AN INVESTIGATION OF SOME PROBLEMS IN THE DESIGN OF TAILLESS AIRPLANES

Thesis

by

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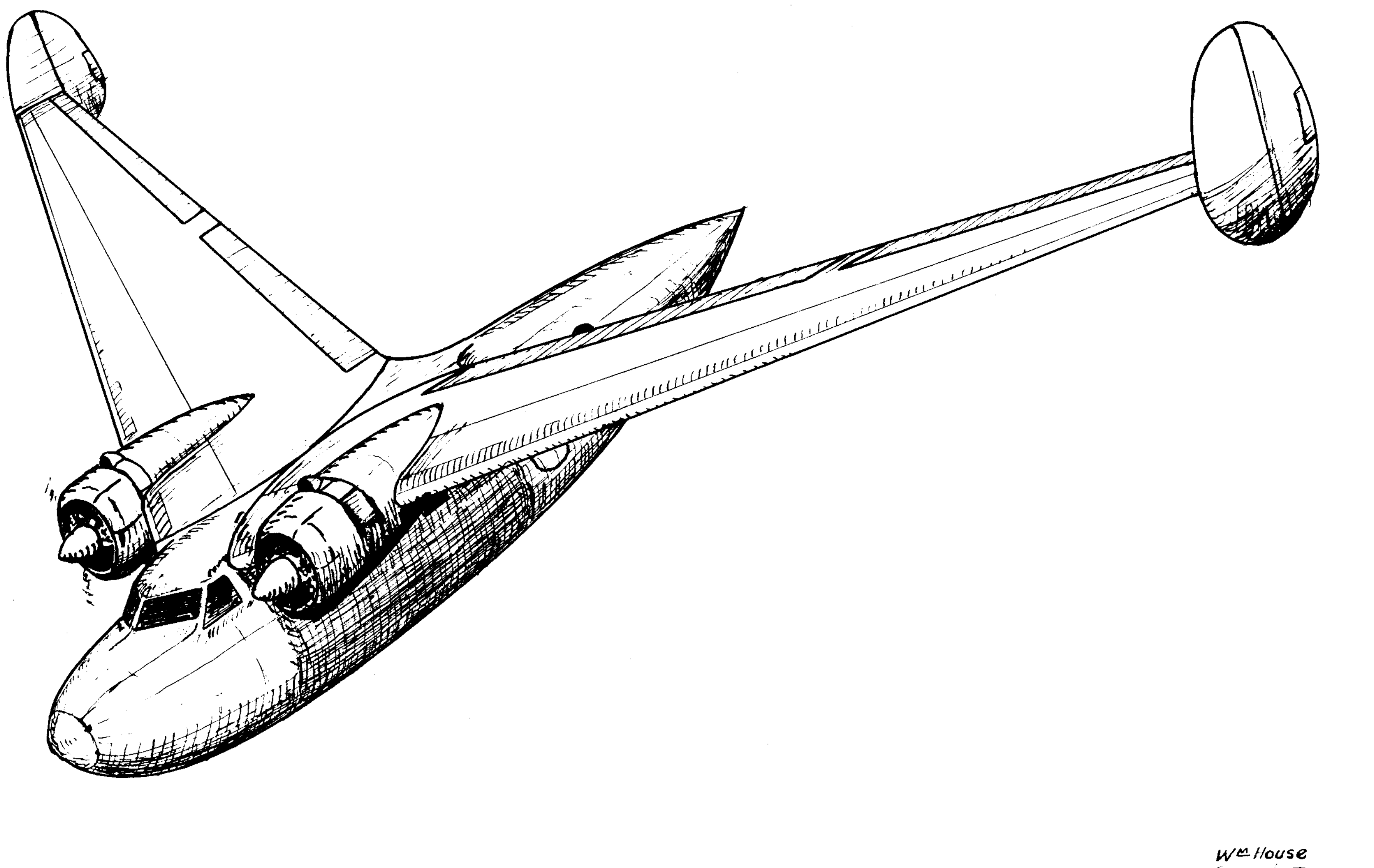
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SUMMARY

Some problems in the design of tailless airplanes are discussed. The conditions essential to the flight of such airplanes are determined, and a survey of methods of calculating the aerodynamic characteristics of plain wings and wings with flaps and ailerons deflected is presented. The effect of adding a fuselage and wing nacelles to a wing to form a practical airplane is discussed.

Consideration is given to the problem of damping of longitudinal oscillations, and a method of calculating the damping in pitch of a sweptback wing alone is presented.

An example is carried through to illustrate the application of all the material covered in the paper. In all respects considered, the tailless airplane is shown to be comparable to normal type airplanes.

INTRODUCTION

There have always been men who wanted to fly, men who felt that their science was equal to the achievements of the birds. And now, after many years of experimentation and endeavor, men do fly. Their flying machines are the cumulative result of achievement in many fields. Even after men believed they knew how to fly, they had to wait till suitable power plants were available. The modern airplane is equally the result of steady progress in engines, methods and materials of construction, and knowledge of aerodynamics.

With our present broad knowledge and understanding of the problems involved in flying, it is time to look back over our progress, to chart the trails we have followed, and to investigate the possibilities of trails which were neglected by or unknown to the early experimenters.

It must be admitted that there is the real possibility that in the early days the progress of aviation was made in some other than the best direction. This is so because the early progress was made by experimenters who followed empirical rules and results. Anything which worked or showed promise of working was eagerly developed. A few pioneers, who had had some success, became leaders. The others followed them. Thus the whole industry grew in the direction in which the first successes were made. This is quite natural, as it is the common attribute of human beings to cling tenaciously to the past, and make at most small improvements on it, rather than to attempt an entirely new approach to the problem after the first bit of progress has been made.

The modern airplane, after years of development along the original lines, flies very well. But it still has a number of faults. Stalls, spins, and fast landings have taken a huge toll of lives.

Admitting this possibility of error in an empirically developed process, and in the light of our modern theoretical knowledge, let us return to the beginning and discuss the possibilities of development of the flying machine.

The airplane is a machine designed to navigate through terrestrial space. Its necessary elements are: a means of lifting it into space; a means of keeping it there in a position of stable equilibrium; a means of propulsion through space; and a means of controlling and directing the lift, propulsion, and equilibrium.

The best airplane is the simplest machine which will fulfill these requirements.

We know that there are available several lifting devices. There is the conventional wing as used on airplanes today, and its variations in the form of the autogyro and the helicopter. Lift may also be obtained by the use of cells of light gas, as in dirigibles.

Propulsion is ordinarily obtained by the use of screw propellers, which operate on the same principle as the wings. Consideration has also been given to the use of jet propulsion, as with rockets.

In the ornithopter, or flapping wing type, an attempt has been made to combine the lift and propulsion into one operation.

While development might be made along any of these lines, we shall here restrict ourselves to a discussion of the possibilities in the use of a fixed wing surface, with whatever auxiliaries may prove necessary, and propelled by a separate means, presumably an engine propeller combination.

Such a fixed wing surface may be varied as to planform and airfoil section. Auxiliaries, such as flaps and external elevators and rudders, may be employed in various ways. It is our purpose to investigate the configuration which shows most promise of development into a flying machine comparable to our present day airplanes. It is especially desired to simplify the airplane by developing a form which requires as few auxiliaries to the main lifting element as possible. This means, in particular, that the horizontal stabilizer is to be eliminated if possible.

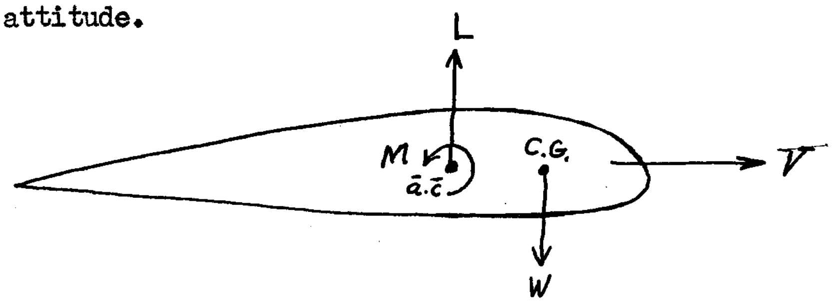
Now let us turn to the modern aerodynamic theory and study the characteristics of wing surfaces. The flow about two dimensional airfoils has been analyzed very carefully. It has been shown theoretically that the air forces acting on such a section can be broken into a lift and a moment, and that there is a point, known as the section aerodynamic center, about which the moment is constant for all angles of attack up to the stall. The manner in which the lift and the moment depend on the angle of attack and on the camber of the section has been demonstrated.

A three dimensional airfoil theory has evolved from this two dimensional analysis. A few simple approximations and the concept of a series of elements of two dimensional airfoils placed side by side, with each element influenced by the vorticity shed from each other element, gave a simple and workable picture.

Mathematical analysis has shown that under these assumptions every three dimensional wing has properties like those of a two dimensional airfoil section. That is, it has an aerodynamic center about which the moment is constant and through which acts a lift force whose magnitude is proportional to the angle of attack of the wing. In addition, the phenomenon of induced drag appears with the three dimensional wing, but it is not necessary to consider it here. As a result of this study, it was established that the moment about the wing aerodynamic center was due to the elemental moments about the sections making up the wing and to what is called the basic lift distribution. This basic lift distribution, the lift distribution for zero total lift on the wing, is dependent on the wing planform and the wing twist. The moment due to this distribution depends on the sweepback of the wing. The location of the wing aerodynamic center and the magnitude of the constant moment about it are susceptible to calculation.

The theory has explained the nature of the forces acting on a wing which is moving through the air, and enables us to calculate the magnitude of these forces, to resolve them into a variable force and a constant moment, and to locate the point through which the variable force acts. By reason of understanding these things and the factors influencing them, we have a degree of control over the location of the aerodynamic center and the magnitude of the moment about it. We can use this knowledge in our attempt to design a simple flying machine.

From the wing alone, we get the lift which was our first requirement for flight. By choosing the moment about the wing aerodynamic center properly, we can satisfy the condition that the machine be in a position of static longitudinal equilibrium in its flying attitude.



We see from the arrangement shown that if a wing with a stalling moment about its aerodynamic center is used, and balance is obtained by requiring that the center of gravity of the system fall ahead of the aerodynamic center, displacements from the equilibrium attitude will introduce lift forces which will tend to restore the system to its original position. Thus a simple wing, properly designed, will easily satisfy two of the vital requirements of a flying machine. Since this is the case, let us investigate the manner in which this wing will meet, or can be made to meet, the other requirements. These, with the exception of propulsion, concern the ability of the plane to maintain itself in flight in a condition of dynamic equilibrium about all three axes, and the possibility of controlling the plane in its flight.

First, what are the variations which lead to the simple wing with a stalling moment about its aerodynamic center? There are three possibilities, sweepforward with twist which increases the angle of attack at the wing tips, sweepback with twist which decreases the angle of attack at the wing tips, and a wing using an airfoil section which normally has a stalling moment about its aerodynamic center. The sweepforward arrangement is neglected from the start, as it is unfavorable from the standpoints of wing tip stalling and directional stability. The straight wing with stable airfoil section is likewise ruled out because of a probable deficiency in damping during pitching oscillations and the small degree of pitching control possible. We are left, then, with the wing with sweepback and aerodynamic twist or a stable airfoil section, or a combination of both.

In order to achieve lateral stability, it may be necessary to equip this wing with some form of vertical fin surfaces. Control will be obtained with wing tip ailerons and rudders on the fins. A combination of flaps and slots may be used to obtain high lift coefficients from the wing.

We have thus arrived at a simple wing arrangement which may possibly prove to have advantages over the conventional airplane as the ultimate in flying machine using a fixed wing surface as the lifting element.

In the present paper the advantages and limitations of this type of aircraft are studied and discussed, and consideration is given to the use of various auxiliary devices designed to broaden the scope of usefulness of the machine.

Coordinate System

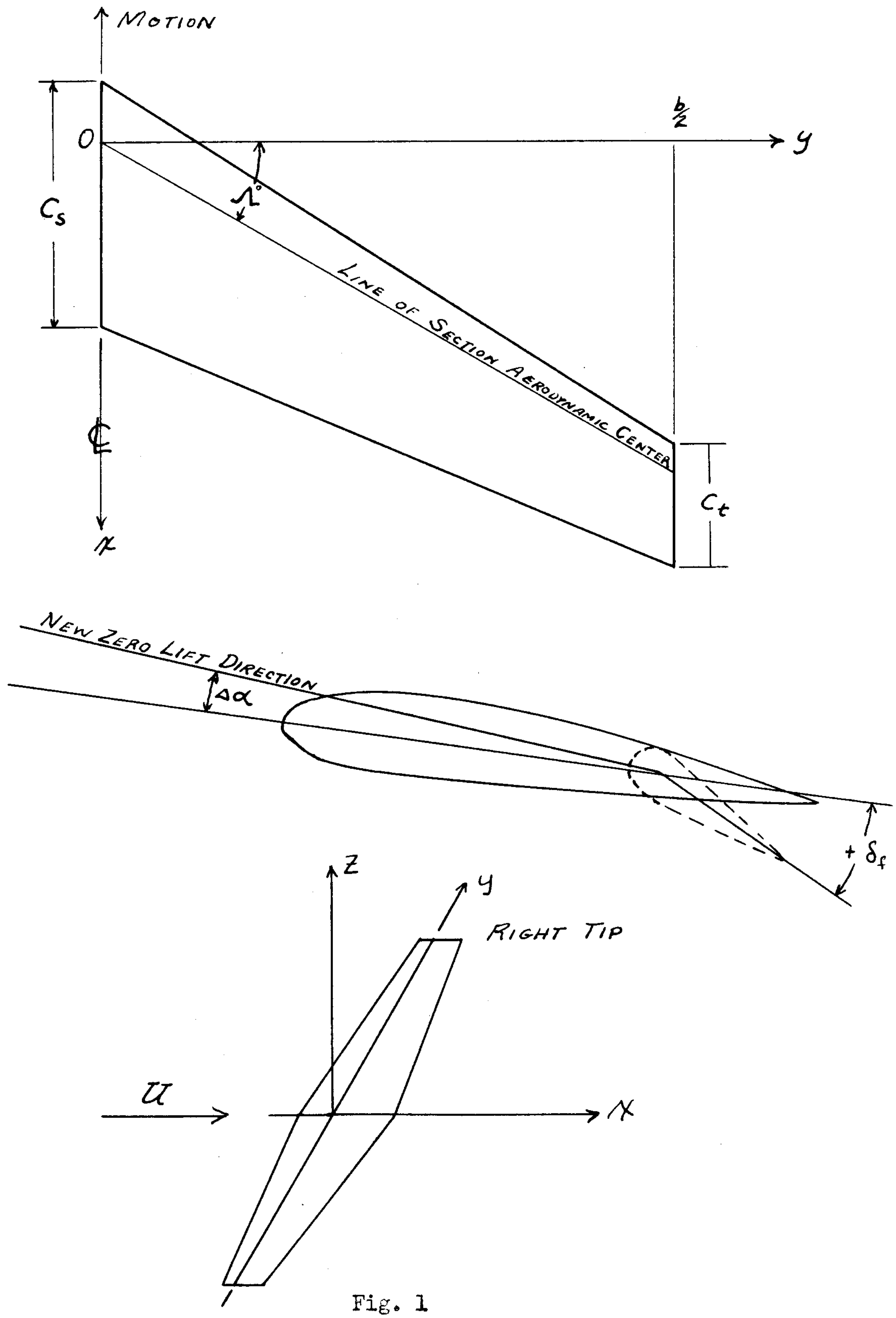


Fig. 1

NOMENCLATURE

b	Wing span, normal to axis of symmetry.
S	Wing area.
AR	Wing aspect ratio.
c	Wing chord.
c_s	Wing chord at root.
c_t	Wing chord at tip.
Λ	Sweepback angle of the line connecting the 25% chord points of the sections.
ϵ	Total aerodynamic twist between root and tip sections. Negative for washout, or tip section at lower angle of attack than the root section.
a_o	Slope of the section lift curve for infinite aspect ratio, per degree.
a	Slope of lift curve for a wing of finite aspect ratio.
C_L	Wing lift coefficient.
$C_{M_{AC}}$	Moment coefficient about the wing aerodynamic center.
c_l	Section lift coefficient.
$c_{m_{ac}}$	Section moment coefficient. Depends on camber.

