AN ANALYTICAL APPROACH TO THE PROBLEM OF LONGITUDINAL STABILITY OF FLYING BOATS IN THE PLANING CONDITION

Thesis by

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INTRODUCTION

The subject of the longitudinal stability, or "porpoising", of flying boats has been the subject of investigation both in this country and abroad. In many cases, recourse to dynamically similar models has been had to determine the stability characteristics of the design in question. In general, however, the bare hull is tested in towing tanks, and the data are presented in the form of curves of resistance, moment, draft, speed, and angle of trim. Conclusions regarding "porpoising" of the full scale airplane cannot be drawn from these tests alone, but it is considered possible to evaluate certain of the hydrodynamic derivatives which in conjunction with aerodynamic derivatives obtained from wind tunnel tests, can be used in the stability equation to determine the behavior of the flying boat in the planing region.

In the analysis of the problem presented here, most of the derivations of the formulae have been omitted. For a more detailed derivation, reference is made to the thesis of Lieutenant George A. Hatton, U. S. N., entitled "The Longitudinal Stability of a Flying Boat in the Planing Condition as Computed from Tank Test Data of a Hull Model."

Throughout the analysis, the effect of a drag has been neglected. A discussion of this omission will be made in the conclusion of the paper.

The first part of the paper will be concerned with a presentation of the formulae and an example of their use with results of calculations made. The second part will cover the dynamic model tests and a discussion of the equipment used. The conclusion will cover the entire paper.

It must be noted that the analysis here is restricted to only the planing condition or that part of the take-off run past the hump, and the formulae are derived for that condition.

METHOD OF DETERMINING STABILITY

BASIC ASSUMPTIONS AND CONDITIONS

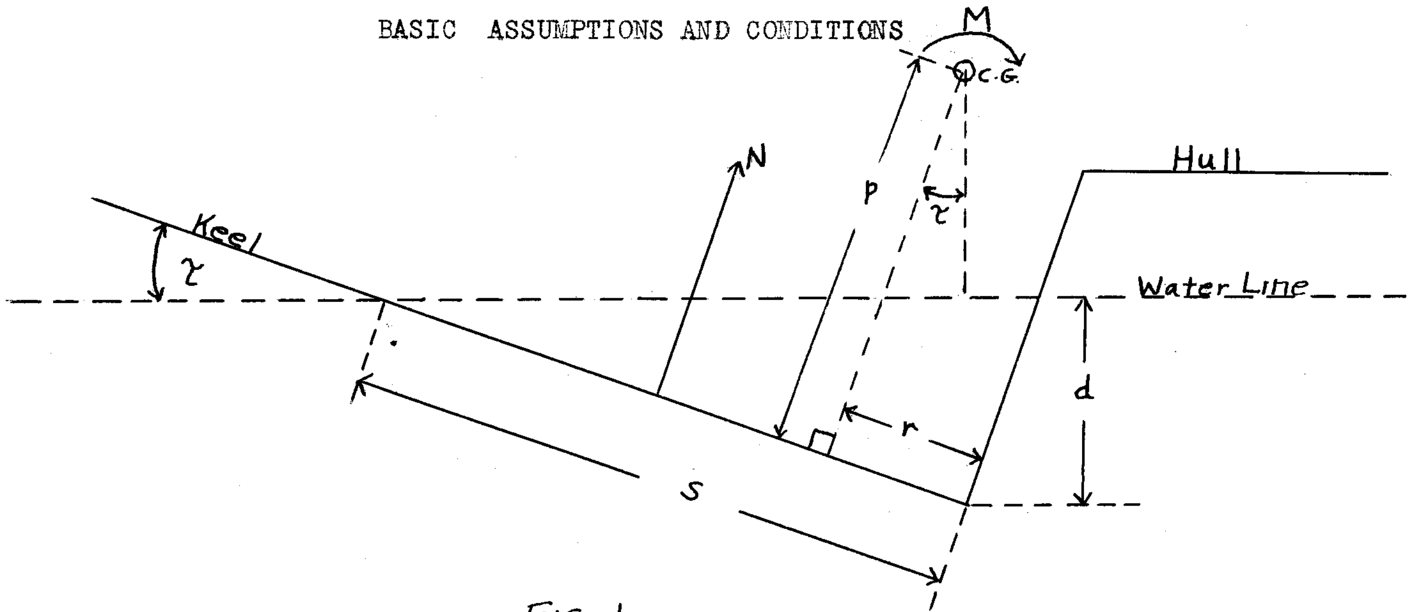


FIG. 1

(1) Replace hull bottom by an equivalent flat planing surface through the keel.

(2) Assume resultant force is normal to the keel = N.

(3) Assume N = -Z.

It is desirable to have an analytical approximation to Z and M for several reasons. The aerodynamic derivatives are similar to those normally used for airplanes, but they are evaluated in terms of beam, trim angle, and other hydrodynamic terms; then the hydrodynamic derivations are deduced also in terms of hull dimensions and attitudes. By these means, the direct addition of hydrodynamic and aerodynamic derivatives for use in the longitudinal stability equation is possible. The criteria for stability are then applied. By the use of the analytical approximations, considerable simplification has been made possible.

A definition of the symbols used will be found at the end of this section of the paper.

Considering the hull bottom as an airfoil, write

$$-Z = \alpha(\tau - \tau_0)\frac{\rho_w}{2}\bar{V}^2 b S$$

where $\alpha(\tau-\tau_0)$ corresponds to an airfoil

$$C_L \begin{cases} \alpha \backsim \frac{dC_L}{d\alpha} \\ \tau - \tau_0 \backsim \alpha - \alpha_{L_0} \end{cases}$$

now $S = \frac{d}{\tau}$, hence

$$-Z = \alpha\frac{\rho_w}{2}\bar{V}^2 b\left(d - \frac{\tau_0 d}{\tau}\right) = \alpha\frac{\rho_w}{2}\bar{V}^2 b^2\left(d_1 - \frac{\tau_0 d_1}{\tau}\right)$$

or writing $d_1 = \frac{d}{b}$

$$-Z = \alpha\frac{\rho_w}{2}\bar{V}^2 b^2\left(d_1 - \frac{\tau_0}{\tau}d_1\right)$$

From a study of tank test data it appears that a modification of this form furnishes a satisfactory framework in which the experimental results may be fitted. This is

(1) $$-Z = \frac{\rho_w}{2}\bar{V}^2 b^2 \alpha(d_1 - \delta)$$

$$\delta = \delta(\tau)$$

Turning now to the pitching moment, M. Glauert assumes that the c.p. of N always lies a definite fraction of S ahead of the step. This leads to

$$M = \frac{\rho_w}{2}\bar{V}^2 b^3 \alpha(d_1 - \delta)\left(\beta\frac{d_1}{\tau} - \frac{r}{b}\right)$$

The tank test data study indicates that a more satisfactory form is

$$(2) \qquad M = \frac{\rho_w}{2} \bar{V}^2 b^3 \alpha (d_1 - \delta) \underbrace{\left(\beta \frac{d_1}{\tau} - \sigma - r_1\right)}_{z}$$

$$\sigma = \sigma(\tau), \quad r_1 = \frac{r}{b}$$

In (1) and (2) there are two undetermined constants α and β and two undetermined functions $\delta(\tau)$ and $\sigma(\tau)$.

All may vary with hull shape and particularly with angle of dead rise. To determine them from normal tank test data giving C_M, C_Δ, d_1 as functions of C_V and τ, proceed as follows:

$$d_1 = \frac{d}{b} \qquad C_V = \frac{\bar{V}}{\sqrt{gb}}$$

$$C_\Delta = \frac{\Delta}{\rho_w g b^3} \qquad C_M = \frac{M}{\rho_w g b^4}$$

Now, let:

$$C_\Delta = \frac{-Z}{\rho_w g b^3} = \frac{\rho_w \bar{V}^2 b^2}{2 \rho_w g b^3} \alpha (d_1 - \delta) = \frac{C_V^2}{2} \alpha (d_1 - \delta)$$

$$(3) \quad \therefore \alpha (d_1 - \delta) = 2 \frac{C_\Delta}{C_V^2}$$

$$C_M = \frac{M}{\rho_w g b^4} = \frac{\rho_w \bar{V}^2 b^3}{2 \rho_w g b^4} \alpha (d_1 - \delta) \left(\beta \frac{d_1}{\tau} - \sigma - r_1\right)$$

$$C_M = \frac{C_V^2}{2} \alpha (d_1 - \delta) \left(\beta \frac{d_1}{\tau} - \sigma - r_1\right) = C_\Delta \left(\frac{\beta}{\tau} d_1 - \sigma - r_1\right)$$

$$(4) \quad \therefore \frac{\beta}{\tau} d_1 - \sigma = \frac{C_M}{C_\Delta} + r_1$$

Hence, if $\frac{C_\Delta}{C_V^2}$ is plotted as function of C_d for a series of τ's, then the slopes of all the τ = Const. lines should be the same and equal to $\frac{\alpha}{2}$, while the intercepts give $\alpha\,\delta(\tau)$.

Similarly, if $\frac{C_M}{C_\Delta} + r_1$ is plotted as function of $\frac{d_1}{\tau}$ all the τ = Const. lines should have the same slope equal to β, and the intercepts give $\sigma(\tau)$.

