TECHNIQUE OF MEASURING TRANSVERSE COMPONENTS OF VELOCITY FLUCTUATIONS IN TURBULENT FLOW

Thesis by

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CONTENTS

NOTATION

x = distance from grid measured downstream along the tunnel.

y,z = horizontal and vertical coordinates respectively.

U = mean air speed in x-direction.

u,v,w = instantaneous values of the x,y, and z components of the fluctuating velocity.

$\frac{\sqrt{\overline{u^2}}}{U}$, $\frac{\sqrt{\overline{v^2}}}{U}$, $\frac{\sqrt{\overline{w^2}}}{U}$ = turbulence levels in the x, y and z directions.

u',v',w' = $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$, $\sqrt{\overline{w^2}}$ respectively.

R,S = correlation coefficients.

$\Lambda = \int_0^\infty R(y)dy$ = scale of turbulence.

M = mesh size of grid.

L = length of a hot-wire.

α = angle between the wires of an X-type v'/U-meter.

K = correction factor by which the observed turbulence values must be multiplied to correct for wire length.

SUMMARY

The mean speed characteristics of several X-type v'/U-meters having widely varied dimensions were measured in order to verify assumptions usually made regarding the range of linear characteristics for meters of this type.

Using the same meters, and other meters with intermediate sizes, values of v'/U were measured in the turbulent fields behind three grids. Comparison of the results gave a qualitative indication of the effect of meter size on turbulence measurements.

A length correction for the X-type meters used was derived and the turbulence values corrected, with the result that all meters gave approximately the same answers.

However, even after application of the length correction, it was found that in the turbulent field behind grids (generally assumed to be isotropic) v'/U was consistently lower than u'/U at a given distance behind the grid. This result is the same as that described by Corrsin in ref. (4) where no length correction was applied.

INTRODUCTION

The technique of measuring velocity fluctuations normal to the direction of the mean velocity in turbulent flow has never become standardized as has that used in the measurement of the fluctuation component parallel to the mean velocity. The techniques commonly employed make use of two general phenomena. One of these utilizes the diffusion of heat, and applications of the method have been made by Schubauer[1] and Reichardt[2]; the other depends on the sensitivity to angular orientation of arrangements of two or more hot-wires, applications having been made by Simmons[3] and by Dryden. Although several arrangements of hot-wires have been proposed, the most common type, and that used at the GALCIT, consists of two wires located in parallel planes parallel to the flow direction and arranged in the form of an X. This type of v'/U-meter has been thought to give good results although a systematic investigation of its characteristics has never been made. The present report describes the results of such an investigation.

The chief stimulus for undertaking the investigation at this time is to be found in the results obtained by Stanley Corrsin[4] during his measurements of the decay of turbulence behind members of a family of similar grids. It is known that the turbulence becomes uniform downstream from a regular grid placed in a uniform airstream, and it has generally been assumed that this turbulence is isotropic. Further, the isotropy has been checked by means of a few (unpublished) measurements of u'/U and v'/U by H. L. Dryden's group and Carl Thiele using hot-wires, by Dryden, Schubauer, and Mock (ref. 6, Fig. 9) using the thermal diffusion method, and by D. C. MacPhail[5] using an indirect method. However, Corrsin found that for every grid he tested, and at all wind speeds, the u'/U level was consistently higher than the v'/U level at all points downstream from the grids. No explanation for the difference was found, and it was concluded that the result should be regarded cautiously in view of the strong experimental evidence for isotropy which had already been collected.

However, because of the consistency of Corrsin's results there seemed to be only two possibilities: either that the turbulence was in fact not isotropic or that the v'/U-meters used had inherent characteristics which led to incorrect results. For example it could be seen that a length correction applied to the v'/U data would at least lessen the discrepency, but no such correction had been worked out.

In view of these results it seemed desirable to make a careful examination of the characteristics of the X-type meters used by Corrsin. The problem was approached along two lines--experimental and theoretical. Experimentally it was hoped to determine the effects of varying the wire lengths while maintaining the angle between them constant, and of varying the angle between wires of the same length. It was also expected that some idea might be gained as to how serious might be the effects of lack of symmetry in the meters, and of misalignment to the mean flow. The theoretical approach to the problem seemed to lead immediately to the computation of a length correction for v'/U-meters similar to that for u'/U-meters worked out by Skramstad[6].

The experimental investigation divided itself roughly into two parts. The first involved an investigation into the mean speed characteristics of X-type meters having various dimensions. This was important since the measurements involved are the same as those necessary to calibrate a meter to be used for measuring v'/U, and because it was desired to know the range in which the assumption of linear characteristics is valid. The second part of the experimental investigation had to do with the measurements of turbulence behind the various grids, and the application of the length correction worked out in another section of this report.

DESCRIPTION OF EQUIPMENT; EXPERIMENTAL PROCEDURE

The mean speed characteristics of meters of the type described above were investigated with the aid of two free jets, one of three inches diameter having fairly low speed and the other a higher speed jet with a diameter of about one inch. The v'/U-meters were initially aligned as nearly as possible with the axis of the jet, and measurements of the difference in the voltage drops across the separate wires were made over a wide range of angles of yaw. This step establishes the sensitivity of the meter to angular orientation and hence to lateral fluctuations in velocity. A meter to be used for measuring the direction of an unknown flow would be calibrated in exactly the same way. At the suggestion of a local aircraft manufacturer desiring a simple yet sensitive yaw meter for use in flight testing, a meter was calibrated in the high speed jet using only a Wheatstone bridge and a ruggedly built galvanometer. These data are included here only to show the effect of mean speed on the v'/U-meter calibration.

The wind tunnel used for the turbulence measurements was of the Eiffel type with a sixteen to one contraction ratio and a working section twenty inches square and twelve feet long. The air enters through a cheesecloth screen backed up by a honey-comb of one-half inch cells. Further downstream is located a seamless precision screen which reduces the free stream turbulence level in the working section to about 0.06 percent. The tunnel is identical with that described in ref. (4) where more details are given. The principal dimensions of the tunnel are given on Fig.11.

The grids used were all of the bi-plane type with the horizontal and vertical rods merely in contact. The grids all had the same rod diameter to mesh size ratio. Further details may be found in ref. (4).

All turbulence measuring instruments were built of Wollaston platinum wire 0.00024 inches in diameter. The silver was etched off before soldering the wires to the sewing needle spindles of the hot-wire holders. Only one wire, about 0.07 inches long, was used for the u'/U measurements. The v'/U-meters used were all of the bi-plane X-type; their dimensions are given in Table I below.

Table I

Dimensions of v'/U-Meters Investigated

Meter number	Wire length, L (inches)	Angle between wires, (degrees)
1	0.14	45
2	0.34	40
3	0.53	40
4	0.04	80
5	0.11	100
6	0.45	100

Detailed descriptions of hot-wire equipment and the principles upon which it operates can be found in the technical literature, for example ref. (7). Constant current methods were used throughout this investigation. The heating circuits used included large chokes which reduced current fluctuations to a minimum. The amplifier had correct characteristics to within ± 3 percent over a frequency range from a few to about 9000 cycles per second. Output readings were taken on a wall galvanometer having a period of about twenty seconds and a full scale deflection of twenty centimeters. The noise level was negligible at the amplification used in measuring turbulence behind the grids but it is certainly appreciable at the amplification necessary to measure free stream turbulence. All instruments were calibrated both for sensitivity and the time-constant, no use being made of average values of the latter as in ref. (4). All turbulence measurements were made at a mean speed of 35 ft/sec (10.7 m/sec).

EXPERIMENTAL RESULTS

The experimental results are summarized in Table II, page 12, and on Figs.1-4 at the end of the report. Since the investigation was concerned primarily with the characteristics of the X-type meters important calibration quantities and the length correction for each meter, as well as the corrected and uncorrected values of v'/U, are included in Table II. The table is divided into three parts each containing the results obtained behind a particular grid. The values of x/M, Λ, and u'/U, for comparison with the v'/U values given, are indicated for each grid. The values in parentheses are corresponding values taken from ref. (4), and serve to indicate the degree to which the present results check those of that investigation. The values of $\frac{\Lambda}{M}$ used were taken from data obtained by Stanley Corrsin at the GALCIT during the summer of 1942 as an extension of his work in ref. (4).

Fig.4 has been prepared from Table II to simplify the comparison of turbulence values obtained using one meter with corresponding values obtained using another. Discussion of this figure is reserved for a later section following the description of the length correction.

Figs.1 and 2 show the results of mean speed calibrations made for meters number 1,5,3, and 6. On Fig.1 are shown two calibrations for the same meter made about two weeks apart. It is apparent that the general characteristics of the meter remain unchanged although the sensitivity is slightly different. This indicates the reason for making a new calibration each such a meter is used. It is to be noted in connection with

TABLE II

METER NUMBER	WIRE DESCRIPTION	v'/U SENSITIVITY - VOLTS PER PERCENT v'/U	u'/U SENSITIVITY - VOLTS PER PERCENT u'/U	L/Λ	K	PERCENT v'/U (UNCORRECTED)	PERCENT v'/U (CORRECTED FOR u'/U SENSITIVITY)	PERCENT v'/U (CORRECTED FOR u'/U SENSITIVITY AND WIRE LENGTH)
				1" GRID				
		$x/M = 20$; $\Lambda/M = .220$; $\Lambda = .220"$; $u'/U = 5.83 (3.41)$*						
1	.14" x 45°	.00144	.00020	0.635	1.109	4.45 (2.50)	3.91	4.33
2	.34" x 40°	.00728	.00021	1.545	1.271	3.51	3.41	4.34
3	.53" x 40°	.01153	.00007	2.410	1.426	3.50	3.48	4.96
4	.04" x 80°	.00015	.00001	0.182	1.032	5.42	5.16	5.32
5	.11" x 100°	.00055	.00007	0.500	1.086	4.16	3.69	4.01
6	.45" x 100°	.00264	.00004	2.045	1.334	4.18	4.12	5.50
				1/2" GRID				
		$x/M = 40$; $\Lambda/M = .286$; $\Lambda = .143"$; $u'/U = 2.09 (2.05)$						
1	.14" x 45°	.00144	.00020	1.021	1.176	1.48 (1.42)	1.30	1.53
2	.34" x 40°	.00728	.00021	2.430	1.425	1.08	1.05	1.50
3	.53" x 40°	.01153	.00007	3.780	1.666	0.98	0.97	1.61
4	.04" x 80°	.00015	.00001	0.286	1.049	1.78	1.66	1.74
5	.11" x 100°	.00055	.00007	0.785	1.131	1.49	1.32	1.50
6	.45" x 100°	.00264	.00004	3.220	1.567	1.17	1.15	1.80
				1/4" GRID				
		$x/M = 80$; $\Lambda/M = .395$; $\Lambda = .099"$; $u'/U = 1.40 (1.30)$						
1	.14" x 45°	.00144	.00020	1.415	1.243	0.92 (0.82)	0.80	0.99
2	.34" x 40°	.00728	.00021	3.440	1.607	0.64	0.62	0.99
3	.53" x 40°	.01153	.00007	5.350	1.940	0.59	0.59	1.14
4	.04" x 80°	.00015	.00001	0.404	1.072	1.21	1.15	1.23
5	.11" x 100°	.00055	.00007	1.111	1.185	1.01	0.90	1.08
6	.45" x 100°	.00264	.00004	4.550	1.753	0.79	0.78	1.37

* THE REASON FOR THE LARGE DIFFERENCE HERE IS GIVEN ON PAGE 25.

these figures that unless the meter is perfectly symmetrical and is exactly aligned with the mean flow direction the voltage difference will not be zero at zero yaw. Since these conditions occur only rarely it has been necessary for easy comparison to shift the curves along the vertical axis until they pass through the origin.