$C_{M_{lb}}$ Wing moment due to basic lift distribution.

C_{M_S} Wing moment due to section moment characteristics of the airfoils used.

$C_{M_{ow}} = C_{M_{lb}} + C_{M_S}$ = Moment of the wing alone.

l_b Section loading due to basic lift distribution.

l_a Section loading depending on wing lift coefficient.

l_e Section loading due to deflecting ailerons or flaps.

l_n Total section loading.

$\Delta\alpha$ Change in zero lift direction of a section when a flap is deflected.

α Angle of attack of any section, measured from its zero lift direction.

ΔC_L Increment in lift coefficient due to deflecting an aileron or flap, the attitude of the wing remaining fixed.

$\Delta_1 C_{M_{AC}}$ Increment in moment coefficient due to the lift forces when an aileron or flap is deflected.

$\Delta_2 C_{M_{AC}}$ Increment in moment coefficient due to changes in section moment coefficients when aileron or flap is deflected.

$\Delta C_{M_{AC}} = \Delta_1 C_{M_{AC}} + \Delta_2 C_{M_{AC}}$

$\delta_f^\circ, \delta_e^\circ$ Angular deflection of flap or aileron, in degrees.

$\bar{a}.\bar{c}.$ Wing aerodynamic center.

q Angular velocity in pitch about the airplane C.G., in radians per second.

α_{as} Angle of attack of a wing measured from the zero lift direction of the root chord.

$\frac{S}{b}$ Mean chord used in determining moments and moment coefficients.

d Distance of the airplane C.G. ahead of the wing aerodynamic center.

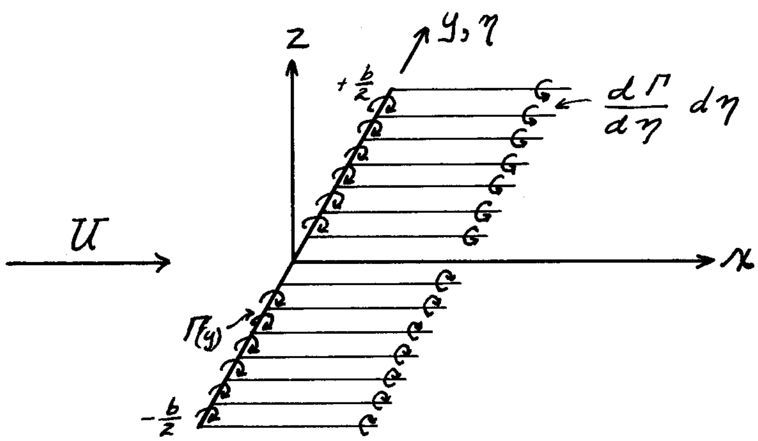
x_{ac} Distance of the wing aerodynamic center behind the quarter chord point of the root chord.

The following quantities are defined in reference 1 and evaluated by means of tables and charts presented therein:

u
v
ω
J
E
G
H
L_a
L_b

THE AERODYNAMIC CHARACTERISTICS OF A WING

The Prandtl wing theory is used as a basis for calculating the characteristics of three dimensional wings. In this theory the wing is replaced by a lifting vortex line with a sheet of trailing vortices behind it. It is assumed that the entire flow is two dimensional, that the trailing vortices follow the direction of the undisturbed stream, and that all vertical velocities are small compared to the forward velocity.



The lift acting on each elemental length is perpendicular to the relative wing over that section; and given in magnitude by $dL = \rho U \Gamma\, dy$, since vertical velocities are small. The trailing vortices induce a downwash over the wing given by

$$w_{(y)} = \frac{1}{4\pi}\int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{\frac{d\Gamma}{d\eta}}{y-\eta}\, d\eta$$

. This downwash acts to change the direction of the lift force acting on the wing, inclining it backwards to produce a component parallel to the undisturbed flow which is known as the induced drag.

The lift at any section can be expressed as:

$$dL = \frac{1}{2}\rho U^2 c\, C_l\, dy = \rho U\, dy$$

and the section lift coefficient can be written as $C_l = m_o \alpha_o$, where α_o is the angle of attack of the relative wind to the section zero lift line. But α_o can be expressed in terms of the undisturbed wind direction and the downwash velocity by $\alpha_o = \alpha - \frac{w}{U}$, where α is the angle of attack of the section with respect to the undisturbed stream.

Combining these relations, the Prandtl integral equation is obtained:

$$\Gamma_{(y)} = \frac{U c\, m_o}{2}\left\{\alpha - \frac{1}{4\pi U}\int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{\frac{d\Gamma}{d\eta}}{y-\eta}\, d\eta\right\}$$

This is the relation that must be satisfied if the lift distribution on any wing is to be found. For the purpose of this analysis it is convenient to assume that the sweepback and dihedral do not affect the spanwise lift distribution over the wing. This has been verified by experiment up to angles of sweepback of about 30 degrees, according to reference 6.

In reference 1, Mr. R. F. Anderson, of the N.A.C.A., uses the Glauert method of expressing the spanwise lift distribution in terms of a Fourier series. In continuing this analysis he finds that the section lift coefficients of the three dimensional wing can be expressed in terms of the coefficients of this Fourier series, and that each coefficient is made up of two parts. One of these parts depends only on the planform of the wing and on the net lift coefficient at which it is acting. The other part depends only on the planform and the distribution of aerodynamic twist across the wing span, and is independent of the angle of attack of the wing as a whole.

This splitting of the coefficients shows that the wing lift can be considered to be made up of a constant and a variable part, which are added linearly as the wing changes its angle of attack.

The additional lift distribution, which is proportional only to the wing angle of attack or net lift coefficient, can be shown to have a centroid through which its resultant acts. This point is known as the wing aerodynamic center.

The basic lift distribution depends only on the twist distribution of the wing. It may be visualized best by considering the lift distribution on a twisted wing at an angle of attack such that there is no net lift. If there is aerodynamic washout, there will be a down load on the tips and an up load on the center of the wing. If there is sweepback, these loads will act as a couple to produce a moment on the wing. Since this basic distribution is not altered by changing the angle of attack of the whole wing, the moment due to it is constant, and, with the sum of the moments due to the section properties of the airfoil sections of the wing, gives a constant moment about the aerodynamic center of the wing.

As a result of this analysis it is possible, conceptually, to replace a three dimensional wing moving through a fluid by a system of forces consisting of a constant moment and a variable lift force acting through the wing aerodynamic center. As long as the planform of the wing is not altered, the position of the aerodynamic center does not move, and a change in the twist of the wing affects only the magnitude of the moment about the aerodynamic center of the wing.

With this as a foundation, the possibilities of designing a wing suitable for use with a tailless flying machine can be studied in an organized manner, it being only necessary to settle upon the combination of planform and twist which give a wing with the desired characteristics.

If the problem of finding the lift distribution for a given wing has been solved, and the section lift coefficients have been broken into the aforementioned two parts, either analytically or by comparing the lift distributions at two different wing lift coefficients, the important characteristics of the wing can be expressed in the following form:

$$C_l = \frac{2\Gamma}{U c}$$

$$C_l = C_{l_b} + C_{l_a} = C_{l_b} + C_{l_{a_1}} \times C_L$$

$$\frac{l_b}{q} = C_{l_b} \times c \qquad \frac{l_a}{q} = C_{l_{a_1}} \times C_L \times c$$

$$l = l_a + l_b$$

$$\alpha_i = \alpha_a - \frac{C_l}{m_o}$$

$$C_L = \frac{1}{S}\int_{-\frac{b}{2}}^{+\frac{b}{2}} C_l \, c \, dy$$

$$C_{D_i} = \frac{1}{S}\int_{-\frac{b}{2}}^{+\frac{b}{2}} \alpha_i \, C_l \, c \, dy$$

$$\frac{x_{ac}}{S/b} = \frac{b}{S^2} \times \frac{1}{C_L} \int_{-\frac{b}{2}}^{+\frac{b}{2}} c_{la}\, c\, x\, dy$$

$$\alpha_{as} = \frac{C_L}{a} + J\epsilon$$

$$C_{m_{lb}} = -\frac{b}{S^2} \int_{-\frac{b}{2}}^{+\frac{b}{2}} c_{l_b}\, c\, x\, dy$$

$$C_{M_S} = -\frac{b}{S^2} \int_{-\frac{b}{2}}^{+b/2} c_{m_{ac}}\, c^2\, dy$$

$$C_{M_{AC}} = C_{M_{lb}} + C_{M_S} = C_{M_{OW}}$$

Other integrations may be made to find the rolling moment and the yawing moment due to the induced drag.

The system of axes used is shown in Fig. 1. The X axis is taken in the direction of the chord line of the section at the axis of symmetry of the wing, and the Y axis extends spanwise from the aerodynamic center of this root chord. The angle of attack of the wing is measured from the chord line of the root chord. This is the same as in reference 1.

Anderson has considered two specific classes of wings, namely those with a linear twist distribution and with elliptical or straight tapered planform. The aerodynamic centers of the sections of all the wings have been assumed to lie on a straight line from the center outboard. He has calculated the characteristics of a number of wings having a large range of values of taper ratio and aspect ratio, and with an arbitrary degree of sweepback, (up to 30°). His results are presented in the following form:

$$C_{l_b} = \frac{\epsilon a_o S}{C b} L_b$$

$$C_{l_{a_1}} = \frac{S}{C b} L_a$$

$$a = f \times \frac{a_o}{1 + \frac{57.3 a_o}{\pi A\!R}}$$

$$C_{D_i} = \frac{C_L^2}{\pi A\!R u} + C_L \epsilon a_o v + (\epsilon a_o)^2 \omega$$

$$C_{m_s} = E\, C_{mac} \qquad \text{FOR } C_{mac} \text{ CONSTANT ALONG SPAN}$$

$$C_{M_{lb}} = -G \epsilon a_o A\!R \tan \Lambda$$

$$\frac{X_{ac}}{S/b} = H A\!R \tan \Lambda$$

Thus:

$$\frac{l_b}{q\left(\frac{b}{2}\right)} = \frac{2}{AR} \epsilon a_0 L_b$$

$$\frac{l_a}{q\left(\frac{b}{2}\right)} = \frac{2}{AR} L_a C_L$$

The values of L_a and L_b are tabulated in reference 1 for the wing planforms investigated. The values of u, v, w, J, E, G and H may be read from curves presented in the report to correspond to any wing of the class. Interpolations are possible.

The report, reference 1, gives examples of the use of the method and shows that the check of theoretical values with experiment is satisfactory.

Reference 2 gives experimental checks of the method with a larger group of wings and discusses more completely the problem of computing the induced drag and the maximum lift coefficient obtainable with a tapered twisted wing.

If it is desired to determine the characteristics of some wing which does not fit into the families described here, it becomes necessary to go back to the basic theory and calculate the Fourier coefficients for the given wing. This procedure, which is rather long and tedious, is described in reference 3. Computing forms are given in this reference and in reference 15 which simplify and guide the work.

Researchers at the California Institute of Technology are at present developing a method which has been applied particularly to wings with straight linear taper, but which can be used with any arbitrary distribution of twist, thus broadening the class of wings which can be investigated readily.

The study from this point on will be concerned primarily with the properties of linear taper wings with straight sweepback. These are the most common in engineering applications.

Using the material which has been discussed so far, it is possible to design a wing which will be suitable for use in a tailless plane moving in some predetermined steady flight condition. It is now necessary to study the effect of control surface deflections and to investigate the range of conditions through which flight is possible.

THE WING WITH ITS CONTROL SURFACES

Control surface deflections have the essential effect of changing the twist distribution on the wing. The effect of arbitrary twist distribution on the lift distribution of straight tapered wings can most easily be studied with the aid of the material in references 4 and 5.

In the first of these Pearson and Jones have presented a series of "influence functions," which, for a group of wings of the class being considered, give the effect at several spanwise stations of the wing of an increment of twist extending from any spanwise station to the right tip of the wing. Any distribution of twist can be built up by a suitable series of these increments, and the resulting lift distribution can be found.

One very interesting fact emerges from the wing theory at this point. It turns out that the effect of a change in twist over a part of the wing on the loading of a section anywhere on the wing is a linear function of the additional twist which is imposed. Also it has been shown theoretically in reference 14 that the change in zero lift direction of a flapped airfoil section is proportional to the flap deflection, and the theoretical factors of proportionality are computed.

This means that, theoretically, the effect of the deflection of an aileron or flap on the lift distribution of a wing is independent of the state of deflection of the other flaps and ailerons and is proportional to the angle of deflection of the surface.

Experiment substantially verifies these results, except that the change in zero lift direction with flap deflection does not quite reach the theoretical value and that the effect decreases still further if the section is already at a high angle of attack.

However, for a general discussion, small control deflections at only moderate wing lift coefficients will be considered. It must then be kept in mind that the results indicate only trends and general orders of magnitude of the derived quantities and cannot be used directly, unless the proper experimental data has been used in the derivation instead of the theoretical constants.

Thus it is possible to compute the effect of a unit deflection of one aileron or flap on the lift coefficient, pitching moment, and rolling moment of a wing, and to find the result of any deflection of all surfaces by a simple superposition of effects. It is to be noted that the drag effects cannot be treated thus, as they are not linear in the lift coefficient.

Now it is necessary to extend the data presented in reference 4 to the calculation of the aforementioned effects. This is done by a procedure which is primarily graphical, though charts are presented in reference 5 which enable one to find directly the effects of deflecting partial span flaps of constant percent chort. The sweepback angle is still assumed not to affect the lift distribution, so it is possible to express all the effects which depend on sweepback in terms of functions of the angle. Thus the calculation of the effects of flap deflection on a straight tapered wing without sweepback is all that is necessary.

The use of the influence functions of reference 4 is as follows: As an example, the effect of an aileron of variable percent chord width on a tapered sweptback wing has been computed.* The calculations and figures are shown in Figs. 3, 4, 5 and 6.

1. The effective aerodynamic twist increment due to aileron deflection is plotted as a function of the span. It may be found from experimental data for similar flaps or calculated approximately from the theory.

2. This distribution can be considered to be made up of a summation, $\sum d\alpha$, each $d\alpha$ extending from some point on the span to the right wing tip. Each of these elements, $d\alpha$, has an effect on the section lift coefficient at every point of the span. The influence curves, figures 4, 5 and 6 of N.A.C.A. Technical Report 635, give these effects at each of several spanwise positions as a function of the distance over the span from the right wing tip covered by each elemental increment of the total twist.

Thus the net value of $C_\ell \frac{c}{c_s}$ at any section is found by summing up $\int \frac{C_\ell \frac{c}{c_s}}{\alpha} d\alpha$ over the entire wing. This is done graphically. A value of $\frac{C_\ell \frac{c}{c_s}}{\alpha}$ is picked from the

* The procedure used here is suggested in reference 4.

charts to correspond to the station being considered and to the distance $d\alpha$ extends across the span. A plot of $\frac{C_\ell \, c/c_s}{\alpha}$ against $\Delta\alpha$ is then made for each spanwise station being considered. An integration gives the value of $C_\ell \frac{c}{c_s}$ at each of these stations.

In brief, $C_\ell \frac{c}{c_s} = \int \frac{C_\ell \, c/c_s}{\alpha} \, d\alpha$ for each spanwise station.

3. To make this notation consistent with that of reference 1, the following substitutions are made:

$$l_c = C_\ell \frac{c}{c_s} \times c_s q \quad or \quad \frac{l_c}{q\left(\frac{b}{2}\right)} = \frac{c_s}{\left(\frac{b}{2}\right)} \int \frac{C_\ell \, c/c_s}{\alpha} \, d\alpha$$

When the value of $\Delta\alpha$ is constant across the flap, as it will be if the flap is of a constant percent chord width, the necessity for a graphical integration is removed, as the expression reduces to:

$$\frac{l_c}{q\left(\frac{b}{2}\right)} = \frac{c_s}{\left(\frac{b}{2}\right)} \times \frac{C_\ell \frac{c}{c_s}}{\alpha} \times \Delta\alpha$$

The value of $\frac{l_c}{q\left(\frac{b}{2}\right)}$ is found for several spanwise stations on the wing and plotted, so that a faired curve can be drawn to represent the additional wing loading caused by deflecting the aileron.

