EQUILIBRIUM TEMPERATURE AND HEAT TRANSFER CHARACTERISTICS OF HOT WIRES IN SUPERSONIC FLOW

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Robert McClellan

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ABSTRACT

An experimental investigation was made of two problems that are fundamental to the application of hot wire anemometry in a supersonic flow. First, the equilibrium temperature of a wire in supersonic flow was established as a function of the flow parameters. Second, the heat loss law for a heated wire in a supersonic flow was determined. In the latter case it was shown that the heat loss, represented by a Nusselt number, is a function of Reynolds number only, with the temperature loading as a parameter. There is no Mach number effect from M = 1.80 to M = 4.54, the range of the present experiments.

The results show that the hot wire should be a satisfactory instrument for the measurement of both mean values and fluctuations of the mass flow and the stagnation temperature.

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I. INTRODUCTION

In the experimental phase of a large number of fluid mechanics problem, two requirements have to be satisfied by the flow measuring instrument. First, the device has to be extremely small in order not to introduce excessive disturbances into the flow; second, it has to respond to rapid fluctuations present in the flow. In a low velocity field the hot wire anemometer satisfies both of these requirements and has been used extensively. Velocity measurements in low speed turbulence are carried out utilizing the hot wire technique almost exclusively. The problem presents itself as to whether the hot wire will be equally useful in the field of compressible flow.

Before any attempt can be made to apply the hot wire as an instrument of research in a supersonic flow, the nature of the relationship between the wire heat loss and the flow parameters must be established. In low speed flow, the heat loss of the wire is governed by the Reynolds number only. In supersonic flow where compressibility is a factor, the Mach number might be an added important parameter.

Several attempts have been made to provide a theoretical analysis of this problem. For low speed flow, King (Ref. 1) gives the correct functional relationship between the variables, although his analysis is based on a nonviscous, incompressible approximation to the flow equations. Two recent papers in the field of compressible flow should be mentioned. Tchen (Ref. 2) studies the case of a line source of heat in an inviscid compressible flow, in an effort to represent the essential aspects of hot wire heat loss without introducing the complications of a boundary layer. Wu and Cole, (Ref. 3) have treated the problem of heat loss from a thin flat plate by linearizing the flow equations of a compressible viscous and heat conducting gas. Both of the above treatments are similar in that they predict a dependence of heat loss on the square root of the Reynolds number and a definite Mach number effect.

The experimental work to date has been characterized by a general lack of agreement with respect to both the results and the point of view to be adapted in presenting the results. Several of the more recent works are listed in References 4 to 7.

Kovasznay and Tormarck show that between the Mach numbers of 1.2 to 2.0 the heat loss from a hot wire is a linear function of only the square root of the Reynolds number; there is no Mach number effect. They also point out that the wire heat loss is not a linear function of the temperature difference, $T_W - T_a$.* Under substantially the same experimental conditions, Lowell shows a definite Mach number effect. Spangenberg extends Kovasznay's Mach number range down to M = .05 and shows a strong Mach number effect in the subsonic and transonic range. At supersonic speeds in the interval of Mach number 1.25 to 1.90, his measurements indicate a slight Mach number effect; however, a re-examination of his data leaves some doubt that this effect has actually been established. Stine tries to correlate his heat loss data with the ratio of the Mach number to the product of Reynolds number and Prandtl number. This is a questionable procedure since most of his experiments are in the continuous flow range where $\frac{M}{Re}$, which is proportional to the ratio of the mean free path of the gas to the wire diameter, is not expected to be the governing parameter.

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* All symbols used in this report are listed on Page 23

Because the successful application of the hot wire technique in the supersonic flow regime requires a reasonably accurate knowledge of the wire heat loss law, and due to the existing disagreement among the various experimental works concerning the nature of this law, a detailed investigation was undertaken at the California Institute of Technology, Jet Propulsion Laboratory and the results are given in this report.

II. EQUIPMENT

Wind Tunnel

The tests were performed in the Jet Propulsion Laboratory's 20-inch supersonic wind tunnel. Briefly, the tunnel is of the continuous flow type with a flexible nozzle providing a Mach number range of 1.33 to 5.00. Tunnel supply pressure can be varied over a range of 15 cm Hg to 330 cm Hg, depending on Mach number.

Hot Wires

The wire material used in this test was an alloy of 90% platinum plus 10% rhodium. Their nominal diameters were .0005" and .00015". The details of the wire holder are shown in Figures 1 and 2. The holder is made of two stainless steel wedge-shaped prongs, insulated electrically from one another by a layer of glass cloth bonded to the metal with a Teflon bonding agent. The wedges have a 7° included angle; the tips to which the wires were bonded have dimensions of about .001" x .015". The wires were attached to the holder with soft solder.

In order to insure an accurate determination of the wire length, it was put on the holder as straight as possible, and then measured with a Kodak Contour Projector. Since experience has shown that the wire holds up better if it has a small amount of sag, the holder tips were bent in slightly after the length measurements were taken.

Since the diameters of the wires were not measured, the nominal values furnished by the manufacturer were assumed to be correct. These values gave consistent data in the final results.

The length to diameter ratio for the wires was approximately 400 for the .0005" diameter wires and 550 for the .00015" wires.

Heating Circuit

The d.c. current for heating the wires was supplied by a 24 volt storage battery. The heating circuit included a Wheatstone bridge for the measurement of the wire resistance. A Brown Null Indicator showed the bridge balance. A Rubicon Portable Precision Potentiometer was used for measuring the potential drop across a one ohm precision resistor in series with the hot wire, thus obtaining the current in the wire. The tunnel supply temperature, as registered by a thermocouple in the tunnel supply section, was recorded on a Brown Potentiometer. A temperature survey made with twenty-six thermocouples located across a section of the supply chamber at the beginning of the contraction indicated a uniform temperature distribution within $1^{o}C$.

Pitot Probes

In order to accurately determine the local Mach number, a pitot probe was placed close to the hot wire. The pitot pressure was measured on a micromanometer to $\pm$ .01 cm Hg.

III. EXPERIMENTAL PROCEDURE

Measurement of the Thermal Coefficient of Resistance

A small air-tight chamber was constructed by using about two inches of the bottom half of a $1\frac{1}{4}$" test tube, plugging this with a rubber stopper, and then inserting a thermometer through the stopper and suspending the hot wire holder and wire inside. This chamber was in turn placed successively in an ice bath, a water bath at room temperature, and one at about 90°C. Temperatures were measured with a precision thermometer with a range of from -1°C to 105°C. The scale divisions were $.1^{\circ}$C. The resistances were measured with the same Wheatstone bridge that was used for the heat loss measurements.

The thermal coefficient of resistance α, based on zero degrees centigrade, was computed from the formula

$$R = R_o\ (1 + \alpha t),$$

R being the resistance of the wire at some temperature t°C. It was felt that within the temperature range covered in the present experiment (0°C to 300°C), the use of a linear resistance-temperature relationship is acceptable.

A typical calibration plot is shown in Figure 3.

Examination of several such calibrations led to the conclusion that for a 90% platinum plus 10% rhodium annealed wire of .0005" diameter, $\alpha = .00175 \pm .4\%$. In the case of smaller wires of .00015" diameter, the value obtained was $\alpha = .00166 \pm 1.5\%$. The percentages refer to the maximum variation of α for the various samples taken from the same spool of wire. The above values of α were used in reducing the heat loss data, except at M = 4.54. In this case, a wire from a different spool with an α = .00169 was used.

Tunnel Operation

The wind tunnel starting procedure posed a fairly serious problem. The tunnel starting loads were appreciably higher than the running loads, and it was not uncommon for the wire to be destroyed by the starting shock. In order to prevent this, a small shield was provided on the tunnel wall, behind which it was hoped that the wire could survive the start. This device was successful in preventing the destruction of the wire by the starting shock. However, when the wire was extended from behind the shield to the free stream, the wake off of the trailing edge of the shield invariably broke the wire. The better procedure was to place the wire within a few thousandths of an inch from the tunnel wall, a position that was deep within the boundary layer, and to start the tunnel with the wire in this position without a shield.

It was soon discovered that survival of the starting loads was no guarantee of long life for the wires. The more usual conclusion of a run was for the wire to break somewhere between its supports after an apparently random amount of running time. Microscopic examination of a few wires that had been run for some time indicated that the tunnel air was not clean, so that solid particles were constantly bombarding the wire in a sort of microscopic shot peening process. Figure 4 shows a comparison between a wire that was run for about five hours and an unused wire. It should be mentioned that unsuccessful efforts to clean the wire shown in Figure 4 indicated that the damage was a physical deformation and not an accumulation of dirt. In order to avoid this difficulty, a paper filter was placed across the tunnel plenum chamber. Figure 5 shows the appreciable improvement in wire condition, again after about five hours of running.

Wire Recovery Temperature Measurements

The measurements consisted of recording the resistance of the unheated wire at various tunnel Mach numbers and pressure levels. It was discovered that during several measurements the wire stretched permanently, presumably due to the starting loads or even due to continuous high air loads, thus increasing its resistance by several percent. This was established by a resistance-temperature recalibration of a few wires that survived a complete run. This occasional stretching effect made it impossible to accurately infer the wire temperature from its measured resistance. It was therefore decided to make a number of very careful measurements of the equilibrium (unheated) wire resistance in the entire Mach number and Reynolds number range. The resistances of all the wires used in this set of experiments were calibrated against temperature before and after the measurements. If the resistances changed more than a few tenths of one percent, the results were discarded. This technique insured consistent data. The length of these wires was in general somewhat smaller than that used for heat loss measurements in order to increase the probability that they survive the test. Some calculations (Appendix I) indicated that in the case of unheated wires the end loss correction is negligible.