For any particular hull shape there can be determined from conventional tank test data:

(5)

$$\alpha, \beta, \delta(\tau), \sigma(\tau)$$

Hence

(6)

$$Z = Z(V, d_1, \tau)$$
$$M = M(V, d_1, \tau)$$

EQUILIBRIUM RELATIONS

For equilibrium relationships the hydrodynamic data, aerodynamic data, and the equilibrium relations for forces and moments must all be used. Using the subscript $(\ )_a$ to denote "aerodynamic" and writing the hydrodynamic lift as Δ, in accordance with usual convention, the conditions for equilibrium are:

$$(7) \qquad \begin{aligned} L_a + \Delta &= mg \\ M_a + M &= 0 \end{aligned}$$

The hydrodynamic relations have been presented, and now turning to the aerodynamic characteristics, write

$$(8) \qquad L_a = \frac{\rho}{2} V^2 S a (\tau - \tau_o)$$

where:

$a = \frac{dC_L}{d\alpha}$ = Slope of lift curve for complete seaplane

$\tau_o^* = \tau$ for $L_a = 0$ (τ_o may be a function of e)

and

$$(9) \qquad M_a = \frac{\rho}{2} V^2 S t \left\{ C_{M_o} - a(\tau - \tau_o)\xi + C_{Me} \right\}$$ Assuming a linear dependance of C_{Ma} on C_L

where

$C_{M_o} = C_{Ma}$ at $C_L = 0$ for $e = 0$

$\xi = -\frac{dC_{Ma}}{dC_L}$ for elevator fixed

$C_{Me} = \Delta C_{Ma}$ due to elevator deflection e (Independent of τ)

Now, turning to the force equation (7):

$$\frac{\rho}{2} V^2 S a (\tau - \tau_o) + C_\Delta \rho_w g b^3 = mg$$

$$\therefore \frac{\rho S b}{2m} \frac{V^2}{gb} a (\tau - \tau_o) + C_\Delta \frac{\rho_w b^3}{m} = 1$$

* Note: τ_o in this case is not the same as τ_o on Page 2. All formulae based on aerodynamic τ_o.

Using relations shown on page 12

$$C_V^2 f(\tau - \tau_0) + 2GC_\Delta = 1$$

$$\therefore C_\Delta = \frac{1 - fC_V^2(\tau - \tau_0)}{2G}$$

Now, turning to the moment equation:

$$C_M \rho_w G b^4 + \frac{\rho}{2} V^2 st \left\{ C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me} \right\} = 0$$

and

$$\frac{\rho_w b^3}{m} C_M + \frac{\rho S b}{2m} \frac{V^2}{gb} \frac{t}{b} \left\{ C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me} \right\} = 0$$

From relations on page 12:

$$2GC_M + JC_V^2 t_1 \left\{ C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me} \right\} = 0$$

$$\therefore C_M = -\frac{Jt_1}{2G} \left\{ C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me} \right\} C_V^2$$

Collecting results:

(10)

(a) $C_\Delta = \dfrac{1 - fC_V^2(\tau - \tau_0)}{2G} = C_\Delta(C_d, C_V, \tau)$ from tank data

(b) $C_M = -\dfrac{Jt_1}{2G} \left\{ C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me} \right\} C_V^2$ from tank data

$= C_M(C_\Delta, C_V, \tau)$

The approximations (3) and (4) may be written

(11) (a) $C_\Delta = C_V^2 \dfrac{\alpha}{2}(d_1 - \delta)$

(b) $C_M = C_V^2 \dfrac{\alpha}{2}(d_1 - \delta)\left(\beta \dfrac{d_1}{\tau} - \sigma - r_1\right)$

Introducing these into (10)

$$d_1 = \delta(\tau) + \frac{1}{\alpha G}\left\{\frac{1}{C_V^2} - f(\tau - \tau_0)\right\}$$

and

$$C_V^2 \frac{\alpha}{2}(C_d - \delta)(\beta \frac{d_1}{\tau} - \sigma - r_1) = -\frac{J t_1}{2G} C_V^2 \left\{C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me}\right\}$$

$$(C_d - \delta)(\beta \frac{d_1}{\tau} - \sigma - r_1) = -\alpha \frac{J}{G} t_1 \left\{C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me}\right\}$$

Hence, corresponding to (10)

(12)

(a) $$d_1 = \delta(\tau) + \frac{1}{\alpha G}\left\{\frac{1}{C_V^2} - f(\tau - \tau_0)\right\}$$

(b) $$(d_1 - \delta)(\beta \frac{d_1}{\tau} - \sigma - r_1) = -\alpha \frac{J}{G} t_1 \left\{C_{M_0} - a(\tau - \tau_0)\Sigma + C_{Me}\right\}$$

In general, one of two fundamental procedures will be followed during the seaplane's run on the water. They are:

(A) The trim angle will be adjusted to a definite value depending on C_V (and possibly C_Δ), i.e. $\tau(C_V, C_\Delta)$ will be determined.

(B) The elevator setting $e(C_V)$ will be predetermined.

In the case of (A) it is necessary to investigate the elevator angles $e(C_V)$ required for moment equilibrium to be sure that they are possible, while in the case of (B) the trim angle variation $\tau(C_V)$ must be calculated.

These two procedures correspond to the additional relations

(13) (A) $\tau = \tau(C_V, C_\Delta)$ given

(B) $C_{Me} = C_{Me}(C_V)$ given

If (A) is used, (10a) and (13A), or (12a) and (13A), give two relations between C_d, C_v, and τ which permit the determination of $C_d\ (C_v)$ and $\tau(C_v)$. Then, from (10b) or (12b), $C_{He}(C_v)$ may be calculated.

If (B) is used, (10) (or 12), and (13B) furnish two relations connecting C_Δ, C_v, and τ or C_d, C_v, and τ (eliminating C_{He}). This permits the determination of $C_\Delta(C_v)$ and $\tau(C_v)$ or of $C_d(C_v)$ and $\tau(C_v)$.

The procedure most commonly followed will probably be (A) in which the seaplane is held at the τ for "best trim" during the run. Best trim, τ_b, is defined as the value of τ giving the minimum C_R for any set of values of C_v, C_Δ. In general, from tank test results τ_b is plotted as a function of C_v and C_Δ. When this procedure is followed it is more convenient to use the graphical tank data for $\tau = \tau_b(C_v, C_\Delta)$ and the exact equation (10a) rather than the approximate form (12a). Given are:

(a) $\tau = \tau_b(C_v, C_\Delta)$ (graphically)

(14)

(b) $C_\Delta = \dfrac{1 - f C_v^2(\tau - \tau_o)}{2\,G}$

In view of the nature of the τ_b curves the following steps lead rapidly to the desired solution:

(a) Assume a C_v and pick a τ_b from the diagram ~ (14a), corresponding to a first choice of C_Δ (for large C_v's τ_b is only slightly changed for considerable variations in C_Δ).

(b) Calculate C_Δ from (14b)

(c) For this C_Δ look up τ_b on the diagram and repeat (a) and (b). This process rapidly converges.

(d) Repeat for other values of C_v giving

$$\tau(C_v) \quad , \quad C_\Delta(C_v)$$

(e) Look up $C_d(C_v, C_\Delta, \tau)$ from the tank test curves giving this relation. If tank data are not available for C_d, calculate C_d from:

$$C_d = d_1 = \delta + \frac{2}{\alpha}\frac{C_\Delta}{C_v^2}$$

where α and $\delta(\tau)$ are the average values chosen.

(f) Determine C_{Me} from

(15) $$C_{Me} = -C_{M_0} + a(\tau - \tau_0)\Sigma - \frac{2G}{Jt_1}\frac{C_M}{C_v^2}$$

where $C_M(C_v, C_\Delta, \tau)$ is taken from tank test curves if these are available. If they are not available.

(16) $$C_{Me} = -C_{M_0} + a(\tau - \tau_0)\Sigma - \frac{2G}{Jt_1}\frac{C_\Delta}{C_v^2}\left(\beta\frac{d_1}{\tau} - \sigma - r_1\right)$$

where β and $\sigma(\tau)$ are the average values chosen.

(g) Finally compare this C_{Me} with the aerodynamic values determined from wind tunnel tests on calculations of $C_{Me}(e)$, and check its reasonableness.

On the next pages are definitions of the symbols and terms used, and the values of the hydrodynamic and aerodynamic derivatives.

Note: For procedure (B) see Example on Page 14.

DEFINITION OF TERMS USED

τ = Trim angle in radians (see sketch)

τ_o = Trim angle in radians for zero aerodynamic lift

p = Distance perpendicularly from keel to center of gravity (See sketch)

r = Distance from step to p (See sketch)

d = Draft (See sketch)

N = Resultant force on keel acting normal to the keel (See sketch)

ϕs = Distance forward of step to center of pressure of the planing surface (See sketch)

s = Wetted length of keel from step forward

Z = Vertical axis, positive down

x = Horizontal axis positive forward

b = Beam of hull

d_1 = $C_d = \frac{d}{b}$

C_v = $\frac{V}{\sqrt{gb}}$

V = Ft./ sec

C_Δ = $\frac{\Delta}{\rho_w g b^3}$

Δ = Load on hull, i.e. static Δ = weight

ρw = 1.964 . Density of water

g = 32.2 ft./sec.

C_m = $\frac{M}{\rho_w g b^4}$

M = Moment about c.G.

