THE LIFETIMES OF THE Λ^0 AND θ^0 PARTICLES

Thesis by

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In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1955

ACKNOWLEDGMENTS

For his supervision of this thesis and his encouragement throughout the experiment, I would like to express my thanks to Professor Carl D. Anderson. Many thanks are also due Professor Robert B. Leighton, who worked closely with me in planning the procedure and critically evaluating the results.

I wish also to express my gratitude to Dr. V. A. J. Van Lint, who originally suggested this project and made several useful suggestions while it was still in the conceptual stage. I am also greatly indebted to Mr. George H. Trilling for countless suggestions and discussions throughout the experiment, and for his aid in making measurements when an independent opinion was necessary. Also greatly appreciated were the many helpful suggestions and opinions which Mr. John D. Sorrels contributed to the completion of this thesis.

ABSTRACT

The maximum likelihood procedure for determining mean lifetimes of unstable particles is applied to cloud chamber photographs of Λ^o and θ^o particles. Selection methods designed to select pure, unbiased samples are discussed in detail. A formulation of selection criteria which enables optimum use of the data without introducing bias is outlined. The techniques which were employed to make the necessary momentum and distance measurements are described.

The mean lifetime of the Λ^o based on 93 cases is $(3.6 \pm 0.6) \cdot 10^{-10}$ sec. The mean Q-value for 82 cases which permit a Q-value calculation is (35.6 ± 1.0) Mev. The errors in these results are briefly discussed.

The mean lifetime of the θ^o is computed using two independent selection techniques. Biases resulting from anomalous θ^o contamination are discussed and a "best value" of $(1.3 \pm 0.3) \cdot 10^{-10}$ sec, based on 60 cases, is given. The errors due to momentum measurements and sample contamination are discussed qualitatively. The mean Q-value for 58 cases is (214 ± 5) Mev.

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I. INTRODUCTION

This study was undertaken for the purpose of measuring the mean lifetimes of the Λ^0 and θ^0 particles. After eighteen months of operation, about 400 neutral V events had been photographed in the cloud chambers in the 48 inch magnet at Pasadena. It was felt that with such a large number of events one could apply sufficiently restrictive selection requirements that pure samples of Λ^0 and θ^0 particles of statistically significant size could be obtained.

Three important considerations were involved in designing the selection criteria: (1) the resulting samples were to be uncontaminated with decays of particles other than that for which the lifetime was being measured, (2) the samples were to be unbiased with respect to lifetime, (3) optimum use was to be made of the data so that the statistical error in the result would be minimized.

The policy adopted to insure purity was to establish selection criteria which would require that a given decay be not only consistent with a certain scheme but also inconsistent with other possible schemes of comparable frequency of observation. The way in which this was administered in the specific cases of Λ^0 and θ^0 particles will be described in the next section.

The bias problem arises from the fact that the volume of chamber in which decays can be observed is finite. To avoid biasing the sample one must attempt to determine a surface such that if the apex of the event had occurred anywhere within the enclosed volume, the efficiency of detection and identification would be uniform.

Sensibly uniform detection efficiency is assured simply by selecting a volume which is well illuminated. However, since the ability to identify an event increases rapidly with increasing visible track length, it is clear that one must choose the volume considerably smaller than the maximum illuminated volume so that in all cases enough length of track can be seen beyond the apex to permit identification.

On the other hand, one must not make the volume too small for two reasons. Firstly, it is obvious that the probability of observing a decay increases with the size of the usable volume, thus a small volume yields a small sample. The second reason arises from consideration of the statistical weights of those cases which are selected. The finite size of the chamber places an upper limit on the lifetime observable at a given velocity. Hence some particles will pass through the chamber and not be detected at all, and some correction must be made for these before a meaningful average can be taken. This correction is made statistically and requires the measurement of a quantity which is referred to as the "gate time," which is that time the particle could have lived and still have been detected and identified. The gate time, then, becomes a measure of the statistical weight of each case, and if the useful volume is small, the statistical significance of all cases is small.

One must make a compromise by selecting a volume large enough to allow good statistics but not so large that one is biased in identification toward those particles decaying early in their flight through the volume. An approach which has been used (1) is to select a fixed volume within which all cases must decay, and within which all gate lengths are measured. To be unbiased such a volume must be chosen so that the most difficult case to identify which is included could still be identified though it should decay just before leaving the volume. This procedure, though unbiased, is extremely wasteful of the data for two reasons: (1) identifiable cases which decay outside the fixed volume must be omitted, (2) the statistical significance of the included cases is smaller than their measurability merits since the gate time is limited by the most unmeasurable case.

It is believed that the following scheme* enables one to make the maximum use of the data consistent with the policy for selecting pure samples, without biasing the selection or weighting of the cases. The selection criteria are formulated in such a manner that a minimum track length required for positive identification is computed. This length may vary from case to case depending on the measurability of the tracks in question. Then all cases are included for which the track in question has an illuminated length in the chamber greater than this minimum. The gate length is taken to be the distance along the line of flight of the neutral from its entrance into the illuminated region to the last point at which it could have decayed and still had the required minimum length fully illuminated. This method not only makes optimum use of the data, but provided the selection requirements are realistic, it automatically guards against bias from the overoptimistic determination of too large a volume.

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* This scheme was suggested to the author by George H. Trilling.

Once the selection is completed in this manner, one must proceed to make sufficient measurements to permit the lifetime to be computed. These measurements may, in some cases, be extremely difficult and inaccurate, but this can in no way bias against their inclusion since the selection had been previously completed using some property of the decay independent of its overall measurability.

With these broad concepts in mind, one can proceed to apply them to the data available with the object of obtaining pure, unbiased samples of Λ^0 and θ^0 particles.

II. SELECTION METHODS

A. General Criteria

The first step in establishing the rules which every case obeys was to determine the surface which would enclose the largest volume such that, should an event occur at any point within, it would almost certainly have been detected. This volume will henceforth be referred to as "the illuminated region," although, for various reasons, considerable volume of chamber is omitted from it which is well illuminated.

To understand the reasons for the surfaces chosen, knowledge of the chamber geometry and optical arrangement is required. Fig. 1 is a schematic view of the chamber geometry and the fiducial surfaces chosen. If distances perpendicular to the piston are given a Z coordinate in centimeters with the piston at $Z = 0$, then two surfaces bounding the usable volume are $Z = 4$ and $Z = 19$. This was determined simply by where the light beam began to cut off as determined by observing the fading of several tracks as they crossed these surfaces. The top and bottom surfaces in each chamber pass through the camera lenses and intersect the piston in horizontal lines near its edge. The left surface passes through the left lens and intersects the piston in a vertical line far enough from the edge of the piston so that all points in the volume viewed from the right lens will be seen against the piston. The mirror of this holds for the right surface. All these choices were made so that all points in the volume are seen against the piston, which is coated with black velvet so as to serve as an optically good background. Actually several decays seen against the

