A PHOTOELASTIC INVESTIGATION OF THE EFFECT OF ELLIPTICAL AND MODIFIED CUTOUTS IN FLAT PANELS SUBJECTED TO COMBINED BENDING AND SHEAR

Thesis by

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TABLE OF CONTENTS

PART I

INTRODUCTION

Quite frequently in an aircraft structure there are occasions when it is necessary or advantagious to make openings or cutouts in the structure. These may be lightening holes in a web or spar or they may be openings in the structure which are necessary for access or other reasons. An analytical solution of the problem of stress distribution around a cut-out in a stressed member is often impossible since the analytical method is limited to problems in which the boundary conditions are simple. The purpose of this investigation was to determine by means of a photo-elastic analysis the amount of stress concentration caused by a cutout in a flat panel and how this concentration varied under different conditions of loading and with various types of cutouts. Simple photoelasticity provides a means of finding the stresses only along the cutout boundary. However since the stress concentrations are a maximen at the boundary, this is the region in which the designer is most interested. Only boundary stresses along the cutout are considered in this paper.

This research is a continuation of the work carried out by Gibbons and Dill at the California Institute of Technology in 1941-42. Their paper entitled "A Photoelastic Investigation of the Effect of Elliptical and Modified Cutouts in Flat Panels Subjected to Combined Bending and Shear" dealt with a similar panel with similar cutouts under much the same conditions of loading as carried out in this research. However the long axis of the cutout in this investigation was horizontal or perpendicular to the shear loading, while the long axis of the cutout in the previous investigation was at 90° to this direction, i.e., vertical.

Tyra and Hollister also did some research at the California Institute of Technology on this subject. Their thesis entitled "A Photoelastic Investigation of the Effect of Cutouts in Panels Subjected to Combined Bending and Shear" dealt with circular and modified circular cutouts. This paper considers an elliptical cutout which was modified by several steps to a rectangular opening, the ratio of the maximum length to width being 2:1.

PART II

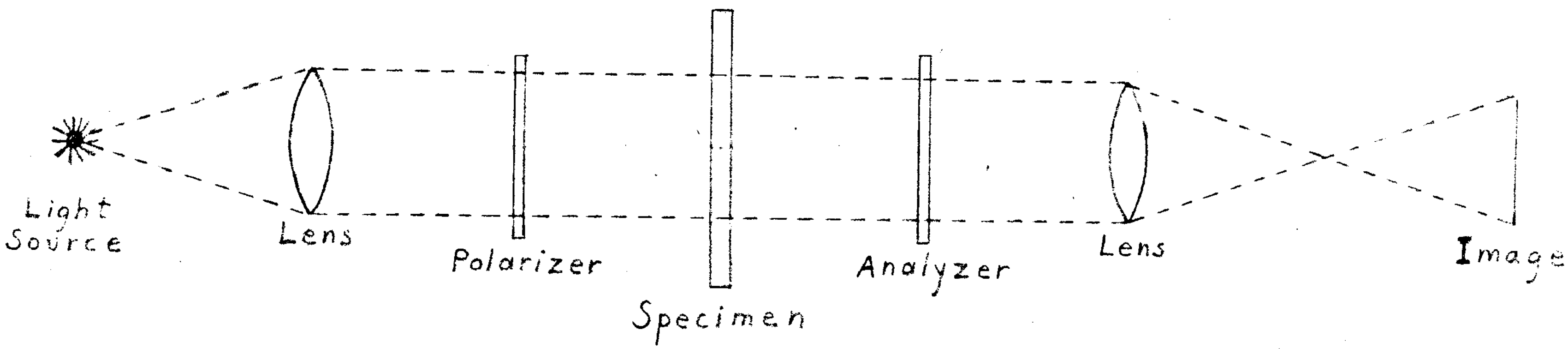
THEORY OF PHOTOELASTICITY

According to the wave theory a beam of light from an ordinary source consists of transverse ether waves of a random nature. By passing the light through a piece of polaroid, which absorbs the components of the light waves in all but one plane, the light emerges with the waves entirely in one plane. This is called plane polarized light. This plane is known as the polarizer plane.

