EXTENDED SUPERGRAVITY WITH A GAUGED CENTRAL CHARGE

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Abstract:

We construct a Lagrangian for the massive scalar multiplet, locally invariant under two types of spinorial transformations (N=2 supersymmetry). Our theory is based on the coupling of the global supermultiplet to N=2 supergravity and corrections generated iteratively in powers of Newton's constant. Consistency of the theory requires the vector field of supergravity to gauge the central charge represented in the massive sector of the multiplet. The same vector may alternatively gauge the internal O(2) symmetry of the two supersymmetry generators. Furthermore, it may even gauge a linear combination of the generators of these two groups; we indicate the grounds for this compatibility.

We discuss the hierarchy of internal symmetries characterizing each sector of the theory, ranging from U(1)xSU(2)xSU(2) down to O(2)xO(2). This internal symmetry imposes tight constraints on the system. For instance, the nonpolynomial structure of the spinless fields at hand is considerably more restricted than that present in the general simple supersymmetric (N=1) theory. Furthermore, the vector field is forced to couple to the matter fields with gravitational strength, to the effect that the resulting Coulomb potential exactly cancels against the Newtonian potential of gravity, in the static limit.

Our theory may be also viewed as a truncation of N=8 supergravity theory, compatibly with the SO(8) breakup scheme into SU(3)xU(1)xU(1). The potential of the spinless fields present has a local minimum at the origin, but further off it is not even bounded from below. However, we point out some indications that the tunneling out of the supersymmetric, metastable vacuum is negligibly small.

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1) INTRODUCTION

1.1) Supersymmetry and Supergravity

In the last few years, a new class of local field theories has been under intensive study. These are characterized by supersymmetry, a fundamental extension of spacetime symmetry. Specifically, the algebra of the Poincaré group is enlarged by the inclusion of spinorial, anticommuting generators which describe supersymmetry. As a result, the ensuing Graded Lie Algebras contain anticommutation relations in addition to commutation relations, thereby circumventing a previous no-go theorem[Co67] that excluded non-trivial, fully relativistic unifications of internal symmetries with the Poincaré group.

Historically, the first instances of supersymmetric field theories appeared in [Go71, Vo73]. Later on, the fermion-boson mixing concepts of the Graded Supergauge Algebras involved in dual models[Ne71,Ra71] were applied to field theoretical realizations[Ge71, We74a, We74b] and were studied on a systematic basis. Reviews and early references may be found in [Zu 75, Fa77].

The novel feature of supersymmetry operations is that they connect fermions with bosons, differing in spin by half a unit. As a result, they allow for (super)multiplet representations which involve particles of several different spins. For example, there exist supersymmetric models in which the Higgs scalars occurring in current phenomenologically realistic field theories belong to the same multiplets as the vectors and the spinors. It is interesting that this is made possible by symmetry principles tightly connected with the structure of spacetime.

Thus emerges the enticing prospect of utilizing supersymmetry as the framework for the unification of all known interactions, strong, weak, electromagnetism, and gravity. In this connection we may cite realistic extensions of Weinberg-Salam type models [Fa77, Fa78a, Fa78b; references on the phenomenological aspects cited therein]. A central problem to be faced in such a program is that of providing a satisfactory mechanism for supersymmetry breaking, since fermi-bose degeneracy does not manifest itself in nature.

An attractive feature of supersymmetric theories is their improved finiteness, due to the systematic cancellation of divergent diagrams interconnected by symmetry. Super-Ward identities reduce the number of renormalization constants necessary[Il74, Na75], and even possibly unrenormalizable theories may yield finite on-shell amplitudes to at least two loops[Van78]. Further similar formal surprises are the vanishing of the Gell-Mann-Low function $\psi(g)$ to two loops in an N=4 supersymmetric Yang-Mills theory[Jo77, Po77], and the complete absence of quantum corrections to the classical spectrum of an N=2 supersymmetric Yang-Mills theory[Wi78]. An extensive investigation of abstract supersymmetric models has been undertaken, in order to clarify the structure of the symmetry, and the constraints, possibilities, and mechanisms inherent in them.

The Poincaré algebra may be enlarged by the addition of up to N=8 different supersymmetry generators, as well as bosonic charges (called central, since they commute with all elements of the algebra). In extended supersymmetry ($N > 1$), the irreducible representations are large, and therefore of particular physical interest.

Simple supersymmetry (N=1) has representations that are not as large, but models in which they interact exhibit most of the drastically new features of supersymmetry and they have been studied more extensively.

In a massless realization, each of the N supersymmetry generators lowers or raises the helicity of a state by 1/2[Sa74, Ha75]. Each generator anticommutes with itself, so that its square is zero. Consequently, starting from the state of highest helicity λ, we may go down to $\lambda - \frac{1}{2}N$ by N steps of 1/2 helicity units. The multiplicity of states at the helicity level $\lambda - \frac{n}{2}$ of the supermultiplet is equal to $\{^N_n\}$, the number of antisymmetric combinations of n generators out of N. The multiplicities $\{^N_o\}=1$, $\{^N_1\}=N$, $\{^N_2\}= \frac{N(N-1)}{2!}$, ..., add up to the total number of states $(1+1)^N=2^N$. We see that the number N of different supersymmetry generators in a theory is bounded from above by 4λ: for $|\lambda|\leq 2$ we have $N\leq 8$; for $\lambda\leq\frac{3}{2}$, $N\leq 6$; for $\lambda\leq 1$, $N\leq 4$; and for $\lambda\leq\frac{1}{2}$, $N\leq 2$. In the massive case there are in general more states, but their number may be reduced in special cases where there is a central charge in the algebra[So78] down to the same number and spin range as in the massless case. When the symmetry is realized linearly, the number of fermion degrees of freedom of the system always equals the number of the boson degrees of freedom.

The supersymmetric invariance of the theory may be either spacetime independent ("global", i.e., "gauge invariance of the first kind"); or else it may be spacetime dependent ("local", i.e., "gauge invariance of the second kind"), by virtue of suitable gauge fields for supersymmetry. The gauge fields transform as the gradient of the infinitesimal (super) transformation parameter so as to allow the construction of (super) covariant derivatives.

The gauge theories of supersymmetry are called supergravity theories. They consist of Einstein's gravity (describing a spin 2 graviton), coupled to the gauge fields of supersymmetry (spin 3/2 gravitinos), as well as other fields of lower spins (for $N>1$), subject to the state counting restrictions already mentioned. For reviews, see [Fr77b, Van78, Fe78a].

Supergravity theories are particularly interesting on several grounds. They rectify previous consistency problems of spin 3/2 theories [Da76], namely ghosts and acausal signal propagation [Ve69]. They have prompted progress in ordinary Einstein gravity, such as the proof of the positivity of energy [Des77, Gr78a]. They furthermore unify other fields with gravity more consistently and naturally than in the past, improving the prospects for a consistent quantum theory of gravity. In particular, as a result of their enhanced symmetry, supergravity amplitudes have finite one and two loop quantum corrections, unlike any other matter theory coupled to gravity [Van78].

It should be pointed out that no theory including gravity is known to be strictly renormalizable, due to the presence of the dimensional coupling constant κ of Newton and the corresponding p^2 momentum dependence of gravitonvertices. The degree of divergence obtained by naive power counting is $2+(n-2)I$, where n is the dimensionality of spacetime and I is the number of loops. For $n>2$ the degree of divergence increases indefinitely with the number of loops. It should be noted though that there is no clear guarantee of the appropriateness of local field theoretical methods at distances of the order of the Planck length.

We indicated above that a major virtue of supergravity theories is the richness of their irreducible representations. The largest of these containing just one graviton is N=8 supergravity. It is possible to embed $SU(3)^c xU(1)xU(1)$ in its internal symmetry group SO(8) and thus break up every spin level of the supermultiplet into representations of this SU(3) color group and the "weak-electromagnetic" U(1)xU(1) group accordingly. The representations are thus identifiable with most of the known elementary fields, and an interesting consequence is that the 2/3, -1/3 charges of the quarks appear naturally in this scheme[Ge77]. (The charge of the spinor identified with the electron comes out three times the fundamental charge in the theory, in accordance with the 3 step descent in helicity levels from 2 to 1/2).

On the face of it, the above example is inadequate, since the electromagnetic-weak group involved is not U(1)xSU(2), so that there is no room for charged W bosons (worse yet, it turns out that there can be no more than four quark flavors at the very best, nor another charged lepton to represent the muon). However, it should be borne in mind that some current speculations on simple, non-supersymmetric gauge models[Ge74] estimate the unification of the strong interactions with the weak ones to take effect at energies of the order of the Planck mass ($\simeq 10^{19}$GeV). Again, as before, there is no reason to trust our intuition as to which fields are elementary or composite at similar energies.

Large supermultiplets (representations of U(N) as opposed to SO(N) groups) could be gotten from Weyl supergravities[Ka77, Van78], which are the gauge theories of supersymmetric extensions of the conformal group. These theories are troubled by instability and non-unitarity problems characteristic of higher derivative field theories. Furthermore, in addition to supersymmetry breaking, mechanisms for the breaking of conformal invariance are also required. Weyl supergravity underlies Poincaré supergravity, and it is mostly useful in clarifying the structure of the latter.

A globally supersymmetric matter theory becomes a locally supersymmetric one by being suitably coupled to supergravity. The quantum amplitudes are then no longer two-loop finite as in the pure supergravity theories mentioned, since the matter and supergravity fields do not belong to a single irreducible multiplet any longer. In some cases, such a system is a consistent truncation of a pure supergravity theory of higher N, and might therefore be a useful step in the construction of the larger theory. For example, the N=1 supermultiplet (3/2, 1) coupled to N=1 supergravity (2, 3/2) may be extended to N=2 supergravity (2, 3/2, 3/2, 1)[Fe76a]. Likewise, the N=1 scalar multiplet (1/2, 0, 0) coupled to N=1 supergravity (2, 3/2) was a useful preliminary to the construction of N=4 supergravity (2, 4x3/2, 6x1, 4x1/2, 0, 0)[Cr77b, Cr77c].

Typically, locally supersymmetric matter theories furnish information on the structure of the local supersymmetry algebra, on the internal symmetries, and on their connection to supersymmetry. The constraints on the interaction terms allowed in these theories determine the (super) symmetry breaking possibilities of the system.

Some examples of locally supersymmetric matter theories are the following: The N=1 Yang-Mills supermultiplet n(1, 1/2) coupled to N=1 supergravity (2, 3/2)[Fr77c]; the N=1 scalar supermultiplet (1/2, 0, 0) with mass and interaction terms made locally supersymmetric[Fe77a, Da77a, Da77b, Cr77b]; the N=2 Yang-Mills supermultiplets n(1, 1/2, 1/2, 0, 0) coupled to N=2 supergravity (2, 3/2, 3/2, 1)[Lu78]; the N=2 scalar multiplet (1/2, 1/2, 0, 0, 0, 0) coupled to N=2 supergravity, formulated here in detail[Za78].

Recently, the gauging of N=1 supermultiplets has been considerably simplified, generalized, and extended through use of a minimal set of auxiliary fields for N=1 supergravity[Fe78b, St78], and the related elegant rules for the composition of several multiplets into new ones, comprising a "tensor calculus"[Fe78c]. These advances have permitted a more complete study of supersymmetry breaking[Cr78c]. Extension to N>1 supersymmetry hasn't been achieved yet. The N=2 scalar multiplet made locally supersymmetric, which we shall consider presently, might facilitate this program[So78b], since the form of a matter theory's coupling to supergravity is characteristic of the underlying auxiliary field structure. It might also provide helpful clues concerning the structure of more ambitious theories like N=8 supergravity[De77, Cr78d], as well as insight into the internal symmetries present in systems with extended supersymmetry.

1.2) Formulation of the Model

Central charges[Ha75] are scalar generators admissible in extended supersymmetry ($N>1$) algebras that have the dimensions of mass. They play an important structural role in supersymmetric model building, since they relate supersymmetry to internal symmetries, thereby constraining the spectrum of the supermultiplets, as well as their local symmetry properties. They have not been plausibly identified with familiar symmetries; instead, they may be thought to represent an intermediate stage in the symmetry breaking hierarchy of larger unified theories.

In a given theory, the central charge may or may not have a trivial action; but if there are any states with nonzero charge, the theory must contain mass, or at least it must allow for some mass scale, set for instance by the expectation value of a scalar field $\langle A \rangle$ (corresponding to the breaking of an internal symmetry[Wi78, Fa78b]), or by a dimensional coupling constant like $1/\kappa$[Fe77b].

Conversely, if a theory contains mass, the central charge serves to reduce the range of helicities spanned. For a massive, N-supersymmetric theory, an analogous counting argument to that of the previous section for the massless representations gives 2^{2N} states, in contrast to the 2^N states specified when there is no mass. The presence of the central charge however may reduce this higher multiplicity 2^{2N} back to 2^N [Fa78b, So78a]. Hence, a massless representation may only acquire a mass in the presence of a central charge in the algebra, so that the resulting theory will have the same multiplicity of states as the original massless one.

In Section 2.1 we review the simplest matter theory containing a central charge, namely the N=2 massive scalar supermultiplet[Fa76, Fe77c] (1/2, 1/2, 0, 0, 0, 0). It consists of two Majorana spinors χ^i (i=1, 2), two scalars A^i, and two pseudoscalars B^i. We briefly demonstrate the necessity of this doubling of fields in the N=2 representation. The Lagrangian is invariant under the two types of global supersymmetry transformations, characterized by constant infinitesimal parameters ε and ζ, which are spinorial (anticommuting). These transformations turn a fermion into a boson and vice versa. The theory is also invariant under the transformation generated by the central charge, which rotates the fields into each other without altering their spin. The transformation parameter is proportional to the mass.

In Section 2.2 we review N=1 supergravity theory which contains a spin 2 graviton $V_\mu{}^a$ and a spin 3/2 gravitino ψ_μ. This theory is invariant under the general coordinate and local Lorentz transformations of general relativity, but also under a <u>local</u> supersymmetry transformation, characterized by a spacetime dependent spinorial parameter $\varepsilon(x)$. This is to say that the strength of the transformation varies over the various points of the manifold; the symmetry is gauged, ψ_μ being the relevant gauge field.

In Section 2.3 we complete the review of background material by extending the above theory to N=2 supergravity. A second gravitino is required to gauge the second supersymmetry ($\zeta(x)$). A vector field A_μ is also present, which may gauge any one of several internal symmetry groups consistently. In this section we arrange for it to gauge the internal symmetry connecting the two supersymmetry generators, thereby introducing an arbitrary coupling constant e[Fr77a].

In Section 2.4 we construct the locally invariant version of the massless scalar multiplet, by coupling the global theory of Section 2.1 to the N=2 supergravity of Section 2.3. Initially we implement iterative techniques, by adding appropriate terms in the Lagrangian and the transformation laws, order by order in κ. At later stages we resort to functional techniques[Da77a, Da77b] and calculate the theory to all orders in κ at once.

In Section 2.5 we repeat the above procedure to construct the massive version of the theory. We establish by direct calculation that for the coupling to be consistent A_μ must gauge the central charge symmetry with a coupling constant κm. Alternatively, A_μ may be used to gauge the internal symmetry of the two supersymmetry generators with another coupling constant e, thereby extending the results of Freedman and Das[Fr77a] (Section 2.3) to matter coupled theories. We then show how these two alternatives interlock consistently into a combined theory. A_μ now gauges the diagonal product of the two groups (whose elements commute with one another), that is, an arbitrary linear combination of their generators.

In Section 3.1 we survey the internal symmetry properties of the theory. The massless sector of Section 2.4 is invariant under U(1)xSU(2)xSU(2). The relevant groups correspond to a duality invariance, the central charge and the two chiral partners of it, and the rotation of the two supersymmetry generators with its chiral partners.

When gauge couplings are introduced the symmetry breaks down to O(2)xSU(2), SU(2)xO(2), or O(2)xO(2), respectively. In each case, whenever an O(2) is gauged by A_μ, its two chiral partners are broken. A discussion of the local supersymmetry algebra and the relevance of the background metric leads to a clarification of the peculiar alternative gauging mentioned above. We conclude this section by commenting on the first order form of the theory, and by introducing a concise SO(4) notation to express the SU(2)xSU(2) part of our theory more elegantly.

In Section 3.2 we discuss some systematic regularities stemming from the symmetry, and comment on related speculations about the origin of the central charge. We illustrate our points by considering an alternative example drawn from the N=2 Yang-Mills supermultiplet coupled to supergravity. In particular, we analyze a characteristic signal for the A_μ gauging of a central charge associated with internal symmetry breaking. We discuss a peculiar feature of the constrained couplings of our model. The vector A_μ couples to the central charge current with strength κm, which leads to a sort of "Antigravity." In the static limit, the classical Coulomb potential of the vector A_μ exactly cancels the Newtonian potential of the gravitational interaction. At higher energies, this balance is distorted and the two interactions exhibit their different spin characteristics. Appendix B contains a more detailed discussion of this phenomenon.

In Section 3.3 we consider in detail how the tight group structure present constrains the nonpolynomial behaviour of the spin-zero fields of the theory into a form far more restricted than for the N=1 case. In this connection, we also outline N=8 supergravity, and indicate the general framework for truncating it to the theory under study. We point out that this truncation is compatible with Gell-Mann's proposed embedding of SU(3)xU(1)xU(1) in SO(8)[Ge77], and consequently the various resulting multiplets can be assigned to definite SU(3) representations. We relegate some conjectural particulars to Appendix C.

Finally, we examine the family of Higgs potentials arising out of the above restrictions in Section 3.4. They exhibit a local minimum at the origin, but further down, past a barrier, there is a negative singularity. Every state will eventually decay into this singularity. A cursory analysis indicates that this pathological theory might have a property we call "virtual stability", in which supersymmetry remains effectively unbroken. In Appendix D we give a sketchy argument indicating that the "false", metastable vacuum at the origin of the potential has a tunneling rate into the singularity that can be vanishingly small. We conclude with a brief summary of the theory and its distinguishing features in Section 3.5.

2) CONSTRUCTION AND INVARIANCE PROOF

2.1) The Global Scalar Multiplet

The Graded Lie Algebra of N = 2 supersymmetry[Sa74,Ha75] consists of the Poincaré group augmented by the additional relations:

$$\{Q^I_\alpha, \bar{Q}^J_\beta\} = \delta^{IJ}\gamma^\mu_{\alpha\beta}P_\mu + \varepsilon^{IJ}\delta_{\alpha\beta}Z \qquad (2.1\text{-}1)$$

$$[Z, (\text{anything})] = 0$$

$$[P_\mu, Q^I_\alpha] = 0$$

$$[M_{\mu\nu}, Q^I_\alpha] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q^I_\beta$$

where α,β are 4-spinor indices, and μ,ν are Lorentz indices. The Q^I_α's are the Majorana spinor generators of the two supersymmetries of ε-type (I = 1) and ζ-type (I = 2), respectively. The anticommutation of two ε or ζ type generators yields a linear combination of momentum components weighted by Dirac gamma matrices, which serve as structure constants. On the other hand, an ε- and a ζ-type generator anticommute to yield a central charge Z[Ha75], which is a generator in the center of the algebra*. (The center of an algebra is the subalgebra whose elements commute with the entire algebra.) Z must have the dimension of P_μ, that is mass.

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*A second permissible central charge term in the Algebra (2.1-1), namely $\gamma_{5\alpha\beta}\varepsilon^{IJ}Z'$ can always be rotated into $\varepsilon^{IJ}Z$ by a chiral transformation, and therefore does not represent a more general structure.

A global realization of this algebra is the $N = 2$ massive scalar supermultiplet[Fa76, Fe77c], which describes two spinors χ^i ($i = 1, 2$), two scalars A^i and two pseudoscalars B^i. The Lagrangian of the system is:

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu A^i \partial^\mu A^i + \frac{1}{2}\,\partial_\mu B^i \partial^\mu B^i + \frac{i}{2}\,\bar{\chi}^i \not{\partial} \chi^i \qquad (2.1\text{-}2)$$

$$- \frac{m^2}{2}\,(A^{i2} + B^{i2}) - \frac{m}{2}\,\bar{\chi}^i \chi^i .$$

Supersymmetry constrains all masses of the theory to be equal, since it commutes with the momentum operator. The Lagrangian is invariant (up to a divergence) under the two supersymmetry transformations with infinitesimal constant spinorial parameters ε and ζ, and the central charge transformation of infinitesimal parameter Λ' (Λ' is dimensionless):

$$\delta A^i = \frac{1}{\sqrt{2}}\,\bar{\varepsilon}\chi^i + \frac{\varepsilon^{ij}}{\sqrt{2}}\,\bar{\zeta}\chi^j + \Lambda' m \varepsilon^{ij} A^j \qquad (2.1\text{-}3)$$

$$\delta B^i = \frac{1}{\sqrt{2}}\,\bar{\varepsilon} i\gamma_5 \chi^i - \frac{\varepsilon^{ij}}{\sqrt{2}}\,\bar{\zeta} i\gamma_5 \chi^j + \Lambda' m \varepsilon^{ij} B^j$$

$$\delta \chi^i = - \frac{i}{\sqrt{2}}\,(\not{\partial} - im)(A^i + i\gamma_5 B^i)\varepsilon + \frac{i}{\sqrt{2}}\,\varepsilon^{ij}(\not{\partial} - im)(A^j - i\gamma_5 B^j)\zeta$$

$$+ \Lambda' m \varepsilon^{ij} \chi^j .$$

The commutator of two supersymmetry transformations on <u>any</u> field ϕ^i of the multiplet is

$$[\delta',\delta]\phi^i = i(\bar{\varepsilon}'\gamma_\mu\varepsilon+\bar{\zeta}'\gamma_\mu\zeta)\partial^\mu\phi^i + m(\bar{\varepsilon}\zeta'-\bar{\zeta}\varepsilon')\varepsilon^{ij}\phi^j \tag{2.1-4}$$

This is consistent with (2.1-1) since

$$-[\bar{\varepsilon}'Q_\varepsilon+\bar{\zeta}'Q_\zeta,[\bar{\varepsilon}Q_\varepsilon+\bar{\zeta}Q_\zeta,\phi^i]] + [\bar{\varepsilon}Q_\varepsilon+\bar{\zeta}Q_\zeta,[\bar{\varepsilon}'Q_\varepsilon+\bar{\zeta}'Q_\zeta,\phi^i]]$$

$$= [[\bar{\varepsilon}Q_\varepsilon+\bar{\zeta}Q_\zeta,\bar{\varepsilon}'Q_\varepsilon+\bar{\zeta}'Q_\zeta],\phi^i] = \tag{2.1-5}$$

$$= [(\bar{\varepsilon}\gamma_\mu\varepsilon'+\bar{\zeta}\gamma_\mu\zeta')P_\mu+(\bar{\varepsilon}\zeta'-\bar{\zeta}\varepsilon')Z,\phi^i].$$

If the theory is massive, we see that the commutator of two different supersymmetry operations gives a central charge transformation of parameter $\Lambda' = \bar{\varepsilon}\zeta' - \bar{\zeta}\varepsilon'$. Thus the central charge is a <u>necessary</u> operator in the algebra, as discussed in the previous section.

