

INVESTIGATION OF DESIGN CRITERIA OF
STIFFENED WOOD CURVED PANELS

Thesis by

Flt. Lt. H. H. Dixon, R.C.A.F.

In partial fulfillment of the
Requirements for the Degree of Aeronautical Engineer

California Institute of Technology
Pasadena, California

1944

Acknowledgments

The author wishes to express his sincere appreciation for the advice given by Dr. E. E. Sechler and Dr. L. G. Dunn of the California Institute of Technology during the research program.

The panel sections used in the test were provided by the Fletcher Aviation Corporation. Appreciation is hereby extended to Mr. Fletcher and Mr. Moseley of that organization for furnishing the panels and for valuable suggestions concerning the test program.

The author takes this opportunity of thanking his friends who have assisted him by suggestions, assistance and reading of manuscript, particularly Mr. Robert Masterson and Structural Laboratory Staff.

Table of Contents

<u>Part</u>	<u>Title</u>	<u>Page</u>
	Summary	
I	Introduction	
II	Symbols	
III	Description of Apparatus and Test Procedure	1
IV	Results	4
V	Discussion	9
VI	Conclusions	12
VII	Appendix	
	A) Summary of Stiffener Design Calculations	13
	B) Effective Width Calculations from Experimental Data.	14
	C) Illustrative Example	15
	D) Critical Buckling Stress of Sheet	18
VIII	References	20
IX	Photograph of Apparatus - Figure A	
X	Illustrative Material	

Summary

A total of 92 stiffened panel specimens of birch plywood of a nominal thickness of one-eighth inch with spruce stiffeners set parallel to the applied load were tested at California Institute of Technology to obtain the stress strain relations and the ultimate compression stress of the panels. The panels had heights of 12, 18, and 24 inches; stiffener spacings of 2, 3, 4, and 5 inches; and radii of curvature of 400, 200, 100, 50, 12, and 6 inches. The experimental data are presented as stress strain and column curves, and in tabular form.

The principal results of the research are as follows:

a) Although the scope of the tests were insufficient to formulate general design data, the results present a guide for design criteria of wood panels. Experimentally it has been shown that the effective width method of analysis is applicable to wooden panels.

b) The percent reinforcement vs. ultimate stress investigation indicates that a minimum of 50 to 60 % reinforcement should be used in stiffened panel construction.

c) A decrease of the radius of curvature ratio gives an increase of ultimate stress approximating a logarithmic variation.
sheet thickness

Introduction

This report presents the results of an investigation on the behavior of wood panel structures subjected to end compression. The information available in the ANC-5 and the ANC Handbook of the design of wood aircraft structures is based on incomplete data and theoretical analysis. That lack of knowledge, causing undue caution to be exercised in the design of curved panels, prompted the investigation to be performed.

Previous work has been carried out by other investigators (reference No. 5) at GALCIT, and by the ANC Committee, published in their Wood Handbook (reference 2). The portion of the field of study covered is the effect of variable panel heights, stiffener spacings, and radii of curvature on the design parameters.

Symbols

A	Area of cross section, square inches.
E_{st}	Modulus of Elasticity of stiffener in compression in direction parallel to the grain is pounds per square inch.
E_{sh}	Modulus of Elasticity of sheet in compression in vertical direction i.e., plus 25 degrees from vertical both sides.
C	End fixity coefficient for columns.
cr	Subscript .. Critical sheet buckling in lbs.
L	Length or height of panel in inches.
e	Strain in inches per inch.
I	Moment of Inertia inches ⁴ .
K	Constant ... generally empirical.
P	Total applied load, lbs.
R	Radius of panel, inches.
T	Subscript .. Total
b	Spacing of stiffeners, inches.
n	Number of stiffeners in panel.
sh	Subscript ... Sheet.
st	Subscript .. Stiffener.
t	Sheet thickness, inches.
W_e	Effective width, inches.
u	Subscript ... Ultimate.
σ	Stress, pounds per square inch.
ρ	Radius of gyration, ins.

Description of Apparatus and Testing Procedure

All panel specimens were tested flat ended in a 300,000 lb. Southwark testing Machine (reference Fig. A). Two face plates were obtained and planed smooth to insure an even load distribution over the panel. The plates were placed between the heads of the testing machine, and the test panel was mounted between them. The Huggenburger extensometers were applied at the stiffener locations, and perpendicular to the sheet. The free edge of the panel was supported only by a stiffener, and the simply supported edges at the plate obtained such restraint as the flat end of the panel offered. These edges of the panel were also restrained to maintain a definite curved radius by templates applied to both edges.

When the load was applied, on account of the non-parallel motion of the movable head of the testing machine with reference to the fixed head, and the fact that the specimens had sanded, not milled edges, the extensometer readings would indicate uneven load distribution. This necessitated shimming of the plates to insure a satisfactory average of readings across the specimen.

After the initial deflection had been applied, and adjustments made, the loading was increased in 10-15 increments until failure occurred. Just previous to failure, the extensometers were removed to avoid damage.

Actual stiffener specimens were compressed at a 6 inch length to obtain the modulus of elasticity, and the Ultimate compressive

stress of the stiffeners alone. The stiffener column curve for longer lengths was obtained by forcing the stiffener to fail along its axis of greatest radius of gyration by a greased groove manufactured from machined 3 inch steel angles. Plywood specimens tested for Young's Modulus were 6 inches long, 6 inches wide and 1 inch thick.

The stiffeners used in the panel construction were routed from spruce. Throughout the research only one stiffener cross section was used (reference Appendix A). The sheet was made up of 2 plies of birch plywood each 1/16 inch thick, bonded together by synthetic cold urea formaldehyde. The grain of the plywood ran at an angle of 25 degrees to the vertical, and the grain direction of the 2 ply was so crossed so that the resultant angle between the grain direction of the 2 plies at any point was 50 degrees.

The panel lengths were so chosen as to cover the range of rib and bulk head spacings that might be encountered in current design practice. The spacing of the stiffeners was varied in order to investigate the effect of that parameter on ultimate strength. The radii under test varied from that required for wing sections down to those required for smaller structural sections. All measurements and areas used throughout the test were nominal, with the variation from specimen to specimen being neglected.

The probable accuracy of the measurements should be fair, as 10 extensometer readings were taken for each panel, and the load scale on the testing machine was adequately large. The inaccuracies of the research may be attributed to the irregular movement of the testing machine head, non-parallelism of the panel edges, and the inherent

variations of the wood and fabrication. The inherent variations in the wood are felt to have offered the widest range of variation of any of the parameters.

Results

Experimental results:

Number of Curve	R	b	L	A _T	P ultimate		
					Experimental	Calculated	
					Tests	Average	
1	400	2	12	6.74	47,000	45,000	43,600
2	"	"	"	"	42,200	45,000	43,600
3	"	"	"	"	45,000	45,000	43,600
4	"	"	18	7.38	45,700	42,300	40,800
5	"	"	24	6.74	31,200	31,000	28,700
6	"	"	"	"	31,300	31,000	28,700
7	"	"	"	"	30,050	31,000	28,700
8	200	"	12	7.38	53,700	49,300	47,800
9	"	"	18	"	39,600	42,300	40,800
10	"	"	24	"	39,000	34,000	31,500
11	100	"	12	"	51,500	49,300	47,800
12	"	"	18	"	42,500	42,300	40,800
13	"	"	24	"	42,500	34,000	31,300
14	400	3	12	5.73	29,200	33,600	32,600
15	"	"	"	"	32,000	33,600	32,600
16	"	"	"	"	32,200	33,600	32,600
17	"	"	18	"	24,000	29,300	29,300
18	"	"	24	"	22,100	24,700	23,000
19	"	"	"	"	23,000	24,700	23,000

20	400	4	12	4.84	24,800	25,600	25,700
21	"	"	"	"	25,200	25,600	25,700
22	"	"	"	"	25,500	25,600	25,700
23	"	"	18	"	22,600	22,300	22,500
24	"	"	24	"	20,200	19,000	17,900
25	"	"	"	"	20,400	19,000	17,900
26	"	"	"	"	18,400	19,000	17,900
27	200	3	12	5.73	28,600	33,600	32,600
28	"	"	18	"	28,500	29,300	29,300
29	"	"	24	"	21,500	24,700	23,000
30	"	4	12	4.84	24,800	25,600	25,700
31	"	"	18	"	22,500	22,300	22,500
32	"	"	24	"	21,000	19,000	17,900
33	100	3	12	5.73	31,000	33,600	32,600
34	"	"	18	"	30,000	29,300	29,300
35	"	"	24	"	26,000	24,700	23,000
36	"	4	12	4.84	26,500	25,600	25,700
37	"	"	18	4.45	20,500	22,300	22,500
38	"	"	24	4.84	20,600	19,000	17,900
39	50	3	12	5.73	31,200		
40	"	"	"	"	33,500		
41	"	"	18	"	31,000		
42	"	"	"	"	33,000		
43	"	"	24	"	28,000		
44	"	"	"	"	28,300		
45	"	4	12	4.84	24,000		

46	50	4	12	4.84	23,400
47	"	"	18	"	23,000
48	"	"	"	"	22,200
49	"	"	24	"	20,700
50	"	"	"	"	19,000
51	"	5	12	4.46	21,500
52	"	"	"	"	21,200
53	"	"	18	"	19,800
54	"	"	"	"	16,600
55	"	"	24	"	16,700
56	"	"	"	"	15,000
57	12	5	24	5.73	26,500
58	"	"	24	"	28,000
59	"	"	18	"	30,000
60	"	"	"	"	32,100
61	"	"	12	"	33,000
62	"	"	"	"	31,400
63	"	4	"	4.84	26,000
64	"	"	"	"	28,500
65	"	"	18	"	24,400
66	"	"	"	"	24,300
67	"	"	24	"	21,300
68	"	"	"	"	20,300
69	"	5	12	4.46	20,700
70	"	"	"	"	20,800
71	"	"	18	"	18,800
72	"	"	"	"	17,500

73	12	5	24	4.46	16,900
74	"	"	"	"	16,000
75	6	3	12	4.22	25,000
76	"	"	"	"	26,900
77	"	4	"	3.97	21,600
78	"	"	"	"	23,600
79	"	5	"	3.46	18,500
80	"	"	"	"	18,300
81	"	3	18	4.22	24,000
82	"	"	"	"	24,100
83	"	4	"	3.97	22,000
84	"	"	"	"	20,900
85	"	5	"	3.46	15,600
86	"	"	"	"	15,400
87	"	3	24	4.22	22,900
88	"	"	"	"	21,500
89	"	4	"	3.97	20,800
90	"	"	"	"	18,200
91	"	5	"	3.46	16,100
92	"	"	"	"	14,700

Note: Load Prediction Calculations were carried out only for 100 to 400 inch radii of curvature. The 100-400 inch radii group were considered together since for all practical purposes they have the same ultimate stress. To approximate the ultimate stress values for smaller radii, The radius effects shown in figures (7) to (10) must be considered.

