00014	.00272	.20633	.07429	-.05592	.03579	.8263	-.62199	-.26374	-6.0889	4.5834
		Fin		.46670			.06530	.11397		-.07999	-.57178	-.02053	-.10488	.05725	.23345	1.7261	-.94227	-1.24756	-9.2242	5.0355
		Wing		1.61279			.08175	.48344		-.10014	-1.97506	-.22497	-.36227	-.06935	.80639	16.1505	3.09171	1.51117	30.2660	5.7939
		Total		2.15126			.15068	.83798		-.17999	-2.54412	-.03917	-.39286	-.06802	1.07563	18.7029	1.52745	0	14.9529	15.4128
	3	Shell	do	.06801	do	do	.00328	.24022	do	.00012	.00258	.20596	.07059	-.05314	.03400	.8261	-.62186	-.25055	-6.0876	4.5825
		Fin		.55092			.09092	.13959		-.11137	-.67467	-.02629	-.12375	.06755	.27546	1.7389	-.92926	-1.47206	-9.2926	5.0729
		Wing		1.53262			.07382	.47551		-.09043	-1.87687	-.22319	-.34425	-.07905	.76631	16.1465	3.70782	1.72266	36.2973	8.3352
		Total		2.15155			.16802	.85532		-.20168	-2.54896	-.04352	-.39741	-.06464	1.07577	18.7115	2.13670	0	20.9171	17.9906
	4	Shell	do	.06478	do	do	.00297	.23991	do	.00011	.00246	.20564	.06724	-.05062	.03239	.8260	-.62174	-.23868	-6.0865	4.5816
		Fin		.62693			.11774	.16641		-.14423	-.76775	-.03233	-.14082	.07687	.31347	1.7523	-.95658	-1.67518	-9.3643	5.1120
		Wing		1.45976			.06697	.46866		-.08204	-1.78765	-.22165	-.32789	-.08782	.72988	16.1431	4.32378	1.91375	42.3272	11.3370
		Total		2.15147			.18768	.87498		-.22616	-2.55294	-.04834	-.40147	-.06157	1.07574	18.7214	2.74546	0	26.8764	21.0306

