MUON CAPTURE IN THE SHELL MODEL

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ABSTRACT

An effective weak interaction Hamiltonian due to Primakoff has been used in conjunction with the shell model of the nucleus to predict μ meson capture rates to the bound states of O^{16}, F^{19} and He^{3}. All three of these nuclei have a slow inverse β decay which should permit detection of μ capture by an experiment which is easier than that necessary when the inverse β decay is fast. The O^{16} and He^{3} capture rates are predictable with sufficient confidence to constitute a test for the presence of the induced pseudoscalar interaction in μ capture, but are not sufficiently sensitive to the presence of the conserved vector current in the weak interaction to decide on its existence. The F^{19} predictions are interesting from the point of view of nuclear structure, in that they are an analogue of "allowed but unfavored" transitions in β decay theory, and the capture rate is reduced due to the different space symmetries of the states involved. Central values of the predicted capture rates to bound states are, in O^{16} $\Gamma c = 20 \times 10^{3}$ sec^{-1}; in He^{3} $\Gamma c = 1250$ sec^{-1}; and in F^{19} $\Gamma c = 1350$ sec^{-1}.

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INTRODUCTION

In the last two years experimental and theoretical advances have stimulated an intense program of research into the problem of the capture of μ-mesons by nuclei. The production of strong μ-meson beams has made possible increased accuracy and scope of experiments, while the establishment of the V-A weak interaction at low momentum transfers has provided a firm foundation for the study of the weak interaction at the high momentum transfers involved in μ-meson capture. As is well known, μ-mesons stopped in matter are captured into Bohr orbits with $n, \ell \sim 15$ from which they cascade down by X-ray and Auger processes to the 1S state in a time of the order of 10^{-10} seconds, which is short compared to the lifetime of the μ meson against capture or decay by the weak interaction. Therefore essentially all μ-mesons stopped in matter decay or are captured by nuclear protons from a 1S Bohr orbit. Ideally one would study the simplest system, μ capture by a proton, to establish the structure of the interaction. For a number of reasons this experiment does not possess the advantages it would appear to. In the first place the capture rate by a single proton is very small and the observation of recoil neutrons is subject to large statistical errors. Also, the system of μ plus a proton is just not simple. Many complicated atomic and molecular processes occur. For example, the (μ-p) system being neutral can penetrate neighboring hydrogen atoms and form (p-μ-p) molecules, changing the capture rate for two reasons. First, the amplitude of the μ wave function at the proton is different in the μ-molecule and the μ-atom, and second, the (μ-p) spin correlation, which has a dramatic effect on the capture rate, is changed by the formation of the molecule. (For a discussion of the interpretation of μ capture rates in liquid hydrogen, see a recent article by Weinberg (1).)

For these and other reasons, it has been the practice to study μ capture in complex nuclei, usually with Z greater than 6. The problem of statistics is alleviated, and the (μ-nucleus) no longer forms a neutral system free to wander about and undergo complicated molecular processes. Basically two experiments have been performed, the total capture rate and the partial capture rate to particular final states.

The total capture rate to all final states has been analyzed by Überall (2) using a Fermi gas model for the nucleus and in a more detailed fashion by Primakoff (3) using a closure approximation to sum the rate over all final nuclear states. This last calculation correctly reproduces the capture rate, the Z^4 dependence and the exclusion principle inhibition of the capture rate due to the fact that the phase space of the capturing nucleon is reduced compared to that for a free nucleon. However, Primakoff's calculation depends on a nucleon-nucleon correlation length within the nucleus, the average neutrino energy, and the proton charge distribution within the nucleus, so that although the overall agreement with experiment is good the uncertainties in the calculation are such as to mask any detailed structure of the interaction.

The experiment of μ meson capture to a particular final state was first done for C^{12}-B^{12}, ground state to ground state transition, by Godfrey (4) and has since been repeated and analyzed by others (5). In this experiment the β decay of the B^{12} ground state ($\tau_{\frac{1}{2}}$ = 20 milliseconds) indicates the μ meson capture to a bound state of B^{12}. There are a number of experimental and theoretical uncertainties involved. The efficiency of counting decay electrons is difficult to establish; decay electrons are easily confused with electrons from the decay of bound μ mesons and with mesonic X-rays. Theoretically it is not clear that only the ground state to ground state transition occurs. There are five other bound states in B^{12} which have a rather obscure structure. It is estimated that approximately ten percent of the transitions go to these higher bound states. The ground state to ground state capture rate can be closely related to the inverse β decay rate, so that a lot of the theoretical uncertainty here can be eliminated. Even so, the experimental uncertainties are large enough to prevent any detailed determination of the interaction. Another attempt to calculate a partial capture rate has been made by Überall (6) for the case of Li^6 to He^6. Unfortunately this system possesses difficulties preventing a good determination of the capture rate. The nuclear radius of Li^6 is so ill-defined as to make the calculation of matrix elements very uncertain, and the capture rate by a nucleus with $Z = 3$ is small. This system possesses the one advantage that He^6 has only one bound state, and the structure of the $A = 6$ system is fairly well understood on the shell model.

An examination of the energy levels of light nuclei (7) reveals that there are two other nuclei for which the relevant bound states are well understood on the shell model, O^{16} and F^{19}. Experimentally these systems should possess a distinct advantage for measuring a partial capture rate. First, they are $Z = 8, 9$ nuclei so that the capture rate should be fairly large. Also, the inverse β decay is slow ($\tau_{\frac{1}{2}}$ for $N^{16} = 7.4$ sec; $\tau_{\frac{1}{2}}$ for $O^{19} = 29$ sec) so that a new type of experiment becomes possible. It is no longer necessary to observe the inverse β decay in the presence of the μ beam with the attendant problems of separating β decay electrons, μ decay electrons and mesonic X-rays, or of determining electron counting efficiencies. Instead, the target (liquid oxygen or fluorine) can be continuously irradiated by the μ beam, and the induced characteristic radioactivity of the gas boiled off measured to determine the capture rate.

On the theoretical side, we shall see that Gamow-Teller type transitions dominate these reactions. The Gamow-Teller coupling constant in the effective interaction Hamiltonian is sensitive to the nature of any induced pseudoscalar interaction, but not very sensitive to the hypothesis we make concerning a conserved vector current so we expect these rates to be able to distinguish various hypotheses concerning the induced pseudoscalar interaction only.

A third example, μ capture by He^3, will also be discussed. This case has been discussed previously and essentially correctly. However, in addition to providing a unified discussion of the problem we will point out an error in the previous discussion. In addition a mechanism not previously discussed for a hyperfine structure effect will be pointed out here.

The μ Capture Hamiltonian

We shall here discuss the interaction Hamiltonian which is constructed (8) to describe the capture of μ mesons by nuclei. The usual weak interaction is expected to be modified by the presence of strong interactions and by the high momentum transfer involved in μ capture.

In the Fermi interaction responsible for μ decay

$$\mu \rightarrow e + \nu + \bar{\nu}$$

the V-A theory of Feynman and Gell-Mann (9) has become established. The interaction Hamiltonian responsible for μ decay is then

$$\mathcal{H} = \sqrt{8}\, G \left\{ \bar{u}_{\mu} \gamma_{\alpha} a\, u_{\nu} \right\}^{+} \left\{ \bar{u}_{e} \gamma_{\alpha} a\, u_{\nu} \right\} + \mathrm{H.C.} \qquad (1)$$

Here H.C. is the Hermitian conjugate,

$$a = \frac{1 + \gamma_5}{2}$$

and G, the weak interaction coupling constant evaluated from the μ decay rate, is $1.01 \times 10^{-5}\, M_p^{-2}$, with M_p the proton mass.

On the hypothesis of a universal Fermi interaction, the same Hamiltonian describes the weak interaction between any four fermions. One has just to replace μ, e, ν, ν by the relevant particles. The hypothesis of universality dictates that the coupling constant is still G and the interaction is still V-A. However, if we admit strong interactions (which do not affect the μ decay process), the virtual pion effects must be expected to alter the effective interaction. In the particular case of nuclear β decay the interaction is known to be modified to an effective interaction (actually an S matrix element in lowest order in the weak interaction)

$$\mathcal{H}_{\beta} = G (\bar{u}_e \gamma_{\alpha} a\, u_{\nu})\, \bar{u}_p \tau^{+} \left\{ g_V \gamma_{\alpha} - g_A \gamma_{\alpha} \gamma_5 \right\} u_N + \mathrm{H.C.} \qquad (2)$$

with g_V evaluated from the β decay of O^{14} = 1.0, or, on the hypothesis of a conserved vector current in the Fermi interaction Lagrangian, exactly equal to one; and g_A evaluated from the β decay of the neutron equal to -1.23.

This β decay interaction is sufficient to describe the situation in the limit of zero momentum transfer. If we think of the virtual pion effects as being expanded in a power series in k/M_N, where k is the momentum transfer in the process and M_N the nucleon mass, then this effective interaction contains renormalization effects of zero order in the momentum transfer, and has an error of the order $k/M_N \approx 1\%$ in a nuclear β decay with a maximum β energy of 10 Mev. In a Fermi interaction involving high momentum transfer, in particular in μ meson capture, the momentum transfer is of the order of the μ meson mass and k/M_N = 10% approximately, and we must seek to keep at least first order corrections in k/M_N to the interaction Hamiltonian. (In fact, we will test the hypothesis that these renormalization terms enter with anomalously large coefficients.)

To this end, consider the most general interaction Hamiltonian which

a) is Lorentz covariant

b) is time reversal invariant

c) is G invariant (i.e. charge independent)

d) reduces to (Eq. 1) in the absence of strong interactions

and e) reduces to (Eq. 2) in the presence of strong interactions at zero momentum transfer.

The particular form relevant to the μ capture process is (10)

$$\mathcal{H} = G\,(\bar{U}_\nu \gamma_\alpha a\, U_\mu)\bar{U}_N \tau^- \Big\{ C_\mu(k^2)\gamma_\alpha + A_\mu(k^2)\gamma_\alpha \gamma_5 + B_\mu(k^2)\frac{k_\alpha}{m_\mu}\gamma_5 + D_\mu(k^2)\frac{k_\rho \sigma_{\alpha\rho}}{i M_p}\Big\} U_p \quad (3)$$

$$+ \text{H.C.}$$

here k_α, the momentum transfer in the interaction, $= p_\alpha - n_\alpha = v_\alpha - \mu_\alpha$ in an obvious notation. Requirements (d) and (e), above, on the effective interaction, specify $C_\mu(0) = 1.0$, $A_\mu(0) = 1.23$. The other two terms do not contribute at low momentum transfer. Consider these terms individually. First the term

$$(\bar{U}_\nu \gamma_\alpha a\, U_\mu)\, B_\mu(k^2)\,\frac{k_\alpha}{m_\mu}\,\gamma_5$$

becomes, when we use the Dirac equation $(\not{P} - m)U(P) = 0$ to eliminate the lepton momenta,

$$-(\bar{U}_\nu \bar{a}\, u_\mu)\, B_\mu(k^2)\,\gamma_5 \quad (4)$$

with $\bar{a} = \frac{1-\gamma_5}{2}$. This is explicity an induced nuclear-pseudoscalar term. Next rewrite the term $C_\mu(k^2)\gamma_\alpha$ in a form separating the spin operator σ. ($D_\mu(k^2)$ will eventually be a renormalization to this spin matrix element proportional to $(\mu_p - \mu_N)$ for the conserved vector current hypothesis.) Use the identity

$$\sigma_{\alpha\rho} k_\rho = -1/2i\,(\gamma_\alpha \gamma_\rho - \gamma_\rho \gamma_\alpha)(p_\rho - n_\rho)$$

or

$$-2i\,\sigma_{\alpha\rho} k_\rho = \gamma_\alpha \not{p} + \not{n} \gamma_\alpha - \not{p} \gamma_\alpha - \gamma_\alpha \not{n} .$$

Now using the free particle Dirac equation for the nucleons, with a free particle proton spinor understood on the right and a neutron spinor on the left, this becomes

$$= 2 M_p \gamma_\alpha - 2 p_\alpha - 2 n_\alpha + \gamma_\alpha \not{p} + \not{n} \gamma_\alpha$$

$$= 4 M_p \gamma_\alpha + 2 k_\alpha - 4 p_\alpha .$$

(The use of a free particle spinor to describe nucleons in a nucleus introduces no error in first order in k/M_p as can be seen from a Foldy-Wouthoyson transformation.) So we have the identity

$$\gamma_\alpha = p_\alpha / M_p - k_\alpha / 2M_p + \frac{\sigma_{\alpha\rho} k_\rho}{2 M_p i} . \tag{5}$$

With these we can rewrite the general interaction Hamiltonian (Eq. 3) as

$$\begin{aligned} \mathcal{H} = \; & G\Big\{ A_\mu(k^2)\,(\bar{u}_\nu \gamma_\alpha a\, u_\mu)(\bar{u}_N \tau^- \gamma_\alpha \gamma_5 u_p) \\ & - B_\mu(k^2)\,(\bar{u}_\nu \bar{a}\, u_\mu)(\bar{u}_N \tau^- \gamma_5 u_p) \\ & + C_\mu(k^2)\,(\bar{u}_\nu \gamma_\alpha a\, u_\mu)(\bar{u}_N \tau^- p_\alpha M_p u_p) \\ & - C_\mu(k^2)\,(\bar{u}_\nu \frac{\gamma_\alpha k_\alpha}{2 M_p} a\, u_\mu)(\bar{u}_N \tau^- u_p) \\ & + \frac{1}{i}\Big(\frac{C_\mu(k^2)}{2} + D_\mu(k^2)\Big)(\bar{u}_\nu \gamma_\alpha a\, u_\mu)(\bar{u}_N \tau^- \frac{\sigma_{\alpha\rho} k_\rho}{M_p} u_p)\Big\} \\ & + \text{H.C.} \end{aligned} \tag{6}$$

In this form the induced couplings $B_\mu(k^2)$ and $D_\mu(k^2)$ appear explicitly as an induced pseudoscalar interaction and as a renormalization to the vector interaction respectively.

Now to evaluate $D_\mu(k^2)$. Notice (11) in (Eq. 6) that

$$C_\mu(k^2)\,\bar{U}_N \tau^- \gamma_\lambda U_N - i D_\mu(k^2)\,\bar{U}_N \tau^- \frac{\sigma_{\lambda\rho} k_\rho}{M_p}\, U_N$$

is the (-1) component of an isovector whose (0) component is similar in structure to the isovector part of the electromagnetic current

$$\mathcal{M}_{EM} = e\bar{U}_N \gamma_\lambda \frac{(1+\tau_z)}{2} U_N - \frac{ie}{2M_p}\bar{U}_N \sigma_{\lambda\rho} k_\rho \left(\frac{\mu_P + \mu_N}{2} + \frac{\mu_P - \mu_N}{2}\tau_z\right) U_N \tag{7}$$

where μ_P and μ_N are the anomalous magnetic moments, in units of one nuclear magneton, of the proton and neutron. On the conserved vector current hypothesis, the weak interaction is the self-interaction of an isovector current, the vector part of which is divergence free, and therefore presumably has the same structure as the isovector part of the electromagnetic current which is known to have zero divergence. Then the weak interaction vector current and the isovector part of the electromagnetic current are two components of the same isovector and should suffer the same renormalization by the strong interactions. If this is the case

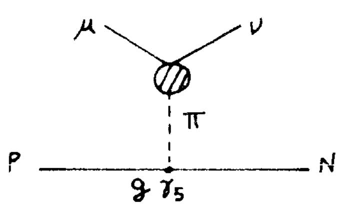
$C_\mu(k^2) : D_\mu(k^2)$ as

$e/2 \; : \; e/2\,\frac{\mu_P - \mu_N}{2}$ so

$$D_\mu(k^2) = \frac{\mu_P - \mu_N}{2} \quad C_\mu(k^2)\,. \tag{8}$$

In this derivation we have neglected the difference at $k^2 \approx m_\mu^2$ between the various electromagnetic form factors. We will compare the predictions of the conserved vector current hypothesis (hereafter referred to as CVC) with the alternative hypothesis that $D_\mu(k^2) = 0$, which is essentially the result obtained (10) with no pion current in the weak vector current.

To estimate the magnitude of the induced pseudoscalar coupling, consider (12) the simplest diagram which could give rise to such an interaction, the decay of a single virtual pion into a μ meson and a neutrino. We can evaluate the effective pion-lepton interaction from the $(\pi \to \mu\nu)$ decay rate with an effective interaction



$$\mathcal{H}_{\pi\mu\nu} = g_{\pi\mu\nu}\,(\bar{U}_\nu \gamma_\alpha a\, U_\mu)\,\frac{P_\alpha}{m_\mu}\,\Pi \tag{9}$$

where $g_{\pi\mu\nu}$ is an effective coupling constant

P_α is the pion momentum $= \nu_\alpha + \mu_\alpha$

Π is the pion wave function .

Using the Dirac equation, $\not{\nu}\, U_\nu = 0$, and $\not{\mu}\, U_\mu = m_\mu U_\mu$ this reduces to

$$\mathcal{H}_{\pi\mu\nu} = g_{\pi\mu\nu}\,(\bar{U}_\nu\, \bar{a}\, U_\mu)\,\Pi\,. \tag{9a}$$

Then the pion decay rate is

$$\Gamma_{\pi\mu\nu} = g^2_{\pi\mu\nu} \int \frac{d^3\mu}{(2\pi)^3} \frac{d^3\nu}{(2\pi)^3} \frac{(2\pi)^4 \, \delta(\mu+\nu-\pi)}{2\, m_\pi} \overline{\mathcal{M}^2}$$

$$= \frac{g^2_{\pi\mu\nu} \, \nu^2 \, \overline{\mathcal{M}^2}}{2\pi \, m_\pi} \quad . \tag{10}$$

Here $\nu = \frac{m_\pi}{2}(1 - x^2)$ is the neutrino momentum for a pion at rest, $x = m_\mu / m_\pi$, and $\overline{\mathcal{M}^2} = 1/2$ so

$$\Gamma_{\pi\mu\nu} = \frac{m_\pi \, g^2_{\pi\mu\nu} \, (1 - x^2)^2}{16\,\pi} \quad . \tag{10a}$$

The total diagram, representing capture of a μ meson by a nucleon through an effective pseudoscalar interaction is

$$g_{\pi\mu\nu} \, g \, (\bar{u}_N \, \tau^- \gamma_5 \, u_p) \, \frac{1}{k^2 + m_\pi^2} \, \frac{k_\kappa}{m_\pi} \, (\bar{u}_\nu \, \gamma_a \, a \, u_\mu) \, .$$

Comparing this with the μ capture effective Hamiltonian, we see that

$$B_\mu^2 (k^2 = m^2) = \frac{g^2 \, g^2_{\pi\mu\nu}}{G^2} \quad \frac{1}{m_\pi^4 (1 + x^2)^2}$$

where we have set $k^2 = m_\mu^2$. In terms of the pion lifetime (Eq. 10a) and the μ lifetime $\tau_\mu^{-1} = \frac{G^2 m_\mu^5}{192\pi^3}$ we get

$$B_\mu^2 (k^2 = m_\mu^2) = \frac{g^2}{4\pi} \quad \frac{1}{3\,\pi} \quad \frac{\tau_\mu}{\tau_\pi} \quad \frac{x^5}{(1 - x^4)^2} \quad . \tag{11}$$

Using τ_μ / τ_π = 87, x = 0.770 and replacing the unrenormalized pion-nucleon coupling constant $g^2/4\pi$ by the renormalized one $G^2/4\pi = 15$, we find $B_\mu(k^2 = m_\pi^2) = 9.85$. This simple argument can of course tell us nothing about the sign of $B_\mu(k^2)$. A dispersion theory argument (13), in which only the nucleon-antinucleon intermediate state in pion decay is retained, yields essentially the above magnitude and predicts a negative sign for $B_\mu(k^2 = m_\mu^2)$. Very general arguments (14) which depend on the assumption that the contribution of the one pion pole at $k^2 = -m_\pi^2$ dominates the matrix element of the axial vector current $\langle p | \mathbb{P}_a | n \rangle$ even at $k^2 = m_\mu^2$ agree with the prediction of a negative sign for $B_\mu(k^2 = m_\mu^2)$. In order to determine the sensitivity of the μ capture rates to the assumptions made concerning the induced pseudo-scalar coupling constant, we will test the hypotheses that

$$B_\mu(k^2 = m_\mu^2) = \pm 8\, g_A \quad \text{or} \quad 0 .$$

So far we have said nothing about the momentum dependence of $A_\mu(k^2)$, $C_\mu(k^2)$ and $D_\mu(k^2)$. The same dispersion theory arguments mentioned above indicate that these are very slowly varying functions of momentum transfer, and in fact differ by only one or two percent at $k^2 = m_\mu^2$ from their values at zero momentum transfer. We will ignore this slight momentum dependence and replace them by their values at $k^2 = 0$, and reduce the final rates accordingly.

Before we proceed to consider specific reactions, let us make a non-relativistic approximation on the nucleons in the interaction Hamiltonian. Also, since the μ meson taking part in the capture reaction is essentially at rest in the lowest Bohr orbit, we can treat the μ non-relativistically. For convenience we will put the neutrino wave function in two component form. Then make the substitutions (which are correct to order p/M even for nucleons bound in the nucleus):

$$u_P \rightarrow \left(1 - \frac{\vec{\gamma} \cdot \vec{P}}{2M_p}\right) w_P$$

$$\overline{u}_N \rightarrow w_N^+ \left(1 - \frac{\vec{\gamma} \cdot \vec{n}}{2M_p}\right)$$

$$a\, u_\nu = \frac{(1 - \sigma \cdot \hat{\nu})}{2\sqrt{2}} e^{i v \cdot x} w_\nu \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$u_\mu = \phi(x)\, w_\mu \begin{pmatrix} 1 \\ 0 \end{pmatrix} .$$

Here u is a relativistic spinor

w is a two component Pauli spinor

M_p is the nucleon mass

p, n, ν, μ designate the particles or their three-momentum whichever is relevant

$\hat{\nu}$ is a unit vector in the direction of the neutrino momentum ν

$\phi(x)$ is the μ meson wave function, reduced (5) from its nominal value by the finite nuclear size effect

$\vec{\nu} \simeq \vec{k}$, the momentum transfer, $= \vec{p} - \vec{n}$.

Using these substitutions and keeping only first order in the various momenta, the nuclear matrix elements occurring in $\mathcal{H}$ become

a) $\bar{U}_N \tau^- \gamma_o \gamma_5 U_p \rightarrow w_N^+ \tau^- \sigma_N \cdot \hat{\nu} \; w_p \left(\frac{\nu}{2 M_p}\right)$

$$- w_N^+ \tau^- \frac{\sigma_N \cdot \vec{P}}{M_p} \; w_p$$

b) $\bar{U}_N \tau^- \gamma_b \gamma_5 U_p \rightarrow - w_N^+ \tau^- \sigma_N^b \; w_p \qquad (b = 1, 2, 3)$

c) $\bar{U}_N \tau^- \gamma_5 U_p \rightarrow -\left(\frac{\nu}{2M_p}\right) w_N^+ \tau^- \sigma_N \cdot \hat{\nu} \; w_p$

d) $\bar{U}_N \tau^- p_o / M_p \, U_p \rightarrow w_N^+ \tau^- w_p$

e) $\bar{U}_N \tau^- p_\mu / M_p \, U_p \rightarrow w_N^+ \tau^- \frac{p_\mu}{M_p} \; w_p \qquad (\mu = 1, 2, 3)$

f) $\bar{U}_N \tau^- U_p \rightarrow w_N^+ \tau^- w_p$

g) $\bar{U}_N \tau^- \frac{\sigma_{a\rho} k_\rho}{M_p} U_p \rightarrow \left(\frac{\nu}{M_p}\right) w_N^+ \tau^- \epsilon^{a \rho \mu} \hat{\nu}_\rho \sigma_N^\mu w_p$

here $\epsilon^{a\rho\mu}$ is the totally antisymmetric tensor, $(\alpha \rho \mu = 1, 2, 3)$

σ_N is the nucleon spin

σ_L the lepton spin .

