COMBUSTION GAS TURBINES FOR AIRPLANES

Thesis by

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DIGEST OF THESIS ON COMBUSTION GAS TURBINES FOR AIRPLANES

This paper contains a theoretical study of various thermodynamic cycles for Gas-Turbine-propeller propulsion units for aircraft.

The object of the study was to evaluate the turbine-propeller engines as a source of power for fast, economical, long range airplanes. Hence, the consumption of fuel and the total weight of fuel and power plant were the main criterion for the evaluation. Since the weights of the particular units were not known, most of the results here are centered around the fuel consumption per horsepower, emphasizing the design criteria and throttle conditions for minimum values.

In order to emphasize the maximum possible performance, formulae for ideal engines were developed for all cases. These relations are compared with the corresponding ones involving unknown efficiencies. A summary table of these comparison formulae is given at the close of the thesis.

In the case of the cycles where there was sufficient experimental data available to estimate the performance of the component parts, the relations for power and specific fuel consumption were written in terms of three parameters which were convenient for graphical use in design purposes.

This published paper is presented with the consent of Mr. Ivan H. Driggs, Chief of the Aviation Design Research Branch, Bureau of Aeronautics, Navy Department. The author wishes to express his appreciation to Mr. R. J. Volluz, who checked the report and inserted the formulae in the vellum copy.

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COMBUSTION GAS TURBINES FOR AIRPLANES

1. Introduction: The purpose of this paper is to study the commercial importance of turbine engines as a source of power for fast, economical long-range airplane cruising at high altitudes. One of the most important items in this connection is specific fuel consumption. This report gives the development of formulae for specific fuel consumption, which are sufficiently accurate for the purpose of engineering exploration of possibilities and which shows the effects of compression ratio, temperature ratio, compressor efficiency and turbine efficiency. Formulae for ideal, useful, and lost horsepower are also developed.

2. Combustion Gas Turbine: A turbine engine is composed of a diffuser, a compressor, a burner, a turbine and a nozzle in sequence. Air enters the motor through a diffuser which reduces its speed, thereby compressing it. It is further compressed by an axial or centrifugal compressor, then is passed through a combustion chamber where it is heated at approximately constant pressure. The heated gases (the air and gases formed by combustion) are then passed through a turbine where the gas pressure is reduced to atmospheric pressure and most of the available energy is converted into mechanical energy in a shaft. Part of the shaft power is used to drive the compressor and the remainder is available to do external work. The remaining available energy, which was not taken out by the turbine, gives the exit gas a a velocity, V_j, which usually exceeds the initial velocity, V_o, of the entering air. When the engine is in an airplane the difference in the kinetic

energy of the entering and exiting gas may be utilized to supply additional jet thrust.

There is one main difference between the jet engine and the turbine engine. As the names indicate the main source of power for the gas jet engine is from the jet but the main source of power for the combustion gas turbine engine is the turbine. In the jet motor the turbine develops only enough power to run the compressor, the thrust is wholly determined by the difference in the momenta of the entering and leaving gases; while in the turbine engine, the turbine develops as much power as possible, only part of this power is used to run the compressor, the remainder is used to do external work, only a small part of the total power is obtained from the jet thrust.

3. <u>Definition of Symbols;</u> In order to understand the developments which follow it is necessary to clearly define the symbols employed.

The points 0, 1, 2, 3, 4, - - - in the enthalpy-entropy diagram represent the properties of the gas at the various stages.

0. Entrance to diffuser.
1. Entrance to compressor.
2. Entrance of burner.
3. Entrance of turbine.
4. Exit of nozzle.
5. Entrance to second turbine.
6. Exit of nozzle after second turbine.
7. Exit of first turbine-entrance to second burner.

8. Exit of refrigerator-entrance to compressor.

9. Exit of compressor in refrigeration cycle.

10. Exit of heat exchanger.

11. Exit of nozzle in jet motor.

12. Entrance to intercooler.

13. Exit of intercooler-entrance to second compressor.

14. Exit of second compressor.

p_i denotes the pressure at the stage i, $i = 0, 1, 2, 3, 4, ----, 14$

T_i denotes the actual temperatures at stage i, $i = 0, 1, 2, 3, 4, -, 14$

T_i' denotes the ideal temperature to obtain the actual pressure at stage i,

$i = 0, 1, 2, 3, 4, ---, 14.$

V_o denotes the entering velocity of air (forward speed of airplane).

V_j denotes the jet velocity.

c_i denotes the velocity of air at stage i, $i = 1, 2, 3, 4, 5, --, 14$.

η_d denotes diffuser efficiency.

η_t denotes turbine efficiency.

η_c denotes compressor efficiency.

η_B denotes burner combustion efficiency.

η_j denotes jet efficiency.

η_f denotes Froude efficiency, $\eta_f = \dfrac{2}{1 + \frac{V_j}{V_o}}$

η_p denotes propeller efficiency.

η_{st} denotes small stage efficiency of turbine.

η_{sc} denotes small stage efficiency of compressor.

η_{sd} denotes small stage efficiency of diffuser.

η_{sj} denotes small stage efficiency of jet.

η_e efficiency of heat exchanger.

η_r efficiency of refrigerator.

η_g efficiency of gears.

W_A = rate of air flow. (lb./sec.)

W_F = fuel rate. (lb./hr.)

g = acceleration due to gravity. (ft./sec.^{2})

J = 788 (ft.lbs./B.T.U.)

c_p = specific heat at constant pressure.

c_v = specific heat at constant volume.

H_s = shaft power. (ft.lbs./sec.)

Q = quantity of heat added. (B.T.U./sec.)

h = enthalpy.

a_i = velocity of sound in air at Temperature T_i.

$M_i = \frac{c_i}{a_i}$ Mach Number at Temperature T_i.

$\gamma = \frac{c_p}{c_v}$

$R = J\,(c_p - c_v)$ gas constant.

$$\rho_D = \frac{P_1}{P_o}, \quad \rho_c = \frac{P_2}{P_1}, \quad \rho_b = \frac{P_3}{P_2}, \quad \rho_t = \frac{P_o}{P_3}, \quad \rho = \frac{P_2}{P_o}.$$

$$\tau = \frac{T_3}{T_o} \qquad \tau_2 = \frac{T_5}{T_o}$$

$$a = \rho^{\frac{\gamma-1}{\gamma}}$$

$$b = 1 + \frac{\gamma-1}{2} M_o^2$$

s.f.c. = Specific fuel consumption for Cycle I.

e_i = ideal power coefficient.

e_u = useful power coefficient for Cycle I.

e_l = $e_i - e_u$, lost horsepower coefficient.

P = total propulsive horsepower.

Roman superscripts on e_u, s.f.c., H_s, W_A, - - - , refer to the cycle number, II, III, IV, - - -

$\approx$ denotes an approximate equality.

4. <u>Thermodynamic Cycle I - Simple Combustion Gas Tu~~be~~rbine</u>

It is assumed that the pressure changes in the diffuser, compressor, and turbine obey the adiabatic law. It is to be noted that an irreversible adiabatic change of state is not ruled out. In the ideal case, where all the components of the engine are 100% efficient, the compression of the gas by ram and by the compressor would raise the temperature from T_0 to T_2', the heat added would raise the temperature to T_3, and the turbine would return the gas to atmospheric pressure at a temperature T_4'. Since the component parts are not 100% efficient, entropy is ~~lost~~ gained during each stage. Hence, the lines on the enthalpy-entropy diagram are to the right of the vertical and less energy is available to do work. Even in the ideal case the efficiency of the engine is not 100% of that of a Carnot Cycle because the air has a higher temperature when it leaves the engine than when it entered. Energy is dissipated. The process is nonreversible.

In the discussion of the problem it is convenient to follow a thermodynamic cycle. Figure 1 gives a graph of the cycle in an enthalpy-entropy diagram.

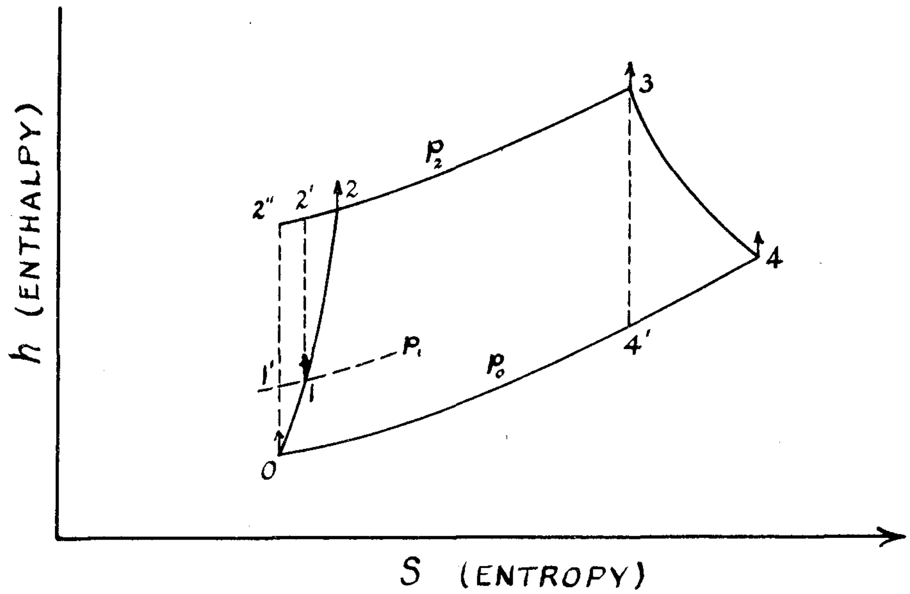

Figure I.

Since energy cannot be created or destroyed, an equation which relates the energy of the entering air and the exit air will reveal the possible output in shaft work such an engine could be expected to deliver. The increase in weight of the exit gas due to combustion is neglected in this report. Thus, the energy equation is

$$(1)\quad W_A \frac{V_0^2}{2g} + h_0 + Q = W_A \frac{V_j^2}{2g} + h_4 + H_S ,$$

or

$$H_S = W_A \left(\frac{V_0^2}{2g} - \frac{V_j^2}{2g} \right) + h_0 - h_4 + Q .$$

Now, for a perfect gas

$$h_0 - h_4 = W_A c_p J (T_0 - T_4),$$

and

$$Q = W_A c_p (T_3 - T_2) \qquad [B.T.U./SEC.],$$

or

$$W_A c_p J (T_3 - T_2) \qquad [FT. LBS./SEC.].$$

Therefore,

$$H_S = W_A \left(\frac{V_0^2}{2g} - \frac{V_j^2}{2g}\right) + W_A c_p J (T_0 - T_4) + W_A c_p J (T_3 - T_2)$$

$$= W_A c_p J \left[(T_0 - T_4) + (T_3 - T_2)\right] + W_A \left(\frac{V_0^2}{2g} - \frac{V_j^2}{2g}\right)$$

$$= W_A c_p J \left[(T_0 - T_1) + (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + (T_3 - T_2)\right] + W_A \left(\frac{V_0^2}{2g} - \frac{V_j^2}{2g}\right),$$

$$(2) \quad H_S = W_A c_p J \left[(T_3 - T_4) - (T_2 - T_1) - (T_1 - T_0)\right] + W_A \left(\frac{V_0^2}{2g} - \frac{V_j^2}{2g}\right).$$

In a moving airplane the shaft power is supplemented by the jet power $W_A \left[\frac{V_j^2}{2g} - \frac{V_0^2}{2g}\right]$. Under the assumption that the propeller efficiency is equal to the Froude efficiency (the propulsive efficiency of the jet) and that the gear efficiency is 100%, the sum of the thrust horsepower of the jet and the shaft yields.

$$(3) \quad \frac{1}{\eta_p} P = \frac{W_A c_p J}{550} \left[(T_3 - T_4) - (T_2 - T_1) - (T_1 - T_0)\right],$$

$$= \frac{W_A c_p J T_0}{550} \left[\frac{T_3}{T_0}\left(1 - \frac{T_4}{T_3}\right) - \frac{T_1}{T_0}\left(\frac{T_2}{T_1} - 1\right) - \left(\frac{T_1}{T_0} - 1\right)\right].$$

This relation divided by $\frac{W_A c_p J T_o}{550}$ gives the power coefficient -

$$(4) \qquad e_u = \eta_p \left[\tau \left(1 - \frac{T_4}{T_3}\right) - \frac{T_1}{T_o}\left(\frac{T_2}{T_1} - 1\right) - \left(\frac{T_1}{T_o} - 1\right)\right].$$

The values of the temperature ratios may be expressed in terms of the pressure ratios. The relations for the diffuser, the compressor, the burner and the turbine are considered in order.

<u>The diffuser:</u>

$$\frac{V_o^2}{2g} - \frac{c_1^2}{2g} = c_p J \, (T_1 - T_o) \quad ,$$

$$\frac{V_o^2}{2g\,c_p J T_o} - \frac{c_1^2}{2g\,c_p J T_1} \cdot \frac{T_1}{T_o} = \frac{T_1}{T_o} - 1 \quad .$$

Now,

$$a_o^2 = g \gamma R T_o \quad ,$$

$$c_p J = \frac{\gamma R}{\gamma - 1} \quad .$$

Thus,

$$\frac{\gamma-1}{2}\left(M_o^2 - M_1^2 \frac{T_1}{T_o}\right) = \frac{T_1}{T_o} - 1 \;,$$

$$b = \frac{T_1}{T_o}\left(1 + \frac{\gamma-1}{2} M_1^2\right) \;..$$

In flight M_1 is usually small compared to M_o, when M_1 is treated as zero, T_1 is the stop temperature (relative to the airplane), under this hypothesis $b = \frac{T_1}{T_o}$. This assumption is made in the following derivations.

By definition

$$\frac{T_1' - T_0}{T_1 - T_0} = \eta_d \ ,$$

$$\frac{T_1'}{T_0} - 1 = \eta_d\left(\frac{T_1}{T_0} - 1\right) \ ,$$

$$\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \eta_d\left(\frac{T_1}{T_0} - 1\right) \approx 1 + \eta_d(b-1) .$$

The Compressor:

Let $$\eta_c = \frac{T_2' - T_1}{T_2 - T_1} .$$

Then $$\frac{T_2}{T_1} - 1 = \frac{1}{\eta_c}\left(\frac{T_2'}{T_1} - 1\right) \ ,$$

$$\frac{T_2}{T_1} - 1 = \frac{1}{\eta_c}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] .$$

Now, $$a = \rho^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$ Thus,

$$\frac{T_2}{T_1} - 1 = \frac{1}{\eta_c}\left[\frac{a}{1+(b-1)\eta_d} - 1\right] .$$

The Burner: There is usually a small drop in pressure over the burner, thus $\rho_b < 1$. Let $\dfrac{1}{\rho_b^{\frac{\gamma-1}{\gamma}}} = f$.

