BURNING OF SINGLE DROPLETS OF HYDROCARBON
FUELS IN OXIDIZING ATMOSPHERES

Thesis by
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The author is indebted to Dr. S. S. Penner for suggesting this investigation and for helpful discussions. He is indebted to Mr. Martin Goldsmith for active assistance with the experimental work and the calculations.
ABSTRACT

Experiments have been conducted in order to determine the burning rates of single droplets of liquid fuels in oxidizing atmospheres. N-heptane and absolute ethyl alcohol were tested in nitrogen-oxygen mixtures for oxygen weight fractions between 0.10 and 0.90. Measurements were made also for benzene and toluene; for oxygen weight fractions greater than for air, a carbon residue formed during the burning of these fuels, which made it impossible to obtain accurate data. For all nitrogen-oxygen mixtures it was found that the mass burning rate of single droplets of fuel is proportional to the first power of the drop diameter.

The theoretical analysis on the burning of single droplets of fuels in an oxidizing atmosphere by Goldsmith and Penner was used to obtain values for the evaporation constants of the fuels tested. The experimentally determined values of the evaporation constants were found to be in good agreement with the calculated values for all fuels tested which did not form carbon.
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NOMENCLATURE

\( r \)  
radial distance from center of drop

\( r \bar{r} \)  
radius of fuel drop

\( r_c \)  
radius of combustion surface

\( T_o \)  
temperature of ambient gas

\( T_c \)  
temperature of combustion

\( Y_{O, o} \)  
weight fraction of oxygen in the oxidizing medium

\( \delta_o \)  
ratio of mass rate of flow of oxygen to the mass rate of flow of fuel vapor

\( \lambda_o \)  
\( \lambda / \rho D_o (c_p)_o \), dimensionless parameter

\( \lambda_k \)  
thermal conductivity of fuel vapor

\( D_o \)  
binary diffusion coefficient of oxygen

\( (c_p)_o \)  
specific heat at constant pressure of gaseous oxygen

\( \rho \)  
density of gas mixture

\( \rho_\bar{r} \)  
density of liquid fuel

\( \Delta l \)  
specific latent heat of vaporization of fuel

\( T_k \)  
normal boiling point of liquid fuel

\( \dot{m}_F \)  
mass rate of flow of fuel vapor

\( a, b \)  
constants in the expression \( (c_p)_F = a + bT \)

\( (c_p)_F \)  
specific heat at constant pressure of the fuel vapor

\( \alpha, \beta, \xi, \phi \)  
computational parameters

\( K' \)  
evaporation constant
I. INTRODUCTION

During the past few years there has been increased effort to determine the mechanism of combustion of individual droplets of fuel. These basic studies were initiated in order to aid in the design of efficient spray injection type burners for heterogeneous combustion. Heterogeneous combustion is of importance in such widely different applications as stationary boilers, Diesel engines, gas turbines, and rocket motors.

Most previous experimental work on the combustion of liquid droplets of fuel has been limited to the use of air as the oxidizing medium. For example, G.A.E. Godsave\(^{(1,2)}\) determined experimentally the burning rates of hydro-carbon fuels such as benzene, ethyl alcohol, and normal heptane at atmospheric pressure, using air as the oxidizing medium. Godsave estimated also the diameter of the flame front surrounding the liquid drop for a number of fuels. The most important conclusion derived from this experimental work was that the mass evaporation rate of the drop is proportional to the first power of the drop diameter.

A. R. Hall and J. Diedericksen\(^{(3)}\) have carried out experimental work on the combustion of drops of liquid fuels. They verified the result that the mass rate of burning is proportional to the first power of the drop diameter. Experiments were conducted also to determine the effect of pressure on the combustion of droplets of fuel. Data were obtained for the burning rate at pressures up to twenty atmospheres. The results showed that the mass rate of burning of fuel droplets is
roughly proportional to the one-fourth power of the pressure.

