DIRECT OUTPUT FEEDBACK CONTROL OF LARGE FLEXIBLE SPACECRAFT

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ABSTRACT

This report addresses the problem of active control of large flexible space structures. The current activities and control schemes in this field are briefly reviewed. A direct output feedback control (DOFC) technique is proposed to control the large flexible space structures. Assuming an N-degree-of-freedom system with n collocated sensor and actuator (S/A) pairs, where N is typically much larger than n, the analysis shows that at least the first n lowest critical vibration modes can be controlled with the system remaining stable. A formula for the selection of the feedback control gain matrix is provided. The DOFC approach is also applicable to the systems with certain types of nonlinearities, as well as systems including sensor/actuator dynamics. A simple criterion for selecting the "optimal" location of collocated sensor and actuator pairs is proposed. Numerical examples are given to illustrate the proposed DOFC technique and the "optimal" location criterion.

CONTENTS

Chapter 1

INTRODUCTION

In recent years, active control of large flexible spacecraft has received widespread attention and interest in the aerospace industry and in the academic community (e.g., [1]-[10]). The precision attitude control, configuration or shape control, and structure vibration suppression of a large communication satellite antenna, solar power station, large telescope, etc., all involve the control of large space structures (LSS). Stringent attitude pointing requirements, along with other mission requirements, make control of LSS quite challenging.

An LSS, like any other continuum, requires an infinite degree-of-freedom mathematical model to characterize precisely its dynamical behavior. However, only a finite dimensional dynamical model can be used for designing a controller or compensator of yet smaller dimension, because the size of the on-board computer is limited and the number of sensors and actuators is usually constrained. Thus, the fundamental problems of controlling a large flexible spacecraft are structure modeling, selection of controlled modes, controller design, sensor and actuator location and number, and the effect on system performance due to truncated modes.

During the last decade, numerous interesting techniques have been proposed. Balas [11], based on a finite N-mode mathematical model, used a typical state estimator and optimal linear feedback control law to investigate the effect of residual modes on the performance of the closed-loop system. Balas suggested the terms "Control Spillover" and "Observation Spillover" to characterize the sluggish response and even instability of the closed loop system due to truncated modes.

Assume that the system can be characterized by the following system of differential equations

$$\dot{x}_c = A_c x_c + B_c u \tag{1.1}$$

$$\dot{x}_s = A_s x_s + B_s u \tag{1.2}$$

with the measurement equation

$$z = H_c x_c + H_s x_s \tag{1.3}$$

where x is the state vector, u is the control input vector, and z is the measurement output vector. Subscripts c and s designate "controlled" and "suppressed" quantities. All vectors and matrices have compatible dimensions. The standard state feedback controller and state estimator can be written as

$$u = -C\,\hat{x}_c \tag{1.4}$$

and

$$\dot{\hat{x}}_c = A_c \hat{x}_c + B_c u + K[z - H_c \hat{x}_c] \tag{1.5}$$

where C is the feedback control gain matrix, $\hat{x}_c$ is the "best" estimate of the controlled state, x_c, and K is the state estimator gain matrix. Both C and K can be obtained either by minimizing certain quadratic cost functions and solving algebraic Riccati equations, or by pole placement technique described in standard modern control textbooks. Define

$$\tilde{x}_c \equiv x_c - \hat{x}_c \tag{1.6}$$

as the estimate error of x_c. Then from (1.1) to (1.6),

$$\dot{x}_c = A_c x_c - B_c C[x_c - \tilde{x}_c]$$

$$\dot{\tilde{x}}_c = [A_c - KH_c]\tilde{x}_c - KH_s x_s$$

$$\dot{x}_s = A_s x_s - B_s C x_c + B_s C\, \tilde{x}_c$$

or

$$\begin{bmatrix} \dot{x}_c \\ \dot{\tilde{x}}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c - B_c C & B_c C & 0 \\ 0 & A_c - KH_c & -KH_s \\ -B_s C & B_s C & A_s \end{bmatrix} \begin{bmatrix} x_c \\ \tilde{x}_c \\ x_s \end{bmatrix} \tag{1.7}$$

The feedback control excites the suppressed states through the term $B_s C$, which is called control spillover and the sensor outputs are contaminated by the suppressed states through the term KH_s, which is called observation spillover. If one ignores the suppressed states x_s, then from (1.7), it is clear that a stable controller and a stable estimator can be designed independently. The closed-loop system represented by (1.7) may become unstable if both control and observation spillover are present. If the observation spillover is zero (i.e., $KH_s = 0$), then the closed-loop eigenvalues of (1.7) are eigenvalues of stable matrices $(A_c - B_c C)$, $(A_c - KH_c)$, and A_s. Thus, the control spillover alone may degrade the performance of the closed-loop system but will not destabilize the system.

The search for a control technique which is immune to the "spill-over problem" has become a matter of intensive research in recent years; e.g., Balas [11] suggested a phase-locked loop prefilter to suppress the spillovers.

Skelton and Likins [12] suggested that an "orthogonal filter" be used, in conjunction with the state estimator, to match the truncation or model error; in this case the dimension of the system is increased.

Sesak et al. [13] proposed a model error sensitivity suppression technique which uses extra sensors and actuators to suppress modeling errors, thereby reducing control and observation spillovers.

Aubrun [14] used a low-gain controller which moderately modifies the structure's characteristics.

Balas [15] introduced a direct velocity feedback control scheme to control a number of lower critical vibration modes. This scheme, using a positive definite feedback gain matrix, will guarantee that all vibration modes remain stable when the active control is in operation; however, the choice of control gain matrix was left open.

Velman [16] used a low order filter to estimate an approximation to a desired linear function of state in conjunction with a high order design model. The filter output, as a function of state is characterized in terms of the transfer function of the estimator, thus permitting the use of classical design methods as well as the recently developed "robust observer" concept by Doyle and Stein [17,18].

Martin, Bryson, and Ashkenazi [19,20] applied a parameter optimization technique to the design of an optimal low order controller for a high order system. In the context of a specific example, these low order controllers were compared with the full order optimal controller and found to be less sensitive to modeling errors.

Tseng and Mahn [21] used the pole placement technique. Schaechter [22] proposed an optimal local control technique that includes feedback of only those state variables that are physically near a particular actuator. Working directly in physical coordinates rather than modal coordinates, a necessary condition has been derived for the solution of the linear quadratic optimal control problem with the constraint of local state feedback. A necessary condition for the optimal estimation of infinite dimensional systems is provided in [23].

Recently, Balas [7] surveyed the current trends in large space structure control theory and related topics in general control science, while Meirovitch et al. [8] made a comparison of control techniques for large flexible systems. Meirovitch et al. compared two active control approaches: coupled controls and independent modal-space controls. The coupled control technique requires fewer sensors and actuators, but requires a large on-board computer to accommodate the state estimator algorithm. The independent modal-space control method permits the design of the control system for each vibration mode separately, since the design takes place in the modal space. As a consequence, the independent modal-space control method demands less computational effort, but it requires more sensors and actuators. Evidently, the number of required actuators is equal to the number of controlled modes.

In this report we propose the direct output feedback control (DOFC) technique for the control of large space structures (LSS). Consider an N-degree-of-freedom structural system with n collocated actuators and sensors (measuring displacements and/or rates) where N is, in general, much greater than n. In this case, the DOFC technique is capable of controlling at least the first n critical vibration modes, while guaranteeing that the system remains stable. It can be shown that the DOFC technique can also be applied to systems with certain types of nonlinearity, and to systems which include the sensor and actuator dynamics.

The remainder of this report is organized as follows. Chapter 2 describes the direct output feedback control technique with some fundamental stability analysis. A method of selecting the feedback control gain matrix is provided. Chapter 3 addresses the issue of "optimal" location of sensor and actuator pairs from the standpoint of system controllability and observability. Concluding remarks, as well as suggestions for further research, are contained in Chapter 4.

Chapter 2

DIRECT OUTPUT FEEDBACK CONTROL

2.1 PROBLEM FORMULATION

The equations of motion of a large space structure (LSS) may be modeled by an N degree-of-freedom linear dynamic system with discrete parameters in matrix notation as

$$M\ddot{y} + D\dot{y} + Ky = f(t) \tag{2.1}$$

where M is an $N \times N$ positive definite symmetric mass matrix, D and K are both $N \times N$ matrices, and are respectively, positive semidefinite symmetric damping and stiffness matrices. (Let us use $\geq$ to mean positive semi-definite and $>$ to mean positive definite so that $M = M^T > 0$, $D = D^T \geq 0$, and $K = K^T \geq 0$.) The variables $y = [y_1 y_2 \cdots y_N]^T$ specify the displacements of each discrete mass related to the equilibrium position of the structure, they are identical to the generalized coordinates of Lagrangian mechanics. And $f(t) \in R^N$ is an external forcing function.

Assume that there are n pairs of collocated actuators and sensors (measuring displacements and/or rates). In reality, both sensors and actuators are dynamic elements which possess finite masses and exhibit certain time delay characteristics in their time responses. For simplicity, the sensors may be tentatively modeled as nondynamic; therefore, the rate measurement vector, $z_1(t)$, and the displacement measurement vector, $z_2(t)$ are directly related to $\dot{y}(t)$ and $y(t)$ as

$$z_1(t) = S\dot{y}(t) \quad , \qquad z_1(t) \in R^n \tag{2.2}$$

$$z_2(t) = Sy(t) \quad , \qquad z_2(t) \in R^n \tag{2.3}$$

The measurement matrix, S, has the following structure,

$$S = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & . & . & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & . & . & . & . & 1 & 0 \end{bmatrix} \tag{2.4}$$

where S, an $n \times N$ rectangular matrix, has all elements equal to zero except those unit entries which correspond to the structure stations where sensors and actuators are located.