Heat Loss Measurements

With the wire in the free stream, its "cold" resistance was measured, that is, the resistance assumed by the wire when there is no heating current in it. Next the wire was heated by passing a current through it until its resistance reached some predetermined value. This current was measured. The cold resistance was measured again and the sequency was repeated, this time for a different predetermined value of resistance. These measurements provided both the heat loss and the temperature of the wire. The maximum resistance to which the wire was heated was usually taken as one and one half times the cold resistance. This increase of one half the cold resistance was usually divided into five parts to provide the increments of resistance for the above sequence of measurements. Simultaneously with these measurements the tunnel total pressure and temperature were recorded and the pressure recovery in the vicinity of the wire was measured. The entire procedure was repeated at several tunnel pressure levels so as to give as wide a variation of Reynolds number as possible. The Mach number in its turn was varied over its entire range.

IV. DATA REDUCTION

The measurement of the wire heat loss involved the recording of the following quantities:

a) <u>the flow parameters</u>: P_t, T_t, P_t' from which the local Mach and Reynolds numbers were obtained. The Mach number was computed from the Rayleigh formula while the Reynolds number was defined by

$$Re = \frac{\rho u d}{\mu} \qquad 1$$

where ρu is the free stream mass flow and μ the viscosity (in cgs units). The viscosity was obtained from (Ref. 8) as

$$\mu = \frac{1.488 \times 10^{-5}\sqrt{T}}{1 + \left[\frac{122.1}{T} \Big/ 10^{5/T}\right]} . \qquad 2$$

μ was based on free stream static temperature, heated wire temperature and unheated or equilibrium wire temperature.

b) <u>the wire parameters</u>: i, R_a, R_w

From the values of R_a measured at various M and Re_s the equilibrium wire temperatures could be computed:

$$T_a = \frac{R_a - R_o}{R_o \alpha} + 273^{\circ}C, \qquad 3$$

where α and R_o were obtained from the resistance-temperature calibrations. Thus the functional relationship

$$\frac{T_a}{T_t} = f\,(M, Re_s) \qquad 4$$

could be established.

Only wires that survived a complete run and showed no change in R_o on recalibration were used in the determination of this relationship. Once established, this relationship permits the computation of T_a from the known values of T_t, M and Re_s.

The wire heat loss was reduced in the form of the Nusselt number. In terms of electrical quantities

$$Nu = \frac{q}{\pi l k(T_w - T_a)} = \frac{i^2 R_w R_a \alpha}{\pi l k(R_w - R_a)[1 + \alpha(T_a - 273)]} \quad \text{--- 5}$$

In the computation of heat conductivity, a formula of the same form as that for viscosity was used. This is given in (Ref.8) as

$$k = \frac{.632 \times 10^{-5}\sqrt{T}}{1 + \left[\frac{245}{T} \Big/ 10^{\frac{12}{T}}\right]} \quad \text{--- 6}$$

k is in cgs units. The values of k were based on the heated wire and the wire equilibrium temperature.

Since R_a was directly measured and T_a was obtained from the previously established experimental relation (Eq. 4) any change in R_o due to wire stretching during the test could be immediately detected, and the correct value of R_o determined. This effect was always less than 3% of the pre-test measured value of R_o.

It was assumed that a change in resistance implied a uniform change in the length and diameter of the wire. The following correction was made to the length to compensate for the wire stretching. In terms of the electrical resistivity, cross-sectional area, and length, the reference resistance R_o is given by

$$R_o = \frac{\sigma_o\, l}{A} \qquad 7$$

The mass of the wire remains constant, as does its electrical resistivity, so that

$$R_o = \frac{\sigma_o l^2}{Al} = \text{const.}\ l^2 \qquad 8$$

Differentiating logarithmically gives

$$\frac{d\,R_o}{R_o} = \frac{2\,dl}{l}. \qquad 9$$

dR_o is the difference between the pre-test measured value of R_o and the correct value as obtained by the method described in this section. Equation (9) then gives the fractional increase in wire length.

No correction was made to the wire diameter.

Due to the fact that the hot wire is of finite length and the holder is at a lower temperature than the heated wire itself, there is heat conduction to the holder as well as heat loss to the airstream by forced convection. This means that the measured heat loss per unit length for a wire of finite length is different than it would be for a wire of infinite length in the same airstream with the same heating current. The correction for this end loss effect was given by King (Ref. 1). Essentially the same method was used here; the technique of computation is due to Kovasznay. The detailed calculations are given in (Ref. 9). The final correction is

$$\frac{Nu_{corr}}{Nu_{meas}} = \frac{\bar{a}_w}{a_w^*}\ \frac{1 + a_w^*}{1 + \bar{a}_w}, \qquad 10$$

where

Nu_{corr} correct value of Nusselt number that would be obtained with no end loss.

Nu_{meas} measured value of Nusselt number.

$\bar{a}_w = \alpha(T_w - T_a)$ mean measured overheating ratio.

a_w^* ideal overheating ratio that would occur if there were no end loss.

The ratio $\frac{\bar{a}_w}{a_w^*}$ is computed from the formula

$$\frac{\bar{a}_w}{a_w^*} = 1 - S\sqrt{\frac{a_w^*}{\bar{a}_w}} \tanh \frac{1}{S}\sqrt{\frac{\bar{a}_w}{a_w^*}}, \qquad (11)$$

where

$$S = \frac{d}{l}\,\frac{\sqrt{1+\bar{a}_w}}{\sqrt{Nu_{meas}}}\sqrt{\frac{K}{k}}. \qquad (12)$$

Since the value of S in the present computation was always less than 0.1, the approximation

$$\frac{\bar{a}_w}{a_w^*} \approx 1 - 1.1\,S, \qquad (13)$$

was used.

All of the Nusselt numbers were corrected according to this method. The corrections are of the order of 5% of the measured value of Nusselt number.

The temperature loading is defined as

$$\tau = \frac{T_w - T_a}{T_a}. \qquad (14)$$

The values of Nusselt number for zero temperature loading were determined by extrapolating to zero the Nusselt number vs. temperature loading curves. A typical example of this procedure is shown in Figures 6 and 7. Figure 8 indicates the variation of Re_W with temperature loading. These curves were used to determine corresponding values of Nusselt number and Reynolds numbers at constant values of temperature loading.

V. RESULTS AND DISCUSSION

An experimental investigation of the laws governing hot wire heat loss in supersonic flow involves the study of the equilibrium temperature assumed by the unheated wire, the heat conduction from the wire to the fluid, and the temperature loading effect.

Equilibrium Temperature

A cylinder placed perpendicular to a supersonic gas stream assumes a temperature that depends upon both the geometry of the body and the flow parameters. There is no adequate theoretical treatment of this problem in the flow regimes in which the present tests were conducted. The Mach number varied from 1.33 to 4.50, the free stream Reynolds number based on wire diameter from 7 to 250.

The experimental measurements consisted of a record of the unheated wire resistance at various Mach and Reynolds numbers. The resistance of the wire at zero degrees centigrade was obtained by a separate calibration. Great care was taken throughout this portion of the test to see that the wires remained undamaged during a run, the criterion being the repeatability of the measurement of R_o before and after the run. The accuracy in the measurement of T_a is believed to be $\pm$ 1%.

A good correlation of T_a with the flow parameters M, Re_s, and T_t was obtained by plotting the ratio of $\frac{T_a}{T_t}$ as a function of $\frac{M}{Re_s}$. These results are shown in Figure 9. It is interesting to note that above a value of $\frac{M}{Re_s}$ of .25, the temperature ratio $\frac{T_a}{T_t}$ is above one.

This suggests that the wire is operating in the slip flow regime, a region between the continuum flow where $\frac{T_a}{T_t} \leq 1$ and free molecular flow where theory predicts recovery temperature ratios larger than one. Stalder, Goodwin, and Creager (Ref. 10) have obtained the same general results in the Ames Laboratory low-density tunnel; the accuracy of the present results, however, is better.

The curve in Fig. 9 shows an upturn for the lower values of $\frac{M}{Re_s}$, corresponding in general to low Mach number, high density operation of the tunnel. This does not seem to be a strain gage effect, because the .00015 inch diameter wires operating under the same tunnel conditions as the .0005 inch diameter wires do not show the increase in $\frac{T_a}{T_t}$, although the stress in the small wires is greater. At present no explanation for this trend can be given.