In conjunction with the experimental studies, there has also been an effort made towards a theoretical understanding of the mechanism of the combustion of single drops of fuel in an oxidizing atmosphere. Godsaver\(^{(1,2)}\) obtained a useful theoretical interpretation of his experimental results on the assumption that the chemical reaction rates do not control the rate of burning. If this is the case, the problem is reduced to a study of heat and mass transfer between the flame front, the fuel droplet, and the surrounding atmosphere. The flame front is formed at some distance from the drop surface. The oxidizer is delivered from the surrounding atmosphere to the region of active combustion by convection and diffusion; the fuel evaporates and diffuses, without chemical change, to the reaction front, which is assumed to be a spherical shell surrounding the droplet. The location of the reaction front is determined empirically. Utilizing the physical model described above, Godsaver\(^{(1,2)}\) was able to derive an explicit expression for the burning rate of liquid fuel droplets which contained, however, two adjustable parameters, viz., the temperature of the flame front and the combustion radius. Godsaver showed that the use of reasonable values for these parameters was consistent with the observed mass burning rates.

Goldsmith and Penner\(^{(4)}\) extended Godsaver's work by using the hypothesis that the position of the flame front is established in such a way that the ratio of fuel to oxidizer corresponds to stoichiometric proportions. In this way, they were able to obtain explicit relations for the radius of the flame front, the flame temperature, and the mass burning rate, which contained no adjustable parameters. As the result
of an improved method of formulating the problem, Goldsmith and Penner were able to solve the equations without introducing invalid approximations concerning the physico-chemical parameters.

There are obvious deficiencies in the assumed model for a burning droplet, which is shown schematically in Fig. 1. The most serious deficiencies are:

1. Contrary to the assumption that a spherical reaction surface exists, photographs of burning droplets indicate that the luminous region is of the form shown in Fig. 2. Godsake measured the diameter of the sphere corresponding to the dotted circle in Fig. 2. The existing flame shape is the result of natural convection currents during burning, which have been assumed to be negligible in the theoretical analysis.

2. The assumption that no chemical reactions occur in non-stoichiometric mixtures is obviously invalid, although a similar hypothesis has been used successfully in the description of gaseous diffusion flames\(^{(5)}\).

3. The calculations of mass burning rate neglect the occurrence of dissociation at the highest temperatures, followed by reassociation at lower temperatures. A more complete study, without an assumed reaction surface, will probably not lead to difficulties of this sort.

Since previous experimental work on the burning of single drops of liquid fuels has been confined to air as the oxidizing medium, the comparison between theory and experiment is not as stringent a test of the theory as might be desired. With this in mind the author has performed experiments on a limited number of fuels in order to
determine the burning rate of single droplets in nitrogen-oxygen mixtures containing varying amounts of oxygen. The evaporation constants (burning rates) determined experimentally are compared in Section IV of this report with theoretical values for the evaporation constant obtained from the equations of Goldsmith and Penner\(^{(4)}\). Although unbelievable high values are calculated for the flame temperature, the observed burning rates are again in accord with the theoretical results, thereby suggesting the occurrence of compensating errors in the analysis.
II. THEORETICAL CALCULATIONS

A. Assumptions

In order to present a clearer picture of the physical model upon which the theoretical analysis of Goldsmith and Penner\(^{(4)}\) is based, the important assumptions are tabulated below.

1. The droplets are spherical.

2. Convection effects may be neglected.

3. The flame front surrounding the drop is represented by a spherical surface concentric with the drop. All reactions take place instantaneously at this surface, at which the delivery rates of fuel to oxidizer are in stoichiometric proportions.

4. Steady-state conditions are assumed for fixed droplet sizes. This restriction greatly facilitates the mathematical treatment. It is reasonable to assume that the solution obtained for a fixed size applies to a drop decreasing in size when it reaches the radius used in the steady-state solution.

5. The effect of heat transfer by radiation is neglected.

6. Mean values are used, when appropriate, for the physical properties.

7. The temperature of the liquid drop is assumed to be uniform and equal to the boiling temperature.

8. The pressure is assumed to be uniform throughout the system.
A schematic diagram of an evaporating and burning fuel droplet in an oxidizing atmosphere is shown in Fig. 1. The radius of the liquid drop is \( r_0 \) and its temperature is the normal boiling point \( T_L \). The radial distance of the combustion surface from the center of the liquid droplet is \( r_\xi \) and its temperature is \( T_c \). The oxygen-inert gas mixture at a large distance from the combustion surface is at the temperature \( T_o \).