Now since $(1 - \sigma_L \cdot \hat{\nu}) = (-\sigma_L \cdot \hat{\nu})(1 - \sigma_L \cdot \hat{\nu})$ we can insert $(-\sigma_L \cdot \hat{\nu})$ for later convenience in separating Fermi and Gamow-Teller terms and bring the interaction Hamiltonian into the form (8)

$$\mathcal{H} = G\,\phi_\mu(x)\,e^{-i\nu\cdot x}\,w_\nu^+\,\frac{(1-\sigma_L\cdot\hat{\nu})}{2\sqrt{2}}\quad w_N^+\,\tau^-\Lambda\,w_\mu\,w_p \tag{12}$$

where

$$\begin{aligned}\Lambda \;=\;& C_\mu(k^2)\,(1+\nu/2M_p) - C_\mu(k^2)\;\sigma_L\cdot\hat{\nu}\;\;\sigma_L\cdot\frac{\vec{P}}{M_p}\\ &+\sigma_L\cdot\sigma_N\,\Big(-A_\mu(k^2) - \frac{\nu}{2M_p}\Big\{C_\mu(k^2)+2D_\mu(k^2)\Big\}\Big) \qquad (13)\\ &+A_\mu(k^2)\;\;\sigma_L\cdot\hat{\nu}\;\;\sigma_N\cdot\frac{\vec{P}}{M}\\ &-\frac{\nu}{2M_p}\;\;\sigma_L\cdot\hat{\nu}\;\;\sigma_N\cdot\hat{\nu}\;\;\Big(B_\mu(k^2)+A_\mu(k^2)-\Big\{C_\mu(k^2)+2D_\mu(k^2)\Big\}\Big)\,.\end{aligned}$$

The various coupling constants we will employ are

$$C_\mu(k^2) = C_\mu(0) = g_\nu = 1$$

$$A_\mu(k^2) = A_\mu(0) = -g_A = 1.23$$

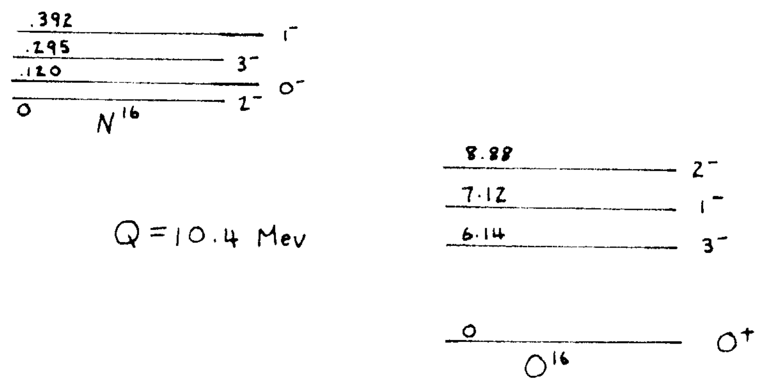
$$B_\mu(k^2) = \pm\,8\,g_A \quad \text{or} \quad 0$$

$$2D_\mu(k^2) = \mu_p - \mu_N \quad \text{or } 0\,.$$

We must now try to find reactions sensitive to the detailed structure of this interaction Hamiltonian. Notice in the last term, that $8\nu/2M_p \approx 1$, so that for a Gamow-Teller type transition, the induced pseudoscalar term can make a contribution of the same order of magnitude as the original Gamow-Teller term. We will indeed find that such transitions are sensitive to the value of $B_\mu(k^2)$. We now proceed to a discussion of some specific reactions.

O^{16} Preliminary Discussion

All four bound states of N^{16} are understood on the shell model (15) as $(1p_{1/2,3/2})$ holes inside the O^{16} core coupled to $(1d_{3/2,5/2}, 2s_{1/2})$ particles outside it to give the $T = 1$, $J = 0^-, 1^-, 2^-, 3^-$ assignments characterizing these states.



Shell Model Levels for A = 16 Nuclei

Consider the μ capture process from the $T = 0$, $J = 0^+$ O^{16} groundstate to these bound states of N^{16}. For a capturing nucleus with zero angular momentum there is of course no worry about the hyperfine structure effect. Then for the

a) $T = 0$, $J = 0^+$ to $T = 1$, $J = 2^-$ transition, which is the groundstate to groundstate transition, the Gamow-Teller matrix element $(\sigma e^{i v \cdot r})$ with $\ell = 1$, $j = 2$ is expected to dominate the rate. The Fermi transition is strictly forbidden. The dominant Gamow-Teller matrix element can be obtained from the inverse β decay so this rate can be determined almost independent of the nuclear model.

b) $T = 0$, $J = 0^+$ to $T = 1$, $J = 0^-$ transition, again the Fermi matrix element is zero and the Gamow-Teller matrix element with $\ell = 1$, $j = 0$ is expected to dominate. This transition should be especially sensitive to assumptions concerning the induced pseudoscalar coupling in the weak interaction.

c) $T = 0$, $J = 0^+$ to $T = 1$, $J = 1^-$ transition, we expect a mixed Fermi and Gamow-Teller transition, each with $\ell = 1$ and $j = 1$. _A priori_, we would expect these matrix elements to be comparable and of the same order of magnitude as the Gamow-Teller matrix elements in the preceding two transitions.

d) $T = 0$, $J = 0^+$ to $T = 1$, $J = 3^-$ transition, which has matrix elements with $\ell = 3$ only, we expect a small contribution to the rate. Even at the large momentum transfer involved in μ capture, radial matrix elements with $\ell = 3$ are very small.

The total rate, then, should be dominated by three Gamow-Teller matrix elements with $\ell = 1$, $j = 0, 1, 2$. The predominantly Gamow-Teller transitions should be sensitive to the assumptions made about the induced pseudoscalar term, and we would hope that the presence of an 0^+ to 0^- transition would make the capture rate particularly sensitive to the presence of a pseudoscalar interaction.

Nuclear Physics

We require the shell model description of the N^{16} bound states in order to calculate the matrix elements entering the μ capture rate. These results are available in the work of Elliott and Flowers (15) but it was decided to repeat the calculations in order to vary the parameters of the nuclear force over a wide range to determine the model sensitivity of the results. The previous results indicate that these states are very close (within ten percent) of the j-j state determined by the $\vec{l}\cdot\vec{s}$ splitting, so the j-j coupling scheme seems to be the natural description of this system and is the one we will use. In addition to describing the bound states of N^{16} we will also test our model against the $T = 0$, odd parity states of O^{16}. (A difficulty arises here in the shell model description of the $T = 0$, $J = 1^-$ state due to center of mass motion, which must be removed. This amounts to a restriction reducing the number of basis states by one and has been included in the previous work. We will ignore this state in our work.)

In the shell model, the degenerate $(1p_{1/2,3/2})^{-1}(1d_{3/2,5/2}, 2s_{1/2})$ configurations of a hole and a particle in a central harmonic oscillator potential are the basis states in which we expand our physical states. The degeneracy is removed and the configuration mixing accomplished by the spin-orbit interaction and the hole-particle interaction. The lowest lying eigenstates of this total Hamiltonian are to be identified with the physical states.

a) The Nuclear Hamiltonian

The spin-orbit interaction in the d shell is determined from the $1d_{5/2} - 1d_{3/2}$ splitting in O^{17} which is 5.08 Mev (in F^{17} it is 4.7 Mev but less well established), so $V_{LS} = \zeta_L \vec{l} \cdot \vec{s}$ with $\zeta_2 = -2.03$ Mev. In the p shell the spin-orbit potential is determined from the $1p_{1/2} - 1p_{3/2}$ splitting of 6.33 Mev in N^{15} (or 6.15 Mev in O^{15}) to be $\zeta_1 = -4.22$ Mev (or -4.10 Mev). In the harmonic oscillator model we will use, the $2s_{1/2}$ level would be degenerate with the $1d_{5/2}$ level in the absence of a spin-orbit interaction, or 2.03 Mev above it with the above $\vec{l} \cdot \vec{s}$ term. Experimentally the splitting in O^{17} is 0.871 Mev (in F^{17} it is 0.50 Mev), so to fit the O^{17} level structure we require a 1d-1s splitting $\Delta_{ds} = 1.16$ Mev.

The simplest assumption to make concerning the two nucleon interaction is that it is a Rosenfeld force with a Yukawa shape and no tensor interaction,

$$V = V_c \frac{\tau_1 \cdot \tau_2}{3} (0.3 + 0.7\, \sigma_1 \cdot \sigma_2) \frac{e^{-m_\pi r}}{m_\pi r} \tag{14}$$

A strength $V_c \simeq 40$ Mev fits the experimental situation in the A = 16 to 19 region. We will in fact consider a more general exchange mixture (15,16)

$$V = V_c \frac{e^{-m_\pi r}}{m_\pi r} \left(W + M P_x - H P_\tau + B P_\sigma \right) \qquad (14a)$$

where
$$P_\tau = \tfrac{1}{2}(1 + \tau_1 \cdot \tau_2) = - P_H$$
$$P_\sigma = \tfrac{1}{2}(1 + \sigma_1 \cdot \sigma_2)$$
$$P_x = - P_\tau P_\sigma$$

so

$$V = V_c \frac{e^{-m_\pi r}}{m_\pi r} \left(V_d + V_\sigma \sigma_1 \cdot \sigma_2 + V_\tau \tau_1 \cdot \tau_2 + V_{\sigma\tau} \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \right) \qquad (14b)$$

with
$$V_d = W + \tfrac{1}{2} B - \tfrac{1}{2} H - \tfrac{1}{4} M$$
$$V_\sigma = \tfrac{1}{2} B - \tfrac{1}{4} M$$
$$V_\tau = - \tfrac{1}{2} H - \tfrac{1}{4} M$$
$$V_{\sigma\tau} = - \tfrac{1}{4} M \ .$$

This leaves us four parameters to vary in the nuclear force, obviously an unmanageable task. Elliott and Flowers, from a consideration of the binding energies of the $A = 18, 19$ nuclei (17), have evaluated the triplet-singlet interaction

$$W + M - H - B = -0.70$$

and the triplet-triplet interaction

$$W - M - H + B = +0.26$$

with the normalization $W + M + H + B = 1$, leaving the singlet-singlet interaction $W - M + H - B = x$ as one free parameter. The Rosenfeld force corresponds to a value $x = 2$, the Visscher-Ferrell force (18) to $x = 0$. We will vary the exchange parameter through $x = 0, 1, 2, 3$

encompassing these two proposed forces and also we will take the strength of the central potential to be 30, 40 and 50 Mev, and the radius parameter b = 1.56 and 1.86 fermis, to determine the model sensitivity of the predicted μ capture rates.

b) The Energy Matrices

We require the matrix elements of the nuclear Hamiltonian

$$\mathcal{H} = \sum_{i=1,2} \xi_i \vec{l}_i \cdot \vec{s}_i + \Delta_{d-s} + V_{12} \qquad (15)$$

in the basis $(1\,p_{1/2}^{-1}\,2\,s_{1/2})$, $(1\,p_{3/2}^{-1}\,2\,s_{1/2})$, $(1\,p_{1/2}^{-1}\,1\,d_{3/2})$, $(1\,p_{3/2}^{-1}\,1\,d_{3/2})$, $(1\,p_{1/2}^{-1}\,1\,d_{5/2})$ and $(1\,p_{3/2}^{-1}\,1\,d_{5/2})$. The spin-orbit and d-s splitting terms are, of course, diagonal. For the matrix elements of V_{12} we need the direct and the exchange matrix elements (see Appendix I). Doing the isotopic spin part of the matrix element directly, we are left with

for T = 1 (16)

$$\langle V \rangle = +V_c \Big\{ (\; + 2\,V_\tau) \langle v(x) \rangle_E + (- \; V_d + V_\tau) \langle v(x) \rangle_D + (\; + 2\,V_{\sigma\tau}) \langle v(x)\, \sigma_1 \cdot \sigma_2 \rangle_E + (- V_\sigma + V_{\sigma\tau}) \langle v(x)\, \sigma_1 \cdot \sigma_2 \rangle_D \Big\}$$

where E(D) denotes exchange (direct) matrix element and

$v(x) = e^{-m_\pi r} / m_\pi r$.

For T = 0

$$V = +V_c\Big\{2V_d\langle v(x)\rangle_E + (-V_d - 3\,V_\tau)\langle v(x)\rangle_D$$
$$+2V_\sigma\langle v(x)\,\sigma_1\cdot\sigma_2\rangle_E + (-V_\sigma - 3\,V_{\sigma\tau})\langle v(x)\,\sigma_1\cdot\sigma_2\rangle_D\Big\}. \qquad (17)$$

The four types of matrix elements occurring are given below. Subscripts m(r) refer to final (initial) particle states and subscripts n(s) to final (initial) hole states. The reduced matrix elements of the Yukawa potential are evaluated in Appendix II. We need (we use the notation and conventions of Edmonds (19))

$$\langle v(x)\rangle_E = (-)^{1+(j_n-\ell_n)+(j_s-\ell_s)}\frac{(\hat{j}_m\hat{j}_n\hat{j}_s\hat{j}_r\hat{\ell}_m\hat{\ell}_r)^{1/2}}{\hat{J}_i}\,\delta_{J_i K}\,\delta_{J_f K}\,\delta_{M_i M_f}\;\times$$
$$\begin{Bmatrix}\ell_m & \ell_n & J_f\\ j_n & j_m & \tfrac{1}{2}\end{Bmatrix}\begin{Bmatrix}\ell_r & \ell_s & J_i\\ j_i & j_r & \tfrac{1}{2}\end{Bmatrix}\langle m_1 s_2\| v_K \| r_2 n_1\rangle \qquad (18)$$

$$\langle v(x)\rangle_D = (-)^{\ell_r+\ell_s+J_i+j_n+j_s+K}\,(\hat{j}_m\hat{j}_s\hat{j}_n\hat{\ell}_m\hat{\ell}_n)^{1/2}\,\delta_{J_f J_i}\,\delta_{M_f M_i}\;\times$$
$$\begin{Bmatrix}K & j_r & j_m\\ \tfrac{1}{2} & \ell_m & \ell_r\end{Bmatrix}\begin{Bmatrix}K & j_s & j_n\\ \tfrac{1}{2} & \ell_n & \ell_s\end{Bmatrix}\begin{Bmatrix}j_n & j_s & K\\ j_r & j_m & J_f\end{Bmatrix}\langle m_1 s_2\| v_K \| r_1 n_2\rangle \qquad (19)$$

$$\langle v(x)\,\sigma_1\cdot\sigma_2\rangle_E = 3(-)^{j_n+j_s+j_m+j_r}\left(\hat{j}_m\hat{j}_s\hat{j}_n\hat{j}_r\,\hat{\tfrac{1}{2}}\,\hat{\tfrac{1}{2}}\,\hat{\ell}_m\hat{\ell}_r\right)^{\frac{1}{2}} \times$$

$$\delta_{J_f J_i}\,\delta_{M_f M_i}\begin{pmatrix} j_n & j_m & J_f \\ \ell_n & \ell_m & K \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} j_s & j_r & J_i \\ \ell_s & \ell_r & K \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}\langle m_1 s_2 \| v_K \| r_2 n_1 \rangle \tag{20}$$

and finally

$$\langle v(x)\sigma_1\cdot\sigma_2\rangle_D = 3(-)^{j_s+j_m+J_i}\left(\hat{j}_m\hat{j}_s\hat{j}_n\hat{j}_r\,\hat{\tfrac{1}{2}}\,\hat{\tfrac{1}{2}}\,\hat{\ell}_m\hat{\ell}_n\right)^{\frac{1}{2}}\delta_{J_f J_i}\,\delta_{M_f M_i} \times$$

$$\sum_{j_1}(-)^{j_1}\hat{j}_1\begin{pmatrix} j_n & j_m & J_f \\ j_r & j_s & j_1 \end{pmatrix}\begin{pmatrix} j_s & j_n & j_1 \\ \ell_s & \ell_n & K \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} j_r & j_m & j_1 \\ \ell_r & \ell_m & K \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix} \times$$

$$\langle m_1 s_2 \| v_K \| r_1 n_2 \rangle . \tag{21}$$

The lowest energy eigenvalues of these energy matrices, and their eigenfunctions, have been found. In Appendix VIII, table 1, these eigenvalues and eigenvectors are listed for $x = 2$, $b = 1.56$ and $V_c = 30$, 40 and 50 Mev. One parameter remains at our disposal. The strength of the harmonic oscillator potential has been chosen so that the average excitation energy of these five states agrees with experiment for $V_c = 40$ Mev. This requires $\hbar\omega = 13.5$ Mev. For comparison, the results of Elliott and Flowers and the j-j limit for these states are also presented.

The β Decay of N^{16}

The ground state of N^{16} has a half life of 7.37 seconds against β decay to various states in O^{16}. The branching ratio for the groundstate to groundstate transition is 26% and the Q value is 10.4 Mev. The 2^- to 0^+ transition is a unique first forbidden Gamow-Teller decay. In terms of the μ decay rate, the β decay rate is

$$\frac{\Gamma_\beta}{\Gamma_\mu} = 96\, g_A^2 \left(\frac{Q}{\mu}\right)^5 \left(\frac{Q}{\nu}\right)^2 (\nu\Theta)^2 \int_{t=0}^{(1-m^2)^{1/2}} dt\, s^2 t^2 (s^2 + t^2)\, F(z,t) \,. \qquad (22)$$

Here $g_A = -1.23$

- m is the electron mass in units of Q
- μ is the μ meson mass = 105 Mev
- ν is the neutrino momentum in μ capture = 95 Mev
- Q is the nuclear Q value = 10.4 Mev
- $s(t)$ is the neutrino (electron) momentum in the β decay in units of Q $\left(s = 1 - (t^2 + m^2)^{1/2}\right)$
- $F(z,t)$ is the coulomb factor for β decay.

$\nu\theta$ is defined for convenience in the μ capture calculation as

$$\begin{pmatrix} \alpha & \beta & M_i \\ 1 & 1 & J_i \end{pmatrix} \sqrt{3}\, \nu\theta = \langle f | \sum_N \frac{\tau^+}{\sqrt{2}} \sigma_\alpha\, \nu x_\beta | i \rangle \,. \qquad (23)$$

Thus $\nu\Theta$ is just the part of G_1 (see Eq. 29b and 35) obtained by replacing $j_1(\nu r)$ by $\frac{1}{3}\nu r$ in the radial matrix element.

We obtain a value of $v\,\Theta = 0.382$, taking the sign from the shell model determination. This is to be compared with a value 0.546 in the j-j limit, 0.411 using the Elliott-Flowers result, and 0.325 using our wave functions. We will postpone a comment on this result until later.

Calculation of the Capture Rate

In the μ capture Hamiltonian, introduce the abbreviations (see Eq. 13)

$$G_V = C_\mu(k^2)\,(1 + v/2\,M_p)$$

$$G_A = A_\mu(k^2) - v/2\,M_p\,(C_\mu(k^2) + 2\,D_\mu(k^2))$$

$$G_p = \left\{ B_\mu(k^2) + A_\mu(k^2) - (C_\mu(k^2) + 2\,D_\mu(k^2)) \right\} \frac{v}{2\,M_p} \quad .$$

Then the matrix element of the interaction becomes

$$\mathcal{H} = G \quad \varphi_\mu(0)\, w_\nu^{+} \, \frac{(1 - \sigma\cdot\hat{v})}{2} \, (A + \sigma\cdot B) \quad w_\mu \quad . \tag{24}$$

Here the spatial dependence of the μ wave function over the nuclear volume has been ignored, and $\varphi_\mu(x)$ replaced by $\varphi_\mu(0)$. The nuclear matrix elements A and B will be defined in a moment. The capture rate is

$$\begin{aligned} \Gamma &= \int \frac{d\,3\,v}{(2\pi)^3}\; 2\,\pi \;\; \delta\,(E_i - E_f)\, G^2 \;\; \varphi_\mu^{\,2}(0) \;\; M^2 \\ &= \frac{v^2\;\, G^2 \;\, \varphi_\mu^{\,2}(0)}{2\,\pi} \quad\; 2\,M^2 \end{aligned} \tag{25}$$

ν is the neutrino momentum, to be taken in the Z direction. $\phi_\mu^2(0)$ is nominally $\frac{(Z\mu e^2)^3}{\pi}$ for a point nucleus. Averaged over initial μ states and summed over final neutrino states, the matrix element is

$$2M^2 = \frac{(1 - \sigma\cdot\hat{\nu})}{2} (A + \sigma\cdot B)(A^* + \sigma\cdot B^*) \frac{(1 - \sigma\cdot\hat{\nu})}{2}$$

$$= AA^* + BB^* . \tag{26}$$

A sum over final and average initial nuclear states is implicit here. Pseudoscalar terms containing $\hat{\nu}$ have been dropped from the rate. The terms A and B can be identified by comparison with $\mathcal{H}$ as

$$A = G_V f - G_V \hat{\nu}\cdot q \tag{27}$$

$$B = G_A g - G_P \hat{\nu}\, \hat{\nu}\cdot g - G_V i \hat{\nu} \times q - g_A \hat{\nu}\, t \tag{28}$$

where the nuclear matrix elements are

$$f = \langle f | \Sigma_N e^{-i\nu\cdot r} \frac{\tau^-}{\sqrt{2}} | i \rangle \tag{29a}$$

$$g_\mu = \langle f | \Sigma_N e^{-i\nu\cdot r} \frac{\tau^-}{\sqrt{2}} \sigma_\mu | i \rangle \tag{29b}$$

$$q_\mu = \langle f | \Sigma_N e^{-i\nu\cdot r} \frac{\tau^-}{\sqrt{2}} \frac{p_\mu}{M} | i \rangle \tag{29c}$$

$$t_{\mu\nu} = \langle f | \Sigma_N e^{-i\nu\cdot r} \frac{\tau^-}{\sqrt{2}} \frac{\sigma_\mu p_\nu}{M} | i \rangle \text{ and } t = t_{\mu\mu} . \tag{29d}$$

The sum over N is a sum over all the nucleons in the nucleus. r, τ^-, σ etc. refer to the nucleon being summed on. In terms of these matrix elements

$$AA^* = G_V^2 f^2 - 2 G_V^2 \operatorname{Re}(f \hat{\nu}\cdot q^*) \tag{30}$$

$$BB^* = G_A^2 g^2 + (G_p^2 - 2\,G_A\,G_p)\,|\hat{\nu}\cdot g|^2$$

$$+ 2\,(G_p - G_A)\,g_A\,\mathrm{Re}\,(\hat{\nu}\cdot g\;t^*) + 2\,G_A G_v\,\mathrm{Im}\,\hat{\nu}\cdot g \times q^*$$

$$+ g_A^2\,t^2 + G_v^2\,q^2\,. \tag{31}$$

Of course, the Hamiltonian is only exact to order k/M so that it is inconsistent to keep the square of the small terms. However, we prefer to treat the approximate model exactly so we will carry these terms along.

Now to evaluate these matrix elements for the transitions to the bound states of N^{16} using our shell model states. The matrix element of $\tau^-/\sqrt{2}$ is common to all these and equal to one, so drop the isotopic spin from further considerations. Then the matrix element of an operator

$$\mathcal{H} = {}_L\mathcal{H}_{\mathcal{L}}^{m_{\mathcal{L}}}\;\;{}_s\mathcal{H}_K^{\mu} \tag{32}$$

where

${}_L\mathcal{H}_{\mathcal{L}}^{m_{\mathcal{L}}}$ is a spherical tensor of rank $(\mathcal{L}, m_{\mathcal{L}})$ operating on space coordinates only and

${}_s\mathcal{H}_K^{\mu}$ is a spherical tensor of rank (K, μ) operating on spin variables only, is

$$<f|\mathcal{H}|i> = C_{j_n j_m J_f}(-)^{j_n - n} \begin{pmatrix} -n & m \\ j_n & j_m \end{pmatrix}\begin{vmatrix} M_f \\ J_f \end{vmatrix} \begin{pmatrix} \delta & d \\ \frac{1}{2} & \ell_n \end{pmatrix}\begin{vmatrix} n \\ j_n \end{vmatrix} \times \quad (33)$$

$$\begin{pmatrix} \alpha & a & \Big| & m \\ \frac{1}{2} & \ell_m & \Big| & j_m \end{pmatrix} \begin{pmatrix} \delta & \mu & \Big| & \alpha \\ \frac{1}{2} & K & \Big| & \frac{1}{2} \end{pmatrix} \begin{pmatrix} d & \mathcal{M}_{\mathcal{L}} & \Big| & a \\ \ell_n & \mathcal{L} & \Big| & \ell_m \end{pmatrix} \times$$

$$<\tfrac{1}{2}\| \mathcal{H}_K \| \tfrac{1}{2}> <\ell_m \| \mathcal{H}_{\mathcal{L}} \| \ell_n> .$$

Here $C_{j_n j_m J_f}$ are the expansion coefficients of our shell model states, subscripts m(n) refer to particle (hole) states. The matrix element summed over magnetic quantum numbers is

$$\langle f|\mathcal{H}|i\rangle = C_{j_n j_m J_f}(-)^{j_m + j_n + J_f} \qquad (\hat{j}_n \hat{j}_m \hat{\tfrac{1}{2}} \hat{\ell}_m)^{\frac{1}{2}} \quad \times \quad (33')$$

$$\begin{pmatrix} \mu & \mathcal{M}_{\mathcal{L}} & \Big| & M_f \\ K & \mathcal{L} & \Big| & J_f \end{pmatrix} \begin{pmatrix} j_n & j_m & J_f \\ \ell_n & \ell_m & \mathcal{L} \\ \frac{1}{2} & \frac{1}{2} & K \end{pmatrix} <\tfrac{1}{2}\| \mathcal{H}_K \| \tfrac{1}{2}> <\ell_m \| \mathcal{H}_{\mathcal{L}} \| \ell_n> .$$

This expression suffices for the large matrix elements f and g. For the momentum type matrix elements, as shown in Appendix V, we need to replace the orbital reduced matrix element by

$$<\ell_m \| \mathcal{H}_{\mathcal{L}} \| \ell_n> \rightarrow \begin{pmatrix} \nu & 0 & \Big| & \mathcal{M}_{\mathcal{L}} \\ 1 & \mathrm{L} & \Big| & \mathcal{L} \end{pmatrix} <\ell_m \| \mathcal{L}, \mathrm{L} \| \ell_n> .$$

The required reduced matrix elements are presented in Appendix III.

