DESIGN OF AN INSTRUMENT FOR MEASURING THE TRUE VERTICAL VELOCITY OF AN AIRPLANE AT MOMENT OF CONTACT IN LANDING

Thesis by

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SUMMARY

In the design of the Vertical Velocity Meter a survey was first conducted of the existing methods used to measure the vertical velocity of an airplane when landing. A study was then made of these methods and of other unsuccessful attempts to design a Vertical Velocity Meter to determine just why they were not satisfactory and in what way they might be improved.

It was decided to develop a design along an entirely different principle than any used before but yet [redacted] one having basic simplicity. The one selected was based on the formula of velocity being equal to a distance divided by the time it took to travel that distance. To accomplish this a device was designed which consisted of two rods of different length which would strike the ground when the plane was landing, thus if the difference in length and the time between the striking of the two rods is known, the velocity at that moment would be determined.

Since it was impracticable to test this device on an airplane, a testing machine was constructed which would test both the strength of the rods and various timing methods. This machine consisted of a wedge mounted on the end of a whirling arm which would strike the rods and simulate the impact received during the landing of an airplane. Two timing methods were tried; a Miller Oscillograph with built in recorder, and a Cathode-Ray Oscilloscope with a camera to record the trace.

INTRODUCTION

Airplane manufacturers and operators have desired, for many years, to be able to measure accurately the vertical impact velocity of landing aircraft in order to know more accurately the landing loads imposed and thus design the structure and landing mechanism accordingly. It is the purpose of this thesis to present a method of determining this true vertical velocity of an airplane as it contacts the ground when landing.

Many methods, devices and instruments have been considered for the determination of the sinking speed of an airplane at the instant of landing. The extent of investigations of the practicability of all of these means is not known. Three methods have been used with varying degrees of success. Two of these methods are photographic, utilizing motion picture cameras on the ground. The other method used a sensitive accelerometer to measure the vertical accelerations up to the time that the wheels contact the ground.

The important features of the photographic methods are as follows:

1. Fixed Camera Method - This method was developed by the Structural Dynamics Section of the Aero Structures Laboratory of the Naval Air Experimental Station for use on landing tests. One or more Mitchell 35 mm motion picture cameras are used depending upon the area in which touchdown is anticipated and how extensive a study of the action subsequent to initial contact is to be made. Special framing masks are used in the cameras to indicate reference planes, normal and parallel to the deck and intersecting at the optical axis of the camera lens. The cameras are leveled visually with the aid of stadia rods.

Pictures of the stadia rods are taken to provide records for correction of errors in camera level and to establish the height of the horizontal reference plane above the deck.

One camera is located approximately 100 feet from the most probable point of touchdown and another approximately 250 feet from this point. Both cameras are placed 50 to 60 feet from the runway centerline. The cameras are adjusted in include a reasonable portion of the landing approach as well as contact so that a study of the approach can be made.

The cameras are operated at a speed of 100 frames/second but since the camera speed cannot be relied upon for an accurate time record, a precision timer with 0.01 seconds divisions of the dial is included in the field of view of each camera. The cameras are remote-controlled for safety of personnel.

Film analysis is done on a Recordak Reader using image size to actual object size (such as wheels, wing span, etc.) relationships to determine the height of the airplane above the horizontal reference plane. The space-time curve thus obtained is differentiated to obtain a sinking speed-time curve.

2. Panned (Rotating) Camera Method - This method was developed by the Grumman Aircraft Engineering Corporation for use on landing tests for the Navy. The method employs an external wire grid with a 16 mm movable camera photographing the airplane through this grid. The camera is pivoted both vertically and horizontally about one of the nodal points of the camera lens system to eliminate errors from camera rotation.

The grid and camera are located to one side of the anticipated landing area at distances up to 500 feet for field tests and on the deck edge for carrier tests. The vertical and horizontal spacing of the grid wires and the distance of the camera from the grid are varied to meet the test requirements and space limitations.

The space-time curve is determined by use of similar right triangle relationships, as the distance from the camera to the grid is known and distance of the airplane from the camera can be found by use of lens formulae. The time record is based on the film speed of 20 frames/second as the camera is driven by a constant speed motor.

Both of the above have proved to be the most satisfactory methods available for field and carrier landing tests. However, the amount of equipment required is rather extensive which reduces the flexibility of both methods. Careful set up of apparatus for both methods is essential for accuracy. Adverse lighting conditions may reduce the quality of the pictures to the point where the accuracy of the methods are seriously affected. Prolonged landing tests of the Lockheed P2V were made by the Lockheed Engineering Corporation, using a method similar to the second one described above. In these tests it was apparent that to obtain any reasonable degree of accuracy, a steady rate of descent must be maintained for quite a distance before contacting the ground. Neither of the above methods can be reliably used where a sudden change in sinking speed is affected just before contact, such as a full stall drop landing or a flare at the last of the landing.

Sensitive accelerometers have been used by several aircraft manufacturers to determine sinking speed during landing tests. How successful this means was is not known, as no reports of the results have been received. However, it is understood that the response of these sensitive instruments to vibrations and to local accelerations, resulting from the flexibility of the airplane structure, may produce erratic records.

The other methods which have been considered generally by the industry and specifically by NAES are as follows:

1. RADAR

(a) Ground Tracking - Rejected because the instantaneous range is not sufficiently accurate and ground reflection interference would produce further errors.

(b) Airborne - Rejected because present equipment is not capable of measuring the small distances involved and the development time of such equipment of usable size and weight would have been too long for use on present landing tests.

2. ALTIMETERS

(a) Sensitive Aneroid - Rejected because of insufficient sensitivity and the instrument lag which would produce serious errors.

(b) Radio - Rejected for the same reasons as Airborne Radar.

(c) Super-Sonic - Rejected because no such equipment was known to be available and development time would have been prohibitive.

3. PHOTO-THEODOLITE

(a) This instrument has not been tried because none have been available to the NAES. It is not known whether any other organization has used this method.

4. MECHANICAL DEVICES

(a) Devices such as telescoping rods, extremely flexible cantilever beams, etc., projecting below the landing gear were not considered practical as the rebond of such mechanisms on striking the ground would produce erratic records resulting in large errors.

5. ANOTHER PHOTOGRAPHIC METHOD

(a) One other photographic method has been considered but not fully investigated. This method used a fixed constant speed motion picture camera located 20 to 30 feet above the ground and to the side of the flight path. A reference plane parallel to the ground at the height of the camera is indicated on the film by a line drawn through the vanishing points of the images of parallel lines on the ground. The chief advantage of this method over the other photographic methods is the reduction in quantity of equipment required.

Many airplane manufacturers have conducted extensive drop tests and other tests and then attempted to correlate this data to the airplane contact velocity. An excellent example of this is found in Lockheed Aircraft Corporation's Report No. 5557, Addendum 1. The object, procedure and summary of this report are given here to illustrate the need for a device that will accurately determine the vertical velocity of the airplane at the moment of contact.

LOCKHEED REPORT NO. 5557 ADDENDUM 1

Object

The purpose of the series of landing tests described in this report was to obtain measurements for determining the performance of the main landing gear shock struts.

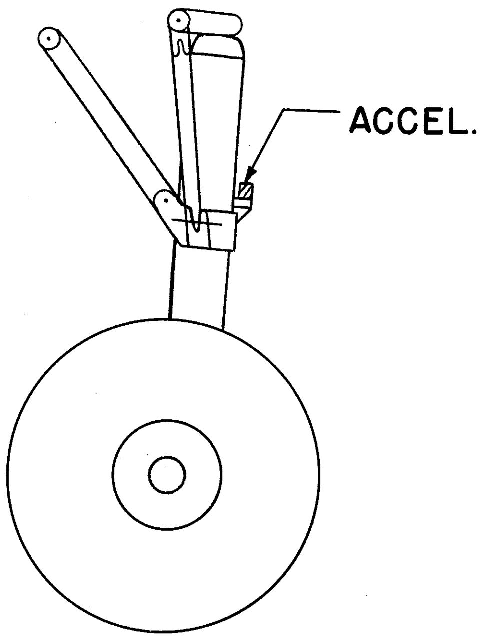
Measurements of landing gear loads, shock strut deflection, vertical acceleration at each main gear, and airplane contact velocity were made to determine if there was any correlation between these factors and also if there was any correlation between measurements made during actual landings and those obtained in drop tests. In an attempt to eliminate the effect of various pilot techniques the landings were made with no flare, that is, a certain rate of descent was set up and that rate maintained right into the ground.

Procedure

Landing gear loads were obtained by measuring the loads in the drag and side struts and calculating the gear loads required to produce the measured strut loads. The drag and vertical loads were assumed to be half compressed.

A positive load is to the left for a side load, up for a vertical load, and aft for a drag load. The gear loads are resolved into load planes parallel and perpendicular to the ground with the airplane in its static position. (F.R.L. tipped back $2\frac{1}{2}$ degrees to the ground line).

During each landing measurements were taken of the side and drag strut loads, strut position, both wheel RPM's on the left gear, and the vertical acceleration at both gears. The location of the accelerometers is shown in the sketch below.



Wheel RPM measurements were made with small D.C. generators geared directly to the wheel. Accelerations were measured with strain gauge, oil damped, beam-type accelerometers and shock strut position was measured with slidewire resistance bridges. The rate of descent of the airplane was measured photographically using the landing and take-off grid.

All measurements except rate of descent were recorded with a Miller recording oscillograph. The output of the strain gauge bridges was connected directly to the oscillograph galvanometers without intermediate amplifiers. A bad feature of the landing measurements results from the fact that the oscillograph was not shock mounted and the trace deflections were affected by acceleration loads on the elements. This affected the accuracy of the gear load and acceleration measurements but did not affect the strut position measurements since these elements were of the high frequency type which are little affected by acceleration. The acceleration affects probably tended to make the load measurements greater. The estimated accuracy of the loads is $\pm$ 20%.

Summary

Preliminary measurements of main landing gear performance were made on Model 49, Serial 1961.

Although the data obtained is insufficient and somewhat inaccurate, certain trends are indicated as listed below:

1. There was some correlation between measurements obtained during actual landings and drop tests. The major difference between the two conditions is the effect of wing lift.

2. The results indicate that the effect of wing lift decreases with increase in severity of landing. The data is insufficient for evaluating the relationship.

3. For a given contact velocity the vertical accelerations measured during an actual landing were greater than those obtained during drop tests. The measured gear vertical load is a function of the incremental acceleration but the increase in load per "g" of acceleration is greater than that calculated ($\Delta P = \frac{WGT}{2} \times \Delta g$).

4. The shock strut deflection for a limit condition landing would be approximately 11 to 13 inches as compared with a deflection of 16 inches obtained for similar conditions during drop tests.

5. There was no shock strut movement while taxiing at medium speeds on the taxi strips at Lockheed Air Terminal.

Information from all sources indicate that THERE IS A DEFINITE NEED FOR A COMPACT DEVICE FOR INDICATION OR RECORDING SINKING SPEED OF AN AIRPLANE IN LANDING AND IT IS DESIRABLE THAT THIS BE THE SINKING SPEED OF THE AIRPLANE WHEELS.

In view of the above it was decided that an entirely new principle should be found to measure this velocity. One method considered was a variation of the accelerometer method described above. This was to measure the deceleration of the plane after contact with the ground and integrate the area under the acceleration-time curve to give the change in vertical velocity and thus the initial velocity at time of contact. This method was discarded because of the same reasons as the original accelerometer method.

The method finally selected for design was a mechanical one, though somewhat of a deviation of the ones previously considered and described above. This consists of a device having a series of fingers or rods pointing downward, the lower ends of which are spaced a known distance above each other. The velocity of impact is then obtained directly by measuring the time differential of the rods striking the ground.

Basically this device may be divided into two main components, namely: the finger attaching and arresting mechanism which may be fastened to either or both of the main landing gears and to the nose or tail gear as the case may be; and the time measuring and recording device. Both are described and discussed in the following pages.

DISCUSSION OF PROBLEMS AFFECTING ANY MECHANICAL DEVICE THAT COULD BE USED TO CONTACT THE GROUND FROM A LANDING AIRPLANE

Any type of telescoping rod or a rod placed as a cantilever beam and fastened to the airplane will, upon striking the ground due to the airplane landing, rebound both from the impact of forward motion and from vertical motion. The amount of damping that would be required to eliminate this rebound would be prohibitive for use on any reasonable sized rod or device. Therefore, it is apparent that if such a mechanical device is to be used in measuring the sinking speed of an airplane at the time of touchdown, it must of necessity be unaffected by any such rebound.

