AN EXPERIMENTAL INVESTIGATION OF THE TRANSFER OF HEAT FROM SMALL WIRES TO A VISCOUS COMPRESSIBLE FLUID.

Thesis by

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In Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1955

ACKNOWLEDGEMENTS

I wish to express my thanks to my adviser, Professor H. W. Liepmann, for his advice and criticism of this research; to Doctors R. L. Chuan and D. Coles for their help and suggestions, and to the staff of the aeronautics machine shop for their part in constructing the more complex portions of the apparatus.

ABSTRACT.

A steam tunnel, suitable for making experimental measurements of the heat transfer from fine wires to a viscous compressible fluid, was developed and constructed.

Measurements of Nusselt numbers and recovery temperatures were carried out using small-diameter (0.00038 to 0.00254 cm.) tungsten wires in steam flow with Reynolds numbers ranging from about 1 to 12 and with nominal Mach numbers of 0.5 to 1.7.

Considerable difference was found between the Nusselt numbers for wires in subsonic and supersonic flow at corresponding Reynolds numbers. The results could be fairly well represented by an available theory based on the assumption that a temperature discontinuity of the fluid existed at the wire surface; however, they did not agree very well with other available data in the same range of Mach and Reynolds numbers.

In supersonic flow, the wire recovery temperatures were found to be consistently higher than the tunnel stagnation temperature.

An experimental procedure for making end-loss corrections to the heat transfer and temperature recovery tests was used and found to give satisfactory correlation of data taken with wires ranging in aspect ratio from 220 to 3040. Some experiments were performed to check the validity of the simple linear theory which is usually used to calculate end-loss corrections; the theory was found to be adequate in the experimental range covered.

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SYMBOLS

		Units
a	first temperature coefficient of thermal conductivity of metal	$1/^{o}C$
A	first temperature coefficient of heat transferred to stream	$watt/cm^{o}C$
AR	aspect ratio of wire, $\frac{2\ell}{d}$	
b	second temperature coefficient of thermal conductivity of metal	$1/^{o}C$
B	second temperature coefficient of heat transferred to stream	$watt/cm^{o}C^{2}$
C	third temperature coefficient of heat transferred to stream	$watt/cm^{o}C^{3}$
d	wire diameter	cm
D	fourth temperature coefficient of heat transferred to stream	$watt/cm^{o}C^{4}$
E	drop in electric potential between ends of a wire	volts
H	power lost from a wire	watts
i	electric current	amperes
k	thermal conductivity of gas	$watts/cm^{o}C$
K	thermal conductivity of metal	$watts/cm^{o}C$
Kn	Knudsen number, $\sqrt{\frac{\pi\gamma}{2}}\;\frac{M}{Re}$	
ℓ	half length of wire	cm
M	Mach number	

Nu	Nusselt number, $\frac{Ei - \text{End loss}}{2\pi k \ell(\theta_m - \theta_r)}$	
p	static pressure in nozzle	dyne/cm^2
p_o	pressure in nozzle supply duct	dyne/cm^2
p_o'	pressure measured by an impact probe facing upstream in the nozzle	dyne/cm^2
Pr	Prandtl number	
r	electrical resistance of a wire	ohms
r_o	electrical resistance of a wire at temperature θ_o	ohms
R	gas constant (assumed to be 4.61 x 10^6 for steam)	ergs/gram^{o}C
Re	Reynolds number, $\frac{\rho V d}{\mu}$	
T	temperature of gas in nozzle	^{o}C or ^{o}K
T_o	temperature of gas in nozzle supply duct	^{o}C or ^{o}K
v	local velocity of sound	cm/sec
V	flow velocity	cm/sec
x	length coordinate of wire, zero at center of wire	cm
α	first temperature coefficient of resistivity of metal	1/^{o}C
β	second temperature coefficient of resistivity of metal	1/^{o}C
γ	ratio of specific heats of a gas	
δ, Δ	Increments	

ζ	third temperature coefficient of resistivity of metal	$1/^{o}C^{3}$
θ	local wire temperature	^{o}C
θ_o	arbitrary reference temperature	^{o}C
θ_1	temperature of wire supports	^{o}C
θ_m	mean wire temperature	^{o}C
θ_r	recovery temperature, the temperature an insulated section of wire of given size and shape would attain if left sufficiently long in a flow of given properties without electrical heating	^{o}C
λ	Thomson coefficient of metal	volt/^{o}C
μ	viscosity coefficient	dyne sec/cm^2
ν	first temperature coefficient of Thomson effect	$1/^{o}C$
ξ	second temperature coefficient of Thomson effect	$1/^{o}C^{2}$
ρ	density	grams/cm^3
σ	electrical resistance per unit length of wire	ohm/cm
σ_o	resistivity at temperature	ohm/cm
ω	cross sectional area of wire	cm^2

I. INTRODUCTION.

At the suggestion of Professor H. W. Liepmann, design was commenced, in the summer of 1950, of a steam tunnel making use of changes in the phase of the working substance. The tunnel was built and used to investigate some problems of viscous compressible flow at low Reynolds numbers. An initial report of the success of the tunnel is given in Reference 1, while a more extensive report of the design philosophy, particular advantages, and usefulness of such a tunnel is given in Reference 2.

At first, the tunnel was somewhat inconvenient to operate. Usually it took about six hours of constant attention to pumps, valves and electrical controls before tests could be started. Once the tunnel was operating satisfactorily, however, the test run could be continued indefinitely.

The flow in the nozzle, however, was not free of pressure gradients and the Mach number of the tunnel was subject to variation if the condenser (downstream) pressure varied. These characteristics have also been observed in other supersonic tunnels designed to operate at low Reynolds numbers[3]. The characteristics are believed to be due to alteration of the effective shape of the nozzle due to changes in the wall boundary layers. In Reference 3 it is reported that an elaborate porous-walled nozzle was used in an effort to control the wall boundary layer thickness. Even so, the tests reported in Reference 3 had to be conducted in a restricted region near the centerline of the nozzle.

In the steam tunnel the thickness of the wall boundary layers was only about 0.25 cm. at a point where the cross section of the nozzle was about 2.0 x 2.0 cm. Nevertheless, the absence of gradient-free flow at a constant Mach number made the tunnel most attractive for research of the sort where the conditions need be known and constant over only a small region of the test section.

Such a problem was the investigation of the loss of heat from a small wire stretched transverse to the fluid flow. In fact, Kovasznay and Tormarck[4] conducted an experiment on the heat loss from hot wires in supersonic flow using a very small tunnel and varied the Mach number by varying the position of the wire in a nozzle having large Mach number gradients.

The prospect of investigating the heat loss from hot wires in a stream flow is attractive for two reasons:

First, considerable of a speculative theoretical nature has been written concerning the heat loss from small wires in fluid flows of diverse character[5 through 11]. In addition, a number of experimental investigations have been made[3, 4, 8, 12, 13, 14]. The general lack of agreement in experimental results has stimulated a search for a correlating function (some dimensionless combination of the variables involved in an experiment on the heat loss from small wires) which would enable the results of experiments to be interpreted as consistent with some law governing heat loss. The functions suggested as being significant usually involve the Prandtl number [12, 13]. There has been little effort, however, to conduct experiments in compressible fluids with Prandtl numbers different from that of air. The Prandtl number of air at ordinary temperatures is about 0.7. Steam under the conditions found in the steam tunnel has a Prandtl number in the vicinity of 1.0[15]. Reference 16, a table of the properties of steam issued by the National Bureau of Standards, specifically disclaims responsibility for defining Prandtl numbers for steam, but simple calculations using the values of viscosity, specific heat, and thermal conductivity given therein also show that the Prandtl number under the steam tunnel conditions is near 1.0. An experimental investigation of heat transfer from small wires in steam would be of interest then because the Prandtl number of steam is different from that of air.

Second, work has been done on heat transfer from wires in monatomic and diatomic gases, but not in triatomic gases like steam. All theoretical work on compressible gas flows shows a dependence of the results on the ratio of specific heats, γ , a number dependent on the molecular structure. Steam at the tunnel conditions has a different γ from that of air.

A feature of the low density steam tunnel which makes it particularly useful for the investigation of the phenomena of flow about small wires is that small Reynolds numbers may be attained at supersonic speeds. One of the criteria in the design of the tunnel was that a model in supersonic flow in the steam tunnel should have a Reynolds number approximately one hundredth of the Reynolds number of an identical model in supersonic flow in the transonic wind tunnel located at this laboratory. This objective of the design was achieved and the Reynolds number of a model with one centimeter characteristic length can be as small as 3000 in supersonic flow. If a wire of 0.001 centimeter diameter were used, the Reynolds number based on this diameter would be 3. Relations between Reynolds number and Mach number of the forms $\frac{M}{Re}$ or $\frac{M^2}{Re}$ are often referred to as being of significance in deciding whether the flow phenomena under consideration can be considered as continuum flow, slip flow, or free-molecule flow. Tsien [17] suggested that it was reasonable to consider a flow as being a continuum flow if the parameter $\frac{M^2}{Re}$ were less than 10^{-4} and that it was reasonable to consider a flow as being a free-molecule flow if the parameter $\frac{M}{Re}$ were greater than 10. Other authors have other opinions, but in general it is considered that flows where $\frac{M^2}{Re}$ is near unity are neither continuum nor free-molecule flow. It is therefore evident that experiments concerning the heat transfer from small wires in supersonic flow in the steam tunnel may be conducted under flow conditions of the transition type between continuum flow and free-molecule flow.

The general purpose of the research described in this paper was to study the transfer of heat from small wires to a rarefied steam flow. The successful completion of this purpose was dependent upon the identification and correction of undesirable characteristics of the equipment used and upon the development of methods of obtaining the necessary data.

In the next section a description will be given of the steam tunnel, the characteristics which made it unsuitable for the research planned, and the characteristics after reconstruction.

Following the description of the tunnel will be a description of the experimental method used to obtain the heat transfer data and a discussion of the results of the experiment. The appendices present some of the considerations bearing on the experiment which were not considered to be of sufficient general interest to warrant their inclusion in the main text.

II. THE STEAM TUNNEL.

When the steam tunnel was first put to use for investigating problems of viscous compressible flow, it had the appearance pictured in fig. 1. The steam used as a working fluid was supplied by boiling water in the bottom of the supply chamber at an absolute pressure of about 3 centimeters of mercury. Early in the tunnel calibration program it was determined, from measurements of the power supplied to boiler and superheater, that part of the superheater power was being used to boil water; probably because water was splashing up onto the superheater elements. After installation of a baffle between the water surface and the superheater, the boiler and superheater operated more nearly as they were intended to. The steam could be superheated to approximately 250 degrees centigrade by some electrically heated rods in the upper part of the supply chamber. The rods, about one half centimeter in diameter, were arranged in two layers and were nominally about ten centimeters apart.

The superheated steam from the supply chamber flowed through a convergent-divergent nozzle into a diffuser and then a condenser. Tests could be conducted in the divergent portion of the nozzle. An absolute pressure of about 1.2 centimeters of mercury could be maintained in the condenser; the latent heat of condensation was extracted by a refrigeration system.

During the course of the tests reported in Reference 2, a number of interesting features of the tunnel operation were observed.

First, the water in the bottom of the supply chamber apparently boiled violently with extremely large volume bubbles breaking the water surface at irregular intervals. The bursting of the bubbles caused a rocking motion of the free water surface which often caused the water in the gage glass of the boiler to surge up and down with an amplitude of about six inches. Explosive boiling of this type may easily be observed in a glass flask if water is boiled at low pressure.

Second, the heights of the liquid columns in the silicone-fluid manometers used to determine pressures at various points of the tunnel system shifted up and down in an erratic manner, making it difficult to determine pressures accurately.

Third, the shock waves from a wedge in the supersonic flow of the nozzle could be observed, on the schlieren system viewing screen, to jump periodically from one inclination to another, the inclinations differing by as much as five degrees.

Fourth, in the measurement of Mach number, which could be determined from pressure readings by two independent methods, it was found that the two methods gave results differing by as much as 0.2 Mach number.

Fifth, it was found that the resistance of an ordinary hot-wire anemometer could not be measured with the instruments ordinarily used for this purpose because of the large and rapid shifts in the apparent resistance.

The combination of the conditions observed made the tunnel an unsatisfactory environment in which to attempt measurements of the heat transfer from small wires.

It was believed that all of the conditions observed were direct consequences of the explosive nature of the boiling process. Consequently, the tunnel was redesigned and rebuilt to eliminate the undesirable effects of the explosive boiling by greatly increasing the boiler operating pressure. The rebuilt tunnel, which is shown in fig. 2, made use of most of the components of the original tunnel. Water could be boiled at slightly above atmospheric pressure in the bottom of the steam generator, the heat being supplied by seven electric immersion heaters controlled by an autotransformer. The pressure was regulated by a valve designed to keep the pressure between 1.000 and 1.0044 atmospheres. Water droplets from the boiling process were removed from the steam by a filter consisting of a 10 cm. thick batt of glass wool held between perforated aluminum plates. The filtered steam was superheated by passage through a bank of three electric heaters consisting of flat spirals of Nichrome wire on Transite supports and incorporating baffle plates designed to cause turbulent mixing. After superheating, the steam was allowed to expand through a needle-type valve into a 20 cm. diameter duct. The duct was obstructed by a baffle plate and a series of five spaced screens of successively smaller mesh size. (2, 4, 12, 30, 80 wires per inch). The pressure and temperature of the steam emerging from the last screen could be measured using appropriate instruments. The steam thus prepared was the supply steam for the convergent-divergent nozzle which was the test section of the tunnel. The pressure could be regulated by adjusting the needle valve and the temperature could be regulated by adjusting an auto-transformer controlling the electric energy supplied to the super-heater.

The other parts of the tunnel were not different from those of the original tunnel.

The nozzle, constructed of brass with two flexible aluminum walls, numerous pressure taps, and access holes, is shown together with the probe traversing mechanism in fig. 3. Two sliding plate valves are provided so that the test section could be isolated from the rest of the tunnel. In this way the test section could be opened to permit changes of the model to be made without admitting air to the condenser or the rest of the tunnel. After a model change the test section could be quickly evacuated and testing resumed.

Downstream of the test section was a diffuser, a duct of 97 cm. length with two walls parallel and two walls diverging by 5 degrees. Under typical tunnel operating conditions (supply section pressure 30 mm. Hg, Mach number 1.7) the pressure recovery in the diffuser was only about 2 mm. of Hg., that is, the condenser pressure would only be about 2 mm. of Hg. greater than the test section static pressure.

The condenser was connected to the diffuser by 12.5 cm. diameter pipe and a conical transition piece. The condenser was a steel tank 61 cm. in diameter and 90 cm. tall with two refrigerant evaporators in the upper portion of the tank. Freon-12 refrigerant was supplied to the evaporators by a compressor driven by a 3.7 kilowatt electric motor. Heat was rejected to water provided by a pump, cooling tower, and storage tank system used by other equipment of the laboratory.

The rebuilt tunnel lacked the faults of the original tunnel for the following reasons: Boiling was carried out at atmospheric pressure, thus eliminating the pressure fluctuations formerly caused by the explosive boiling at low pressure. The steam was superheated at atmospheric pressure in three stages with thorough mixing between stages. The steam was further mixed by expanding it through a needle valve and a series of baffles and screens so that it would have only very small scale temperature and velocity fluctuations before being admitted to the nozzle.

When the rebuilt tunnel was to be operated, approximately the following procedure was followed: With the pressure in boiler and condenser equal, water was transferred by gravity from the condenser to the boiler until there was an adequate supply in the boiler for a run of the expected duration. The needle valve separating the supply duct from the steam generator was closed and the portion of the tunnel downstream of the valve was evacuated. The pressure could be lowered, in about an hour, to about 5 cm. of mercury using a single small aspirator. At the same time that the downstream portion of the tunnel was being evacuated, the temperature of the condenser was being lowered by operating the refrigerator and the temperature of the water in the steam generator was being raised to the boiling point. The condenser pressure was further lowered using a pair of positive displacement mechanical vacuum pumps connected in parallel. The pumps were protected from water vapor by a water trap consisting of some Pyrex glass tubes immersed in a bath of pulverized solid carbon dioxide and liquid methyl cellusov. After the condenser pressure had been lowered to the saturation pressure of water at condenser temperature and after the steam generator had been ejecting steam to the atmosphere through the pressure regulating valve for several minutes, the needle valve could be opened and flow established in the nozzle. Usually the tunnel was allowed to operate for about an hour before commencing experimental measurements in order to assure that steady conditions had been reached.

The range of operating conditions which could be covered was approximately as follows: Supply section pressure between 18 mm. Hg. and 30 mm. Hg. if the flow in the test section was to be supersonic, 10 mm. Hg. to 30 mm. Hg. if the flow in the test section was to be subsonic. Supply section temperature between 140 and 325 degrees centigrade if the test section Mach number was 1.7 and between 98 and 290 degrees centigrade if the Mach number was 0.5. The limits of the range of operating conditions arose in the following manners: Maximum supply section pressure was the maximum pressure which could conveniently read on the manometers used. So long as the pressures to be read were 30 mm. Hg. or lower, no water would condense in the manometer system, provided that the ambient temperature in the vicinity of the manometers was 30 degrees centigrade or higher.

The minimum supply section pressure was determined by the condenser pressure. In order to operate the tunnel with a given test section Mach number, the supply section pressure must be in some definite ratio to the condenser pressure. In rebuilding the tunnel two vacuum pumps rather than one were used in order to keep condenser pressure as low as possible. Actually there would be no permanent need for vacuum pumps if the tunnel were perfectly sealed, but the use of cast off machinery and materials in the tunnel construction resulted in a structure with some leakage. In the original tunnel the leakage was sufficient that a Cenco Hyper-vac-20 pump was unable to lower the condenser pressure to less than about 12 mm. mercury. In the rebuilt tunnel most of the pumping capacity was needed to remove gases which apparently entered the system with the supply steam. Two vacuum pumps were sufficient to lower the condenser pressure to about 5 mm. of mercury if the mass flow of steam through the tunnel was small. The ratio of supply pressure to condenser pressure which was necessary to run the tunnel with test section Mach number 1.7 was about 2.5; hence, if the condenser pressure was 7 mm. Hg., the supply pressure would have to be 18 mm. Hg. or more in order to maintain supersonic flow at Mach number 1.7 in the test section. When the tunnel was run with subsonic flow in the test section the flow was stabilized and isolated from disturbances in the diffuser by an orifice at which the flow would reach Mach number 1.0 immediately downstream of the test section. In order to insure that the flow reach Mach number one at this orifice the supply pressure was maintained at twice the condenser pressure.

If tests were being conducted at Mach number 1.7, the supply section temperature was limited to a maximum of 325 degrees centigrade by the power which could be supplied to the superheaters and to a minimum of 140 degrees centigrade by the requirement that the steam not become saturated in the test section. If the tests were to be subsonic, the maximum temperature, 290 degrees centigrade, was also determined by the power which could be supplied to the superheaters and the minimum temperature, 98 degrees centigrade, was determined by the temperature of the steam generated at atmospheric pressure. In each case the temperature of the steam leaving the superheater was higher than that measured at the supply section. This would be a direct result of the removal of heat from the flow by the needle valve, duct walls, baffle, and screens which the flow must pass before reaching the supply section. The loss in flow temperature between superheaters and supply section was dependent on the mass flowing through the tunnel and the temperature of the flow after the superheaters. In some tests where no energy was supplied to the superheaters, the losses were enough to lower the temperature from that of saturated steam at atmospheric pressure (99 + degrees centigrade) to about 98 degrees centigrade at the supply section.

The response of the tunnel supply section temperature to a sudden change in the power supplied to the superheater was quite slow; approximately five minutes would elapse before any change in supply section temperature could be detected and about an hour would elapse before the temperature would again be steady if the controls were left alone. Once these characteristics were known it was possible to make supply section temperature changes of the order of 60 degrees centigrade in about 20 minutes by proper manipulation of the autotransformer controlling the power supplied to the superheaters.

The pressure measurements necessary to establish the Mach and Reynolds numbers of the flow in the test section were made using U-tube manometers. Because of the small differences (at Mach number 1.7, if the supply section pressure was 30 mm. of Hg., the minimum pressure would be the static pressure in the test section, 6.2 mm. of Hg.) it was desirable to use a low density fluid in the manometers designed to read differences in pressure between various points in the tunnel. A manometer having six tubes and a common reservoir filled with Dow Corning silicone fluid of nominal density 0.94 gram/cm.^{3} was used. The manometer tubes, about 0.3 cm. inside diameter and 30 cm. long were provided with scales divided in millimeters and the height of the liquid columns were easily readable to an accuracy of $\pm$ 1/2 millimeter. Differences between the pressures at various points in the tunnel therefore could be read with an accuracy of $\pm$ 0.1 millimeters of mercury. The pressures at various points in the tunnel were compared, using the silicone fluid manometers, to the pressure in the condenser, an ideal reference since this pressure was not subject to rapid fluctuations and the reference pressure prevailed in a relatively large volume (1.5×10^5 cm.^{3}).

The absolute pressure in the condenser was determined (by comparison to the vapor pressure of mercury) using a U-tube mercury manometer which could be read with an accuracy of $\pm$ 0.2 mm. of mercury.

When tests were being conducted pressures were determined in the supply section, in the condenser, at two points on the nozzle walls, and on impact and static pressure probes in the immediate vicinity of the model being tested. Mach number could be determined from the supply section pressure and static pressure using the isentropic flow equation

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{1}$$

The Mach number could also be determined from the impact and static pressures using the Rayleigh formula.

$$\frac{p}{p_0'} = \left(\frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \left(\frac{\gamma+1}{2} M^2\right)^{\frac{-\gamma}{\gamma+1}} \tag{2}$$

When the value 1.33 was used for the ratio of specific heats, γ, the Mach numbers as determined from the two expressions above were found in every case to be in agreement within 0.05 Mach number. These results are quite different from the results obtained in the original steam tunnel where the Mach numbers calculated using the two above expressions usually differed by about 0.2 Mach number and neither method gave Mach numbers which consistently agreed with Mach numbers determined by optical methods.

