MINIMUM DRAG DUE TO LIFT FOR A DELTA WING WITH SONIC LEADING EDGES

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ABSTRACT

Using linearized theory a close approximation to the optimum camber and twist distribution of a Delta wing of zero thickness with sonic leading edges has been obtained.

The optimum wing has a camber line that is negative over the front quarter of the total planform area and is positive over the remaining area. A certain amount of washout is exhibited.

Compared to the flat plate a saving in induced drag of slightly over 7 percent is shown. The leading edges are relieved of a considerable portion of the total lift. It is shown that this lift has been re-distributed over the central part of the wing.

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EXPLANATION OF SYMBOLS

C_L = wing lift coefficient

C_D = wing drag coefficient

ℓ = $\frac{C_L^2}{C_D}$

$\alpha(x, y)$ = angle of attack distribution (slope in x-direction of mean camber line)

L = lift

D' = drag

D = normalized drag = $\frac{D'}{4qS}$

p = local static pressure

p_∞ = static pressure at infinity

$\Delta p'(x, y)$ = local lift distribution = $2(p - p_\infty)$

Δp = normalized local lift distribution = $\frac{\Delta p'}{4qS}$

β = basic angle of attack distribution

$P_n(u)$ = nth Legendre polynomial

$\phi(x, y)$ = perturbation velocity potential

x, y, u, v = rectilinear coordinates

ξ, η, μ, ν = dummy rectilinear coordinates

$\theta, \overline{\theta}, \phi, \overline{\phi}$ = variables

$$I_{m,p} = \int_{\pi}^{\pi - \overline{\theta}} \sin (2m+1) \frac{\theta}{2} P_p (\cos \theta) \, d\theta$$

$$K_{m,n,p,q} = \frac{1}{\pi} \int_{\pi}^{0} \sin (2n+1) \frac{\overline{\theta}}{2} P_q (\cos \overline{\theta}) \, I_{m,p}(\overline{\theta}) \, d\overline{\theta}$$

$\left.\begin{matrix} b_2, b_3, b_4, \\ c_3, c_4, d_4 \end{matrix}\right\}$ = constants

$\eta(x, y)$ = camber and twist distribution

Subscripts identify particular wings. Subscript opt stands for optimum (min. drag) configuration of the four orthogonal wings considered.

I. INTRODUCTION

It is well known that in supersonic flow using linearized theory it is permissible to consider drag due to lift and drag due to thickness separately and obtain the total pressure drag by superposition. The drag due to lift is computed from the zero thickness mean camber wing at angle of attack and the wave drag is found by considering the symmetric thickness wing at zero angle of attack. This thesis is concerned with the problem of minimizing the drag due to lift of a wing of a specific planform.

In 1952 E. W. Graham (Ref. 1) developed a method of drag reduction using superposition of "orthogonal" loadings. Since this is the method used in this thesis a very brief description of the theory is given below.

Consider a free stream Mach number $M > 1$ and a wing planform of zero thickness with a certain angle of attack distribution. For this wing it is then possible to calculate pressure, lift, drag, moment, etc. If another wing of the same planform, but with a different kind of angle of attack distribution in a stream of the same Mach number is superimposed on the first wing, a new wing is obtained which has angle of attack distribution, pressure and lift equal to the sum of the two angle of attack distributions, pressures and lifts. The drags, however, do not add up in this manner. Since the resulting drag is the integral over the planform area of the product of the sum of the two pressures and the sum of the two angle of attack distributions, cross product terms (interference drag) appear. For some special loadings these cross product terms will disappear. By definition such loadings are called orthogonal. If the convention is made only to consider superposition of mutually orthogonal loadings it now becomes possible to maximize the non-dimensional parameter

$$\ell = \frac{C_L^{\,2}}{C_D}$$

which is a characteristic of the type of angle of attack distribution of the wing. If the first wing is defined by the subscript 1, the second wing by the subscript 2, and so on it can be shown that by superimposing several of these orthogonal wings in a manner so as to obtain minimum drag for a given lift, the resulting optimum wing has a parameter ℓ as follows:

$$\ell_{opt} = \ell_1 + \ell_2 + \ell_3 + \dots\dots\dots$$

Furthermore Rodriguez, Lagerstrom and Graham in 1954 (Ref. 2) showed that for the wing thus obtained

$$\alpha_{opt} = \frac{C_L}{\ell_{opt}} \sum_{k=1}^{\infty} \frac{L_k}{D_k} \, d_k$$

and $$\Delta p'_{opt} = \frac{C_L}{\ell_{opt}} \sum_{k=1}^{\infty} \frac{L_k}{D_k} \, \Delta p'_k$$

where α and $\Delta p'$ are defined as respectively the angle of attack and pressure distribution over the planform.

II. STATEMENT OF THE PROBLEM

The problem discussed in this thesis may be stated as follows: Given a Delta wing of zero thickness and sonic leading edges at a certain lift coefficient, find the camber and twist distribution for minimum drag. For convenience the Mach number has been chosen equal to $\sqrt{2}$. In order to have sonic leading edges the leading edge sweepback is 45°. By the Prandtl-Glauert rule the results can be applied to any sweepback angle or Mach number as long as the leading edges are sonic. To obtain the optimum angle of attack distribution it would be necessary to superimpose an infinite number of orthogonal loadings. Since it is not possible to find the sum of the series of orthogonal loadings used due to the somewhat complex expression of each loading, a true optimum cannot be obtained. It was decided to employ only four orthogonal loadings, this being a compromise between the amount of labor involved and the desirability of obtaining a large drag reduction. The camber and twist distribution found is therefore not a true optimum. It would be correct, however, to call it a restricted optimum since it represents the best possible combination of only four orthogonal loadings selected from an infinite number of orthogonal loadings.

Knowing the angle of attack distribution of a particular planform the pressures, lift, drag, moment, etc. can in theory be calculated using linearized supersonic wing theory. However, in practical application it is often very difficult to perform the required integrations analytically.

For the planform discussed in the present report the analytical work is comparatively simple. A complete set of basic angle-of-attack distributions may be found, using Legendre polynomials, such that lift, drag and interference drag may be calculated in closed form. This method is due to A. M. Rodriguez.

These basic distributions will be discussed in the following section.

III. BASIC DISTRIBUTION

The coordinates x, y and u, v (and the corresponding dummy variables of integration ξ, η and μ, ν) are shown in Fig. I. Let the basic angle of attack distribution be defined as

$$\beta_{mn} = \frac{1}{2}\left[P_m(u)P_n(v) + P_n(u)\, P_m(v)\right]$$

From supersonic wing theory the perturbation velocity potential (ϕ) corresponding to the flow of free stream velocity U over this wing is (from Ref. 4)

$$\phi(x, y) = -\frac{U}{\pi}\iint_R \frac{\beta_{mn}\, d\xi\, d\eta}{\sqrt{(x-\xi)^2-(y-\eta)^2}} \qquad \text{for } M = \sqrt{2}$$

where R = region of the forward Mach cone from the point (x, y).