B. Equations Necessary to Determine the Evaporation Constant

The equations derived by Goldsmith and Penner\(^{(4)}\) for determining the flame temperature \( T_c \), the combustion radius \( r_\xi \), and the mass burning rate \( \dot{m}_F \) are summarized below:

\[
T_c = \frac{\beta T_o - \alpha \left[ \left( Y_o/\gamma_o \right)^{(\beta/\gamma_o)} - 1 \right]}{\beta \left[ 1 + (Y_o/\gamma_o)^{(\beta/\gamma_o)} \right]}, \quad (1)
\]

\[
\frac{r_\xi}{r_0} = 1 + \left( \frac{c_p_0}{b} \phi \right) \left[ \frac{1}{\beta} (T_o - T_c) - \frac{\alpha}{\beta^2} \ln \left( \frac{\alpha + \beta T_o}{\alpha + \beta T_c} \right) \right]^{-1} \quad (2)
\]

where

\[
\phi = \ln \left\{ 1 + \left[ \frac{(T_c - T_\xi)}{\Delta \xi} \right] \left[ a + \left( \frac{b}{2} \right) (T_c + T_\xi) \right] \right\}
- \frac{a}{\xi} \ln \left( \frac{a + b T_c - \xi}{a + b T_c + \xi} \right) \left( \frac{a + b T_\xi - \xi}{a + b T_\xi + \xi} \right), \quad (3)
\]

and

\[
\dot{m}_F = \frac{4\pi \lambda_\xi r_\xi}{b T_\xi \left[ 1 - r_\xi/r_\xi \right]} \phi \quad (4)
\]

The symbols used in Equations (1) through (4) are defined in the Table of Nomenclature.
The linear functional relation between \( \dot{m}_F \) and \( \xi \) is customarily used to obtain the following expression for the variation of droplet diameter with time: \(^{(1, 2, 4)}\)

\[
d^2 = d_o^2 - K't
\]

(5)

Here \( d \) is the droplet diameter at time \( t \), \( d_o \) is the initial droplet diameter, and the evaporation constant \( K' \) is defined by the following relation:

\[
K' = 2 \frac{\dot{m}_F}{\pi \eta \rho_k}
\]

(6)

Since \( K' \) is independent of \( \xi \), it is a convenient parameter for comparing burning rates of different fuels for arbitrary droplet sizes.

The procedure for calculating the various quantities involves the following steps:

(a) For suitably chosen values of the physico-chemical parameters a value of \( T_c \) is obtained from Eq. (1).

(b) The computational parameter \( \Phi \) is next obtained from Eq. (3).

(c) The quantity \( \xi / \xi_k \) is obtained from Eq. (2).

(d) \( \dot{m}_F \) is determined from Eq. (4).

(e) Finally the evaporation constant \( K' \) is calculated from Eq. (6).

Listed in Table 1 are appropriate values of the physico-chemical parameters used to obtain theoretical values for the evaporation constant.
### TABLE 1. PHYSICO-CHEMICAL PARAMETERS

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$T_0$ (°K)</th>
<th>$\gamma_0$</th>
<th>$T_\lambda$ (°K)</th>
<th>$\alpha$ (g/m²)</th>
<th>$\Delta \lambda$ (cal/gm)</th>
<th>$a$ (cal/gm-°K)</th>
<th>$b$ (cal/[gm-(°K)³])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>300</td>
<td>3.08</td>
<td>353</td>
<td>0.84</td>
<td>94.0</td>
<td>0.34</td>
<td>$0.25 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ethyl Alcohol</td>
<td>300</td>
<td>2.09</td>
<td>351</td>
<td>0.79</td>
<td>204.0</td>
<td>0.44</td>
<td>$0.26 \times 10^{-3}$</td>
</tr>
<tr>
<td>N-Heptane</td>
<td>300</td>
<td>3.52</td>
<td>371</td>
<td>0.68</td>
<td>75.6</td>
<td>0.44</td>
<td>$0.40 \times 10^{-3}$</td>
</tr>
<tr>
<td>Toluene</td>
<td>300</td>
<td>3.13</td>
<td>384</td>
<td>0.84</td>
<td>87.0</td>
<td>0.37</td>
<td>$0.25 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

### TABLE 1 (Continued)

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$\lambda_\lambda$ (cal/cm·sec·°K)</th>
<th>$\gamma_0$</th>
<th>$(c_p)_0$ (cal/gm·°K)</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>$55 \times 10^{-6}$</td>
<td>0.77</td>
<td>0.26</td>
<td>2.43</td>
<td>-38,375</td>
<td>0.37</td>
</tr>
<tr>
<td>Ethyl Alcohol</td>
<td>$55 \times 10^{-6}$</td>
<td>0.77</td>
<td>0.26</td>
<td>2.79</td>
<td>-25,680</td>
<td>0.42</td>
</tr>
<tr>
<td>N-Heptane</td>
<td>$55 \times 10^{-6}$</td>
<td>0.77</td>
<td>0.26</td>
<td>3.63</td>
<td>-42,180</td>
<td>0.54</td>
</tr>
<tr>
<td>Toluene</td>
<td>$55 \times 10^{-6}$</td>
<td>0.77</td>
<td>0.26</td>
<td>2.61</td>
<td>-38,300</td>
<td>0.42</td>
</tr>
</tbody>
</table>
C. Results