Another factor which must be considered is that of the unevenness of runways, both as to gentle up and down slopes and to small roughnesses or irregularities. In considering the slope of the runway, it is well to ask just what is required in the measurement of sinking speed; the speed relative to the true horizontal plane or the speed of approach to the ground. Since it is the impact that is desired, it is, therefore, the normal speed of approach to the ground that is desired and any mechanical device will, of necessity, be dependent upon this latter approach speed. In regard to the small surface irregularities, the only way to use a mechanical device and not introduce large errors because of these irregularities is to take some form of an average reading.

A third problem to be investigated or discussed is that of the angle of attack of the airplane at the instant of touchdown when the measurements would be made. It seems reasonable to expect that for any series of landings, the angle of attack of the airplane at touchdown can be predicted within plus or minus five degrees of the actual value. Since the cosine of five degrees is 0.99619, the possible error arising from this factor would only be of the order of 0.4%

It was primarily the first two factors that dictated the basic design of the device to be used in measuring the vertical velocity at landing. To eliminate the first factor of rebound, it was decided to use a rod and to measure its initial contact with the ground. To provide a measure of the sinking speed, it was then necessary to provide a second similar rod [redacted] spaced slightly above the first rod and then measure the time differential of the rods striking the ground. The second factor of irregularities in the runway prompted the use of more than two rods in order to obtain an average value by measuring the time between any two consecutive rods.

Having decided upon the basic system, many new problems arose. One of the most important of these was the question of how much impact the rods will stand without taking a permanent deformation. No direct answer to this problem could be found in literature so a theoretical analysis was made which, although not providing the final answer, provided information which would be of help in the tests of the rod strength. This theoretical analysis is presented as a supplement to this report.

Another problem which can not be completely solved, except as regards a particular type of plane or landing, is the problem of arresting the rod after the impact of landing. If the rod were spring loaded in the down position at all times, it would be worn off at the end and thus cause inaccuracies in the time measurements. If a cocking system is used with which the rod can be placed in the extended position after take-off and before the landing and which will let the rod "ride high" after the landing; then some thought must be given to the possibility of a second or multiple contact with the ground on one approach.

The latter would be especially true in carrier landings of airplanes equipped with tricycle gear in which the bounce landing may be much harder than the initial contact, especially on the nose gear. A suggestion as to the answer to this problem is made later in the discussion of the design to be used on an average airplane.

The final major problem, regarding the rods, is the aerodynamic effect of the airstream both from the vibration and the deflection standpoint. The answer to this was found in the NACA Technical Note No. 468, from which the drag coefficient to be used in calculations and the rod spacing was obtained.

AERODYNAMIC CONSIDERATIONS OF THE RODS

The determination of the drag coefficient to be used in the calculations of rod size, deflections and forces was made from NACA Technical Report No. 468.

By interpolation for rods between 0.25 and 0.50 inches in diameter, the drag coefficient is selected as 1.20 for a single rod. The results of the Report indicate that to hold down vibration and to keep the interference drag as low as possible, the rod spacing should be at least three diameters between centerlines. For three diameters spacing, the interference drag coefficient is 0.15. Therefore, the total drag coefficient to be used with each of the rods, assuming more than two, is 1.35.

The drag, D, of each rod is then 1.35 q d L

where q is the dynamic pressure

d is the diameter of the rod

L is the length of the rod

or we can say that D/L = 1.35 q d

We may arbitrarily specify that for 120 mph the maximum deflection at the end of the rod should be held to 0.10 inches or less in the transverse direction. Then the drag per unit length, D/L = 0.345 d.

The deflection at the end of the rod, acting as a cantilever beam, is given by

$$\delta = (D/L)(L^4/8EI)$$

where $I = \pi d^4/64$ for a solid rod and nearly so for a thick walled tube.

For the longest rod selected, which is 20 inches and gives an L of 15 inches exclusive of the rod holder which will have a negligible deflection, the diameter necessary to hold the deflection within the limit will be given by

$$d = \sqrt[3]{0.001482/\delta} \quad = 0.246 \text{ inches}$$

For the tubular rod this dimension must be increased slightly, so we should use either a one-quarter inch or a 9/32 inch diameter rod.

We might also arbitrarily say that the rod should be held in the extended position up to an airspeed of 180 mph, though not necessarily holding within the above end deflection limit. Using the above drag coefficient and a rod diameter of 9/32 of an inch, the drag per unit length, D/L is 0.218 #/in. The moment about the supported upper end is then,

$$M = (0.218)(15)(12.5) = 41 \text{ inch pounds.}$$

To hold the rod in the extended position under the above moment, a volute spring producing a moment of at least 41 inch pounds should be placed at the upper end of the rod; or a helical spring producing a force of 20.5 pounds at an effective moment arm of two inches can be used either directly or under mechanical advantage.

STRENGTH CONSIDERATIONS OF THE RODS

Since the calculations for the required strength of the rods is very involved and is presented in the supplement, only the interpreted results and specified materials are given here.

The results indicated that the highest strength material available should be used. Also considering the corrosion problem, the material specified is a chrome-vanadium steel, SAE 6145 to 6150, heat treated to a tensile yield strength of as near 300,000 psi as practicable. The results of the supplement also indicate that a tubular rod should be used. However, from the standpoint of local buckling strength and local impact strength, the walls should be relatively thick.

DISCUSSION OF THE TIME MEASUREMENTS

Since it will be necessary to measure the time increment between the various length rods striking the ground, it is will to consider just what the order of magnitude of that time is. The aviation industry in conjunction with the operators have established a free drop height for the airplane to give a vertical velocity equivalent to the highest encountered in an actual landing. For most airplanes, exclusive of fighters, the equivalent drop height has been set at 42 inches. For fighter type aircraft the height is generally set at 48 inches. However, due to recent failures of the landing gear particularly those of the tricycle type during carrier landings, it is indicated that the equivalent drop height should be closer to 60 inches.

The free body drop height is equivalent to about 75% of the airplane drop height due to the air effect on the wings, itc. Therefore, we may assume that the highest speed that the equipment will be required to measure will be that equivalent to an airplane drop height of 60 inches or a free body drop of 48 inches. The velocity resulting from a free drop from 48 inches is

$$v = \sqrt{2gh} = 192.5 \text{ in/sec}$$

and the time necessary to drop one inch (the selected vertical distance between the rods) is

$$t = 0.0052 \text{ seconds}$$

Therefore, if it is desired to hold the error of the time measurement below 2%, it will be necessary to record the time to 0.0001 seconds. From the Miller Oscillograph recording it is possible to read the time to the nearest 0.0002 second. If it is deemed necessary for any series of landing tests, to hold the error to less than 4% for the hardest landings, a more accurate method of measuring and recording the time must be used.

One suggested method is the use of telemetering equipment in the airplane to send a pulse at the time of impact of each rod, these impulses to fire a cathode-ray oscilloscope and the trace to be photographed.

DESCRIPTION OF THE TESTING MACHINE USED FOR STRENGTH AND TIME CALIBRATION TESTS

From the theoretical analysis, it was apparent that tests must be made to determine if rods could be found or made that would not take a permanent set under the impact loadings received when the airplane was landing. This loading would be due to the resultant of the forward and vertical components of velocity of the airplane. It is evident that the rods should not take a permanent set if they were to be used again or if the device was to be capable of measuring multiple contact landings.

Calibration tests of the time measuring accuracy of the equipment must also be made. Since it was impossible, in the time available, to design and construct the device and actually make the tests on an airplane, it was decided to make a single testing machine to do both the calibration and strength tests. By using a testing machine, the component parts could be tested as they were completed and thus many difficulties could be eliminated prior to the completion of the equipment. It was also thought that a truer picture of the actual conditions could thus be obtained than in any other way.

Since only a limited space was available in which to set up the test machine, it was felt that some sort of a whirling device would be best. Thus, a machine was devised which consisted of an aluminum wedge, the face of which was at the proper angle to give an average combination of forward and vertical velocity as might occur in an actual landing. This wedge was placed at the end of a 36 inch arm which was made to rotate at the proper speed, it was desired to have a variable speed motor to drive this arm, but since none could be obtained, it was thought that the necessary speed increments could be obtained by using various pulley combinations. A rod assembly, using only two rods but otherwise similar to one that could be used on the actual airplane, was then placed so that the rods would be hit by the wedge as it swung and thus very closely approximate the conditions that would occur in the actual airplane landing.

The entire rod assembly was made to rotate slightly at the proper time in relation to the wedge rotation, by means of a spring and a solenoid and thus remain outside the orbit of the arm and wedge until the arm had attained its momentum. When the reading or test was ready to be made, a coil spring lever arm was raised into position; this was struck by the wedge arm as it rotates and threw a switch which energized the solonoid; this, in turn, pulled a pin which allowed the spring loaded rod assembly to swing into position, and the rods were struck by the wedge as it completed its next revolution. The photographs of the test machine and rod assembly show these details, as do the drawings of the rod assembly as used in these tests.

Figures I to VIII inclusive show the machine as used in the strength tests.

Figures IX and X show the additions made to the test machine for the calibration tests. Two micro switches were attached to the forward part of the rod assembly for the first tests. When the rods were in position, the rod holders held the switches closed, resulting in a closed circuit; when the wedge struck the rods the switches were opened and the circuit was broken. This was to be the basis of measuring the time increment between the striking of the two rods. For the next series of tests the micro-switches were modified to remain in the open position until the rods were struck at which time the circuit was closed. A third method was also tried using small diameter insulated wire tied around the rods, this wire to be broken when the rods were struck, thus breaking or opening the circuit. The actual measurements were to be recorded on a Miller Oscillograph; an alternate method of measuring the time increment was to use a cathode-ray oscilloscope, the trace to be started when the first rod was struck and deflected when the wedge struck the second rod. A set time scale was superimposed on the trace by an oscillator and the entire picture recorded by a camera focused on the scope.

To check these methods of measuring the time increment, the time was first calculated analytically by measuring the arm length, the distance the wedge traveled in operating both switches or breaking both wires, as the case might be, and the speed of the rotating arm by using a Strobotac. The time calibration tests would therefore determine whether or not these methods were capable of measuring the small time increments accurately enough to be used for the determination of the sinking speed of the airplane.

RESULTS OF STRENGTH TESTS ON THE RODS

The specified material being unobtainable, the tests were made using a plain carbon steel which was heat treated to fairly high allowable yield stresses.

Tests were first made using one-quarter inch diameter solid rods, heat treated to between 120,000 and 140,000 psi yield strength. This material did not permanently deform at the low wedge speed of 94.5 feet per second but did deform slightly at the high wedge speed of 153 feet per second. No tests were made at intermediate speeds to determine just where this deformation would start. The amount of deformation for each impact was of the order of one-sixteenth of an inch for a twenty inch rod length.

The next tests were made using 9/32 inch diameter drill rod heat treated to an estimated yield of between 220,000 and 250,000 psi. (This estimate was based on a Rockwell C hardness test). Repeated tests were made at a wedge speed of 153 feet per second, using the slant face of the wedge, and after five impacts the deformation could barely be discerned. No further deformation was measurable after twenty-five additional impacts. A final test was made with one of these same rods, using the flat or butt end of the wedge, at a speed of 153 feet per second. As a result of this test a deformation of approximately 3/32 of an inch was measured for the twenty inch rod. However, it is extremely unlikely that any such impact would occur in an actual landing, and if such was encountered, the rod could be considered expendable.

The results from the above described tests indicate that solid rods can be used if the yield strength exceeds 250,000 psi. However, it is recommended, for continued use of the same rods, that they should be hollow and have a yield strength of over 250,000 psi.

RESULTS OF TIME CALIBRATION TESTS

Since it was impossible to obtain and assemble all of the time calibration test equipment until a very late date, sufficient time was not available to satisfactorily complete the time calibration tests due to the unavailability of new equipment found necessary after preliminary tests were conducted. However, enough additional tests were made, using makeshift equipment, to establish trends and to also determine the type of switches and circuits necessary to make the time measurements that would be required in the actual airplane landing tests. It must also be noted here, that the calibration tests were not primarily to test the accuracy of the overall design, since this would of necessity have to be done on the actual airplane installation, but rather to develop the time measuring and recording technique. At first it was not anticipated that trouble would be experienced with the switches since high quality micro-switches were used, though this later proved to be the cause of most of our trouble.

The wedge speed and shape, in relation to the length increment of the rods, was adjusted to give a shorter increment of time than was expected to occur in the actual installation. After the first tests were made it was apparent that the micro-switches, which had flat contact points, were not holding a definite contact in the closed position in the instant just before the rods were struck by the wedge; this "chattering" was caused by vibrations set up when the rod assembly snapped into position. Considerable time and effort was spent trying to remedy this trouble and it was finally decided that the points had a relative sliding action which caused the breaks in the circuit. However, some improvement was made in their action and it was decided to continue the tests using the same switches since a better type could not be immediately obtained.