In an isotropic medium there is only one wave velocity. However in a crystalline material, a light beam is broken into two beams which are polarized in two mutually perpendicular planes each of which has a different velocity of propogation through the crystal. Thus the emerging waves of light are not only polarized in two perpendicular planes but these two waves are out of phase, one lagging behind the other. Such a material is "doubly refracting".

Materials used for photoelastic models are bakelite, gelatin, celluloid and other amorphous substances. Bakelite is the most commonly used. Such materials are normally isotropic in the unstressed state. However, when these particular materials are loaded and become stressed, each point acts like a doubly refracting crystal. The light at each point is polarized in the two directions of principal stress, which are mutually perpendicular. One of these two waves of light lags behind the other and it has been experimentally proven that the degree of retardation is directly proportional to the stress existing at that point and, also, of course, to the thickness of the material.

Fig. 1 shows a diagramatic sketch of a polariscope. A mono-



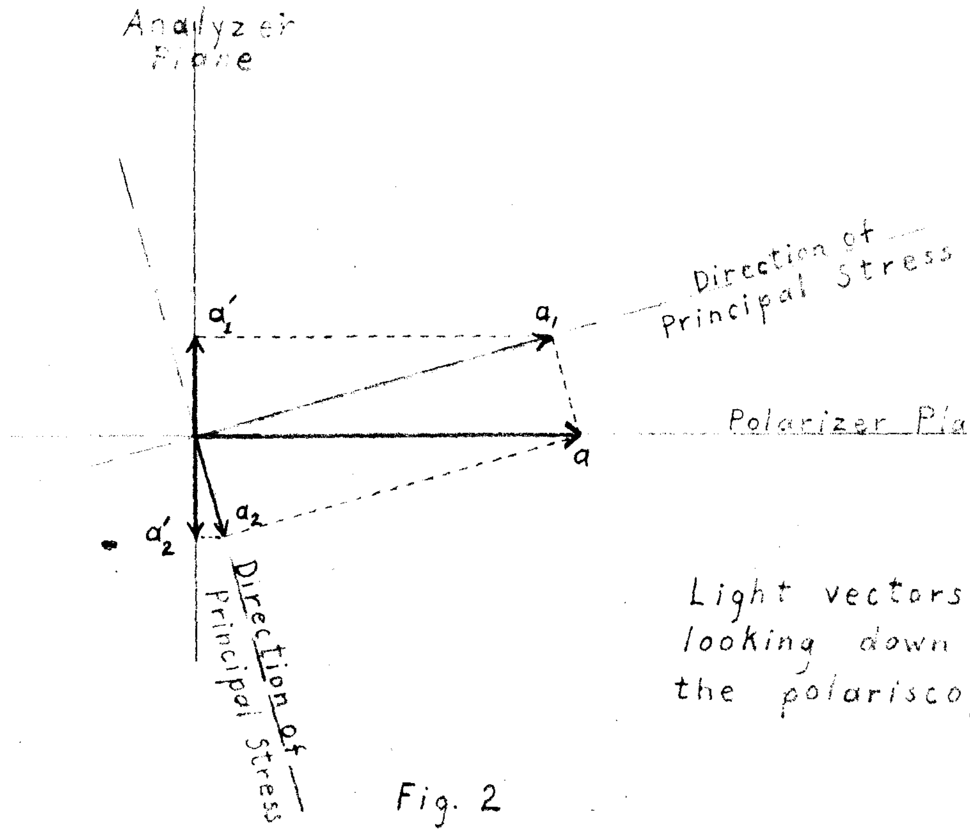
DIAGRAMATIC SKETCH OF A POLARISCOPE

Fig 1

MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	TOLERANCES ± .010 OR $\frac{1}{64}$ UNLESS OTHERWISE NOTED
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			NAME				DRAWING NO.