It might appear from the counting of states given in Section 1.1 that an adequate realization for N = 2 supersymmetry could consist of only one spinor χ, one scalar A, and one pseudoscalar B (just as for the N = 1 case) and thus the doubling of fields in the theory so far described would be unnecessary. However, it turns out that without this doubling, there is not enough room to accommodate the higher symmetry of the N = 2 algebra. This can be proved by considering the most general linear supersymmetry transformations on A and χ (taken for simplicity to be massless), with real coefficients a, a', b, c, ...

$$\delta A = \frac{1}{\sqrt{2}} (a\bar{\varepsilon}\chi + a'\bar{\varepsilon} i\gamma_5\chi + b\bar{\zeta}\chi + c\bar{\zeta} i\gamma_5\chi), \ \delta B = \ldots \qquad (2.1\text{-}6)$$

$$\delta\chi = -\frac{1}{\sqrt{2}} \not{\partial} A(d\varepsilon + ei\gamma_5\varepsilon + f\zeta + gi\gamma_5\zeta) + \text{terms linear in B.}$$

In a chirally invariant theory, we can rotate away the a' term. An inescapable constraint on any representation is closure of the transformations on any field (e.g., A) into the N = 2 algebra $[\delta',\delta]A = i(\bar{\varepsilon}'\gamma^\mu\varepsilon + \bar{\zeta}'\gamma^\mu\zeta)\partial_\mu A$. Closure of the transformations (2.1-6) fixes their coefficients to:

$$1 = da, \ 1 = cg + bf, \ 0 = af + bd + ce, \ 0 = ag - eb + cd. \qquad (2.1\text{-}7)$$

The last three of these relations yield $d(a^2+b^2+c^2) = 0$ which cannot be satisfied for $a \neq 0$, $d \neq 0$, as required by the first relation. Consequently, (χ, A, B) do not constitute an adequate set, and they must be doubled to represent the N = 2 algebra by a field theory.

The Noether currents of the theory in (2.1 - 2) are:

$$J^\varepsilon_\mu = \frac{1}{\sqrt{2}} [\partial^\nu(A^i + i\gamma_5 B^i)\gamma_\nu\gamma_\mu\chi^i + im(A^i + i\gamma_5 B^i)\gamma_\mu\chi^i] \qquad (2.1\text{-}8)$$

$$J^\zeta_\mu = \frac{\varepsilon^{ij}}{\sqrt{2}} [\partial^\nu(A^i - i\gamma_5 B^i)\gamma_\nu\gamma_\mu\chi^j + im(A^i - i\gamma_5 B^i)\gamma_\mu\chi^j]$$

$$J^{\Lambda'}_\mu = \varepsilon^{ij}[\partial_\mu A^i A^j + \partial_\mu B^i B^j + \tfrac{i}{2}\bar{\chi}^i\gamma^\mu\chi^j]$$

and they are conserved on the mass shell. The current for translations P_ν is the energy-momentum tensor:

$$T_{\mu\nu}= \tfrac{1}{2}\partial_\mu A^i \partial_\nu A^i + \tfrac{1}{2}\partial_\mu B^i \partial_\nu B^i + \tfrac{i}{4}\,\bar{\chi}^i(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\chi^i - \tfrac{1}{2}g_{\mu\nu}\mathcal{L}\,. \tag{2.1-9}$$

These currents can be seen to transform as:

$$\delta_{\Lambda'}J^\varepsilon_\mu = 0,\ \delta_\varepsilon J^{\Lambda'}_\mu = \ldots,\ \delta_\varepsilon J^\varepsilon_\mu = -i\gamma^\nu T_{\mu\nu}\varepsilon + \ldots \tag{2.1-10}$$

$$\bar{\zeta}\delta_\varepsilon J^\zeta_\mu = -\bar{\varepsilon}\delta_\zeta J^\varepsilon_\mu = -m\bar{\varepsilon}\zeta J^{\Lambda'}_\mu + \ldots$$

where ... indicate terms K_μ which are conserved ($\partial_\mu K^\mu = 0$) and chargeless ($\int d^3\chi K_o = 0$) bilinears of the fields in the theory. Consequently, the space integrals of the zeroth component of the above transformations (2.1-10) reconstitute the Graded Lie Algebra (2.1-1), as they should*.

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* All currents could be "improved" by the addition of chargeless, conserved terms K_μ. It has been noted in the massless N = 1 theory[Fe75] that the supercurrent J^ε_μ, the energy momentum tensor $T_{\mu\nu}$, and one out of several allowed chiral currents $J^{(5)}_\mu$ ($= A\partial_\mu B - B\partial_\mu A + \frac{i}{4}\bar{\chi}\gamma_5\gamma_\mu\chi$) may be suitably improved so as to give a supermultiplet. This amounts to their transforming into each other under supersymmetry, through use of the equations of motion. For instance, $\delta_\varepsilon J^{\varepsilon\,(imp)}_\mu = -i\gamma^\nu T^{(imp)}_{\mu\nu}\varepsilon + \frac{i}{6}\,\partial_\nu J^{(5)(imp)}_\kappa(\varepsilon^{\nu\kappa\tau\lambda}\gamma_\tau g_{\lambda\mu} + i\gamma_5\gamma^\nu\delta^\kappa_\mu)\varepsilon$. When mass is introduced, new improvements are necessary, and more currents enter into the supermultiplet. To close the N = 2 supermultiplet, an even larger number of extra bilinears is needed[So78b].

In converting the theory of this section into a locally supersymmetric one, we shall condense the notation, to express the formulas more compactly:

$$Z^i \equiv A^i + i\gamma_5 B^i, \qquad \bar{Z}^i \equiv A^i - i\gamma_5 B^i \qquad (2.1\text{-}11)$$

$$u \equiv \kappa^2 \bar{Z}^i Z^i, \qquad a \equiv 1 - \frac{1}{2} u$$

where the identity matrix in spinor space implicit in the definition of u is dropped when u is written outside a spinor product.

2.2) N = 1 Supergravity

The algebra (2.1-1) may be restricted to the algebra of N = 1 supersymmetry:

$$\{Q_\alpha,\ \bar{Q}_\beta\} = \gamma^\mu_{\alpha\beta} P_\mu \tag{2.2-1}$$

$$[P_\mu,\ Q_\alpha] = 0$$

$$[M_{\mu\nu},\ Q_\alpha] = \frac{1}{2}\ \sigma_{\mu\nu\alpha}{}^{\beta} Q_\beta$$

which contains only one spinorial generator.

The gauge theory for this algebra is N = 1 supergravity[Fr76a, Des76, Fr76b]. It describes a spin 2 graviton V^a_μ with interactions specified by Einstein's theory of gravity, coupled to a spin 3/2 gravitino ψ_μ through the gravitational coupling constant κ ($\approx 5.7 \cdot 10^{-33}$ cm).

The graviton is represented by the vierbein ("the square root of the metric": $g_{\mu\nu} = V^a_\mu V^b_\nu \eta_{ab}$), which connects the flat tangent space indices (Latin) to the Greek indices of the curved spacetime (base space). The vierbein is essential for accommodating the fermion present in the system, because the group of general coordinate transformations GL(4,R) does not have a representation which transforms like a spinor under its Lorentz subgroup SO(3,1). By use of the vierbein, we can write every world tensor as a Lorentz tensor in the tangent space, where spinors are easily defined. Thus a set of Latin indices denotes a tangent space tensor, a set of Greek indices a world tensor, and a spinor is a tangent space spinor (and also a world scalar).

N=1 supergravity theory may be thought of as a gauge theory with the vierbein V^a_μ, the vierbein connection ω^{ab}_μ, and the gravitino ψ_μ regarded as the gauge potentials for translations, Lorentz rotations and supersymmetry transformations, respectively. Thus, these invariances of the theory are local, and we call the corresponding infinitesimal spacetime-dependent parameters $\xi^\mu(x)$, $\Lambda^{ab}_{(x)}$, $\varepsilon(x)$. In analogy to pure general relativity, MacDowell and Mansouri[Ma77] have emphasized the quasi-geometrical nature of this theory by gauging the orthosymplectic group OSp (4,1). The bosonic part of this graded group is Sp (4), and it covers the de Sitter group SO (3,2), which in turn can be Wigner-Inonü contracted to the Poincaré group IO (3,1).

In the Cartan-Palatini first order formalism[Des76], both $V_{\mu a}$, ψ_μ <u>and</u> the tangent space connection ω^{ab}_μ are treated as independent fields. The scalar density Lagrangian is:

$$\mathcal{L} = -\frac{V}{4\kappa^2} R - \frac{1}{2}\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho . \qquad (2.2\text{-}2)$$

It is generally covariant, Lorentz invariant, and supersymmetric with the infinitesimal transformations:

$$\delta V_{\mu a} = -i\kappa\bar{\varepsilon}\gamma_a\psi_\mu \qquad (2.2\text{-}3)$$

$$\delta\psi_\rho = \frac{1}{\kappa} D_\rho \varepsilon$$

$$\delta\omega_{\mu ab} = -\frac{\kappa}{2V}[(B_{\mu ab}-V_{\mu b}B_{ca}{}^{c}) - (a\leftrightarrow b)]$$

$$B_c{}^{\sigma\tau} = \varepsilon^{\sigma\tau\nu\rho}\bar{\varepsilon}\gamma_5\gamma_c D_\nu\psi_\rho$$

where $V \equiv \det V_{\mu a}$, and the curvature is $R \equiv V^{a\mu}V^{b\nu}R_{\mu\nu ab}(V,\omega)$. The Riemann tensor is $R_{\mu\nu ab} = \partial_\mu\omega_{\nu ab} + \omega_{\mu a}{}^{c}\omega_{\nu cb} - (\mu\leftrightarrow\nu)$.

The derivative of the spinor $D_\nu\psi_\rho$ is Lorentz covariant, but not world covariant[Fr76b]:

$$D_\nu \quad \psi_\rho \quad \equiv (\partial_\nu + \frac{1}{2}\,\omega_{\nu ab}\sigma^{ab})\,\psi_\rho . \qquad (2.2\text{-}4)$$

Hence $D_\nu\psi_\rho$ is not a world covariant tensor, in contrast to $D_\nu\psi_\rho - \Gamma^\sigma_{\nu\rho}\psi_\sigma$. However, the curl structure in the Lagrangian $(D_\nu\psi_\rho - D_\rho\psi_\nu)$ is a tensor, since it differs from the curl of the fully covariant derivatives by $\Gamma^\sigma_{\rho\nu} - \Gamma^\sigma_{\nu\rho} \equiv 2S^\sigma_{\rho\nu}$, the torsion, which <u>is</u> a tensor.

The first term of the Lagrangian (2.2-2) is Einstein's Lagrangian. The second term is the Rarita Schwinger Lagrangian for ψ_μ which describes a massless spin 3/2 field. The fermionic gauge invariance present in this piece of the Lagrangian and the absence of $\partial_o\psi_o$ in it enable us to eliminate 12 out of the 16 degrees of freedom contained in the spinor vector field ψ_μ by $\psi_o = 0$, $\vec{\nabla}\cdot\vec{\psi} = 0$, and $\vec{\gamma}\cdot\vec{\psi} = 0$[Da76]. The remaining 4 degrees of freedom reduce to 2 by virtue of the Majorana condition, and they describe the 2 polarization states of the massless spin 3/2 field. The propagators obtained by conventional gauge field quantization techniques have been shown[Da76] to be ghost free and causal.

From the metric postulate $D_\mu^{[covariant]} V_\nu^a = D_\mu V_\nu{}^a - \Gamma_{\mu\nu}{}^\rho V_\rho{}^a = 0$, we note that:

$$D_\mu V_\nu^a - D_\nu V_\mu^a = 2S_{\mu\nu}^a \qquad (2.2\text{-}5)$$

and hence:

$$\omega_{\mu ab} = \omega_{\mu ab}(S = 0) + K_{\mu ab} \qquad (2.2\text{-}6)$$

$$\omega_{\mu ab}(S = 0) = \frac{1}{2}\,[V_a^\nu(\partial_\mu V_{\nu b} - \partial_\nu V_{\mu b}) + V_a^\rho V_b^\sigma(\partial_\sigma V_{c\rho})V_\mu^c - (a \leftrightarrow b)]$$

(the torsionless connection) and

$$K_{\mu\nu\rho} = -S_{\mu\nu\rho} + S_{\nu\rho\mu} - S_{\rho\mu\nu} \quad \text{(the contorsion)}.$$

The $\omega_\mu{}^{ab}$'s are thus auxiliary, nonpropagating fields.

Using the fundamental formula for the curvature:

$$[D_\mu, D_\nu]\text{(spinor)} = \frac{1}{2}\, R_{\mu\nu ab}\sigma^{ab}\text{(spinor)} \qquad (2.2\text{-}7)$$

we may read off the $\omega_\mu{}^{ab}$ content of R, and thus obtain the equation of motion $\frac{\delta I}{\delta \omega_{\mu ab}} = 0$ in the form

$$\frac{1}{4}\,\varepsilon^{\lambda\rho\nu\mu}\bar{\psi}_\lambda\gamma_5\gamma_\nu\sigma_{ab}\psi_\rho = -\frac{1}{4\kappa^2}\,D_\nu[V(V^\mu_a V^\nu_b - V^\mu_b V^\nu_a)] \qquad (2.2\text{-}8)$$

which specifies

$$S_{\mu\nu\rho} = -\frac{i\kappa^2}{2}\,\bar{\psi}_\mu\gamma_\rho\psi_\nu . \qquad (2.2\text{-}9)$$

Consequently, if we separate the contortion out of the connection ω_μ^{ab}, and if we then interpret $\omega_{\mu ab}$ in the tangent space covariant derivative as the torsionless function $\omega_{\mu ab}(S=0)=\omega_{\mu ab}(V)$ of (2.2-6), we can reexpress the above theory in the second order form [Fr76a], (which was the first one to appear historically):

$$\mathcal{L} = -\frac{V}{4\kappa^2}\,R(V,\omega(V)) - \frac{1}{2}\,\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho \qquad (2.2\text{-}10)$$

$$-\frac{\kappa^2 V}{16}\,[\bar{\psi}^\lambda\gamma^\mu\psi^\rho(\bar{\psi}_\lambda\gamma_\mu\psi_\rho + 2\bar{\psi}_\mu\gamma_\lambda\psi_\rho) - 4(\bar{\psi}\cdot\gamma\psi_\sigma)^2]$$

$$\delta V_{a\mu} = -i\kappa\bar{\varepsilon}\gamma_a\psi_\mu$$

$$\delta\psi_\rho = \frac{1}{\kappa}\,D_\rho\varepsilon - \frac{i\kappa}{4}\,[2\bar{\psi}_\mu\gamma_\nu\psi_\rho + \bar{\psi}_\mu\gamma_\rho\psi_\nu]\sigma^{\mu\nu}\varepsilon .$$

The action is locally Lorentz invariant as is manifest in view of the appropriate saturation of all Latin indices and the covariance of all derivatives under tangent space transformations. The latter is achieved by the presence of the gauge field ω_μ^{ab} for the Lorentz transformation: $\delta\omega_\mu^{ab} = -D_\mu\Lambda^{ab}(x)$.

It is also invariant under general coordinate transformations (of parameter ξ^μ) because covariant tensors transform as

$$\delta T^{\mu\cdots}_{\nu\cdots} = \xi^\lambda\partial_\lambda T^{\mu\cdots}_{\nu\cdots} + \partial_\nu\xi^\lambda T^{\mu\cdots}_{\lambda\cdots} - \partial_\lambda\xi^\mu T^{\lambda\cdots}_{\nu\cdots} + \ldots \quad (2.2\text{-}11)$$

and all indices are saturated properly. We remind the reader that, since the vierbein determinant V is a scalar density, $\delta V = \partial_\lambda(\xi^\lambda V)$. For any scalar ϕ we obtain $\delta(\phi V) = \partial_\lambda(\xi^\lambda V)\phi + V\xi^\lambda\partial_\lambda\phi = \partial_\lambda(\xi^\lambda V\phi)$, a total divergence which amounts to no increment in the action (assuming there are no problems with surface terms). All scalar pieces in the Lagrangian should, therefore, be multiplied by a V, except the ones containing $\varepsilon^{\mu\nu\kappa\lambda}$, which is a tensor density itself, so $\frac{\varepsilon^{\mu\nu\kappa\lambda}}{V}$ is a tensor.

The proof of local supersymmetry invariance of the theory can be considerably simplified by the introduction of the so-called "1.5 order" formalism[To77]. This is a shortcut which combines the simplicity of the Lagrangian and transformation laws of the 1st order formalism, with the 2nd order formalism's advantage of not requiring the complicated $\omega_{\mu ab}$ variations of the 1st order language. Analytically, the 1st order action is varied:

$$\delta I = \frac{\delta I}{\delta\omega_{\mu ab}}\,\delta\omega_{\mu ab} + \frac{\delta I}{\delta V^{a\mu}}\,\delta V^{a\mu} + \delta\bar\psi_\mu\,\frac{\delta I}{\delta\bar\psi_\mu}. \quad (2.2\text{-}12)$$

Subsequently, the equation of motion $\frac{\delta I}{\delta\omega_{\mu ab}} = 0$ allows for $\omega_{\mu ab}$ to be solved in terms of $V_{\mu a}$ and ψ_μ, as given in (2.2-9) and (2.2-6). As a result, if we substitute for $\omega_{\mu ab}$ in terms of the remaining fields in the variation (2.2-12), the first term vanishes automatically, while the remaining two terms become simpler than the corresponding second order expressions. To prove invariance, we must show that the second term cancels against the third. The second term is:

$$\frac{\delta I}{\delta V^{\mu}_{a}} \delta V^{\mu}_{a} = -i\kappa\bar{\psi}_{a}\gamma^{\mu}\varepsilon\left(\frac{V}{4\kappa^{2}} V^{a}_{\mu}R - \frac{V}{2\kappa^{2}} R^{a}_{\mu} + \frac{1}{2} \varepsilon^{\lambda\rho a\nu}\bar{\psi}_{\lambda}\gamma_{5}\gamma_{\mu}D_{\nu}\psi_{\rho}\right). \tag{2.2-13}$$

The last term on the right hand side becomes, upon a Fierz rearrangement (Appendix A), and use of (2.2-9):

$$-\frac{i\kappa}{2} \varepsilon^{\lambda\rho\sigma\nu}\bar{\varepsilon}\gamma^{\mu}\psi_{\sigma}\bar{\psi}_{\lambda}\gamma_{\mu}\gamma_{5}D_{\nu}\psi_{\rho} = -\frac{1}{2\kappa} \varepsilon^{\lambda\sigma\nu\rho}S^{a}_{\lambda\sigma}\bar{\varepsilon}\gamma_{5}\gamma_{a}D_{\nu}\psi_{\rho}. \tag{2.2-14}$$

Finally, the third term in δI is:

$$\delta\bar{\psi}_{\mu} \frac{\delta I}{\delta\bar{\psi}_{\mu}} = \frac{1}{\kappa} \bar{\varepsilon}\overleftarrow{D}_{\mu}\varepsilon^{\mu\nu\rho\lambda}\left(-\gamma_{5}\gamma_{\nu}D_{\rho}\psi_{\lambda} - \frac{1}{2} \gamma_{5}S^{a}_{\rho\nu}\gamma_{a}\psi_{\lambda}\right). \tag{2.2-15}$$

Integration by parts, judicious splittings of the world covariant derivatives and use of (2.3-5), (2.2-6), (2.2-7), along with the metric postulate, allow us to recast (2.2-15) in the form:

$$\delta\bar{\psi}_{\mu} \frac{\delta I}{\delta\bar{\psi}_{\mu}} = \frac{iV}{4\kappa} \left(2\bar{\varepsilon}\gamma^{\mu}\psi_{a}R^{a}_{\mu}-\bar{\varepsilon}\gamma\cdot\psi R\right) + \frac{1}{2\kappa} \varepsilon^{\mu\nu\rho\lambda}S^{a}_{\mu\nu}\bar{\varepsilon}\gamma_{5}\gamma_{a}D_{\rho}\psi_{\lambda}. \tag{2.2-16}$$

This exactly cancels (2.2-13) as reexpressed using (2.2-14) and hence the supergravity action is invariant under local supersymmetry transformations.

2.3) N = 2 Supergravity

The gauge theory for the algebra (2.1-1) is N = 2 supergravity (2, 3/2, 3/2, 1)[Fe76a]. It contains N = 1 supergravity, plus a second gravitino field ϕ_μ and a vector field A_μ. The geometrical formulation of the theory[To77] focuses on the gauging of OSp (4,2), which has a bosonic part Sp(4) x SO(2). The two gravitinos ψ_μ and ϕ_μ may be interpreted as the gauge fields for ϵ and ζ type supersymmetries, respectively. A_μ can be arranged to gauge the central charge, as we shall demonstrate in the next section, but here we shall have it gauge the "gravity O(2)", which rotates the two supersymmetry generators into each other.