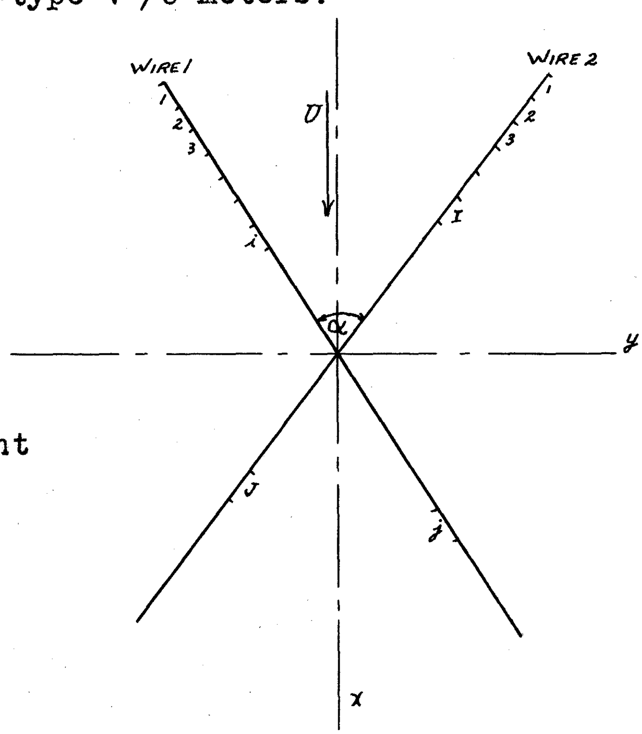
Fig.3 contains the results of the calibration of a yaw meter similar to v'/U-meter number 5. It is seen that the sensitivity of such a meter depends on the mean speed and decreases with increasing speed. This characteristic is due chiefly to the fact that the cooling is proportional to the square-root of the velocity, but also results from lack of symmetry in the meter. It shows that a v'/U-meter is more or less sensitive to u'/U fluctuations as well. The use of such an instrument as a yaw meter where varying speed conditions must be met would appear to be limited due to the rather elaborate calibration required and the ease with which the wires may be broken, in the event of which a completely new calibration is necessary.

In connection with the turbulence measurements it is worth mentioning that several points were checked for w'/U simply by rotating the X-wires ninety degrees about the longitudinal axis. No appreciable difference was found between v'/U and w'/U values at the same point.

LENGTH CORRECTION FOR X-TYPE v'/U-METERS

In the measurements of intensity of turbulence the hot-wires used are usually sufficiently long so that velocity fluctuations on one part of the wire are not completely correlated with those on another part. In Part IV of N.A.C.A. Technical Report 581 H.K. Skramstad has shown that the lack of correlation causes the root-mean-square voltage fluctuation across the wire to be reduced an amount that depends upon the rate of falling off of correlation along the wire. In that report Skramstad obtains a correction factor K, by which u'/U values, measured by the usual method using a single wire, must be multiplied in order to obtain the true values. In what follows the attempt is made to derive in a similar manner a length correction factor K for X-type v'/U-meters.

Consider the arrangement of hot-wires shown in the sketch. It is assumed that the meter is symmetrical about the axes and that it is perfectly aligned with the mean velocity U. The wires of length L carry a constant current. In the equations which follow lower-case subscripts refer to wire 1; upper-case subscripts refer to wire 2.



For the case of complete correlation of velocity fluctuations at all points of the wires, the fluctuating output voltages are for wires 1 and 2 respectively,

$$e_1 = (au + bv)L \tag{1}$$

$$E_1 = (au - bv)L \tag{2}$$

Here u and v are the fluctuating velocities, and a and b are constants depending on the dimensions of the wires, their orientation with respect to the mean velocity, their resistance and temperature coefficient of resistance, the current through the wires, the mean speed U, and the amplification.

Subtracting eqn.(2) from eqn.(1) there results

$$(e-E)_1 = 2bvL \tag{3}$$

The output meter gives indications proportional to the mean square of the output voltage given by

$$\overline{(e-E)_1^2} = 4b^2L^2\overline{v^2} \tag{4}$$

Now consider the case in which the velocity fluctuations at various points along the wires are not completely correlated. Divide the wire into segments of length L. The voltage drop in the ith element is

$$\Delta e_i = (au_i + bv_i)\Delta L_i \tag{5}$$

and that in the Jth element is

$$\Delta E_J = (au_J - bv_J)\Delta L_J \tag{6}$$

The total voltage drop in wire 1 is

$$e = \sum_{i=1}^{n}(au_i + bv_i)\,\Delta L_i \tag{7}$$

and in wire 2 it is

$$E = \sum_{J=1}^{N}(au_J - bv_J)\Delta L_J \tag{8}$$

The voltage difference is

$$(e-E) = \sum_{i=1}^{n}(au_i + bv_i)\Delta L_i - \sum_{J=1}^{N}(au_J - bv_J)\Delta L_J \tag{9}$$

Since all the L's are the same the subscripts on this quantity may be omitted. In what follows mean values are understood, but the bars are omitted for convenience.

From eqn.(9)

$$(e-E)^2 = \left\{\left[\sum_{i=1}^{n}(au_i + bv_i)\right]^2 - 2\sum_{i=1}^{n}(au_i + bv_i)\sum_{J=1}^{N}(au_J - bv_J) + \left[\sum_{J=1}^{N}(au_J - bv_J)\right]^2\right\}\Delta L^2 \tag{10}$$

The first term here can be written in the form

$$\left[\sum_{i=1}^{n}(au_i + bv_i)\Delta L\right]^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(au_i + bv_i)(au_j + bv_j)\Delta L^2 \tag{11}$$

Expanding the right hand side, and omitting terms involving cross-products of u and v which are zero for zero turbulent shear (this condition is fulfilled if isotropy prevails)

$$\left[\sum_{i=1}^{n}(au_i + bv_i)\Delta L\right]^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(a^2 u_i u_j + b^2 v_i v_j)\Delta L^2 \tag{12}$$

Similarly the second and third terms of eqn.(10) become, respectively,

$$-2\sum_{i=1}^{n}(au_i + bv_i)\Delta L\sum_{J=1}^{N}(au_J - bv_J)\Delta L = -2\sum_{i=1}^{n}\sum_{J=1}^{N}(a^2 u_i u_J - b^2 v_i v_J)\Delta L^2 \tag{13}$$

$$\left[\sum_{J=1}^{N}(au_J - bv_J)\Delta L\right]^2 = \sum_{I=1}^{N}\sum_{J=1}^{N}(a^2 u_I u_J + b^2 v_I v_J)\Delta L^2 \tag{14}$$

The correlation coefficient R between any two velocity fluctuations u_r and u_s is defined as

$$R = \frac{\overline{u_r u_s}}{\sqrt{\overline{u_r^2}}\sqrt{\overline{u_s^2}}} \tag{15}$$

and similarly for velocity fluctuations v_r and v_s. Since the mean square of the velocity fluctuations along a wire is constant

$$\overline{u_r^2} = \overline{u_s^2} = \overline{u^2}$$

$$\overline{v_r^2} = \overline{v_s^2} = \overline{v^2} \qquad (16)$$

Using these results the following expressions are obtained,

$$\overline{u_i u_j} = \overline{u^2} R_{ij} \qquad \overline{u_I u_J} = \overline{u^2} R_{IJ} \qquad \overline{u_i u_J} = \overline{u^2} R_{iJ}$$

$$\overline{v_i v_j} = \overline{v^2} S_{ij} \qquad \overline{v_I v_J} = \overline{v^2} S_{IJ} \qquad \overline{v_i v_J} = \overline{v^2} S_{iJ} \qquad (17)$$

where R denotes a correlation coefficient between u velocity fluctuations, and S denotes a correlation coefficient between v velocity fluctuations.

Substituting expressions (17) into eqns.(12), (13), and (14), and these back into eqn.(10) it is found that

$$\overline{(e-E)^2} = a^2\overline{u^2}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 + \sum_{I=1}^{N}\sum_{J=1}^{N} R_{IJ}\,\Delta L^2\right\}$$

$$+ b^2\overline{v^2}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij}\,\Delta L^2 + \sum_{I=1}^{N}\sum_{J=1}^{N} S_{IJ}\,\Delta L^2\right\}$$

$$- 2a^2\overline{u^2}\sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2 + 2b^2\overline{v^2}\sum_{i=1}^{n}\sum_{J=1}^{N} S_{iJ}\,\Delta L^2 \qquad (18)$$

If it is now assumed that the correlation between the velocity fluctuations at two points is a function only of the distance between the points, eqn.(18) can be shortened somewhat since

$$R_{ij} = R_{IJ}$$

$$S_{ij} = S_{IJ} \qquad (19)$$

Consider now a single term of eqn.(18), namely,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\Delta L^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L_i\,\Delta L_j \tag{20}$$

Skramstad shows that the correlation coefficient R_{ij} may be quite accurately represented by

$$R_{ij} = \mathcal{E}^{-\frac{|j-i|\Delta L}{\Lambda}} \tag{21}$$

where $\mathcal{E}$ = the base of natural logarithms

$|j-i|\Delta L$ = the absolute value of the distance between the ith and jth elements