4. The lift, pitching moment, and rolling moment caused by this lift distribution can then be found from the following formulas by a series of graphical integrations.

$$\Delta C_L = \frac{AR}{4} \int_{-1}^{+1} \frac{l_c}{q(\frac{b}{2})} \, d \frac{y}{(\frac{b}{2})}$$

$$\Delta_1 C_{MAC} = -\frac{AR^2 \tan \Lambda}{8} \int_{-1}^{+1} \frac{l_c}{q(\frac{b}{2})} \left(\frac{y}{(\frac{b}{2})} - 2H \right) d \frac{y}{(\frac{b}{2})}$$

$$\Delta_2 C_{MAC} = \frac{AR^2}{8} \int_{-1}^{+1} \Delta_2 C_{mac} \left[\frac{c}{(\frac{b}{2})} \right]^2 d \frac{y}{(\frac{b}{2})}$$

$$C_{M_{ROLL}} = -\frac{AR}{8} \int_{-1}^{+1} \frac{l_c}{q(\frac{b}{2})} \, \frac{y}{(\frac{b}{2})} \, d \frac{y}{(\frac{b}{2})} = \frac{M_{ROLL}}{q S b}$$

It is convenient to express the dimensions of a straight tapered wing with sweepback in terms of its dimensionless parameters and the semi-span. Doing this, we get the following relationships. The system of coordinates shown in Fig. 1 applies here also.

$$S = \frac{4}{AR} \left(\frac{b}{2} \right)^2$$

$$\frac{S}{b} = \frac{2}{AR} \left(\frac{b}{2} \right)$$

$$C_s = \frac{4}{AR(1+\lambda)} \qquad \lambda = \frac{C_t}{C_s}$$

$$c = c_s \left[1 - (1-\lambda) \frac{y}{\left(\frac{b}{2}\right)} \right]$$

$$\frac{x}{\left(\frac{b}{2}\right)} = \frac{y}{\left(\frac{b}{2}\right)} \tan \Lambda$$

As a result of the analysis this far, the lift and moment characteristics of a plain tapered sweptback wing can be computed, and the additional effects caused by control surface or flap deflections can be determined for a group of wings with a given set of values of aspect ratio and taper ratio.

If it is desired to determine the characteristics of the complete wing at an angle of attack and with surfaces deflected; for instance, to compute the induced drag or the maximum lift coefficient, the following formulas may be used:

$$l_n = l_a + l_b + l_e$$

$$c_{l_n} = \frac{l_n}{qc} = \frac{l_n}{q\left(\frac{b}{2}\right)} \times \frac{1}{\frac{c}{\left(b/2\right)}}$$

$$C_{D_i} = \frac{AR}{4} \int_{-1}^{+1} \left(\alpha - \frac{c_{l_n}}{m_o} \right) c_{l_n} \times \frac{c}{\left(\frac{b}{2}\right)} \; d\frac{y}{\left(\frac{b}{2}\right)}$$

$$C_{M_{YAW}} = -\frac{AR}{8} \int_{-1}^{+1} \frac{c}{\left(\frac{b}{2}\right)} \times \frac{y}{\left(\frac{b}{2}\right)} \left(\alpha - \frac{c_{l_n}}{m_o} \right) c_{l_n} \; d\frac{y}{\left(b/2\right)}$$

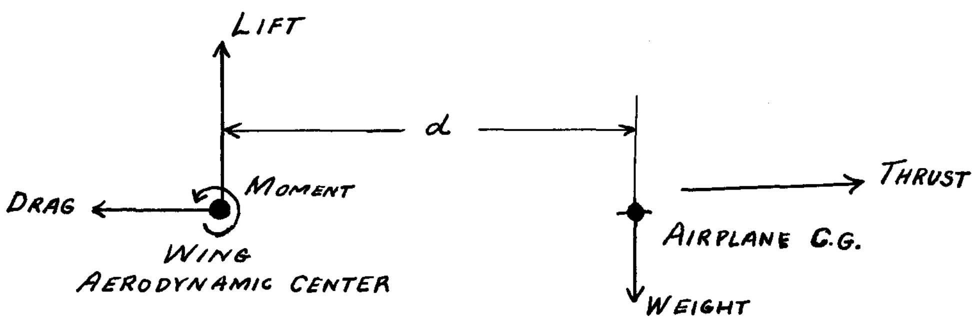
$$C_L = C_{L_W} + \Delta C_L$$

$$C_{M_{AC}} = C_{M_{O_W}} + \Delta_1 C_{M_{AC}} + \Delta_2 C_{M_{AC}}$$

THE AIRPLANE AS A WHOLE

The tailless plane will, in general, consist of a stable wing and other necessary accessories. These will perhaps include a fuselage, engine nacelles on the wing or body, and vertical surfaces at the wing tips. The characteristics of the wing alone will be denoted here by quantities with the subscript "w". Similarly the influence of the other parts of the airplane will be indicated by the subscript "a".

In level flight, the airplane is acted upon by the forces shown below:



$$L = \frac{1}{2} \rho U^2 S C_L$$

$$D = \frac{1}{2} \rho U^2 S C_D$$

$$M_{AC} = \frac{1}{2} \rho U^2 S \left(\frac{S}{b}\right) C_{M_{AC}}$$

When the ailerons and flaps are in the undeflected condition, the dimensionless coefficients from which the forces and moments are derived are made up as follows:

$$C_L = C_{L_W} + C_{L_A}$$

$$C_D = C_{DP} + C_{Di}$$

$$C_{DP} = C_{DP_W} + C_{DP_A}$$
$$C_{Di} = C_{Di_W} + C_{Di_A}$$

$$C_{M_{AC}} = C_{M_{0W}} + C_{M_A} + \left(\frac{dC_M}{dC_L}\right)_A \times (C_{L_W} + C_{L_A})$$

$$C_{M_{CG}} = -C_L \times \frac{d}{S/b} + C_{M_{AC}}$$

In these equations, C_{L_W} is the lift coefficient of the wing with the control surfaces neutral at the given attitude and C_{L_A} is the increment in lift coefficient due to the change in zero lift angle as a result of adding the fuselage and nacelles. It is necessary to include the value of the quantity $\left(\frac{dC_M}{dC_L}\right)_A$ to account for small changes which the nacelles and fuselage may make in the position of the aerodynamic center of the combination, as compared with that of the wing alone.

If the airplane is in a position of equilibrium as shown, the moment of the forces about the center of gravity of the plane must be equal to zero. Thus the position of the C.G. relative to the wing aerodynamic center is determined to be:

$$\frac{d}{S/b} = \frac{C_{MAC}}{C_{L_{TRIM}}}$$

The effect of the drag and thrust forces on the moment are neglected here. If this is not permissible in an actual airplane, the effect of the eccentricities and magnitudes of these forces can easily be included in the analysis.

The static stability of the airplane is then found to be:

$$\frac{dC_{M_{CG}}}{dC_L} = -\frac{d}{S/b} + \left(\frac{dC_M}{dC_L}\right)_A = -\frac{C_{MAC}}{C_{L_{TRIM}}} + \left(\frac{dC_M}{dC_L}\right)_A$$

If, now, the ailerons and flaps are deflected, there will be increments to the wing lift and moment coefficients. The equilibrium equations then become:

$$W = \frac{1}{2}\rho U^2 S \left(C_{L_W} + C_{L_A} + \Delta C_L\right)$$

$$M_{CG} = -Wd + \frac{1}{2}\rho U^2 S \frac{S}{b}\left[C_{M_{O_W}} + C_{M_A} + \left(\frac{dC_M}{dC_L}\right)_A C_L + \Delta C_{MAC}\right]$$

The location of the center of gravity relative to the wing aerodynamic center for static equilibrium in level flight becomes:

$$\frac{d}{S/b} = \frac{\left[C_{M_{O_W}} + C_{M_A} + \left(\frac{dC_M}{dC_L}\right)_A \left(C_{L_W} + C_{L_A} + \Delta C_L\right) + \Delta C_{MAC}\right]}{\left(C_{L_W} + C_{L_A} + \Delta C_L\right)}$$

The condition that the airplane remain in trim after a control deflection is that this ratio remains constant. If the additional effects of auxiliary surface deflections can be expressed analytically, this fact can be utilized to calculate the net lift coefficient at which trim is obtained for any combination of control surface deflections.

The maximum lift coefficient which the wing will attain can be found by comparing the section lift coefficient at various attitudes and surface settings with the section maximum lift coefficients. The maximum lift coefficient which any section will reach depends on the basic section and the angle through which its flap is deflected.

A method of determining the maximum lift coefficient at which trim can be maintained with a given flap deflection is shown in the example analysis included in this thesis.

For calculating performance, it is necessary to know the drag of the airplane. Experiments indicate that the increments in drag due to fuselage or nacelles are almost constant over the flying range of an airplane. For more exact calculations, the values of the additional profile drag and induced drag of these items can be estimated or determined from wind tunnel tests.

The calculation of the dynamic longitudinal stability of a tailless airplane follows the general analysis given in reference 7. However, due to the absence of a tail, new expressions must be found to replace those given for the quantities

m_q and μm_α . The new expressions are:

$$m_q = \frac{\frac{1}{2} \bar{U} \, S/b}{K_y^2} \times \frac{d C_{M_{CG}}}{d q} \qquad \text{and} \qquad \mu m_\alpha = \frac{\frac{1}{2} m \, S/b}{\rho S K_y^2} \times \frac{d C_{M_{CG}}}{d \alpha}$$

where K_y is the radius of gyration of the airplane in pitch and q is the pitching velocity of the plane in radians per second. The value of K_y may be calculated from a weight diagram of the airplane or estimated from a knowledge of the values for conventional airplanes. The calculation of the quantity $\frac{d C_{M_{CG}}}{d q}$ is the subject of a separate part of this thesis.

DAMPING DUE TO PITCHING VELOCITY

The expression presented here for the damping in pitch of a sweptback wing is obtained by superposing three effects from the stationary airfoil theory. It does not involve the use of the non-stationary airfoil theory; i.e., it neglects the induction effects of the spanwise vortices of the wake. This approximation is believed to be justified for the low frequency oscillations which occur in the longitudinal motions of an airplane. The analysis presupposes a straight taper wing with linear sweepback, as shown in Fig. 1.

It has been impossible to check the result by experiment, and until this is done it is recommended that it be applied only to wings having a considerable amount of sweepback, say from 15 to 30 degrees.

The general procedure followed in the analysis is indicated below. The details are presented in a later section of this thesis.

The motion of any part of the wing due to rotation about the center of gravity of the airplane is broken into three components:

1. Uniform vertical motion of the wing.
2. Vertical motion of each section of the wing proportional to its distance from the plane of symmetry of the wing.
3. Rotation of each section of the wing about its 75% chord point.

The effect of these motions is:

1. An additional lift at the wing aerodynamic center due to:

   a. The uniform vertical motion.

   b. The change in the zero lift direction of the wing due to the effective twist corresponding to the vertical motion proportional to the distance of the sections from the plane of symmetry.

2. A moment about the aerodynamic center of the wing due to the basic lift distribution caused by the effective twist.

3. A moment at each section due to the rotation of the section about its 75% chord point.

The lift and moment about the wing aerodynamic center due to the additional and basic lift and the change in zero lift direction induced by the translatory components of the motion of the wing can be calculated by the use of the formulas and charts in reference 1.

The effect of the section rotation about the 75% chord point is found by considering each section as part of a two dimensional airfoil and using Munk Integrals to calculate the lift and moment per unit span of the airfoil. It is found that there is no lift produced by rotation about this point of the chord. The moment contributions of each section of the wing are summed up to give a moment for the entire wing.

The moment of the lift at the wing aerodynamic center about the center of gravity of the airplane is found and all the moments are expressed in coefficient form and then differentiated with respect to the pitching velocity to give the sought after damping derivative.

Summary of Damping Moments

$$\left.\frac{dC_m}{dq}\right)_1 = -57.3\,\frac{S/b}{U}\,\frac{d}{S/b}\,f\,\frac{a_0}{1+\frac{57.3\,a_0}{\pi AR}}\left[\frac{1}{1+\lambda}+\frac{d}{S/b}-H\,AR\tan\Lambda - J\,\frac{AR}{2}\left(\tan\Lambda-\frac{2(1-\lambda)}{AR(1+\lambda)}\right)\right]$$

due to lift forces

$$\left.\frac{dC_m}{dq}\right)_2 = -\frac{57.3}{2}\,\frac{S/b}{U}\,AR^2\,a_0\,G\tan\Lambda\left[\tan\Lambda-\frac{2}{AR}\,\frac{(1-\lambda)}{(1+\lambda)}\right]$$

due to effective twist

$$\left.\frac{dC_m}{dq}\right)_3 = -\frac{\pi}{4}\,\frac{S/b}{U}\,\frac{(1+\lambda^2)}{(1+\lambda)^2}$$

Sum of section moments due to rotation.

WHERE:

f, J, G, H from T.R. 572

$\lambda = \frac{C_t}{C_s}$

Λ = sweepback

a_0 = slope per degree for ∞ AR

$\frac{d}{S/b} = \frac{C_M}{C_{L_{TRIM}}}$ for airplane in the trim position.

DISCUSSION OF EXAMPLE DESIGN PROBLEM

In order to illustrate the use of the formulas and methods described here, a sample design problem has been carried through. To permit comparison with existing normal type airplanes the plane was designed to have the same power and load carrying capacity as a standard land transport, the Douglas D.C.-3.

It was decided that the design should be as shown in Fig. 2 having a sweptback wing with a large cabin suspended from it in which the load is carried, and two outboard nacelles on the leading edge of the wing for the power plants. In these respects it is much like the Douglas plane, except for the absence of the tail surfaces and the different form of the wing. A pair of rudders are mounted on the wing tips for directional stability and control.

The wing area of the example is chosen as roughly equal to the sum of the wing and tail areas of the D.C.-3. It is assumed that the gross weight of both planes is the same. The aspect ratio was chosen as high as it was believed possible to build the wing; and the taper ratio was chosen high for structural reasons, yet not so high that troubles with tip stalling and lateral control could be anticipated. The sweepback angle of 30 degrees was chosen as a compromise of longitudinal stability and damping with structural problems and induced drag.

The drag and moment effects of the fuselage and nacelles were estimated and converted to coefficients based on the wing area of the plane. A value of the static stability in the cruising condition with controls neutral was picked, and the necessary value of the wing moment coefficient about its aerodynamic center was computed. This enabled the determination of the amount of linear twist necessary. The method of reference 1 was used.

After the twist was found, the basic load distribution and the shape of the additional load distribution were found. Fig. 3 shows these results.

It was then necessary to determine upon the flap and aileron geometry of the wing. An aileron with a varying percent chord width and a flap with a constant percent chord width were chosen in order to more completely illustrate the methods of calculating the additional effects they produce.

The steps in the calculation of the increments of lift and moment about the wing aerodynamic center due to aileron and flap deflection are shown in Figs. 3, 4 and 5, and in the appended calculations. The theoretical factors given in reference 14 were used to determine the change in zero lift direction and section moment coefficient with flap angle. For this reason the actual deflections necessary to produce the calculated results will be higher than those assumed.

One oft-mentioned disadvantage of the tailless airplane is the fact that the deflection of the flaps introduces a diving moment which must be counteracted by deflecting the ailerons upward.

This process decreases the proportion of the wing which can reach a high angle of attack, and thus reduces the maximum lift coefficient obtainable. This low maximum lift has been one of the most serious drawbacks to tailless design.

In the present design, the aspect ratio and the sweepback angle were made as high as possible. Theory shows that this combination will give better aileron control and greater positive pitching moment at zero lift with a smaller penalty in induced drag than any other combination. The result of this choice was that the aerodynamic center of the wing was aft of the centroid of the additional lift due to flap deflection, and so far aft that the moment of the additional lift was greater than the section diving moment induced by the flap deflection. Thus the net effect of increasing the flap angle is an increase in lift and a stalling moment. This effect is, of course, contributed to by the short flap span, 40 percent of the wing span.

The important result is that it seems possible to increase the maximum lift coefficient obtainable with such wings by choosing the aspect ratio, sweepback, and flap span properly. It also is apparent from a study of the lift distributions caused by the various control deflections that a wing of this sort is definitely center-stalling, unless, possibly, a tip is stalled in the attempt to execute a banked turn at slow speed.