Also from linearized theory (Ref. 4)

$$p-p_\infty = -\rho U\, \frac{\partial\phi}{\partial x}$$

Let the local lift, that is the differential pressure between top and bottom surface of the wing (positive in the positive lift direction), be written as

$$\Delta p'_{mn} = 2(p-p_\infty) = \frac{2\rho U^2}{\pi}\, \frac{\partial}{\partial x}\iint_R \frac{\beta_{mn}\, d\xi\, d\eta}{\sqrt{(x-\xi)^2-(y-\eta)^2}}$$

Since from Fig. I:

$$x = \frac{1}{\sqrt{2}}\,(u+v)$$

$$y = \frac{1}{\sqrt{2}}\,(u-v)$$

it can be easily verified that

$$\Delta p'_{mn}(u,v) = -\frac{\rho U^2}{\pi}\left(\frac{\partial}{\partial u}+\frac{\partial}{\partial v}\right)\int_{-1}^{u}\int_{-1}^{v}\frac{\beta_{mn}(\mu,\nu)\,d\mu\,d\nu}{\sqrt{u-\mu}\,\sqrt{v-\nu}}$$

A new set of variables ($\theta, \bar{\theta}$ and $\phi, \bar{\phi}$) for ease of integration is now introduced:

$$u = \cos\theta \qquad \mu = \cos\phi$$

$$v = \cos\bar{\theta} \qquad \nu = \cos\bar{\phi}$$

The following expression is then obtained

$$\Delta p'_{mn}(\theta,\bar{\theta}) = -\frac{\rho U^2}{\pi}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\right)\int_{\bar{\theta}}^{\pi}\int_{\theta}^{\pi}\frac{\beta_{mn}\sin\phi\sin\bar{\phi}\,d\phi\,d\bar{\phi}}{\sqrt{\cos\theta-\cos\phi}\sqrt{\cos\bar{\theta}-\cos\bar{\phi}}}$$

The Legendre polynomials may be expressed as

$$P_n(\cos\phi) = \frac{\sqrt{2}}{\pi}\int_0^{\phi}\frac{\cos(2n+1)\frac{\theta}{2}\,d\theta}{\sqrt{\cos\phi-\cos\theta}}$$

which when substituted into $\Delta p'_{mn}$ yields

$$\Delta p_{mn}(\theta,\bar{\theta}) = \frac{1}{2\pi}\left\{\frac{1}{\sin\theta}\left[\frac{\cos(2n+1)\frac{\theta}{2}\sin(2m+1)\frac{\bar{\theta}}{2}}{2n+1}+\frac{\cos(2m+1)\frac{\bar{\theta}}{2}\sin(2n+1)\frac{\theta}{2}}{2m+1}\right]\right.$$
$$\left.+\frac{1}{\sin\bar{\theta}}\left[\frac{\sin(2n+1)\frac{\bar{\theta}}{2}\cos 2(m+1)\frac{\theta}{2}}{2m+1}+\frac{\sin(2m+1)\frac{\bar{\theta}}{2}\cos(2n+1)\frac{\theta}{2}}{2n+1}\right]\right\}$$

The above expression has been normalized by dividing by $4\rho U^2$ (or $4qS$ since $S = 2$). This Δp_{mn} is now the normalized pressure distribution of a wing with angle of attack distribution β_{mn}. In Appendix I the four chosen basic loadings and their pressure distributions are given. It may be noted that β_{01} and β_{10} are not used since they give zero lift and are hence useless for drag reduction.

Since the cross product terms previously mentioned must be set equal to zero to satisfy the condition of orthogonality, it is advantageous to obtain a general expression for the pressure distribution of one loading acting on the angle of attack distribution of another loading. Let the interference drag between the angle of attack distribution β_{mn} and β_{pq} be denoted by D'(m, n;p, q). This quantity is then evaluated as:

$$D'(m,n;p,q) = \int_S \Delta p'_{mn}\, \beta_{pq}\, dS \equiv 4qS\, D(m,n;p,q)$$

where S = total planform area. Note that when m = p and n = q, D(m, n;m, n) is the normalized drag of the β_{mn} loading. Then

$$D(m,n;p,q) = \frac{1}{2\pi} \int_{\pi}^{0} \int_{\pi}^{\pi-\overline{\theta}} \left[\frac{\cos(2n+1)\frac{\overline{\theta}}{2}\sin(2m+1)\frac{\theta}{2}}{2n+1} + \frac{\cos(2m+1)\frac{\overline{\theta}}{2}\sin(2n+1)\frac{\theta}{2}}{2m+1} \right]$$

$$\times \left[P_p(\cos\theta)P_q(\cos\overline{\theta}) + P_q(\cos\theta)P_p(\cos\overline{\theta}) \right] \sin\overline{\theta}\, d\theta\, d\overline{\theta}$$

Performing the integration the following expression is obtained:

$$D(m,n;p,q) = \frac{1}{4}\left\{ \frac{1}{2n+1}\left[K_{m,n+1,p,q} + K_{m,n+1,q,p} + K_{m,-n,p,q} + K_{m,-n,p,q} + K_{m,-n,q,p} \right] + \frac{1}{2m+1}\left[K_{n,m+1,p,q} + K_{n,m+1,q,p} + K_{n,-m,p,q} + K_{n,-m,q,p} \right] \right\}$$

where by definition

$$K_{m,n,p,q} = \frac{1}{\pi} \int_{\pi}^{0} \sin(2n+1)\frac{\overline{\theta}}{2} P_q(\cos\overline{\theta})\, I_{m,p}(\overline{\theta})d\overline{\theta}$$

and

$$I_{m,p}(\overline{\theta}) = \int_{\pi}^{\pi-\overline{\theta}} \sin(2m+1)\frac{\theta}{2} P_p(\cos\theta)d\theta$$

The recurrence formulas for the K's and I's are:

$$K_{m,n,p,q} = \frac{2q-1}{2q}\left[K_{m,n+1,p,q-1} + K_{m,n-1,p,q-1}\right] - \frac{q-1}{q}K_{m,n,p,q-2}$$

$$I_{m,p}(\overline{\theta}) = \frac{2p-1}{2p}\left[I_{m+1,p-1} + I_{m-1,p-1}\right] - \frac{p-1}{p}I_{m,p-2}$$

It is seen that if the first few K's are found by regular integration the rest can be obtained from the recurrence formula. A "dictionary" of the values of the K's required to solve for the cross product drags of the four orthogonal loadings considered here is found in Table I.

IV. ORTHOGONALIZATION

So far no orthogonal loadings have been considered. It may be mentioned that the concept of orthogonality is not at all necessary for drag reduction calculations. For the special case of conical α-distributions S. H. Tsien (Ref. 3) has minimized the drag of a Delta wing with sonic L.E. without using orthogonal loadings. It is, however, believed that by using orthogonality the work involved in obtaining drag reduction is considerably lowered.