Λ = the scale of the turbulence $(\Lambda = \int_0^\infty R(y)dy)$

A similar expression is assumed to hold for S_{ij}. Using eqn.(21) the term considered in expression (20) becomes

$$\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\Delta L^2 = \Delta L^2 \sum_{i=1}^{n}\sum_{j=1}^{n} \mathcal{E}^{-\frac{|j-i|\Delta L}{\Lambda}}$$

$$= 2\Delta L^2 \sum_{j=1}^{n}\sum_{i=0}^{j-1} \mathcal{E}^{-\frac{(j-i)\Delta L}{\Lambda}} \tag{22}$$

$$= 2\Delta L^2 \sum_{j=1}^{n} \mathcal{E}^{-\frac{j\Delta L}{\Lambda}} \sum_{i=0}^{j-1} \mathcal{E}^{\frac{i\Delta L}{\Lambda}}$$

$$= 2\Delta L^2 \sum_{j=1}^{n} \mathcal{E}^{-\lambda j} \sum_{i=0}^{j-1} \mathcal{E}^{\lambda i} \tag{23}$$

where

$$\lambda = \frac{\Delta L}{\Lambda} \tag{24}$$

The sum of a geometric series is known to be

$$a\sum_{i=0}^{n-1} r^i = a + ar + ar^2 + \cdots\cdots ar^{n-1} \equiv \frac{a(1-r^n)}{1-r} \tag{25}$$

and from this it is easy to show that

$$a\sum_{i=1}^{n} r^i = a\left\{\frac{1-r^n}{1-r} - 1 + r^n\right\} \tag{26}$$

Using eqn.(25) in eqn.(23)

$$\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 = 2\Delta L^2 \sum_{j=1}^{n} \varepsilon^{-\lambda j}\left\{ \frac{1-\varepsilon^{\lambda(j-1)}}{1-\varepsilon^{\lambda}} \right\} \quad (27)$$

$$= \frac{2\Delta L^2}{1-\varepsilon^{\lambda}} \sum_{j=1}^{n} \left\{ \varepsilon^{-\lambda j} - \varepsilon^{-\lambda} \right\}$$

$$= \frac{2\Delta L^2}{(1-\varepsilon^{\lambda})^2} \sum_{j=1}^{n} \left\{ \varepsilon^{-\lambda j}(1-\varepsilon^{\lambda}) \right\} - \frac{2\Delta L}{(1-\varepsilon^{\lambda})}\varepsilon^{-\lambda} \sum_{i=1}^{n} \Delta L \quad (28)$$

Now using eqn.(26) in eqn.(28)

$$\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 = \frac{2\Delta L^2}{(1-\varepsilon^{\lambda})^2} \left\{ \left(\frac{1-\varepsilon^{-\lambda n}}{1-\varepsilon^{-\lambda}} - 1 + \varepsilon^{-\lambda n} \right)(1-\varepsilon^{\lambda}) \right\}$$

$$- \frac{2\Delta L}{\varepsilon^{\lambda}(1-\varepsilon^{\lambda})} \sum_{i=1}^{n} \Delta L \quad (29)$$

$$= \frac{2\Delta L^2}{(1-\varepsilon^{\lambda})^2} \left\{ \frac{\varepsilon^{-\lambda}(1-\varepsilon^{-\lambda n})}{(1-\varepsilon^{-\lambda})}(1-\varepsilon^{\lambda}) \right\} - \frac{2\Delta L}{\varepsilon^{\lambda}(1-\varepsilon^{\lambda})} \sum_{i=1}^{n} \Delta L \quad (30)$$

$$= \frac{2\Delta L^2}{(1-\varepsilon^{\lambda})^2} \left\{ -1 + \varepsilon^{-\lambda n} \right\} - \frac{2\Delta L}{\varepsilon^{\lambda}(1-\varepsilon^{\lambda})} \sum_{i=1}^{n} \Delta L \quad (31)$$

Now consider

$$\lim_{\Delta L \to 0} \frac{\Delta L}{1-\varepsilon^{\lambda}} = \lim_{\Delta L \to 0} \frac{\Delta L}{1-\varepsilon^{\frac{\Delta L}{\Lambda}}}$$

Using the series

$$\varepsilon^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\frac{\Delta L}{1-\varepsilon^{\frac{\Delta L}{\Lambda}}} = \frac{\Delta L}{1-1-\frac{\Delta L}{\Lambda}-\frac{\Delta L^2}{2\Lambda^2}\cdots} = -\Lambda \quad (32)$$

where terms involving ΔL^2, ΔL^3, etc. have been dropped.

Similarly

$$\frac{\Delta L}{\varepsilon^{\frac{\Delta L}{\Lambda}}(1-\varepsilon^{\frac{\Delta L}{\Lambda}})} = \frac{\Delta L}{\varepsilon^{\frac{\Delta L}{\Lambda}} - \varepsilon^{\frac{+2\Delta L}{\Lambda}}} = \frac{\Delta L}{1+\frac{\Delta L}{\Lambda}+\frac{\Delta L^2}{2\Lambda^2}\cdots - 1 - \frac{2\Delta L}{\Lambda} - \frac{4\Delta L^2}{2\Lambda^2}\cdots}$$

$$= -\Lambda \quad (33)$$

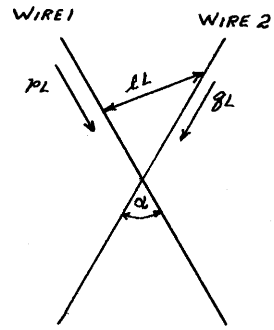
Substituting the results given by eqns.(32) and (33) into eqn.(31)

$$\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 = 2(-\Lambda)^2\left\{-1+\mathcal{E}^{-n\frac{\Delta L}{\Lambda}}\right\} - 2(-\Lambda)L$$

$$= 2\Lambda^2\left\{\mathcal{E}^{-\frac{L}{\Lambda}} - 1 + \frac{L}{\Lambda}\right\} \qquad (34)$$

Eqn.(34) gives the final expression for terms having the form of $\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2$. This is identical with the result obtained by Skramstad using a somewhat different method.

Now terms of the form $\sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2$ must be considered. Choose coordinates qL and pL along the wires as indicated in the sketch, and assume as before that



$$R_{iJ} = \mathcal{E}^{-\frac{\ell L}{\Lambda}} \qquad (35)$$

Then

$$\sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2 = \sum_{i=1}^{n}\sum_{J=1}^{N} \mathcal{E}^{-\frac{\ell L}{\Lambda}}\,\Delta p\,\Delta q\,(L^2) \qquad (36)$$

Considering the wires to be in the same plane, that is neglecting the separation of the wires in the direction normal to the plane of the sketch, the distance between any two points may be expressed by the cosine law as

$$(\ell L)^2 = \left(\frac{L}{2} - pL\right)^2 + \left(\frac{L}{2} - qL\right)^2 - 2\left(\frac{L}{2} - pL\right)\left(\frac{L}{2} - qL\right)\cos\alpha \qquad (37)$$

Hence

$$\sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2 = L^2\int_0^1 dq\int_0^q \mathcal{E}^{-\frac{L}{\Lambda}\left[\left(\frac{1}{2}-p\right)^2+\left(\frac{1}{2}-q\right)^2-2\left(\frac{1}{2}-p\right)\left(\frac{1}{2}-q\right)\cos\alpha\right]^{\frac{1}{2}}}\,dp \qquad (38)$$

Returning to eqn.(18) and using the simplification expressed in (19)

$$(e-E)^2 = 2a^2u^2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 - \sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2\right\}$$

$$+2b^2v^2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij}\,\Delta L^2 + \sum_{i=1}^{n}\sum_{J=1}^{N} S_{iJ}\,\Delta L^2\right\} \qquad (39)$$

The correction factor sought is given by

$$K^2 = \frac{(e-E)_1^2}{(e-E)^2} \qquad (40)$$

where K is the correction factor.

Substituting

$$K^2 = \frac{4b^2v^2L^2}{2a^2u^2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 - \sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2\right\} + 2b^2v^2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij}\,\Delta L^2 + \sum_{i=1}^{n}\sum_{J=1}^{N} S_{iJ}\,\Delta L^2\right\}} \qquad (41)$$

To simplify this and enable one to write the result more compactly note that these terms may be written

$$\sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij}\,\Delta L^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij}\,\Delta L^2 = I_1$$

$$\sum_{i=1}^{n}\sum_{J=1}^{N} R_{iJ}\,\Delta L^2 = \sum_{i=1}^{n}\sum_{J=1}^{N} S_{iJ}\,\Delta L^2 = I_2 \qquad (42)$$

Note that I_1 and I_2 are given by eqns.(34) and (38) respectively.

The expression for K^2 becomes

$$K^2 = \frac{2b^2v^2L^2}{a^2u^2(I_1-I_2) + b^2v^2(I_1+I_2)} \qquad (43)$$

A form in which all terms are dimensionless is obtained by multiplying numerator and denominator by $\frac{1}{2\Lambda^2}$. To permit numerical calculations the turbulence is assumed isotropic so that $u^2 = v^2$.

Now

$$K^2 = \frac{b^2 \frac{L^2}{\Lambda^2}}{a^2(J_1 - J_2) + b^2(J_1 + J_2)} \tag{44}$$

where, using eqns.(34) and (38) respectively,

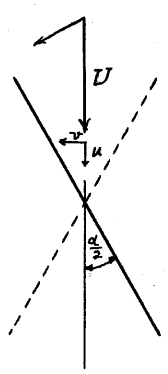
$$J_1 = \left\{ \varepsilon^{-\frac{L}{\Lambda}} - 1 + \frac{L}{\Lambda} \right\} \tag{45}$$

$$J_2 = \frac{1}{2}\frac{L^2}{\Lambda^2} \int_0^1 dq \int_0^q \varepsilon^{-\frac{L}{\Lambda}\left[(\frac{1}{2}-p)^2 + (\frac{1}{2}-q)^2 - 2(\frac{1}{2}-p)(\frac{1}{2}-q)\cos\alpha\right]^{1/2}} dp \tag{46}$$

The calculations are carried out more easily if K^2 is written finally in the form

$$K^2 = \frac{\dfrac{\frac{L^2}{\Lambda^2}}{J_1 + J_2}}{1 + \dfrac{a^2}{b^2}\dfrac{(J_1 - J_2)}{(J_1 + J_2)}} = \frac{K_1^2}{1 + \dfrac{a^2}{b^2}\dfrac{(J_1 - J_2)}{(J_1 + J_2)}} \tag{47}$$

The ratio $\frac{a^2}{b^2}$ is easily obtained. Consider a hot-wire at an angle to the mean velocity as in the sketch. Due to the v fluctuations the component of velocity normal to the wire is increased by the amount v cos $\alpha/2$, and due to the u fluctuations it is increased by the amount u sin $\alpha/2$. The voltage fluctuation



in the wire was taken as $e = au + bv$. Hence a is proportional to sin $\alpha/2$, and b is related to cos $\alpha/2$ by the same proportionality constant. Hence

$$K^2 = \frac{K_1^2}{1 + \left(\dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\right)^2 \dfrac{(J_1 - J_2)}{(J_1 + J_2)}} \tag{48}$$

where

$$K_1^2 = \frac{\frac{L^2}{\Lambda^2}}{J_1 + J_2} \tag{49}$$

The quantities J_1 and J_2 are given by eqns.(45) and (46).