Summary of Graphical Results:

- Figure (1-A) Spruce stiffener stress-strain curves for the computation of Youngs Modulus. This group of results checks closely with panel stress strain relations.
- (1-B) Same as (1-A), but this group evidently contains a different type of spruce, and the results are recorded to show probable scatter of Youngs Modulus values, and are not used in report.
- (2) Spruce Stiffener Column curve with the stiffener forced to fail along its axis of greatest radius of gyration (the manner in which it fails in a panel) The coefficient of end fixity obtained is assumed to be constant throughout the panel tests at $C = 2.0$
- (3) Birch plywood Stress-Strain curve for Youngs Modulus.
- (4) Ultimate stress vs. percent reinforcement values were plotted for the values of radii of 100-400 in.
- (5) Illustrative example curve to indicate method of computation off effective width.
- (6) Curve for determining the effective width was calculated from curves similar to figure No. 5 for each stiffener spacing.
- (7) to
- (10) Ultimate stress of curved sheet panels with various stiffener spacing.
- (11) to
- (15) Stress strain curves for stiffened panels.

Discussion

This investigation is based on the classical column stability formula of Johnson and Euler types being applicable to the stiffener column curve as the basis of Effective Width analysis. It is shown in the results that the effective width method will give close approximation to the experimental values. Emphasis must be placed on the fact that these structures can easily have a 15% variation in results due to the anisotropy of wood. This is particularly true with regards to stiffener Youngs Modulus values, and care must be taken to insure that the correct value is chosen in panel design. Consequently, throughout the investigation, several tests have been performed for each set of parameters, with the final results being a statistical mean.

The stiffener section selected was torsionally stable, and careful inspection showed no tendencies to fail in torsion. When the sheet buckled, the stiffener exerted a restraint on the sheet, preventing its collapse. The equal restraint of the sheet on the stiffener did not appear to influence the stiffener stability, for the wave form of the buckled sheet was not consistent, and distinctly not symmetrical.

In most cases the stability of the sheet between stiffeners reaches its limit before any bending takes place. When the critical buckling stress of the sheet is higher than the stiffener stress, it has been proved that the total sheet area is effective. This

experimental relation is plotted in Figure (8), and it has higher values than those shown by the NACA in (reference (2), Figure 2.21). The curves are of the same type, and the discrepancies may be explained as follows:

- 1) The NACA curves are the lower of the average test values.
- 2) The experimental curve was based on the average values of failure, and the origin is at $\sigma_{ST}/\sigma_{cr} = 1$ not 0, as with the NACA curve. This curve is of the same general type as reference (1), Figure 6.2.

No consistent variation of the effective width could be detected with a change in panel length.

The Ultimate Stress vs. Percent Reinforcement indicates that design values should, for short bulkhead or rib spacings, be 60%; and for longer spacings to be a minimum of 50%. Figure (4) indicates almost linear variations below these values, with consequent weight penalties. The lower part of the curve is taken from Reference (5), suitably adjusted for the differences in material. The highest percent-reinforcement point on each curve was calculated by the effective width method, and it agrees well with the expected value.

The stress values from the above curve were used as origins at the 400" radius, of the ultimate stress v. ρ radius of curvature graphs. Referring to figures 4 and 8 it is evident that for the 3" stiffener spacing, and 12" panel length, the ultimate stress values are below the average curve. This is probably caused by some of the lower strength stiffeners (reference Figure 1-B) in

the panel.

The results of curves of ultimate stress of curved sheet panels with various stiffener spacing indicate that the ultimate stress varies as the panel curvature. The slopes of all curves of ultimate stress vs. curvature of panel are the same, and for the radii of 400 to 12 inches follow an approximate logarithmic variation.

Additional investigation should be conducted as follows:

- 1) To determine the critical buckling stress for the number of plies in the sheet for straight and curved sections.
- 2) To verify the ultimate stress values for the smaller radii regime where the R/t ratio is less than 400.
- 3) To determine the suitability of presenting wood design data as in Chapter 8, Reference No. 1.
- 4) To determine the effect of sheet thickness on the strength of stiffened panels.

Conclusions

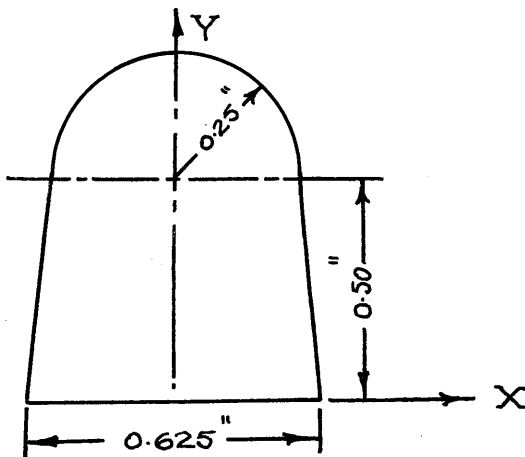
The purpose of the investigation was to obtain a better understanding of the behavior of stiffened wood panels as used in aircraft construction. The scope of the tests has been limited, but the results are offered as a guide in design structural analysis.

The investigations have been purely experimental, with data presented compatible with current design methods. No attempt has been made to allude to any theoretical development, as it has been presented in Reference (3).

Appendix "A"

Summary of Stiffener Design Calculations

Cross Section of Stiffener .



1) Area = 0.38 sq. ins.

2) Position of the center of gravity -

$$\bar{Y} = 0.3353 \text{ ins.}$$

$$\bar{X} = 0 \text{ ins.}$$

3) Moment of Inertia about the C.G. , $I = 0.0160 \text{ ins.}^4$

4) Radius of gyration $\rho = 0.205 \text{ ins.}$

Appendix "B"

Effective Width Calculations from Experimental Data

(For a more detailed discussion see Reference (1) Chap. 6)

The effective width of any stiffened panel may be calculated for any given load if the stiffener stress is known. The stiffener stress up to the proportional limit can be directly obtained from extensometer readings by means of the equation -

$$\sigma_{ST} = K T E \quad \text{where } K = \text{Extensometer constant}$$
$$T = \quad \quad \quad \text{reading}$$

To obtain values above the proportional limit, flat end compression test data on actual specimens were obtained. The stiffener load becomes,

$$P_{st} = \sigma_{ST} A_{st} n$$

The load carried by the sheet is,

$$P_{sh} = P - P_{st}$$
$$= 2 W_e \sigma_{ST} t (n - 1)$$

And the effective width acting with each stiffener is

$$W_e = P_{sh} / (2 \sigma_{ST} t (n - 1))$$
$$= (P - n A_{st} \sigma_{ST}) / 2 \sigma_{ST} t (n - 1)$$

Appendix "C"

Illustrative Example

(Assume a panel of 400" R, 12" L, 2" b, and 11 n)

No appreciable change in the value of ultimate stress can be noted for the radii of 400 to 100 inches inclusive. To obtain an average stress-strain relation for the 13 specimens, from 12 to 24 inches long, in that radii range, the results were plotted on one curve (Reference Figure (5)). By statistical average it was found that $E_{st} = 2.42 \times 10^6$ and $E_{sh} = 1.5 \times 10^6$. Thus the effective modulus of elasticity E' of the composite section, up to the buckling stress of the sheet, is,

$$E' = (E_{st} A_{st} n + E_{sh} A_{sh}) / (A_{st} n + A_{sh})$$

The values of the panel cross-sectional areas except for indicated exceptions were as follows:

b	n	n A _{st}	A _{sh}	A _t	n A _{st} /A _t %
2	11	4.18	2.56	6.74	62.0
3	8	3.04	2.69	5.73	53.0
4	6	2.28	2.56	4.84	47.2
5	5	1.90	2.56	4.46	42.6

Thus $E' = 10^6 (2.42 \times 4.18 + 1.5 \times 2.56) / 6.74 = 10^6 \times 2.075$

The specimen curves were then corrected by one or both of the following processes;

- 1) Moved laterally to parallel the statistical average curve, , or , and ,
- 2) If the specimen curve below buckling was not parallel to the average curve, slope offsets were made by means of the formula,

$$e_{avg} = \frac{E_{specimen} e_{specimen}}{E_{average}}$$

By means of these corrected curves, a mean curve was projected for each stiffener spacing, i.e. Fig. 5. From Fig. (4),

$$\sigma_u = 6680 \text{ lbs. per sq. in.}$$

$$\text{Thus } P_u = 6680 \times 6.74 = 45,000 \text{ lbs.}$$

From Appendix "D", for 2" spacing, $\sigma_{cr} = 7250 \text{ psi.}$

From Fig. (5) $e_u = 32.5 \times 10^{-4} \text{ ins.}$

$$\text{whence } \sigma_{sr} = 2.42 \times 10^6 \times 32.5 \times 10^{-4}$$

$$\sigma_{sr} = 7860 \text{ psi}$$

$$\text{Below buckling } \frac{W_e}{b} = 1/2 E_{sh}/E_{st} = 0.310$$

From Appendix "B"

$$W_e = \frac{45,000 - 11(.38)(7860)}{2(7860)(1/8)(11-1)} = 0.616 \text{ ins.}$$

$$\frac{W_e}{b} = 0.308$$

This was plotted as point No. 1 on Fig. No. 6.

To compute "P calculated". (Assume 400 "R, 4" b, 12" L)

From Fig. No. 2 $\sigma_{sr} = 7200 \text{ psi}$

$$\frac{\sigma_{sr}}{\sigma_{cr}} = \frac{7200}{1810} = 4.0$$

From Fig. No. 6 $\frac{W_e}{b} = 0.225$

If calculations are continued, Ref. (1) Fig. 6.10, the final values will be $\frac{W_e}{b} = 0.221$, and $\sigma = 7600$ psi

$$\begin{aligned} \text{Whence } P &= \sigma (A_{st} n + 2 W_e t (n-1)) \\ &= 7600 (2.28 + \frac{2}{8} (.221)(4)(5)) \\ &= 25,700 \text{ lbs.} \end{aligned}$$

Appendix "D"

Critical Buckling Stress of Sheet

The deviations of the panel stress-strain curve from the linear begin at the critical buckling stress of the sheet. It was desired to obtain experimental verification of the calculations of the buckling stress; so a limited number of tests were completed. The test was a sheet simply supported on all 4 sides, with a record taken of the head deflection while the sheet was axially loaded. The results were unsatisfactory for the following reasons:

1) The slotted tubes at the edges were not stiff enough and at high stresses the tendency to buckle separated the slots.

2) The Dial Test Indicator readings of the head movement was not a sufficiently accurate method. Tests should have been conducted as in Ref. No. 4. This inaccuracy prevented satisfactory determination when buckling commenced, especially at lower stress values.

3) Initial warping of the plywood caused serious eccentricities.

The results were as follows:

b	σ_{cr}	
	Test	Calculated
2	4000	7250
3	3600	3220
4	3000	1810
5	2500	1160

The test values definitely did not follow the equation,

$$\sigma_{cr} = K E \left(\frac{t}{b} \right)^2$$

and hence were rejected because of poor experimental technique.

Reference Fig. (5), by inspection σ_{cr} is at $e = 0.003''$, whence $\sigma_{cr} = 7250$ psi. Substituting this value in the above formula, the values obtained for other spacings agreed with their mean curves fairly well, although the calculated σ_{cr} was always lower. This is to be expected, since at the lower stress values the change in slope of the curve is small.