Heat Loss

The problem of the heat loss from a wire in a viscous, compressible flow has not yet been solved satisfactorily from the theoretical point of view. Dimensional analysis, however, furnishes some knowledge of the pertinent parameters upon which to base an experimental investigation. This analysis is carried out by Kovasznay and Tormarck (Ref. 4) and the following functional relationship is obtained:

$$Nu = Nu\,(M, Re, Pr, \gamma, \tau, d/l) \qquad (15)$$

The Prandtl number and the ratio of specific heats of the gas may be considered constant under the conditions of the present test. The ratio of d/l is the geometrical factor which determines that amount of the wire heat loss that goes to the wire holder by conduction. This quantity is small if d/l is small. A correction is made for this so-called end loss effect. The important parameters in Eq. 15 are then the Mach number, Reynolds number and the temperature loading. Experimentally, one looks for a functional relationship of the type

$$Nu = Nu\ (M, Re, \tau) \qquad 16$$

However, any effort to correlate the Nusselt number with the flow parameters is beset with a fundamental difficulty. The Reynolds number involves the viscosity, and the Nusselt number the heat conductivity. Both viscosity and heat conductivity are functions of the temperature only, and the choice of the proper temperature on which to base these quantities is not a priore obvious. In the present test, the Nusselt number and the Reynolds number were computed, a) with the viscosity and heat conductivity based on heated wire temperature, b) with these quantities based on the wire equilibrium temperature. Other choices may prove equally plausible, the only justification for the above selections is the manner in which the data correlates. The density and velocity, which appear only in the Reynolds number as the product ρu, were computed at free stream conditions. The Mach number is also based on free stream conditions.

The results of the heat loss measurements are shown in Figures 10 and 11. The Nusselt number is plotted as a function of the square root of the Reynolds number, with the temperature loading as a parameter. Within the accuracy of the experimental measurements, for a Mach number range of 1.79 to 4.54, there is no Mach number effect. The Nusselt number is a function of the Reynolds number and temperature loading only. For Reynolds numbers above about 10, the Nusselt number for constant temperature loading is a linear function of the square root of the Reynolds number. Kovasznay and Tormarck also found the linear relationship between Nusselt number and the square root of Reynolds number, and the lack of Mack number effect. However, their results do not agree in absolute value with those of the present test. Spangenberg's data for M = 1.25 to M = 1.90 agree with the results of this test within the scatter of his experiments.

Below a value of Reynolds number of about 10, the linear relationship between Nusselt number and the square root of Reynolds number breaks down. This is to be expected since the wires in this region are in a slip flow, where the Reynolds number would not be expected to be the only governing parameter.

Temperature Loading Effect

In the definition of Nusselt number appears a "characteristic temperature difference" ΔT, that controls the flow of heat. This characteristic temperature difference should satisfy two conditions. First, the heat transfer should go to zero as ΔT goes to zero, and second, there should be a linear dependence of heat transfer on ΔT. The first of these conditions is satisfied by the definition $\Delta T = T_W - T_a$, which was used in this test. This definition does not satisfy the second condition of linear dependence of heat transfer on temperature difference. The resulting non-linearity is the so-called temperature loading effect. However, the above definition of ΔT is recommended by its simplicity.

In Figures 12 and 13, the ratio of Nusselt number at some temperature loading τ to the Nusselt number at zero temperature loading is plotted as a function of temperature loading, with the Reynolds number as a parameter. This data was taken from the faired curves in Figures 10 and 11.

In Figure 12, where the data is reduced on the basis of the heated wire temperature, no systematic variation of the temperature loading effect with Reynolds number can be detected. The maximum variation obtained in the Reynolds number range of from 4 to 100 is indicated on the plot, and amounts to about $\pm$ 1.5%.

In Figure 13, the Nusselt numbers and Reynolds numbers are based on the wire equilibrium temperature. For the values of Reynolds numbers from 16 to 100, the results lie within the uncertainty of the data. Below a Reynolds number of 16, there appears to be a Reynolds number effect. However, there is not enough data in this region to definitely establish the trend. For this reason the curves in Figure 13 for Reynolds numbers below 16 are shown dotted.

VI. CONCLUSIONS

The results of this investigation may be summarized as follows:

1) For a hot-wire in supersonic flow, the temperature ratio $\frac{T_a}{T_t}$ is a unique function of the ratio $\frac{M}{Re_s}$ in the range M = 1.33 to 4.54, Re_s = 7 to 250.

2) In the Mach number range M = 1.80 to 4.54, there is no Mach number effect on the Nusselt number. It is a function of the Reynolds number and temperature loading only. For values of Re_w and Re_a above approximately 10, the Nusselt number is a linear function of the square root of the Reynolds number.

3) The temperature loading effect is independent of Reynolds number and Mach number, except possibly for values of Re_a below 16.

4) The wires used in these experiments were in a slip flow when operated at low values of Reynolds numbers or high values of $\frac{M}{Re_s}$. This is shown by the increase of $\frac{T_a}{T_t}$ above one for high values of $\frac{M}{Re_s}$.

5) The hot-wire should be a satisfactory instrument for measuring both mean values and fluctuations of mass flow and stagnation temperatures up to M = 4.54.

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SYMBOLS

A	cross-sectional area of wire
d	diameter of wire
i	current
k	heat conductivity of air
K	heat conductivity of wire
l	length
M	Mach number
Nu	Nusselt number, $\frac{q}{\pi l k \Delta T}$ (for a cylinder)
P_t	supply pressure
P_t'	pressure recovery
Pr	Prandtl number
q	heat loss
R	resistance
Re	Reynolds number
T	temperature, degrees centigrade absolute
T_t	total temperature
t	temperature, degrees centigrade
u	velocity
α	thermal coefficient of resistance
γ	ratio of specific heats
σ	resistivity
θ	temperature, using infinite wire temperature as zero reference
ρ	density
τ	temperature loading $\frac{T_w - T_a}{T_a}$
β	$\sqrt{\frac{Nu\ k \pi}{AK}}$

Subscripts

a unheated wire temperature

r hot-wire holder recovery temperature

s free stream conditions

w heated wire temperature

o refers to conditions at zero reference temperature ($0^{o}C$ unless otherwise specified)

wire of infinite length

APPENDIX I

The Effect of a Hot Wire Holder on the Temperature Distribution of the Unheated Wire

For an unheated wire in an airstream, the equation governing heat conduction is

$$AK \frac{d^2\theta}{dx^2} + \frac{q}{l} = 0, \qquad A1$$

$$q = - Nu\, \pi k l (\theta - \theta_\infty) \qquad A2$$

For simplicity, let θ_∞ be the zero reference temperature and denote $\frac{Nu k \pi}{AK}$ by β^2, then equations A1 and A2 give

$$\frac{d^2\theta}{dx^2} - \beta^2 \quad = 0. \qquad A3$$

With the boundary conditions $\theta = \theta_r$ at x = 0, x = 1, the solution of equation A3 is (Ref. 11)

$$\theta = \frac{\theta_r \left[\sinh\beta (1-x) + \sinh\beta x \right]}{\sinh\beta 1}. \qquad A4$$

The resistance of a wire with a given temperature distribution is

$$R = \int_0^\ell \frac{\sigma_o (1 + \alpha \theta)}{A} dx = R_o + \int_0^\ell \frac{\alpha \sigma_o \theta \, dx}{A}, \qquad A5$$

where α, σ_o and R_o are all based on the same reference temperature. If equation A4 is substituted into A5 and the integration is carried out, the unheated resistance of the wire is

$$R_a = R_o \left\{ 1 + \frac{2\alpha \theta_r}{\beta 1} \tanh \frac{\beta 1}{2} \right\}. \qquad A6$$

The average temperature of an unheated wire, θ_a, is obtained from its measured resistance by

$$R_a = R_o \,(1 + \alpha(\theta_a)). \qquad \text{A7}$$

Comparison of equations A6 and A7 give the relationship between θ_r and θ_a,

$$\theta_a = \frac{2\,\theta_r}{\beta\, l} \tanh \frac{\beta\, l}{2}. \qquad \text{A8}$$

Remembering that θ_∞ has been chosen as the zero reference temperature, the results can be expressed in terms of degrees centigrade absolute as follows

$$\left.\begin{aligned} \theta_a &= T_a - T_\infty \\ \theta_r &= T_r - T_\infty \end{aligned}\right\}. \qquad \text{A9}$$

Equations A8 and A9 give

$$T_a - T_\infty = (T_r - T_\infty)\,\frac{2}{\beta\, l}\, \tanh \frac{\beta\, l}{2}. \qquad \text{A10}$$

A conservative estimate of the difference between the measured temperature T_a and the equilibrium temperature T_∞ of an infinite wire can be obtained in the following manner. Assume for the tip of the hot wire holder a temperature recovery factor of .88, then

$$\frac{T_r - T_s}{T_t - T_s} = .88. \qquad \text{A11}$$

Further, assume that

$M = 4.54$

$T_t = 300^\circ C$ abs.

$Nu = 1.0$

$k = 6 \times 10^{-5}$ (cgs units)

$K = .072$ "

Wire Diameter = .00127 cm

Wire Length = .2 cm

$$\frac{T_\infty}{T_t} = .95$$

then

$$\beta = 45$$

$$\frac{\beta l}{2} = 4.5$$

and very closely

$$\tanh \frac{\beta l}{2} = 1.$$

Equation A10 then gives $T_a - T_\infty = 3^\circ C$, since the order of magnitude of T_a is $300^\circ C$ absolute, the percentage error in assuming that $T_a = T_\infty$ is small.