The temperature in the supply section was determined from the reading of a calibrated thermocouple. The hot junctinn of the 0.013 cm. diameter copper and Advance wires was cantilevered out into the flow of the duct about 1.25 cm. from a supporting structure of ceramic tubing. A traversing mechanism was provided so that measurements of the flow temperature could be made on a line extending from one wall to the centerline of the duct. The thermocouple wires were carried through pressure seals to the exterior of the tunnel and to a cold junction in a melting ice bath. The thermoelectric potential was measured using a Leeds and Northrup K-2 potentiometer and galvanometer. Calibration of the thermocouple in the range between 100 and 300 degrees centigrade was made in a bath of molten wax by comparison to the readings of a mercury-in-glass 3-inch-immersion thermometer. Surveys of the flow temperature under various tunnel operating conditions showed that the temperature indicated by the thermocouple was constant within $\pm$ 1.5 degrees centigrade over the portions of the duct farther than 1.6 cm. from the duct wall.

Some preliminary tests were made to determine the general suitability of the tunnel flow for the work planned, a study of the steady state heat transfer from fine wires. For this purpose a hot-wire probe constructed of 0.0013 cm. diameter tungsten wire about 0.7 cm. long was mounted on a traversing mechanism and was used to explore the flow under various tunnel operating conditions. The hot wire current was adjusted to give a mean wire temperature about 100 degrees centigrade above the tunnel supply section temperature, the output signal was connected to an amplifier and the amplified signal observed on an oscilloscope. When the test section flow was supersonic no signal was observed with the probe either upstream or downstream of the throat except in the nozzle wall boundary layer. When the flow in the test section of the same nozzle was supersonic no signals were observed when the probe was upstream of the throat, but large signals were observed when the probe was downstream of the throat.

III. PROCEDURE OF THE INVESTIGATION OF THE LOSS OF HEAT FROM A WIRE.

The loss of heat from a wire immersed in a fluid flowing normal to the wire axis was first investigated by L. V. King[6] in 1914. King not only performed experiments, but also gave theoretical methods for predicting the heat loss and suggested the useful application of the knowledge, the hot-wire anemometer.

The usefulness of the hot-wire anemometer depends on the understanding of a number of physical phenomena. The measureable quantities involved are: the physical dimensions of the wire, the electrical resistance of the wire, and the electric current passing through the wire. From these quantities, some information about the temperature, velocity, or density of the fluid passing the wire is to be determined. In the simplest application of the hot wire anemometer, the measurement of fluctuations of velocity in an airflow with small velocity, the instrument is used as a calibrated device. A given current is passed through a given hot-wire anemometer; the electric resistance of the wire is determined as a function of velocity by placing the wire in an airstream whose velocity may be varied and measured by some other instrument. Thereafter the velocity of similar flows may be deduced from the resistance of the calibrated hot-wire anemometer. The simple relation of resistance to velocity obtained in the case mentioned is a consequence of being able to neglect density and temperature changes in the flow. If a ot-wire anemometer is to be used in flows where density and temperature are not constant, the deduction of useful information from the readings of current and resistance of the wire depend on the understanding of a large number of physical relationships.

The relation between wire electric resistance and temperature should be known. From the wire total resistance the mean wire temperature may be deduced. From the electric resistance and current the power being dissapated by the wire may be calculated. How the heat transfer from a wire of known temperature varies with flow velocity, density, viscosity, thermal conductivity, and temperature should be known. Then if the length, diameter, and thermal conductivity of the wire and the temperature of the supports to which it is attached are known and the ways in which these quantities affect the power conducted to the supporting structure are known, there is hope of using the hot-wire anemometer for finding out something of the properties of the flow passing the wire.

Typical of the experiments which have been performed is the study reported by Kovasznay and Tormarck[(4)]. The resistances of tungsten wires of small diameter were determined at various temperatures. Then the hot-wire anemometers were placed in a supersonic air-flow of known characteristics and the relation between power supplied to the wire and wire resistance was determined. The heat flow from the wire to the stream was taken to be nearly equal to the power supplied to the wire; a small correction factor based on the theory presented by L. V. King[(6)] in 1914 was used to account for the flow of heat to the wire supports. The wire mean temperature could be related to the wire resistance and the recovery temperature was taken to be that temperature associated with the resistance of the wire when the heating current was vanishingly small. The data obtained in this fashion was collected for flows with various Mach and Reynolds numbers and for various wire temperatures.

It became evident after some preliminary tests that the method outlined above, which was followed in the experiments of Kovasznay and Tormarck[4], could not be used in the steam tunnel. The apparent wire recovery temperature was found to be much lower than the tunnel supply section temperature and in some cases even lower than the calculated flow temperature. This behavior was evidently an effect of the wire support temperatures being much lower than the tunnel supply section temperature. The uncertainty as to the true value of the recovery temperature resulted in an uncertainty in the Nusselt number, the two quantities being connected by the expression

$$Nu = \frac{Ei - \text{End loss}}{2\pi k \ell (\theta_m - \theta_r)} \qquad (3)$$

where

Nu = Nusselt number

Ei = power supplied to wire

k = thermal conductivity of gas

ℓ = half length of wire

θ_m = wire mean temperature

θ_r = wire recovery temperature

To eliminate the uncertainties in recovery temperature and Nusselt number, it was decided to build an apparatus whereby the wire support temperature could be varied and measured at will. The principle involved is that the flow of heat from the wire to the supports would be zero if the gradients of wire temperature along the wire axis were zero where the wire joins the supports. Hence, if the support temperature were adjusted to be equal to the mean wire temperature, then the flow of heat to the wire supports should be small and the mean wire temperature should be very nearly the temperature the wire would assume in the absence of supports. The method of heating supports to eliminate the flow of heat from model to supports was used in the experiments of Stalder, Goodwin, and Creager [3], a study of heat transfer in a very rarefied gas stream. It should be evident that if the gas flowing by the wire has a very small Reynolds number, the Nusselt number will also be small and the exchange of heat between wire and stream might be small compared to the exchange of heat between the wire model and its supporting structure unless special precautions were taken. The small Reynolds numbers prevailing in the steam tunnel and in the tunnel used in the experiments reported in Reference 3 necessitated the special precaution that the support temperature be adjustable in order that the quantity to be measured, heat exchange between model and flow, not be small compared to the heat exchange between model and supports.

The wires to be tested in the steam tunnel were supported by the model holder shown in fig. 4. The wires were attached by spot welding to two wedge-shaped stainless steel members projecting from the sidewalls of the tunnel. The stainless steel members were heated by Nichrome wire resistance elements imbedded in the steel in the fashion of an ordinary electric soldering iron, but were electrically insulated from the sidewalls by Transite spacers. Each supporting member had two leads to supply heating current, a lead for supplying hot-wire current, two thermocouple leads for measuring support emperature, and a lead for measuring the electric potential of the end of the hot wire. All leads were carried through the side cover plates on pins insulated and pressure sealed by synthetic rubber 0-rings. Heating currents for the two supports were individually controlled by autotransformers.

The resistivity as a function of temperature of the wire to be used in the tests was determined by testing samples about 6 cm. long in a molten wax bath. The samples were cut from rolls of wire considered to be homogeneous in diameter over a large length by reason of weight per unit length tests, and were attached to heavy copper electrodes. A Nichrome wire heating element drawing alternating current power from an autotransformer heated the bath and a small propeller driven by an electric motor also controlled by an autotransformer stirred the bath. Temperatures were determined by a 3-inch-immersion mercury-in-glass thermometer; for the tests the temperatures ranged between 90 and 310 degrees centigrade.

The results of the resistivity as function of temperature tests were similar to those found by other investigators. The values of the first temperature coefficient of resistivity was found to be considerably lower than values listed in handbooks. A typical calibration is shown in fig. 5.

The thermocouple used to determine the temperature of the steam in the supply section of the tunnel and the thermocouples attached to the heated supports for the wire models were also checked in the calibration bath.

Wires cut from the calibrated samples were cleaned in benzene and mounted on the stainless steel supports by spot welding. A commercial apparatus which furnished the welding current by discharging capacitors was used. The wire mounting operation was carried out under a binocular microscope, the wires being manipulated with tweezers and bits of wax, but otherwise no special equipment was found to be necessary for this work. After mounting the wire was again cleaned, its length measured, and its electrical resistance at room temperature measured.

To determine the resistance at room temperature, the potential drop across the hot wire and the potential drop across a one-ohm resistor carrying the heating current were recorded for heating currents ranging from 0.5 to 10.0 milliamperes. A graph of the calculated resistance as a function of heating current was plotted and the resulting straight line was extrapolated to determine the resistance to be expected if there were no current.

Heating current for the hot wire was furnished by a series of storage batteries with a nominal potential of 36 volts. The current, adjusted by four on-off switches capable of changing the current by ten approximately equal steps, was read approximately by a milliameter and more exactly by reading the potential drop across a one-ohm resistor carring the current. A diagram of the circuit is shown in fig. 6.

The calibrated wire and model support system were installed in the test section and the tunnel started according to the method outlined in Section II.

After the tunnel operating pressures and temperatures were steady, the temperature of the hot-wire was raised and lowered by adjusting the heating current for the wire. The hot wire current, potential drop across the wire, and temperatures of the supply section and wire supports were recorded, along with the pressure readings necessary to determine the tunnel flow conditions. Next, the temperature of the wire supports was changed by adjusting the current supplied for support heating and the wire temperature was again raised and lowered by adjusting the wire current.

After the data had been gathered for a number of different support temperatures, the tunnel flow conditions would be changed. Usually the amount of power supplied to the superheaters was adjusted, thereby changing the tunnel supply section temperature. An hour would be allowed to elapse before commencing the data recording in order to assure that the new tunnel operating conditions would be steady.

IV. OPERATIONS PERFORMED TO REDUCE THE RAW DATA TO USEFUL FORM.

1. Flow Data.

The Mach number was calculated from readings of the static pressure and supply pressure using the isentropic flow relation given previously as expression (1). As a check the Mach number in supersonic flow was also calculated from the impact pressure and static pressure using the Rayleigh formula given previously in expression (2).

The stagnation temperature, T_0, was measured by a thermocouple in the supply duct.

Isentropic flow equations were then applied to calculate conditions in the test section.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{4}$$

The knowledge of pressure and temperature at a point is sufficient to determine the density through the perfect gas relation

$$\frac{p}{\rho} = RT \tag{5}$$

The viscosity and heat conductivity are functions of temperature; the curves used in reducing the data are those given in fig. 7. There is considerable disagreement between various experimental investigatinns of these two properties of steam so the results of the investigation are given in tabular form in sufficient detail that the final data may be recalculated using other viscosity and heat conductivity curves if so desired.

The flow velocity may be determined through the two expressions

$$V = Mv \tag{6}$$

$$v = \sqrt{\gamma RT} \tag{7}$$

Having established density, velocity, and viscosity, the Reynolds number may be calculated.

$$Re = \frac{\rho V d}{\mu} \tag{8}$$

The expressions listed above are sufficient to determine the well-known flow parameters. As other parameters are mentioned in discussion of results they will be defined.

2. Wire Data.

The current through the potential drop across the wire model was measured. Thus the power dissipated by the wire could be calculated.

$$H = Ei \tag{9}$$

The resistance of the wire was calculated

$$r = \frac{E}{i} \tag{10}$$

The mean temperature of the wire may be deduced from the total resistance. The resistance as function of temperature might be represented

$$r = r_o \left[1 + \alpha (\theta_m - \theta_o) \right] \tag{11}$$

where r_o is the resistance of the wire at temperature θ_o and the mean temperature is

$$\theta_m = \frac{1}{2\ell}\int_{-\ell}^{\ell}\theta dx \tag{12}$$

A limitation on the validity of this representation is given in Appendix I.

Therefore, a suitable value of θ_0 was chosen and values of $(\theta_m - \theta_0)$ were established from (11). The values of $(\theta_m - \theta_0)$ were corrected for drift in the stagnation temperature during a run according to the expression

$$\Delta(\theta_m - \theta_0) = \Delta T_0 \tag{13}$$

A further correction to $(\theta_m - \theta_0)$ was that due to end temperature drif'

$$\delta(\theta_m - \theta_0) = \frac{\partial(\theta_m - \theta_0)}{\partial(\theta_1 - \theta_0)} \quad \delta(\theta_1 - T_0) \tag{14}$$

The partial derivative was evaluated by observing the mean temperature of the wire at several end temperatures.

After the drift corrections had been applied to $(\theta_m - \theta_0)$, the values θ_m were plotted as abscissae and the values of H were plotted as ordinates on a graph similar to fig. 8.

The power dissipated as a function of mean wire temperature is shown for a number of end temperatures. It was assumed that there would be no end loss if the mean wire temperature were equal to the end temperature. Thus, in fig. 8, the locus of points where mean wire temperature and end temperature are equal is considered to show the variation of power dissipated with variation in the mean wire temperature considering that there is no flow of heat out of the ends of the wire.

The slope of the "no-end-loss" line in fig. 8 may be related to the Nusselt number

$$Nu = \frac{1}{2\pi \ell k}\left(\frac{\partial H}{\partial \theta_m}\right)_{\text{no end loss}} \tag{15}$$

for a small range of temperature near the recovery temperature. The intersection of the "no-end-loss" line with the zero power line determines the recovery temperature of the wire, the temperature that the wire would assume if there were no end loss and no heating current.

V. RESULTS.

1. Temperature Recovery.

The results of the temperature recovery and heat transfer tests are summarized in Table I. Listed are the diameter and aspect ratio of the wire, Mach number, Reynolds number based on the wire diameter and free flow properties, Knudsen number, stagnation pressure, tunnel supply section or stagnation temperature, wire recovery temperature, Nusselt number, and the symbol used to represent the data on the Figures. Subsonic data will be represented by crosses and supersonic data by points enclosed by simple geometric figures.

It may be noted that in supersonic flow the wire recovery temperature was consistently higher than the stagnation temperature of the tunnel, while in subsonic flow the recovery temperature was scattered with most points below the stagnation temperature.

Consider some possible explanations for the observed results.

First, consider that the results might be a consequence of the experimental method used. The thermocouple used to measure the supply section or stagnation temperature loses heat by conduction to the exterior of the tunnel and therefore reads low. If this were an important consideration, the results should be strongly influenced by the difference between stagnation and room ambient temperatures; however, fig. 9 testing this possible error source, shows no significant effect. Hence it is supposed that the high apparent recovery temperatures are not caused by an error in measuring stagnation temperature.

It is also possible that the data reduction method, a graphical method of making end corrections, may be in error. In the discussion of end correction methods in Appendix II it will be shown (Expressions (39) and (42) of Appendix II) that the importance of end corrections is a function of a parameter $\left(\frac{\tanh P\ell}{P\ell}\right)$ which may be approximated by $\frac{1}{AR\sqrt{\frac{k}{K}Nu}}$. In fig. 10, the discrepancy between stagnation and recovery temperatures is shown as a function of the end correction parameter. Inasmuch as the tunnel operating conditions could not be varied over a very wide range, the spread in values of $\left(\frac{\tanh P\ell}{P\ell}\right)$ is almost entirely due to variation of aspect ratio. It may be surmised then that the graphical end correction procedure adequately correlates the recovery temperatures taken with wires of small aspect ratio ($74 \leqslant AR \leqslant 910$).

The scatter in the data of figs. 9 and 10 is considerable, but does not obscure the large difference between the results obtained in subsonic and supersonic flow. Since the observed results may not be readily shown to be consequences of obvious possible faults in experimental technique, some other explanation must be sought.

Wire recovery temperatures greater than tunnel stagnation temperatures have been obtained before in experiments in rarefied supersonic flow [3 and 8]. The experiments mentioned covered a wide range of Knudsen numbers (on the order of from 0.025 to 40) in attempts to establish the boundaries of the slip flow and free-molecule flow regimes. Among the conclusions reached in Reference 3 were: (a). The temperature recovery factor for cylinders in high-speed rarefied gas flow is primarily dependent on Knudsen number. (b). The recovery temperature is greater than tunnel stagnation temperature for Knudsen numbers greater than 0.2. (c). Fully developed free-molecule flow occurs for Knudsen numbers of about 2 and higher, a conclusion reached upon observing the agreement between the experimental recovery temperatures and the free-molecule theory predictions for recovery temperature.

Judged on the basis of the conclusions of Reference 3, all the data taken in supersonic flow should have shown recovery temperature higher than tunnel stagnation temperature. Table I shows that the Knudsen number of the data ranged from 0.218 to 0.822 in supersonic flow and from 0.072 to 0.720 in subsonic flow. (Knudsen number is the ratio of mean-free molecular path to the wire diameter. It is described in Reference 3 as being nearly $\sqrt{\frac{\pi\gamma}{2}}\ \frac{M}{Re}$ in a rarefied perfect gas.) Using the method described in Reference 8, calculations were made of the expected recovery temperatures of wires in a free-molecule flow of a gas with triatomic molecules. The results of the calculation are shown in fig. 11. The experimental data is shown plotted against Knudsen number in fig. 12 along with some of the theoretical values taken from fig. 11. The loci of the supersonic data points are such that a straight line connecting $\frac{\theta_r}{T_o} = 1.0$ at $Kn = 0.2$ and the free-molecule flow theory prediction for $\frac{\theta_r}{T_o}$ at $Kn = 2.0$ passes reasonably close to the experimental points. In subsonic flow, however, the majority of the data points in the transition region are far from what might be expected from free-molecule flow theory.

The results of the recovery temperature tests may be summarized as follows:

Recovery temperatures in supersonic flow were consistently greater than tunnel stagnation temperature. Recovery temperatures in subsonic flow ranged from near tunnel stagnation temperature to about four percent below stagnation temperature for the range of Knudsen numbers from 0.1 to 0.7.

2. Heat Transfer.

The results of the heat transfer tests are presented in Table I in the form of Nusselt numbers. The Nusselt number is usually plotted against the square root of the Reynolds number; fig. 14 shows this relationship. A well defined spread between the data taken in subsonic and supersonic flow is evident.

The graphical end correction procedure was apparently sufficient to collect all data except that taken with a wire of aspect ratio 74 into a single scatter band. It is not surprising that the method did not work very well on the wire of aspect ratio 74. The end correction for this wire under steam tunnel conditions is about the same as would be necessary on a similar wire of aspect ratio 74 in atmospheric air at 0^o centigrade and 10 meters per second velocity. Of the power supplied for heating such a wire, approximately 24 percent was found to be conducted to the wire supports when the heated supports were maintained at wire recovery temperature.

If an experiment be conducted in supersonic flow the density, pressure, velocity, temperature, viscosity, and heat conductivity may vary greatly from point to point in the flow field. Hence, dimensionless ratios such as Reynolds numbers and Nusselt numbers could be constructed which would take account of the fluid properties prevailing at diverse points of the flow. Kovasznay and Tormarck[4] chose to define Reynolds number and Nusselt number in the special fashions given,

$$Re_{10} \equiv \frac{\rho V d}{\mu_o} \tag{16}$$

$$Nu_o \equiv \frac{H}{k_o \pi \ell (\theta_m - \theta_r)} \tag{17}$$

(the subscript "o" signifies conditions prevailing in the tunnel supply duct) because these definitions would be useful when the information on heat loss was eventually applied to the investigation of supersonic turbulence. This representation has been used since by other investigators in the presentation of their data [3 and 13]. Fig. 14 shows the data of fig. 13 replotted using the definitions of expressions (16) and (17). Whereas Kovasznay and Tormarck found that these special definitions would correlate their data taken over a range of Mach numbers from 1.16 to 2.03, fig. 14 shows that the subsonic data is not collected into a single line with the supersonic data by using these particular special definitions, a finding in agreement with the results of Spangenberg[13].

It would be advantageous to find a method of presenting the results of heat transfer experiments which would collect into a single line the data taken in subsonic and supersonic flow since hot wires are most probably going to be used to explore such mixed flows as occur in boundary layers and around bluff bodies.

It has been suggested by Stine[12] that the test results at subsonic and supersonic Mach numbers could be collected into a single line if Nusselt number were plotted as a function of Knudsen number; fig. 15 shows the data of Table I plotted with Nusselt number as a function of Knudsen number. It may be noted that there remains a well defined spread between the data taken in subsonic and supersonic flow, contrary to the finding of Stine. The data taken here, however, agrees very well with that taken by Spangenberg[13, fig. 15].

In order to demonstrate the relation between the data taken here and that taken by some other investigators, fig. 16 is presented. Data from References 3, 4, and 13 are shown using the parameters of Kovasznay and Tormarck, Expressions 16 and 17. At $Re_{10} = 60$ there are shown three sets of data with a scatter of Nusselt number from 2.1 to 3.9, a ratio of 1.8. The largest Nusselt numbers were obtained by Kovasznay and Tormarck and the smallest were obtained by Stalder, Goodwin, and Creager. The paper by Stalder, Goodwin, and Creager[3] discusses this discrepancy at some length. At first it was believed that the discrepancy might be due to some tunnel blocking phenomenon in the tests by Stalder, Goodwin, and Creager since a large model (0.126" dia.) was tested in a fairly small jet. The model was retested in the tunnel at the University of California (Berkeley) which had a larger space available for testing than the tunnel at Ames Aeronautical Laboratories and the results confirmed the results of the Ames tests. It was suggested in Reference 3 that the discrepancy between the results reported in References 3 and 4 might be due to a difference in the turbulence levels in the streams of the tunnels used, possibly arising from the radically different density levels in the tunnels.

The data obtained here may likewise be compared to that obtained by Stalder, Goodwin and Creager; at $Re_{10} = 10$ in supersonic flow the spread of Nusselt numbers is from 0.60 to 1.25, a ratio of 2.1, with the data of Stalder, Goodwin, and Creager again being at the low end of the scatter range. If the reason for the discrepancy between the results of the tests reported in References 3 and 4 were due to differences in the turbulence levels of the tunnels used, a similar set of circumstances could account for the discrepancy between the results reported here and those reported in Reference 3. It should be noted that, of the tunnels mentioned, only those at Ames Aeronautical Laboratories and at the University of California (Berkeley) did not have screens in the supply duct upstream of the test section. Note also the range of Reynolds numbers (Re_{10} based on a 1 cm. characteristic length) which could be obtained in the various tunnels; 145, 000 to 273, 000 in the tunnel used by Kovasznay and Tormarck; 1, 100 to 2, 600 in the tunnel used here; and 60 to 210 in the tunnel used by Stalder, Goodwin, and Creager. It is plausible, as suggested by Reference 3, that the discrepancy in results of heat transfer tests conducted at identical Mach and Reynolds numbers in two different tunnels could be due to some difference in the tunnel flow characteristics such as turbulence level. It is plausible because the large variations in structure and operating conditions in the various tunnels considered could be the cause of differences in the turbulence level and because of the fact that turbulence is known to greatly influence the heat transfer from heated objects in low speed flow. In fact, a method of evaluating turbulence levels in jets was worked out by Loiziansky and Schwab(18) which made use of the influence of turbulence on the heat transfer from small spheres operating at less-than-critical Reynolds numbers.