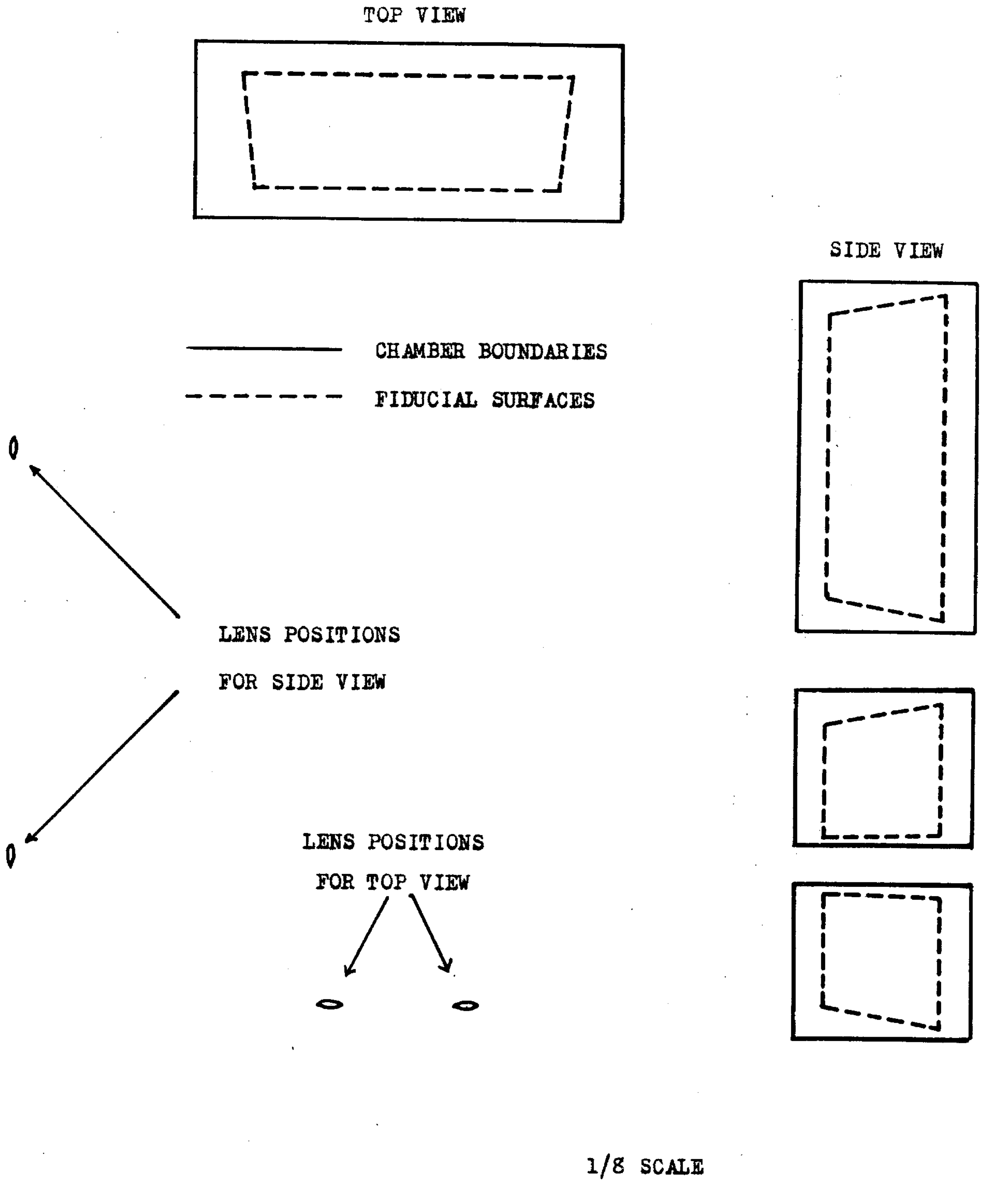
TOP VIEW
SIDE VIEW
CHAMBER BOUNDARIES
FIDUCIAL SURFACES
LENS POSITIONS
FOR SIDE VIEW
LENS POSITIONS
FOR TOP VIEW
1/8 SCALE

Figure 1.

chromium sides of the chamber are easily visible, but none of these were used since the detection efficiency in these regions is not comparable to that in the volume used.

One next considers by what methods he is to select pure samples. The first consideration should be how one is to distinguish the broad category of neutral V events from charged V events. In this study, this separation was done for the most part in a subjective manner by the original scanners; decay events were classified as neutral or charged depending primarily on their general appearance. Since the scanners were competent persons trained in data analysis and interpretation, and since the large majority of cases are clearly similar to one or the other of two greatly different characteristic decays, the author has confidence that this procedure is entirely adequate. All those cases for which the decision was not considered obvious were examined in more detail, and only those which were inconsistent with an interpretation as any known charged V type were included in the neutral V events of this study. This doubtful group, which included, for example, all large angle ($>90^{\circ}$) decays, and those cases where the direction of flight of the neutral deviated markedly from the usual downward trend, constituted less than five per cent of the total number of neutral V events. Since these were subjected to further selection tests, it is extremely unlikely that even one case of a charged V event found its way into the final sample.

B. Selection Criteria for the Λ^0

Given a collection of neutral V events, one must then proceed on some basis to separate these into homogeneous groups. The first assumption made on which the selection criteria used in this study were based is that the only neutral V event which has a proton secondary is the two body Λ^0 which decays according to the scheme

$$\Lambda^0 \longrightarrow p + \pi^- + Q, \quad Q = (35 \pm 1) \text{ Mev.}$$ [2]

It is true that some observations of neutral decays with proton secondaries have been made where the Q-value calculated for the above reaction appears to be inconsistent with 35 Mev.[3] These, however, seem to constitute a very small fraction of the total number of observations. Among the data taken in the 48 inch magnet, there exists no convincing exception to the above decay scheme. There are a number of cases which would be more consistent with other Q-values: several from 20 to 28 Mev, and a few between 50 and 60 Mev; but the general spread of Q-values is completely consistent with a gaussian distribution of error with a probable error smaller than that which one would expect from the assigned errors. On the basis of these considerations, the assumption is made reasonable. That one can justify this assumption for the purpose at hand is extremely fortunate. To distinguish between two different modes of decay on the basis of Q-value alone would require such a high degree of measurability from each case that a statistical study would be impossible.

With the above assumption, it then becomes sufficient for identifying a Λ^0 particle to identify the positive secondary as a proton. To aid in this identification a second assumption is made. It is assumed that no neutral V event has a secondary with mass intermediate between that of a proton and a π meson. Here again decay schemes which have such intermediate or K mesons as secondaries have been proposed[4] as possible explanations for certain observations, but here, even more than with anomalous Q-value Λ^0 decays, it seems their occurrence, if they exist at all, is so rare as to assure a negligible contamination of the sample if they are neglected entirely.

The result of these two assumptions, and a third one denying the existence of neutral V events with secondaries heavier than a proton, is that one has identified a Λ^0 particle if he can show that the positive secondary is consistent with a proton and inconsistent with a π meson. The measurements one has at his disposal to make a separation of positive tracks into protons and π mesons are curvature measurements and ionization estimates. If the ionization of the track is above minimum one can make a positive statement about the velocity of the particle, and from the curvature, the momentum is known. A combination of these two items yields information about the mass, hence a separation of protons from mesons should be possible, for such cases.

The first criterion, therefore, which all cases must obey if they are to be identified as Λ^0 particles is that they must be heavily ionizing. It is worth noting that this restriction not only aids in positive identification, but also selects cases with low velocity. This is an extremely useful thing to do, since for a given length of chamber available, the gate time is longer for slow cases, hence the statistical weight is greater. To allow some margin of error in determining a heavily ionizing track, only positives with ionization greater than 1.5 times minimum were used. The next section will contain descriptions of how these measurements were made.

In addition to being heavily ionizing, the positive secondaries must also satisfy a curvature requirement if they are to be identified as protons; they must have a radius of curvature too large to be consistent with a π meson of the estimated ionization. Since there is likely to be a certain error in any curvature measurement as a result of distortions due to thermal convection currents, to be certain one has selected protons he should require the difference of the curvatures of a proton and a π meson of the estimated ionization to be greater than the expected error.

Experience in analyzing pictures from the 48 inch magnet has led to the adoption of a constant error in the sagitta of a track equal to about one-third of a track width, or 0.03 cm. Thus the ability to distinguish protons from mesons will depend on the length of the track. We wish to find what minimum length of chord is required to enable one to say that the difference in the sagittas of a proton and π meson of ionization I times minimum is greater than the probable error for the difference. The following notation is defined:

δ_p is the sagitta of a proton of ionization I times minimum and chord length L,

δ_m is the sagitta of a π meson of ionization I times minimum and chord length L,

δ_d is the probable error in a sagitta measurement due to distortion; taken as 0.03 cm,

ρ_p is the radius of curvature corresponding to δ_p,

ρ_m is the radius of curvature corresponding to δ_m,

P_p is the momentum corresponding to ρ_p,

P_m is the momentum corresponding to ρ_m,

ΔA is the probable error in the quantity A.

Our requirement states that

$$\delta_m - \delta_p > \Delta(\delta_m - \delta_p) = \sqrt{2}\ \delta_d. \qquad (1)$$

From the formula relating sagitta, chord length, and radius of curvature this can be written as

$$\frac{L^2}{8}\left(\frac{1}{\rho_m} - \frac{1}{\rho_p}\right) > \sqrt{2}\ \delta_d \qquad (2)$$

The relationship between radius of curvature and momentum in a uniform field of magnetic induction B is

$$P = 3 \cdot 10^{-4}\ B\rho \qquad (3)$$

where $B\rho$ is in gauss-centimeters and P is in Mev/c. The average field in the 48 inch magnet is about 8000 gauss, so neglecting corrections due to non-uniformities of field and effects of conical projection onto the film,[5] the momentum of a track in Mev/c is proportional to the measured radius of curvature in cm as follows:

$$P = 2.4\rho \, . \tag{4}$$

Using (4) and (2) and the value for δ_d we get for L in cm

$$L^2 \left(\frac{1}{P_m} - \frac{1}{P_p}\right) > 0.14$$

or

$$L^2 > \frac{0.14 \; P_m}{(1 - P_m/P_p)} \quad . \tag{5}$$

P_m/P_p is a constant independent of I, equal to 0.15, therefore (5) becomes

$$L^2 > 0.17 \; P_m \quad . \tag{6}$$

Equation (6) could now be used to find the minimum L required to distinguish protons from mesons for any ionization greater than 1.5. However, since the velocity varies approximately inversely as the square root of the ionization, one would expect L to vary roughly inversely as the fourth root of the ionization, thus quite slowly. Also, for the majority of cases the ionizations are not large, therefore at the cost of a small amount of gate length in a few cases, one can greatly simplify the procedure by taking the minimum L for all cases the same as for the worst possible case. Since the momentum of a π meson with ionization 1.5 times minimum is about 150 Mev/c, the requirement becomes

$$L > 5 \text{ cm.}$$

Thus the selection criteria for Λ^0 particles are:

(1) ionization of positive secondary greater than 1.5 times minimum,

(2) positive secondary too straight to be consistent with a π meson of the estimated ionization as determined by observation on at least 5 cm of track in the illuminated region.