The theoretical calculations for the determination of the combustion and temperature \( T_C \), the radius of the reaction zone \( r_C \), and the evaporation constant \( K' \) were carried out by Goldsmith and Penner for combustion in air (for which the weight fraction of oxygen, \( y_{O_2,0} \), has the value 0.23). Similar calculations have been performed by the author for oxygen weight fractions \( y_{O_2,o} \) of 0.10, 0.37, 0.50, 0.70, and 0.90. The results of these calculations are listed in Table 2, together with the known results for air.

Reference to Table 2 indicates the expected trend of the data, namely, the calculated rate of burning increases with an increase in the concentration of oxygen. Furthermore, with a decreased concentration of diluent inert gases, the combustion surface moves closer to the liquid drop. For a weight fraction of oxygen smaller than that for air, the opposite behavior is noted, i.e., for \( y_{O_2,o} = 0.10 \), the combustion surface moves farther from the drop and the rate of burning is decreased.

The values obtained for the temperature of the combustion zone, \( T_C \), are, of course, fictitious. As was stated in the introduction, the high flame temperatures are a direct consequence of deficiencies in the assumed physical model. Goldsmith and Penner\(^4\) noted that proper allowance for dissociation and reassociation of the reaction products would greatly decrease the value of \( T_C \) but would probably not change materially the temperature gradient near the droplet surface, which determines the burning rate. The success of the theory must be ascribed to this type of compensation, at least until a satisfactory theoretical model can be analyzed quantitatively.
### TABLE 2. THEORETICAL VALUES OF $T_c$, $\zeta/\zeta_0$, AND $K'$

FOR DIFFERENT VALUES OF $Y_{O, o}$

<table>
<thead>
<tr>
<th></th>
<th>$Y_{O, o} = 0.10$</th>
<th>$Y_{O, o} = 0.23$</th>
<th>$Y_{O, o} = 0.37$</th>
<th>$Y_{O, o} = 0.50$</th>
<th>$Y_{O, o} = 0.70$</th>
<th>$Y_{O, o} = 0.90$</th>
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<tr>
<td>N-Heptane</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_c$ (°K)</td>
<td>1694</td>
<td>3225</td>
<td>4555</td>
<td>5575</td>
<td>6810</td>
<td>7755</td>
</tr>
<tr>
<td>$\zeta/\zeta_0$</td>
<td>19.02</td>
<td>8.58</td>
<td>4.51</td>
<td>3.88</td>
<td>2.91</td>
<td>2.39</td>
</tr>
<tr>
<td>$K'$ (cm$^2$/sec)</td>
<td>$4.85 \times 10^{-3}$</td>
<td>$8.58 \times 10^{-3}$</td>
<td>$12.35 \times 10^{-3}$</td>
<td>$14.58 \times 10^{-3}$</td>
<td>$18.52 \times 10^{-3}$</td>
<td>$22.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ethyl Alcohol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_c$ (°K)</td>
<td>1687</td>
<td>3110</td>
<td>4280</td>
<td>5110</td>
<td>6080</td>
<td>6780</td>
</tr>
<tr>
<td>$\zeta/\zeta_0$</td>
<td>8.70</td>
<td>5.30</td>
<td>3.62</td>
<td>2.82</td>
<td>2.30</td>
<td>1.99</td>
</tr>
<tr>
<td>$K'$ (cm$^2$/sec)</td>
<td>$3.88 \times 10^{-3}$</td>
<td>$7.91 \times 10^{-3}$</td>
<td>$11.64 \times 10^{-3}$</td>
<td>$14.95 \times 10^{-3}$</td>
<td>$19.47 \times 10^{-3}$</td>
<td>$24.0 \times 10^{-3}$</td>
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<tr>
<td>Benzene</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$T_c$ (°K)</td>
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<td>3455</td>
<td>6160</td>
<td>8900</td>
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<tr>
<td>$\zeta/\zeta_0$</td>
<td>18.10</td>
<td>9.64</td>
<td>4.39</td>
<td>2.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K'$ (cm$^2$/sec)</td>
<td>$5.37 \times 10^{-3}$</td>
<td>$10.02 \times 10^{-3}$</td>
<td>$17.25 \times 10^{-3}$</td>
<td>$26.4 \times 10^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toluene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_c$ (°K)</td>
<td>1745</td>
<td>3365</td>
<td>5975</td>
<td>8590</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta/\zeta_0$</td>
<td>19.98</td>
<td>9.54</td>
<td>4.41</td>
<td>2.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K'$ (cm$^2$/sec)</td>
<td>$4.62 \times 10^{-3}$</td>
<td>$8.72 \times 10^{-3}$</td>
<td>$14.99 \times 10^{-3}$</td>
<td>$23.15 \times 10^{-3}$</td>
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</table>
III. EXPERIMENTAL WORK