The Turbine:

Let $$\eta_t = \frac{T_4 - T_3}{T_4' - T_3} .$$

Then,
$$1 - \frac{T_4}{T_3} = \eta_t \left(1 - \frac{T_4'}{T_3}\right) .$$

Now,
$$\rho_D \; \rho_c \; \rho_b \; \rho_t = 1 \quad ,$$

$$\rho_t^{\frac{\gamma-1}{\gamma}} = \frac{1}{\rho^{\frac{\gamma-1}{\gamma}} \cdot \rho_b^{\frac{\gamma-1}{\gamma}}} = \frac{f}{a} \quad ,$$

$$\frac{T_4'}{T_3} = \rho_t^{\frac{\gamma-1}{\gamma}} \quad ,$$

$$1 - \frac{T_4}{T_3} = \eta_t \left(1 - \frac{f}{a}\right) .$$

Thus, the power coefficient is

$$\text{(5)} \qquad e_u = \eta_p \left[\eta_t \tau \left(1 - \frac{f}{a}\right) - \frac{b}{\eta_c}\left(\frac{a}{1 + (b-1)\eta_d} - 1\right) - (b-1) \right] ,$$

or
$$e_u = \eta_p \left[\eta_t \tau \left(1 - \frac{f}{a}\right) - \frac{a-f}{\eta_c} + \epsilon_1 \right] ,$$

where,
$$\epsilon_1 = \frac{a-f}{\eta_c} - \frac{b}{\eta_c}\left(\frac{a}{1 + (b-1)\eta_d} - 1\right) - (b-1)$$

$$= \frac{1-f}{\eta_c} + \frac{b-1}{\eta_c\left[1 + (b-1)\eta_d\right]} \Big[(a-1)(\eta_d - 1) - \eta_c(1 - \eta_d)$$

$$+ \eta_d \, b \, (1 - \eta_c) \Big] ,$$

In order to gain analytic simplicity it is convenient to write an approximate expression for e_u by omitting ϵ_1. The approximate value is then -

$$(6)\qquad \boxed{e_u \approx \frac{a-f}{\eta_c a}\left(\tau\eta_c\eta_t - a\right)\eta_p}$$

If $f = \eta_d = 1$, ϵ_1 is positive and (6) gives a conservative estimate for the power coefficient. With the usual expected efficiencies, ϵ_1 may be either positive or negative and (6) may be in error as much as 4%.

When $\eta_p = \eta_c = \eta_b = \eta_d = f = 1$

relations (5) and (6) yield,

$$(7)\qquad \boxed{e_i = \frac{a-1}{a}\left(\tau - a\right)}\ .$$

This is the maximum value of the power coefficient and is called the ideal power coefficient. The expression for it is identical with the ideal power coefficient for jet motors, discussed in Bollay's report on "Performance Analysis of Gas-Turbine-Jet-Propulsion Units for Aircraft".

The difference between e_u and e_i shall be called the lost horsepower coefficient and is denoted by e_l.

$$(8)\qquad \boxed{e_l \approx \frac{a-1}{a}\left(\tau - a\right) - \frac{\eta_p\left(\tau\eta_c\eta_t - a\right)\left(a-f\right)}{a\,\eta_c}}\ .$$

What are the values of τ and a which give the maximum value for e_u ? It is evident from relations (5) and (6) that the power coefficient increases linearly with τ . From a practical point of view there is an upper limit for the values of τ , hence it is logical to ask what should be the design value of a which will make e_u a maximum for a fixed τ ?

The logarithmic derivative of (6), with respect to a, yields

$$\frac{\frac{\partial e_u}{\partial a}}{e_u} = -\frac{1}{a} + \frac{1}{a-f} + \frac{\tau \eta_t \frac{\partial \eta_c}{\partial a} + \tau \eta_c \frac{\partial \eta_t}{\partial a} - 1}{A - a}, \text{ where } A = \eta_c \eta_t \tau .$$

At the design values for a, η_c and η_t have their maximum, thus

$$\frac{\partial \eta_c}{\partial a} = \frac{\partial \eta_t}{\partial a} = 0 .$$

A necessary condition for a maximum value of e_u is $\frac{\partial e_u}{\partial a} = 0$

and when $e_u \neq 0$, this implies

$$-\frac{1}{a} + \frac{1}{a-f} - \frac{1}{A-a} = 0 ,$$

$$\frac{f}{a(a-f)} = \frac{1}{A-a} , \qquad a = \sqrt{Af} .$$

On the interval $1 < a < A$, $e_u > 0$

and $e_u = 0$, when $a = f$ and $a = A$;

thus, $a = \sqrt{Af}$ gives a maximum value for e_u . It is

$$e_u = \eta_p \frac{(\sqrt{A} - \sqrt{f})^2}{\eta_c} .$$

Specific Fuel Consumption:

The specific fuel consumption is defined to be the ratio of the rate of fuel per hour to the useful horsepower developed: i.e.,

(10) $$s.f.c. = \frac{550\ W_F}{c_p J T_o W_A e_u}$$

Under the hypothesis that $\eta_b = 1$, and the lower combustion value of the fuel is 19,000 B.T.U. per lb.,

$$19{,}000\ W_F = 3600\ c_p W_A (T_3 - T_2)$$

$$= 3600\ c_p W_A T_o \left(\frac{T_3}{T_o} - \frac{T_2}{T_o}\right)$$

(11) $$= 3600\ c_p W_A T_o \left[\tau - b\left(1 + \frac{1}{\eta_c}\left\{\frac{a}{1+(b-1)\eta_d} - 1\right\}\right)\right]$$

(11') $$= 3600\ c_p W_A T_o \left[\tau - b - \frac{a-b}{\eta_c} + \frac{a(b-1)(\eta_d - 1)}{\eta_c[1+(b-1)\eta_d]}\right]$$

Again, an approximation is made by deleting the term $\frac{a(b-1)(\eta_d-1)}{\eta_c[1+(b-1)\eta_d]}$.

Since this term is greater than or equal to zero, the true value of the fuel consumption will be greater than or equal to the approximate value

(11'') $$W_F \approx \frac{36\ W_A c_p T_o}{190}\left[\tau - b - \frac{a-b}{\eta_c}\right].$$

Therefore, an approximate value of the specific fuel consumption is the ratio of expression (11') to the propulsive power, or

$$\text{s.f.c.} = \frac{1}{\eta_p}\,\frac{(55)(36)}{(19)(778)}\,\frac{\tau - b - \frac{a-b}{\eta_c}}{\frac{a-f}{a\,\eta_c}\left(\tau\,\eta_c\,\eta_t - a\right)} \;.$$

Or,

(12) $$\text{s.f.c.} = \frac{.134}{\eta_p}\,\frac{a\,(B-a)}{(a-f)(A-a)} \;,$$

where

(13) $$\begin{cases} A = \tau\,\eta_c\,\eta_t \\ B = \tau\,\eta_c + (1-\eta_c)\,b \end{cases}$$

In the ideal case, where $f = \eta_p = \eta_c = \eta_t = \eta_d = 1$,

(14) $$(\text{s.f.c.})_i = .134\,\frac{a}{a-1} \;.$$

For economical cruising the specific fuel consumption must be low. It is natural to ask, "What should be the design values of a for optimum s.f.c.?" Again, it is assumed that the $\dfrac{\partial \eta_t}{\partial a} = 0 = \dfrac{\partial \eta_c}{\partial a}$,

that is, the efficiencies have their maximum values at the design value of a yet to be determined. The logarithmic derivative of (12) with respect to a is:

$$(15)\qquad \frac{\dfrac{\partial(s.f.c.)}{\partial a}}{s.f.c.} = \frac{1}{a} - \frac{1}{a-f} - \frac{1}{B-a} + \frac{1}{A-a} \; .$$

$\frac{\partial(s.f.c.)}{\partial a} = 0$, $s.f.c. \neq 0$, implies the right hand side of equation (15) vanishes. This relation coupled with relations (13) leads to a quadratic equation in x, where $X = \frac{B-a}{A-a}$.

The verification follows:

$$\frac{1}{a} - \frac{1}{a-f} - \frac{1}{B-a} + \frac{1}{A-a} = 0 \; ,$$

$$(15')\qquad -\frac{f}{a(a-f)} - \frac{1}{B-a} + \frac{1}{A-a} = 0 \; ,$$

$$1 - \frac{A-a}{B-a} = \frac{f(A-a)}{a(a-f)} \; ,$$

or

$$(16)\qquad \left(1 - \frac{1}{X}\right) = \frac{f(A-a)}{a(a-f)} \; .$$

From relations (13)

$$B = \frac{A}{\eta_t} + (1-\eta_c)b \; ,$$

$$\eta_t(B-a) = (A-a) + a(1-\eta_t) + (1-\eta_c)\eta_t b \; ,$$

$$\eta_t \frac{B-a}{A-a} = 1 + \frac{a(1-\eta_t) + (1-\eta_c)\eta_t b}{A-a} \; .$$

Replacing (A-a) by its value in (16) yields -

$$(\eta_t x - 1) = \frac{a(1-\eta_t) + (1-\eta_c)\eta_t b}{\left(1-\frac{1}{x}\right)\frac{a(a-f)}{f}} \quad ,$$

$$(\eta_t x - 1)\left(1-\frac{1}{x}\right) = C' \quad ,$$

where

$$C' = \frac{a(1-\eta_t) + (1-\eta_c)\eta_t b}{\frac{a(a-f)}{f}} \quad ,$$

(17) $$\eta_t x^2 - (1+\eta_t + C')x + 1 = 0 \quad .$$ Q.E.D.

Thus, the minimum value for the specific fuel consumption, for a given τ and b is given by the relation:

(18) $$\boxed{s.f.c. = \frac{.134\ a x}{\eta_p (a-f)}} \quad .$$

where x is the larger root of (17). When $\eta_t = \eta_c = 1$, then $C' = 0$, $x = 1$ and the specific fuel consumption reduces to its limiting value (14).

What is the effect of a change in τ upon the specific fuel consumption for the design value of a which gives the minimum s.f.c. for a certain τ ?

$$\frac{\frac{\partial(s.f.c.)}{\partial \tau}}{s.f.c.} = \frac{\eta_c}{B-a} - \frac{\eta_t \eta_c}{A-a} \quad ,$$

(19) $$\frac{\partial(s.f.c.)}{\partial \tau} = \frac{\eta_c (s.f.c.)}{B-a}\left[1 - \eta_t \frac{B-a}{A-a}\right] .$$

Thus, the s.f.c. increases or decreases with τ according as $\left[1-\eta_t \frac{B-a}{A-a}\right]$ is greater than or less than zero.

From (17) $$\eta_t X\left[X-1\right] - \left[1+C'\right]X + 1 = 0,$$

$$\eta_t X = 1 + C' + \frac{C'}{X-1} .$$

Now, $C' > 0$ and for a design value $X > 1$, thus

$$\eta_t X > 1 .$$

Consequently, $$\frac{\partial(s.f.c.)}{\partial \tau} < 0 .$$

Thus, for a fixed <u>a</u> the s.f.c. decreases with an increase in τ.

It is customary to estimate the efficiencies η_t and η_c from their "small stage efficiencies", η_{ts} and η_{cs} respectively, by the following relations:

(21) $$\begin{cases} \eta_c = \Phi\left(\frac{a}{b}\right) \approx \dfrac{\frac{a}{b} - 1}{\left(\frac{a}{b}\right)^{1/\eta_{cs}} - 1} , & \eta_{cs} \approx .84 \\ \eta_t = \Phi(a) \approx \dfrac{1 - \left(\frac{1}{a}\right)^{\eta_{ts}}}{1 - \frac{1}{a}} , & \eta_{ts} \approx .86 . \end{cases}$$

These are the maximum values for η_c and η_t for a given design and the relations do not account for the variations in the efficiency off the design point. This topic will be taken up later.

What is the effect of changes of η_c and η_t on the s.f.c.?

From (12),

$$\text{(22)} \qquad \frac{\dfrac{\partial(\text{s.f.c.})}{\partial \eta_t}}{\text{s.f.c.}} = -\frac{\tau\,\eta_c}{A-\alpha} \,,$$

$$\text{(23)} \qquad \frac{\dfrac{\partial(\text{s.f.c.})}{\partial \eta_c}}{\text{s.f.c.}} = -\frac{\tau\,\eta_t}{A-\alpha} + \frac{\tau-b}{B-\alpha} \,.$$

Whence,

$$\text{(24)} \qquad \frac{\dfrac{\partial(\text{s.f.c.})}{\partial \eta_t}}{\dfrac{\partial(\text{s.f.c.})}{\partial \eta_c}} = \frac{\eta_c}{\eta_t - \dfrac{1-b/\tau}{X}} \,.$$

Thus, the s.f.c. decreases with an increase in either η_t or η_c. However, the rate of decrease is greater for an increase in η_t than for the same increase in η_c. In the range of operations -

$$\frac{1-b/\tau}{X} \approx \tfrac{1}{2}\,\eta_t \,, \quad \text{or}$$

$$\text{(25)} \qquad \frac{\dfrac{\partial(\text{s.f.c.})}{\partial \eta_t}}{\dfrac{\partial(\text{s.f.c.})}{\partial \eta_c}} \approx \frac{2\,\eta_c}{\eta_t} \,.$$

Or in words, the change in specific fuel consumption with a change in η_t is about twice as great as that for a corresponding change in η_c in the range of operations.

This ratio increases with a and τ and as a and τ decrease to one the ratio decreases to η_c/η_t.

The change in s.f.c. with b is so small it is not considered here.

5. Thermodynamic Cycle II: - Two Turbines with Second Burner Between Them.

An increase in the top temperature produces both of the desired results; an increase in power and a decrease in s.f.c. Unfortunately, this top temperature is limited by the physical properties of the unit, so the question arises; what can be done to increase the power? Many modifications in design which might produce such a desired result may be studied. Of these the most desirable designs would maintain as nearly as possible this top temperature. An application of heat between each stage of the turbine would produce an approximation to the desired condition. A simple design which gives a partial effect is one containing two burners and two turbines; a burner in front of each turbine, the first turbine delivering power for the compressor only, with the second turbine delivering the power for the propeller. To summarize: the design consists of a diffuser, a compressor, a burner, a turbine (which runs compressor), a burner and a turbine (which runs propeller) in series.

The corresponding enthalpy entropy diagram follows in Figure 2.

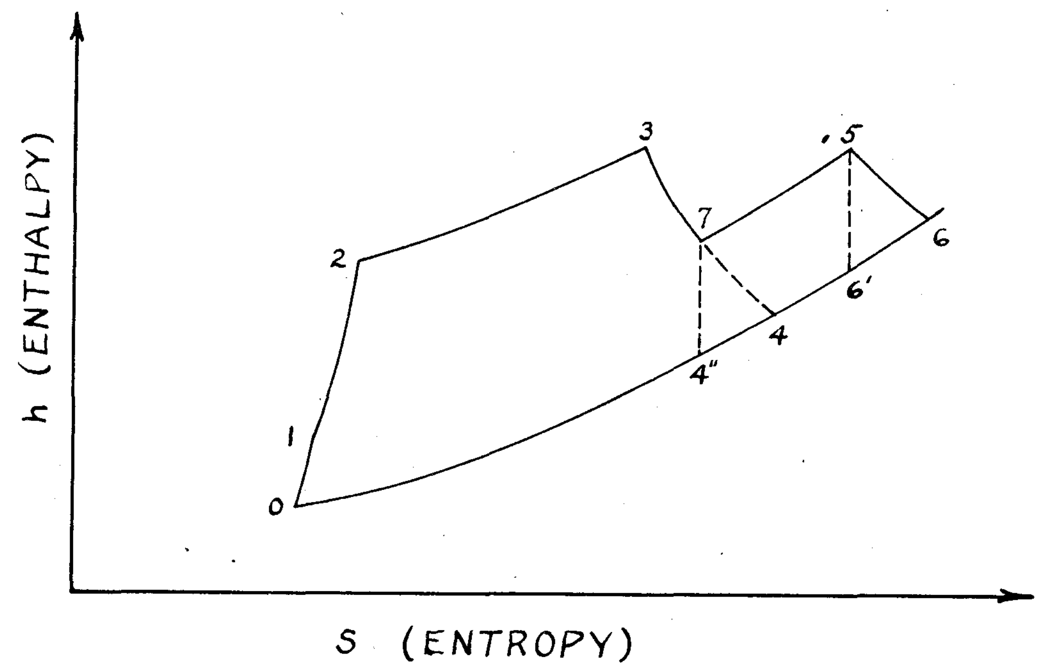

Figure 2.