References

- (1) Sechler and Dunn - Airplane Structural Analysis and Design. John Wiley and Sons, Inc., 1942.
- (2) ANC Handbook of the Design of Wood Aircraft Structures. July, 1942.
- (3) Timoshenko - Theory of Elastic Stability, McGraw Hill Book Co., Inc, 1936.
- (4) Dunn - An Investigation of Sheet Stiffener Panels subjected to Compression Loads with Particular Reference to Torsionally Weak Stiffeners. N.A.C.A., T.N. 752.
- (5) Sweeney and Bridgland - Investigation of the Design Criteria of Stiffened Wooden Panels. C.I.T. Thesis 1943.

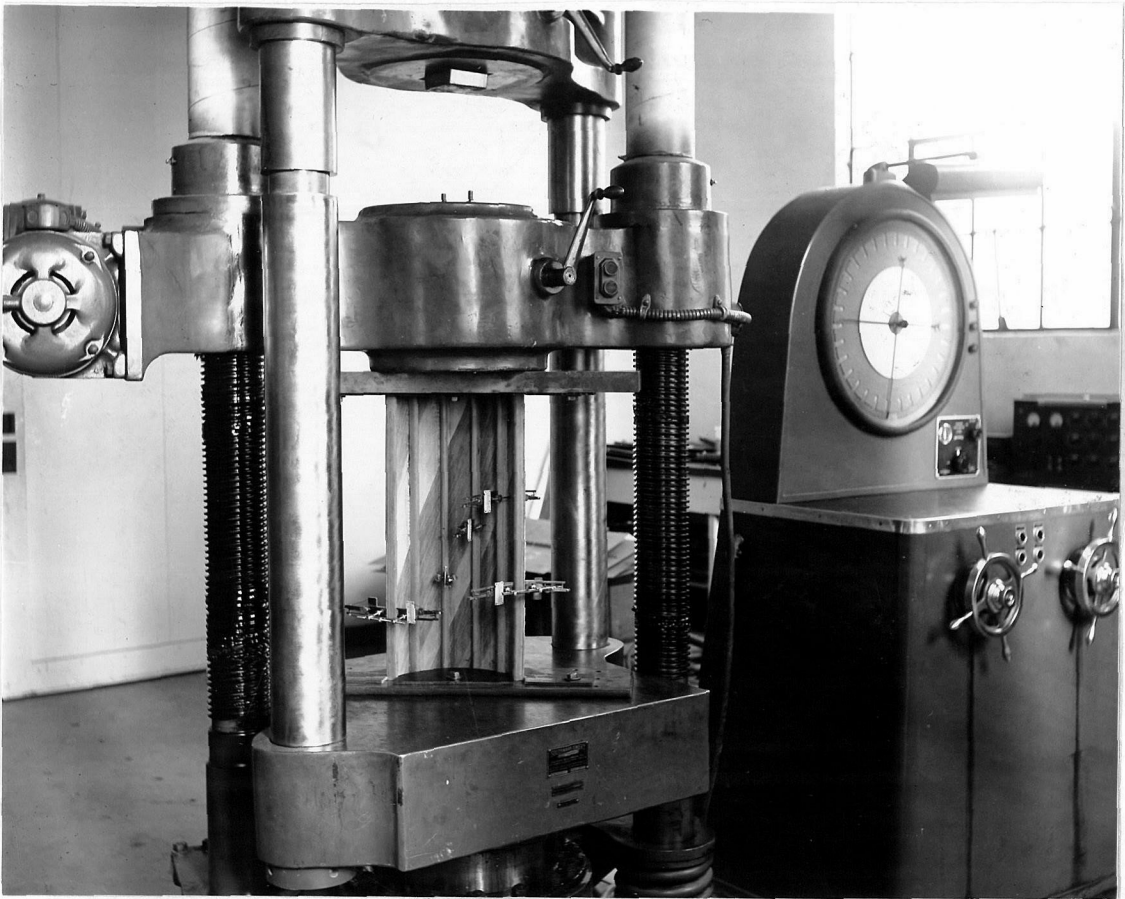


Figure A

STRENGTH PROPERTIES OF SPRUCE STIFFENERS (Compression)