The first tests were recorded on a Miller Oscillograph, see test Nos. 4, 5, and 6. Additional tests were made on a Cathode-Ray Oscilloscope but results could not be determined, due to excessive chatter in the circuit, and were not recorded. The recorded tests on the Miller Oscillograph were analyzed and the time increment determined. However, it is doubtful if the time could have been measured if the characteristics of the switches had not been known or if the time increments had varied widely.

It was thought that if an open circuit was used which would be closed when the wedge struck the rod, the pre-impact chatter would be avoided and though it was realized that chatter would occur after the switch closed, it was thought that the first contact could be recorded and measured. Therefore, the micro-switches were modified to operate in this manner and Test Nos. 7 to 12 inclusive were then made using the Miller Oscillograph to record the time. The results of these tests were completely unreliable since the first point of the switch closing could not be determined. Tests were then made using the Cathode-Ray Oscilloscope for time measuring and recording. The first of these tests, Test Nos. 1 to 6, were unreliable but after changing the oscilloscope setting and using a higher frequency time scale, it was quite easy to measure the time of the first point of switch contact. A typical oscilloscope photograph is shown in Figure XIV which is made of Test No. 15. These results appeared quite reliable.

All of the above tests indicate that the recorded time is, in general, higher than the analytically calculated time and furthers the suspicion that the wedge moves slightly downward between the time of hitting the first and second rod.

It was then definitely decided, that further tests were fruitless from a time calibration standpoint unless the rotating arm and wedge were modified to hold the wedge deflection to a smaller value or, preferably, to eliminate it entirely; however, tests should be continued to develop a satisfactory circuit from which the time could be readily measured and recorded with the Miller Oscilloscope.

An unsuccessful effort was made to locate a snap action switch having a knife switch type of point or any other type of switch that would be free from chatter due to vibration. As a last resort a circuit was made using an insulated wire looped around each of the rods and which would be broken as each of the rods was hit by the wedge. This method could be used, in principle, on the actual airplane where only one contact with the ground is to be made for a given landing.

This method is not recommended but does give a very accurate reading on the Miller Oscillograph. Figures Nos. XI, XII and XIII show typical recordings of tests using each of the above described circuits.

CALIBRATION TESTS USING THE CATHODE-RAY OSCILLOSCOPE

Test No.	Oscillator Frequency	Type Circuit	Recorded* Time (seconds)	Calibrated Time (seconds)	Remarks
1	2000	Open Switch	No Results	0.00277	It is believed that 2000 cycles is too low a setting for accurate results.
2	2000	Open Switch	0.00424	0.00277	
3	2000	Open Switch	0.00372	0.00277	
4	2000	Open Switch	0.00477	0.00277	
5	2000	Open Switch	0.00424	0.00277	
6	2000	Open Switch	0.00343	0.00277	
7	5000	Open Switch	No Results	0.00277	
8	5000	Open Switch	No Results	0.00277	
9	5000	Open Switch	No Results	0.00277	
10	5000	Open Switch	0.00275	0.00277	
11	5000	Open Switch	No Results	0.00277	
12	5000	Open Switch	0.002845	0.00277	
13	10000	Open Switch	0.002850	0.00277	
14	10000	Open Switch	No Results	0.00277	
15	10000	Open Switch	0.002800	0.00277	
16	10000	Open Switch	0.003055	0.00277	

* Tests marked "No Results", were failures because of either solenoid timing error or because of an incomplete circuit and were no fault of the equipment.

CALIBRATION TESTS USING THE MILLER OSCILLOGRAPH

Test No.	Type Circuit	Recorded* Time (seconds)	Calculated Time (seconds)	Remarks
1	Closed Switch	No Results	0.00422	Miss on second rod
2	Closed Switch	No Results	0.00422	Miss on second rod
3	Closed Switch	No Results	0.00422	Miss on second rod
4	Closed Switch	0.0029	0.00251	Results hard to read because of chatter in micro-switch
5	Closed Switch	0.0031	0.00251	
6	Closed Switch	0.0029	0.00251	
7	Open Switch	0.0056	0.00279	
8	Open Switch	0.0035	0.00279	
9	Open Switch	0.0030	0.00279	
10	Open Switch	0.0044	0.00279	
11	Open Switch	0.0034	0.00279	
12	Open Switch	0.0029	0.00279	
13	Closed Wire	No Results	0.00292	Miss
14	Closed Wire	0.0033	0.00292	
15	Closed Wire	0.0034	0.00292	
16	Closed Wire	0.0035	0.00292	

* Tests marked "No Results", were failures because of either solenoid timing error or because of an imcomplete circuit and were no fault of the equipment.

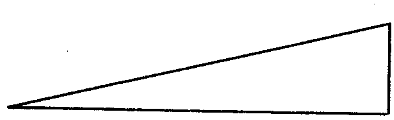
CONCLUSIONS

It is indicated that the basic design, as presented in the following section, is usable as a method of measuring the vertical velocity of an airplane at the moment of contact with the ground.

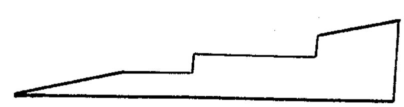
The rods, which contact the ground, should be tubular and of a high strength alloy steel. It is unlikely that any trouble will be experienced from permanent deformation of the rods provided they are heat treated to a tensile yield strength of over 250,000 psi.

Results indicate that the time lag of the switches and, in general, an overall time calibration test of the equipment, after installation on the airplane, should be made by drop test or some other suitable method.

If further time calibration tests are to be made, using the existing or similar simulated test equipment, it is suggested that a stepped wedge, instead of the slanting-face wedge, should be used to eliminate the wedge deflection error. See sketch below.



Present Wedge



Suggested Wedge

Further tests could well be made on the strength analysis of the rods under impact, using various size and shape rods and various rod lengths. The problem of rods under impact loading is very involved and although a considerable knowledge of their action was gained in conducting the foregoing tests, there is a great deal more information to be gained by a more thorough investigation of that problem alone. It is then likely that the strength criteria, as set forth in this report, could be modified to some extent.

BASIC DESIGN OF THE EQUIPMENT NECESSARY TO MEASURE THE SINKING SPEED OF AN AVERAGE SIZED AIRPLANE

The conclusions from the strength and calibration tests indicate that the following basic design of the device, for use on an average sized airplane, can be expected to give good results:

The rod assembly should consist of six tubular rods mounted on a common axis in the manner of the two used in the strength and calibration tests. It is suggested that the rods be of chrome-vanadium steel, SAE 6145 to 6160, heated treated to at least 250,000 psi yield strength. The outside diameter should be 9/32 of an inch and the inside diameter should be 5/32 of an inch. The entire box or rod assembly should be mounted on the axle alongside the airplane wheel so that the bottom of the box is just above the flat tire ground line of the wheel. It should be mounted so that the rods, when in their extended position, make an angle of sixty degrees with the fore and aft axis of the airplane. That is, the rods slant rearward thirty degrees from the vertical. The rods should slant thus so that when the airplane is at its highest angle of attack there will be no component of relative velocity to the ground in the upward direction along the rod. The thirty degree slant is arrived at by assuming the highest angle of attack of the airplane will be sixteen degrees upon contacting the ground and that the highest vertical to forward velocity ratio will be 1 to 4. If either differs materially in the actual case, the angle can be changed accordingly.

The rods should vary in length by increments of not less than one inch, the shortest being just long enough to touch the ground line of the undeflected tire when the airplane is in its expected landing attitude. The actual distance between the ends of the rods should be measured when they are installed, this measurement to be made along a line perpendicular to the ground line of the plane in its expected touchdown attitude.

Each rod should be held in the extended position by a volute spring which developes a moment of 44 inch pounds. Each rod should be equipped with a ratchet to hold it in the retracted position unless released by the pilot or operator. Thus the rods could be left in the retracted position during take-off to avoid wear on the rods. Where only one ground contact will be made during the landing, the ratchet can be left engaged and thus catch and hold the rod in the retracted position after the contact. However, if more than one contact is expected during the landing due to a bounce, and it is desired to measure each one, then the ratchet may be left disengaged until such time as the final contact is made. In that case the rod will not be caught in the extreme retracted position but will be pushed by the up and down motion of the plane into a position to clear the ground most of the time.

A snap action knife switch for each rod should be used in place of the micro switches used in the calibration tests, the switch to remain open when the rod is in the extreme extended position and to close when the rod leaves this position. These switches should be very positive and fast acting.

A circuit through each of these switches should be connected to one channel of a Miller Oscillograph, equipped with high frequency elements. The Oscillograph should be set to run at its maximum speed. Thus the difference between the times when the various rods strike the ground and close the switches can be measured from the oscillograph record to about the nearest .0002 second. Since the actual height, in inches, between the ends of the rods is known, the vertical velocity of the airplane can be obtained by dividing the heighth difference of the rods by the time difference for the corresponding rods and then taking the average between the six rods to be the true vertical velocity of the airplane relative to the ground line at the point of touchdown.

S U P P L E M E N T

STRESS ANALYSIS OF RODS UNDER LATERAL IMPACT

No direct information could be obtained as to the magnitude of stresses of rods under impact. Therefore, the following analysis is made, not as a solution to the very involved problem, but to be used as an evaluation method for results that can be obtained experimentally, and as a method of more accurately determining the most desirable size, shape, and material of rod to withstand the forces of impact under consideration.

Much of the analysis is taken from Chapter VIII of "Theory of Sound" by Lord Rayleigh, with the proper application to the problem at hand. This seemed to be the most basic material and as easily applicable as any material available.

Equation of Motion and Solution Thereof:

In general this analysis applies to a rod the section of which is symmetrical about a neutral axis perpendicular to the plane of bending. The equation of motion, if we neglect the rotary moment of inertia of the cross-section, in the plane of bending of such a rod is

$$EI \frac{d^4y}{dx^4} + \rho A \frac{d^2y}{dt^2} = 0 \qquad \text{. . . . .(1)}$$

where A = area of the cross section

ρ = density of the material

We may assume the harmonic form of y as,

$$y = u \sin \left(\frac{kb}{L^2} m^2 t\right) \qquad \text{. . . . .(2)}$$

where k = radius of gyration of the cross section

$b = \sqrt{E/\rho}$ = speed of transmission of longitudinal waves

L = Length of the rod

m = abstract number depending on the mode of vibration

Substituting in (1), we obtain

$$\frac{d^4u}{dx^4} = \frac{m^4}{L^4}$$

the solution of which is

$$u = A\cos(mx/L) + B\sin(mx/L) + Ce^{\frac{mx}{L}} + De^{-\frac{mx}{L}} \quad \ldots\ldots\ldots (3)$$

From the boundary conditions at the ends we can determine the ratio of A:B:C:D. We then have an equation from which we may solve for a series of values of m, and the corresponding u is then determined except for a constant multiplier which in our case may be found by bringing in one more boundary condition, namely, the strength of the rod in bending.

Equation (3) may be rewritten as

$$u = A\cos(mx/L) + B\sin(mx/L) + E\cosh(mx/L) + F\sinh(mx/L) \quad \ldots\ldots\ldots (4)$$

from the relation of

$$\cosh(mx/L) = \tfrac{1}{2}(e^{\frac{mx}{L}} + e^{-\frac{mx}{L}})$$

$$\sinh(mx/L) = \tfrac{1}{2}(e^{\frac{mx}{L}} - e^{-\frac{mx}{L}})$$

For the evaluation of the constants we need

$$\frac{du}{dx} = \frac{m}{L}(-A\sin\frac{mx}{L} + B\cos\frac{mx}{L} + E\sinh\frac{mx}{L} + F\cosh\frac{mx}{L})$$

$$\frac{d^2u}{dx^2} = \frac{m^2}{L^2}\left(- A \cos\frac{mx}{L} - B \sin\frac{mx}{L} + E \cosh\frac{mx}{L} + F \sinh\frac{mx}{L}\right)$$

$$\frac{d^3u}{dx^3} = \frac{m^3}{L^3}\left(A \sin\frac{mx}{L} - B \cos\frac{mx}{L} + E \sinh\frac{mx}{L} + F \cosh\frac{mx}{L}\right)$$

The solution of the problem of a rod one of whose end conditions can be represented as a nodal point of a rod of twice its length, can best be handled by considering the longer rod. Therefore, for the rod having one end free and the other simply supported, we shall consider the even modes of a rod free at both ends and having a nodal point in the center corresponding to the simply supported end of the shorter rod.