C. Selection Criteria for the θ^0

One is now interested in establishing selection criteria for the θ^0, which decays according to the scheme

$$\theta^0 \longrightarrow \pi^+ + \pi^- + Q, \quad Q + (214 \pm 5) \text{ Mev.}$$ (6)

In attempting to distinguish this decay from all other known modes for neutral V events, one finds the situation considerably different from that for Λ^0 particles. For one thing, in a cloud chamber one cannot distinguish π mesons from μ mesons except in rare instances, therefore, one cannot rule out the possibility that any observed decay apparently yielding π mesons might actually give μ mesons instead. This statement actually applies to the Λ^0 particle also, but is not pertinent in that connection, since no one has any reason to believe that a μ meson has ever been paired with a proton as a secondary. For the θ^0 there is some reason to believe that such confusion could exist.

Unstable particles with light meson secondaries which do not fit the above decay scheme definitely have been observed. These exhibit anomalous behavior in that the Q-value, as calculated assuming the above scheme, is absolutely inconsistent with 214 Mev. In the data from the 48 inch magnet alone, at least ten good cases exist which are not Λ^0 particles and yield anomalous Q-values when treated as θ^0 particles. The computed Q-values for these cases show some bunching between 29 and 62 Mev, but excellent cases at 10 and 150 Mev also exist. This, plus the fact that several cases exhibit good origins which do not line up with the visible tracks, strongly suggests a three-body decay if one is to find a single scheme consistent with all the anomalous cases.(7)

These "low-Q" cases do not distinguish themselves from ordinary θ^0 particles in any way except Q-value calculations or the equivalent. Yet to select a sample by computing Q-values would require such a high degree of measurability from each case that the result would be a sample of no statistical significance. A compromise had to be made, therefore, between a statistically significant sample and a pure sample. The procedure used was as follows.

All cases which could be shown to be not Λ^0 particles were called θ^0 particles, where these then were mixtures of normal θ^0 particles and anomalous θ^0 decays. From this group, all cases were eliminated for which further measurement clearly demonstrated that they were inconsistent with the normal θ^0 decay scheme. In this connection, "inconsistent" was defined as meaning that the Q-value, as computed for a normal θ^0 particle, differed from 214 Mev by more than twice the assigned error. Since there are some cases for which Q-values cannot be computed, and several for which the errors of measurement are very large, this process cannot remove all the anomalous θ^o contamination from the sample. In addition, it is felt that the assigned errors are conservatively large so that some cases which are apparently highly consistent with an energy release of 214 Mev may be anomalous. It is also entirely possible that anomalous decays could yield a Q-value of 214 Mev, as would be the case for the three body decay proposed by Van Lint.(7) The result of this procedure is thus a sample of θ^o particles with some contamination. In a later section an attempt will be made to estimate the number of anomalous cases present.

One is now faced with the problem of selecting cases which are inconsistent with a Λ^o particle from a collection of neutral V events. From the way in which the Λ^o particles were selected, an obvious method of obtaining θ^o particles is to select cases for which the positive secondary is clearly inconsistent with a proton. One has the inverse problem that he had for protons: in that case one looked for heavily ionizing, straight tracks, while here he must look for highly curved, minimum ionizing tracks. In the previous case, the determining measurement was the ionization and only a lower limit on the radius of curvature was necessary. For the present situation, only an upper limit can be established for the ionization, and the decisive measurement must be the curvature. The following notation is defined:

δ is the measured sagitta for a chord length L,

δ_p is the maximum sagitta for a proton corresponding to the maximum ionization of the track for a chord length L,

δ_d is the probable error in the sagitta measurement as a result of distortion; taken as 0.03 cm,

ρ is the radius of curvature corresponding to δ,

ρ_p is the radius of curvature corresponding to δ_p,

P is the momentum corresponding to ρ,

P_p is the momentum corresponding to ρ_p.

Then the requirement is

$$\delta - \delta_d > \delta_p. \tag{1}$$

Using the relationship relating radius of curvature, sagitta, and chord length one has

$$\frac{L^2}{8}\left(\frac{1}{\rho} - \frac{1}{\rho_p}\right) > \delta_d. \tag{2}$$

Using (4) from part B, (2) becomes

$$L^2\left(\frac{1}{P} - \frac{1}{P_p}\right) > 0.1$$

or

$$L^2 > \frac{0.1P}{(1 - P/P_p)} \quad . \tag{3}$$

All but an exceptional case have ionization less than 1.5 times minimum at which a proton has momentum

$$P_p = 1000 \text{ Mev/c},$$

therefore

$$L^2 > \frac{0.1P}{1 - 0.001P} = \frac{100}{\frac{1000}{P} - 1} \quad . \qquad (4)$$

The selection criteria for a θ^0 particle on the basis of observations on the positive secondary alone become:

(1) an illuminated track length such that the chord length, L, and the momentum, P, satisfy inequality (4);

(2) ionization less than 1.5 minimum or too small to be consistent with the measured curvature.

There is, however, another useful way of finding cases which are not Λ^0 particles. For any given two-body decay, there is an upper limit for the quantity $P_- \sin\theta$,[8] where θ is the total angle between the two secondaries and P_- is the momentum of the negative secondary. For the Λ^0 particle this limit is given by

$$P_- \sin\theta \leq 115 \text{ Mev/c},^*$$

while for the θ^0 particle the corresponding relationship is

$$P_- \sin\theta \leq 715 \text{ Mev/c}.$$

Thus any neutral V event for which $P_- \sin\theta$ is greater than 115 Mev/c is not a Λ^0 particle; and moreover, since it can be shown[8] that the distribution of this quantity is highly skewed toward the upper limit, a large fraction of θ^0 particles can be selected by this method.

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* This inequality is modified to read $P_- < 115$ Mev if $\theta > 90^0$, as will be shown in a later section.

We wish to have a margin of safety of one probable error in applying this criterion, therefore we write, using notation previously defined,

$$(P_- - \Delta P_-) \sin \theta > 115,$$

$$P_- - \Delta P_- = 2.4(\rho_- - \Delta\rho_-) = \frac{2.4\, L^2}{8(\delta_- + \delta_d)} > \frac{115}{\sin \theta} ,$$

$$L^2 > \frac{8 \cdot 115}{2.4 \sin \theta}(\delta_- + \delta_d) = \frac{115}{\sin \theta}\left(\frac{L^2}{P_-} + \frac{8\,\delta_d}{2.4}\right) ,$$

$$L^2\left(1 - \frac{115}{P_- \sin \theta}\right) > \frac{8 \cdot 115\,\delta_d}{2.4 \sin \theta} = \frac{11.5}{\sin \theta} ,$$

$$L^2 > \frac{11.5}{\sin \theta\left(1 - \frac{115}{P_- \sin \theta}\right)} = \frac{0.1\, P_-}{\frac{P_- \sin \theta}{115} - 1} .$$

This last inequality determines the length of the negative secondary required to identify as a θ^o particle an event which has an angle θ and measured momentum of negative secondary P_-.

It will be noted that the two methods described above for identifying θ^o particles are mutually exclusive as far as the measurable quantities of the decay are concerned: one uses the positive secondary, the other the negative secondary and angle, thus they are independent methods of selection. There will in general, however, be a group of cases common to the samples selected by both methods, thus the lifetimes calculated for the two samples are not strictly independent. It would not be correct, however, to simply mix all cases selected by either method, since the cases receive different statistical treatment for the two methods of selection. A discussion of the relative merits of these two selection methods will be deferred to a later section.

III. MEASUREMENT TECHNIQUES

The purpose of this section is to examine each quantity which must be known in order to calculate the lifetime, and describe how it was obtained.

The lifetime of each unstable particle is a quantity of prime importance and it is computed from a formula which is derived below. Define the notation:

t is the lifetime of a particle measured in its own rest system,

x is the distance it travels in the laboratory system,

βc is the velocity of the particle measured in the laboratory system, where c is the velocity of light,

γ is equal to $(1 - \beta^2)^{-1/2}$,

M is the mass of the particle in energy units,

P is the momentum of the particle in energy units.

The Einstein time dilatation formula gives for t,

$$t = \frac{1}{\gamma} \; \frac{x}{\beta c} = \frac{xM}{cP} \; .$$

M is known from the known masses of the secondaries and the Q-value. Thus only x and P must be measured.