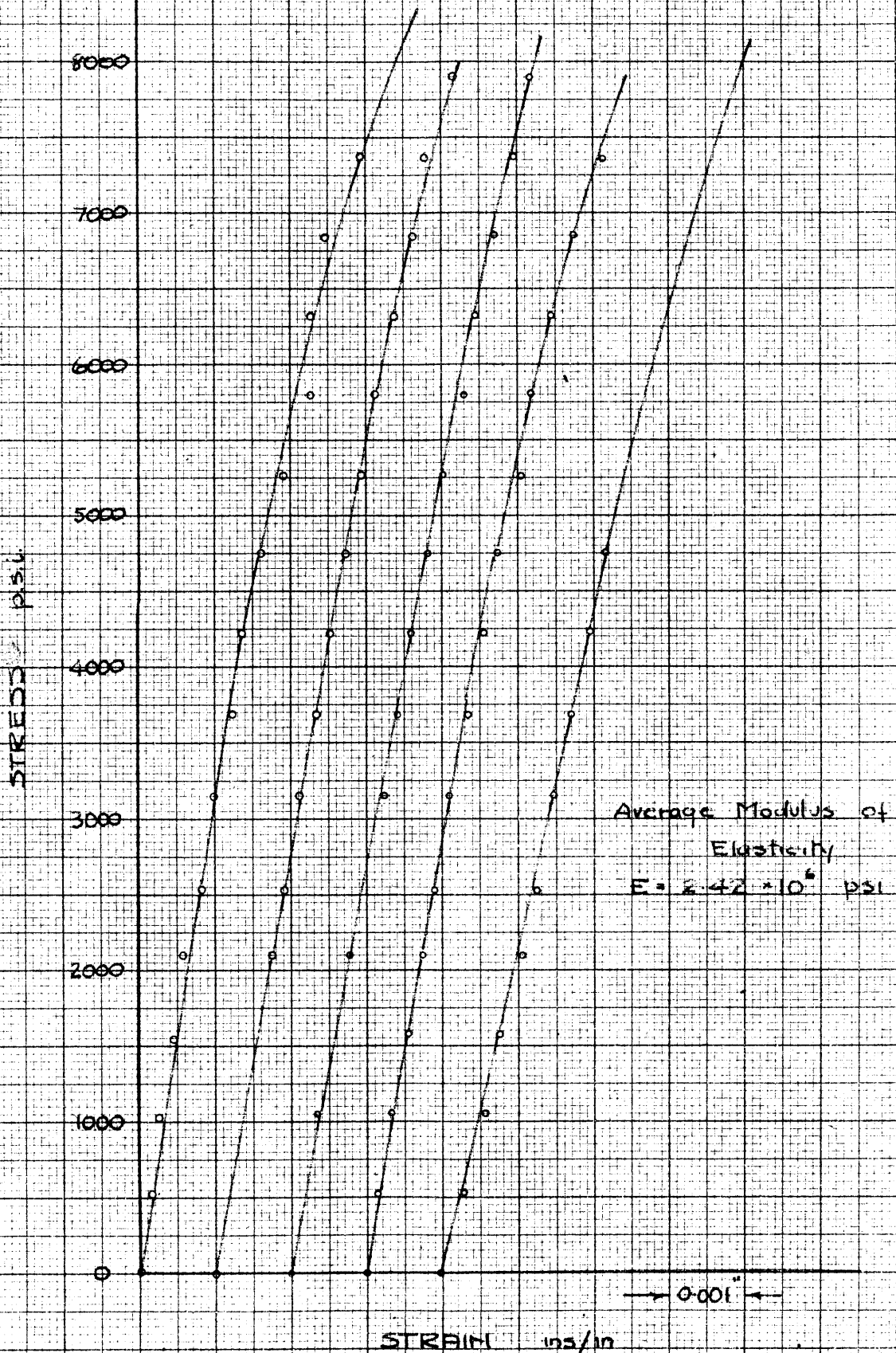


Fig 1-A

STRENGTH PROPERTIES OF SPRUCE STIFFENERS

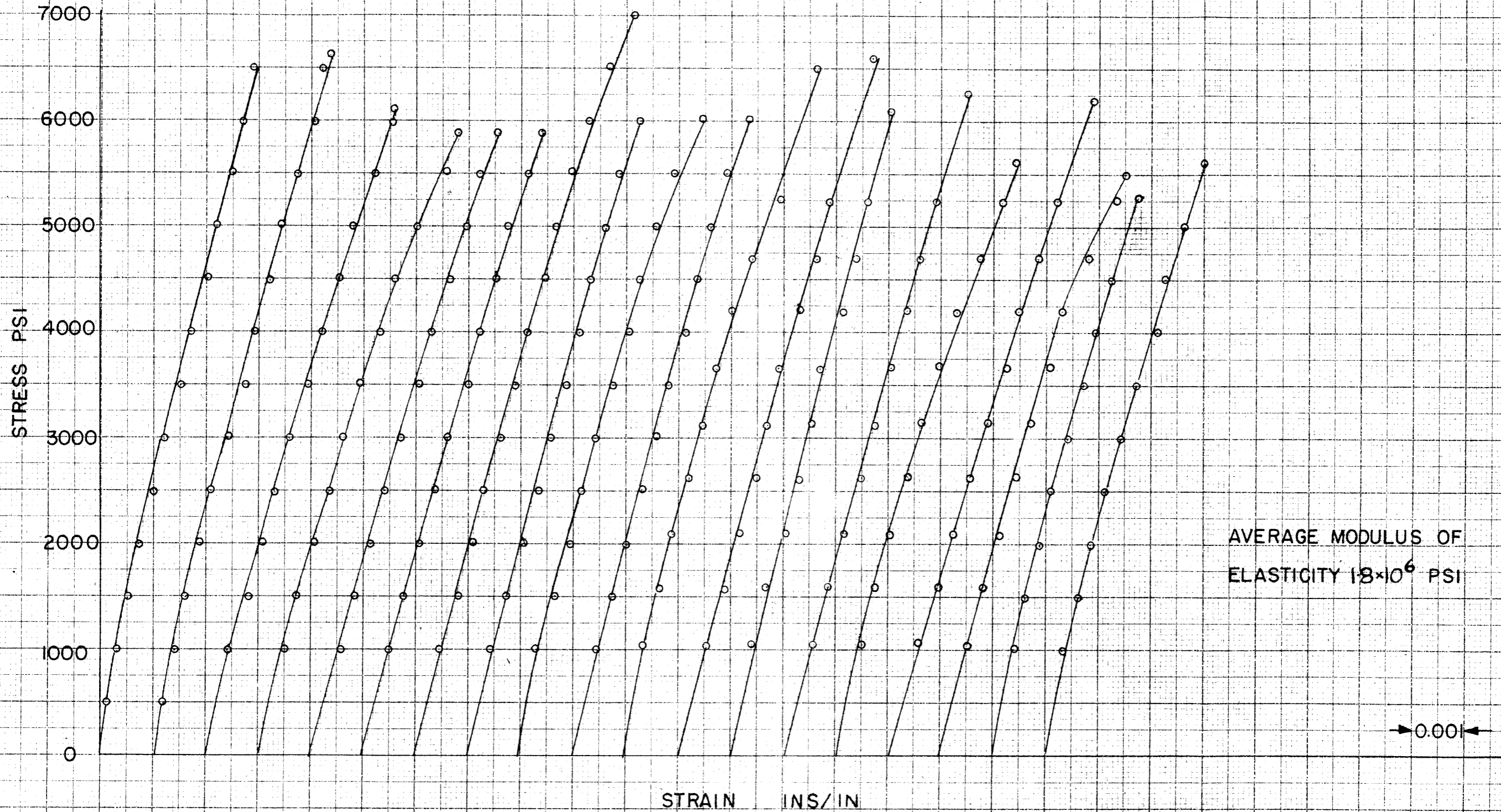
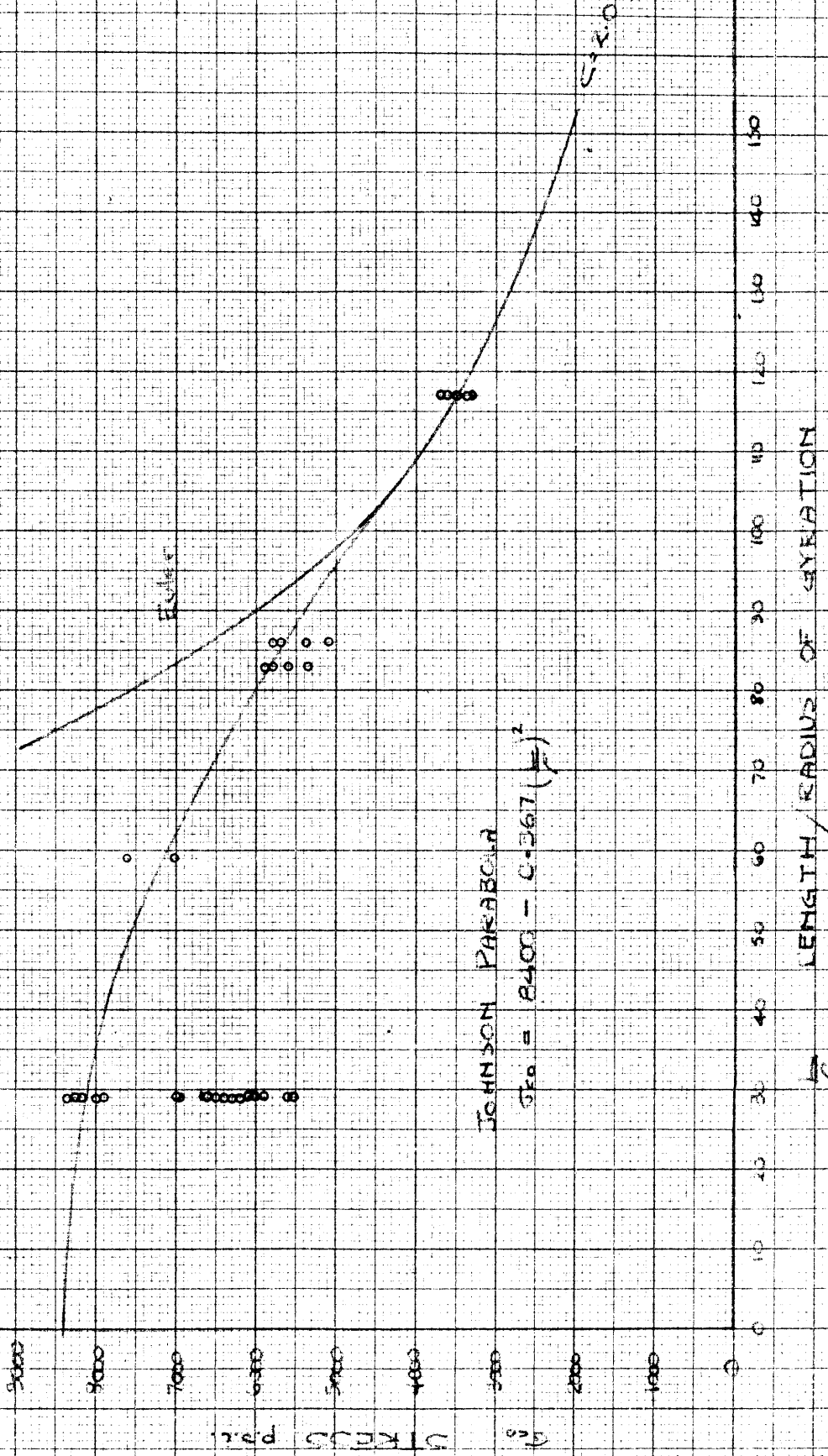