The condition that trim be maintained in flight at all attitudes and with any flap and aileron angles is used, and aileron deflection is calculated as a function of net wing lift coefficient for trim with the flaps at an angle of 20 degrees and plotted in Fig. 7.

To find the maximum lift of the wing under these conditions, it is necessary to set up the criteria for the stalling of the sections of the wing along the span. It is assumed in this case that the wing will stall when the center of the unflapped portion of the wing, at station $\frac{y}{b/2} = .45$, reaches a section lift coefficient of 1.50. A check of the lift coefficient distribution when this condition is satisfied shows that the assumption is reasonable, and that the plane can easily fly in a trimmed condition at the required attitude. A wing maximum lift coefficient of 1.67 is thus obtained with the flaps at 20°. Fig. 8 shows these results.

Since this lift coefficient is based upon a greater wing area than a normal plane of the same capacity has, it is apparent that, in this case, we have succeeded in obtaining a maximum lift comparable to that which is obtained by an airplane with a horizontal tail surface.

In order to check the analysis of damping due to pitching velocity, the period and the time to damp to half amplitude for longitudinal oscillations were computed for the example airplane in its cruising condition. The method outlined in reference 7 was used. The moment of inertia in pitch was conservatively chosen as 7/8 of that of the D.C.-3. The time to damp to half amplitude turned out to be just a little more than one period, showing that the plane is quite stable. This, however, is not conclusive, as instability is more likely to appear at high lift coefficients than at the cruising condition.

If a tailless airplane is to be practical, its performance must compare favorably with that of conventional planes. For this reason some preliminary calculations were made to determine the cruising speed of the example airplane under the same cruising power conditions as the D.C.-3. The results showed a speed of a few miles per hour higher than the advertized cruising speed of the parent plane. The landing speed, based on the previously determined maximum lift coefficient, was also just a bit lower than the advertized landing speed of the parent plane.

As far as the comparison has been carried, it has been shown that the tailless airplane of the type described compares favorably with the conventional type plane. The chief difficulties are the structural problem of a high aspect ratio wing which can support endplates and rudders at the tips, and the question of directional control and stability.

An advantage which the tailless plane may prove to have over the airplane equipped with horizontal tail surfaces is the reduced effect of power on stability. The tailless plane has no trimming or stabilizing surfaces which are in the propeller slipstream, and its static stability should be more nearly independent of the power applied. This problem is growing quite serious with normal airplanes.

As a means of extending the range of usefulness and insuring that trim can be attained at high lift coefficients, the use of a forward trimming surface has been suggested. Reference 8 describes a device of this kind which floats in front of a swept-back wing and applies a constant moment to the airplane. The magnitude of the moment can be changed by tabs located at the trailing edge of the surface. Control for all normal flight conditions is obtained with ailerons and rudders on the wing. Devices of this kind show promise, and if the stable wing by itself does not prove to be adaptable enough to practical operating conditions, it may be necessary to incorporate them into the design. It may even be possible that a fixed or floating surface which is retractable, and would be used for landing conditions only, might prove useful.

RECOMMENDATIONS FOR FUTURE WORK

The problems in the design of tailless airplanes have been investigated only in their very general aspects in this paper. A few suggestions for further study are listed below:

1. The effect of sweepback on lift distribution and the induced drag of a wing.

2. A better procedure for determining the change in effective angle of attack of a surface when a flap is deflected. This probably requires an extensive analysis of experimental data for different types of flap slots and gaps.

3. A more refined method of computing the maximum lift coefficient which a wing can reach with flaps and ailerons deflected.

4. An analysis of the directional and lateral stability and control characteristics of tailless planes and the best means of achieving this control.

5. A check of this work by wind tunnel experiments.

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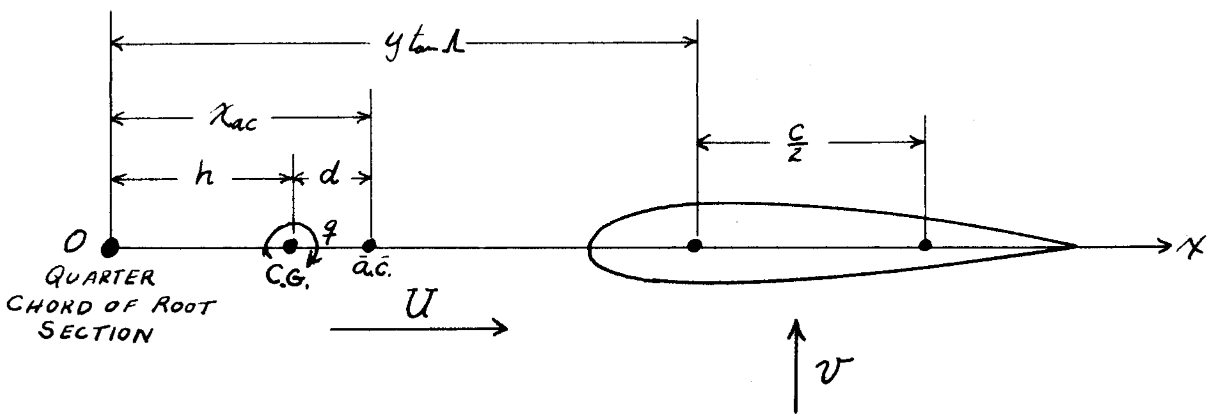
DAMPING DUE TO PITCH (DETAILED ANALYSIS)



The rotation of any section about the C.G. can be broken into a translation and rotation about the 75% point of the chord section.

$$v = q\left(y \tan \Lambda + \frac{c}{2} - h\right) + q\left(x - y \tan \Lambda - \frac{c}{2}\right)$$

Let $\quad c = P - Qy \qquad P = \frac{4}{AR(1+\lambda)}\left(\frac{b}{2}\right) \qquad Q = \frac{4(1-\lambda)}{AR(1+\lambda)}$

$$\left(\frac{b}{2}\right) = \frac{AR}{2} \cdot \frac{S}{b}$$

$$v = q\left(y \tan \Lambda - h + \frac{P}{2} - \frac{Q}{2} y\right) + q\left(x - y \tan \Lambda - \frac{c}{2}\right)$$

$$v = q\left(\frac{P}{2} - h\right) + q y\left(\tan \Lambda - \frac{Q}{2}\right) + q\left(x - y \tan \Lambda - c/2\right)$$

CONSTANT VERTICAL VELOCITY → CHANGE IN ANGLE OF ATTACK

VARIABLE VERTICAL VELOCITY → TWIST

ROTATION ABOUT 75% CHORD POINT

The following quantities enter into the calculations:

$$a = f \times \frac{a_0}{1 + \frac{57.3\, a_0}{\pi\, AR}}$$

a_0 = infinite AR slope of lift curve in 1/degrees.

$$\alpha_{s_{L=0}} = \alpha_{\ell o s} + J\epsilon$$

ϵ = twist in degrees (increase in angle at tip)

$$X_{ac} = \frac{S}{b}\, AR\, H \tan\Lambda$$

$$C_{m_{\ell b}} = -G\,\epsilon\, a_0\, AR \tan\Lambda$$

Where f, J, H, and G are factors from charts in T.R. 572.

The uniform velocity, $q\left(\frac{P}{2} - h\right)$ may be considered as an increase in angle of attack of the wing of;

$$\Delta\alpha_1 = q\left(\frac{P}{2} - h\right) \times \frac{57.3}{U} \quad \text{degrees}$$

The vertical relative velocity proportional to the span results in an effective linear twist, with a resultant moment and a lift due to the change in zero lift angle. The effective twist is:

$$\epsilon = \frac{q\left(\frac{b}{2}\right)\left(\tan\Lambda - \frac{Q}{2}\right) \times 57.3}{U} \quad \text{degrees at the tip.}$$

Following T.R. 572, this results in a moment of

$$C_{m_{\ell b}} = -G\,\epsilon\, a_0\, AR \tan\Lambda$$

and an effective increase in angle of attack of

$$\Delta\alpha_2 = -J\epsilon$$

Moment due to Lift Forces

The lift due to these increases in angle of attack acts at the wing aerodynamic center and has a moment about the C.G. of

$$M_{CG}\Big)_1 = -\frac{1}{2}\rho U^2 S\, a\,(\Delta\alpha_1 + \Delta\alpha_2)\times d$$

$$M_{CG}\Big)_1 = -\frac{1}{2}\rho U^2 S\,\frac{S}{b}\,\frac{d}{S/b}\,a_0\,\frac{f}{1+\frac{57.3a_0}{\pi AR}}\left[\left(\frac{P}{2}-h\right) - J\frac{b}{2}\left(\tan\Lambda - \frac{a}{2}\right)\frac{57.3}{U}q\right.$$

$$\frac{dC_m}{dq}\Big)_1 = -\frac{57.3}{U}\,\frac{d}{S/b}\times\frac{a_0 f}{1+\frac{57.3a_0}{\pi AR}}\left[\frac{1}{1+\lambda} - \left(\frac{X_{ac}}{S/b} - \frac{d}{S/b}\right) - J\frac{AR}{2}\left(\tan\Lambda - \frac{2(1-\lambda)}{AR(1+\lambda)}\right)\right]$$

$$\frac{dC_m}{dq}\Big)_1 = -\frac{57.3\,S/b}{U},\,\frac{d}{S/b}\times\frac{a_0 f}{1+\frac{57.3a_0}{\pi AR}}\left[\frac{1}{1+\lambda} - HAR\tan\Lambda + \frac{d}{S/b} - J\frac{AR}{2}\left(\tan\Lambda - \frac{2(1-\lambda)}{AR(1+\lambda)}\right]\right.$$

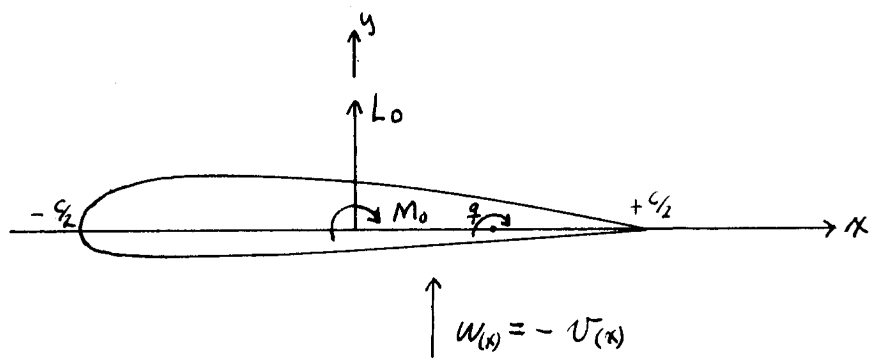
Moment due to Effective Twist

$$C_M\Big)_2 = -G\,\epsilon\,a_0\,AR\tan\Lambda$$

$$C_M\Big)_2 = -G\,AR\tan\Lambda\;a_0\left(\tan\Lambda - \frac{Q}{2}\right)\times\frac{q}{U}\times 57.3\times\frac{AR}{2}\times S/b$$

$$\frac{dC_M}{dQ}\Big)_2 = -\frac{57.3}{2U}\,AR^2\,\frac{S}{b}\,G\,a_0\tan\Lambda\left[\tan\Lambda - \frac{2(1-\lambda)}{AR(1+\lambda)}\right]$$

Effect of Rotation about 75% of Section Chord



Munk's Integrals give the lift and moment at the 50% chord point due to any arbitrary external velocity imposed on the airfoil.

$$L_o = -\rho U c\int_0^{\pi} v_{(x)}\,(1+\cos\tau)\,d\tau$$

$$M_o = -\rho U\frac{c^2}{2}\int_0^{\pi} v_{(x)}\sin^2\tau\,d\tau$$

where: $x = \frac{c}{2}\cos\tau$

so: $v_{(x)} = -w_{(x)} = -q\left(x-\frac{c}{4}\right) = -q\,\frac{c}{2}\left(\cos\tau - \frac{1}{2}\right)$

$$L_o = +\rho U q \frac{c^2}{2}\int_0^{\pi}(\cos\tau - \tfrac{1}{2})(1+\cos\tau)\,d\tau$$

$$= \rho U q \frac{c^2}{2}\int_0^{\pi}\left(\cos^2\tau + \frac{\cos\tau}{2} - \frac{1}{2}\right)d\tau$$

$$= \rho U q \frac{c^2}{2}\left[\frac{\tau}{2} + \frac{\sin^2 2\tau}{4} + \frac{1}{2}\sin\tau - \frac{\tau}{2}\right]_0^{\pi} = 0$$

<u>Moment due to Section Rotation</u>

$$M_o = \rho U q \frac{c^3}{4}\int_0^{\pi}(\cos\tau - \tfrac{1}{2})\sin^2\tau\,d\tau$$

$$= \rho U q \frac{c^3}{4}\int_0^{\pi}\left(\cos\tau\sin^2\tau - \frac{1}{2}\sin^2\tau\right)d\tau$$

$$= \rho U q \frac{c^3}{4}\left[\frac{\sin^3\tau}{3} - \frac{1}{2}\left(\tau/2 - \frac{\sin 2\tau}{4}\right)\right]_0^{\pi}$$

$$M_o = -\frac{\pi}{16}\rho U q c^3 \quad \text{per unit span}$$

The total moment on the wing is :

$$M\Big)_3 = 2\int_0^{b/2} M_0\, dy = -2\int_0^{b/2} \frac{\pi}{16}\rho U q\,(P-Qy)^3\, dy$$

$$M\Big)_3 = +\frac{\pi}{8}\rho U q \left.\frac{(P-Qy)^4}{4Q}\right]_0^{b/2}$$

$$M\Big)_3 = \frac{\pi}{32}\rho U q \frac{1}{\frac{4(1-\lambda)}{AR(1+\lambda)}}\left[\left(P-Q\tfrac{b}{2}\right)^4 - P^4\right]$$

$$M\Big)_3 = \frac{\pi}{32}\rho U q \frac{1}{\frac{4(1-\lambda)}{AR(1+\lambda)}} \times \left[\frac{4}{AR(1+\lambda)}\left(\frac{b}{2}\right)\right]^4 \left(\lambda^4-1\right)$$

$$M\Big)_3 = -\frac{\pi}{32}\rho U q\left(\frac{4}{AR(1+\lambda)}\right)^3\left(\frac{b}{2}\right)^4\frac{(1-\lambda^4)}{(1-\lambda)}$$

$$\frac{dC_M}{dq}\Big)_3 = -\frac{\pi}{16}\,\frac{1}{US\,S/b} \times \frac{4^3}{AR^3(1+\lambda)^3} \times \frac{AR^2}{2^4} \times \left(\frac{S}{b}\right)^4 \frac{(1-\lambda^4)}{(1-\lambda)}$$

$$\frac{dC_M}{dq}\Big)_3 = -\frac{\pi}{4}\,\frac{S/b}{U}\,\frac{(1+\lambda^2)}{(1+\lambda)^2}$$

The total damping is obtained by combining these three damping effects. See page 35.