For the case of the free rod, we have the boundary conditions at both ends;

at $x = 0$ $\quad \frac{d^2u}{dx^2} = 0 \quad$ so $A = E$

$\quad \frac{d^3u}{dx^3} = 0 \quad$ so $B = F$

At $x = L$ $\quad \frac{d^2u}{dx^2} = 0 \quad$ so $A(-\cos m + \cosh m) + B(-\sin m + \sinh m) = 0$

$\quad \frac{d^3u}{dx^3} = 0 \quad$ so $A(\sin m + \sinh m) + B(-\cos m + \cosh m) = 0$

Solving gives

$$(\cosh m - \cos m)^2 = \sinh^2 m - \sin^2 m$$

but $\cosh^2 m - \sinh^2 m = 1$

so $(\cos m)(\cosh m) = 1$

This gives A:B as $(-\cos m + \cosh m):(-\sin m - \sinh m)$

And now

$$u = C \left\{ (\sin m - \sinh m)(\cos \frac{mx}{L} + \cosh \frac{mx}{L}) - (\cos m - \cosh m) \right.$$

$$\left. (\sin \frac{mx}{L} + \sinh \frac{mx}{L}) \right\} \quad \ldots\ldots\ldots (5)$$

The solution of $\cos m \cosh m = 1$ to a sufficient degree of accuracy may be solved by successive approximations. The values as given in Rayleigh are,

$$m_1 = 4.7300$$
$$m_2 = 7.8532$$
$$m_3 = 10.9956$$
$$m_4 = 14.1371$$
$$m_r = \tfrac{1}{2}(2n + 1)\pi, \quad n = \text{an integer above } 5$$

We can readily see that for the first mode and successive odd modes there is no nodal point at the center. Therefore, for our case of a simply supported rod, we need the value of m only for the even modes.

By referring to equation (5), we see that there is a value of C for each mode. We need to determine the ratio of the C's to each other. We shall at this point introduce subscripts for use with the C's, u's, etc., these subscripts to refer to the mode under consideration and correspond to those used with m as previously given. Assume $(\ )_r$ corresponds to any general mode.

The ratio of C_2 to C_4 etc. may be determined by a method analagous to one used in Rayleigh, sec. 168. The value of y may be written,

$$y = C_2 u_2' \sin(\frac{kb}{L^2} m_2^2 t) + C_4 u_4' \sin(\frac{kb}{L^2} m_4^2 t) + \ldots + C_r u_r' \sin(\frac{kb}{L^2} m_r^2 t)$$

where only the even subscripts are used denoting the nodal point in the center. And where $u' = u/C$, as given in equation (5). Initially $y = 0$, for a rod initially at rest and excited by a sudden blow. Then

$$\dot{y}_o = (kb/L^2)(m_2^2 C_2 u_2' + m_4 C_4 u_4' + \ldots +) \qquad \ldots\ldots\ldots (6)$$

Multiply (6) by u_r' and integrate over the length of the rod.

$$\text{Then} \quad \frac{L^2}{kb} \int \dot{y}_o\, u_r'\, dx = m_r\, C_r \int (u_r')^2\, dx \qquad \ldots\ldots\ldots (7)$$

For a rod free at $x = 0$

$$\int (u_r')^2\, dx = \tfrac{1}{4} L\, [u_r'\,(0)\,]^2$$

and calling $\int \dot{y}_o\, \rho\, A dx$ the whole momentum, Y, of the blow which for our case will be applied at $x = 0$; then

$$C_r = \frac{L^2 Y}{kb\rho A} \; \frac{u_r'(x)}{m_r^2\, \frac{1}{4}\, L\,[\,u_r'(0)\,]^2} \qquad \ldots\ldots\ldots (7a)$$

$$= \frac{4YL}{kb\rho A} \; \frac{1}{m_r^2 \;\; u_r'(0)} \qquad \text{where } x = 0 \qquad \ldots\ldots\ldots (7b)$$

$$\text{the ratio of } \frac{C_2}{C_r} = \frac{m_r^2 \;\, u_r'(0)}{m_2^2 \;\, u_r'(0)} \qquad \ldots\ldots\ldots (8)$$

Equation (6) now becomes.

$$\dot{y}_o = (kb/L^2) \left\{ m_2^2 C_2 u_2'(0) + m_4^2 \, \frac{C_2 m_2^2 u'(0)}{m_4^2 u_4'(0)} \, u_4'(0) + \ldots\ldots \right.$$

$$\left. + \, m_r^2 \, \frac{C_2 m_2^2 u_2'(0)}{m_r^2 u_r'(0)} \, u_r' \right\}$$

$$= (kb/L^2) \left\{ m_2^2 C_2 u_2'(0) \right\} \; n \qquad \ldots\ldots\ldots (9)$$

where n = number of modes considered or excited.

In the general term of u,

$$\dot{y}_o = (kb/L^2)(m_2^2\, u_2(0)\, n \qquad \ldots\ldots\ldots (9a)$$

We need the value of $u_2(0)$ and also the ratio of $u_r(0)$ to $u_2(0)$ for the evaluation of (9a) and C_2/C_r needed later. The equation of u as given in equation (5) is simplified for numerical calculation to the following form. (Rayleigh, sec. 177)

$$u = C\left\{\sqrt{2}\cos r\pi \sin\left[mx/L - \tfrac{1}{4}\pi + (-1)^r\frac{\beta}{2}\right] + \sin\frac{\beta}{2}\, e^{\frac{mx}{L}} - \cos r\pi\cos\frac{\beta}{2}\, e^{-\frac{mx}{L}}\right\}$$

where β is a small angle whose value is given below.

For our case r is always even valued. Therefore $\cos r\pi = +1$ and $(-1)^r = +1$. Thus u may be further reduced and small valued term neglected as follows:

$$u = C\left\{\sqrt{2}\left(\sin\frac{mx}{L}\cos\frac{\pi}{4} - \cos\frac{mx}{L}\sin\frac{\pi}{4}\right)\cos\frac{\beta}{2} - \cos\frac{\beta}{2}\, e^{-\frac{mx}{L}} + \sqrt{2}\left(\cos\frac{mx}{L}\cos\frac{\pi}{4} + \sin\frac{mx}{L}\sin\frac{\pi}{4}\right)\sin\frac{\beta}{2} + \sin\frac{\beta}{2}\, e^{\frac{mx}{L}}\right\}$$

For u_2, $\beta = 2'\ 40''$ of arc and very much smaller for u_4 and above.

Therefore,

$$\cos\frac{\beta}{2} = 1$$

$$\sin\frac{\beta}{2}_{(u_2)} = .0004$$

$$\sin\frac{\beta}{2}_{(u_r)} = 0$$

and so to a very close degree of accuracy

$$u_2 = C_2\left\{\left(\sin\frac{m_2x}{L} - \cos\frac{m_2x}{L}\right) - e^{-\frac{m_2x}{L}} + \left(\cos\frac{m_2x}{L} + \sin\frac{m_2x}{L}\right)(.0004) + .0004\, e^{\frac{m_2x}{L}}\right\} \qquad \ldots\ldots\ldots (10)$$

and $$u_r = C_r \left\{ \sin\frac{mx}{L} - \cos\frac{mx}{L} - e^{-\frac{mx}{L}} \right\} \quad \ldots\ldots\ldots (11)$$

Therefore,

$$u_2\ (0) = C_2(0 - 1 - 1 + .0004 + .0004) = 1.9992\ C_2$$

or nearly $2\ C_2$

and $u_r\ (0) = 2\ C_2$

Equation (8) now becomes

$$\frac{C_2}{C_r} \times \frac{m_r^2}{m_2^2} \qquad \text{or} \quad C_2\ m_2^2 = C_r\ m_r^2 = \text{constant} \quad \ldots .(11a)$$

<u>Bending Moment</u>:

From equation (10) and (11),

$$\frac{d^2u_2}{dx^2} = C_2\left(\frac{m_2}{L}\right)^2 \left[- \sin\frac{mx}{L} + \cos\frac{mx}{L} - e^{-\frac{mx}{L}} + .0004\ e^{\frac{mx}{L}}\right] \ldots .(12)$$

$$\frac{d^2u_r}{dx^2} = C_r\left(\frac{m_r}{L}\right)^2 \left[- \sin\frac{mx}{L} + \cos\frac{mx}{L} - e^{-\frac{mx}{L}}\right]$$

$$= C_2\left(\frac{m_2}{L}\right)^2 \left[- \sin\frac{mx}{L} + \cos\frac{mx}{L} - e^{-\frac{mx}{L}}\right] \quad \ldots\ldots\ldots (13)$$

From M, the bending moment, $= EI\ \frac{d^2u}{dx^2}$; $\sigma = \frac{Mc}{I}$;

and letting $R = \frac{ML^2}{E\ m_2^2\ c\ R_{max}}$

$$C_2 = \frac{\sigma L^2}{E\ m_2^2\ c\ R_{max}} \quad \ldots\ldots\ldots (14)$$

where $R_{max} \approx M_{max}$

and from (9a)

$$\dot{y}_o = v = 2\ \frac{kb}{L^2}\ m_2^2\ C_2\ n$$

TABLE I

Bending Moments for Rods under Impact

(1)	(2)	(3)	(4)	(5)	2nd Mode	4th Mode	6th Mode
$\frac{mx}{L}$	$-\sin\frac{mx}{L}$	$\cos\frac{mx}{L}$	$-e^{-mx/L}$	R	$(\frac{x}{L})_2$	$(\frac{x}{L})_4$	$(\frac{x}{L})_6$
.4	- .389	.921	-.67	- .137	.051	.028	.02
.6	- .565	.825	-.549	- .288	.076	.057	
.8	- .717	.697	-.449	- .468	.102	.057	.039
1.0	- .842	.540	-.368	- .669	.127		
1.2	- .932	.362	-.301	- .870	.153	.085	.059
1.4	- .985	.17	-.247	-1.06	.178		
1.6	-1.000	-.029	-.202	-1.233	.204	.113	.078
1.8	- .974	-.227	-.165	-1.364	.229		
2.0	- .909	-.416	-.135	-1.457	.254	.141	.098
2.2	- .808	-.590	-.111	-1.505	.280		
2.3	- .745	-.667	-.100	-1.512	.293	.163	.112
2.4	- .674	-.739	-.091	-1.499	.305	.170	.117
2.6	- .514	-.858	-.074	-1.441	.331		
2.8	- .333	-.943	-.061	-1.331	.356	.198	.137
3.0	- .14	-.990	-.050	-1.172	.382		
3.2	.06	-.998	-.041	- .969	.407	.226	.157
3.4	.257	-.966	-.033	- .730	.433		
3.6	.444	-.896	-.027	- .464	.458	.255	.176
3.8	.613	-.790	-.022	- .181	.484		
4.0	.758	-.652	-.018	.088		.283	.196
4.4	.952	-.305	-.012	.635		.311	.215
4.8	.996	.089	-.008	1.077		.339	.235
5.2	.881	.471	-.005	1.347		.368	.254
5.6	.629	.778	-.004	1.403		.396	.274
6.0	.276	.961	-.002	1.235		.424	.293
6.4	- .120	.993	-.002	.871		.452	.313
6.8	- .497	.868	-.001	.370		.481	.333
7.2	- .796	.606	-.001	- .191			.352
8.0	- .989	-.149	0	-1.138			.391
8.8	- .581	-.814		-1.395			.430
9.6	.179	-.984		- .805			.47

Where terms effecting the 4th decimal place are neglected.

where v = velocity imparted to the end of the rod under the momentum, Y.

then $$v = \frac{2\,k\,b\,\sigma}{E\,c} \quad \frac{n}{R_{max}}$$

From table I, we see that $R_{max} = 1.512$ for each mode of vibration. Therefore, if we consider that only the first even mode of vibration was excited we would have from equation (15) that the greatest velocity that could be momentarily imparted to the free end of a rod simply supported at the other end and not cause permanent deformation, would be given by

$$v = \frac{2kb\sigma}{Ec}\,\frac{1}{|1.512|} = 1.323\,\frac{kb\sigma}{Ec} = 1.323\,\left(\frac{k}{c}\right)\frac{\sigma}{\sqrt{E\rho}} \quad \ldots\ldots\ldots (16)$$

Special Considerations for More Than One Mode:

In considering the case of more than one mode of vibration, we need to consider first the following items:

1. Although each mode of vibration has one maximum value of R of $\pm$ 1.512 and one or more other points along the length of the rod where $|1.414| \leqslant |R| < |1.512|$ (the lower limit of 1.414 being obtained by the highest additive value of the $\sin \frac{mx}{L} + \cos \frac{mx}{L}$ in the region where $e^{-\frac{mx}{L}} \doteq 0$), no two of these maximum values for the different modes occur at the same point (until the much higher order modes are reached) along the length of the rod. Therefore, when considering two or more modes (n in equation 9a) R_{max} will be somewhat less than 1.512 times n.

