THE EFFECT OF RECOMBINATION ON THE PRIMARY PHOTO-ELECTRIC CURRENT FROM A CRYSTAL

by

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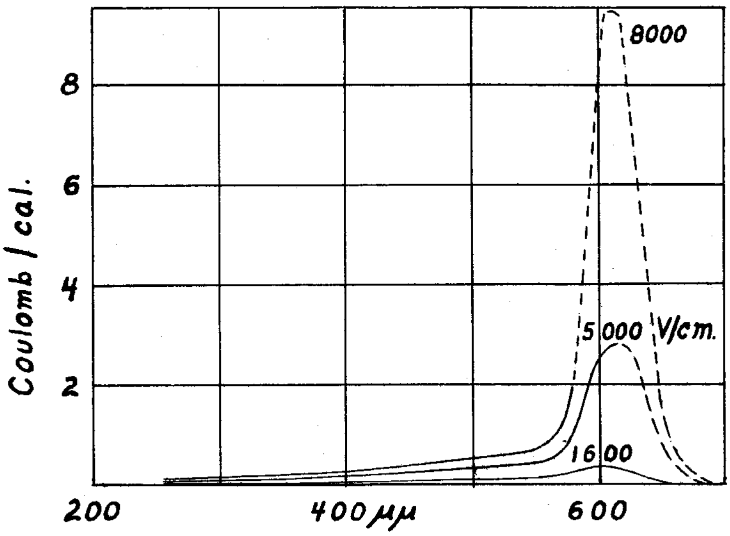
The Effect of Recombination on the Primary Photo-electric Current from a Crystal.

Many crystals that are very good insulators in the dark show an appreciable conductance when illuminated. The crystals exhibiting this phenomenon have been the subject of numerous researches. Of the facts reported by the workers in this field those that are most important in connection with this investigation are:

There is a particular wavelength to which the crystal is photo-electrically most sensitive. Often the sensitivity curve shows more than one peak. The sharpness of a maximum is increased by an increase in the applied voltage, (fig. 1). A decrease in temperature usually shifts the peak towards the ultraviolet.

Only crystals with a high index of refraction ($n > 2$) are photo-electrically active.* However, a high index of refraction does not necessarily mean conductance, there are exceptions, and some crystals with low index of refraction can be made conducting by exposing them to X rays. Rock salt is an example of such a crystal.

The photo-electric current is of two kinds,* (a) the primary current consisting of electrons liberated by the light; (b) the secondary current representing the motion of the positive ions towards the cathode. Under suitable experimental conditions the current is all of the first or primary kind. As ions naturally move much slower than electrons, if short illumination periods are used only a negligible portion of the positive ions formed have time to reach the cathode. Heat or infrared radiation hastens the motion of ions. If, therefore, only ultraviolet light is used for excitation fig.1

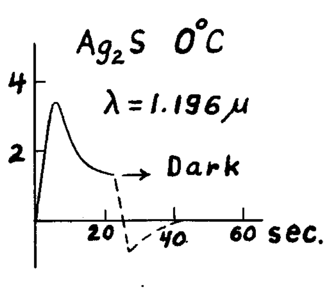


Gudden and Pohl

Zeitschrift fur Physik

page 369, vol.2, 1920

fig.2

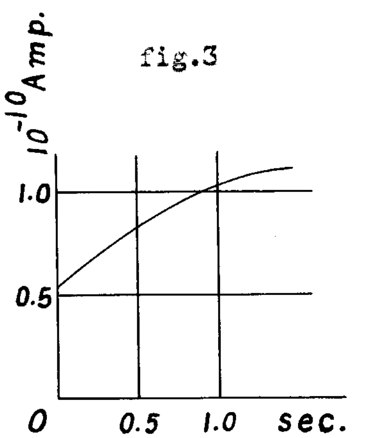


W. W. Coblentz

Bureau of Standards

page 595, vol.16, 1920

fig.3



Gudden and Pohl

Naturwissenschaften

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and the temperature is kept low enough, no positive current should flow. The latter method was used by Gudden and Pohl in their rock salt experiments

Quantitative measurements of the light absorbed and primary current obtained show that within experimental error each $h\nu$ absorbed gives rise to one electron if the wavelength of the absorbed light is greater than and not too near to that of the light most strongly absorbed. On approaching the critical frequency from the long wavelength side the primary current drops rapidly and almost vanishes just as the critical frequency is reached (fig. 11)

The total photo-electric current changes with time in various ways depending upon the crystal and the experimental conditions. If the mobility of the electrons is rather low it starts from zero and rises to a maximum, as shown in fig. 2 . If the mobility of the electrons is high it starts with the initial value of the primary current and the secondary current is added on more gradually, (fig. 3). If there is no secondary current the primary current starts to decrease immediately on account of the positive space charge left behind (fig. 8).

When there is no dark current and no positive current electrons must accumulate on the cathode and neutralize the space charge so far as the applied voltage is concerned. Due to the fact that the crystal is not an infinite plane this condition is never fully realized. In the ideal case of an infinite plane crystal the net effect of the space charge would be to accelerate the progress of the electrons towards the anode, the additional accelerating force being a maximum at the cathode and decreasing to zero at the anode. When, however, an electron gets close enough to an ion it must recombine.** The ions of the space charge therefore reduce the primary current by capturing electrons on their way to the anode. It is the purpose of this investigation to calculate to what extent recombination may reduce the primary current.

I. The experiments are of two classes; in the first the light is perpendicular to the electric field (fig. 4); in the second the light is parallel and opposite to the electric field* (fig. 9).

If the light is parallel to the electric field the effect of lateral diffusion is to flatten out the original density distribution $e^{-\mu x}$ given by the light. Those electrons diffusing from regions of greater electron density (and consequent greater ion density) to regions of lesser electron density (and consequent lesser ion density) have a better chance of escaping recombination with some ion and reaching the anode than those electrons which do not diffuse. On the other hand those electrons which do not diffuse have a smaller chance of reaching the anode than they would have, did not some others diffuse, because they must now escape not only the ions normally in their path but also those ions which the diffusing electrons escape. The effect of diffusion is thus somewhat self compensating and quite a change in the distribution of electrons from this cause need not, so far as can be seen without actually taking it into account in the calculations, cause a material change in the primary current. As it is not an easy matter to take diffusion into account in calculating the current to be expected it will be shown instead that, for the experiments to which the expression derived for the current is to be applied, diffusion does not sensibly alter an original distribution while it traverses the crystal. This is done in section II. In section III the primary current is computed for the case of a field at right angles to the incident light and in section IV it is computed for the case of a field parallel and opposite to the incident light.

II. Let free electrons be present in a crystal and be distributed according to the law $e^{-\mu x}$ which is the law by which they are photo-electrically liberated. μ is the absorption constant for the light used. Suppose now that these

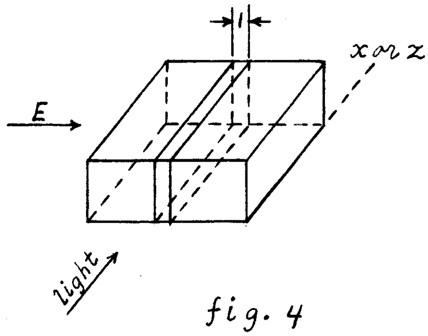


fig. 4

electrons are uniformly distributed in a unit layer under the curve $e^{-\mu x}$. The problem before us is to find out how much diffusion has flattened out this original distribution $e^{-\mu x}$ by the time the field E has pulled the unit layer over to the anode.