P can be computed from the usual cosine law formula once the momenta of the secondaries, P_+ and P_-, and the included angle θ are known. θ was directly and accurately obtained from the direction cosines of the secondaries; only one exception to this occurred in this study, an event with one very short secondary, but an excellent origin provided another equally useful angle measurement. P_+ and P_- were calculated from the measured radii of curvature whenever these could be measured. There were, however, several cases where these measurements could not be made due to tracks being too short, too straight, or too distorted, so that other methods of finding the momenta had to be devised.

For the Λ^0 particles, four categories will be considered:

(1) cases for which the curvature of both tracks is measurable,

(2) cases for which the curvature of the positive secondary is not measurable,

(3) cases for which the curvature of the negative secondary is not measurable,

(4) cases for which the curvature of neither track is measurable.

Group (1) needs no comment. For group (2), the momentum of the positive was determined from ionization estimates. The ionizations of these cases and all those for which the ionization was not obviously greater than 1.5 times minimum were estimated independently by three different observers. These estimates were ranges in which the observer felt the correct value almost surely lay. The maximum range which was contained within all three estimates was determined, and if the lower bound of this was less than 1.5 the case was rejected. The center of this overlap range was used as the ionization from which the momenta of the protons was obtained for the cases of group (2). The mass, momentum, ionization relationship was taken from standard graphs of the Bethe-Bloch formula for Argon.

For the cases of group (3), the momentum of the negative secondary was computed assuming the Q-value of 35 Mev and using the measurements of P_+ and θ. The derivation of the appropriate formula will be included here since it is not only of interest in this connection, but also one can obtain the P_- sin θ upper limit as a by-product from it. The following notation will be used:

W_o, P_o, M_o	the total energy, momentum, and mass of the primary particle,
$W_{1,2}$, $P_{1,2}$, $M_{1,2}$	the total energies, momenta, and masses of the two secondaries,
θ	the angle included between the secondaries.

The conservation laws for energy and momentum are

$$W_o = W_1 + W_2 \ ,$$

$$P_o^2 = P_1^2 + P_2^2 + 2P_1P_2 \cos\theta \ .$$

Use

$$W_i^2 = P_i^2 + M_i^2 \text{ for } i = 0, 1, 2,$$

eliminate P_o, and solve for P_2. The result is

$$P_2 = \frac{M^2P_1 \cos\theta \pm W_1\sqrt{M^4 - 4M_2^2\,(M_1^2 + P_1^2 \sin^2\theta)}}{2(M_1^2 + P_1^2 \sin^2\theta)}$$

where

$$M^2 \equiv M_o^2 - M_1^2 - M_2^2 \ ,$$

and M_o can be found from

$$M_o = M_1 + M_2 + Q \ .$$

If one identifies P_2 with P_-, this can be used to find P_- from P_+, θ, and Q for the cases of group (3).

If one requires that P_2 be real, then

$$P_1 \sin\theta \leq \frac{\sqrt{M^4 - 4M_1{}^2M_2{}^2}}{2M_2} ,$$

and if one now identifies P_1 with P_- then this becomes

$$P_- \sin\theta \leq 115 \text{ Mev.}$$

If one also requires that P_2 be positive, then for $\theta > 90^\circ$,

$$W_1 \sqrt{M^4 - 4M_2{}^2\left(M_1{}^2 + P_1{}^2 \sin^2\theta\right)} > \left|M^2P_1 \cos\theta\right| ,$$

$$\left(P_1{}^2 + M_1{}^2\right)\left[M^4 - 4M_2{}^2 \left(M_1{}^2 + P_1{}^2 \sin^2\theta\right)\right] > M^4P_1{}^2 \cos^2\theta ,$$

$$M^4\left(M_1{}^2 + P_1{}^2 \sin^2\theta\right) - 4M_2{}^2 \left(P_1{}^2 + M_1{}^2\right)\left(M_1{}^2 + P_1{}^2 \sin^2\theta\right) > 0 ,$$

$$M^4 - 4M_2{}^2 \left(P_1{}^2 + M_1{}^2\right) > 0 ,$$

$$P_1 < \frac{\sqrt{M^4 - 4M_1{}^2M_2{}^2}}{2M_2} .$$

Thus for $\theta > 90^\circ$ the rule simplifies to

$$P_- < 115 \text{ Mev.}$$

Use will be made of this addition to the inequality without further mention.

The cases of group (4) are handled by a combination of the methods of groups (3) and (4). The errors here are likely to be large since an inaccurately measured positive is used to compute the negative, but fortunately there were only two such Λ^{o} particles in this study.

For the θ^{o} particles also, four categories will be considered:

(1) cases where the curvature of both tracks is measurable,

(2) cases where the curvature of only one track is measurable but an origin for the θ^{o} particle can be located,

(3) cases where the curvature of only one track is measurable and no origin can be located,

(4) cases where the curvature of neither track is measurable.

Group (1) again needs no comment. For the cases of group (2), the momentum of the unmeasurable track was computed assuming momentum balance about the line of flight determined by the origin and the apex of the decay. The momentum of the unmeasurable track for the cases of group (3) was computed from the measured momentum and the included angle assuming a Q-value of 214 Mev.

Only two cases came under group (4), and both of these had poorly located origins. The line of flight suggested by these origins approximately bisected the decay angle, therefore the momenta of the secondaries were assumed equal, which is the most likely situation for a symmetric scheme such as the θ^{o} decay. This assumption plus knowledge of the Q-value permits calculation of the momenta.

It should be mentioned here that in applying the selection criteria for θ^{o} particles, momenta obtained indirectly as for the unmeasurable tracks of groups (2), (3), and (4) were never used; for this purpose the best possible limits as measured directly were used. This is in keeping with the spirit of selecting cases only on the merits of one feature, not the sum total of information available.

Once all the momenta are known, one need only measure x to be able to compute t. It is not necessary, as one might at first think, to find an origin for each case and take x as the distance from this origin to the apex. Since the entry of the particle into the chamber occurs at a random time in its life, the distribution of lifetimes after entry into the chamber is identical to the distribution of lifetimes after production. Thus x is taken as the distance along the line of flight from the point of entry into the illuminated region to the apex. The line of flight is constructed using the known momenta of the secondaries.

The only remaining quantity required to calculate the mean lifetime is the gate length. This is the distance the primary could have gone and still had the length of secondary required for identification by the selection criteria within the visible region. Location of the last point along the line of flight where the selection requirements are satisfied is in general a complicated task in projective geometry. It is true, however, that for tracks which are parallel to the piston, the projection of the tangent onto the piston has the same direction regardless of where along the line of flight the secondary may have originated. This makes a very convenient simplification in the construction necessary to locate the critical point. Since the large majority of cases have a small direction cosine with the z-axis, the error in using this simplified construction is small, therefore it was used to find the gate lengths in all cases. Once the gate length is measured, it can be converted to a gate time using the formula

$$T = \frac{MD}{Pc}$$

where D and T are the gate length and gate time. This formula is an obvious analogue of the previously derived formula for the lifetime of each particle.

IV. THE DATA

The purpose of this section is to list the data as it was measured for each case by the techniques described in the previous section.

The notation used as column headings will be redefined here:

I is the estimated ionization of the positive secondary, in units of minimum ionization,

P_+ is the momentum of the positive secondary,

P_- is the momentum of the negative secondary,

θ is the angle included between the two secondaries,

P_o is the momentum of the primary,

Q is the energy release as computed from P_+, P_-, θ, and the masses of the secondaries,

ΔQ is the assigned error in the Q-value,

x is the distance the neutral travels in the chamber before decay,

D is the distance the neutral could have traveled and still have satisfied the selection criteria,

t is the time to travel distance x as measured in the rest system of the particle,

T is the time to travel distance D measured in the rest system of the particle.

Table I contains the data for Λ^o particles except the direct lifetime measurements.

TABLE I

Identification No.	I+	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_0 (Mev/c)	Q (Mev)	ΔQ (Mev)
2175	2-2.5	550	149	47.5	660	32	5
2264		475	189	29.5	645	35	7
2466		620	175	31.4	775	26	7
2772	4-4.5	299	71**	138.4	250		
3223	4.5-7	378	154	34.4	513	28	11
2427	8-9	308	81**	107.0	295		
3456	3-4	480*	210	29.0	670	42	15
3844	3-4	480*	130	55.6	564	32	10
4173		475	140	46.0	581	28	6
4410		350	159	37.4	484	33	3
4609		309	154	40.5	438	34	7
4769	6.5-7.5	305*	171	26.5	464	36	16
5369		200	132	58.1	292	35	3
6260		440	86	87.7	451	34	5
6898	3.5-6	400*	193**	19.0	586		
7384	2-4	550*	229	10.8	670	34	12
7509		550	78	84.3	564	38	8
7563		252	129	65.2	327	36	3.5

* These momenta were obtained from ionization estimates.