To evaluate the individual terms in M^2, it is useful to use the identities for vector products in terms of spherical components (19),

$$a \cdot b^* = (-)^{1+\nu} \sqrt{3} \begin{pmatrix} \mu & \nu & \big| & 0 \\ 1 & 1 & \big| & 0 \end{pmatrix} a_\mu (b_{-\nu})^*$$

and $$(a \times b^*)_\rho = (-)^\nu \frac{\sqrt{2}}{i} \begin{pmatrix} \mu & \nu & \big| & \rho \\ 1 & 1 & \big| & 1 \end{pmatrix} a_\mu (b_{-\nu})^* .$$

Now to the individual terms in the rate. The Fermi matrix element f has $K = 0$, $\langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle = 1$, and

$$\langle \ell_m \| \mathcal{H}_\mathcal{L} \| \ell_n \rangle = \langle \ell_m \| e^{-i\nu \cdot r}_{\mathcal{L}} \| \ell_n \rangle .$$

With $\hat{\nu}$ in the $\hat{Z}$ direction, $\mathcal{M}_\mathcal{L} = 0$ and we can write

$$f = (\hat{J}_f)^{\frac{1}{2}} \; \delta M_f, 0 \; \delta J_f, \mathcal{L} \; F_\mathcal{L}$$

and $$f^2 = \hat{J}_f F_\mathcal{L}^2 \; \delta J_f, \mathcal{L} . \tag{34}$$

We can identify $F_\mathcal{L}$ from the general expression above.

The Gamow-Teller matrix element, with $K = 1$, can be written

$$g_\mu = \begin{pmatrix} \mu & 0 & \big| & M_f \\ 1 & \mathcal{L} & \big| & J_f \end{pmatrix} \sqrt{\hat{\mathcal{L}}} \; G_\mathcal{L}$$

and

$$g^2 = \hat{J}_f \sum_\mathcal{L} G_\mathcal{L}^2 . \tag{35}$$

Also $$|\hat{\nu} \cdot g|^2 = \hat{J}_f \left| \sum_\mathcal{L} \begin{pmatrix} 0 & 0 & \big| & 0 \\ 1 & \mathcal{L} & \big| & J_f \end{pmatrix} \sqrt{\frac{\hat{\mathcal{L}}}{\hat{J}_f}} \; G_\mathcal{L} \right|^2 . \tag{36}$$

The momentum type matrix element q can be written

$$q_\mu = \begin{pmatrix} \nu & 0 \\ 1 & \mathbb{L} \end{pmatrix}\begin{vmatrix} M_f \\ J_f \end{vmatrix} \; \delta_{J_f \mathcal{L}} \quad \delta_{m_\mathcal{L} M_f} \sqrt{\hat{\mathbb{L}}} \quad Q_{\mathcal{L}\mathbb{L}} \; .$$ $Q_{\mathcal{L}\mathbb{L}}$ is

to be identified from (33') with K = 0 and $< \ell_m \| \mathcal{H}_\mathcal{L} \| \ell_n > = < \ell_m \| \mathcal{L}, \mathbb{L} \| \ell_n >$.

Then

$$q^2 = \hat{J}_f \sum_{\mathbb{L}} Q^2_{\mathcal{L}\mathbb{L}} \; \delta_{J_f \mathcal{L}} \; . \tag{37}$$

The cross terms involving q are

$$\text{Ref } \hat{\nu} \cdot q^* = \hat{J}_f \sum_{\mathbb{L}} \begin{pmatrix} 0 & 0 \\ 1 & \mathbb{L} \end{pmatrix}\begin{vmatrix} 0 \\ J_f \end{vmatrix} \sqrt{\frac{\hat{\mathbb{L}}}{\hat{J}_f}} \quad \delta_{J_f \mathcal{L}} \quad \text{Re } F_\mathcal{L} \, Q^*_{\mathcal{L}\mathbb{L}} \tag{38}$$

and

$$I_m \; \hat{\nu} \cdot g \times q^* = \hat{J}_f \sum_{\mathcal{L}\mathbb{L}} \sqrt{2 \hat{\mathcal{L}} \hat{\mathbb{L}}'} \quad (-)^{J_f} \begin{pmatrix} 0 & 0 \\ \mathbb{L}' & \mathcal{L} \end{pmatrix}\begin{vmatrix} 0 \\ 1 \end{vmatrix} \times$$

$$\begin{pmatrix} \mathbb{L}' & \mathcal{L} & 1 \\ 1 & 1 & J_f \end{pmatrix} \delta_{\mathcal{L}' J_f} \quad \text{Re } G_\mathcal{L} \, Q^*_{\mathcal{L}' \mathbb{L}'} \; . \tag{39}$$

The momentum type matrix element $t_{\mu\nu}$ can be expressed

$$t_{\mu\nu} = \begin{pmatrix} \mu & m_\mathcal{L} \\ 1 & \mathcal{L} \end{pmatrix}\begin{vmatrix} M_f \\ J_f \end{vmatrix} \begin{pmatrix} \nu & 0 \\ 1 & \mathbb{L} \end{pmatrix}\begin{vmatrix} m_\mathcal{L} \\ \mathcal{L} \end{vmatrix} \frac{\hat{J}_f}{\hat{\mathcal{L}}^{1/2}} \quad T_{\mathcal{L}\mathbb{L}}$$, as we see from

(33') with K = 1. The contraction is

$$t = \sum_{\mathcal{L}\mathbb{L}} (\hat{J}_f)^{1/2} \; (-)^{\mathcal{L}+\mathbb{L}} \delta_{M_f, 0} \quad \delta_{J_f \mathbb{L}} \quad T_{\mathcal{L}\mathbb{L}} \; .$$

Then the terms required are

$$t^2 = \hat{J}_f \left| \sum_{\mathcal{L} \mathbb{L}} (-)^{\mathcal{L} + \mathbb{L}} \delta_{J_f \mathbb{L}} \quad T_{\mathcal{L}\mathbb{L}} \right|^2 \tag{40}$$

and the cross term

$$\text{Re } t\, \hat{\nu} \cdot g^* = \hat{J}_f \sum_{\mathcal{L} \mathbb{L} \mathcal{L}'} (-)^{\mathcal{L} + \mathbb{L}} \delta_{J_f \mathbb{L}} \sqrt{\frac{\hat{\mathcal{L}}'}{\hat{J}_f}} \begin{pmatrix} 0 & 0 & 0 \\ 1 & \mathcal{L}' & J_f \end{pmatrix} \times$$

$$\text{Re } G_{\mathcal{L}'} \; T_{\mathcal{L}\mathbb{L}}^* \; . \tag{41}$$

Individual Transitions

O^+ to O^- - the terms entering this rate are g^2, $\hat{\nu} \cdot g$ ^{2}, t^2 and Re $t\, \hat{\nu} \cdot g^*$, for which two matrix elements are required, namely G_1 and T_{10}.

O^+ to 1^- - the terms entering here are f^2, g^2, q^2, Re $f\, \hat{\nu} \cdot q^*$ and $Im\, \hat{\nu} \cdot g \times q^*$. The individual matrix elements required are F_1, G_1, Q_{10} and Q_{12}.

O^+ to 3^- - the same terms enter here as did in the O^+ to 1^- transition. The matrix elements required are F_3, G_3, Q_{32} and Q_{34}.

O^+ to 2^- - here we require the terms g^2, q^2, $\hat{\nu}\cdot g$ ^{2} t^2, Re t $\hat{\nu}\cdot g^*$ and $Im\,\hat{\nu}\cdot g \times q^*$. The individual matrix elements entering are G_1, G_3, Ω_{22} and the combination $T = -T_{12} + T_{22} - T_{32}$.

All these matrix elements have been evaluated using the shell model wave functions for a range of parameters. Results are presented in Appendix IV, table 2.

The first forbidden Gamow-Teller matrix element G_1 in the O^+ to 2^- transition can be expressed as $G_1 = R(\nu\Theta)$, where $\nu\Theta$ is the β decay matrix element obtained from the 2^- to O^+ β decay of N^{16}. The matrix element is determined in magnitude and its sign is chosen to agree with the shell model prediction. R is the ratio of the calculated values of G_1 and $\nu\Theta$ which is very nearly constant and equal to 0.789 for b = 1.56.

The total capture rate to the bound states of N^{16} calculated with these matrix elements and with the hypotheses $g_p = \pm 8\, g_A$, 0 with and without CVC, is also presented in Appendix IV, table 3,4.

Results and Discussion

The nominal capture rate (eqn. 25)

$$\Gamma_c = G^2 \quad \frac{v^2 (Ze^2 \mu)^3}{2\pi^2} \qquad (2M^2)$$

must be reduced for three reasons - to take account of the finite mass of the nucleus (3), the finite size of the nucleus (5), and the momentum dependence of the weak interaction coupling constants (13). The finite mass of the nucleus introduces a factor approximately equal to

$$(1 - 5\mu/AM_p)$$

so this causes a reduction in the rate of 3.5%. According to the numerical calculations of Ford and Flamand, the finite size of the nucleus causes a 6% reduction in $\varphi_\mu^2(0)$. We reduce the rate 5% to account for the momentum dependence of the coupling constants. Then

$$\frac{\Gamma_c}{\Gamma_\mu} = 42.6 \times 10^{-3} (2M^2) . \qquad (25')$$

A central value of $2M^2 = 1$ gives a capture rate to the bound states of $\Gamma_c = 19.2 \times 10^3 \text{sec}^{-1}$.

Now we must try to estimate the precision with which we can predict the matrix element $2M^2$. From Appendix IV, table 4, we see that $2M^2$ varies by less than one percent as we vary V_c through 30, 40 and 50 Mev. Additional calculations show that $2M^2$ is equally independent of x, the exchange parameter in the nuclear force. As x is varied through 0, 1, 2 and 3, $2M^2$ increases by less than one percent.

Also, as the nuclear radius parameter is increased from b = 1.56 fermis to b = 1.86 fermis, the rate decreases by about four percent. The reason for the lack of dependence of the rate upon the nuclear force parameters is the fortuitous circumstance that the capture transition is predominantly (~80%) to the groundstate of N^{16}, and the dominant matrix element G_1 can be determined experimentally from the inverse β decay rate. It is clear then that the uncertainty in the choice of nuclear parameters is not the significant source of error in the predicted rate.

Here it is necessary to discuss the discrepancy which persists between our description of the A = 16 shell model states and that of Elliott and Flowers. The difference is seen to be small but it has a significant effect upon the μ capture rate. On the basis of comparison to experiment there appears to be little to choose between them. Our T = 0 levels are in the correct order. The T = 1 levels are grouped within 3 Mev, compared to a grouping within 1 Mev for EF and 392 kev experimentally. Our predicted value of $v\,\Theta$ agrees with experiment at x = 2, V_c = 30 Mev. The EF result is 5% high at x = 2, V_c = 40 Mev. Both of these are satisfactory results, since $v\,\Theta$ is particularly sensitive to a cancellation between the contributions from the $(1\,p_{1/2}^{-1}\,1\,d\,5/2)$ and $(1\,p_{3/2}^{-1}\,1\,d\,5/2)$ components which dominate the $J = 2^-$ T = 1 state. Actually, the really significant difference between the two results as far as predicting the μ capture rate is concerned is just one, namely the structure of the $J = 0^-$ T = 1 state. Both results are within ten percent of the jj limit but the admixture of $(1\,p_{1/2}^{-1}\,1\,d\,3/2)$ is of opposite sign in the two cases. The matrix element of $G_1(T_{10})$ to the $(1\,p_{3/2}^{-1}\,1\,d\,3/2)$ state is roughly twice as large as to the $(1\,p_{1/2}^{-1}\,1\,d_{1/2})$ state so that this difference in sign causes a considerable enhancement (above the jj limit) in these two matrix elements for EF, whereas they are considerably reduced in our case. This single feature is primarily responsible for the different predictions of the two sets of wave functions. A careful examination of the energy matrices supports our conclusions. We are forced to accept our results over those of EF.

Since roughly eighty percent of the rate is accounted for by the experimentally determined matrix element G_1, we can safely make a liberal assumption about the uncertainty in the "predicted" part of the rate. We will consider the possibility of a 30% uncertainty in this part, or a 6% uncertainty in the total rate.

Conclusions

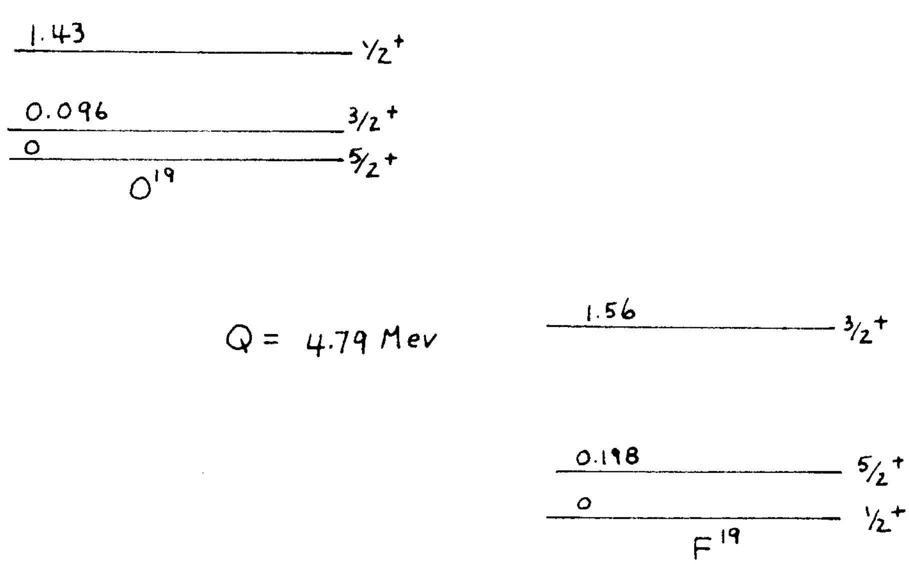
The predicted rates for the various hypotheses including a six percent uncertainty are

Case 1	$2M^2$ =	1.00 - 0.88
2		1.70 - 1.50
3		1.29 - 1.13
4		.81 - .71
5		1.56 - 1.38
6		1.09 - .97

It is seen that it will not be possible to tell, even within these predictions, whether or not the conserved vector current is present. In fact, within these uncertainties, there is a slight overlap between cases 1 and 6 which have $g_p = +8\,g_A$ and zero.

F^{19} Preliminary Discussion

F^{19} and O^{19} provide another example of nuclei all of whose known normal (positive) parity bound states are understood on the basis of the intermediate coupling shell model (17). In this model the nucleus is treated as a closed O^{16} shell plus three extra core nucleons in 1d or 2s states. The configuration mixing is accomplished by extra core nucleon-nucleon interactions, plus d-s splitting and $\vec{l} \cdot \vec{s}$ splitting for each nucleon. The structure of the relevant nuclear states obtained by diagonalizing this interaction is presented in appendix VI (17 and 20). The notation used is that of Jahn (21) and we use his scheme to classify states of three particles in the d-shell. d^2s states are classified by their d^2 parents, ds^2 states by their s^2 parents, and the s^3 state is unique.



Shell Model States for A = 19 Nuclei

Let us examine the μ capture process from the T = 1/2, J = $1/2^+$ ground state of F^{19} to the T = 3/2, J = $5/2^+$, $3/2^+$, $1/2^+$ ground and excited states of O^{19}. First of all, we notice that the inverse β decay from J = $5/2^+$ to J = $1/2^+$ is second-forbidden; the β decay proceeds to excited states of F^{19} so it will not be possible to replace any calculated matrix element by an experimentally determined one. Consider the capture reactions intuitively, on the basis of the large Fermi ($e^{iv \cdot r}$) and Gamow-Teller ($\sigma e^{iv \cdot r}$) terms in the interaction Hamiltonian.

a) $1/2\, 1/2^+ - 3/2\, 1/2^+$ transition: in β decay, we would say that a Fermi transition between such states is forbidden by the change in isotopic spin, since we can factor out a coordinate type matrix element $\langle 1 \rangle$ leaving the matrix element $\langle T^- \rangle$ which vanishes between states of different isotopic spin. In μ capture, a similar situation exists. To the extent that the matrix element $\langle j_o(vr) \rangle$ is independent of the basis states, it can be factored too, leaving the matrix element $\langle T^- \rangle$ which vanishes as in β decay. In fact, $\langle j_o(vr) \rangle$ is not exactly independent of the basis states, so we would expect the Fermi matrix element to be a small difference between the contributions from various basis states. No such factorization is possible in the Gamow-Teller matrix element. Off-hand, one would expect this term to dominate the whole transition rate. (In fact, we shall see that this is not the case, due to the different dominant space symmetries of these states.)

b) $1/2\ 1/2^+$ - $3/2\ 3/2^+$: the Fermi matrix element has $L = 2$ only. With the large momentum transfer, this matrix element could be large. However, experience shows that large changes in L are not favored and we expect this transition, too, to be dominated by the Gamow-Teller matrix element with $L = 0$.

c) $1/2\ 1/2^+$ - $3/2\ 5/2^+$: a priori, it is not possible to say anything very specific regarding this transition. We do expect this rate to be small compared to the rates for the transitions with smaller changes in angular momentum though.

It is worth remarking here about the effect of possible higher excited bound states in O^{19}. The work of Zimmerman (22) indicates that there are no other bound states at excitation energies less than 2.4 Mev. Above this energy there is no experimental information. The shell model predicts the next positive parity states to be $J = 7/2^+$, $9/2^+$ and $11/2^+$. States of such high angular momentum would make a very small contribution to the capture rate, and could be neglected or corrected for using just their dominant shell model states for an approximate calculation. Another possibility is that there might be negative parity states of low angular momentum. A description of such states in the shell model is not available, and they are usually interpreted in the cluster model (23) as being, for example, states of an alpha particle bound to a C^{15} negative parity core. Intuitively, we expect the weak interaction to be very inefficient in causing such a large rearrangement of nucleons. The expectation is borne out, for example, by the absence of a first forbidden Gamow-Teller transition from the $5/2^+$ groundstate of O^{19} to the $1/2^-$ first excited state in F^{19} in competition with the slow allowed but unfavored transitions that do proceed. Finally, if positive parity bound states of low angular momentum are found in O^{19}, this work will have to be supplemented by their inclusion using their shell model descriptions.

On the basis of these considerations, we expect the total capture rate to the bound states of O^{19} to be dominated by the Gamow-Teller terms in the Hamiltonian. As mentioned in the introduction, such transitions are expected to be sensitive to the presence of an induced pseudoscalar term in the interaction Hamiltonian.

The Hyperfine Structure Effect

μ capture by F^{19} is complicated by the fact that the nucleus has non-zero spin, in this case spin 1/2. The mesonic atom will exist in hyperfine states with total angular momentum $F = 1$ or zero and in these states there will be a different correlation between the μ meson spin and the nuclear angular momentum J. Since the spin of the extra core proton will be correlated with the angular momentum J, then there will be a different correlation between the μ spin and the spin of the extra core proton in the two hyperfine states. This correlation will not affect the μ capture by the protons within the O^{16} core but will for capture by the outside proton. An examination of the capture Hamiltonian reveals two terms dependent on the μ-proton spin correlation, namely $\sigma_L \cdot \sigma_N$ and $\sigma_L \cdot \hat{\nu}\ \ \sigma_N \cdot \hat{\nu}$ (24). A direct calculation for μ capture from F = 0 and F = 1 hyperfine states by a single proton shows that with the Primakoff Hamiltonian

$$\frac{\Gamma_{F=0}}{\Gamma_{F=1}} \approx 50 .$$

This large dependence of the capture rate on the μ-proton spin correlation makes it necessary to treat the two hyperfine levels independently, and to correct for any transfer between these levels. Even in a complex nucleus we expect this hyperfine effect to necessitate a separate treatment of the individual hyperfine levels.

The hyperfine splitting in a μ-mesic atom is

$$\Delta \nu = \left(\frac{m_\mu}{m_e} \right)^2 Z^3 \ \Delta \nu_H \ \approx \ 6 \times 10^{13} \ Z^3 \ \sec^{-1}$$

(where $\Delta \nu_H = 1.4 \times 10^9 \sec^{-1}$ is the HFS splitting in hydrogen). The finite width of these states arises from (3)

a) μ capture and decay for which

$$\Gamma \approx 10^6 \sec^{-1}$$

and conversion between these states by

b) collisional de-excitation (This is small except for the highly mobile neutral hydrogen μ-mesic atom.),

c) magnetic dipole radiation with a rate

$$\Gamma = 10\, m_\mu (e^2)^{13} Z^9 \left(\frac{m_\mu}{m_p} \right)^3 \simeq 10^3 \text{ sec}^{-1}, \text{ for } F^{19}, \text{ and}$$

d) Auger electron ejection with a rate

$$\Gamma = 10\, m_\mu (e^2)^6 \; Z \left(\frac{m_e}{m_\mu} \right)^3 \approx 10^5 \text{ sec}^{-1}.$$

Thus, in estimating the effect of the HFS effect, we may neglect the overlap of these two states and assume that they decay independently, starting with an initially statistical population. The triplet hyperfine level lies highest since the magnetic moment of F^{19} is positive. Now let

$\lambda_{\pm}$ by the total μ disappearance rate from the hyperfine level with $F = J \pm 1/2$

$\lambda_c^{\pm}$ be the μ capture rate to bound states from the HFS level with $F = J \pm 1/2$

λ_T be the conversion rate from $F = J + 1/2$ to $F = J - 1/2$

$N_{\pm}$ be the probability that a μ is in the HFS level $F = J \pm 1/2$ at time t after its arrival in the $^2S\ 1/2$ level of the μ atom, with the initial values $N_{+o} = 3/4 \quad N_{-o} = 1/4$

$N_c^{\pm}$ be the probability that a μ has been captured to a bound state from the HFS level $F = J \pm 1/2$, with the initial values $N_c^{\pm} = 0$. From the solutions of the differential equations governing this process we find, as $t \to \infty$,

$$N_c^+ = \frac{\lambda_c^+}{\lambda_+ + \lambda_T} \frac{3}{4} \quad N_c^- = \frac{\lambda_c^-}{\lambda_-} \left(\frac{1}{4} + \frac{3}{4} \frac{\lambda_T}{\lambda_+ + \lambda_T} \right) .$$

Let us treat these corrections - the HFS effect and the conversion effect - to first order only. Introduce the average disappearance rate $\bar{\lambda} = 3/4\, \lambda^+ + 1/4\, \lambda^-$ and define $\delta = \frac{\lambda^+ - \lambda^-}{\bar{\lambda}}$ so that $\lambda^+ = \bar{\lambda}(1 + \frac{\delta}{4})$ and $\lambda^- = \bar{\lambda}(1 - \frac{3}{4}\delta)$. To first order in δ and $\lambda_T/\bar{\lambda}$ we get from the above that the total probability of capture to bound states is

$$N_c = N_c^+ + N_c^- = \frac{1}{\bar{\lambda}} \left\{ \frac{3}{4} \lambda_c^+ + \frac{1}{4} \lambda_c^- + \left(\frac{3}{16} \delta + \frac{3}{4} \frac{\lambda_T}{\bar{\lambda}} \right) (\lambda_c^- - \lambda_c^+) \right\}. \quad (43)$$

Here $\lambda = \frac{3}{4} \lambda_c^+ + \frac{1}{4} \lambda_c^-$ is the capture rate for no correlation between μ and proton spins, or for a statistical population of these levels.