The first order form of the theory is[Fr77d]:

$$\mathcal{L} = \frac{-V}{4\kappa^2} R - \frac{1}{2} \epsilon^{\lambda\rho\mu\nu}(\bar{\psi}_\lambda \gamma_5 \gamma_\mu D_\nu \psi_\rho + \bar{\phi}_\lambda \gamma_5 \gamma_\mu D_\nu \phi_\rho) - \frac{1}{4} V F_{\mu\nu} F^{\mu\nu} \qquad (2.3\text{-}1)$$
$$- \kappa \bar{\psi}_\mu (V F^{\mu\nu} - \frac{1}{2} \gamma_5 \tilde{F}^{\mu\nu}) \phi_\nu - \frac{\kappa^2}{2} \bar{\psi}_\mu \phi_\nu [V(\bar{\psi}^\mu \phi^\nu - \bar{\phi}^\mu \psi^\nu) - i\epsilon^{\mu\nu\kappa\lambda} \bar{\psi}_\kappa \gamma_5 \phi_\lambda]$$

This Lagrangian is invariant under

$$\delta V_{\mu a} = -i\kappa(\bar{\varepsilon}\gamma_a\psi_\mu + \bar{\zeta}\gamma_a\phi_\mu) \tag{2.3-2}$$

$$\delta A_\mu = -\bar{\varepsilon}\phi_\mu + \bar{\zeta}\psi_\mu$$

$$\delta\begin{pmatrix}\psi_\rho \\ \phi_\rho\end{pmatrix} = \frac{1}{\kappa} D_\rho \begin{pmatrix}\varepsilon \\ \zeta\end{pmatrix} - \frac{i}{2}\sigma^{\mu\nu}[F_{\mu\nu} + 2\kappa\bar{\psi}_\mu\phi_\nu]\gamma_\rho\begin{pmatrix}\zeta \\ -\varepsilon\end{pmatrix}$$

$$\delta\omega_{\mu ab} = -\frac{\kappa}{2V}[(B_{\mu ab} - V_{\mu b}B_{ca}{}^{c}) - (a\leftrightarrow b)]$$

$$B_c{}^{\sigma\tau} = \varepsilon^{\sigma\tau\nu\rho}(\bar{\varepsilon}\gamma_5\gamma_c D_\nu\psi_\rho + \bar{\zeta}\gamma_5\gamma_c D_\nu\phi_\rho).$$

The torsion now has contributions from both gravitino fields:

$$S_{\mu\nu\rho} = -\frac{i\kappa^2}{2}(\bar{\psi}_\mu\gamma_\rho\psi_\nu + \bar{\phi}_\mu\gamma_\rho\phi_\nu). \tag{2.3-3}$$

We prefer to use the second order form of this theory, which exhibits all powers of κ explicitly and is thereby better suited to the gauging procedure of the next section, carried out order-by-order in κ. The second order Lagrangian and transformation laws are:

$$\mathcal{L}_0 = \frac{-V}{4\kappa^2} R - \frac{1}{2} \epsilon^{\lambda\rho\mu\nu} (\bar{\psi}_\lambda \gamma_5 \gamma_\mu D_\nu \psi_\rho + \bar{\phi}_\lambda \gamma_5 \gamma_\mu D_\nu \phi_\rho) - \frac{1}{4} V F_{\mu\nu} F^{\mu\nu}$$
$$- \kappa \bar{\psi}_\mu (V F^{\mu\nu} - \frac{1}{2} \gamma_5 \tilde{F}^{\mu\nu}) \phi_\nu - \frac{\kappa^2}{2} \bar{\psi}_\mu \phi_\nu [V(\bar{\psi}^\mu \phi^\nu - \bar{\phi}^\mu \psi^\nu) - i\epsilon^{\mu\nu\kappa\lambda} \bar{\psi}_\kappa \gamma_5 \phi_\lambda]$$
$$- \frac{\kappa^2 V}{16} [(\bar{\psi}^\lambda \gamma^\mu \psi^\rho + \bar{\phi}^\lambda \gamma^\mu \phi^\rho)(\bar{\psi}_\lambda \gamma_\mu \psi_\rho + 2\bar{\psi}_\mu \gamma_\lambda \psi_\rho + \bar{\phi}_\lambda \gamma_\mu \phi_\rho + 2\bar{\phi}_\mu \gamma_\lambda \phi_\rho)$$
$$- 4(\bar{\psi}\cdot\gamma\psi_\sigma + \bar{\phi}\cdot\gamma\phi_\sigma)^2]$$

$$\delta_0 V_{a\mu} = - i\kappa(\bar{\epsilon}\gamma_a \psi_\mu + \bar{\zeta}\gamma_a \phi_\mu)$$

$$\delta_0 A_\mu = - \bar{\epsilon}\phi_\mu + \bar{\zeta}\psi_\mu \qquad (2.3 - 4)$$

$$\delta_0 \begin{pmatrix} \psi_\rho \\ \phi_\rho \end{pmatrix} = \frac{1}{\kappa} D_\rho \begin{pmatrix} \epsilon \\ \zeta \end{pmatrix} - \frac{i\kappa}{4} [2\bar{\psi}_\mu \gamma_\nu \psi_\rho + \bar{\psi}_\mu \gamma_\rho \psi_\nu + 2\bar{\phi}_\mu \gamma_\nu \phi_\rho + \bar{\phi}_\mu \gamma_\rho \phi_\nu] \sigma^{\mu\nu} \begin{pmatrix} \epsilon \\ \zeta \end{pmatrix}$$
$$- \frac{i}{2} \sigma^{\mu\nu} (F_{\mu\nu} + 2\kappa \bar{\psi}_\mu \phi_\nu) \gamma_\rho \begin{pmatrix} \zeta \\ -\epsilon \end{pmatrix}$$

A convenient quantity is the supercovariant field strength[Fe76b]

$$\hat{F}_{\mu\nu} \equiv F_{\mu\nu} + \kappa(\bar{\psi}_\mu \phi_\nu - \bar{\phi}_\mu \psi_\nu). \qquad (2.3\text{-}5)$$

The defining feature of the supercovariant derivative of a field is that its variation is free of derivatives of the supersymmetry transformation parameters. This is achieved by introducing minimal couplings of the gauge fields ψ_μ, ϕ_μ to the transform of the field acted upon by the derivative - recall that ψ_μ and ϕ_μ transform as the gradient of the infinitesimal parameters ε and ζ. To give an example, the transformation of the supercovariant field strength is:

$$\delta\hat{F}_{\mu\nu} = [-\bar{\varepsilon}D_\mu\phi_\nu + \frac{i\kappa}{2}\bar{\psi}_\mu\hat{F}\cdot\sigma\gamma_\nu\varepsilon \qquad (2.3\text{-}6)$$

$$+ \bar{\zeta}D_\mu\psi_\nu + \frac{i\kappa}{2}\bar{\phi}_\mu\hat{F}\cdot\sigma\gamma_\nu\zeta] - (\mu\leftrightarrow\nu)$$

and its use results in simplifications in the invariance proof of the theory. Upon the introduction of this notation, the last three terms of (2.3-1) reduce to

$$-\frac{V}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{\kappa i}{2}\hat{F}^{\mu\nu}\bar{\psi}_\mu\gamma_5\phi_\nu - \frac{\kappa^2}{2}\bar{\psi}_\mu\phi_\nu\varepsilon^{\mu\nu\kappa\lambda}\bar{\psi}_\kappa i\gamma_5\phi_\lambda \quad (2.3\text{-}7)$$

and the $\delta\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix}$ term of (2.3-2) in the brackets is just $\hat{F}_{\mu\nu}$.

At this point, it is worth mentioning that the theory presented in this section possesses an SU(2) symmetry, an O(2) subgroup of which is particularly useful. We call it "gravity" O(2), and it rotates the two supersymmetry generators into each other:

$$\delta_{\Lambda_2}\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix} = \Lambda_2\begin{pmatrix}\zeta\\ -\varepsilon\end{pmatrix}, \quad \delta_{\Lambda_2}\begin{pmatrix}\psi_\mu\\ \phi_\mu\end{pmatrix} = \Lambda_2\begin{pmatrix}\phi_\mu\\ -\psi_\mu\end{pmatrix}. \qquad (2.3\text{-}8)$$

All of the above expressions in this section could be shortened significantly by use of a "gravity" O(2) tensor notation, but we defer this until Section 3.1, in order to avoid confusion of the several groups involved.

Gravity SO(2) may be gauged by A_μ [Fr77a], with an arbitrary, dimensionless, coupling constant e. The extra pieces of the theory are:

$$\mathcal{L}_e = -\frac{eV}{\kappa}(\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu+\bar{\phi}_\mu\sigma^{\mu\nu}\phi_\nu) + \frac{3}{2}V\frac{e^2}{\kappa^4} \qquad (2.3\text{-}9)$$

$$\delta_e\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \frac{ie}{2\kappa^2}\gamma_\rho\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix} .$$

In addition, all derivatives of (2.3-4) must be regarded as O(2) covariant ones, by completing them as follows: $D_\mu\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix}$ now contains a term $-eA_\mu\begin{pmatrix}\phi_\rho\\ -\psi_\rho\end{pmatrix}$ and $D_\mu\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix}$ picks up a term $-eA_\mu\begin{pmatrix}\zeta\\ -\varepsilon\end{pmatrix}$. It should be remembered that an O(2) covariant derivative does not commute with supersymmetry:

$$[\delta,D_\mu]\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = e(\bar{\varepsilon}\phi_\mu-\bar{\zeta}\psi_\mu)\begin{pmatrix}\phi_\rho\\ -\psi_\rho\end{pmatrix} . \qquad (2.3\text{-}10)$$

The resulting theory (2.3-4) augmented by (2.3-9) is invariant under (2.3-8), interpreted as a <u>local</u> transformation, and extended by $\delta_{\Lambda_2}A_\mu = \frac{1}{e}\partial_\mu\Lambda_2$. The gravitino mass-like terms $(M=\frac{e}{\kappa})$ and the cosmological term in eq. (2.3-9) will be interpreted in Sec. 3.3.

The gravity O(2) symmetry discussed is very useful in checking the correctness of the expressions. More significantly, it reduces the number of relevant terms in the local supersymmetry invariance proof by half. This is so because any transformation term containing ζ may be transformed into a term containing ε, through a finite O(2) rotation by $\pi/2$. Consequently, only the overall cancellation of the ε variations needs to be checked. The proof of supersymmetry invariance, which we omit, follows the same lines as that for N = 1 supergravity. There are now extra variations such as bilinears in $F_{\mu\nu}$ generated from $\frac{\delta I}{\delta V_{a\mu}} \delta V_{a\mu}$ and $\delta\bar{\phi}_\mu \frac{\delta I}{\delta\bar{\phi}_\mu}$, canceling each other; also, terms linear in $F_{\mu\nu}$ and others trilinear in the spinors, and others yet of O(e) and $O(e^2)$[Fr77a], each sector canceling separately.

2.4) The Local Massless Scalar Multiplet

We shall couple the massless sector of the global supermultiplet (2.1-2) and (2.1-3) to N = 2 supergravity (2.3-4), to obtain a locally supersymmetric theory $\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$, $\delta = \delta_o + \delta_1 + \delta_3$. The new system contains $\mathcal{L}_0$ of (2.3-4) augmented by the additional part

$$\mathcal{L}_1 = \frac{V}{2a^2} g^{\mu\nu}\partial_\mu \bar{Z}^i \partial_\nu Z^i + \frac{i}{2} V\bar{\chi}^i\gamma^\mu D_\mu \chi^i \tag{2.4-1}$$

$$\mathcal{L}_2 = -\frac{\kappa V}{a\sqrt{2}} \bar{\psi}_\mu \partial_\nu Z^i \gamma^\nu\gamma^\mu\chi^i - \frac{\kappa V}{a\sqrt{2}} \bar{\phi}_\mu \partial_\nu \bar{Z}^i \varepsilon^{ij}\gamma^\nu\gamma^\mu\chi^j + \frac{\kappa}{2} F_{\mu\nu}\varepsilon^{ij}\bar{\chi}^i\sigma^{\mu\nu}\chi^j$$

$$\mathcal{L}_3 = -\frac{\kappa^2}{2a}\varepsilon^{\lambda\rho\mu\nu}[\tfrac{1}{4}(\bar{\psi}_\lambda\gamma_5\gamma_\mu \bar{Z}^i \overleftrightarrow{\partial}_\rho Z^i\psi_\nu - \bar{\phi}_\lambda\gamma_5\gamma_\mu \bar{Z}^i\overleftrightarrow{\partial}_\rho Z^i\phi_\nu) + \bar{\psi}_\mu\gamma_5\gamma_\lambda\varepsilon^{ij}\bar{Z}^i\partial_\rho\bar{Z}^j\phi_\nu]$$

$$+ \frac{i\kappa^2}{16}\varepsilon^{\mu\lambda\nu\rho}\bar{\chi}^i\gamma_5\gamma_\rho\chi^i(\bar{\psi}_\mu\gamma_\lambda\psi_\nu + \bar{\phi}_\mu\gamma_\lambda\phi_\nu) + \frac{\kappa^2 V}{8}\bar{\chi}^i\gamma_5\gamma_\rho\chi^i(\bar{\psi}_\mu\gamma_5\gamma_\rho\psi^\mu + \bar{\phi}_\mu\gamma_5\gamma_\rho\phi^\mu)$$

$$- \frac{\kappa^2 V}{4}\varepsilon^{ij}\bar{\chi}^i\sigma_{\kappa\lambda}\chi^j(\bar{\psi}_\mu i\gamma_5\phi_\nu V^{-1}\varepsilon^{\mu\nu\kappa\lambda} + 2\bar{\psi}_\mu\sigma^{\kappa\lambda}\phi^\mu + 2\bar{\psi}^\lambda\phi^\kappa) - \frac{\kappa^2 V}{8}(\varepsilon^{ij}\bar{\chi}^i\sigma_{\mu\nu}\chi^j)^2$$

$$- \frac{\kappa^2 V}{32}(\bar{\chi}^i\gamma_5\gamma_\rho\chi^i)^2 - \frac{\kappa^2 iV}{4a}\bar{\chi}^i Z^i \overleftrightarrow{\not\partial} Z^j\chi^j + \frac{\kappa^2 iV}{8a}\bar{\chi}^i Z^j \overleftrightarrow{\not\partial} Z^j\chi^i$$

$$\delta_1\begin{pmatrix}A^i\\B^i\end{pmatrix} = \frac{a}{\sqrt{2}}\bar{\varepsilon}\begin{pmatrix}1\\i\gamma_5\end{pmatrix}\chi^i + \frac{a}{\sqrt{2}}\bar{\zeta}\varepsilon^{ij}\begin{pmatrix}1\\-i\gamma_5\end{pmatrix}\chi^j$$

$$\delta_1\chi^i = \frac{-i}{a\sqrt{2}}(\not\partial Z^i\varepsilon - \varepsilon^{ij}\not\partial\bar{Z}^j\zeta)$$

$$\delta_3\chi^i = \frac{i\kappa}{2}(\gamma^\mu\varepsilon\bar{\chi}^i\psi_\mu + \gamma^5\gamma^\mu\varepsilon\bar{\chi}^i\gamma_5\psi_\mu + \gamma^\mu\zeta\bar{\chi}^i\phi_\mu + \gamma^5\gamma^\mu\zeta\bar{\chi}^i\gamma_5\phi_\mu)$$

$$+ \frac{i\kappa}{2}\varepsilon^{ij}(\gamma^\mu\varepsilon\bar{\chi}^j\phi_\mu - \gamma^5\gamma^\mu\varepsilon\bar{\chi}^j\gamma_5\phi_\mu - \gamma^\mu\zeta\bar{\chi}^j\psi_\mu + \gamma^5\gamma^\mu\zeta\bar{\chi}^j\gamma_5\psi_\mu)$$

$$+ \frac{\kappa^2}{2\sqrt{2}} [\gamma_5 \chi^i \bar{\chi}^j \gamma_5 (Z^j \epsilon + \epsilon^{kj} \bar{Z}^k \zeta) - \gamma_5 \chi^j (\bar{\epsilon}\gamma_5 Z^j \chi^i + \bar{\epsilon}\gamma_5 Z^i \chi^j + \bar{\zeta}\gamma_5 \epsilon^{kj} \bar{Z}^k \chi^i + \bar{\zeta}\gamma_5 \epsilon^{ki} \bar{Z}^k \chi^j)$$

$$- \chi^j (\bar{\epsilon} Z^j \chi^i - \bar{\epsilon} Z^i \chi^j + \bar{\zeta} \epsilon^{kj} \bar{Z}^k \chi^i - \bar{\zeta} \epsilon^{ki} \bar{Z}^k \chi^j)]$$

$$\delta_3 \begin{pmatrix} \psi_\rho \\ \phi_\rho \end{pmatrix} = - \frac{\kappa}{4a} \bar{Z}^i \overleftrightarrow{\partial}_\rho Z^i \begin{pmatrix} \epsilon \\ -\zeta \end{pmatrix} + \frac{\kappa}{2a} \epsilon^{ij} \begin{pmatrix} \bar{Z}^i \partial_\rho \bar{Z}^j \zeta \\ -Z^i \partial_\rho Z^j \epsilon \end{pmatrix}$$

$$- \frac{i\kappa}{4} \gamma_5 \sigma_{\tau\rho} \begin{pmatrix} \epsilon \\ \zeta \end{pmatrix} \bar{\chi}^i \gamma_5 \gamma^\tau \chi^i - \frac{i\kappa}{2} \gamma_5 \gamma^\tau \begin{pmatrix} \zeta \\ -\epsilon \end{pmatrix} \epsilon^{ij} \bar{\chi}^i \gamma_5 \sigma_{\rho\tau} \chi^j$$

$$- \frac{\kappa^2}{2\sqrt{2}} \gamma_5 \begin{pmatrix} \psi_\rho \\ -\phi_\rho \end{pmatrix} (\bar{\epsilon}\gamma_5 Z^i \chi^i + \bar{\zeta}\gamma_5 \epsilon^{ij} \bar{Z}^j \chi^i) - \frac{\kappa^2}{2\sqrt{2}} \gamma_5 \begin{pmatrix} \phi_\rho \\ \psi_\rho \end{pmatrix} (\bar{\epsilon}\gamma_5 \epsilon^{ij} \bar{Z}^i \chi^j + \bar{\zeta}\gamma_5 Z^i \chi^i)$$

$$- \frac{\kappa^2}{2\sqrt{2}} \begin{pmatrix} \phi_\rho \\ -\psi_\rho \end{pmatrix} (\bar{\epsilon} \epsilon^{ij} \bar{Z}^i \chi^j - \bar{\zeta} Z^i \chi^i)$$

The construction of this theory follows the conventional lines of order-by-order coupling[Fe77a]. The theory of (2.1-2) as it stands has a <u>local</u> supersymmetry variation (in addition to the usual total divergence):

$$\delta\mathcal{L} = \partial_\mu \bar{\epsilon} J^{\mu\epsilon} + \partial_\mu \bar{\zeta} J^{\mu\zeta}. \tag{2.4-2}$$

To cancel these terms, we start by adding the minimal Noether coupling $-\kappa(\bar{\psi}_\mu J^{\mu\epsilon} + \bar{\phi}_\mu J^{\mu\zeta})$ to the Lagrangian. This achieves $O(\kappa^o)$ invariance, since all fields transform through pieces of $O(\kappa^o)$ or higher, except for the gradient in the transformation law of the gravitino. Since we are coupling a theory to gravity, all expressions we deal with must be written in world covariant language.

The extra piece added to the Lagrangian generates, in turn, $O(\kappa)$ terms in $\delta\mathcal{L}$. To cancel these, we add $O(\kappa)$ corrections to the variations of the fields and $O(\kappa)$, $O(\kappa^2)$ pieces to the Lagrangian. This procedure can be made more systematic by use of the consistency condition $D_\mu \frac{\delta I}{\delta\bar{\psi}_\mu} = 0$[Des76, Fr77c], which holds modulo equations of motion. The reason this condition holds is that no other field variation besides $\delta\psi_\mu$ contains gradients of ε. This method was used to obtain terms in the Lagrangian such as the Pauli moment term in $\mathcal{L}_2$*. Such a coupling has previously been found only in N = 8 supergravity[De77]. It could be reexpressed, through integration by parts, as a linear coupling of κA_μ to a "current": $\varepsilon^{ij}\bar{\chi}^i\sigma^{\mu\nu}\partial_\nu\chi^j$, conserved and chargeless. Thus it bears some structural similarity to a possible "improvement" term for the central charge current, in the sense of Section 2.1. It is an independent "current" necessary to close the supermultiplet of N = 2 currents[So78b]. Unlike the central charge current coupling of the next Section 2.5, this coupling lacks a crucial coefficient proportional to the mass in the theory, so that it (evidently) doesn't decouple in the massless limit.

We proceed to the next level of invariance $O(\kappa^2)$. To achieve this, we add $O(\kappa^2)$ pieces to the variations, and $O(\kappa^2)$, $O(\kappa^3)$ pieces to the Lagrangian, and so on.

---

* I thank Dr. Fayet for reminding me that matter fields may couple to $F_{\mu\nu}$ in extended supersymmetry theories [Lu78].

Our task is facilitated by the internal symmetries present in the theory. Invariances respected by the first step of our iterative construction will propagate on to the rest of the theory. They serve to constrain the form of the corrections to be added, and they also reduce the number of cancellations to be checked explicitly among the appropriate pieces in the variations.

For instance, we can see that the central charge (2.1-3) invariance of the Noether couplings (2.4-2) constrains all new pieces of the theory to be central charge scalars, which amounts to saturation of internal symmetry indices.

Similarly, the "gravity" O(2) (2.3-8) of the previous section may have its domain extended to include the spinless fields of the matter theory (2.1-2, 2.1-3), in such a way that both the original matter theory and the Noether couplings are gravity O(2) invariant. Specifically, a transformation meeting this requirement is $\delta_{\Lambda_2} Z^i = -\Lambda_2 \varepsilon^{ij} \bar{Z}^j$, for real Λ_2, so that the scalar A^i is rotated in the opposite sense to the pseudoscalar B^i. This results in the simplifications already encountered in the previous section, namely extra constraints on the candidate correction terms and the possibility of ignoring all ζ-type variations.

We may further extend the above O(2) x O(2) symmetry by the addition of chiral invariances to SU(2) x SU(2) $\sim$ SO(4). This will be considered in detail in Section 3.1.

The expansion in ever higher powers of κ continues indefinitely, but at the $O(\kappa^2)$ invariance level we observe that all new terms are combinations of scalars multiplying terms that have appeared previously. We thus resort to the functional techniques of Das et al.[Da77a, Da77b], which determine the theory to every order in κ all at once.

Specifically, $O(\kappa^2)$ invariance selects out only bilinears of the scalar fields that mesh together in the SO(4) singlet combination $u = \kappa^2 Z^i \bar{Z}^i$. For example, SO(4) invariance allows more complicated structures, like terms of the form $\frac{\kappa^2}{4} \partial_\mu (Z^i \bar{Z}^i) \partial^\mu (Z^j \bar{Z}^j)$ in the Lagrangian, and $\delta \begin{pmatrix} A^i \\ B^i \end{pmatrix} \sim \kappa^2 \begin{pmatrix} A^i \\ B^i \end{pmatrix} \bar{\varepsilon} Z^k \chi^k$, $\delta\chi^i \sim \kappa^2 \not\partial (\bar{Z}^k Z^k) Z^i \varepsilon$ in the transformation laws. However, their coefficients are fixed to zero by the cancellation of variations of the type $[\kappa^2 \varepsilon\chi\partial\partial ZZZ]$ and $[\kappa^2 \varepsilon\psi\partial\partial ZZZ]^*$. We are led to assume that, <u>a fortiori</u>, higher powers of the scalars will appear in the same combination u. We proceed to probe the nonpolynomial behavior of the theory by postulating six arbitrary functions of u: $(a_1)^{-2}$, $(a_2)^{-1}$, $(a_3)^{-1}$, (a_4), $(a_5)^{-1}$ and $(a_6)^{-1}$. They are assigned as coefficients to the kinetic term of the scalars, the Noether couplings, the biscalar-bispinor couplings, the scalar transformation laws, the spinor transformations and the gravitino transformations, respectively. It can be seen that the coefficients of all other terms in the theory must be constants.