$(\ )_1$ = $\frac{(\ )}{b}$

$(\ )_a$ = Aerodynamic quantities

$C_R \quad = \dfrac{R}{\rho_w g b^3}$

R = Resistance

$\Sigma \quad = -\dfrac{dC_{M_a}}{dC_L}$

C_{M_e} = ΔC_{M_a} due to elevator deflection e

C_{M_o} = C_{M_a} at $C_L = 0$ for $e = o$

e = Elevator angle

ρ = Density of air

DEFINITION OF HYDRODYNAMIC AND AERODYNAMIC DERIVATIVES

HYDRODYNAMIC

$$Z_z = \alpha G$$

$$Z_w = \alpha G \left(\frac{d_1}{\tau} - \delta'\right)$$

$$Z_\theta = \alpha G (r_1 - p_1 \tau - \delta')$$

$$Z_q = -\alpha G \frac{d_1}{\tau}\left\{\xi\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1 \tau \left(1 - 2\frac{\delta}{d_1}\right)\right\}$$

$$m_z = -\alpha H \left\{\xi + \frac{\beta}{\tau}(d_1 - \delta)\right\}$$

$$m_w = -\alpha H \frac{d_1}{\tau}\left\{\xi\left(1 - \frac{\delta'\tau}{d_1}\right) - (d_1 - \delta)\frac{\sigma'\tau}{d_1}\right\}$$

$$m_\theta = -\alpha H \left\{\xi(r_1 - p_1\tau - \delta') + (d_1 - \delta)\left[\frac{\beta}{\tau}\left(r_1 - p_1\tau - \frac{d_1}{\tau}\right) - \sigma'\right]\right\}$$

$$m_q = +\alpha H \frac{d_1}{\tau}\xi \left\{\xi\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right) - \sigma'\tau\left(1 - \frac{\delta}{d_1}\right)\right\}$$

Where :-

$$G = \frac{\rho_w b^3}{2m} = \frac{1}{2C_{\Delta R}} \qquad (\ )_1 = \frac{(\ )}{b} \qquad d_1 = C_d$$

$$H = \frac{G}{K_1^2} \qquad \xi = \frac{\beta}{\tau} d_1 - \sigma - r_1 \qquad G = (\tfrac{1}{2}G)_{SP}$$

$$J = \frac{\rho S b}{2m} = \frac{\rho}{\rho_w}\frac{S}{b^2}\frac{1}{2C_{\Delta R}} = \frac{\rho}{\rho_w}\frac{(b_0/b)^2}{AR} G$$

AERODYNAMIC

$$Z_z = 0 \qquad m_z = 0 \qquad f = J\frac{dC_L}{d\alpha}$$

$$Z_w = f \qquad m_w = h \qquad h = -J\frac{t_1}{K_1^2}\frac{dC_L}{d\alpha}\frac{dC_M}{dC_L}$$

$$Z_\theta = f \qquad m_\theta = h \qquad j = J K \eta_t \frac{l_1^2}{K_1^2}\frac{S_t}{S}\frac{dC_{Lt}}{d\alpha_t}$$

$$Z_q = 0 \qquad m_q = j$$

write :-

$$\frac{dC_L}{d\alpha} = a$$

$$\frac{dC_{Lt}}{d\alpha_t} = a_t$$

ALTERNATE FORM

$$Z_z = \alpha G$$

$$Z_w = \alpha G \frac{d_1}{\tau}\left(1 - \frac{\delta'\tau}{d_1}\right)$$

$$Z_\theta = \alpha G (r_1 - p_1\tau - \delta')$$

$$Z_q = -\alpha G \frac{d_1}{\tau}\left\{\xi\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right)\right\}$$

$$m_z = -\alpha H \left\{\xi + \frac{\beta}{\tau} d_1 \left(1 - \frac{\delta}{d_1}\right)\right\}$$

$$m_w = -\alpha H \frac{d_1}{\tau}\left\{\xi\left(1 - \frac{\delta'\tau}{d_1}\right) - \sigma'\tau\left(1 - \frac{\delta}{d_1}\right)\right\}$$

$$m_\theta = -\alpha H \left\{\xi(r_1 - p_1\tau - \delta') + \left(1 - \frac{\delta}{d_1}\right)\left[\frac{\beta}{\tau} d_1 \left(r_1 - p_1\tau - \frac{d_1}{\tau}\right) - \sigma' d_1\right]\right\}$$

$$m_q = +\alpha H \frac{d_1}{\tau}\xi\left\{\xi\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right) - \sigma'\tau\left(1 - \frac{\delta}{d_1}\right)\right\}$$

(Note: $Z_w m_q - Z_q m_w = -\alpha^2 G H d_1^2 p_1 \sigma' \left(1 - 2\frac{\delta}{d_1}\right)\left(1 - \frac{\delta}{d_1}\right)$

DISCRIMINANT FOR STABILITY

$B = Z_w + m_q$

$C = Z_z + m_\theta + Z_w m_q - Z_q m_w$

$D = Z_z m_q - Z_q m_z + Z_w m_\theta - Z_\theta m_w$

$E = Z_z m_\theta - Z_\theta m_z$

$R = BCD - D^2 - B^2 E$

Condition for stability is that B, C, D, and E are positive, and that R be positive.

EXAMPLE OF USE OF PROCEDURE

As an example of the use of the analysis, the Consolidated Model 31 flying-boat has been used. In the example a run with elevators set at 15° down has been chosen.

The first procedure is the reduction of the tank test data* of the hull alone. These are plotted as shown in figures (2) and (3). From these curves, values of σ and δ are found and plotted as shown in figure (4).

Table I contains the needed information concerning dimensions of the airplane for the computations of terms in the derivatives. The aerodynamic data have been collected from wind tunnel tests for the configuration used.

Procedure and Order of Calculations

(1) Angles of τ chosen and corresponding values of σ and δ found from figure (4)

(2) Values of C_V solved for in the following manner:

$$(d_1-\delta)\left(\beta\frac{d_1}{\tau}-\sigma-r_1\right) = -\alpha\frac{J}{G}t_1\left\{C_{M_0}-a(\tau-\tau_0)\Sigma+C_{M_e}\right\}$$

and

$$(d_1-\delta)\left\{(d_1-\delta)-\left(\frac{\sigma+r_1}{\beta}\right)\tau+\delta\right\} = -\frac{\tau}{\beta}\frac{J}{G}\alpha t_1\left\{C_{M_0}-a(\tau-\tau_0)\Sigma+C_{M_e}\right\}$$

Now, let $(d_1-\delta) = X$

Then:

$$X^2-\left(\frac{\sigma+r_1}{\beta}\tau-\delta\right)X+\frac{\tau\alpha}{\beta}\frac{J}{G}t_1\left\{C_{M_0}-a(\tau-\tau_0)\Sigma+C_{M_e}\right\}=0$$

* Consolidated tank tests on Model 31.
N.A.C.A. Model 87-B. November, 1938.

Let

$$\eta = \left\{ C_{M_o} - a(\tau - \tau_o)\Sigma + C_{M_e} \right\}$$

Then the equation becomes:

$$(17) \quad d_1 - \delta = \frac{1}{2}\left\{ \left(\frac{\sigma + r_1}{\beta}\tau - \delta\right) \pm \sqrt{\left(\frac{\sigma + r_1}{\beta}\tau - \delta\right)^2 - 4\,\frac{\alpha J t_1}{\beta G}\tau\eta} \right\}$$

With the relation:

$$d_1 - \delta = \frac{1}{\alpha G}\left\{\frac{1}{C_v^2} - f(\tau - \tau_o)\right\}$$

The evaluation of C_v is accomplished by the substitution of the values of (17) into the following form:

$$(18) \quad \frac{1}{C_v^2} = \alpha G(d_1 - \delta) + f(\tau - \tau_o)$$

The solution of (17) and (18) gives the equilibrium relations as shown for the case 50,000 gross weight, 30% C.G., and elevators down 15° in figure (6) in which the curve of τ vs. C_v is plotted.

(3) From the curve of τ vs. C_v select the τ corresponding to assumed values of C_v, i.e.

C_v	4	5	6	7	8
τ	10.1°	7.8°	5.7°	3.7°	2.0°

(4) From figures (4) and (5) pick off values of σ, δ, σ' and δ' corresponding to the angles of τ. (In some instances it has been found necessary to extrapolate the curves for some angles of τ).

(5) For these values of C_v and τ determine d_1 from equation (12a).

(6) With this information the hydrodynamic and aerodynamic derivatives can be calculated. See page 12. It is advisable to use column form for the solutions.

(7) The aerodynamic derivatives and the hydrodynamic derivatives are added together algebraically. See Table II.

(8) With these values the constants B, C, D, and E of the Routh's Discriminant R are determined. See page which gives terms in B, C, D, E, and R. Results of the example are shown in Table II.

(9) The plot of the discriminant vs. C_V is shown in figure 9.

TABLE I

INFORMATION ON AIRPLANE

Gross weight = 50,000 lbs.; C. G. = 30% M. A. C.; e = 15°

$AR = 11.55$	$St/S = 0.154$	$t_o = -11^\circ = -0.192$
$a = 6.2$	$l/t = 4.39$	$\Sigma = 0.17$
$\frac{ba}{b} = 12$	$l_1 = 4.93$	$C_{M_o} = 0.090$
$t_1 = 1.122$	$AR_t = 3.02$	$C_{M_e} = 0.29$
$\rho w = 1.964$	$\eta_t = .75$	$\alpha = 0.70$
$\rho = .002378$	$A_t = 3.6$	$\beta = 0.576$
	$K = 1.25$	

b = 13.75" for 1/8 scale

b = 9.17 ft. full scale

DATA VARYING WITH W and C. G. LOCATION

W = 50,000 lbs. $k_1 = 1.236$ $p_1 = 1.274$ $r_1 = 0.281$

$= \frac{50,000}{64 \times 9.17} = 1.02$; G = 0.493; H = 0.322

$J = .00121 \times \frac{144}{11.55} \times .493 = 0.00744$; $\alpha G = 0.343$; $+H = 0.225$

$V = 11.7C_v$ (mph)

AERODYNAMIC TERMS

$f = .00744 \times 6.2 = 0.0461$

$h = -.00744 \times 6.2 \times \frac{1.122}{1.236} \times (-.17) = 0.00574$

$j = .00744 \times 1.25 \times .75 \times \left(\frac{4.93}{1.236}\right)^2 \times .154 \times 3.6 = 0.0614$

f = 0.0461

h = 0.00574

j = 0.0614

TABLE II

C_v	Z_z	Z_w	Z	Z_q	m_z	m_w	m	m_q
4	.3450	1.0000	.7600	+.0134	-.1066	-.0805	-.0060	.0019
	0	.0461	.0461	0	0	.0057	.0057	.0614
	.3450	1.0461	.8061	.0134	-.1066	-.0748	-.0003	.0633
5	.3450	.7120	.4050	-.0606	-.0896	-.0901	-.0179	.0080
	0	.0461	.0461	0	0	.0057	.0057	.0614
	.3450	.7581	.4511	-.0606	-.0896	-.0844	-.0122	.0694
6	.3450	.6350	.2285	-.1149	-.0871	-.1100	-.0073	.0181
	0	.0461	.0461	0	0	.0057	.0057	.0614
	.3450	.6811	.2746	-.1149	-.0871	-.1043	-.0016	.0795
7	.3450	.7820	.1615	-.2070	-.0995	-.1940	-.0149	.0476
	0	.0461	.0461	0	0	.0057	.0057	.0614
	.3450	.8281	.2076	-.2070	-.0995	-.1883	-.0092	.1090
8	.3450	1.2820	.1471	-.5200	-.1408	-.4110	+.0522	.1635
	0	.0461	.0461	0	0	.0057	.0057	.0614
	.3450	1.3281	.1932	-.5200	-.1408	-.4053	.0579	.2249

TERMS OF DISCRIMINANT

C_v	B	C	D	E	R
4	1.1094	.4098	.0833	.0857	-.0745
5	.8275	.3798	.0474	.0362	-.0121
6	.7506	.3761	.0448	.0234	-.0026
7	.9371	.3870	.0485	.0174	-.0001
8	1.5530	.3761	.1594	.0472	-.0455

EXPERIMENTAL TESTS

In the course of the research it was deemed advisable to check the theoretical calculations by experimental results. To do this a dynamically similar model of the Consolidated Model 31 was used. Various center of gravity positions and various gross weights were tested. A sample of the results obtained is shown in figure 10.