Due to diffusion each element of volume $e^{-\mu x}\,dx$ in the unit layer will in time assume the distribution $k\,e^{-au^2}$ where k and a are determined by the condition.

$$k\int_{-\infty}^{\infty} e^{-au^2}\,du = \text{Area under the elementary probability curve}$$
$$= k\sqrt{\frac{\pi}{a}} = e^{-\mu x}\,dx \qquad \cdots (1)$$

Since an electron cannot diffuse from the surface of the crystal into the air the probability curve must be folded back at the surface of the crystal. The contribution of an element of volume $e^{-\mu x}\,dx$ to the electrons at any other point in the same unit layer after the redistribution is thus from two parts of the little probability

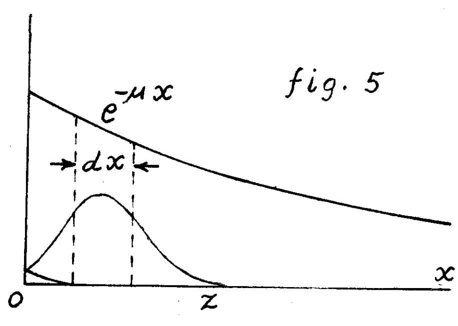


fig. 5

curve distant $2x$ apart. The number of electrons at a point z due to the diffusion of $e^{-\mu x}\,dx$ electrons starting from the point x is therefore

$$d\rho = k\, e^{-a(x-z)^2} + k\, e^{-a(x+z)^2}$$

But from (1) we have it that

$$k = \sqrt{\frac{a}{\pi}}\; e^{-\mu x}\, dx$$

and consequently that

$$d\rho = \sqrt{\frac{a}{\pi}} \left(e^{-a(x-z)^2} + e^{-a(x+z)^2} \right) e^{-\mu x}\, dx$$

The total number of electrons between z and $z+1$ is then

$$\rho = \sqrt{\frac{a}{\pi}} \int_0^\infty \left(e^{-a(x-z)^2} + e^{-a(x+z)^2} \right) e^{-\mu x}\, dx \qquad \cdots (2)$$

This new distribution ρ satisfies the differential equation

$$\frac{d^2 \rho}{d z^2} = \mu^2 \rho - 2\mu \sqrt{\frac{a}{\pi}}\; e^{-a z^2} \qquad \cdots (3)$$

The properties of ρ are not apparent from (2) but (3) shows that ρ does have the characteristics to be expected from the nature of the problem: for if $a \doteq \infty$ meaning that there is no diffusion, (3) reduces to $\frac{d^2\rho}{d z^2} = \mu^2 \rho$ of which the solution is $\rho = e^{-\mu z}$ which is the original distribution law; as z becomes very large (3) again reduces to $\frac{d^2 \rho}{d z^2} = \mu^2 \rho$ with the solution $\rho = e^{-\mu z}$ which means that if the original distribution is quite uniform, as it is for large values of z, then diffusion does not affect it.

The expression (2) has yet to be put in a form suitable for computation.

$$\begin{aligned}
\rho &= \sqrt{\frac{a}{\pi}} \int_0^\infty \left\{ e^{-a(x-z)^2} + e^{-a(x+z)^2} \right\} e^{-\mu x}\, dx \\
&= \sqrt{\frac{a}{\pi}} \int_0^\infty \left\{ e^{-ax^2 - a(x+\frac{\mu}{2a}-z)^2 + a(\frac{\mu}{2a}-z)^2} + e^{-ax^2 - a(x+\frac{\mu}{2a}+z)^2 + a(\frac{\mu}{2a}+z)^2} \right\} dx \\
&= \sqrt{\frac{a}{\pi}}\; e^{-az^2} \left[e^{a(\frac{\mu}{2a}-z)^2} \int_{\frac{\mu}{2a}-z}^\infty e^{-as^2}\, ds + e^{a(\frac{\mu}{2a}+z)^2} \int_{\frac{\mu}{2a}+z}^\infty e^{-as^2}\, ds \right]
\end{aligned}$$

$$= \sqrt{\frac{a}{\pi}}\Big[e^{\mu(\frac{\mu}{4a}-z)}\Big(\tfrac{1}{2}\sqrt{\tfrac{\pi}{a}}+\int_{\frac{\mu}{2a}-z}^{0} e^{-as^2}ds\Big) + e^{\mu(\frac{\mu}{4a}+z)}\Big(\tfrac{1}{2}\sqrt{\tfrac{\pi}{a}}+\int_{\frac{\mu}{2a}+z}^{0} e^{-as^2}ds\Big)\Big]$$

$$= e^{\frac{\mu^2}{4a}}\Big[\cosh\mu z - \sqrt{\tfrac{a}{\pi}}\Big(e^{-\mu z}\int_0^{\frac{\mu}{2a}-z} e^{-as^2}ds + e^{\mu z}\int_0^{\frac{\mu}{2a}+z} e^{-as^2}ds\Big)\Big]$$

$$\rho = e^{\frac{\mu^2}{4a}}\Big[\cosh\mu z - \sqrt{\tfrac{a}{\pi}}\Big(e^{\mu z}\int_0^{z+\frac{\mu}{2a}} e^{-as^2}ds - e^{-\mu z}\int_0^{z-\frac{\mu}{2a}} e^{-as^2}ds\Big)\Big] \cdot\cdot (4)$$

If the electron can be looked upon as having a mean free path l then

$$a = \frac{4^2}{\pi^2}\frac{1}{nl^2}$$

where n is the number of collisions it has made

$\mu = \frac{1}{2}\frac{el}{mv} =$ mobility of the electron

$V = \mu E = \frac{eEl}{2mv} =$ velocity with which it proceeds through the crystal

$\frac{V}{v} = \frac{eEl}{2mv^2} =$ ratio of forward progress to total random motion

$n = \frac{v}{V}d\frac{1}{l} = \frac{2d}{eEl^2}mv^2 =$ number of collisions made in advancing a distance d

$$\therefore a = \frac{4^2}{\pi^2}\frac{1}{nl^2} = \frac{4^2}{\pi^2}\frac{eE}{4d\frac{1}{2}mv^2} = \frac{4eE}{\pi^2 d\times K.E.}$$

$$= \frac{4}{\pi^2 d}\,\frac{4.774\times10^{-10}E_v}{300}\,\frac{273}{5.621\times10^{-14}\,T} = 3130\,\frac{E_v}{Td}$$

where E_v is the electric field in volts per cm. and T is the absolute temperature. $T = 306°$; take $d = .15$ cm. = half the crystal thickness; assume $E_v = 117.5$ and then $a = 8100.$

As μ only affects the abscissa scale when $e^{-\mu x}$ is plotted let $\mu = 1$.