EXAMPLE DESIGN CALCULATIONS

The airplane is to be equivalent to the Douglas D.C.-3

$S = 1300 \text{ ft.}^2$
$AR = 10$
$\Lambda = 30°$
$\lambda = .25$
$W = 24{,}000^{\#}$
$b = 114 \text{ ft.}$
$S/b = 11.4 \text{ ft.}$

FROM T.R. 572

$f = .999$
$J = -.385$
$G = .0244$
$H = .205$

$C_s = 18.25 \text{ ft.}$

$C_t = 4.56 \text{ ft.}$

$C_s/(\frac{b}{2}) = .320$

$C_t/(\frac{b}{2}) = .080$

$\frac{X_{ac}}{(\frac{b}{2})} = .237$

$a_0 = .099$ (assumed)

The effect of adding a fuselage is:

$C_{D\pi} = .07 \qquad S_\pi = 53 \text{ ft}^2$

$\Delta C_{D_{P_{fus}}} = .0029$

$\Delta C_{M_{0_{fus}}} = -.028$

$\Delta \left.\frac{dC_M}{dC_L}\right)_{fus} = +.028$

The effect of two leading edge nacelles is:

$$C_{D_{\pi}\,nacelles} = .105 \qquad S_{\pi} = 2 \times 19.6 = 39.2\ ft.^2$$

$$\Delta C_{D_P\,nacelles} = .0032$$ (including cooling drag)

$$\Delta C_{M_O\,nacelles} = -.014$$

$$\Delta \left.\frac{dC_M}{dC_L}\right)_{nacelles} = +.036$$

The two wing tip fins will have a combined area of 15% of the wing area

$$.15 \times 1300 = 195\ ft^2$$

$$C_{D_{\pi}\,fins} = .010$$

$$\Delta C_{D_P\,fins} = .0015$$

The wing with flaps and ailerons is assumed to have a drag given by

$$C_{D_{P\,wing}} = .010$$

The net effect of all the extra parts is then :

$$C_{L_A} = 0$$

$$C_{D_{P_A}} = .0076$$

$$C_{M_{O_A}} = -.042$$

$$\left.\frac{dC_M}{dC_L}\right)_A = .064$$

It is desired that the airplane cruise at a lift coefficient of

$C_L = .20$ and have a static stability of

$$\frac{dC_{M_{CG}}}{dC_L} = -.12$$

Since: $$\frac{dC_{M_{CG}}}{dC_L} = -\frac{C_{M_{AC}}}{C_{L_{TRIM}}} + \left.\frac{dC_M}{dC_L}\right)_a$$

we have $$-.12 = -\frac{C_{M_{AC}}}{.20} + .064$$

and $$C_{M_{AC}} = +.0368$$

but $$C_{M_{AC}} = C_{M_{0_W}} + C_{M_A} + \left.\frac{dC_m}{dC_L}\right)_A \times C_L$$

so $$C_{M_{0_W}} = .0368 + .042 - .0128 = +.066$$

The wing section is assumed symmetrical, so there is no section moment.

Then $$C_{M_{0_W}} = -G\,\epsilon\, a_0\, AR \tan\Lambda$$

$$\epsilon = \frac{-.066}{.0245 \times .099 \times .577 \times 10} = -4.72^\circ$$

$$\frac{d}{s/b} = \frac{C_{M_{AC}}}{C_L} = \frac{.0368}{.2} = .184$$

$$\frac{d}{(\frac{b}{2})} = .0368$$

The chord distribution of the ailerons is shown in the following table, along with the change in zero lift direction and the change in moment at the sections corresponding to an aileron deflection of 1 radian. The theoretical factors from reference 14 are used to obtain these values.

		For: δ_e = 1 radian	
Station	Aileron % Chord	$\Delta\alpha$	$\Delta_2 C_{m_{ac}}$
.50	.20	.546	-.641
.60	.2285	.581	-.650
.70	.265	.623	-.655
.80	.312	.670	-.630
.90	.392	.738	-.592
1.00	.50	.815	-.499

In order to plot the various loadings, we have:

$$\frac{l_b}{q(\frac{b}{2})} = \frac{2}{AR}\epsilon a_0 L_b = \frac{1}{5}(-4.72)\times .0991\, L_b = -.0935 L_b$$

$$\frac{l_a}{q(\frac{b}{2})} = \frac{2}{AR} C_L L_a = .2 L_a \quad (\text{for } C_L = 1.00)$$

$$\frac{l_e}{q(\frac{b}{2})} = \frac{c_s}{(\frac{b}{2})}\int \frac{c_l\, c/c_s}{\alpha}\, d\alpha = .320\int \frac{c_l\, c/c_s}{\alpha}\, d\alpha$$

Where $\frac{c_l\, c/c_s}{\alpha}$ is the influence function of reference 4 and the integration gives the effect at a particular spanwise station of the increments of twist making up the effect of the aileron displacement. The integration is carried out over this range of angle increments.

L_a and L_b for this wing are tabulated in the following table. The values of $\frac{C_\ell\, c/c_s}{\alpha}$ needed to find the loading due to the aileron deflection by the method described in the body of the thesis are also tabulated.

STATION $\frac{y}{(b/2)}$ =	L_b	L_a	$\frac{\ell_b}{q(b/2)}$	$\frac{\ell_{a_1}}{q(b/2)}$ for $C_L = 1.00$
0	-.3225	1.442	.0302	.288
.2	-.2220	1.323	.0208	.265
.4	-.006	1.157	.00056	.231
.6	.1335	.938	-.0125	.188
.8	.1830	.678	-.0171	.135
.9	.175	.506	-.0164	.101
.95	.145	.378	-.0136	.0755
.975	.0975	.259	-.0091	.0518
1.00	0	0	0	0

Table of $\frac{C_\ell\, c/c_s}{\alpha}$ for Aileron Deflection of 1 Radian

$\Delta\alpha$	Location where Increment starts	STATION .875	.75	.625	.50	.375	.25	0
0	.50	1.72	2.12	2.26	—	.40	.25	.12
.546	.50	1.72	2.12	2.26	1.27	.40	.25	.12
.581	.60	1.66	1.90	1.76	.33	.25	.12	—
.623	.70	1.56	1.56	.38	.15	—	.06	—
.670	.80	1.38	.25	—	.06	—	—	—
.738	.90	.40	.06	—	—	—	—	—
.815	1.00	0	0	0	—	0	0	0
Area under curve		1.203	1.350	1.347	.7250	.2250	.1425	.0675
$\frac{\ell_c}{q(b/2)}$ at the station		.385	.432	.430	.232	.072	.046	.0216

ONE AILERON DEFLECTED ONE RADIAN

STATION	$\frac{l_c}{q(\frac{b}{2})}$	$\left(\frac{y}{(\frac{b}{2})} - 2H\right)$	Product: $\frac{l_c}{q(\frac{b}{2})} \times \left(\frac{y}{\frac{b}{2}} - 2H\right)$
$y/(b/2)$ = .95	.272	.54	.147
.90	.356	.49	.174
.80	.422	.39	.165
.70	.438	.29	.127
.60	.420	.19	.0797
.55	.385	.14	.054
.50	.230	.09	.0207
.45	.120	.04	.0048
.35	.061	-.06	-.00366
.20	.039	-.21	-.0082
0	.024	-.41	-.00984
- .20	.016	-.21	-.00336
- .40	.010	-.06	-.0006
-1.00	0	0	0

Station	$\Delta_z C_{m_{ac}}$	$c/(\frac{b}{2})$	$\left[\frac{c}{(\frac{b}{2})}\right]^2$	$\Delta_z C_{m_{ac}} \times \left[\frac{c}{(\frac{b}{2})}\right]^2$
.50	-.641	.20	.04	-.0256
.60	-.650	.175	.0306	-.0199
.70	-.655	.151	.0228	-.01491
.80	-.630	.128	.0164	-.01032
.90	-.592	.102	.0104	-.00615
1.00	-.499	0	0	0

Tables are also given of the quantities required to plot the curves and perform the graphical integrations from which the effect of aileron deflection on the wing moment can be determined. (Page 55)

The areas under the various curves are found and the following quantities calculated for one aileron deflected one radian.

$$\Delta C_L = \frac{AR}{4} \times .228 = +.57$$

$$\Delta_1 C_{M_{AC}} = -\frac{AR^2}{8} \tan \Lambda \int_{-1}^{+1} \frac{l_c}{q\left(\frac{b}{2}\right)} \left(\frac{y}{(b/2)} - 2H\right) d \frac{y}{\left(\frac{b}{2}\right)}$$

$$= -\frac{100}{8} \times .577 \times .062 = -.447$$

$$\Delta_2 C_{M_{AC}} = \frac{AR^2}{8} \int_{-1}^{+1} \Delta_2 C_{m_{ac}} \left[\frac{c}{\left(\frac{b}{2}\right)}\right]^2 d \frac{y}{\left(\frac{b}{2}\right)}$$

$$= \frac{100}{8} \times .0064 = -.080$$

$$\Delta C_{M_{AC}} = -.447 - .080 = -.527 \text{ per aileron per radian}$$

The flaps are 20% chord width and extend from the center of the wing to the 40% spanwise station. When the flaps are deflected together one radian, the value of the quantities is found as before, but are constant across the flapped portion

$$\Delta\alpha = .547$$

$$\Delta_2 C_{m_{ac}} = -.641$$

$$\frac{l_c}{q\left(\frac{b}{2}\right)} = \frac{4}{AR(1+\lambda)} \frac{c_e\, c/c_s}{\alpha} \times \Delta\alpha = .175 \frac{c_e\, c/c_s}{\alpha}$$

The tabular quantities required to plot the curves and perform the integrations are presented below.

Station	$\frac{c}{(\frac{b}{2})}$	$\frac{\text{Flap Chord}}{(\frac{b}{2})}$	$\left[\frac{c}{(\frac{b}{2})}\right]^2$
0	.320	.064	.1023
.1	.297	.0594	.0881
.2	.271	.0541	.0732
.3	.248	.0496	.0616
.4	.223	.0446	.0498

Station	$\frac{C_L\, c/c_s}{\alpha}$	$\frac{l_c}{q(\frac{b}{2})}$	$\left(\frac{y}{(\frac{b}{2})} - 2H\right)$	Product: $\frac{l_c}{q(\frac{b}{2})} \cdot \left(\frac{y}{(\frac{b}{2})} - 2H\right)$
± .875	.06	.0105	.465	.004885
± .75	.13	.0227	.34	.00771
± .625	.32	.056	.215	.01206
± .50	.70	.1225	.09	.01103
± .375	2.62	.4585	-.035	-.01605
± .25	3.60	.630	-.16	-.1009
± .125	4.00	.700	-.285	-.1998
0	4.25	.7435	-.41	-.305

The area under the curves is found and the effect of the flaps calculated as:

$$\Delta C_L = \frac{10}{4} \times .586 = 1.468$$

$$\Delta_1 C_{M_{AC}} = -\frac{100}{8} \times .577(-.105) = .757$$

$$\Delta_2 C_{M_{AC}} = \frac{100}{8} \times (-.641) \times .030 = -.2405$$

$\Delta C_{M_{AC}} = +.757 - .2405 = +.516$ per radian for both flaps acting together.

To find aileron deflection for trim at different lift coefficients, we have:

$\frac{d}{S/b} = \frac{C_{M_{AC}}}{C_L} = .184 =$ constant as long as the plane remains trimmed and the C.G. does not shift.

With flaps at zero:

$$\frac{d}{S/b} = \frac{C_{M_{O_W}} + \Delta C_M + \left.\frac{dC_M}{dC_L}\right)_A \times C_L + \Delta C_{M_{AC}}}{C_L}$$

So: $.184 = .066 - .042 + .064\, C_L - .013\, \delta_c^\circ$

from which $C_L = .200 - .1084\, \delta_c^\circ$ (Flaps Zero)

Similarly, when the flaps are at 20°

$$.184 = \frac{.066 - .042 + .064\, C_L + .18 - .013\, \delta_c^\circ}{C_L}$$

and $$C_L = 1.70 - .1084\, \delta_c^\circ \qquad (\text{Flaps at } 20^\circ)$$

Estimation of Maximum Lift

Flaps at 20°

Assume that the wing stalls when C_ℓ at Station $\frac{y}{(b/2)} = .45$ is 1.50

The section lift coefficient at this station is made up of 4 parts:

$$\frac{\ell_a}{q(\frac{b}{2})} = .221\, C_{L_w}$$

$$\frac{\ell_b}{q(\frac{b}{2})} = -.022$$

$$\left.\frac{\ell_c}{q(\frac{b}{2})}\right)_{flaps} = +.063$$

$$\left.\frac{\ell_e}{q(\frac{b}{2})}\right)_{ailerons} = \frac{.132}{57.3}\, \delta_c^\circ$$

when $$C_\ell = 1.5 \; ; \; \frac{\ell_a}{q(\frac{b}{2})} = C_\ell \times \frac{c}{(\frac{b}{2})} = 1.5 \times .212 = .318$$

From the condition that trim be maintained, we have

$$C_L = 1.70 - .1084\,\delta_e^\circ$$

Since
$$C_L = C_{L_w} + \Delta C_L)_{flap} + \Delta C_L)_{aileron}$$

$$1.70 - .1084\,\delta_e^\circ = C_{L_w} + .512 + .0199\,\delta_e^\circ$$

or
$$C_{L_w} = 1.19 - .128\,\delta_e^\circ$$

Summing up the contributions to the section loading at the given station:

$$.318 = .221\,(1.19 - .128\,\delta_e^\circ) - .002 + .063 + .002305\,\delta_a^\circ$$

from which $\delta_e^\circ = +.231$

and $C_{L_{MAX.}} = 1.67$ and $C_{L_w} = 1.16$

Knowing the flap and elevator deflections and the wing alone lift coefficient for this maximum lift condition, the total loading at all spanwise stations can be computed and the section lift coefficients found. The following table gives the quantities required to plot the curves of Fig. 8.

Loading at Maximum Lift

$\delta_f = 20°$ $\delta_c = +.23°$ $C_{L_w} = 1.16$

Sta.	$\frac{l_a}{q(\frac{b}{2})}$ ($C_L = 1.00$)	$\delta = 1$ radian: $\left.\frac{l_c}{q(\frac{b}{2})}\right]_{flap}$	$\delta = 1$ radian: $\left.\frac{l_c}{q(\frac{b}{2})}\right]_{aileron}$	$\frac{l_e}{q(\frac{b}{2})}$	$\frac{l_b}{q(\frac{b}{2})}$	$\left.\frac{l_c}{q(\frac{b}{2})}\right]_{flap}$	$\left.\frac{l_c}{q(\frac{b}{2})}\right]_{aileron}$
0	.289	.750	.050	.335	.030	.261	0
.1	.283	.700	.050	.328	.027	.244	0
.2	.267	.665	.052	.310	.020	.232	0
.3	.251	.560	.064	.291	.011	.195	0
.4	.230	.330	.098	.265	.002	.115	0
.5	.210	.122	.235	.243	-.007	.043	.001
.6	.188	.063	.430	.218	-.014	.022	.002
.7	.163	.036	.440	.189	-.019	.013	.002
.8	.135	.020	.426	.156	-.021	.007	.002
.9	.099	.008	.359	.115	-.017	.003	.001
.95	.075	.004	.258	.087	-.013	.001	.001
1.0	0	0	0	0	0	0	

Sta.	$\frac{l_n}{q(\frac{b}{2})}$	$c/(\frac{b}{2})$	c_{l_n}
0	.626	.320	1.955
.1	.599	.297	2.02
.2	.562	.271	2.07
.3	.497	.248	2.01
.4	.382	.223	1.72
.5	.280	.200	1.40
.6	.228	.175	1.30
.7	.185	.151	1.22
.8	.144	.128	1.13
.9	.102	.102	1.00
.95	.076	.90	.84
1.0	0	0	0

Dynamic Stability Analysis

Required quantities:

$W = 24000^{\#}$

$S = 1300\ ft^2$

$V = 190\ mph = 278.5\ ft/sec$

$C_L = .20$

$C_D \backsim .02$

$$\left.\begin{aligned} \frac{dC_L}{d\alpha} &\backsim 4.83 \\ \frac{dC_D}{d\alpha} &\backsim .077 \end{aligned}\right\}\ \text{per radian}$$

$-m_q = 1.818$

$-\mu m_\alpha = 7.34$

$C_R^2 = .0404$

$S/b = 11.4\ ft.$

Assume: $K_y^2 = 108\ ft^2 = .865\ (K_y^2)_{D.C.3.}$

$$\frac{dC_M}{d\alpha} = \frac{dC_M}{dC_L} \times \frac{dC_L}{d\alpha} = -.12 \times .084 \times 57.3 = -.577$$

$$m = \frac{W}{g} = \frac{24000}{32.2} = 745.0\ \text{slugs}$$

$$-\mu m_\alpha = \frac{-\frac{1}{2} m\, S/b}{K_y^2 \rho S} \frac{dC_M}{d\alpha} = -\frac{1}{2} \times \frac{745 \times 11.4\ (-.577)}{108 \times .002378 \times 1300} = +7.34$$

Damping in Pitch (See page 35)

$$\left.\frac{dC_m}{dq}\right)_1 = -\frac{57.3\times 11.4}{278.5}\times .184\times .0844\left[\frac{1}{1.25}+.184-10\times .205\times .577+.385\times\frac{10}{2}\left(.577-\frac{2}{10}\times\frac{.75}{1.25}\right)\right]$$

$$= -\frac{57.3\times 11.4}{278.5}\times .184\times .0844\left[.80+.184-1.182+.869\right]$$

$$= -\frac{57.3\times 11.4\times .184\times .0844\times .671}{278.5} = -.0240$$

$$\left.\frac{dC_m}{dq}\right)_2 = -\frac{57.3}{2}\times\frac{11.4}{278.5}\times 100\times .10\times .0244\times .577\times .457$$

$$= -.0755$$

$$\left.\frac{dC_m}{dq}\right)_3 = -\frac{\pi}{4}\times\frac{11.4}{278.5}\times\frac{(1+.25^2)}{1.25^2} = -.0219$$

total $\dfrac{dC_m}{dq} = -.1234$

and $-m_q = \dfrac{-\frac{1}{2}V\,{}^{S}/_{b}}{k_y^2}\times\dfrac{dC_m}{dq} = \dfrac{1}{2}\times\dfrac{278.5\times 11.4}{108}\times .1234 = 1.818$

Substituting in Equations (17) of T.R. 521

$$B = 1.818+\tfrac{1}{2}(.06+4.83) = 1.818+\tfrac{1}{2}(4.89) = 1.818+2.45 = 4.28$$

$$C = 1.818\times 2.45+7.34+\tfrac{1}{2}(.02\times 4.83-.2\times .077+.0404)$$
$$= 4.45+7.34+.0608 = 11.85$$

$$D = 1.818 \times .0608 + \frac{3}{2} \times .02 \times 7.34 = .1105 + .220 = .3305$$

$$E = \frac{7.34 \times .0404}{2} = .1483$$

Then:

$$\zeta' = -\frac{1}{2}\left(\frac{.3305}{11.85} - \frac{4.28 \times .1483}{11.85^2}\right) = -\frac{1}{2}\left(.0279 - .004520\right)$$

$$= -\frac{1}{2} \times .0234 = -.0117$$

$$\psi' = \sqrt{\frac{.1483}{11.85} - \frac{1}{4}\left(\frac{.3305}{11.85} - \frac{4.28 \times .1483}{11.85^2}\right)^2} = \sqrt{.01251 - .0001369}$$

$$= \sqrt{.01237} = .1111$$

$$T = \frac{-.313}{-.0117}\sqrt{\frac{24000}{1300} \times .20} = \frac{.313}{.117} \times 1.921 = 51.4 \text{ sec}$$

* Time to damp to $\frac{1}{2}$ amplitude.

$$P = \frac{2.83}{.1111} \times 1.921 = 48.95 \text{ sec. period}$$

Since the time to damp to half amplitude is only slightly more than the period, the stability under these conditions is satisfactory.