The experimental set-up consisted of a speed boat with a model mounted at the end of a horizontal 12-foot boom. The model was free in pitch and rise, being restrained only in roll and yaw. The boat was started from rest and accelerated throughout the test run. Pictures of the motion were taken with a 16-millimeter motion picture camera. A horizontal reference line was mounted on the boat and was leveled by a member of the crew who sighted through a cross hair arrangement at the shore. A reference line parallel to the base line of the model was drawn on the side of the hull. A stop watch and a light were mounted at the top of the hull. Stakes were arranged on the shore every ten yards apart. As the boat passed the stakes, the member of the crew tending the leveling of the reference line, pushed a button which flashed the light, thus giving an indication of distance traveled. The camera mounted in the cockpit of the boat was focused on the hull and thus recorded the horizontal reference line, the reference line on the hull, the stop watch, and the blinking light.

After the tests were run and the pictures developed, the film was re-run through a projector and angles of trim versus speed were recorded. By this means, it is possible to plot angle of trim versus speed coefficient and any "porpoising" is evident on the resulting curve.

In conjunction with any model tests, it should be remembered that the results of the tests on the model cannot be applied directly to full scale conditions. The methods of extrapolation to full scale of wind tunnel data are well known. However, it is also necessary that acknowledgment be made of the fact that model conditions and full scale conditions do not exactly agree in the case of dynamic model testing. The relations between model and full scale conditions in this case are not as well known as wind tunnel extrapolations. For that reason, it was deemed advisable to put into this paper some of the relationships between model and full scale conditions for dynamically similar models.

To begin with, it is necessary to consider the laws of dynamic similarity of the model and the full scale airplane. During the motion of a float on the free surface of water, exterior forces determining the process of motion appear. A considerable part of the energy transmitted from the airplane to the water is used up in the resulting system of waves, which is under the influence of gravity. In addition to the forces of inertia occasioned by the masses of water set into motion and the spray, as well as the forces of friction, forces of gravity are also active. In model testing, a perfect dynamic similarity is impossible, since the three types of forces acting require the application of Froude's and Reynold's Model Laws in addition to Newton's General Law of Similarity. However, since the forces of gravity are of the same magnitude as the forces of friction, and since the latter, with the coefficient of friction used as a constant, is at least approximately correct, it suffices to conform to the conditions of Froude's Model Law. That is, if Froude's number is constant for model and full scale airplanes.

For instance, the landing speed used on a model for dynamically similar landing is:

$$V_{LM} = \frac{V_{LH}}{\sqrt{\lambda}}$$

Neglecting the influence of Reynold's Law upon air resistance and properties of the airfoil, with geometrically similar design of the model with similar total weight

$$W_M = \frac{W_H}{\lambda^3}$$

and with the location of the c.g. being similar, there are, according to Newton's General Law of Similarity, lift and resistance of the model similar to lift and resistance of the full scale airplane.

Full Scale Airplane	Model
$L = C_L \times A \times \rho/2 \times V_H^2$	$\frac{L}{\lambda^3} = C_L \times \frac{A}{\lambda^2} \times \rho/2 \times \left(\frac{V_H}{\sqrt{\lambda}}\right)^2$
$D = C_D \times A \times \rho/2 \times V_H^2$	$\frac{D}{\lambda^3} = C_D \times \frac{A}{\lambda^2} \times \rho/2 \times \left(\frac{V_H}{\sqrt{\lambda}}\right)^2$

The gliding angle is the same for the model and the full scale airplane.

$$\tan \beta = \frac{D}{L} = \frac{C_D}{C_L}$$

Upon these assumptions, there immediately arises the question of scale effect upon the lift coefficient. Examination of aerodynamic data shows there exists a pronounced scale effect upon C_L max. This effect becomes of importance when models are tested in the speed range covering landing and take off. To have the model conform more closely to full scale conditions, it is necessary to resort to the use of high lift devices. These devices, in general, consist in the addition of flaps and slots across the span of the model. This will give the same lift coefficient for the model and full scale airplane.

For progressive motion, all requirements are met by the condition that model acceleration is equal to full scale acceleration. For rotational motion, there is an additional requirement for similarity.

$$J_M = \frac{J_H}{\lambda^5}$$

which is sufficiently met by:

$$J_{y_M} = \frac{J_{y_H}}{\lambda^5}$$

since, at a normal landing, only rotation about the transverse axis occurs.

Furthermore, there are the following relations:

$$\omega_M = \omega_H \times \sqrt{\lambda}$$

$$n_M = n_H \times \sqrt{\lambda}$$

$$\varepsilon_M = \varepsilon_H \times \lambda$$

For light models, complete similarity concerning the moment of inertia can at present not yet be attained for all types of airplanes. Therefore, in transposing the rotational speeds and accelerations and the corresponding frequencies from the model to the full scale airplane, the following correction has to be made. If:

$$J_M' = K \times J_M$$

represents the moment of inertia of the model not completely meeting the requirements for similarity, there is, in applying the basic dynamic equation under the influence of a given movement of rotation

$$M_M = \frac{M_H}{\lambda^4}$$ * with free oscillation

$$M_M = J' \times \frac{d\omega_M'}{dt}$$

whereas, with a similar moment of inertia, there would be:

$$M_M = J_M \times \frac{d\omega_M}{dt}$$

Therefore:

$$J_M \times \frac{d\omega_M}{dt} = J_M' \times \frac{d\omega_M'}{dt} = K \times J_M \times \frac{d\omega_M'}{dt}$$

*Note: For the free oscillation case, corrections cannot be given with oscillations occurring on the surface of the water because of the unknown quantity of water oscillating simultaneously.

and derived therefrom:

$$\varepsilon_M = K \times \varepsilon_M'$$

$$\omega_M = K \times \omega_M'$$

In the testing of the dynamically similar model, a method of recording the data must be used. The most general method lies in the use of motion pictures. After taking pictures of the process of motion, by changing the speed of the reel during the projection of the film, it is possible to make the angular velocity of the model ω_M and the coefficient of progression,

$$f_M = \frac{V_M}{\ell_M}$$

(which is the number indicating how many times its own length the model travels in one secone) equal to the same values on the full scale airplane. At a certain distance, the impression of direct observation of the full scale airplane is created.

For: with the model going through oscillation of the frequencies n_M at a progressive velocity of V_M , or a coefficient of progression:

$$f_M = \frac{V_M}{\ell_M}$$

the corresponding frequencies of the full scale airplane are:

$$n_H = \frac{n_M}{\sqrt{\lambda}}$$

at a progressive velocity of:

$$V_H = V_M \times \sqrt{\lambda}$$

and a coefficient of progression of:

$$f_H = \frac{V_H}{l_H} = \frac{V_M \times \sqrt{\lambda}}{l_M \times \lambda} = \frac{V_M}{l_M \times \sqrt{\lambda}}$$

Reducing the speed of the reel at a ratio of $\frac{1}{\sqrt{\lambda}}$, the frequencies of the model n_M change to:

$$\frac{n_M}{\sqrt{\lambda}} = n_H$$

and the coefficient of progression f_M to:

$$\frac{f_M}{\sqrt{\lambda}} = \frac{V_M}{l_M \times \sqrt{\lambda}} = f_H$$

as was intended.

In projecting the film, a speed of sixteen pictures per second is the lower limit advisable to attain a picture free from flickering. Therefore, it is advisable to take the pictures at a speed corresponding to a number $> 16 \times \sqrt{\lambda}/s$.

If, with the consideration of the dissimilarity of the moment of inertia of the model, the frequency is to be equal to that of the full scale airplane (which is desirable for judging the process of oscillation), the speed of the reel during the projection of the film must be reduced to a ratio of $\frac{K}{\sqrt{\lambda}}$, whereby the coefficient of progression becomes:

$$f_H \times K$$

Consequently, the speed appears K times too high.