The general theory of orthogonality has been amply described by E. W. Graham (Ref. 1) and will not be repeated here.

Now from the basic angle of attack distribution it is desired to construct orthogonal loadings. Let by definition

$$\alpha_1 = \beta_{00}$$

$$\alpha_2 = \beta_{00} + b_2\beta_{11} \qquad \alpha_4 = \beta_{00} + b_4\beta_{11} + c_4\beta_{20} + d_4\beta_{21}$$

$$\alpha_3 = \beta_{00} + b_4\beta_{11} + c_3\,\beta_{20}$$

where the α's are the orthogonal loadings and the constants b_2, b_3, b_4, c_3, c_4 and d_4 are to be determined from the condition that each α distribution must be orthogonal to any other α distribution.

Having defined the angle of attack distributions as shown above it immediately follows that:

$$\Delta p_1 = \Delta p_{00}$$

$$\Delta p_2 = \Delta p_{00} + b_2\Delta p_{11}$$

$$\Delta p_3 = \Delta p_{00} + b_3\Delta p_{11} + c_3\Delta p_{20}$$

$$\Delta p_4 = \Delta p_{00} + b_4\Delta p_{11} + c_4\Delta p_{20} + d_4\Delta p_{21}$$

where Δp_1 is the pressure distribution of the wing with angle of attack distribution α_1; Δp_2 is due to α_2, etc.

As an illustration of how the constants above are determined consider the superposition of the first two loadings (α_1 and α_2). The resulting angle of attack and pressure distributions are respectively $(\alpha_1 + \alpha_2)$ and $(\Delta p_1 + \Delta p_2)$. Then the drag of the combination is

$$\int_S (\Delta p_1+\Delta p_2)(\alpha+\alpha_2)ds = \int_S \Delta p_1\alpha_1 ds + \int_S \Delta p_2\alpha_2 ds + \int_S \Delta p_1\alpha_2 ds + \int_S \Delta p_2\alpha_1 ds$$

where it is understood that the products as before are integrated over the planform area. The last two terms on the right-hand side represent the interference drag which must be set equal to zero. This means that

$$\int_S \Delta p_{00}(\beta_{00}+b_2\beta_{11})ds + \int_S (\Delta p_{00}+b_2\Delta p_{11})\beta_{00}ds = 0$$

or collecting terms:

$$[D(0,0;1,1)+D(1,1;0,0)]\, b_2 = -2D(0,0;0,0)$$

Hence b_2 is determined.

Similarly b_3 and c_3 are determined from the following two linear equations:

1) $[D(0,0;1,1)+D(2,1;0,0)]\, b_3 + [D(0,0;2,0)+D(2,0;0,0)]\, c_3 = -2D(0,0;0,0)$

2) $[D(0,0;1,1) + D(1,1;0,0) + 2b_2\, D(1,1;0,0)]\, b_3$
$\{+ D(0,0;2,0) + D(2,0;0,0) + b_2\, [D(1,1;2,0)$
$+D(2,0;1,1)]\}\, c_3 = -2D(0,0;0,0) - b_2\, [D(0,0;1,1)$
$+D(1,1;0,0)]$

For the determination of b_4, c_4 and d_4 the equations below must be solved.

$$1)\ \left[D(0,0;1,1) + D(1,1;0,0)\right] b_4 + \left[D(0,0;2,0) + D(2,0;0,0)\right] c_4 + \left[D(0,0;2,1) + D(2,1;0,0)\right] d_4 = -2D(0,0;0,0)$$

$$2)\ \left[D(0,0;1,1) + D(1,1;0,0) + 2b_2 D(1,1;1,1)\right] b_4 + \left\{D(0,0;2,0) + D(2,0;0,0) + b_2 \left[D(1,1;2,0) + D(2,0;1,1)\right]\right\} c_4 + \left\{D(0,0;2,1) + D(2,1;0,0) + b_2 \left[D(1,1;2,1) + D(2,1;1,1)\right]\right\} d_4 = -2D(0,0;0,0) - b_2 \left[D(0,0;1,1) + D(1,1;0,0)\right]$$

$$3)\ \left\{D(0,0;1,1) + D(1,1;0,0) + 2b_3 D(1,1;1,1) + c_3 \left[D(1,1;2,0) + D(2,0;1,1)\right]\right\} b_4 + \left\{D(0,0;2,0) + D(2,0;0,0) + b_3 \left[D(1,1;2,0) + D(2,0;1,1)\right] + 2c_3 D(2,0;2,0)\right\} c_4 + \left\{D(0,0;2,1) + D(2,1;0,0) + b_3 \left[D(1,1;2,1) + D(2,1;1,1)\right] + c_3 \left[D(2,0;2,1) + D(2,1;2,0)\right]\right\} d_4 = -2D(0,0;0,0) - b_3 \left[D(0,0;1,1) + D(1,1;0,0)\right] - c_3 \left[D(0,0;2,0) + D(2,0;0,0)\right]$$

With the aid of the general expression for $D(m,n;p,q)$ and Table I, Table 2 has been prepared. Using Table 2 it is now easy to show that the above system of equations become

$$-10b_2 = -240$$

$$\left\{\begin{array}{ll} -10b_3 + 15c_3 & = -240 \\ 470b_3 + 87c_3 & = 0 \end{array}\right\}$$

$$\left\{\begin{array}{ll} -10b_4 + 15c_3 + 19d_4 & = -240 \\ 470b_4 + 87c_3 - 149d_4 & = 0 \\ \qquad 417c_4 - 74d_4 & = 0 \end{array}\right\}$$

which upon solution give these results:

$$\begin{array}{lll} b_2 = 24 & b_3 = 29/11 & b_4 = -3.62385 \\ & c_3 = -470/33 & c_4 = -2.26300 \\ & & d_4 = -12.75229 \end{array}$$

The first four mutually orthogonal loadings are now completely determined, i.e.:

$$a_1 = \beta_{00}$$

$$a_2 = \beta_{00} + 24\beta_{11}$$

$$a_3 = \beta_{00} + 29/11\,\beta_{11} - 470/33\,\beta_{20}$$

$$a_4 = \beta_{00} - 3.62385\beta_{11} - 2.263\beta_{20} - 12.75229\beta_{21}$$

From these the corresponding pressure differences across the wings are determined:

$$\Delta p_1 = \Delta p_{00}$$

$$\Delta p_2 = \Delta p_{00} + 24\Delta p_{11}$$

$$\Delta p_3 = \Delta p_{00} + 29/11\ \Delta p_{11} - 470/33\ \Delta p_{20}$$

$$\Delta p_4 = \Delta p_{00} - 3.62385\ \Delta p_{11} - 2.263\ \Delta p_{20} - 12.75229\ \Delta p_{21}$$