In calculations of intensity of turbulence the square-root of the output meter reading enters as a multiplying factor. Hence to obtain the true value for the intensity, the measured value must be multiplied by the factor K given by eqn.(48).

The calculation of K from eqn.(48) offers rather considerable difficulties due to the integral J_2 . This integral cannot be evaluated in terms of elementary functions so that graphical methods were resorted to. The large amount of labor involved in this undertaking, and the limited time available, prevented the calculation of as many points as might be desirable, but a sufficient number was computed so that, as a result of careful fairing, the curves of K vs. L/Λ for constant α shown on Fig.5 are felt to represent eqn.(48) with accuracy sufficient for all practical purposes.

Details of the calculation procedure, and sample calculations, will be found in the Appendix.

DISCUSSION

The most important results of this investigation are to be found in Fig.4. Here are plotted the v'/U values obtained with the several v'/U-meters used together with the values of u'/U. Note that three sets of v'/U values are plotted, namely, uncorrected values, values corrected for the u'/U sensitivity# of the meter, and final values corrected both for u'/U sensitivity and for wire length using the correction factor K given in Fig.5. Since all measurements were made at the same distance behind the grids, but using three different grid sizes, three distinct turbulence levels appear. For each grid points corresponding to the same approximation to v'/U are connected with straight lines in order to aid the eye and better enable one to see the data as a whole. The shapes of the curves are of no significance except in so far as they represent the pattern of the points obtained with different meters. The u'/U

Perhaps it should be pointed out explicitly that a v'/U-meter's sensitivity to u'/U has two sources. First of all there is the sensitivity to mean speed variation resulting from the unsymmetry of the meter. It is the correction for this lack of symmetry that is referred to here. Secondly there is the sensitivity to u'/U which even a symmetrical meter would have due to the lack of complete correlation between all elements of the wires. This latter correction is included in the length correction factor K.

turbulence levels behind the grids are indicated by horizontal lines.

This method of representing the data is convenient for the following reason. It is assumed that the correction to be applied due to u'/U sensitivity is valid, and there seems to be no question of this. If the length correction derived in the preceeding section is correct one should expect to obtain a horizontal line representing the final corrected values of v'/U since one purpose of the correction is to compensate for differences in meter dimensions. In other words the dashed lines in Fig.4 should be horizontal, while the solid lines may show deviations due to the fact that they represent uncorrected results. Further, if the turbulence is isotropic behind the grids the dashed lines representing the final corrected values should be the same as the lines indicating the u'/U levels.

Omitting for the moment consideration of the data for the 1" grid, it can be seen from Fig.4 that the application of the length correction to the v'/U values does indeed reduce the variation from the mean value. Some variation remains, but it should be pointed out that the range of meter dimensions is very large leading to correction factors K all the way from about 1.03 for the smallest meter to 1.94 for the largest. With a variation in K of almost 100% while the maximum deviation from the mean value of v'/U is not more than about 10% it would seem that the correction factor is not far from correct.

The turbulence level measured behind the one-inch grid is somewhat higher than that measured by Corrsin in ref.(4). This is not considered of much importance since the measurements were made at a value of $x/M = 20$ where the slope of the decay curve is very steep. For this same reason the data are not considered to be too reliable and it is believed that they must contain a large amount of experimental scatter. However, the general results described in the preceeding paragraph still appear.

The general conclusion at this point is that in a given turbulent field the value of v'/U measured will not depend to a large extent upon the meter used if the corrections for its u'/U sensitivity and length are applied. (This of course is assumed to be true for u'/U measurements) However, even with the length correction the result is found, confirming Corrsin's measurements described in ref. (4), that v'/U is less than u'/U behind the grids. On Figs.6 and 7 are shown u'/U decay curves taken from Fig.5 of ref. (4), and v'/U decay curves corrected for u'/U sensitivity, but not for length, taken from Figs. 7 and 8 of that reference. The length correction derived in the preceeding section has been applied to the uncorrected v'/U curves and the corrected curves are indicated by dashed lines. It is apparent that even with the length correction u'/U is higher than v'/U. If the final decay curves are plotted on logarithmic paper, straight lines result. This has done on Fig.8. Here it can be seen that behind the 1" grid the turbulence is tending toward isotropy, the expected result occuring where the lines intersect almost 1000 mesh lengths downstream from the grid. The turbulence behind the ½" grid seems also to be tending toward isotropy though at a much slower rate than that behind the 1" grid.

The present writer is not qualified to argue the question of whether or not the turbulence behind a grid is isotropic. A strong experimental group, as mentioned previously, has checked the isotropy using the heat diffusion and indirect methods. So far as is known the only attempt to check it directly using hot-wires was that of Dryden and Thiele at the GALCIT in 1940 in which isotropy was found to prevail behind the single grid used. What has been done concerning length corrections for v'/U-meters is not known, although the matter does not appear to have received much attention. This is suprising in view of the rather large correction indicated by the analysis in this report. For a u'/U-meter and a v'/U-meter having wires of the same length the correction for the v'/U-meter will be slightly larger, and will depend on the angle between the wires. In practice the corrections for v'/U-meters will usually be larger than those for u'/U-meters for the reason that it is difficult to build v'/U-meters having wires as short as the wire of an average u'/U-meter. Schubauer's heat diffusion method is, according to G. I. Taylor(8), open to criticism, but it has been extensively used. At the same time it should be true that any arrangement of hot-wires having linear characteristics with respect to angular orientation should be suitable for measuring v'/U.

That the X-type meter used in these investigations does have linear characteristics is shown by Figs.1 and 2. It may be seen that as the angle of yaw nears the half angle of the meter (i.e. $\alpha/2$) the sensitivity increases, but that over a considerable range the calibration curve is quite straight. Of course, if the meter were to be used in a situation in which the amplitude of some of the velocity fluctuations was so large that the meter worked at times on that part of the calibration curve beyond the peak, the average difference in the voltage drops of the wires would be decreased and the turbulence value measured would be too low. However, this is not thought to be the explanation for the results descibed here.

One might summarize this discussion by saying that with the application of the length correction, shown to be at least approximately correct since with it all meters give the same answer, to Corrsin's[4] results and those obtained in the present investigation, all loop-holes in the GALCIT measurements seem to have been closed. Still the isotropy behind regular grids, so generally believed to exist, is not confirmed. No explanation is offered since none is known. One can only point out that the problem appears to be a very interesting one, and one deserving further and careful investigation. Having now established the fact that X-type meters may be counted upon for consistent results if all the corrections are applied, the next step would seem to be to measure v'/U in a given turbulent field using X-type meters, the thermal diffusion method of Schubauer[1], and the method described by Reichardt in ref. (2), and to compare these results.

CONCLUSIONS

The most important conclusions reached in the present investigation are listed below.

1) The length correction for an X-type v'/U-meter is quite appreciable, and will usually be larger than the corresponding correction for a u'/U meter since the difficulty of building an X-type meter results in the use of longer wires than commonly found in a u'/U-meter.

2) The length correction for v'/U meters derived here is thought to be approximately correct since by its use all meters of a group having widely varying dimensions are found to give nearly the same results.

3) The X-type meters with which the experimental part of this investigation was chiefly concerned are shown to have linear characteristics over a range more than sufficient for normal turbulence measurements.

4) The isotropy of turbulence behind a regular grid is not verified, although there is insufficient evidence to say that it is disproved.

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APPENDIX-- Sample Calculations of Length Correction Factor K.

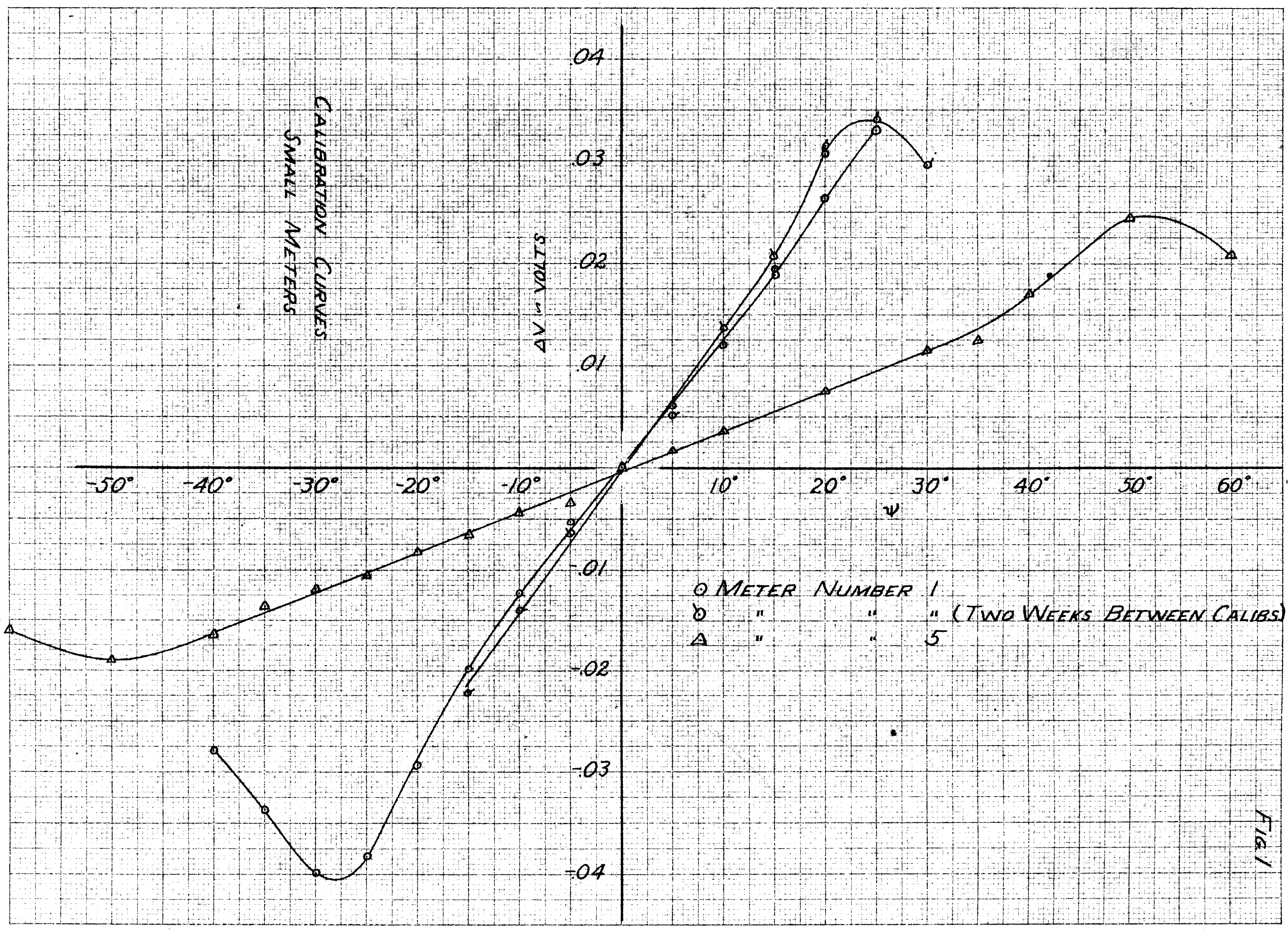
The numerical evaluation of the length correction factor K involves the calculation of the quantities J_1 and J_2 . The first of these offers no difficulty. However, since the integral J_2 cannot be evaluated analytically graphical methods must be resorted to. Since the integral is a double one the process is an exceptionally tedious one. The main steps involved are outlined very briefly in the next paragraph.