TABLE IV-D DYNAMIC STABILITY DERIVATIVES - CASE IV

Mach No.	n	Component	$\frac{\partial C_L}{\partial \alpha}$	C_L	C_{D_f}	C_{D_w}	$\frac{\partial C_D}{\partial \alpha}\alpha^2$	C_D	$\frac{M^2}{2\beta}\frac{\partial}{\partial \beta}C_{D_w}$	$\frac{M^2\alpha^2}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	$\frac{M^2\alpha}{2\beta}\frac{\partial}{\partial \beta}\left(\frac{\partial C_L}{\partial \alpha}\right)$	x_u	z_u	m_u	x_w	z_w	m_w	x_q	z_q	m_q
1.7	1	Shell	1.445	.07565	.12743	.08950	.00396	.22089	-.01986	.00019	.03367	.20122	.07933	-.06099	.03783	.8329	-.64033	-.28471	-6.2684	4.8191
		Fin	2.017	.17607	.00416	.02689	.01537	.04642	-.02056	-.01175	-.13462	.01411	.04145	-.02263	.08804	1.0319	-.56331	-.47049	-5.5145	3.0104
		Wing	19.230	1.00686	.03635	.31331	.05268	.40234	-.23956	-.03133	-.59883	.13145	.40803	.06252	.05343	9.8162	1.50414	.75515	14.7243	2.2563
		Total	22.692	1.25859	.16784	.42970	.07201	.66965	-.27998	-.04289	-.72978	.34678	.52881	-.02109	.62930	11.6810	.30050	0	2.9414	10.0858
	2	Shell	do	.07164	do	do	.00355	.22048	do	.00017	.00348	.20079	.07512	-.05775	.03582	.8327	-.64017	-.26958	-6.2669	4.8179
		Fin		.23339			.02701	.05806		-.02065	-.17844	.01685	.05495	-.03000	.11669	1.0378	-.56654	-.62359	-5.5460	3.0276
		Wing		.95340			.04723	.39689		-.02809	-.55704	.12924	.38636	.07396	.47670	9.8134	1.87860	.89334	18.3903	3.5205
		Total		1.25843			.07779	.67543		-.04857	-.74200	.34686	.51643	-.01379	.62921	11.6839	.67169	0	6.5774	11.3660
	3	Shell	do	.06803	do	do	.00320	.22013	do	.00015	.00330	.20042	.07133	-.05484	.03401	.8325	-.64002	-.25596	-6.2654	4.8168
		Fin		.28496			.04026	.07131		-.03078	-.21787	.01997	.06709	-.03662	.14246	1.0441	-.56997	-.76141	-5.5797	3.0459
		Wing		.90533			.04259	.39225		-.02533	-.53844	.12736	.36689	.08425	.45267	9.8111	2.26298	1.01760	22.0554	5.0647
		Total		1.25832			.08605	.68369		-.05596	-.75301	.34775	.50531	-.00721	.62916	11.6877	1.04299	0	10.2103	12.9274
	4	Shell	do	.06478	do	do	.00290	.21983	do	.00014	.00314	.20011	.06792	-.05227	.03239	.8324	-.63994	-.24377	-6.2646	4.8162
		Fin		.33164			.05453	.08558		-.04169	-.25357	.02333	.07807	-.04262	.16582	1.0513	-.57390	-.88614	-5.5181	3.0669
		Wing		.86206			.03862	.38828		-.02297	-.51271	.12575	.34935	.09357	.43103	9.8091	2.62728	1.13016	25.7195	6.8687
		Total		1.25848			.09605	.69369		-.06452	-.76314	.34919	.49534	-.00127	.62924	11.6928	1.41344	0	13.8358	14.7718
1.3	1	Shell	1.412	.08908	.12744	.10950	.00562	.24256	-.03438	.00021	.00338	.20839	.09246	-.06960	.04454	.8273	-.62274	-.32822	-6.0962	4.5889
		Fin	3.338	.34735	.00416	.04451	.03614	.08481	-.05451	-.04427	-.42537	-.01397	-.07802	.04259	.17368	1.7115	-.93431	-.92815	-9.1463	4.9930
		Wing	27.185	1.71499	.03635	.51854	.10819	.66308	-.63501	-.07548	-1.19617	-.04741	.51882	.07764	.85750	13.9239	2.08371	1.25624	20.3985	3.0527
		Total	31.935	2.15142	.16795	.67255	.14995	.99045	-.72390	-.11954	-1.61816	.14701	.53326	.05063	1.07572	16.4627	.52666	0	5.1560	12.6346
	2	Shell	do	.08437	do	do	.00604	.24198	do	.00019	.00320	.20779	.08757	-.06592	.04219	.8270	-.62252	-.31090	-6.0942	4.5873
		Fin		.44268			.05870	.10737		-.07190	-.64211	-.01904	-.09943	.05428	.22134	1.7227	-.94042	-1.18284	-9.2061	5.0256
		Wing		1.62436			.09706	.65195		-.06771	-1.13295	-.05077	.49141	.09407	.81216	13.9166	2.66442	1.49360	25.5961	4.8999
		Total		2.15142			.16080	1.00130		-.13942	-1.67186	.13798	.47955	.08243	1.07571	16.4682	1.10148	0	10.2958	14.5128
	3	Shell	do	.08014	do	do	.00455	.24149	do	.00017	.00304	.20728	.08318	-.06261	.04007	.8267	-.62229	-.29528	-6.0920	4.5856
		Fin		.52844			.08365	.13232		-.10246	-.64713	-.02465	-.11869	.06479	.26422	1.7351	-.94719	-1.41199	-9.2724	5.0618
		Wing		1.54283			.08755	.64245		-.06109	-1.07609	-.05365	.46674	.10718	.77142	13.9137	3.19514	1.70715	30.7910	7.0708
		Total		2.15142			.17575	1.01626		-.16338	-1.72018	.12898	.43123	.10936	1.07571	16.4755	1.62566	0	15.4266	16.7182
	4	Shell	do	.07631	do	do	.00412	.24106	do	.00016	.00290	.20684	.07921	-.05963	.03816	.8265	-.62215	-.28120	-6.0905	4.5846
		Fin		.60602			.11001	.15868		-.13476	-.74214	-.03059	-.13612	.07431	.30301	1.7483	-.95440	-1.61928	-9.3429	5.1003
		Wing		1.46908			.07939	.63428		-.05539	-1.02465	-.05612	.44443	.11904	.73454	13.9096	3.72555	1.90025	35.9841	9.6380
		Total		2.15142			.19352	1.03402		-.18999	-1.76389	.12013	.38752	.13372	1.07571	16.4844	2.14900	0	20.5507	19.3229