Assuming collocation of sensors and actuators, the forcing function $f(t)$ on the right hand side of (2.1) can be expressed as

$$f(t) = S^T u(t) \tag{2.5}$$

where S^T is the transpose of S, and $u(t) \in R^n$ is the actuator output vector.

The direct output feedback control (DOFC) means that the actuator output $u(t)$ is directly proportional to the measurements, assuming the actuators to be nondynamic, thus

$$u(t) = -C_1 z_1(t) - C_2 z_2(t) \tag{2.6}$$

where $C_1 \varepsilon R^{nxn}$ and $C_2 \varepsilon R^{nxn}$ are the rate and displacement feedback control gain matrices,which are at the designer's discretion. Substituting (2.2) and (2.3) into (2.6) yields

$$u(t) = -C_1 S\dot{y}(t) - C_2 Sy(t) \tag{2.7}$$

Using (2.1), (2.5), and (2.7), the closed-loop system description is

$$\begin{aligned} M\ddot{y} + D\dot{y} + Ky &= -S^T C_1 S\dot{y} - S^T C_2 Sy \\ &= -B_1\dot{y} - B_2 y \end{aligned} \tag{2.8}$$

with $B_1 \equiv S^T C_1 S$ and $B_2 \equiv S^T C_2 S$.

The gain matrices C_1 and C_2 can be chosen such that the system described by (2.3) remains stable and possesses sufficient damping and stiffness to meet certain performance specifications. In (2.8), it is apparent that the effect of the rate feedback and displacement feedback on the system response may be investigated independently.

The following sections explore certain stability conditions related to the system of (2.8) for various cases and propose a procedure for selecting the proper feedback control gain matrices C_1 and C_2.

2.2 STABILITY ANALYSIS

This section addresses the critical issue of stability of the DOFC system. Specifically, we are considering the stability of the following four systems with DOFC,

(i) linear system

(ii) nonlinear system

(iii) system with time delay due to sensor/actuator dynamics

(iv) system including sensor/actuator masses

It is worth noting that for attitude control or rigid body mode control the displacement feedback is a necessity and that, in reality, all nonrigid body modes possess some finite, but small, structural damping. In the sequel, we will consider the system stability due to both displacement and rate feedback.

Theorem 2.1

Consider an N degree-of-freedom linear dynamic system with displacement and rate feedback described by

$$M\ddot{y} + D\dot{y} + Ky = -B_1\dot{y} - B_2 y \tag{2.9}$$

with initial conditions

$$y(0) = y_o \quad , \qquad \dot{y}(0) = \dot{y}_o$$

where the $N \times N$ matrices M, D, and K are symmetric, M is positive definite, while D and K are at least nonnegative definite; i.e., $M = M^T > 0$, $D = D^T \geq 0$, $K = K^T \geq 0$. If $B_1 = B_1^T \geq 0$, $B_2 = B_2^T \geq 0$, and $(D + B_1) > 0$, $(K + B_2) > 0$, then all solutions of (2.9) are asymptotically stable.

Proof

Consider the Lyapunov function $V(y,\dot{y})$

$$\begin{aligned} V &= \frac{1}{2}\,[\dot{y}^T M\dot{y} + y^T(K + B_2)y] \geq 0 \\ V &= 0 \qquad \text{iff} \qquad y = 0 \ \text{ and } \ \dot{y} = 0 \end{aligned} \tag{2.10}$$

Differentiating V with respect to time,

$$\dot{V} = \dot{y}^T[M\ddot{y} + (K+B_2)y] \tag{2.11}$$

Substituting (2.9) into (2.11) yields,

$$\dot{V} = -\dot{y}^T(D+B_1)\dot{y} \leq 0 \tag{2.12}$$

Since $(D+B_1)$ is positive definite,

$$\dot{V} = 0 \qquad \text{iff} \quad \dot{y} = 0 \tag{2.13}$$

But from (2.9), $\dot{y} = 0$ means that $\ddot{y} \neq 0$, if $y \neq 0$. Thus, $\dot{y} = 0$ exists only for an instant. We conclude that

$$\dot{V} < 0$$

Therefore, according to Lyapunov's second (or direct) stability theorem (e.g., [24]), all solutions of (2.9) are asymptotically stable.

Theorem 2.2

Consider an N-degree-of-freedom nonlinear dynamic system with displacement and rate feedback as follows:

$$M\ddot{y} + D\dot{y} + Ky + h(y) = -B_1\dot{y} - B_2 y \tag{2.14}$$

with

$$y(0) = y_0 \quad , \quad \dot{y}(0) = \dot{y}_0$$

where $M = M^T > 0$, $D = D^T \geq 0$, $K = K^T \geq 0$, and $h(y) = \nabla_y H(y)$, with $H(y) > 0$ and $H(0) = 0$ (∇_y is a vector gradient operator; $H(y)$ is a potential function). If $B_1 = B_1^T \geq 0$, $B_2 = B_2^T \geq 0$, and $(D+B_1) > 0$, $(K+B_2) > 0$, then all solutions of (2.14) are asymptotically stable.

Proof

Consider a Lyapunov function $V(y,\dot{y})$ of the form

$$V = \frac{1}{2}[\dot{y}^T M\dot{y} + y^T(K+B_2)y] + H(y) \geq 0 \tag{2.15}$$

$$V = 0 \qquad \text{iff} \qquad y = 0 \text{ and } \dot{y} = 0$$

Then

$$\dot{V} = \dot{y}^T[M\ddot{y} + (K+B_2)y] + \dot{y}^T\, h(y) \tag{2.16}$$

With (2.14), (2.16) becomes

$$\dot{V} = -\dot{y}^T(D+B_1)\dot{y} \leq 0 \tag{2.17}$$

$$\dot{V} = 0 \qquad \text{iff} \qquad \dot{y} = 0$$

Thus, applying the same argument as in Theorem 2.1, we conclude that all solutions of (2.14) are asymptotically stable.

Theorem 2.3

Consider an N degree-of-freedom linear dynamic system with displacement and rate feedback. Assume there are time delays associated with measurements. Then

$$M\ddot{y} + D\dot{y} + Ky = -B_1 z_1 - B_2 z_2 \tag{2.18}$$

$$y(0) = y_o\ , \quad \dot{y}(0) = \dot{y}_o$$

and

$$I\dot{z}_1 + \beta I z_1 = \beta I\dot{y} \tag{2.19}$$

$$I\dot{z}_2 + \beta I z_2 = I\dot{y} + \beta I y \tag{2.20}$$

where $M = M^T > 0$, $D = D^T \geq 0$, $K = K^T \geq 0$, and z_1 and z_2 are rate and displacement measurements, respectively. Assume that all measurements have identical time delays with time constant of $1/\beta$, $\beta > 0$. If $B_1 = B_1^T \geq 0$, $B_2 = B_2^T \geq 0$, and $(D + B_1) > 0$, $(K + B_2) > 0$, then all solutions of (2.18) are asymptotically stable.

<u>Proof</u>

Consider a Lyapunov function $V(y,\dot{y})$ of the form

$$V = \frac{1}{2}\,[\dot{y}^T M \dot{y} + y^T K y + \frac{1}{\beta}\,I\,z_1^T B_1\, z_1 + z_2^T B_2\, z_2] \geq 0 \qquad (2.21)$$

$$V = 0 \qquad \text{iff} \quad y = \dot{y} = z_1 = z_2 = 0$$

Then,

$$\dot{V} = \dot{y}^T[M\ddot{y} + Ky] + \dot{z}_2^T B_2\, z_2 + \frac{1}{\beta}\,I\dot{z}_1^T\, B_1\, z_1 \qquad (2.22)$$

Using (2.19) and (2.20), and assuming that one can adjust $z_2(0)$ such that $z_2(0) = y(0)$, it follows that

$$\frac{1}{\beta}\,I\,(\dot{z}_1 - \dot{y}) = -Iz_1 \qquad (2.23)$$

$$z_2 = y \qquad (2.24)$$

Substituting (2.18), (2.23), and (2.24) into (2.22),

$$\begin{aligned} \dot{V} &= -\,\dot{y}^T D\dot{y} - \dot{y}^T B_1 z_1 + \frac{1}{\beta_1}\,I\,\dot{z}_1^T B_1\, z_1 \\ &= -\,\dot{y}^T D\dot{y} - z_1^T B_1 z \qquad (2.25) \\ &= -\frac{1}{\beta^2}\,I\dot{z}_1^T\, D\dot{z}_1 - z_1^T(D + B_1)z_1 \leq 0 \end{aligned}$$

and

$$\dot{V} = 0 \qquad \text{iff} \qquad z_1 = \dot{z}_1 = 0 \quad \text{or} \quad \dot{y} = 0 \tag{2.26}$$

Therefore, applying the same argument as in Theorem 2.1, we conclude that all solutions of (2.18) are asymptotically stable.

Theorem 2.4

Consider an N degree-of-freedom linear dynamic system,

$$M\ddot{y} + D\dot{y} + Ky = f(t) = S^T u$$

with

$$y(0) = y_o , \quad \dot{y}(0) = \dot{y}_o \tag{2.27}$$

Assume there are rate measurements only and the actuator dynamics are second order. One can eliminate the effect of the actuator dynamics on the stability of the closed-loop system by letting

$$I\ddot{u} + [\text{\`{}}\beta_1\text{\textasciitilde}]\dot{u} + [\text{\`{}}\beta_2\text{\textasciitilde}]u = -C_1 S\dddot{y} - [\text{\`{}}\beta_1\text{\textasciitilde}]\, C_1 S\ddot{y} - [\text{\`{}}\beta_2\text{\textasciitilde}]C_1 S\dot{y} \tag{2.28}$$

where $M = M^T > 0$, $D = D^T \geq 0$, $K = K^T \geq 0$, I is the identity matrix, and S is defined in (2.4); C_1 is a positive definite rate feedback gain matrix. Assume identical dampings and natural frequencies for each actuator and $\ddot{y}$ is available for measurement. (Note that β_1 and β_2 are positive and known quantities associated with the actuators.) Define $B_1 = S^T C_1 S$. If $B_1 = B_1^T \geq 0$ and $(D + B_1) > 0$, then all solutions of (2.27) are asymptotically stable.