Using Table 2 it is then just a simple calculation to obtain lift and drag of the orthogonal wings. Note that since $\beta_{00} = 1$, $D(m, n; 0, 0)$ = lift of the m, n loading. Thus

$$L_1 = L_2 = L_3 = 4qS$$

$$L_4 = 4qS \int_S \Delta p_4 ds = D(0,0;0,0) - 12.75229\, D(2,0;0,0) = (.14985) 4qS$$

Similarly

$$D_1' = D(0,0;0,0) = 4qS$$

$$D_2' = D_2\, 4qS = D(0,0;0,0) + (24)^2 D(1,1;1,1) + 24\; D(1,1;0,0)$$

$$+ D(0,0;1,1) \quad = (47)\, 4qS$$

$$D_3' = 24\, (4qS)$$

$$D_4' = 1.78448\, (4qS)$$

V. RESULTS

Percent Drag Reduction

The four orthogonal loadings are now combined so as to give the minimum drag using the non-dimensional parameter ℓ as defined on page 2. Since

$$\ell_1 = \frac{(4qS)^2 [D(0,0;0,0)]^2}{qS\ 4qS\ D(0,0;00)} = 4$$

$$\ell_2 = 4/47 = .08511$$

$$\ell_3 = 4/24 = .16667$$

$$\ell_4 = .05035$$

the parameter for the wing of minimum drag is

$$\ell_{opt} = \ell_1 + \ell_2 + \ell_3 + \ell_4 = 4.30213$$

This means that the percentage reduction in drag from the flat plate (α_1) drag is

$$1 - \frac{\ell_1}{\ell_{opt}} = 1 - \frac{4}{4.30213} = 7.023\ {}^o/o$$

Since ℓ_3 is larger than ℓ_2 and ℓ_4 the α_3 loading gives the most drag reduction.

Optimum camber and twist distribution

The optimum angle of attack distribution is given by

$$\alpha_{opt} = \frac{C_L}{\ell_{opt}} \sum_{k=1}^{4} \frac{L_k}{D_k}\ \alpha_k = \frac{C_L qS}{\ell_{opt}} \sum_{k=1}^{4} \frac{\ell_k}{L_k}\ \alpha_k$$

$$= \frac{C_L}{4.30213} (\alpha_1 + 1/47\ \alpha_2 + 1/24\ \alpha_3 + .084\ \alpha_4)$$

which in terms of x, y coordinates become

$$\frac{\alpha_{opt}}{C_L} = .35765 + .088x - .09007x^2 - .18311y^2 - .132x^3 + .132xy^2$$

and is found plotted for various spanwise stations in Fig. 2.

The expressions for α_1, α_2 and α_3 are given in Appendix 2. In order to obtain the camber and twist distribution of the optimum wing the angle of attack distribution was integrated with respect to the x variable. Let $\eta(x, y)$ be the camber and twist distribution. Then

$$\eta(x, y) = -\int_0^x \alpha_{opt}\, dx + f(y)$$

where f(y) is a constant of integration that has been determined (=0) by requiring the wing trailing edge to lie in the plane z = 0 (z being the axis perpendicular to x, y). The above integration results in

$$\frac{\eta(x, y)}{C_L} = -x(.35765 + .044x - .03002\, x^2 - .18311\, y^2 - .033x^3 + .066xy^2)$$

which is found plotted in Fig. 3 for several constant values of y. It is noted that the wing has washout which is in accordance with the usual aerodynamic practice. The wing has S camber over the center section.

Optimum pressure distribution

Similarly the optimum pressure distribution is given by

$$\Delta p_{opt} = \frac{C_L qS}{\ell_{opt}} \sum_{k=1}^{4} \frac{\ell_k}{L_k} \Delta p_k$$

$$= \frac{C_L}{4.30213} (\Delta p_1 + 1/47\, \Delta p_2 + 1/24\, \Delta p_3 + .084\, \Delta p_4)$$

which can be written as

$$\frac{\Delta p'_{opt}}{q} = \frac{4u_{opt}}{U} = \frac{8C_1}{4.30213}\ (\Delta p_1 + 1/47\ \Delta p_2 + 1/24\ \Delta p_3 + .084\ \Delta p_4)$$

or

$$\frac{u_{opt}}{U} = \frac{C_L}{2.15107}\ (\Delta p_1 + 1/47\ \Delta p_2 + 1/24\ \Delta p_3 + .084\ \Delta p_4)$$

In the u, v coordinates this becomes

$$\frac{C_L u_{opt}}{\pi\ U} = .0077481 \left\{ \sqrt{\frac{1+v}{1+u}}\ (21.60304 - 6.34892\,u \right.$$

$$+14.07988v + 2.04256\,uv - 12.7969u^2 - 11.12586\,v^2$$

$$-21.4162u^2v - 12.8497uv^2) + \sqrt{\frac{1+u}{1+v}}\ (21.60304$$

$$-6.34892v + 14.07988u + 2.04256uv - 12.7969v^2$$

$$-11.12586u^2 - 21.4162uv^2 - 12.8497u^2v)$$

where

$$u = \frac{1}{\sqrt{2}}\ (x+y) \qquad \text{and} \qquad v = \frac{1}{\sqrt{2}}\ (x-y)$$

Fig. 4 shows the distribution of local lift along the wing trailing edge compared with the flat plate distribution and the distribution obtained by S. H. Tsien (Ref. 3) by the conical flow theory.

In Fig. 5 the distribution of local lift along the root chord has been plotted. The flat plate lift distribution over the root chord is also shown.

VIII. CONCLUDING REMARKS

It has been shown that a reduction of induced drag of 7.023 percent can be obtained by suitably cambering and twisting the flat Delta wing. The procedure used can be continued to obtain even higher drag reduction by taking additional basic loading into account. For conical distribution S. H. Tsien (Ref. 3) obtained a drag reduction of $8^{o}/o$. By combining this loading with the non-conical loading of this report it should be possible to find an appreciable drag reduction. Due to lack of time, this has not been done here. However, the present report contains all the basic material needed and only some numerical work is required.

In Figs. 2 to 5 the angle of attack, local lift and the camber and twist distribution of the optimum wing are shown. The wing has washout and has negative camber over approximately the front quarter wing area, while the rest carries positive camber. The leading edge droops down and reaches a maximum droop at a point about 65 percent along the half span from the root chord.

Compared to the flat plate the optimum wing carries a higher lift distribution over the center section and less along the leading edges.