The first step is to prepare, for a given wire angle α , a table of values of $\left[(\frac{1}{2}-p)^2+(\frac{1}{2}-q)^2-2(\frac{1}{2}-p)(\frac{1}{2}-q)\cos\alpha\right]^{\frac{1}{2}}$ like the sample shown on the next page. Then for a given value of one may plot curves of $\mathcal{E}^{-\frac{L}{\Lambda}\left[(\frac{1}{2}-p)^2+(\frac{1}{2}-q)^2-2(\frac{1}{2}-p)(\frac{1}{2}-q)\cos\alpha\right]^{\frac{1}{2}}}$ vs. p for constant values of q. The result is a family of curves like that at the bottom of Fig.9. Fortunately symmetry makes it necessary to plot only one-half of the complete family. The area under each of these curves is then measured with a planimeter, or equivalent means, and these in turn plotted against q as shown at the top of Fig.9. The area under this last named curve is proportional to the value of the integral It is necessary to repeat these steps for every pair of values of α and $\frac{L}{\Lambda}$ for which the correction factor is desired.

Values of $\left[\left(\frac{1}{2}-p\right)^2+\left(\frac{1}{2}-q\right)^2-2\left(\frac{1}{2}-p\right)\left(\frac{1}{2}-q\right)\cos\alpha\right]^{\frac{1}{2}}$

$\alpha = 45°$

p \ q	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.382	.356	.358	.386	.435	.500	.576	.657	.745	.834	.925
.1	.356	.306	.283	.295	.336	.400	.476	.560	.649	.740	.834
.2	.358	.283	.230	.212	.241	.300	.377	.464	.555	.649	.744
.3	.386	.295	.212	.152	.148	.200	.280	.370	.464	.560	.657
.4	.435	.336	.241	.148	.077	.100	.148	.280	.377	.476	.567
.5	.500	.400	.300	.200	.100	0	.100	.200	.300	.400	.500
.6	.576	.476	.377	.280	.184	.100	.077	.148	.241	.336	.435
.7	.657	.560	.464	.370	.280	.200	.148	.152	.212	.295	.386
.8	.744	.649	.555	.464	.377	.300	.241	.212	.230	.283	.358
.9	.834	.740	.649	.560	.476	.400	.336	.295	.283	.306	.356
1.0	.925	.834	.745	.657	.576	.500	.435	.386	.358	.356	.382


CALIBRATION CURVES
SMALL METERS
ΔV ~ VOLTS
.04
.03
.02
.01
-.01
-.02
-.03
-.04
-50°
-40°
-30°
-20°
-10°
10°
20°
30°
40°
50°
60°
ψ
⊙ METER NUMBER 1
" " " (TWO WEEKS BETWEEN CALIBS)
△ " " 5
FIG. 1

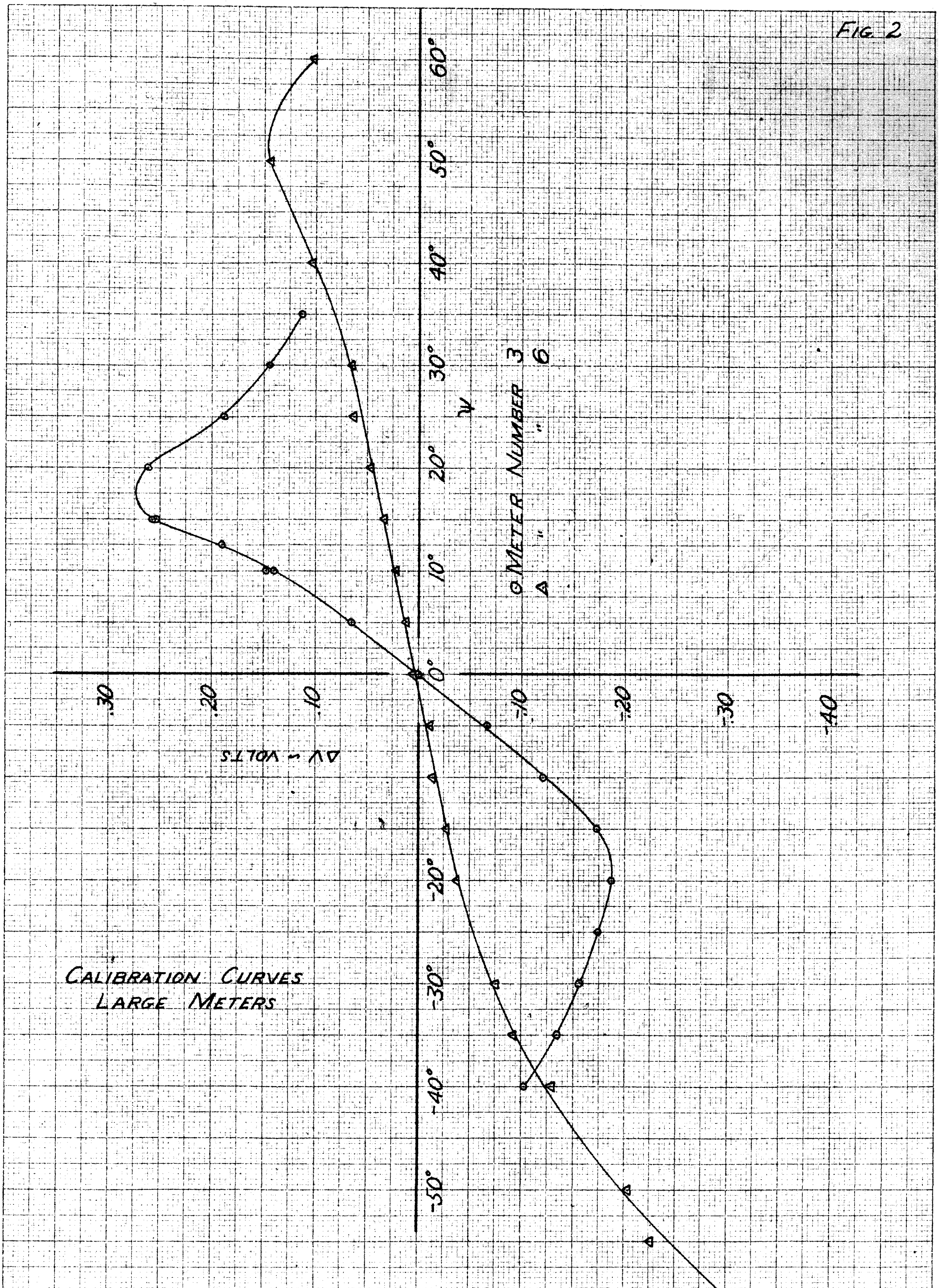
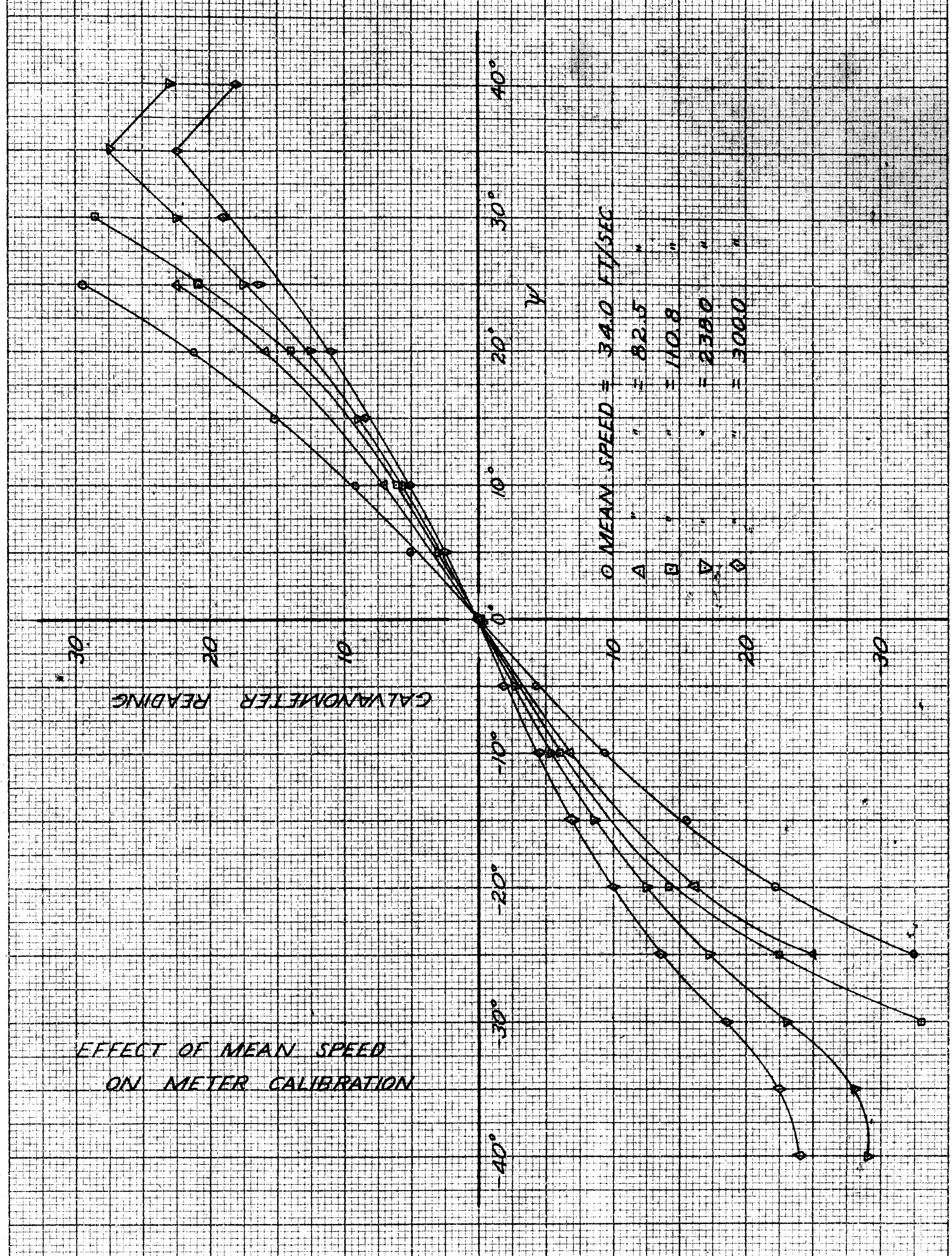