Since this modification in design does not effect the weight of air flow, the effect of the change on the power may be determined by considering the power coefficient which shall be denoted by e_u^{II}. The only factors to consider in the power coefficient are the adiabatic heads of the second turbine and of the ram, because the compressor is just balanced by the first turbine. Thus,

$$(26) \qquad e_u^{II} = \eta_p \left[\frac{(T_5 - T_6)}{T_0} - \frac{(T_1 - T_0)}{T_0} \right] ,$$

$$e_u^{II} = \eta_p \left[\frac{T_5}{T_0}\left(1 - \frac{T_6}{T_5}\right) - \left(\frac{T_1}{T_0} - 1\right) \right] .$$

Now, $\dfrac{T_7 - T_4}{T_7 - T_4''} = \eta_t$, $\dfrac{T_5 - T_6}{T_5 - T_6'} = \eta_{t2}$,

$$\left(1 - \frac{T_6}{T_5}\right) = \frac{\eta_{t2}}{\eta_t}\left(1 - \frac{T_4}{T_7}\right) ,$$

and $\left(1 - {}^{T_4}/_{T_7}\right)$ may be determined from e_u ,*

$$e_u = \frac{\eta_p}{T_o}\left[(T_3 - T_4) - (T_2 - T_1) - (T_1 - T_o)\right] ,$$

Since, neglecting bearing loss, $\dfrac{T_2 - T_1}{T_o} = \dfrac{T_3 - T_7}{T_o}$,

$$e_u = \frac{\eta_p}{T_o}\left[(T_3 - T_4) - (T_3 - T_7) - (T_1 - T_o)\right]$$

$$= \frac{\eta_p}{T_o}\left[(T_7 - T_4) - (T_1 - T_o)\right]$$

$$= \eta_p\left[\frac{T_7}{T_o}\left(1 - \frac{T_4}{T_7}\right) - \left(\frac{T_1}{T_o} - 1\right)\right] ,$$

$$\eta_p\left(1 - \frac{T_4}{T_7}\right) = \frac{T_o}{T_7}\left[e_u + \eta_p(b-1)\right] .$$

But $\dfrac{T_3 - T_7}{T_o} = \dfrac{T_2 - T_1}{T_o} = \dfrac{b}{\eta_c}\left[\dfrac{a}{1 + (b-1)\eta_d} - 1\right]$,

$$\frac{T_3}{T_o} - \frac{T_7}{T_o} = \frac{b}{\eta_c}\left[\frac{a}{1 + (b-1)\eta_d} - 1\right] ,$$

$$\frac{T_7}{T_o} = \tau - \frac{b}{\eta_c}\left[\frac{a}{1 + (b-1)\eta_d} - 1\right] .$$

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*This estimate of $\left(1 - {}^{T_4}/_{T_7}\right)$ is not exactly correct, for η_t depends upon the pressure drop. The exact relation will be considered later when discussing stage efficiencies.

Thus, $$\eta_p\left(1-\frac{T_4}{T_7}\right) = \left[e_u + \eta_p(b-1)\right]\frac{\eta_c}{\eta_c\tau - b\left[\frac{a}{1+(b-1)\eta_d}-1\right]}$$

Consequently,

$$e_u^{II} = \frac{\eta_{t2}}{\eta_t}\,\frac{\tau_2\,\eta_c}{\eta_c\tau - b\left[\frac{a}{1+(b-1)\eta_d}-1\right]}\left[e_u+\eta_p(b-1)\right] - \eta_p(b-1),$$

$$\frac{e_u^{II}}{e_u} = \frac{\eta_{t2}}{\eta_t}\,\frac{\tau_2}{\tau - \frac{b}{\eta_c}\left[\frac{a}{1+(b-1)\eta_d}-1\right]} + \frac{\eta_p(b-1)}{e_u\,\eta_c}\left[\frac{\eta_{t2}}{\eta_t}\,\frac{\tau_2}{\tau-\frac{b}{\eta_c}\left[\frac{a}{1+(b-1)\eta_d}-1\right]}-1\right].$$

Or, when $\eta_d = 1$

(27) $$\frac{e_u^{II}}{e_u} = \frac{\tau_2}{\tau - \frac{a-b}{\eta_c}}\cdot\frac{\eta_{t2}}{\eta_t} + \epsilon \quad ,$$ where

$$\epsilon = \frac{\eta_p(b-1)}{e_u\,\eta_c}\left[\frac{\eta_{t2}}{\eta_t}\cdot\frac{\tau_2}{\tau-\frac{a-b}{\eta_c}} - 1\right],$$ and the

subscript 2 on τ and η_t refers to the values for the second turbine.

It is evident from relation (27) that the modified design will give an increase in power if $\tau_2 = \tau$. What about the specific fuel con-consumption?

$$s.f.c.^{II} = .134\,\frac{(T_3-T_2)+(T_5-T_7)}{T_0\,e_u^{II}}$$

$$s.f.c.^{II} = .134\,\frac{\left(\tau-\frac{T_2}{T_0}\right)+\left(\tau_2-\frac{T_7}{T_0}\right)}{e_u^{II}}$$

$$(28) \qquad s.f.c.^{II} = .134\, \frac{\tau_2 - b}{e_u^{II}}$$

How does this compare with s.f.c.?

$$s.f.c. = .134\, \frac{\tau - b - \frac{1}{\eta_c}(a-b)}{e_u}$$

Hence, if $\eta_d = 1$ as in the approximate value of s.f.c.

$$\frac{s.f.c.^{II}}{s.f.c.} \approx \frac{e_u}{e_u^{II}} \cdot \frac{\tau_2 - b}{\tau - b - \frac{a-b}{\eta_c}}$$

Now, if in relation (27) $\eta_{t_2} = \eta_t$ and ϵ is neglected.

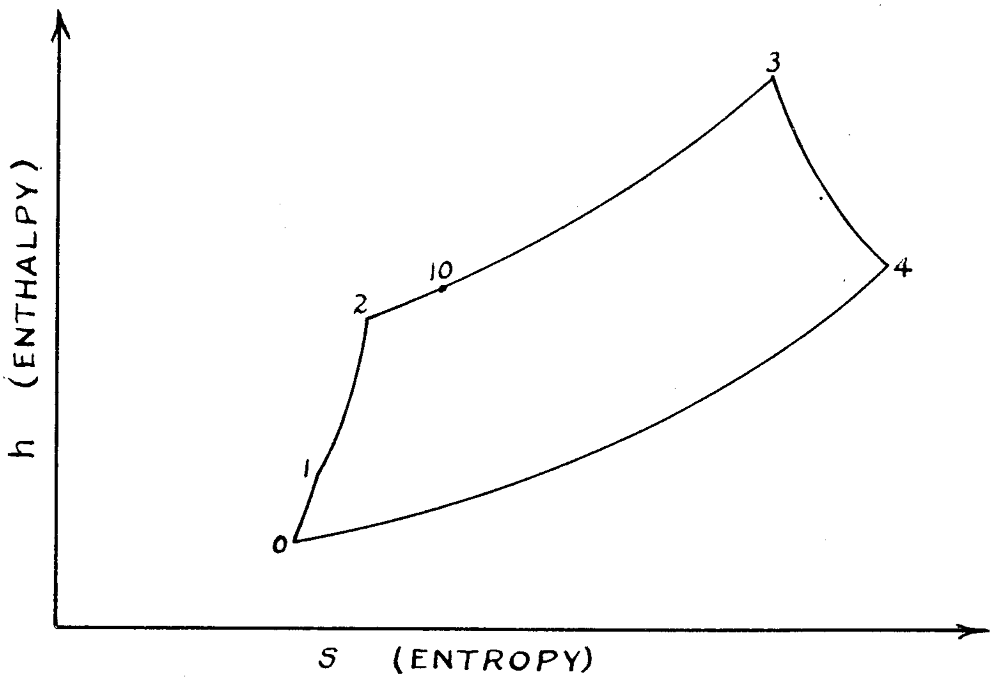
$$\frac{s.f.c.^{II}}{s.f.c.} \approx \frac{\tau - \frac{a-b}{\eta_c}}{\tau_2} \cdot \frac{\tau_2 - b}{\tau - b - \frac{a-b}{\eta_c}} ,$$

$$(29) \qquad \boxed{\frac{s.f.c.^{II}}{s.f.c.} \approx \frac{1 - \frac{b}{\tau_2}}{1 - \frac{b}{\tau - \frac{(a-b)}{\eta_c}}}}$$

Since $\tau_2 \geq \frac{T_7}{T_0} \approx \tau - \frac{a-b}{\eta_c}$ under the above hypothesis, $s.f.c.^{II} \geq s.f.c.$ This also follows in the general case. Although the ϵ term omitted increases $\frac{s.f.c.^{II}}{s.f.c.}$, it approaches 0 as τ_2 approaches $\frac{T_7}{T_0}$.

Thus, the modified design affords a unit with much greater power and a higher specific fuel consumption.

6. <u>Thermodynamic Cycle III - Heat Exchanger:</u> (The effect of a heat exchanger can be seen by inserting one in a simple gas combustion turbine cycle.) In a heat exchanger the warm exhaust gas is passed in a counterflow manner over the air between the compressor and the burner. The thermodynamic cycle in this case is given in the diagram. The air is heated from T_2 to T_{10} by the exhaust gas, then from T_{10} to T_3 by the burner.



Under these conditions, where the reduction in V_j is assumed converted into heat energy.

$$H_S^{III} = W_A^{III} c_p J\left[(T_3-T_{10})-(T_4-T_0)+(T_{10}-T_2)\right] - W_A^{III}\left(\frac{V_j^2}{2g}-\frac{V_0^2}{2g}\right)$$

$$= W_A^{III} c_p J\left[(T_3-T_4)-(T_2-T_1)-(T_1-T_0)\right] - W_A^{III}\left(\frac{V_j^2}{2g}-\frac{V_0^2}{2g}\right)$$

$$= W_A^{III} c_p J T_0\left[\tau\eta_t\left(1-\frac{f}{a}\right)-\frac{b}{\eta_c}\left(\frac{a}{1+(b-1)\eta_d}-1\right)-(b-1)\right] - W_A^{III}\left(\frac{V_j^2}{2g}-\frac{V_0^2}{2g}\right).$$

Thus, the only change in the horsepower developed will be due to the change in the values of W_A and f, where f now, and in the following cycles, includes the loss in pressure due to the heat exchanger, both in the burner and in the exhaust gases.

Now,

$$19{,}000\ W_F^{III} = c_p W_A^{III}(3600)(T_3 - T_{10}) \ ,$$

$$W_F^{III} = \frac{3.6}{19} c_p W_A^{III} T_o \left[\frac{T_3}{T_o} - \frac{T_{10}}{T_o}\right] .$$

Let,

$$\frac{T_{10} - T_2}{T_4 - T_2} = \eta_e \ , \quad then \quad T_{10} - T_2 = \eta_e (T_4 - T_2) .$$

Thus,

$$\frac{T_{10}}{T_o} = \frac{T_2}{T_o} + \eta_e\left(\frac{T_4}{T_o} - \frac{T_2}{T_o}\right)$$

$$= \eta_e \frac{T_4}{T_o} + \frac{T_2}{T_o}(1 - \eta_e)$$

Whence,

$$W_F^{III} = \frac{3.6}{19} c_p W_A^{III} T_o \left[\tau - \tau\eta_e + \tau\eta_e\eta_t\left(1 - \frac{f}{a}\right) + b(1-\eta_e)\left(1 + \frac{1}{\eta_c}\left\{\frac{a}{1+(b-1)\eta_d} - 1\right\}\right)\right]$$

$$(30) \quad s.f.c.^{III} = \frac{.134}{\eta_p}\,\frac{\tau - \frac{T_{10}}{T_o}}{\tau - \frac{T_4}{T_o} - \frac{T_2}{T_o} + 1} = \frac{.134}{\eta_p}\,\frac{\tau - \frac{T_2}{T_o} - \eta_e\left(\frac{T_4}{T_o} - \frac{T_2}{T_o}\right)}{\tau - \frac{T_4}{T_o} - \frac{T_2}{T_o} + 1} = s.f.c. - \frac{.134\,\eta_e\left[\frac{T_4}{T_o} - \frac{T_2}{T_o}\right]}{\eta_p\left[\tau - \frac{T_4}{T_o} - \frac{T_2}{T_o} + 1\right]}$$

This reveals what is known from a practical point of view, that an heat exchanger as described cannot be of any value unless $T_4 > T_2$, or

$$\frac{T_4}{T_o} > \frac{T_2}{T_o}$$

$$\tau\left[1 - \eta_t\left(1 - \frac{f}{a}\right)\right] > b + \frac{b}{\eta_c}\left[\frac{a}{1+(b-1)\eta_d} - 1\right]$$

Under the assumption that $f = 1 = \eta_d$, then

$$\tau - \tau\eta_t\left(\frac{a-1}{a}\right) > b + \frac{a-b}{\eta_c} \ .$$

This later inequality does not ensure the former but the two are approximately equivalent,

$$a(\tau - \tau\eta_t)\eta_c + \tau\eta_t\eta_c > ab(\eta_c - 1) + a^2 \ ,$$

$$a^2 + a\left[b(\eta_c - 1) - \tau\eta_c(1-\eta_t)\right] - \tau\eta_t\eta_c < 0 \ .$$

This shows that a heat exchanger is valuable, provided a satisfies the inequality,

$$1 < a < \frac{\tau\eta_c(1-\eta_t) + b(1-\eta_c)}{2} + \sqrt{\left[\frac{\tau\eta_c(1-\eta_t)+b(1-\eta_c)}{2}\right]^2 + \tau\eta_c\eta_t} \ .$$

At an altitude of 15,000 ft., speed 450 mph, $T_3 = 1660$, if

$\eta_c = .84$, $\eta_t = .86$, the inequality reduces to

$$1 < a < 1.929.$$

The more accurate values obtained with $\eta_d = .9$, $f = 1.01$ are

$$1 < a < 1.925.$$

Approximations, similar to those made in the study of thermodynamic Cycle I, gives

$$s.f.c.^{\text{III}} \approx \frac{.134}{\eta_p}\,\frac{a(B-a)(1-\eta_e) + A\eta_e(a-f)}{(a-f)(A-a)}$$

$$\frac{\frac{\partial(s.f.c.^{III})}{\partial a}}{s.f.c.^{III}} \approx -\frac{1}{a-f}+\frac{1}{A-a}+\frac{-a(1-\eta_e)+(B-a)(1-\eta_e)+A\eta_e}{a(B-a)(1-\eta_e)+A\eta_e(a-f)}$$

From which it follows, s.f.c.^{III} is a minimum when

$$a=\frac{(2\eta_e-1)fA+\sqrt{(1-2\eta_e)^2(fA)^2-[(1-\eta_e)(B-A-f)+A\eta_e][-ABf(1-\eta_e)+f^2A\eta_e]}}{(1-\eta_e)(B-f-A)+\eta_e A}$$

When η_e = .75 the above assumed values give

a = 1.468

Whereas, in Cycle I, ($\eta_e = 0$)

$$a=\frac{-fA+\sqrt{(fA)^2+ABf(B-A-f)}}{B-A-f}$$

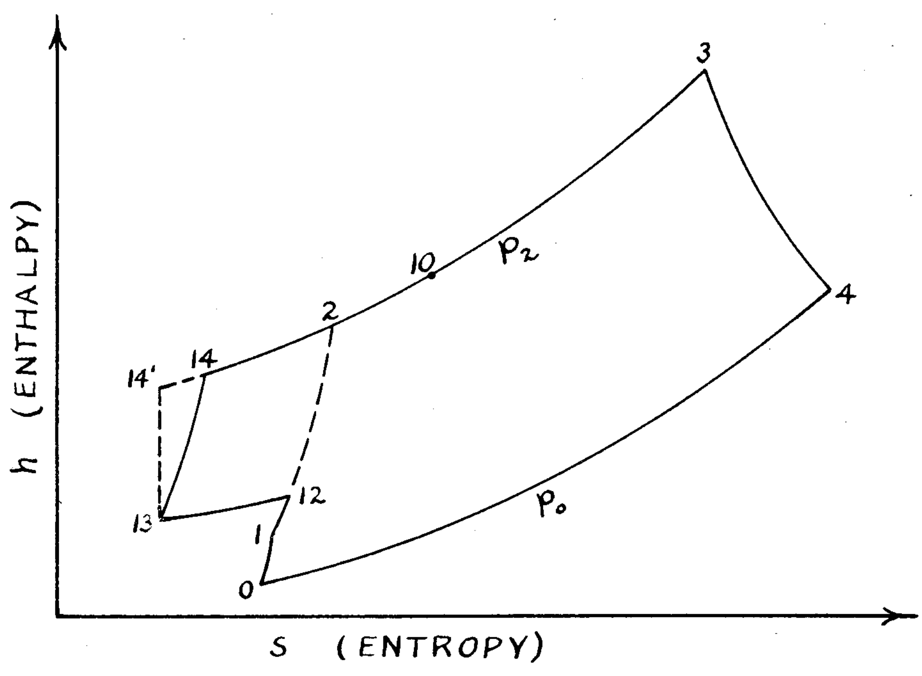
and for the preceding assumed values,

a = 1.872

Not only does a heat exchanger decrease the s.f.c., but at the same time it permits a lower compression ratio.