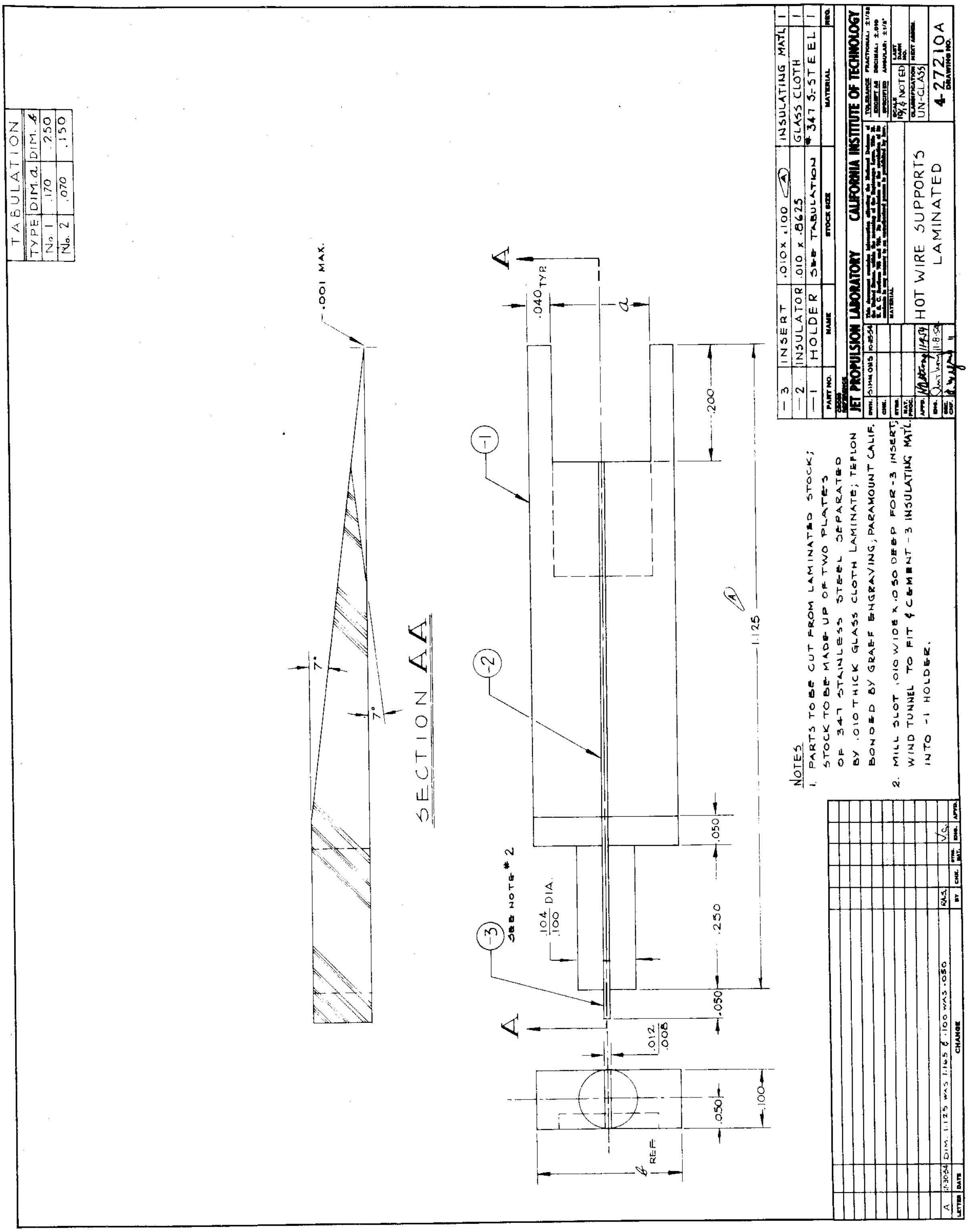

TABULATION
TYPE | DIM. a | DIM. b
No. 1 | .170 | .250
No. 2 | .070 | .150
.001 MAX.
SECTION AA
7°
7°
A
A
.040 TYP.
a
.200
1.125
.050
.250
.050
104/.100 DIA.
SEE NOTE # 2
.012/.008
.050
.100
b REF.
NOTES
1. PARTS TO BE CUT FROM LAMINATED STOCK; STOCK TO BE MADE UP OF TWO PLATES OF 347 STAINLESS STEEL SEPARATED BY .010 THICK GLASS CLOTH LAMINATE; TEFLON BONDED BY GRAFF ENGRAVING, PARAMOUNT CALIF.
2. MILL SLOT .010 WIDE X .050 DEEP FOR -3 INSERT; WIND TUNNEL TO FIT & CEMENT -3 INSULATING MAT'L INTO -1 HOLDER.
-3 INSERT .010X .100 INSULATING MAT'L 1
-2 INSULATOR .010 x .8625 GLASS CLOTH 1
-1 HOLDER SEE TABULATION #347 S-STEEL 1
PART NO. NAME STOCK SIZE MATERIAL REQ.
JET PROPULSION LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY
HOT WIRE SUPPORTS LAMINATED
UN-CLASS
4-27210A
DRAWING NO.
A 11-30-54 DIM. 1.125 WAS 1.165 & .100 WAS .050
CHANGE

FIGURE I.