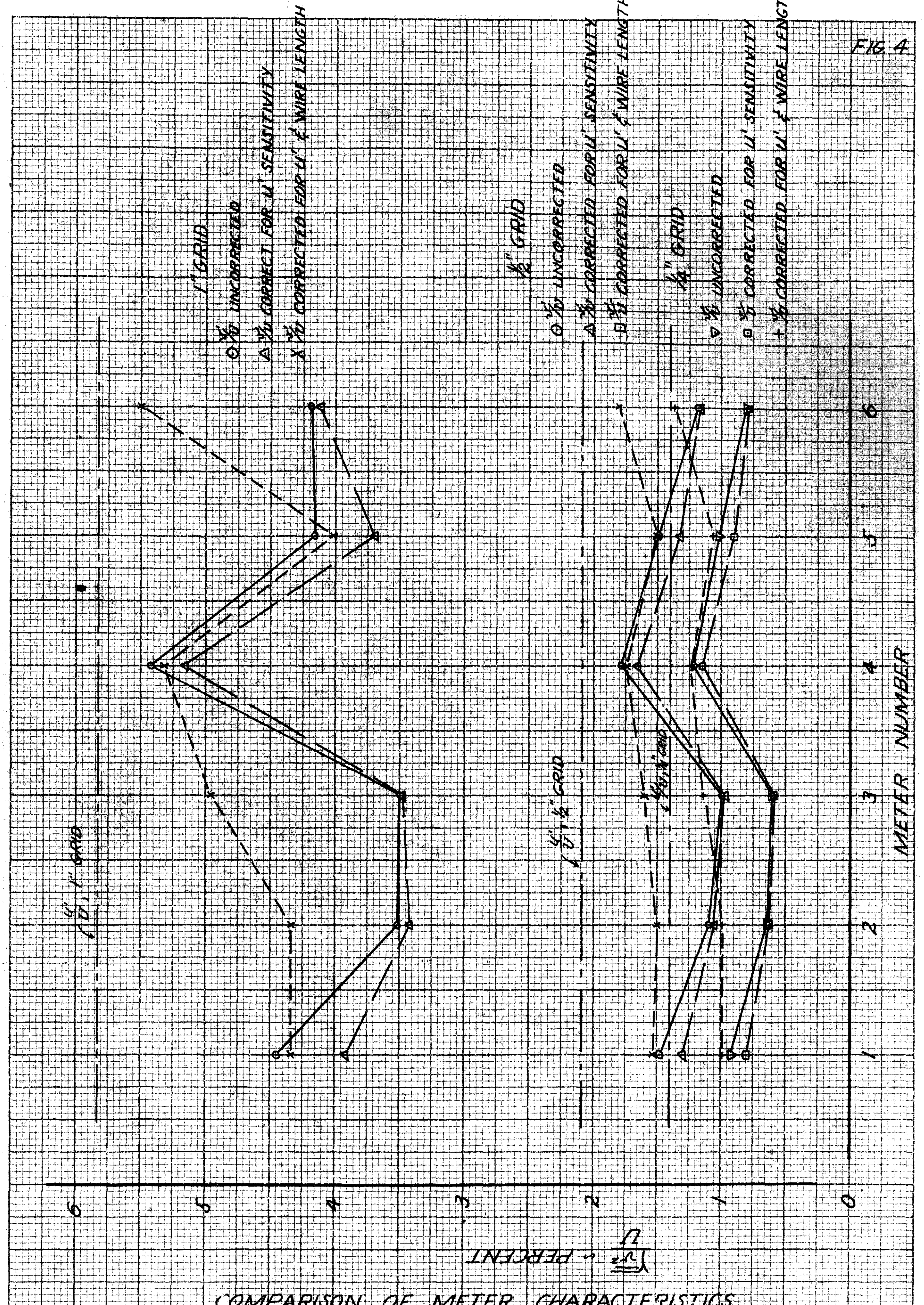
FIG 2

CALIBRATION CURVES
LARGE METERS

FIG. 3



EFFECT OF MEAN SPEED
ON METER CALIBRATION



COMPARISON OF METER CHARACTERISTICS

FIG. 5

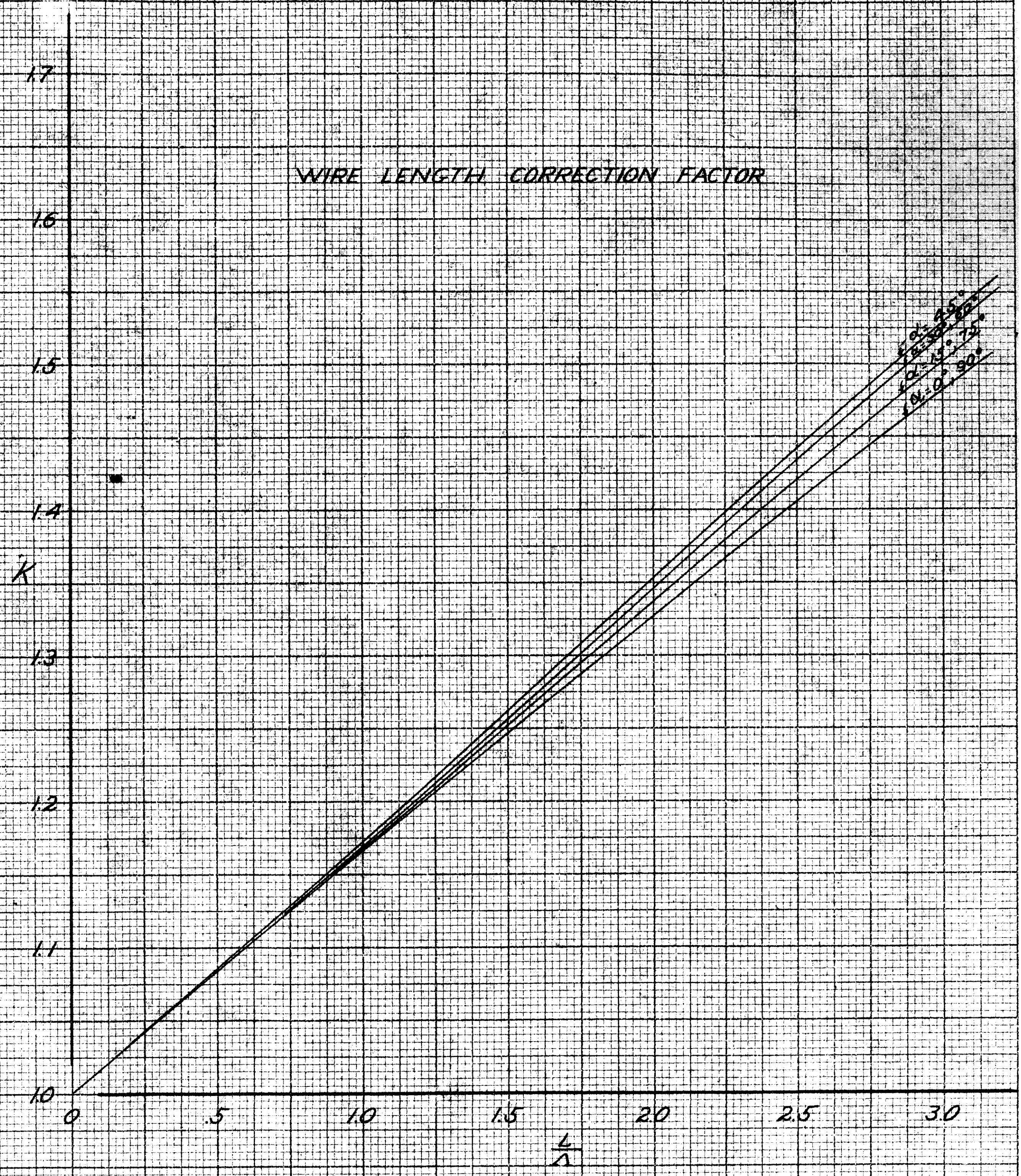

WIRE LENGTH CORRECTION FACTOR
K
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
.5
1.0
1.5
2.0
2.5
3.0
L/λ
α = 45°
α = 30°, 60°
α = 15°, 75°
α = 0°, 90°