Soderberg and Smith indicated that a value of η_e = .75 is not unreasonable. A design value of a = 1.5, η_e = .75 and the above values give s.f.c.^{III} = .409/ η_p as compared to s.f.c. = .545/ η_p without a heat exchanger, a decrease of 25 percent.

7. <u>Thermodynamic Cycle IV. - Cooler and Heat Exchanger:</u> As was noted above, the importance of a heat exchanger becomes more pronounced when there is a greater temperature difference between the exhaust temperature and the temperature of the air after compression. One means of increasing this temperature difference is to introduce a cooling system which cools the air before compression or cools the air after partial compression by an absorption refrigeration system, which is operated on the heat energy from the exhaust gases. This method of cooling the air would be quite attractive if it were not for the low efficiencies and the additional weight. After the temperature of the air has been increased by some compression it may be cooled down by the outside air (never quite as low as outside air) by means of an intercooler. Since this means of cooling the air is less complicated mechanically the cycle containing an intercooler and heat exchanger is treated first. Precisely, the unit would consist of a diffuser, a compressor, an intercooler, a second compressor, a burner, a turbine and a heat exchanger.



It is assumed that the heat transfer is possible as long as there is Y° temperature difference.

Let $T_{13} = T_0 + Y^{\circ}$,

$$d = \frac{T_{12}'}{T_1} ,$$

then $$a = \left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{14}'}{T_{13}} \cdot d \cdot \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} ,$$

$$\therefore \quad \frac{T_{14}'}{T_{13}} = \frac{a}{d\left(1+[b-1]\eta_d\right)} .$$

Now, $$e_u^{IV} = \frac{\eta_p}{T_0}\left[(T_3 - T_4) - (T_{14} - T_{13}) - (T_{12} - T_0)\right] ,$$

$$\text{s.f.c.}^{IV} = \frac{.134}{\eta_p}\left[\frac{T_3 - T_{10}}{T_3 - T_4 - T_{14} + T_{13} - T_{12} + T_0}\right] .$$

Let,

$$\frac{T_{10} - T_{14}}{T_4 - T_{14}} = \eta_e , \quad \frac{T_{10}}{T_0} = \frac{T_{14}}{T_0}(1 - \eta_e) + \eta_e \frac{T_4}{T_0} ,$$

$$\frac{T_{14}' - T_{13}}{T_{14} - T_{13}} = \eta_c , \quad \frac{T_{14}}{T_{13}} - 1 = \frac{1}{\eta_c}\left(\frac{T_{14}'}{T_{13}} - 1\right) ,$$

$$\frac{T_{12}' - T_1}{T_{12} - T_1} = \eta_c , \quad \frac{T_{12}}{T_1} - 1 = \frac{1}{\eta_c}\left(\frac{T_{12}'}{T_1} - 1\right) .$$

Thus, $$\frac{e_u^{IV}}{\eta_p} = \tau\eta_t\left(1 - \frac{f}{a}\right) - \frac{1}{\eta_c}\left(1 + \frac{Y}{T_0}\right)\left(\frac{a}{d[1+(b-1)\eta_d]} - 1\right) - \frac{b}{\eta_c}(d-1) - (b-1) ,$$

$$\text{s.f.c.}^{IV} = \frac{.134}{\eta_p}\left[\frac{\tau - \tau\eta_e\left\{1 - \eta_t\left(1 - \frac{f}{a}\right)\right\} - (1-\eta_e)\left(1 + \frac{Y}{T_0}\right)\left(1 + \frac{1}{\eta_c}\left\{\frac{a}{d[1+(b-1)\eta_d]} - 1\right\}\right)}{\tau\eta_t\left(1 - \frac{f}{a}\right) - (b-1) - \frac{b}{\eta_c}(d-1) - \frac{1}{\eta_c}\left(1 + \frac{}{T_0}\right)\left(\frac{a}{d[1+(b-1)\eta_d]} - 1\right)}\right] .$$

The optimum value for d for a minimum fuel consumption is about $\sqrt{a}/b$.

For a design to operate at an altitude if 15,000 feet, speed of 450 m.p.h., $T_3 = 1660$; if $\eta_c = .84$, $\eta_t = .86$, $f = 1.01$, $\eta_d = .9$, $Y = 50°$ a $= 1.5$, and d $= 1.15$, then $\tau = 3.57$,

$$\text{s.f.c.}^{IV} = \frac{.384}{\eta_p}$$

while the corresponding s.f.c. for the simple cycle is $\frac{.5427}{\eta_p}$. Thus, under the assumed conditions the specific fuel consumption of Cycle IV is 29% less than that of the simple cycle.

If the intercooler is replaced by an absorption refrigeration system cooling the air before compression, then,

$$\frac{e_u^{V}}{\eta_p} = \frac{1}{T_o}\left[(T_3 - T_4) - (T_{14} - T_{13}) - (T_1 - T_o)\right], \quad T_{12} = T_1,$$

$$\text{s.f.c.}^{V} = \frac{.134}{\eta_p}\left[\frac{\tau - \frac{T_{10}}{T_o}}{\tau - \frac{T_4}{T_o} - \frac{T_{14}}{T_o} + \frac{T_{13}}{T_o} - \left(\frac{T_1}{T_o} - 1\right)}\right],$$

where

$$\eta_e^2 \eta_r (T_{14} - T_o - Y) = T_1 - T_{13}, \quad \text{and}$$

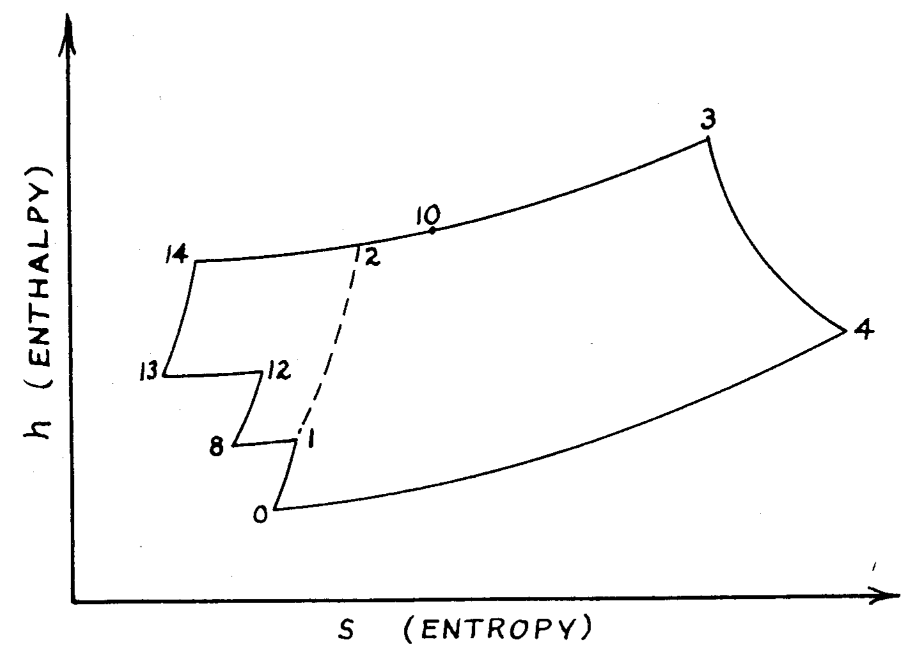
$$\frac{T_{14}}{T_{13}} = 1 + \frac{1}{\eta_c}\left[\frac{a}{1 + (b-1)\eta_d} - 1\right].$$

Whence,

$$\frac{T_{13}}{T_o} = \frac{1 + \frac{Y}{T_o} + \frac{b}{\eta_e^2 \eta_r}}{\frac{1}{\eta_e^2 \eta_r} + 1 + \frac{1}{\eta_c}\left[\frac{a}{1 + (b-1)\eta_d} - 1\right]}.$$

With an assumed efficiency of $\eta_r = .3$ and $\eta_e^2 = (.75)^2$ and other values assumed in evaluating the s.f.c. for the intercooler-heat exchanger cycle, the cycle with an absorption refrigeration gives an s.f.c. of $.384/\eta_p$.

Naturally, both types of coolers may be introduced into the same cycle and, thereby, further reduce the s.f.c.



$$\frac{e_u^{VI}}{\eta_p} = \frac{1}{T_o}\left[T_3 - T_4 - T_{14} + T_{13} - T_{12} + T_8 - T_1 + T_o\right] ,$$

$$s.f.c.^{VI} = \frac{.134}{\eta_p}\left[\frac{T_3 - T_{10}}{T_3 - T_4 - T_{14} + T_{13} - T_{12} + T_8 - T_1 + T_o}\right] .$$

Where

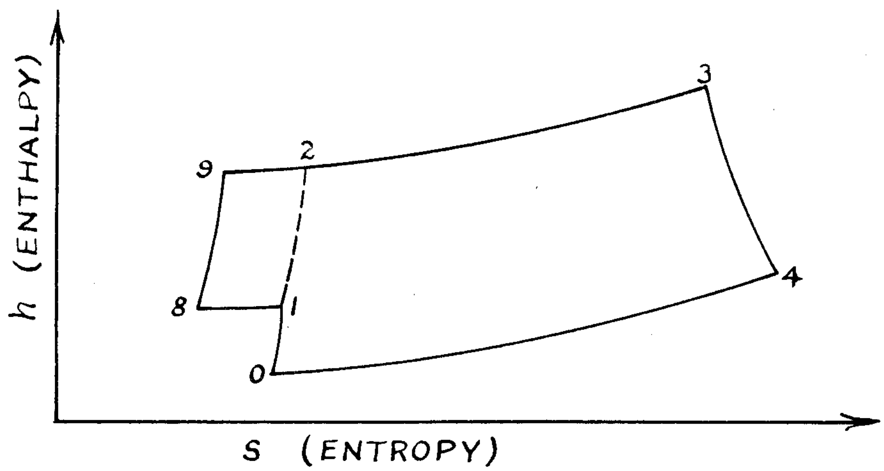
$$T_{13} = T_o + Y^\circ ,$$

$$\frac{T_{12}'}{T_8} = d ,$$

$$T_1 - T_8 = \eta_e^2 \eta_r (T_{14} - T_o - Y^\circ) .$$

It is clear that an appropriate choice of d will give a low theoretical value for s.f.c.

In order to clarify the effect of the absorption cooler, the following results for a cycle with an absorption cooler without a heat exchanger are given:



$$P^- = W_A^- c_p J\left[(T_3-T_9)-(T_4-T_o)\right] - W_A^- c_p J\left[T_1-T_8\right]$$
$$= W_A^- c_p J T_o\left[\tau - \frac{T_9}{T_o} - \frac{T_4}{T_o} + 1\right] - W_A^- c_p J T_o\left[\frac{T_1}{T_o} - \frac{T_8}{T_o}\right] ,$$

$$19{,}000\ W_F^- = 3{,}600\ W_A^- c_p\left(T_3 - T_9\right) ,$$

$$W_F^- = \frac{3.6}{19} W_A^- c_p T_o\left(\tau - \frac{T_9}{T_o}\right) ,$$

$$s.f.c.^- = \frac{.134}{\eta_p}\,\frac{\tau - \frac{T_9}{T_o}}{\tau - \frac{T_9}{T_o} - \frac{T_4}{T_o} + 1 - \frac{T_1}{T_o} + \frac{T_8}{T_o}} ,$$

$$\frac{T_4}{T_o} = \frac{T_4}{T_3}\cdot\frac{T_3}{T_o} = \tau\left[1 - \eta_t\left(1 - \frac{f}{a}\right)\right] ,$$

$$\frac{T_9}{T_o} = \frac{T_9}{T_8}\cdot\frac{T_8}{T_o} .$$

If it is assumed that 75% of the heat of the gas is recovered by counter-flow, the refrigerator is 30% efficient and 75% of the refrigeration is transferred to the entering gas, by counter flow exchanger, then

$$T_1 - T_8 = (.75)^2(.3)(T_4 - T_0) \quad ,$$

$$\frac{T_1}{T_0} - \frac{T_8}{T_0} = .16875\left(\frac{T_4}{T_0} - 1\right) \quad ,$$

$$\frac{T_8}{T_0} = b - .16875\left(\frac{T_4}{T_0} - 1\right) \quad .$$

Now,

$$\eta_c = \frac{T_9' - T_8}{T_9 - T_8} \quad ,$$

$$\frac{T_9}{T_8} = 1 + \frac{1}{\eta_c}\left(\frac{T_9'}{T_8} - 1\right) \quad ,$$

but

$$\frac{T_9'}{T_8} = \frac{T_2'}{T_1} \quad .$$

Thus,

$$\frac{T_9}{T_8} = 1 + \frac{1}{\eta_c}\left[\frac{a}{1+(b-1)\eta_d} - 1\right] \quad ,$$

$$\frac{T_9}{T_0} = \left[1 + \frac{1}{\eta_c}\left(\frac{a}{1+(b-1)\eta_d} - 1\right)\right]\left[b - .16875\left(\frac{T_4}{T_0} - 1\right)\right] \quad .$$

Now, in the simple cycle without the refrigeration

$$s.f.c. = \frac{.134}{\eta_P}\;\frac{\tau - \frac{T_2}{T_0}}{\tau - \frac{T_2}{T_0} - \frac{T_4}{T_0} + 1} \quad .$$

Therefore,

$$(31)\qquad \frac{\overline{s.f.c.}}{s.f.c.} = 1 - \frac{\left(\frac{T_4}{T_0} - 1\right)\left(\frac{1}{\tau - \frac{T_2}{T_0}} - \frac{1.16875}{\tau - \frac{T_9}{T_0}}\right)}{1 - \frac{\left(\frac{T_4}{T_0} - 1\right)(1.16875)}{\tau - \frac{T_9}{T_0}}}$$

At an altitude of 15,000 feet, speed of 450 m.p.h., $T_3 = 1660$, if $\eta_c = .84$, $f = 1.01$, $\eta_d = .9$, $a = 1.8$, $\eta_t = .86$.