Performance Analysis of Example Plane

$W = 24000^{\#}$
$S = 1300 \; ft^2$
$b = 114 \; ft$

$C_{D_p} = .0176 \qquad f = 22.9 \; ft^2$

$e = .76$ (assumed)

Also assume that trim attitude can be changed by small amounts without changing the value of "f".

For the cruising condition:

$$l_p = \frac{W}{f} = \frac{24000}{22.9} \approx 1049$$

$$l_s = \frac{W}{e b^2} = \frac{24000}{.76 \times 114^2} = 2.43$$

Cruising Power = 585 H.P. at 10,000 ft. per engine

Constant speed propeller ; $R.P.M = 1900 \times \frac{11}{16} = 1308$ R.P.M.

Prop Diameter $= 11.5$ ft.

$\sigma = .736$ at 10,000 ft.

$$C_p = \frac{50 \times 585}{.736 \times (1.308)^3 \times 11.5^5} = .0882$$

The Performance Equation is:

$$C_h = \frac{33,000}{l_t} - \frac{.22520 \sigma V^3}{l_p} - 10,944 \frac{l_s}{\sigma V}$$

J	η	l_t	$\frac{33,000}{l_t}$	$\frac{.22525\sigma V^3}{l_p}$	$\frac{10,944}{\sigma V} l_s$	ch	V	β
.938	.805	25.5	1295	648	225.8	421	160	25°
1.055	.820	25.0	1320	920	200.8	209	180	26.4°
1.172	.820	25.0	1320	1266	180.6	-127	200	28.0°
1.290	.830	24.7	1338	1690	164.2	-516	220	30°

where: $J = \frac{88V}{nD} = \frac{88\ V}{1308 \times 11.5} = .00586\ V$

$$l_t = \frac{W}{P\eta} = \frac{24000}{2 \times 585 \times \eta} = \frac{20.5}{\eta}$$

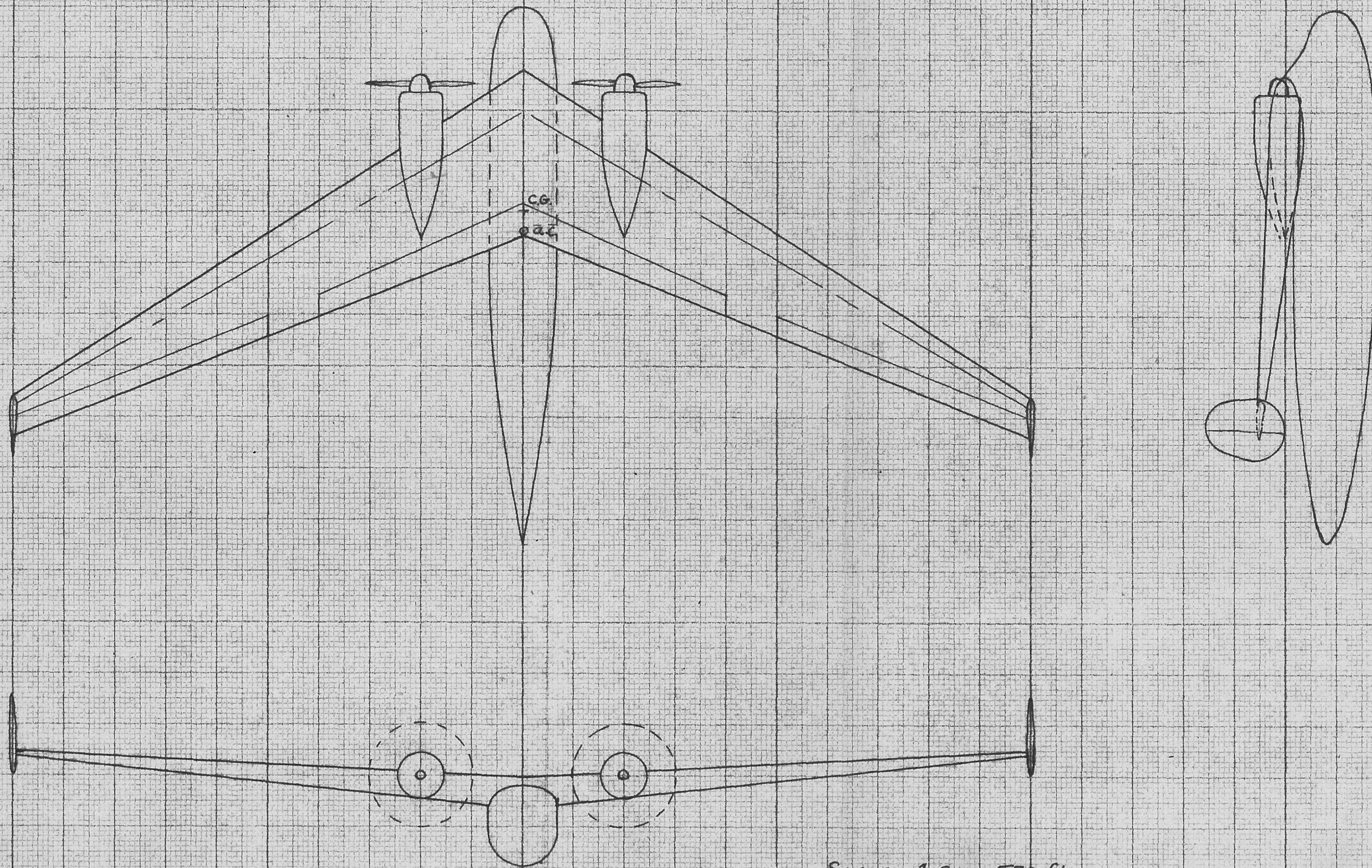
The prop. efficiencies are chosen as 2% less than those from the charts for prop 5868-9
There are no tip speed losses at cruising speed.

Cruising Speed = 192 mph (ch = 0 at this speed)
at 10,000 ft.

Stalling Speed: $C_{L_{Max}} = 1.67$

$$V_{stall} = 19.78 \sqrt{\frac{W}{S\,C_L}} = 19.78 \sqrt{\frac{24000}{1300 \times 1.67}} = 65.6 \text{ mph at sea level.}$$

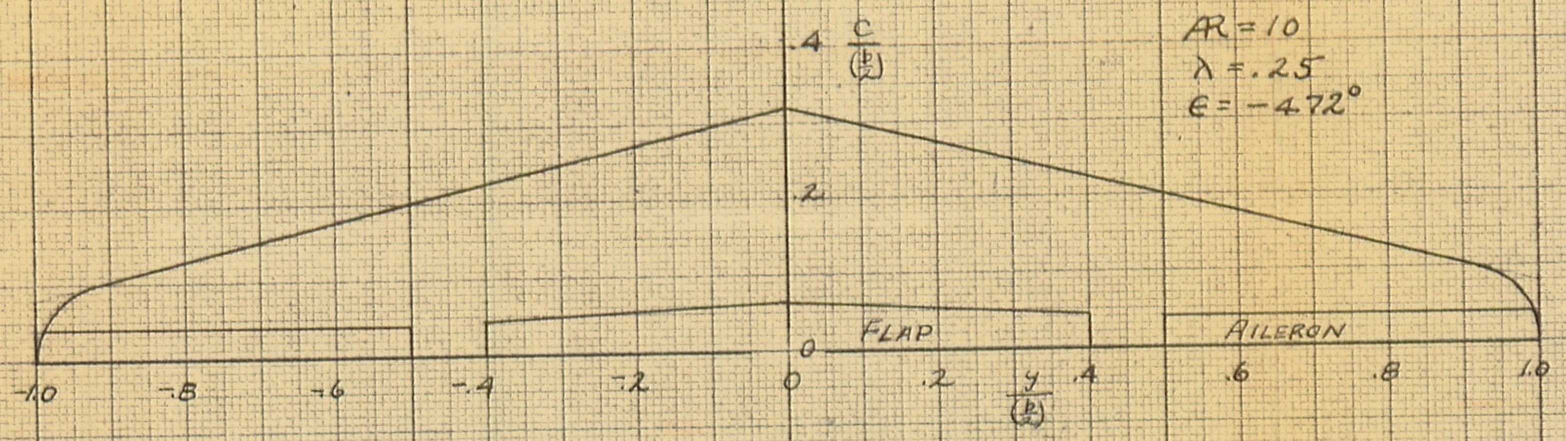
FIG. 2



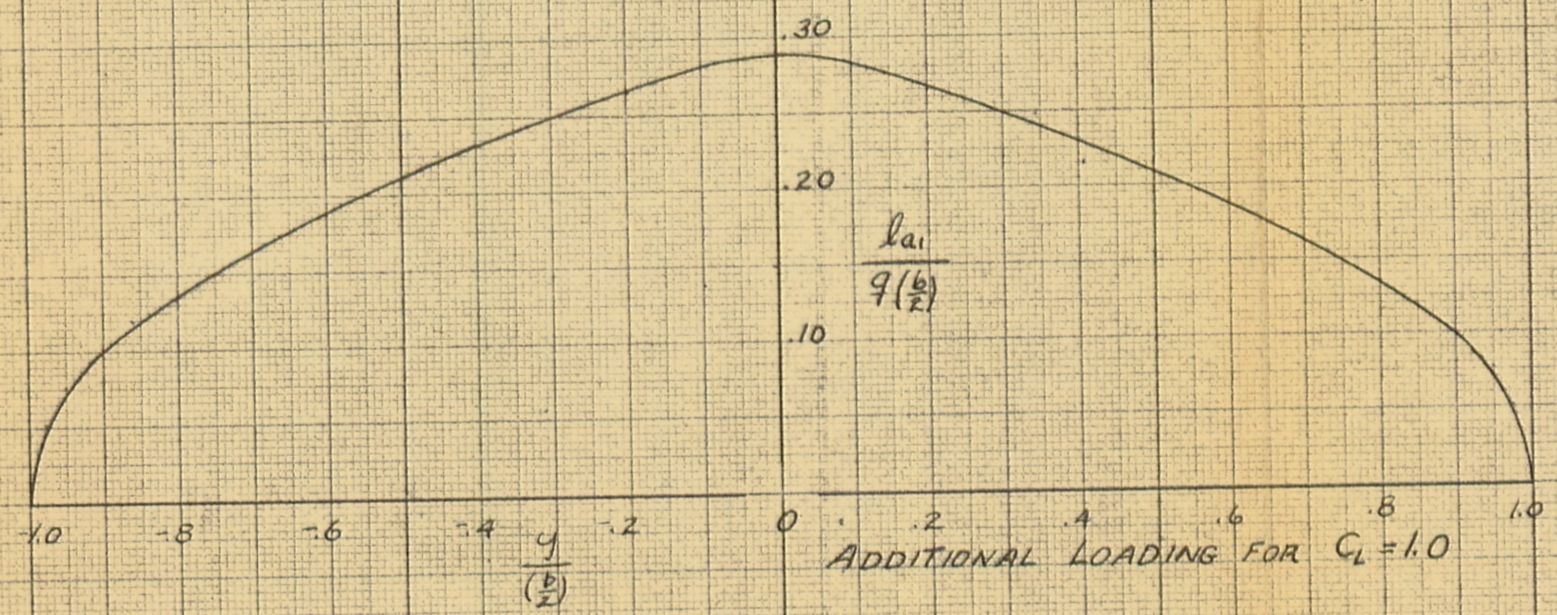
SCALE: 1 cm = 5.70 ft.