The value of the results of the experiments would be enhanced if it could be shown that the results are consistent with some theory. A number of theories were examined but the only one considered applicable over the range of conditions covered by the experiment was that of Sauer and Drake(9). This theory may be used to predict the effect of Mach number changes on the Nusselt numbers for small wires provided that the Nusselt numbers for the wires are known in incompressible flow. Briefly, the work consisted of solutions to a steady, two-dimensional, simplified energy equation for the heat transferred from a circular cylinder to a non-viscous, incompressible fluid under two sets of boundary conditions. First, a solution was found for continuum-type boundary condition; the temperature of the fluid adjacent to the cylinder surface was assumed to equal the cylinder temperature, and the solution was made to match experimental data for continuum flow by adjustment of one arbitrary constant. Next, a solution was found with a slip-flow temperature-jump boundary condition of the type found by Smoluchovski; the difference between the temperature of the cylinder surface and the temperature of the fluid adjacent to the cylinder surface was assumed to be proportional to the temperature gradient in the fluid adjacent to the cylinder surface. The arbitrary constant which was found to make continuum theory agree with experimental results was used in the slip-flow theory, and a chart of Nusselt numbers as functions of Reynolds number for various Mach numbers was constructed. Air was used as a working fluid in the calculations by Drake and Sauer, but the results may easily be converted to apply for steam. The results of the calculations are shown in fig. 13 along with the experimental data. It may be seen that there is fair agreement both as to trends in the data and in actual location of the data points.

Some results of free-molecule flow theory have also been plotted in fig. 13. The Nusselt number in a free-molecule steam flow was calculated using the methods outlined by Oppenheim[10]. It may be seen that the free-molecule flow theory agrees reasonably well with the slip flow theory in the region where both should be applicable. The limits of free-molecule flow theory as suggested in Reference 3 are also shown in fig. 13.

The results of the heat transfer tests may be summarized as follows:

a. Nusselt numbers in subsonic flow near Mach number 0.5 were found to be considerably larger than the Nusselt numbers at corresponding Reynolds numbers in supersonic flow near Mach number 1.7.

b. A good approximation to the results obtained as furnished by the theory of Drake and Sauer[9].

c. The Nusselt numbers obtained were considerably larger than those obtained at corresponding Reynolds numbers by Stalder, Goodwin, and Creager[3] but were in fair agreement with those obtained by Spangenberg[13].

3. End Corrections.

In fig. 8 a typical set of the data taken to determine recovery temperature and heat transfer is presented. The differences between the slope of the "no end loss line" and the slopes of the recorded data lines must be accounted for by end loss.

The end loss usually is calculated by assuming some differential equation for the distribution of temperature along the wire, solving the equation, calculating the temperature gradients at the ends of the wire, and computing what power would be conducted from the wire to the supports if the temperature gradients calculated were correct.

As has been demonstrated by Reference 19, the inclusion of the "non-linear" terms in the differential equation alters the calculated distribution of temperature along the wire and hence also alters the end correction calculated using the equation.

That end loss corrections are necessary in this experimental investigation of the heat loss from wires is demonstrated by inspection of the data shown in fig. 17. This figure shows both the uncorrected Nusselt numbers and corrected Nusselt numbers as function of Reynolds number for wires in supersonic flow. As would be expected, the apparent (uncorrected) Nusselt number is greatly affected by the length of the wire. It may be seen that the scatter of data is less for the corrected data than for the uncorrected data and that the end loss corrections are appreciable even for the longest of the wires.

It may also be demonstrated that end corrections are necessary if a wire is to be used as a temperature measuring device. The mean temperature as determined from the resistance of a wire placed in the stream may be expected to depend on the temperature of the wire supports and on the length-to-diameter ratio of the wire. Fig. 18 shows the difference between apparent (uncorrected) recovery temperature and recovery temperature as determined from the method discussed previously plotted as a function of the difference between support temperature and recovery temperature.

Having established that end corrections to both the recovery temperature and heat transfer measurements were important to the investigations carried out here, an attempt was made to derive useful methods of calculating end corrections. This study is reported in Appendix II. The significant results of the study were as follows:

a. A fairly simple differential equation describing the balance of heat flow in and out of a fine wire was described and its solutions were used to predict some of the features of the operation of hot wires which could be checked by the experiments.

b. The predictions of the simple end correction theory were found to agree fairly well with the experiments in the case of the variations of mean wire temperature with variations of supporting structure temperature and with variations in the heating current supplied to the wire.

c. Two more complex forms of the differential equation describing the heat balance of the wire were investigated but found not to be enough advantage in predicting behavior of hot wires under the restricted range of conditions covered by the experiments to warrant the added complexity involved in their usage.

VI. CONCLUSIONS.

A steam tunnel suitable for making experimental measurements of the heat transfer from fine wires to a viscous compressible fluid was developed and constructed.

Measurements of Nusselt numbers and recovery temperatures were carried out using fine wires in steam flow with Reynolds numbers ranging from about 1 to 12 and with nominal Mach numbers of 0.5 and 1.7.

In supersonic flow, the wire recovery temperature was found to be consistently higher than the tunnel stagnation temperature.

Considerable difference was found between the Nusselt numbers for wires in subsonic and supersonic flow at corresponding Reynolds numbers. The results could be well represented by an available theory; however, they did not agree very well with other available data in the same range of Mach and Reynolds numbers.

An experimental procedure for making end-loss corrections to the heat transfer and temperature recovery tests was used and found to give satisfactory correlation of data taken with wires ranging in aspect ratio from 220 to 3040. Some experiments were performed to check the validity of the simple linear theory which is usually used to calculate end-loss corrections; the theory was found to be adequate in the experimental range covered.

TABLE I.

RESULTS OF HEAT TRANSFER AND TEMPERATURE RECOVERY TESTS.

Symbol	Wire Diameter Cm. d	Aspect Ratio Æ	Mach Number M	Reynolds Number Re	Knudsen Number Kn	Stagnation Pressure mm.Hg. p_o	Stagnation Temperature °Centigrade T_o	Recovery Temperature °Centigrade θ_r	Nusselt Number Nu
⊡	.00254	74	1.66	4.89	.490	17.9	286.4	298.2	.991
			1.63	6.88	.339	22.4	252.6	258.7	1.148
			1.71	9.70	.255	28.2	213.5	223.0	1.355
⅄	.00254	220	.41	1.87	.319	9.8	213.6	202.0	.613
			.43	2.93	.213	16.7	244.6	236.7	.854
			.44	4.16	.153	24.1	255.5	252.2	1.040
			.43	6.36	.098	25.8	151.5	149.6	1.294
◈	.00254	220	1.68	7.30	.331	25.2	267.7	280.4	.929
			1.75	11.65	.218	27.1	154.7	166.6	1.157
×	.00254	455	.46	2.35	.279	11.6	218.2	203.7	.688
			.47	3.02	.223	16.8	259.3	246.7	.815
			.48	4.15	.165	24.9	285.6	278.5	.909
			.47	4.53	.149	24.2	248.2	244.8	.985
			.47	5.69	.120	25.3	199.5	198.3	1.098
			.46	5.97	.111	22.6	149.1	150.4	1.128
			.47	6.45	.106	25.1	163.2	159.8	1.172
			.46	8.10	.083	25.0	104.2	106.3	1.191
⊙	.00254	455	1.56	4.25	.529	14.9	278.2	297.6	.803
			1.73	4.96	.500	19.3	296.8	311.3	.823
			1.77	5.76	.440	23.5	311.5	326.8	.882
			1.71	5.86	.418	21.3	281.9	292.6	.870
			1.76	5.99	.422	23.0	293.3	309.4	.840
			1.78	6.24	.410	26.3	323.4	348.1	.830
			1.79	6.60	.390	24.6	289.9	303.1	.860

	d	AR	M	Re	Kn	p_o	T_o	θ_r	Nu
⊙	.00254	455	1.78	6.62	.386	25.8	294.9	313.8	.882
			1.68	6.69	.360	24.6	285.6	298.4	.881
			1.87	6.71	.401	23.9	257.1	271.4	.951
			1.70	6.99	.352	22.5	243.9	254.0	.955
			1.76	7.09	.357	24.5	259.9	268.0	.967
			1.69	7.31	.333	22.8	237.6	247.6	.892
			1.78	7.41	.344	26.2	265.0	286.1	.856
			1.87	7.47	.361	23.5	220.9	231.4	1.021
			1.69	7.52	.322	25.2	257.1	278.6	.817
			1.79	7.60	.338	25.9	252.1	266.3	.935
			1.77	8.04	.319	24.1	218.4	230.1	.955
			1.79	8.45	.305	25.3	218.2	229.5	1.000
			1.67	9.40	.264	24.5	196.1	205.9	.961
			1.77	9.50	.277	25.9	199.5	208.7	1.067
			1.70	9.80	.264	25.3	196.1	206.5	.954
			1.87	9.92	.272	24.5	156.5	167.4	1.178
			1.72	10.21	.245	25.2	169.7	180.4	1.009
			1.80	10.40	.250	26.4	169.8	177.8	1.237
			1.77	10.50	.243	25.7	164.0	173.2	1.077
			1.77	10.67	.239	25.7	159.8	167.5	1.116
			1.78	10.78	.239	26.1	160.4	168.8	1.089
			1.68	10.90	.224	24.4	151.1	159.3	1.064
x	.00254	690	.42	1.95	.308	9.9	206.3	183.2	.622
			.43	3.01	.207	16.0	225.8	208.0	.868
			.45	4.02	.160	26.9	289.6	274.6	.887
			.43	4.51	.138	25.3	242.0	221.4	1.095
			.43	8.65	.072	27.9	99.3	91.9	1.383
✧	.00254	690	1.71	6.36	.386	25.6	315.7	317.0	.840
			1.86	8.85	.302	29.3	240.5	245.3	1.048
			1.84	9.24	.288	27.1	205.2	205.7	.984
			1.79	10.51	.247	26.4	158.8	159.4	1.126

	d	Æ	M	Re	Kn	p_o	T_o	θ_r	Nu
ƒ	.00127	440	.39	.85	.650	9.6	213.5	191.5	.407
			.45	1.38	.465	15.5	249.1	249.3	.522
			.46	1.99	.330	25.5	287.2	269.0	.650
▽	.00127	440	1.69	4.39	.555	29.6	258.1	281.7	.713
			1.75	6.05	.416	30.3	171.0	188.2	.804
+	.00127	910	.38	.75	.720	8.2	202.0	187.0	.351
			.49	1.06	.656	9.9	215.6	199.3	.367
			.54	1.74	.446	17.6	265.0	247.6	.503
			.52	2.14	.346	24.0	278.5	261.7	.580
			.58	2.39	.347	24.5	286.6	267.3	.626
			.51	2.47	.297	21.2	206.4	196.5	.650
			.49	4.25	.167	24.2	99.1	99.3	.801
			.46	4.30	.156	26.0	99.3	95.3	.871
△	.00127	910	1.77	3.08	.822	23.3	287.7	294.4	.556
			1.75	3.09	.813	24.1	296.9	307.3	.513
			1.76	3.31	.791	22.6	267.7	287.6	.447
			1.77	3.47	.734	24.2	261.9	279.0	.532
			1.74	3.79	.662	21.3	202.5	210.1	.590
			1.72	4.09	.610	25.6	235.3	258.8	.570
			1.76	4.19	.606	23.3	231.6	247.4	.554
			1.74	4.81	.522	23.9	169.7	180.7	.698
			1.74	4.84	.516	24.2	172.8	194.2	.635
			1.80	4.85	.535	24.7	170.6	182.8	.652
∽	.00038	480	.46	1.10	.609	22.0	98.2	100.8	.351
✱	.00038	3040	.46	1.13	.546	22.3	99.0	105.0	.362

APPENDIX I.

RELATION OF THE RESISTANCE OF A WIRE TO ITS TEMPERATURE.

Consider a metallic material, the resistivity of which may be suitably related to the temperature by an expression of the form,

$$\sigma = \sigma_0 \left[1 + \alpha(\theta - \theta_0) + \beta(\theta - \theta_0)^2 + \zeta(\theta - \theta_0)^3 + \ldots \right] \qquad (18)$$

where the "o" subscript specifies some arbitrary reference temperature.

In the International Critical Tables[20] the values of α, β, ζ for tungsten are listed as

$$\alpha = 5.238 \times 10^{-3} \text{/degree centigrade}$$

$$\beta = 0.70 \times 10^{-6} / \text{(degree centigrade)}^2$$

$$\zeta = 0.062 \times 10^{-9} / \text{(degree centigrade)}^3$$

for a range from - 80° C to 3300° C, $\theta_0 = 0°$centigrade. If the temperature is presumed to be a function of the linear dimension of the wire, then, to terms of order $(\theta - \theta_0)^3$,

$$r = 2\ell\sigma_0 + \alpha\sigma_0 \int (\theta - \theta_0)dx + \beta\sigma_0 \int (\theta - \theta_0)^2 dx + \zeta\sigma_0 \int (\theta - \theta_0)^3 dx$$

$$-\ell \leq x \leq \ell$$

But $2\ell\sigma_0 = r_0$ so

$$\frac{r}{r_0} - 1 = \alpha \left[(\theta - \theta_0)\right]_{mean} + \beta\left[(\theta - \theta_0)^2\right]_{mean} + \zeta\left[(\theta - \theta_0)^3\right]_{mean}$$

Assuming $(\theta - \theta_0) = 100$ at all points on the wire then $\frac{r}{r_0} - 1 =$ $5.238 \times 10^{-1} + 7.0 \times 10^{-3} + 6.2 \times 10^{-5}$ and the mean $(\theta - \theta_0)$ would be determined within 1.4 percent by ignoring the higher order dependence of resistivity on temperature even in this extreme case. Thus it may be seen that it is sufficiently accurate to consider that

$$r = r_0 \left[1 + \alpha(\theta_m - \theta_0) \right] \tag{19}$$

provided that r_0 is chosen such that $(\theta_m - \theta_0)$ is never larger than, say, 50 degrees centigrade.

APPENDIX II.

A STUDY OF THE THERMAL INTERACTION BETWEEN A HEATED WIRE AND ITS SUPPORTING STRUCTURE.

A study of the equilibrium of energy flowing in and out of a section of a wire will be made. The method to be followed is somewhat like that of Reference 21.

A. Heat Conducted Along the Wire.

Consider a wire immersed in a gas flowing normal to the wire axis. For a portion of the wire of length Δx, one end of which is at wire length coordinate x, the outflows of heat from the ends of the element are

$$K\omega \frac{d(\theta-\theta_o)}{dx} \quad \text{and} \quad -K\omega \frac{d(\theta-\theta_o)}{dx} - \omega \frac{d(K\frac{d\theta}{dx})}{dx}\Delta x$$

where K = thermal conductivity of the metal

ω = cross sectional area of the wire

θ = temperature of the wire

θ_o = some arbitrary reference temperature

When the expressions above are written, the assumptions that the temperature is constant across any cross section of the wire and the shape and cross-sectional area of the wire are constant throughout its length are included. The argument usually presented in support of the former assumption is that the thermal conductivity of metal is so large compared to the thermal conductivity of gases that no large temperature gradients may be supported in the metal due to the uneven transfer of small amounts of heat to the gas at the metal-gas boundary. The assumption of uniform wire shape and cross-sectional area will be discussed below. The thermal conductivity of the metal may be represented as being dependent on temperature.

$$K = K_0\left[1 + a(\theta-\theta_0) + b(\theta-\theta_0)^2 + \ldots\right] \qquad (20)$$

The net outflow of heat per second from the element by conduction along the wire then is

$$-\omega \frac{d\left(K\frac{d\theta}{dx}\right)}{dx} \Delta x \qquad (21)$$

B. Heat Transferred to the Fluid Stream and Surroundings.

The amount of heat transferred to the fluid stream and surroundings is assumed to be a function of the fluid flow parameters, the shape and cross-sectional area of the wire, the wire and fluid temperatures, the temperature of the surroundings, and the length element.

The heat lost to the stream and surroundings may be written

function (Flow Parameters, Shape, Size, Temperatures) = 0 (22)

Restrictive assumptions as to the form of this functional dependence usually are made to simplify the mathematical treatment of the problem. Among the assumptions are: That the dependence of heat loss on the flow parameters is the same at all points along the length of the wire, the shape and cross sectional area are constant along the length of the wire, the heat loss is linearly dependent on the element length, and the heat loss depends on the elevation of wire temperature above some equilibrium temperature. With the wire immersed in a given flow, the heat transferred in unit time to fluid stream and surroundings might be represented as

$$\left[A(\theta-\theta_r) + B(\theta-\theta_r)^2 + C(\theta-\theta_r)^3 + D(\theta-\theta_r)^4 + \ldots\right] \Delta x \qquad (23)$$

An examination of the assumptions mentioned will be made.

The flow parameters together with the shape of the wire determine the details of the flow about the wire; the laminar or turbulent character of the flow, the presence or absence of wakes, eddies, and shock waves, and the velocity, density, pressure, and temperature distributions in the vicinity of the wire. Anything which altered the local details of the flow would be presumed to invalidate the assumption that the dependence of heat loss on flow parameters was independent of position along the length of the wire. In particular, the assumption would be invalid if the character of the flow were found to depend on the wire temperature. Also, the assumption would be expected to be invalid in those regions of the wire where the presence of the supporting structure would alter the character of the flow through the action of boundary layers or shock waves. Both possible objections to the assumption usually are disposed of by manhandling. Since little is known of the character of the flow about very small wires, either the effect wire temperature might have on the character may be ignored, or the experiment may be conducted so that the variation of temperature along the wire is small. Experiments usually are conducted so that the bulk of the wire is outside the region affected by the wire supporting structure.

Microscopic examination of small wires by optical methods have not revealed much information about the uniformity of the wire diameter or the surface conditions. Electron microscope pictures of 0.00038 cm. diameter tungsten wire are described in Reference 4 as revealing that the surface conditions are "surprisingly good". The assumption of uniform surface conditions and diameter must be based on an argument that the wire size and shape should change very little along a short length of wire cut from a very long continuous wire.

The assumption that the heat loss is linearly dependent on the element length has in effect been challenged in Reference 24 for wires in certain flow conditions. Without this assumption, however, a differential equation, expressing the balance of heat flow in and out of an element of the wire, whose solution expresses the distribution of temperature along the length of the wire, could not be written. The justification for making the assumption lies in the results; the solution of the differential equation may be used to predict certain features of the behavior of hot-wires under varying conditions. That the predictions agree in some measure with experimentally observed results is justification for making the assumption but not proof that the assumption is correct.

The assumption that the heat loss depends on the elevation of the wire temperature above some equilibrium temperature is related to the concept of recovery temperature. If a section of wire carrying no current could be insulated so that no heat could flow out the ends of the section by conduction along the wire, then the section would assume a temperature such that the net outflow of heat from the element would be zero. This temperature is assumed to be a function of the flow parameters and the shape of the wire. That the heat loss depends on the elevation of wire temperature above equilibrium temperature is a well established concept. The definition of Nusselt number emphasizes this idea. (Nusselt number is defined in Expression 3). It has been found by many investigators that the dependence of heat loss on the temperature elevation is not linear, for example see References 4, 12, and 13. Reference 19 develops equations for the temperature distribution along a wire under the assumption that second order dependence of heat loss on temperature elevation must be considered. If fourth order terms are retained, the radiation losses may be included in the equations.

C. Heat Generated by Electrical Means.

The wire is assumed to be heated by an electric current. The resistance of the wire per unit length is assumed to be a function of temperature.

$$\sigma = \sigma_0 \left[1 + \alpha(\theta - \theta_0) + \beta(\theta - \theta_0)^2 + \ldots \right] \tag{24}$$

The Joule heat developed in an element by the passage of a current is

$$i^2 \sigma_0 \left[1 + \alpha(\theta - \theta_0) + \beta(\theta - \theta_0)^2 + \ldots \right] \tag{25}$$

In addition, heat is liberated or absorbed according to the Thomson effect. The heat absorbed per unit time is

$$\int_{\theta}^{\theta + \frac{d\theta}{dx}\Delta x} i \lambda \, d\theta = i \lambda \frac{d\theta}{dx} \Delta x \tag{26}$$

λ = Thomson coefficient, positive when heat is absorbed as current flows in the direction of decreasing temperature

$$\lambda = \lambda_0 \left[1 + \nu(\theta-\theta_0)^2 + \xi(\theta-\theta_0)^2 + \ldots.. \right] \tag{27}$$

D. The Heat Balance Equation.

The steady state net heat outflow from an element of the wire must be zero. This is expressed by the differential equation

$$\begin{aligned} & i^2\sigma_0 \left[1 + \alpha(\theta-\theta_0) + \beta(\theta-\theta_0)^2 + \ldots \right] \\ & + i^2\lambda_0 \left[1 + \nu(\theta-\theta_0) + \xi(\theta-\theta_0)^2 + \ldots \right] \frac{d\theta}{dx} \\ & - \left[A(\theta-\theta_r) + B(\theta-\theta_r) + \ldots \right] \\ & + \omega K_0 \left[1 + a(\theta-\theta_0) + b(\theta-\theta_0)^2 + \ldots \right] \frac{d^2\theta}{dx^2} \\ & + \omega K_0 \left[a + 2b(\theta-\theta_0) + \ldots \right] \left(\frac{d\theta}{dx} \right)^2 = 0 \end{aligned} \tag{28}$$

E. A Simplified Linear Heat Balance Equation.

The differential equation developed above is quite complex; however, it can be shown that simpler forms of the equation may be usefully applied in an experimental investigation of a finite length hot wire.