A. Apparatus

The apparatus used for photographing single burning drops suspended on a silica filament is shown in Fig. 3. The object of the investigations was to record the decrease in drop size with time during combustion. The combustion tank shown in Fig. 3 was utilized so that the composition of the oxidizing medium surrounding the fuel droplet could be varied. The tank is cylindrical, 42 inches long, with an inside diameter of 6-1/2 inches and an outside diameter of 8 inches. It can be pressurized to approximately 500 psi and can be heated to a temperature of about 1000°F. There are a total of ten windows, 2 inches in diameter, that is, five windows on each side diametrically opposed. The tank was constructed by M. Goldsmith for experiments on falling drops, combustion under pressure, and combustion at elevated temperatures.

A fuel injection system is built into the head of the combustion tank. This system is provided with a fuel reservoir, a fuel inlet valve for filling the reservoir, and with means for pressurizing the reservoir. The fuel system is connected through a ball check valve to a hollow brass needle which extends approximately 8 inches down into the tank.

A fine silica filament was cemented to the brass needle. This filament was of such length that its tip extended vertically down so as to be in the line of sight of the top windows of the tank. It was necessary to thicken the end of the filament in order to retain the liquid drop on the filament. The diameter of the thickened end of the filament was approximately 0.30 mm.
The drops were suspended on the filament by forcing fuel through the brass needle and onto the silica filament. The suspended drops were found to be between 1.5 and 1.8 mm in diameter. As can be seen from Fig. 4, the suspended drops were reasonably spherical during the major part of their life.

The suspended drops were ignited by means of an electric spark. A six-volt storage battery was connected through a spark coil and an on-off switch to a spark plug screwed into the head of the combustion tank. A welding rod was used to lengthen the center pole of the spark plug to a length commensurate with the tip of the filament. Another rod was extended down from the head forming a spark gap approximately 2 cm in length, across the filament tip. This method of ignition was found to be satisfactory for oxygen concentrations greater than about 20 percent in oxygen-nitrogen mixtures. At $Y_{O_2} = 0.10$ it was impossible to ignite a drop for the four fuels tested.

An electrically driven Bell and Howell "Eyemo" 32 mm movie camera was used to photograph the burning drops. The drops were photographed in silhouette by providing strong background illumination, and using a specially selected lens in order to obtain as large an image as possible of the drop on each frame of the film. The lens used was a six inch F/4.5 Cooke Teleking Anastigmat connected to the camera through a 10 inch adapter tube. A lens setting of f16 was determined to be the most satisfactory setting. The background illumination was provided by a 100 watt light bulb shining directly into the tank window opposite that into which the camera was focused. Kodak Plus X 32 mm film was used.
The drops were photographed at a camera setting of 24 frames per second. The actual camera speed at this setting was checked by photographing a calibrated stop watch. The camera timing was found to be 24,60 frames per second (1 frame per 0.04065 seconds).

A 3/32 inch ball bearing was photographed at the beginning and end of each 100 foot roll of film used. This calibration was carried out under the same camera focusing conditions as for the burning drops, thereby providing an accurate reference measurement for determining the magnification of the photographs and, consequently, the actual size of the drops.