$$\rho = e^{\frac{1}{32400}}\Big[\cosh z - \tfrac{90}{\sqrt{\pi}}\Big(e^{z}\int_0^{z+\frac{1}{16200}} e^{-as^2}ds - e^{-z}\int_0^{z-\frac{1}{16200}} e^{-as^2}ds\Big)\Big]$$

Let $90s = t$ and then if $s = z \pm \frac{1}{16200}$ $\quad t = 90z \pm \frac{1}{180}$

$$\rho = \cosh z - \tfrac{1}{\sqrt{\pi}}\Big(e^{z}\int_0^{90z+\frac{1}{180}} e^{-t^2}dt + e^{-z}\int_0^{90z-\frac{1}{180}} e^{-t^2}dt\Big)$$

The greatest change due to diffusion must occur at the origin $x=0$ for it is here that the slope of the curve $e^{-\mu x}$ is greatest (x positive). Placing therefore $x=0$ we calculate

$$\rho_0 = 1 - \frac{1}{\sqrt{\pi}} \int_{-\frac{1}{180}}^{\frac{1}{180}} e^{-t^2} dt = 1 - .0062 = .9938$$

or that the average maximum change in an ordinate, under the conditions stated, is 0.6% or less; less because the average electron travels less than half the crystal thickness.

From these considerations it is seen to be permissible to assume in the mathematical work of deriving the expression for the primary current that the electrons move only in the direction of the electric field.

III It is assumed that an electron reaches the anode unless it recombines with an ion and that the radiation emitted when recombination occurs does not liberate another electron before it reaches the surface of the crystal and escapes. Were this condition not completely fulfilled the effect would merely be that diffusion and the distance between ions would both appear to be greater than normal for the disappearance of an electron in one place and its appearance in another could be looked upon as diffusion and by this process one collision with an ion has been lost.

The mean free path of the electrons, referred to ions, is taken to be $\lambda = \frac{1}{N\pi\sigma^2 f}$ where N is the number of ions per c.c., σ is the radius of an ion and f may be a function of the mobility of the electrons, of the field, and consequently a function of the concentration of ions but is assumed to be a constant. This point will be brought up again.

λ is thus so defined that the chance that an electron will collide with an ion in going a distance ds is $\frac{ds}{\lambda}$

The chance that an electron will go a distance S and then collide with an ion in the distance ds is

$$e^{-\frac{S}{\lambda}} \frac{ds}{\lambda}$$

where λ is the mean free path, provided λ is a constant. If λ is not a constant it is easily seen that we must have

$$\lim_{n \doteq \infty} e^{-\frac{ds}{\lambda_1}} e^{-\frac{ds}{\lambda_2}} \cdots\cdots e^{-\frac{ds}{\lambda_n}} \frac{ds}{\lambda_n} = \lim_{n \doteq \infty} e^{-\sum_1^n \frac{ds}{\lambda_i}} \frac{ds}{\lambda_n} = e^{-\int_{S_1}^{S_n} \frac{ds}{\lambda}} \frac{ds}{\lambda_n}$$

Then the chance that an electron liberated a distance z from the cathode will go to a point distant y

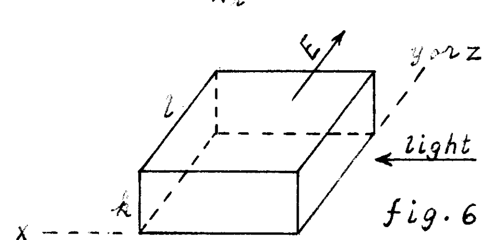


fig. 6

from the cathode and recombine in the distance dy is

$$e^{-\int_z^y \frac{ds}{\lambda(s)}} \frac{dy}{\lambda(y)}$$

provided we may neglect the change in λ at any point during the time it takes an electron to go from z to y. If this is not permissible we must write instead

$$e^{-\int_x^y \frac{ds}{\lambda(s,t-\frac{y-s}{\mu E})}} \frac{dy}{\lambda(y,t)}$$

where μ is the mobility, or velocity in a unit field.

The rate at which electrons are being liberated is a function of x only, $F(x)$. Then from the element of volume $k\,dx\,dz$ there are liberated $k\,dx\,dz\,F(x)$ electrons per sec. of which the number recombining in the element of volume $k\,dx\,dy$ is

$$k \cdot dx \cdot dz \cdot F(x)\, e^{-\int_z^y \frac{ds}{\lambda(s,t-\frac{y-s}{\mu E})}} \frac{dy}{\lambda(y,t)}$$

at the time t if $t \geqslant \frac{y-z}{\mu E}$

If there are n electrons per sq. cm. per sec. leaving the cathode, the number of these which combine with ions in the element of volume $k\,dx\,dy$ is

$$k \cdot dx \cdot n \cdot e^{-\int_0^y \frac{ds}{\lambda(s,t-\frac{y-s}{\mu E})}} \frac{dy}{\lambda(y,t)}$$

per sec. at the time t. In the experiments to which this work is to be applied $n=0$ but it will be retained for the present.

During the time $t < \frac{l}{\mu E}$ the total number of liberated electrons which recombine in $k\,dx\,dy$ is

$$k \cdot dx \cdot dy \cdot F(x) \frac{1}{\lambda(y,t)} \int_{y-\mu E t}^{y} e^{-\int_z^y \frac{ds}{\lambda(s,t-\frac{y-z}{\mu E})}} dz$$

per sec.

So up to the time $t = y/\mu E$ the total number of electrons recombining per sec. in $k \cdot dx \cdot dy$ is

$$k \cdot dx \cdot dy \, \frac{1}{\lambda(y,t)} \left[n \, e^{-\int_{y-\mu E t}^{y} \frac{ds}{\lambda(s,\, t - \frac{y-s}{\mu E})}} + F(x) \int_{y-\mu E t}^{y} e^{-\int_{z}^{y} \frac{ds}{\lambda(s,\, t - \frac{y-s}{\mu E})}} dz \right]$$

while the number being produced per sec. is

$$k \cdot dx \cdot dy \cdot F(x)$$

The difference must be the net rate of increase of ions which is

$$k \cdot dx \cdot dy \cdot \frac{\partial N}{\partial t} = k \cdot dx \cdot dy \, \frac{1}{\pi \sigma^2 f} \frac{\partial}{\partial t} \frac{1}{\lambda(y,t)}$$

and hence we have

$$\frac{1}{\pi\sigma^2 f} \frac{\partial}{\partial t} \frac{1}{\lambda} = F(x) - \frac{1}{\lambda} \left[n \, e^{-\int_{y-\mu E t}^{y} \frac{ds}{\lambda}} + F(x) \int_{y-\mu E t}^{y} e^{-\int_{z}^{y} \frac{ds}{\lambda}} dz \right] \cdots (1)$$

as the defining equation for $\lambda = \lambda(x, y, t)$ up to $t = y/\mu E$

The second term on the right has the lower limit $y - \mu E t$ for the integral because the chance that the $n k \, dx$ electrons reach the point $y - \mu E t$ is 1 if $t < y/\mu E$. If $t > y/\mu E$ we must write instead of (1)

$$\frac{1}{\pi\sigma^2 f} \frac{\partial}{\partial t} \frac{1}{\lambda} = F(x) - \frac{1}{\lambda} \left[n \, e^{-\int_{0}^{y} \frac{ds}{\lambda}} + F(x) \int_{0}^{y} e^{-\int_{z}^{y} \frac{ds}{\lambda}} dz \right] \cdots (2)$$