Then $\tau = 3.57$,

$$\frac{T_4}{T_0} = 3.57\left[1 - .86\left(1 - \frac{1.01}{1.8}\right)\right] = 2.223 ,$$

$$\tau - \frac{T_2}{T_0} = 3.57 - 1.0785\left[1 + \frac{1}{.84}\left(\frac{1.8}{1+.0707} - 1\right)\right] = 1.62 ,$$

$$\tau - \frac{T_9}{T_0} = 3.57 - \left[1 + \frac{1}{.84}\left(\frac{1.8}{1.0707} - 1\right)\right]\left[1.0785 - .16875(1.223)\right]$$

$$= 1.988 .$$

Whence,

$$\frac{\overline{s.f.c.}}{s.f.c.} = 1 - 1.223\left[\frac{.617 - 1.16875(.503)}{1 - \frac{1.223(1.16875)}{1.988}}\right] = 1 - .127$$

$$= .873 ;$$

$$s.f.c. = .5427 ,$$

$$\overline{s.f.c.} = .474 ,$$

8. Other Thermodynamic Cycles:

The thermodynamic cycles I and II are just special cases of Gas Combustion Turbine Motors. More complicated engines may be constructed by introducing coolers and heat exchangers into the system. In outline form a constant-pressure gas turbine cycle consists of

(1) 1 or more compressors,

(2) 0 or more coolers,

a. There must be one intercooler between compressors and there might be a cooler before compressing,

(3) 1 or more burners at $\approx$ constant pressure,

(4) 1 or more turbines, and

(5) 0 or one heat exchanger to recover the exhaust heat.

The power coefficient and the specific fuel consumption for any one of these cycles can be easily computed.

A unit which contains each of the five basic elements is representative of the entire group. The following schematic representation of a unit showing the thermal and mechanical hook-up was given by Soderberg and Smith.*

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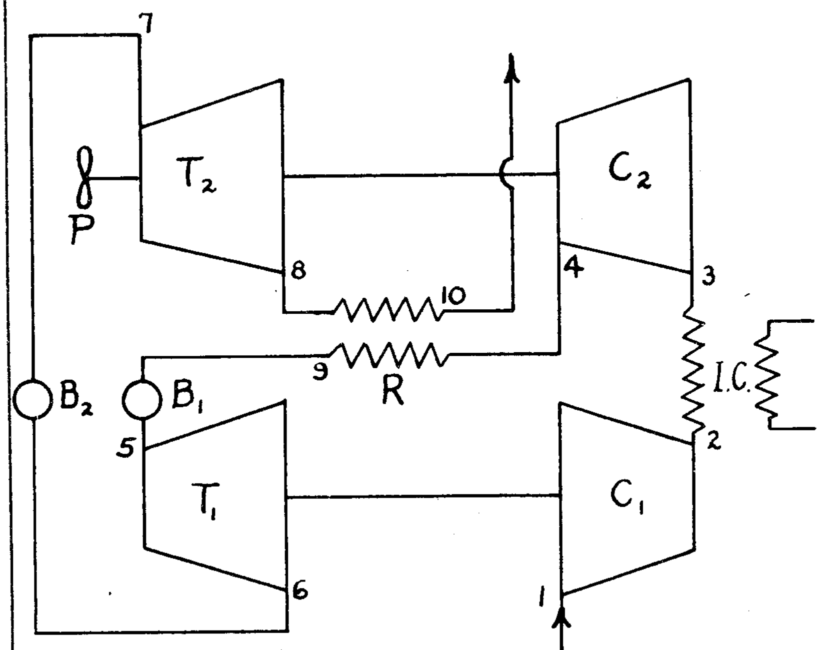
* "The Gas Turbine for Use on Ships", Marine Engineering, Feb., March, 1944.



T_1 = High Pressure Turbine

T_2 = Low Pressure Turbine

c_1 = Low Pressure Compressor

c_2 = High Pressure Compressor

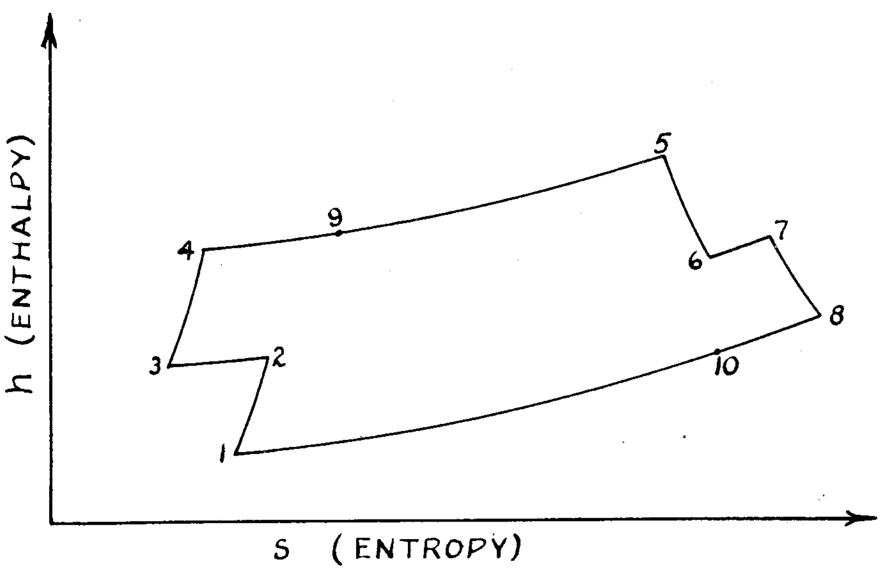
I.C. = Intercooler

B_1 = High Pressure Combustion Chamber

B_2 = Low Pressure Combustion Chamber

R = Regenerator

P = Propeller



Typical Combustion Gas Turbine.

The unit in the above diagram has two shafts, this is not necessary, all of the elements may be on the same shaft.

"In order to facilitate the identification of different cycles, the following nomenclature, originally suggested by Lysholm, will be helpful. Each cycle is identified by three numbers -

(a, b, c), where,

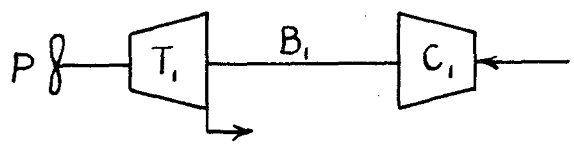
(a) is the number of combustion chambers, which is also equal to the number of turbine expansions,

(b) is the number of intercoolers, which is also equal to the number of compressors less one,

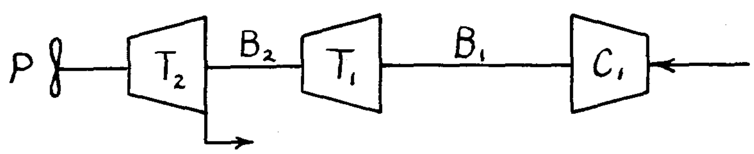
(c) is the regenerator efficiency."

In terms of these symbols, the typical turbine would be characterized as (2, 1, 0.75).

Thermodynamic Cycle I = (1, 0, 0) and the representation is



Thermodynamic Cycle II = (2, 0, 0) and



9. <u>Throttling:</u> By a variation of the design or a change in the size of the unit, or both, a motor may be produced which will give any desired power. Regardless of the design of the unit when it is placed in operation it must develop varying amounts of power. How should one produce this variation in power? Or, in other words, what should be changed in order to vary the power output and still keep the specific fuel consumption a minimum? In the calculations of s.f.c. and e_u, the efficiencies of the component parts of the motor enter as factors. A slight change in anyone of these efficiencies produces an appreciable change in both s.f.c. and e_u. Unfortunately, the efficiencies of compressors and turbines are not the same for all points in the operating range. If a turbine or compressor is designed to operate at a given pressure ratio the efficiency is less for any other pressure ratio.

The power developed is proportional to the weight of air flow W_A as well as e_u. Hence, it is necessary to have an estimate of W_A before the question of throttling can be answered. From empirical considerations, the quantity may be written as follows:

$$(32) \qquad W_A = K\frac{\delta}{\sqrt{\theta}}(a-b) \, ,$$

where K is a constant, δ and θ the ratio of the pressure and temperature at flight altitude to the pressure and temperature, respectively, at sea level (according to the Standard Atmosphere Table).

From (3) and (6)

$$P = \frac{W_A c_p J T_o}{550}\left(\frac{a-f}{a\eta_c}\right)\left(\tau \eta_c \eta_t - a\right) \, , \qquad \text{or}$$

(33) $$P = \frac{c_p J K_1 \delta\sqrt{\theta}}{550}(a-b)\left(\frac{a-f}{a\eta_c}\right)\left(\tau\eta_c\eta_t - a\right),$$

where $K_1 = T_{oo}K$, T_{oo} = the temperature at sea level.

In terms of the above definition where only power developed by the engine (not considering power necessary for the airplane) and the s.f.c. are considered, the best way to throttle is to increase the altitude, keeping the R.P.M. and top temperature constant. Relation (33) reveals the power becomes less with an increase in altitude; although the increase in τ increases the power, the decrease in the product $\delta\sqrt{\theta}$ is the dominating effect, and the total power is decreased. Moreover, the increase in τ with altitude decreases the s.f.c. Hence, both the power and s.f.c. may be reduced by increasing the flight altitude. Of course, there is a limit to the amount the altitude may be increased when the engine is in an airplane for then the power developed must equal the power required.

When the altitude and the velocity of the plane is a constant, P and s.f.c. are functions of a and τ alone, i.e.

(34) $$\begin{cases} P = \Phi(a,\tau), \\ s.f.c. = \xi(a,\tau). \end{cases}$$ and

The minimum specific fuel consumption for a constant value of P, will occur only if $\frac{d(s.f.c.)}{d\tau} = 0$, or

(35) $$\frac{d(s.f.c.)}{d\tau} = \frac{\partial(s.f.c.)}{\partial\tau} + \frac{\partial(s.f.c.)}{\partial a}\cdot\frac{da}{d\tau} = 0.$$

Now,

(36) $$0 = \frac{\partial P}{\partial \tau} + \frac{\partial P}{\partial a} \cdot \frac{da}{d\tau} \quad .$$

These two relations are consistent if, and only if,

(37) $$\frac{\partial(s.f.c.,\, P)}{\partial(a, \tau)} = 0 \ .$$

If equations (34) are viewed as a family of curves in the s.f.c. - P plane, one curve for each value of τ, each given in parametric representation in <u>a</u>; then, equations (34) and (37) give a parametric representation of the <u>envelop curve.</u> (See Appendix A for the proof.) <u>This curve can be easily constructed, and</u>, since it happens to be the minimum values (not maximum, or other stationary values) <u>it is the throttling curve for a fixed altitude</u>. That is, for a given P the corresponding s.f.c. on this envelope curve is the minimum s.f.c. which may be obtained for a fixed altitude and speed. The values of <u>a</u> and τ which determine the R.P.M. and the top temperature T, may be obtained by solving the equations (34) simultaneously. This analytic relation between <u>a</u> and τ is given in Appendix B.

10. <u>Formulas for Design Purposes:</u> In estimating the possibilities of fuel consumption for combustion gas turbines it was convenient to combine the shaft power and the jet power. However, when it comes to the problems of practical design it is necessary to separate these two quantities.

From relation (2),

$$H_S = W_A c_p J \left[(T_3-T_4)-(T_2-T_1)-(T_1-T_0)\right] + W_A\left[\frac{V_0^2}{2g}-\frac{V_j^2}{2g}\right] ,$$

and since $\dfrac{V_0^2}{2g\,Jc_p} = (T_1 - T_0)$,

$$(50) \quad H_S = W_A c_p J T_0 \left[\frac{T_3}{T_0}\left(1-\frac{T_4}{T_3}\right) - \frac{T_1}{T_0}\left(\frac{T_2}{T_1}-1\right)\right] - W_A \frac{V_j^2}{2g} .$$

This relation was developed under the hypothesis of 100% gear efficiency. The introduction of a gear efficiency and further simplification yields

$$\frac{H_S}{W_A\,\theta} = \eta_g\left[c_p J T_{00}\left(\tau\eta_t\left\{1-\frac{f}{a}\right\} - \frac{b}{\eta_c}\left\{\frac{a}{1+(b-1)\eta_d}-1\right\}\right) - \frac{\left(V_j/\sqrt{\theta}\right)^2}{2g}\right] ,$$

or

$$(51) \quad \boxed{\frac{H_{S\,HP}}{A_\omega\,\delta\sqrt{\theta}} = 176.4\,\eta_g\left(\tau\beta - C\right) - \eta_g\frac{\left(V_j/\sqrt{\theta}\right)^2}{1100\,g}} ,$$

where

$$\beta = \eta_t\left(1-\frac{f}{a}\right) ,$$

$$C = \frac{b}{\eta_c}\left(\frac{a}{1+[b-1]\,\eta_d}-1\right) ,$$

$$H_{S\,HP} = \frac{H_S}{550}$$

$$A_\omega\frac{\delta}{\sqrt{\theta}} = W_A .$$

It is assumed that the ratio of the jet velocity under operating condition to the jet design velocity may be determined from the temperature ratio of the exit gas to the design exit gas. Under this assumption, the development is as follows. (Quantities with the subscript D denote design values. The V's denote volumes and the c's velocities.)

$$P_4 V_4 = R T_4 \quad ; \quad P_{4_D} V_{4_D} = R T_{4_D} \quad ,$$

$$W_A = \frac{c_4 A}{V_4} \quad ; \quad W_{A_D} = \frac{c_{4_D} A}{V_{4_D}} \quad ,$$

Note: V's here are volumes.
c's are velocities.

$$\frac{c_4}{c_{4_D}} = \frac{W_A V_4}{W_{A_D} V_{4_D}} \quad ; \quad \frac{T_4}{T_{4_D}} = \frac{P_4 V_4}{P_{4_D} V_{4_D}}$$

$$\therefore \quad \frac{c_4}{c_{4_D}} = \frac{W_A}{W_{A_D}} \cdot \frac{T_4 \, P_{4_D}}{T_{4_D} \, P_4} \quad ,$$

$$\frac{c_4}{c_{4_D}} = \frac{A_\omega \frac{\delta}{\sqrt{\theta}} \, \theta \, \frac{T_4}{T_o} \, \delta_D}{A_{\omega_D} \frac{\delta_D}{\sqrt{\theta_D}} \, \theta_D \, \frac{T_{4_D}}{T_{o_D}} \, \delta} \quad ,$$

$$\frac{c_4}{c_{4_D}} = \frac{A_\omega \, \tau_n \sqrt{\theta}}{A_{\omega_D} \, \tau_{n_D} \sqrt{\theta_D}} \quad ,$$

where $\tau_n = \frac{T_4}{T_o}$, $\tau_{n_D} = \frac{T_{4_D}}{T_{o_D}}$.