FIG. 1-B

STIFFER COLUMN CURVE



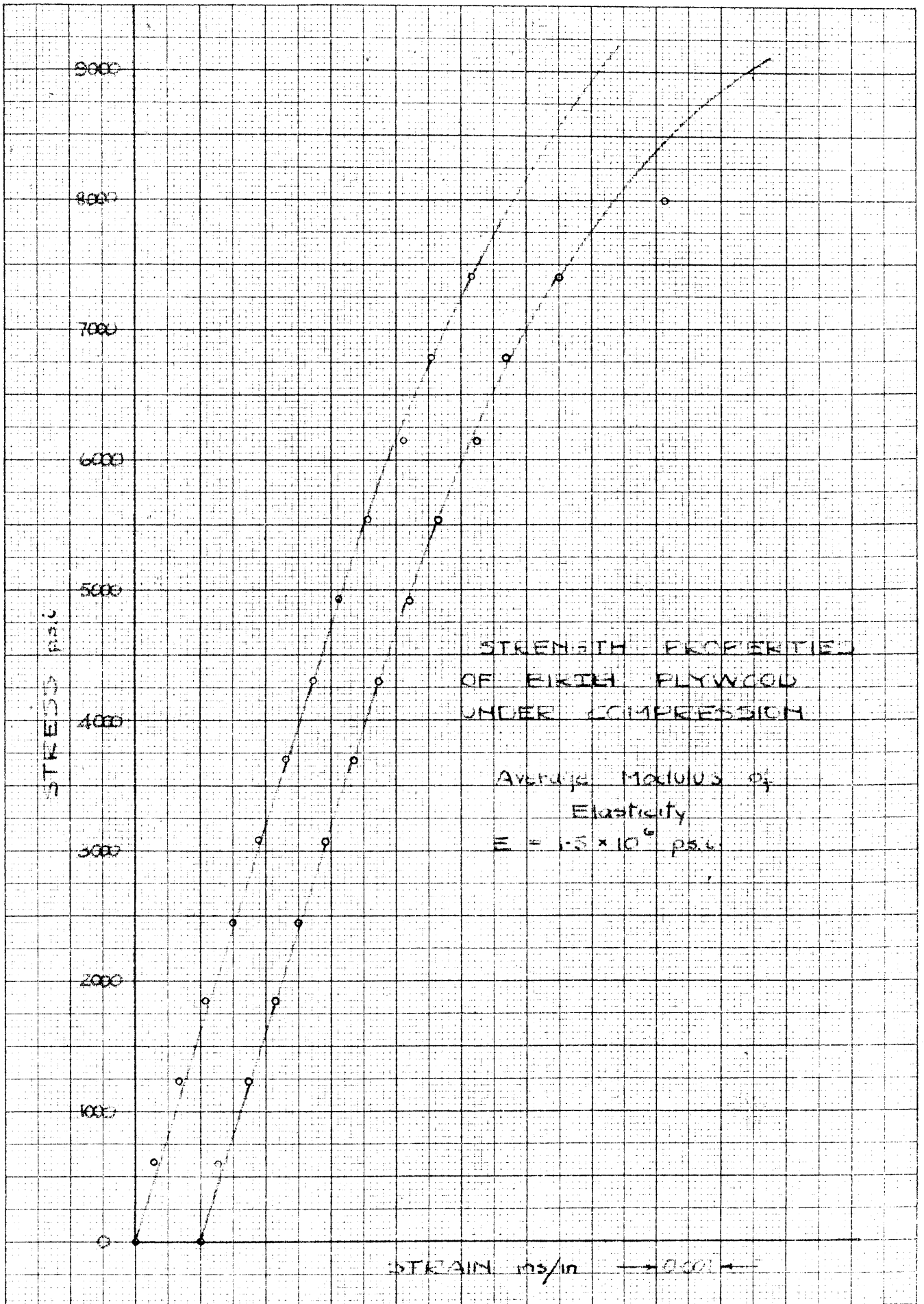


Fig 5

ULTIMATE STRESS VS. PERCENT REINFORCEMENT

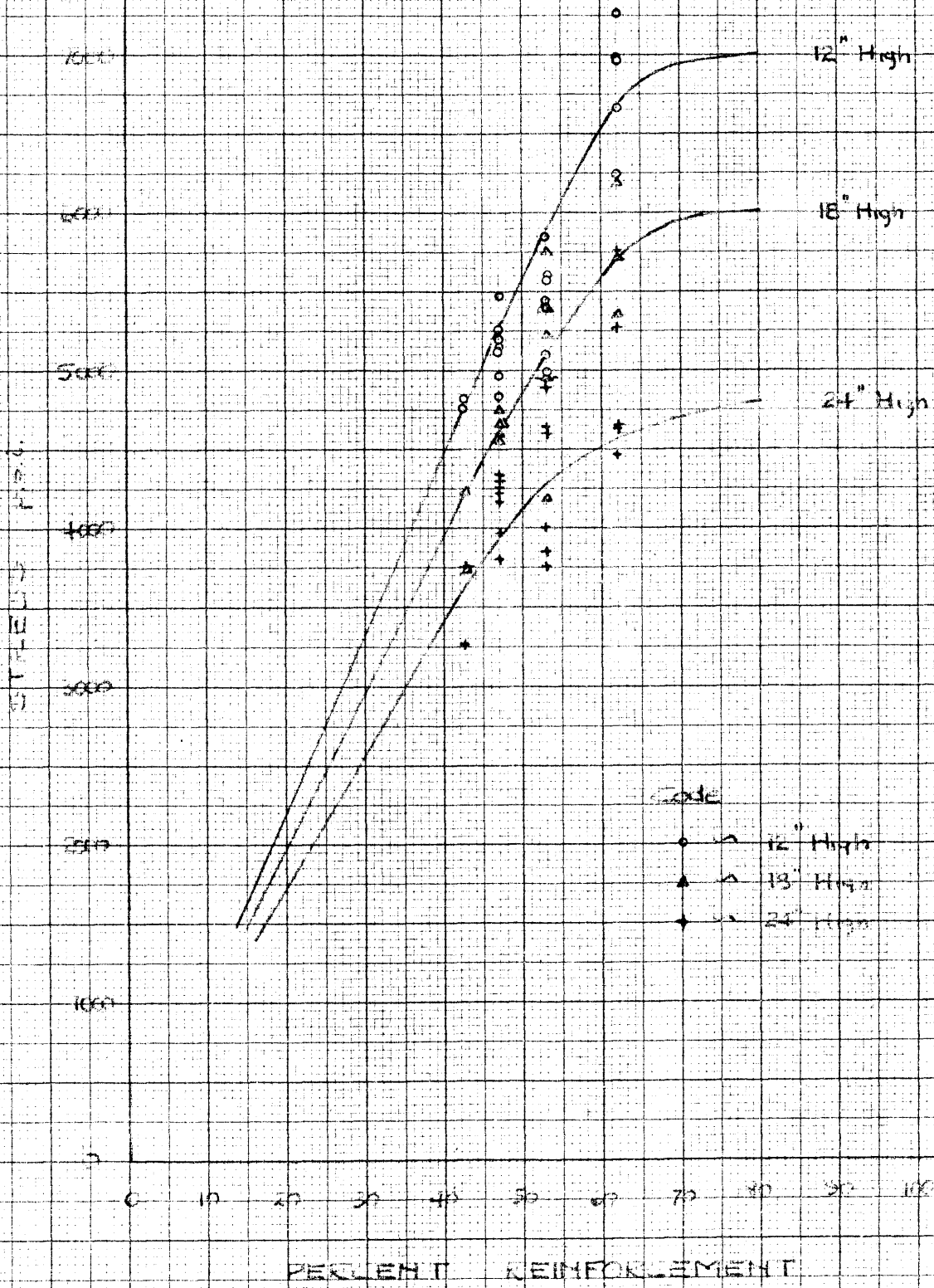
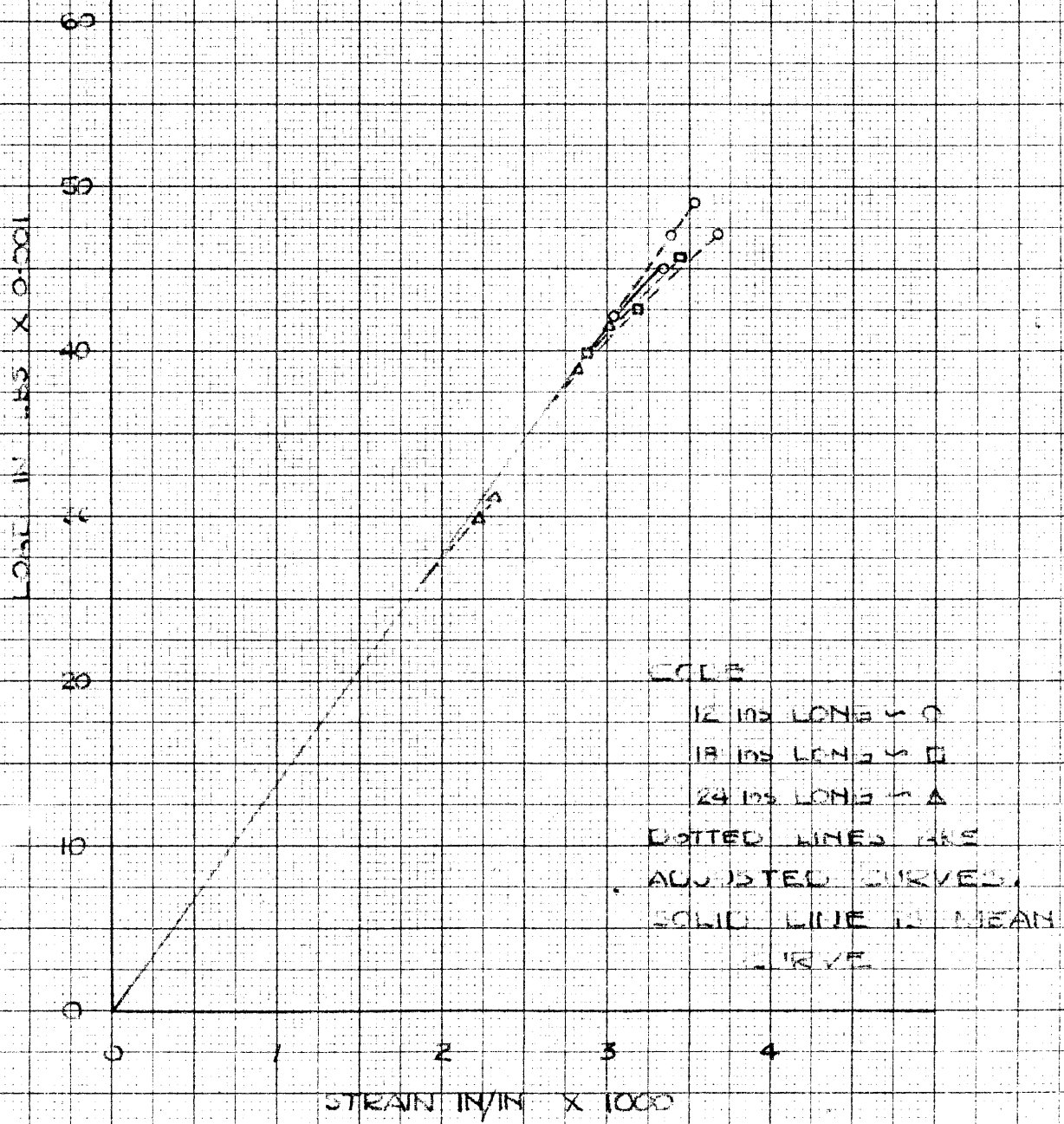


Fig. 1

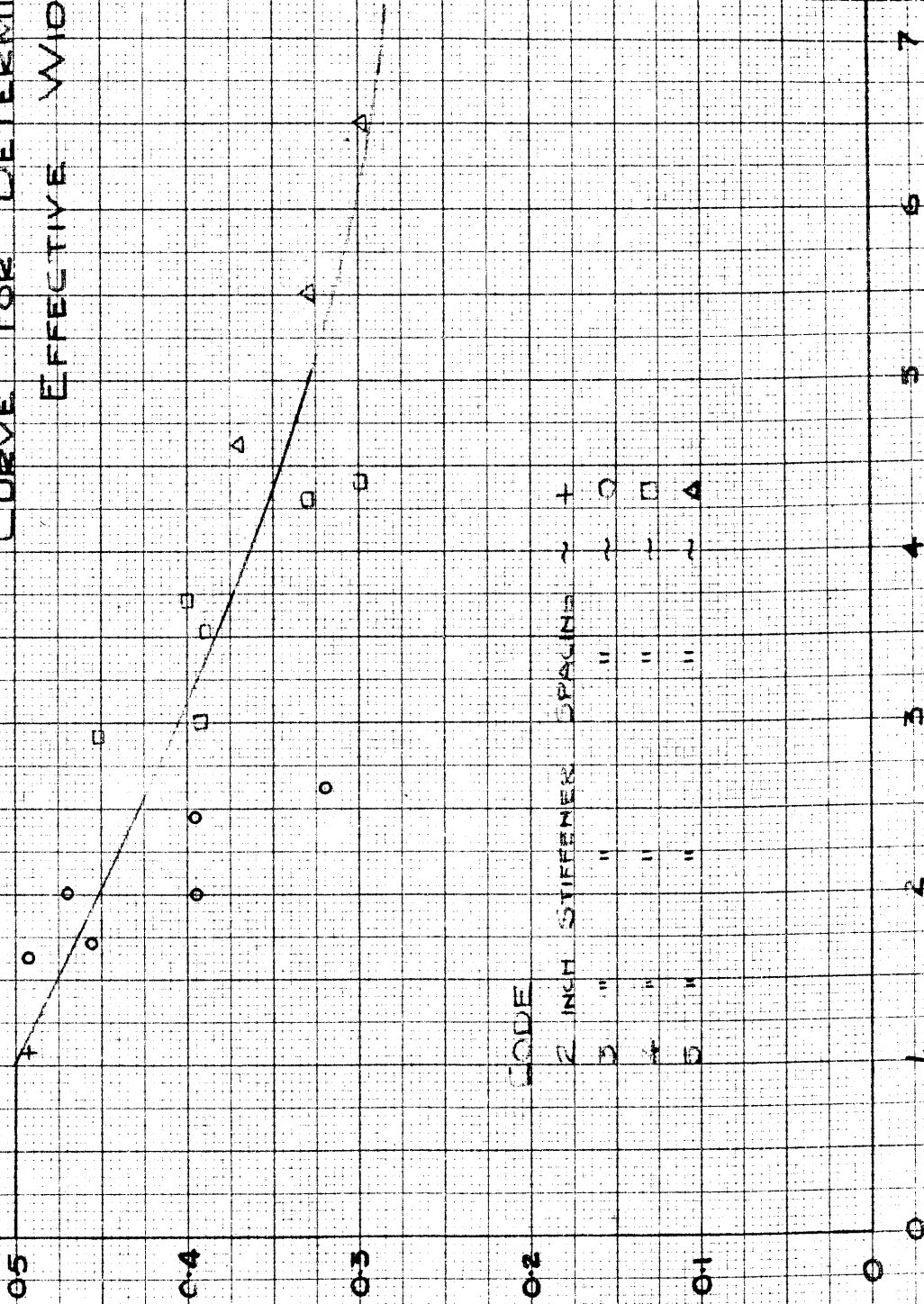
ILLUSTRATIVE EXAMPLE CURVE



12 LBS LONG \circ
 18 LBS LONG \square
 24 LBS LONG \triangle
 DOTTED LINES ARE
 ADJUSTED CURVES.
 SOLID LINE IS MEAN
 CURVE.