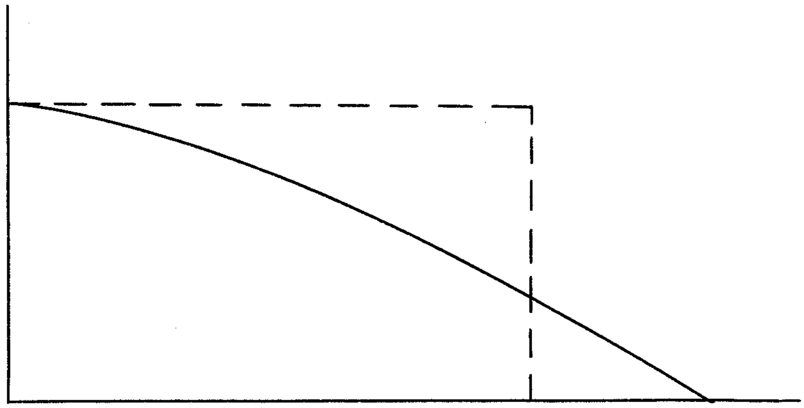
2. The period of each vibration varies as $1/m^2$ (see equation 2) or we can say that the period, T, $= K/m^2$. To a sufficient degree of accuracy to illustrate the point to be made, m is equal to $\frac{1}{2}\pi\,(2n + 1)$, and is accurate to the second decimal place in the first even mode, accurate to the fifth place in the second even mode, and to at least the seventh

decimal place in the higher modes. Therefore,

$$T = \frac{K}{m^2} = \frac{K}{[\frac{1}{2}\pi(2n + 1)]^2}$$

From this we can see that with respect to time, the first condition that the various vibrations reinforce each other in the maximum amount occurs when the first even mode has gone $(2n + 1)^2/4 = 25/4 = 6\frac{1}{4}$ cycles, the $\frac{1}{4}$ coming in because the maximum bending is reached on the first and third quarter points of each cycle. The second even mode will have gone $81/4 = 20\frac{1}{4}$ and the third even mode $169/4 = 42\frac{1}{4}$ cycles etc., at this same time. Due to the inherent damping properties of all materials the higher modes will be partly or completely damped before they reach this first maximum reinforcing position. This will thus further reduce the maximum R value as n is increased.

3. At this point it is perhaps proper to investigate just how much we can increase n. In equation 7b it is assumed that $x = 0$ in the term, $u_r'(x)$ of equation 7a. However, the blow cannot be considered to act exactly at the end since due to the local deflection of the material the blow is effectively distributed over a portion of the end of the rod, probably to a distance of two or three diameters. As u_r' is taken nearer to a nodal point its value lessens and becomes zero at the nodal point. See equation (11). The higher the vibration the nearer to the end is the nodal point. (For a rod having L/d ratio of 120, the nodal point occurs within 3 diameters from the end on the seventh even mode.) Therefore, equation (11a) which shows that $C_2 m_2^2 = C_r m_r^2 =$ constant is not strictly true expecially for the higher vibration. If $C_r m_r^2$ is plotted against n, the curve will have somewhat the general shape.



We can get nearly the same results if we assume that $C_r m_r^2$ does not diminish gradually but is suddenly reduced to zero at some particular value of "n" as shown by the dashed lines in the figure. Then we can say that $C_2 m_2^2 = C_r m_r^2$ = constant up to a certain value of n, and equation (9a) will be valid, but n will have a finite value of probably about 6.

Interpretations:

From the preceding analysis, expecially considering the final three paragraphs, we can make the following conclusions.

A. Although we do not know the exact value of the numerical constant as given in equation (16) we do know the properties of the rod on which "v" depends. We can also pick a shape to give as high a value of k/c, the only geometric properties involved, as possible, of course the limiting value being $k/c = 1$. A solid rod gives k/c of 1/2 and a thin tubular cylinder gives k/c of .707. Since $b = \sqrt{\frac{E}{\rho}}$, $(\frac{b\sigma}{E})$, the remaining terms, are equal to $\frac{\sigma}{\sqrt{E\rho}}$

For aluminum $\sqrt{E\rho} = 51$

For steel $\sqrt{E\rho} = 148.7$

Since the top limit of σ for aluminum is about 60,000 psi. yield, whereas the limit for steel is over 200,000 psi. for such shapes as tubing and close to 300,000 psi. for drawn wire up to about 3/8" diameter, the ratio of $\frac{\sigma}{\sqrt{E\rho}}$ will be higher for steel than aluminum.

B. As stated in A we do not know the exact value of the numerical constant of equation (16) we do know that it has a minimum value of 1.323 as given.

C. In general we can make "n" in equation (9a) higher by using a higher L/d ratio. This is really the only way in which the length of the rod could effect the limiting velocity.

D. By reference to consideration number two above, we see that if damping is high, the higher vibrations can be considerably reduced before they reach the first reinforcing condition. Therefore, this indicates that a material having a high damping ability or if necessary, damping material such as rubber can be wrapped around the rod or better still inserted in the center of a tubular rod if aerodynamic smoothness is required on the outside.

Length of Time That The Momentum is Applied:

In the preceding analysis it has been assumed that the "force" at a constant velocity has been applied only for the time necessary to impart the vibrations in the rod. However, if the blow or force acts indefinitely at the given velocity there is a correction to be made. This can best be shown by an energy analysis of the rod under the blow.

By making use of the energy relation,

$$U = \frac{1}{2}\int_L \frac{M^2}{EI}\,dx = \frac{EI}{2}\int_L \left(\frac{d^2u}{dx^2}\right)^2 dx$$

and considering only the first even mode of vibration, we obtain for the total allowable internal strain energy in the system under this type of bending,

$$U = \frac{EI}{2}\frac{C^2m^4}{L^4}\Big[\frac{L}{2m}(.00000016\ e^{\frac{2mx}{L}} + e^{-\frac{2mx}{L}}) + \frac{L}{m}(\cos^2\frac{mx}{L} + .0004\ e^{\frac{mx}{L}}$$

$$\cos\frac{mx}{L} - e^{-\frac{mx}{L}}\cos\frac{mx}{L} - e^{-\frac{mx}{L}}\cos\frac{mx}{L} + .0004e^{\frac{mx}{L}}\cos\frac{mx}{L})\Big]_0^L$$

$$= \frac{EI}{2}\frac{C^2m^3}{L^3}\ 7.83 = 3.415\ \frac{EI\,C^2m^3}{L^3}$$

But $C = \dfrac{\sigma L^2}{E\,m^2 cR_{max}}$ from equation (14)

so $U = 2.26\ \left(\frac{k}{c}\right)^2 \frac{\sigma^2 A}{Em}\ L$ . . . . (17)

By equating internal strain energy to external work done, we can say $W = U = F\ v\ t$ where F = the average force acting during an arbitrary time, t.

Since this same force, F, acts to accelerate the rod about the supported end, this acceleration will be given by

$$\alpha = \frac{FL}{I_{end}} = \frac{3F}{\rho A L^2}$$

The velocity of the mean centerline of the rod at the free end due to this acceleration after time, t, above will be

$$V = \alpha L t = \frac{3Ft}{\rho AL}$$

but $F\ t = W/v$ from above

so $V = \dfrac{3}{\rho AL}\ 2.26\ \left(\frac{k}{c}\right)^2 \dfrac{\sigma^2 AL}{E\,mv} = \dfrac{6.78}{\rho}\ \left(\frac{k}{c}\right)^2 \dfrac{\sigma^2}{E\,m}\dfrac{1}{v}$ . . . . (18)

where v as given by equation (16) is the Maximum vibratory velocity of the end of the rod and where the instantaneous vibratory velocity of the rod is given by $\dot{y} = v \cos (\frac{kb}{L^2} m^2 t)$. If the blow acts at a velocity, S, continually, then the algebraic sum of V and $\dot{y}$ must be equal to or greater than S, or $V \pm v \geqslant S$. The smallest value of V that satisfies this condition is when V = 2v.

Equation (18) now becomes

$$v^2 = \frac{6.78}{2\rho} \left(\frac{k}{c}\right)^2 \frac{\sigma^2}{Em} = .432 \left(\frac{k}{c}\right)^2 \frac{\sigma^2}{E\rho}$$

$$\text{or } S = v = .657 \left(\frac{k}{c}\right) \frac{\sigma}{\sqrt{E\rho}}$$

which is the same form as equation (16) but has a different numerical constant and most of the discussion relation thereto concerning higher vibrations holds.

If the velocity of the blow exceeds that as given above for a given material the limiting internal strain energy of the rod will be exceeded and must be dissapated by damping material if the rod is not to take permanent set.

It is well to consider the maximum possible value of internal strain energy due to bending under any number of modes of vibrations. We may obtain the limit of this by considering every element of length bent to the maximum amount in which case

$$U = \frac{1}{2} \int_0^L \frac{M^2}{EI} dx = \frac{1}{2} \frac{\sigma^2 A k^2}{c^2 E} L$$

It is hoped that some correlation may be made between this material and the test results.

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REPORT NO. AML NAM 2486 PART I

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DEVELOPMENT OF DROP TESTING TECHNIQUE AND INSTRUMENTATION-

DYNAMIC REACTION PLATFORMS- EVALUATION OF

NACA REPORT NO. 468

THE INTERFERENCE BETWEEN STRUTS IN VARIOUS COMBINATIONS

by David Biermann and William H. Herrnstein, Jr.

"THEORY OF SOUND"

by Lord Rayleigh

REPORT NO. 5968

Lockheed Aircraft Corporation's Report on

MEASURED MAIN LANDING GEAR SCISSOR LOADS - MODEL 49

REPORT NO. 5557 ADDENDUM 1

Lockheed Aircraft Corporation's Report on

MEASURED LANDING GEAR LOADS ON MODEL 49

APPENDIX I

APPENDIX I

LOCKHEED WATER JET METHOD

The latest method of measuring the vertical velocity of an airplane when landing was developed by the Lockheed Aircraft Corporation within the past two or three months. This is the so-called "water jet" method which consists of a jet of water directed vertically downward from a point on the landing gear and a gun camera mounted on the opposite gear which photograhs this stream of water.

As the plane settles in for a landing the vertical stream of water strikes the runway and diminishes in length as the gear approaches the ground. This diminishing length is photographed by the gun camera mounted on the opposite gear. From these pictures a plot is made of length vs. time and it is then a simple matter to calculate the vertical velocity of the airplane as it lands.

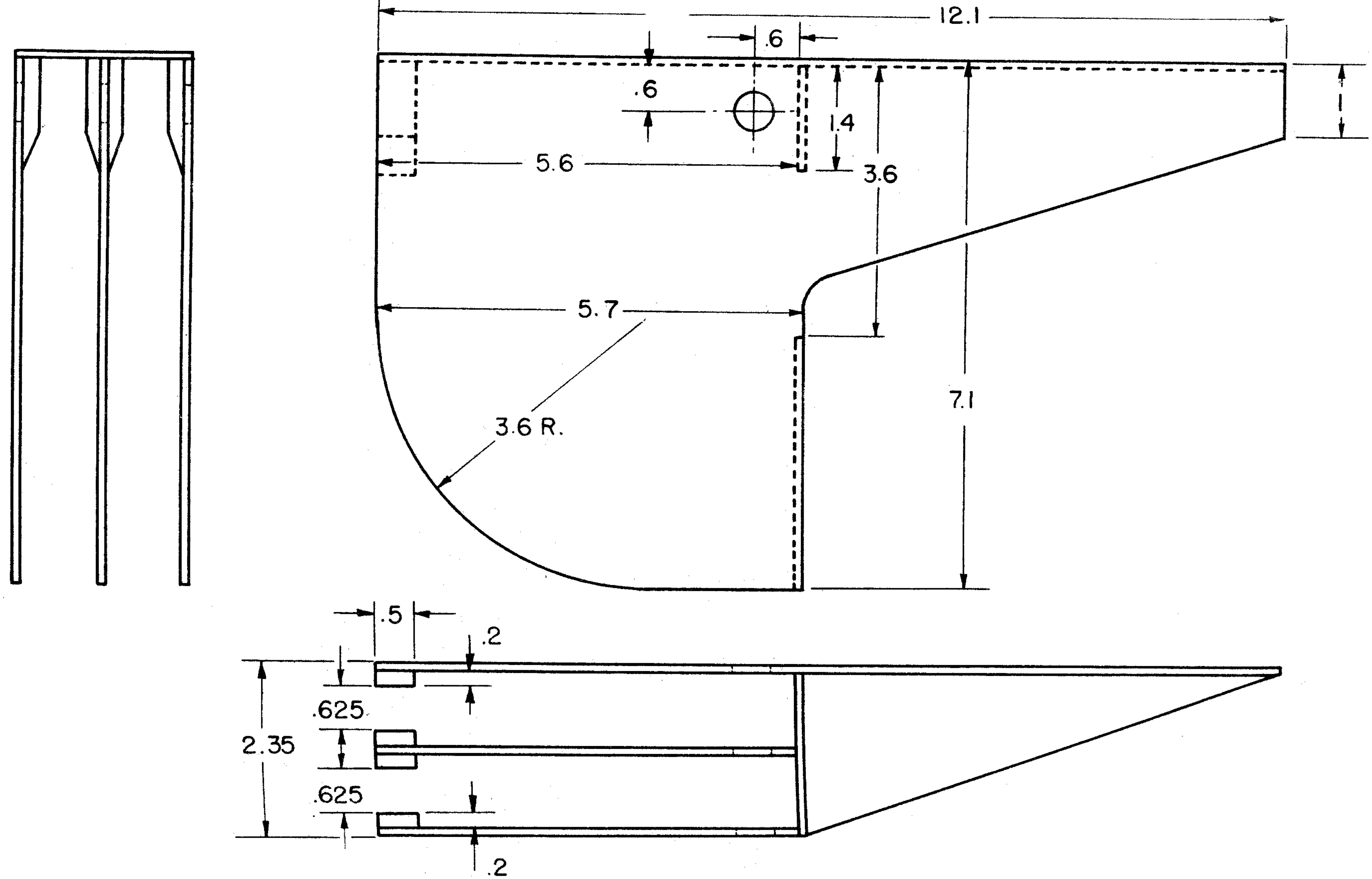
Although no written material was available on this method, a visual inspection of an actual installation was allowed and, from this, plus discussions with Lockheed test personnel, the following description is presented.