Assume then that the resistivity of the metal is a linear function of temperature, the Thomson effect is negligible, the heat lost to the stream is a linear function of temperature, and that the thermal conductivity of the metal is not a function of temperature. A simplified linear differential equation then may be written

$$i^2\sigma_0 \left[1 + \alpha(\theta-\theta_0) \right] - A\left[(\theta-\theta_0) - (\theta_r-\theta_0) \right] + K\omega \frac{d^2\theta}{dx^2} = 0 \tag{29}$$

The solution of this equation with the boundary condition $\theta = \theta_1$ at $x = \pm \ell$ is

$$\theta - \theta_0 = \frac{Q}{P^2} - \frac{\cosh Px}{\cosh P\ell}\left[\frac{Q}{P^2} - (\theta_1 - \theta_0)\right] \tag{30}$$

where

$$P^2 = \frac{A - i^2\sigma_0\alpha}{K\omega} \tag{31}$$

$$Q = \frac{i^2\sigma_0 + A(\theta_r - \theta_0)}{K\omega} \tag{32}$$

and θ_1 is the temperature of the wire supports. The solution may be related to the resistance of the wire, a quantity which may be measured in an experiment, by the expression

$$r = \sigma_0 \int_{-\ell}^{\ell} \left[1 + \alpha(\theta - \theta_0)\right] dx = r_0 + \alpha r_0 \left\{\frac{Q}{P^2} - \frac{\tanh P\ell}{P\ell}\left[\frac{Q}{P^2} - (\theta_1 - \theta_0)\right]\right\} \tag{33}$$

The portion of Expression (33) enclosed between braces may be recognized according to Expression (12) as the mean value of $(\theta - \theta_0)$;

$$(\theta_m - \theta_0) = \frac{Q}{P^2} - \frac{\tanh P\ell}{P\ell}\left[\frac{Q}{P^2} - (\theta_1 - \theta_0)\right] \tag{34}$$

Subject to the assumptions on which the linearized theory is based, the temperature of the wire is constant along the wire and equal to the end temperature if the end and wire mean temperatures are equal, for from Expression (34), if $(\theta_1 - \theta_0) = (\theta_m - \theta_0)$ then

$$(\theta_1 - \theta_0) = (\theta_m - \theta_0) = \frac{Q}{P^2} \tag{35}$$

and from (30)

$$\theta - \theta_0 = \frac{Q}{P^2} \tag{36}$$

for $-\ell \leq x \leq \ell$. Furthermore, the wire temperature obtained under these conditions is equal to that which would be attained by an element of the wire thermally insulated from the other elements for if in the linear differential equation (29) it is assumed that the section of wire under consideration be thermally insulated from the rest of the wire, then

$$i^2\sigma_o\left[1+\alpha(\theta-\theta_o)\right]-A\left[(\theta-\theta_o)-(\theta_r-\theta_o)\right]=0 \qquad (37)$$

and

$$(\theta-\theta_o)=\frac{i^2\sigma_o+A(\theta_r-\theta_o)}{A-i^2\sigma_o\alpha}=\frac{Q}{P^2} \qquad (38)$$

The value of θ defined by (38) will be designated θ_e .

It is to be concluded, then, that subject to the assumptions of the linear theory, no heat flows from the wire to the supports if the mean temperature and the support temperature are equal. An identical conclusion may be reached without including all the restrictive assumptions of the linear theory.

In order to illustrate that the linear differential equation describes fairly accurately the balance of heat in a finite length hot wire, the linear theory will be used to calculate some effects which may also be determined by experiment.

It is important to know how the wire mean temperature varies when end temperature is varied if the wire is to be used as a temperature measuring device. The partial derivative of wire mean temperature with end temperature may be obtained directly from Expression (34)

$$\frac{\partial(\theta_m - \theta_o)}{\partial(\theta_1 - \theta_o)} = \frac{\tanh P\ell}{P\ell} \tag{39}$$

It may be seen from the definition of Nusselt number, (3), and the discussion leading to the linear equatinn, (29), that

$$Nu = \frac{A}{\pi k} \tag{40}$$

If the heating current is adjusted so that $i^2\sigma_o\alpha$ is small in comparison to A, then it may be seen from Expression (31) that

$$P\ell \doteq AR\sqrt{Nu\frac{k}{K}} \tag{41}$$

If, in addition, $P\ell \geq 3.0$ then $\tanh P\ell \doteq 1.0$ and

$$\frac{\partial(\theta_m - \theta_o)}{\partial(\theta_1 - \theta_o)} \doteq \frac{1}{AR\sqrt{Nu\frac{k}{K}}} \tag{42}$$

In Expressions (35) to (38) it was shown that $(\theta_m - \theta_o)$ and $(\theta_1 - \theta_o)$ would be equal to each other at some value $(\theta_e - \theta_o)$.

Therefore

$$(\theta_m - \theta_e) = (\theta_1 - \theta_e)\frac{\tanh P\ell}{P\ell} \tag{43}$$

From Expression (38) $\theta_e = \theta_r$ if $i = 0$; hence

$$(\theta_m - \theta_r) = (\theta_1 - \theta_r)\frac{\tanh P\ell}{P\ell} \tag{44}$$

applies provided $i = 0$.

The validity of Expression (44) has been tested by plotting the predicted values of $(\theta_m - \theta_r)$ against the experimental values as shown in fig. 19. It may be noted that there is fair agreement of the theoretical values with the experimental values but that there is considerable scatter. The unequal scattering of points on either side of the line of true agreement indicates that the theory tends to overestimate somewhat. Many of the points (the equilateral triangles with apices upward) on the fringes of the scatter pattern were obtained from tests of a 0.00127 cm. diameter wire of aspect ratio 910 in supersonic flow. It is not known why the results of the tests made with this wire were more erratic than the results of the tests made with other wires.

The linear theory may also be used to predict what part of the power supplied is lost to the supports by conduction from the ends of the wire. Expression (30) may be differentiated to determine the temperature gradients at the ends of the wire.

$$\frac{d(\theta-\theta_0)}{dx} = -P\,\frac{\sinh Px}{\cosh P\ell}\left[\frac{Q}{P^2}-(\theta_1-\theta_0)\right] \tag{45}$$

At $x = \pm\ell$

$$\frac{d(\theta-\theta_0)}{dx} = \mp\, P\tanh P\ell\left[\frac{Q}{P^2}-(\theta_1-\theta_0)\right] \tag{46}$$

The power loss from the two ends of the wire is therefore

$$\text{End loss} = 2K\omega\, P\tanh P\ell\left[\frac{Q}{P^2}-(\theta-\theta_0)\right] \tag{47}$$

In an experiment, however, an easily measured quantity is the partial derivative of power supplied with respect to wire mean temperature, $\frac{\partial\,(i^2r)}{\partial\,(\theta_m-\theta_0)}$. (See fig. 8.) Inasmuch as the Nusselt number is related to the quantity A by Expression (40) and to $(i^2r - \text{End loss})$ by (3), then

$$\frac{\partial\,(i^2 r)}{\partial\,(\theta_m - \theta_o)} = 2A\ell + \frac{\partial\,(\text{End loss})}{\partial\,(\theta_m - \theta_o)} \tag{48}$$

and it would be convenient to know the variation of end loss with wire mean temperature. From Expressions (34) and (47)

$$\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)} =$$

$$2K\omega P^2 \ell \frac{\left(\frac{P^2+\alpha Q}{P^2}\right)\left(\frac{\tanh P\ell}{P\ell}\right) - \frac{\alpha}{2}\left[\frac{Q}{P^2} - (\theta_1 - \theta_o)\right]\left[\operatorname{sech}^2 P\ell + \frac{\tanh P\ell}{P\ell}\right]}{\left(\frac{P^2+\alpha Q}{P^2}\right)\left(1 - \frac{\tanh P\ell}{P\ell}\right) + \frac{\alpha}{2}\left[\frac{Q}{P^2} - (\theta_1 - \theta_o)\right]\left[\operatorname{sech}^2 P\ell - \frac{\tanh P\ell}{P\ell}\right]} \tag{49}$$

Under the special conditions that $i = 0$ and the reference temperature is chosen equal to recovery temperature $(\theta_r = \theta_o)$ the expression simplifies to

$$\frac{\partial\,(\text{End loss})}{\partial\,(\theta_m - \theta_o)} =$$

$$2A\ell \frac{\left\{\frac{\tanh P\ell}{P\ell} + \frac{\alpha}{2}(\theta_1 - \theta_r)\left[\operatorname{sech}^2 P\ell + \frac{\tanh P\ell}{P\ell}\right]\right\}}{1 - \frac{\tanh P\ell}{P\ell} - \frac{\alpha}{2}(\theta_1 - \theta_r)\left[\operatorname{sech}^2 P\ell - \frac{\tanh P\ell}{P\ell}\right]} \tag{49a}$$

This prediction of the linear theory has also been checked against values obtained from the experiment; the results are shown in fig. 20.

Inspection of fig. 20 reveals that the points are grouped on the plot. The horizontal character of the grouping of data points taken with one diameter and aspect ratio indicates that the experimental values of $\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)}$ tend to vary more widely than the values predicted by the linear theory. This is probably due to experimental error. Plots, of which fig. 8 is representative, were used to determine the experimental values of $\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)}$. The procedure was to measure the difference between the slope of the no-end-loss line and the slope of a curve faired through the data points; hence, the probable error in the experimental value of $\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)}$ is quite large. In an effort to reduce scatter, the slopes of the lines faired through the data points of a series of plots similar to fig. 8 were determined by the mathematical procedure of finding that line for which the sum of the squares of the distances of the data points from the line was a minimum. The scatter of the experimental values of $\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)}$ was about the same whether the data was faired by eye or faired statistically.

The extent to which the linear theory is able to correctly predict the behavior of hot wires may be further examined by splitting $\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)}$ into two parts.

The partial derivatives of Expressions (34) and (47) with respect to current squared may be divided to give Expression (49)

$$\frac{\partial(\text{End loss})}{\partial(\theta_m - \theta_o)} = \frac{\dfrac{\partial(\text{End loss})}{\partial(i^2)}}{\dfrac{\partial(\theta_m - \theta_o)}{\partial(i^2)}}$$

But $\frac{\partial(\text{End loss})}{\partial(i^2)}$ and $\frac{\partial(\theta_m - \theta_o)}{\partial(i^2)}$ may be easily determined from the experimental data, $\frac{\partial(\theta_m - \theta_o)}{\partial(i^2)}$ directly and $\frac{\partial(\text{End loss})}{\partial(i^2)}$ through Expression (50) obtained from (48) and (19).

$$\frac{\partial(\text{End loss})}{\partial(i^2)} = r - 2\ell\left[A - i^2\sigma_0\alpha\right]\frac{\partial(\theta_m - \theta_0)}{\partial(i^2)} \tag{50}$$

The predictions of the simple theory for these two quantities are

$$\frac{\partial(\theta_m - \theta_0)}{\partial(i^2)} = \frac{\sigma_0}{K\omega P^2} \times$$

$$\left\{\left(\frac{P^2+\alpha Q}{P^2}\right)\left(1 - \frac{\tanh P\ell}{P\ell}\right) + \frac{\alpha}{2}\left[\frac{Q}{P^2} - (\theta_1 - \theta_0)\right]\left[\operatorname{sech}^2 P\ell + \frac{\tanh P\ell}{P\ell}\right]\right\} \tag{51}$$

$$\frac{\partial(\text{End loss})}{\partial(i^2)} = 2\sigma_0\ell \times$$

$$\left\{\left(\frac{\tanh P\ell}{P\ell}\right)\left(\frac{P^2+\alpha Q}{P^2}\right) - \frac{\alpha}{2}\left[\frac{Q}{P^2} - (\theta_1 - \theta_0)\right]\left[\operatorname{sech}^2 P\ell + \frac{\tanh P\ell}{P\ell}\right]\right\} \tag{52}$$

In the case that $i = 0$ and θ_0 is set equal to θ_r the expressions are simplified to

$$\left.\frac{\partial(\theta_m - \theta_0)}{\partial(i^2)}\right|_{i=0} =$$

$$\frac{\sigma_r}{A}\left\{1 - \frac{\tanh P\ell}{P\ell} + \frac{\alpha}{2}(\theta_1 - \theta_r)\left(-\operatorname{sech}^2 P\ell + \frac{\tanh P\ell}{P\ell}\right)\right\} \tag{51a}$$

$$\left.\frac{\partial(\text{End loss})}{\partial(i^2)}\right|_{i=0} = \tag{52a}$$

$$2\sigma_r\ell\left\{\frac{\tanh P\ell}{P\ell} + \frac{\alpha}{2}(\theta_1 - \theta_r)\left(\operatorname{sech}^2 P\ell + \frac{\tanh P\ell}{P\ell}\right)\right\}$$

These two expressions have been used to calculate the theoretical values which are plotted for comparison with the experimental values in figs. 21 and 22. The values of $\frac{\partial(\theta_m - \theta_o)}{\partial(i^2)}$ calculated using (51a) agree very well with the experimental values. The agreement between theoretical and experimental values of $\frac{\partial(\text{End loss})}{\partial(i^2)}$ is fair, but the plotted points of fig. 22 are quite scattered; again, probably due to the inaccuracy of determining end losses from the experimental data.

F. Other Forms of the Heat Balance Equation.

In an attempt to find a theory which would give better agreement with experiment than the simple linear theory two other forms of the heat balance equation were considered.

In the discussion of assumptions under part B of this section it was noted that several investigators had concluded that the heat lost to the stream was dependent on the second power of the temperature elevation as well as the first power. Assume then, that the resistivity of the metal is a linear function of temperature, the Thomson effect is negligible, the heat lost to the stream is a quadratic function of temperature elevation, and the thermal conductivity of the metal is not a function of temperature. Expression (28) then is reduced to

$$K\omega \frac{d^2\theta}{dx^2} + (\theta - \theta_o)\left[i^2\sigma_o\alpha - A + 2B(\theta_r - \theta_o) - B(\theta - \theta_o)\right] + i^2\sigma_o + A(\theta_r - \theta_o) - B(\theta_r - \theta_o)^2 = 0 \qquad (53)$$

A first integral of this expression was obtained and suitable boundary conditions introduced so that expression (54) could be obtained.

$$\frac{\partial(\text{End loss})}{\partial(i^2)}\Bigg|_{i=0} = 2\sigma_r \frac{1-\frac{\alpha}{2}(\theta_r-\theta_1)}{\frac{1}{K\omega}\left[A-\frac{2}{3}B(\theta_r-\theta_1)\right]} \qquad (54)$$

The value of B which may be deduced from the experiment of Kovasznay and Tormarck[4] is

$$B = -\frac{0.18A}{\theta_r} \qquad (55)$$

where θ_r is in degrees Kelvin.

Calculations of the values of $\frac{\partial(\text{End loss})}{\partial(i^2)}\Big|_{i=0}$ using Expressions (54) and (55) were found to differ from those calculated using expression (52a) by less than two percent in every case. The small difference in the results obtained by the two methods does not indicate that the more elaborate method is not useful. The small difference might be due to the fact that the experiments were deliberately arranged to eliminate the dependence of heat transfer on the second power of the temperature difference as an important factor in the experiments. The mean wire temperature was limited to vary over temperature ranges of several hundred degrees and hence the dependence of heat transfer on the temperature difference was found to be an important factor.

A study was also made of a simplified heat balance equation in which account was taken of the variation of thermal conductivity of the metal with temperature.

The equation may be treated by the same method as was Expression (53). Using the equation

$$\omega K_o \left[1 + a(\theta - \theta_o) \frac{d^2\theta}{dx^2}\right] + (\theta - \theta_o) \left[i^2 \sigma_o \alpha - A\right]$$

$$+ i^2 \sigma_o + A(\theta_r - \theta_o) + \omega a K \left(\frac{d\theta}{dx}\right)^2 = 0 \tag{56}$$

it is possible to obtain

$$\left.\frac{\partial\,(\text{End loss})}{\partial\,(i^2)}\right|_{i=0} = \frac{2\sigma_r}{\sqrt{\frac{A}{K\omega}}} \frac{\left[1 - \frac{\alpha}{2}(\theta_r - \theta_1) - \frac{a}{2}(\theta_r - \theta_1) + \frac{2}{3}(\theta_r - \theta_1)^2\right]}{\sqrt{1 - \frac{2}{3}\, a(\theta_r - \theta_1)}} \tag{57}$$

This expression may be compared to Expressions (52a) and (54) obtained from the two other simplified forms of the heat balance equation.

For the purposes of calculation, a value of - 1.0 x 10^{-4} per degree centigrade was assumed for the coefficient "a". This value is listed in Reference 20 and is designated as applicable for a range of temperature from 0 to 100 degrees centigrade.

Calculations of $\frac{\partial(\text{End loss})}{\partial(i^2)}$ using Expression (57) were made for comparison to the experimentally determined values of this quantity. In each case the result obtained using Expression (57) differed from the result obtained using Expression (52a) by less than one percent.

G. Discussion of End Corrections.

Part A of the text above has been written in an attempt to demonstrate that an acceptable method of making end corrections was vital to the success of the particular experiments which are described in this paper. It was shown that both the apparent Nusselt number and the apparent recovery temperature were considerably influenced by end effects. In other experiments there has not been much emphasis placed on the fact that the apparent recovery temperature could be influenced by end effects. The reason for this lack of emphasis is probably that the experiments in supersonic flow are interpreted in the same light as those in subsonic flow. In a typical hot wire experiment in subsonic flow both the wire recovery temperature and the wire support temperatures would be found to be equal to the stagnation temperature of the flow, hence there would be no reason to question whether or not the measured recovery temperature was correct. In supersonic flow it is not necessarily true that either the wire recovery temperature or the wire support temperatures be equal to the stagnation temperature. From examination of Expressions (31), (32), (33), and (40) above it may be seen that the Nusselt number cannot be accurately determined from measurable data (i , r , ℓ , d) unless both the support temperature (θ_l) and recovery temperature (θ_r) are known; that is to say, the Expressions represent relationships between three unknowns. With a little care the value of the support temperature could be measured or estimated quite accurately and, if any prior knowledge of Nusselt number were available, say from other experiments, the three expressions could be used to determine fairly accurately the true wire recovery temperature.

To illustrate the procedure outlined, sample calculations showing the magnitude of the end corrections to be applied to recovery temperature measurements are made for two typical cases.

Assume a hot-wire with 0.00076 cm. diameter and aspect ratio 500 is operated in an airflow of Mach number $\sqrt{3}$ and 3 which are issuing from a reservoir with a pressure of one atmosphere and a temperature of 20° centigrade. Assume that the supports for the hot wire are conical-pointed needles and that the needles are enclosed by laminar boundary layers. Also, assume the following properties for air:

$$\gamma = 1.40$$

$$R = 2.87 \times 10^{6} \ \frac{\text{erg}}{\text{gram}\ ^{\circ}\text{C}}$$

and that the wire is of tungsten with thermal conductivity of

$$1.60 \ \frac{\text{watts}}{\text{cm}^{\circ}\ \text{c}} .$$

The results of the calculations are listed below.

		Reservoir	Test Section	
Mach number	(M)	0	1.73	3.00
Pressure, $\frac{dyne}{cm^2}$	(p)	1.013×10^6	1.96×10^5	2.75×10^4
Temperature ^{o}Kelvin	(T)	293	183	105
Density, $\frac{gram}{cm^3}$	(ρ)	1.21×10^{-3}	3.73×10^{-4}	9.24×10^{-5}
Flow velocity, $\frac{cm}{sec}$	(V)		4.70×10^4	6.15×10^4
Viscosity, $\frac{gram}{cm\ sec}$	(μ)	2.25×10^{-4}	1.21×10^{-4}	7.21×10^{-5}
Thermal conductivity $\frac{watt}{cm^oC}$	(k)	2.47×10^{-4}	1.66×10^{-4}	9.70×10^{-5}
Reynolds number	(Re)		110	60
Nusselt number estimated from Kovasznay's work	(Nu)		5.25	4.42
Pℓ			11.65	8.20
Recovery temperature of supports, ^{o} Kelvin	(θ_1)		276	264
Apparent recovery temperature (assumed), ^{o} Kelvin	(θ_m)		293	293
True recovery temperature minus apparent recovery temperature, ^{o} C	($\theta_r - \theta_m$)		1.6	4.0

The pressure and temperature in the reservoir were assumed. The density was calculated from (5). Pressure and temperature in the test section were calculated for the two assumed Mach numbers from expressions (1) and (4) and density in the test section again calculated from (5). The velocity of flow was calculated from (6) and (7) and tabulated values of viscosity were used so that Reynolds numbers could be calculated from (8). Having determined the Reynolds number, the Nusselt number was estimated using the findings of Kovasznay and Tormarck[4] as a basis. The quantity was estimated from expression (41) assuming that the heating current for the wire was zero. Assume that the conical-needle wire supports attain a temperature such that

$$\frac{\theta_1 - T}{T_0 - T} = \sqrt{P_r} \tag{58}$$

an expression verified by the experiments of Eber[22]. For the sake of the calculations assume that the apparent recovery temperature θ_m was equal to the tunnel stagnation temperature.

From expression (44) it may be seen that

$$(\theta_r - \theta_m) \doteq \frac{\theta_m - \theta_1}{P\ell - 1} \tag{59}$$

Under the influence of all the assumptions made the value of $(\theta_r - \theta_m)$ has been calculated from expression (59).

The values found do not seem large in comparison to such temperature differences as $(T_0 - T)$; however, the magnitude could be important in the case that an accurate determination of the recovery temperature of wires was a principal object of an experimental investigation. A simple experimental technique such as was used by Kovasznay and Tormarck (the ends of the wire were heavily copper plated leaving only a small unplated section of the wire in the middle) could be used to eliminate most of the error calculated above.

The conclusions to be drawn from the studies of end losses are as follows:

a. In the experiment conducted, end losses were of such importance that neither Nusselt numbers nor recovery temperatures could be determined without special precautions.

b. A simple linear theory for the steady-state heat balance of an electrically heated wire is sufficient to predict some of the manifestations of end losses, such as the variation in apparent recovery temperature with variation in support temperature.

c. Under the circumstances of the experiment, a main feature of which was that the wire temperature was varied only over a relatively small range of about 50 degrees centigrade, there was nothing to be gained by using either of two more complex end correction procedures which were investigated.

d. End corrections to the recovery temperature could be important in a critical investigation of recovery temperature of fine wires in supersonic flow.