Proof

Let the vector ε be

$$\varepsilon = u + C_1 S\dot{y} \tag{2.29}$$

From (2.28),

$$I\ddot{\varepsilon} + [\ulcorner\beta_1\lrcorner]\,\dot{\varepsilon} + [\ulcorner\beta_2\lrcorner]\,\varepsilon = 0 \tag{2.30}$$

(2.30) has the solution

$$\varepsilon(t) = \overset{*}{U}(t)\;\varepsilon(0) + \overset{*}{V}(t)\;\;\dot{\varepsilon}(0) \tag{2.31}$$

where $U^*(t)$ and $V^*(t)$ are the principal matrix solutions of the following system

$$\ddot{U}^* + [\ulcorner\beta_1\lrcorner]\,\dot{U}^* + [\ulcorner\beta_2\lrcorner]\,U^* = 0$$

$$\ddot{V}^* + [\ulcorner\beta_1\lrcorner]\,\dot{V}^* + [\ulcorner\beta_2\lrcorner]\,V^* = 0 \tag{2.32}$$

with

$$U^*(0) = I \qquad \dot{U}^*(0) = 0$$

$$V^*(0) = 0 \qquad \dot{V}^*(0) = I$$

It is obvious that both $\overset{*}{U}(t)$ and $\overset{*}{V}(t)$ are bounded. From (2.29) and (2.31),

$$u = -C_1S\dot{y} + \overset{*}{U}(t)\;\varepsilon(0) + \overset{*}{V}(t)\;\;\dot{\varepsilon}(0) \tag{2.33}$$

Substituting (2.33) into (2.27) yields

$$M\ddot{y} + D\dot{y} + Ky = -B_1\dot{y} + S^T[U^*(t)\;\varepsilon(0) + V^*(t)\;\;\dot{\varepsilon}(0)] \tag{2.34}$$

Since the forcing terms in (2.34) are bounded and tend to zero as $t\to\infty$, the stability of (2.34) is completely determined by the unforced system

$$M\ddot{y} + D\dot{y} + Ky = -B_1\dot{y} \tag{2.35}$$

Then according to Theorem 2.1, all solutions of (2.27) are asymptotically stable.

Note that for the case of position measurements only, the stability of the system can be analyzed in the same fashion.

Theorem 2.5

Consider an N degree-of-freedom linear dynamic system with displacement and rate feedback. Assume the sensor and actuator pairs have identical lumped masses m. Then,

$$\begin{aligned} &(M + mS^T S)\ddot{y} + D\dot{y} + Ky = -B_1\dot{y} - B_2 y \\ &y(0) = y_o \qquad \text{and} \qquad \dot{y}(0) = \dot{y}_o \end{aligned} \tag{2.36}$$

where S is defined in (2.4), $M = M^T > 0$, $D = D^T \geq 0$, $K = K^T \geq 0$. If $B_1 = B_1^T \geq 0$, $B_2 = B_2^T \geq 0$, and $(D + B_1) > 0$, $(K + B_2) > 0$, then all solutions of (2.36) are asymptotically stable.

Proof

Consider a Lyapunov function $V(y,\dot{y})$ as

$$\begin{aligned} &V = \frac{1}{2}\,[\dot{y}^T(M + mS^T S)\dot{y} + y^T(K + B_2)y] \geq 0 \\ &V = 0 \qquad \text{iff} \qquad y = 0 \ \text{and} \ \dot{y} = 0 \end{aligned} \tag{2.37}$$

Then

$$\begin{aligned} &\dot{V} = -\dot{y}^T(D + B_1)\dot{y} \leq 0 \\ &\dot{V} = 0 \qquad \text{iff} \qquad \dot{y} = 0 \end{aligned} \tag{2.38}$$

Therefore, all solutions of (2.36) are asymptotically stable.

2.3 SELECTION OF FEEDBACK CONTROL GAIN MATRICES

Assume an LSS system, represented by the homogeneous portion of (2.1), possesses classical normal modes [25,26]; then there exists a so-called $N \times N$ modal-shape matrix Φ such that

$$\Phi^T M \Phi = I_N = \text{an } N \times N \text{ identity matrix} \tag{2.39}$$

$$\Phi^T D \Phi = \Lambda = \begin{bmatrix} \ddots & & 0 \\ & 2\xi_i\omega_i & \\ 0 & & \ddots \end{bmatrix} = \text{an } N \times N \text{ diagonal matrix} \tag{2.40}$$

$$\Phi^T K \Phi = \Omega = \begin{bmatrix} \ddots & & 0 \\ & \omega_i^2 & \\ 0 & & \ddots \end{bmatrix} = \text{an } N \times N \text{ diagonal matrix} \tag{2.41}$$

where ξ_i is the damping ratio of the i^{th} vibration mode, and ω_i is its modal frequency.

Let

$$y = \Phi x \qquad \text{so that} \qquad \dot{y} = \Phi\dot{x} \tag{2.42}$$

Substitute (2.42) into (2.8) and premultiply both sides of the equation by Φ^T,

$$\Phi^T M \Phi \ddot{x} + \Phi^T D \Phi \dot{x} + \Phi^T K \Phi x = -\Phi^T B_1 \Phi \dot{x} - \Phi^T B_2 \Phi x \tag{2.43}$$

where

$$B_1 = S^T C_1 S \qquad \text{and} \qquad B_2 = S^T C_2 S \tag{2.44}$$

Using (2.39), (2.40), and (2.41), (2.43) becomes

$$I\ddot{x} + \Lambda\dot{x} + \Omega x = -\Phi^T B_1 \Phi\dot{x} - \Phi^T B_2 \Phi x$$

$$\equiv -\bar{\Lambda}\dot{x} - \bar{\Omega}x \tag{2.45}$$

$$\bar{\Lambda} \equiv \Phi^T B_1 \Phi \quad ; \quad \bar{\Omega} \equiv \Phi^T B_2 \Phi$$

where x is the modal displacement vector.

Equation (2.45) reveals that the structural system represented by the unforced portion of (2.45) is uncoupled, but the feedback control makes the closed-loop system coupled. Furthermore, (2.45) shows that the rate and displacement feedback control gain matrices may be selected separately. In the sequel we will investigate the selection of rate feedback control gain matrix only; the selection of displacement feedback control gain matrix can be handled in exactly the same fashion. Rewrite (2.45) as

$$I\ddot{x} + \Lambda\dot{x} + \Omega x = -\Phi^T B_1 \Phi\dot{x}$$

$$\equiv -\bar{\Lambda}\dot{x} \tag{2.46}$$

Let

$$\Phi = [\Phi_I \quad \Phi_{II}] \tag{2.47}$$

where

$$\Phi_I \in R^{N\mathrm{x}n} \quad , \quad \Phi_{II} \in R^{N\mathrm{x}(N-n)}$$

Define

$$\Theta = S\Phi = [S\Phi_I \quad S\Phi_{II}]$$

$$= [\Theta_I \quad \Theta_{II}] \tag{2.48}$$

where

$$\Theta_I \in R^{nxn} \quad , \qquad \Theta_{II} \in R^{nx(N-n)}$$

Thus,

$$\bar{\Lambda} = \Phi^T B_1 \Phi = \Phi^T S^T C_1 S \Phi \quad = \begin{bmatrix} \Theta_I^T \, C_1 \Theta_I & \Theta_I^T \, C_1 \Theta_{II} \\ \\ \Theta_{II}^T C_1 \Theta_I & \Theta_{II}^T C_1 \Theta_{II} \end{bmatrix} \tag{2.49}$$

Define

$$\begin{aligned} \bar{\Lambda}_{II} &= \Theta_I^T C_1 \Theta_I \\ \bar{\Lambda}_{I\,II} &= \Theta_I^T C_1 \Theta_{II} \\ \bar{\Lambda}_{II\,I} &= \Theta_{II}^T C_1 \Theta_I \\ \bar{\Lambda}_{II\,II} &= \Theta_{II}^T C_1 \Theta_{II} \end{aligned} \tag{2.50}$$

Thus (2.49) becomes

$$\bar{\Lambda} = \Phi^T B_1 \Phi \;\; = \Phi^T S^T C_1 S \Phi = \begin{bmatrix} \bar{\Lambda}_{I\,I} & \bar{\Lambda}_{I\,II} \\ \bar{\Lambda}_{II\,I} & \bar{\Lambda}_{II\,II} \end{bmatrix} \tag{2.51}$$

where $\bar{\Lambda}_{II} \in R^{nxn}$, $\bar{\Lambda}_{I\,II} \in R^{nx(N-n)}$, $\bar{\Lambda}_{II\,I} \in R^{(N-n)xn}$,

and $\bar{\Lambda}_{II\,II} \in R^{(N-n)x(N-n)}$.