The system consists essentially of a water tank, pumps, lines or tubing, a short length of tubing to produce the stream of water and a gun camera. The water is contained in a drum or other suitable container, the capacity of which varies from 10 to 50 gallons, depending upon the number of landings desired to be made before refilling. The device at present averages 5 to 6 gallons of water used per installation per landing. From the drum the water goes through electrically driven pumps where the pressure in the line is raised to approximately 275 psi; fuel pumps driven by an electric motor have been successfully used. From the pump or pumps, the water is carried through three-quarter inch rubber hose to the landing gear where the size is reduced to about one-eighth of an inch by means of a short piece of copper tubing. This one-eighth inch stream of water is then projected vertically downward from a point just above the flat tire line. A device such as this is mounted on each component of the landing gear; left gear, right gear and nose wheel. It takes from 3 to 4 seconds to get up pressure in the lines and start the jet of water flowing.

A gun camera is mounted opposite from each water jet to record the landing operation. These cameras are run at various speeds depending on the type of landing expected but generally either at 32 frames per second or at 64 frames per second. The frame speed of the gun cameras was checked and at 32 frames per second it was found to actually be 31.8 frames per second which gives a reasonably high degree of accuracy. To further insure accuracy an electrical timing device is used in connection with the camera which has a moving pointer that appears in the pictures. Knowing the time that the pointer appears per second, the frame speed can be checked.

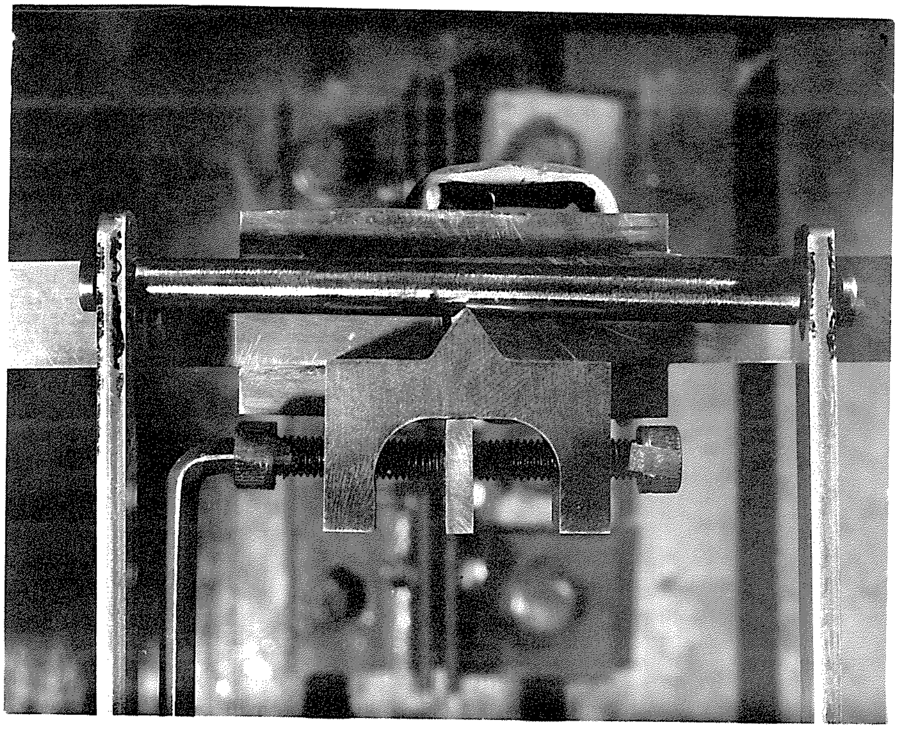
Although the device is new, preliminary landing tests show that it is the best and most accurate method used to date. None of the reduced data was available to incorporate in this report but personnel at Lockheed seem satisfied with the results so far obtained. One undesirable feature of this method is the fact that the data cannot be recorded on something like the Miller Oscillograph along with other test data but must be recorded separately with the usual problems of camera recording.

Figure XV is a photograph taken by the gun camera as the plane was landing. Figure XVI shows a schematic drawing of the installation.

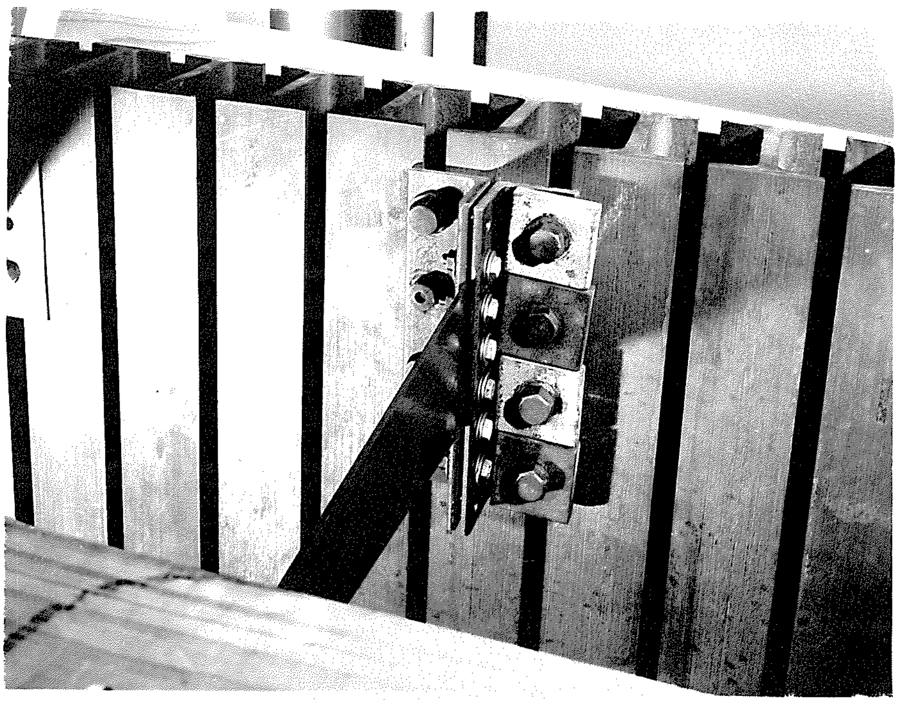


ALL DIMENSIONS IN INCHES

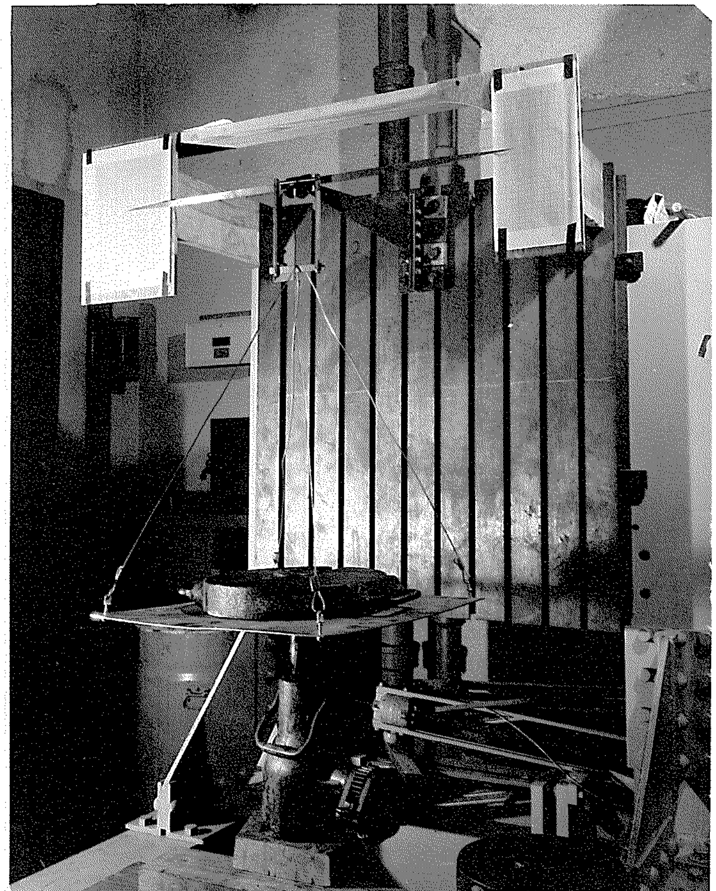
							TOLERANCES ± .010 OR $\frac{1}{64}$ UNLESS OTHERWISE NOTED
							SCALE
MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	REF.
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			ROD ASSEMBLY BOX				FIGURE I
			NAME				DRAWING NO.