** These momenta were computed from P_+, θ, and Q.

TABLE I (Continued)

Identification No.	I_+	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_o (Mev/c)	Q (Mev)	ΔQ (Mev)
7678		625	189	33.8	791	35	7
7939		590	90	105.7	572	49	10
8346		430	150	53.6	533	45	11
9150		650	112	59.5	712	34	7
9207	3-4	380	133**	60.2	461		
9464	2-4	580*	215	22.8	844	34	10
9899	1.5-2	865*	38	78.6	873	43	30
10144	4-6	380*	105	36.6	468	13	8
10252	5.5-10	300*	96	59.5	358	20	8
10257	2-3	630*	123	17.2	748	4.5	21
10450	1.5-1.7	930*	308	27.9	1210	56	56
10670	3	530*	110	48.4	609	20	8
11036	10-12	214	140	53.0	318	37	10
11118	3-5	900	355	8.5	937	46	20
11957		395	160	31.2	539	28	5
12073		380	170	51.5	503	48	14
12181		510	150	41.6	630	27	3
12986	3-5	444	116	58.0	515	28	12
13087	2-4	500	211	25.2	696	38	23

* These momenta were obtained from ionization estimates.

** These momenta were computed from P_+, θ, and Q.

TABLE I (Continued)

Identification No.	I_+	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_0 (Mev/c)	Q (Mev)	ΔQ (Mev)
13147		305	127	59.5	385	33	5
13386	1.5-3	760*	125	45.6	853	36	10
14316	3	530*	157	26.5	674	19	13
14979		110	141	12.5	249	25.5	2.5
15874	1.5-1.8	900*	262	21.8	1148	32	17
15896	1.5-2	860	108	48.5	936	29	6
16003	1.5-2	865*	350	13.1	1210	54	34
16460	2-2.5	670*	72	87.1	678	44	14
16863	5	247	126**	66.2	320		
17609		500	231	35.7	702	56	12
18268	1.5-2	800	118	50.5	880	33	6
18290		427	126	65.3	494	36	3
19232		732	260	2.0	992	27	20
19606	2.5-3	570*	213**	26.7	766		
20968		660	108	76.5	712	51	13
22012	1.5-2	680	305	20.0	971	57	34
22055		310	122	71.5	368	36.5	2.5
22425	1.7-3	700*	224	35.0	893	44	30
22451		272	154	44.6	397	38	4

* These momenta were obtained from ionization estimates.

** These momenta were computed from P_+, θ, and Q.

TABLE I (Continued)

Identification No.	I_+	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_o (Mev/c)	Q (Mev)	ΔQ (Mev)
22815		785	178	35.1	938	28	3
22971	1.7-3	468	81**	91.0	474		
23023		530	235	7.2	763	36	4
24004		625	151	42.1	745	28	6
24114		320	153	48.9	437	38	5
24420	2-3	630*	88	69.0	667	32	21
24537	1.5-2.5	830	102**	56.7	890		
24694	1.7-2	815	78	78.1	838	50	30
26797	1.5-2	865*	248	29.0	1089	38	18
27137		625	175	58.4	732	57	16
27552		645	142	51.9	741	35.5	2.5
27801	1.8-2	780*	112	61.6	840	42	17
27811	2-3.5	700	183**	39.3	850		
28175	1.5-2	710	158	38.2	839	26	7
28212	1.5-1.7	1065	309	16.8	1370	32	14
28381	3.5-5	425	167**	40.1	564		
28650	2.5-5	465	140	49.5	568	31	13
29101	4-6	666	134	57.4	746	40	12
29129	6-7	276	131	55.5	366	33	14

* These momenta were obtained from ionization estimates.

** These momenta were computed from P_+, θ, and Q.

TABLE I (Continued)

Identification No.	I_+	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_o (Mev/c)	Q (Mev)	ΔQ (Mev)
29569	4-5	608	155	47.6	721	36	30
29833	3	652	193	21.2	835	22	16
29903	3-5	450*	88	88.5	462	35	9
32087		275	144	50.9	383	37	3
32170		542	218	32.4	735	45	9
32267		349	177	38.0	500	42	11
33280		364	140	55.5	458	36	9
33603		600	145	53.5	696	36	6
33755		390	232	30.0	602	59	13
34528		440	182	34.4	599	41	7
34845		406	177**	33.0	562		
34955		745	250	27.0	975	42	4
35265		514	50	92.5	514	27	5
35363		415	56	122.4	417	35	6
35386		470	195	21.8	654	33	8
35665		485	208	36.1	664	47	11
35800		228	147	59.9	328	44	14
35912		490	274	24.3	745	66	19

* These momenta were obtained from ionization estimates.

** These momenta were computed from P_+, θ, and Q.

Table II contains a similar set of data for the θ^0's except I_+, which was necessary only for the Λ^0's, is omitted and P_- sin θ, an important parameter for the selection of θ^0's, is included.

TABLE II

Identification No.	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_0 (Mev/c)	P_- sin θ (Mev/c)	Q (Mev)	ΔQ (Mev)
2469	270	1120	38.5	1340	698	205	45
4417	390	1100*	43.6	1410	760	300	70
4559	475	1250*	35.7	1659	730	290	100
4688	480	710**	40.9	1120	655		
5205	562	58	39.6	607	37	127	90
5828	900	1500	20.5	2370	525	218	70
5979	1400***	1400***	16.7	2760	400		
6233	370	270	70.4	526	254	180	45
6482	400	590	45.6	915	422	195	90
6700	330	3000**	10.3	3325	536		
7651	985	326	43.6	1240	225	255	70
8814	196	725*	61.2	837	635	226	70
8895	350	175	86.6	398	174	170	35
9369	320	2200	12.2	2530	465	170	70
9494	750	315	52.6	975	250	259	45

* These momenta were obtained from momentum balance about an origin.

** These momenta were computed from the known momentum, angle, and Q-value.

*** These momenta were computed assuming $P_+ = P_-$ from θ and Q.

TABLE II (Continued)

Identification No.	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_0 (Mev/c)	$P_- \sin\theta$ (Mev/c)	Q (Mev)	ΔQ (Mev)
9529	138	890	31.8	1010	469	150	50
9630	265	513	57.6	691	434	186	38
10782	272	88	150.0	201	88	169	37
10909	3020**	855	73.0	3855	192		
10930	148	400	169.4	254	400	291	100
11753	322	579*	49.0	826	436	181	170
12633	103	560	85.1	579	558	200	30
12786	1290	720	30.0	1946	360	296	80
13075	1980**	820	17.6	2770	248		
15110	81	750	54.4	800	610	183	60
15230	1000	600	25.1	1568	254	156	50
16393	680	1150	29.2	1775	561	236	44
16776	790	60	33.0	840	33	175	44
17556	510	600	36.4	1055	357	155	35
18393	1390**	950	20.1	2310	326		
18932	1290	731	25.8	1975	318	238	36

* These momenta were obtained from momentum balance about an origin.

** These momenta were computed from the known momentum, angle, and Q-value.

*** These momenta were computed assuming $P_+ = P_-$ from θ and Q.

TABLE II (Continued)

Identification No.	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_0 (Mev/c)	$P_- \sin\theta$ (Mev/c)	Q (Mev)	ΔQ (Mev)
19065	1460*	1460*	16.0	2900	403		
19143	375	820	46.0	1112	590	238	30
19731	640	630	47.5	1195	465	279	70
21645	554	895	19.3	1430	296	100	86
22332	620	481	44.5	1022	338	215	17
22638	800	1720	16.4	2500	485	167	38
23035	1120	3280	13.7	4400	776	273	96
23569	142	900**	20.6	1035	316	117	50
23586	1120	4100	8.9	5320	634	176	44
23855	290	2180	14.8	2460	556	191	106
24097	550	700	42.9	1120	476	253	74
24107	613	58	49.1	652	44	148	51
24287	1570	1950	15.4	3490	518	228	45
24551	880	4300	5.6	5160	420	135	50
24740	735	35	87.2	735	35	220	14
27136	560	1450	37.2	1925	877	374	110
27448	1000	3880**	14.2	4855	950	320	76
27502	210	770**	37.1	946	465	140	60

* These momenta were computed assuming $P_+ = P_-$ from θ and Q.

** These momenta were obtained from momentum balance about an origin.

TABLE II (Continued)

Identification No.	P_+ (Mev/c)	P_- (Mev/c)	θ (degrees)	P_o (Mev/c)	$P_- \sin\theta$ (Mev/c)	Q (Mev)	ΔQ (Mev)
28138	300	179*	91.7	346	179	157	100
28181	1900	235	35.9	2095	138	316	155
28484	510	1860	15.4	2360	494	158	40
28544	552	288*	74.8	663	278	286	100
29634	382	2370**	17.4	2735	709		
29772	203	336	102.0	356	336	216	60
32961	1063	870	23.7	1891	350	202	17
33280	229*	705	57.8	850	596	222	19
33509	487	1650	20.2	2115	570	176	111
34007	1200	480	33.3	1622	263	252	72
34606	836	490	47.0	1225	358	302	120
34944	418	1600	23.2	1990	630	200	68
35104	772	500	36.0	1213	294	196	36
35473	655	1300	24.0	1916	529	204	72
35508	1020	1650	14.8	2650	421	154	112
35784	78	950	19.6	1024	318	175	93
35889	895**	303	42.5	1136	204		
35898	1900	2030	10.0	3920	352	121	62

* These momenta were obtained from momentum balance about an origin.