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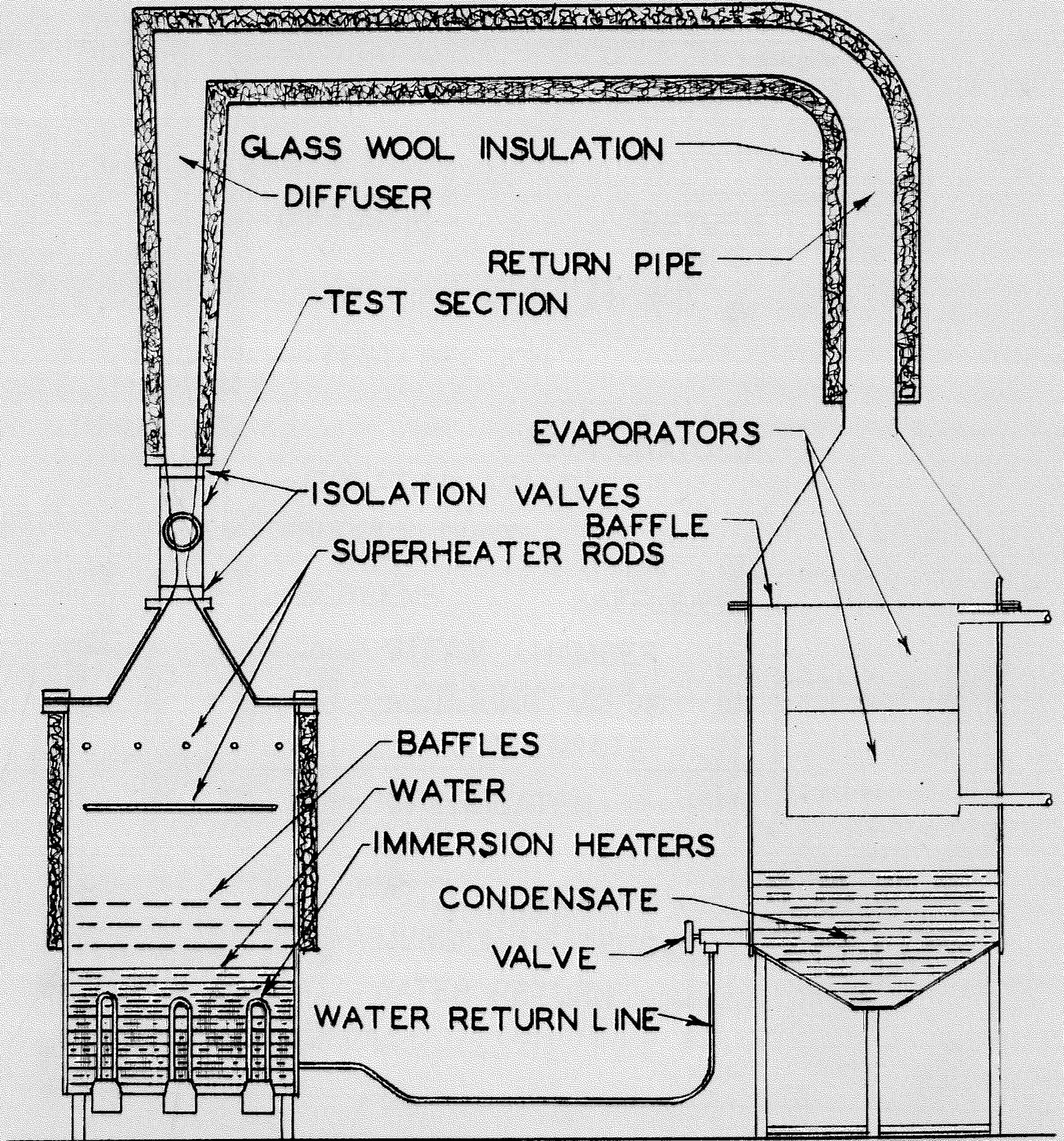
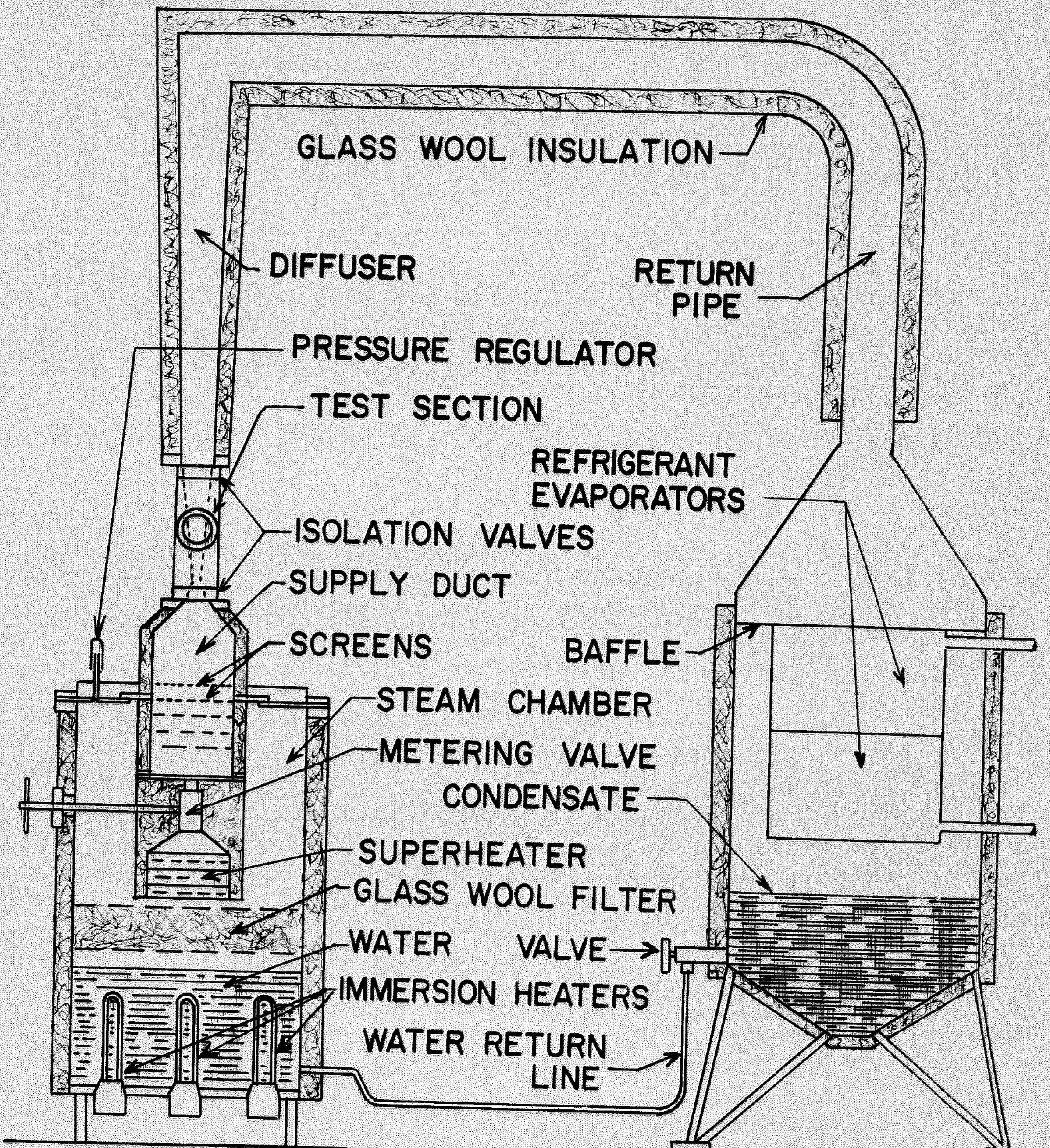

GLASS WOOL INSULATION
DIFFUSER
RETURN PIPE
TEST SECTION
EVAPORATORS
ISOLATION VALVES
BAFFLE
SUPERHEATER RODS
BAFFLES
WATER
IMMERSION HEATERS
CONDENSATE
VALVE
WATER RETURN LINE

FIGURE 1



SKETCH OF STEAM TUNNEL

1/16 FULL SIZE

FIG. 2

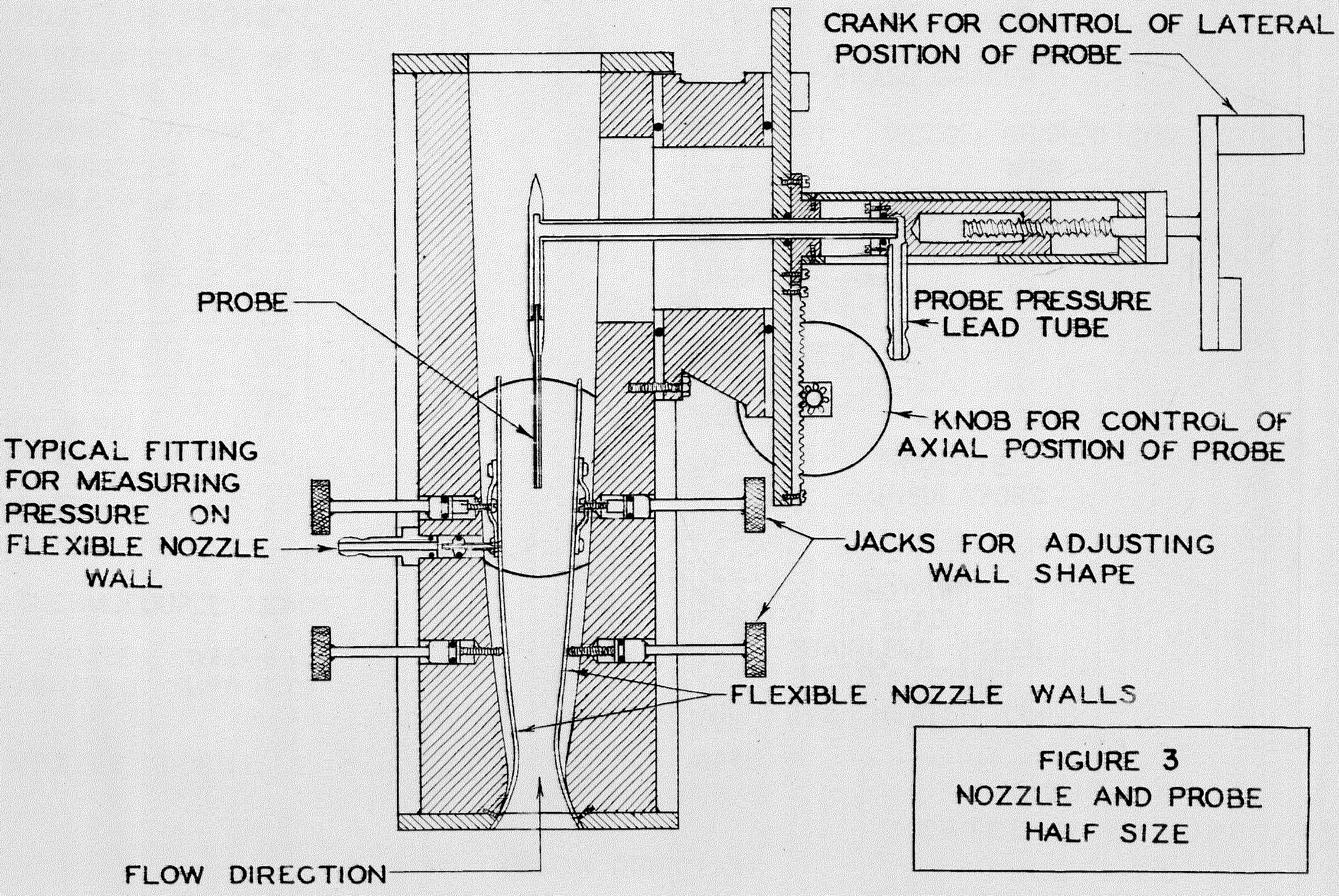


FIGURE 3
NOZZLE AND PROBE
HALF SIZE

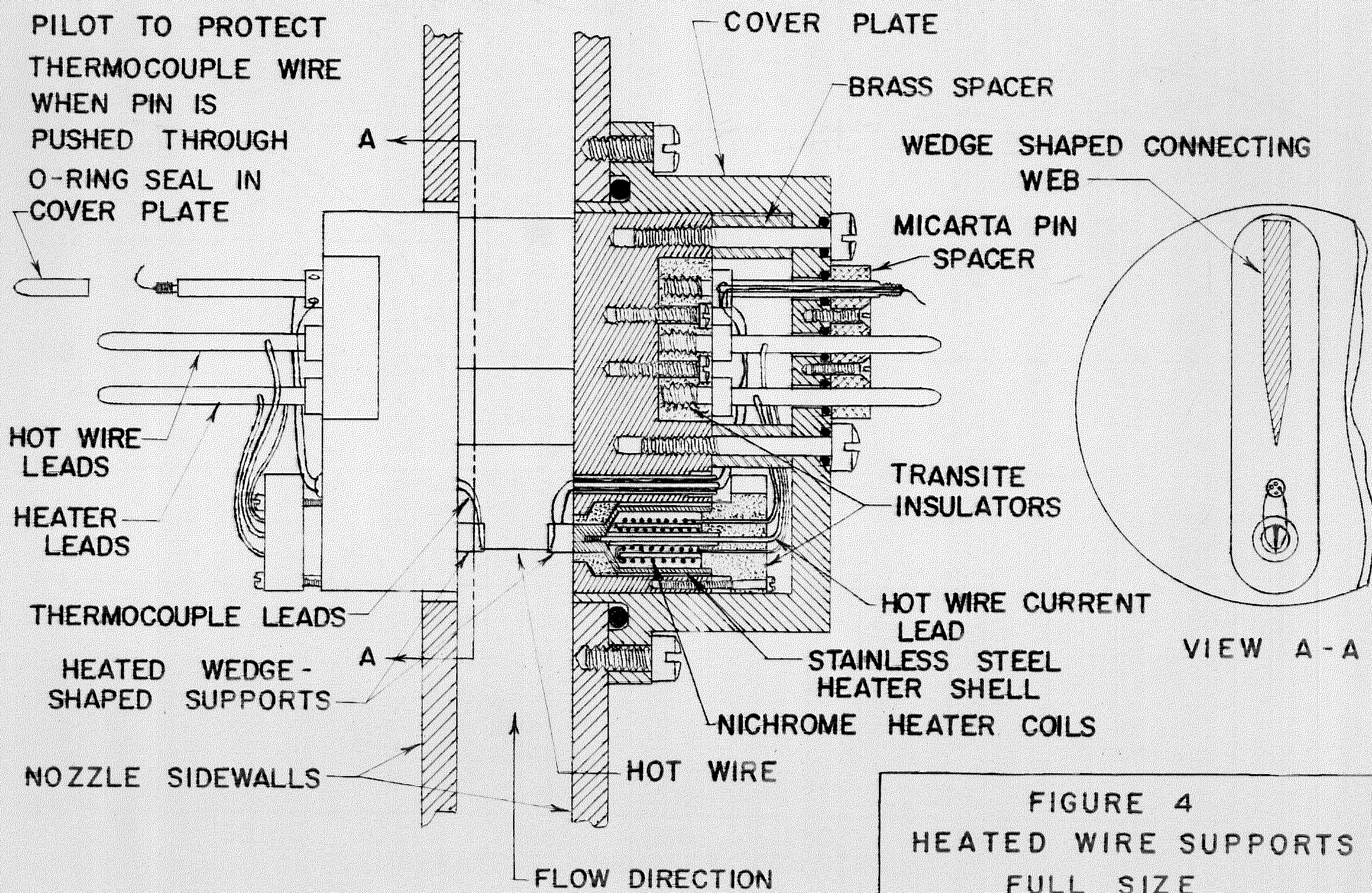
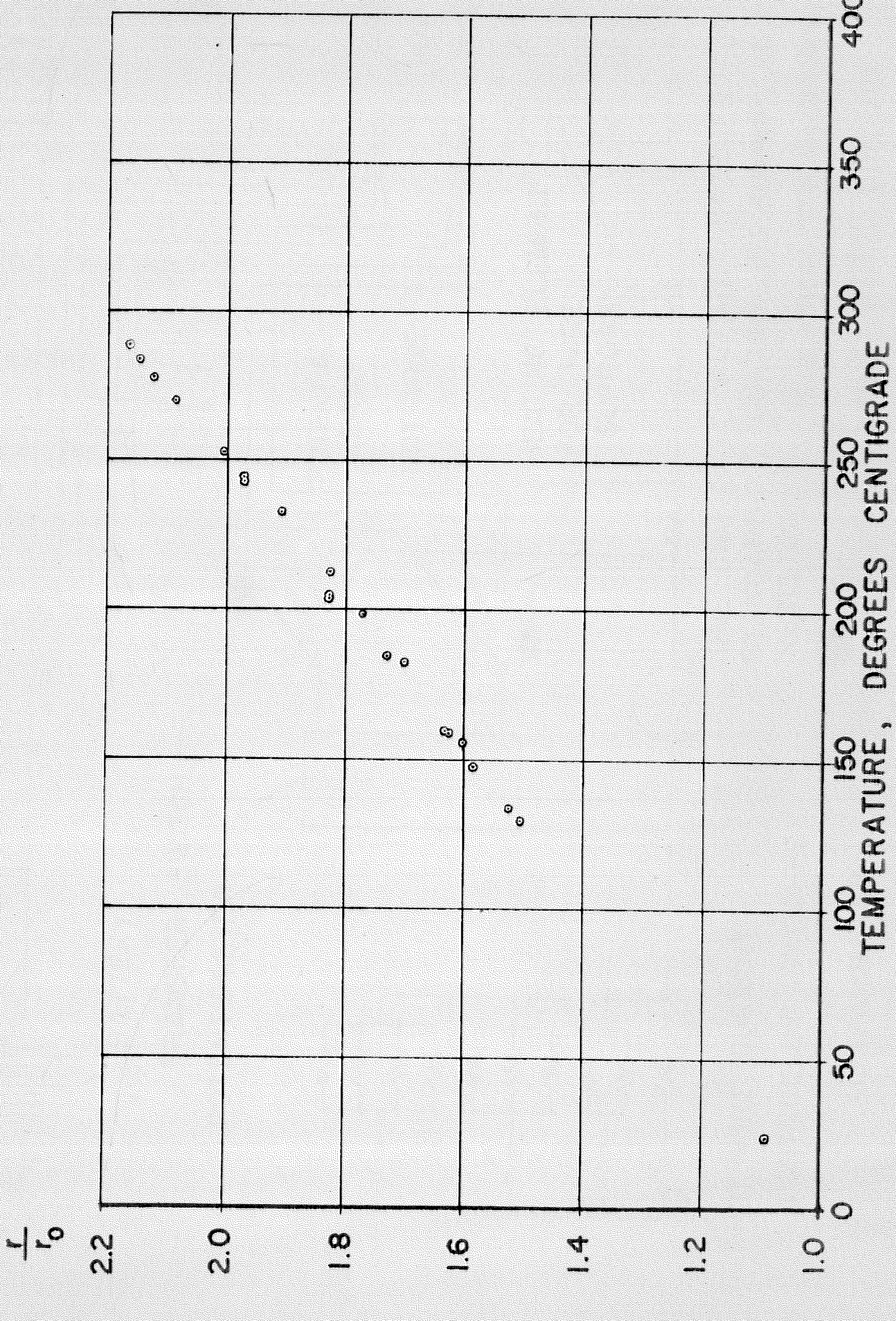
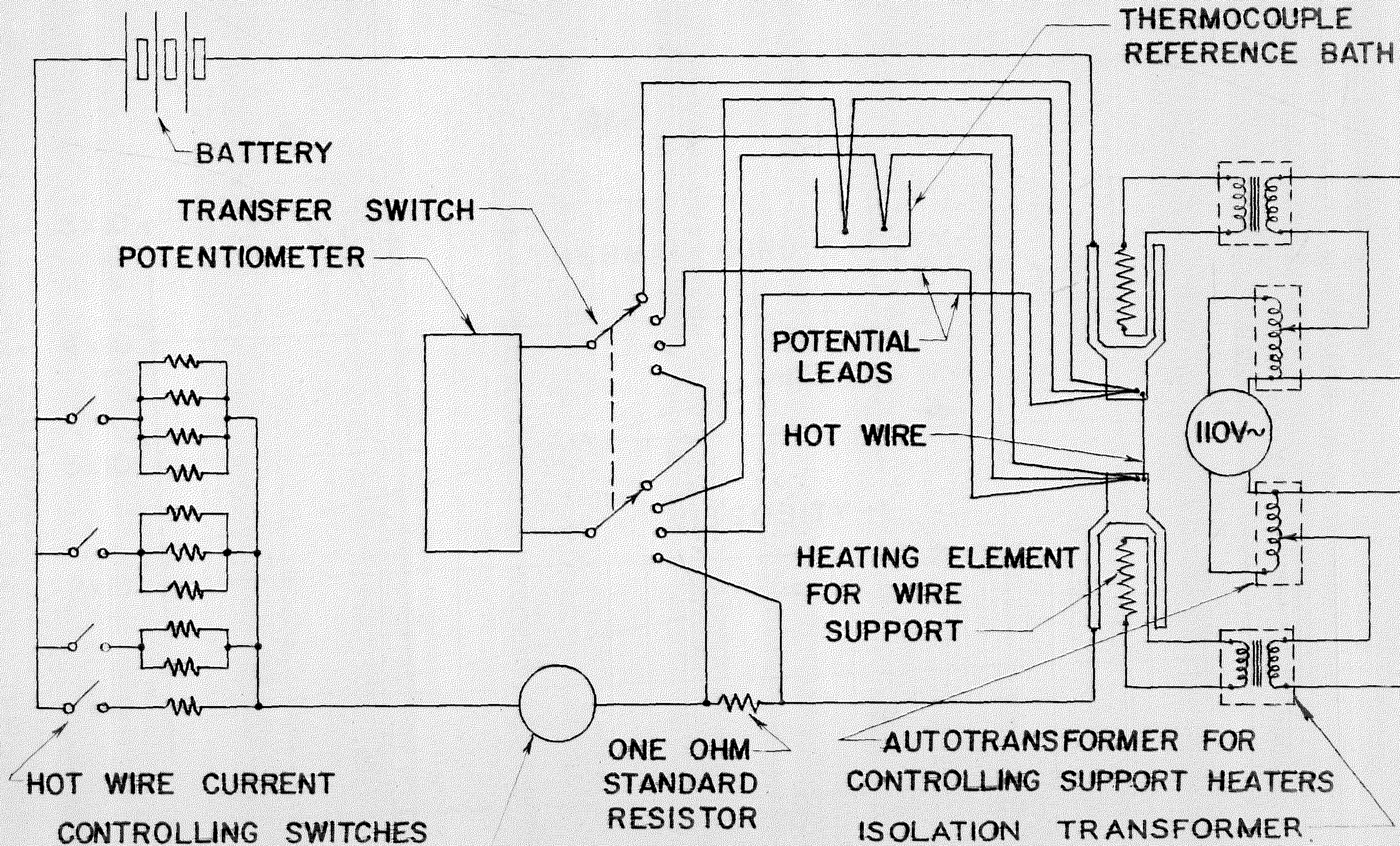