Fig 5

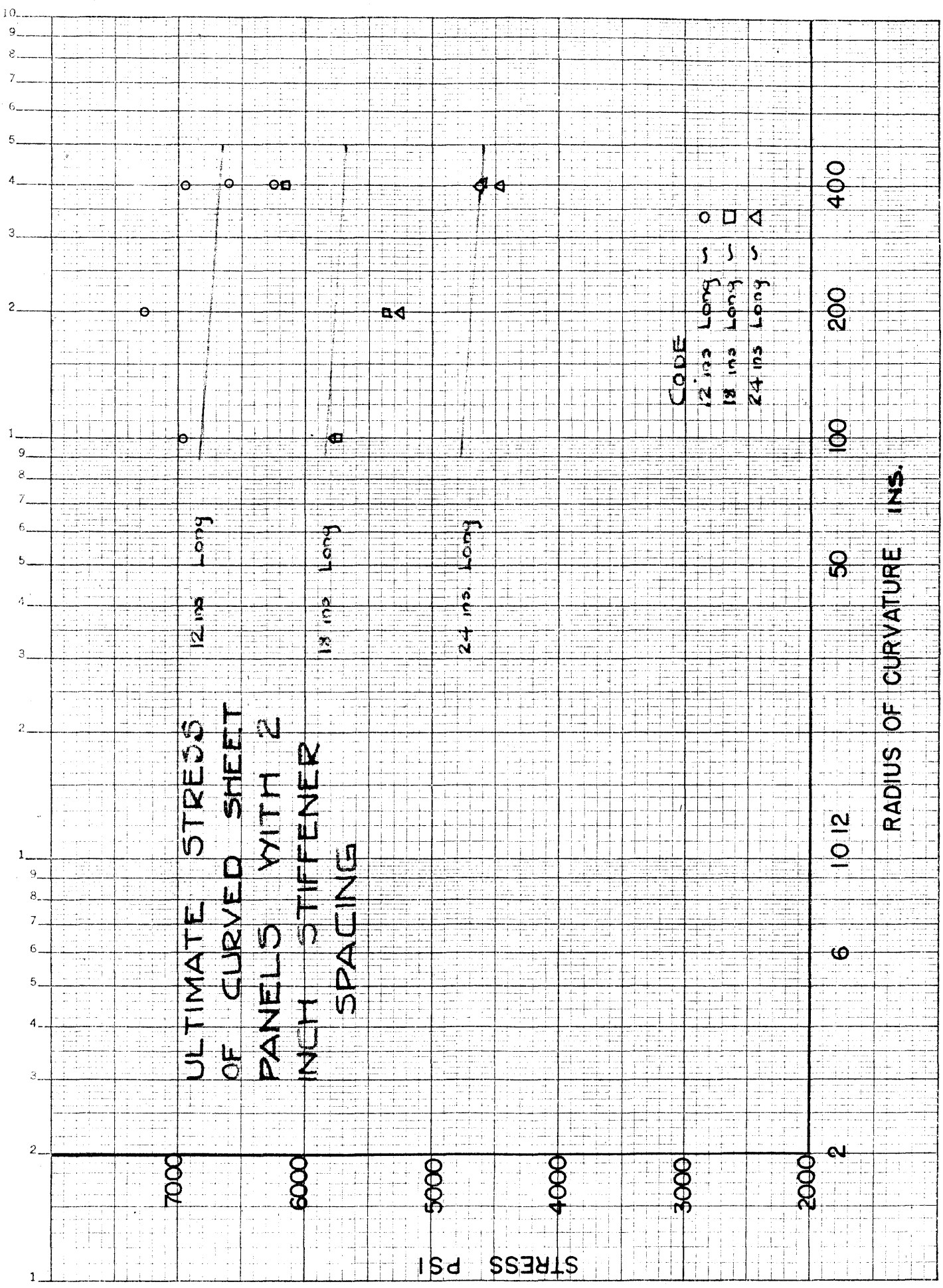
CURVE FOR DETERMINING EFFECTIVE WIDTH



CODE	2 INCH STIFFENER SPACING	+	O	□	Δ
1	"				
2	"				
3	"				
4	"				
5	"				

$\frac{W_e}{W}$
 $\frac{I_s}{I}$

FIG. 6



ULTIMATE STRESS OF CURVED SHEET PANELS WITH 3 INCH STIFFENER SPACING

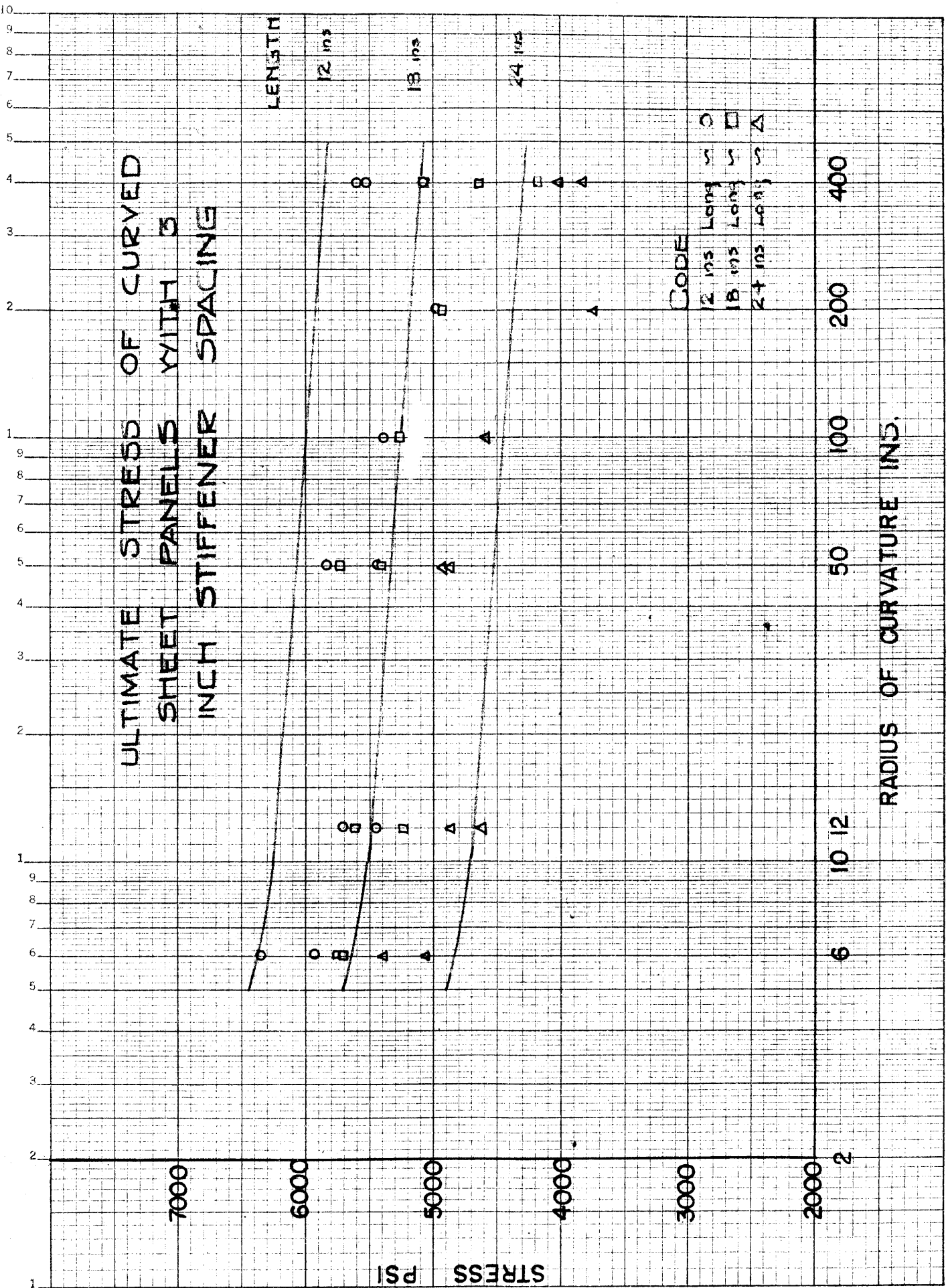


FIG 3

ULTIMATE STRESS OF CURVED SHEET PANELS WITH 4 INCH STIFFENER SPACING

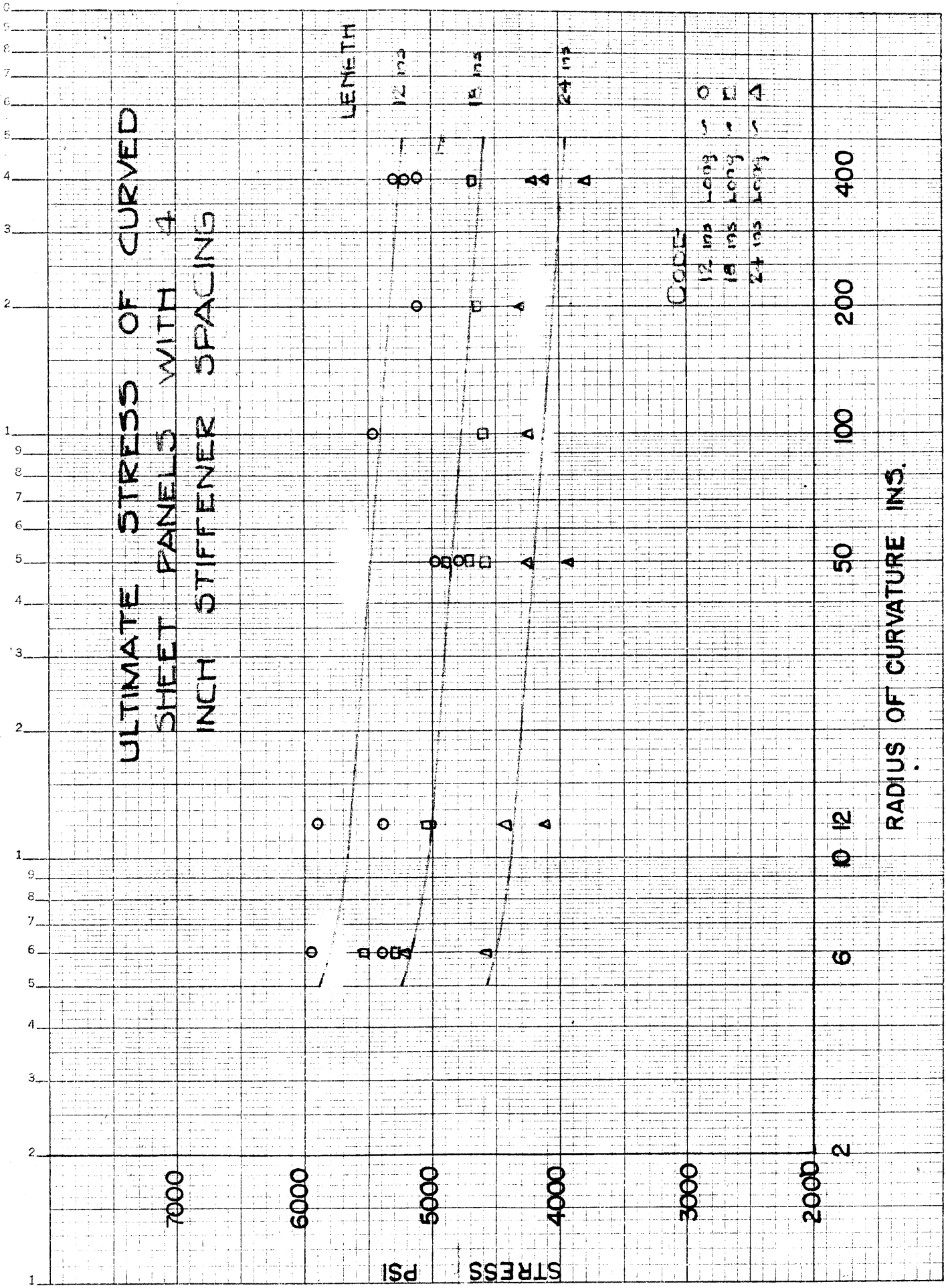
LENGTH
12 ins
18 ins
24 ins

CODE

12 ins Long
18 ins Long
24 ins Long

STRESS
PSI

RADIUS OF CURVATURE INS.