Assume that the system performance specification requires that each of the first n vibration modes possess a damping ratio of $\bar{\xi}_i$, $i = 1,\cdots,n$ in addition to the system natural damping of ξ_i, $i = 1,\cdots,N$. Since ω_i and Φ are assumed known and that S has been specified. (S depends on the location of sensors and actuators; the selection of S will be discussed in the next chapter), we then can choose $\bar{\Lambda}_{II}$ to be

$$\bar{\Lambda}_{II} = \Lambda^*_{II} = \begin{bmatrix} \ddots & & 0 \\ & 2\bar{\xi}_i\omega_i & \\ 0 & & \ddots \end{bmatrix} = \text{an nxn diagonal matrix} \qquad (2.52)$$

where Λ^*_{II} is a <u>given</u> nxn positive definite matrix. From (2.50), with both $\bar{\Lambda}_{II}$ and Θ_I nonsingular, the rate feedback control gain C_1 can be computed as

$$C_1 = \Theta_I^{-T} \Lambda^*_{II} \Theta_I^{-1} \qquad (2.53)$$

Therefore, if S is given, Λ^*_{II} can be specified arbitrarily (within some admissible set constrained by the size of the actuator) by the designer. Equations (2.53) and (2.49) indicate that C_1 is positive definite and $\Phi^T B_1 \Phi$ is at least positive semidefinite, since $\bar{\Lambda}_{II}$ is positive definite. Therefore, according to Theorem 2.1 of Section 2.2, the rate feedback control system (2.46) is stable. Define

$$x = [x_I^T \quad x_{II}^T]^T \qquad (2.54)$$

where

$$x_I \varepsilon R^{nx1} \quad , \quad x_{II} \varepsilon R^{(N-n)x1}$$

Combining (2.46), (2.49), and (2.52),

$$\ddot{x}_i + 2\xi_i\omega_i\dot{x}_i + \omega_i^2 x_i = -2\bar{\xi}_i\omega_i\dot{x}_i - (\bar{\Lambda}_{I\,II}\dot{x}_{II})_i \qquad (2.55)$$

$$\ddot{x}_j + 2\xi_j\omega_j\dot{x}_j + \omega_j^2 x_j = -(\bar{\Lambda}^T_{I\,II}\dot{x}_I)_j - (\bar{\Lambda}_{II\,II}\dot{x}_{II})_j \qquad (2.56)$$

where $i \varepsilon (1,n)$, $j \varepsilon (n+1,N)$, and $(\bar{\Lambda}_{I\,II}\dot{x}_{II})_i$ is the i^{th} element of $(\bar{\Lambda}_{I\,II}\dot{x}_{II})$. $(\bar{\Lambda}^T_{I\,II}\dot{x}_I)_j$ and $(\bar{\Lambda}_{II\,II}\dot{x}_{II})_j$ are the j^{th} element of $(\bar{\Lambda}^T_{I\,II}\dot{x}_I)$ and

$(\bar{\Lambda}_{II\,II}\,\dot{x}_{II})$, respectively.

The effect of displacement feedback on the system can be analyzed in exactly the same fashion. The feedback control gain matrix, C_1, can be selected according to (2.52) and (2.53). Equations (2.55) and (2.56) reveal that although the complete system is stable, the x_{II} modes will affect the response of the first x_I modes; therefore, the resulting closed-loop damping of the x_I modes may not be the same as those prescribed. For small feedback gain, Aubrun [14] used Jacobi's root perturbation formulas to estimate the discrepancy between the prescribed and resulting closed-loop system eigenvalues. A more straightforward way to remedy the problem of discrepancy between the prescribed and resulting closed-loop system eigenvalues is to use a few extra pairs of sensors and actuators, such that the x_I modes can be decoupled from the first few modes in x_{II}.

2.4 EXAMPLES

In this section three examples are used to illustrate the merit of the DOFC approach described in the previous sections.

Example 2.1

Consider a simply supported shear beam which is represented by a discrete model with ten identical lumped masses connected by springs, as shown in Figure 2.1. Assume all lumped masses have the same value of one, and the spring constants, k_1, are also identical, with the value of ten. The equation of motion is

$$M\ddot{y} + Ky = f(t) \tag{2.57}$$

where

$M =$ a 10×10 identity matrix

$$K = \begin{bmatrix} 20 & -10 & & & & & \\ -10 & 20 & -10 & & & & \\ & -10 & 20 & -10 & & & \\ & & & \ddots & & & \\ & & & -10 & 20 & -10 & \\ & & & & -10 & 20 & -10 \\ & & & & & -10 & 20 \end{bmatrix} \tag{2.58}$$

The modal frequencies are easily computed and are listed in the first column of Table 2-1.

Assume that there are two pairs of collocated rate sensors and actuators placed at stations 2 and 9, and that the structure possesses a damping ratio of 0.005 for each vibration mode. Assume the design specification requires that the first and second modes have induced damping time constants of 20.22 sec ($\frac{1}{\xi_1\omega_1} = \frac{1}{.05 \times .9} = 20.22$). Hence, $N = 10$, $n = 2$, and

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.59}$$

$$\overline{\Lambda}_{II} = \begin{bmatrix} .09 & 0 \\ 0 & .09 \end{bmatrix} \tag{2.60}$$

In modal coordinates, the equation of motion is

$$I\ddot{x} + \Lambda\dot{x} + \Omega x = -\Phi^T S^T C_1 S\Phi\dot{x} = -\overline{\Lambda}\dot{x} \tag{2.61}$$

where Φ is the modal-shape matrix of the system, which is easily computed by a standard eigenvalue-eigenvector subroutine, and

$$\Lambda = \begin{bmatrix} \ddots & & \\ & 2 \times 0.005 \times \omega_i & \\ & & \ddots \end{bmatrix} \tag{2.62}$$

$$\Omega = \begin{bmatrix} \ddots & & \\ & \omega_i^2 & \\ & & \ddots \end{bmatrix} \tag{2.63}$$

The control gain matrix C_1 can be computed, according to (2.53), as

$$C_1 = \begin{bmatrix} 0.5729 & 0.2738 \\ 0.2738 & 0.5729 \end{bmatrix} \tag{2.64}$$

Since the modal response of a shear beam due to a step excitation is inversely proportional to its modal frequency, let us assume the following initial conditions,

$$x_i(0) = \frac{\omega_1^2}{\omega_i^2} \quad , \qquad i \varepsilon (1,10) \tag{2.65}$$

The total energy E(t) can be represented as

$$E(t) = \sum_{i=1}^{10} [\omega_i^2 x_i^2(t) + \dot{x}_i^2(t)] \tag{2.66}$$

With C_1 in (2.64) and the initial conditions in (2.65), (2.61) can be completely solved.

The total energy, E(t), is plotted in Figure 2.2,which clearly shows that E(t) decreases with respect to time, as one expects of a stable system. The time response of all ten vibration modes with the

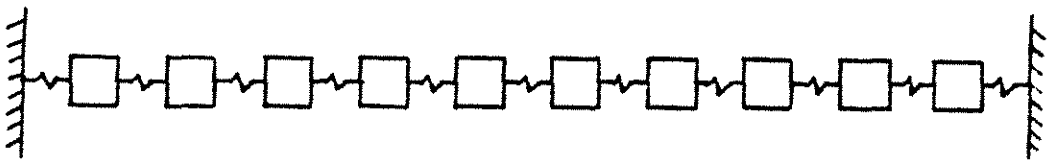

Figure 2.1 Discrete Model of a Shear Beam

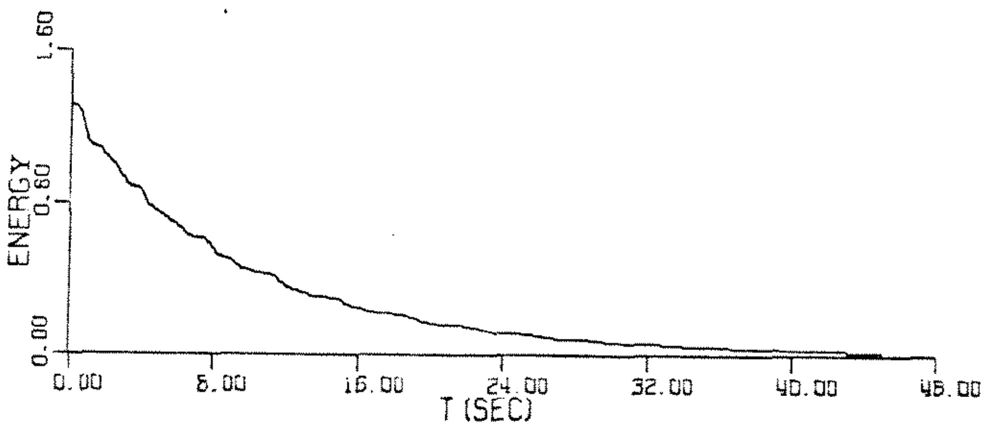

Figure 2.2 Total Energy

the same vertical scale and different vertical scales are shown in Figures 2.3 and 2.4, respectively. It is interesting to note that the rate feedback control provides not only the prescribed damping of first and second vibration modes, but it also introduces some damping in all the higher modes as well. The eigenvalues corresponding to the 20th order system (2.61) are listed in the second column of Table 2-1; they all have negative real parts.