LAYOUT OF EXAMPLE AIRPLANE

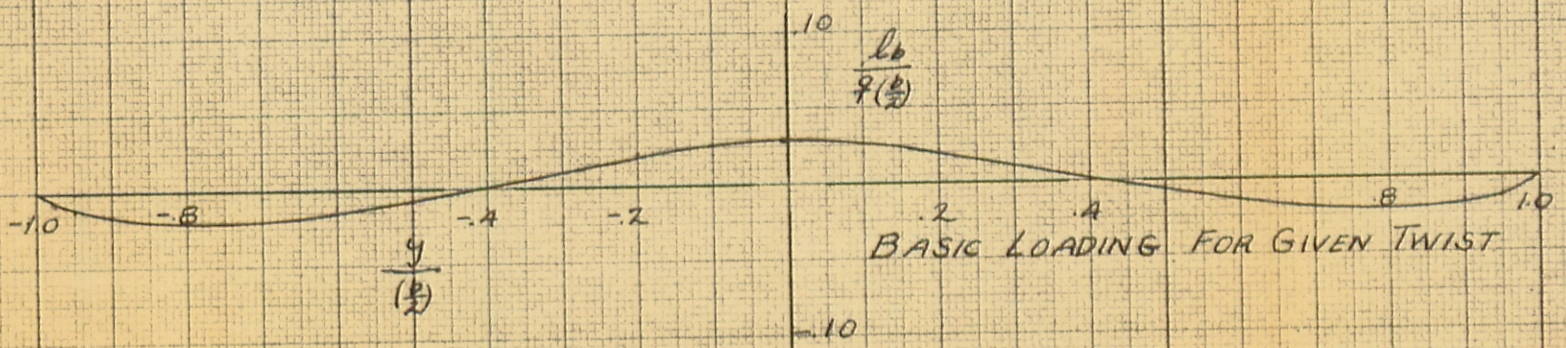
FIG. 3



CHORD DISTRIBUTION
FLAP AND AILERON PROPORTIONS

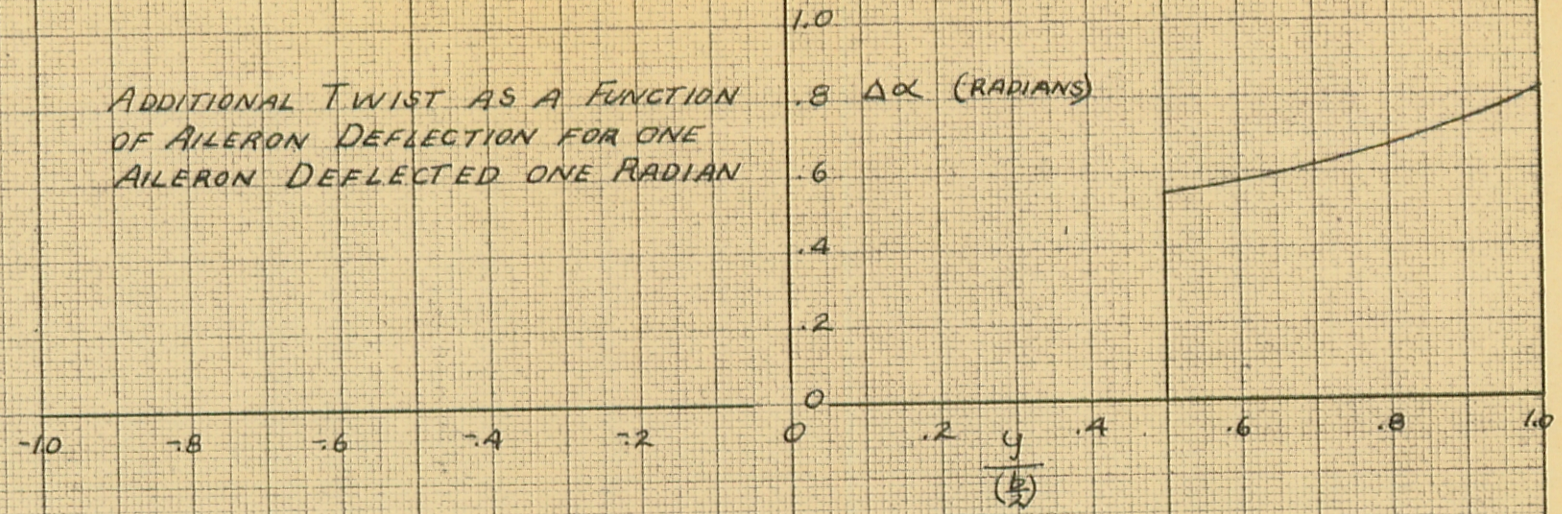


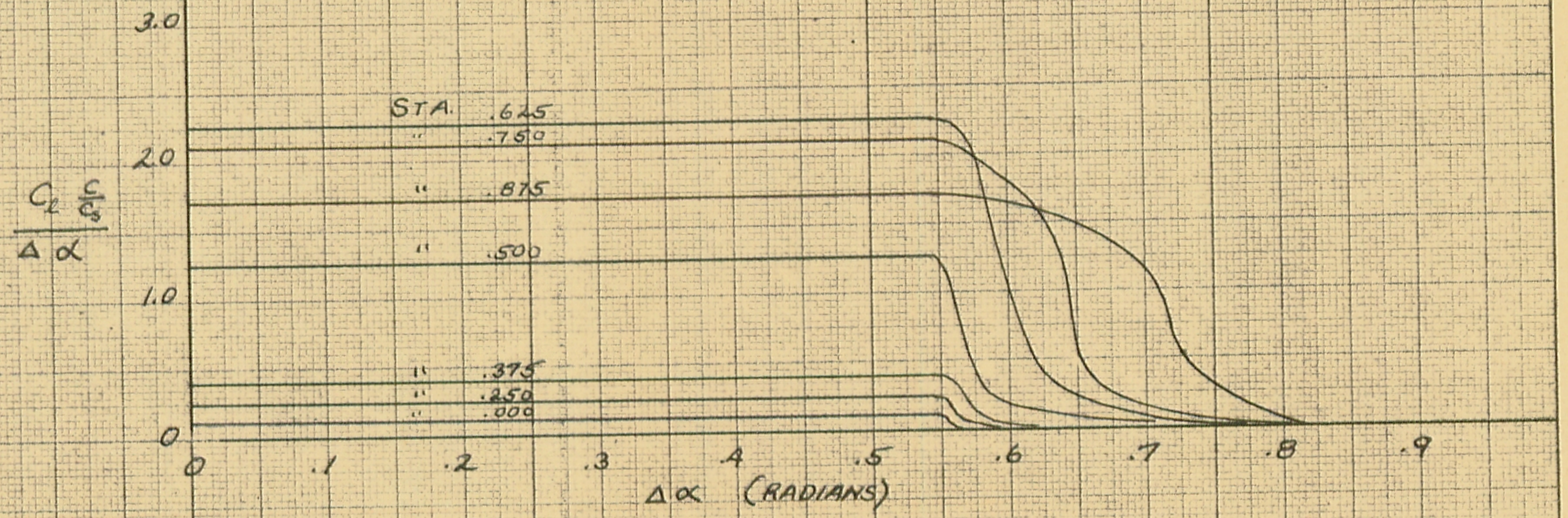
ADDITIONAL LOADING FOR $C_L = 1.0$



BASIC LOADING FOR GIVEN TWIST

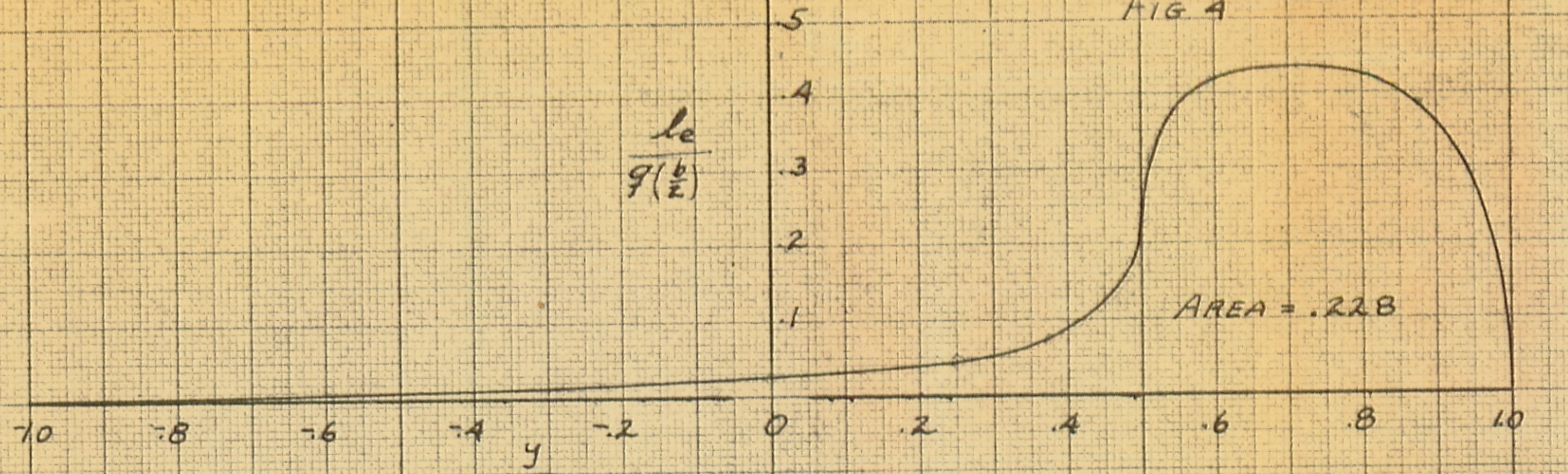
ADDITIONAL TWIST AS A FUNCTION OF AILERON DEFLECTION FOR ONE AILERON DEFLECTED ONE RADIAN



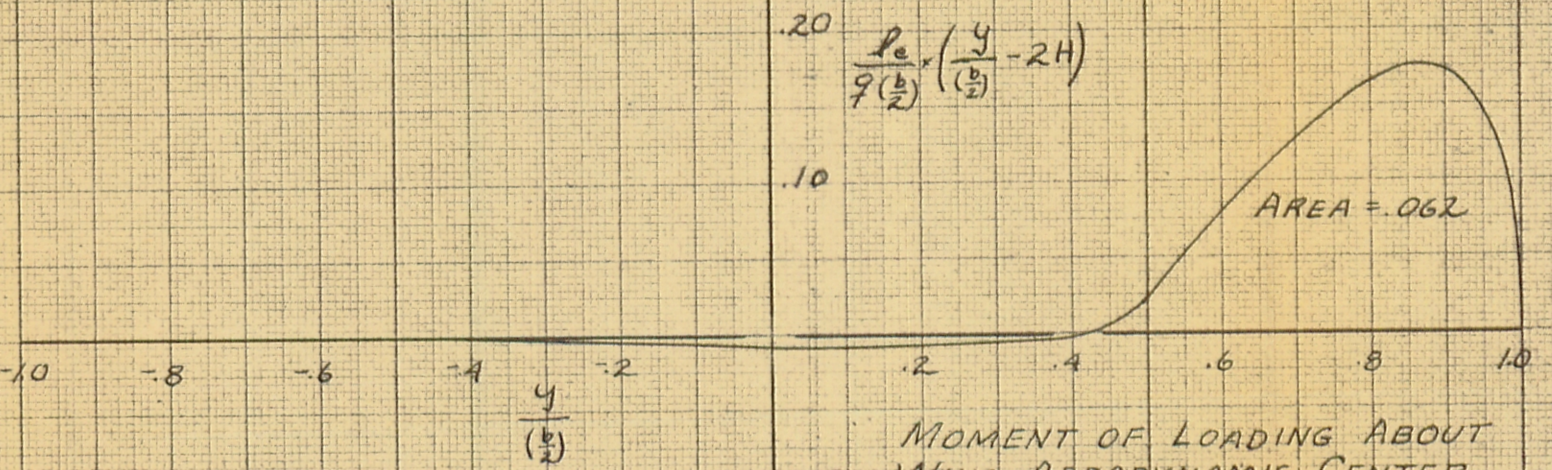


DETERMINATION OF ADDITIONAL LOADING DUE TO AILERON DEFLECTION AT SEVERAL SPANWISE STATIONS

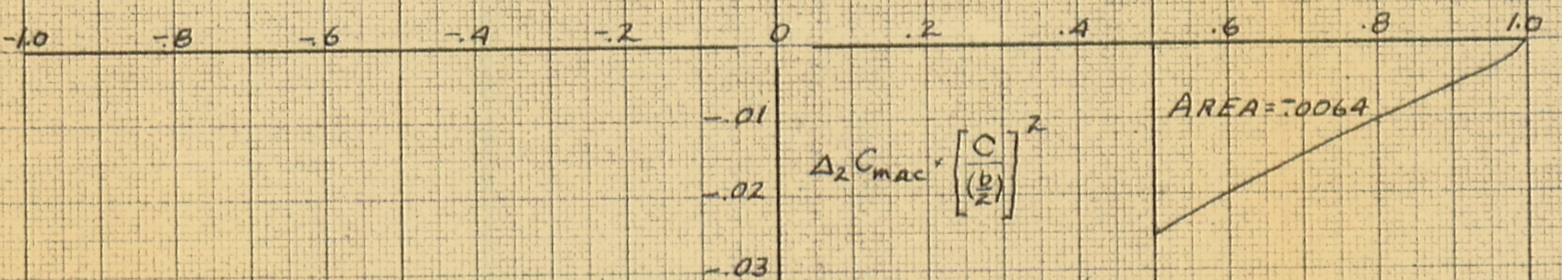
STATION	AREA UNDER CURVE
.000	.0675
.250	.1425
.375	.2250
.500	.7250
.625	1.347
.750	1.350
.875	1.203



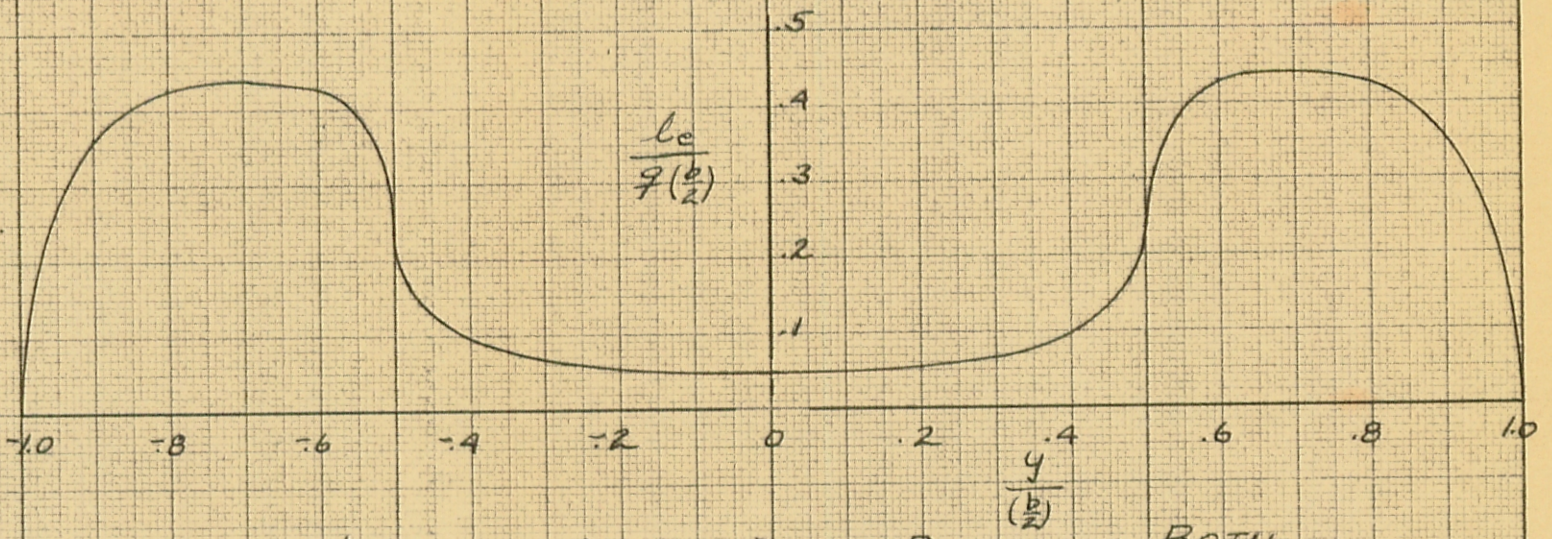
ADDITIONAL LOADING DUE TO DEFLECTING ONE AILERON ONE RADIAN



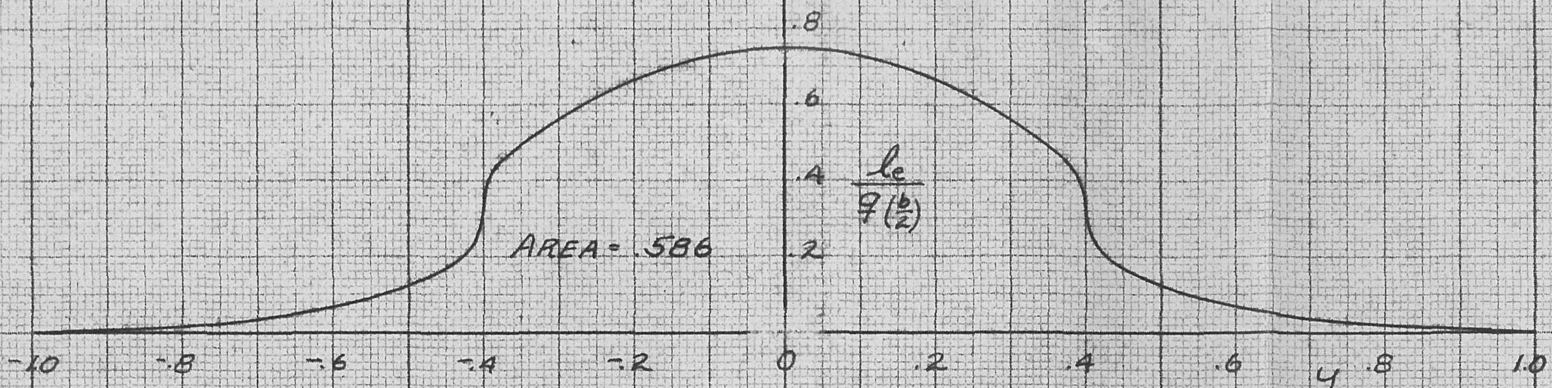
MOMENT OF LOADING ABOUT WING AERODYNAMIC CENTER



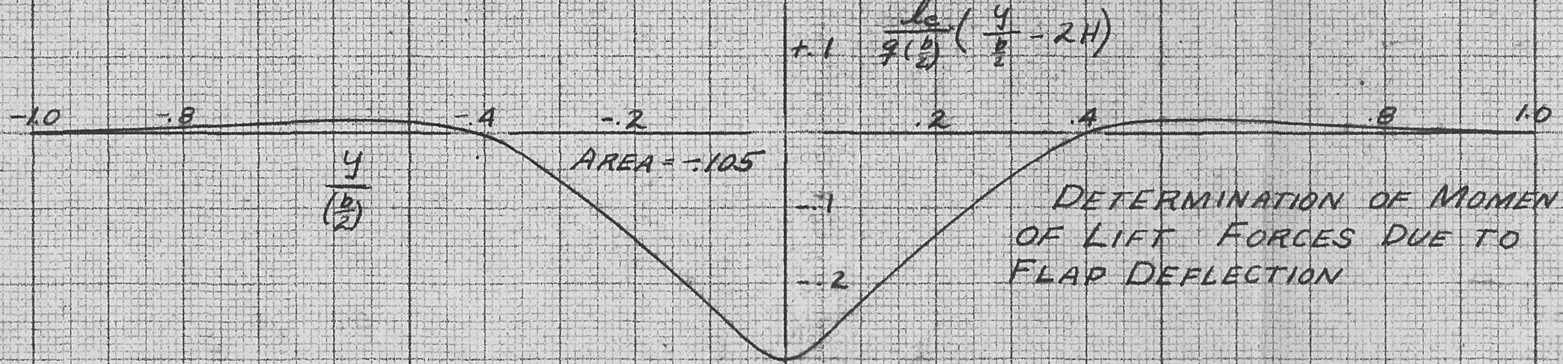
EFFECT OF CHANGE IN SECTION ON WING MOMENT