For a crude order of magnitude estimate, take $\delta = 0.10$ $\lambda_T/\bar{\lambda} = 0.10$ and $\lambda_c^- - \lambda_c^+ \approx \pm \lambda$. The correction due to HFS and conversion effects is then of the order of ten percent.

We can make a more detailed estimate of the HFS effect using a result presented by Primakoff (3). From this work,

$$\delta = \frac{b}{a} \quad \frac{1}{2Z} \quad \langle \sigma_p \cdot J \rangle \quad \frac{2J+1}{J(J+1)} \tag{44}$$

where J is the nuclear angular momentum, Z the nuclear charge (for an odd Z, odd A nucleus) and $\langle \sigma_p \cdot J \rangle$ is the ground state expectation value of the scalar product of J with the spin of the odd proton. b/a is a ratio of coupling constants equal to -0.95. This result rests on the closure approximation and the approximation that the exclusion principle inhibition factor is the same for the core protons as for the extra core proton. It also includes the assumption that the μ decay rate is the same as the μ capture rate. For the case of F^{19}, then, $\delta = -.14 \ \langle \sigma_p \cdot J \rangle$. With the assumption of a single particle model for F^{19}, $\langle \sigma_p \cdot J \rangle = 3/2$ and $\delta = -.21$.

The single particle model for F^{19}, in which the ground state is taken to be a $2\,S_{1/2}$ proton, has the justification that the magnetic moment of this state is almost exactly equal to the single particle value (2.63 compared to 2.79). This is really no guarantee that other properties coincide with their single particle model predictions. For example, consider the orbital angular momentum of the extra core proton. On the single particle model $\langle \ell_p^2 \rangle$ = zero. Using the complete shell model description of the ground state (see Appendix VI, use the V = 40 eigenvector), we find a 48% probability that the proton is in a d-state and 52% probability that it is in an s-state, so

$\langle \ell_p^2 \rangle = 2.87$. One might thus expect $\langle \sigma_p \cdot J \rangle$ to be considerably less than 3/2. The matrix element $\langle \sum_i \frac{1}{2} (1 + \tau_3^i) \sigma^i \cdot J \rangle$ has been evaluated using the methods to be discussed later, and the shell model states in Appendix VI. In fact, these detailed results differ but little from the single particle model. We obtain

	V=30	V=40	V=50
$\langle \sigma_p \cdot J \rangle$	1.34	1.39	1.43

which reduces our estimate of δ to -.20.

More detailed estimates of λ_T, the transfer rate between hyperfine states, and of δ have recently been made by Überall (25). His prediction for λ_T is essentially the one above, but he finds, using closure and the complete shell model wave function for F^{19}, that Primakoff's approximations are not very good. In particular, he finds that the exclusion principle inhibition factor is considerably larger for core protons than for the extra core proton so that in Primakoff's estimate Z should be reduced and δ thus increased to $\delta = -0.60$.

Calculation of the Capture Rate

From the preceding discussion for the case of O^{16}, the capture rate is (see Eq. 25)

$$\Gamma = \frac{G^2 (Z e^2)^3 \mu^3 \nu^2}{2 \pi^2} \, 2 M^2 \tag{45}$$

where M is the matrix element between μ and initial nuclear states and neutrino and final nuclear states of the interaction Hamiltonian (with $\varphi_\mu^2(0)$ removed), which we will write $M = \langle f \mid \mathcal{H} \mid i \rangle$. In the case of F^{19}, it is necessary to sum M^2 over final states (neutrino and nuclear states independently) and average over initial states, but in this average the HFS states $F = J \pm 1/2$ must be treated separately. Denote these as $\lambda_c^+ = M_+^2$ (averaged over HFS states with F = 1) and $\lambda_c^- = M_-^2$ (averaged over HFS states with F = 0). In order to carry out a sum over μ and initial nuclear states independently rather than coupled, insert projection operators P+ and P- with the properties

$$P+ \mid F = 1 \rangle = \mid F = 1 \rangle \qquad P- \mid F = 0 \rangle = \mid F = 0 \rangle$$

$$P+ \mid F = 0 \rangle = 0 \qquad P- \mid F = 1 \rangle = 0 \,.$$

By analogy with the two particle case, the required operators are

$$P- = \frac{1}{4} (1 - 2 J \cdot \sigma_L) \qquad P+ = \frac{1}{4} (3 + 2 J \cdot \sigma_L)$$

where σ_L is the lepton spin and J the nuclear angular momentum.

Then

$$\lambda_c^+ = \frac{1}{3} \langle f \mid \mathcal{H} \, P+ \mid i \rangle \langle i \mid \mathcal{H} \mid f \rangle$$ and

$$\lambda_c^- = \langle f \mid \mathcal{H} \, P- \mid i \rangle \langle i \mid \mathcal{H} \mid f \rangle$$ where we sum over

uncoupled initial states. Two rates are required:

$$\lambda = \frac{3}{4}\lambda_c^+ + \frac{1}{4}\lambda_c^- = \frac{1}{4}\left|\langle f|\mathcal{H}|i\rangle\right|^2 \qquad (46a)$$

since P+ + P- = 1, and

$$\lambda_D = \lambda_c^- - \lambda_c^+ = -\frac{8}{3}\left\{\frac{1}{4}\langle f|\mathcal{H}\sigma_L\cdot J|i\rangle\langle i|\mathcal{H}|f\rangle\right\} \qquad (46b)$$

with $P- - \frac{P+}{3} = -\frac{8}{3}\,\frac{\sigma_L\cdot J}{4}$.

Calculation of λ_D will be postponed until after the discussion of λ. From the previous work $2\lambda = AA^* + BB^*$ where (see Eq. 30 and 31)

$$AA^* = G_V^2 f^2 - 2G_V^2\, \mathrm{Re}\,(f\,\hat{v}\cdot q^*) \qquad (47)$$

$$BB^* = G_A^2 g^2 + (G_P^2 - 2G_AG_P)\,|\hat{v}\cdot g|^2 + 2(G_P - G_A)g_A\,\mathrm{Re}(\hat{v}\cdot g\, t^*)$$
$$+ 2G_AG_V\,\mathrm{Im}\,(\hat{v}\cdot g\times q^*) + g_A^2t^2 + G_V^2q^2 \quad . \qquad (48)$$

As in the previous work, the approximate model is being treated exactly. The notation is the same as that used previously. f, g, q and t are the nuclear matrix elements defined in equation (29a-d).

Nuclear Physics

There follows a discussion of the evaluation of the various nuclear matrix elements using the complete shell model specification of the nuclear states. In a later section the single particle model predictions will be compared with those of the shell model.

The antisymmetrical shell model wave functions for three particles in the (1d, 2s) shells are required. The phase conventions used are those of Elliott and Flowers (17) and Jahn (21). The notation is

subscript 123:	-	label the three extra core nucleons
$TLS\mathscr{S}$	-	completely specify the three particle state
$t\ell\Delta\alpha$	-	completely specify the two particle state
$\langle d^2; t\ell\Delta\alpha \| \} d^3; TLS\mathscr{S}\rangle$	-	the fractional percentage coefficient (FPC) for expanding a three particle state in terms of its two particle parents
$\|d^2;\ t\ell\Delta\alpha\rangle_{12}$	-	antisymmetric state of two particles (with magnetic quantum numbers suppressed) .

The wave functions, with summation over all magnetic quantum numbers except Tz and Jz understood, are, in LS coupling:

<u>1d^3</u>

$$|d^3;\ TLSJ\mathscr{S}\rangle = \langle d^2;\ t\ell\Delta\alpha |\} d^3;\ TLS\mathscr{S}\rangle \ |d^2;\ t\ell\Delta\alpha\rangle_{12}\ |d\rangle_3$$

$$\begin{pmatrix} \mu & t_z & T_z \\ \frac{1}{2} & t & T \end{pmatrix} \begin{pmatrix} \gamma & \ell_z & L_z \\ 2 & \ell & L \end{pmatrix} \begin{pmatrix} \beta & \Delta_z & S_z \\ \frac{1}{2} & \Delta & S \end{pmatrix} \begin{pmatrix} S_z & L_z & J_z \\ S & L & J \end{pmatrix} \tag{49a}$$

<u>(1d)^2 (2s)</u>

$$|d^2 s;\ TLSJ\alpha\rangle = \frac{1}{\sqrt{3}}\left\{|d^2;\ t\ell s \alpha\rangle_{12}\, |s\rangle_3 + (-)^P\, \mathbb{P}\right\}$$

$$\begin{pmatrix} \mu & t_z & T_z \\ \frac{1}{2} & t & T \end{pmatrix}\begin{pmatrix} 0 & \ell_z & L_z \\ 0 & \ell & L \end{pmatrix}\begin{pmatrix} \beta & s_z & S_z \\ \frac{1}{2} & s & S \end{pmatrix}\begin{pmatrix} S_z & L_z & J_z \\ S & L & J \end{pmatrix}$$

(49b)

(where $(-)^P\, \mathbb{P}$ is a sum over permutations of the three particles to antisymmetrize the state)

<u>(1d) (2s)^2</u>

$$|ds^2;\ TLSJ\alpha\rangle = \frac{1}{\sqrt{3}}\left\{|s^2;\ to s\alpha\rangle_{12}\, |d\rangle_3 + (-)^P\, \mathbb{P}\right\}$$

$$\begin{pmatrix} \mu & t_z & T_z \\ \frac{1}{2} & t & T \end{pmatrix}\begin{pmatrix} \gamma & 0 & L_z \\ 2 & 0 & L \end{pmatrix}\begin{pmatrix} \beta & s_z & S_z \\ \frac{1}{2} & s & S \end{pmatrix}\begin{pmatrix} S_z & L_z & J_z \\ S & L & J \end{pmatrix}$$

(49c)

<u>(2s)^3</u>

$$|s^3;\ TL{=}0\,SJ\delta\rangle = \langle s^2;\ t\ell = 0\, s\alpha \| s^3;\ TL{=}0S\,\delta\rangle\ |s^2;\ t\ell = 0\, s\alpha\rangle_{12}\, |s\rangle_3$$

$$\begin{pmatrix} \mu & t_z & T_z \\ \frac{1}{2} & t & T \end{pmatrix}\begin{pmatrix} \beta & s_z & S_z \\ \frac{1}{2} & s & S \end{pmatrix}\begin{pmatrix} S_z & 0 & J_z \\ S & 0 & J \end{pmatrix}. \quad (49d)$$

The $(2s)^3$ state is unique. However, the phase of the FPC is not given in either of the previous works. The phase used here was chosen by analogy to the FPC of the similar configuration for $(1d)^3$. Thus, since $\langle {}^{13}D \| \} d^3 \, [3]\, {}^{22}S \rangle = -\frac{1}{\sqrt{2}}$ and $\langle {}^{31}D \| \} d^3 \, [3]\, {}^{22}S \rangle = +\frac{1}{\sqrt{2}}$, we have taken $\langle {}^{13}S \| \} s^3 [3]\, {}^{22}S \rangle = -\frac{1}{\sqrt{2}}$ and $\langle {}^{31}S \| \} s^3 [3]\, {}^{22}S \rangle = +\frac{1}{\sqrt{2}}$.

In the matrix elements, the particles within the closed shells can be ignored. The interaction Hamiltonian is then $\mathcal{H} = \sum_{i=1}^{3} \mathcal{H}_i$ for the three extra core nucleons only. A matrix element of this operator between a state in O^{19} and a state in F^{19} is

$$\langle O^{19} | \mathcal{H} | F^{19} \rangle = \sum_{\alpha\beta} 3 \langle O^{19} | \alpha \rangle \langle \beta \mid F^{19} \rangle \langle \alpha | \mathcal{H}_3 | \beta \rangle . \tag{50}$$

The basis states $|\alpha\rangle$, $|\beta\rangle$ and the expansion coefficients are given in Appendix VI. The following types of matrix elements of the single particle operator $\mathcal{H}_3$ occur:

a) $\langle d^3 | \mathcal{H}_3 | d^3 \rangle' = (\text{FPC})(\text{FPC})' \;\; {}_{12}\langle d^2 | d^2 \rangle'_{12} \;\; {}_3\langle d | \mathcal{H}_3 | d \rangle'_3$

b) $\langle d^2 s | \mathcal{H}_3 | d^3 \rangle' = \frac{1}{\sqrt{3}} (\text{FPC})' \;\; {}_{12}\langle d^2 | d^2 \rangle'_{12} \;\; {}_3\langle s | \mathcal{H}_3 | d \rangle'_3$

c) $\langle d^3 | \mathcal{H}_3 | d^2 s \rangle' = \frac{1}{\sqrt{3}} (\text{FPC}) \;\; {}_{12}\langle d^2 | d^2 \rangle'_{12} \;\; {}_3\langle d | \mathcal{H}_3 | s \rangle'_3$

d) $\langle d^2 s | \mathcal{H}_3 | d^2 s \rangle' = \frac{1}{3} \Big\{ {}_{12}\langle d^2 | d^2 \rangle'_{12} \;\; {}_3\langle s | \mathcal{H}_3 | s \rangle'_3 \; +$
$\qquad 2 \;\; {}_{13}\langle d^2 | \mathcal{H}_3 | d^2 \rangle'_{13} \;\; {}_2\langle s | s \rangle'_2 \Big\}$

e) $\langle ds^2|\mathcal{H}_3|d^2s\rangle' = \frac{2}{3} \langle s^2_{32}\, d_1|\mathcal{H}_3|d^2_{13}\, s_2\rangle'$

f) $\langle ds^2|\mathcal{H}_3|s^3\rangle' = \frac{1}{\sqrt{3}}\ (\mathrm{FPC})'_{12}\ \langle s^2|s^2\rangle'_{12}\ {}_3\langle d|\mathcal{H}_3|s\rangle'_3$.

In order to evaluate sums on magnetic quantum numbers in these matrix elements, expand $\mathcal{H}_3 = {}_L\mathcal{H}_{\mathcal{L}}^{m_\mathcal{L}}\ {}_s\mathcal{H}_K^{\mu}\ \tau^-$ where

${}_L\mathcal{H}_{\mathcal{L}}^{m_\mathcal{L}}$ operates only on space coordinates and is a spherical tensor of order $(\mathcal{L}, m_\mathcal{L})$,

${}_s\mathcal{H}_K^{\mu}$ operates only on spin coordinates and is a spherical tensor of order (K, μ).

Also introduce reduced matrix elements (see Appendix V) of these operators, for example the reduced matrix element $\langle \ell_f \| \mathcal{H}_\mathcal{L} \| \ell_i \rangle$ is defined by

$$\langle \ell_f\ m_f|{}_L\mathcal{H}_{\mathcal{L}}^{m_\mathcal{L}}|\ell_i\ m_i\rangle = \begin{pmatrix} m_i & m_\mathcal{L} & \big| & m_f \\ \ell_i & \mathcal{L} & \big| & \ell_f \end{pmatrix} \langle \ell_f \| \mathcal{H}_\mathcal{L} \| \ell_i \rangle . \tag{51}$$

Following are the various types of matrix elements occurring in the μ capture. Primed quantities refer to final states. Certain summations are implicit. (In all these a common term

$$(\hat{J}\,\hat{J}')^{\frac{1}{2}}\ (-)^{J-J_z} \begin{pmatrix} -J_z & J'_z & \big| & m_1 \\ J & J' & \big| & j_1 \end{pmatrix} \begin{pmatrix} \mu & m_\mathcal{L} & \big| & m_1 \\ K & \mathcal{L} & \big| & j_1 \end{pmatrix}$$

has been omitted. It will later go out in a sum over nuclear states.) In the previous order

$$\langle d^3 | \mathcal{H}_3 | d^3 \rangle = \langle d^2; t\ell \mathcal{A}\alpha \| d^3; TLS\mathcal{S} \rangle \langle d^2; t\ell\mathcal{A}\alpha \| d^3; T'L'S'\mathcal{S}' \rangle \times$$

$$(60\, \hat{T}\, \hat{S}\, \hat{L}\, \hat{S}'\hat{L}')^{\frac{1}{2}} (-)^{t+T+\mathcal{A}+\ell+L'+S'+1} \times$$

$$\left(\begin{matrix} T_z & -1 \\ T & 1 \end{matrix}\middle| \begin{matrix} T_z' \\ T \end{matrix}\right) \begin{pmatrix} T & 1 & T' \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \mathcal{A} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} L & \mathcal{L} & L' \\ 2 & \ell & 2 \end{pmatrix} \begin{pmatrix} S & L & J \\ S' & L' & J' \\ K & \mathcal{L} & j_1 \end{pmatrix} \times$$

$$\langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle d \| \mathcal{H}_{\mathcal{L}} \| d \rangle$$

(52a)

$$\langle d^2 s | \mathcal{H}_3 | d^3 \rangle = \frac{1}{\sqrt{3}} \langle d^2; t\ell\mathcal{A}\alpha \| d^3; TLS\mathcal{S} \rangle \times$$

$$(\tfrac{12}{5}\, \hat{T}\, \hat{S}\, \hat{L}\, \hat{S}')^{\frac{1}{2}} (-)^{t+T+\mathcal{A}+S'+\ell+L+1} \times$$

$$\left(\begin{matrix} T_z & -1 \\ T & 1 \end{matrix}\middle| \begin{matrix} T_z' \\ T' \end{matrix}\right) \begin{pmatrix} T & 1 & T \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \mathcal{A} & \frac{1}{2} \end{pmatrix} \delta_{\mathcal{L}2}\, \delta_{\ell L'} \begin{pmatrix} S & L & J \\ S' & L' & J' \\ K & \mathcal{L} & j_1 \end{pmatrix} \times$$

$$\langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle s \| \mathcal{H}_{\mathcal{L}} \| d \rangle$$

(52b)

$$\langle d^3 | \mathcal{H}_3 | d^2 s \rangle = \frac{1}{\sqrt{3}} \langle d^2; t\ell\mathcal{A}\alpha \| d^3; T'L'S'\mathcal{S}' \rangle \times$$

$$(\tfrac{12}{5}\, \hat{T}\, \hat{S}\, \hat{L}\, \hat{S}')^{\frac{1}{2}} (-)^{t+T+\mathcal{A}+S'+1} \times$$

$$\begin{pmatrix} T_z & -1 & T_z' \\ T & 1 & T' \end{pmatrix} \begin{pmatrix} T & 1 & T \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \mathcal{A} & \frac{1}{2} \end{pmatrix} \delta_{\mathcal{L}2}\, \delta_{\ell L} \begin{pmatrix} S & L & J \\ S' & L' & J' \\ K & \mathcal{L} & j_1 \end{pmatrix} \times$$

$$\langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle s \| \mathcal{H}_{\mathcal{L}} \| d \rangle$$

(52c)

In this case, there are two types of terms, s-s and d-d:

$$\langle d^2 s | \mathcal{H}_3 | d^2 s \rangle_{s-s} = \frac{1}{3} \left(\frac{12 \hat{S} \hat{S}' \hat{T}}{\hat{K}} \right)^{\frac{1}{2}} (-)^{t+T+\Delta+S+K+S'+L+J'+1} \times$$

$$\begin{pmatrix} T_z & -1 & \big| & T_z' \\ T & 1 & \big| & T' \end{pmatrix} \begin{pmatrix} T & 1 & T' \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \Delta & \frac{1}{2} \end{pmatrix} \begin{pmatrix} J & J' & K \\ S' & S & L \end{pmatrix} \times$$

$$\delta_{\ell L'} \; \delta_{\ell L} \quad \langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle s \| \mathcal{H}_{\mathcal{L}} \| s \rangle$$

(52d)

$$\langle d^2 s | \mathcal{H}_3 | d^2 s \rangle_{d-d} = \frac{2}{3} \left(60 \hat{t} \hat{t}' \hat{\Delta} \hat{\Delta}' \; \hat{S} \hat{S}' \; \hat{L} \hat{L}' \right)^{\frac{1}{2}} \times$$

$$(-)^{t+t'+T'+\Delta+\Delta'+S+L+K} \begin{pmatrix} T_z & -1 & \big| & T_z' \\ T & 1 & \big| & T' \end{pmatrix} \times$$

$$\begin{pmatrix} t & 1 & t' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} T & 1 & T' \\ t' & \frac{1}{2} & t \end{pmatrix} \begin{pmatrix} \Delta & K & \Delta' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \Delta' & \frac{1}{2} & \Delta \end{pmatrix} \begin{pmatrix} L & \mathcal{L} & L' \\ 2 & 2 & 2 \end{pmatrix} \times$$

$$\delta_{\ell' L'} \; \delta_{\ell L} \begin{pmatrix} S & L & J \\ S' & L' & J' \\ K & \mathcal{L} & j_1 \end{pmatrix} \langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle d \| \mathcal{H}_{\mathcal{L}} \| d \rangle$$

(52d')

$$\langle ds^2 | \mathcal{H}_3 | d^2 s \rangle = \frac{2}{3} \left(\frac{12}{5} \; \hat{t} \hat{t}' \hat{T} \; \hat{\Delta} \hat{\Delta}' \hat{S} \hat{S}' \hat{L} \right)^{\frac{1}{2}} \times$$

$$(-)^{t+T+\Delta+L+S'+L'+\mathcal{L}+1} \begin{pmatrix} T_z & -1 & \big| & T_z' \\ T & 1 & \big| & T' \end{pmatrix} \delta_{\ell L} \; \delta_{L' 2} \; \delta_{\mathcal{L} 2} \times$$

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & t' & T' \\ t & \frac{1}{2} & T \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & K \\ \frac{1}{2} & \Delta' & S' \\ \Delta & \frac{1}{2} & S \end{pmatrix} \langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle s \| \mathcal{H}_{\mathcal{L}} \| d \rangle$$

(52e)

and finally

$$\langle ds^2 | \mathcal{H}_3 | s^3 \rangle = \frac{1}{\sqrt{3}} \langle s^2 ; t\,o\,\Delta\,\kappa \,|\}\, s^3 ; T\,O\,S\,\mathcal{S} \rangle \times$$

$$\left(\frac{12}{5}\, \hat{T}\, \hat{S}\, \hat{S}'\hat{L}\right)^{\frac{1}{2}} (-)^{t+T+\Delta+S'+1} \begin{pmatrix} T_z & -1 & T_z' \\ T & 1 & T' \end{pmatrix} \times$$

$$\begin{pmatrix} T & 1 & T' \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \Delta & \frac{1}{2} \end{pmatrix} \delta_{\ell 2}\, \delta_{L0}\, \delta_{L'2} \begin{pmatrix} S & O & J \\ S' & L' & J' \\ K & \ell & j_1 \end{pmatrix} \times$$

$$\langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle s \| \mathcal{H}_\ell \| d \rangle .$$

(52f)

The Rate λ

Before we evaluate M^2 averaged over initial nuclear states and summed over final ones remove a factor $\frac{3}{\sqrt{2}}$ from each matrix element. Thus

$$f = \langle f | \sum_{i=1}^{3} e^{-iv\cdot r_i} \frac{\tau_i^-}{\sqrt{2}} | i \rangle$$

$$= \frac{3}{\sqrt{2}} \langle f | e^{iv\cdot r_3} \tau_3^- | i \rangle \rightarrow \frac{3}{\sqrt{2}} f .$$

Take this factor 9/2 right out of M^2. The μ capture Hamiltonian is now in the form of the operator $\mathcal{H}_3$. For the Fermi matrix element $K = 0$ and

$$f = (\hat{J}\,\hat{J}')^{\frac{1}{2}}\,(-)^{J-J_z} \begin{pmatrix} -J_z & J_z' \\ J & J' \end{pmatrix}\begin{matrix} \mathcal{M}_\mathcal{L} \\ \mathcal{L} \end{matrix}\Bigg) F_\mathcal{L}$$

. Averaged over Jz and summed over J_z' (with $\mathcal{M}_\mathcal{L}=0$ fixed)

$$f^2 = \hat{J}' \sum_\mathcal{L} F_\mathcal{L}^2 \quad . \tag{53}$$

$F_\mathcal{L}$ is to be evaluated from the above general matrix elements using

$$\langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle = 1 \quad \text{and} \quad \langle \ell_f \| \mathcal{H}_\mathcal{L} \| \ell_i \rangle = \langle \ell_f \| e^{iv\cdot r}{}_\mathcal{L} \| \ell_i \rangle .$$

The Gamow-Teller matrix element has K = 1,

$$g_\mu = (\hat{J}\,\hat{J}')^{\frac{1}{2}}\,(-)^{J-J_z} \begin{pmatrix} -J_z & J_z' & \Big| & m_1 \\ J & J' & \Big| & j_1 \end{pmatrix} \begin{pmatrix} \mu & \mathcal{M}_\mathcal{L} & \Big| & m_1 \\ 1 & \mathcal{L} & \Big| & j_1 \end{pmatrix} G_{j,\mathcal{L}}$$

and

$$g^2 = \hat{J}' \sum_{j,\mathcal{L}} \frac{\hat{j}_1}{\hat{\mathcal{L}}} G_{j,\mathcal{L}}^2 \quad . \tag{54}$$