** These momenta were computed from the known momentum, angle, and Q-value.

Table III contains the data for the Λ^0's which pertain directly to the lifetime computation.

TABLE III

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
2175	1.00	13.46	.075	0.56	7.57
2264	2.08	12.46	.168	1.20	7.17
2466	0.34	15.03	.022	0.16	7.20
2772	2.35	11.73	.200	3.49	17.4
3223	0.82	15.89	.052	0.59	11.5
3427	5.53	20.80	.266	6.95	26.2
3456	5.94	17.82	.334	3.28	9.88
3844	0.96	12.15	.079	0.63	8.00
4173	3.78	17.48	.216	2.41	11.1
4410	1.47	21.30	.069	1.13	16.4
4609	5.40	12.40	.435	4.58	10.5
4769	0.44	5.98	.073	0.35	4.79
5369	1.26	3.56	.353	1.60	4.52
6260	3.71	13.39	.277	3.04	11.0
6898	0.34	2.12	.160	0.22	1.34
7384	1.74	12.28	.142	0.96	6.80
7509	4.52	12.60	.359	2.98	8.30
7563	3.94	13.55	.290	4.46	15.4
7678	2.92	12.17	.240	1.37	5.70

TABLE III (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
7939	1.50	2.14	.700	0.97	1.39
8346	2.80	14.16	.198	1.95	9.86
9150	2.11	10.14	.208	1.10	5.29
9207	2.02	7.26	.279	1.62	5.85
9464	8.25	12.40	.665	3.63	5.45
9899	3.87	14.93	.259	1.64	6.35
10144	11.37	14.41	.789	9.02	11.4
10252	11.79	15.06	.783	12.21	15.6
10257	9.11	12.05	.756	4.52	5.98
10450	10.00	11.64	.859	3.07	3.57
10670	5.86	12.05	.486	3.58	7.36
11036	4.90	12.30	.398	5.71	14.3
11118	1.71	15.96	.108	0.68	6.32
11957	0.83	12.25	.067	0.57	8.45
12073	8.64	13.01	.664	6.38	9.61
12181	1.65	4.11	.400	0.97	2.42
12986	5.98	14.37	.416	4.30	10.3
13087	3.98	8.26	.486	2.12	4.40
13147	0.13	2.29	.058	0.12	2.21
13386	11.07	14.98	.740	4.82	6.53
14316	0.26	1.55	.170	0.14	0.86

TABLE III (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
14979	0.96	13.09	.074	1.43	19.5
15874	3.71	41.80	.089	1.20	13.5
15896	16.37	37.14	.440	6.49	14.8
16003	18.91	29.62	.640	5.80	9.09
16460	0.84	13.29	.064	0.46	7.28
16863	2.98	15.01	.199	3.46	17.4
17609	2.78	5.40	.514	1.47	2.85
18268	4.34	17.98	.242	1.83	7.59
18290	5.52	26.30	.210	4.15	19.8
19232	7.76	42.75	.181	2.90	16.0
19606	5.80	7.48	.776	2.81	3.62
20968	1.27	13.80	.092	0.66	7.20
22012	2.27	12.94	.175	0.87	4.95
22055	1.17	9.38	.125	1.18	9.45
22425	5.50	11.50	.479	2.29	4.79
22451	2.67	11.83	.226	2.50	11.1
22815	11.74	33.38	.352	4.65	13.2
22971	2.19	13.64	.160	1.72	10.7
23023	6.30	32.91	.192	3.06	16.0
24004	5.04	11.45	.440	2.51	5.70
24114	9.58	35.96	.266	8.13	30.5

TABLE III (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
24420	7.32	11.97	.613	4.07	6.65
24537	5.20	18.31	.284	2.17	7.64
24694	7.19	15.45	.465	3.18	6.85
26797	3.39	14.25	.238	1.16	4.85
27137	1.46	10.27	.142	0.74	5.20
27552	7.34	34.19	.215	3.67	17.1
27801	3.22	7.69	.420	1.42	3.40
27811	0.60	4.58	.131	0.26	2.00
28175	0.22	22.87	.010	0.10	10.1
28212	4.19	37.67	.111	1.13	10.2
28381	1.52	4.15	.367	1.00	2.74
28650	8.41	12.83	.655	5.50	8.40
29101	1.30	21.50	.060	0.65	10.7
29129	2.18	12.04	.181	2.21	12.2
29569	2.72	9.10	.300	1.40	4.68
29833	2.43	10.02	.243	1.08	4.45
29903	2.20	4.63	.475	1.77	3.72
32087	1.26	12.05	.105	1.22	11.67
32170	0.12	30.93	.004	0.06	15.61
32267	1.14	7.00	.163	0.85	5.19
33280	3.24	11.17	.291	2.62	9.05

TABLE III (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
33603	0.02	6.72	.003	0.01	3.58
33755	1.73	12.95	.134	1.07	7.98
34528	8.80	16.88	.521	5.45	10.45
34845	4.18	16.66	.251	2.76	11.00
34955	4.90	37.34	.131	1.86	14.21
35265	1.44	20.43	.070	1.04	14.75
35363	3.71	13.81	.268	3.30	12.29
35386	5.20	8.36	.621	2.95	4.74
35665	1.66	11.77	.141	0.93	6.58
35800	8.49	13.05	.650	9.60	14.76
35912	2.97	13.05	.228	1.48	6.50

Table IV contains the lifetime data for those θ^0's which were selected on the basis of the positive being identified as a meson.

TABLE IV

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
2469	8.55	16.96	.505	1.05	2.08
4417	2.76	10.52	.262	0.32	1.23
4559	11.64	18.35	.635	1.16	1.82
4688	9.57	10.45	.915	1.41	1.54
5205	4.24	7.69	.553	1.15	2.09
6233	7.66	15.00	.511	2.40	4.70
6700	11.07	16.05	.690	0.55	0.80
8814	0.73	13.60	.054	0.14	2.68
8895	0.31	5.38	.058	0.13	2.23
9369	2.21	10.26	.216	0.14	0.67
9494	4.34	8.23	.528	0.73	1.39
9529	5.61	12.65	.444	0.91	2.06
9630	2.91	18.80	.155	0.69	4.48
10782	3.66	11.16	.328	3.00	9.14
10930	4.25	14.70	.289	2.76	9.53
12633	3.47	21.40	.162	0.99	6.09
15110	22.40	36.68	.610	4.61	7.55
16393	0.96	27.87	.034	0.09	2.59
16776	4.90	5.08	.964	0.96	1.00
17556	3.04	10.41	.292	0.47	1.63

TABLE IV (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
19143	0.82	27.37	.030	0.12	4.05
21645	5.17	7.08	.731	0.60	0.82
22332	4.10	29.04	.141	0.66	4.68
22638	12.57	16.45	.764	0.83	1.08
23569	9.84	13.28	.741	1.57	2.11
24107	5.39	5.51	.976	1.36	1.39
24551	5.75	15.90	.361	0.18	0.51
24740	5.22	26.11	.200	1.17	5.85
27136	3.09	10.70	.289	0.26	0.92
27502	5.55	12.03	.462	0.97	2.09
28138	1.79	16.15	.111	0.85	7.69
28484	7.55	12.75	.591	0.53	0.89
28544	2.36	6.77	.348	0.59	1.68
29634	7.52	9.91	.759	0.45	0.60
29772	0.75	7.08	.105	0.35	3.28
33509	6.66	26.40	.252	0.52	2.06
34944	15.89	30.00	.530	1.31	2.48
35104	5.16	18.55	.278	0.32	1.15
35473	7.75	8.15	.951	0.67	0.70
35784	3.43	12.86	.268	0.55	2.07

Table V contains the lifetime data for those θ^{0}'s which were selected on the basis of having $P_{-} \sin\theta > 115$.