It is convenient to rewrite equ. (1) as

$$A \frac{\partial}{\partial t} \frac{1}{\lambda} = 1 - \frac{1}{\lambda} \left[B \, e^{-\int_{y-\mu E t}^{y} \frac{ds}{\lambda}} + \int_{y-\mu E t}^{y} e^{-\int_{z}^{y} \frac{ds}{\lambda}} dz \right] \cdots (3)$$

where $A = \left[\pi \sigma^2 f \, F(x) \right]^{-1}$ and $B = n / F(x)$

or $$-A \frac{\partial}{\partial t} \log \lambda = \lambda - \left[B \, e^{-\int_{y-\mu E t}^{y} \frac{ds}{\lambda}} + \int_{y-\mu E t}^{y} e^{-\int_{z}^{y} \frac{ds}{\lambda}} dz \right] \cdots (4)$$

Now let $\lambda = 1 \Big/ \frac{\partial}{\partial s} \log v(s, t - \frac{y-s}{\mu E})$ under the integral signs

$$\equiv 1 \Big/ \frac{\partial}{\partial y} \log v(y,t) \quad \text{outside the integral signs}$$

where $s = y$

Since $\lambda = \infty$ if $t = 0$ $v(y,0) = \text{const.} = v_0$

With this change of variable equ. (4) becomes

$$-A\left(\frac{1}{v}\frac{\partial v}{\partial t} - \frac{1}{\frac{\partial v}{\partial y}}\frac{\partial^2 v}{\partial y \partial t}\right) - \frac{v}{\frac{\partial v}{\partial y}} = -B\frac{v_0}{v} - \frac{1}{v}\int_{y-\mu E t}^{y} v(z, t - \frac{y-z}{\mu E})\, dz$$

or

$$A\left(\frac{\partial v}{\partial t} - \frac{v}{\frac{\partial v}{\partial t}}\frac{\partial^2 v}{\partial y \partial t}\right) + \frac{v^2}{\frac{\partial v}{\partial y}} = B v_0 + \int_{y-\mu E t}^{y} v(z, t - \frac{y-z}{\mu E})\, dz \qquad \cdots (5)$$

If we differentiate (5) with respect to y and t and then add the results we get the following differential equation

$$\frac{\partial}{\partial y}\left[A\left(\frac{\partial v}{\partial t} - \frac{v}{\frac{\partial v}{\partial y}}\frac{\partial^2 v}{\partial y \partial t}\right) + \frac{v^2}{\frac{\partial v}{\partial y}}\right] + \frac{1}{\mu E}\frac{\partial}{\partial t}\left[A\left(\frac{\partial v}{\partial t} - \frac{v}{\frac{\partial v}{\partial y}}\frac{\partial^2 v}{\partial y \partial t}\right) + \frac{v^2}{\frac{\partial v}{\partial y}}\right] = v \quad \cdots (6)$$

which must be solved for v if we are to know how the current rises when the light is turned on.

The current passing any point.

The part of the $k \cdot dx \cdot dz \cdot F(X)$ electrons liberated per second from the element of volume $k \cdot dx \cdot dz$ that reaches the point y is

$$k \cdot dx \cdot dz \cdot F(X)\; e^{-\int_z^y \frac{ds}{\lambda(s, t - \frac{y-s}{\mu E})}}$$

Of the n electrons passing 1 sq. cm. per sec. at the point $y - \mu E t$ at the time $t = 0$ the number passing 1 sq. cm. per sec. at the point y at the time t is

$$n\; e^{-\int_{y-\mu E t}^{y} \frac{ds}{\lambda(s, t - \frac{y-s}{\mu E})}}$$

up to the time $t = y/\mu E$

Then the total number of electrons passing 1 sq. cm. per sec. at time t at the point y is

$$\frac{\delta I_y}{e} = M = F(x) \int_{y-\mu E t}^{y} e^{-\int_z^y \frac{ds}{\lambda(s, t-\frac{y-s}{\mu E})}} dz + n\, e^{-\int_{y-\mu E t}^{y} \frac{ds}{\lambda(s,t-\frac{y-s}{\mu E})}}$$

Placing now $\lambda = 1 \Big/ \frac{\partial}{\partial s} \log v(s, t - \frac{y-s}{\mu E})$ as before we get

$$M = F(x) \left\{ B \frac{v_0}{v} + \frac{1}{v} \int_{y-\mu E t}^{y} v(z, t - \frac{y-z}{\mu E})\, dz \right\}$$

or

$$M = \frac{F}{v} \left\{ B v_0 + \int_{y-\mu E t}^{y} v(z, t - \frac{y-z}{\mu E})\, dz \right\} = \frac{F}{v} \left\{ A \left(\frac{\partial v}{\partial t} - \frac{v}{\frac{\partial v}{\partial y}} \frac{\partial^2 v}{\partial y \partial t} \right) + \frac{v^2}{\frac{\partial v}{\partial y}} \right\} \quad \cdots (7)$$

From (6) then we have it that

$$\frac{\partial}{\partial y}\left(\frac{M v}{F}\right) + \frac{1}{\mu E} \frac{\partial}{\partial t}\left(\frac{M v}{F}\right) = v$$

or

$$\frac{\partial^2}{\partial y^2}\left(\frac{M v}{F}\right) - \frac{\partial v}{\partial y} = \frac{1}{\mu^2 E^2} \frac{\partial^2}{\partial t^2}\left(\frac{M v}{F}\right) - \frac{1}{\mu E} \frac{\partial v}{\partial t} \quad \cdots (8)$$

In all the experiments of Gudden and Pohl the current leaps immediately to a maximum and then decreases. This indicates that μE is very large and consequently that the time taken for an electron to cross the crystal is small compared to the time required for the distribution of ions to change sensibly. We have then to deal with (2) and with (6) rewritten as

$$\frac{\partial}{\partial y}\left[A \left(\frac{\partial v}{\partial t} - \frac{v}{\frac{\partial v}{\partial y}} \frac{\partial^2 v}{\partial y \partial t} \right) + \frac{v^2}{\frac{\partial v}{\partial y}} \right] = v$$

or $$\left(A \frac{\partial^2 v}{\partial y \partial t} - v \right) - \frac{\partial}{\partial y} \left[\frac{v}{\frac{\partial v}{\partial y}} \left(A \frac{\partial^2 v}{\partial y \partial t} - v \right) \right] = 0$$

or $$\frac{\partial^2 v}{\partial y \partial \tau} = v \qquad \text{where} \quad \tau = \frac{t}{A} = \pi \sigma^2 b\, F(x)\, t \quad \cdots (9)$$