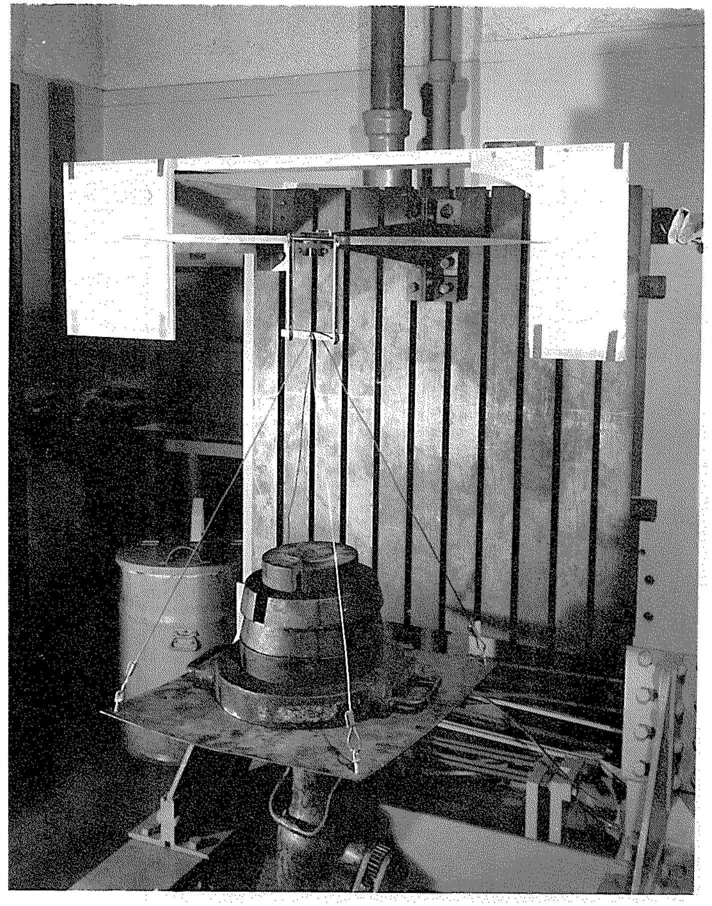
1. Detail of knife edge



2. Detail of fixed end



3. General experimental setup



4. Setup under load

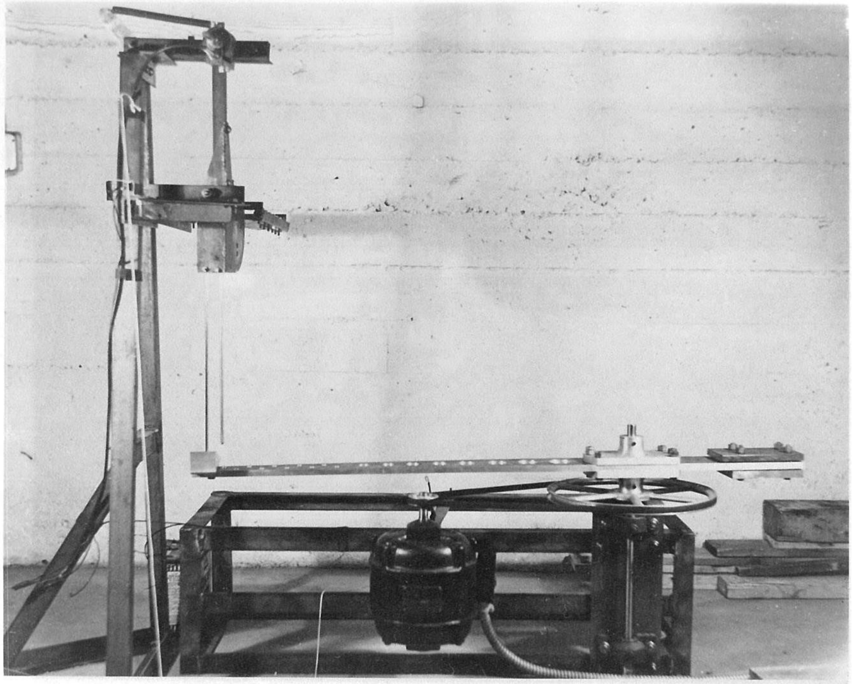


Figure III

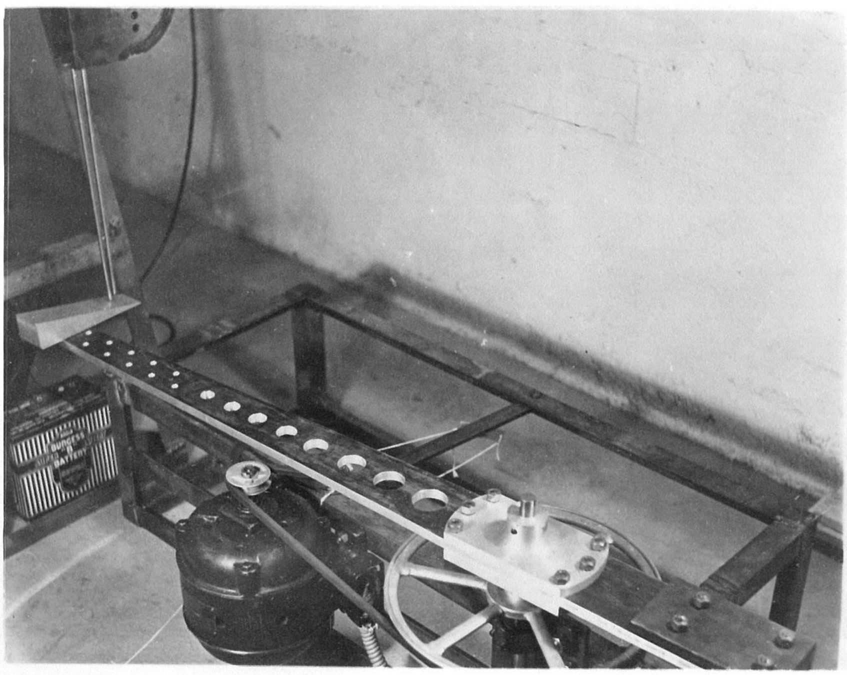


Figure IV

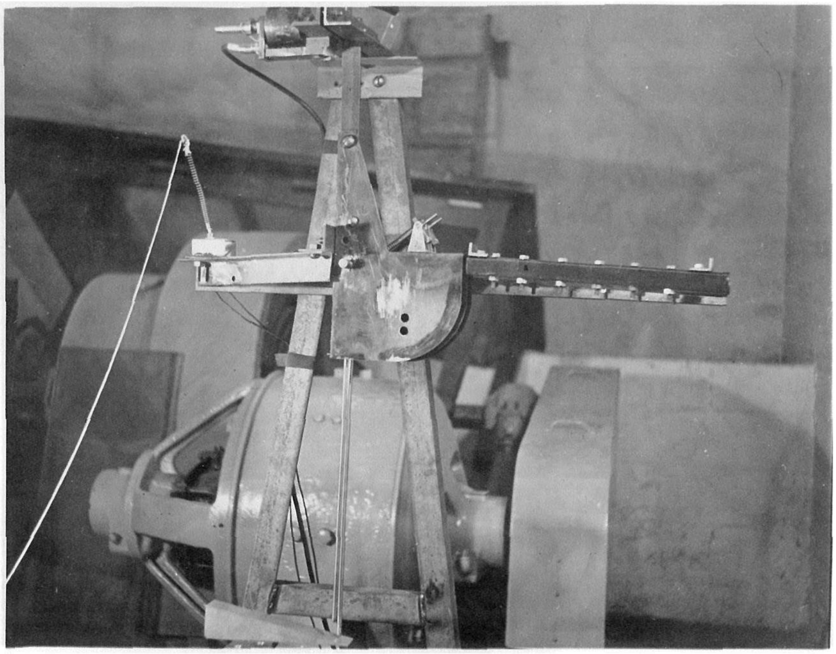


Figure V

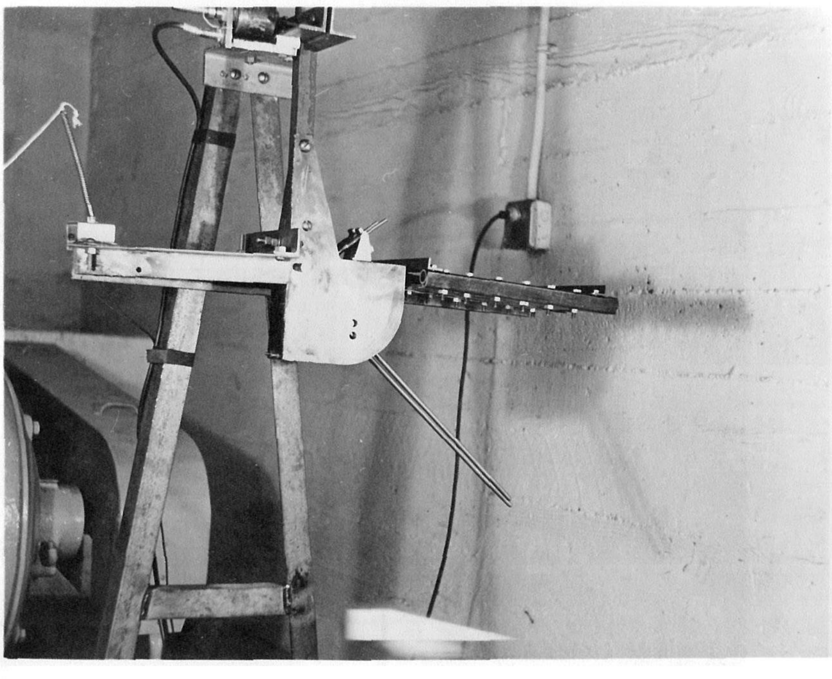
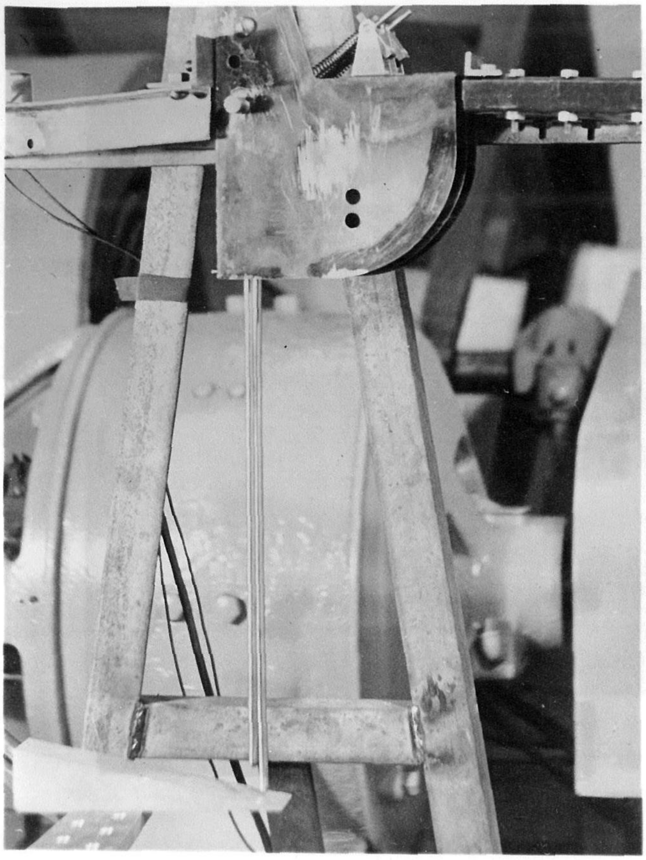


Figure VI



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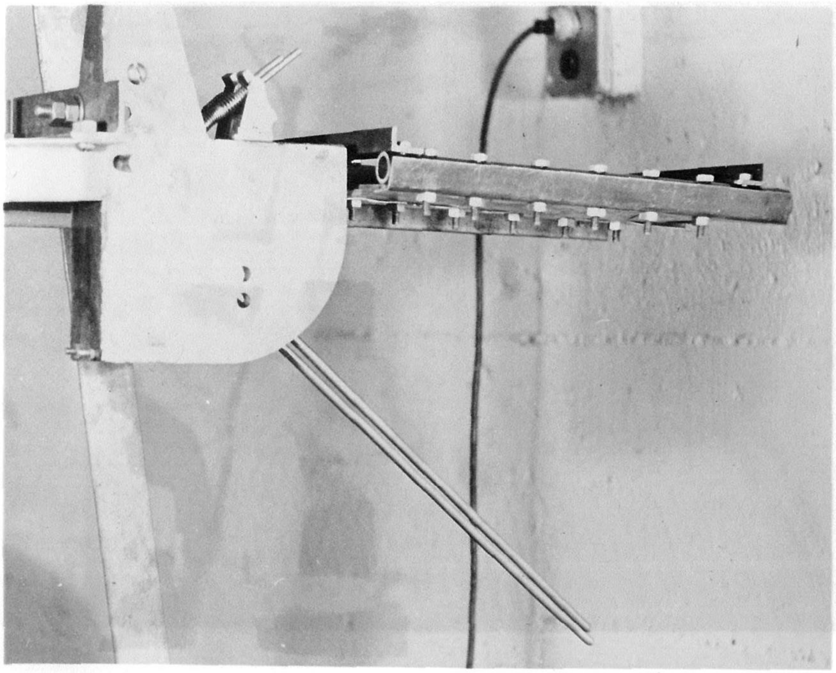