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APPENDIX I

$$\beta_{00} = 1$$

$$\beta_{11} = \cos\theta\cos\bar{\theta}$$

$$\beta_{20} = \frac{1}{4}\left[3\cos^2\theta + 3\cos^2\bar{\theta} - 2\right]$$

$$\beta_{21} = \frac{1}{4}\left[3\cos^2\theta\cos\bar{\theta} + 3\cos\theta\cos^2\bar{\theta} - \cos\theta - \cos\bar{\theta}\right]$$

$$\Delta p_{00} = \frac{1}{\pi}\left[\frac{\cos\frac{\bar{\theta}}{2}\sin\frac{\theta}{2}}{\sin\theta} + \frac{\sin\frac{\bar{\theta}}{2}\cos\frac{\theta}{2}}{\sin\bar{\theta}}\right]$$

$$\Delta p_{11} = \frac{1}{3\pi}\left[\frac{\cos 3\frac{\bar{\theta}}{2}\sin 3\frac{\bar{\theta}}{2}}{\sin\theta} + \frac{\sin 3\frac{\bar{\theta}}{2}\cos 3\frac{\theta}{2}}{\sin\bar{\theta}}\right]$$

$$\Delta p_{20} = \frac{1}{10\pi}\left[\frac{5\cos\frac{\bar{\theta}}{2}\sin 5\frac{\theta}{2} + \cos 5\frac{\bar{\theta}}{2}\sin\frac{\theta}{2}}{\sin\theta} + \frac{\sin\frac{\bar{\theta}}{2}\cos 5\frac{\theta}{2} + 5\sin 5\frac{\bar{\theta}}{2}\cos\frac{\theta}{2}}{\sin\bar{\theta}}\right]$$

$$\Delta p_{21} = \frac{1}{30\pi}\left[\frac{5\cos 3\frac{\bar{\theta}}{2}\sin 5\frac{\theta}{2} + 3\cos 5\frac{\bar{\theta}}{2}\sin 3\frac{\theta}{2}}{\sin\theta} + \frac{3\sin 3\frac{\bar{\theta}}{2}\cos 5\frac{\theta}{2} + 5\sin 5\frac{\bar{\theta}}{2}\cos 3\frac{\theta}{2}}{\sin\bar{\theta}}\right]$$

APPENDIX II

$a_1 = 1$

$a_2 = 1 + 24uv = 1 + 12(x^2 - y^2)$

$a_3 = 8.1212 + 2.6364\, uv - 10.6818(u^2 + v^2)$

$\quad = 8.1212 - 9.3636x^2 - 12y^2$

$a_4 = 2.1315 - 3.62385uv - 1.69725\,(u^2 + v^2)$

$\quad - 9.56422\, uv\,(u+v) + 3.18807(u+v)$

$\quad = 2.1315 + 4.50857x - 3.00918x^2 - 0.38532y^2$

$\quad + 6.76286xy^2 - 6.76286x^3$

TABLE I

$K_{m,n,p,q} = 0$ except as stated below

$K_{m,n,0,0}$	$= \frac{(-1)^m}{2m+1}$	when $m-n = 0$
	$= -\frac{(-1)^m}{2m+1}$	$m+n = -1$
$K_{m,n,1,0}$	$= -\frac{(-1)^m}{2(2m-1)}$	when $m-n = 1$
	$= -\frac{(-1)^m}{2(2m+3)}$	$m-n = -1$
	$= \frac{(-1)^m}{2(2m-1)}$	$m+n = 0$
	$= \frac{(-1)^m}{2(2m+3)}$	$m+n = -2$
$K_{m,n,2,0}$	$= \frac{3(-1)^m}{8(2m-3)}$	when $m-n = 2$
	$= \frac{(-1)^m}{4(2m+1)}$	$m-n = 0$
	$= \frac{3(-1)^m}{8(2m+5)}$	$m-n = -2$
	$= -\frac{3(-1)^m}{8(2m-3)}$	$m+n = 1$
	$= -\frac{(-1)^m}{4(2m+1)}$	$m+n = -1$
	$= -\frac{3(-1)^m}{8(2m+5)}$	$m+n = -3$
$K_{m,n,0,1}$	$= \frac{(-1)^m}{2(2m+1)}$	when $m-n = \pm 1$
	$= \frac{-(-1)^m}{2(2m+1)}$	$\left\{ \begin{array}{l} m+n = 0 \\ m+n = -2 \end{array} \right.$

$$K_{m,n,1,1} = -\frac{(-1)^m}{4}\left[\frac{1}{2m+3}+\frac{1}{2m-1}\right] \quad \text{when } m-n = 0$$

$$= -\frac{(-1)^m}{4}\frac{1}{(2m-1)} \quad m-n = 2$$

$$= -\frac{(-1)^m}{4(2m+3)} \quad m-n = -2$$

$$= \frac{(-1)^m}{4}\left[\frac{1}{2m+3}+\frac{1}{2m-1}\right] \quad m+n = -1$$

$$= \frac{(-1)^m}{4(2m-1)} \quad m+n = 1$$

$$= \frac{(-1)^m}{4(2m+3)} \quad m+n = -3$$

$$K_{m,n,2,1} = \frac{3(-1)^m}{16(2m-3)} \quad \text{when } m-n = 3$$

$$= \frac{3(-1)^m}{16(2m-3)} + \frac{(-1)^m}{8(2m+1)} \quad m-n = 1$$

$$= \frac{3(-1)^m}{16(2m+5)} + \frac{(-1)^m}{8(2m+1)} \quad m-n = -1$$

$$= \frac{3(-1)^m}{16(2m+5)} \quad m-n = -3$$

$$= \frac{-3(-1)^m}{16(2m-3)} \quad m+n = -2$$

$$= -\frac{(-1)^m}{8(2m+1)} - \frac{3(-1)^m}{16(2m-3)} \quad m+n = 0$$

$$= \frac{-3(-1)^m}{16(2m+5)} - \frac{(-1)^m}{8(2m+1)} \quad m+n = -2$$

$$= -\frac{3(-1)^m}{16(2m+5)} \quad m+n = -4$$

$$K_{m,n,0,2} = \frac{3(-1)^m}{8(2m+1)} \quad \text{when } m-n = \pm 2$$

$$= \frac{(-1)^m}{4(2m+1)} \quad m-m = 0$$

$$= -\frac{3(-1)^m}{8(2m+1)} \quad m+n = \begin{cases} 1 \\ -3 \end{cases}$$

$$= -\frac{(-1)^m}{4(2m+1)} \quad m+1 = -1$$

$$K_{m,n,1,2} = -\frac{3(-1)^m}{16(2m-1)} \qquad \text{when } m-n = 3$$

$$= -\frac{3(-1)^m}{16(2m+3)} - \frac{(-1)^m}{8(2m-1)} \qquad m-n = 1$$

$$= -\frac{(-1)^m}{8(2m+3)} - \frac{3(-1)^m}{16(2m-1)} \qquad m-n = -1$$

$$= -\frac{3(-1)^m}{16(2m+3)} \qquad m-n = -3$$

$$= \frac{3(-1)^m}{16(2m-1)} \qquad m+n = 2$$

$$= \frac{(-1)^m}{8(2m-1)} + \frac{3(-1)^m}{16(2m+3)} \qquad m+n = 0$$

$$= \frac{(-1)^m}{8(2m+3)} + \frac{3(-1)^m}{16(2m-1)} \qquad m+n = -2$$

$$= \frac{3(-1)^m}{16(2m+3)} \qquad m+n = -4$$

TABLE II

m	n	p	q	D(m, n;p, q)
0	0	0	0	1
0	0	1	1	-1/12
0	0	2	0	1/8
0	0	2	1	11/120
1	1	0	0	0
1	1	1	1	1/12
1	1	2	0	1/24
1	1	2	1	-23/840
2	0	0	0	0
2	0	1	1	-1/60
2	0	2	0	1/8
2	0	2	1	-1/84
2	1	0	0	1/15
2	1	1	1	-13/420
2	1	2	0	-5/168
2	1	2	1	7/240