We now check invariance to arbitrary order in u, and hence to all orders in κ. Cancellation of terms in the variation

---

* Here, we have assumed that there are no functions of scalars multiplying the spinor kinetic term $\bar{\chi}^i \not\partial \chi^i$, after suitable rescalings. This is based on intuition drawn from the N = 1 theory.

containing one spinor [$\varepsilon\chi\partial\partial Z$, $\varepsilon\psi\partial\partial Z$, $\kappa\varepsilon\chi\partial ZF$, $\kappa^2\varepsilon\phi\partial ZZF$] already fixes all a_n's to the form $a \equiv 1 - \frac{1}{2}u$. This form is subsequently verified in the vanishing of selected variations containing three spinors [$\kappa^2\varepsilon\chi\phi\phi Z\partial$, $\kappa^2\varepsilon\chi\chi\chi Z\partial$]. Due to their considerable complexity, not all such variations have been checked, but there is no indication of unwelcome mismatches at any point. At this level, the calculations make ample use of the O(2) index transposition identities, derived in Appendix A, which enable us to rearrange the internal symmetry groupings in a form suitable for cancellation.

To ensure the absence of transformations quartic in the fermion fields, we finally carry out checks for the vanishing of numerous variations containing five spinors. By virtue of Fierz transposition symmetry, several groupings of terms vanish automatically (e.g., $\frac{\delta(\frac{\kappa^2}{8}\bar{\chi}^i\gamma_5\gamma_\lambda\chi^i\bar{\psi}_\mu\gamma_5\gamma^\lambda\psi^\mu)}{\delta\psi_\rho}\delta_3\psi_\rho = 0$ for $\delta_3\psi_\rho$ proportional to $\gamma_5\psi_\rho$).

This completes the construction of the massless sector of the locally supersymmetric theory.

2.5) The Mass and Interaction Sectors

The massive sector of the global supermultiplet (2.1-2, 2.1-3) may be also included in the gauging procedure of the previous section. We obtain the sector $\mathcal{L}_4$, δ_4 given here, which may be appended to (2.3-4) and (2.4-1), to yield a massive, locally supersymmetric theory:

$$
\begin{aligned}
\mathcal{L}_4 = & - \frac{m^2 V}{2\kappa^2 a^2}\left(u - \frac{3}{4}u^2\right) - \frac{mV}{2a}\bar{\chi}^i\chi^i - \frac{i\kappa m V}{a\sqrt{2}}\left(\bar{\psi}_\mu z^i\gamma_\mu\chi^i + \bar{\phi}_\mu\varepsilon^{ij}\bar{z}^i\gamma_\mu\chi^j\right) \\
& - \frac{\kappa^2 m V}{2a}\left(\bar{\psi}_\mu\sigma^{\mu\nu}z^i z^i\psi_\nu + \bar{\phi}_\mu\sigma^{\mu\nu}\bar{z}^i\bar{z}^i\phi_\nu + 2\bar{\psi}_\lambda\sigma^{\lambda\nu}\varepsilon^{ij}\bar{z}^i z^j\phi_\nu\right) + \frac{i\kappa m V}{2}A_\mu\varepsilon^{ij}\bar{\chi}^i\gamma^\mu\chi^j \\
& - \frac{\kappa m V}{2a^2}A^\mu\varepsilon^{ij}z^i\overleftrightarrow{\partial}_\mu\bar{z}^j + \frac{\kappa^2 m^2 V}{2a^2}A_\mu A^\mu\bar{z}^i z^i + \frac{\kappa^2 m V}{a\sqrt{2}}A_\nu\left(\bar{\psi}_\mu\varepsilon^{ij}z^i\gamma^\nu\gamma^\mu\chi^j - \bar{\phi}_\mu\bar{z}^i\gamma^\nu\gamma^\mu\chi^i\right) \\
& - \frac{i\kappa^3 m V}{2a}A^\mu\left(\bar{\chi}^i z^i\gamma_\mu\varepsilon^{jk}z^k\chi^j + \frac{1}{2}\bar{\chi}^i z^k\varepsilon^{jk}\gamma_\mu z^j\chi^i\right) \qquad (2.5\text{-}1) \\
& + \frac{\kappa^3 m}{4a}A_\rho\varepsilon^{\lambda\rho\mu\nu}\left(\bar{\psi}_\lambda\gamma_5\gamma_\mu\varepsilon^{ij}z^i\bar{z}^j\psi_\nu - \bar{\phi}_\lambda\gamma_5\gamma_\mu\varepsilon^{ij}z^i\bar{z}^j\phi_\nu + 2\bar{\psi}_\mu\gamma_5\gamma_\lambda\bar{z}^i z^i\phi_\nu\right).
\end{aligned}
$$

$$
\begin{aligned}
\delta_4\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = & \frac{i\kappa m}{4a}\gamma_\rho z^i z^i\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix} + \frac{i\kappa m}{4a}\gamma_\rho\varepsilon^{ij}\bar{z}^i z^j\begin{pmatrix}\zeta\\ \varepsilon\end{pmatrix} + \frac{\kappa^2 m}{2a}A_\rho\varepsilon^{ij}z^i\bar{z}^j\begin{pmatrix}\varepsilon\\ -\zeta\end{pmatrix} \\
& - \frac{\kappa^2 m}{2a}A_\rho\begin{pmatrix}\bar{z}^i\bar{z}^i\zeta\\ -z^i z^i\varepsilon\end{pmatrix}.
\end{aligned}
$$

$$
\delta_4\chi^i = -\frac{m}{a\sqrt{2}}\left(z^i\varepsilon - \varepsilon^{ij}\bar{z}^j\zeta\right) - \frac{i\kappa m}{a\sqrt{2}}\not{A}\left(\varepsilon^{ij}z^j\varepsilon + \bar{z}^i\zeta\right)
$$

The crucial feature of this sector is that local supersymmetry invariance is forcing the central charge to be gauged by the vector of supergravity A_μ. This has been anticipated by the authors of [Fe77b, Fe77c], from a study of the local supersymmetry algebra in pure N = 2 supergravity (2.3-4). They observed that the commutator of two supersymmetry transformations acting on the vector field gives a gauge transformation $[\delta', \delta] A_\mu = -\frac{1}{\kappa}\partial_\mu(\bar{\varepsilon}\zeta'-\bar{\zeta}\varepsilon') + \ldots$, which suggests that A_μ is the gauge field for the central charge. More accurately, A_μ must gauge any bosonic charge (central or not) occurring in the commutator of two supersymmetry transformations as we shall explain in Section 3.1.

The vector A_μ couples to the central charge current $J_\mu^{\Lambda'}$ (2.1-8). The parameter Λ' of the central charge transformation (2.1-3) is now spacetime dependent and

$$\delta_{\Lambda'}A_\mu = -\frac{1}{\kappa}\partial_\mu\Lambda'(x). \qquad (2.5\text{-}2)$$

All derivatives of the matter fields may be extended to covariant derivatives, in the standard gauge theory form:

$$D_\mu\begin{pmatrix} Z^i \\ \chi^i \end{pmatrix} = [\partial_\mu\delta^{ij}+\kappa m A_\mu\varepsilon^{ij}]\begin{pmatrix} Z^j \\ \chi^j \end{pmatrix} \qquad (2.5\text{-}3)$$

where ∂_μ is implied to incorporate the tangent space corrections (2.2-4). All A_μ's in (2.5-1) may thus be absorbed into the derivatives in(2.1-2) and (2.1-3),that is A_μ couples minimally to matter with a fixed, dimensionless coupling constant κm. Covariant derivatives allow for shortcuts in the invariance proof, since entire sectors of the variation containing A_μ cancel among themselves automatically, following the cancellations of the ∂_μ's. In particular, $\delta(D_\mu\phi^i) = D_\mu(\delta\phi^i) + \kappa m\delta A_\mu\varepsilon^{ij}\phi^j$, and hence only the last terms on the right hand side need be considered, while the ones hidden in the covariant derivatives may be considered as $O(m^0)$ and, therefore, ignored. We shall next apply this technique in the construction of $\mathcal{L}_5$, $\mathcal{L}_6$, δ_5, δ_6.

To construct the above sector, we append the mass terms of (2.1-2) and (2.1-3) to the construction of Section 2.4 and then we extend our requirement of local invariance to the O(m) and $O(m^2)$ sectors. The gauging of the central charge is signaled by the necessity of a term $\kappa m A_\mu J^{\mu\Lambda'}$ in $\mathcal{L}_4$, at the $O(\kappa)$ invariance check. As in the previous section, we assign six arbitrary functions of u to multiply the standard terms of the theory, and we subsequently fix these functions through the general invariance check to all orders in u. We have worked out the cancellation of all terms containing one spinor, and selected types of terms containing 3 spinors. Clearly, there are no variations containing 5 spinors in the O(m) and $O(m^2)$ sectors.

It is not obvious that the nonpolynomial behavior should be described only by a function of the SO(4) singlet u, because the introduction of mass breaks SO(4) down to O(2) x SU(2). This form was postulated due to its simplicity, and in view of its meshing naturally with the massless sectors via the covariant derivatives. Nevertheless, we cannot guarantee that ours is the unique solution - this question might be easier to answer through a systematic study of interaction terms analogously to [Cr78c], after the development of the N = 2 tensor calculus. The same remark applies to the sectors we construct next.

In Section 2.3), A_μ gauges the gravity O(2) group (which commutes with the central charge) with an arbitrary coupling constant e. It is possible to extend this gauging to the matter-coupled theory consistently, even in the presence of mass. The full, locally supersymmetric, massive, interacting extension of (2.4-1) is:

$$\mathcal{L}_4 = -\frac{m^2V}{2\kappa^2a^2}\left(u-\frac{3}{4}u^2\right)-\frac{mV}{2a}\bar{\chi}^i\chi^i \tag{2.5-4}$$

$$-\frac{i\kappa mV}{\sqrt{2}a}\left(\bar{\psi}_\mu z^i\gamma_\mu\chi^i+\bar{\phi}_\mu\varepsilon^{ij}\bar{z}^i\gamma_\mu\chi^j\right)$$

$$-\frac{\kappa^2mV}{2a}\left(\bar{\psi}_\mu\sigma^{\mu\nu}z^iz^i\psi_\nu+\bar{\phi}_\mu\sigma^{\mu\nu}\bar{z}^i\bar{z}^i\phi_\nu+2\bar{\psi}_\lambda\sigma^{\lambda\nu}\varepsilon^{ij}\bar{z}^iz^j\phi_\nu\right)$$

$$\mathcal{L}_5 = \frac{3}{2}\frac{e^2V}{\kappa^4a^2}\left(1-\frac{u}{3}\right)-\frac{eV}{a\kappa}\left(\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu+\bar{\phi}_\mu\sigma^{\mu\nu}\phi_\nu\right)$$

$$-\frac{ieV}{\sqrt{2}a}\left(\bar{\psi}_\mu\gamma^\mu z^i\chi^i+\bar{\phi}_\mu\gamma^\mu\varepsilon^{ik}\bar{z}^i\chi^k\right)$$

$$-\frac{e\kappa V}{4a}\left(\bar{\chi}^i\varepsilon^{ij}\bar{z}^j\bar{z}^l\varepsilon^{kl}\chi^k+\bar{\chi}^iz^iz^j\chi^j\right)$$

$$\mathcal{L}_6 = \frac{emV}{4\kappa a^2}\left(z^iz^i+\bar{z}^i\bar{z}^i\right)$$

$$\delta_4\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \frac{i\kappa m}{4a}\gamma_\rho z^iz^i\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix}+\frac{i\kappa m}{4a}\gamma_\rho\varepsilon^{ij}\bar{z}^iz^j\begin{pmatrix}\zeta\\ \varepsilon\end{pmatrix}$$

$$\delta_4\chi^i = -\frac{m}{\sqrt{2}a}\left(z^i\varepsilon-\varepsilon^{ij}\bar{z}^j\zeta\right)$$

$$\delta_5\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \frac{ie}{2\kappa^2 a}\,\gamma_\rho\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix}$$

$$\delta_5\chi^i = -\frac{e}{\sqrt{2}\kappa a}\,(\bar{Z}^i\varepsilon-\varepsilon^{ij}Z^j\zeta)$$

All derivatives of (2.4-1) are now generalized to covariant ones

$$D_\mu\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \partial_\mu\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} - eA_\mu\begin{pmatrix}\phi_\rho\\ -\psi_\rho\end{pmatrix}$$

$$D_\mu\chi^i = \partial_\mu\chi^i + \kappa m A_\mu\varepsilon^{ij}\chi^j \qquad (2.5\text{-}3)$$

$$D_\mu Z^i = \partial_\mu Z^i + A_\mu(\kappa m\varepsilon^{ij}Z^j + e\varepsilon^{ij}\bar{Z}^j).$$

In this theory, the vector A_μ gauges a linear combination of the generators for the central charge and the gravity O(2) group. The charges carried by the fields are: e for the gravitinos, κm for the spinors, $\kappa m + e$ for the scalars and $\kappa m - e$ for the pseudoscalars.

We have derived (2.5-4) by the usual methods. We first add (2.3-9) to (2.4 - 1) for $m = 0$, and then extend the supersymmetry invariance to the new sectors O(e) and O(e^2). This is a relatively simple task using gauge covariant derivatives, as mentioned above. Having established the basic structure of $\mathcal{L}_5$, we assign arbitrary functions of u as coefficients of the terms at hand, and then determine these functions by checking invariance to all orders in u. We have checked all variations containing one spinor, and several containing three spinors. The latter step provides a check for the absence of terms in the new pieces of the Lagrangian containing four spinors and terms in the transformation law containing two spinors. Consequently there are no new variations containing five spinors.

Finally, in order to introduce the mass, we add $\mathcal{L}_5$ to $\mathcal{L}_4$ and observe that $\mathcal{L}_6$ is then sufficient to cancel O(me) variations containing one spinor. There are no O(me) variations with three or more spinors.

In summary, the most general Lagrangian density we have obtained is $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$, locally supersymmetric under $\delta = \delta_o + \delta_1 + \delta_3 + \delta_4 + \delta_5$. We shall discuss its properties in the next chapter. For convenience, we list the entire theory below. All dependence on A_μ is now absorbed in covariant derivatives (2.5 - 5) and field strengths:

$$\mathcal{L} = \sum_{i=o}^{6} \mathcal{L}_i \tag{2.5-6}$$

$$\mathcal{L}_0 = \frac{-V}{4\kappa^2} R - \frac{1}{2} \epsilon^{\lambda\rho\mu\nu} (\bar{\psi}_\lambda \gamma_5 \gamma_\mu D_\nu \psi_\rho + \bar{\phi}_\lambda \gamma_5 \gamma_\mu D_\nu \phi_\rho) - \frac{1}{4} V F_{\mu\nu} F^{\mu\nu}$$

$$- \kappa \bar{\psi}_\mu (V F^{\mu\nu} - \frac{1}{2} \gamma_5 \tilde{F}^{\mu\nu}) \phi_\nu - \frac{\kappa^2}{2} \bar{\psi}_\mu \phi_\nu [V(\bar{\psi}^\mu \phi^\nu - \bar{\phi}^\mu \psi^\nu) - i \epsilon^{\mu\nu\kappa\lambda} \bar{\psi}_\kappa \gamma_5 \phi_\lambda]$$

$$- \frac{\kappa^2 V}{16} [(\bar{\psi}^\lambda \gamma^\mu \psi^\rho + \bar{\phi}^\lambda \gamma^\mu \phi^\rho)(\bar{\psi}_\lambda \gamma_\mu \psi_\rho + 2\bar{\psi}_\mu \gamma_\lambda \psi_\rho + \bar{\phi}_\lambda \gamma_\mu \phi_\rho + 2\bar{\phi}_\mu \gamma_\lambda \phi_\rho)$$

$$- 4(\bar{\psi} \cdot \gamma \psi_\sigma + \bar{\phi} \cdot \gamma \phi_\sigma)^2]$$

$$\mathcal{L}_1 = \frac{V}{2a^2} g^{\mu\nu} D_\mu \bar{z}^i D_\nu z^i + \frac{1}{2} V \bar{\chi}^i \gamma^\mu D_\mu \chi^i$$

$$\mathcal{L}_2 = -\frac{\kappa V}{a\sqrt{2}}\bar{\psi}_\mu D_\nu z^i\gamma^\nu\gamma^\mu\chi^i - \frac{\kappa V}{a\sqrt{2}}\bar{\phi}_\mu D_\nu \bar{z}^i\varepsilon^{ij}\gamma^\nu\gamma^\mu\chi^j + \frac{\kappa}{2}F_{\mu\nu}\varepsilon^{ij}\bar{\chi}^i\sigma^{\mu\nu}\chi^j$$

$$\mathcal{L}_3 = -\frac{\kappa^2}{2a}\varepsilon^{\lambda\rho\mu\nu}[\frac{1}{4}(\bar{\psi}_\lambda\gamma_5\gamma_\mu\bar{z}^i\overleftrightarrow{D}_\rho z^i\psi_\nu - \bar{\phi}_\lambda\gamma_5\gamma_\mu\bar{z}^i\overleftrightarrow{D}_\rho z^i\phi_\nu) + \bar{\psi}_\mu\gamma_5\gamma_\lambda\varepsilon^{ij}\bar{z}^i D_\rho \bar{z}^j\phi_\nu]$$

$$+\frac{i\kappa^2}{16}\varepsilon^{\mu\lambda\nu\rho}\bar{\chi}^i\gamma_5\gamma_\rho\chi^i(\bar{\psi}_\mu\gamma_\lambda\psi_\nu+\bar{\phi}_\mu\gamma_\lambda\phi_\nu) + \frac{\kappa^2 V}{8}\bar{\chi}^i\gamma_5\gamma_\rho\chi^i(\bar{\psi}_\mu\gamma_5\gamma_\rho\psi^\mu+\bar{\phi}_\mu\gamma_5\gamma_\rho\phi^\mu)$$

$$-\frac{\kappa^2 V}{4}\varepsilon^{ij}\bar{\chi}^i\sigma_{\kappa\lambda}\chi^j(\bar{\psi}_\mu i\gamma_5\phi_\nu V^{-1}\varepsilon^{\mu\nu\kappa\lambda}+2\bar{\psi}_\mu\sigma^{\kappa\lambda}\phi^\mu+2\bar{\psi}^\lambda\phi^\kappa) - \frac{\kappa^2 V}{8}(\varepsilon^{ij}\bar{\chi}^i\sigma_{\mu\nu}\chi^j)^2$$

$$-\frac{\kappa^2 V}{32}(\bar{\chi}^i\gamma_5\gamma_\rho\chi^i)^2 - \frac{\kappa^2 iV}{4a}\bar{\chi}^i z^i\overleftrightarrow{\not{\partial}}z^j\chi^j + \frac{\kappa^2 iV}{8a}\bar{\chi}^i z^j\overleftrightarrow{\not{\partial}}z^j\chi^i$$

$$\mathcal{L}_4 = -\frac{m^2 V}{2\kappa^2 a^2}(u-\frac{3}{4}u^2) - \frac{mV}{2a}\bar{\chi}^i\chi^i$$

$$-\frac{i\kappa mV}{\sqrt{2}a}(\bar{\psi}_\mu z^i\gamma_\mu\chi^i+\bar{\phi}_\mu\varepsilon^{ij}\bar{z}^i\gamma_\mu\chi^j)$$

$$-\frac{\kappa^2 mV}{2a}(\bar{\psi}_\mu\sigma^{\mu\nu}z^i z^i\psi_\nu+\bar{\phi}_\mu\sigma^{\mu\nu}\bar{z}^i\bar{z}^i\phi_\nu+2\bar{\psi}_\lambda\sigma^{\lambda\nu}\varepsilon^{ij}\bar{z}^i z^j\phi_\nu)$$

$$\mathcal{L}_5 = \frac{3}{2}\frac{e^2 V}{\kappa^4 a^2}(1-\frac{u}{3}) - \frac{eV}{a\kappa}(\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu+\bar{\phi}_\mu\sigma^{\mu\nu}\phi_\nu)$$

$$-\frac{ieV}{\sqrt{2}a}(\bar{\psi}_\mu\gamma^\mu z^i\chi^i+\bar{\phi}_\mu\gamma^\mu\varepsilon^{ik}\bar{z}^i\chi^k)$$

$$-\frac{e\kappa V}{4a}(\bar{\chi}^i\varepsilon^{ij}\bar{z}^j\bar{z}^l\varepsilon^{kl}\chi^k+\bar{\chi}^i z^i z^j\chi^j)$$

$$\mathcal{L}_6 = \frac{emV}{4\kappa a^2}(z^i z^i+\bar{z}^i\bar{z}^i)$$

$$\delta_1\begin{pmatrix}A^i\\B^i\end{pmatrix} = \frac{a}{\sqrt{2}}\bar{\varepsilon}\begin{pmatrix}1\\i\gamma_5\end{pmatrix}\chi^i + \frac{a}{\sqrt{2}}\bar{\zeta}\varepsilon^{ij}\begin{pmatrix}1\\-i\gamma_5\end{pmatrix}\chi^j$$

$$\delta_1\chi^i = \frac{-i}{a\sqrt{2}}(\not{\partial}z^i\varepsilon-\varepsilon^{ij}\not{\partial}\bar{z}^j\zeta)$$

$$\delta_3\chi^i = \frac{i\kappa}{2}\,(\gamma^\mu\epsilon\bar{\chi}^i\psi_\mu+\gamma^5\gamma^\mu\epsilon\bar{\chi}^i\gamma_5\psi_\mu+\gamma^\mu\zeta\bar{\chi}^i\phi_\mu+\gamma^5\gamma^\mu\zeta\bar{\chi}^i\gamma_5\phi_\mu)$$

$$+\frac{i\kappa}{2}\,\epsilon^{ij}(\gamma^\mu\epsilon\bar{\chi}^j\phi_\mu-\gamma^5\gamma^\mu\epsilon\bar{\chi}^j\gamma_5\phi_\mu-\gamma^\mu\zeta\bar{\chi}^j\psi_\mu+\gamma^5\gamma^\mu\zeta\bar{\chi}^j\gamma_5\psi_\mu)$$