Returning to the V_j notation,

$$(52) \quad \boxed{\frac{V_j}{\sqrt{\theta}} = \left(\frac{V_j}{\sqrt{\theta}}\right)_D \cdot \frac{A_\omega}{A_{\omega_D}} \cdot \frac{\tau_n}{\tau_{n_D}}} \quad .$$

Now, $$\frac{T_4}{T_0} = \frac{T_3}{T_0} \cdot \frac{T_4}{T_3} = \tau\left(1 - \eta_t\left[1 - \frac{f}{a}\right]\right) = \tau(1-\beta) .$$

Thus,

(53) $$\boxed{\frac{V_j}{\sqrt{\theta}} = \left(\frac{V_j}{\sqrt{\theta}}\right)_D \cdot D \cdot \frac{\tau(1-\beta)}{\tau_D(1-\beta_D)}} \quad , \qquad \text{where } D = \frac{A_\omega}{A_{\omega_D}} .$$

Samaras has shown that the condition for maximum total thrust is

(54) $$\left(\frac{V_j}{V_0}\right)_D = \frac{1}{\eta_g \eta_p} .$$

From (51), (53) and (54),

(55) $$\boxed{\frac{H_{SHP}}{(A_\omega)_D\,\delta\sqrt{\theta}} = 176.4\, D\eta_g(\tau\beta - C) - \frac{D^3\eta_g\,\tau^2(1-\beta)^2}{1100g\,\tau_D^2(1-\beta_D)^2}\left(\frac{V_0}{\sqrt{\theta}}\right)_D^2\left(\frac{1}{\eta_g\eta_p}\right)^2} .$$

In relation (11), the efficiency of the burner was assumed 100%, the introduction of η_b in this relation yields

$$W_F = \frac{3.6}{19}\,\frac{W_A c_p T_0}{\eta_b}\left[\tau - b - \frac{b}{\eta_c}\left(\frac{a}{1+(b-1)\eta_d} - 1\right)\right] ,$$

or

(56) $$\boxed{\frac{W_F}{A_{\omega_D}\,\delta\sqrt{\theta}} = \frac{23.6\,D}{\eta_b}\left[\tau - (b + C)\right]} ,$$

$$\text{Jet Thrust} = \frac{W_A}{g}\left(V_j - V_o\right) ,$$

$$\frac{\text{Jet Thrust}}{A_{\omega_D}\,\delta} = \frac{D}{g}\left(\frac{V_j}{\sqrt{\theta}} - \frac{V_o}{\sqrt{\theta}}\right) .$$

But, $\dfrac{V_o}{\sqrt{\theta}} = \dfrac{V_o}{\dfrac{\sqrt{T_o\,\gamma\,R g}}{\sqrt{T_{oo}\,\gamma\,R g}}} = M_o\, a_{oo}$ where a_{oo} is velocity of sound at sea level. Thus, from (53) and (54)

$$\frac{\text{Jet Thrust}}{A_{\omega_D}\,\delta} = \frac{D}{g}\left[\frac{D}{\eta_g\eta_p}\left(\frac{V_o}{\sqrt{\theta}}\right)_D \frac{\tau(1-\beta)}{\tau_D(1-\beta_D)} - M_o\, a_{oo}\right] . \tag{57}$$

Empirical relations obtained from Westinghouse data:

$$A_\omega = \left(.00411 + .000365\ M_o\right)\frac{N}{\sqrt{\theta}} , \tag{58}$$

$$\frac{\rho_c}{\rho_{c_D}} = \left[\frac{N/\sqrt{\theta}}{\left(N/\sqrt{\theta}\right)_D}\right]^{1.4} , \tag{59}$$

$$\frac{\eta_c}{\eta_{c_D}} = \left[\frac{a/b}{\left(a/b\right)_D}\right]^{.021 \text{ or } -.18} \tag{60}$$

where the exponent is $\gtrless 0$ according as $a/b \gtrless \left(a/b\right)_D$,

and

$$\frac{\eta_t}{\eta_{t_D}} = \left[\frac{a}{a_D}\right]^{.064 \text{ or } -.105} \tag{61}$$

11. Small Stage Efficiency:

The author is of the opinion that efficiencies customarily used in the reports on combustion gas turbines, viz., the ratio of temperature differences, are not the ones which should be employed. The natural efficiencies to employ for this type of problem are defined as the ratio of the difference of the logarithms of the temperatures. This premise is based upon the usual hypothesis that the actual processes are polytropic and the ideal processes are isentropic. Here efficiencies are defined in terms of energy, then these relations reduced to the definition in terms of logarithms, and thence shown that these definitions are equivalent to the small stage efficiencies mentioned in the previous sections of this report. (This discussion is based upon the usual assumption, that C_p is a constant.)

$$(62) \quad \eta_{sc} = \frac{\text{enthalpy added - heat lost}}{\text{enthalpy added}} = \frac{\text{energy retained}}{\text{energy supplied}} ,$$

$$\eta_{sc} = \frac{c_p(T_2 - T_1) - c_n(T_2 - T_1)}{c_p(T_2 - T_1)} ,$$

$$(63) \quad \eta_{sc} = 1 - \frac{c_n}{c_p} ,$$

where, c_n as defined by Faires is the specific heat of a polytropic process,

$$(64) \quad c_n = c_v\left(\frac{\gamma - n}{1 - n}\right) .$$

Thus,
$$\eta_{sc} = 1 - \frac{c_v}{c_p}\left(\frac{\gamma - n}{1-n}\right) ,$$
$$\eta_{sc} = \frac{n(1-\gamma)}{\gamma(1-n)} ,$$

or

$$\left(\frac{n-1}{n}\right)\eta_{sc} = \frac{\gamma-1}{\gamma} . \tag{65}$$

Now, if T_1 denotes the initial temperature, T_2 and T_2' denote the final temperatures of the polytropic and isentropic processes which produce the same pressure ratio, then

$$\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1} , \qquad \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2'}{T_1} ,$$

and
$$\left(\frac{P_2}{P_1}\right)^{\eta_{sc}\left(\frac{n-1}{n}\right)} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} ;$$

thus,

$$\left(\frac{T_2}{T_1}\right)^{\eta_{sc}} = \frac{T_2'}{T_1} , \tag{66}$$

whence,

$$\boxed{\eta_{sc} = \frac{\log T_2' - \log T_1}{\log T_2 - \log T_1}} . \tag{67}$$

In the definition of η_c used in the first sections of this report

$$\eta_c = \frac{T_2' - T_1}{T_2 - T_1} \,,$$

and by (21)

$$\eta_c = \frac{\rho_c^{\frac{\gamma-1}{\gamma}} - 1}{\left(\rho_c^{\frac{\gamma-1}{\gamma}}\right)^{1/\eta_{sc}} - 1} \,,$$

thus,

$$\frac{\rho_c^{\frac{\gamma-1}{\gamma}} - 1}{\rho_c^{\frac{1}{\eta_{sc}}\left(\frac{\gamma-1}{\gamma}\right)} - 1} = \frac{T_2' - T_1}{T_2 - T_1} \,; \quad \frac{\frac{T_2'}{T_1} - 1}{\left(\frac{T_2'}{T_1}\right)^{1/\eta_{sc}} - 1} = \frac{\frac{T_2'}{T_1} - 1}{\frac{T_2}{T_1} - 1} \,,$$

$$\left(\frac{T_2'}{T_1}\right)^{1/\eta_{sc}} = \frac{T_2}{T_1} \,,$$

which is equivalent to (66) from which (67) follows immediately.

Turbine: In the case of the turbine, define η_{st} as follows:

(68) $$\eta_{st} = \frac{\text{enthalpy drop}}{\text{enthalpy drop + heat loss}}$$

If, according to the usual notation above, T_3 is the entering temperature, T_4 and T_4' are the exit temperatures for the polytropic and isentropic processes, respectively, then from the above definition:

$$\eta_{st} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_4) + c_n(T_4 - T_3)}, \quad \text{in this process}$$

$$\eta_{st} = \frac{c_p}{c_p - c_n} \,,$$

$$(69) \qquad \frac{1}{\eta_{st}} = 1 - \frac{c_n}{c_p} ,$$

and, as above,

$$(70) \qquad \frac{n-1}{n} = \left(\frac{\gamma-1}{\gamma}\right)\eta_{st} ,$$

$$(71) \qquad \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_3}{T_4} , \qquad \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4'} .$$

Thus, it follows

$$(72) \qquad \frac{T_3}{T_4} = \left(\frac{T_3}{T_4'}\right)^{\eta_{st}} ,$$

or

$$(73) \qquad \boxed{\eta_{st} = \frac{\log T_3 - \log T_4}{\log T_3 - \log T_4'}} .$$

Now, from the two definitions of η_t

$$\frac{1 - \left(\frac{1}{\rho^{\frac{\gamma-1}{\gamma}}}\right)^{\eta_{st}}}{1 - 1/\rho^{\frac{\gamma-1}{\gamma}}} = \frac{T_3 - T_4}{T_3 - T_4'} ,$$

$$\frac{1 - \left(T_4'/T_3\right)^{\eta_{st}}}{1 - T_4'/T_3} = \frac{1 - T_4/T_3}{1 - T_4'/T_3} ,$$

$$\left(\frac{T_4'}{T_3}\right)^{\eta_{st}} = \frac{T_4}{T_3} ,$$

from whence (73) follows.

Diffuser: In an analogous manner the efficiency of the diffuser may be defined as

(74) $$\frac{T_1}{T_0} = \left(\frac{T_1'}{T_0}\right)^{1/\eta_{sd}}$$

The above definitions of efficiencies have a big advantage over the temperature difference ratio definition. When a process is carried out in two steps or stages, with equal efficiency as defined above, the efficiency for the two stages is equal to the efficiency of each stage. This is not true for the temperature difference ratio definition. In particular, the logarithmic definition is most applicable when one desires to compare the jet and the turbine motors, for then the turbine efficiency for the entire enthalpy drop may be employed to compute the enthalpy drop which may be removed by a jet. Moreover, in the design of a turbine, the problem of reheat is eliminated if η_{st} is known.

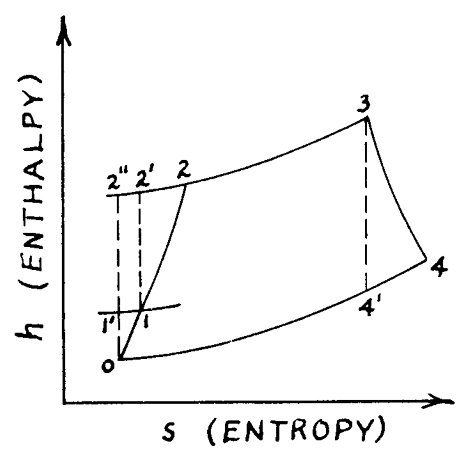
In terms of these stage efficiencies the formulae for e_u and s.f.c. for various cycles of the combustion gas turbine and of the jet-turbine are developed.

Thermodynamic Cycle I.

$$e^{I}_{u_t} = \eta_p \frac{1}{T_0}\left(T_3 - T_4 - T_2 + T_0\right)$$

$$= \eta_p\left(\frac{T_3}{T_0} - \frac{T_4}{T_3}\cdot\frac{T_3}{T_0} - \frac{T_2}{T_1}\cdot\frac{T_1}{T_0} + 1\right)$$

$$= \eta_p\left[\tau - \tau\left(\frac{f}{a}\right)^{\eta_{st}} - \left(\frac{T_2'}{T_1}\right)^{1/\eta_{sc}} b + 1\right].$$



Now, since $\dfrac{T_2'}{T_2''} = \dfrac{T_1}{T_1'}$,

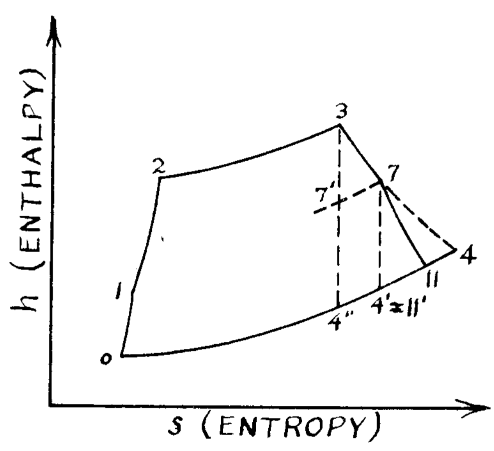
$$\frac{T_2'}{T_1} = \frac{T_2''}{T_0}\cdot\frac{T_0}{T_1}\cdot\frac{T_2'}{T_2''} = \frac{T_2''}{T_0}\cdot\frac{T_0}{T_1}\cdot\frac{T_1}{T_1'} = a\, b^{-\eta_{sd}} ,$$

(75) $$e^{I}_{u_t} = \eta_p\left[\tau - \tau\left(\frac{f}{a}\right)^{\eta_{st}} - a^{1/\eta_{sc}}\, b^{1-\eta_{sd}/\eta_{sc}} + 1\right] ,$$

$$\text{s.f.c.} = \frac{.134}{\eta_p}\cdot\frac{T_3 - T_2}{T_3 - T_4 - T_2 + T_0} ,$$

$$\text{s.f.c.} = \frac{.134}{\eta_p}\;\frac{\tau - a^{1/\eta_{sc}}\, b^{1-\frac{\eta_{sd}}{\eta_{sc}}}}{\tau\left[1-\left(\frac{f}{a}\right)^{\eta_{st}}\right] - a^{1/\eta_{sc}}\, b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + 1} ,$$

(76) $$\text{s.f.c.} = .134\left[\frac{1}{\eta_p} + \frac{-1 + \tau\left(f/a\right)^{\eta_{st}}}{e^{I}_{u_t}}\right] .$$



<u>Jet Motor:</u> $e_{u_j} = \eta_f \dfrac{1}{T_0}\left[T_3 - T_{11} - T_2 + T_0\right]$,

$$e_{u_j} = \eta_f \frac{1}{T_0}\left[(T_3 - T_4) - (T_2 - T_0) + (T_4 - T_{11})\right]$$
$$= \eta_f\left[\frac{e_{u_t}}{\eta_p} + \frac{T_4}{T_0}\left(1 - \frac{T_{11}}{T_4}\right)\right] .$$

It is assumed that $T_3 - T_7 = T_2 - T_1$

then, replacing the jet with an imaginary turbine, $e_{u_t} = \eta_p\left[\dfrac{1}{T_0}(T_7 - T_4) - b + 1\right]$,

$$\frac{1}{\eta_p} e_{u_t} + (b-1) = \frac{T_4}{T_0}\left(\frac{T_7}{T_4} - 1\right) ; \quad \frac{(1/\eta_p)\, e_{u_t} + b - 1}{\tau\,(f/a)^{\eta_{st}}} = \left(\frac{T_7}{T_4'}\right)^{\eta_{st}} - 1 ,$$

$$\left[1 + \frac{(1/\eta_p)\, e_{u_t} + b - 1}{\tau\,(f/a)^{\eta_{st}}}\right] = \left(\frac{T_7}{T_{11}'}\right)^{\eta_{st}} ,$$

$$\left[1 + \frac{(1/\eta_p)\, e_{u_t} + b - 1}{\tau\,(f/a)^{\eta_{st}}}\right]^{\eta_{sj}/\eta_{st}} = \frac{T_7}{T_{11}} \quad ,$$

where by definition

$$\frac{T_7}{T_{11}} = \left(\frac{T_7}{T_{11}'}\right)^{\eta_{sj}} \quad , \qquad \text{or} \quad \eta_{sj} = \frac{\log T_7 - \log T_{11}}{\log T_7 - \log T_{11}'} \, .$$

Now, $\dfrac{T_{11}}{T_4} = \dfrac{T_7/T_4}{T_7/T_{11}}$.