chromatic (single wave length) source of light is used and this light is passed through a polaroid lens called the polarizer, the light then becoming plane polarized. This polarized light is represented by the vector "a" in Fig. 2. Then the polarized light passes through the bakelite specimen, a temporarily doubly refracting material. At each point in the specimen the polarized light beam is broken into two mutually perpendicular polarized components. They are represented by the vectors "a_1" and "a_2". These components are in the direction of the principal stresses at that point. These light waves, "a_1" and "a_2", are also out of phase, the amount being directly proportional to the stress existing at that point and the thickness of the specimen. The light then passes through another polaroid lens similar to the polarizer and known as the analyzer. The analyzer will again pass only the components of the light waves in one plane. Thus the only light going through the analyzer are the components "a_1'" and "a_2'". These are in the same plane. However, they are out of phase and if this phase difference is one half a wave length, a dark interference fringe will result. These dark lines are called isochromatic lines or fringes.

There is another means of obtaining a dark line when using plane polarized light. This is illustrated in Fig. 3. Suppose that the direction of one of the principal stresses is parallel to the polarized light vector, "a", from the polarizer. This vector "a" is unchanged by the doubly refracting specimen since it lies completely in one of the planes of principal stress. However when it reaches the analyzer, no part of "a" can pass since no component of it lies in the plane which the analyzer allows to pass. Therefore, a dark fringe is formed where-ever the principal stresses are parallel to the polarizer plane. This is known as an isoclinic line.



Light vectors as seen looking down axis of the polariscope.

Fig. 2

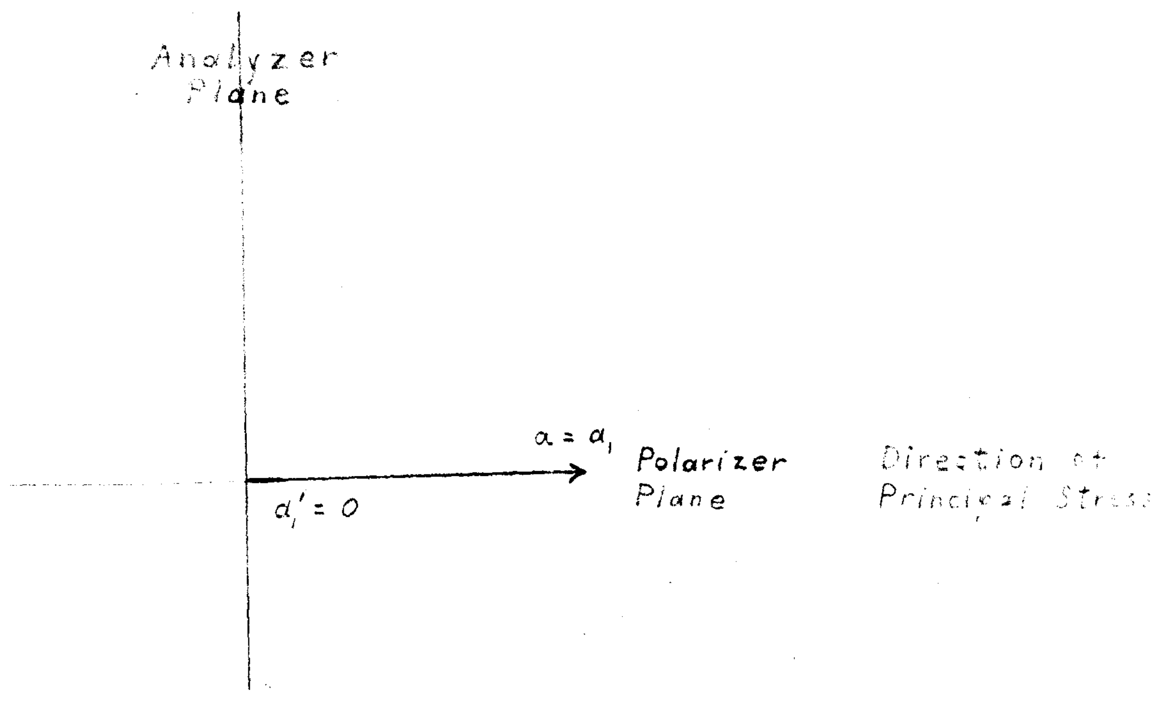


Fig. 3.