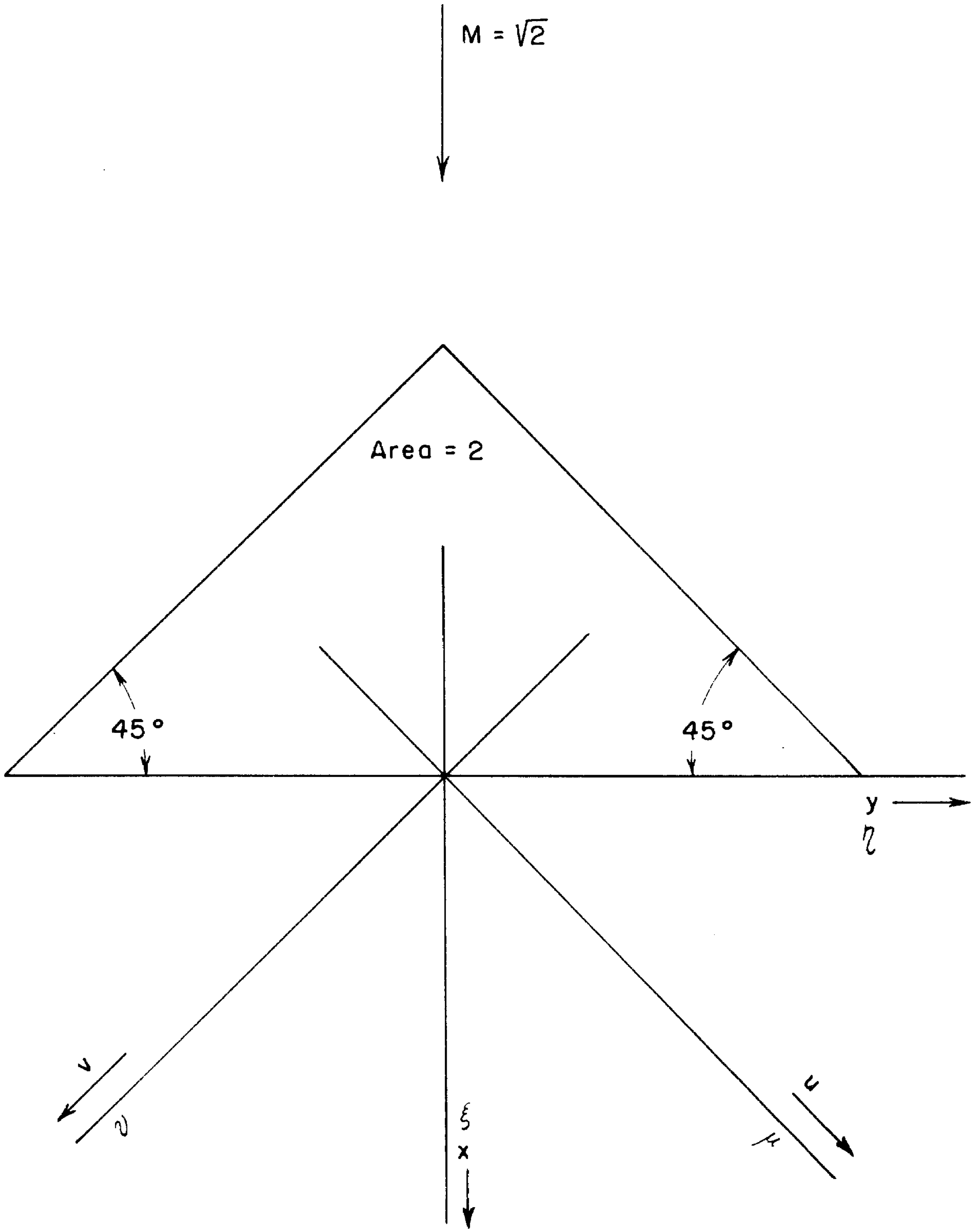

FIG. 1 — WING PLANFORM AND AXIS SYSTEM

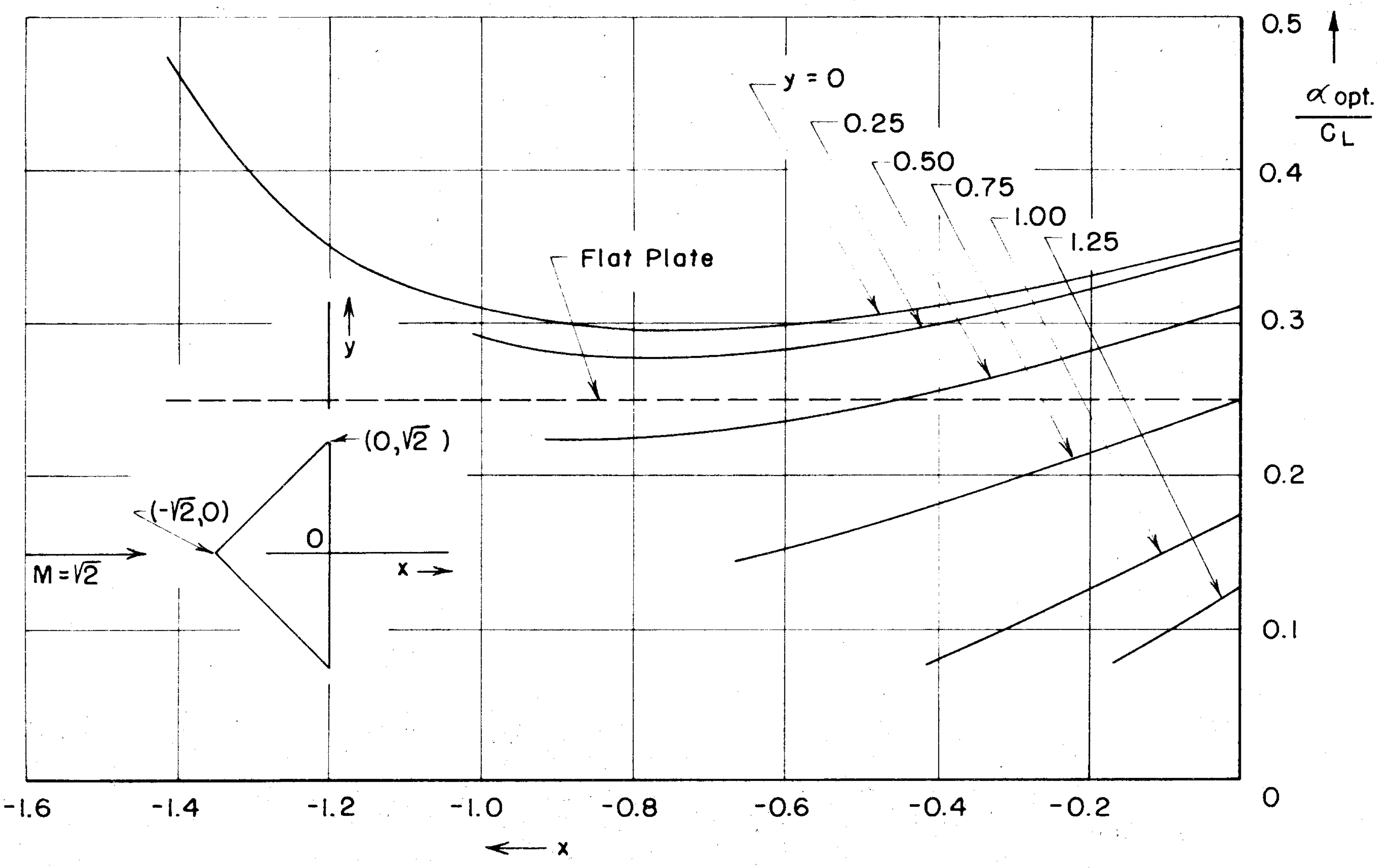

FIG. 2 - RESTRICTED OPTIMUM