FIGURE 4
HEATED WIRE SUPPORTS
FULL SIZE

FIGURE 5


r/r₀
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
50
100
150
200
250
300
350
400
TEMPERATURE, DEGREES CENTIGRADE



HOT WIRE CIRCUIT

FIGURE 6

FIGURE 7

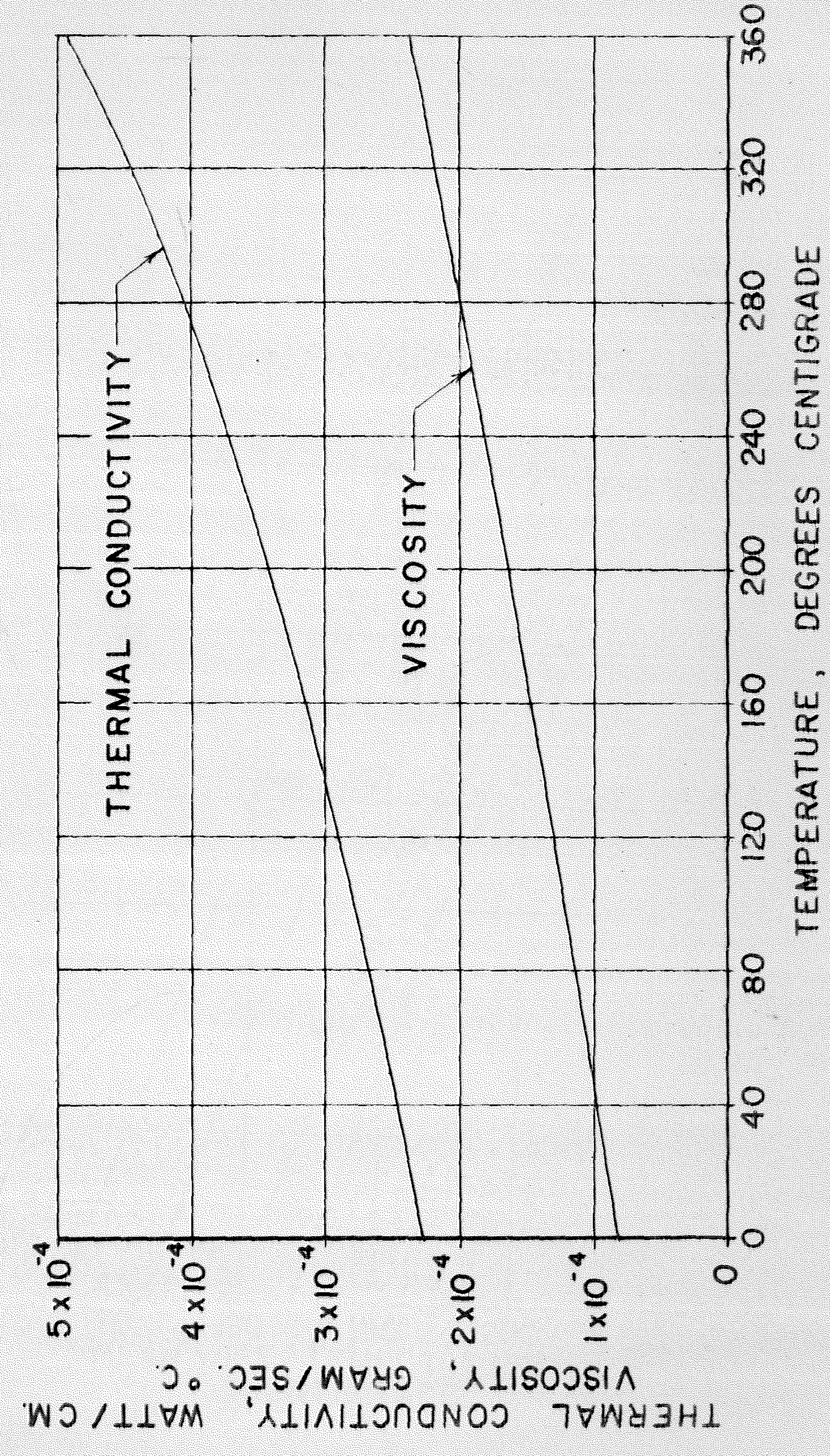
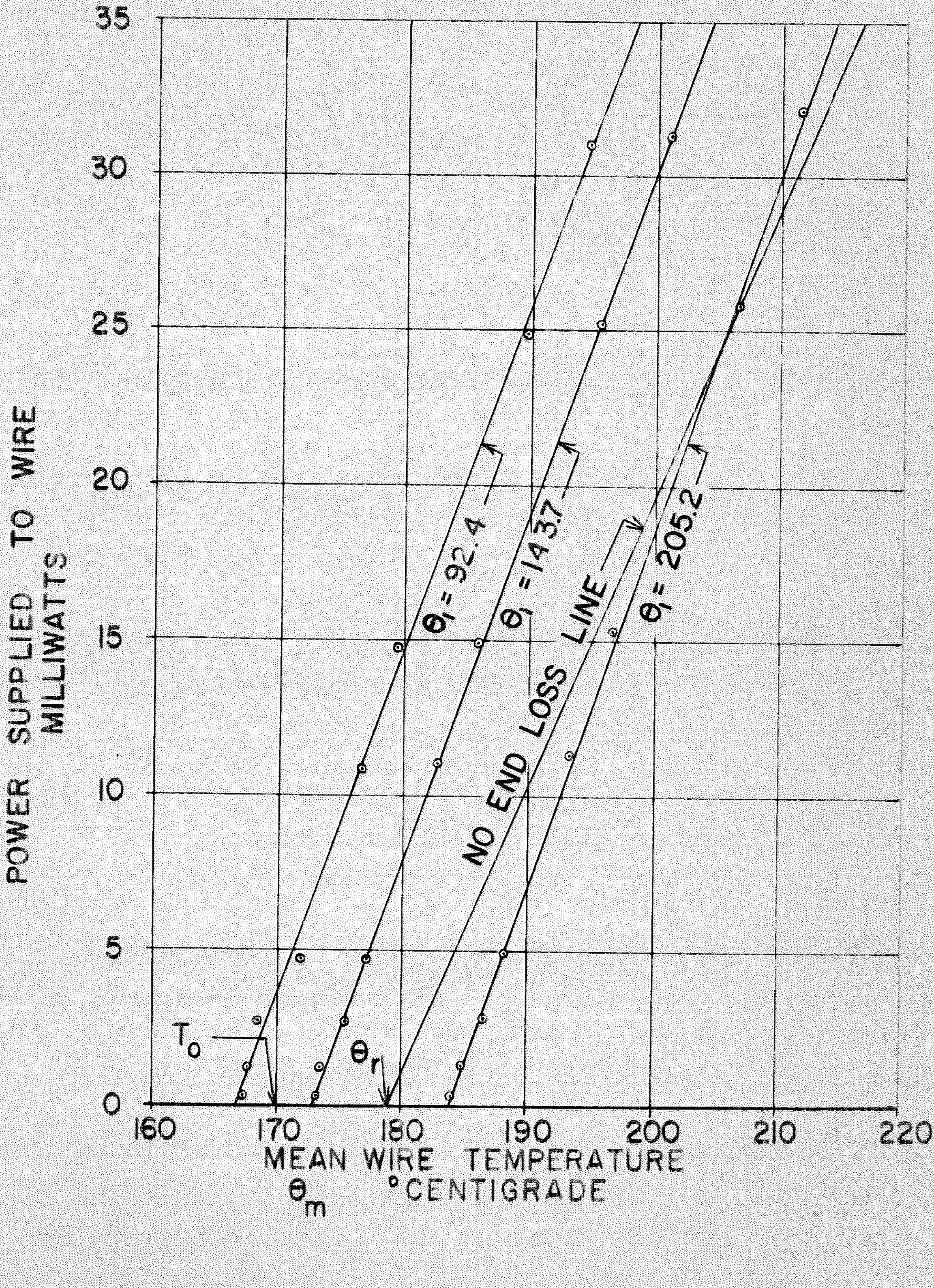

THERMAL CONDUCTIVITY
VISCOSITY
TEMPERATURE, DEGREES CENTIGRADE
0
40
80
120
160
200
240
280
320
360
THERMAL CONDUCTIVITY, WATT/CM. °C.
VISCOSITY, GRAM/SEC. °C.
5x10^-4
4x10^-4
3x10^-4
2x10^-4
1x10^-4
0

FIGURE 8



35
30
25
20
15
10
5
0
160
170
180
190
200
210
220
POWER SUPPLIED TO WIRE
MILLIWATTS
MEAN WIRE TEMPERATURE
θm °CENTIGRADE
θ1 = 92.4
θ1 = 143.7
NO END LOSS LINE
θ1 = 205.2
To
θr

FIGURE 9

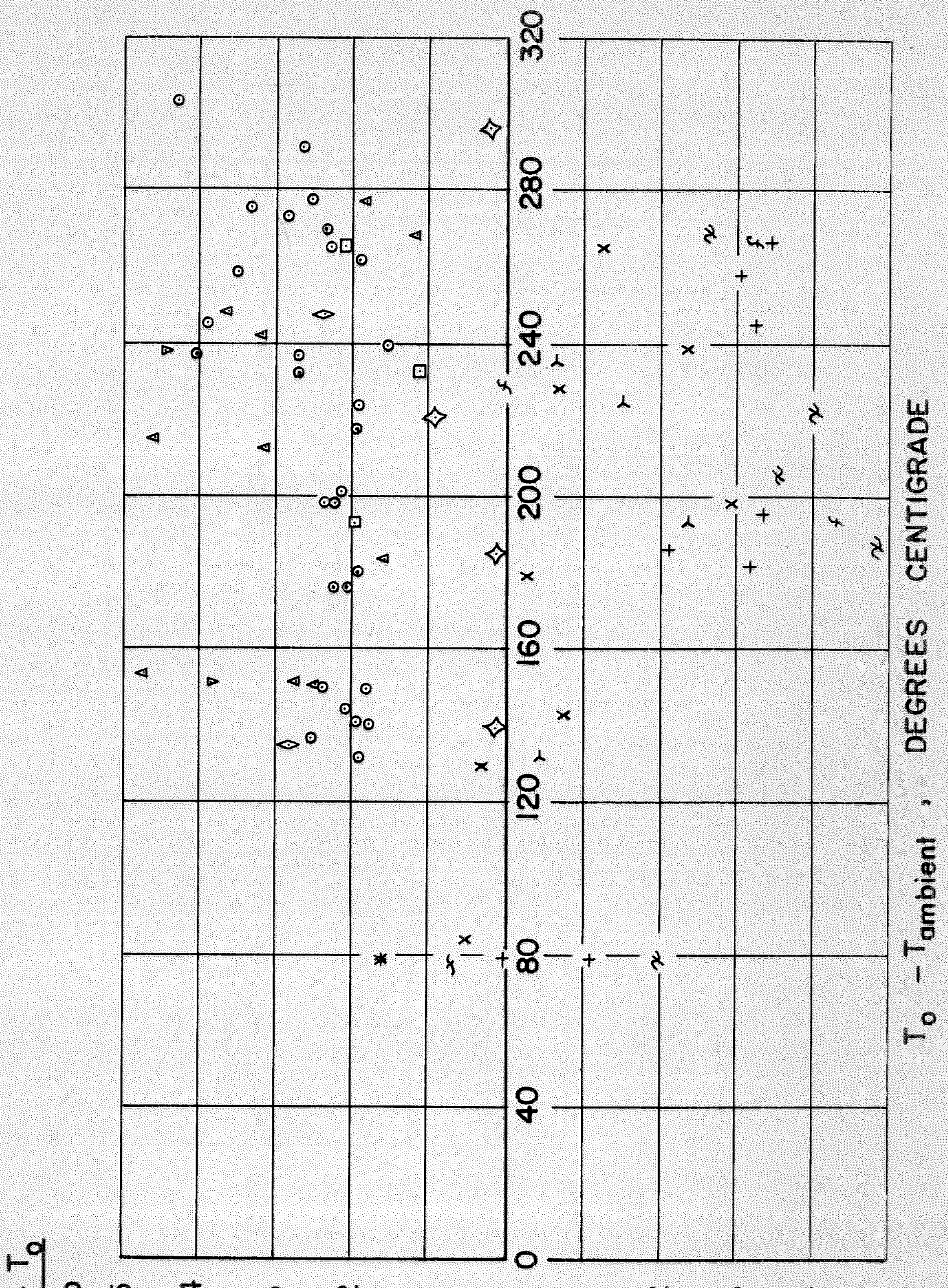
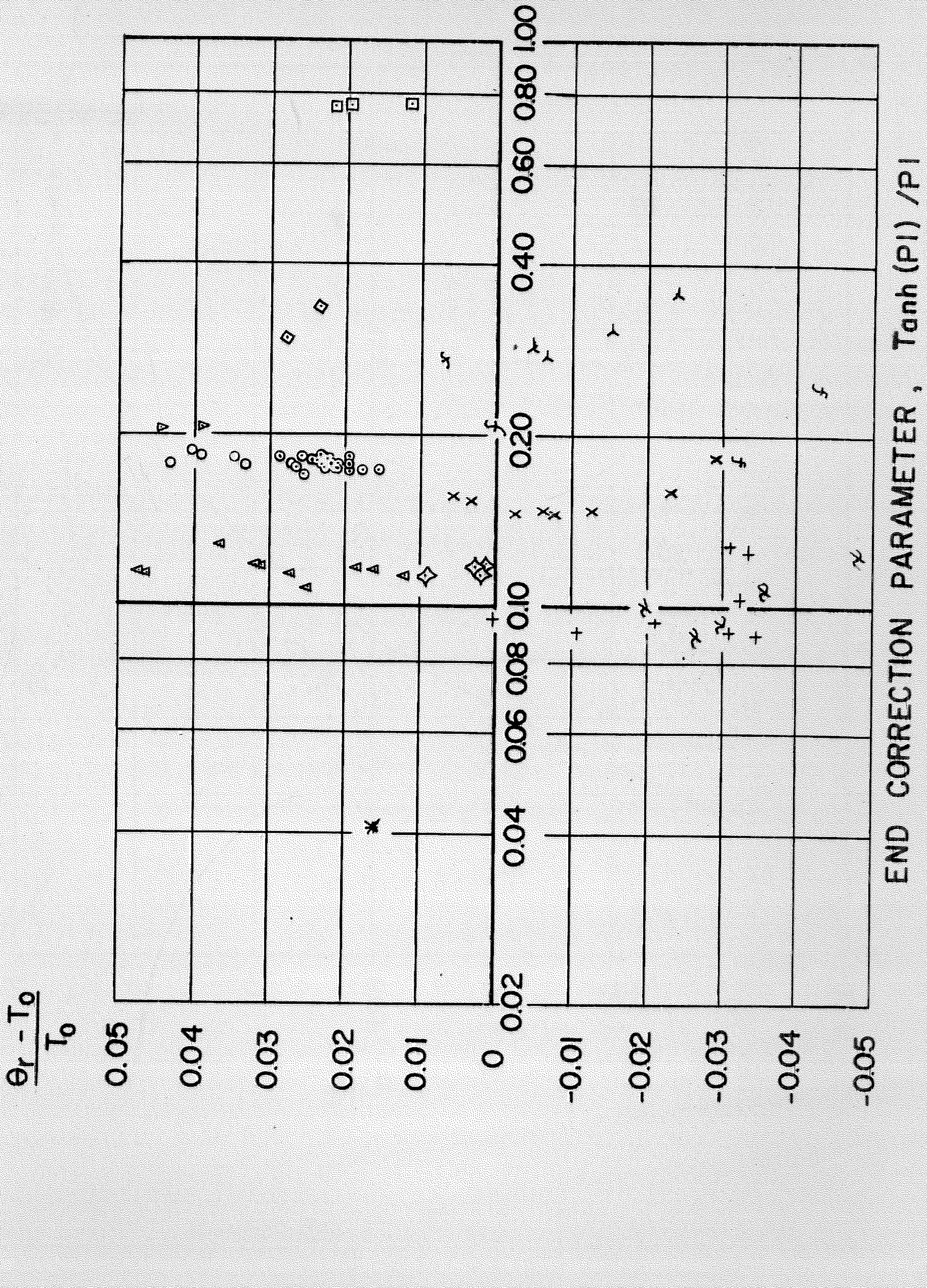

$\frac{\theta_r - T_a}{T_o}$
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
40
80
120
160
200
240
280
320
$T_o - T_{ambient}$, DEGREES CENTIGRADE

FIGURE 10



$\frac{\theta_r - T_0}{T_0}$
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0.02
0.04
0.06
0.08
0.10
0.20
0.40
0.60
0.80
1.00
END CORRECTION PARAMETER, Tanh (PI) /PI

FIGURE 11

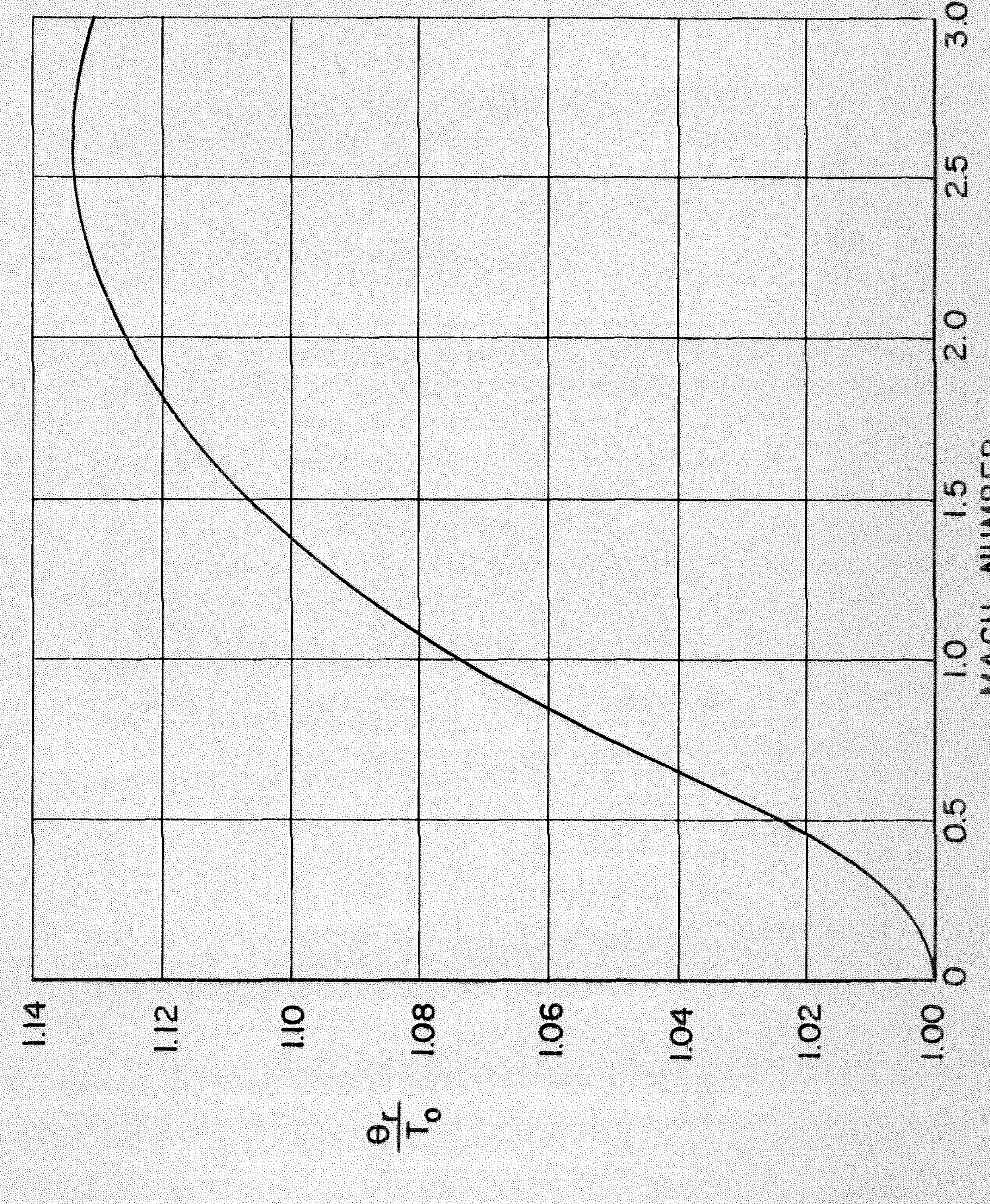

1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
0
0.5
1.0
1.5
2.0
2.5
3.0
MACH NUMBER
$\frac{\theta_r}{T_o}$