$$+\frac{\kappa^2}{2\sqrt{2}}\,[\gamma_5\chi^i\bar{\chi}^j\gamma_5(Z^j\epsilon+\epsilon^{kj}\bar{Z}^k\zeta)\;-\;\gamma_5\chi^j(\bar{\epsilon}\gamma_5Z^j\chi^i+\bar{\epsilon}\gamma_5Z^i\chi^j+\bar{\zeta}\gamma_5\epsilon^{kj}\bar{Z}^k\chi^i+\bar{\zeta}\gamma_5\epsilon^{ki}\bar{Z}^k\chi^j)$$

$$-\chi^j(\bar{\epsilon}Z^j\chi^i-\bar{\epsilon}Z^i\chi^j+\bar{\zeta}\epsilon^{kj}\bar{Z}^k\chi^i-\bar{\zeta}\epsilon^{ki}\bar{Z}^k\chi^j)]$$

$$\delta_4\chi^i = -\frac{m}{a\sqrt{2}}\,(Z^i\epsilon-\epsilon^{ij}\bar{Z}^j\zeta) \qquad\qquad \delta_5\chi^i = -\frac{e}{\sqrt{2}\kappa a}\,(\bar{Z}^i\epsilon-\epsilon^{ij}Z^j\zeta)$$

$$\delta_0 V_{a\mu} = -i\kappa(\bar{\epsilon}\gamma_a\psi_\mu+\bar{\zeta}\gamma_a\phi_\mu) \qquad\qquad \delta_0 A_\mu = -\bar{\epsilon}\phi_\mu+\bar{\zeta}\psi_\mu$$

$$\delta_0\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \frac{1}{\kappa}\,D_\rho\begin{pmatrix}\epsilon\\ \zeta\end{pmatrix} - \frac{i\kappa}{4}\,[2\bar{\psi}_\mu\gamma_\nu\psi_\rho+\bar{\psi}_\mu\gamma_\rho\psi_\nu+2\bar{\phi}_\mu\gamma_\nu\phi_\rho+\bar{\phi}_\mu\gamma_\rho\phi_\nu]\sigma^{\mu\nu}\begin{pmatrix}\epsilon\\ \zeta\end{pmatrix}$$

$$-\frac{1}{2}\,\sigma^{\mu\nu}(F_{\mu\nu}+2\kappa\bar{\psi}_\mu\phi_\nu)\gamma_\rho\begin{pmatrix}\zeta\\ -\epsilon\end{pmatrix}$$

$$\delta_3\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = -\frac{\kappa}{4a}\,\bar{Z}^i\overleftrightarrow{D}_\rho Z^i\begin{pmatrix}\epsilon\\ -\zeta\end{pmatrix} + \frac{\kappa}{2a}\,\epsilon^{ij}\begin{pmatrix}\bar{Z}^i D_\rho\bar{Z}^j\zeta\\ -Z^i D_\rho Z^j\epsilon\end{pmatrix}$$

$$-\frac{i\kappa}{4}\,\gamma_5\sigma_{\tau\rho}\begin{pmatrix}\epsilon\\ \zeta\end{pmatrix}\bar{\chi}^i\gamma_5\gamma^\tau\chi^i - \frac{i\kappa}{2}\,\gamma_5\gamma^\tau\begin{pmatrix}\zeta\\ -\epsilon\end{pmatrix}\epsilon^{ij}\bar{\chi}^i\gamma_5\sigma_{\rho\tau}\chi^j$$

$$-\frac{\kappa^2}{2\sqrt{2}}\,\gamma_5\begin{pmatrix}\psi_\rho\\ -\phi_\rho\end{pmatrix}(\bar{\epsilon}\gamma_5Z^i\chi^i+\bar{\zeta}\gamma_5\epsilon^{ij}\bar{Z}^j\chi^i) - \frac{\kappa^2}{2\sqrt{2}}\,\gamma_5\begin{pmatrix}\phi_\rho\\ \psi_\rho\end{pmatrix}(\bar{\epsilon}\gamma_5\epsilon^{ij}\bar{Z}^i\chi^j+\bar{\zeta}\gamma_5Z^i\chi^i)$$

$$-\frac{\kappa^2}{2\sqrt{2}}\begin{pmatrix}\phi_\rho\\ -\psi_\rho\end{pmatrix}(\bar{\epsilon}\epsilon^{ij}\bar{Z}^i\chi^j-\bar{\zeta}Z^i\chi^i)$$

$$\delta_4\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \frac{i\kappa m}{4a}\,\gamma_\rho Z^iZ^i\begin{pmatrix}\epsilon\\ \zeta\end{pmatrix} + \frac{i\kappa m}{4a}\,\gamma_\rho\epsilon^{ij}\bar{Z}^iZ^j\begin{pmatrix}\zeta\\ \epsilon\end{pmatrix} \qquad\qquad \delta_5\begin{pmatrix}\psi_\rho\\ \phi_\rho\end{pmatrix} = \frac{ie}{2\kappa^2 a}\,\gamma_\rho\begin{pmatrix}\epsilon\\ \zeta\end{pmatrix}$$

3) PROPERTIES OF THE THEORY

3.1) The Internal Symmetry Structure

The massless theory (2.3-4) and (2.4-1) possesses a "gravity" SU(2) invariance. In particular,

the O(2) already discussed (τ_2) together with two chiral invariances of the theory (τ_1 and τ_3) form an SU(2) group. The infinitesimal transformation laws are:

(3.1-1)

$$\tau_1: \quad \delta\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix} = \Lambda_1 i\gamma_5\begin{pmatrix}\zeta\\ \varepsilon\end{pmatrix} \qquad \delta\begin{pmatrix}\psi_\mu\\ \phi_\mu\end{pmatrix} = \Lambda_1 i\gamma_5\begin{pmatrix}\phi_\mu\\ \psi_\mu\end{pmatrix} \qquad \delta\begin{pmatrix}Z^i\\ \bar{Z}^i\end{pmatrix} = \Lambda_1 i\gamma_5\varepsilon^{ij}\begin{pmatrix}\bar{Z}^j\\ -Z^j\end{pmatrix}$$

$$\tau_2: \quad \delta\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix} = \Lambda_2\begin{pmatrix}\zeta\\ -\varepsilon\end{pmatrix} \qquad \delta\begin{pmatrix}\psi_\mu\\ \phi_\mu\end{pmatrix} = \Lambda_2\begin{pmatrix}\phi_\mu\\ -\psi_\mu\end{pmatrix} \qquad \delta\begin{pmatrix}Z^i\\ \bar{Z}^i\end{pmatrix} = -\Lambda_2\varepsilon^{ij}\begin{pmatrix}\bar{Z}^j\\ Z^j\end{pmatrix}$$

$$\tau_3: \quad \delta\begin{pmatrix}\varepsilon\\ \zeta\end{pmatrix} = \Lambda_3 i\gamma_5\begin{pmatrix}\varepsilon\\ -\zeta\end{pmatrix} \qquad \delta\begin{pmatrix}\psi_\mu\\ \phi_\mu\end{pmatrix} = \Lambda_3 i\gamma_5\begin{pmatrix}\psi_\mu\\ -\phi_\mu\end{pmatrix} \qquad \delta\begin{pmatrix}Z^i\\ \bar{Z}^i\end{pmatrix} = -\Lambda_3 i\gamma_5\begin{pmatrix}Z^i\\ -\bar{Z}^i\end{pmatrix}.$$

The infinitesimal parameters ε and ζ are numbers, so that they do not transform under this SU(2) group. However, if we formally rotate them according to the above formula, the parameter saturated supersymmetry transformation $\bar{\varepsilon}Q_\varepsilon + \bar{\zeta}Q_\zeta$ transforms as an SU(2) singlet. Thus the supersymmetry transformation formulas in (2.4-1) commute with the formulas of (3.1-1).

Similarly, we may find two different chiral invariances present (τ_1' and τ_3') which close together with the central charge (τ_2') into what we shall call an "internal symmetry" SU(2). This group commutes not only with supersymmetry, but also with the SU(2) in (3.1-1). Its field transformations are:

$$\tau_1': \quad \delta\chi^{1 \atop 2} = \Lambda_1' i\gamma_5 \chi^{2 \atop 1} \qquad \delta\bar{Z}^{1 \atop 2} = \Lambda_1' i\gamma_5 \bar{Z}^{2 \atop 1} \tag{3.1-2}$$

$$\tau_2': \quad \delta\chi^i = \Lambda_2' \varepsilon^{ij} \chi^j \qquad \delta\bar{Z}^i = \Lambda_2' \varepsilon^{ij} \bar{Z}^j$$

$$\tau_3': \quad \delta\chi^{1 \atop 2} = \pm\Lambda_3' i\gamma_5 \chi^{1 \atop 2} \qquad \delta\bar{Z}^{1 \atop 2} = \pm\Lambda_3' i\gamma_5 \bar{Z}^{1 \atop 2}.$$

In addition to the above symmetries, the theory is also invariant under a "duality" transformation (an extension to the matter-coupled theory of an invariance present in pure supergravity)[Fe77c]. It commutes with both of the above SU(2)'s, but not with supersymmetry:

$$\delta F^{\mu\nu} = -\frac{\Lambda''}{2} \tilde{F}^{\mu\nu} - i\Lambda''\kappa\varepsilon^{ij}\bar{\chi}^i\gamma_5\sigma^{\mu\nu}\chi^j \tag{3.1-3}$$

$$- \Lambda''\kappa(\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\rho\phi_\sigma + i\bar{\psi}^\mu\gamma_5\phi^\nu - i\bar{\psi}^\nu\gamma_5\phi^\mu)$$

$$\delta\begin{pmatrix}\psi_\mu \\ \phi_\mu\end{pmatrix} = \Lambda'' i\gamma_5 \begin{pmatrix}\psi_\mu \\ \phi_\mu\end{pmatrix} \qquad \delta\chi^i = -\Lambda'' i\gamma_5 \chi^i.$$

We see that it consists of a chiral transformation on the half integral spin fields, and a rotation of the vector fields which connects the dual $(F_{\mu\nu} + \frac{\tilde{F}_{\mu\nu}}{2})$ and the antidual $(F_{\mu\nu} - \frac{\tilde{F}_{\mu\nu}}{2})$ sectors to each other. Properly speaking, this is not an invariance of the Lagrangian, but only of the equations of motion (the manifold of solutions). Specifically, if one is interested in the transformation law for the canonical field A_μ, the solvability condition for obtaining δA_μ from $\delta F_{\mu\nu}$ turns out to be precisely Maxwell's equations. This symmetry has been used in finiteness proofs for N = 2 supergravity amplitudes[Gr76], and there are indications that its validity extends beyond the classical path and into the quantum domain, to at least one loop[Gr78b]. Thus, it is possible that its marginal status is only an artifact of the canonical fields not having been defined in an optimal way.

The resulting overall symmetry is SU(2) x SU(2) x U(1), isomorphic to SO(4) x U(1). The fields transform under SU(2) x SU(2) as follows: $Z^i(1/2, 1/2)$, $\chi^i(0, 1/2)$, ψ_μ and ϕ_μ (1/2, 0), and the generators Q_ε and Q_ζ parallel the behavior of their gauge fields. This symmetry is broken in stages upon each successive enlargement of the theory. $\mathcal{L}_4$ introduces spinor mass terms, and thus it breaks the U(1) along with the chiral generators (τ_1', τ_3') of the internal SU(2). At the same time, the central charge (τ_2') is gauged by A_μ. Alternatively, $\mathcal{L}_5$ introduces masslike terms for the gravitino, and it breaks duality as well as the chiral parts (τ_1, τ_3) of the gravity SU(2), while A_μ gauges τ_2. In the widest extension $\mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$, all chiral groups are broken, while A_μ gauges a linear combination of the surviving O(2) s $[(\kappa m)\tau_2' + e\tau_2]$, and the orthogonal combination $[\kappa m\tau_2 - e\tau_2']$ remains as a global symmetry. Table 3.1 summarizes this stepwise reduction of symmetry.

Table 3.1: The Hierarchical Breaking of Internal Symmetries.

(V denotes that a symmetry is valid, X that it is broken, and G that it is gauged by A_μ. G/√ denotes gauging of a particular linear combination with another generator).

Sector of the Theory	Gravity SU(2) x τ_1	τ_2	τ_3	Internal SU(2) x τ_1'	τ_2'	τ_3'	Duality U(1) T
$\mathcal{L}_0+\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3$	√	√	√	√	√	√	√
" $+\mathcal{L}_4$	√	√	√	X	G	X	X
" $+\mathcal{L}_5$	X	G	X	√	√	√	X
" $+\mathcal{L}_4+\mathcal{L}_5+\mathcal{L}_6$	X	G/√	X	X	G/√	X	X

Note that the breaking of the chiral pieces of an SU(2) goes together with the gauging of the remaining O(2).

We have seen that the vector A_μ can gauge the gravity O(2), the central charge, or even a linear combination of the two, consistently with local supersymmetry. It is demonstrated below that this is possible because of the symmetry present in the system[Fe77d], and that, as a result, the gravity O(2) is (in some sense) a central charge as well.

In order to maintain supersymmetry in $\mathcal{L}_5$, we needed to introduce a cosmological term with a positive sign $\frac{3e^2}{2\kappa^4}$ ($= \frac{\lambda}{2\kappa^2}$), gravitino masslike terms, and a constant term in the variation of the gravitino fields ($e\gamma_\mu\varepsilon$). All of these might be misconstrued[Fr77a] as signals of spontaneous supersymmetry breaking. In fact, supersymmetry is not broken as we show in Section 3.4. Closer examination of the theory reveals that the cosmological term implies a spacetime symmetry different from the Poincaré group IO(3,1), namely the De Sitter group SO(3,2)[Fr77a], which can be contracted to the Poincaré group by letting the De Sitter radius $(\frac{\kappa}{e})$ go to infinity. The background metric we should expand around is not the Minkowskian one, but the De Sitter solution. As a result, the "mass" of the gravitinos turns out to be only a symptom of the peculiar field propagation in De Sitter space[De77a]. In fact, it is necessary in order that the gravitino has two degrees of freedom, as a massless spin 3/2 particle should.

The De Sitter radius $(\frac{\kappa}{e})$ described by $\mathcal{L}_0 + \mathcal{L}_5$ would be of the order of the Planck length for e of the order of unity. Comparison to astronomical lower bounds on a possible De Sitter radius for the universe yields $e^2 < 10^{-120}$ [Fr77a], as compared to electromagnetism: $\frac{e^2}{4\pi} = 1/137$. In any case, there is hardly any incentive to make an identification of e with familiar coupling constants, since the spinors χ do not couple to the vector A_μ with this charge anyway.

The property of the gravity O(2) crucial to its gauging is that it appears in the anticommutation of the two supersymmetries which belong to the graded extension of the De Sitter group. The symplectic group in Sp(4) x SO(2) covers the De Sitter group in SO(3,2) x SO(2). Sp(4) x SO(2) is the Bose subalgebra of OSp(4,2), which contains 10 + 1 bosonic and 2 fermionic generators. This graded algebra, suitably rescaled but not yet contracted, is a modification of (2.1-1) through the extra pieces[To77]:

$$\{Q^I_\alpha, Q^J_\beta\} = \delta^{IJ}\gamma_{\alpha\beta}\cdot P + \epsilon^{IJ}\delta_{\alpha\beta}\Lambda - \frac{2e}{\kappa}\,\delta^{IJ}\sigma_{\mu\nu}M^{\mu\nu} \qquad (3.1\text{-}4)$$

$$[P_\mu, P_\nu] = -i\,\frac{e^2}{\kappa^2}\,M_{\mu\nu}$$

$$[P^\mu, Q^I_\alpha] = \frac{1}{2}\,\frac{e}{\kappa}\,\gamma^\mu_{\alpha\beta}Q^I_\beta$$

$$[\Lambda, Q^I_\alpha] = \frac{e}{\kappa}\,\epsilon^{IJ}Q^J_\alpha.$$

Upon contraction ($e \to 0$), the algebra reduces to (2.1-1), and Λ becomes a central charge Z. However, before contraction, we see that Λ represents the gravity O(2), and yet it also appears in the commutator of two supersymmetry transformations. The algebra (3.1-4) may be extended consistently, by the addition of another boson generator Z which commutes with all elements of the algebra. Then defining $\Lambda = \Lambda_2 + Z$, we have

$$[Z, (\text{anything})] = 0 \qquad (3.1\text{-}5)$$

$$\{Q^I_\alpha, \bar{Q}^J_\beta\} = \delta^{IJ}\gamma_{\alpha\beta}\cdot P + \epsilon^{IJ}\delta_{\alpha\beta}(\Lambda_2 + Z).$$

The gravity O(2) is then identified with Λ_2 and the central charge with Z. Their sum is the generator of the algebra gauged by the field A_μ.

The commutator of two infinitesimal local supersymmetry transformations on a field yields the sum of a general coordinate transformation, a Lorentz transformation, a supersymmetry transformation, and a gauge transformation (whose parameters are field dependent). This is the local version of (2.1-4).

A systematic list of the composition rules for the parameters of these transformations may be found in [Br78]. We may easily see that the gauge transformation component of the commutator of two supersymmetry transformations acting on the vector field is:

$$[\delta',\delta]A_\mu = -\frac{1}{\kappa}\partial_\mu[(\bar{\varepsilon}\zeta'-\bar{\zeta}\varepsilon') - i\kappa(\bar{\varepsilon}\not{A}\varepsilon'+\bar{\zeta}\not{A}\zeta')] + \ldots \tag{3.1-6}$$

Consequently, A_μ gauges $\Lambda_2 + Z$ by virtue of the position this operator occupies in the algebra (3.1-5). The other fields transform proportionately to their respective charges:

$$[\delta',\delta]A^i = \frac{(m\kappa+e)}{\kappa}[(\bar{\varepsilon}\zeta'-\bar{\zeta}\varepsilon') - i\kappa(\bar{\varepsilon}\not{A}\varepsilon'+\bar{\zeta}\not{A}\zeta')]\varepsilon^{ij}A^j + \ldots \tag{3.1-7}$$

$$[\delta',\delta]B^i = \frac{(m\kappa-e)}{\kappa}[(\bar{\varepsilon}\zeta'-\bar{\zeta}\varepsilon') - i\kappa(\bar{\varepsilon}\not{A}\varepsilon'+\bar{\zeta}\not{A}\zeta')]\varepsilon^{ij}B^j + \ldots$$

and so on. As an aside, we note that the field dependent portion of the gauge parameter in the brackets may be absorbed into the translation component of the commutator, $-i(\bar{\varepsilon}\gamma^\mu\varepsilon'+\bar{\zeta}\gamma^\mu\zeta')\partial_\mu$, to convert the plain derivative into a gauge covariant one, as specified in (2.5-5).

The SO(4) symmetric part of the theory $(\mathcal{L}_0+\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3)$ may be written more compactly using an SU(2) x SU(2) covariant notation. Recall that $\begin{pmatrix}\bar{Z}^1\\ Z^2\end{pmatrix}$ is an internal SU(2) doublet, while $\begin{pmatrix}\bar{Z}^1\\ -Z^2\end{pmatrix}$ is a gravity SU(2) doublet. Thus we may define the

SO(4) matrix which transforms an internal SU(2) doublet into a gravity SU(2) doublet:

$$Z \equiv \begin{pmatrix} z^1 & z^2 \\ -\bar{z}^2 & \bar{z}^1 \end{pmatrix} . \qquad (3.1\text{-}8)$$

We further denote $\psi_\mu \equiv \begin{pmatrix} \psi_\mu \\ \phi_\mu \end{pmatrix}$, $\varepsilon \equiv \begin{pmatrix} \varepsilon \\ \zeta \end{pmatrix}$, $\chi = \begin{pmatrix} \chi^1 \\ \chi^2 \end{pmatrix}$, and the antihermitean Pauli matrices $\tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\tau_1 = i\gamma_5 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau_3 = i\gamma_5 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$*. Contractions within the relevant SU(2)'s are implied to accompany the contractions in spinor space. For any two spinors υ and ω, the cross product $\bar{\upsilon}\tau_2\omega$ is an SU(2) invariant, sometimes referred to as a "Majorana mass" type term ($e^{\vec{\alpha}\cdot\vec{\tau}^T}\tau_2 e^{\vec{\alpha}\cdot\vec{\tau}} = \tau_2$). The SO(4) part of our theory may now be presented as follows:

$$\mathcal{L}_0 = -\frac{VR}{4\kappa^2} - \frac{1}{2}\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho - \frac{V}{4}F_{\mu\nu}F^{\mu\nu} \qquad (3.1\text{-}9)$$

$$-\frac{\kappa}{4}\bar{\psi}_\mu\tau_2(2VF^{\mu\nu} - i\gamma_5\tilde{F}^{\mu\nu})\psi_\nu$$

$$-\frac{\kappa^2}{4}\bar{\psi}_\mu\tau_2\psi_\nu[V\bar{\psi}^\mu\tau_2\psi^\nu - \frac{i}{2}\varepsilon^{\mu\nu\kappa\lambda}\bar{\psi}_\kappa\gamma_5\tau_2\psi_\lambda]$$

$$-\frac{\kappa^2 V}{16}[\bar{\psi}^\lambda\gamma^\mu\psi^\rho(\bar{\psi}_\lambda\gamma_\mu\psi_\rho + 2\bar{\psi}_\mu\gamma_\lambda\psi_\rho) - 4(\bar{\psi}\cdot\gamma\psi_\sigma)^2]$$

$$\mathcal{L}_1 = \frac{V}{2a^2}\frac{\mathrm{Tr}}{2}(g^{\mu\nu}\partial_\mu Z\partial_\nu\bar{Z}^T) + \frac{iV}{2}\bar{\chi}\gamma^\mu D_\mu\chi$$

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* They incorporate the i's of the transformation laws.