ANGLE OF ATTACK DISTRIBUTION

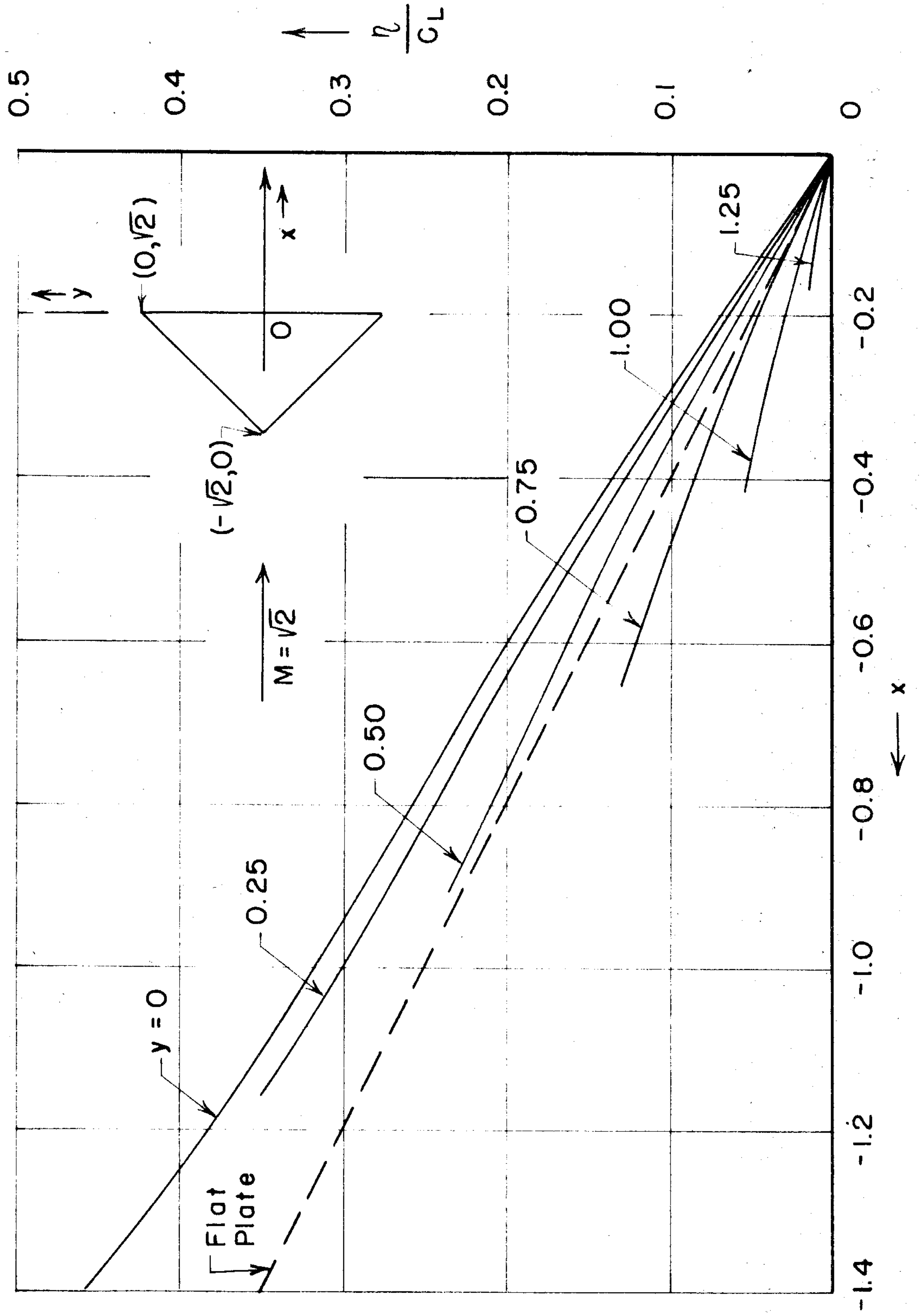

FIG. 3 - CAMBER AND TWIST DISTRIBUTION η

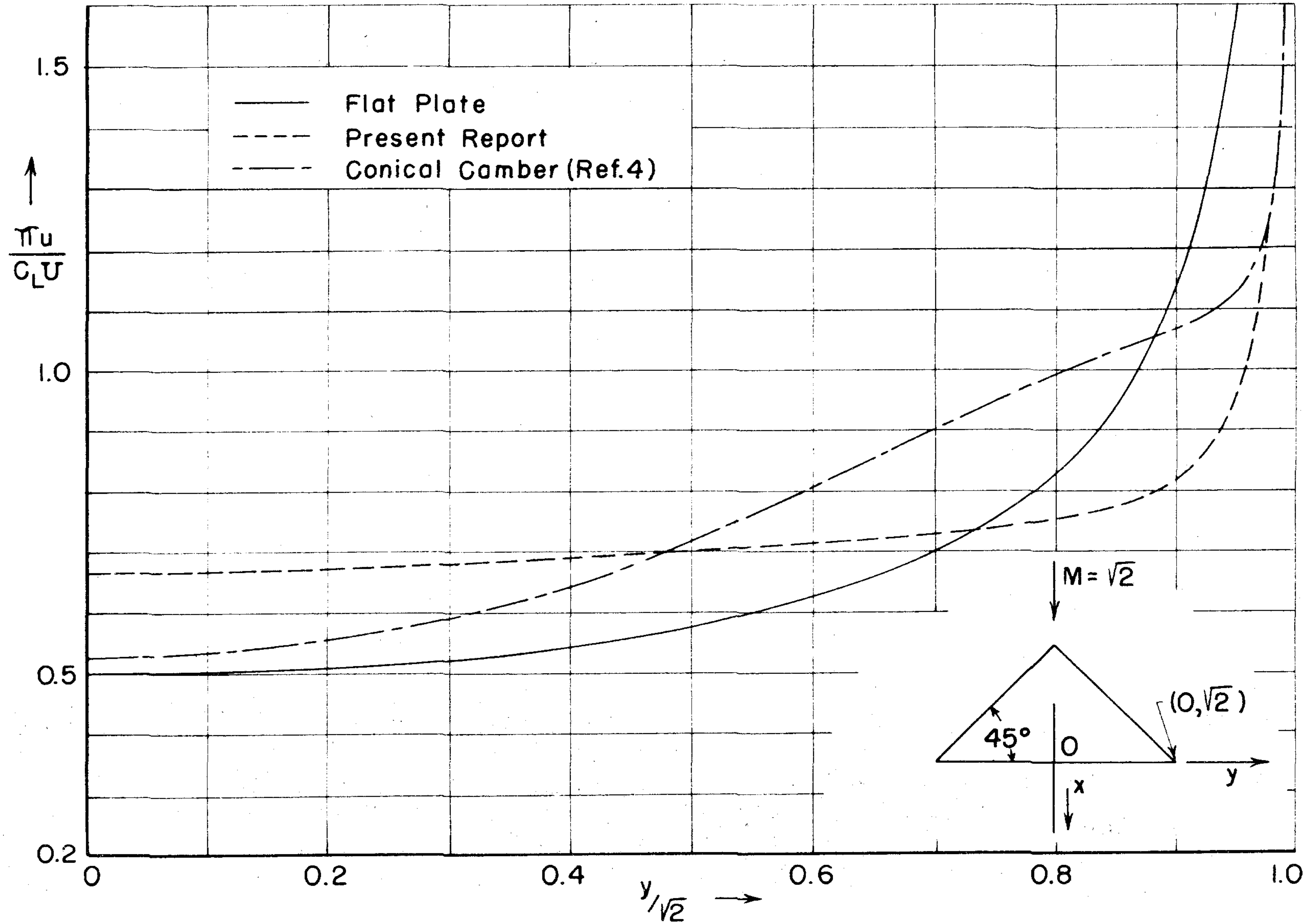