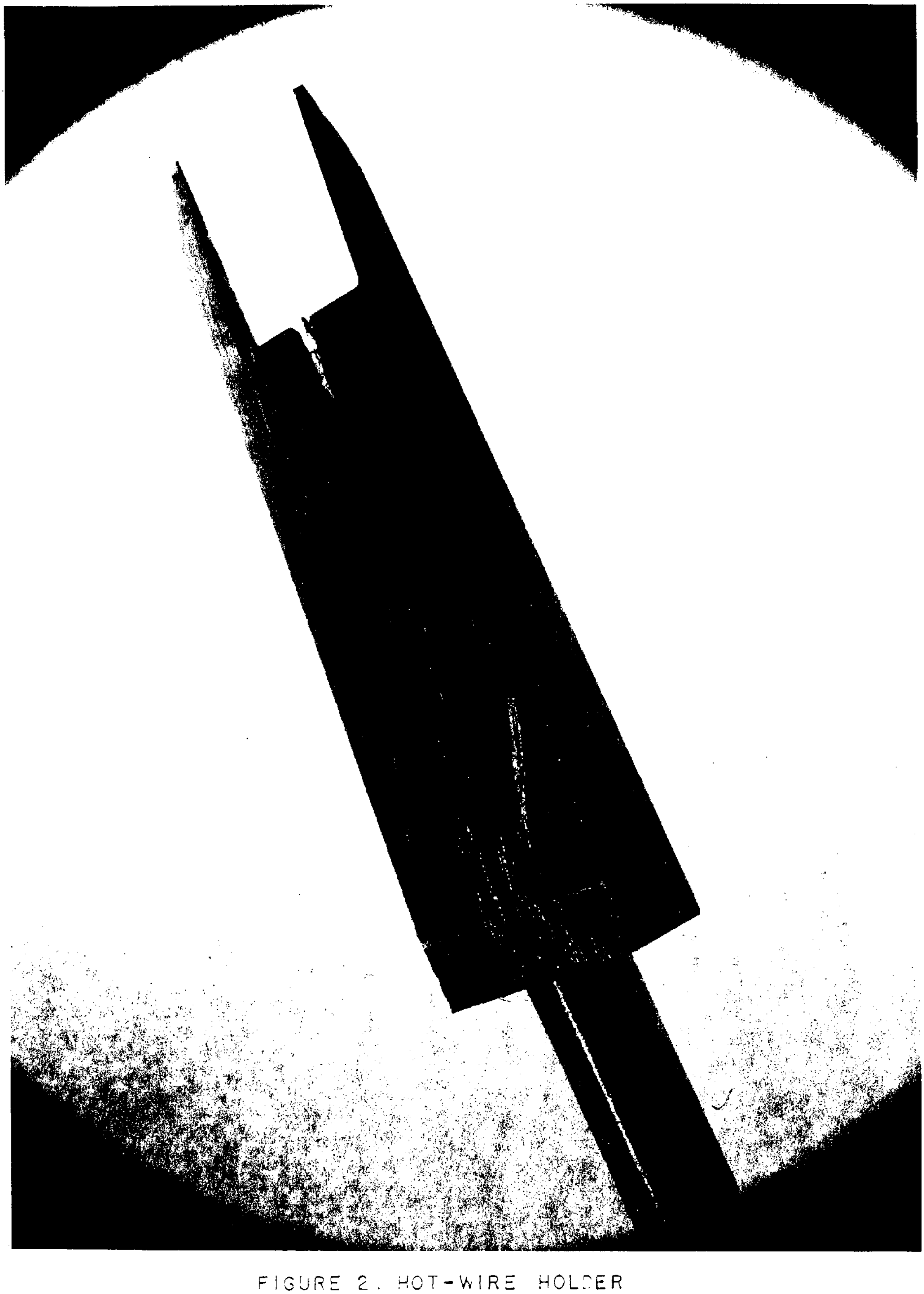


FIGURE 2. HOT-WIRE HOLDER

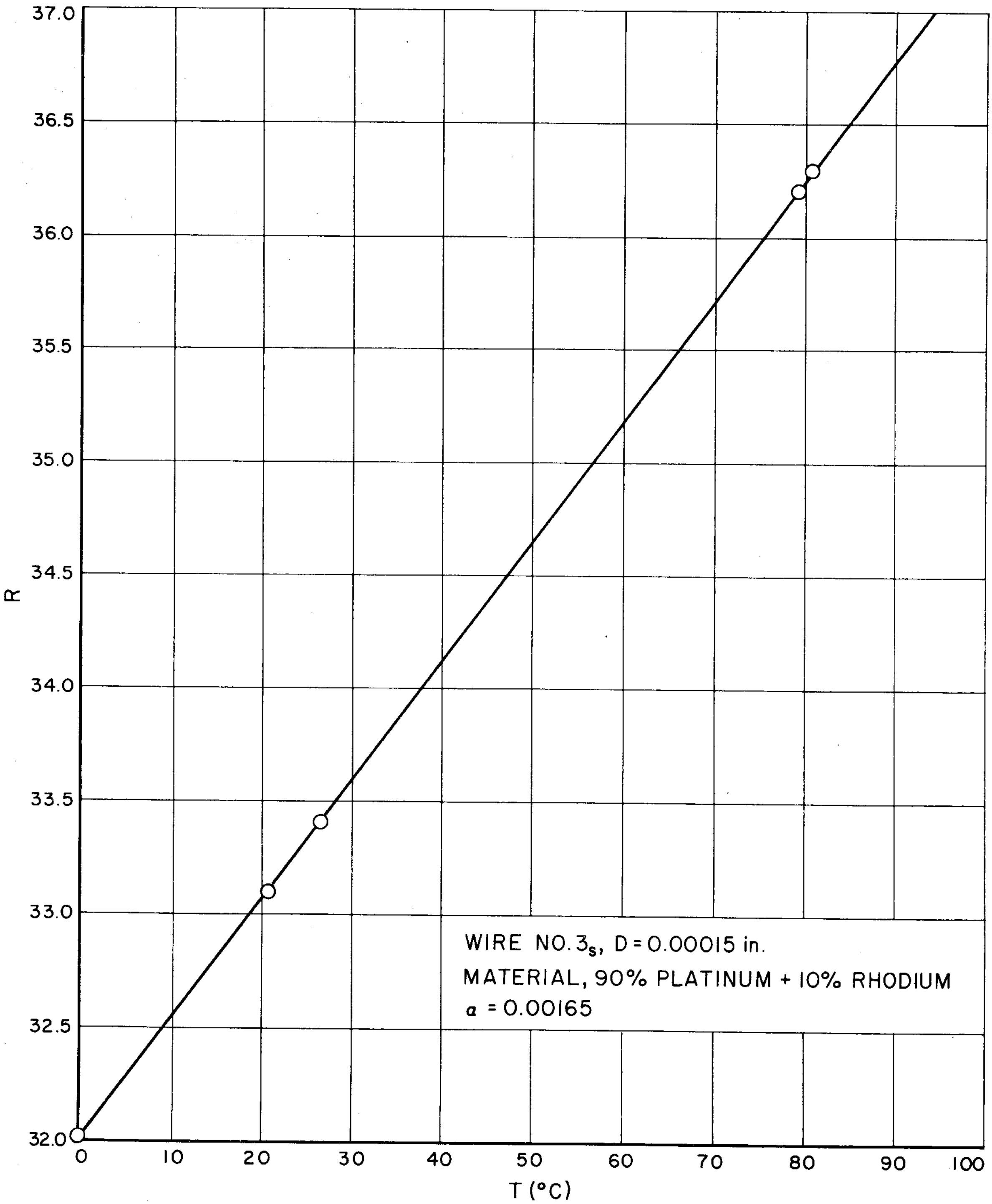
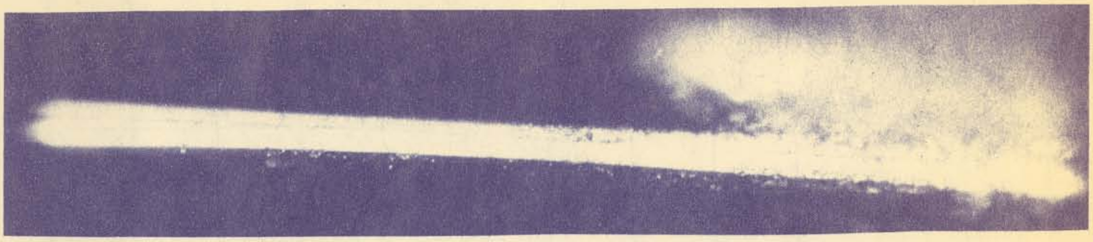
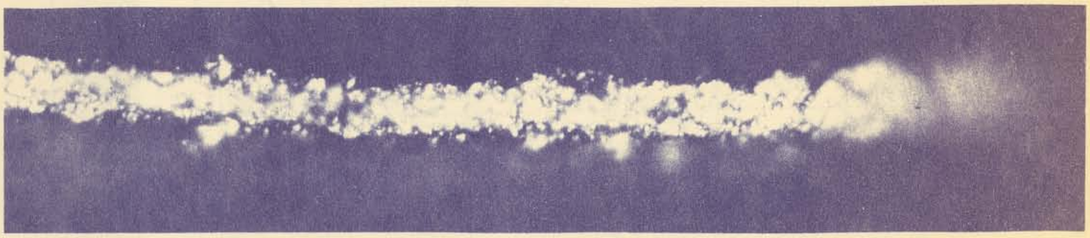