$$\mathcal{L}_2 = -\frac{\kappa V}{a\sqrt{2}}\,\bar{\psi}_\mu \partial_\nu Z\gamma^\nu\gamma^\mu\chi + \frac{\kappa}{2}\,F_{\mu\nu}\bar{\chi}\sigma^{\mu\nu}\tau_2'\chi$$

$$\mathcal{L}_3 = -\frac{\kappa^2}{8a}\,\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\overline{Z}\overleftrightarrow{\partial}_\rho Z^T\psi_\nu$$

$$+ \frac{i\kappa^2}{16}\,\varepsilon^{\mu\lambda\nu\rho}\bar{\chi}\gamma_5\gamma_\rho\chi\bar{\psi}_\mu\gamma_\lambda\psi_\nu + \frac{\kappa^2 V}{8}\,\bar{\chi}\gamma_5\gamma_\rho\chi\bar{\psi}_\mu\gamma_5\gamma^\rho\psi^\mu$$

$$- \frac{\kappa^2 V}{4}\,\bar{\chi}\tau_2'\sigma_{\kappa\lambda}\chi\left(\frac{\varepsilon^{\mu\nu\kappa\lambda}}{2V}\,\bar{\psi}_\mu i\gamma_5\tau_2\psi_\nu + \bar{\psi}_\mu\sigma^{\kappa\lambda}\tau_2\psi^\mu + \bar{\psi}^\lambda\tau_2\psi^\kappa\right)$$

$$- \frac{\kappa^2 V}{8}\,(\bar{\chi}\sigma_{\mu\nu}\tau_2'\chi)^2 - \frac{\kappa^2 V}{32}\,(\bar{\chi}\gamma_5\gamma_\rho\chi)^2 - \frac{i\kappa^2 V}{8a}\,\bar{\chi}Z^T\overleftrightarrow{\not\partial}Z\chi$$

$$\delta_1 Z = \frac{a}{\sqrt{2}}\,[(\bar{\varepsilon}\chi)\mathbb{1} + (\bar{\varepsilon}\vec{\tau}\chi)\cdot\vec{\tau}]$$

$$\delta_1\chi = -\frac{i}{a\sqrt{2}}\,\not\partial Z^T\varepsilon$$

$$\delta_3\chi = -\frac{i\kappa}{4}\,[\gamma^\mu\gamma^\nu\chi\bar{\psi}_\mu\gamma_\nu\varepsilon + \gamma_5\gamma^\mu\gamma^\nu\chi\bar{\psi}_\mu\gamma_5\gamma_\nu\varepsilon$$

$$+ 2\tau_2'\sigma^{\kappa\lambda}\chi\bar{\psi}_\mu\tau_2\sigma_{\kappa\lambda}\gamma^\mu\varepsilon - \tau_2'\gamma_\nu\chi\bar{\psi}_\mu\tau_2\gamma^\nu\gamma^\mu\varepsilon$$

$$+ \tau_2'\gamma_5\gamma_\nu\chi\bar{\psi}_\mu\tau_2\gamma_5\gamma^\nu\gamma^\mu\varepsilon]$$

$$+ \frac{\kappa^2}{4\sqrt{2}}\,[3\gamma_5\chi\bar{\chi}\gamma_5 Z\varepsilon - \chi\bar{\chi}Z\varepsilon - 2\sigma_{\mu\nu}\chi\bar{\chi}\sigma^{\mu\nu}\varepsilon + \gamma_5\gamma_\mu Z\varepsilon\bar{\chi}\gamma_5\gamma^\mu\chi]$$

$$\delta V_{a\mu} = -i\kappa\bar{\varepsilon}\gamma_a\psi_\mu$$

$$\delta_o A_\mu = -\bar{\varepsilon}\tau_2\psi_\mu$$

$$\delta_o\psi_\rho = \frac{1}{\kappa} D_\rho\varepsilon - \frac{i\kappa}{4} (2\bar{\psi}_\mu\gamma_\nu\psi_\rho+\bar{\psi}_\mu\gamma_\rho\psi_\nu)\sigma^{\mu\nu}\varepsilon$$

$$- \frac{i}{2} \sigma^{\mu\nu}(F_{\mu\nu}+\kappa\bar{\psi}_\mu\tau_2\psi_\nu)\gamma_\rho\tau_2\varepsilon$$

$$\delta_3\psi_\rho = - \frac{\kappa}{4a} \overline{Z}\overleftrightarrow{\partial}_\rho Z^T\varepsilon - \frac{i\kappa}{4} \gamma_5\sigma_{\tau\rho}\varepsilon\bar{\chi}\gamma_5\gamma^\tau\chi$$

$$- \frac{i\kappa}{2} \gamma_5\gamma^\tau\tau_2\varepsilon\bar{\chi}\gamma_5\sigma_{\rho\tau}\tau_2'\chi + \frac{\kappa^2}{2\sqrt{2}} \vec{\tau}\psi_\rho\cdot(\bar{\varepsilon}\vec{\tau}Z\chi).$$

Our theory in the above form could be further reduced into a more elegant form using the first order formalism. We can retrace the steps of Section 2.2 leading from (2.2-2) to (2.2-10), now in the reverse direction. That is, we convert second order covariant derivatives D_μ into first order ones by completing them through a term $\frac{1}{2} \sigma^{\nu\rho}K_{\mu\nu\rho}$. At the same time, we subtract what we have just added from the fermion contact terms, thereby annihilating several of them, but not all.

The contorsion resulting out of the $\frac{\delta I}{\delta\omega_{\mu ab}} = 0$ equations of motion now has an extra contribution from the spinor kinetic terms:

$$K_{\mu\nu\rho} = \frac{i\kappa^2}{2} (\bar{\psi}_\mu\gamma_\rho\psi_\nu-\bar{\psi}_\nu\gamma_\mu\psi_\rho+\bar{\psi}_\rho\gamma_\nu\psi_\mu) - \frac{\kappa^2}{4} \varepsilon_{\mu\nu\rho\sigma}\bar{\chi}\gamma_5\gamma^\sigma\chi. \tag{3.1-10}$$

Consequently, the form of (3.1-9) changes by the addition to the Lagrangian of a term[Fr77d]:

$$\frac{\kappa^2 V}{16}\,[\bar{\psi}^{\lambda}\gamma^{\mu}\psi^{\rho}(\bar{\psi}_{\lambda}\gamma_{\mu}\psi_{\rho}+2\bar{\psi}_{\mu}\gamma_{\lambda}\psi_{\rho}) - 4(\bar{\psi}\cdot\gamma\psi_{\sigma})(\bar{\psi}\cdot\gamma\psi^{\sigma}) \tag{3.1-11}$$

$$-\frac{3}{2}\,(\bar{\chi}\gamma_5\gamma_{\sigma}\chi)(\bar{\chi}\gamma_5\gamma^{\sigma}\chi) + i\varepsilon^{\lambda\mu\rho\sigma}\bar{\chi}\gamma_5\gamma_{\sigma}\chi\bar{\psi}_{\lambda}\gamma_{\mu}\psi_{\rho}].$$

This addition cancels the gravitino contact terms in the last bracket of $\mathcal{L}_0$, and it also modifies the coefficients of $(-\frac{\kappa^2}{32})V(\bar{\chi}\gamma_5\gamma_{\rho}\chi)^2$, and $(\frac{i\kappa^2}{16})\varepsilon^{\mu\lambda\nu\rho}\bar{\chi}\gamma_5\gamma_{\rho}\chi\bar{\psi}_{\mu}\gamma_{\lambda}\psi_{\nu}$ in $\mathcal{L}_3$ to $(-\frac{\kappa^2}{8})$, and $(\frac{i\kappa^2}{8})$, respectively. The presence of residual terms of this type indicates that the fermion contact terms of the theory are not "pure torsion" (in contrast to the truncation of SO(4) supergravity to an N = 1 scalar multiplet coupled to supergravity[Cr77b], for which the above procedure eliminated all contact terms). Finally, the terms $-\frac{i\kappa}{4}(2\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\rho} + \bar{\psi}_{\mu}\gamma_{\rho}\psi_{\nu})\sigma^{\mu\nu}\varepsilon$ and $-\frac{i\kappa}{4}\gamma_5\sigma_{\tau\rho}\varepsilon\bar{\chi}\gamma_5\gamma^{\tau}\chi$ are removed from $\delta_0\psi_{\rho}$ and $\delta_3\psi_{\rho}$, respectively, leaving the theory in first order form.

3.2) The Implications of the Internal Symmetries.

In this section we discuss several mechanisms present in the theory (2.5-6) which are connected to the internal symmetries analyzed in the previous section, and we compare them with formal analogs occuring in similar theories. In our discussion, we follow two current (and mutually complementary) viewpoints concerning the origins of the central charge, outlined below.

P. Fayet has studied a general class of theories with extended supersymmetry[Fa78b] posessing internal gauge symmetries which may break without affecting supersymmetry. The breaking of some symmetries in a given theory is achieved by assigning non-vanishing vacuum expectation values to several scalar fields. As the scalar fields are shifted appropriately, certain field-dependent gauge transformations in the supersymmetry algebra get promoted to central charge transformations, provided they are preserved through the breaking and all symmetries that did not commute with them have been broken. The mass scale of the central charges is set by the vacuum expectation values of the scalar fields.

In the previous section we showed that, in the model constructed here (2.5-6), the introduction of a sector where the central charge is represented as a gauge symmetry is accompanied by the explicit breaking of internal symmetries. The above mechanism is illustrated by a similar phenomenon we have observed in the other existing N = 2 gauged matter model[Lu78], namely the N = 2 Yang-Mills supermultiplet n (1, 1/2, 1/2, 0, 0) coupled to N = 2 supergravity (2, 3/2, 3/2, 1).

In this multiplet, the matter vectors* $B_\mu^{\ \alpha}$ (field strength $G_{\mu\nu}^{\ \ \alpha}$) gauge the internal symmetry group G with a coupling constant g'. The matter fields are massless, and they belong to the n-dimensional adjoint representation of G, with indices α, β, γ, etc. The vector A_μ of supergravity gauges the gravity O(2) which rotates the two spinors as well as the two gravitinos into each other, in this particular theory. As in (2.5-6), gravitino "mass" terms and a cosmological constant are necessary for supersymmetry invariance.

The Lagrangian of this Yang-Mills supermultiplet coupled to supergravity contains a term: $-\kappa(A^\alpha G_{\mu\nu}^{\ \ \alpha}F^{\mu\nu} - B^\alpha G_{\mu\nu}^{\ \ \alpha}\tilde{F}^{\mu\nu})$, where A^α and B^α are the scalar and pseudoscalar fields present. Integrating by parts, we obtain $-2\kappa A_\mu \partial_\nu (A^\alpha G^{\mu\nu\alpha} - B^\alpha \tilde{G}^{\mu\nu\alpha})$, which is a coupling of the vector of gravity to a current. The charge of the current is a surface term

$$Z' = \int d^3x\ \partial_\nu (A^\alpha G^{o\nu}_{\ \ \alpha} - B^\alpha \varepsilon^{o\nu\kappa\lambda} G_{\kappa\lambda}^{\ \ \alpha}) \qquad (3.2\text{-}1)$$

which is not necessarily vanishing[Wi78]. In particular, if we break the gauge group G (taken for instance to be O(3)) by giving a vacuum expectation value to a scalar field A , the integral has a non-zero value. Z' has the dimensions of mass, whose scale is set by $\langle A^\alpha \rangle$, and, according to Witten and Olive[Wi78], it appears in the commutator of two supersymmetry transformations, so that it may be identified with the central charge**. Thus we see that the breaking of the internal symmetry G promotes a generator of an unbroken symmetry to a central

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*Our notation differs slightly from that of the original reference[Lu78] for purposes of uniformity.

**For cautionary remarks, see [Fa78b].

charge Z' which is gauged by the vector of gravity.

In the second approach to understanding the origins of the central charge, theories with a central charge are obtained from theories in a space with n extra compact dimensions which have been shrunk away[Sc75, Sc78]. The central charge results from a displacement generator in a compactified dimension [O179] and the vector field A_μ that gauges it arises out of the vielbein field associated with this dimension.

Dimensional reduction has been used to derive 4-dimensional theories from theories in 4+n dimensions.* The n compact dimensions are suppressed by reducing the scale of their space (e.g. if they close into circles, the radii of the circles are set equal to zero). In the process, the fields of the original theory split up into representations of the 4-dimensional Lorentz group, and lose their spacetime indices that correspond to the n compact dimensions. Studies in several models[Sc78, G177, Br77, Cr78d] reveal that the vectors of supergravity A_μ arise out of components of the vielbeins with indices in the n extra dimensions. The momentum operators P_i corresponding to the n compact dimensions transmutate into charges with the dimension of mass[Sc78, Fa78b, O179], and they may appear at the site occupied by the central charge in the 4-dimensional supersymmetry algebra (2.1-1). For example, the fifth component of the "fünfbein" $V_m{}^4$ occurring in N=2 supergravity in five dimensions[Br79]

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* or, in general, D-dimensional theories from D+n dimensional ones[Sc78]. This method featured in efforts to account for the extra dimensions occurring in string models, and was later applied to 6, 10, and 11-dimensional supersymmetric field theories[G177, Br77, Cr78d].

may be identified with A_μ in four dimensions, and it is seen to transform as the gradient of $\bar{\varepsilon}'\gamma^4\varepsilon$ under the anticommutation of two supersymmetry transformations. The parameter $\bar{\varepsilon}'\gamma^4\varepsilon$ of the translation in the fifth dimension may be reexpressed in four dimensional form $(\bar{\varepsilon}'\zeta - \bar{\zeta}'\varepsilon)$, which is immediately recognizable as the transformation parameter of the central charge.

This interpretation of the central charges is in agreement with the formula connecting the mass to the central charge in several theories[Wi78, Fa78b, So78a, Ol79]:

$$M^2 = \Sigma_{i=1}^{n} Z_i^2 \tag{3.2-2}$$

In our particular theory (2.1-3), the central charge Z (defined to be hermitean: $im\varepsilon^{ij}$) has eigenvalues $\pm$ m . Formula (3.2-2) originates in the vanishing of the scalar product of the 4+n dimensional momenta $\mathbb{P}$:

$$0 = \mathbb{P}\cdot\mathbb{P} = P_\mu P^\mu - \sum_{i=1}^{n} P_i P^i = M^2 - \sum_{i=1}^{n} Z_i^2 \qquad (3.2\text{-}3).$$

The connections discussed above are not free of ambiguity, but they might serve to illuminate the underlying equivalence between the vielbeins of compact dimensions to the vector of gravity A_μ on one hand, and the momentum operators in these dimensions to the central charge (gauged by A_μ), on the other. This correspondence manifests itself in our theory (2.5-1) through "antigravity". The coupling of A_μ to all matter fields in $\mathcal{L}_4$ has strength κm. In the static limit this gives a Coulomb potential which has exactly the same strength as the Newtonian 1/r potential, gotten from the coupling of gravity to the matter fields. A derivation of these static forces from the one particle exchange amplitudes is given in Appendix B. In the static limit, the vector force is attractive or repulsive for oppositely or similarly charged fields respectively. As a result, any stationary particles with charges of the same sign will not interact with each other, since their gravitational attraction is exactly cancelled by their vector repulsion.

Needless to say, any two oppositely charged particles attract each other with twice the gravitational force. At higher energies, and in the quantum domain, this balance no longer holds, as the spin difference of the two interactions manifests itself. In the ultrarelativistic limit, the vector exchange amplitudes go as $m^2(1+\frac{2s}{t})$ (P wave), whereas the graviton exchange amplitudes go as $\frac{s^2+st}{t}$ (D wave). Hence, at sufficiently high energies, gravity will dominate the picture, being a "harder" force.

An analogous cancellation is observed in a different model[0179], in which the classical Coulomb potential cancels against the static limit of the amplitude due to the exchange of a Higgs field. It is shown in this model that the scalar field involved in this effect may be interpreted as an extra component of the vector potential in a higher dimensional theory; the corresponding momentum in the extra dimension is identified with the "electric" charge of the model.

3.3) Non-polynomial Behaviour and Constraints

The additional symmetry of our theory constrains its form considerably, in comparison to N = 1 analogs. The gauge coupling of A_μ to the central charge is determined by the mass, and the coupling to the gravity O(2) by the cosmological term. In the previous section, we saw a manifestation of these constraints in the form of "antigravity". Another manifestation is the restriction of the nonpolynomial behaviour of the spinless fields in the system.

In order to study its nonpolynomial behaviour, we truncate the N = 2 theory (2, 3/2, 3/2, 1) x (1/2, 1/2, 0, 0, 0, 0) to an N = 1 scalar multiplet coupled to supergravity (2, 3/2) x (1/2, 0, 0). In this truncation of $\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$ the Q_ζ supersymmetry is eliminated since its gauge field $\phi_\mu \to 0$, also the vector $A_\mu \to 0$, and one of the two matter multiplets A^2, B^2, $\chi^2 \to 0$. This is consistent in the sense that the variations of all the fields set equal to zero also vanish*.

The N = 1 theory obtained by truncation is then a particular representative of the family of N = 1 scalar multiplets coupled to supergravity surveyed by Das et al.[Da77b]. Their analysis characterizes

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* Our particular theory (2.5-6) cannot be restricted to N = 1 supergravity coupled to two scalar multiplets. We must set half the scalar fields equal to zero, otherwise $\delta_3\phi_\mu \neq 0$. Such a doubled local multiplet is obtainable from other theories, like N = 1 local super-Q.E.D. [Da77b], which may be truncated into it upon suppression of e, ξ, λ, and A_μ. Alternatively, two (n=2) N = 2 vector multiplets coupled to supergravity [Lu78] can be truncated into it upon supression of ε^2, ψ^2, λ^2, A_μ, B_μ; upon truncation to n=1 (further halving the matter fields), the nonpoly-nomial behaviour is identical to (3.3-3) [Cr78b].

the nonpolynomial behaviour of the massless, non-interacting sector by a fundamental function $\Omega(u)$. The coefficients of all terms in the Lagrangian and the transformation laws depend on this function in a definite way. For instance, the coefficient of the kinetic term for the scalar field is $\Theta(u) \equiv (u\Omega')'$. These functions turn out to have unique forms in the truncation of $\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_{\frac{2}{2}} + \mathcal{L}_3$:

$$\Theta(u) = 1/(1 - \tfrac{\kappa}{2}Z\bar{Z})^2 = 1/a^2 \qquad \text{where } a = 1 - u/2 \qquad (3.3\text{-}1)$$

$$\text{hence} \quad \Omega' = 1/a, \text{ so that } \quad \Omega = -2\ln a$$

Thus $e^{\Omega/2} = 1/a$, so that there are no exponential fields in the theory.

In the analysis of Das et al.[Da77b], the mass and interaction sectors depend on two functions $\Lambda(Z)$ and $M(Z)$ which may be expressed in terms of $\Omega(u)$ and another fundamental function $\Phi(Z)$:

$$\Lambda(Z) = \frac{1}{\sqrt{\Theta}}\left[\frac{\partial\Phi(Z)}{\partial Z} + \kappa^2\bar{Z}\Omega'\Phi(Z)\right] \qquad (3.3\text{-}2)$$

$$M(Z) = \frac{1}{\sqrt{\Theta}}\left[\frac{\partial\Lambda(Z)}{\partial Z} + \kappa^2\,\bar{Z}\Lambda(Z)\,\left(\Omega' - \frac{1}{2}\frac{\Theta'}{\Theta}\right)\right]$$

$\Phi(Z) = \sum_{s=0}^{\infty} c_s Z^s$ is a power series in Z with real constant coefficients c_s of dimension κ^{s-3}. These three functions also have unique forms in the truncation of $\mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$:

$$\Phi(Z) = \frac{e}{\kappa^3} + \frac{m}{2}Z^2 \quad \Lambda(Z) = mZ + \frac{e}{\kappa}\bar{Z} \qquad M(Z) = m + \frac{e\kappa}{2}\bar{Z}\bar{Z} \quad (3.3\text{-}3)$$

Other theories with higher symmetry exhibit a unique non-polynomial behaviour when truncated into an N=1 theory. For example, SO(4) supergravity [Cr77b]* yields a dependence on scalar fields different from (3.3-4), namely:

$$\Theta = \frac{1}{(1-u)^2}\ , \quad \Omega = -\ln(1-u), \qquad (3.3\text{-}4)$$

$$\Phi(Z) = \frac{e}{\kappa^3}\ , \qquad \Lambda(Z) = \frac{e}{\kappa}\ \bar{Z}$$

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*(2,4x 3/2, 6x1, 4x 1/2, 0, 0). This theory has only two spinless fields and thus cannot contain (2.5-6) which has four spinless fields.

This nonpolynomial dependence specifies the terms of the theory quartic in χ to be "pure torsion", as mentioned at the end of Section 3.1. It has been noted [Cr77b] that this is natural for a theory which originates in a supergravity multiplet, and hence the uniqueness of the form (3.3-4) is justified. The uniqueness of this form has also been linked to an additional invariance of the SO(4) supergravity theory[Cr78a], namely an SU(1,1) of dual type, analogous to (3.1-3). SU(1,1) is also present in the supersymmetric non-linear σ-model coupled to supergravity[Cr78b], where it is utilized to rotate away a scalar degree of freedom. This leads to a redefinition of the fields in the system, through which the analog of $(1-u)^2$ appears in the denominator of the kinetic terms, and other appropriate sites. In a similar way, the SU(1,1) realized non-linearly in SO(4) supergravity reflects a redefinition of scalar fields[Cr78a, Cr78b].

We have not been able to find a satisfactory non-compact invariance in our theory that would explain its nonpolynomial character (3.3-1), (3.3-2). Nevertheless, we may regard the form of Φ in (3.3-2) as more plausible, if we consider its compatibility with internal symmetry, like the central charge O(2). Due to its origin in an O(2) symmetric theory, Φ cannot contain odd powers of Z. The reason is that, before truncation from N=2 to N=1, it is not possible to saturate the O(2) indices of an odd power of Z to form an O(2) singlet as needed in the N=2 Lagrangian. Thus Φ may only contain <u>even</u> powers of Z. It is conceivablethat the theory (2.5-6) could be generalized to contain interactions Φ featuring a quartic term in Z *, or even higher powers of Z, but it

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*For instance, the N=1 truncation would contain $\Phi(Z)=gZ^4$, where g has dimensions of length, so $\Lambda(Z)=gZ^3(4-u)$ and hence $M=g\bar{Z}^2\,(12-2u)$. The extra pieces to the truncated theory would be: $\delta_g\chi = \frac{g}{\sqrt{2}}\frac{Z^3(4-u)}{a}\epsilon$, $\delta_g\psi_\mu = -\frac{ig\kappa}{2a}\gamma_\mu Z^4\epsilon$

$$\mathcal{L}_g = -g\,\frac{6-u}{a}\,\bar{\chi}Z^2\chi - \frac{\kappa^2}{6}\,\frac{u^3(4-u)^2-3u^4}{2a^2} - \frac{ig\kappa}{\sqrt{2}a}(4-u)\bar{\psi}\cdot\gamma Z^3\chi - \frac{\kappa^2 g}{a}\,\bar{\psi}_\mu\sigma^{\mu\nu}Z^4\psi_\nu$$

is not clear how A_μ would couple to the extra pieces.

If the theory (2.5-6) could be obtained as a restriction of N=8 supergravity[De77], it would illuminate the structure and truncation mechanisms of the N=8 theory into smaller ones. SO(8) supergravity (2, 8x 3/2, 28 x 1, 56 x 1/2, 70 x 0) splits up into the following collection of massless N=2 supersymmetric theories: supergravity (2, 3/2, 3/2, 1) coupled together with up to six spin 3/2 multiplets (3/2, 1, 1, 1/2), up to fifteen vector multiplets (1, 1/2, 1/2, 0, 0)[Lu78], and up to ten scalar multiplets (1/2, 1/2, 0, 0, 0, 0). The rules for counting the states were given in the introduction. The number of fields in the fragments must match the number in the original SO(8) supergravity, at any given spin level.