FIG. 6

TURBULENCE DECAY
1" GRID

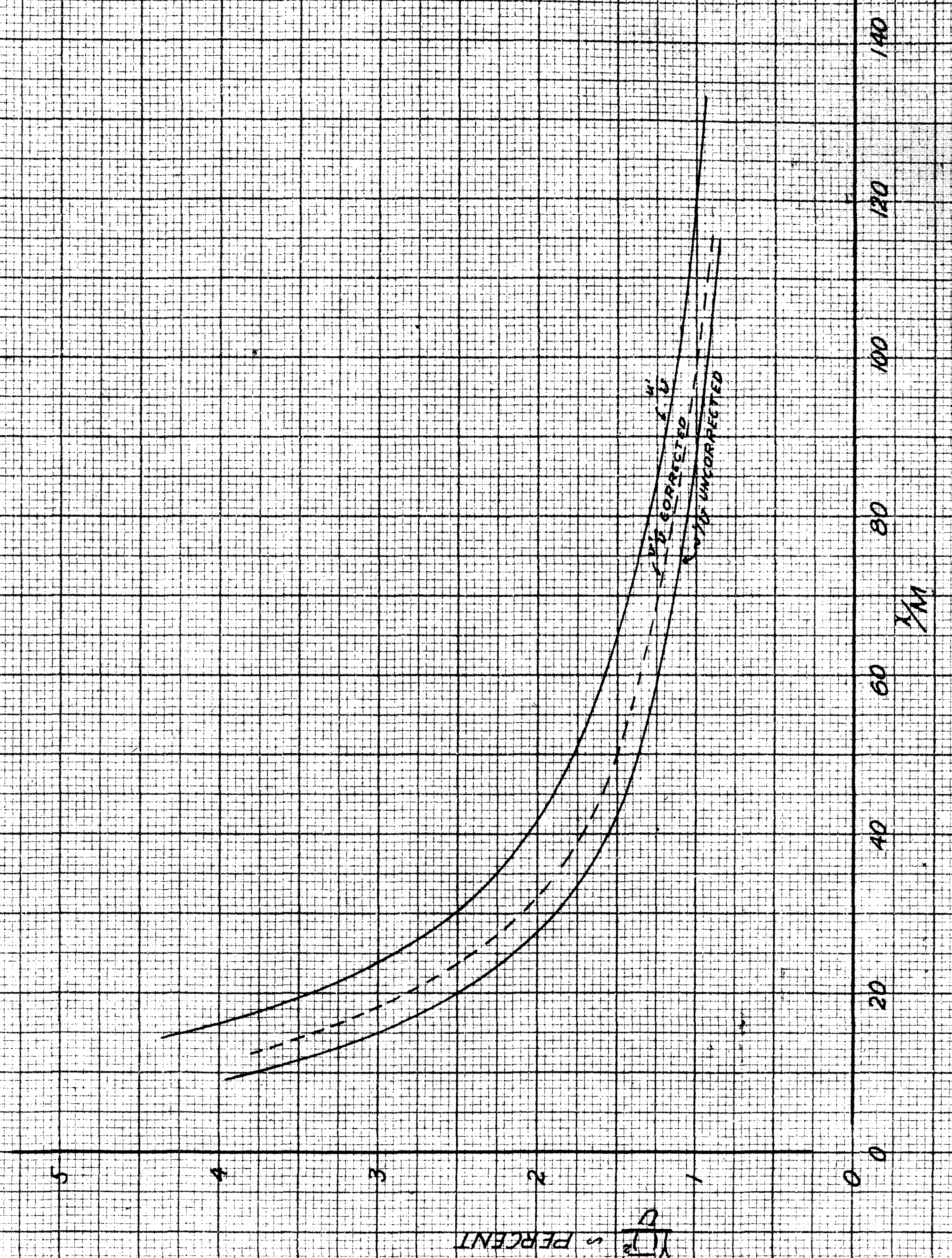
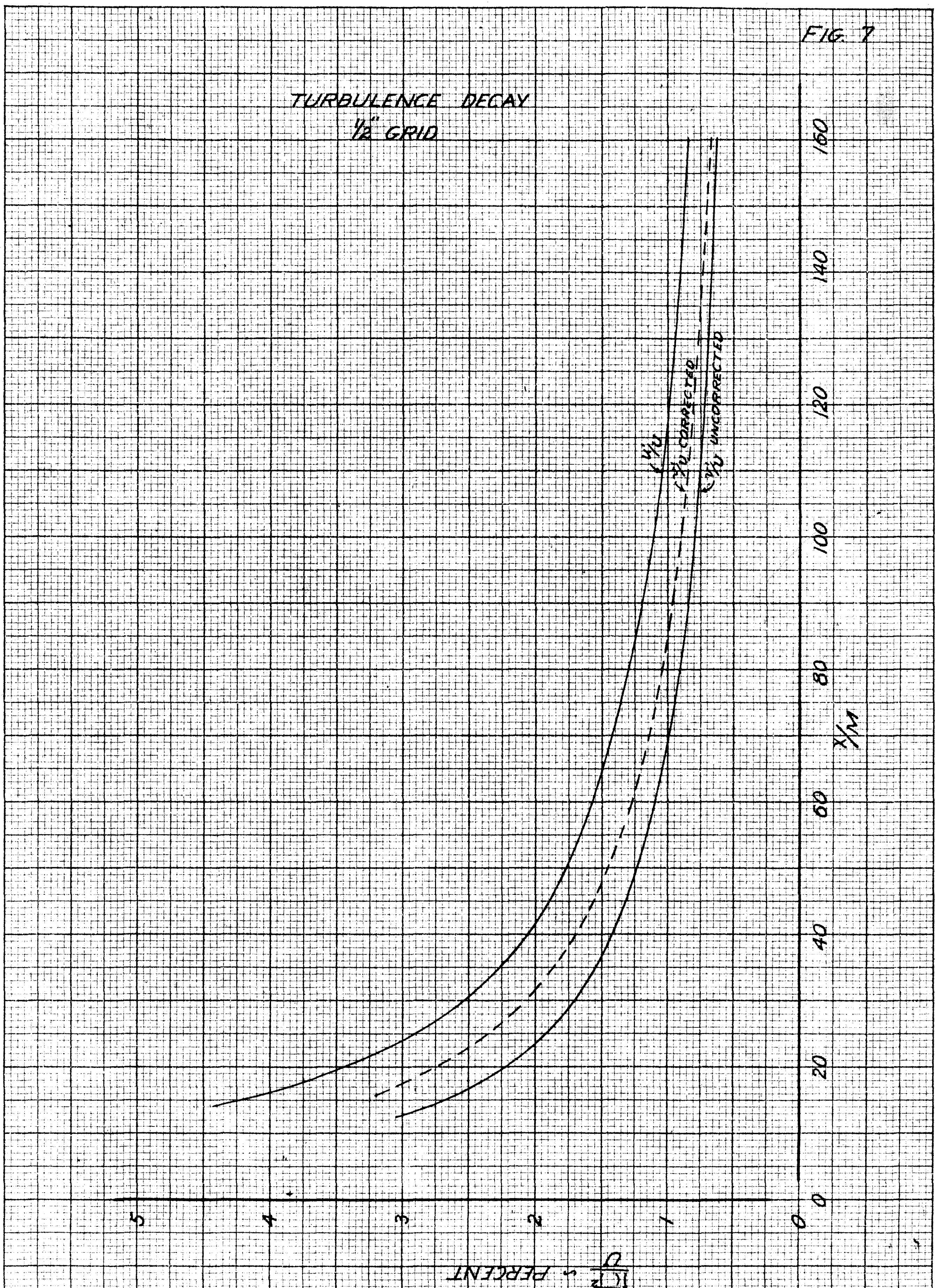
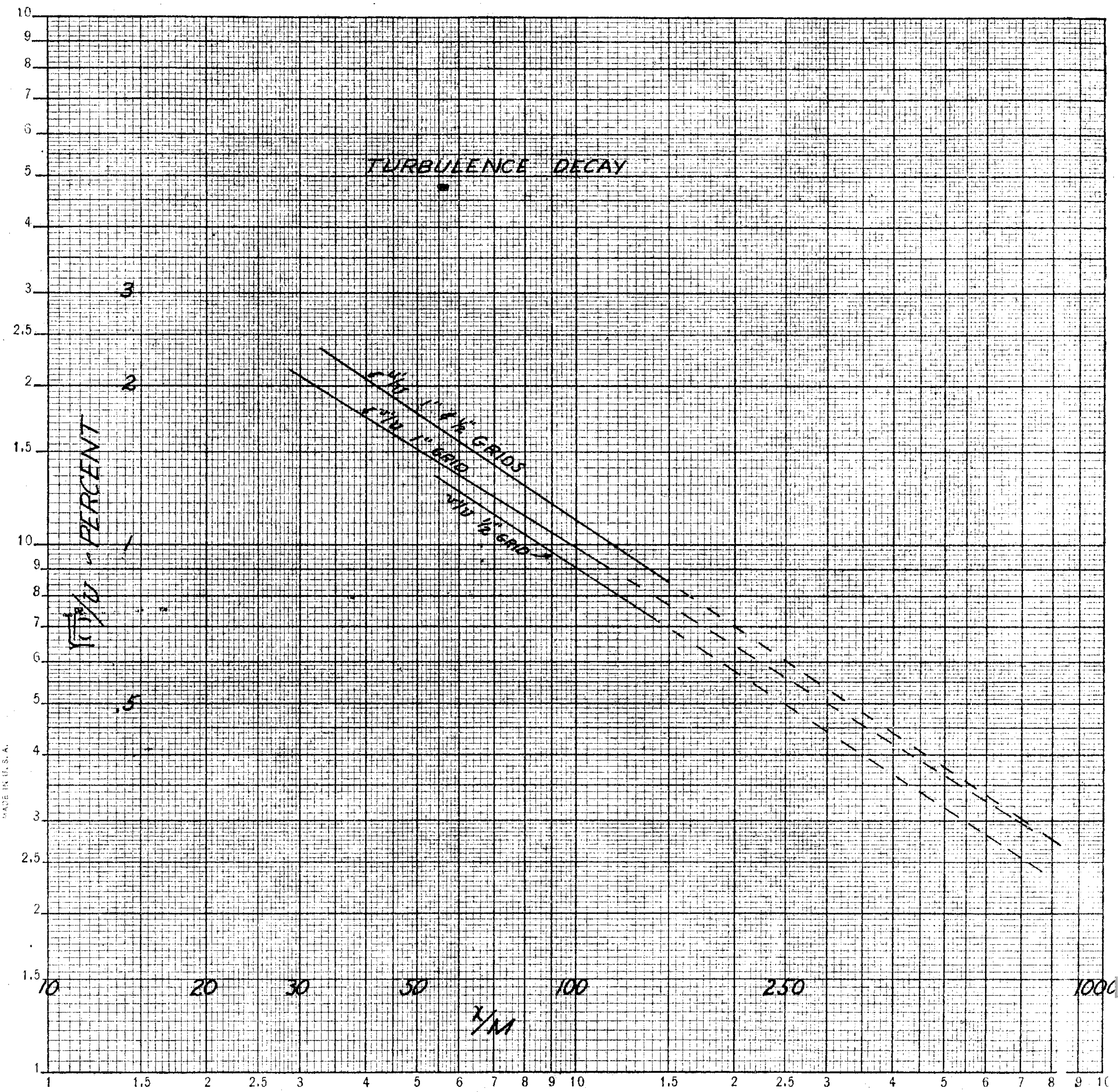