FIGURE 3. CALIBRATION PLOT FOR α



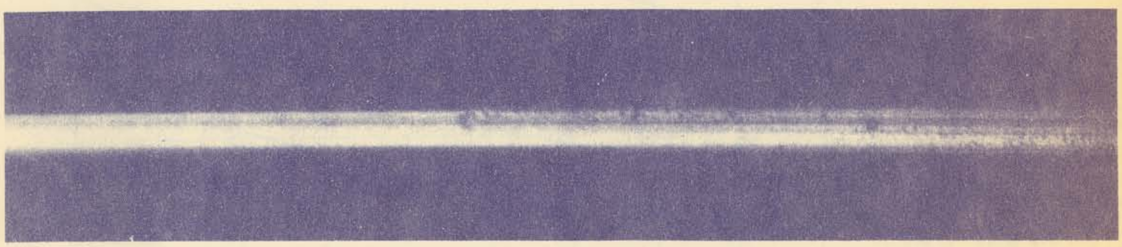
UNUSED WIRE, 0.0005 in. D. 500 X



WIRE RUN 5 HOURS, TUNNEL AIR NOT FILTERED

0.0005 in. D 500 X

Figure 4



WIRE RUN 5 HOURS, TUNNEL AIR FILTERED

0.0005 in. D 500 X

Figure 5

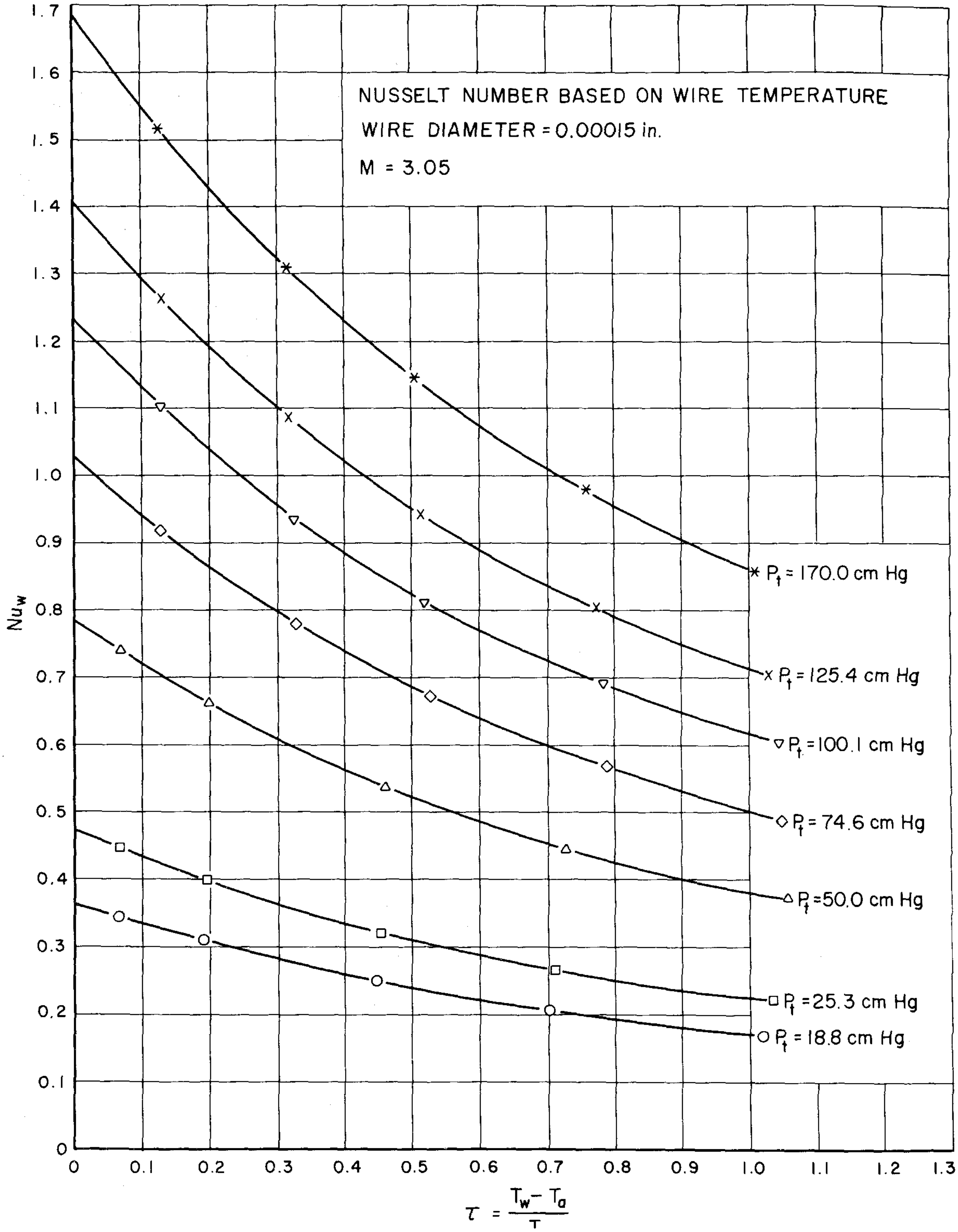


FIGURE 6. Nu_w VS TEMPERATURE LOADING AT CONSTANT SUPPLY PRESSURE