The size of the burning drops was determined by using a technique very similar to that described by Godsone (1). The film was measured with the aid of a 32 mm microfilm reader (to produce further magnification) and a steel scale graduated to 1/100th of an inch. Two measurements were made on each frame, namely, the two perpendicular diameters inclined at 45° to the major and minor axes in the plane of observation. The mean of these two measurements was recorded as the "effective diameter" of the drop. If the major and minor axes do not differ greatly, as was the case in our tests, then it is easily shown that the volume of a sphere with the measured effective diameter is not greatly different from that of the prolate spheroid which actually corresponds to the shape of the drop.

In most cases, measurements of each drop were taken over a range of size extending from ignition down to approximately one half or the original drop diameter. For burning in air, every fifth frame of the film on each drop was measured. As the burning rate increased, with increase in oxygen concentration, every two to four frames of the
film for any given drop were measured.

In order to vary the weight fraction of the oxygen \((Y_{O_2})\) in the atmosphere inside the combustion tank, a bottle of compressed oxygen was connected through a regulator valve to the tank. Then, for example, for \(Y_{O_2} = 0.50\), the combustion tank was pressurized to a gage pressure of 7.1 psi. The tank was kept at the required pressure for an extended time in order to allow complete mixing of the oxygen with the original air in the tank. The pressure in the tank was slowly decreased to the operating pressure, a drop of fuel suspended on the filament, the camera started, and the drop ignited. After each run the combustion tank was flushed with air before repeating the above process for another run. All combustion experiments were carried out at atmospheric pressure. It should be noted that for \(Y_{O_2} = 0.10\) the combustion tank was pressurized with nitrogen to a gage pressure of 19.9 psi before bleeding to atmospheric pressure. However, ignition at this reduced oxygen concentration was impossible and no experimental data were obtained.

B. Experimental Results

A systematic study was made to determine the evaporation constant, \(K'\), for different oxygen concentrations and for the following fuels: n-heptane, absolute ethyl alcohol, benzene, and toluene. The experimental results gave directly the droplet diameter, \(d\), as a function of the time, \(t\). The plots of \(d^2\) against \(t\) were found to be linear. Typical experimental plots for n-heptane and ethyl alcohol for \(Y_{O_2} = 0.50\), and for benzene and toluene at \(Y_{O_2} = 0.23\) are shown in
Figs. 5 to 8, respectively. Thus the linear functional relation between drop diameter $d$ and time $t$, $d^2 = d_0^2 - K't$, has been verified again. Here $d_0$ is the diameter at time $t = 0$ (corresponding to the beginning of steady burning) and $K'$ is the evaporation constant. The value of $K'$ is determined directly from the slope of plots of $d^2$ vs $t$ and has the dimensions $\text{cm}^2 \text{sec}^{-1}$.

The experimentally determined values of $K'$ for the four fuels tested at different oxygen weight fractions are listed in Table 3. The results of five separate experiments were examined for each fuel at each value of $Y_{O_2/o}$. The tabulated values of the evaporation constant are the mean values for each set of five runs. The reproducibility in each set of runs was good; the maximum spread of experimental data was found to be less than $\pm 1$ percent.

With the lighting used for photography during the experiments, it was impossible to obtain pictures of the flames suitable for measurement of flame surface diameter. For n-heptane and ethyl alcohol the luminous flame surface was not sufficiently well defined to permit conclusions regarding flame shape and flame size. For benzene and toluene burning in air, the luminous region could be seen but was not sufficiently clear to yield accurate measurement. However, it was possible to determine qualitatively that for the latter cases the apparent values of $\nu_c/\nu_c^*$ are considerably smaller than the theoretical values listed in Table 2. This noticeable difference between the observed and calculated values for $\nu_c/\nu_c^*$ has been considered in the paper by Goldsmith and Penner(4), and should be ascribed to deficiencies in the theoretical treatment, which does not make proper allowance for the influence of convection currents
TABLE 3. EXPERIMENTAL VALUES OF $K'(\text{cm}^2\text{sec}^{-1})$ FOR SEVERAL FUELS AND WEIGHT FRACTIONS OF OXYGEN