For $\hat{v}\cdot g$, take $\hat{v}$ in the z direction so that $\mu = \mathcal{M}_\mathcal{L} = m_1 = 0$. Then

$$\hat{v}\cdot g = (\hat{J}\,\hat{J}')^{\frac{1}{2}}\,(-)^{J-J_z} \begin{pmatrix} -J_z & J_z' & \Big| & 0 \\ J & J' & \Big| & j_1 \end{pmatrix} \begin{pmatrix} 0 & 0 & \Big| & 0 \\ 1 & \mathcal{L} & \Big| & j_1 \end{pmatrix} G_{j,\mathcal{L}}$$

and the square is

$$|\hat{v}\cdot g|^2 = \hat{J}' \sum_{j_1} \left\{ \sum_\mathcal{L} \begin{pmatrix} 0 & 0 & \Big| & 0 \\ 1 & \mathcal{L} & \Big| & j_1 \end{pmatrix} G_{j,\mathcal{L}} \right\}^2 \quad . \tag{55}$$

The momentum type matrix element with K = 1 can be written (see Appendix V)

$$t_{\mu\nu} = (\hat{J}\,\hat{J}')^{\frac{1}{2}} (-)^{J-J_z} \begin{pmatrix} -J_z & J'_z & m_1 \\ J & J & j_1 \end{pmatrix} \begin{pmatrix} \nu & \mathcal{M}_{\mathcal{L}} & m_1 \\ 1 & \mathcal{L} & j_1 \end{pmatrix} \begin{pmatrix} \mu & 0 & \mathcal{M}_{\mathcal{L}} \\ 1 & L & \mathcal{L} \end{pmatrix} T_{\mathcal{L} L j_1}$$

and the contraction

$$t = (\hat{J}\,\hat{J}')^{\frac{1}{2}} (-)^{J-J_z} \begin{pmatrix} -J_z & J'_z & 0 \\ J & J' & L \end{pmatrix} (-)^{\mathcal{L}+L} \sqrt{\frac{\hat{\mathcal{L}}}{\hat{L}}}\, \delta_{j_1 L} \quad T_{\mathcal{L} L j_1} .$$

$T_{\mathcal{L} L j_1}$ involves the reduced matrix element $\langle \ell_f \| \mathcal{L}\, L \| \ell_i \rangle$, but aside from this difference has the same matrix elements as $G_{j_1 \mathcal{L}}$. The terms in λ involving t are

$$t^2 = \hat{J}' \sum_L \left\{ \sum_{\mathcal{L} j_1} (-)^{\mathcal{L}} \sqrt{\frac{\hat{\mathcal{L}}}{\hat{L}}} \quad \delta_{j_1 L} \quad T_{\mathcal{L} L j_1} \right\}^2 \tag{56}$$

and

$$\mathrm{Re}\, \hat{v}\cdot g\, t^* = \hat{J}'\, \mathrm{Re} \sum_{L j_1} (-)^L \quad \delta_{j_1 L} \left\{ \sum_{\mathcal{L}'} \begin{pmatrix} 0 & 0 & 0 \\ 1 & \mathcal{L}' & j_1 \end{pmatrix} \quad G_{j_1 \mathcal{L}'} \right\} \times \tag{57}$$

$$\left\{ \sum_{\mathcal{L}} (-)^{\mathcal{L}} \sqrt{\frac{\hat{\mathcal{L}}}{\hat{L}}} \quad T^*_{\mathcal{L} L j_1} \right\} .$$

For the momentum type matrix element with K = 0 we can write

$$q_\mu = (\hat{J}\,\hat{J}')^{\frac{1}{2}} (-)^{J-J_z} \begin{pmatrix} -J_z & J'_z & m_1 \\ J & J' & j_1 \end{pmatrix} \begin{pmatrix} 0 & \mathcal{M}_{\mathcal{L}} & m_1 \\ 0 & \mathcal{L} & j_1 \end{pmatrix} \begin{pmatrix} \mu & 0 & \mathcal{M}_{\mathcal{L}} \\ 1 & L & \mathcal{L} \end{pmatrix} Q_{\mathcal{L} L j_1} .$$

$Q_{\mathcal{L} L j_1}$ involves the reduced matrix element $\langle \ell_f \| \mathcal{L}\, L \| \ell_i \rangle$ but otherwise has the same matrix elements as $F_{\mathcal{L}}$. The three terms required are

$$q^2 = \hat{j}' \sum_{\mathcal{L} L j_1} \frac{\hat{\mathcal{L}}}{\hat{L}} \delta_{j_1 \mathcal{L}} Q^2_{\mathcal{L} L j_1} , \tag{58}$$

$$\mathrm{Re}\, f\, \hat{v}\cdot q^* = \hat{j}'\, \mathrm{Re} \sum_{\mathcal{L} L j_1} F_{\mathcal{L}} \left\{ \begin{pmatrix} 0 & 0 \\ 1 & L \end{pmatrix}\!\!\left|\begin{matrix} 0 \\ \mathcal{L} \end{matrix}\right) Q^*_{\mathcal{L} L j_1} \right\} \delta_{j_1 \mathcal{L}} , \tag{59}$$

and finally

$$\mathrm{Im}\; \hat{v}\cdot g \times q^* = \hat{j}'\; \mathrm{Re} \sum_{L \mathcal{L} \mathcal{L}' j_1} \sqrt{2}\; \hat{j}_1 (-)^{\mathcal{L}+L+j_1+1} \delta_{j_1 \mathcal{L}'} \times \tag{60}$$

$$\begin{pmatrix} L & \mathcal{L} & 1 \\ 1 & 1 & j_1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ L & \mathcal{L} \end{pmatrix}\!\!\left|\begin{matrix} 0 \\ 1 \end{matrix}\right) G_{j_1 \mathcal{L}} Q^*_{\mathcal{L}' L j_1'} .$$

Numerical Results

a) $1/2\; 1/2^+$ to $3/2\; 1/2^+$ transition: The matrix elements required here are F_0 (and Q_{010} which differs only in the reduced matrix element), G_{10} (and T_{011}), G_{12} (and T_{211}), T_{111} and Q_{111}. These have been evaluated using the states in Appendix VI. Numerical results are presented in Appendix VII.

b) $1/2\; 1/2^+$ to $3/2\; 3/2^+$ transition: Here we need the matrix elements F_2 (and Q_{212}, Q_{232}), G_{10} (and T_{011}), G_{12} (and T_{211}), G_{22}, T_{111} and Q_{111}. Results are presented in Appendix VII.

c) $1/2\; 1/2^+$ to $3/2\; 5/2^+$ transition: The matrix elements that enter are F_2 (and Q_{212}, Q_{232}), G_{22}, G_{32} (and T_{233}), G_{34} (and T_{433}), T_{333} and Q_{333}. These results too are presented in Appendix VII.

In addition to the individual matrix elements, Appendix VII contains also the separate terms in the rate λ summed over the three bound state transitions. The coupling constants for the various hypotheses are in Appendix VII, table 2, and the rate λ calculated with these is listed in Appendix VII, table 3.

Comparison with the Single Particle Model

It is interesting to know just how closely the single particle model corresponds to the shell model for the A=19 nuclei. In the single particle model, the A =19 nuclei are considered as a single particle coupled to an (L = O, S = O, T = 1, Tz = -1) A = 18 core. The $J = 1/2^+$ states are supposed to be a 2 $s_{1/2}$ single particle and the $J = 3/2^+, 5/2^+$ states are 1 $d_{3/2, 5/2}$ single particle states. We will calculate the μ capture matrix elements on this simple model. The matrix element of a single particle operator $\mathcal{H} = {}_L\mathcal{H}_{\mathcal{L}}^{m_{\mathcal{L}}}{}_s\mathcal{H}_K^{\mu}\frac{\tau^-}{\sqrt{2}}$ between these single particle states (again omitting a factor

$$(\hat{J}\,\hat{J}')^{\frac{1}{2}}(-)^{J-J_z}\begin{pmatrix}-J_z & J_z & \Big| & m_1 \\ J & J' & \Big| & j_1\end{pmatrix}\begin{pmatrix}\mu & m_{\mathcal{L}} & \Big| & m_1 \\ K & \mathcal{L} & \Big| & j_1\end{pmatrix}$$

) is

$$\langle f|\mathcal{H}|i\rangle = \sqrt{\frac{2}{3\hat{L}_f}}\ \delta_{J,1/2}\ \delta_{\mathcal{L}L_f}\begin{pmatrix}J & J' & j_1 \\ \mathcal{L} & K & \frac{1}{2}\end{pmatrix}(-)^{K+\mathcal{L}+j_1}\ \times \tag{61}$$

$$\langle 0\|\mathcal{H}_{\mathcal{L}}\|L_f\rangle\langle 1/2\|\mathcal{H}_K\|1/2\rangle.$$

The matrix elements for μ capture in this model are listed in Appendix VII. A factor $\left(3/\sqrt{2}\right) \times 10^{-2}$ has been taken out as in the previous work.

There are a number of major differences in the predictions of these two models.

a) In the $1/2\ 1/2^{+}$-$3/2\ 1/2^{+}$ transition, the single particle model does not (in fact, cannot, as it is constructed) reflect the isotopic spin inhibition of the Fermi matrix element F_o, which the full shell model does quite explicitly. Neither does the single particle model reproduce the cancellations which in the shell model results in a much reduced Gamow-Teller matrix element G_{10}, so even if we were to arbitrarily drop the Fermi term, the single particle model results would in no way correspond to those of the shell model for this transition.

b) In the $1/2\ 1/2^{+}$-$3/2\ 3/2^{+}$ transitions, the matrix elements F_2, G_{12} and G_{22} correspond fairly well (except for a common sign) in the two models. G_{10}, the dominant matrix element in the shell model, does not occur in the single particle model.

c) For the $1/2\ 1/2^{+}$-$3/2\ 5/2^{+}$ transitions, the predictions of the two models differ almost by an order of magnitude. We have to conclude a posteriori that the single particle model is not adequate and the shell model is required. It is true, of course, that the amount of work required in the shell model calculation could be greatly reduced by keeping only the dominant states.

The Rate λ_D

In order to correct for the HFS effect and for transfer between the HFS states, we need to know

$$\lambda_D = \lambda_c^- - \lambda_c^+ = -\frac{8}{3}\left\{ \frac{1}{4} \langle f | \mathcal{H} \sigma_L \cdot J | i \rangle \langle i | \mathcal{H} | f \rangle \right\}. \qquad (46b)$$

In the previous notation (Eq. 26), this rate is

$$-\frac{3}{8}(2\ \lambda_D) = \frac{1}{2}\ \mathrm{tr}\ (1 - \sigma_L \cdot \nu)(A + \sigma_L \cdot B)\ \sigma_L \cdot J(A^* + \sigma_L \cdot B^*)$$

$$= 2\,\mathrm{Re}\,A\ J \cdot B^* + i\ B \times J \cdot B^* \qquad (62)$$

where nuclear matrix elements are implicit and pseudoscalar terms have been dropped. The terms that enter $2\,\mathrm{Re}\,AJ \cdot B^*$, in terms of the nuclear matrix elements previously defined, are:

$$2\,\mathrm{Re}\,G_V\ G_A\, f\, J \cdot g^* = 2\,\mathrm{Re}\,G_V G_A (-)^{J'+J+1+j_1'}\ \hat{J}'\,\hat{j}_1 \times$$

$$\sqrt{\frac{3}{2\hat{\mathcal{L}}}} \begin{pmatrix} \mathcal{L} & j_1' & 1 \\ J & J & J' \end{pmatrix} \delta_{\mathcal{L}\mathcal{L}'}\ F_{\mathcal{L}}\ G^*_{j_1'\mathcal{L}'} \qquad (63a)$$

$$-2\,\mathrm{Re}\,G_V\ G_P\ \ f\, J \cdot \hat{v}(\hat{v} \cdot g)^* = 2\,\mathrm{Re}\,G_V\ G_p (-)^{J+J'+j_1'}\ \hat{J}' \sqrt{\frac{3}{2}\ \hat{j}_1'} \times$$

$$\left(\begin{matrix} 0 & 0 \\ 1 & \mathcal{L} \end{matrix}\middle|\begin{matrix} 0 \\ j_1' \end{matrix}\right) \left(\begin{matrix} 0 & 0 \\ 1 & \mathcal{L}' \end{matrix}\middle|\begin{matrix} 0 \\ j_1' \end{matrix}\right) \begin{pmatrix} \mathcal{L} & j_1' & 1 \\ J & J & J' \end{pmatrix} F_{\mathcal{L}}\ G^*_{j_1'\mathcal{L}'} \qquad (63b)$$

$$2\,\mathrm{Re}\,G_V^2\, f\, J \cdot i\, (\hat{v} \times q)^* = 2\,\mathrm{Re}\,G_V^2\, (-)^{J+J'+\mathcal{L}}\ 3\,\hat{J}'\,\hat{j}'\ \ \delta_{j'\mathcal{L}'} \times$$

$$\begin{pmatrix} \mathcal{L} & j_1' & 1 \\ J & J & J' \end{pmatrix} \begin{pmatrix} j' & L' & 1 \\ 1 & 1 & \mathcal{L} \end{pmatrix} \left(\begin{matrix} 0 & 0 \\ 1 & L' \end{matrix}\middle|\begin{matrix} 0 \\ \mathcal{L} \end{matrix}\right) F_{\mathcal{L}}\ Q^*_{\mathcal{L}'L'j_1'} \qquad (63c)$$

$$-2\,\mathrm{Re}\;\; G_V\, g_A\, f\, \mathbf{J}\cdot\hat{v}\, t^* = 2\,\mathrm{Re}\; G_V\, g_A\, (-)^{J+J'+1+\mathcal{L}'}\;\hat{J}' \qquad \times$$

$$\sqrt{\tfrac{3}{2}\hat{\mathcal{L}}'}\;\; \begin{pmatrix} \mathcal{L} & L' & 1 \\ J & J & J' \end{pmatrix} \delta_{j_1' L'}\;\; F_{\mathcal{L}}\;\; T^*_{\mathcal{L}' L' j_1'} \qquad \text{(63d)}$$

$$-2\,\mathrm{Re}\; G_V\;\; G_A\;\; \hat{v}\cdot q\;\; \mathbf{J}\cdot \mathbf{g}^* = 2\,\mathrm{Re}\; G_V\;\; G_A\;\; (-)^{J+J'+j_1'}\;\; \sqrt{\tfrac{3}{2}}\;\hat{J}'\,\hat{j}_1' \qquad \times$$

$$\begin{pmatrix} j_1 & j_1' & 1 \\ J & J & J' \end{pmatrix} \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & L & j_1 \end{array}\right) \delta_{j_1 \mathcal{L}}\;\; \delta_{j_1 \mathcal{L}'}\;\;\; Q_{\mathcal{L} L j_1}\, G^*_{j_1' \mathcal{L}'}$$

$$\text{(63e)}$$

$$2\,\mathrm{Re}\;\; G_V\, G_P\;\; \hat{v}\cdot q\;\;\; \mathbf{J}\cdot\hat{v}\;(\hat{v}\cdot \mathbf{g})^* = 2\,\mathrm{Re}\; G_V\;\; G_P\;\; (-)^{J+J'+1+j_1'}\;\hat{J}'\;\sqrt{\tfrac{3}{2}\;\;\hat{j}_1} \qquad \times$$

$$\begin{pmatrix} j_1 & j_1' & 1 \\ J & J & J' \end{pmatrix} \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & L & j_1 \end{array}\right) \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & \mathcal{L}' & j_1' \end{array}\right) \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & j_1 & j_1' \end{array}\right) \times$$

$$\delta_{j_1 \mathcal{L}} \qquad Q_{\mathcal{L} L j_1}\;\; G^*_{j_1' \mathcal{L}'} \qquad \text{(63f)}$$

$$-2\,\mathrm{Re}\; G_V^2\;\; \hat{v}\cdot q\;\; \mathbf{J}\cdot i\;(\hat{v}\times q)^* = 2\,\mathrm{Re}\; G_V^2\;\; (-)^{J+J'+L'}\;\; 3\hat{J}'\;\hat{j}_1' \qquad \times$$

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & L & j_1 \end{array}\right) \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & L' & j_1' \end{array}\right) \begin{pmatrix} j_1 & j_1' & 1 \\ J & J & J' \end{pmatrix} \begin{pmatrix} j_1 & j_1' & 1 \\ 1 & 1 & L \end{pmatrix} \times$$

$$\delta_{j_1 \mathcal{L}}\;\;\; \delta_{j_1' \mathcal{L}'}\;\;\; Q_{\mathcal{L} L j_1}\;\;\; Q^*_{\mathcal{L}' L' j_1'} \qquad \text{(63g)}$$

$$2\,\mathrm{Re}\; G_V\;\; g_A\;\;\; \hat{v}\cdot q\;\; \mathbf{J}\cdot\hat{v}\;\; t^* = 2\,\mathrm{Re}\; G_V\; g_A\; (-)^{J+J'+\mathcal{L}'}\;\hat{J}'\;\sqrt{\tfrac{3}{2}\,\hat{\mathcal{L}}'} \qquad \times$$

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & L & j_1 \end{array}\right) \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & L' & j_1 \end{array}\right) \begin{pmatrix} j_1 & j_1' & 1 \\ J & J & J' \end{pmatrix} \qquad \times$$

$$\delta_{j_1 \mathcal{L}} \qquad \delta_{j_1' L'} \qquad Q_{\mathcal{L} L j_1}\;\;\; T^*_{\mathcal{L}' L' j_1'} \qquad \text{(63h)}$$

In $i\mathbf{B} \times \mathbf{J}\cdot\mathbf{B}^*$ the following terms are required:

$$i\,G_A^2 \;\; \mathbf{g}\times\mathbf{J}\cdot\mathbf{g}^* = G_A^2\;(-)^{J'+J+\mathcal{L}}\;\frac{3\hat{j}_1\,\hat{j}_1'}{\hat{\mathcal{L}}}\,\hat{J}'\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix}\begin{pmatrix} j_1 & j_1' & 1\\ 1 & 1 & \mathcal{L}\end{pmatrix}\times$$

$$\delta_{\mathcal{L}\mathcal{L}'}\quad G_{j_1\mathcal{L}}\quad G^*_{j_1'\mathcal{L}'} \tag{64a}$$

$$i\,G^2\,(\hat{v}xq)\times\mathbf{J}\cdot(\hat{v}xq)^* = G_V^2\;(-)^{J+J'+1+L+j_1'}\;18\,\hat{j}_1\,\hat{j}_1'\,\hat{J}'\hat{K}\quad\delta_{j_1\mathcal{L}}\quad\delta_{j_1'\mathcal{L}'}\quad\times$$

$$\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix}\begin{pmatrix} 0 & 0 & 0\\ L & L' & t\end{pmatrix}\begin{pmatrix} 0 & 0 & 0\\ 1 & 1 & t\end{pmatrix}\times$$

$$\begin{pmatrix} 1 & 1 & 1\\ 1 & j_1' & j_1\\ 1 & K & L\end{pmatrix}\begin{pmatrix} K & 1 & L\\ j_1' & 1 & L'\\ 1 & 1 & t\end{pmatrix}\quad Q_{\mathcal{L}Lj_1}\quad Q^*_{\mathcal{L}'L'j_1'}$$

$$\tag{64b}$$

$$2\,\mathrm{Re}\left\{-i\,G_A\,G_p\,(gxJ)\cdot\hat{v}(\hat{v}\cdot g)^*\right\} = 2\,\mathrm{Re}\,G_A\,G_p\;3\,\hat{J}'\,\hat{j}_1(-)^{J+J'+j'}\qquad\times$$

$$\begin{pmatrix} 0 & 0 & 0\\ 1 & \mathcal{L} & j_1'\end{pmatrix}\begin{pmatrix} 0 & 0 & 0\\ 1 & \mathcal{L}' & j_1'\end{pmatrix}\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix}\begin{pmatrix} j_1 & j_1' & 1\\ 1 & 1 & \mathcal{L}\end{pmatrix}\times$$

$$G_{j_1\mathcal{L}}\quad G^*_{j_1'\mathcal{L}'} \tag{64c}$$

$$-2\,\mathrm{Re}\,G_V\,G_A(gxJ)\cdot(\hat{v}xq)^* = 2\,\mathrm{Re}\,G_V\,G_A\;(-)^{J+J'+1+j_1'}\;3\,\hat{j}\;\hat{j}'\;\hat{J}'\qquad\times$$

$$\sqrt{\frac{3\,\hat{J}}{\hat{\mathcal{L}}}}\begin{pmatrix} 0 & 0 & 0\\ 1 & L' & \mathcal{L}\end{pmatrix}\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix}\begin{pmatrix} j_1 & j_1' & 1\\ 1 & 1 & \mathcal{L}\end{pmatrix}\begin{pmatrix} 1 & 1 & 1\\ j_1' & L' & \mathcal{L}\end{pmatrix}\times$$

$$\delta_{j_1'\mathcal{L}'}\qquad G_{j_1\mathcal{L}}\quad Q^*_{\mathcal{L}'L'j_1'} \tag{64d}$$

$$2\,\mathrm{Re}\left\{-i\,G_A\,g_A(g\times J)\cdot\hat{v}\,t^*\right\} = 2\,\mathrm{Re}\,G_A\,g_A\,(-)^{J+J'+1+\mathcal{L}}\,3\,\hat{J}'\hat{j}_1 \times$$

$$\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix}\begin{pmatrix} j_1 & j_1' & 1\\ 1 & 1 & \mathcal{L}\end{pmatrix}\left(\begin{array}{cc|c}0&0&0\\1&L'&\mathcal{L}\end{array}\right)\delta_{j_1'L'} \times$$

$$G_{j_1\mathcal{L}}\quad T^*_{\mathcal{L}'L'j_1'} \qquad\qquad (64e)$$

$$2\,\mathrm{Re}\left\{G_P\,G_V(\hat{v}\cdot g\,\hat{v}xJ)\cdot(\hat{v}xq)^*\right\} = 2\,\mathrm{Re}\,G_P\,G_V\,(-)^{1+j_1'+J+J'} \times$$

$$3\,\hat{J}'\hat{j}_1'\,\sqrt{6\,\hat{j}_1}\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix}\left(\begin{array}{cc|c}0&0&0\\1&\mathcal{L}&j_1\end{array}\right) \times$$

$$\left(\begin{array}{cc|c}0&0&0\\1&j_1&K\end{array}\right)\left(\begin{array}{cc|c}0&0&0\\1&L'&K\end{array}\right)\begin{Bmatrix}1&1&1\\1&j_1'&j_1\\1&L'&K\end{Bmatrix}\delta_{j_1'\mathcal{L}'} \times$$

$$G_{j_1\mathcal{L}}\quad Q^*_{\mathcal{L}'L'j_1'} \qquad\qquad (64f)$$

and finally

$$-2\,\mathrm{Re}\left\{G_V\,g_A\,(\hat{v}xq)x\,J\cdot\hat{v}\,t^*\right\} = 2\,\mathrm{Re}\,G_V\,g_A\,(-)^{J+J'+\mathcal{L}'} \times$$

$$3\,\hat{J}'\hat{j}_1\,\sqrt{6\,\hat{\mathcal{L}}'}\;\delta_{j_1'L'}\begin{pmatrix} j_1 & j_1' & 1\\ J & J & J'\end{pmatrix} \times$$

$$\left(\begin{array}{cc|c}0&0&0\\1&L&K\end{array}\right)\left(\begin{array}{cc|c}0&0&0\\1&j_1'&K\end{array}\right)\begin{Bmatrix}1&1&1\\1&j_1'&j_1\\1&K&L\end{Bmatrix} \times$$

$$Q_{\mathcal{L}Lj_1}\quad T^*_{\mathcal{L}'L'j_1'}\,. \qquad\qquad (64g)$$

In all these, primed quantities refer to the final state, and unprimed to the initial state and summation over repeated indices is implicit.

Numerical results for these terms are given in Appendix VII, Table 1. The rate λ_D according to the various hypotheses appears in Appendix VII, Table 3.

An examination of these results indicates that λ_D is dominated by two terms. The first is the Gamow-Teller term G_{10} leading to capture to the 3/2 $3/2^+$ state from the triplet HFS state. This term is compensated by a large cross term between G_{10} and Q_{111}. The rate λ_D is never larger than one half λ, so the HFS and transfer correction to the rate, using the estimates for δ and λ_T in the preliminary discussion, is going to be in the neighborhood of ten percent.