FIG. 12

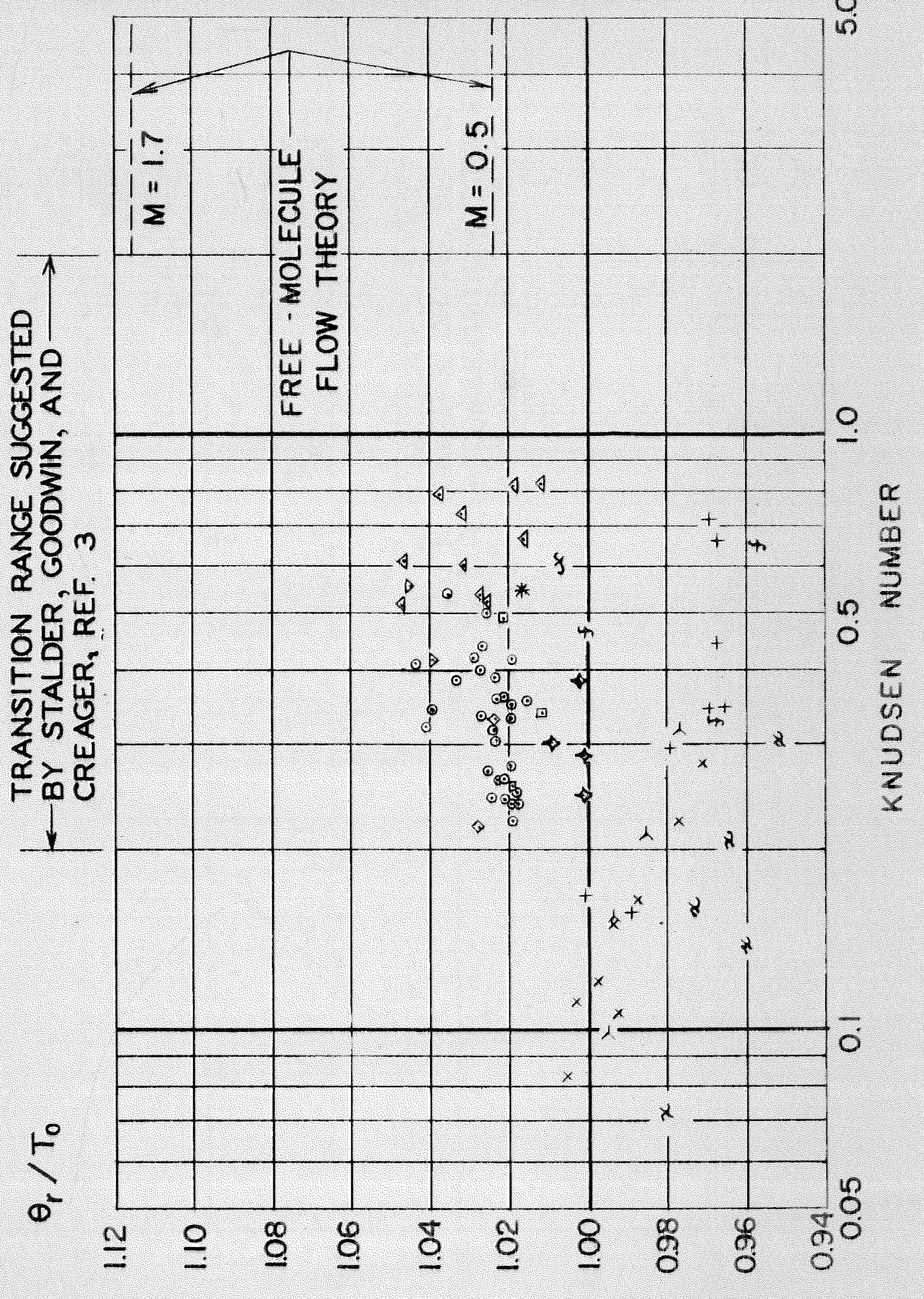

TRANSITION RANGE SUGGESTED BY STALDER, GOODWIN, AND CREAGER, REF. 3
FREE - MOLECULE FLOW THEORY
M = 1.7
M = 0.5
θ_r / T_0
1.12
1.10
1.08
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.05
0.1
0.5
1.0
5.0
KNUDSEN NUMBER

FIGURE 13

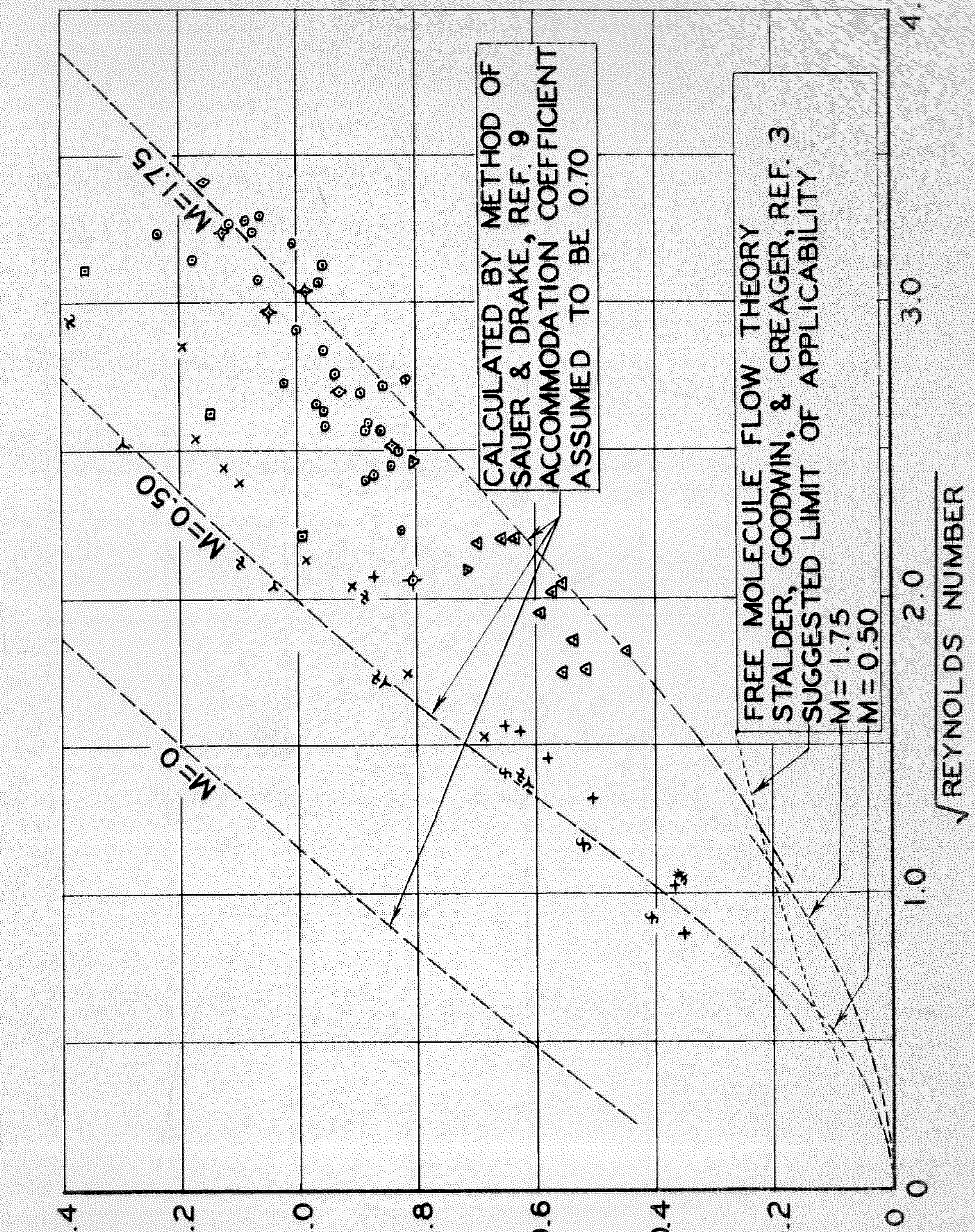

CALCULATED BY METHOD OF
SAUER & DRAKE, REF. 9
ACCOMMODATION COEFFICIENT
ASSUMED TO BE 0.70
FREE MOLECULE FLOW THEORY
STALDER, GOODWIN, & CREAGER, REF. 3
SUGGESTED LIMIT OF APPLICABILITY
M= 1.75
M= 0.50
M=1.75
M=0.50
M=0
NUSSELT NUMBER
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
√REYNOLDS NUMBER
0
1.0
2.0
3.0
4.0

FIGURE 14

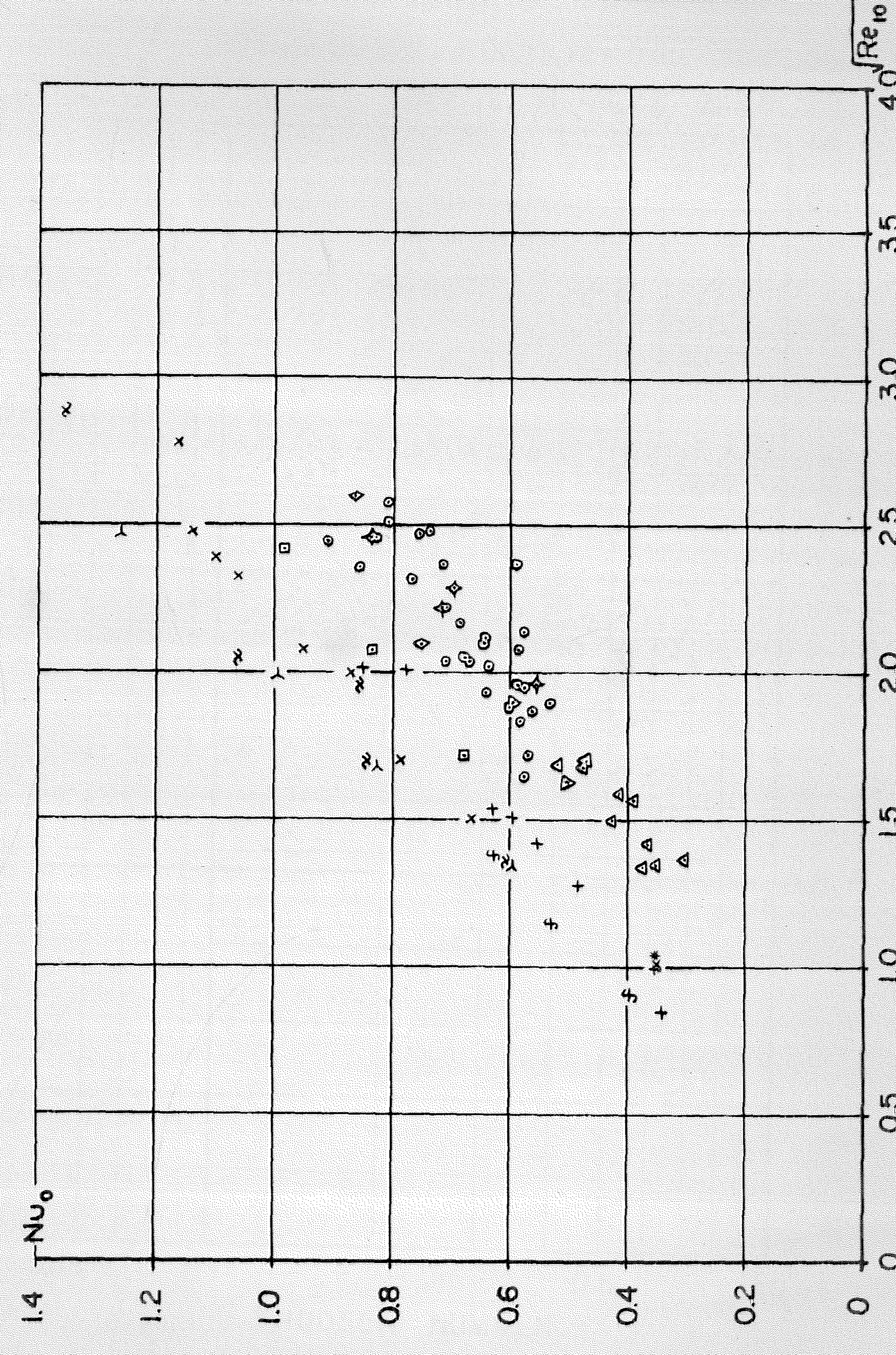


FIGURE 15

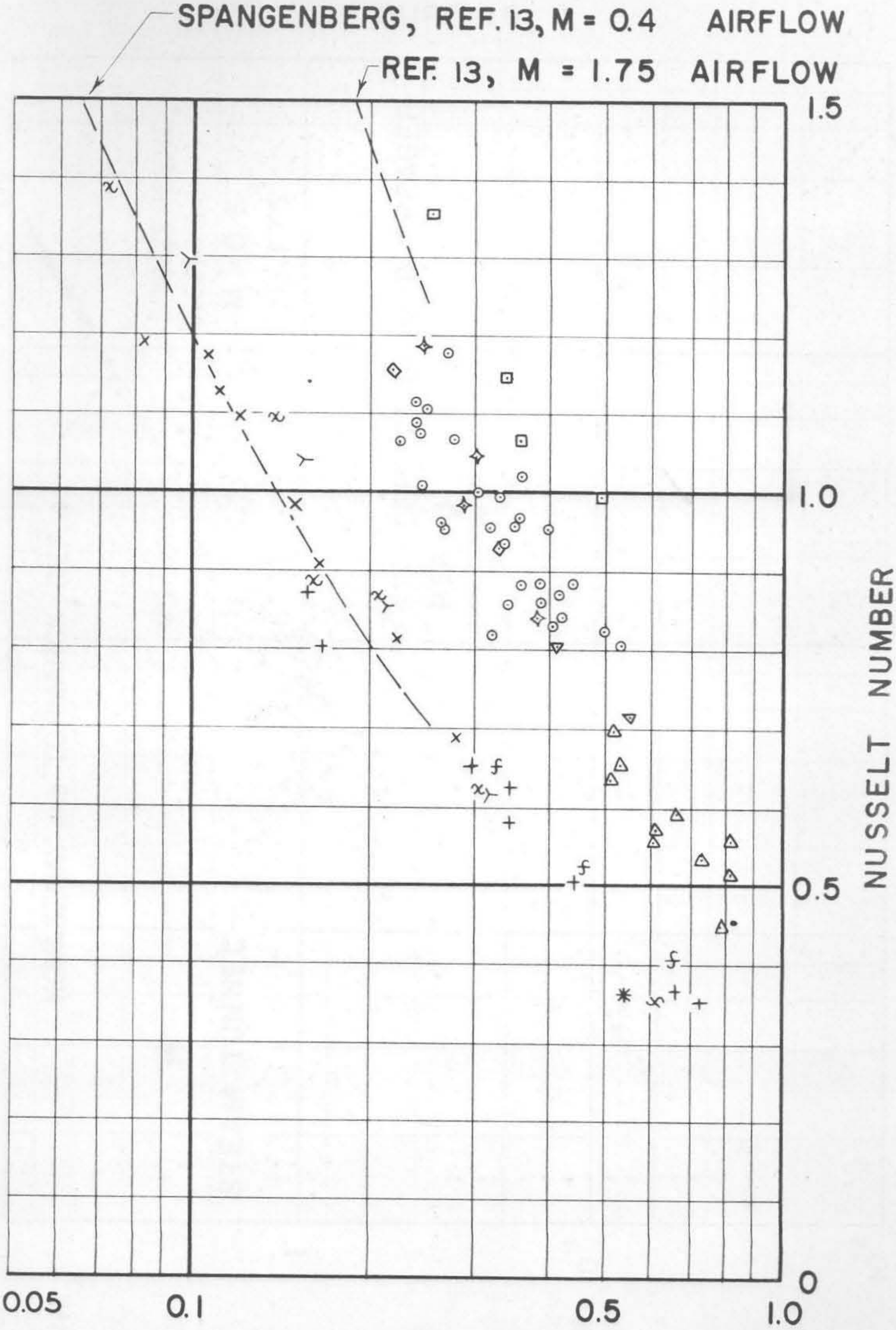
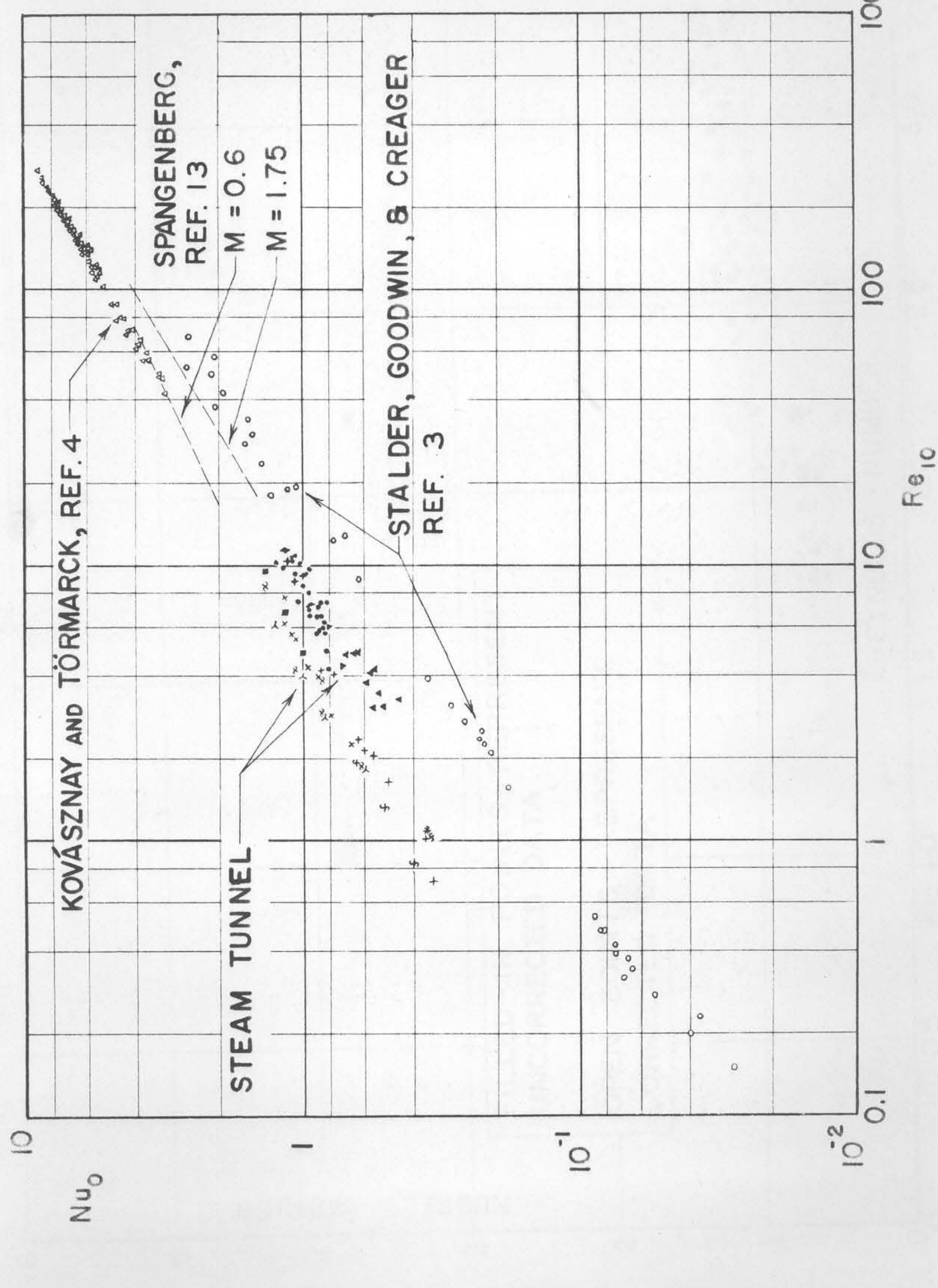

SPANGENBERG, REF. 13, M = 0.4 AIRFLOW
REF. 13, M = 1.75 AIRFLOW
1.5
1.0
0.5
0
NUSSELT NUMBER
0.05
0.1
0.5
1.0
KNUDSEN NUMBER

FIGURE 16



KOVÁSZNAY AND TÖRMARCK, REF. 4
SPANGENBERG, REF. 13
M = 0.6
M = 1.75
STEAM TUNNEL
STALDER, GOODWIN, & CREAGER
REF. 3
Nu_0
Re_{10}
10
1
10^{-1}
10^{-2}
0.1
1
10
100
1000

FIGURE 17

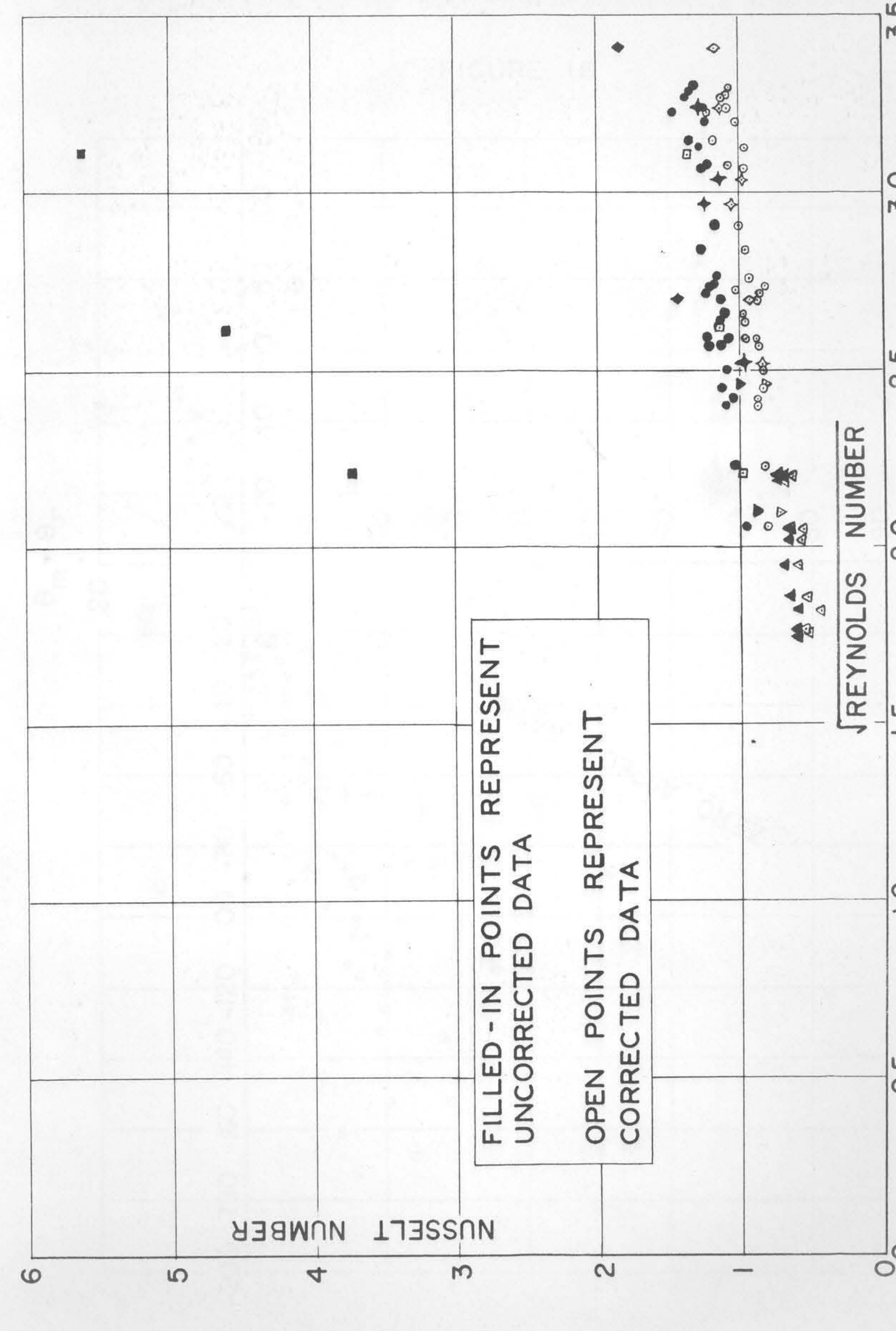

NUSSELT NUMBER
√REYNOLDS NUMBER
FILLED-IN POINTS REPRESENT UNCORRECTED DATA
OPEN POINTS REPRESENT CORRECTED DATA
0
1
2
3
4
5
6
0.5
1.0
1.5
2.0
2.5
3.0
3.5