TABLE V - COEFFICIENTS OF THE STABILITY QUARTIC AND ROUTH'S DISCRIMINANT

Mach No.	n	Case	B		C				D					E		R
			$x_u+z_w+m_q$	$m_q(z_w+x_u)$	μm_w [1]	C = ⑤ + ⑥	$m_q(x_u z_w - x_w z_u)$	$\mu m_w x_u$	$-\mu x_w$	$-\frac{1}{2}C_L\mu$	m_u(⑩ + ⑪)	D = ⑧ + ⑨ + ⑫	$m_w z_u$	$-m_u z_w$	E = $\frac{1}{2}C_L$(⑭ + ⑮)	$\times 10^{-6}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1.7	1	I	22.24	123	6399	6522	66	3773	-5524	-14069	-856	2983	.2724	-.5062	-3289	425
		II	22.53	125	6481	6606	96	5469	-7385	-14069	-666	4899	.2519	-.3602	-1524	706
		III	21.98	120	6516	6639	31	1859	-14069	-14069	1331	3221	.1043	.5496	9200	455
		IV	22.11	121	6718	6843	37	2330	-14069	-14069	593	2960	.1589	.2464	5702	436
	2	I	23.51	138	14624	14762	75	8607	-5978	do	-1260	7422	.5998	-.7284	-1809	2522
		II	23.80	141	14729	14870	108	12401	-7738		-1044	11465	.5524	-.5559	-49	3926
		III	23.26	135	14770	14910	35	4228	-14069		1260	5523	.2342	.5206	10619	1879
		IV	23.40	137	15021	15160	42	5211	-14069		388	5641	.3470	.1611	7148	1965
	3	I	25.07	157	22847	23004	85	13436	-6385	do	-1638	11883	.9048	-.9287	-336	6712
		II	25.36	161	22966	23127	122	19320	-8056		-1394	18048	.8332	-.7323	1420	10259
		III	24.82	154	23012	23170	40	6626	-14069		1196	7862	.3619	.4944	12047	4452
		IV	24.96	156	23318	23470	48	8109	-14069		203	8360	.5270	.0843	8600	4822
	4	I	26.90	179	31039	31218	97	18279	-6754	do	-1992	16384	1.1909	-1.1100	1138	13489
		II	27.19	184	31195	31379	140	26244	-8345		-1720	24684	1.0975	.8920	2891	20433
		III	26.66	176	31245	31420	46	9063	-14069		1139	10248	.4877	.4709	13487	8470
		IV	26.81	178	31600	31780	56	11035	-14069		36	11127	.7001	.0148	10058	9349
1.3	1	I	25.85	166	1523	1689	102	928	-6997	-24049	-5381	-4351	.1029	-2.2782	-52314	-174
		II	26.39	173	2925	3098	112	2113	-8602	-24049	-5027	-2802	.1768	-2.0868	-45933	-205
		III	31.96	248	20511	20759	-3	-729	-24049	-24049	3452	2720	-.3558	1.3422	23722	1773
		IV	29.24	210	11775	11986	23	1731	-24049	-24049	-2435	-681	.2809	-.8336	-13292	-228
	2	I	27.24	185	10441	10626	111	6980	-7897	do	-7380	-289	.6563	-3.0383	-57285	-41
		II	27.83	194	12191	12385	123	8627	-9418		-6932	1818	.6840	-2.8089	-51102	704
		III	34.08	288	34149	34440	-4	-1338	-24049		3272	1930	-.6001	1.2722	16160	2243
		IV	31.12	241	24626	24870	25	3398	-24049		-3965	-542	.5282	-1.3575	-19940	-400
	3	I	28.93	209	19334	19543	123	12667	-8708	do	-9269	3521	1.1336	-3.7234	-62282	2030
		II	29.50	217	21437	21654	135	14875	-10151		-8720	6290	1.1192	-3.4597	-56287	4027
		III	36.66	336	47770	48110	-7	-2079	-24049		3109	1023	-.8491	1.2095	8667	1792
		IV	33.32	278	36345	36620	28	4688	-24049		-5260	-544	.7010	-1.8018	-26470	-635
	4	I	30.93	236	28211	28447	136	18129	-9440	do	-11048	7217	1.5461	-4.3443	-67294	6362
		II	31.57	247	30669	30916	151	20883	-10815		-10399	10635	1.4931	-4.0498	-61486	10328
		III	39.70	392	61380	61770	-10	-2968	-24049		2961	-17	-1.1022	1.1527	1214	-44
		IV	35.93	321	48045	48370	30	5772	-24049		-6432	-630	.8328	-2.2043	-32983	-1053

[1] μ = 22,360.