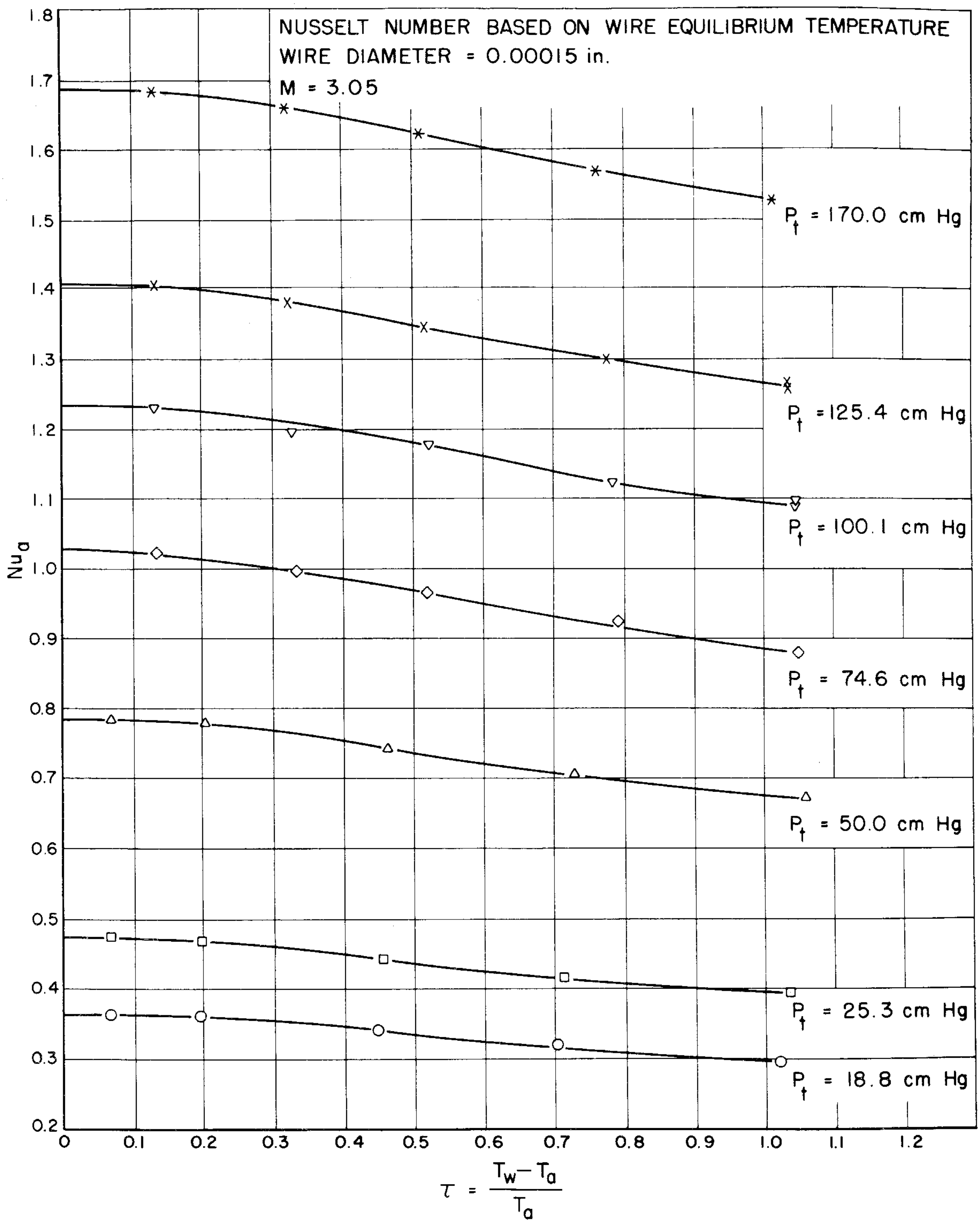


FIGURE 7. Nu_a VS TEMPERATURE LOADING AT CONSTANT SUPPLY PRESSURE

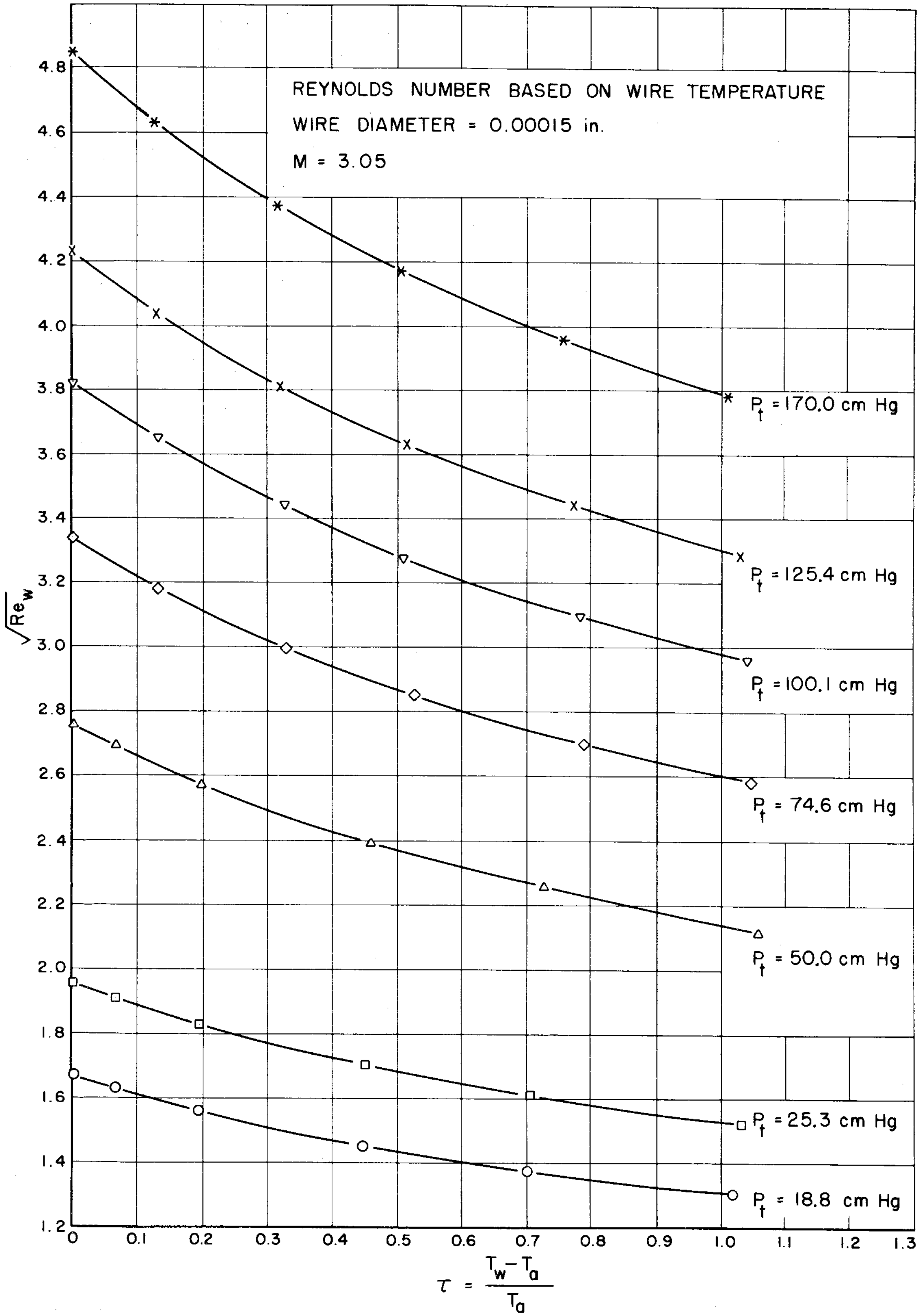


FIGURE 8. $\sqrt{Re_w}$ VS TEMPERATURE LOADING AT CONSTANT SUPPLY PRESSURE

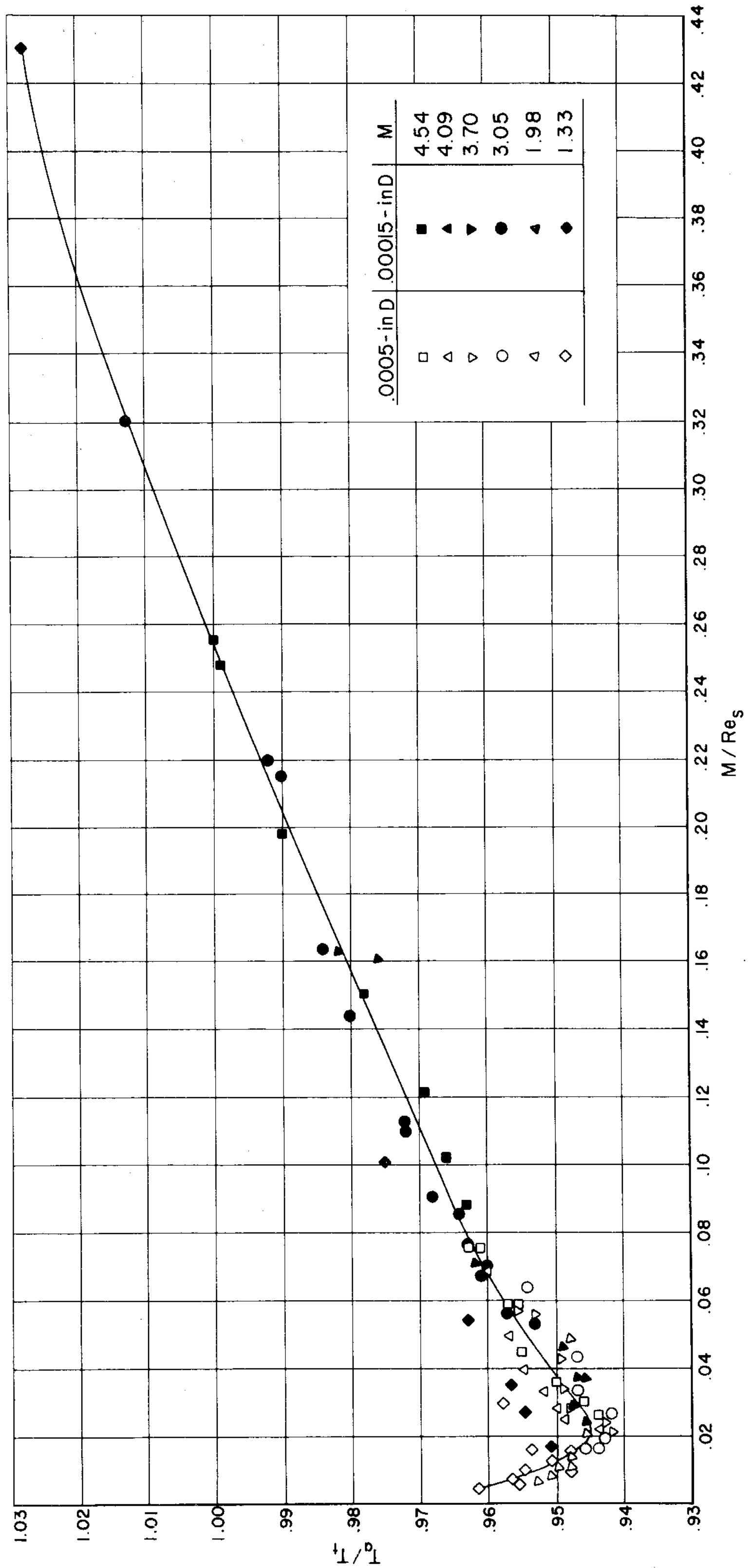


FIGURE 9. TEMPERATURE RATIO VS $\frac{M}{Re_s}$

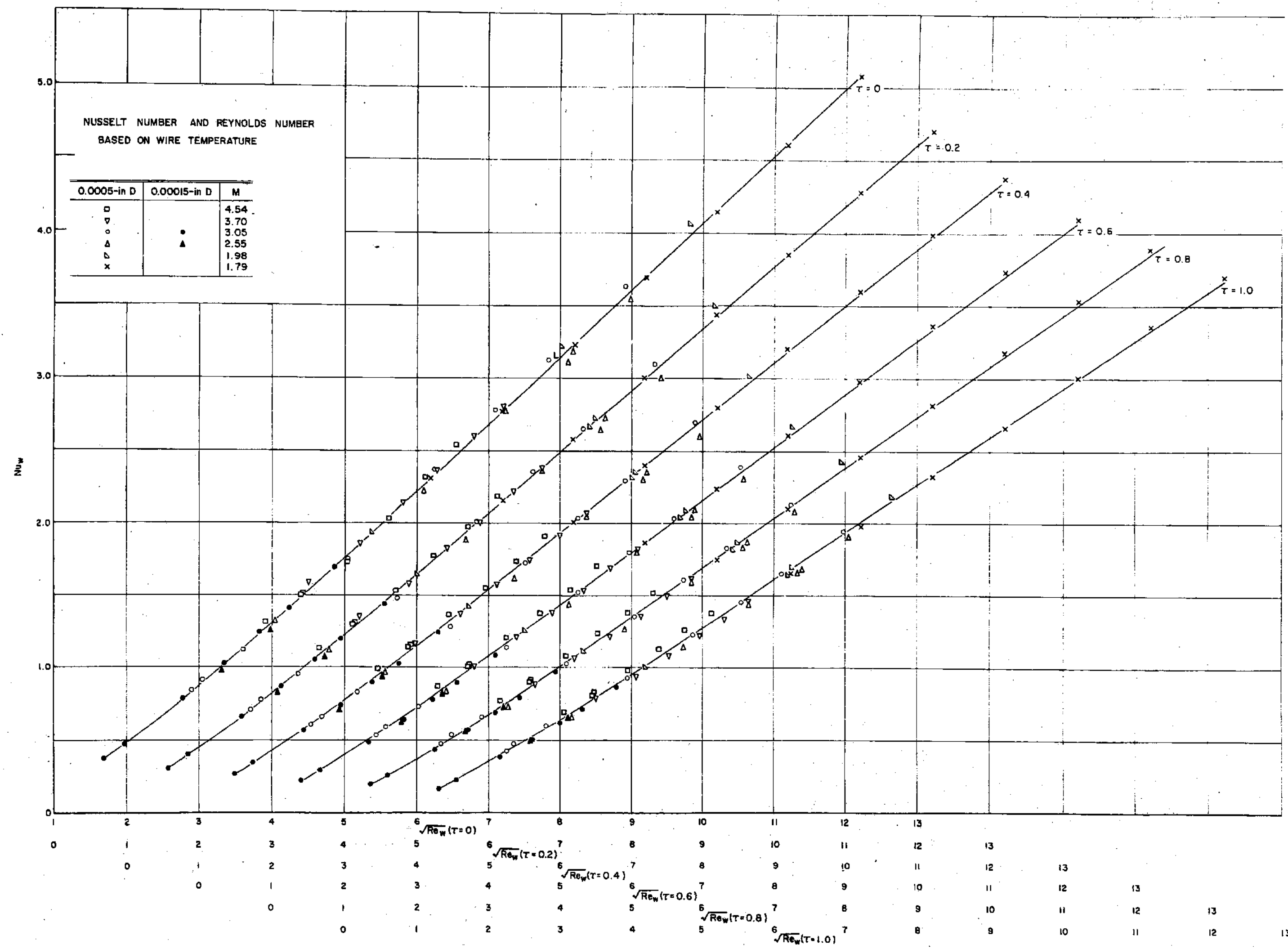