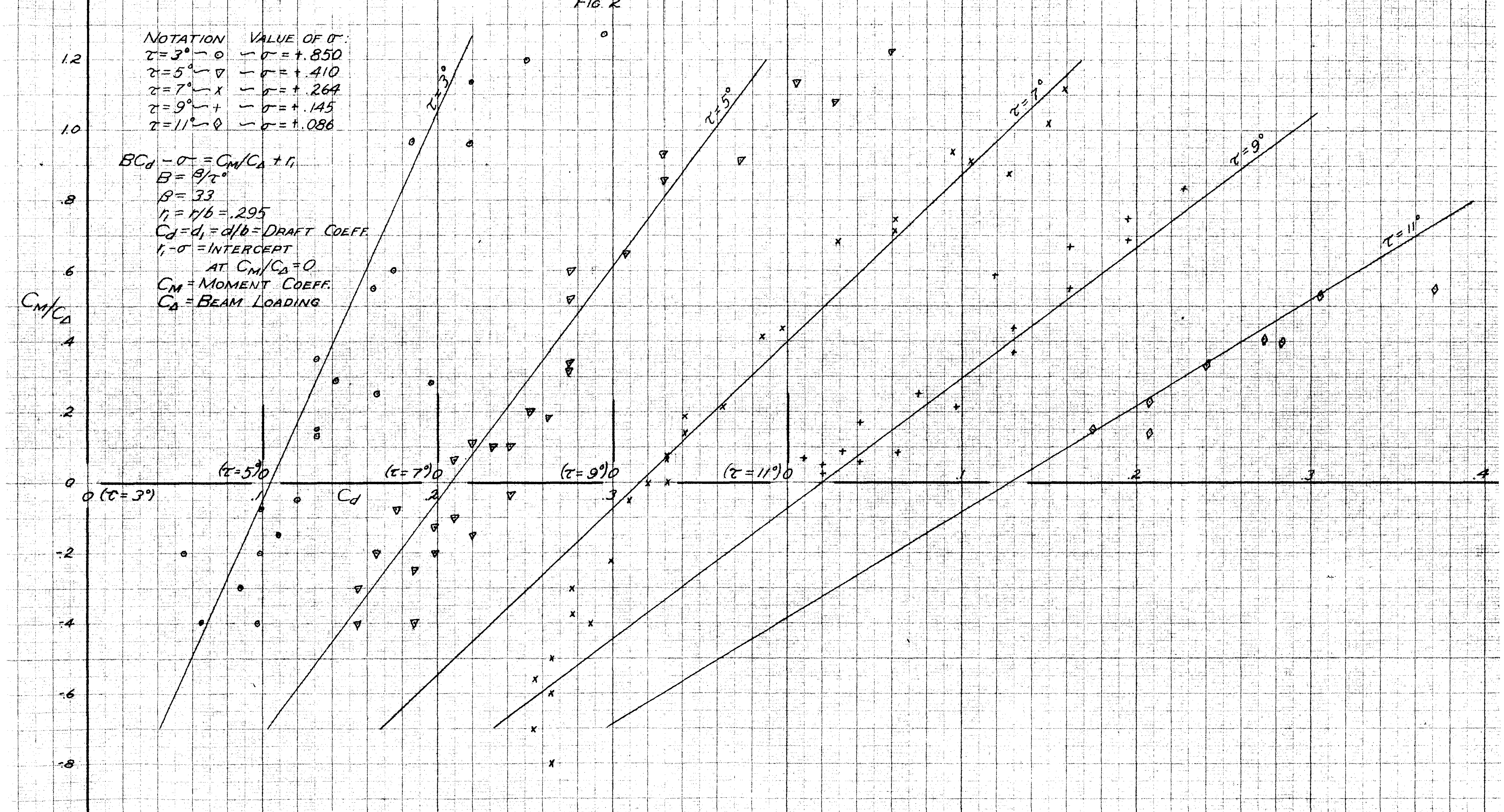
In the design of a dynamically similar model, it is necessary to take into consideration the strength requirements. Since this paper is concerned only with actual testing, the subject of design of the model will exist, as in the case of similar motion.

Definitions of the terms used in the discussion of experimental testing follow:

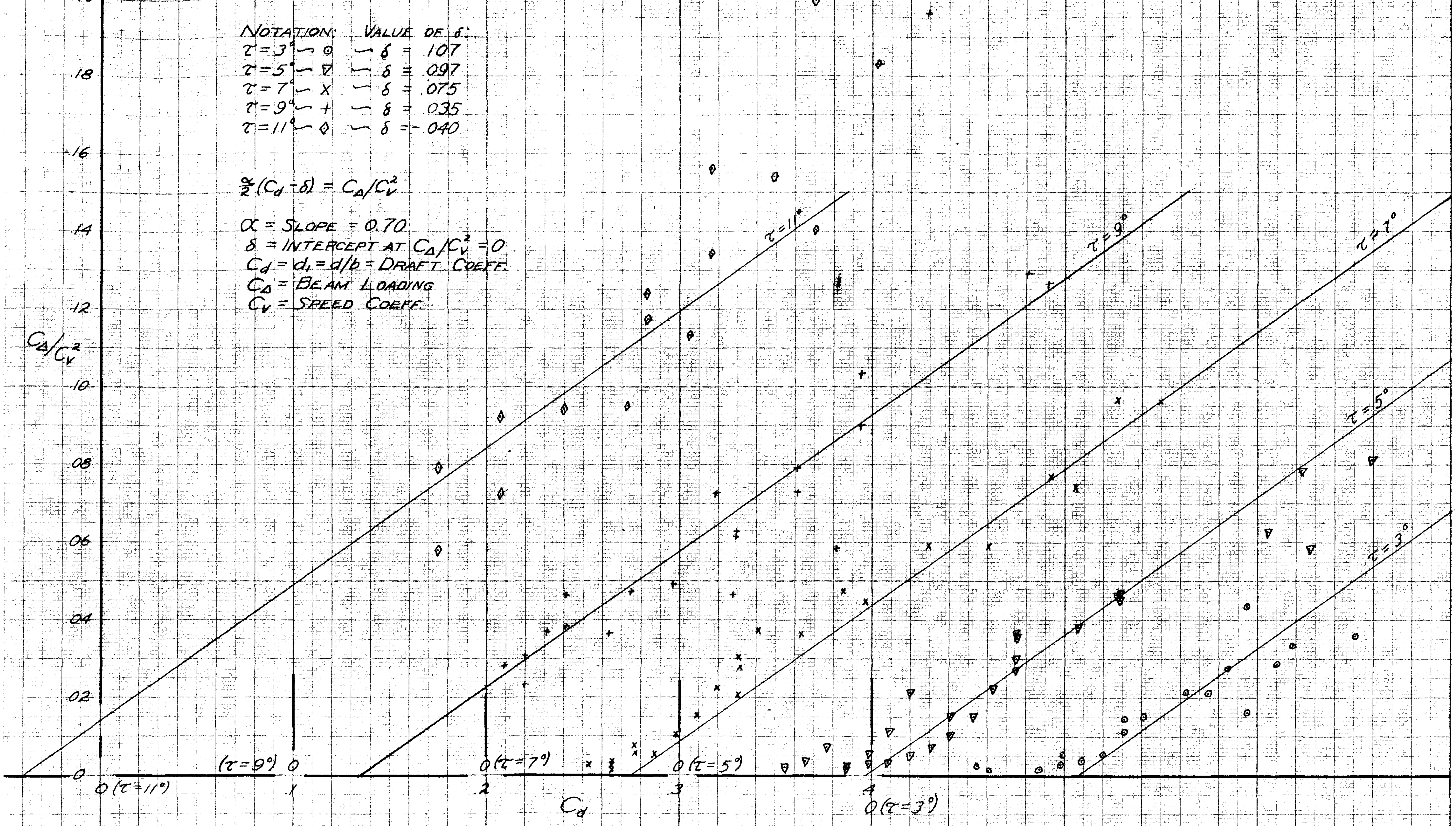
$( )_M$	Model
$( )_H$	Full scale
F	Froude's number Where: $F = \frac{V_H}{\sqrt{g \times l_H}} = \frac{V_M}{\sqrt{g \times l_M}}$ = constant
R	Reynold's number
f	Factor of progression
V	Horizontal component of velocity
V_L	Landing speed
S	Travel
t	Time
a	Acceleration
W	Weight
L	Lift
D	Resistance or drag
A	Wing area
C_L	Coefficient of lift
C_D	Coefficient of drag

J	Moment of inertia
M	Moment of forces tending to capsize
λ	Scale of Model = $\frac{lH}{lM}$
l	Characteristic length
β	Gliding angle
ω	Velocity of rotation
ε	Acceleration of rotation
η	Frequency
κ	Coefficient of correction
ρ	Density of air