FIG. 4 - LOCAL LIFT DISTRIBUTION ALONG TRAILING EDGE

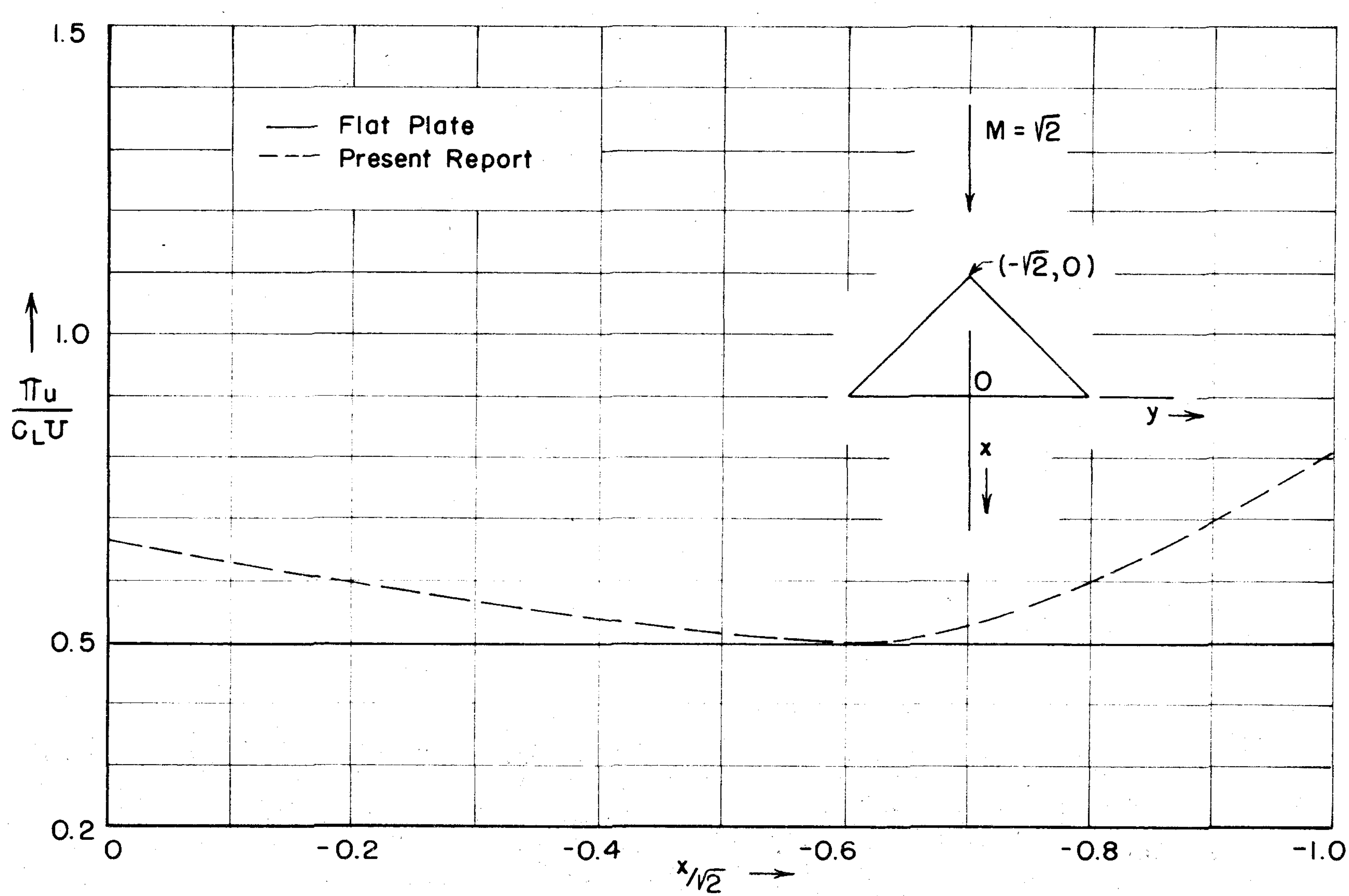

FIG. 5 - LOCAL LIFT DISTRIBUTION ALONG ROOT CHORD