ULTIMATE STRESS OF CURVED
SHEET PANELS WITH 5
INCH STIFFENER SPACING

STRESS PSI

LENGTH

12 ins

18 ins

24 ins

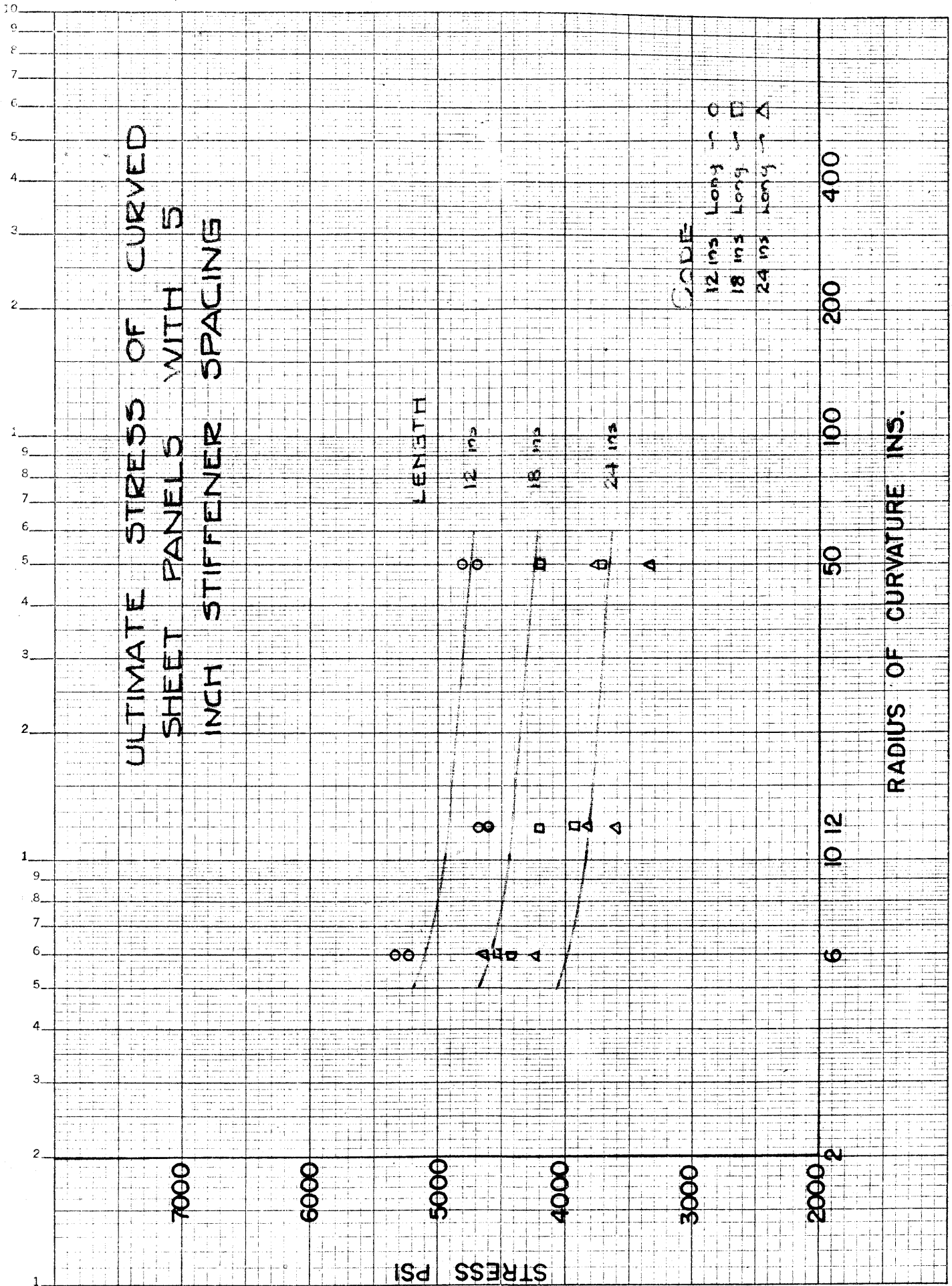
CODE

12 ins Long - O

18 ins Long - □

24 ins Long - △

RADIUS OF CURVATURE INS.