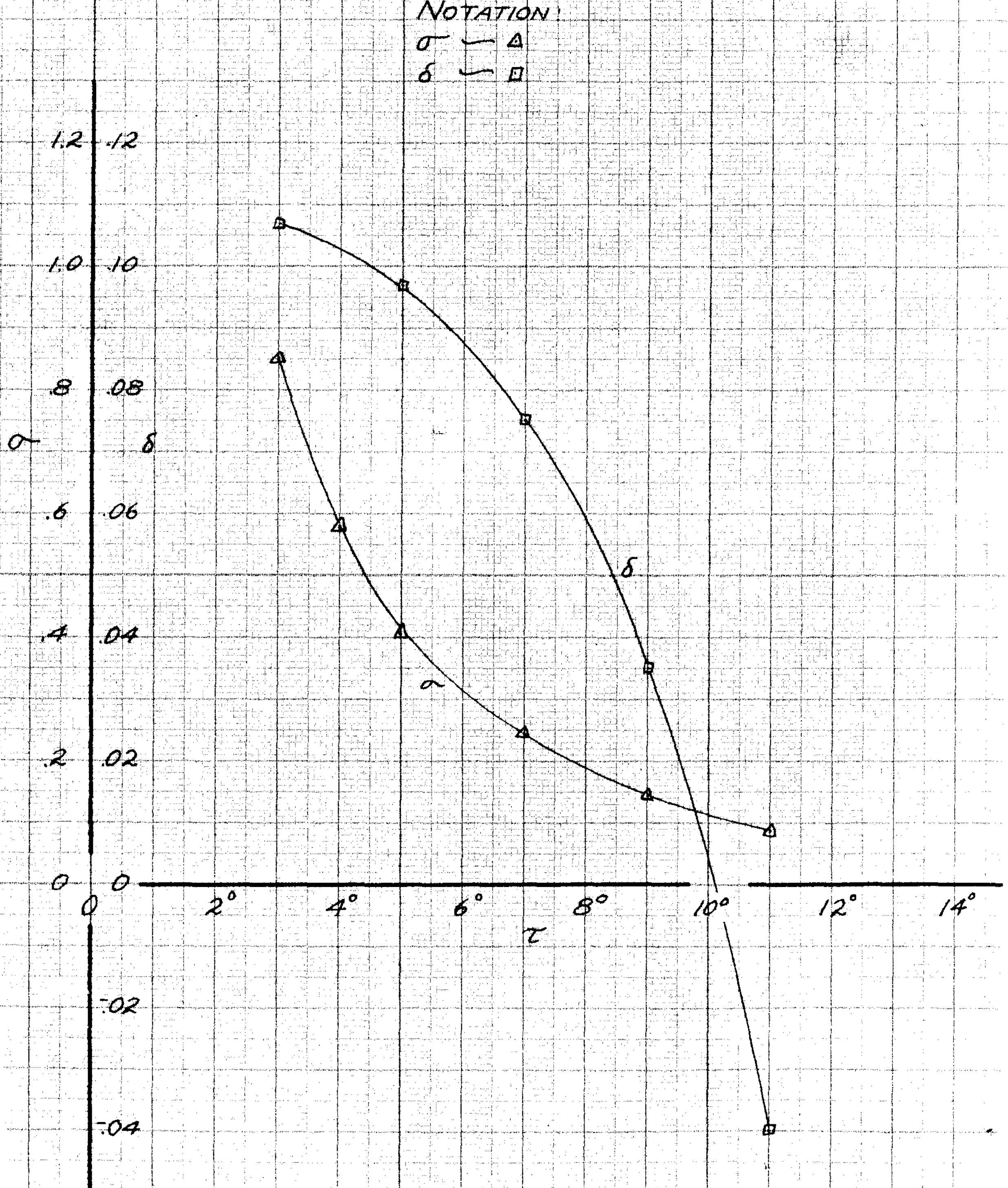
FIG. 2



CURVES FOR DETERMINATION OF B AND σ
AT VARIOUS ANGLES OF τ°
FROM CONSOLIDATED MODEL 31
TANK TESTS

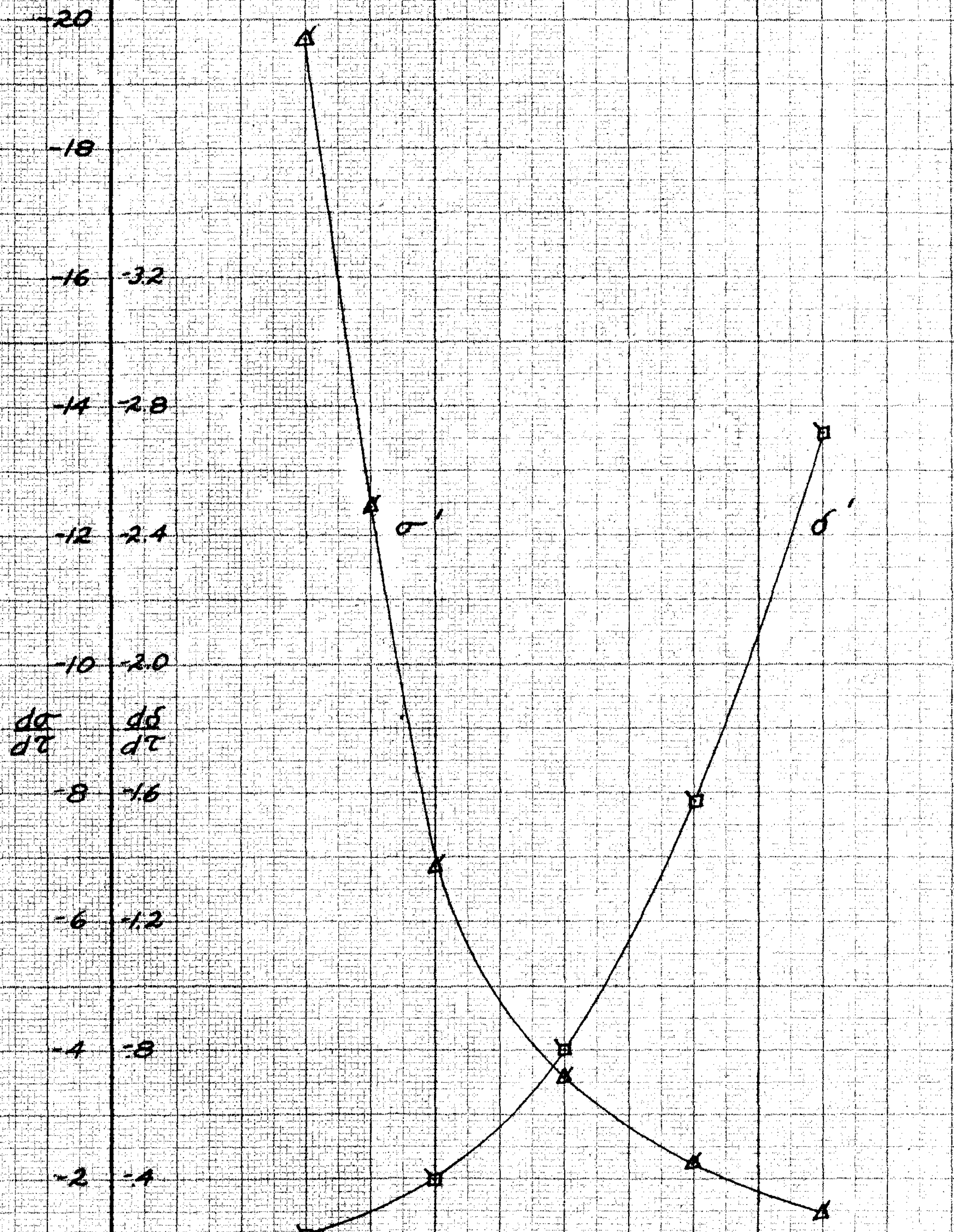


CURVES FOR THE DETERMINATION OF A AND δ
AT VARIOUS ANGLES OF τ°
FROM CONSOLIDATED MODEL 31
TANK TESTS



PLOT OF σ AND δ VS. ANGLES OF τ°

FIG. 5



PLOT OF σ' AND δ' VS. ANGLES OF τ
DERIVATIVES TAKEN FOR τ IN RADIANS

FIG 6

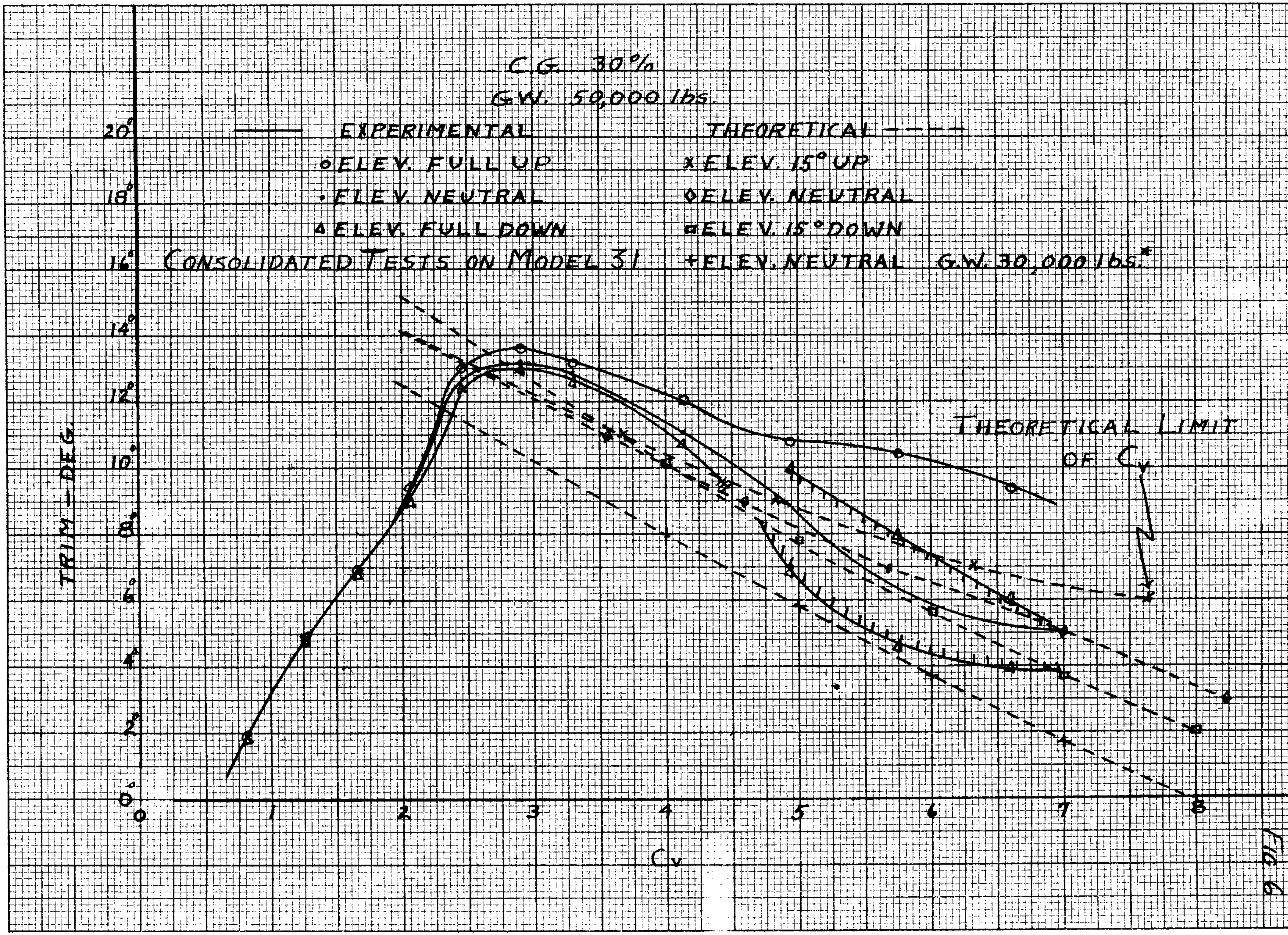

C.G. 30%
G.W. 50,000 lbs.
EXPERIMENTAL
THEORETICAL
ELEV. FULL UP
ELEV. NEUTRAL
ELEV. FULL DOWN
ELEV. 15° UP
ELEV. NEUTRAL
ELEV. 15° DOWN
ELEV. NEUTRAL G.W. 30,000 lbs.*
CONSOLIDATED TESTS ON MODEL 31
THEORETICAL LIMIT OF C_V
TRIM - DEG.
C_V
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
20°
0
1
2
3
4
5
6
7
8
9