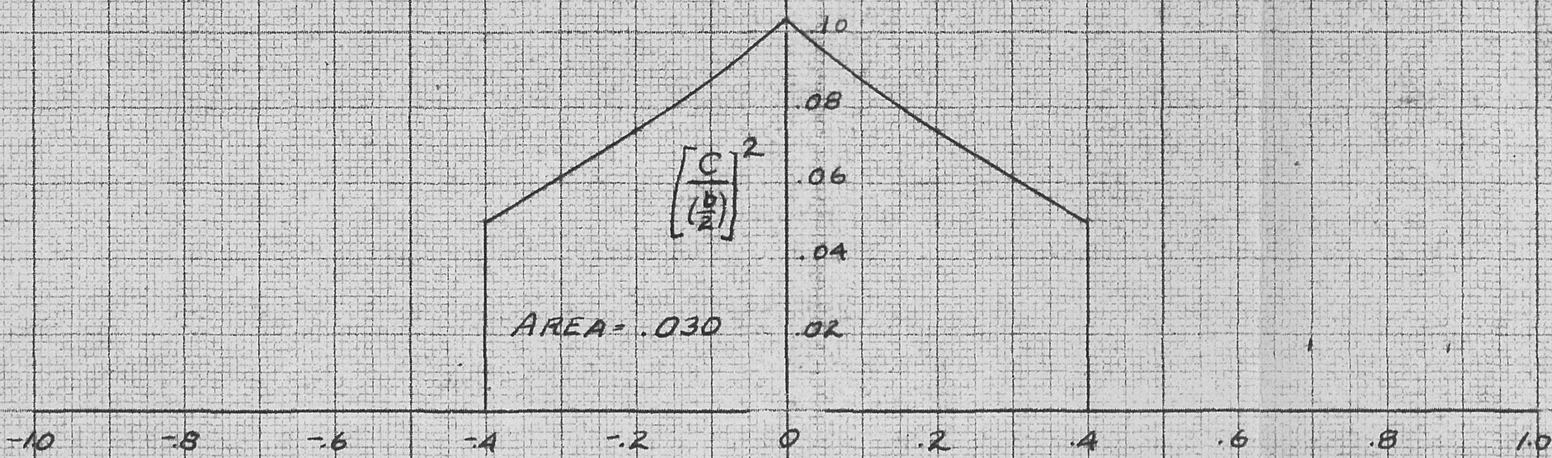
ADDITIONAL LOADING DUE TO DEFLECTING BOTH AILERONS ONE RADIAN



ADDITIONAL LOADING DUE TO DEFLECTING BOTH FLAPS TOGETHER ONE RADIAN

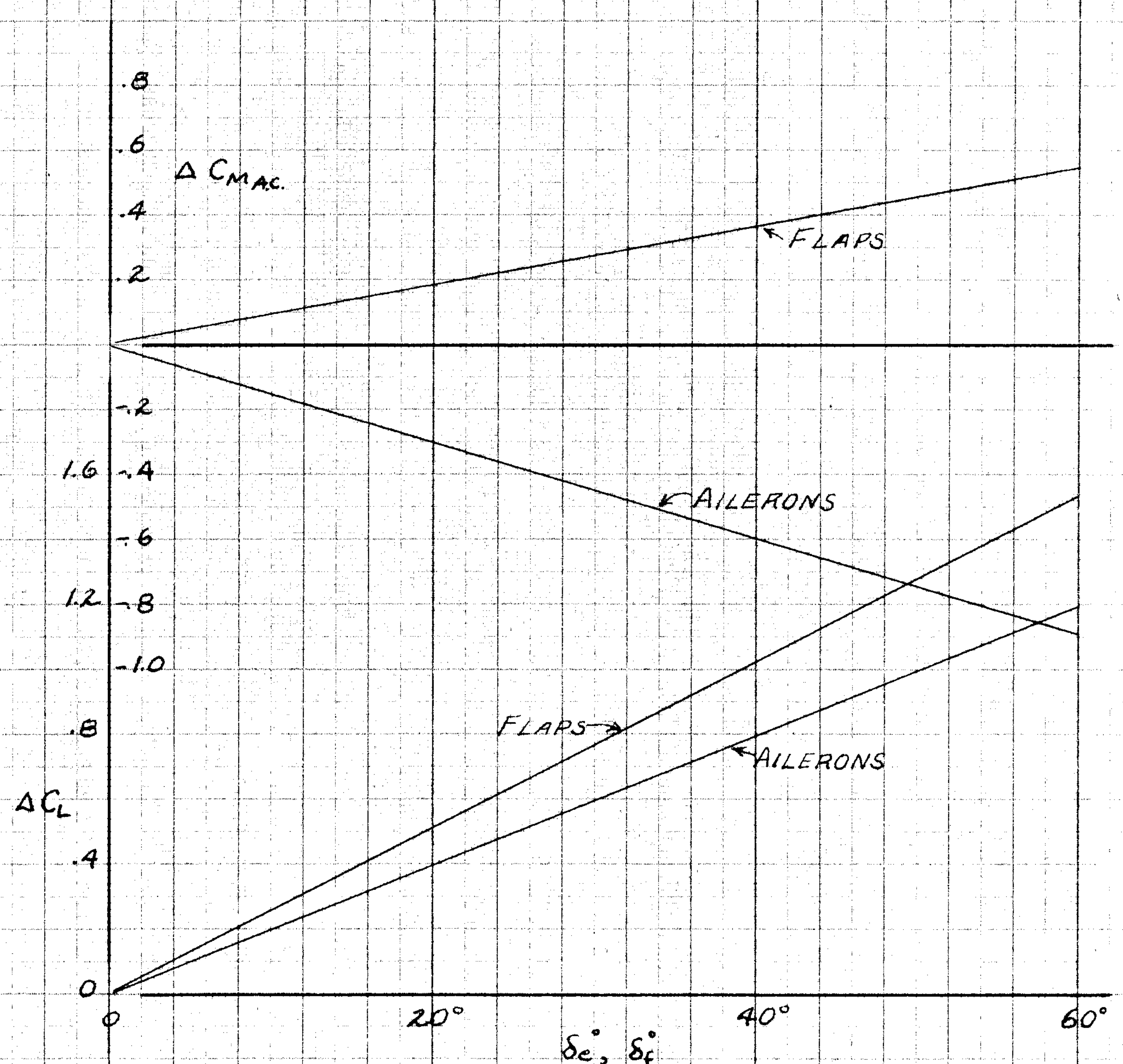


DETERMINATION OF MOMENT OF LIFT FORCES DUE TO FLAP DEFLECTION

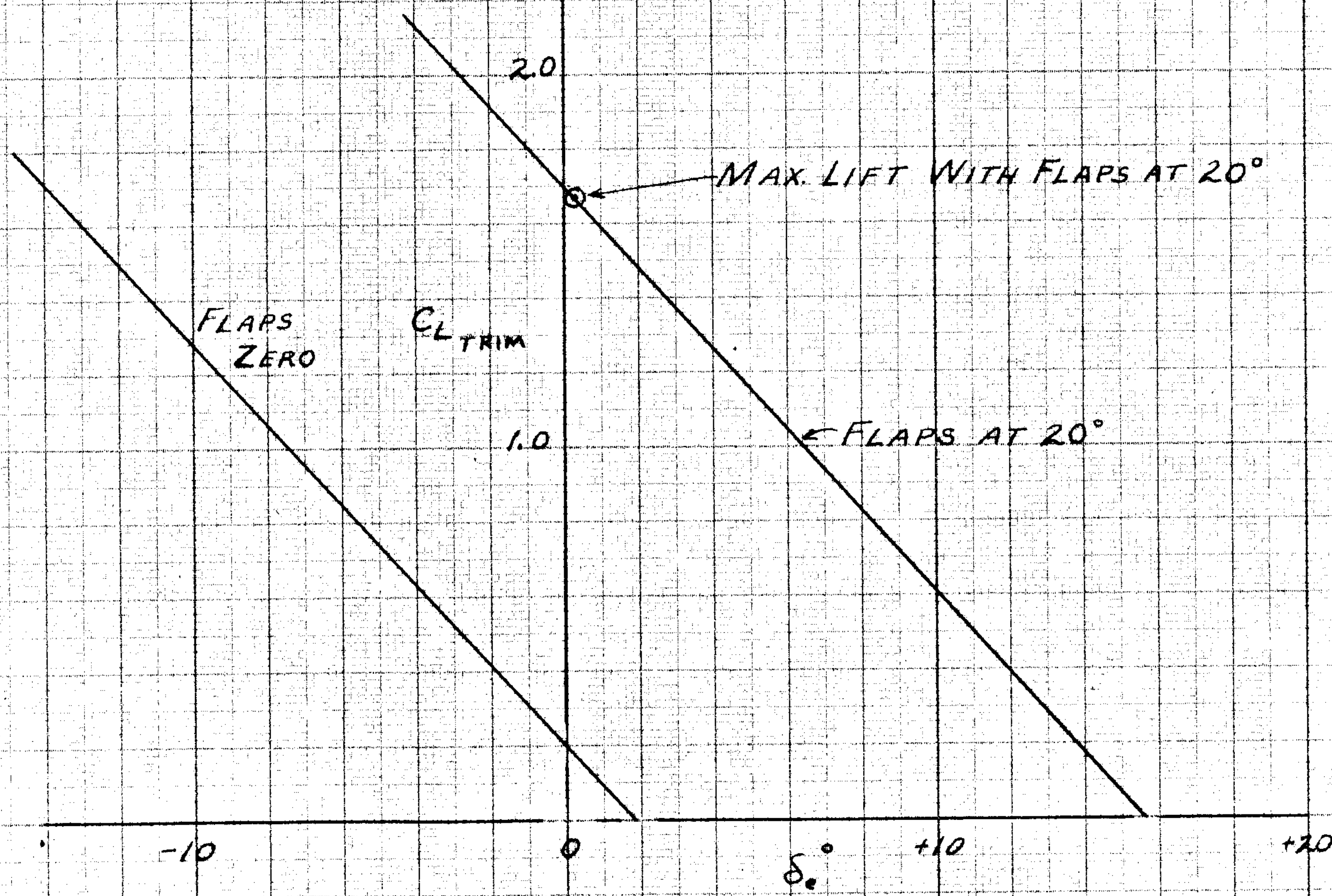


DETERMINATION OF CHANGE IN WING MOMENT DUE TO CHANGE IN SECTION.

SURFACE	ΔC_L	$\Delta C_{M_{AC}}$
FLAPS	$.0256\ \delta_f^\circ$	$.0090\ \delta_f^\circ$
AILERONS	$.0199\ \delta_a^\circ$	$-.0183\ \delta_a^\circ$

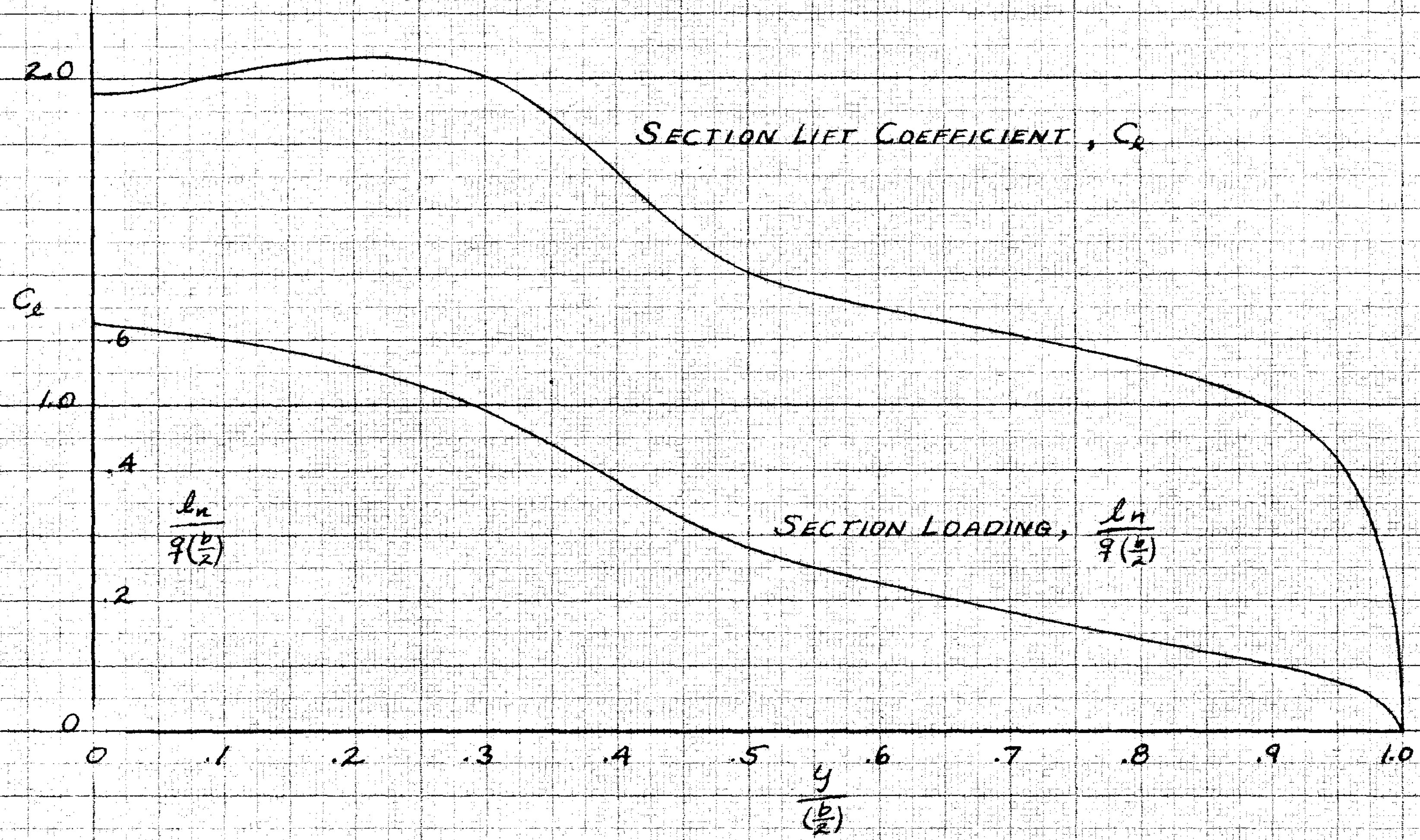


EFFECT OF AILERON AND FLAP DEFLECTIONS ON WING CHARACTERISTICS.



AILERON DEFLECTION FOR TRIM AT VARIOUS LIFT COEFFICIENTS

FIG. 8



SECTION LIFT COEFFICIENT AND LOADING
AT MAXIMUM LIFT COEFFICIENT.
FLAPS AT 20° AILERONS AT ±23°
$C_L = 1.67$