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Weight Fractions of Oxygen</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_{O_2} = 0.23$</td>
<td>$Y_{O_2} = 0.37$</td>
<td>$Y_{O_2} = 0.50$</td>
<td>$Y_{O_2} = 0.70$</td>
<td>$Y_{O_2} = 0.90$</td>
</tr>
<tr>
<td>N-Heptane</td>
<td>$8.35 \times 10^{-3}$</td>
<td>$13.80 \times 10^{-3}$</td>
<td>$15.01 \times 10^{-3}$</td>
<td>$18.56 \times 10^{-3}$</td>
<td>$22.34 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ethyl Alcohol</td>
<td>$8.58 \times 10^{-3}$</td>
<td>$11.11 \times 10^{-3}$</td>
<td>$15.04 \times 10^{-3}$</td>
<td>$18.41 \times 10^{-3}$</td>
<td>$20.36 \times 10^{-3}$</td>
</tr>
<tr>
<td>Benzene</td>
<td>$9.85 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td>CARBON FORMATION</td>
</tr>
<tr>
<td>Toluene</td>
<td>$7.70 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td>CARBON FORMATION</td>
</tr>
</tbody>
</table>

on flame shape, and which assumes a discontinuous reaction surface without proper allowance for dissociation.

C. Carbon Formation

For oxygen weight fractions exceeding $Y_{O_2} = 0.23$, formation of carbon residue was obtained during the burning of benzene and toluene. Photographs of toluene burning at $Y_{O_2} = 0.50$ are shown in Fig. 4(b). Reference to Fig. 4(b) shows that the carbon formed during combustion at $Y_{O_2} = 0.50$ remained on the filament after all of the liquid fuel had burned. This carbon residue was inspected and found to be of a dry, brittle, consistency. An effort was made to determine the value of $Y_{O_2}$ at which carbon formation is first noticeable. However, no definite conclusions could be drawn since any reasonable change in the oxygen concentration above that of normal air resulted in carbon formation. The formation of carbon in heterogeneous combustion of aromatic fuels has been observed by other investigators and merits additional quantitative study.
For oxygen weight fraction \( Y_{O_2} = 0.90 \), transitory formation of carbon occurs during the early stages of combustion of droplets of benzene and toluene, but the carbon is consumed by the time combustion is completed, i.e., there are no deposits of the residue remaining on the filament at the end of burning.

Because of carbon formation during the combustion of benzene and toluene, it was impossible to determine accurately the burning rates of these fuels. The shroudlike formation of carbon caused considerable distortion of the drop, and very shortly after ignition the carbon completely surrounded and obscured the drop of fuel.

D. Possibility of Utilizing Diameter Vs. Time Data for Studying the Ignition Process

The experimental data yield not only information on steady burning but provide information also on the transitory ignition processes prior to steady burning (Fig. 9). Thus a change of burning rate takes place during the time that the temperature of the liquid drop is raised from its initial ambient temperature to the steady-burning temperature. It is possible to study the pre-steady state burning in a manner similar to that employed for determining the steady burning rates of drops of fuel. However, spark ignition should not be used for the study of ignition phenomena. With the electric spark, there is considerable distortion of the drop at the time of ignition and, in extreme cases, there is some loss of the liquid fuel.
IV. COMPARISON BETWEEN THEORY AND EXPERIMENT FOR
THE EVAPORATION CONSTANT IN OXYGEN-ENRICHED
NITROGEN-OXYGEN MIXTURES

The calculated and observed evaporation constants for different oxygen weight fractions are contrasted in Table 4. The satisfactory agreement between calculated and observed values of $K'$ suggests that the physical model upon which the theoretical analysis is based represents a useful first approximation for predicting the burning rates of single droplets of the fuels considered. The success of the theory must be ascribed to a nearly complete compensation between obvious deficiencies in the analysis: the temperature gradient near the drop surface which determines the burning rate, will probably be similar to the calculated profile if proper allowance is made for distributed chemical reactions, dissociation, and reassociation.