Results and Discussion

The capture rate in units of the decay rate of the μ is

$$\frac{\Gamma_c}{\Gamma_\mu} = 96\,\pi\left(\frac{v}{\mu}\right)^2 (Ze^2)^3 (2\lambda) \quad . \tag{65}$$

According to the numerical results of Ford and Flamand (5), $\varphi_\mu^2(0)$ should be reduced from its nominal value by 8% due to the effect of finite nuclear size. We also reduce the rate another 5% to account for the momentum dependence of the coupling constants. Then

$$\frac{\Gamma_c}{\Gamma_\mu} = 0.306 \times 10^{-4} (2\lambda) .$$

For example, for $2\lambda = 100$ this gives a capture rate to the bound states of O^{19} of $\Gamma_c = 1.35 \times 10^3 \text{ sec}^{-1}$.

We are now faced with the difficult task of estimating the uncertainty in our result. One source, the HFS effect and the transfer effect, can be corrected for. The dependence on the nuclear radius is slight since the dominant matrix element is G_{10}. The troublesome source of uncertainty is the shell model calculation itself. There is the ground state magnetic moment of F^{19} to check against experiment, and as well there are three β decay rates in the A = 19 system which we can compare to determine the effectiveness of the shell model description of these states.

The magnetic moment predicted is 2.80 - 2.87 nuclear magnetons compared to the experimental value of 2.63. This is an error of 7% in a matrix element, or 14% in a rate. A serious feature of this result is that the calculated value equals the experimental value for no values of the nuclear force parameter V_c.

The Ne^{19}-F^{19} β decay, a superallowed transition, agrees well with its shell model value. The shell model result is slowly varying and never differs by more than 20% from its experimental value for $V_c = 0$ to 50 Mev. At $V_c = 30$ Mev the error is -12%.

The O^{19}-F^{19} (3/2) β decay rate predicted varies from log ft = 3.4 at $V_c = 0$ to log ft = 4.7 at $V_c = 50$ Mev. The predicted rate agrees with experiment at V_c = 25 Mev and is 15% high at V_c = 30 Mev.

The O^{19}-F^{19} (5/2) β decay rate predicted varies from log ft = 4.2 to log ft = 8 as V_c varies from 0 to 50 Mev. The shell model prediction agrees with experiment at V_c = 30 Mev.

There are some encouraging features about these results. In the first place the predicted results are within 15% of the experimental results for all rates at V_c = 30 Mev. This gives us a feeling that an error estimate of 15-20% in our results at V_c = 30 Mev is reasonable. The μ capture rate λ, however, is remarkably independent of V_c, and since we expect the theoretical value to intersect the experimental one for some value of V_c, we could be even more conservative in our error estimates. However, we will consider a possible error of 20% in the rate as a liberal estimate.

A 20% uncertainty in our results completely obscures the difference between the assumptions of a CVC or not. However, the difference between the various hypotheses concerning the induced pseudoscalar still remain. If we include a 10% experimental error, or a total uncertainty of 30%, then the predictions are

for g_P = +8 g_A	2λ = 65-140
-8 g_A	200-350
0	120-230 .

Concerning the individual matrix elements involved, there are some points worth closer examination. First, the Fermi matrix element F_o in the $1/2\ 1/2^+ - 3/2\ 1/2^+$ transition is greatly reduced over its single particle value due to the isotopic spin change inhibition as we expect. A surprising result which can be understood in retrospect is the drastic cancellation in the Gamow-Teller matrix element G_{10} in this transition. An examination of the dominant space symmetries in the shell model description of the A = 19 states (17) reveals the explanation for this reduction of G_{10}. The dominant (90%) space symmetry of the T = 1/2 states of A = 19 is the symmetry [3] (i.e., a space wave function totally symmetric under interchange of any two particles), whereas the dominant (again close to 90%) space symmetry of the T = 3/2 states is the space symmetry [2 1] (i.e., a space wave function symmetric under interchange of particles one and two, but antisymmetric under interchange of one and three or two and three). States of different space symmetry are of course orthogonal, so that the matrix element $\langle 1 \rangle$ between such states vanishes. This gives us a more detailed understanding of the mechanism of the isotopic spin change inhibition of the matrix element F_o. But now we can understand also the reduction of G_{10}, which is essentially the matrix element $\langle \sigma\, j_o(\nu r) \rangle$, or, to the extent that $\langle j_o(\nu r) \rangle$ is factorable, it is just the matrix element $\langle \sigma \rangle$ which vanishes between states of different space symmetry.

The $T = 3/2$, $J = 3/2^+$, $5/2^+$ states also belong predominantly to the space symmetry type [21] . This gives us an explanation of the reduction from the single particle value of the matrix element G_{10} in the $1/2^+ - 3/2^+$ transition. The small value of other matrix elements can also be explained by this inability fo span different space symmetry types. Even for matrix elements with $\ell = 2$, $\langle j_2(\nu r)\rangle$ is roughly independent of basis states and can in a crude sense be factored leaving a vanishing matrix element $\langle 1 \rangle$. (This same "unfavored" character of the matrix elements between $T = 3/2$ and $T = 1/2$ states is responsible for the slowness of the allowed β decays from O^{19} to F^{19}.)

This reduction in the μ capture matrix elements due to their unfavored nature is fortunately not sensitive to the assumptions concerning the nuclear force strength parameter V_c, so does not cause a wide variation in the predicted capture rate for $V_c = 30$, 40 and 50 Mev. It is true that the symmetry of these states, and hence the cancellations in the matrix elements, might be sensitive to the exchange mixture used in the nuclear force. The only reassurance we have on this score is the success of the Elliott-Flowers wave functions (which use a Rosenfeld mixture) in predicting the unfavored β decay rates.

There are two features of λ_D that deserve comment, although there is no clear explanation for them. The first feature is the sensitive dependence of λ_D on the nuclear force parameter V_c. Fortunately, the rate λ does not have such a wide variation. The second feature is the fact that λ_D does not demonstrate the dominance of the capture by the F = 0 state which one would expect for capture by a single proton (and which is demonstrated by the total capture rate to all final states (25)). The reason for this must be in a secondary effect overwhelming the primary HFS effect. The mechanism that must be responsible is an exclusion principle effect inhibiting creation of a neutron in a bound state from μ capture in the F = 0 state.

Conclusion

If we can believe the shell model within twenty percent, and the β decay rates indicate that we can, then the capture rate to the bound states of O^{19} provides a distinction between the hypotheses concerning the induced pseudoscalar interaction.

It is clear of course that a work such as this which is so dependent upon a model cannot provide a definitive test of the coupling involved. Instead, the interest in this work must be from the point of view of nuclear physics rather than particle physics, so that eventually with a well established μ capture interaction, these results will provide a test of the shell model and in fact supplement the study begun by Elliott and Flowers.

He^3 Introduction

An example of μ capture which has gained increased interest recently is μ capture in He^3 to the groundstate of H^3(26). The wave functions of the A = 3 groundstates are known with some certainty (27), so that the capture rate can be predicted with considerable confidence. This problem has in fact been treated before (28). Our results are in close agreement with the previous treatment. The purpose of this discussion will be to complete the previous treatment by including momentum type matrix elements and L = 2 matrix elements previously omitted. It turns out that these terms, which we might expect to make approximately ten percent contributions, in fact only amount to about two percent of the rate. In addition to these slight improvements we discuss the HFS effect (3) in μ capture by He^3, an even Z nucleus.

Experimentally this case poses a difficult problem, that of determining the counting efficiency with which the inverse β decay electrons are detected. The maximum electron energy in the H^3-He^3 transition is only 53 kev. The long half life (12.4 years) makes it possible to detect the inverse β decay away from the μ beam background, but also serves to reduce the counting rate.

On the theoretical side there are a number of uncertainties in the predicted capture rate, none of which appear to preclude about five percent accuracy of our results however. These are the mean square radius (and to a lesser extent the shape) of the charge distribution of the A = 3 bound states, the wave functions of the A = 3 groundstates (which may have D state admixtures due to the tensor force as in the case of the deuteron), meson exchange effects and the HFS effect.

Information on the charge radius of the A = 3 system is available from the coulomb energy difference between H^3 and He^3 which is 0.76 Mev and can be completely ascribed to the coulomb energy. This gives $R = 1.6/\mu = 2.24$ fermis for a uniform spherical distribution. Werntz (28) gives an extensive discussion on the effect of various shapes and sizes of charge distribution. Here we will content ourselves with using the above as a central value for the root mean square radius to determine the length parameter in harmonic oscillator wave functions with which we will evaluate radial matrix elements.

The He^3, H^3 groundstates are $T = 1/2$, $J = 1/2^+$ mirror states for which the only compatible assignments are ${}^{22}S_{1/2}$, ${}^{22}P_{1/2}$, ${}^{24}P_{1/2}$ and ${}^{24}D_{1/2}$. Verde (27) summarizes arguments which conclude that a groundstate wavefunction which is 96% ${}^{22}S_{1/2}$ (symmetric in space coordinates, totally antisymmetric in spin and isotopic spin coordinates) plus 4% ${}^{24}D_{1/2}$ can explain the groundstate magnetic moments and (by the exclusion of the P states) the anomalously small deuteron neutron and proton capture cross sections to these states. We will use this assignment. A difference between our treatment and that of

Werntz will be the fact that cross terms between these S and D components do occur in the capture rate, so that the unknown phase of the D state admixture introduces an uncertainty in the capture rate (which amounts to only one percent however).

Meson exchange effects are presumed to explain three features of the A = 3 system; the discrepancy of 0.27 which still remains in the magnetic moments even with a 4% ${}^{24}D_{1/2}$ admixture (which corrects the sum of the magnetic moments of H^3 and He^3 but not the individual moments), the neutron capture cross section (which would vanish for a pure ${}^{22}S_{1/2}$ (space symmetric) state, and cannot be explained by an electric quadrupole matrix element to the ${}^{24}D_{1/2}$ component of the groundstate), and also the ft value for the superallowed H^3 to He^3 β decay (29) which deviates by 2% from that for neutron decay. Primakoff (8) argues that meson exchange currents will make contributions of the order of 1% to the capture rate, but this remains to be convincingly demonstrated. We will ignore these effects here.

The HFS effect exists for μ capture by He^3 due to a mechanism indirectly related to that discussed by Primakoff (3). Consider a simple model. Let ↑ (↑ ↑) be a neutron (proton, μ meson) with spin up. Then for capture in the F = 0 state the configuration which is antisymmetric for like particles is (↓ ↑ ↑ ↓) and for capture in the F = 1 configuration it is (↓ ↑ ↑ ↑). For a V-A interaction neglect μ capture by a proton with parallel spin. Then, if A is the spin-flip amplitude for μ capture by a proton with opposite spin, and B is the non-spin-flip amplitude we have symbolically

$$(\bar{\downarrow} \; \underline{\uparrow} \; \underset{\bullet}{\uparrow} \; \downarrow) = A (\bar{\downarrow} \; \overset{\bullet}{\downarrow} \; \underset{\bullet}{\uparrow}) + \nu$$

$$(\bar{\downarrow} \; \underline{\uparrow} \; \underset{\bullet}{\uparrow} \; \uparrow) = B (\overset{\bullet}{\downarrow} \; \underline{\uparrow} \; \underset{\bullet}{\uparrow}) + \nu \quad ,$$

and there results a hyperfine effect if $B^2 \neq A^2$.

For a pure V-A interaction the spin-flip and non-spin-flip rates are equal, and there would be no HFS effect. For a renormalized V-A interaction we expect no more than, say, twenty percent difference between the $F = 0$ and $F = 1$ rates. In addition, for He^3 there does not exist a mechanism to disturb the statistical population of the HFS levels in a time comparable to the lifetime of the μ meson against decay. The mechanism responsible in hydrogen is proton exchange in close collisions between neutral $(\mu - p)$ atoms and protons. This mechanism is not important here because the coulomb barrier prevents $(\mu\text{-}He^3)$ atoms making close collisions with He^3 nuclei, the overlap of the nuclear wave functions is small and the exchange amplitude reduced.

Wave Functions and Matrix Elements

Here we can borrow the notation and techniques used in the shell model discussion of F^{19}. From equation(49d)we have the wave function for a state $^{22}S_{1/2}$ [3] (i.e., a state symmetric in space, antisymmetric in spin and isotopic spin),

$$\frac{^{22}S_{1/2}}{\left| s^3; T\, L = O\, S\, J\, \delta \right\rangle} = \left\langle s^2; t\, \mathcal{s}; \alpha \right|\left\} s^3; T\, S; \delta \right\rangle \; x$$

$$\begin{pmatrix} \alpha & t_z & T_z \\ \frac{1}{2} & t & T \end{pmatrix} \begin{pmatrix} \beta & \mathcal{s}_z & S_z \\ \frac{1}{2} & \mathcal{s} & S \end{pmatrix} \begin{pmatrix} S_z & 0 & J_z \\ S & 0 & J \end{pmatrix} \quad x$$

$$\left| s^2; t\, \mathcal{s}\, \alpha \right\rangle_{12} \left| s \right\rangle_3 \qquad (66)$$

where $T = S = J = 1/2$

$T_z = +1/2$ for He^3, $-1/2$ for H^3

and the FPC are $-\frac{1}{\sqrt{2}}$ for $t = 0$, $\mathcal{s} = 1$

and $+\frac{1}{\sqrt{2}}$ for $t = 1$, $\mathcal{s} = 0$.

As it is written this wave function implies a separable space wave function but in fact such a restriction is not necessary. For the $^{24}D_{1/2}$ state, since it enters only with a weight of 4%, it suffices to take a simple wave function which is separable. From equation(49c) we have for the (ds^2) configuration, which is the simplest one for this assignment,

$$\frac{^{24}D_{1/2}}{\left| ds^2; T\, L\, S\, J\, \alpha \right\rangle} = \frac{1}{\sqrt{3}} \left\{ \left| s^2; t\, o\, \mathcal{s}\, \alpha \right\rangle_{12} \left| d \right\rangle_3 + (-)^P \, \mathbb{P} \right\}$$

$$\begin{pmatrix} \mu & t_z & T_z \\ \frac{1}{2} & t & T \end{pmatrix} \begin{pmatrix} \gamma & 0 & L_z \\ 2 & 0 & L \end{pmatrix} \begin{pmatrix} \beta & \mathcal{s}_z & S_z \\ \frac{1}{2} & \mathcal{s} & S \end{pmatrix} \begin{pmatrix} S_z & L_z & J_z \\ S & L & J \end{pmatrix}$$

(67)

where $T = 1/2$, $S = 3/2$, $L = 2$, $J = 1/2$

and $s = 1$ for $S = 3/2$

$t = 0$ for $|s^2; t\,o\,\Delta\alpha\rangle_{12}$ to be antisymmetric.

Now we want matrix elements of an operator $\mathcal{H} = \sum_{i=1}^{3} \mathcal{H}_i$

where $\mathcal{H}_i = {}_L\mathcal{H}_{\mathcal{L}}^{m_\ell}\;{}_S\mathcal{H}_K^{\mu}\frac{\tau^-}{\sqrt{2}}$, in our former notation. These are

$$\langle {}^{22}S_{1/2} | \mathcal{H} | {}^{22}S_{1/2} \rangle =$$

$$\langle s^2; t\Delta\alpha | \} s^3; T'S'\mathcal{S}'\rangle \langle s^2; t\,\Delta\,\alpha | \} s^3; T\,S\,\mathcal{S}\rangle$$

$$\sqrt{\frac{54\,\hat{T}}{\hat{K}}} \begin{pmatrix} T & 1 & T' \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \left(\begin{matrix} T_z & -1 \\ T & 1 \end{matrix}\middle|\begin{matrix} T_z' \\ T' \end{matrix}\right) (-)^{t+T+\Delta+S+1}$$

$$\delta_{SJ}\,\delta_{S'J'}\,\delta_{\mathcal{L},0} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \Delta & \frac{1}{2} \end{pmatrix} \langle \tfrac{1}{2} \| \mathcal{H}_K \| \tfrac{1}{2} \rangle \langle s \| \mathcal{H}_{\mathcal{L}} \| s \rangle \; . \tag{68}$$

Here we've omitted a term $(-)^{J-J_z}(\hat{J}\,\hat{J}')^{\frac{1}{2}} \left(\begin{matrix} -J_z & J_z' \\ J & J' \end{matrix}\middle|\begin{matrix} m \\ j \end{matrix}\right) \left(\begin{matrix} \mu & m_{\mathcal{L}} \\ K & \mathcal{L} \end{matrix}\middle|\begin{matrix} m_1 \\ j_1 \end{matrix}\right)$

which will go out in the sum over final states and average over initial states. Primed quantities refer to the final state, unprimed to the initial state. Again, the reduced matrix element $\langle s \| \mathcal{H}_{\mathcal{L}} \| s \rangle$ does not require a separable space wave function although we will use one to evaluate it.

For matrix elements between the ${}^{22}S_{1/2}$ and the ${}^{24}D_{1/2}$ configurations (where now we do need to use separable space wave functions)

$$\langle {}^{24}D_{1/2} | \mathcal{H} | {}^{22}S_{1/2} \rangle = \langle s^2; t\Delta\alpha \| \} s^3; T\,0\,S; \delta \rangle$$

$$\left(\frac{18}{5} \hat{T}\hat{S}\hat{S}'\hat{L}'\right)^{\frac{1}{2}} (-)^{t+T+\Delta+S'+1}$$

$$\begin{pmatrix} T_z & -1 & \Big| & T_z' \\ T & 1 & \Big| & T' \end{pmatrix} \begin{pmatrix} T & 1 & T' \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \Delta & \frac{1}{2} \end{pmatrix} \begin{pmatrix} S & O & J \\ S' & L' & J' \\ K & \mathcal{L} & j_1 \end{pmatrix}$$

$$\langle 1/2 \| \mathcal{H}_K \| 1/2 \rangle \; \langle s \| \mathcal{H}_{\mathcal{L}} \| d \rangle \qquad (69)$$

(and a similar expression for $\langle {}^{22}S_{1/2} | \mathcal{H} | {}^{24}D_{1/2} \rangle$),

also we need

$$\langle {}^{24}D_{1/2} | \mathcal{H} | {}^{24}D_{1/2} \rangle = (30\, \hat{T}\,\hat{S}\,\hat{L}\,\hat{S}'\hat{L}')^{\frac{1}{2}} \; (-)^{t+T+\Delta+S'+1+\mathcal{L}}$$

$$\begin{pmatrix} T & 1 & T' \\ \frac{1}{2} & t & \frac{1}{2} \end{pmatrix} \begin{pmatrix} T_z & -1 & \Big| & T_z' \\ T & 1 & \Big| & T' \end{pmatrix} \begin{pmatrix} S & K & S' \\ \frac{1}{2} & \Delta & \frac{1}{2} \end{pmatrix} \begin{Bmatrix} S & L & J \\ S' & L' & J' \\ K & \mathcal{L} & j_1 \end{Bmatrix}$$

$$\langle 1/2 \| \mathcal{H}_K \| 1/2 \rangle \langle d \| \mathcal{H}_{\mathcal{L}} \| d \rangle . \qquad (70)$$

The matrix elements for s-s transitions between the ${}^{24}D_{1/2}$ configurations vanish because this would be a $t = 0$ to $t = 0$ transition.

The Rate

From our results for μ capture in F^{19} we have the capture rate (Eq. 45) for a statistical population of the HFS levels

$$\Gamma = \frac{G^2 (Ze^2)^3 \mu^3 v^2 \quad (2\lambda)}{2\pi^2} \qquad (71)$$

where (see Eq. 46a et seq.)

$$2\lambda = AA^* + BB^*$$

with $AA^* = G_V^2 f^2 - 2 G_V^2 \, Re(f \; \hat{v} \cdot q^*)$

and $BB^* = G_A^2 g^2 + (G_P^2 - 2 G_A G_P) \, |\hat{v} \cdot g|^2 + 2 (G_P - G_A) g_A \, Re(\hat{v} \cdot g \; t^*)$

$$+ 2 G_A G_V \, Im (\hat{v} \cdot g \times q^*) + g_A^2 t^2 + G_V^2 q^2 \; .$$

In the $1/2^+$ to $1/2^+$ transition for μ capture in He^3 to the groundstate of H^3 the nonvanishing matrix elements are (Eq. 53-60) F_0, G_{10}, G_{12}, T_{011}, Q_{010}, T_{111}, T_{211} and Q_{111}. We will evaluate the large matrix elements (f and g) between states including a D state admixture

$$\sqrt{1 - \epsilon^2} \;\; {}^{22}S_{1/2} + \epsilon \, {}^{24}D_{1/2}$$

where $\epsilon^2 \simeq 0.04$ and $|\epsilon| \simeq 0.20$. We will just include the contribution to the momentum type matrix elements (t and q) from the dominant configuration ${}^{22}S_{1/2}$, so that to this approximation we only keep T_{011} and Q_{010}. Using equations 58, 69 and 70 we obtain

$$F_0 = \frac{1}{\sqrt{2}} (1 - \epsilon^2) \langle s \| e^{-is \cdot r}{}_{\mathcal{L}=0} \| s \rangle + \epsilon^2 \langle d \| e^{-is \cdot r}{}_{\mathcal{L}=0} \| d \rangle \qquad (72a)$$

$$G_{10} = \frac{1}{\sqrt{2}} (1 - \epsilon^2) \langle s \| e^{-is \cdot r}{}_{\mathcal{L}=0} \| s \rangle + \frac{\epsilon^2}{3} \langle d \| e^{-is \cdot r}{}_{\mathcal{L}=0} \| d \rangle \qquad (72b)$$

$$G_{12} = \frac{2}{\sqrt{15}} \, \epsilon \sqrt{1 - \epsilon^2} \, \langle s \| e^{-is \cdot r}{}_{\mathcal{L}=2} \| d \rangle - \sqrt{\frac{7}{90}} \, \epsilon^2 \langle d \| e^{-is \cdot r}{}_{\mathcal{L}=2} \| d \rangle$$

$$(72c)$$

$$Q_{010} = \frac{1}{\sqrt{2}} \langle s \| \mathcal{L} = 0 \;\; \mathbb{L} = 1 \| s \rangle \qquad (72d)$$

and

$$T_{011} = \frac{1}{\sqrt{2}} \langle s \,\|\, \mathcal{L} = 0 \quad \mathbb{L} = 1 \,\|\, s \rangle \ . \tag{72e}$$

The terms entering the rate (2λ) are (Eq. 53 et seq.)

$$f^2 = 2 F_o^2 \tag{73a}$$

$$g^2 = 2 \left(3 G_{10}^2 + \frac{3}{5} G_{12}^2\right) \tag{73b}$$

$$\mathrm{Re}(f \,\hat{v}\cdot q^*) = -\frac{2}{\sqrt{3}} \mathrm{Re}\left(F_o Q^*_{010}\right) \tag{73c}$$

$$t^2 = \frac{2}{3} T_{011}^2 \tag{73d}$$

$$q^2 = \frac{2}{3} Q_{010}^2 \tag{73e}$$

$$\mathrm{Re}\, \hat{v}\cdot g\, t^* = -\frac{2}{\sqrt{3}} \mathrm{Re}\left(G_{10} - \sqrt{\frac{2}{5}}\, G_{12}\right) T^*_{011} \tag{73f}$$

$$|\hat{v}\cdot g|^2 = 2\left(G_{10} - \sqrt{\frac{2}{5}}\, G_{12}\right)^2 \ . \tag{73g}$$

We include the square of small terms (Eq. 73d, e) in spite of the inconsistency of doing so merely for the sake of completeness. Terms of first order in the nucleon momentum (Eq. 73c, f) which were omitted previously are also included. The term $|\hat{v}\cdot g|^2$ (Eq. 73g) was treated incorrectly by Werntz (28) who averaged over neutrino directions assuming only $\mathcal{L} = 0$ matrix elements and lost the cross term between G_{10} and G_{12}. This term is linear in the D state amplitude whose sign is unknown and this constitutes an extra source of uncertainty in the calculation of the rate.

In Appendix VIII we evaluate the reduced matrix elements required using the simplest space wave functions with a length parameter b = 0.80, 1.0 and 1.2 fermis (compared to the usual criterion b = 0.468 R = 1.05 derived from the coulomb energy). In addition we present the rates (2λ) and ($2\lambda_D$) (see Eq. 62) calculated with the various hypotheses on the coupling constants.

Conclusions

The capture rate (Eq. 45), including effects of finite nuclear mass (3), and a 5% reduction due to coupling constant form factors (10) is

$$\Gamma = 250\,(2\lambda)\ \text{sec}^{-1} \tag{74}$$

With a central value of 2λ = 5, we get a capture rate 1250 sec^{-1}.