Example 2.2

Consider the same system as in example 2.1, with the same rate feedback gain matrix C_1. Assume the collocated sensor and actuator pairs have a combined mass of m. Then the equation of motion becomes

$$(M + mS^T S)\ddot{y} + Ky = - S^T C_1 S\dot{y} \tag{2.67}$$

which can be expressed in state variable form as

$$\dot{x} = Ax \tag{2.68}$$

where

$$x \equiv \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

$$A \equiv \begin{bmatrix} 0 & I \\ -(M + mS^T S)^{-1}(S^T C_1 S) & -(M + mS^T S)^{-1} K \end{bmatrix}$$

$$x \in R^{20} \quad , \quad A \in R^{20 \times 20}$$

The eigenvalues of the system matrix A, for the cases of $m = 0.1$ and $m = 1$, are listed in columns 3 and 4 of Table 2-1, respectively. The

Table 2-1. SYSTEM EIGENVALUES $(-\xi_i\omega_i \pm j\omega_i)$

MODE NO.	Without DOFC	With DOFC		
	M = I, m = 0	M = I, m = 0	M = I, m = 0.1	M = I, m = 1
1	-.005 x .9 ± j.90	-.055 x .9 ± j.90	-.055 x .9 ± j.90	-.058 x .9 ± j.90
2	-.005 x 1.78 ± j1.78	-.030 x 1.78 ± j1.78	-.031 x 1.75 ± j1.75	-.032 x 1.54 ± j1.54
3	-.005 x 2.63 ± j2.63	-.063 x 2.62 ± j2.62	-.062 x 2.58 ± j2.58	-.052 x 2.26 ± j2.26
4	-.005 x 3.42 ± j3.42	-.014 x 3.42 ± j3.42	-.014 x 3.38 ± j3.38	-.011 x 3.16 ± j3.16
5	-.005 x 4.14 ± j4.14	-.008 x 4.14 ± j4.14	-.009 x 4.14 ± j4.14	-.005 x 4.09 ± j4.09
6	-.005 x 4.78 ± j4.78	-.006 x 4.78 ± j4.78	-.006 x 4.77 ± j4.77	-.006 x 4.71 ± j4.71
7	-.005 x 5.32 ± j5.32	-.022 x 5.32 ± j5.32	-.022 x 5.26 ± j5.26	-.014 x 4.89 ± j4.89
8	-.005 x 5.75 ± j5.75	-.014 x 5.75 ± j5.75	-.014 x 5.65 ± j5.65	-.008 x 5.12 ± j5.12
9	-.005 x 6.07 ± j6.07	-.025 x 6.04 ± j6.04	-.020 x 5.97 ± j5.97	-.007 x 5.78 ± j5.78
10	-.005 x 6.26 ± j6.26	-.008 x 6.26 ± j6.26	-.006 x 6.23 ± j6.23	-.0052 x 6.18 ± j6.18

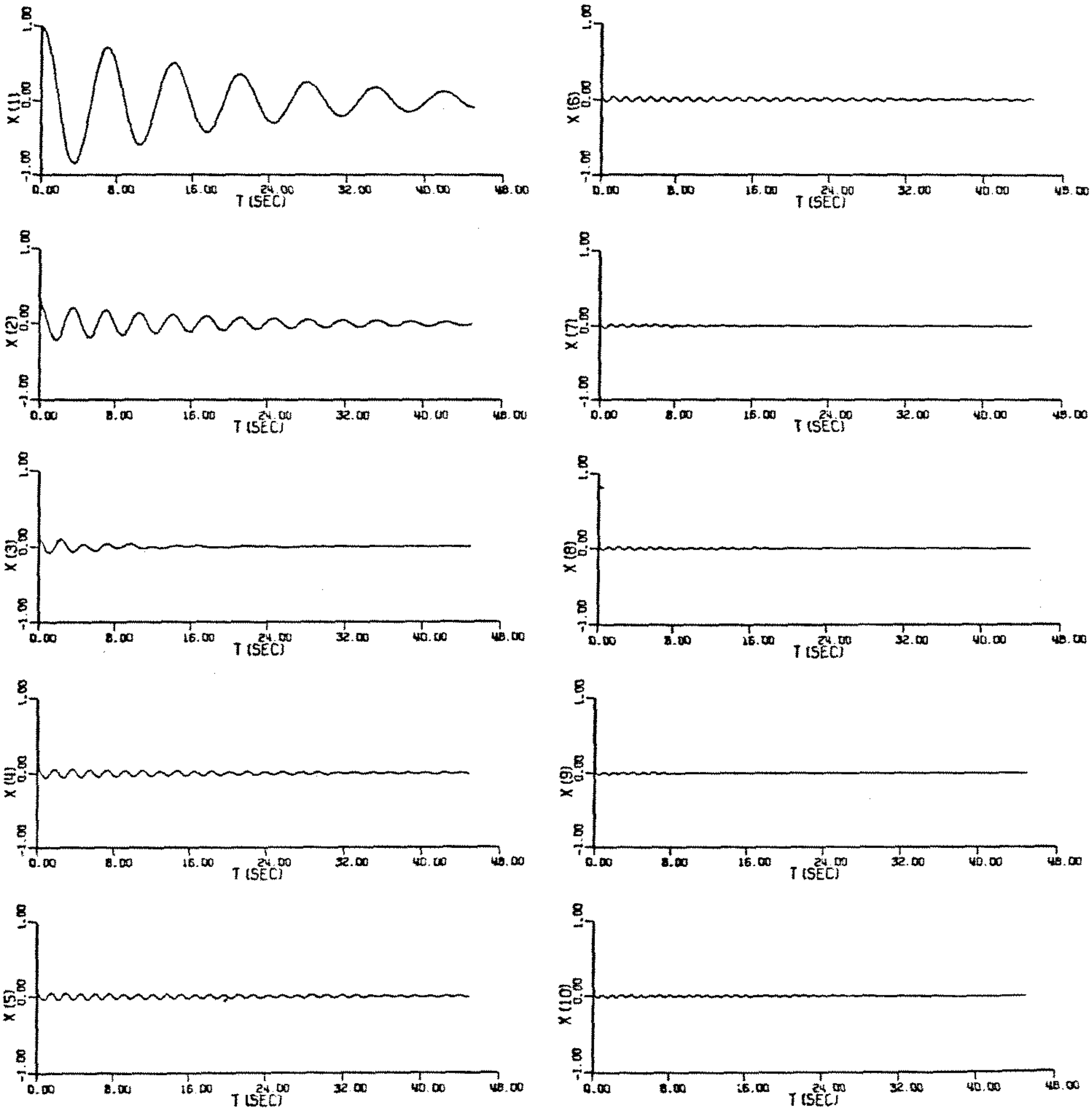

Figure 2.3 Modal Response (same vertical scale)

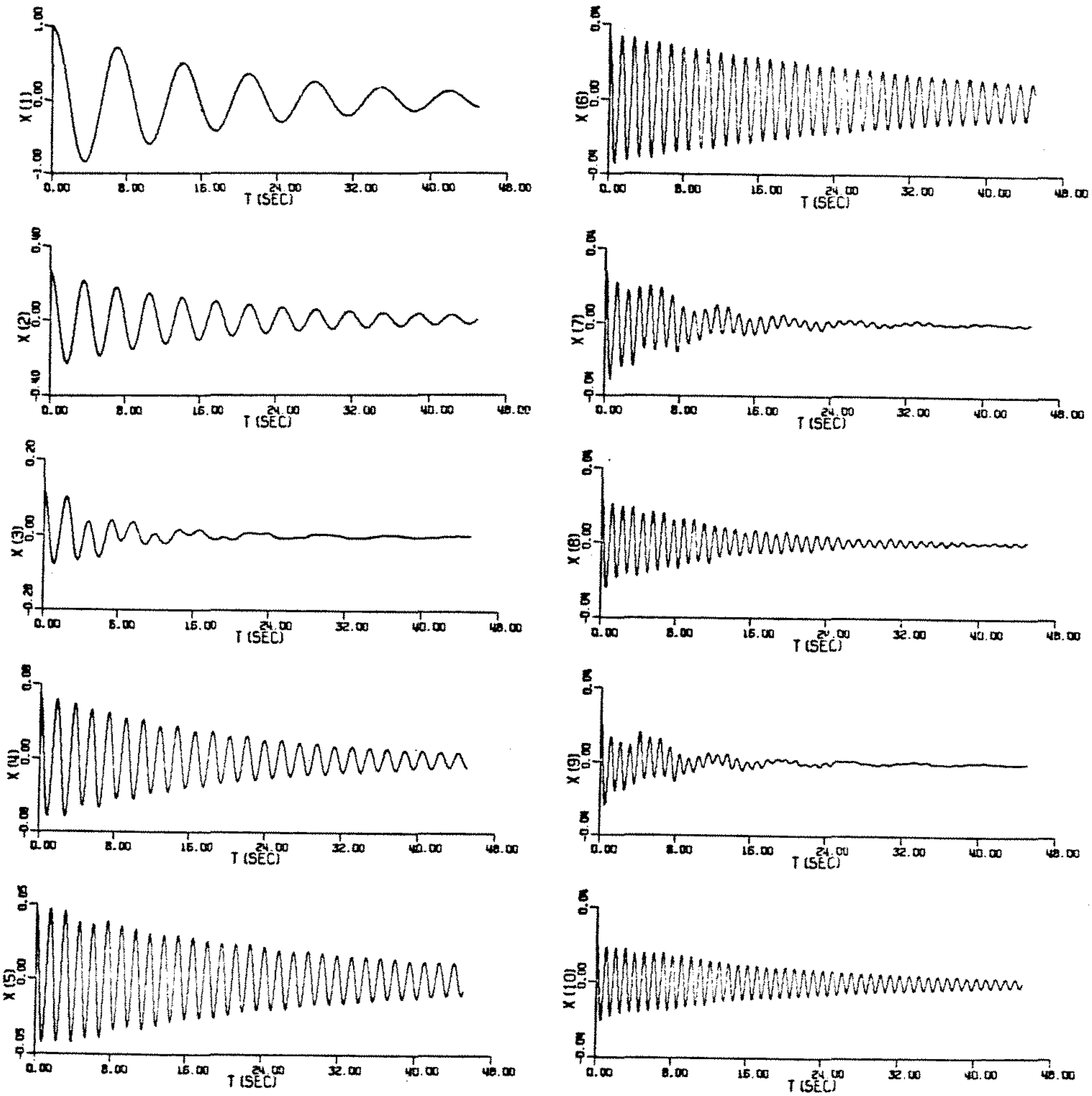

Figure 2.4 Modal Response (different vertical scales)

table shows that all the eigenvalues of A have negative real parts; thus, the system is stable, which is in agreement with Theorem 2.5 in Section 2.2.