Further comparison between theory and experiment is shown in Figs. 10 and 11 for n-heptane and ethyl alcohol. These plots indicate the calculated trend of the evaporation constant, $K'$, (and hence of $m_F$) with increasing oxygen concentration in the oxidizing medium, and also the satisfactory agreement between theoretical and experimentally determined values.
<table>
<thead>
<tr>
<th>Fuel</th>
<th>( Y_{O, o} = 0.23 )</th>
<th>( Y_{O, o} = 0.37 )</th>
<th>( Y_{O, o} = 0.50 )</th>
<th>( Y_{O, o} = 0.70 )</th>
<th>( Y_{O, o} = 0.90 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-Heptane</td>
<td>( 8.58 \times 10^{-3} )</td>
<td>( 12.35 \times 10^{-3} )</td>
<td>( 14.58 \times 10^{-3} )</td>
<td>( 18.52 \times 10^{-3} )</td>
<td>( 22.50 \times 10^{-3} )</td>
</tr>
<tr>
<td>Theory Experiment</td>
<td>( 8.35 \times 10^{-3} )</td>
<td>( 13.80 \times 10^{-3} )</td>
<td>( 15.01 \times 10^{-3} )</td>
<td>( 18.56 \times 10^{-3} )</td>
<td>( 22.34 \times 10^{-3} )</td>
</tr>
<tr>
<td>Ethyl Alcohol</td>
<td>( 7.91 \times 10^{-3} )</td>
<td>( 11.64 \times 10^{-3} )</td>
<td>( 14.95 \times 10^{-3} )</td>
<td>( 19.47 \times 10^{-3} )</td>
<td>( 24.00 \times 10^{-3} )</td>
</tr>
<tr>
<td>Theory Experiment</td>
<td>( 8.58 \times 10^{-3} )</td>
<td>( 11.11 \times 10^{-3} )</td>
<td>( 15.04 \times 10^{-3} )</td>
<td>( 18.41 \times 10^{-3} )</td>
<td>( 20.36 \times 10^{-3} )</td>
</tr>
<tr>
<td>Benzene</td>
<td>( 10.02 \times 10^{-3} )</td>
<td>( 11.02 \times 10^{-3} )</td>
<td>( 15.04 \times 10^{-3} )</td>
<td>( 18.41 \times 10^{-3} )</td>
<td>( 20.36 \times 10^{-3} )</td>
</tr>
<tr>
<td>Experiment</td>
<td>( 9.85 \times 10^{-3} )</td>
<td>( 11.11 \times 10^{-3} )</td>
<td>( 15.04 \times 10^{-3} )</td>
<td>( 18.41 \times 10^{-3} )</td>
<td>( 20.36 \times 10^{-3} )</td>
</tr>
<tr>
<td>Toluene</td>
<td>( 8.72 \times 10^{-3} )</td>
<td>( 7.70 \times 10^{-3} )</td>
<td>( 7.70 \times 10^{-3} )</td>
<td>( 7.70 \times 10^{-3} )</td>
<td>( 7.70 \times 10^{-3} )</td>
</tr>
</tbody>
</table>
V. SUGGESTIONS FOR FUTURE WORK

Additional work on the burning of single droplets of fuel in an oxidizing atmosphere should include the following problems:

1. A detailed investigation of carbon formation during heterogeneous burning of aromatic chemicals.

2. Comparison between theory and experiment for the effects of pressure on the burning rates of liquid fuels.

3. Effect of temperature of the oxidizing atmosphere on the burning rate of liquid fuels.

4. A detailed study of ignition phenomena.

5. A detailed study of the effects of convection and turbulence on the gaseous phase of the fuel.
REFERENCES


FIGURE 1. SCHEMATIC DIAGRAM OF BURNING FUEL DROP
FIGURE 2. SCHEMATIC DIAGRAM OF FLAME FRONT SURROUNDING A BURNING DROPLET
FIGURE 3. PHOTOGRAPH OF APPARATUS.
4a. N-Heptane burning at $\chi_{0,c}=0.50$.

4b. Toluene burning at $\chi_{0,c}=0.50$.

FIGURE 4. PHOTOGRAPHS OF BURNING DROPS OF FUEL.
FIGURE 5. Plot of $d^2$ Vs. $t$ for N-Heptane and $Y_{0,0}=0.50$
($K' = 15.01 \times 10^{-3}$ cm$^2$/sec$^{-1}$)

FIGURE 6. Plot of $d^2$ Vs. $t$ for Ethyl Alcohol
and $Y_{0,0}=0.50$  ($K' = 15.04 \times 10^{-3}$ cm$^2$/sec$^{-1}$)
FIGURE 7. Plot of $d^2$ Vs. $t$ for Toluene and $Y_0, c = 0.23$
($K' = 7.70 \times 10^{-3} \, \text{cm}^2 \, \text{sec}^{-1}$)

FIGURE 8. Plot of $d^2$ Vs. $t$ for Benzene and $Y_0, c = 0.23$
($K' = 9.85 \times 10^{-3} \, \text{cm}^2 \, \text{sec}^{-1}$)
FIGURE 9. EXAMPLE OF TRANSITORY IGNITION PROCESS.
(ETHYL ALCOHOL BURNING IN AIR)
FIGURE 10. COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF $K'$ FOR N-HEPTANE.
FIGURE 11. COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF K' FOR ETHYL ALCOHOL.