TABLE V

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
2469	8.55	16.00	.535	1.05	1.97
4417	2.76	13.91	.198	0.32	1.62
4559	11.64	13.28	.878	1.16	1.32
4688	9.57	14.32	.666	1.41	2.11
5828	13.08	17.39	.753	0.91	1.21
5979	11.22	15.70	.715	0.67	0.94
6233	7.66	18.01	.425	2.40	5.64
6482	0.80	1.68	.474	0.14	0.30
6700	11.07	15.14	.730	0.55	0.75
7651	4.30	13.04	.330	0.57	1.73
8814	0.73	6.80	.107	0.14	1.34
8895	0.31	10.62	.029	0.13	4.39
9369	2.21	8.95	.253	0.14	0.58
9494	4.34	14.70	.296	0.73	2.48
9529	5.61	11.51	.488	0.91	1.88
9630	2.91	16.44	.177	0.69	3.92
10909	2.55	15.36	.166	0.11	0.66
10930	8.55	14.53	.588	5.54	9.42
11753	2.12	13.90	.152	0.42	2.77
12633	5.28	19.01	.277	1.50	5.41

TABLE V (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
12786	6.20	19.52	.318	0.52	1.65
13075	6.02	15.36	.392	0.36	0.91
15110	22.40	35.23	.635	4.61	7.25
15230	0.89	10.90	.081	0.09	1.14
16393	0.96	36.68	.026	0.09	3.40
17556	3.04	17.47	.174	0.47	2.73
18393	4.39	17.19	.255	0.31	1.23
18932	2.20	36.59	.060	0.18	3.05
19065	7.13	8.82	.809	0.40	0.50
19143	0.82	36.20	.023	0.12	5.36
19731	2.66	13.94	.191	0.37	1.92
21645	5.17	10.08	.514	0.60	1.16
22332	4.10	37.91	.108	0.66	6.11
22638	12.57	33.18	.379	0.83	2.19
23035	1.97	33.59	.059	0.07	1.26
23586	4.18	34.41	.121	0.13	1.07
23855	14.55	21.62	.674	0.97	1.45
24097	14.90	21.40	.697	2.19	3.15
24287	0.99	33.58	.029	0.05	1.58
24551	5.75	28.39	.202	0.18	0.91

TABLE V (Continued)

Identification No.	x (cm)	D (cm)	$\frac{x}{D}$	t (10^{-10}sec)	T (10^{-10}sec)
27136	3.09	6.61	.467	0.26	0.57
27448	1.19	13.01	.092	0.04	0.44
27502	5.55	13.20	.421	0.97	2.30
28181	5.41	29.70	.182	0.43	2.33
28484	7.55	27.07	.278	0.53	1.89
28544	2.36	3.20	.738	0.59	0.79
29634	7.52	8.44	.892	0.45	0.51
29772	0.75	7.64	.098	0.35	3.53
32961	0.13	35.83	.004	0.01	3.12
33280	1.20	38.80	.031	0.23	7.52
33509	6.66	28.60	.233	0.52	2.27
34007	1.42	13.43	.106	0.14	1.36
34606	3.94	4.74	.831	0.53	0.64
34944	15.89	29.75	.535	1.31	2.46
35104	2.27	12.65	.179	0.31	1.72
35473	7.75	20.05	.385	0.67	1.72
35508	5.16	18.55	.278	0.32	1.15
35784	3.43	6.73	.506	0.55	1.08
35889	4.15	19.22	.216	0.60	2.79
35898	8.65	29.21	.296	0.36	1.23

V. THE RESULTS

A. The Λ^0 Lifetime

There were 93 Λ^0 decays identified by the selection criteria previously discussed. For 82 of these it was possible to make enough additional measurements so that a Q-value could be computed which can be compared with the assumed value of 35 Mev. The weighted mean of these cases yields a value

$$Q = (35.6 \pm 1.0) \text{ Mev.}$$

This average was computed assuming that the percentage error in Q is independent of Q, and therefore the error in log Q is normally distributed. Thus the logarithm of Q was weighted inversely as the square of the percentage error. The error in the mean is half statistical and half systematic, where a 0.5 per cent error in the value of the magnetic field is the basis for estimating the systematic error.

As previously stated, the distribution around the mean is consistent with a gaussian distribution of error with a probable error even smaller than the assigned errors would indicate. This is demonstrated by the fact that only 22 cases were more than one assigned error from the mean, and only two cases were more than two assigned errors from the mean.

The mean lifetime of the Λ^0 particle based on these 93 cases is

$$\tau = (3.6 \pm 0.6) \cdot 10^{-10} \text{ sec.}$$

The lifetime was computed from the formula

$$\tau = \frac{1}{N} \sum_{i=1}^{N} \left(t_i + \frac{T_i}{e^{T_i/\tau} - 1} \right) , \qquad (9)$$

which is the result of applying the maximum likelihood procedure(10) to the data. It is seen that this is not explicitly solved for τ, but is in a convenient form for iterative computation.

The error quoted here is purely statistical and is computed from a formula which gives the number of cases, n, of infinite gate length which would be statistically equivalent to the N cases of the sample:

$$n = \sum_{i=1}^{N} \left(1 - \frac{T_i^2}{\tau^2} \frac{e^{T_i/\tau}}{\left(e^{T_i/\tau} - 1\right)^2} \right) \qquad (9), (10)^*$$

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* Alford(9) has pointed out that this formula is a function of T_i/τ and therefore requires knowledge of τ, while the distribution of $t_i/T_i = x_i/D_i$ is a measure of significance which can be found directly from the experimental data. In particular he states that the mean value $\langle x/D \rangle$ must be significantly less than one-half if one is to calculate a lifetime from the data, since the distribution of decay points throughout the chamber would be uniform if τ were large compared to the gate times. This assumes that the gate times are more or less the same for all cases. In particular, this is not true in this experiment where two different sizes of cloud chambers were used. The value of $\langle x/D \rangle$ could be very near one-half because of a preponderance of cases in the small chambers, but a few good cases in the large chamber would still permit a meaningful lifetime to be computed. For this reason, although a column of x/D values is listed in the data tables, no use has been made of the distribution of this quantity.

From the assumptions made in obtaining the selection criteria, and from the consistency of the data, it is felt that at most two cases are not Λ^0 particles. Unless these should have extreme lifetimes, their effect on the result would be negligible.

Some additional uncertainty arises from the limitations in accuracy inherent in the measurement techniques. The length measurements can be made to greater precision than necessary to assure no error from this source. The graphical constructions required to make these measurements introduce some uncertainty but with sufficient care the accuracy of the line segments to be measured is probably limited by the momentum measurements necessary to construct the line of flight.

The momentum measurements are also essential to convert distances to times. For the majority of tracks the momenta are directly measurable to an accuracy of ten to thirty per cent. In order to avoid biases, a number of cases were included where less measurability prevailed. In 23 cases the momentum of the positive secondary was obtained from ionization estimates good, for the most part, to no better than thirty to fifty per cent. In 11 cases the momentum of the negative was computed from that of the proton and the assumed Q-value. This procedure yields errors comparable with the input momentum since it is approximately true on the average that

$$\frac{\partial P_-}{\partial P_+} \approx \frac{P_-}{P_+} \quad .^*$$

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* Note that the inverse procedure, calculating the momentum of the proton from that of the meson and the Q-value, is subject to large errors and hence was avoided.

Thus in the majority of cases momenta were measured to better than thirty per cent, and seldom was a momentum determined to worse than fifty per cent. From these considerations it seems one is justified in concluding that the effect of these errors on the mean lifetime is small compared to that which results from the limited statistics.

B. The θ^0 Lifetime

There were 76 cases identified as being not Λ^0 particles. Of these, nine cases were clearly anomalous as determined by a Q-value, computed as for a normal θ^0, which differed by more than twice the assigned error from 214 Mev. Of the remaining 67 cases, sufficient additional measurements were possible in 58 cases to compute a Q-value. The mean Q-value computed by the same method as for the Λ^0 particle is

$$Q = (214 \pm 5) \text{ Mev.}$$

There was, of course, some bias toward 214 Mev from the way anomalous cases were eliminated, nevertheless the agreement between this result and the accepted value as measured by Thompson(6) is striking. The error here includes about 3 Mev statistical uncertainty and 2 Mev systematic error due to field measurements.

Only 17 cases had Q-values which differed by more than one assigned error from the mean. This does not count those nine cases which were removed from consideration for being more than two assigned errors away from 214 Mev.

There were 40 cases which could be identified as having a light meson positive secondary. The lifetime computed for these cases is

$$\tau = (2.1 \pm 0.8) \cdot 10^{-10} \text{ sec.}$$

There were 60 cases which could be identified on the basis of P_- sin θ and the lifetime computed from these cases is

$$\tau = (1.3 \pm 0.3) \cdot 10^{-10} \text{ sec.}$$

The difference between these two determinations, while not so large as to be statistically unlikely, is sufficiently large so that a careful examination of the other sources of error is in order. One would like to understand any situations which might cause one of these selection methods to be superior to the other, with a view toward determining the best value based on all the information available.

With regard to distance measurements and graphical constructions, the same remarks made for Λ^0 particles apply to the θ^0 particles. Essentially, it is felt that no error is introduced from this source which is not entirely overshadowed by the errors in momentum measurements.