The above truncation preserves Gell-Mann's assignment of "color" and "charge" to the states of N=8 supergravity[Ge77]. According to this scheme, SU(3) x U(1) x U(1) is embedded in SO(8), and at every spin level the fields falling into SO(8) representations break up into SU(3) multiplets (their charge is given in parentheses):

$$\text{Gravitinos:}\quad \underset{\sim}{8} = \underset{\sim}{3}(-1/3) + \underset{\sim}{\bar{3}}(1/3) + \underset{\sim}{1}(0) + \underset{\sim}{1}(0) \qquad (3.3\text{-}4)$$

$$\text{Vectors:}\quad \underset{\sim}{28} = (\underset{\sim}{8}\times\underset{\sim}{8})_A = \underset{\sim}{\bar{3}}(-2/3) + \underset{\sim}{3}(2/3) + \underset{\sim}{8}(0) + \underset{\sim}{1}(0) + $$
$$+ \underset{\sim}{3}(-1/3) + \underset{\sim}{3}(-1/3) + \underset{\sim}{\bar{3}}(1/3) + \underset{\sim}{\bar{3}}(1/3) + \underset{\sim}{1}(0)$$

and so on for the spinors and the scalars.

Since two of the eight gravitinos do not carry charge and are SU(3) singlets, two supersymmetry generators commute with SU(3) and are neutral; therefore the fields they connect fall into the same SU(3) multiplet and have a common charge. Since we may choose to break the six charged supersymmetry generators, truncation of the theory to N=2 supersymmetry is compatible with the above color scheme, in contrast to any truncation to N>2 supersymmetry.

As a necessary consequence, N=2 supergravity is an SU(3) singlet; the spin 3/2 multiplets are charged triplets; the vector multiplets exemplify several representations, charged or not; and all scalar multiplets are charged, consistently to the necessity of doubling the fields in a representation, demonstrated in(2.1-7). The situation is summarized in Table 3.3, on the next page.

In Appendix C, we give the general methodology of truncating SO(8) supergravity down to the N=2 scalar multiplet coupled to supergravity. We show how O(2)xO(2), that is the central charge and the "gravity rotation", may be identified with two generators of the SO(8) group. Unfortunately, only the first few orders of the SO(8) theory are available at present in explicit form[De77], so that we cannot draw any meaningful inferences on the nonpolynomial structure of our theory. For instance, we do not know if this structure is related to the consistency requirements of the truncation process - after all, the N=2 vector multiplet coupled to supergravity has a different nonpolynomial behaviour, as mentioned above, although it should result out of a restriction of the SO(8) theory as well.

These problems could be resolved using recent progress in the construction of the entire SO(8) theory. Specifically, simple supergravity in 11 dimensions may be dimensionally reduced to a four dimensional theory with the spectrum of SO(8) Supergravity[Cr78d]: The "elfbein" splits into a Vierbein, 7 vectors, and 28 scalars. The gravitino of 11 dimensions splits into 8 four dimensional gravitinos and 56 spinors. Finally, the 11-dimensional completely antisymmetric gauge po-

TABLE 3.3: Breakup of SO(8) Supergravity into N=2 supermultiplets, consistent with SU(3) symmetry.

Supermultiplet character	SU(3) representation (charge)	Multiplicity of fields Spin: 2	3/2	1	1/2	0
Supergravity	1 (0)	1	2	1		
Spin 3/2 multiplets	3 (-1/3)		1	2	1	
	$\bar{3}$ (1/3)		1	2	1	
Vector multiplets	1 (0)			1	2	2
	3 (2/3)			1	2	2
	$\bar{3}$ (-2/3)			1	2	2
	8 (0)			1	2	2
Scalar multiplets	1 (1)				1	2
	1 (-1)				1	2
	3 (-1/3)				1	2
	$\bar{3}$ (1/3)				1	2
	6 (1/3)				1	2
	$\bar{6}$ (-1/3)				1	2

tential $A_{\mu\nu\rho}$ (necessary to match the fermionic degrees of freedom within the framework of supersymmetry) splits up into 21 vectors and 35 + 7 scalars. The manifest internal symmetry of the compactified theory is SO(7), but the fields may be redefined to exhibit SO(8). The presence of non-compact invariances observed in this theory could be connected to the nonpolynomial structure of the scalar fields.

3.4) Study of the Potential

The full derivative independent part of the interaction read off from (2.5-4), is:

$$U = \frac{1}{2a^2\kappa^2}\left[-\frac{1}{2}\frac{e}{\kappa}m\kappa^2(Z^iZ^i + \bar{Z}^i\bar{Z}^i) - 3\frac{e^2}{\kappa^2}(1 - u/3) + m^2(u - \frac{3}{4}u^2)\right] \tag{3.4-1}$$

Defining $c \equiv \frac{e}{\kappa}\frac{1}{m}$, it may be reexpressed as:

$$U = \frac{m^2}{2\kappa^2 a^2}\left[(u - \frac{3}{4}u^2) - 3c^2(1 - \frac{u}{3}) - cu + 2c\kappa^2 B^iB^i\right] \tag{3.4-2}$$

U has a singularity at u = 2 with a negative residue, so that the theory is unstable for all values of c. (This singularity prevents any functional integral approach). We now scan the potential U across the plane $B^i=0$ (for $c \geq 0$; otherwise we should go to the plane $A^i=0$) in Fig. 3.4 - 1. We observe that the only local minimum is at the origin u = 0, for all $0 \leq c < 1/2$. Beyond the singularity, located at u = 2, there is no local minimum. For small enough masses, namely $c \geq 1/2$, there is no (maximum) bump to contain even a metastable ground state at the origin. We see in the figure that as c increases from zero, the maximum to the left of the singularity gradually vanishes, while past $c = \frac{\sqrt{17} - 3}{2}$ a maximum reappears to the right of the singularity (see c = 5.5 in Fig. 3.4 - 1).

It turns out therefore that there is no supersymmetry breaking in the theory. The cosmological term and the gravitino mass terms present for $e \neq 0$ are required in De Sitter space, and are not due to a super-Higgs effect, as pointed out in Section 3.1. In particular, the term $\frac{ie}{2\kappa^2}\gamma_\mu\varepsilon$ in the transformation law for the gravitino can be regarded as part of a "De Sitter

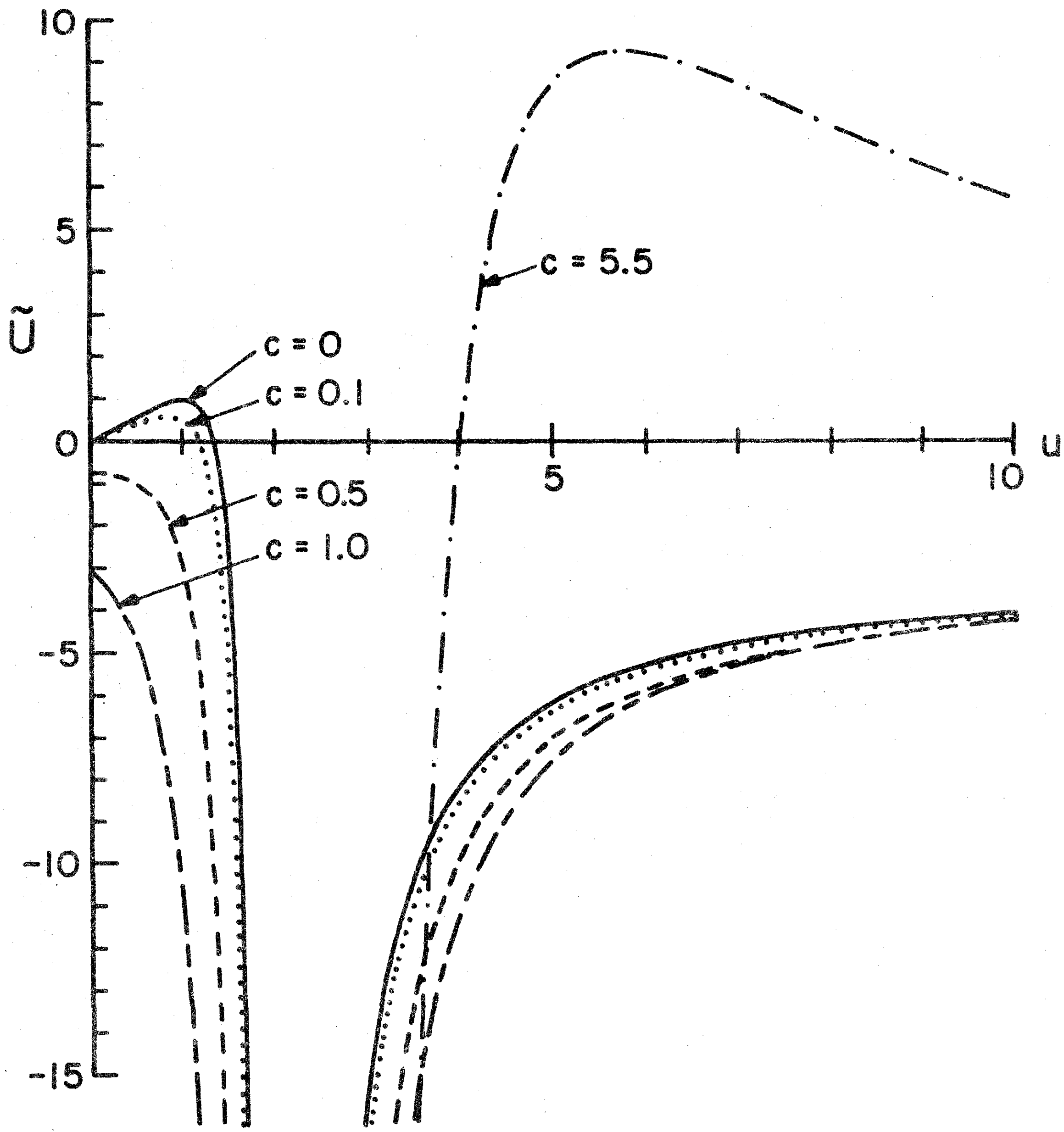

Fig. 3.4-1 : $\tilde{U} \equiv (u - \frac{3}{4} u^2 - 3c^2 + c^2 u - cu)/(1 - \frac{u}{2})^2$ plotted versus u, for various values of c.

covariant derivative" and cannot be used by itself to argue that Q_ε doesn't annihilate the vacuum, which would be a signal for spontaneous breaking of supersymmetry. Thus in fact supersymmetry remains unbroken.

The theory under investigation is unstable, i.e. only long lived at best. To explore its lifetime, we shall consider the "best" potential , namely c = 0, which is the one with the highest potential barrier for containing the ground state at the origin ($A^i=0$, $B^i=0$). If the lifetime is sufficiently long, we might consider it as a physical theory, despite the instability.

At first glance, the kinetic terms of the scalar fields would suggest the presence of derivative dependent parts to the interaction. However, as we pointed out before, the singularities of the spinless fields reflect, in a way, the non-optimal way they have been defined. In fact, P. Fayet* has suggested a redefinition of one scalar field A^1 which eliminates the derivative dependent part of the potential and pushes the singularity out from u = 2 to infinity in terms of the redefined field. Unfortunately we have not been able to extend this to include the pseudoscalar fields B^i or even to include both scalar fields instead of just one.

We shall thus have to proceed along the line $B^i= 0$, $A^2 = 0$, ignoring the extra spinless degrees of freedom. We hope this parallels the actual situation in a schematic way. The field redefinitions are

$$\frac{\kappa\ A}{\sqrt{2}} = \tanh \frac{\kappa\ \phi}{\sqrt{2}}\ , \quad \frac{\kappa\phi}{\sqrt{2}} = -\ \frac{1}{2}\ \ln\ \left(\frac{1\ -\ \kappa\ A/\sqrt{2}}{1\ +\ \kappa\ A/\sqrt{2}}\right) \qquad (3.4\ -\ 3)$$

As a result, the part of the Lagrangian containing only this scalar field simplifies to :

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* Private communication.

$$\frac{1}{2}\frac{\partial_\mu A\,\partial^\mu A}{(1-\frac{\kappa^2}{2}A^2)^2} - \frac{m^2(\kappa^2A^2 - 3\kappa^4A^4/4)}{2\,\kappa^2(1-\kappa^2A^2/2)^2} = \tag{3.4 - 4}$$
$$= \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi + \frac{m^2}{32\kappa^2}\left(e^{4\kappa\phi/\sqrt{2}} + e^{-4\kappa\phi/\sqrt{2}} - 12e^{2\kappa\phi/\sqrt{2}} - 12e^{-2\kappa\phi/\sqrt{2}} + 22\right)$$

In terms of dimensionless variables

$$U \equiv \frac{m^2}{32\,\kappa^2}\,\mathcal{U}(w)\ , \text{ where } \quad w \equiv \frac{2\kappa\phi}{\sqrt{2}}\ , \text{ and} \tag{3.4-5}$$

$$\mathcal{U}(w) = -(e^{2w} + e^{-2w} - 12e^{w} - 12e^{-w} + 22)$$

We plot this potential in Fig. 3.4 - 2. It has a maximum at $w=\text{arcosh}3$ and a zero at $w=\ln(\frac{\sqrt{3/2}+1}{\sqrt{3/2}-1})$, while the singularity is moved to $w = \infty$.*

The minimum at the origin of the potential (Fig. 3.4 - 2) is only a local one. Consequently, a quantum state **whose** wavefunction is concentrated in this region at t=0 will eventually tunnel through the barrier quantum mechanically, and fall down the singularity classically. Physically, this would mean that all masses and couplings will be rapidly shifted to preposterous magnitudes, which would be the end of the supersymmetric "universe" under consideration - in fact the theory is not well defined. Nevertheless, a rough semiclassical estimate yields a rate for such a false vacuum decay of the order of $\exp(-1/\kappa^2m^2)$. This is negligible for mass scales of the order of 1 GeV. We give the details of this WKB calculation in Appendix D.

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* The above is an analysis of the part of our theory below the singularity u = 2. The (less interesting) part above the singularity can be displayed in the same way by relations complementary to (3.4-3), needed for a full covering of the domain [0, ∞] of A. They are $\kappa A/\sqrt{2} = \coth(\kappa\phi/\sqrt{2})$ and $\kappa\phi/\sqrt{2} = -\frac{1}{2}\ln(\frac{1+\kappa A/\sqrt{2}}{1-\kappa A/\sqrt{2}})$, and they yield a potential $\mathcal{U}$ which ranges from $-3m^2/2\kappa^2$ to $-\infty$ without extrema:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m^2}{32\kappa^2}\left(e^{4\kappa\phi/\sqrt{2}} + e^{-4\kappa\phi/\sqrt{2}} + 12e^{2\kappa\phi/\sqrt{2}} + 12e^{-2\kappa\phi/\sqrt{2}} + 22\right)$$

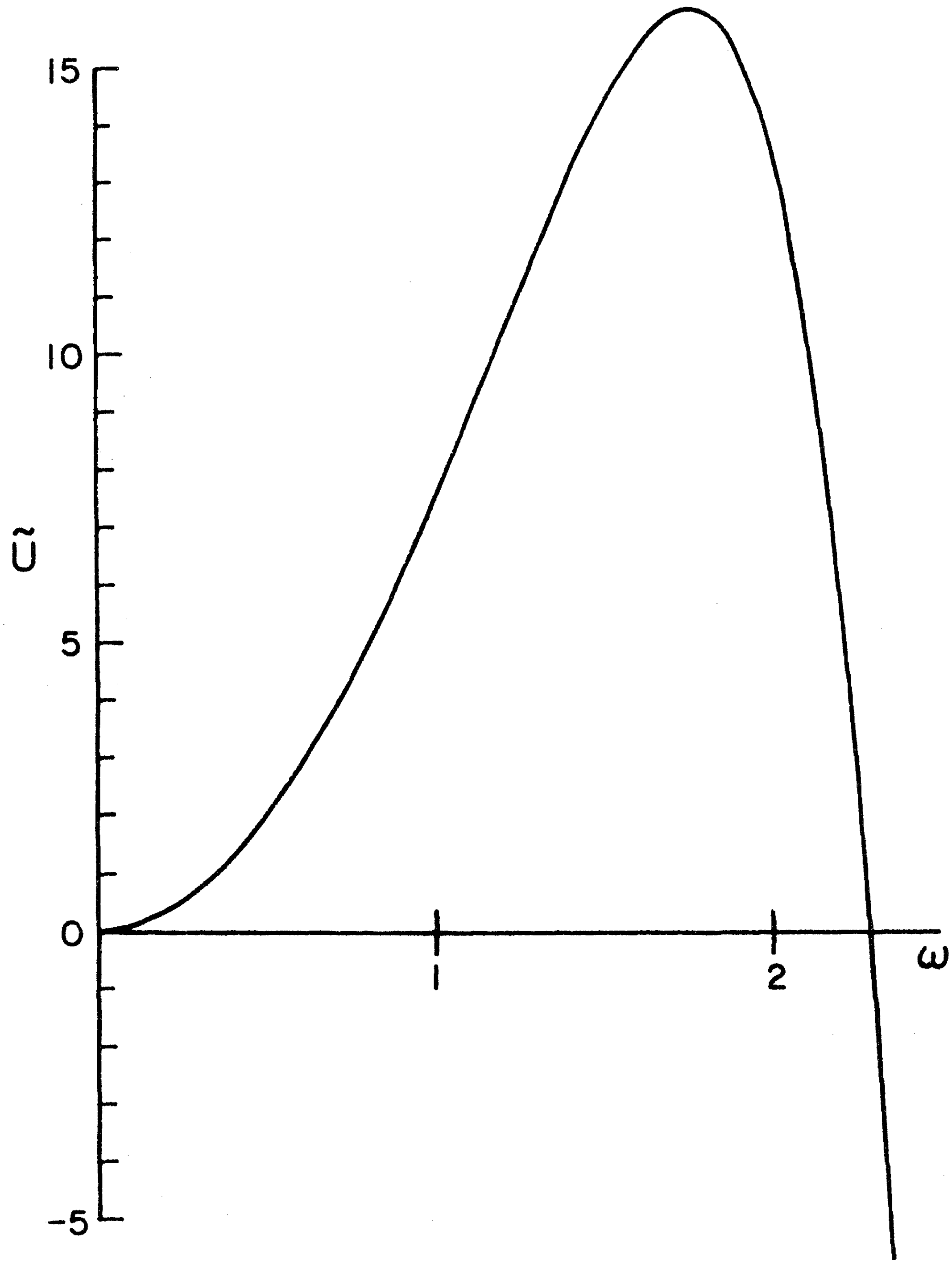


Fig. 3.4-2 : $\tilde{U} = -(e^{2w} + e^{-2w} - 12\ e^{w} - 12 e^{-w} + 22)$

plotted versus w.

3.5) Conclusions

In summary, we have constructed a theory that is invariant under local N = 2 supersymmetry transformations. The matter fields of the theory have mass and interactions, but we cannot claim we have exhausted all possibilities concerning the form of interaction consistent with supersymmetry invariance. The extended supersymmetry of the system restricts the spin zero fields to a particular behaviour. As a result, the permissible potentials of the spinless fields are not bounded from below, and therefore the theory is unstable. However, the significance of the theory presented does not lie in its physical applications, but rather in its exploration of the algebraic possibilities in supersymmetric systems.

In particular, we have shown that the vector of supergravity couples to the current of the central charge with the same strength as the gravitinos couple to the supercurrents and the graviton couples to the energy momentum tensor. Thus, every generator of the N = 2 supersymmetry algebra is gauged by a supergravity field.

We have observed an intricate hierarchy of symmetries, which may be explicitly broken piecewise by introducing further sectors to the theory. We have also shown that the vector of gravity is sufficient to gauge an internal symmetry different from (and commuting with) the central charge. As a result of this gauging, cosmological and gravitino"mass" terms appear, which do not signal supersymmetry breaking, but are instead symptoms of a De Sitter spacetime background.

We hope this study will contribute to a better understanding of extended supersymmetry and off-shell supergravity, as well as of the interplay between supersymmetry and internal symmetries. These are central aspects of the construction of field theories which attempt to unify all interactions including gravity.

APPENDIX A: LIST OF NOTATIONS, CONVENTIONS, AND IDENTITIES

We follow the conventions of [Fr77, Da77a, Da77b, Cr77b], and many others, namely:

$$\eta^{ab} = (+, -, -, -), \qquad \varepsilon^{0123} = 1 = -\varepsilon_{0123}, \tag{A-1}$$
$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \sigma^{ab} = \tfrac{1}{4}[\gamma^a, \gamma^b].$$

All spinors used throughout the text are Majorana spinors: $\psi = C\bar{\psi}^T$. For internal symmetry indices we are using ε^{ij} and δ^{ij}, where $\varepsilon^{12} = 1$, and $i_{ij} = 1,2$. For indices of the "gravity group" we use δ^{IJ} and ε^{IJ}, where $I,J = 1,2$.

κ denotes the gravitational coupling constant ($= \sqrt{4\pi G} \simeq$ $\simeq 5.7 \cdot 10^{-33}$ cm; the Planck mass is $1/\kappa \simeq 3.4 \cdot 10^{18}$ GeV).

$Q_\alpha^{\ I}$ denotes a (spinorial) supersymmetry generator.

ε, ζ denote the two infinitesimal transformation parameters of $N = 2$ supersymmetry. They are spacetime dependent Majorana spinors.

Z denotes the central charge.

m denotes the mass, common to all matter fields.

$V^a_{\ \mu}$ denotes the vierbein, whose square gives the metric tensor $g_{\mu\nu}$: $V^a_{\ \mu} V^b_{\ \nu} \eta_{ab} = g_{\mu\nu}$, while $g^{\mu\nu} V^a_{\ \mu} V^b_{\ \nu} = \eta^{ab}$. It is employed to interchange Greek indices (which describe the base space / curved spacetime) with Latin indices(which describe the flat tangent space).

$$V = \det V_{a\mu} = \sqrt{-g}, \text{ so that } \delta V = V\, V^{a\mu} \delta V_{\mu a}. \tag{A-2}$$

A_μ denotes the vector field of N=2 supergravity. Its field strength is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and its dual is $\tilde{F}_{\mu\nu} = \quad \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$.

ψ_μ, ϕ_μ denote the two gravitino fields of N=2 supergravity, namely spin 3/2 fields with a curved space-time index and an implicit spinor index.

χ^i denotes the two spinor fields of the N=2 matter multiplet.

A^i denotes the two scalar fields of the same multiplet.

B^i denotes the two pseudoscalar fields of the same multiplet.