An examination of the terms entering the rate indicates that momentum type matrix elements reduce the rate by about 2%. Inclusion of the D state reduces the rate (due to the reduced Gamow-Teller matrix element between S = 3/2 states) by about 6% from that calculated with a pure $^{22}S_{1/2}$ configuration. The term linear in the D state amplitude amounts to about 1% of the total rate, and must constitute a source of error. We can assign a 5% uncertainty to the results due to the uncertainty in the nuclear radius. If the nuclear radius can be determined more precisely, and if the experiment can be done with one or two percent accuracy, it might then be interesting to do the radiative corrections to the capture rate and to make a more careful study of meson exchange effects.

APPENDICES

APPENDIX I. Matrix Elements for Hole-Particle States

In the shell model, the groundstate of O^{16} is a double closed shell and the odd parity states of O^{16} and N^{16} are considered to be states of one hole in the closed shell plus one particle outside the closed shell. Here we derive the required matrix elements for transitions between these hole-particle states and between a hole-particle state and the groundstate. We also present the matrix elements of two particle operators between hole-particle states.

Denote a state of n fermions by

$$\Psi = \prod_{\lambda} a_{\lambda}^{+} \Phi_{o} \qquad (I.1)$$

where Φ_o is the vacuum state, and the creation-destruction operators satisfy the anticommutation relations $\left(a_{\lambda}, a_{\mu}^{+}\right)_{+} = \delta_{\mu\lambda}$. The product runs over the states λ occupied by the n fermions. Denote a closed shell by the state

$$\Psi_o = \prod_{\lambda \in K} a_{\lambda}^{+} \Phi_o \qquad (I.2)$$

where λ runs over all the states within the closed shell, which are numbered up to K. Then a state with one hole in a closed shell is

$$\Psi = a_m \Psi_o \qquad \text{where } m < K \qquad (I.2a)$$

and a state with one particle outside a closed shell is

$$\Psi = a_m^+ \Psi_o \qquad \text{where } m > K . \tag{I. 2b}$$

Now introduce creation-destruction operators b_m^+ and b_m for holes by the correspondence $a_m \rightarrow b_m^+$ and $a_m^+ \rightarrow b_m$ for $m < K$. States with one particle outside a closed shell and one hole within are

$$\Psi = a_n^+ b_m^+ \Psi_o \qquad \text{with } n > K > m . \tag{I. 2c}$$

Let us evaluate matrix elements for various physical operators in the second quantized representation. A single particle operator, with creation-destruction operators antisymmetrized, is

$$\mathcal{O} = \sum_{rs} \langle \varphi_r(x) | \mathcal{O}(x) | \varphi_s(x) \rangle \; 1/2 \; (a_r^+ a_s - a_s a_r^+) \tag{I. 3}$$

where r, s run over all states. If we make the artificial separation into holes and particles, the operator becomes

$$\mathcal{O} = \mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 \qquad \text{where}$$

$$\mathcal{O}_1 = \sum_{r\, s > K} \langle \varphi_r(x) | \mathcal{O}(x) | \varphi_s(x) \rangle \; 1/2 \; (a_r^+ a_s - a_s a_r^+) \tag{I. 3a}$$

$$\mathcal{O}_2 = \sum_{r\, s < K} \langle \varphi_r(x) | \mathcal{O}(x) | \varphi_s(x) \rangle \; 1/2 \; (b_r b_s^+ - b_s^+ b_r)$$

$$\mathcal{O}_3 = \sum_{\substack{r > K \\ s < K}} \langle \varphi_r(x) | \mathcal{O}(x) | \varphi_s(x) \rangle \times (a_r^+ b_s^+ + b_r a_s)$$

which are, respectively, particle-particle, hole-hole and particle-hole or hole-particle transitions. We will want the matrix element of such a single particle operator between states of one particle and one hole. The matrix element of $\mathcal{O}_3$ vanishes between such states. The remaining two result in matrix elements for particle-particle and hole-hole transitions, so that, if a, c denote particle states and b, d hole states, the matrix element is

$$\langle \Psi_{ab} | \mathcal{O} | \Psi_{cd} \rangle = \delta_{bd} \langle \varphi_a(x) | \mathcal{O}(x) | \varphi_c(x) \rangle - \delta_{ac} \langle \varphi_d(x) | \mathcal{O}(x) | \varphi_b(x) \rangle . \tag{I.4}$$

Had we not antisymmetrized the creation-destruction operators in $\mathcal{O}$ there would have been diagonal matrix elements of the form $\delta_{ac} \ \delta_{bd} \ \sum_{rs} \langle \varphi_r(x) | \mathcal{O}(x) | \varphi_s(x) \rangle$ which would have contributed to the energy of the state Ψ_{ab} and which we could have removed by lumping into the central "single particle-closed shell" interaction.

We will employ this shortcut to eliminate similar diagonal matrix elements of two particle operators, without doing all the extra work of antisymmetrizing the operator. Then, using this shortcut, the two particle operator is

$$\mathcal{H} = 1/2 \sum_{ij;k\ell} \mathcal{H}_{ij;k\ell} \quad a_i^+ a_j^+ a_k a_\ell \tag{I.5}$$

where

$$\mathcal{H}_{ij;k\ell} = \langle \varphi_i(1) \ \varphi_j(2) | \mathcal{H}_{12}(x) | \varphi_k(2) \ \varphi_\ell(1) \rangle$$

$$= \mathcal{H}_{ji;\ell k} \quad \text{since} \quad \mathcal{H}_{12} = \mathcal{H}_{21}.$$ Here $ijk\ell$ run over all states, and we have yet to transfer to hole-particle notation.

A matrix element taken between states of one hole and one particle is

$$\langle \Psi_{mn} | \mathcal{H} | \Psi_{rs} \rangle = \sum \tfrac{1}{2} \mathcal{H}_{ij;k\ell} \langle \Psi_0 | b_n a_m (a_i^+ a_j^+ a_k a_\ell) a_r^+ b_s^+ | \Psi_0 \rangle$$

where (m, r) are particle states, (n, s) hole states, and $ijk\ell$ refer to both. There are sixteen possible combinations, $ijk\ell$ greater or less than K, of which the only ones with non-vanishing matrix elements are

$$a_i^+ b_j a_k b_\ell^+ ,\ a_i^+ b_j b_k^+ a_\ell ,\ b_i a_j^+ a_k b_\ell^+ ,\ b_i a_j^+ b_k^+ a_\ell ,\ \text{and}\ b_i b_j b_k^+ b_\ell^+ .$$

All told, the matrix element of these operators is

$$\langle \Psi_{mn} | \mathcal{H} | \Psi_{rs} \rangle = + \mathcal{H}_{ms;rn} - \mathcal{H}_{ms;nr} + \delta_{ns} \Lambda_{mr} + \delta_{mr} \Lambda_{ns} .$$

The single particle (single hole) parts of the matrix element $\delta_{ns} \Lambda_{mr}$ ($\delta_{mr} \Lambda_{ns}$) could have been eliminated by using ordered operators in $\mathcal{H}$ or can now be lumped into the central potential. In any case, we will drop these terms from the two particle matrix element and retain

$$\langle \Psi_{mn} | \mathcal{H} | \Psi_{rs} \rangle = + \mathcal{H}_{ms;rn} - \mathcal{H}_{ms;nr} \tag{I. 6}$$

which is just an exchange minus a direct matrix element.

One other type of matrix element arises, that of a one particle operator between the closed shell and a one hole, one particle state. Now only the $\mathcal{O}_3$ part of $\mathcal{O}$ has a non-vanishing matrix element, and the groundstate transition matrix element is

$$\langle \Psi_o | \mathcal{O} | \Psi_{mn} \rangle = \langle \Psi_o | \langle \varphi_r(x) | \mathcal{O}(x) | \varphi_s(x) \rangle \, b_r a_s a_m^+ b_n^+ \, | \Psi_o \rangle$$

$$= \quad \langle \varphi_n(x) | \mathcal{O}(x) | \varphi_m(x) \rangle \, . \qquad (I.7)$$

There remains only the problem of coupling hole and particle states -eigenstates of J_P^2, J_H^2, J_{PZ}, J_{HZ} (as well as of isotopic spin) - to form eigenstates of the total angular momentum J^2 and J_Z where $J = J_P - J_H$. This problem can be treated by using the time reversed state (19), for which $J \rightarrow -J$, for the hole state. The time reversed, or now the hole state, is

$$u\,(j, m)_{\text{hole}} = (-)^{j+m} \, u(j, -m) \, . \qquad (I.8)$$

The isotopic spin of the hole state also introduces a factor $(-)^{t+t_z}$.

Finally, let us write down the complete matrix elements, between hole-particle states which are eigenstates of J, J_z, T and T_z. We use jj coupling and consistently denote particle states by the subscripts (m, r) and hole states by the subscripts (n, s). Subscripts f and i refer to final and initial states.

a) For one particle operators

$$\langle \Psi_{mn} | \mathcal{O} | \Psi_{rs} \rangle = (-)^{(j_n - m_n)+(j_s - m_s)+(t_n - t_{z_n})+(t_s - t_{z_s})} \times$$

$$\left(\begin{array}{cc|c} -m_n & m_m & M_f \\ j_n & j_m & J_f \end{array}\right) \left(\begin{array}{cc|c} -m_s & m_r & M_i \\ j_s & j_r & J_i \end{array}\right) \left(\begin{array}{cc|c} -t_{z_n} & t_{z_m} & T_{z_f} \\ t_n & t_m & T_f \end{array}\right) \left(\begin{array}{cc|c} -t_{z_s} & t_{z_r} & T_{z_i} \\ t_s & t_r & T_i \end{array}\right) \times$$

$$\left\{ \delta_{ns} \langle \varphi_m(x) | \mathcal{O}(x) | \varphi_r(x) \rangle - \delta_{mr} \langle \varphi_s(x) | \mathcal{O}(x) | \varphi_n(x) \rangle \right\}$$

(I.9)

b) For two particle operators

$$\langle \Psi_{mn} | \mathcal{H} | \Psi_{rs} \rangle = (-)^{(j_n - m_n)+(j_s - m_s)+(t_n - t_{z_n})+(t_s - t_{z_s})} \times$$

$$\left(\begin{array}{cc|c} -m_n & m_m & M_f \\ j_n & j_m & J_f \end{array}\right) \left(\begin{array}{cc|c} -m_s & m_r & M_i \\ j_s & j_r & J_i \end{array}\right) \left(\begin{array}{cc|c} -t_{z_n} & t_{z_m} & T_{z_f} \\ t_n & t_m & T_f \end{array}\right) \left(\begin{array}{cc|c} -t_{z_s} & t_{z_r} & T_{z_i} \\ t_s & t_r & T_i \end{array}\right) \times$$

$$\left\{ + \langle \varphi_m(1)\ \varphi_s(2) | \mathcal{H}_{12} | \varphi_r(2)\ \varphi_n(1) \rangle \right.$$

$$\left. - \langle \varphi_m(1)\ \varphi_s(2) | \mathcal{H}_{12} | \varphi_n(2)\ \varphi_r(1) \rangle \right\}$$

(I.10)

and

c) For matrix elements in groundstate transitions

$$\langle \Psi_{mn} | \mathcal{O} | \Psi_0 \rangle = (-)^{(j_n - m_n)+(t_n - t_{z_n})} \times$$

$$\left(\begin{array}{cc|c} -m_n & m_m & M_f \\ j_n & j_m & J_f \end{array}\right) \left(\begin{array}{cc|c} -t_{z_n} & t_{z_m} & T_{z_f} \\ t_n & t_m & T_f \end{array}\right) \langle \varphi_m(x) | \mathcal{O}(x) | \varphi_n(x) \rangle .$$

(I.11)

APPENDIX II. Evaluation of the Talmi Integrals.

The reduced matrix elements of the two body interactions are readily evaluated in the model where we use harmonic oscillator wave functions and a Yukawa shaped potential. In harmonic oscillator units (where the unit of length is $b = (2Mw)^{-1/2} = 1$) with $\hbar = 1$, the normalized radial functions R_ℓ are

$$R_0 = (8/3\sqrt{\pi})^{1/2} (x^2 - 3/2)\, e^{-x^2/2}$$
$$R_1 = (8/3\sqrt{\pi})^{1/2} (x)\, e^{-x^2/2} \qquad \text{(II.1)}$$
$$R_2 = (\tfrac{2}{5}\, 8/3\sqrt{\pi})^{1/2} (x^2)\, e^{-x^2/2}$$

and the Yukawa potential for the two body interaction is

$$V(r_{12}) = e^{-\mu r_{12}}/\mu r_{12} = \sum_{kq} v_k(x;y)\, Y_\kappa^q(\hat{x})\, Y_\kappa^{q\,*}(\hat{y}),$$

with $\vec{r}_{12} = \vec{x} - \vec{y}$. (II.2)

The reduced matrix elements of the two body interaction defined by

$$\langle \varphi_A(m_A;1)\, \varphi_B(m_b;2) \,|\, v(r_{12}) \,|\, \varphi_c(m_c;1)\, \varphi_D(m_D;2) \rangle =$$
$$\begin{pmatrix} m_c & q & m_A \\ \ell_c & \kappa & \ell_A \end{pmatrix} \begin{pmatrix} m_B & q & m_D \\ \ell_B & \kappa & \ell_D \end{pmatrix} \langle \varphi_A(1)\, \varphi_B(2) \,\|\, v_\kappa \,\|\, \varphi_c(1)\, \varphi_D(2) \rangle$$

(II.3)

are

$$\langle \varphi_A(1)\varphi_B(2)\| v_K \|\varphi_C(1)\,\varphi_D(2)\rangle = \frac{\hat{K}}{4\pi}\sqrt{\frac{\hat{\ell}_C\ \hat{\ell}_B}{\hat{\ell}_A\ \hat{\ell}_D}}\ \times$$

$$\begin{pmatrix} 0 & 0 & 0 \\ \ell_C & K & \ell_A \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ \ell_B & K & \ell_D \end{pmatrix}\times$$

$$2\pi\hat{K}\,I \qquad \text{(II. 4)}$$

where

$$2\pi\hat{K}\,I = \int x^2 dx\, y^2 dy\, R_A(x)\, R_c(x)\, v_K(x;y)\, R_B(y)\, R_D(y)$$

$$= \int d^3x\, d^3y\, R_A(x)\, R_c(x)\, R_B(y)\, R_D(y)\;\; V(r_{12})\;\; P_K(\hat{x}\cdot\hat{y})\ .$$

(II. 5)

The change of variables

$$\vec{x} = \vec{R} + 1/2\,\vec{r}$$

$$\vec{y} = \vec{R} - 1/2\,\vec{r}$$

with the substitutions

$$d^3x\, d^3y = d^3R\, d^3r$$

$$x^2 = R^2 + \frac{1}{4}\, r^2 + Rrt \quad \text{with } t = \cos\angle\vec{R},\vec{r}$$

$$y^2 = R^2 + \frac{1}{4}\, r^2 - Rrt$$

$$\hat{x}\cdot\hat{y} = \frac{R^2 - 1/4\, r^2}{xy}$$

allows us to write (30)

$$I = \int R^2\, dR\, r^2 dr\, dt\, \left\{R_A(x)\, R_c(x)\right\}\left\{R_D(y)\, R_B(y)\right\}\; V(r)\;\; P_K(\hat{x}\cdot\hat{y})\ .$$

(II. 5')

The use of harmonic oscillator radial functions makes it possible to do the integrals over R and t directly leaving integrals over r of the type

$$\int_0^\infty r\,dr\,e^{-(r^2/2+\mu r)}\,r^{2n} = e^{1/2\,\mu^2}\,I_n \qquad \text{where}$$

the integrals $I_n = \int_\mu^\infty dx\,e^{-x^2/2}\,(x-\mu)^{2n+1}$ must be done numerically. The required integrals for the two values of the nuclear radius used are

I_n	b = 1.56 f $m_\pi b = \mu = 1.10$	b = 1.86 f $m_\pi b = 1.30$
I_0	0.17274	0.11388
I_1	0.17809	0.10583
I_2	0.42989	0.23361
I_3	1.6889	0.85055

and the reduced matrix elements required are

	b=1.56 fermis	b=1.86 fermis
$\langle \ell_1 = 0\ \ell_2 = 1 \Vert K = 0 \Vert \ell_1 = 0\ \ell_2 = 1 \rangle$	+0.1011	+0.06256
$\langle \ell_1 = 2\ \ell_2 = 1 \Vert K = 0 \Vert \ell_1 = 2\ \ell_2 = 1 \rangle$	+0.09751	+0.06597
$\langle \ell_1 = 2\ \ell_2 = 1 \Vert K = 2 \Vert \ell_1 = 2\ \ell_2 = 1 \rangle$	+0.3301	+0.2505
$\langle \ell_1 = 0\ \ell_2 = 1 \Vert K = 2 \Vert \ell_1 = 2\ \ell_2 = 1 \rangle$	-0.2899	-0.2158
$\langle \ell_1 = 0\ \ell_2 = 1 \Vert K = 1 \Vert \ell_1 = 1\ \ell_2 = 0 \rangle$	+0.3741	+0.3307
$\langle \ell_1 = 0\ \ell_2 = 1 \Vert K = 1 \Vert \ell_1 = 1\ \ell_2 = 2 \rangle$	+0.2124	+0.1545
$\langle \ell_1 = 0\ \ell_2 = 1 \Vert K = 1 \Vert \ell_1 = 1\ \ell_2 = 2 \rangle$	-0.1003	-0.07014
$\langle \ell_1 = 2\ \ell_2 = 1 \Vert K = 3 \Vert \ell_1 = 1\ \ell_2 = 2 \rangle$	+0.3151	+0.2450

APPENDIX III. Reduced Matrix Elements for μ Capture in O^{16}

There are two types of radial matrix elements arising in the μ capture calculation in O^{16},

$$\text{a)} \quad I_{L\ell} = \int_0^\infty R_L^*(r)\; j_\ell(vr)\; R_1(r)\, r^2\, dr$$

and

$$\text{b)} \quad \Phi_{L\ell 0} = -\frac{1}{bM} \int_0^\infty j_\ell(vr)\, R_L^*(r) \left(\frac{d}{dr} + \frac{2}{r}\right) R_1(r)\, r^2 dr$$

$$\Phi_{L\ell 2} = -\frac{1}{bM} \int_0^\infty j_\ell(vr)\, R_L^*(r) \left(\frac{d}{dr} - \frac{1}{r}\right) R_1(r)\, r^2 dr.$$

Here b = 1.56 fermis and 1.86 fermis

M = nucleon mass. bM = 7.24 and 8.64

v = neutrino momentum in units of b

= 0.700 for b = 1.56 fermis

= 0.835 for b = 1.86 fermis

These integrals are easily evaluated using a series expansion for the spherical Bessel functions. Even with the large values of v, adequate results can be got with two or three terms in such an expansion. Following is a table of these integrals.

	b = 1.56 fermis	b = 1.86 fermis
I_{01}	.181	.194
I_{21}	.310	.347
I_{23}	.0159	.0254
Φ_{000}	.116	.0888
Φ_{010}	.0306	.0278
Φ_{220}	.00224	.00202
Φ_{222}	.0210	.0260
Φ_{242}	.000872	.00149
Φ_{202}	.163	.120
Φ_{022}	.0184	.0208

APPENDIX III.

The reduced matrix elements required, $\langle \ell_m \| \mathcal{H}_\ell \| \ell_n \rangle$ and $\langle \ell_m \| \mathcal{L}, \mathbb{L} \| \ell_n \rangle$ are (all these are times a factor -i)

	b = 1.56 fermis	b = 1.86 fermis
$\langle 0 \| 1 \| 1 \rangle$	.544	.582
$\langle 2 \| 1 \| 1 \rangle$	-.588	-.658
$\langle 2 \| 3 \| 1 \rangle$	-.0565	-.0900
$\langle 0 \| 1, 0 \| 1 \rangle$	-.116	-.0888
$\langle 0 \| 1, 2 \| 1 \rangle$	-.0582	-.0659
$\langle 2 \| 1, 0 \| 1 \rangle$	+.103	+.0758
$\langle 2 \| 1, 2 \| 1 \rangle$	+.0232	+.0280
$\langle 2 \| 2, 2 \| 1 \rangle$	+.0243	+.0310
$\langle 2 \| 3, 2 \| 1 \rangle$	+.0126	+.0145
$\langle 2 \| 3, 4 \| 1 \rangle$	+.00356	+.00610

APPENDIX IV

TABLE 1 Structure of the A=16 States on the Shell Model.

The results presented are for x = 2, b = 1.56 fermis and V_c = 30, 40, 50 Mev. For comparison, the results of Elliott and Flowers (EF) as well as the (jj) limit are presented. The experimental (Ex) and calculated (E_T) energies are given fitted at V_c = 40 Mev with $\hbar\omega$ = 13.5 Mev.

APPENDIX I.

Table 1.

		$^1p_{1/2}^{-1}2s_{1/2}$	$^1p_{3/2}^{-1}2s_{1/2}$	$^1p_{1/2}^{-1}1d_{3/2}$	$^1p_{3/2}^{-1}1d_{3/2}$	$^1p_{1/2}^{-1}1d_{5/2}$	$^1p_{3/2}^{-1}1d_{5/2}$	E_T
	Vc= 30	.995			-.100			12.4
T=1 J=0⁻	40	.993			-.118			13.4
Ex=10.5	50	.992			-.132			14.4
	EF	.995			+.050			
	jj	1.						
	Vc=30	.966	+.015	+.155	+.043		+.198	11.7
T=1 J=1⁻	40	.895	+.068	+.222	+.080		+.221	12.0
Ex=10.8	50	.890	+.147	+.318	+.124		+.264	12.3
	EF	.98	-.16	-.01	+.02		+.08	
	jj	1.						
	Vc= 30		-.136	+.281	-.020	+.904	+.210	10.6
T=1 J=2⁻	40		-.175	+.367	-.077	+.8[illegible]5	+.284	10.6
Ex=10.4	50		-.209	+.430	-.133	+.784	+.371	10.5
	EF		-.06	-.10	+.09	+.98	+.14	
	jj					1.		
	Vc=30				-.220	+.885	-.410	11.4
T=1 J=3⁻	40				-.258	+.828	-.497	11.8
Ex=10.7	50				-.283	+.782	-.556	12.0
	EF				+.06	+.98	-.18	
	jj					1.		
	Vc= 30		-.107	-.172	+.220	+.929	+.221	8.20
T=0 J=2⁻	40		-.085	-.187	+.273	+.870	+.355	7.64
Ex=8.88	50		-.043	-.189	+.311	+.788	+.496	6.88
	EF		-.02	+.27	-.13	+.87	+.40	
	jj					1.		
	Vc= 30				-.113	+.893	+.436	3.44
T=0 J=3⁻	40				-.194	+.8[illegible]2	+.551	1.84
Ex=6.14	50				-.267	+.730	+.629	-.49
	EF				-.33	+.87	+.37	
	jj					1.		

The matrix elements for μ capture in O^{16} are presented below, for nuclear parameters b = 1.56 fermis, x = 2 and Vc = 30, 40, 50 Mev. For comparison results obtained using the wave functions of Elliott and Flowers (EF) are presented, as well as those obtained in the (jj) limit.

Table 2.

		Vc=30	40	50	EF	jj
O^+-O^-	G_1	-.192	-.182	-.172	-.286	-.256
	T_{10}	+.130	+.125	+.120	+.196	+.164
O^+-1^-	F_1	-.0388	-.0302	-.0474	+.080	+.148
	G_1	+.117	+.115	+.105	+.190	+.209
	Q_{10}	+.00392	+.00116	+.00705	-.0319	-.0546
	Q_{12}	-.00740	-.00720	-.00674	-.00916	-.0122
O^+-3^-	F_3	-.0111	-.00971	-.00865	-.0111	-.0147
	G_3	-.00938	-.00737	-.00587	-.0172	-.0170
	Q_{12}	+.00296	+.00259	+.00232	+.00294	+.00390
	Q_{34}	+.00046	+.00040	+.00036	+.00046	+.00061
O^+-2^-	G_1	+.302	+.257	+.197	+.324	+.430
	G_3	+.00012	-.00100	-.00203	+.00629	+.00450
	Q_{12}	-.00950	-.0108	-.0121	-.00557	-.00531
	T	-.00316	-.00398	-.00399	-.0012	+.0029
	$v\Theta$	+.383	+.325	+.246	+.411	+.546
	R	.789	.790	.800	+.788	+.788

APPENDIX 4.

Table 3. The individual terms in the rate for μ capture in O^{16}.