TABLE VI MOTION CHARACTERISTICS FOR STABLE CONFIGURATIONS

Mach No.	n	Case	Short Oscillation $\lambda_{1,2}$	Short Oscillation $t_{\frac{1}{2}s}$	Short Oscillation T_s	Phugoid Oscillation $\lambda_{3,4}$	Phugoid Oscillation $t_{\frac{1}{2}p}$	Phugoid Oscillation T_p
1.7	4	I	-.4070 ± 5.230i	1.703	1.201	-.002492 & -.01339	87.28	
	3	II	-.3837 ± 4.475i	1.806	1.404	-.002686 & -.02093	58.69	
	1	III	-.3325 ± 2.443i	2.084	2.571	-.007270 ± .03487i	95.32	180.2
	1	IV	-.3345 ± 2.481i	2.072	2.532	-.006503 ± .02684i	106.57	234.1
1.3	1	III	-.4835 ± 4.332i	1.427	1.450	-.001956 ± .02684i	354.3	194.6

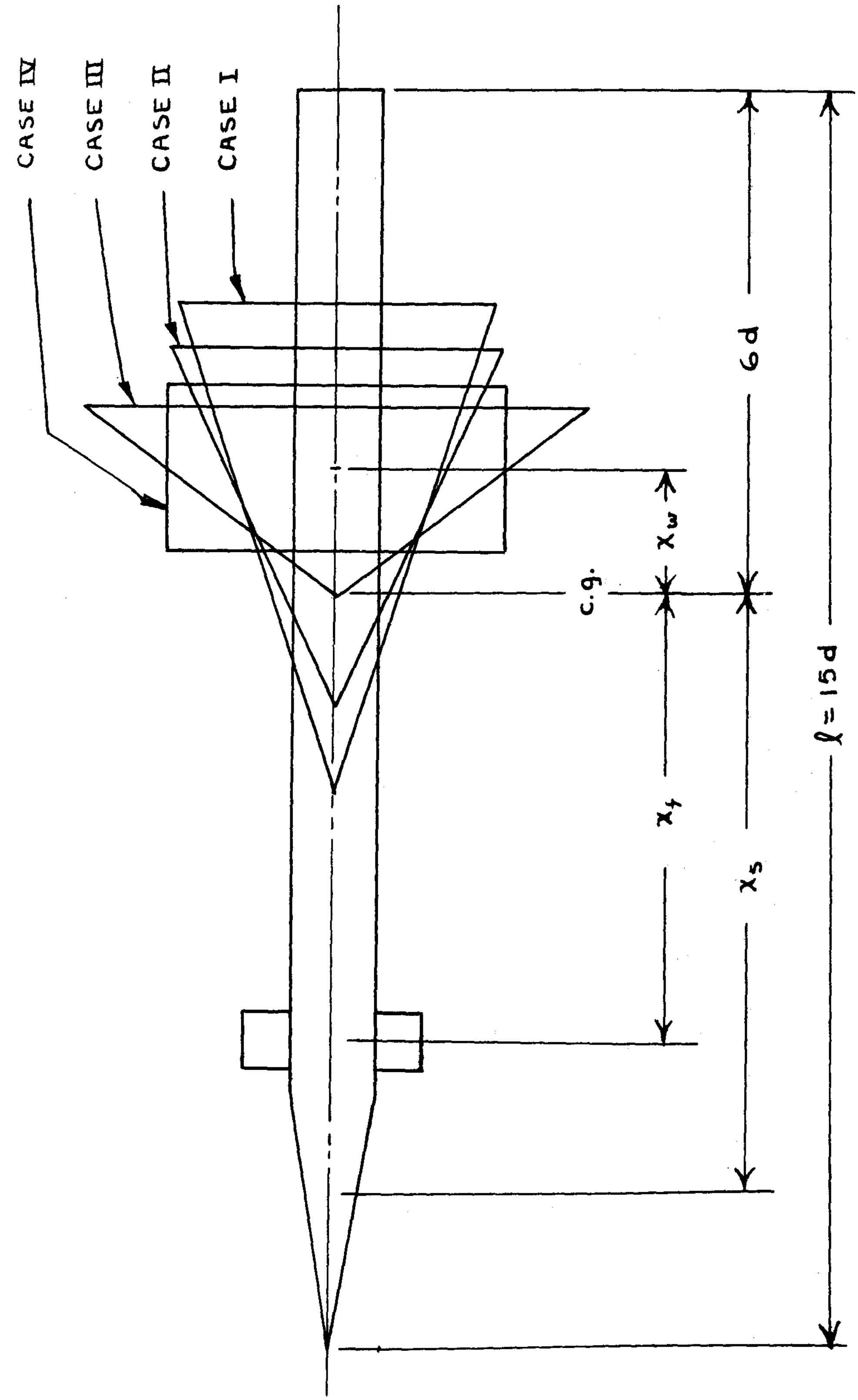


FIGURE 1. AIRPLANE CONFIGURATION

FIGURE 2. VARIATION OF QUARTIC COEFFICIENT "D" WITH STATIC STABILITY

$$D = m_q(x_u z_w - x_w z_u) + \mu m_w x_u - \mu m_u (x_w - \tfrac{1}{2} C_L)$$

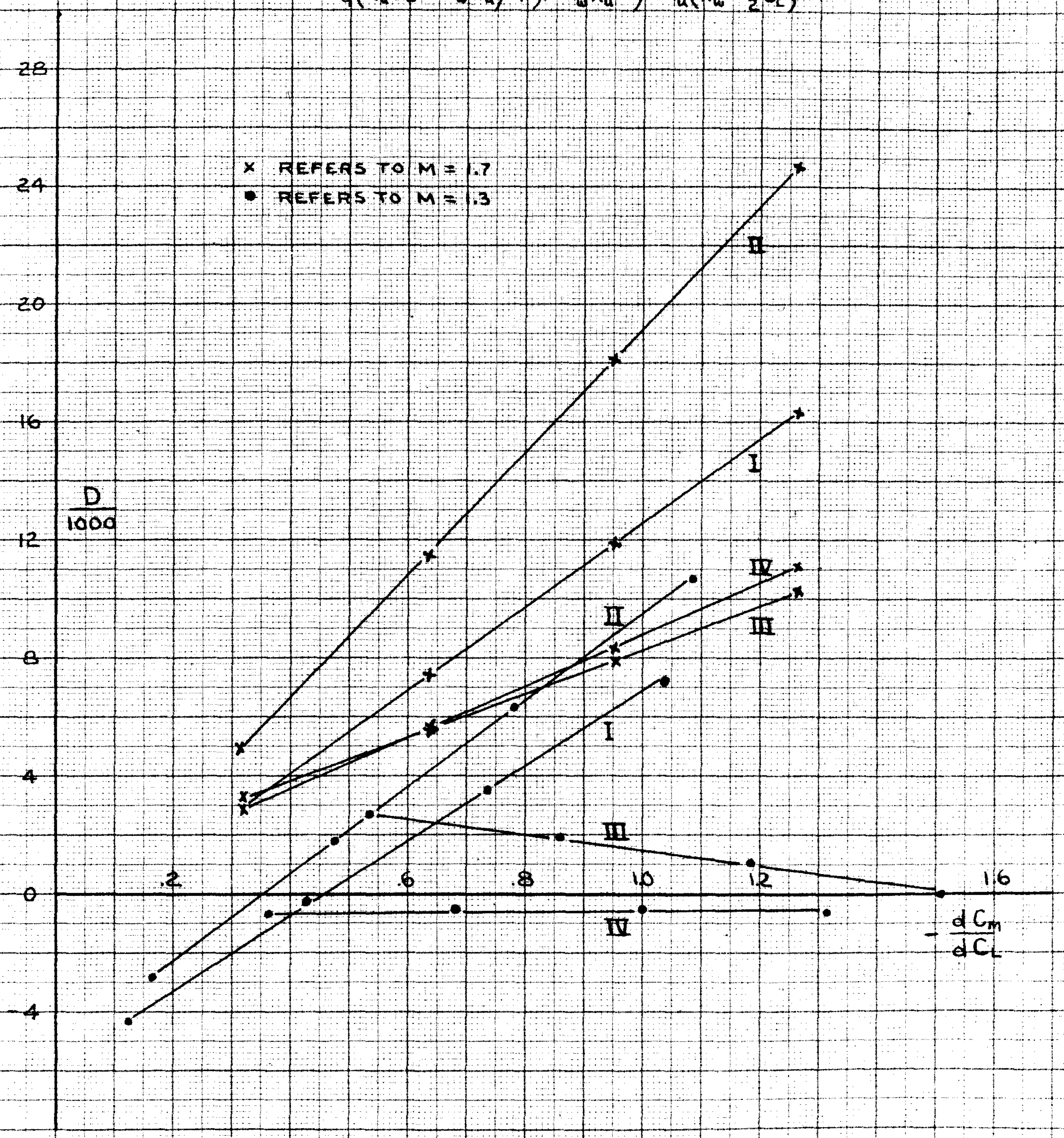


FIGURE 3. VARIATION OF QUARTIC COEFFICIENT "E" WITH STATIC STABILITY