Λ denote infinitesimal parameters for internal symmetry transformations.

We define

$$Z^i \equiv A^i + i\gamma_5 B^i \tag{A-3}$$

a matrix in spinor space, and

$$u \equiv \kappa^2 \bar{Z}^i Z^i = \kappa^2 (A^i A^i + B^i B^i), \tag{A-4}$$

a unit matrix in spinor space; written outside a spinor product, u denotes just its magnitude (without the unit matrix). Finally

$$a \equiv 1 - \frac{1}{2} u \tag{A-5}$$

Some useful Dirac algebra identities are:

$$\begin{aligned} \varepsilon^{\nu\mu\rho\sigma}\varepsilon_{\nu\kappa\lambda\tau} &= -\delta^\mu_\kappa\delta^\rho_\lambda\delta^\sigma_\tau + \delta^\mu_\lambda\delta^\rho_\kappa\delta^\sigma_\tau - \delta^\mu_\tau\delta^\rho_\kappa\delta^\sigma_\lambda \\ &\quad +\delta^\mu_\kappa\delta^\rho_\tau\delta^\sigma_\lambda - \delta^\mu_\lambda\delta^\rho_\tau\delta^\sigma_\kappa + \delta^\mu_\tau\delta^\rho_\lambda\delta^\sigma_\kappa \end{aligned} \tag{A-6}$$

and therefore $\varepsilon^{\mu\nu\kappa\lambda}\varepsilon_{\mu\nu\rho\sigma} = -2(\delta^\kappa_\rho\delta^\lambda_\sigma - \delta^\kappa_\sigma\delta^\lambda_\rho)$; as a consequence $\tilde{\tilde{F}}^{\mu\nu} = -4F^{\mu\nu}$.

$$\frac{\varepsilon^{\mu\nu\kappa\lambda}}{V}\gamma_5\gamma_\nu = i\{\gamma^\mu, \sigma^{\kappa\lambda}\} \tag{A-7}$$

$$\frac{\varepsilon^{\mu\nu\kappa\lambda}}{V}\sigma_{\kappa\lambda} = -2i\gamma_5\sigma^{\mu\nu}$$

$$\gamma_\mu\gamma_\lambda\gamma_\rho = -\frac{i\varepsilon_{\mu\lambda\rho\tau}}{V}\gamma_5\gamma^\tau + g_{\lambda\rho}\gamma_\mu + g_{\lambda\mu}\gamma_\rho - g_{\mu\rho}\gamma_\lambda$$

$$\gamma_\mu\sigma_{\kappa\lambda}\gamma_\nu - \gamma_\nu\sigma_{\kappa\lambda}\gamma_\mu = g_{\mu\kappa}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\kappa} - i\gamma_5\frac{\varepsilon_{\mu\kappa\lambda\nu}}{V}$$

$$[\sigma^{\mu\lambda}, \gamma^\nu] = g^{\nu\lambda}\gamma^\mu - g^{\mu\nu}\gamma^\lambda$$

$$\sigma^{\mu\lambda}\gamma_\lambda = \gamma_\lambda\sigma^{\lambda\mu} = \frac{3}{2}\gamma^\mu$$

$$\sigma^{\mu\nu}\sigma^{\rho\lambda}\sigma_{\mu\nu} = \sigma^{\rho\lambda}.$$

For any two Majorana spinors χ and ψ, the vector and tensor bilinear products are transposition odd ($\bar{\chi}\gamma^{\mu}\psi = -\bar{\psi}\gamma^{\mu}\chi$, $\bar{\chi}\sigma^{\mu\nu}\psi = -\bar{\psi}\sigma^{\mu\nu}\chi$), while the scalar, pseudoscalar, and axial vector bilinears are even ($\bar{\chi}\psi = \bar{\psi}\chi$, $\bar{\chi}\gamma_5\psi = \bar{\psi}\gamma_5\chi$, $\bar{\chi}\gamma_5\gamma_\mu\psi = \bar{\psi}\gamma_5\gamma_\mu\chi$). To interchange spinors among the trilinear products:

$$\begin{aligned} & S\chi\,\bar{\psi}\phi + V\gamma_\mu\chi\,\bar{\psi}\gamma_\mu\phi + T\sigma_{\mu\nu}\chi\,\bar{\psi}\sigma^{\mu\nu}\phi + A\gamma_5\gamma_\mu\chi\,\bar{\psi}\gamma_5\gamma^{\mu}\phi \\ & \qquad + P\gamma_5\chi\,\bar{\psi}\gamma_5\phi = \\ & = S'\phi\,\bar{\psi}\chi + V'\gamma_\mu\phi\,\bar{\psi}\gamma^{\mu}\chi + T'\sigma_{\mu\nu}\phi\,\bar{\psi}\sigma^{\mu\nu}\chi + A'\gamma_5\gamma_\mu\phi\,\bar{\psi}\gamma_5\gamma^{\mu}\chi \\ & \qquad + P'\gamma_5\phi\,\bar{\psi}\gamma_5\chi. \end{aligned} \tag{A-8}$$

The Fierz matrix

$$\lambda = -\frac{1}{4}\begin{pmatrix} 1 & 1 & -2 & -1 & 1 \\ 4 & -2 & 0 & -2 & -4 \\ -3 & 0 & -2 & 0 & -3 \\ -4 & -2 & 0 & -2 & 4 \\ 1 & -1 & -2 & 1 & 1 \end{pmatrix} \tag{A-9}$$

relates the above coefficients through $\lambda^{T}\begin{pmatrix} S \\ V \\ T \\ A \\ P \end{pmatrix} = \begin{pmatrix} S' \\ V' \\ T' \\ A' \\ P' \end{pmatrix}$.

Clearly $\lambda^2 = 1$.

Some consequences are: $\chi\bar{\chi}\gamma_5\chi = -\gamma_5\chi\bar{\chi}\chi$, $\chi\bar{\chi}\gamma_5\gamma_\mu\chi = -\gamma_5\gamma_\mu\chi\bar{\chi}\chi$.

Whence: $$\bar{\chi}\gamma_5\chi\bar{\chi}\chi = \bar{\chi}\gamma_5\gamma_\mu\chi\bar{\chi}\chi = 0. \tag{A-10}$$

The following identities for O(2) group indices are used:

$$\varepsilon^{mn}\varepsilon^{kl} = \delta^{mk}\delta^{nl} - \delta^{ml}\delta^{nk}. \qquad \text{(A-11)}$$

Contracting this with ε^{mj}, we obtain:

$$\delta^{ij}\varepsilon^{kl} + \delta^{ik}\varepsilon^{lj} + \delta^{il}\varepsilon^{jk} = 0.$$

Multiplying or contracting these two identities with ε's, we obtain numerous other identities permuting several group indices, e.g.,

$$\delta^{ij}\delta^{mk}\delta^{nl} - \delta^{ij}\delta^{ml}\delta^{nk} + \delta^{ik}\delta^{ml}\delta^{nj} - \delta^{ik}\delta^{mj}\delta^{nl} +$$

$$+ \delta^{il}\delta^{mj}\delta^{nk} - \delta^{il}\delta^{mk}\delta^{nj} = 0 \qquad \text{(A-13)}$$

and so forth.

APPENDIX B: CANCELLATION OF STATIC FORCES

In Section 3.2 we point out that the vector-induced Coulomb potential equals in magnitude the graviton-induced Newtonian static potential. Here, we give a derivation of the classical potentials from the lowest order (κ^2), one particle exchange amplitudes[Ve76]. For simplicity, we study the scattering of a scalar A off a pseudoscalar B, so as to avoid the exchange diagram, but it can be checked that the same result holds in general.

We define the complex fields

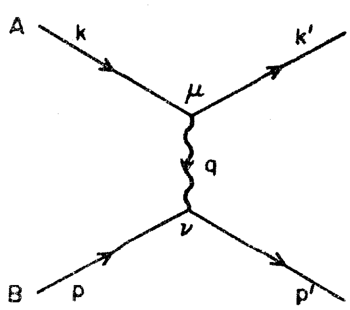
$$A \equiv A^1 - iA^2 \quad \text{and} \quad B \equiv B^1 - iB^2 \ . \tag{B-1}$$

The coupling of these charged fields to the vector is read off from (2.5-1):

$$-\frac{\kappa m}{2} A^\mu \, \varepsilon^{ij} \, Z^i \overleftrightarrow{\partial}_\mu Z^j = -\frac{i\kappa m}{2} A^\mu (A^* \overleftrightarrow{\partial}_\mu A + B^* \overleftrightarrow{\partial}_\mu B) \tag{B-2}$$

and consequently the "electric charge" is κm.

The vector exchange amplitude on the mass shell is then:



$$(2\pi)^4 \, (i\kappa m)^2 \, (k_\mu + k'_\mu) \, \frac{i \, \eta^{\mu\nu}}{q^2 + i\varepsilon} \, (p_\nu + p'_\nu) \tag{B-3}$$

We study the classical behaviour in the static, non-relativistic limit, where all momenta are negligible in comparison to the common mass m in the system. Thus $(k_\mu + k'_\mu)(p^\mu + p^{\mu\prime}) \simeq 4m^2$ and $q^2 \simeq -|\vec{q}|^2$, since $q_o = O(\vec{q}^2)$.

The Breit potential of the interaction is obtained from the three dimensional Fourier transform of the static limit of the amplitude, suitably normalized:

$$4\pi\ V = \frac{1}{(2\pi)^4\ 4im^2} \int d^3q\ \ e^{iq\cdot r}\ \left(\frac{4im^2\ (2\pi)^4 (\kappa m)^2}{|q|^2} \right) = \tag{B-4}$$

$$= \frac{(\kappa m)^2}{r}$$

The potential has a positive sign, since A and B carry the same charge. It would become attractive (minus sign), if we substituted A* for A (or B* for B), as we may see from the coupling (B-2) which is antisymmetric under charge conjugation.

In contrast to the above, all particles couple identically to gravity through their energy-momentum tensor. We see this from the lowest order coupling (κ) of the gravitons to the spinless fields, obtained from the relevant part of our Lagrangian

$$\mathcal{L} = -VR/4\kappa^2 + \frac{V}{2} g^{\mu\nu} \partial_\mu \bar{z}^i \partial_\nu z^i - \frac{Vm^2}{2} \bar{z}^i z^i + \dots \tag{B-5}$$

We linearize the metric around a Minkowskian background:

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} \tag{B-6}$$

We substitute this into (B-5), to read off the kinetic term of the graviton excitation $h_{\mu\nu}$ and and its coupling to the spinless fields. Indices are implicitly raised by the Minkowski metric - not the full $g^{\mu\nu}$:

$$\mathcal{L}_{kin} = -\frac{1}{4}\left[-\frac{1}{2}\partial_\lambda h^\lambda{}_\mu \partial^\mu h_\nu{}^\nu + \frac{1}{2}\partial_\lambda h^\lambda{}_\mu \partial_\nu h^{\nu\mu} - \frac{1}{4}\partial_\lambda h_{\nu\mu}\partial^\lambda h^{\mu\nu} + \frac{1}{4}\partial_\lambda h_\nu{}^\nu \partial^\lambda h_\mu{}^\mu\right]$$

$$= \frac{1}{16}\left[\partial_\lambda h_{\mu\nu}\, \partial^\lambda\!\left(h^{\mu\nu} - \frac{\eta^{\mu\nu}}{2} h_\rho{}^\rho\right) - 2\partial_\lambda \left(h^{\mu\lambda} - \frac{\eta^{\mu\lambda}}{2} h_\rho{}^\rho\right)\partial_\nu \left(h_\mu{}^\nu - \frac{\delta_\mu^\nu}{2} h^\sigma{}_\sigma\right)\right]$$

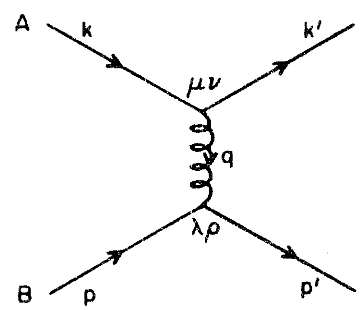
$$\mathcal{L}_I = -\frac{\kappa}{2} h^{\mu\nu}\left[\partial_\mu A^* \partial_\nu A + \partial_\mu B^* \partial_\nu B - \frac{\eta_{\mu\nu}}{2}\left(\partial_\lambda A^* \partial^\lambda A + \partial_\lambda B^* \partial^\lambda B - m^2 A^* A - m^2 B^* B\right)\right]. \tag{B-7}$$

In the harmonic gauge

$$\partial^{\nu} h_{\mu\nu} = -\frac{1}{2}\,\partial_{\mu} h_{\nu}{}^{\nu} \tag{B-8}$$

only $[\,-\frac{1}{4}\,\partial_{\lambda} h_{\mu\nu}\partial^{\lambda}\, h^{\mu\nu} + \frac{1}{8}\partial_{\lambda} h_{\mu}{}^{\mu}\;\; \partial^{\lambda} h_{\nu}{}^{\nu}\,]$ survives in $\mathcal{L}_{kin}$, from which we may read the graviton propagator. We note that, due to general coordinate invariance, all particles couple to gravity through the energy-momentum tensor.

We write down the on-mass shell amplitude for AB scattering via graviton exchange:



(B-9)

$$(2\pi)^4 (i\kappa)^2\,[\,k_{\mu} k_{\nu}' - \frac{\eta_{\mu\nu}}{2}\,k\cdot k' + \frac{\eta_{\mu\nu}\,m^2}{2}\,]\,(-4i)\,\cdot$$
$$\cdot\,[\frac{\eta^{\mu\lambda}\eta^{\nu\rho} + \eta^{\mu\rho}\eta^{\nu\lambda} - \eta^{\mu\nu}\eta^{\lambda\rho}}{q^2 + i\varepsilon}]\;\cdot\,[p_{\lambda} p_{\rho}' - \frac{\eta_{\lambda\rho}}{2}\,p\cdot p' + \frac{\eta_{\lambda\rho} m^2}{2}\,]$$

which has a static limit:

$$-\,\frac{4i\;\;(2\pi)^4\;\kappa^2\;\;m^4}{|\vec{q}|^{\,2} - i\varepsilon} \tag{B-10}$$

This is the negative of the amplitude in (B-4), hence the corresponding Newtonian potential has a minus sign.

Beyond the static limit, the two different forces manifest their separate character in their velocity-dependent (Breit) potentials. In the relativistic limit ($m \to 0$), for example, the graviton exchange amplitude (B-9) goes as $\frac{s^2 + st}{t}$, whereas the vector exchange amplitude (B-3) goes as $m^2(1 + \frac{2s}{t})$, corresponding to D wave and P wave exchanges, respectively. s and t stand for the Mandelstam variables $(k+p)^2$ and $(k'-k)^2$.

APPENDIX C: TRUNCATION OF SO(8) SUPERGRAVITY

As promised in Section 3.3, we give below the systematics of an allowable truncation of the part of SO(8) Supergravity available so far[De77]. This theory, written in the 1st order formalism, contains a graviton $V_{a\mu}$; eight gravitinos ψ^i_μ (i = 1 to 8); 28 vectors A^{ij}_μ (ij antisymmetric); 56 spinors χ^{ijk} (ijk antisymmetric); 35 scalars A^{ijkl} and 35 pseudoscalars B^{ijkl}, antisymmetric, self-dual and anti-self-dual respectively in their internal indices: $A^{ijkl} = -\frac{\varepsilon^{ijklmnpq}}{24} A^{mnpq}$, $B^{ijkl} = \frac{1}{24}\tilde{B}^{ijkl}$.

To lowest order in the spin zero fields, we may restrict the above fields as follows:

$$\begin{array}{ll} A^{12}_\mu \rightarrow A_\mu, & \psi^1_\mu \rightarrow \psi_\mu,\ \psi^2_\mu \rightarrow \phi_\mu, \\ \chi^{345} = \chi^{378} \rightarrow \chi^1, & \chi^{678} = \chi^{645} \rightarrow \chi^2 \\ Z^{1345} = \bar{Z}^{2678} = Z^{1378} = \bar{Z}^{2645} \rightarrow -Z^1 & \\ Z^{1678} = -\bar{Z}^{2345} = Z^{1645} = -\bar{Z}^{2378} \rightarrow -Z^2 & \end{array} \qquad (C-1).$$

We also restrict the transformation laws $\varepsilon^1 \rightarrow \varepsilon$, $\varepsilon^2 \rightarrow \zeta$.
All other fields are set equal to zero - it can be checked that their variations vanish after the restriction given, so that this truncation is a consistent one, and the resulting theory is N = 2 supersymmetric and capable of identification with (2.3-4), (2.4-1).

Note that directions 4,5 are now indistinguishable from 7,8. Directions 1,2 and 3,6 are used to tag the gravity O(2) and the central charge O(2), respectively, in such a way that the self-duality relations in SO(8) bring out the opposite relative senses of rotation between our scalars $A^{1,2}$ and our pseudoscalars $B^{1,2}$ under the gravity O(2).

Upon the restriction proposed above, the only terms surviving in the SO(8) Lagrangian and transformation laws are of the type to be found in (2.3-4), (2.4-1), modulo conventions and field normalizations. Conversely, all terms of (2.3-4), (2.4-1) are found among the lot of these survivors - note that some of them are hidden in the torsion of the first order formalism of the SO(8) truncation. Remarkably, for instance, the Pauli moment term emerges out of $\kappa\varepsilon^{ijklmnpq}\,\bar{\chi}^{ijk}\sigma^{\mu\nu}\chi^{lmn}F^{pq}_{\mu\nu}$ in the SO(8) theory.

This version of the SO(8) theory has not been worked out to higher orders sufficient for specifying the nonpolynomial behavior of the spinless fields. As a consequence, the fields of the proposed truncation can always be scaled by functions of the group singlet u so as to coincide with the corresponding first few orders of (2.2-9), (2.3-1). For example, a simple rescaling $Z^i \to Z^i(1 - u/6 + \ldots)$ transforms the $[\bar{Z}Z\partial\partial + \kappa^2\bar{Z}\bar{Z}ZZ\partial\partial]$ terms of the truncated theory to those of (2.3-1). Similarly, if a few terms of higher order were worked out, they could be absorbed into redefinitions of the spinors $\chi^i \to \chi^i f(u)$, without really providing essential information on the nonpolynomial structure.

APPENDIX D: VACUUM TUNNELING

We outline a rough estimate of the decay rate for the metastable ground state introduced in Section 3.4, which represents a temporary, false vacuum. After this state penetrates through the barrier of Fig. 3.4-2, it is no longer the ground state of the theory (and in fact the theory is meaningless). The WKB approximation for barrier penetration may be generalized to a system with an infinite number of degrees of freedom, thus yielding a decay rate which is the imaginary part of the energy of the configuration under study[Co77, Ca77]. For a field theory with a scalar field ϕ and no derivative interactions ($S = \int d^4x\ (\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\ - U(\phi)\ )$, the tunneling rate per unit (three) volume Γ/V of the "false" ground state in a potential U has been shown to be:

$$\frac{\Gamma}{V} = A\ e^{-B} \qquad \text{(D -1)}$$

B is the minimized action of the associated Euclidean variational problem, with the sign of the potential of the original problem reversed, and suitable boundary conditions. This expression arises from the dominant (stationary) contributions to the functional integral $\int[d\phi]\ e^{-S[\phi]}$ governing the transition amplitude sought. If the problem Possesses (hyper)spherical symmetry in the Euclidean variables ($\rho^2 = \vec{x}^2 - (ix_4)^2$), it may be written in a simple form:

$$B = 2\pi^2 \int_0^\infty d\rho\quad \rho^3 [\frac{1}{2}(\frac{d\phi}{d\rho})^2 + U] \qquad \text{(D- 2)}$$

$$\left.\frac{d\phi}{d\rho}\right|_0 = 0 \quad \text{and } \phi(\rho=\infty) = \text{(its value at the false vacuum)}$$

The solution of the associated problem(D-2) called "the bounce" (due to an analogy from particle mechanics), and the trivial solution $\phi = 0$ is not acceptable, because it turns out to contribute no imaginary part to the energy at all. V has its origin in the integration over all locations of the contributing solutions. A results from the functional determinants of the semiclassical correction (Gaussians) to the action and the normalizations in configuration space. It is generally a complicated expression with the dimensions of $(\text{energy})^4$, but it is related to B^2 times the characteristic energy scale of the particular problem.

Inserting our expressions (3.4 - 4), (3.4 - 5) into (D - 2), we obtain.

$$B = 2\pi^2 \int_0^\infty d\rho \;\; \rho^3 [\frac{1}{4\kappa^2} (\frac{dw}{d\rho})^2 - \frac{m^2}{32\kappa^2}(e^{2w}+e^{-2w}-12e^{w}-12e^{-w}+22)$$

$$w(\rho = \infty) = 0 \qquad \text{and} \qquad \left.\frac{dw}{d\rho}\right|_0 = 0 \qquad\qquad (D - 3)$$

We may define $t \equiv \frac{64}{m^2\rho^2}$, to get:

$$B = \frac{64\pi^2}{\kappa^2 m^2} \int_0^\infty dt \;\; [(\frac{dw}{dt})^2 - \frac{2}{t^3} (e^{2w} + e^{-2w} - 12e^{w} - 12e^{-w} + 22)]$$

$$w(t = 0) \qquad \text{and} \; \frac{dw}{dt} \; (t=\infty) = 0 \qquad\qquad (D - 4)$$

The minimization of (D-4) is a highly non-linear, irregular singular, second order boundary value problem. We have not been able to find a solution. Nevertheless, we may observe that if the minimum value for the integral is nonzero, then Γ is controlled by the exponential e^{-B} and it is negligible for all conventional values of $m << 1/\kappa$. For instance, $m \sim 1$ GeV yields $B \sim 10^{40} \cdot 6.4 \cdot$ (minimum value of integral). The probability of decay of our metastable vacuum since the beginning of the universe (10^{10} years ago $\sim 10^{41}$ f), and for a factor $A \sim B^2/f^4$ is roughly:

$$P \sim 10^{164} \; B^2 \; e^{-B} \sim 0 \qquad\qquad (D - 5)$$

and thus the false vacuum is effectively stable.

This might be suggestive of the actual situation prevailing in the full-fledged theory with all four spinless fields and gravity. Superficially, we would expect the extra spinless degrees of freedom to introduce numerical factor modifications to the above, without altering its basic form. Furthermore, gravity enters through R/κ^2, so that it can absorb the same rescalings as the spinless fields and conform to the above conclusions, if it can be carelessly introduced into the associated Euclidean problem. In principle, we should include loop corrections to the effective potential[Ca77] for the next order in $\hbar$, as well as renormalized quantities and the appropriate counterterms to the action - refinements beyond the reach of our non-renormalizable theory.

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