	$V_c=30$	40	50	EF	jj
f^2	.00537	.00340	.00727	.0200	.0672
g^2	.531	.529	.531	.643	.651
$\lvert\hat{v}\cdot g\rvert^2$	.218	.217	.220	.254	.241
t^2	.0169	.0156	.0145	.0385	.0270
q^2	.00067	.00074	.00101	.00565	.00917
$\mathrm{Re}\, f\, \hat{v}\cdot q^*$	-.00109	-.00069	-.00144	-.00278	-.00973
$\mathrm{Im}\, \hat{v}\cdot g\times q^*$	-.0116	-.0136	-.0136	-.0241	-.0384
$\mathrm{Re}\, \hat{v}\cdot g\, t^*$	-.0280	-.0265	-.0245	-.0571	-.0392

Table 4. The rate $2M^2$ on the various hypotheses.

	$V_c=30$	40	50	EF	jj
Case 1	.941	.936	.936	1.28	1.38
2	1.60	1.59	1.60	2.12	2.13
3	1.21	1.21	1.21	1.63	1.70
4	.761	.756	.757	1.05	1.14
5	1.47	1.46	1.46	1.94	1.94
6	1.03	1.02	1.03	1.40	1.45

APPENDIX V. Reduced Matrix Elements for μ Capture in F^{19}

We refer to the general formulae for matrix elements in the $A = 19$ system. The reduced matrix elements for the spin variables are

a) $K = 0$ $\quad {}_s\mathcal{H}_K^{\mu} = 1 \quad \langle 1/2 \| K = 0 \| 1/2 \rangle = 1$

b) $K = 1$ $\quad {}_s\mathcal{H}_K^{\mu} = \sigma_\mu \quad \langle 1/2 \| K = 1 \| 1/2 \rangle = \sqrt{3}$.

To calculate the reduced matrix elements in the space variables we use the harmonic oscillator wave functions

$$\psi_s = (8/3\sqrt{\pi}\ )^{1/2} \ (x^2 - 3/2)\, e^{\frac{-x^2}{2}}\ Y_0^0$$
$$\psi_D = (2/5\ \tfrac{8}{3\sqrt{\pi}}\ )^{1/2}\ x^2\ e^{\frac{-x^2}{2}}\ Y_2^{\gamma} \qquad (V-1)$$

and also the expansion

$$e^{-is\cdot r} = (-)^{\ell}\ 4\pi\ i^{\ell}\ j_{\ell}(vx)\ Y_{\ell}^{m_\ell}(\hat{x})\ \ Y_{\ell}^{m_\ell *}(\hat{v}). \qquad (V-2)$$

Here we understand summation over repeated indices, and we have

$x = r/b$ $\qquad v = bs$

$b = 1.7$ fermis $\qquad s = 100$ Mev $\qquad v = 0.850$.

Then, for example,

$$\langle d |\, e^{-is\cdot r}\, | d \rangle = \begin{pmatrix} \mu & m_\ell & \gamma \\ 2 & \ell & 2 \end{pmatrix} \langle d \| e^{-is\cdot r}_{\ \ell} \| d \rangle \qquad (V-3)$$

defines reduced matrix elements of $e^{-is\cdot r}$ with $\mathcal{L} = 0$, 2 and 4. There are five such reduced matrix elements required, which we evaluate for $\hat{s}$ in the $\hat{z}$ direction. They are

a) $\langle d \| e^{-is\cdot r} \; \mathcal{L} = 0 \| d \rangle = 0.642$

b) $\langle d \| e^{-is\cdot r} \; \mathcal{L} = 2 \| d \rangle = 0.347$

c) $\langle d \| e^{-is\cdot r} \; \mathcal{L} = 4 \| d \rangle = 0.034$

d) $\langle s \| e^{-is\cdot r} \; \mathcal{L} = 0 \| s \rangle = 0.652$

e) $\langle s \| e^{-is\cdot r} \; \mathcal{L} = 2 \| d \rangle = -.580$.

We also need reduced matrix elements of momentum type matrix elements

$$\langle f \,|\, e^{-is\cdot r} \quad p_\mu / M \,|\, i \rangle$$

and to apply the general expressions derived for these matrix elements we need to write this as a reduced matrix element depending on $(l_f m_f)$ and $(l_i m_i)$ through $\begin{pmatrix} m_i & m_{\mathcal{L}} & m_f \\ \ell_i & \mathcal{L} & \ell_f \end{pmatrix}$. In order to apply the gradient formula write

$$|\ell_f m_f\rangle = \Psi_{\ell_f} \, Y_{\ell_f}^{m_f} \qquad \text{then}$$

$$e^{-is\cdot r} \,|\, \ell_f m_f \rangle = i^{\mathbb{L}} \, (4\pi \mathbb{L})^{1/2} \, j_{\mathbb{L}}(vx) \quad \Psi_{\ell_f} \; x$$

$$\left(\frac{\hat{\ell}_f \; \hat{\mathbb{L}}}{4\pi \; \hat{K}} \right)^{1/2} \begin{pmatrix} 0 & 0 & 0 \\ \mathbb{L} & \ell_f & K \end{pmatrix} \begin{pmatrix} 0 & m_f & \omega \\ \mathbb{L} & \ell_f & K \end{pmatrix} \; Y_K^{\omega} \tag{V-4}$$

where we sum over L, K, ω and take $\hat{s}$ in the $\hat{z}$ direction. The matrix element then is

$$\left\langle f \left| e^{-is\cdot r} \quad P_\mu / M \right| i \right\rangle = \frac{1}{bMi} \; (-i)^{\mathbb{L}} \; (4\pi \mathbb{L})^{1/2} \left(\frac{\hat{\ell}_f \; \hat{\mathbb{L}}}{4\pi \; \hat{K}} \right)^{1/2} \times$$

$$\begin{pmatrix} 0 & 0 & \big| & 0 \\ \mathbb{L} & \ell_f & \big| & K \end{pmatrix} \begin{pmatrix} 0 & m_f & \big| & \omega \\ \mathbb{L} & \ell_f & \big| & K \end{pmatrix} \times$$

$$\int d^3x \left\{ j_{\mathbb{L}} (vx) \; \Psi_{\ell_f} Y_K^{\omega} \right\}^* \nabla_\mu \left\{ Y_{\ell_i}^{m_i} \; \Psi_{\ell_i} \right\} \quad \text{(V-5)}$$

and using the gradient formula (19) this becomes

$$= \frac{1}{bMi} \quad (-i)^{\mathbb{L}} \quad (4\pi \hat{\mathbb{L}})^{1/2} \left(\frac{\hat{\ell}_f \; \hat{\mathbb{L}}}{4\pi \; \hat{K}} \right)^{1/2} \begin{pmatrix} 0 & 0 & \big| & 0 \\ \mathbb{L} & \ell_f & \big| & K \end{pmatrix} \begin{pmatrix} 0 & m_f & \big| & \omega \\ \mathbb{L} & \ell_f & \big| & K \end{pmatrix} \begin{pmatrix} m_i & \mu & \big| & \omega \\ \ell_i & 1 & \big| & K \end{pmatrix} \times$$

$$\left\{ \delta_{K, \ell_i + 1} \left(\frac{\ell_i + 1}{2\ell_i + 3} \right)^{1/2} \Phi^{(+)}_{\ell_f \mathbb{L} \ell_i} - \delta_{K, \ell_i - 1} \left(\frac{\ell_i}{2\ell_i - 1} \right)^{1/2} \Phi^{(-)}_{\ell_f \mathbb{L} \ell_i} \right\} .$$

(V-5')

Here we have introduced the notation

$$\Phi^{(+)}_{\ell_f \mathbb{L} \ell_i} = \int_0^\infty x^2 \, dx \;\; j_{\mathbb{L}}(vx) \quad \Psi_{\ell_f} \left(\frac{d}{dx} - \frac{\ell_i}{x} \right) \Psi_{\ell_i} \qquad \text{and}$$

$$\Phi^{(-)}_{\ell_f \mathbb{L} \ell_i} = \int_0^\infty x^2 \, dx \; j_{\mathbb{L}} (vx) \;\; \Psi_{\ell_f} \left(\frac{d}{dx} + \frac{\ell_i + 1}{x} \right) \Psi_{\ell_i} \quad .$$

To fit the pattern of our general formulae we need

$$\begin{pmatrix} 0 & m_f \\ \mathbb{L} & \ell_f \end{pmatrix}\!\left|\begin{matrix}\omega \\ K\end{matrix}\right) \begin{pmatrix} m_i & \mu \\ \ell_i & 1 \end{pmatrix}\!\left|\begin{matrix}\omega \\ K\end{matrix}\right) = \hat{R}\sqrt{\frac{\hat{\mathcal{L}}}{\hat{\ell}_f}}\,(-)^{\ell_f+\ell_i+1} \begin{Bmatrix} \ell_i & \mathcal{L} & \ell_f \\ \mathbb{L} & K & 1 \end{Bmatrix} \times$$

$$\left(\begin{matrix} m_i & \mathcal{M}_{\mathcal{L}} \\ \ell_i & \mathcal{L} \end{matrix}\right|\left.\begin{matrix} m_f \\ \ell_f \end{matrix}\right)\left(\begin{matrix} \mu & 0 \\ 1 & \mathbb{L} \end{matrix}\right|\left.\begin{matrix} \mathcal{M}_{\mathcal{L}} \\ \mathcal{L} \end{matrix}\right). \tag{V-6}$$

Then we can define reduced momentum type matrix elements by

$$\left\langle f \left| e^{-is\cdot r}\; P_\mu /M \right| i \right\rangle = \left(\begin{matrix} m_i & \mathcal{M}_{\mathcal{L}} \\ \ell_i & \mathcal{L} \end{matrix}\right|\left.\begin{matrix} m_f \\ \ell_f \end{matrix}\right)\left(\begin{matrix} \mu & 0 \\ 1 & \mathbb{L} \end{matrix}\right|\left.\begin{matrix} \mathcal{M}_{\mathcal{L}} \\ \mathcal{L} \end{matrix}\right) \left\langle \ell_f \| \mathcal{L}\ \mathbb{L} \| \ell_i \right\rangle. \tag{V-7}$$

There are ten such elements, not all of which are significantly large.

The integrals required are

$\Phi^{+}_{212} = -.724$ $\qquad\qquad$ $\Phi^{-}_{212} = +.374$

$\Phi^{+}_{010} = -.307$

$\Phi^{+}_{012} = -.624$ $\qquad\qquad$ $\Phi^{-}_{012} = -.012$

$\Phi^{+}_{232} = -.074$ $\qquad\qquad$ $\Phi^{-}_{232} = +.012$

$\Phi^{+}_{252} = -.0031$

and the reduced matrix elements are

$$\langle 2 \| \mathcal{L}=0 \ \mathbb{L}=1 \| 2 \rangle = -.0620$$

$$\langle 2 \| \mathcal{L}=1 \ \mathbb{L}=1 \| 2 \rangle = -.143$$

$$\langle 2 \| \mathcal{L}=2 \ \mathbb{L}=1 \| 2 \rangle = +.0074$$

$$\langle 2 \| \mathcal{L}=2 \ \mathbb{L}=3 \| 2 \rangle = -.0160$$

$$\langle 2 \| \mathcal{L}=3 \ \mathbb{L}=3 \| 2 \rangle = -.0144$$

$$\langle 2 \| \mathcal{L}=4 \ \mathbb{L}=3 \| 2 \rangle = -.00025$$

$$\langle 2 \| \mathcal{L}=4 \ \mathbb{L}=5 \| 2 \rangle = -.00153$$

$$\langle 0 \| \mathcal{L}=0 \ \mathbb{L}=1 \| 0 \rangle = -.0665$$

$$\langle 0 \| \mathcal{L}=2 \ \mathbb{L}=1 \| 2 \rangle = -.0037$$

$$\langle 0 \| \mathcal{L}=2 \ \mathbb{L}=3 \| 2 \rangle = +.0069$$

APPENDIX VI. Structure of the Required States for A = 19

In the following table we give the structure of the states in F^{19} and O^{19} which enter our work. The states are given for three values of the central two particle (Yukawa with a Rosenfeld exchange mixture) interaction (30, 40 and 50 Mev), and refer to a standard convention (17) for relative phases. The notation requires a bit of explanation. For example,

a) $d^3 [3] (10)\, {}^2D$ means 3(d) particles in a state totally symmetric in space, belonging to the representation (10) of the subgroup (R 5) of the permutation group, with total spin 1/2 and orbital angular momentum 2.

b) $d^2 s \quad 13 \ {}^2S$ means a parent state of two (d) particles in a singlet isotopic spin state, triplet spin state, with zero orbital angular momentum plus an (s) particle LS coupled to this parent state to give the relevant final state.

The ds^2 and s^3 states have a similar designation.

State	T=1/2 J=1/2 30	40	50	T=3/2 J=1/2 30	40	50	T=3/2 J=3/2 30	40	50	T=3/2 J=5/2 30	40	50
d^3 [3] (10) ^2D												
(30) ^2S	-.319	-.310	-.312									
^2F												
[21] (10) ^2D							.025	.040	.049	.760	.770	.768
^4D												
(21) ^2P	.126	.100	.079	-.0543	-.070	-.0780						
^4P	-.126	-.110	-.093									
^2D							-.510	-.480	-.462	-.206	-.200	-.189
^4D	-.039	-.030	-.019									
^2F										.425	.410	.388
^4F												
^4G												
[111] (11) ^2P	-.039	-.030	-.018									
^4P				-.0136	-.0140	-.0149	.256	.220	.193	-.287	-.250	-.222
^2F												
^4F							-.135	-.120	-.111	-.208	-.190	-.168
d^2s 13 ^2S	-.498	-.530	-.545									
^4S												
31 ^2S	.518	.510	.503	.849	.860	.876						
13 ^4D	-.049	-.030	-.025									
31 ^2D							-.344	-.380	-.405	-.064	-.060	-.056
13 ^4G												
11 ^2P	.444	.120	.097									
33 ^2P	-.126	-.100	-.086	-.389	-.380	-.379	.377	.430	.468			
^4P	-.217	-.180	-.155	-.354	-.320	-.287	-.083	-.080	-.079	.0260	.020	.017
11 ^2F												
33 ^2F										.045	.050	.055
^4F							.094	.100	.095	.006	.000	.003
ds^2 13 ^2D												
^4D	-.002	.000	-.001									
31 ^2D							-.022	-.030	-.039	.257	.320	.373
s^3 [3] ^2S	.511	.540	.544									

APPENDIX VII. Numerical Results for F^{19}.

In the following table are presented the matrix elements for μ capture by F^{19}. These are evaluated with the shell model wave functions of Elliott and Flowers (17) given in Appendix VI, as a function of the two nucleon interaction V. Also, for comparison, matrix elements evaluated on the single particle (SP) model are presented.

Recall that a factor $(3/\sqrt{2}) \times 10^{-2}$ has been removed from all these.

TABLE I

Term		V=30	V=40	V=50	SP
$1/2\,1/2^+-3/2\,1/2^+$					
	F_0	-.133	-.145	-.141	-17.7
	G_{10}	-.444	-.802	-1.086	+17.7
	G_{12}	+.198	+.146	+.125	
	Q_{010}	+.0599	+.0654	+.0634	+ 1.81
	T_{011}	+.00512	+.0378	+.0648	- 1.81
	T_{211}	+.00447	+.00338	+.00284	
	T_{111}	-.178	-.198	-.243	
	Q_{111}	-.716	-.741	-.708	
$1/2\,1/2^+-3/2\,3/2^+$					
	F_2	+.609	+.601	+.602	- 3.20
	G_{10}	-2.687	-2.661	-2.577	
	G_{12}	+1.505	+1.532	+1.576	- 5.05
	G_{22}	+.805	+.877	+.949	- 3.91
	Q_{212}	+.00536	+.00516	+.00521	+ .0199
	Q_{232}	-.0114	-.0101	-.00941	+ .0376
	T_{011}	+.261	+.260	+.253	
	T_{211}	+.0329	+.0327	+.0331	+ .0316
	T_{111}	+.618	+.639	+.660	
	Q_{111}	-.950	-.997	-1.038	
$1/2\,1/2^+-3/2\,5/2^+$					
	F_2	+.190	+.253	+.289	+ 3.17
	G_{22}	-.113	-.256	-.366	- 2.60
	G_{32}	-.649	-.820	-.932	- 4.12
	G_{34}	-	-	-	
	Q_{212}	+.00353	+.00379	+.00404	- .0199
	Q_{232}	-.00427	-.00492	-.00549	- .0374
	T_{233}	+.0115	+.01295	+.01357	

Table I (continued)

Term		V=30	V=40	V=50	SP
1/2 1/2-3/2 5/2$^+$ (continued)					
	T_{433}	-	-	-	
	T_{333}	.00129	-.000972	-.000757	
	Q_{333}	-.000027	+.000113	+.000208	
Total	f^2	1.735	1.870	1.992	
	g^2	99.61	103.6	104.8	
	$\lvert\hat{v}\cdot g\rvert^2$	54.66	56.05	55.98	
	t^2	.793	.901	1.220	
	q^2	4.643	5.077	5.317	
$\mathrm{Re}\, f\, \hat{v}\cdot q^*$		+.0449	.0468	.0474	
$\mathrm{Im}\, \hat{v}\cdot g x q^*$		12.56	13.84	14.21	
$\mathrm{Re}\, \hat{v}\cdot g\, t^*$		-5.97	-6.14	-6.38	
$i(g x J)\cdot g^*$		33.03	28.50	21.81	
$i(\hat{v} x q) x J\cdot(\hat{v} x q)^*$		.330	.35	.45	
$-i(g x J)\cdot\hat{v}\,(\hat{v}\cdot g)^*$		-7.65	-5.60	-2.95	
$-(g x J)\cdot(\hat{v} x q)^*$		13.32	13.43	13.64	
$-i(g x J)\cdot\hat{v}\, t^*$		.552	.756	1.04	
$(\hat{v}\cdot g\, \hat{v} x J)\cdot(\hat{v} x q)^*$		-4.31	-4.17	-4.05	
$-(\hat{v} x q)\, x J\cdot\hat{v}\, t^*$		-.76	-.86	-.98	
$f\, J\cdot g^*$		3.48	3.97	4.46	
$-f\, J\cdot\hat{v}\, \hat{v}\cdot g^*$		-3.76	-3.92	-4.04	
$+i\, f\, J\cdot(\hat{v} x q)^*$		-.93	-.98	-.100	
$-f\, J\cdot\hat{v}\, t^*$		+.58	+.60	+.62	
$-\hat{v}\cdot q\, J\cdot g^*$		-.118	-.079	-.066	
$\hat{v}\cdot q\, J\cdot\hat{v}\,(\hat{v}\cdot g)^*$		+.068	+.056	+.046	
$-i\,\hat{v}\cdot q\, J\cdot(\hat{v} x q)^*$		+.041	+.046	+.043	
$\hat{v}\cdot q\, J\cdot\hat{v}\, t^*$		-.017	-.019	-.021	

TABLE 2

The coupling constants in the μ capture Hamiltonian are listed. Cases 1, 2, 3 assume $g_p = +8\, g_A$, $-8\, g_A$, 0 and a conserved vector current. Cases 4, 5, 6 assume $g_P = +8\, g_A$, $-8\, g_A$, 0 and no conserved vector current.

	1	2	3	4	5	6
G_V	1.053	1.053	1.053	1.053	1.053	1.053
G_A	-1.480	-1.480	-1.480	-1.283	-1.283	-1.283
G_P	- .705	+ .340	- .183	- .510	+ .586	+ .012

TABLE 3

The rates λ and λ_D for the various hypotheses 1 to 6 calculated with the matrix elements of Table 1 are given below:

		V_C=30	V=40	V=50
2λ	Case 1	111.6	115.2	118.2
	2	275.6	285.5	286.9
	3	178.6	184.0	187.2
	4	91.4	94.1	96.5
	5	266.1	278.8	276.1
	6	158.2	162.6	165.3
$2\lambda_D$	Case 1	-46.14	-27.92	-10.64
	2	-62.24	-26.71	+32.64
	3	-45.21	-18.68	+18.77
	4	-27.08	- 6.74	+15.33
	5	-30.58	+ .568	+42.48
	6	-26.11	- 3.20	+28.30

APPENDIX VIII.

The reduced matrix elements required in μ capture by He^3, evaluated with the harmonic oscillator wave functions

$$\psi_S = \sqrt{\frac{4}{\sqrt{\pi}}} \; e^{-x^2/2} \; Y_0^0$$

$$\psi_D = \sqrt{\frac{2}{5} \frac{8}{3\sqrt{\pi}}} \; x^2 e^{-x^2/2} \; Y_2^{\gamma}$$

where $x = r/b$, b = 0.8, 1.0 and 1.2 fermis

are, for s = 100 Mev,

Table 1

	b=0.8	1.0	1.2
$\langle s \| e^{-is\cdot r} \; \mathcal{L}=0 \; \| s \rangle$	.960	.938	.920
$\langle d \| e^{-is\cdot r} \; \mathcal{L}=0 \; \| d \rangle$	.908	.860	.804
$\langle s \| e^{-is\cdot r} \; \mathcal{L}=2 \; \| d \rangle$	-.102	-.154	-.217
$\langle d \| e^{-is\cdot r} \; \mathcal{L}=2 \; \| d \rangle$	.097	.146	.201
$\langle s \| \; \mathcal{L}=0 \; \mathbb{L}=1 \; \| s \rangle$	-.043	-.042	-.041

The matrix elements entering the rate, for a D state amplitude

$\epsilon = \pm 0.20$ are

Table 2

	b=0.8	1.0	1.2
$2 \; F_0$	.958	.934	.915
$2 \; G_{10}$	.934	.911	.894
$2 \; \Omega_{010}$	-.043	-.042	-.041
$2 \; T_{011}$	-.043	-.042	-.041
$G_{12} \; (\epsilon = +.20)$	-.012	-.018	-.025
$G_{12} \; (\epsilon = -.20)$	+.009	+.014	+.020

Here we present the individual terms entering the rates (2λ) and $(2\lambda_D)$. For the terms in (2λ) we also present the results for $\epsilon = 0$, b = 1.0 fermis. In calculating $(2\lambda_D)$ we have taken ϵ to be positive.

Table 3

	b=0.8	1.0	1.2	ϵ =0 b=1.0
(2λ)				
f^2	.912	.872	.837	.880
g^2	2.616	2.490	2.397	2.639
t^2	.0006	.0006	.0006	.0006
q^2	.0006	.0006	.0006	.0006
Re $f\,\hat{v}\cdot q^*$	.024	.023	.022	.023
Re $\hat{v}\cdot g\, t^*$	.023	.022	.021	.022
$\lvert\hat{v}\cdot g\rvert^2$ ϵ =+.20	.892	.859	.839	.880
ϵ =-.20	.857	.807	.767	
$(2\lambda_D)$				
$i(g\times J)\cdot g^*$	-2.616	-2.490	-2.397	-2.639
$-i(g\times J)\cdot\hat{v}(\hat{v}\cdot g)^*$	+.872	+.830	+.799	+.880
$-i(g\times J)\cdot\hat{v}\, t^*$	.023	.022	.021	.023
$f\, J\cdot g^*$	-1.342	-1.276	-1.227	-1.320
$-f\, J\cdot\hat{v}(\hat{v}\cdot g)^*$	.775	.737	.708	.762
$-f\, J\cdot\hat{v}\, t^*$	-.021	-.020	-.019	-.020
$-\hat{v}\cdot q\, J\cdot g^*$	+.034	+.033	+.032	+.034
$\hat{v}\cdot q\, J\cdot\hat{v}\,(\hat{v}\cdot g)^*$	-.012	-.011	-.011	-.011
$\hat{v}\cdot q\, J\cdot\hat{v}\, t^*$	-.0003	-.0003	-.0003	-.0003

The rates (2λ) and $(2\lambda_D)$ calculated with the coupling constants of Appendix VII are presented in table 4. We take $\epsilon = +.20$.

Table 4

		b=0.8	1.0	1.2	b=1.0, ϵ=0
2λ	Case 1	5.23	4.96	4.75	5.26
	2	7.60	7.24	6.98	7.60
	3	6.17	5.86	5.63	6.19
	4	4.27	4.05	3.83	4.29
	5	6.78	6.49	6.26	6.88
	6	5.21	4.95	4.76	5.21
$2\lambda_D$	Case 1	2.23	2.12	2.04	2.53
	2	4.92	4.69	4.53	5.29
	3	3.56	3.40	3.27	4.05
	4	.800	.768	.739	.997
	5	2.66	2.54	2.46	2.99
	6	1.68	1.60	1.54	1.93

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