Thus,

$$\frac{T_{11}}{T_4} = \frac{1 + \dfrac{(1/\eta_p)\, e_{u_t} + b - 1}{\tau\,(f/a)^{\eta_{st}}}}{\left[1 + \dfrac{(1/\eta_p)\, e_{u_t} + b - 1}{\tau\,(f/a)^{\eta_{st}}}\right]^{\eta_{sj}/\eta_{st}}} = \left[1 + \frac{(1/\eta_p)\, e_{u_t} + b - 1}{\tau\,(f/a)^{\eta_{st}}}\right]^{1 - \frac{\eta_{sj}}{\eta_{st}}} .$$

Therefore,

$$e_{u_j} = \eta_f \left\{ \frac{e_{u_t}}{\eta_p} + \tau\left(\frac{f}{a}\right)^{\eta_{st}} - \tau\left(\frac{f}{a}\right)^{\eta_{st}} \left[\frac{(1/\eta_p)\, e_{u_t} + \tau\,(f/a)^{\eta_{st}} + b - 1}{\tau\,(f/a)^{\eta_{st}}}\right]^{1 - \frac{\eta_{sj}}{\eta_{st}}} \right\} ,$$

(77) $$e_{u_j} = \eta_f \left\{ \tau - a^{1/\eta_{sc}}\, b^{1 - \frac{\eta_{sd}}{\eta_{sc}}} + 1 - \tau^{\eta_{sj}/\eta_{st}} \left(\frac{f}{a}\right)^{\eta_{sj}} \left[\tau - a^{1/\eta_{sc}}\, b^{1 - \frac{\eta_{sd}}{\eta_{sc}}} + b\right]^{1 - \frac{\eta_{sj}}{\eta_{st}}} \right\} ,$$

(78) $$s.f.c. = .134 \left[\frac{1}{\eta_f} + \frac{-1 + \tau^{\eta_{sj}/\eta_{st}} \left(f/a\right)^{\eta_{sj}} \left[\tau - a^{1/\eta_{sc}}\, b^{1 - \frac{\eta_{sd}}{\eta_{sc}}} + b\right]^{1 - \frac{\eta_{sj}}{\eta_{st}}}}{e_{u_j}}\right] .$$

Relations (75), (76), (77), and (78) show that the power coefficient and the specific fuel consumption for the gas combustion turbine and the jet motor would be practically equivalent if $\eta_p = \eta_f$, since $\eta_{sj} \approx \eta_{st}$.

Since η_f is much smaller than η_p the jet motor develops less thrust power and has a higher fuel consumption than the combustion gas turbine. Whenever the speed of the airplane is sufficiently great to give an η_f greater than η_p it would be advantageous to eliminate the propeller completely and employ only the jet.

Thermodynamic Cycle II: Two Turbines with Second Burner Between Them.

$$\frac{1}{\eta_p} e_u^{II} = \frac{1}{T_0}\left[T_5 - T_6 - T_1 + T_0\right] ,$$

$$\frac{1}{\eta_p} e_u^{II} = \frac{T_5}{T_0}\left[1 - \frac{T_6}{T_5}\right] - \frac{T_1}{T_0} + 1 ,$$

$$= \tau_2\left[1 - \left(\frac{T_6'}{T_5}\right)^{\eta_{st_2}}\right] - b + 1$$

$$= \tau_2\left[1 - \left(\frac{T_4'}{T_7}\right)^{\eta_{st_2}}\right] - b + 1 .$$

But,

$$\frac{1}{\eta_p} e_{u_t} = \frac{1}{T_0}\left(T_7 - T_4\right) - b + 1 ,$$

$$\frac{1}{\eta_p} e_{u_t} + b - 1 = \frac{T_7}{T_0}\left(1 - \frac{T_4}{T_7}\right) ,$$

$$\frac{1}{\eta_p} e_{u_t} + b - 1 = \frac{T_7}{T_0}\left[1 - \left(\frac{T_4'}{T_7}\right)^{\eta_{st}}\right] .$$

Since

$$\frac{T_7}{T_0} - \frac{T_3}{T_0} = \frac{T_1}{T_0} - \frac{T_2}{T_0} \quad ,$$

$$\frac{T_7}{T_0} = \tau - a^{1/\eta_{sc}}\, b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + b \quad ,$$

$$\frac{\tau - a^{\frac{1}{\eta_{sc}}}\, b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + b}{\tau\,(f/a)^{\eta_{st}}} = \left(\frac{T_7}{T_4'}\right)^{\eta_{st}} .$$

Thus,

$$(79) \quad e_u^{II} = \eta_p\left[\tau_2\left\{1-\left(\frac{\tau\,(f/a)^{\eta_{st}}}{\tau - a^{1/\eta_{sc}}\, b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + b}\right)^{\frac{\eta_{st_2}}{\eta_{st}}}\right\} - b + 1\right] \quad ,$$

$$s.f.c.^{II} = .134\,\frac{\frac{T_3}{T_0} - \frac{T_2}{T_0} + \frac{T_5}{T_0} - \frac{T_7}{T_0}}{e_u^{II}} = .134\,\frac{\frac{T_5}{T_0} - \frac{T_1}{T_0}}{e_u^{II}}$$

$$= .134\,\frac{\tau_2 - b}{e_u^{II}}$$

$$(80) \quad s.f.c.^{II} = .134\left[\frac{1}{\eta_p} + \frac{-1 + \tau_2\left[\frac{\tau\,(f/a)^{\eta_{st}}}{\tau - a^{1/\eta_{sc}}\, b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + b}\right]^{\frac{\eta_{st_2}}{\eta_{st}}}}{e_u^{II}}\right] .$$

As was pointed out in a foot-note the first estimates of e_u^{II} and s.f.c.^{II} were only approximately correct due to the interpretation of the value of T_7 based upon a value of η_t for a temperature drop from T_3 to T_4 instead of the η_t based upon the temperature drop from T_3 to T_7. As a result of the use of small stage efficiencies in the development the true values for e_u^{II} and s.f.c.^{II} are determined.

Thermodynamic Cycle III. - Heat Exchanger.

In this and the following developments it is assumed that at least Y ° temperature drop is necessary for heat transfer.*

$$(81) \quad e_u^{III} = \eta_p \left[\tau - \tau \left(f/a \right)^{\eta_{st}} - a^{1/\eta_{sc}} b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + 1 \right] ,$$

$$W_F^{III} = \frac{3.6}{19} c_p W_A^{III} \left(T_3 - T_{10} \right) \; ; \; \eta_e = \frac{T_{10} - T_2}{(T_4 - Y) - T_2} \; ;$$

$$W_F^{III} = \frac{3.6}{19} c_p W_A T_0 \left[\tau - \eta_e \frac{T_4}{T_0} - (1-\eta_e) \frac{T_2}{T_0} \right] + \frac{3.6}{19} c_p W_A^{III} Y \eta_e ,$$

$$= \frac{3.6}{19} c_p W_A^{III} T_0 \left[\tau - \eta_e \tau \left(\frac{f}{a} \right)^{\eta_{st}} - (1-\eta_e) a^{\frac{1}{\eta_{sc}}} b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + \eta_e \left(\frac{Y}{T_0} \right) \right] ,$$

$$(82) \quad s.f.c.^{III} = .134 \left[\frac{1}{\eta_p} + \frac{-1 + (1-\eta_e) \tau \left(\frac{f}{a} \right)^{\eta_{st}} + \eta_e a^{1/\eta_{sc}} b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + \eta_e \left(\frac{Y}{T_0} \right)}{e_u^{III}} \right] .$$

Thermodynamic Cycle IV - Coolers and Heat Exchanger.

$$\frac{1}{\eta_p} e_u^{IV} = \tau - \frac{T_4}{T_0} - \frac{T_{14}}{T_0} + \frac{T_{13}}{T_0} - \frac{T_{12}}{T_0} + 1 ,$$

$$s.f.c.^{IV} = \frac{.134}{\eta_p} \left[\frac{\tau - \frac{T_{10}}{T_0}}{\tau - \frac{T_4}{T_0} - \frac{T_{14}}{T_0} + \frac{T_{13}}{T_0} - \frac{T_{12}}{T_0} + 1} \right] .$$

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* This assumption for a heat exchanger was not assumed in Section 5. It was only assumed for an intercooler.

Now, $$\left(\frac{T_{14}'}{T_{13}}\right)^{\frac{1}{\eta_{sc}}} = \frac{T_{14}}{T_{13}} \quad , \quad \left(\frac{T_{12}'}{T_1}\right)^{1/\eta_{sc}} = \frac{T_{12}}{T_1} \quad ,$$

$$\frac{T_{13}}{T_o} = 1 + \frac{Y}{T_o} \quad ,$$

$$\frac{T_{10}}{T_o} = \frac{T_{14}}{T_o}(1-\eta_e) + \eta_e \frac{T_4}{T_o} - \eta_e \frac{Y}{T_o} \quad ,$$

(83) $$\frac{1}{\eta_p} e_u^{IV} = \tau - \tau\left(\frac{f}{a}\right)^{\eta_{st}} - \left(1+\frac{Y}{T_o}\right)\left[\left(\frac{a}{d\,b^{\eta_{sd}}}\right)^{\frac{1}{\eta_{sc}}} - 1\right] - b\,d^{\frac{1}{\eta_{sc}}} + 1 \quad ,$$

(84) $$s.f.c.^{IV} = \frac{.134}{\eta_p}\left[\frac{\tau - \left(1+\frac{Y}{T_o}\right)(1-\eta_e)\left(\frac{a}{d\,b^{\eta_{sd}}}\right)^{\frac{1}{\eta_{sc}}} - \eta_e \tau\left(\frac{f}{a}\right)^{\eta_{st}} + \eta_e \frac{Y}{T_o}}{\tau - \tau\left(\frac{f}{a}\right)^{\eta_{st}} - \left(1+\frac{Y}{T_o}\right)\left[\left(\frac{a}{d\,b^{\eta_{sd}}}\right)^{1/\eta_{sc}} - 1\right] - b\,d^{1/\eta_{sc}} + 1}\right] \quad .$$

(84') $$s.f.c.^{IV} = .134\left[\frac{1}{\eta_p} + \frac{-1 + \tau(1-\eta_e)\left(\frac{f}{a}\right)^{\eta_{st}} - \left(1+\frac{Y}{T_o}\right)(1-\eta_e)\left(\frac{a}{d\,b^{\eta_{sd}}}\right)^{1/\eta_{sc}} + b\,d^{1/\eta_{sc}} + \eta_e \frac{Y}{T_o}}{e_u^{IV}}\right]$$

The corresponding formulae for the absorption refrigeration without the heat exchanger are:

$$\frac{e_u^-}{\eta_p} = \tau - \tau\left(\frac{f}{a}\right)^{\eta_{st}} - a^{\frac{1}{\eta_{sc}}} b^{1-\frac{\eta_{sd}}{\eta_{sc}}} + 1 + \eta_r \eta_e^2 \left(a^{\frac{1}{\eta_{sc}}} b^{-\frac{\eta_{sd}}{\eta_{sc}}} - 1\right)\left(\tau\left(f/a\right)^{\eta_{st}} - 1 - \frac{Y}{T_o}\right) ,$$

$$s.f.c.^- = .134\left[\frac{1}{\eta_p} + \frac{-1 + \tau\left(\frac{f}{a}\right)^{\eta_{st}} + \eta_r \eta_e^2\left(\tau\left(\frac{f}{a}\right)^{\eta_{st}} - 1 - \frac{Y}{T_o}\right)}{e_u^-}\right] \quad .$$

12. Summary Table: The following table gives a summary of the formulae for the e_u, s.f.c., e_i, and s.f.c.$_i$ for several cycles. A careful study of these expressions will show the relative merits of the various cycles. All efficiencies are small stage efficiencies. It has been assumed that the loss in energy, due to a reduction of the jet velocity by passing the gas through a heat exchanger, has been converted into available heat energy; that $Y°$ drop in temperature is necessary for every heat transfer except in the ideal cases; that the Froude efficiency for the jet on the combustion gas turbines is equal to the propeller efficiency. It is further assumed that there is no loss in pressure during the cooling processes.

CYCLE	ϵ_i	[illegible]	$\frac{e_u}{\eta_p}$	$\eta_p\left(\frac{s.f.c.}{.134}\right)$
SIMPLE JET MOTOR.	$\frac{a-1}{a}(\tau - a)$	$\frac{a}{a-1}$	$\frac{\eta_f}{\eta_p}\left\{(\tau-E)\left[1-\left(\frac{\tau(1-\Lambda)}{\tau-E}\right)^{\frac{\eta_{sj}}{\eta_{st}}}\right]-b+1\right\}$	$\frac{\eta_p}{\eta_f}\left\{\frac{\tau-E-b}{(\tau-E)\left[1-\left(\frac{\tau(1-\Lambda)}{\tau-E}\right)^{\eta_{sj}/\eta_{st}}\right]-b+1}\right\}$
I SIMPLE TURBINE.	$\frac{a-1}{a}(\tau - a)$	$\frac{a}{a-1}$	$\tau\Lambda - E - b + 1$	$\frac{\tau - E - b}{\tau\Lambda - E - b + 1}$
II TWO BURNERS, TWO TURBINES.	$\frac{\tau_2\left[\frac{a-1}{a}(\tau-a)+b-1\right]}{\tau - a + b} - b + 1$ WHERE $\tau_2 \geq \tau - a + b$	$\frac{(\tau_2 - \ )(\tau - a + b)}{\frac{a-1}{a}(\tau - a)\tau_2 + (b-1)(\tau_2 - \tau + a - b)}$	$\tau_2\left[1-\left(\frac{\tau(1-\Lambda)}{\tau-E}\right)^{\frac{\eta_{st_2}}{\eta_{st}}}\right]-b+1$ WHERE $\tau_2 \geq \tau - E$	$\frac{\tau_2 - b}{\tau_2\left[1-\left(\frac{\tau(1-\Lambda)}{\tau-E}\right)^{\eta_{st_2}/\eta_{st}}\right]-b+1}$
III SIMPLE TURBINE, HEAT EXCHANGER.	$\frac{a-1}{a}(\tau - a)$	$\frac{\tau}{\tau - a}$ WHERE $\tau \geq a^2$	$\tau\Lambda - E - b + 1$ WHERE $\tau(1-\Lambda) \geq E + b + K - 1$	$\frac{\tau - E - b - \eta_e[\tau(1-\Lambda) - E - b - K + 1]}{\tau\Lambda - E - b + 1}$
IV HEAT EXCHANGER, INTERCOOLER.	$\tau\left(\frac{a-1}{a}\right) - \frac{a}{bd} - bd + 2$ WHERE $\frac{a}{b} \geq d$; $\tau \geq \frac{a^2}{bd}$	$\frac{\tau\left(\frac{a-1}{a}\right)}{\tau\left(\frac{a-1}{a}\right) - \frac{a}{bd} - bd + 2}$	$\tau\Lambda - K\Omega - bd^{1/\eta_{sc}} + 1$ WHERE $a/b^{\eta_{sd}} \geq d$	$\frac{\tau - K(\Omega+1) - \eta_e[\tau(1-\Lambda) - K(\Omega+1) - K + 1]}{\tau\Lambda - K\Omega - bd^{1/\eta_{sc}} + 1}$ WHERE $\tau(1-\Lambda) \geq K(\Omega+1) + K - 1$
V HEAT EXCHANGER, REFRIGERATOR COOLER.	$\tau\left(\frac{a-1}{a}\right) - (b+1)\frac{a-b}{a+b} - b + 1$	$\frac{\tau\left(\frac{a-1}{a}\right)}{\tau\left(\frac{a-1}{a}\right) - (b+1)\frac{a-b}{a+b} - b + 1}$ WHERE $\tau/_1 \geq \frac{(b+1)(a-b)}{a+b}$	$\tau\Lambda - \Gamma\left(\frac{E}{b}\right) - b + 1$	$\frac{\tau - \Gamma\left(\frac{E}{b}+1\right) - \eta_e\left[\tau(1-\Lambda) - \Gamma\left(\frac{E}{b}+1\right) - K + 1\right]}{\tau\Lambda - \Gamma(E/b) - b + 1}$ WHERE $\tau(1-\Lambda) \geq \Gamma\left(\frac{E}{b}+1\right) + K - 1$
VI REFRIGERATOR COOLER, INTERCOOLER, HEAT EXCHANGER.	$\tau\left(\frac{a-1}{a}\right) + (d-2)\left(\frac{a}{bd} - 1\right) - bd + 1$ WHERE $\frac{a}{b} > d$	$\frac{\tau\left(\frac{a-1}{a}\right)}{\tau\left(\frac{a-1}{a}\right) + (d-2)\left(\frac{a}{bd} - 1\right) - bd + 1}$ WHERE $\tau \geq a^2/bd$	$\tau\Lambda - K\Omega\left[1 - \eta_e^2\eta_r\left(d^{1/\eta_{sc}} - 1\right)\right] - bd^{1/\eta_{sc}} + 1$ WHERE $a/b^{\eta_{sd}} > d$	$\frac{\tau - K(\Omega+1) - \eta_e[\tau(1-\Lambda) - K(\Omega+1) - K + 1]}{\tau\Lambda - K\Omega\left[1 - \eta_e^2\eta_r\left(d^{1/\eta_{sc}} - 1\right)\right] - bd^{1/\eta_{sc}} + 1}$ WHERE $\tau(1-\Lambda) \geq K(\Omega+1) + K - 1$
VII INTERCOOLER, TWO BURNERS, HEAT EXCHANGER.	$\frac{\tau_2\left[\tau\left(\frac{a-1}{a}\right) - \frac{a}{bd} - bd + b + 1\right]}{\tau - \frac{a}{bd} - bd + b + 1} - b + 1$ WHERE $\tau_2 \geq \tau - a/bd - bd + b + 1$	$\frac{\tau_2\left[1 - \frac{\tau/_1}{\tau - a/bd - bd + b + 1}\right] + \frac{a}{bd} + bd - b - 1}{\tau_2\left[1 - \frac{\tau/_1}{\tau - a/bd - bd + b + 1}\right] - b + 1}$ AND $a/b > a$	$\tau_2\left[1 - \left\{\frac{\tau(1-\Lambda)}{\tau - K\Omega - b\left(d^{1/\eta_{sc}} - 1\right)}\right\}^{\frac{\eta_{st_2}}{\eta_{st}}}\right] - b + 1$ WHERE $\tau_2 \geq \tau - K\Omega - b\left(d^{1/\eta_{sc}} - 1\right)$	$\frac{\tau_2 - K + b\left(d^{1/\eta_{sc}} - 1\right) - \eta_e\left[\tau_2\left\{\frac{\tau(1-\Lambda)}{\tau - K\Omega - b\left(d^{1/\eta_{sc}} - 1\right)}\right\}^{\frac{\eta_{st_2}}{\eta_{st}}} - K(\Omega+1) - K + 1\right]}{\tau_2\left[1 - \left\{\frac{\tau(1-\Lambda)}{\tau - K\Omega - b\left(d^{1/\eta_{sc}} - 1\right)}\right\}^{\eta_{st_2}/\eta_{st}}\right] - b + 1}$
VIII REFRIGERATOR COOLER, INTERCOOLER, TWO BURNERS, HEAT EXCHANGER.	$\frac{\tau_2\left[\tau\left(\frac{a-1}{a}\right) + \frac{a}{b} + b - d(b+1) - \frac{2a}{bd} + 2\right]}{\tau + \frac{a}{b} + b - d(b+1) - \frac{2a}{bd} + 2} - b + 1$ WHERE $\tau_2 \geq \tau + \frac{a}{b} + b - d(b+1) - \frac{2a}{bd} + 2$	$\frac{\tau_2\left[1 - \frac{\tau/a}{\tau + \frac{a}{b} + b - d(b+1) - \frac{2a}{bd} + 2}\right] - \frac{a}{b} - b + d(b+1) + \frac{2a}{bd} - 2}{\tau_2\left[1 - \frac{\tau/a}{\tau + \frac{a}{b} + b - d(b+1) - \frac{2a}{bd} + 2}\right] - b + 1}$ AND $a/b > d$	$\tau_2[1 - G] - b + 1$ WHERE $\tau_2 \geq \tau - K\Omega\left[1 - \eta_e^2\eta_r\left(d^{1/\eta_{sc}} - 1\right)\right] + b\left(d^{1/\eta_{sc}} - 1\right)$	$\frac{\tau_2 - K + \left[b - \eta_e^2\eta_r K\Omega\right]\left[d^{1/\eta_{sc}} - 1\right] - \eta_e\left[\tau_2 G - K(\Omega+1) - K + 1\right]}{\tau_2[1 - G] - b + 1}$ AND $\tau_2 G \geq K(\Omega+1) + K - 1$