Example 2.3

Consider a free-free shear beam which is represented by a discrete model with ten lumped masses connected by springs. The equation of motion is

$$M\ddot{y} + Ky = f(t) \tag{2.69}$$

where

M = a 10 x 10 identity matrix

$$K = \begin{bmatrix} 10 & -10 & & & & & & & & \\ -10 & 20 & -10 & & & & & & & \\ & -10 & 20 & -10 & & & & 0 & & \\ & & -10 & 20 & -10 & & & & & \\ & & & -10 & 20 & -10 & & & & \\ & & & & -10 & 20 & -10 & & & \\ & 0 & & & & -10 & 20 & -10 & & \\ & & & & & & -10 & 20 & -10 & \\ & & & & & & & -10 & 20 & -10 \\ & & & & & & & & -10 & 10 \end{bmatrix}$$

Assume there are displacement and rate measurements, and two pairs of collocated sensors and actuators are located at stations 1 and 10. If one is to control the first (rigid body) and second modes such that $\omega_1 = 0.5$, $\xi_1 = 0.1$; $\omega_2 = 1.1085$, $\xi_2 = 0.0451$, i.e.,

$$\overline{\Lambda}_{II} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \quad \text{and} \quad \overline{\Omega}_{II} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

Then, using the same procedure as in example 2.1, one obtains

$$C_1 = \text{rate feedback gain} = \begin{bmatrix} 0.378 & 0.122 \\ 0.122 & 0.378 \end{bmatrix}$$

$$C_2 = \text{displacement feedback gain} = \begin{bmatrix} 0.945 & 0.305 \\ 0.305 & 0.945 \end{bmatrix}$$

Both C_1 and C_2 are positive definite; thus, the closed-loop system is stable. The resulting first and second modal dampings and frequencies are listed in Table 2-2.

Table 2-2 TWO PAIRS OF S/A

	ω_1	ξ_1	ω_2	ξ_2
Prescribed	0.5	0.1	1.108	0.045
Calculated	0.464	0.08	1.104	0.042

The result shows that the DOFC technique can provide certain prescribed damping and stiffness to the system, at least approximately. In this case, the resulting damping is 20 percent off from the prescribed value for the rigid body mode.

If one uses two extra pairs of sensors and actuators, and locates them, say, at stations 4 and 9, with the intent to decouple the first and second modes from the third and fourth modes with prescribed $\overline{\Lambda}_{II}$ and $\overline{\Omega}_{II}$ as follows:

$$\bar{\Lambda}_{II} = \begin{bmatrix} 0.1 & & & 0 \\ & 0.1 & & \\ & & 0.5 & \\ 0 & & & 0.5 \end{bmatrix}$$

and

$$\bar{\Omega}_{II} = \begin{bmatrix} 0.25 & & & 0 \\ & 0.25 & & \\ & & 0.5 & \\ 0 & & & 0.5 \end{bmatrix}$$

then

$$C_1 = \text{rate feedback gain} = \begin{bmatrix} .465 & -.356 & -.554 & .603 \\ -.356 & .801 & .643 & -.678 \\ -.554 & .643 & 5.983 & -5.190 \\ .603 & -.678 & -5.190 & 4.814 \end{bmatrix}$$

and

$$C_2 = \begin{array}{l}\text{displacement} \\ \text{feedback gain}\end{array} = \begin{bmatrix} .618 & -.195 & -.616 & .588 \\ -.195 & 1.087 & 1.035 & -.904 \\ -.616 & 1.035 & 7.794 & -6.003 \\ .588 & -.904 & -6.003 & 5.192 \end{bmatrix}$$

The resulting first and second modal dampings and frequencies are listed in Table 2-3, which shows substantial improvement relative to Table 2-2 in matching the prescribed values, but the penalty for this selection is two extra pairs of sensors and actuators.

Table 2-3 FOUR PAIRS OF S/A

	ω_1	ξ_1	ω_2	ξ_2
Prescribed	0.5	0.1	1.108	0.045
Calculated	0.48	0.093	1.104	0.043

Chapter 3

"OPTIMAL" LOCATION OF SENSOR AND ACTUATOR PAIRS

3.1 PROBLEM FORMULATION

Consider an N degree-of-freedom linear nondissipative dynamic system with discrete parameters, expressed in matrix notation as

$$M\ddot{y}(t) + Ky(t) = f(t) = S^T u(t) \tag{3.1}$$

Assume there are n pairs of collocated sensors and actuators, such that the measurement z(t) is

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} S\dot{y}(t) \\ Sy(t) \end{bmatrix} \tag{3.2}$$

The output vector z(t) often corresponds to the displacement vector $z_2(t)$ alone or the rate vector $z_1(t)$ alone.

The control u(t) is chosen to be

$$u(t) = -C_1 S\dot{y}(t) - C_2 Sy(t) \tag{3.3}$$

All variables and matrices in (3.1), (3.2), and (3.3) were defined in the previous chapter. Define

$$y(t) \equiv M^{-1/2}\,\tilde{y}(t) \tag{3.4}$$

Then (3.1) and (3.2) become

$$\ddot{\tilde{y}} + M^{-1/2} K M^{-1/2} \tilde{y}(t) = M^{-1/2} S^T u(t) \tag{3.5}$$

$$z = \begin{bmatrix} SM^{-1/2}\,\dot{\tilde{y}} \\ SM^{-1/2}\,\tilde{y} \end{bmatrix} \tag{3.6}$$

Let

$$\tilde{y}(t) \equiv \tilde{\Phi}\, q(t) \qquad (3.7)$$

where

$$\tilde{\Phi}^T\tilde{\Phi} = I \qquad (3.8)$$

$$\tilde{\Phi}^T M^{-1/2} K M^{-1/2}\, \tilde{\Phi} = \Omega = \begin{bmatrix} \diagdown & & \\ & \omega_i^2 & \\ & & \diagdown \end{bmatrix} \qquad (3.9)$$

The matrix $M^{-1/2}\,K\,M^{-1/2}$ is symmetric; $\tilde{\Phi}$ is the eigenvector matrix of $M^{-1/2}\,K\,M^{-1/2}$; and Ω is a diagonal matrix which has elements equal to the eigenvalues of $M^{-1/2}\,K\,M^{-1/2}$.

Substituting (3.7), (3.8), and (3.9) into (3.5) and (3.6) yields

$$I\ddot{q}(t) + \Omega q(t) = \tilde{\Phi}^T\, M^{-1/2}\, S^T\, u(t) \qquad (3.10)$$

$$z(t) = \begin{bmatrix} SM^{-1/2}\,\tilde{\Phi}\,\dot{q} \\ SM^{-1/2}\,\tilde{\Phi}\,q \end{bmatrix} \qquad (3.11)$$

Define

$$\Phi \equiv M^{-1/2}\,\tilde{\Phi} = \text{normalized modal-shape matrix} \qquad (3.12)$$

Then from (3.8) and (3.9),

$$\Phi^T\, M\, \Phi = I \qquad (3.13)$$

$$\Phi^T\, K\, \Phi = \Omega \qquad (3.14)$$

and (3.10) and (3.11) can be written as

$$I\ddot{q}(t) + \Omega q(t) = \Phi^T S^T u(t)$$

$$= (S\Phi)^T u(t) \tag{3.15}$$

$$z(t) = \begin{bmatrix} S\Phi\ \dot{q}(t) \\ S\Phi\ q(t) \end{bmatrix} \tag{3.16}$$

Equations (3.15) and (3.16) can be represented in state space form as

$$\dot{x}(t) = A\ x(t) + B\ u(t) \tag{3.17}$$

$$z(t) = H\ x(t) \tag{3.18}$$

where

$$x = [q^T \quad \dot{q}^T]^T$$

$$A = \begin{bmatrix} 0 & I \\ -\Omega & 0 \end{bmatrix}$$

$$B = [0 \quad (S\Phi)]^T \tag{3.19}$$

$$H = \begin{bmatrix} S\Phi & 0 \\ 0 & S\Phi \end{bmatrix}$$

and

$$x \in R^{2N}, \quad A \in R^{2N \times 2N}, \quad B \in R^{2N \times n}, \quad H \in R^{2n \times 2N}.$$

In general, the matrices Ω and Φ can be obtained by the finite element method without any difficulty, but the selection of the S matrix, which directly depends on the location of sensor and actuator pairs, remains to be explored. In this chapter we will examine the controllability and observability of the system governed by (3.17) and (3.18). A qualitative criterion is then proposed to determine the "optimal" location of collocated sensor and actuator pairs.