FIG. 7

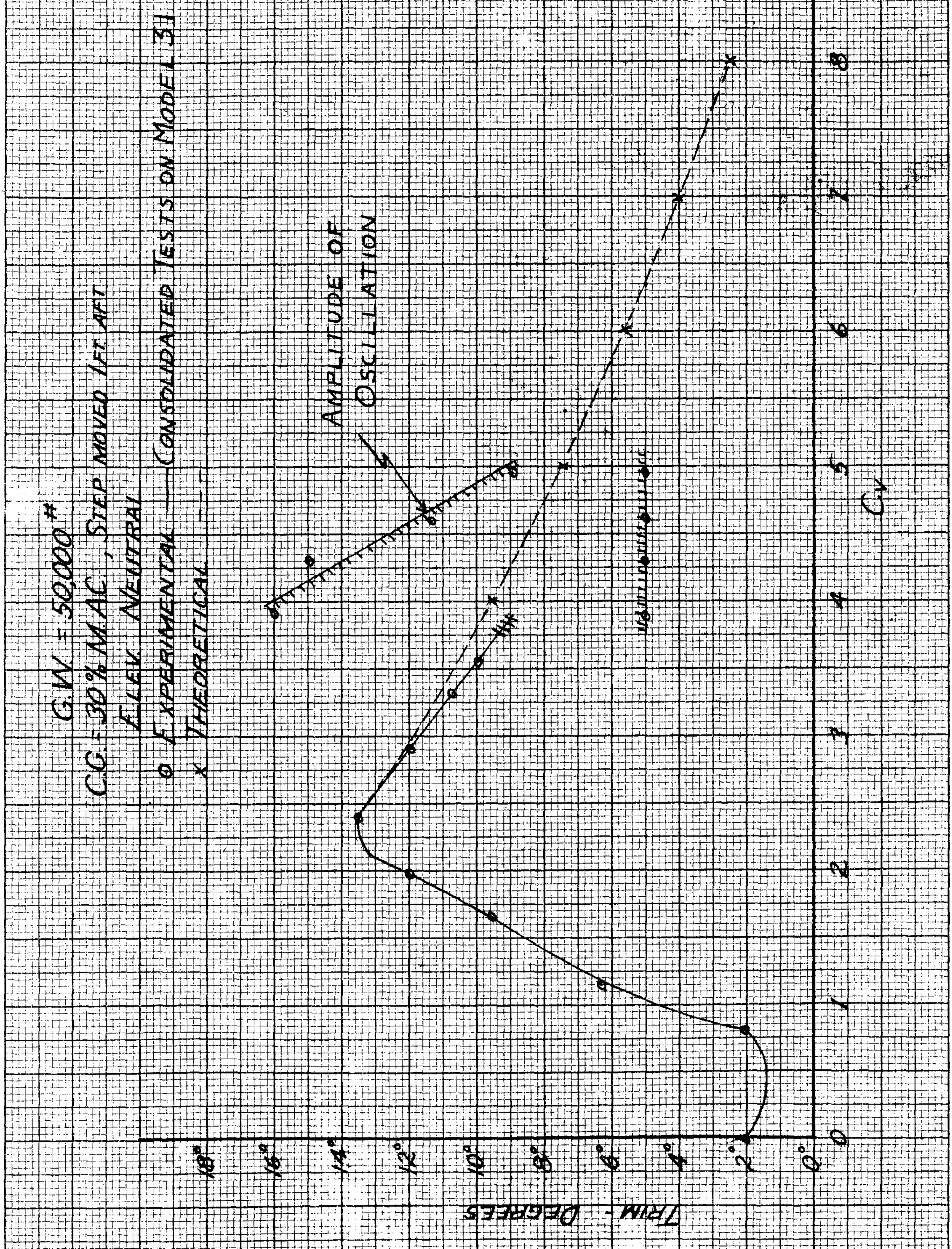

G.W. = 50,000 #
C.G. - 30% M.A.C., STEP MOVED 1 FT. AFT
ELEV. NEUTRAL
o EXPERIMENTAL — CONSOLIDATED TESTS ON MODEL 31
x THEORETICAL - - -
AMPLITUDE OF OSCILLATION
TRIM - DEGREES
18°
16°
14°
12°
10°
8°
6°
4°
2°
0°
0
1
2
3
4
5
6
7
8
C_V

FIG. 8

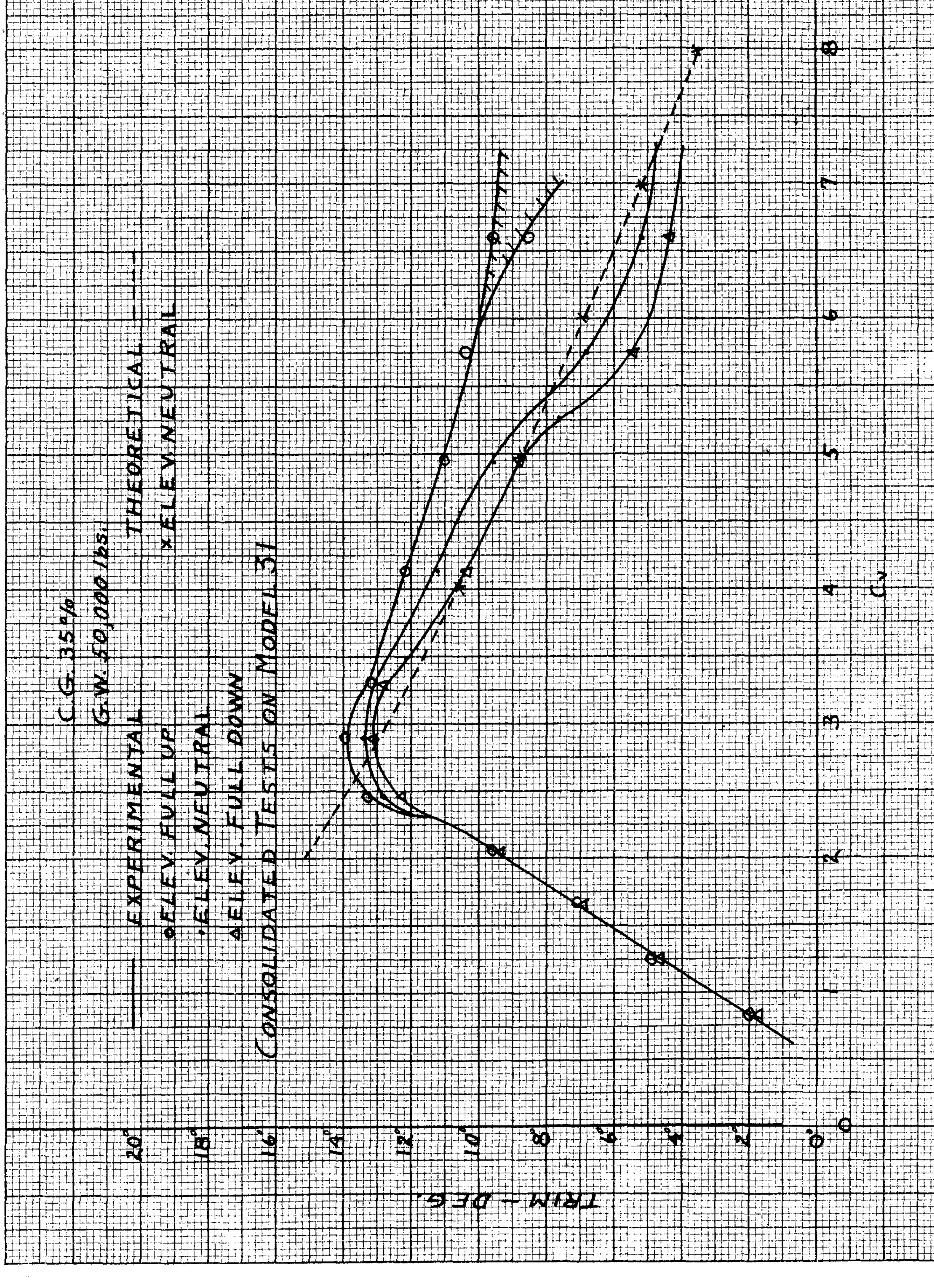
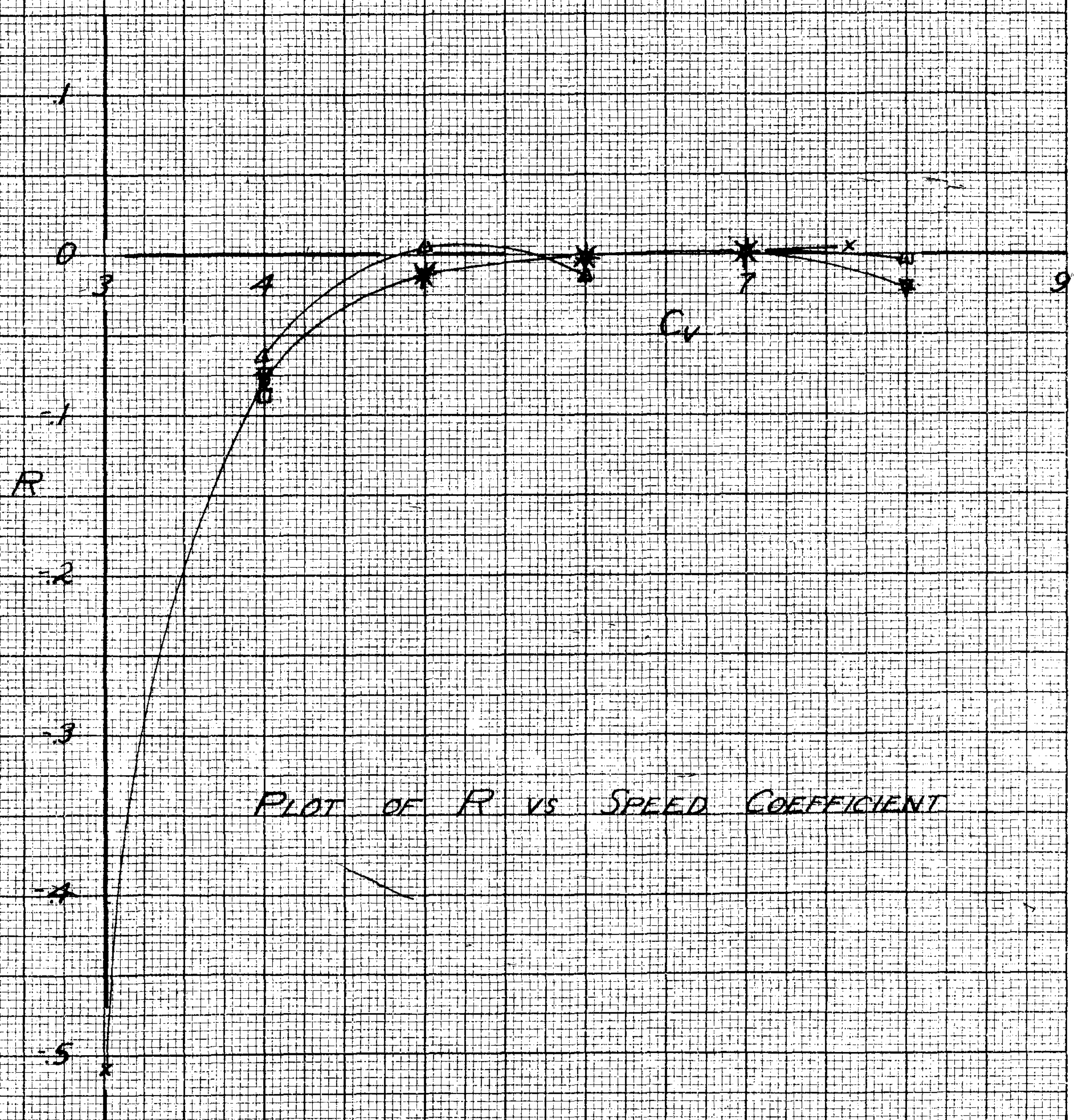