DEFINITIONS

$$\Lambda = 1 - \left(\frac{f}{a}\right)^{\eta_{st}}$$

$$\Omega = \left(\frac{a}{d\,b^{\eta_{sd}}}\right)^{1/\eta_{sc}} - 1$$

$$E = a^{1/\eta_{sc}}\, b^{1 - \frac{\eta_{sd}}{\eta_{sc}}} - b$$

$$G = \left[\frac{\tau(1-\Lambda)}{\tau - K\Omega\left[1 - \eta_e^2\eta_r\left(d^{1/\eta_{sc}} - 1\right)\right] - b\left(d^{1/\eta_{sc}}\right.}\right.$$

$$K = 1 + \frac{Y}{T_0}$$

$$\Gamma = \frac{b + \eta_e^2\eta_r}{1 + \eta_e^2\eta_r\left(a/b^{\eta_{sd}}\right)^{1/\eta_{sc}}}$$

NOTE:
THE FORMULAE FOR V CANNOT BE OBTAINED FROM VI BY LETTING $a/b = d$. THIS DOE NOT ELIMINATE THE INTER-COOLER.
THE AIR IS COOLED AND RE HEATED AT THE SAME PRES-SURE.

APPENDIX A.

Theorem: A necessary condition for

$$(1) \quad \begin{cases} P = \phi(\tau) , \\ s.f.c. = \psi(\tau) , \end{cases}$$

to be the envelope of the family of curves

$$(2) \quad \begin{cases} s.f.c. = \xi(a,\tau) , \\ P = \Phi(a,\tau) , \end{cases}$$

is that the values (1) satisfy simultaneously the relations (2) and

$$\frac{\partial(P, s.f.c.)}{\partial(a,\tau)} = 0 .$$

Proof: A curve is defined to be an envelope of a family of curves provided it intersects all curves of the family, at points where its tangents coincide with the tangents of the curves of the family.

First,

$$\phi(\tau) = \Phi(a,\tau) \quad ; \quad \psi(\tau) = \xi(a,\tau) .$$

Thus,

$$\phi'(\tau) = \frac{\partial \Phi}{\partial a}\cdot\frac{da}{d\tau} + \frac{\partial \Phi}{\partial \tau} \quad , \quad \psi'(\tau) = \frac{\partial \xi}{\partial a}\cdot\frac{da}{d\tau} + \frac{\partial \xi}{\partial \tau}$$

And the slope of the tangent is the ratio of these two quantities. On the other hand, the slope of the tangent to a curve of the family (τ = Const.) is ratio dP to $d(s.f.c.)$, thus

$$dP = \frac{\partial P}{\partial a}\, da \quad , \quad d(s.f.c.) = \frac{\partial (s.f.c.)}{\partial a}\, da .$$

Thus, if (1) is the envelope -

$$\frac{\partial(s.f.c.)}{\partial a}\cdot\frac{da}{d\tau}+\frac{\partial(s.f.c.)}{\partial\tau}=K\,\frac{\partial(s.f.c.)}{\partial a},$$

$$\frac{\partial P}{\partial a}\cdot\frac{da}{d\tau}+\frac{\partial P}{\partial\tau}=K\,\frac{\partial P}{\partial a}.$$

These relations are consistent if, and only if,

$$(5)\quad \frac{\partial(P,\,s.f.c.)}{\partial(a,\,\tau)}=\frac{\partial P}{\partial a}\cdot\frac{\partial(s.f.c.)}{\partial\tau}-\frac{\partial P}{\partial\tau}\cdot\frac{\partial(s.f.c.)}{\partial a}=0.$$

If $\frac{\partial(P,\,s.f.c.)}{\partial(a,\tau)}=0$, and if $\left(\frac{\partial P}{\partial a}\right)^2+\left(\frac{\partial(s.f.c.)}{\partial a}\right)^2\neq 0$,

and $[\phi'(\tau)]^2+[\psi'(\tau)]^2\neq 0$,

then, (1) is the envelope of (2). For under these conditions the derivatives or their reciprocal exist and are equal.

APPENDIX B

Analytic relations between a and τ on the throttling curve for constant power at a constant altitude; (f is considered constant, which is essentially correct)

$$\begin{cases} P = \dfrac{K(a-b)(a-f)(A-a)}{\eta_c\, a}, \\[2ex] s.f.c. = \dfrac{.134}{\eta_p}\,\dfrac{a(B-a)}{(a-f)(A-a)}, \end{cases}$$

$$\frac{\partial P}{\partial \tau}\Big/ P = \frac{\partial A}{\partial \tau}\Big/ A-a\;; \quad \frac{\partial P}{\partial a}\Big/ P = \frac{1}{a-b} + \frac{1}{a-f} - \frac{1}{a} - \frac{\partial \eta_c/\partial a}{\eta_c} + \frac{\partial A/\partial a - 1}{A-a},$$

$$\frac{\partial (s.f.c.)}{\partial \tau}\Big/ s.f.c. = \frac{\partial B/\partial \tau}{B-a} - \frac{\partial A/\partial \tau}{A-a}\;; \quad \frac{\partial (s.f.c.)}{\partial a}\Big/ s.f.c. = -\frac{1}{a-f} + \frac{1}{a} + \frac{\partial B/\partial a - 1}{B-a} - \frac{\partial A/\partial a - 1}{A-a},$$

$$0 = \frac{\partial (P, s.f.c.)}{\partial (a, \tau)} = \begin{vmatrix} \dfrac{\frac{\partial A}{\partial \tau}}{A-a} & ;\; \dfrac{1}{a-b} + \dfrac{1}{a-f} - \dfrac{1}{a} - \dfrac{\partial \eta_c/\partial a}{\eta_c} + \dfrac{\partial A/\partial a - 1}{A-a} \\[3ex] \dfrac{\frac{\partial B}{\partial \tau}}{B-a} - \dfrac{\frac{\partial A}{\partial \tau}}{A-a} & ;\; -\dfrac{1}{a-f} + \dfrac{1}{a} + \dfrac{\partial B/\partial a - 1}{B-a} - \dfrac{\partial A/\partial a - 1}{A-a} \end{vmatrix},$$

$$0 = \begin{vmatrix} \dfrac{\frac{\partial A}{\partial \tau}}{A-a} & ;\; \dfrac{1}{a-b} + \dfrac{f}{a(a-f)} - \dfrac{\partial \eta_c/\partial a}{\eta_c} + \dfrac{\partial A/\partial a - 1}{A-a} \\[3ex] \dfrac{\frac{\partial B}{\partial \tau}}{B-a} & ;\; \dfrac{1}{a-b} - \dfrac{\partial \eta_c/\partial a}{\eta_c} + \dfrac{\partial B/\partial a - 1}{B-a} \end{vmatrix},$$

$$0 = \frac{\eta_c}{(A-a)(B-a)} \begin{vmatrix} \eta_t & ;\; \dfrac{A-a}{a-b} + \dfrac{f(A-a)}{a(a-f)} + \dfrac{a}{\eta_c}\dfrac{\partial \eta_c}{\partial a} + \tau \eta_c \dfrac{\partial \eta_t}{\partial a} - 1 \\[3ex] 1 & ;\; \dfrac{B-a}{a-b} - \dfrac{b}{\eta_c}\dfrac{\partial \eta_c}{\partial a} + \dfrac{a}{\eta_c}\dfrac{\partial \eta_c}{\partial a} - 1 \end{vmatrix},$$

$$0 = \begin{vmatrix} 1 \, ; & \dfrac{A-a}{\eta_t(a-b)} + \dfrac{f(A-a)}{\eta_t a(a-f)} + \dfrac{a}{\eta_c\eta_t}\dfrac{\partial\eta_c}{\partial a} + \tau\dfrac{\eta_c}{\eta_t}\dfrac{\partial\eta_t}{\partial a} - \dfrac{1}{\eta_t} \\ 1 \, ; & \dfrac{B-a}{a-b} - \dfrac{b}{\eta_c}\dfrac{\partial\eta_c}{\partial a} + \dfrac{a}{\eta_c}\dfrac{\partial\eta_c}{\partial a} - 1 \end{vmatrix} ,$$

$$0 = \frac{\eta_t(B-a)-(A-a)}{\eta_t(a-b)} - \frac{f(A-a)}{\eta_t a(a-f)} + \frac{a}{\eta_c}\left(1-\frac{1}{\eta_t}\right)\frac{\partial\eta_c}{\partial a} - \frac{b}{\eta_c}\frac{\partial\eta_c}{\partial a} - \tau\frac{\eta_c}{\eta_t}\frac{\partial\eta_t}{\partial a} + \frac{1}{\eta_t} - 1 ,$$

$$0 = \frac{b(1-\eta_c)}{a-b} - \frac{f(A-a)}{\eta_t a(a-f)} + \frac{1}{\eta_c}\left[a\left(1-\frac{1}{\eta_t}\right)-b\right]\frac{\partial\eta_c}{\partial a} - \tau\frac{\eta_c}{\eta_t}\frac{\partial\eta_t}{\partial a} + \frac{1}{\eta_t} - 1 ,$$

$$0 = \frac{\eta_t b(1-\eta_c)}{a-b} - \frac{f(A-a)}{a(a-f)} + \frac{\eta_t}{\eta_c}\left[a\left(1-\frac{1}{\eta_t}\right)-b\right]\frac{\partial\eta_c}{\partial a} - \tau\eta_c\frac{\partial\eta_t}{\partial a} - \eta_t + 1 .$$

Now, $\dfrac{P}{P_D} = \dfrac{K(a-b)(a-f)(A-a)a_D}{K(a_D-b)(a_D-f)(A_D-a_D)a}$, WHERE $A_D = \tau_D\,\eta_{c_D}\eta_{t_D}$,

$$\frac{P}{P_D}\cdot\frac{fs}{(a-f)^2(a-b)} = \frac{f(A-a)}{a(a-f)} , \quad \text{WHERE} \quad s = \frac{(a_D-b)(a_D-f)(A_D-a_D)}{a_D} ,$$

$$\tau = \frac{a}{\eta_c\eta_t}\left[\frac{Ps + P_D(a-f)(a-b)}{P_D(a-f)(a-b)}\right] .$$

Thus,

$$0 = \frac{\eta_t b(1-\eta_c)}{a-b} - \frac{Psf}{P_D(a-f)^2(a-b)} + \frac{\eta_t}{\eta_c}\left[a\left(1-\frac{1}{\eta_t}\right)-b\right]\frac{\partial\eta_c}{\partial a} - \frac{a}{\eta_t}\left[\frac{Ps+P_D(a-f)(a-b)}{P_D(a-f)(a-b)}\right]\frac{\partial\eta_t}{\partial a} - \eta_t + 1 .$$

Since efficiency may be closely approximated by a quadratic in a , we have an equation of the 10th degree in a .

(6) $$0 = \eta_t^2\eta_c P_D(a-f)^2\left[b(2-\eta_c)-a\right] - \eta_t\eta_c\left[Psf - P_D(a-f)^2(a-b)\right] + \eta_t P_D(a-f)^2(a-b)\left[a(\eta_t-1)-b\right]\frac{\partial\eta_c}{\partial a} - \eta_c a(a-f)\left[Ps + P_D(a-f)(a-b)\right]\frac{\partial\eta_t}{\partial a}$$

Denote (6) by $F(a) = 0$, then by Newton's method of solution if a is an approximate root, then a_1 is a closer approximation -,

$$a_1 = a - \frac{F(a)}{F'(a)}$$

, where $F'(a)$ denotes the derivative of $F(a)$ with respect to a.

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