Following the suggestion of (8) that t and y may enter into v in the same way, we assume $v = v[(\tau+g)(y+h)]$ and find that (9) reduces to the total differential equation (the other simple and most obvious assumption $v = v(\tau + y)$ does not yield a suitable form of solution):

$$(\tau+g)(y+h)\, v'' + v' = v$$

Letting $(\tau+g)(y+h) = -\frac{\alpha^2}{4}$ this becomes

$$v'' + \frac{1}{\alpha} v' + v = 0$$

$$\therefore \quad v = J_0(\alpha) = J_0\left(2i\sqrt{(\tau+g)(y+h)}\right)$$

Using now the fact that $n = 0$ and μE is large we may rewrite (3) in the following form

$$A \frac{\partial}{\partial t} \frac{1}{\lambda} = 1 - \frac{1}{\lambda}\int_0^y e^{-\int_z^y \frac{ds}{\lambda}}\, dz$$

or

$$\frac{\partial v}{\partial \tau} = \int_0^y v(z, \tau)\, dz \qquad \cdots (10)$$

where we have used (9) and $\frac{1}{\lambda} = \frac{\partial}{\partial y} \log v$

The condition $\lambda = 1/\frac{\partial}{\partial y} \log v = \infty$ if $t = 0$ makes $g = 0$ for only then will v be independent of y when $t = 0$. Substituting $J_0\left(2i\sqrt{\tau(y+h)}\right)$ into (10) we find that $h = 0$.

$$\therefore \lambda = 1/\frac{\partial}{\partial y} \log J_0\left(2i\sqrt{\tau y}\right)$$

Substituting (9) in (7) we find

$$\delta I_y = eF\,\frac{1}{v}\frac{\partial v}{\partial \tau} = \frac{e}{\pi\sigma^2 b}\,\frac{1}{v}\frac{\partial v}{\partial t} = e\,y\,F\,\frac{1}{v}\frac{dv}{d(\tau y)} = e\,y\,F\,\frac{v'}{v}$$

$$\therefore I_l = e\,h\,l\int_0^w \frac{v'(\tau l)}{v(\tau l)}\, F(x)\, dx$$

Wherein we have taken account of the facts that the crystal is h cm.

high and that the current is measured at the anode where $y = l$. The ratio $\frac{v'}{v}$ may be expanded into a power series and then

$$I = ekl\int_0^w \{1 + a_1 Tl + a_2 (Tl)^2 + a_3 (Tl)^3 + \cdots\} F(x)\, dx$$

If Q_0 ergs of energy enter 1 sq. cm. of the surface each second the energy remaining in this square beam at a depth x in the crystal is $Q_1 = Q_0 e^{-\mu x}$. Of the energy remaining in the beam when it reaches the rear face a fraction r is reflected. The reflected energy $Q_0 e^{-\mu w} r$ contributes an amount $Q_2 = Q_0 r e^{-\mu w} e^{-\mu(w-x)}$ at the depth x. Obviously the total energy at any point is an infinite series $Q = \sum_1^\infty Q_i$

$$Q = Q_0 e^{-\mu x} + Q_0 r e^{-2\mu w} e^{\mu x} + Q_0 r^2 e^{-2\mu w} e^{-\mu x} + Q_0 r^3 e^{-4\mu w} e^{\mu x} + \cdots$$

$$= \frac{Q_0}{1 - r^2 e^{-2\mu w}} \left(e^{-\mu x} + r e^{-2\mu w} e^{\mu x}\right)$$

$$= Q_0 A \cosh\{\mu(x - w) + \log\sqrt{r}\}$$

where $A = 2\sqrt{r}\, e^{-\mu w} / (1 - r^2 e^{-2\mu w})$

The rate at which electrons are being liberated at the depth is

$$F(x) = \frac{-1}{h\nu}\frac{dQ}{dx} = -\frac{Q_0\mu}{h\nu} A \sinh\{\mu(x - w) + \log\sqrt{r}\}$$

So
$$I = ekl\int_0^w \{1 + a_1(\pi\sigma^2 btl)F(x) + a_2(\pi\sigma^2 btl)^2 F^2(x) + \cdots\} F(x)\, dx$$

$$= ekl\frac{Q_0}{h\nu} A \{B_1 + a_1(\theta A) B_2 + a_2(\theta A)^2 B_3 + \cdots\} \qquad \cdots (11)$$

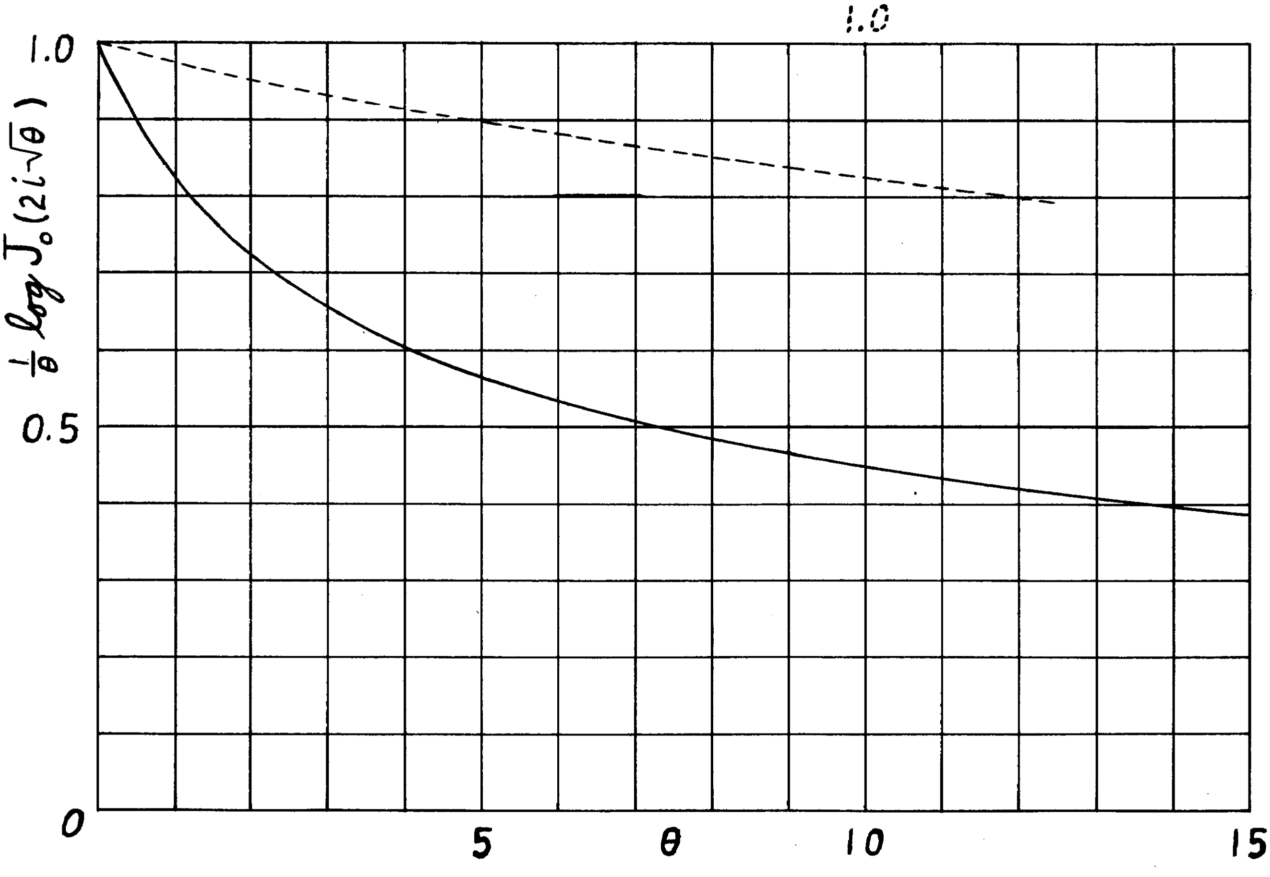
where $\theta = \pi\sigma^2 btl\, Q_0 \mu / h\nu$