Figure VIII

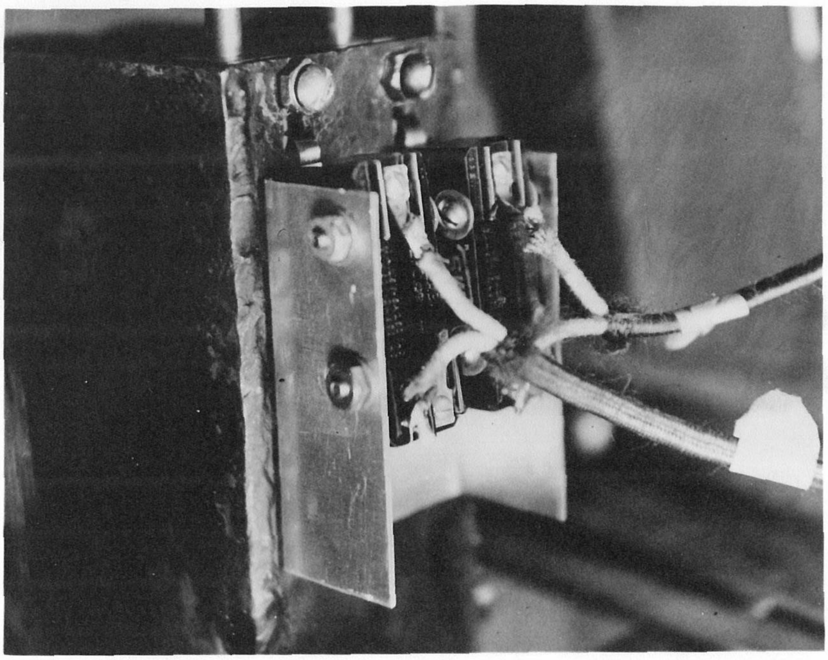


Figure IX

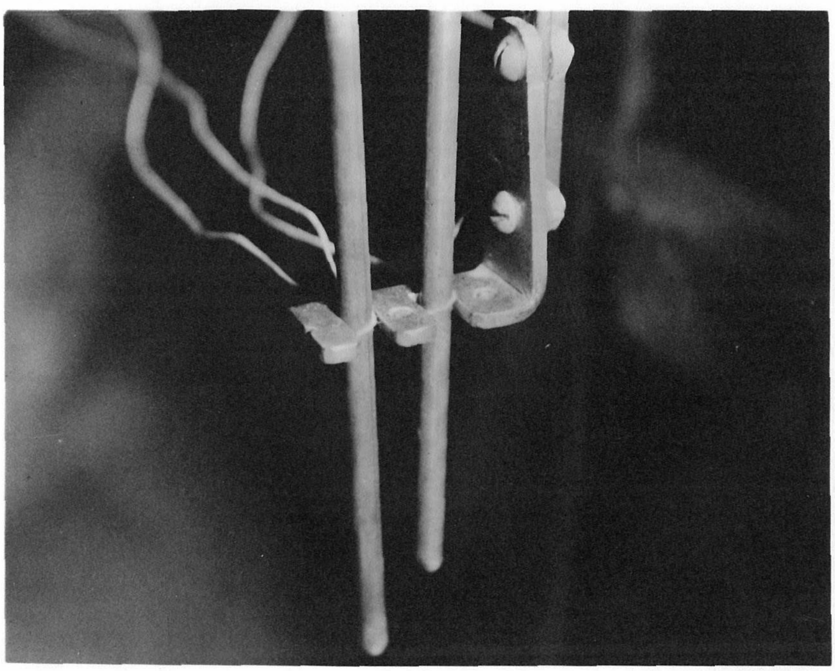


Figure X

Figure XI

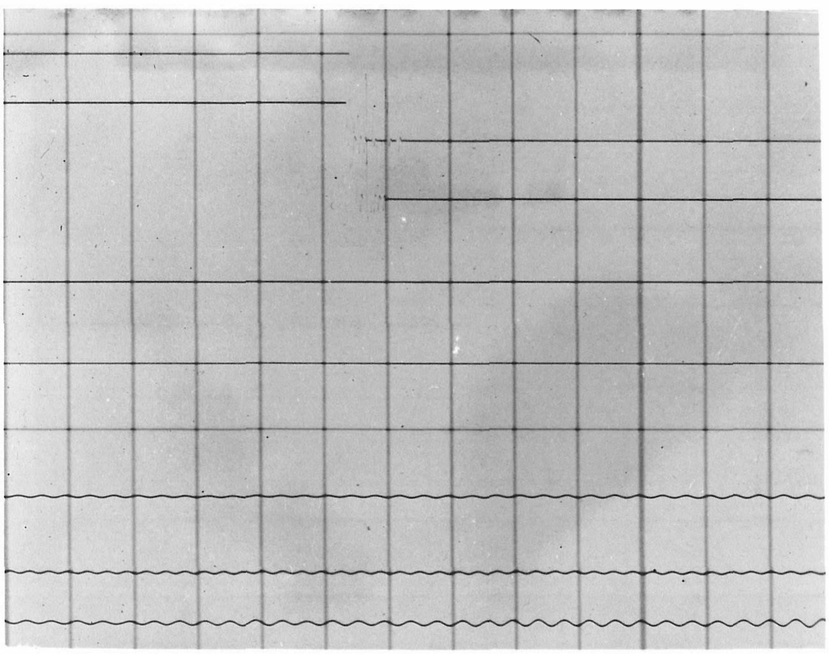


Figure XII

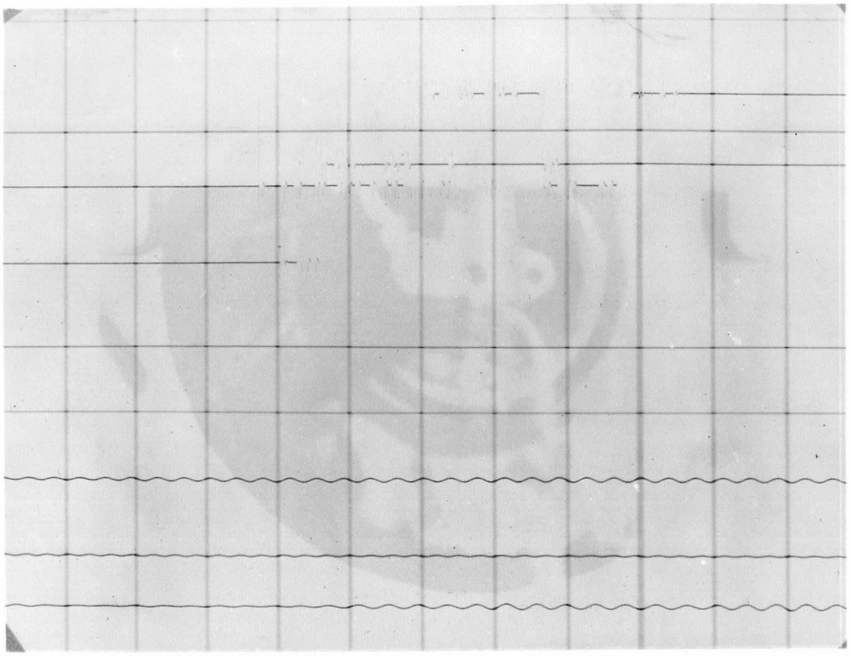


Figure XIII

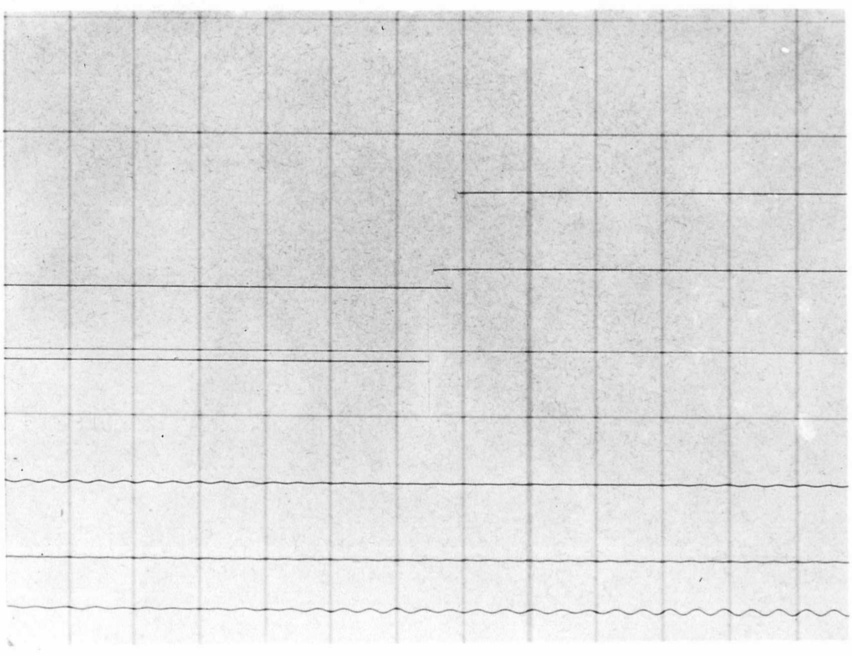


Figure XIV

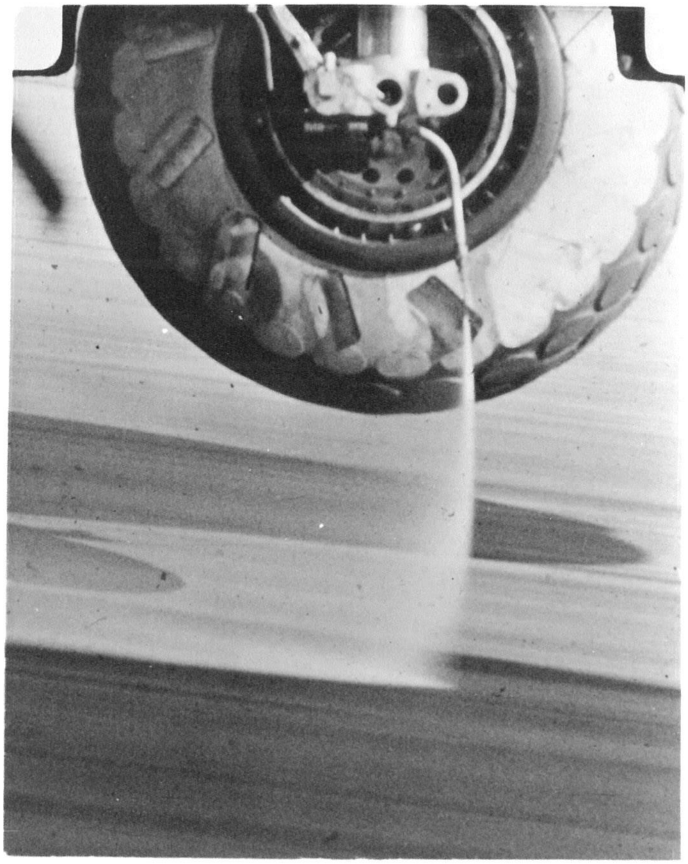
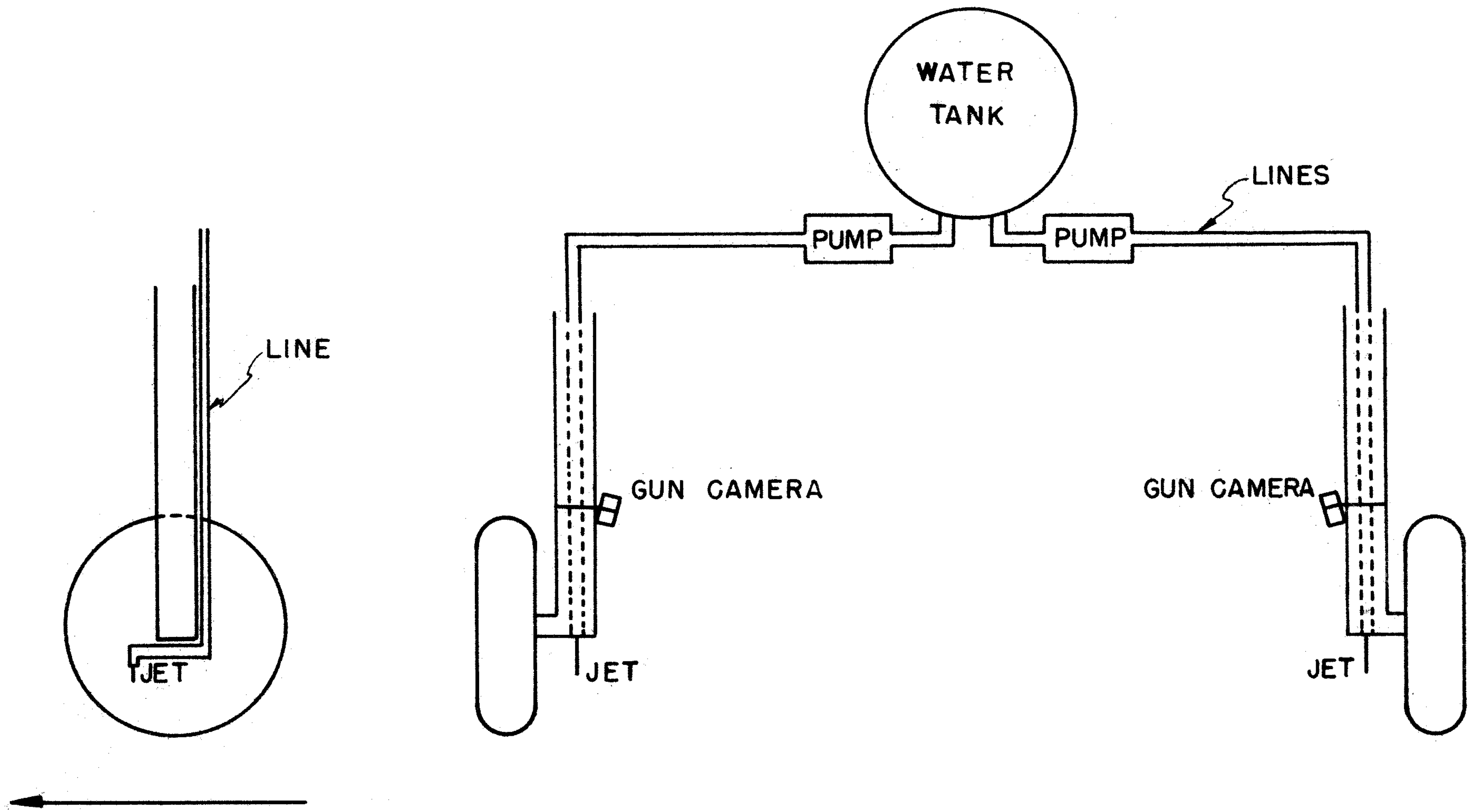


Figure XV



MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	TOLERANCES ± .010 OR $\frac{1}{64}$ UNLESS OTHERWISE NOTED

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SCHEMATIC DRAWING OF LOCKHEED WATER JET METHOD

NAME

SCALE

REF.

FIGURE XVI

DRAWING NO.