3.2 SYSTEM CONTROLLABILITY AND OBSERVABILITY

The system of (3.17) is controllable iff

$$\mathrm{RANK}[B \quad AB \quad A^2B \quad \cdots\cdots \quad A^{2N-1}B] = 2N \tag{3.20}$$

Define

$$\overline{B} = (S\Phi)^T \quad , \qquad \overline{B} \in R^{Nxn} \tag{3.21}$$

Then

$$B = [0 \quad \overline{B}^T]^T \tag{3.22}$$

Expanding the left-hand side of (3.20),

$$\mathrm{RANK}\begin{bmatrix} 0 & \overline{B} & 0 & -\Omega\overline{B} & \cdots\cdots\cdots\cdots & 0 & (-\Omega)^{N-1}\overline{B} \\ \overline{B} & 0 & -\Omega\overline{B} & 0 & \cdots\cdots\cdots\cdots & (-\Omega)^{N-1}\overline{B} & 0 \end{bmatrix} = 2N \tag{3.23}$$

Since the sign of a row does not affect the rank of a matrix, (3.23) is equivalent to

$$\mathrm{RANK}[\overline{B} \quad \Omega\overline{B} \quad \Omega^2\overline{B} \cdots\cdots\cdots\cdots\cdots \Omega^{N-1}\overline{B}] \;=\; N \tag{3.24}$$

Assume that Ω has distinct diagonal elements, and let

$$\overline{B} \equiv \begin{bmatrix} b_1^T \\ b_2^T \\ \vdots \\ b_N^T \end{bmatrix} \quad , \qquad b_i \in R^{nx1} \; , \qquad \forall\, i \tag{3.25}$$

Therefore, the system represented by (3.17) is controllable iff

$$C_i \equiv \|b_i\| > 0 \quad , \qquad \forall \quad i \in (1,N) \tag{3.26}$$

where C_i is the controllability measure and $\|\cdot\|$ is the vector Euclidean norm.

Thus, the N vibration modes can be ranked in terms of their controllability measures according to the size of C_i.

Next, let us investigate the observability of the system governed by (3.17) and (3.18), which is observable iff

$$\mathrm{RANK}[H^T \quad A^T H^T \quad (A^T)^2 H^T \;\cdots\cdots\; (A^T)^{2N-1} H^T] = 2N \tag{3.27}$$

Expanding the left-hand side yields,

$$\mathrm{RANK}\begin{bmatrix} \overline{B} & 0 & 0 & -\Omega\overline{B} & -\Omega\overline{B} & 0 & 0 & \Omega^2\overline{B} & \cdots\cdots & 0 & 0 & (-\Omega)^N\overline{B} \\ 0 & \overline{B} & \overline{B} & 0 & 0 & -\Omega\overline{B} & -\Omega\overline{B} & 0 & & (-\Omega)^{N-1}\overline{B} & (-\Omega)^{N-1}\overline{B} & 0 \end{bmatrix} = 2N \tag{3.28}$$

In other words,

$$\mathrm{RANK}\begin{bmatrix} \overline{B} & 0 & -\Omega\overline{B} & 0 & \cdots\cdots & (-\Omega)^{N-1}\overline{B} & 0 & (-\Omega)^N\overline{B} \\ 0 & \overline{B} & 0 & -\Omega\overline{B} & & & (-\Omega)^{N-1}\overline{B} & 0 \end{bmatrix} = 2N \tag{3.29}$$

which can be written as

$$\mathrm{RANK}[\overline{B} \quad \Omega\overline{B} \quad \Omega^2\overline{B} \;\cdots\cdots\; \Omega^{N-1}\overline{B}] = N \tag{3.30}$$

This observability condition (3.30) is identical to the controllability condition of (3.24). In the case of either displacement measurement alone or rate measurement alone, the same observability condition of (3.30) can be easily proven. We have proven:

Theorem 3.1

If Ω has distinct diagonal elements, the system of (3.17) and (3.18) is controllable and observable iff

$$C_i = O_i = \|b_i\| > 0 \qquad \forall \quad i \in (1,N) \tag{3.31}$$

where C_i and O_i are controllability and observability measures, respectively.

The system of (3.17) and (3.18) is a nondissipative one. In reality, any LSS system possesses at least certain relative weak dissipative mechanisms. It has been pointed out in [27,28] that the controllability and observability of a nondissipative system are preserved under small perturbations of damping. Therefore, a sufficiently small dissipation will not ruin the controllability and observability properties established for a nondissipative system.

Thus, it is concluded that for either displacement or rate feedback, or both, the conditions for the system of (3.17) and (3.18) to be controllable and observable are identical; that is, (3.31) should be satisfied. It is this unique condition which permits us to provide a simple qualitative criterion in the next section for the selection of an "optimal" location of sensor and actuator pairs.

3.3 "OPTIMAL" LOCATION CRITERION FOR SENSOR AND ACTUATOR PAIRS

The placement of sensors and actuators on the structure is directly related to the system stability and performance. One may select the location of sensor and actuator pairs such that during certain time intervals the total energy is minimum; that is, the total energy has a maximal decay rate. Although lower modes of the structural system possess a large portion of the total energy, in certain structural systems the higher modes may be close together in the frequency spectrum; thus, selecting the total energy decay as a criterion for placing sensors and actuators may not guarantee that the first few modes have sufficient control. Furthermore, the energy criterion involves very complicated and expensive computation for an LSS; for example, linear dynamic programming is required for the continuous system and integer dynamic programming for the discrete system.

Taking advantage of the simple criterion for system controllability and observability of the previous section, and the direct output feedback control technique, we propose the following simple criterion for placing sensor and actuator pairs.

From the previous sections, the system can be characterized by

$$\dot{x} = Ax + Bu \tag{3.32}$$

$$z = Hx \tag{3.33}$$

where

$$A = \begin{bmatrix} 0 & I \\ -\Omega & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ \overline{B} \end{bmatrix} \quad , \qquad \overline{B}^T = S(M^{-1/2}\tilde{\Phi}) = S\Phi \tag{3.34}$$

Suppose there are rate measurements only, so that

$$H = [0 \quad \overline{B}^T] \tag{3.35}$$

Let

$$\overline{B} = \begin{bmatrix} b_1^T \\ b_2^T \\ \vdots \\ b_N^T \end{bmatrix} \quad , \qquad b_i \in R^{n\times 1} \quad , \forall\ i \tag{3.36}$$

Then the controllability measure, C_i, and observability measure, O_i, are the same, i.e.,

$$C_i = O_i = \|b_i\| > 0 \quad , \forall\ i \tag{3.37}$$

Since matrices Φ and Ω are known, the only unknown quantity yet to be determined is the S matrix, which is directly dependent on the location of sensor and actuator pairs.

One qualitative and practical method for placement of the collocated sensor and actuator pairs involves finding an S matrix such that all vibration modes have a certain amount of control. Since the S matrix has elements equal to zero, except those unit entries which correspond to the structure stations where sensor and actuator pairs are located, the criterion for selecting the S matrix can be determined as follows.

Procedures for Selecting S Matrix

Step 1. Find the smallest nonzero element of each row of the $N \times N$ Φ matrix, i.e.,

$$\min_j |\phi_{ij}| = \phi_i \quad , \qquad \forall\ i,j \in (1,N)$$

Step 2. Find the n largest values among the ϕ_i, say ϕ_i^k, $k \in (1,n)$, where n is the number of collocated sensor and actuator pairs. The locations of these n entries in the ϕ_i vector are the desirable locations of sensor and actuator pairs.

One may designate a performance index J as

$$J = \sum_{k=1}^{n} \; k^{th} \text{ largest value of } \max_{i} \min_{j} |\Phi_{ij}| \tag{3.38}$$

This criterion can be implemented with very little computational effort; in fact, a desirable S matrix can be selected by direct inspection of the Φ matrix. Step 1 guarantees that all vibration modes are controllable and observable. Step 2 makes sure that the controlled system has a relatively high level of controllability and observability, so that the control gain matrix is kept small, thus avoiding the saturation of actuator outputs. This simple location criterion, in conjunction with the DOFC technique, assures that the closed loop system will have some specified damping and stiffness for at least the first n modes and that the remaining modes will have a certain amount of control.

In the next section, a few examples are given to demonstrate the merit of this simple location criterion.

3.4 EXAMPLES

Example 3.1. Consider the same 10 degree-of-freedom system as in Example 2.1. The normalized modal-shape matrix Φ is as follows:

$$\Phi = \begin{bmatrix} .12 & -.23 & .32 & -.39 & .42 & -.42 & .39 & -.32 & .23 & -.12 \\ .23 & -.39 & .42 & -.32 & .12 & .12 & -.32 & .42 & -.39 & .23 \\ .32 & -.42 & .23 & .12 & -.39 & .39 & -.12 & -.23 & .42 & -.32 \\ .39 & -.32 & -.12 & .42 & -.23 & -.23 & .42 & -.12 & -.32 & .39 \\ .42 & -.12 & -.39 & .23 & .32 & -.32 & -.23 & .39 & .12 & -.42 \\ .42 & .12 & -.39 & -.23 & .32 & .32 & -.23 & -.39 & .12 & .42 \\ .39 & .32 & -.12 & -.42 & -.23 & .23 & .42 & .12 & -.32 & -.39 \\ .32 & .42 & .23 & -.12 & -.39 & -.39 & -.12 & .23 & .42 & .32 \\ .23 & .387 & .42 & .32 & .12 & -.12 & -.32 & -.42 & -.39 & -.23 \\ .12 & .23 & .32 & .39 & .42 & .42 & .39 & .32 & .23 & .12 \end{bmatrix} \qquad (3.39)$$

Suppose there are two pairs of collocated sensors and actuators. Applying the criterion (3.38),

$$J = 0.12 + 0.12 = 0.24 \qquad (3.40)$$

In fact, J = 0.24 regardless of where one places these two pairs of sensors and actuators; this is due to the symmetry of the system.