The majority of tracks have directly measurable curvatures. However, the average momentum of the θ^0 particles of this study is larger than that of the Λ^0 particles, therefore, since the percentage error in a momentum measurement is approximately proportional to the momentum, the errors here are larger than for the Λ^0 particles. In ten cases the momentum of one track is found from momentum balance about an origin. These origins are frequently difficult to locate accurately because of the necessity for extending tracks considerably beyond the visible region, hence large errors in the angles the tracks make with the line of flight result and cause large errors in the computed momentum. In nine cases no origin could be located and the unmeasurable momentum was computed using knowledge of the Q-value. Since clearly the approximate equality

$$\frac{\partial P_+}{\partial P_-} \approx \frac{P_+}{P_-}$$

is true on the average for θ^o decays from symmetry, the computed momentum has about the same percentage error as the measured momentum. From the standpoint of accuracy, this procedure would seem preferable to using momentum balance, however, it was considered important to use all available information to compute Q-values whenever one could, rather than assume this knowledge, in order to eliminate as many anomalous θ^o decays as possible.

For two cases so little information was available that one resorted to guessing that the relation between the momenta of the two secondaries would be the most likely one, namely $P_+ = P_-$. The errors in such a procedure are obviously likely to be very large.

On the whole it is felt that the errors in determining the momenta, especially for those high momentum cases which are frequently selected by the P_- sin θ criterion, are large enough to have some influence on the result. For lack of a good quantitative estimate, and since this source is less likely to cause a really serious error than sample contamination, this error is not included in the quoted results.

The really insidious source of error in the lifetime of the θ^0 particle is the contamination from anomalous cases. As previously discussed, no effective means for their elimination which would not disastrously reduce the sample size could at this time be devised. The full effect of contamination cannot be estimated since the mean lifetime for the anomalous decays is not known. One can only suppose what might happen if their lifetime were comparable to that for normal θ^0 particles.

Firstly, if the anomalous θ^0 decays are three body decays, as has been previously suggested, it is immediately obvious that one will underestimate the momentum of the primary by computing it from the momenta of the visible secondaries. Hence the computed lifetimes for these cases will be too high. Again, if one constructs lines of flight using only the visible momenta, these are likely to have entirely the wrong directions, thus grossly distorting the measurements of x, D, and even x/D. In general this would tend to overestimate x and D since any deviation from the usual downward direction will usually increase the length of passage through the chamber. One would thus expect, from both these considerations, that the real lifetime of the anomalous cases would be overestimated by this method, and if their lifetime is comparable to that for normal θ^0 particles, their presence in the sample will cause the value calculated to be too high.

One next reviews the selection methods to see if either method biases for or against the selection of anomalous θ^0 decays. On reflection, it seems likely that for roughly the same reason that the Q-value for anomalous cases is low, P_- sin θ will also be low. In support of this proposition, the following table lists the Q-values and the values of P_- sin θ for the nine identified anomalous cases.

TABLE VI

Identification No.	Q (Mev)	ΔQ (Mev)	P_- sin θ (Mev/c)
4480	51	29	112
15329	45	25	210
20411	57	15	139
19143	41	5	56
24746	128	35	276
25927	150	15	125
16847	61	50	190
31855	10	1	40
35045	17	17	100

When these values of P_- sin θ are compared with those for the sample of θ^0 particles as a whole, the average of which is 419 Mev/c, it is clear that a correlation does exist between low Q and low P_- sin θ.

From this it would seem to follow that selection on the basis of high P_- sin θ not only eliminates Λ^0 particles, but also biases strongly against anomalous θ^0 decays. Thus the sample selected on the basis of having a meson positive secondary, not enjoying this favorable bias, would contain a higher percentage of anomalous cases. This could explain, at least in part, why the lifetime computed for this sample was high compared to the more accurately computed value based on P_- sin θ selection. Indeed, one might expect that the cases selected by identification of the positive secondary which do not have high P_- sin θ would have a high concentration of anomalous θ^0 decays. There were only seven such cases, and their Q-values, all calculable to some degree, are listed below for inspection.

TABLE VII

Identification No.	Q (Mev)	ΔQ (Mev)
10782	169	37
16776	175	44
24740	220	14
28138	157	100
23569	117	50
5205	127	90
24107	148	51

As is readily seen, all but one case is below 214 Mev, and at least three cases seem suspiciously low. In addition it is worth noting from table IV that the average lifetime for these seven cases is above the overall average.

One concludes, from these arguments, that the sample selecting high P_- sin θ is superior both statistically and from the standpoint of contamination. Since the primary goal has been to obtain pure samples, this apparent aid to that end is considered sufficiently important to warrant naming the mean lifetime as computed by that method of selection the most likely θ^0 lifetime based on the observed data.

One can now estimate an upper limit for the number of anomalous cases to be found in the sample of 60 cases selected for high P_- sin θ. If one deducts from the total number of cases identified as not Λ^0 particles the nine cases with completely unmeasurable Q-values, and a like number of other poorly measurable cases, one is left with 58 cases among which were found nine anomalous events, or about one in six. Thus, not considering that among the high P_- sin θ cases the ratio should be considerably less, one would expect about three anomalous decays among the poorly measured cases. Unless there is a maximum in the anomalous θ^0 Q-value distribution around 214 Mev, this should mean an upper limit of five per cent contamination in the sample. It is felt that unless the lifetime of this five per cent differs greatly from that for normal θ^0 particles, their effect on the result would be smaller than the statistical error.

VI. CONCLUSIONS

The lifetime of the Λ^0 particle has been measured to be $(3.6 \pm 0.6) \cdot 10^{-10}$ sec. It is felt that unique characteristics of this decay have enabled a pure sample to be selected. It is also believed that the selection system was objective and unbiased, and that the relatively large number of slow cases selected enabled the errors of measurement to be small compared to the statistical error due to the smallness of the sample.

Since the labor involved in treating on the order of 100 cases by this method is considerable, it is doubtful that the accuracy of this result should be substantially improved by going to larger samples. It is true, however, that the error using 100 cases could be reduced to nearly ten per cent by using larger chambers and slower Λ^0 particles, thus increasing the gate times. If the lifetime is to be measured more precisely than this, methods must be devised which do not require for each statistic the careful analysis demanded by a cloud chamber photograph.

The most probable lifetime for the θ^0 particle based on the data of this study is believed to be $(1.3 \pm 0.3) \cdot 10^{-10}$ sec. The error stated here is only the statistical error inherent in the limited sample size, and should, perhaps, be increased somewhat to account for errors in momentum measurements and sample contamination. It is hoped that the selection system used to obtain this result, while designed to distinguish θ^0 particles from Λ^0 particles, has eliminated a substantial number of anomalous cases, and an upper limit of five per cent contamination has been estimated.

With a better understanding of the anomalous decays it should be possible to devise more effective ways to eliminate them from a sample of θ^o particles. When this is done and one has confidence in the purity of the sample, it may be worthwhile to make use of a larger number of cases both to improve the statistics and to enable more restrictive selection criteria to be used so as to increase the accuracy of the momentum measurements. Here again, however, ten per cent is considered a practical lower limit on the error attainable by the use of cloud chambers.

TABLE VIII. PREVIOUS LIFETIME DETERMINATIONS

OF THE Λ^0 AND θ^0 PARTICLES

Experimenters	Particle	Mean Lifetime (10^{-10} sec)	No. of Cases	Reference
Rochester and Butler	V^0	$5 \cdot 10^2$	1	(11)
Seriff, et al	V^0	(3 ± 2)	30	(12)
Alford and Leighton	Λ^0	(2.5 ± 0.7)	74	(13)
	Λ^{0*}	(2.9 ± 0.8)	37	
	Λ^{0**}	(1.3 ± 0.5)	20	
Fretter, et al	Λ^0	(10 ± 7)	18	(14)
	θ^0	(4 ± 3)	11	
Bridge, et al	Λ^0	(3.5 ± 1.2)	24	(15)
	θ^0	(1.2 ± 0.5)	4	
J. P. Astbury	θ^0	$(1.6 {}^{+1.6}_{-0.6})$	11	(16)
Deutshmann	Λ^0	$(4.8 {}^{+2.6}_{-1.3})$	22	(17)
	θ^0	$(2.3 {}^{+2.1}_{-0.7})$	9	(18)
Page and Newth	Λ^0	$(3.7 {}^{+3.9}_{-1.3})$	26	(17)
D. B. Gayther	Λ^0	$(4.0 {}^{+3.7}_{-1.2})$	21	(18)
	θ^0	$(1.2 {}^{+0.8}_{-0.3})$	8	
D. I. Page	Λ^0	$(3.6 {}^{+1.1}_{-0.7})$	23	(19)
	θ^0	$(0.7 {}^{+0.3}_{-0.2})$	14	(20)

* Cases with measured energy release $Q < 50$ Mev

** Cases with measured energy release $Q > 50$ Mev

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