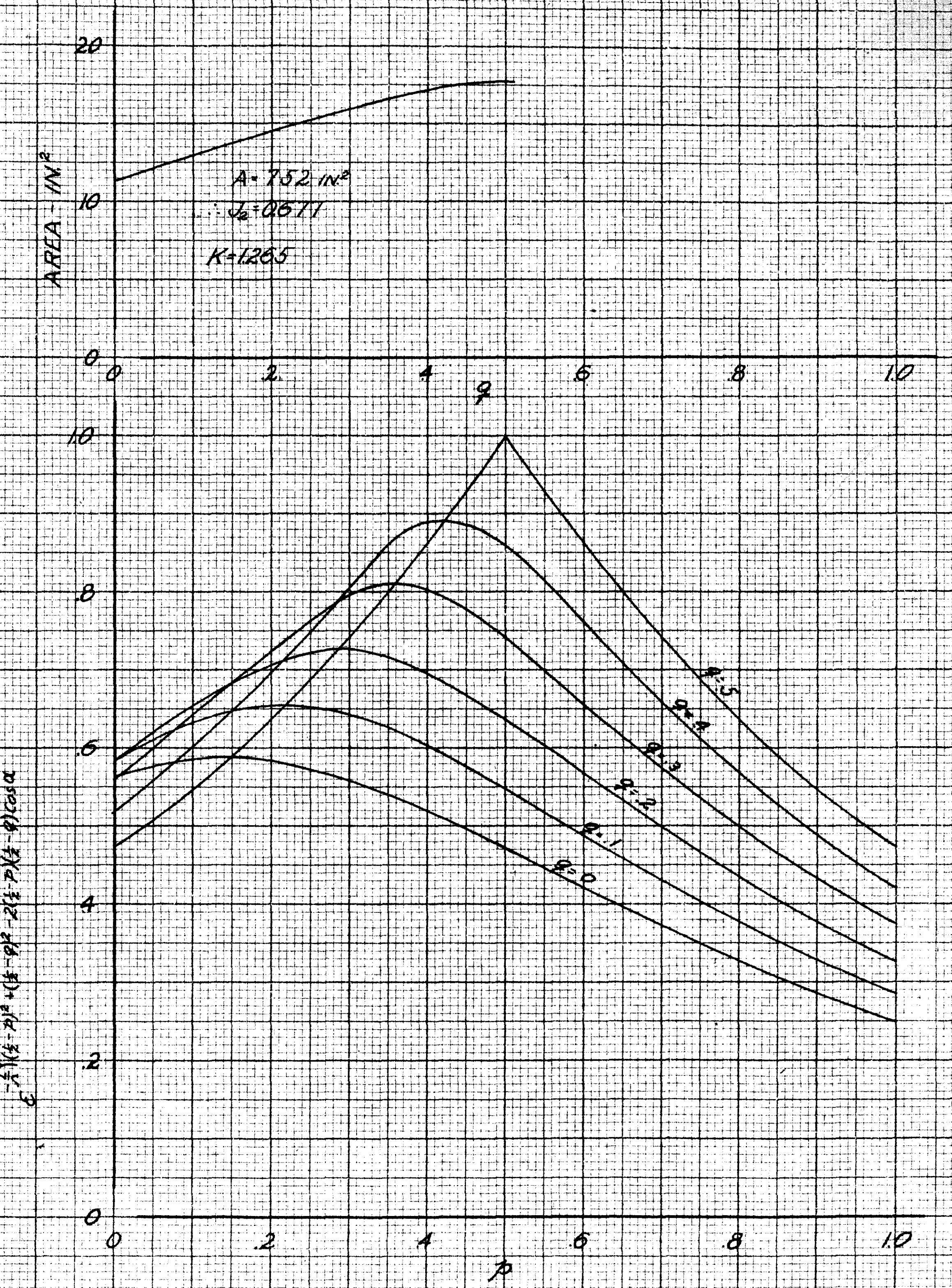
FIG. 7



TURBULENCE DECAY
1/2" GRID
u'/U
v'/U CORRECTED
v'/U UNCORRECTED
X/M
0
20
40
60
80
100
120
140
160
0
1
2
3
4
5
PERCENT

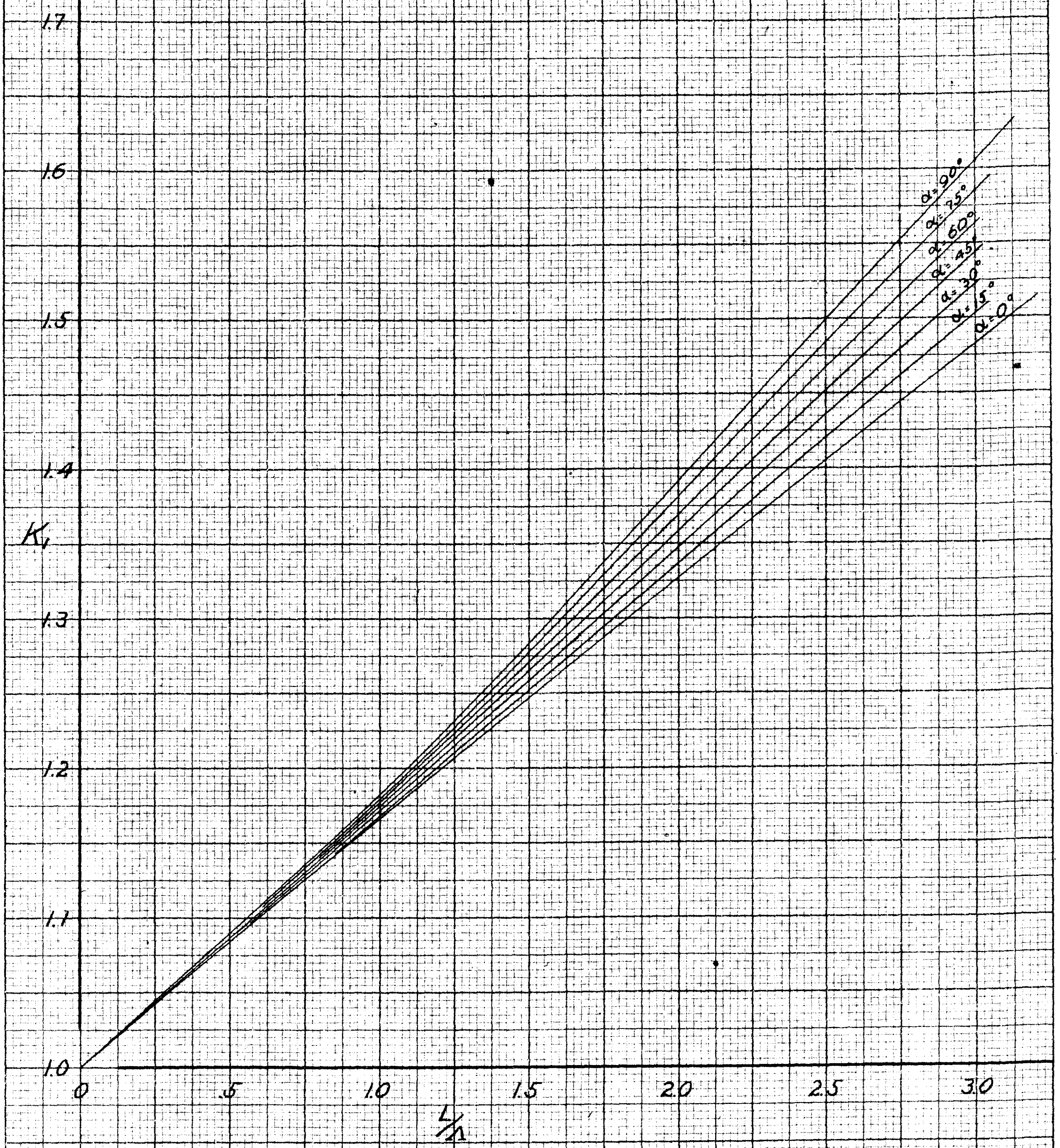
FIG. 8





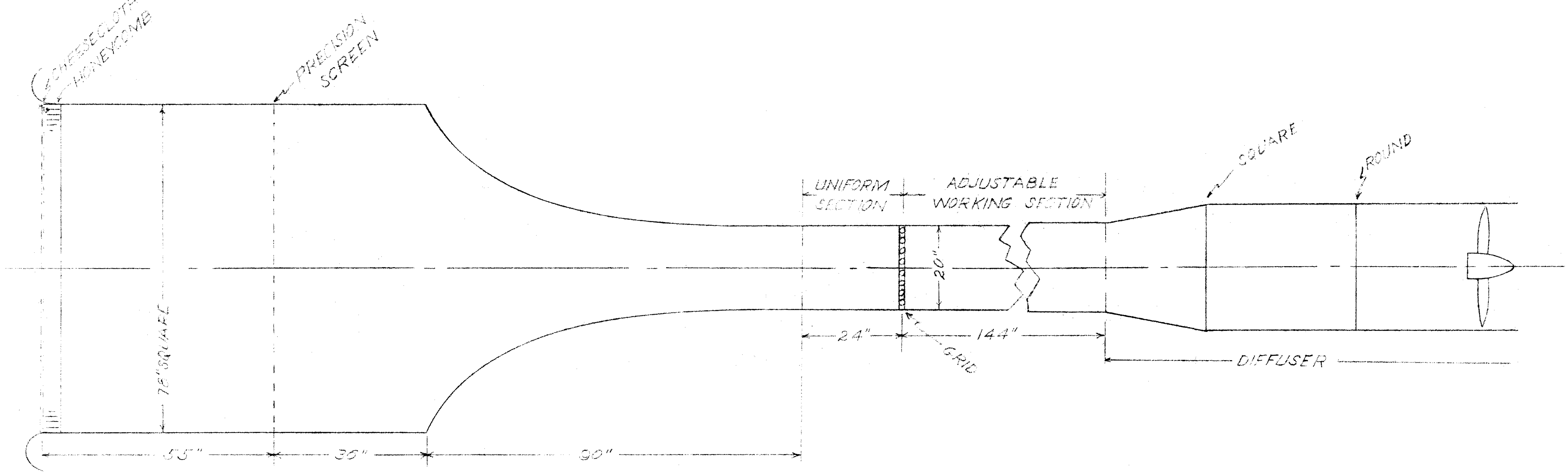
SAMPLE INTEGRATION – $\frac{L}{\Lambda} = 1.5$; $\alpha = 45°$

FIG. 10



1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
K₁
0
.5
1.0
1.5
2.0
2.5
3.0
L/λ
α=90°
α=75°
α=60°
α=45°
α=30°
α=15°
α=0°

FIG. 11



SKETCH OF WIND TUNNEL

MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	TOLERANCES ± .010 OR 1/64 UNLESS OTHERWISE NOTED
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			NAME				DRAWING NO.