$$B_n = \int_0^w (-1)^n \sinh^n\{\mu(x - w) + \log\sqrt{r}\}\, \mu\, dx$$

If r is negligible then $Q = Q_0 e^{-\mu x}$ and $F(x) = \frac{Q_0\mu}{h\nu} e^{-\mu x}$ and (11) becomes

$$I = ekl\frac{Q_0}{h\nu}\left\{(1 - e^{-\mu w}) + \tfrac{1}{2} a_1 \theta (1 - e^{-2\mu w}) + \cdots\right\} \qquad \cdots (12)$$

fig. 7



If most of the light is absorbed on the first trip across the crystal and r is small, it will be sufficiently accurate to correct for the light passing clear through by making w infinite. With w infinite (12) becomes

$$I = e\,k\,l\,\frac{Q_0}{h\nu}\,f(\theta) \qquad \cdots (13)$$

where

$$\begin{aligned} f(\theta) &= 1 + a_1 \tfrac{1}{2}\theta + a_2 \tfrac{1}{3}\theta^2 + a_3 \tfrac{1}{4}\theta^3 + \cdots \\ &= \frac{1}{\theta}\left(\theta + a_1 \tfrac{1}{2}\theta^2 + a_2 \tfrac{1}{3}\theta^3 + a_3 \tfrac{1}{4}\theta^4 + \cdots\right) \\ &= \frac{1}{\theta}\int_0^\theta \left(v'(\theta)/v(\theta)\right) d\theta = \frac{1}{\theta}\log v(\theta) \\ &= \frac{1}{\theta}\log J_0(2i\sqrt{\theta}) \end{aligned}$$

$$\lim_{\theta \doteq \infty} f(\theta) = \lim_{\theta \doteq \infty} \frac{\log v(\theta)}{\theta} = \lim_{\theta \doteq \infty} \frac{v'(\theta)}{v(\theta)} = \lim_{\theta \doteq \infty} \frac{1}{i\sqrt{\theta}}\,\frac{J_1(2i\sqrt{\theta})}{J_0(2i\sqrt{\theta})}$$

for $J_1(2i\sqrt{\theta}) < J_0(2i\sqrt{\theta})$

The theory thus says that, ν and Q_0 being constant, the current should drop with time as shown in fig. 7 : for θ is a linear function of the time.

This is just what Gudden and Pohl have observed in NaCl#. With intense illumination they got a drop such as shown. With weak illumination they got very little drop in the same length of time and this is to be expected from the form of θ . If Q_0 is small it takes a long time for θ to become large.

If the experimental conditions were such that most of the light was absorbed in the rock salt it is permissible to compare $f(\theta)$ with the experimental curves, otherwise it would be necessary to use the more exact expression for I. As b is not known the test of the theory will consist of seeing if a constant b can be so chosen that the theoretical curve will lie along the experimental curve. The following figure shows how closely $f(\theta)$ can fit the experimental curve.

In view of the approximations introduced in deriving $f(\theta)$ and in applying it to the experiments the fit seems very good and appears to warrant the conclusion that recombination is sufficient to account for the decrease in the current with time and that the assumption $b = const.$ was practically correct.

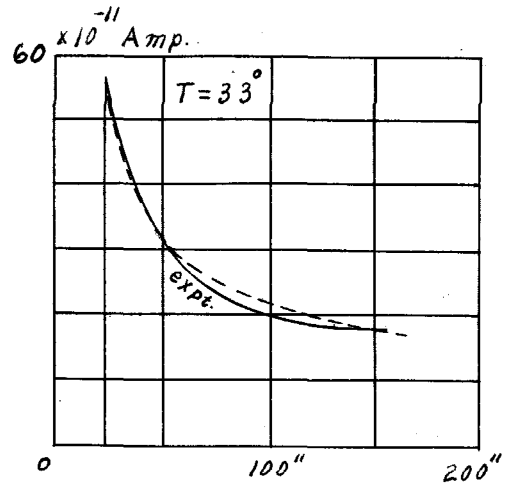


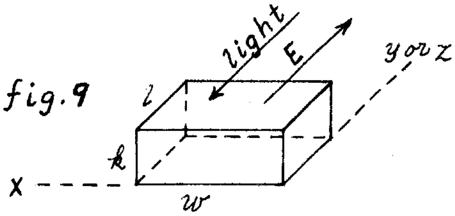
fig. 8

If current per unit intensity of entering light is plotted against wavelength a sensitivity curve having a peak is always obtained. The low current on the red side of the peak is accounted for by the small amount of light absorbed. A possible explanation of the low current on the violet side is furnished by the relation $\lim_{\theta \doteq \infty} f(\theta) = 0$. On the violet side of the peak μ is large, consequently θ is large and therefore $f(\theta)$ is small. However, as is shown in the next section, this is only a partial explanation and there is some other phenomenon which also reduces the current on the violet side of the peak.

IV. In the experiments on diamond, ZnS and HgS the light was, as shown in the accompanying figure, parallel and opposite to the electric field. The rate of production of electrons is now not a function of x but of y

$$F(y) = \frac{Q_0 \mu}{h \nu} e^{-\mu(l-y)}$$

fig.9



Reflection at the rear face of the crystal is neglected.

It having been found possible and necessary to neglect the change in the density of ions while an electron is crossing the crystal, that simplification is now made at the start.

The total number of electrons recombining in the element of volume $k \cdot dx \cdot dy$ is got by integrating $k \cdot dx \cdot dz \cdot F(z)\, e^{-\int_z^y \frac{ds}{\lambda}} \frac{dy}{\lambda(y)}$ from $z = 0$ to $z = y$ and so is

$$k \cdot dx \cdot dy \cdot \frac{Q_0 \mu}{h \nu} \frac{1}{\lambda(y)} \int_0^y e^{-\mu(l-z)} e^{-\int_z^y \frac{ds}{\lambda(s)}} dz \quad \text{per sec.}$$

The number of electrons liberated per sec. in the same element of volume is $k\, dx \cdot dy \cdot \frac{Q_0 \mu}{h \nu} e^{-\mu(l-y)}$ and taking the difference we get, as before, the net rate of production to be

$$k \cdot dx \cdot dy \frac{Q_0 \mu}{h \nu} \left[e^{-\mu(l-y)} - \frac{1}{\lambda} \int_0^y e^{-\mu(l-z)} e^{-\int_z^y \frac{ds}{\lambda(s)}} dz \right]$$

which must equal

$$k \cdot dx \cdot dy \frac{\partial N}{\partial t} = k \cdot dx \cdot dy \frac{1}{\pi \sigma^2 f} \frac{\partial}{\partial t} \frac{1}{\lambda}$$

$$\therefore \frac{1}{\pi \sigma^2 f} \frac{\partial}{\partial t} \frac{1}{\lambda} = \frac{Q_0 \mu}{h \nu} e^{-\mu l} \left[e^{\mu y} - \frac{1}{\lambda} \int_0^y e^{\mu z - \int_z^y \frac{ds}{\lambda(s)}} dz \right]$$

or $$\frac{\partial}{\partial \tau} \frac{1}{\lambda} = e^{\mu y} - \frac{1}{\lambda} \int_0^y e^{\mu z - \int_z^y \frac{ds}{\lambda(s)}} dz$$

where $T = \pi\sigma^2 b \frac{Q_0 \mu}{k\nu} e^{-\mu l} t$ defines λ as a function of t and y.