FIGURE 10. Nu_w VS $\sqrt{Re_w}$ FOR CONSTANT TEMPERATURE LOADINGS

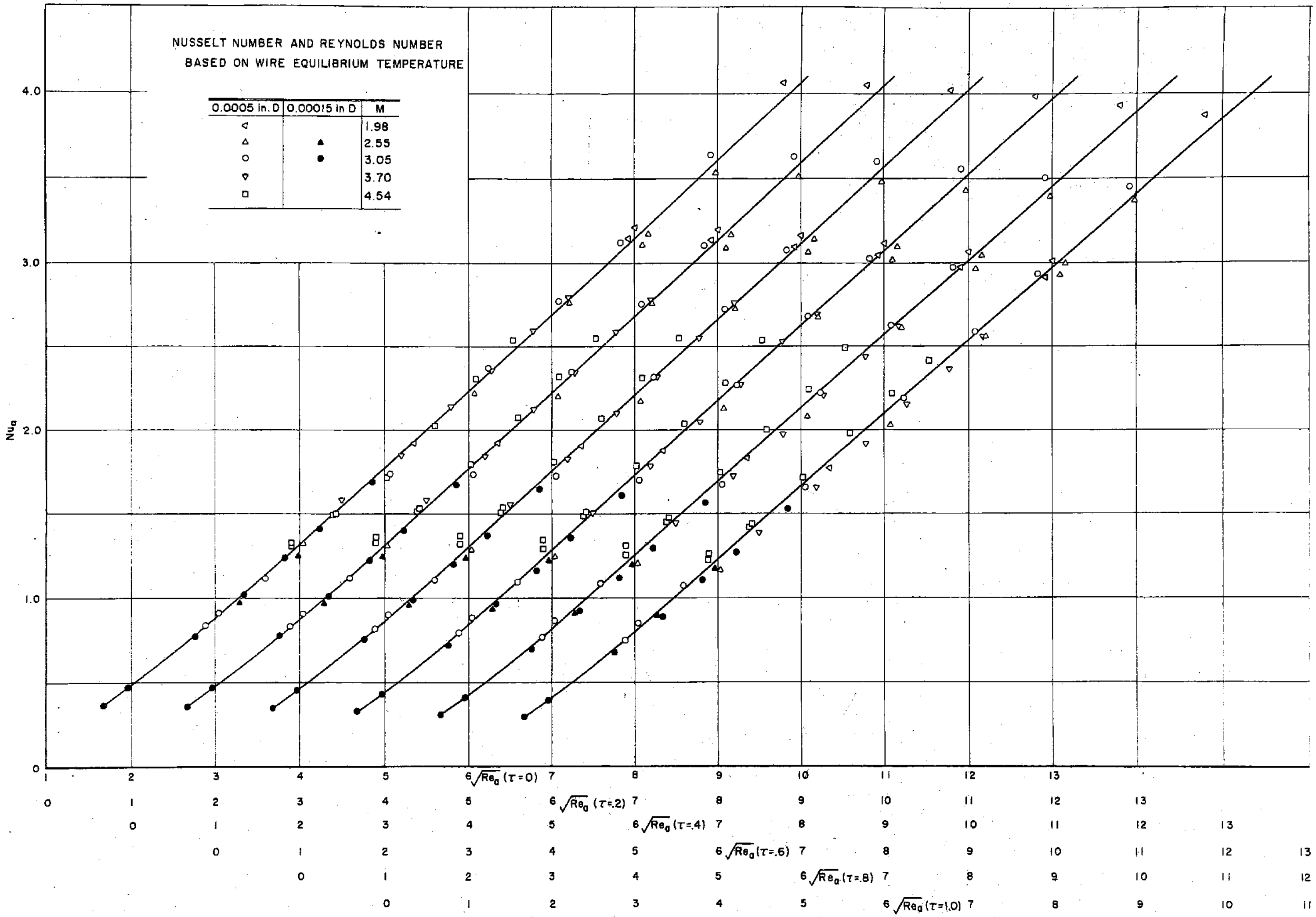


FIGURE 11. Nu_a VS $\sqrt{Re_a}$ FOR CONSTANT TEMPERATURE LOADINGS

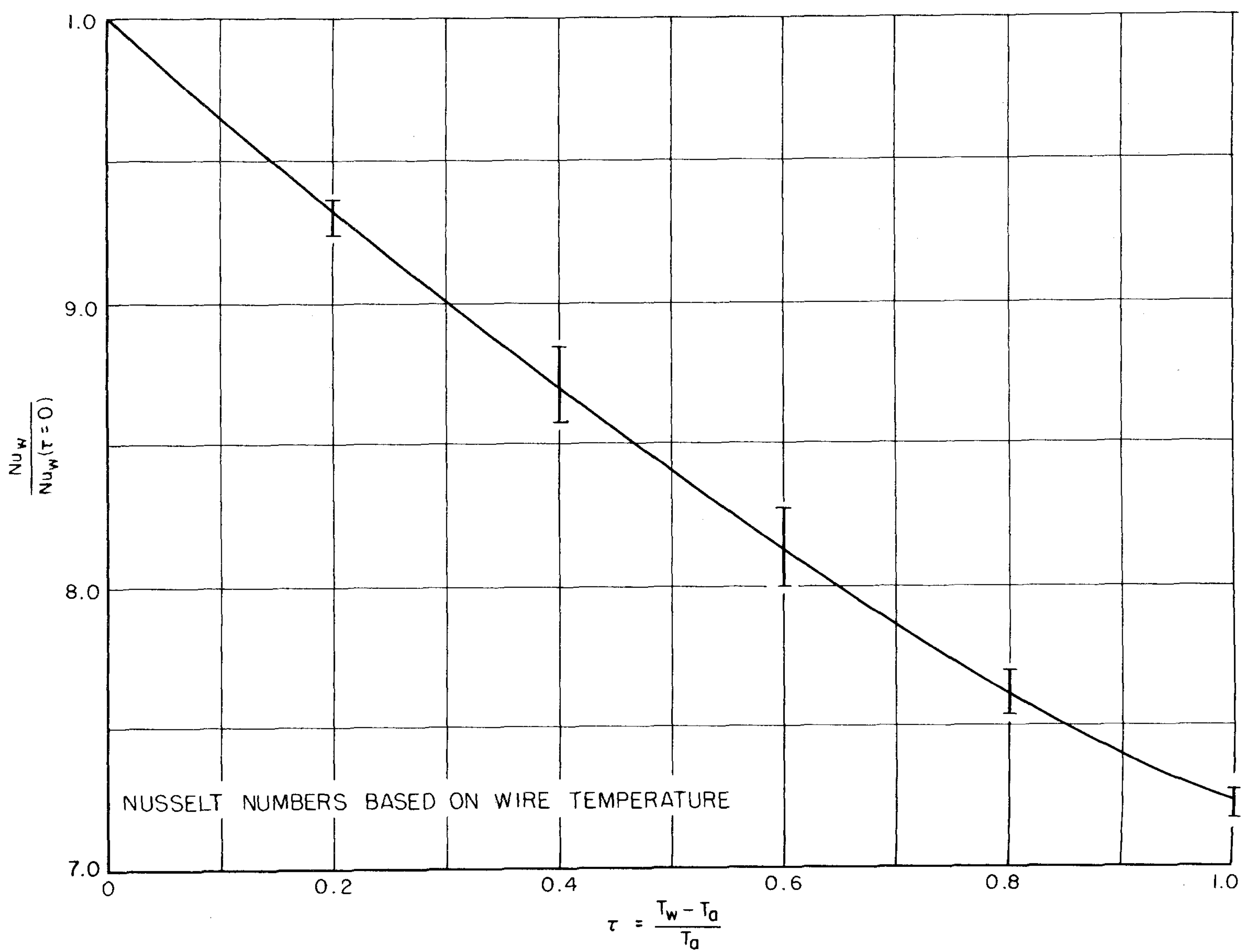


FIGURE 12. TEMPERATURE LOADING EFFECT

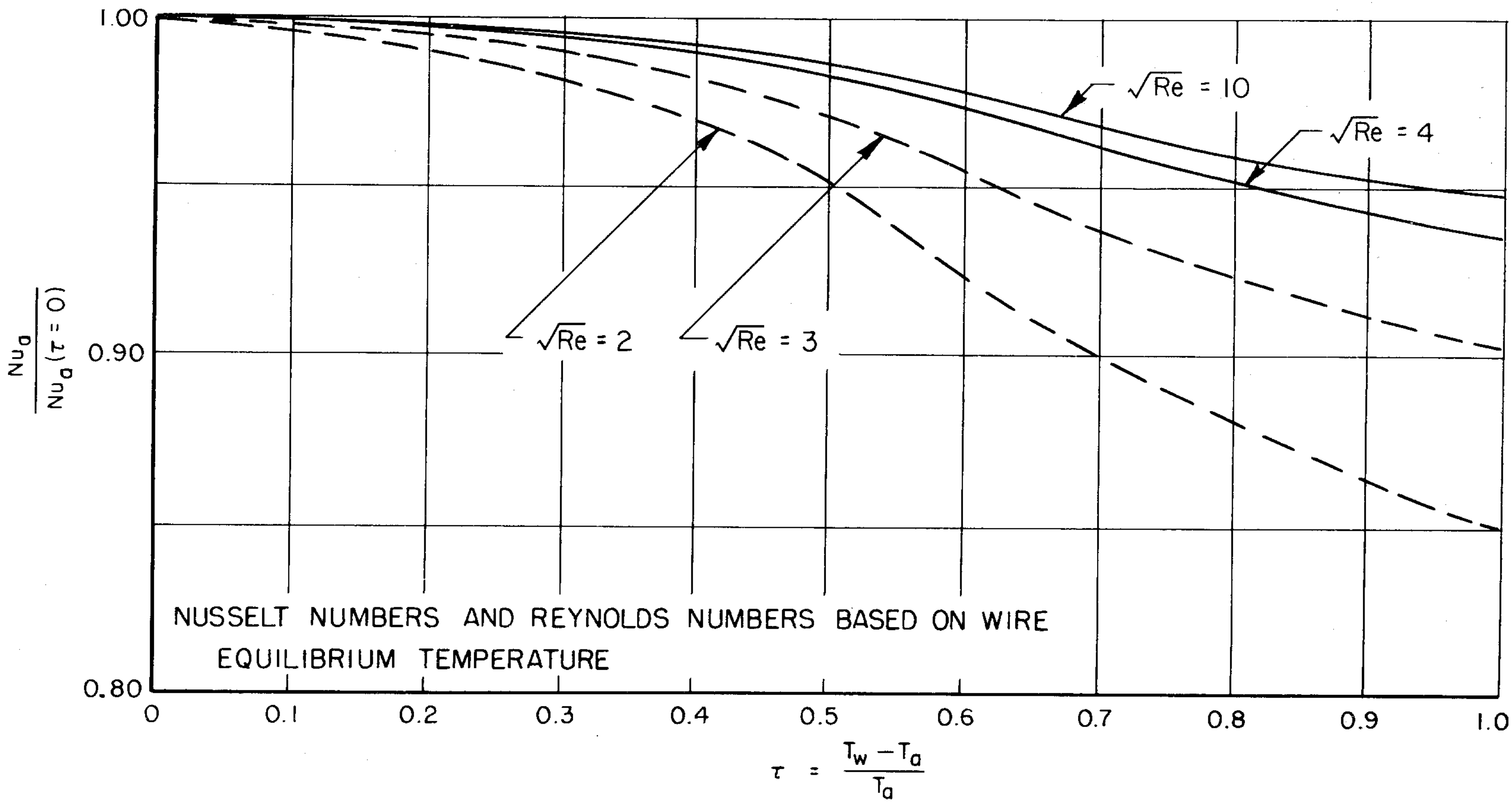


FIGURE 13. TEMPERATURE LOADING EFFECT