STRESS-STRAIN CURVES FOR STIFFENED PANELS

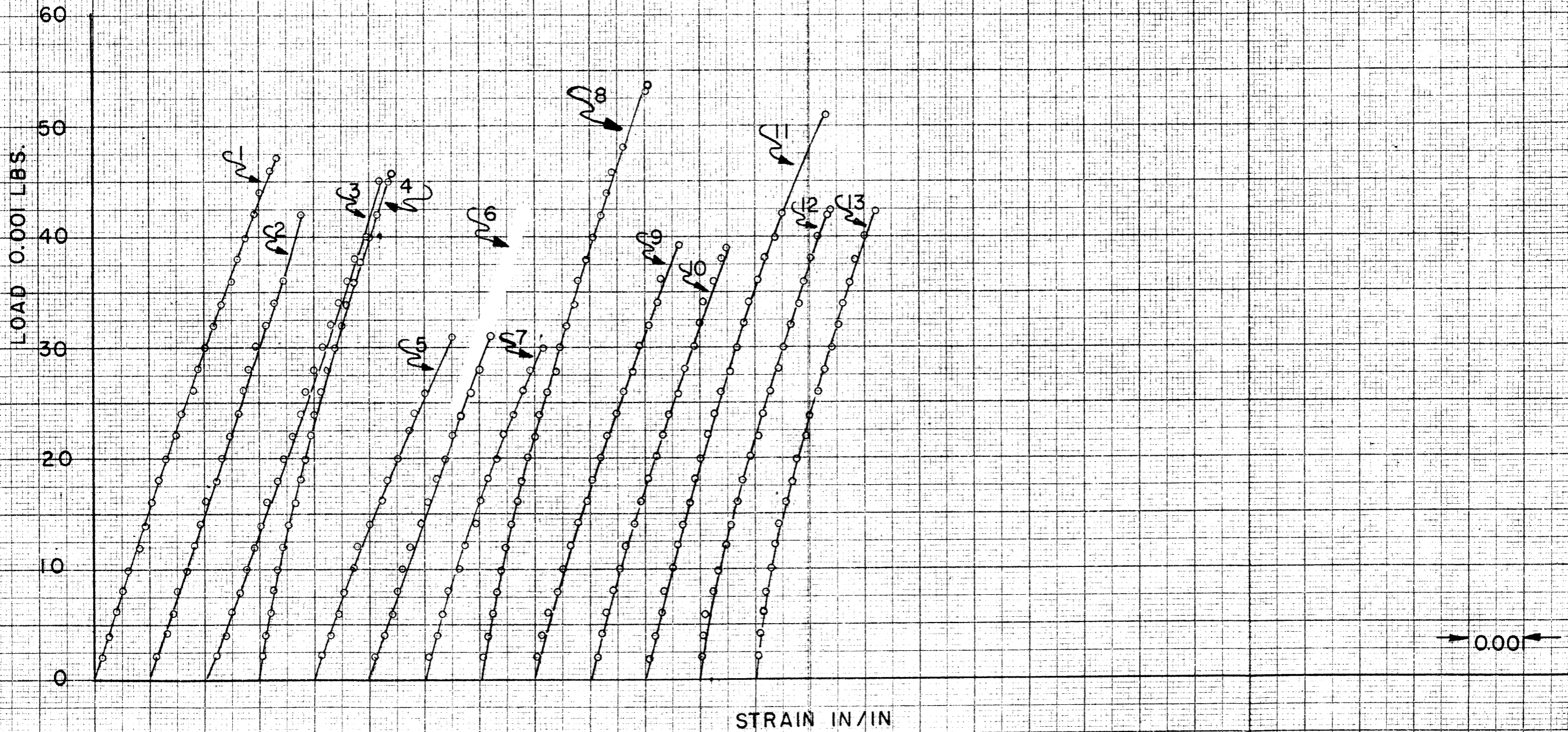


FIG 11

STRESS - STRAIN CURVES FOR STIFFENED PANELS

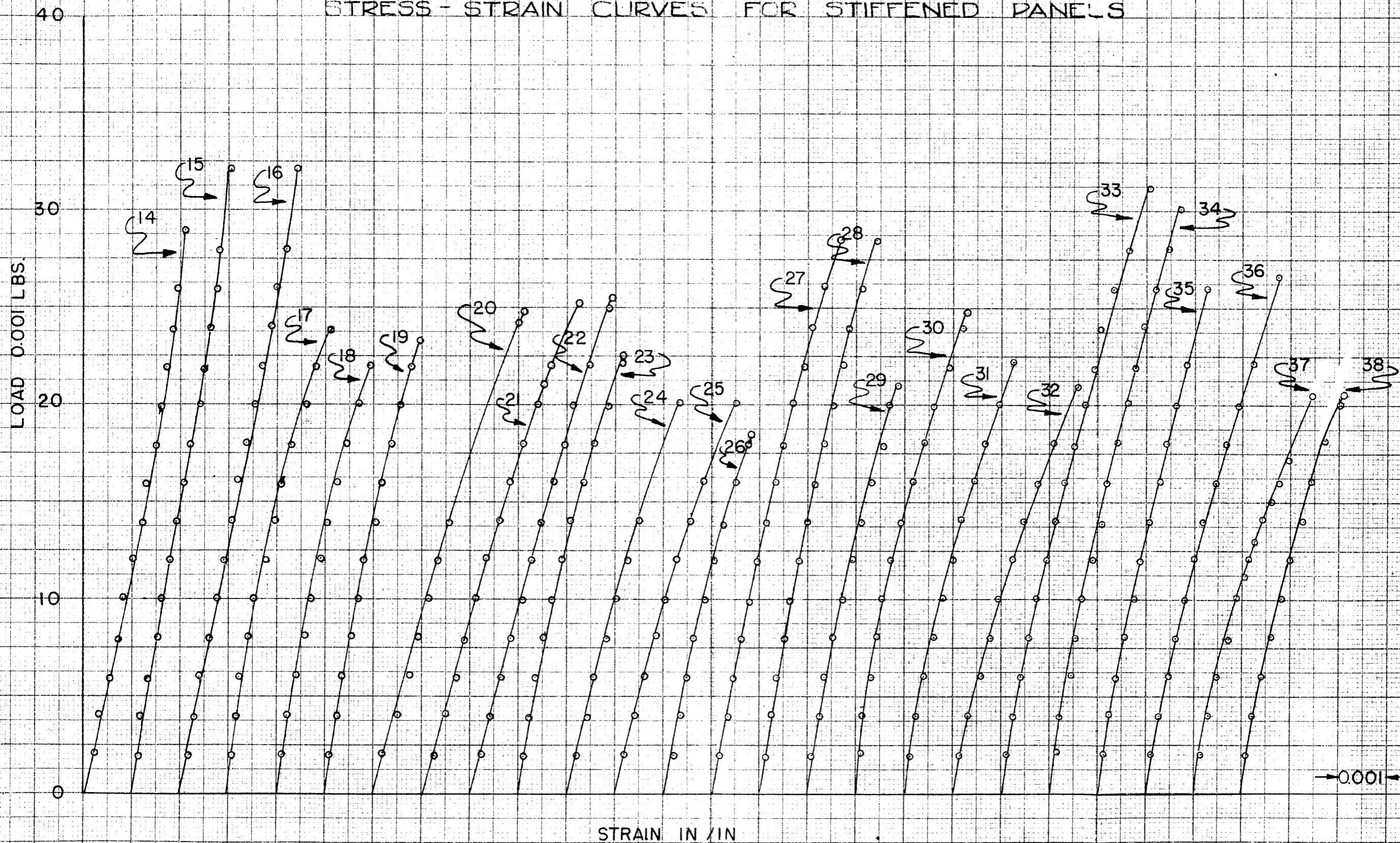


FIG 12

STRESS-STRAIN CURVES FOR STIFFENED PANELS

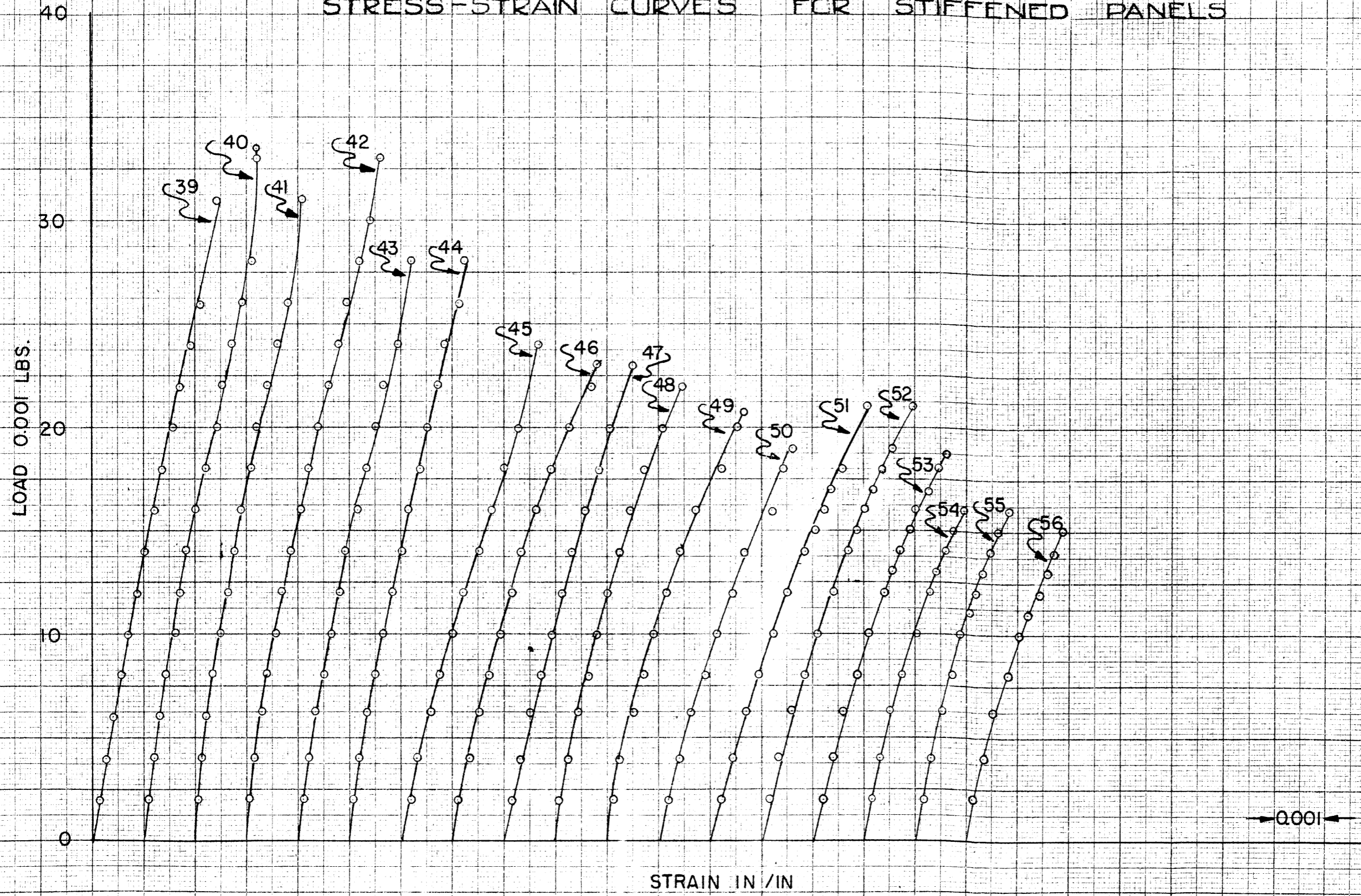


FIG 13

STRESS- STRAIN CURVES FOR STIFFENED PANELS

40

30

20

10

0

LOAD 0.001 LBS.

STRAIN IN / IN

→ 0.001 ←

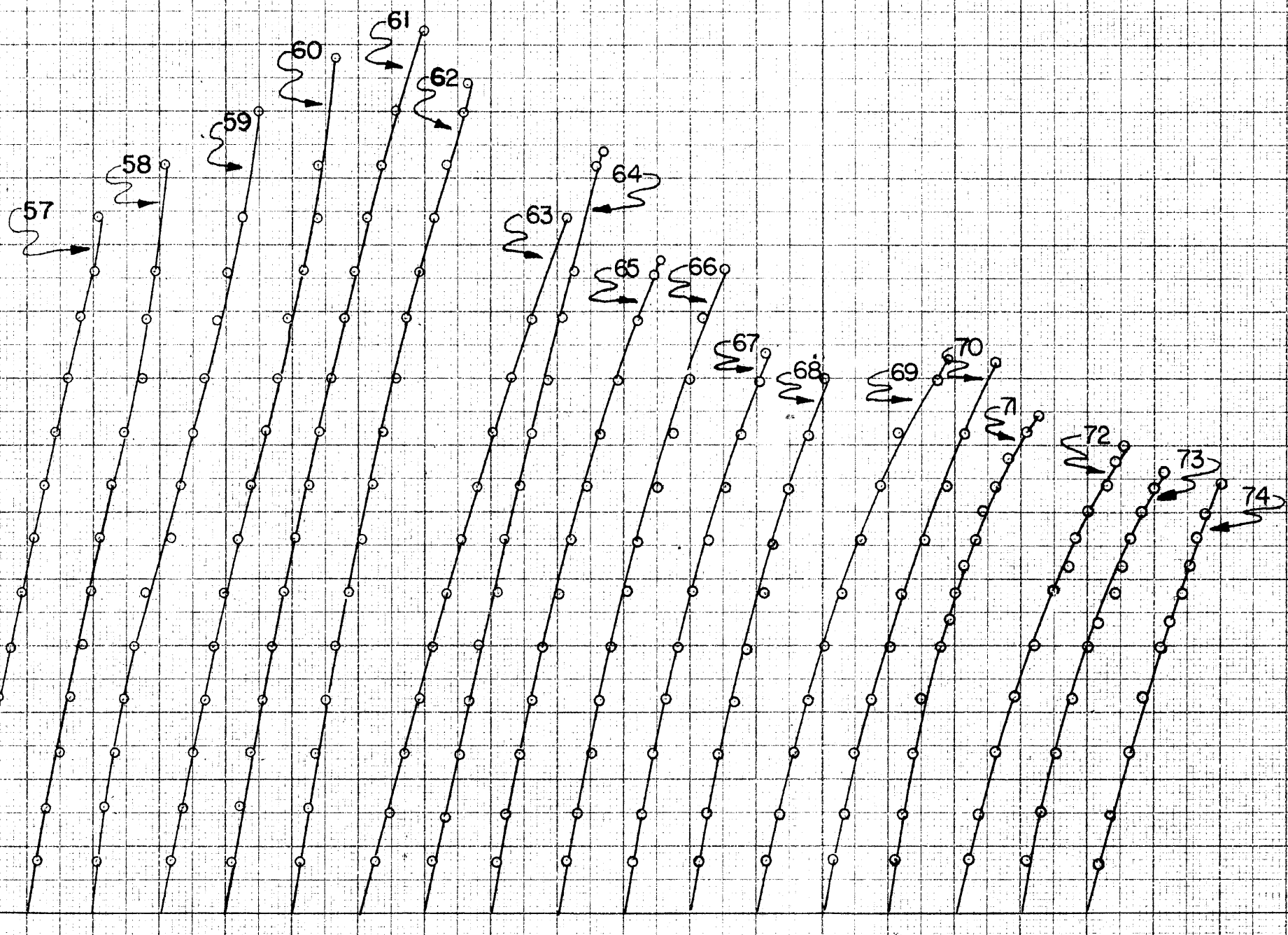


FIG 14

STRESS - STRAIN CURVES FOR STIFFENED PANELS

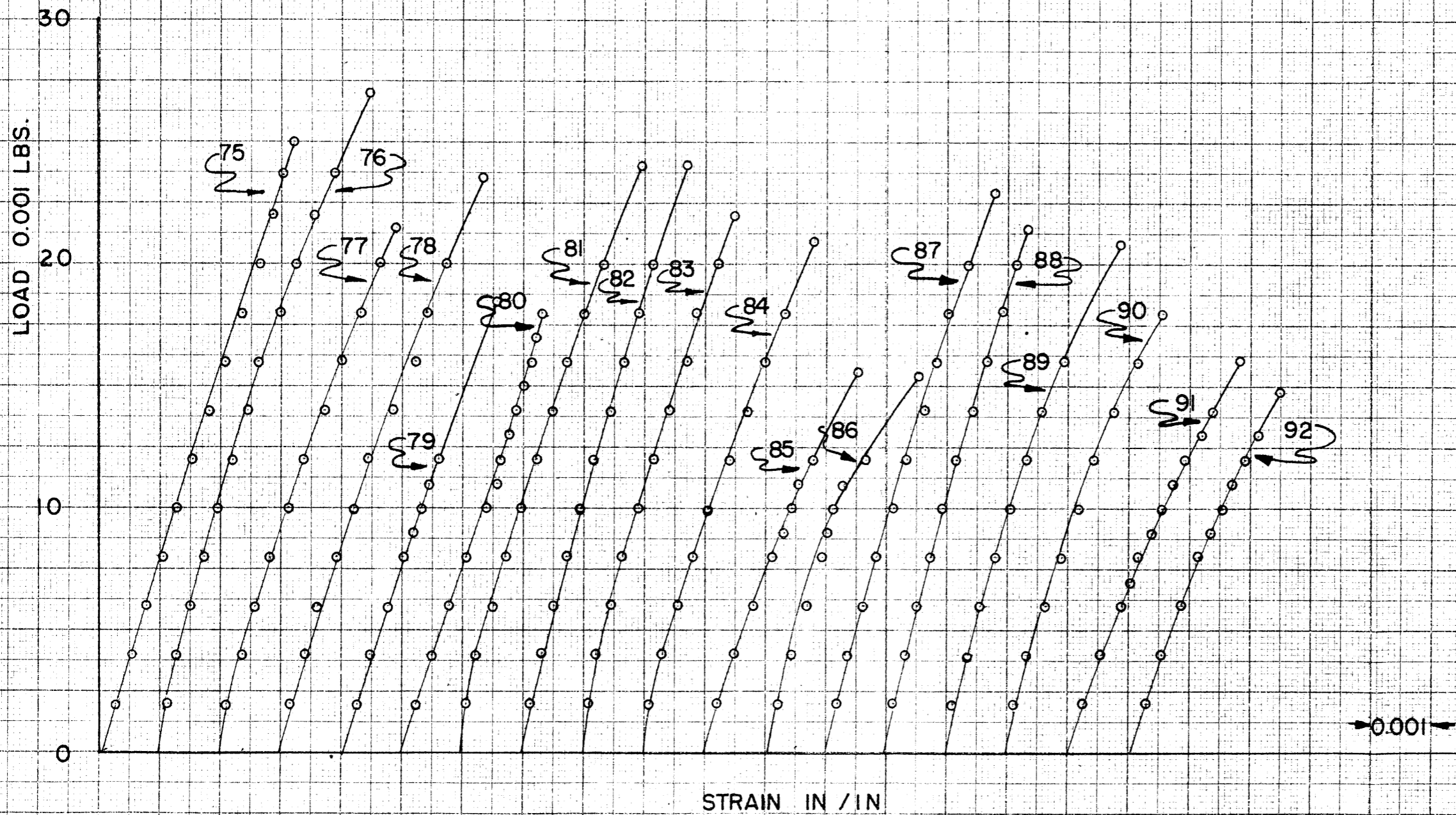


FIG 15