This last equation may also be written

$$-\frac{\partial}{\partial T} \log \lambda = \lambda e^{\mu y} - \int_0^y e^{\mu z - \int_z^y \frac{ds}{\lambda(s)}} dz \qquad \cdots (1)$$

Differentiating once with respect to y we have

$$-\frac{\partial^2}{\partial y \partial T} \log \lambda = e^{\mu y}\left(\frac{\partial \lambda}{\partial y} + \mu\lambda\right) - \left\{ e^{\mu y} - \frac{1}{\lambda}\int_0^y e^{\mu z - \int_z^y \frac{ds}{\lambda}} dz \right\}$$

$$= e^{\mu y}\left(\frac{\partial \lambda}{\partial y} + \mu\lambda\right) - \frac{\partial}{\partial T}\frac{1}{\lambda}$$

Let $e^{\mu y} = \zeta$, $\frac{\partial}{\partial y} = \frac{\partial}{\partial \zeta}\frac{d\zeta}{dy} = \mu\zeta\frac{\partial}{\partial \zeta}$ and get

$$-\mu\zeta\frac{\partial^2}{\partial\zeta\partial T}\log\lambda = \zeta\left(\mu\zeta\frac{\partial\lambda}{\partial\zeta} + \mu\lambda\right) - \frac{\partial}{\partial T}\frac{1}{\lambda}$$

or $$\frac{\partial^2}{\partial\zeta\partial T}\log\lambda = \frac{1}{\mu\zeta}\frac{\partial}{\partial T}\frac{1}{\lambda} - \left(\zeta\frac{\partial\lambda}{\partial\zeta} + \lambda\right)$$

$$= \frac{\partial}{\partial T}\frac{1}{\mu\zeta\lambda} - \frac{\partial}{\partial\zeta}(\zeta\lambda)$$

Placing $\lambda = e^{\xi}$, $\frac{1}{\mu\zeta} = \beta$, $\zeta = \alpha$ this takes the standard form

$$-\frac{\partial^2 \xi}{\partial\zeta\partial T} = \frac{\partial}{\partial\zeta}\left(\alpha e^{\xi}\right) - \frac{\partial}{\partial T}\left(\beta e^{-\xi}\right)$$

of which the solution is known to be

$$e^{\xi} = \lambda = v/u$$

where v and u must satisfy the equations

$$\alpha v = \frac{\partial u}{\partial T} \quad \text{and} \quad \beta u = \frac{\partial v}{\partial \zeta}$$

or $\zeta v = \frac{\partial u}{\partial T}$ and $\frac{1}{\mu\zeta} u = \frac{\partial v}{\partial\zeta}$

which can be combined into the one equation

$$v = \mu\frac{\partial^2 v}{\partial\zeta\partial T} \quad \text{or} \quad v = \frac{\partial^2 v}{\partial\zeta\partial T_1} \qquad \text{where } T_1 = T/\mu$$

If we place $e^{\mu y} = \xi$ in the original integral equation (1) it becomes

$$-\frac{\partial}{\partial\tau}\log\lambda = \mu\xi - \frac{1}{\mu}\int_1^{\xi} e^{\int_{z_1}^{\xi}\frac{d\xi}{\mu\xi\lambda}}\,dz_1$$

Now $\lambda = \frac{v}{u} = v/\mu\xi\frac{\partial v}{\partial\xi}$ or $\frac{1}{\mu\xi\lambda} = \frac{\partial}{\partial\xi}\log v$ whence by substitution

$$\frac{v}{\mu\frac{\partial v}{\partial\xi}} - \frac{1}{v}\frac{\partial v}{\partial\tau} = \frac{v}{\mu\frac{\partial v}{\partial\xi}} - \frac{1}{\mu v}\int_1^{\xi} v(\tau, z_1)\,dz_1$$

or

$$\frac{\partial v}{\partial\tau_1} = \int_1^{\xi} v(\tau, z_1)\,dz_1$$

We need then a solution of $v = \frac{\partial^2 v}{\partial\xi\partial\tau_1}$ that satisfies the conditions

$$\frac{\partial v}{\partial\tau_1} = \int_1^{\xi} v(\tau, z_1)\,dz_1 \quad \text{and} \quad \lambda = \infty \text{ if } t = 0$$

It has already been found that a solution of $v = \frac{\partial^2 v}{\partial\xi\partial\tau_1}$ is

$$v = J_0\left(2i\sqrt{(\tau_1+g)(\xi+h)}\right)$$

$$= 1 + \frac{1}{(1!)^2}(\tau_1+g)(\xi+h) + \frac{1}{(2!)^2}(\tau_1+g)^2(\xi+h)^2 + + \cdots\cdots$$

Obviously the second condition is satisfied by taking $g = 0$ and the first by taking $h = -1$, we then have

$$\lambda = 1/\mu\xi\log J_0\left(2i\sqrt{(\xi-1)\tau_1}\right)$$

$$I = \text{current} = e\,k\int_0^{w}dx\int_0^{l}\frac{\partial N}{\partial t}\,dy = e\,k\,w\int_1^{e^{\mu l}}\frac{\partial N}{\partial t}\frac{d\xi}{\mu\xi}$$

But $\frac{\partial N}{\partial t} = \frac{1}{\pi\sigma^2 k}\frac{\partial}{\partial t}\frac{1}{\lambda} = \frac{1}{\pi\sigma^2 k}\frac{\partial}{\partial t}\left[\mu\xi\frac{\partial}{\partial\xi}\log J_0\left(2i\sqrt{(\xi-1)\tau_1}\right)\right]$

$$\therefore I = \frac{e\,k\,w}{\pi\sigma^2 k}\frac{\partial}{\partial t}\int_1^{e^{\mu l}}\frac{\partial}{\partial\xi}\log J_0\left(2i\sqrt{(\xi-1)\tau_1}\right)d\xi$$

$$= \frac{e\,k\,w}{\pi\sigma^2 k}\frac{\partial}{\partial t}\log J_0\left(2i\sqrt{(e^{\mu l}-1)\tau_1}\right)$$