Example 3.2. Consider a cantilever beam shown in Figure 3.1:

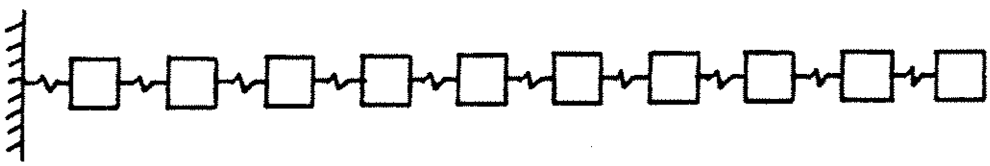

Figure 3.1

which can be modelled by a 10 degree-of-freedom discrete system as

$$M\ddot{y} + Ky = f(t) \qquad (3.41)$$

where

$$M = \begin{bmatrix} 2.0 & & & & & & & & & \\ & 1.9 & & & & & 0 & & & \\ & & 1.8 & & & & & & & \\ & & & 1.7 & & & & & & \\ & & & & 1.6 & & & & & \\ & & & & & 1.5 & & & & \\ & & & & & & 1.4 & & & \\ & & & & & & & 1.3 & & \\ & 0 & & & & & & & 1.2 & \\ & & & & & & & & & 1.1 \end{bmatrix} \tag{3.42}$$

$= 10 \times 10$ mass matrix

and K is the 10×10 stiffness matrix

$$K = \begin{bmatrix} 20 & -10 & & & & & & & & \\ -10 & 20 & -10 & & & & & & & \\ & -10 & 20 & -10 & & & & 0 & & \\ & & -10 & 20 & -10 & & & & & \\ & & & -10 & 20 & 10 & & & & \\ & & & & -10 & 20 & -10 & & & \\ & & & & & -10 & 20 & -10 & & \\ & 0 & & & & & -10 & 20 & -10 & \\ & & & & & & & -10 & 20 & -10 \\ & & & & & & & & -10 & 10 \end{bmatrix} \tag{3.43}$$

The associated matrix Ω of natural frequencies is

$$\Omega = \left[\ulcorner \omega_i^2 \lrcorner\right] = \begin{bmatrix} .163 & & & & & & & & & \\ & 1.292 & & & & & & & 0 & \\ & & 3.439 & & & & & & & \\ & & & 6.409 & & & & & & \\ & & & & 9.925 & & & & & \\ & & & & & 13.648 & & & & \\ & & & & & & 17.202 & & & \\ & & & & & & & 20.339 & & \\ & 0 & & & & & & & 23.634 & \\ & & & & & & & & & 28.608 \end{bmatrix} \tag{3.44}$$

The normalized modal-shape matrix Φ is

$$\Phi = \begin{bmatrix} .059 & .163 & .252 & -.315 & .342 & .326 & -.257 & .131 & -.026 & -.001 \\ .116 & .284 & .331 & -.226 & .005 & -.238 & .370 & -.270 & .071 & .004 \\ .167 & .336 & .194 & .138 & -.342 & -.185 & -.212 & .373 & -.150 & -.012 \\ .217 & .309 & -.064 & .343 & -.078 & .326 & -.137 & -.350 & .268 & .035 \\ .259 & .214 & -.284 & .174 & .317 & .082 & .339 & .137 & -.390 & -.088 \\ .295 & .075 & -.348 & -.173 & .209 & -.338 & -.118 & .178 & .427 & .193 \\ .323 & -.078 & -.232 & -.354 & -.210 & -.066 & -.270 & -.324 & -.270 & -.353 \\ .344 & -.217 & -.005 & -.217 & -.337 & .332 & .229 & .096 & -.074 & .516 \\ .357 & -.320 & .225 & .101 & -.029 & .141 & .216 & .262 & .349 & -.533 \\ .364 & -.373 & .361 & .341 & .314 & -.281 & -.242 & -.212 & -.218 & .248 \end{bmatrix} \tag{3.45}$$

Assume that there is natural structural damping of 0.5 percent for each mode and that there are two pairs of collocated rate sensors and actuators. Suppose the design specification asks for the additional damping of 5 percent for the first mode and 1.78 percent for the second mode, thus

$$\Lambda_{II}^{*} = \begin{bmatrix} 2\times\omega_1\times .05 & \\ 0 & 2\times\omega_1\times .05 \end{bmatrix} = \begin{bmatrix} 2\times\omega_1\times .05 & 0 \\ 0 & 2\times\omega_2\times .0178 \end{bmatrix} = \begin{bmatrix} .0404 & 0 \\ 0 & .0404 \end{bmatrix}$$

Then from (3.45) and (3.38), the optimal locations of rate sensor and actuator pairs are at stations 5 and 10; the performance index J is

$$J = .082 + .212 = .294$$

The rate feedback gain matrix and closed-loop eigenvalues are shown in Table 3-1. Fixing Λ_{II}^{*} and varying the location of rate sensor and actuator pairs will result in different rate feedback gain matrices. A comparison of results is given in Table 3-1, where it is shown that, compared with case 1, cases 2 and 3 provide better damping for higher modes, but that the gain matrices are substantially larger than the gain matrix of case 1.

Table 3.1 EFFECT OF S/A LOCATION ON CLOSED-LOOP SYSTEM EIGENVALUES

Case No.		1	2	3	Open Loop
S/A Location Stations		No.5 & No.10	No.1 & No.10	No.5 & No.6	
Closed-Loop Eigenvalues (ξ_i, ω_i)	1	.055, .4037	.055, .4042	.0551, .4040	.005, .4042
	2	.023, 1.137	.0227, 1.138	.0223, 1,1396	.005, 1.1368
	3	.019, 1.854	.052, 1.864	.0207, 1.857	.005, 1.8545
	4	.010, 2.532	.032, 2.538	.043 , 2.580	.005, 2.537
	5	.0125, 3.1498	.0498, 3.1436	.0112, 3.1527	.005, 3.1504
	6	.007, 3.694	.0249, 3.6774	.0304, 3.8153	.005, 3.6943
	7	.0114, 4.147	.0213, 4.1213	.0366, 4.3123	.005, 4.1476
	8	.0066, 4.5092	.0073, 4.5067	.228, 4.3194	.005, 4.5100
	9	.011, 4.8595	.0062, 4.860	.0664, 4.512	.005, 4.8625
	10	.0062, 5.348	.0060, 5.348	.0081, 5.314	.005, 5.3487
Rate Feed-back Gain Matrix (2x2)		$\begin{bmatrix} .3599 & -.0190 \\ -.0196 & .1497 \end{bmatrix}$	$\begin{bmatrix} 1.661 & .242 \\ .242 & .184 \end{bmatrix}$	$\begin{bmatrix} 1.9669 & -1.9668 \\ -1.9668 & 2.4037 \end{bmatrix}$	
Performance Index		.294	.213	.157	

Chapter 4

CONCLUSION

4.1 CONCLUDING REMARKS

This report has investigated a direct output feedback control (DOFC) technique for large flexible space structures. By considering an N degree-of-freedom system with n pairs of collocated sensors and actuators, where N is typically much larger than n, our analysis shows that at least the n lowest vibration modes can be effectively controlled in a prescribed manner, with the overall system remaining stable. A method for selecting the feedback control gain matrices was proposed. The DOFC approach is applicable to systems with a certain type of nonlinearity and with sensor/actuator dynamics. The DOFC technique possesses the merits of both coupled control and independent modal-shape control.

The issue of the "optimal" location of sensors and actuators was considered. A simple qualitative criterion was proposed. By investigating the modal-shape matrix directly, the "optimal" location of sensors and actuators can be obtained. The criterion guarantees that all vibrational modes have a certain minimal amount of control. A few examples were given to demonstrate the use of the DOFC technique, as well as the application of the selection criterion for the location of sensor and actuator pairs.

4.2 SUGGESTED SUBJECTS FOR FURTHER RESEARCH

The control of large flexible structures constitutes a vast research area involving many disciplines. This report has reviewed some approaches related to this difficult but interesting subject, and has proposed the direct output feedback control technique, as well as one criterion to select the "optimal" location of the sensor and actuator pairs. The approaches are all based on the assumption of a finite-dimensional discrete system model and a collocated arrangement of sensors and actuators. From the results of this investigation, it is natural to propose the following subjects for further research.

<u>System Stability Analysis for Uncollocated Arrangement of Sensors and Actuators (S/A)</u>

The stability theorems in Chapter 2 were based on the assumption that the S/A were collocated; furthermore, the stability theorems only provided sufficient, but not necessary conditions. It is worthwhile to pursue further the possibilities of:

(1) Proving that the theorems in Chapter 2 are also necessary conditions.

(2) Determining whether, if (1) does not apply, it is necessary to have certain pairs, or at least one pair, of sensors and actuators collocated to secure system stability.

(3) Determining the conditions under which the system is stable, if the sensors and actuators are not collocated. As an example: for a system of the form of (2.8) without gyroscopic terms being stable it is essential to prove that the symmetric portions of ${S_a}^T C_1 S_s$ and ${S_a}^T C_2 S_s$ are positive definite, where S_a is the matrix associated with actuator output, S_s is the matrix associated with the measurement, and C_1 and C_2 are rate and displacement feedback control gain matrices, respectively.

Continuous System Stability Analysis which Includes the Sensor and Actuator Dynamics

The stability theorems in Chapter 2 can be easily extended to include continuous systems, except that for the infinite-dimensional case the interaction between the higher vibration modes and the S/A dynamics is not clearly understood. One may assume that the S/A dynamics exhibit a drastic reduction in magnitude beyond a certain frequency range and can be ignored. However, the possible phase delay associated with S/A dynamics in a high frequency range must be taken into consideration. Could the phase delay in S/A dynamics cause a closed-loop system designed with the DOFC technique to become unstable?

Slewing of Large Flexible Structures

Recently, Breakwell [29] considered the problem of optimal feedback slewing of flexible spacecraft. Using a system model which only includes the rigid body mode and a first vibration mode, the problem becomes essentially a minimal fuel or minimal time optimal control problem. The effect of truncated modes on the system stability and system performance was not considered in [26]. How to incorporate the DOFC technique with a minimal time or minimal fuel problem constitutes an interesting research subject.

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