FIGURE 18

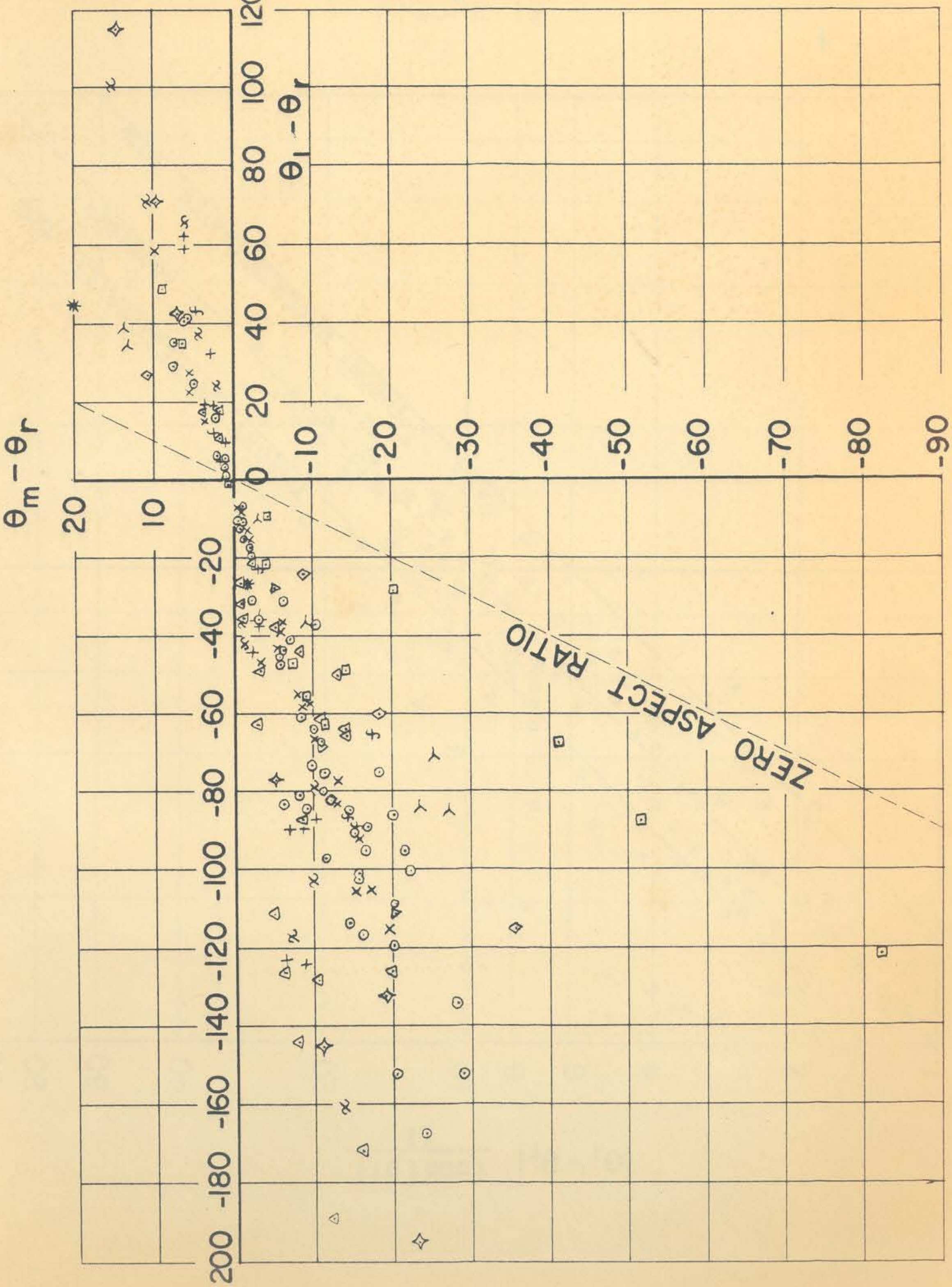

$\theta_m - \theta_r$
$\theta_l - \theta_r$
ZERO ASPECT RATIO
20
10
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
120
100
80
60
40
20
0
-10
-20
-30
-40
-50
-60
-70
-80
-90

FIGURE 19

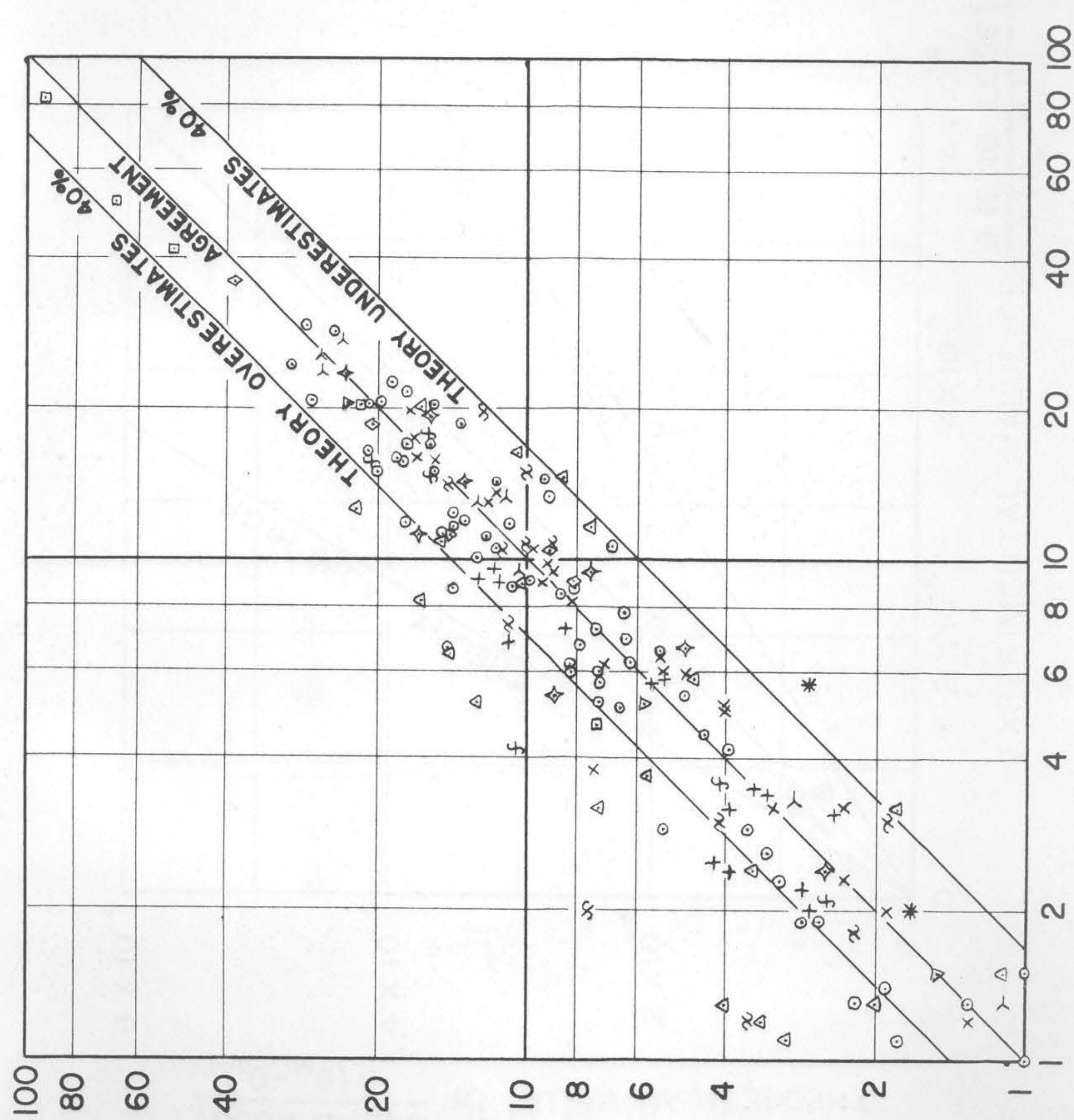

THEORY UNDERESTIMATES
40%
AGREEMENT
THEORY OVERESTIMATES
40%
1 2 4 6 8 10 20 40 60 80 100
$|\theta_m - \theta_r|$
$|\theta_1 - \theta_r| \frac{\tanh(pl)}{pl}$

FIGURE 20

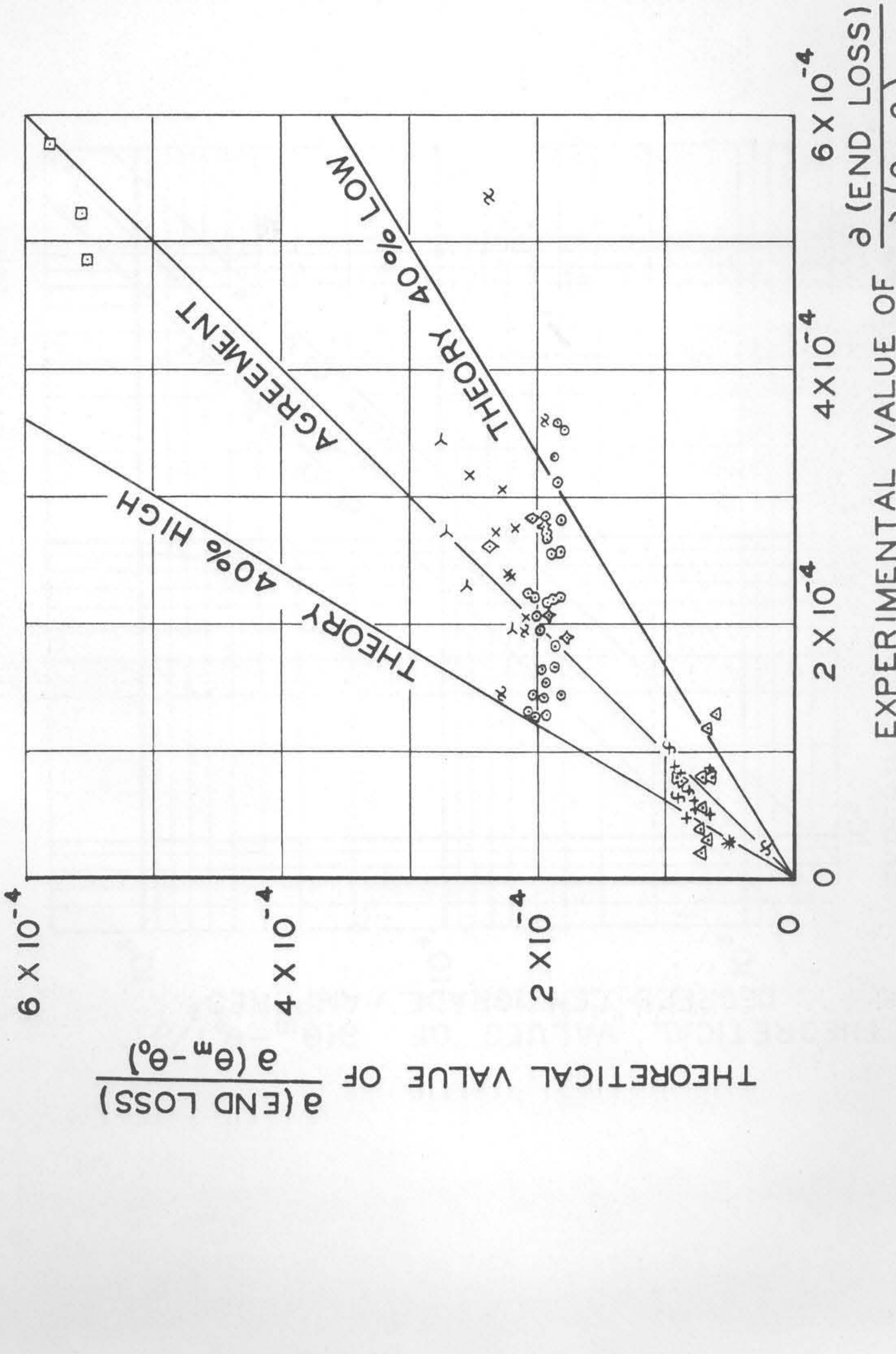

THEORY 40% LOW
AGREEMENT
THEORY 40% HIGH
0
2 X 10^-4
4 X 10^-4
6 X 10^-4
EXPERIMENTAL VALUE OF ∂ (END LOSS) / ∂ (θm-θo)
THEORETICAL VALUE OF ∂ (END LOSS) / ∂ (θm-θo)

FIGURE 21

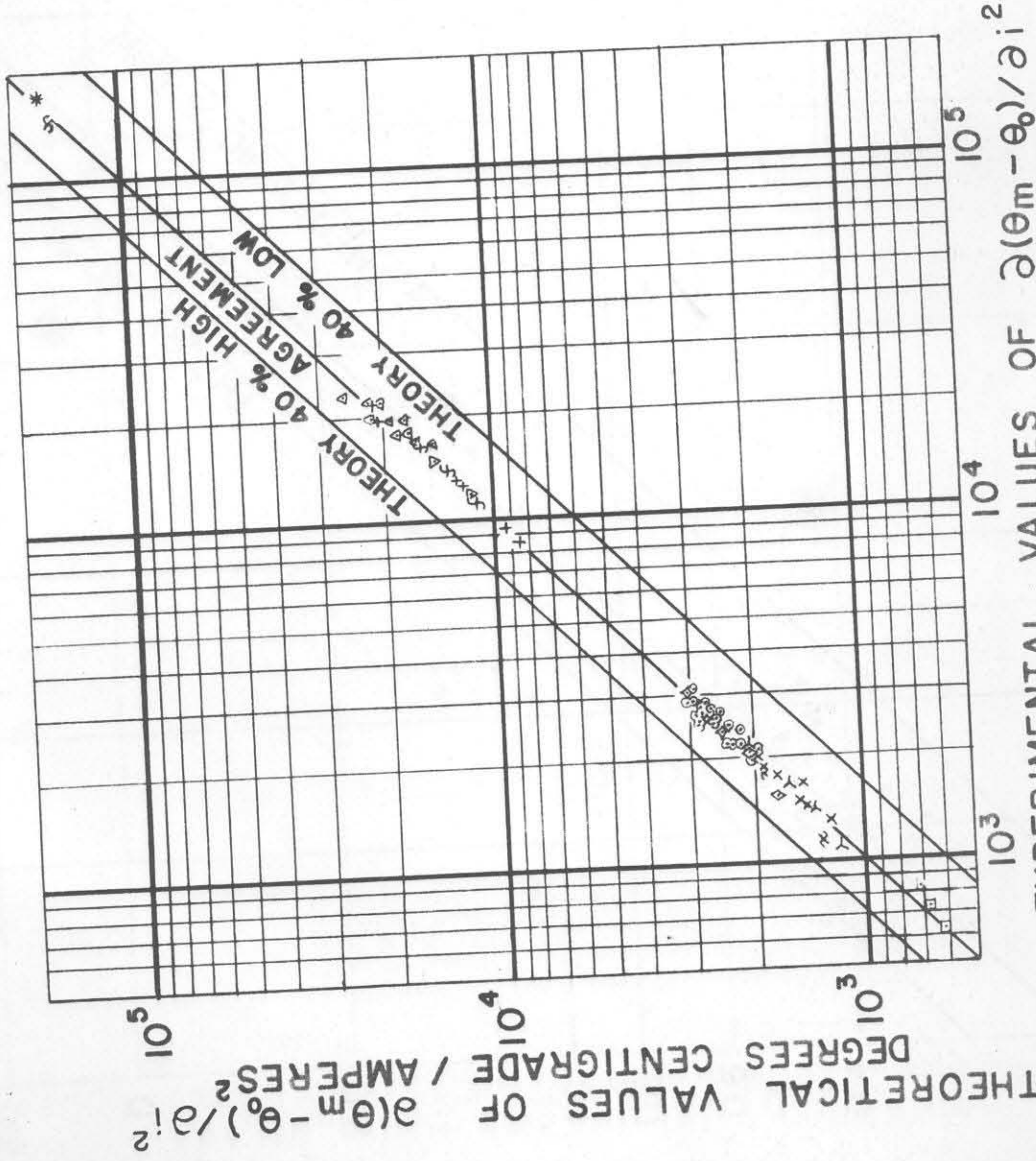
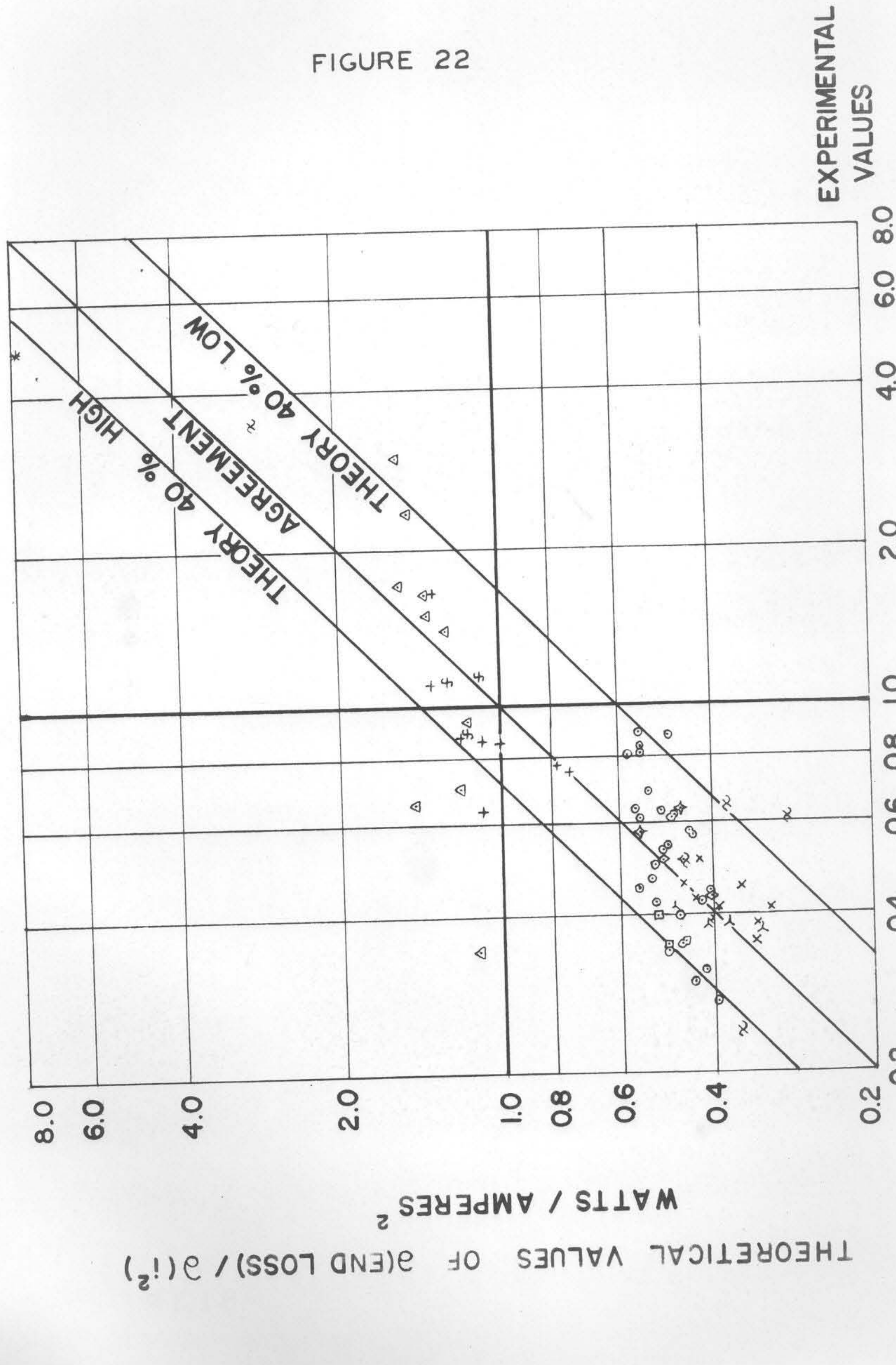

THEORY 40 % LOW
AGREEMENT
THEORY 40 % HIGH
10^3
10^4
10^5
EXPERIMENTAL VALUES OF ∂(θm − θo)/∂i²
10^5
10^4
10^3
THEORETICAL VALUES OF ∂(θm − θo)/∂i²
DEGREES CENTIGRADE / AMPERES²

FIGURE 22


EXPERIMENTAL VALUES
THEORY 40 % LOW
AGREEMENT
THEORY 40 % HIGH
0.2
0.4
0.6
0.8
1.0
2.0
4.0
6.0
8.0
THEORETICAL VALUES OF ∂(END LOSS)/∂(i²)
WATTS / AMPERES²