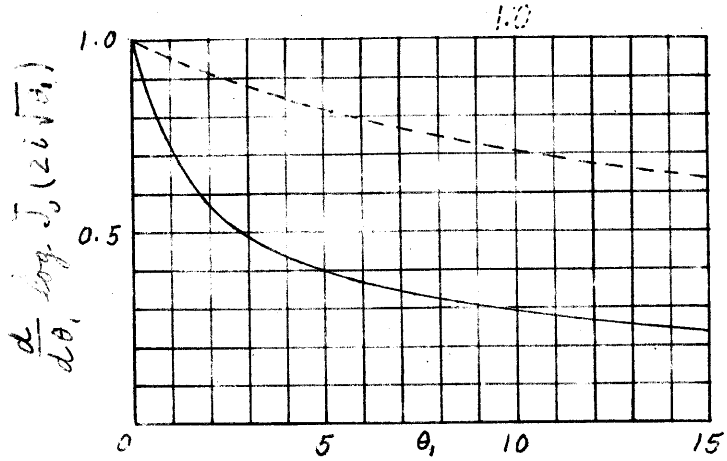
$$= e\,k\,w\frac{Q_0}{\lambda\nu}(1-e^{-\mu l})\frac{d}{d\theta_1}\log J_0(2i\sqrt{\theta_1}) \qquad \cdots(2)$$

where $\theta_1 = \pi \sigma^2 b \frac{Q_0}{h\nu}(1 - e^{-\mu l}) t$

or $$I = e k w \frac{Q_0}{h\nu}(1 - e^{-\mu l}) \frac{J_1(2i\sqrt{\theta_1})}{J_0(2i\sqrt{\theta_1})} \frac{1}{i\sqrt{\theta_1}}$$

$$\lim_{t \doteq \infty} I = \lim_{\theta_1 \doteq \infty} I = 0$$

fig. 10



This shows how recombination should make the current drop with time when the light is parallel and opposite to the electric field.

The quantity of electricity passing to the anode between the times $t = 0$ and $t = t$ is

$$C = \int_0^t I\,dt = \int_0^t \frac{ekw}{\pi\sigma^2 b} \frac{\partial}{\partial t} \log J_0(2i\sqrt{(e^{\mu l}-1)\tau_1})\,dt$$

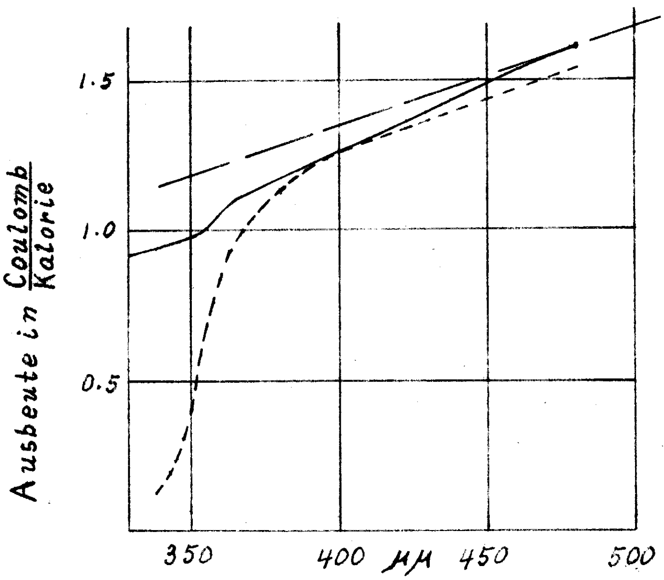
$$= \frac{ekw}{\pi\sigma^2 b} \log J_0(2i\sqrt{(1 - e^{-\mu l})\pi\sigma^2 b \frac{Q_0}{h\nu} t})$$

$$= e k w \frac{Q_0}{h\nu}(1 - e^{-\mu l}) \cdot t \cdot \frac{1}{\theta_1} \log J_0(2i\sqrt{\theta_1}) \qquad \cdots (3)$$

For short observation periods (t small) the quantity C is thus seen to be proportional to t for $f(0) = 1$. This is in agreement with the measurements of Gudden and Pohl#.

If recombination is neglected, i. e. if $f(\theta_1)$ is made equal to one, $C = e k w \frac{Q_0}{h\nu} t$ which is a straight line when plotted against $\frac{1}{\nu}$. The factor $(1-e^{-\mu l})$ is left off to correct for the light that goes clear through and is lost. It is this C which Gudden and Pohl measured in their experiments on diamond, HgS, and ZnS.

The dotted curve in fig. 11 gives the results of the experiments on ZnS in terms of C/Q_0 . The dashed straight line is the C/Q_0 which would have been obtained had every $h\nu$ liberated one electron which also reached the anode. It was located by multiplying the ordinate at $405\,\mu\mu$ by $\frac{7}{6.55}$. Neglecting recombination, the experimental value of h is here 7×10^{-27} instead of 6.55×10^{-27} . The discrepancy is said to be within experimental error.



Zeitschrift für Physik, Vol. 17, page 345.

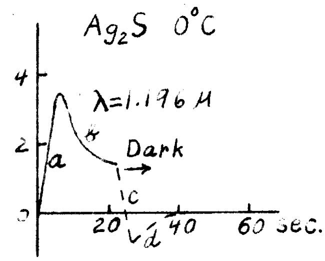
fig. 11

The full curve is the C/Q_0 given by equ. (3). It was calculated with the aid of the curves given by Gudden and Pohl# for μ in ZnS, taking $l = .6$ mm. and so choosing b as to make the two curves coincide at $\lambda = 400\,\mu\mu$. It represents then the "Ausbeute" to be expected if recombination were the only phenomenon tending to reduce the current, and provided the phenomenon other than recombination which causes the large drop at $350\,\mu\mu$ is inoperative at and beyond $400\,\mu\mu$. Judging from the shape of the dotted curve this is the case.

The peculiar drop is such as would be caused by a rapid increase in b between 400 and $340\,\mu\mu$, this does not seem at all likely. If it is caused by initial recombination or some other reason amounting to the same thing it is equivalent to a decrease in Q_0 , and we must have $Q_0 = Q_0(\nu)$. This does not affect the validity of equations (2) and (3) because the integrations were only with respect to space and time.

The effect of the unknown process causing the peculiar drop in current when the light is most strongly absorbed is thus seen to be less than it would seem to be were recombination neglected. By determining b where the effect is inappreciable we see how big the effect is at the shorter wave lengths.

Recombination is able to explain qualitatively some otherwise unintelligible current-line curves published by W. W. Coblentz. This is a sample. The curve is superposed on the dark current which is the axis of abscissas. Along a the current rises because it takes an electron an appreciable time to cross the



crystal so that up to the time required for an electron to cross the crystal the anode is receiving electrons from increasingly greater distances from the anode. Along b the current drops because of the formation of a space charge. Along c the current drops because the supply of photo-electrons is cut off when the light is and in the time required for an electron to cross the crystal they cease arriving at the anode. The current drops below the dark current because part of the dark current must go to neutralizing the space charge. When the space charge has been all neutralized the dark current will have returned to its original value.

It does not seem possible, however, to get a quantitative explanation of these curves because the differential equations representing the different portions of the curves, for example III 6 , are quite unmanageable.

** Physikalische Zeitschrift, vol. 26, page 481.

* Naturwissenschaften, vol. 11, page 354.

Zeitschrift für Physik, vol. 17, page 331.