C.G. 35%
G.W. 50,000 lbs.
EXPERIMENTAL
THEORETICAL
○ELEV. FULL UP
·ELEV. NEUTRAL
△ELEV. FULL DOWN
×ELEV. NEUTRAL
CONSOLIDATED TESTS ON MODEL 31
TRIM - DEG.
C_V
20°
18°
16°
14°
12°
10°
8°
6°
4°
2°
0°
0
1
2
3
4
5
6
7
8

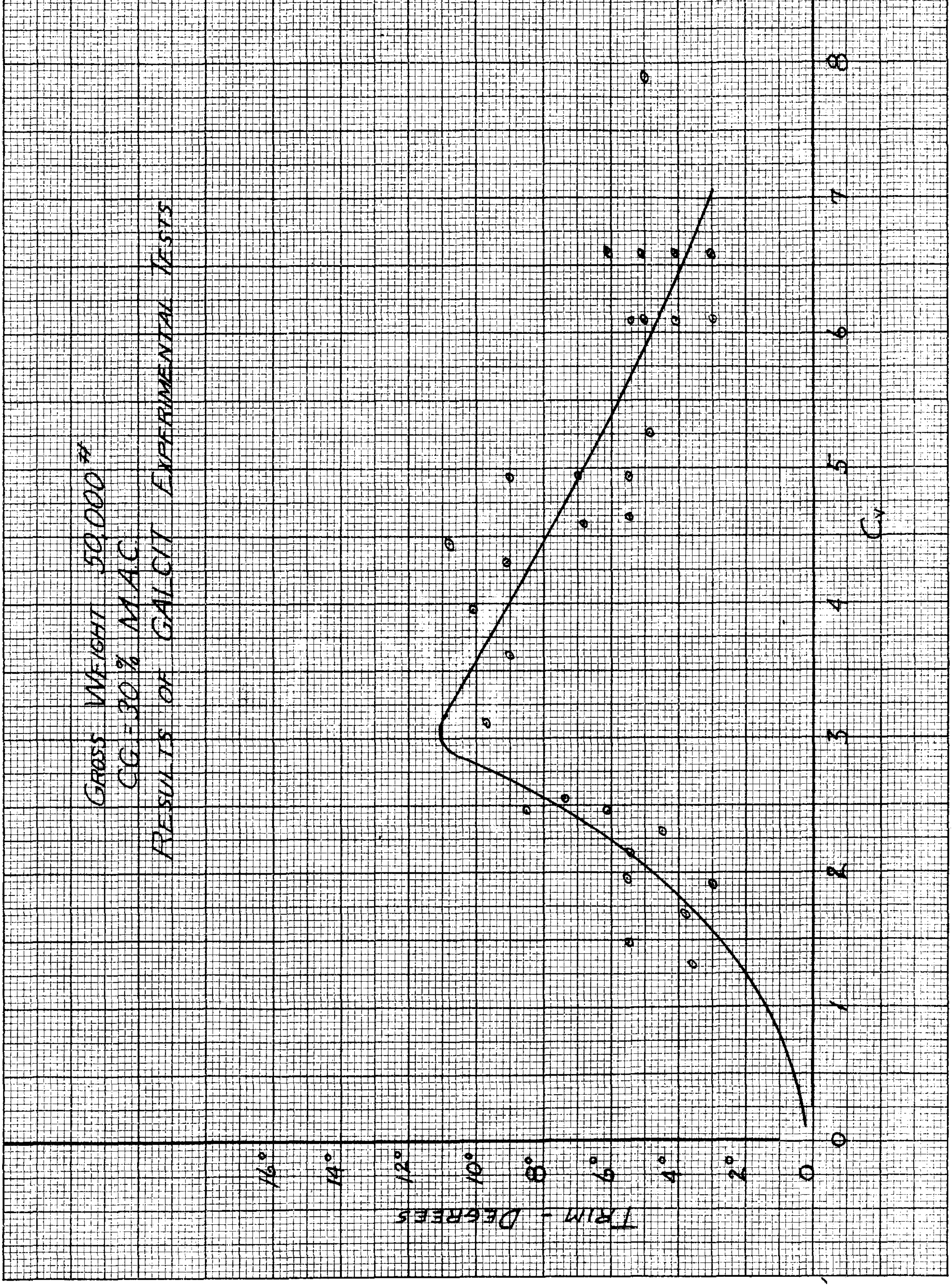
FIG. 9

o G.W.=50,000#, C.G.=30% M.A.C., e=0°
x " " , " " " , e=-15°
▽ " " , " " " , e=+15°
+ " " , " " " , e=0°
(STEP MOVED ONE FOOT AFT)
□ G.W.=50,000#, C.G.=35% M.A.C., e=0°
△ G.W.=30,000#, C.G.=30% M.A.C., e=0



PLOT OF R VS SPEED COEFFICIENT

FIG. 10


GROSS WEIGHT 50,000 #
CG = 30% M.A.C.
RESULTS OF GALCIT EXPERIMENTAL TESTS
C_V
0
1
2
3
4
5
6
7
8
TRIM - DEGREES
0
2°
4°
6°
8°
10°
12°
14°
16°

CONCLUSION

Throughout the paper an attempt has been made to simplify the whole procedure in order that it can be used easily by the person wishing to make this check on the stability of his flying boat in the planing condition. It is believed that a thorough investigation of the value of the method has been made. Examination of curves 6, 7, and 8 show that there is relatively good agreement between the equilibrium condition computed theoretically and the experimental equilibrium conditions. It is to be noted that the theoretical values agree much better in the higher speed range than in the lower speed range past the hump. This would seem to indicate that something in the equilibrium relationship has been neglected. Investigation of the resistance curves at this lower speed range shows that the resistance is quite high and falls off to a certain point and then begins to rise again with increasing speed. Throughout the analysis, the effect of resistance has been neglected because of calculations previously made by Glauert in which he proved that the effect of resistance is not important. However, so many of Glauert's computations with respect to the stability of flying boats have been found wrong that it seems there is cause of doubt concerning this assumption. It is recommended that investigation along these lines be made.

Another point that has been brought to light upon investigation of the computations is the fact that the distance of the vertical c.g. has not been properly taken care of.

Turning now to the results of the Routh's Discriminant, Figure 9, it is seen that in all cases investigated, the discriminant proved to be negative in much of the speed range which means that that particular configuration is unstable. However, actual tests made upon the airplane indicate that some of these are not unstable. Therefore, this means there is something inherently in error in the fundamentals of the derivations of the stability quartic.

Another point of interest is the fact that of all the cases investigated, none varied very far from the others, so that the results of all the cases investigated could be drawn as one faired curve. This again indicates something inherently in error.

It is recommended that a more thorough analysis of the basic assumptions and conditions affecting the stability of the flying boat be made. It is also firmly believed that basically, the idea of an analytical solution to the problem of longitudinal stability of flying boats in the planing region is feasable. Thought was given to the possibility that the condition might be that of an unstationary case in which event the attempt to represent the problem as a stationary flow phenomena would not apply. However, it is generally agreed that the wave length of the induced waves as compared to the wetted length of the hull itself is not sufficiently close for the system to be in resonance. It might be advisable to investigate this possibility more thoroughly.

Turning now to the experimental results obtained from testing of the dynamic model, it is to be noted that great scatter exists in the data gathered. It is believed that this is primarily due to the dispersion of the factors entering into proper recording of the data. Therefore, it is recommended that a more satisfactory method of recording the horizontal reference line be made. This can be done by providing the camera with a wide angle lense which will take in not only the model, but the shore line as well. With the use of this wide angle lense it would be possible to include in the picture also the stakes on the shore which would eliminate the use of the blinking light, thus eliminating another possible error in human judgment. Another recommendation to be made is concerned with the actual running of the tests themselves. If possible, a camera that is self-winding should be used and more slowly accelerated runs made. This will eliminate many of the high acceleration forces imposed upon the system and tend for more accurate results.

Although the results that have been derived from this year's work on the problem have been negative, it is believed that the work os of value and will form a foundation for continued study along this line, and that eventually the problem can be solved satisfactorily.

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