Obviously it is impossible to differentiate between an isoclinic line and an isochromatic line. However, the isoclinic line may be eliminated if circularly polarized light is used instead of plane polarized. If the plane polarized light is thought of as a vector lying in one plane, then circularly polarized light may be thought of as a rotating vector, the tip of which forms a helix. Circularly polarized light is obtained by installing a plate of doubly refracting material after the polarizer and of such a thickness that the two polarized components from the plate are one fourth of a wave length out of phase. This plate is known as a quarter wave plate. After circularly polarized light has passed through a doubly refracting medium it has the property of always being divided into two mutually perpendicular pomponents as was plane polarized light, but these two components are always equal. Because these components are always equal regardless of the orientation of the axis of principal stress in the stressed specimen, no isoclinic lines can exist. It is necessary to install a second quarter wave plate just ahead of the analyzer to restore the light to plane polarized. The isochromatic lines, which appear because of the relative retardation between the light wave components from the doubly refracting specimen, are formed with the circularly polarized light just the same as with plane polarized light.

It was stated that the relative retardation between the two light components leaving the stressed specimen was directly proportional to the stress and specimen thickness. The exact expression is: $R_t = \frac{(\sigma_1 - \sigma_2)\, b}{K}$

R_t	- relative retardation
σ_1, σ_2	- principal stresses
b	- specimen thickness
K	- a constant

As previously explained, this relative retardation produces dark lines or fringes when the two light components are out of phase by one half wave length. It can be proven that: $N = \frac{(\sigma_1 - \sigma_2)\ b}{C}$ where

N = number of consecutive fringe interference line

C = photoelastic constant

Since $\frac{\sigma_1 - \sigma_2}{2} = \tau$ max, these fringe lines are actually loci of constant shearing stress.

For a polariscope using plane polarized light, the axes of the polarizer and analyzer must be set so as to be either perpendicular or parallel. If the axes are perpendicular to each other the specimen image field is known as a "dark field", since the polarized light which is not affected by the specimen cannot pass through the analyzer and shows as a dark area. In this case the dark fringe lines are whole integers: 0, 1, 2, 3, etc. The lines of light correspond to half fringe orders: 1/2, 3/2, etc. Unstressed portions of the specimen show as black area and are the zero order. If the polarizer and analyzer axes are parallel, the field of the polariscope is a "light field". This follows from the fact that light from the polarizer which is unaffected by the specimen is completely passed by the analyzer and shows up as a bright area. The lines of light are then the whole integers in N and the dark fringes will be the 1/2 fringe orders.

To obtain a dark field for a polariscope using circularly polarized light, the polarizer and analyzer axes are set parallel. The quarter wave plate axes are set parallel to each other and at 45° to the axes of the polarizer and analyzer. For a light field it is only necessary to change one quarter wave plate so that the two plates have their axes at right angles. If the polarizer or analyzer and its corresponding quarter wave plate are rotated together, there will be no change in the type of field obtained.

Therefore, if the fringe order is determined at any point on the specimen the value of $(\sigma_1 - \sigma_2)$ can be found providing the constant C is known. The photoelastic constant is easily determined. Along a free boundary one principal stress, σ_2, is zero. The specimen may be loaded in tension, compression or bending, then the fringe order determined along a free boundary where the stress may be easily calculated. All of the components of the formula, $N = \frac{(\sigma - 0)b}{C}$, are known except C.

At points on the specimen other than along a free boundary, another relationship between σ_1 and σ_2 is necessary to solve for the principal stresses. There are several methods for obtaining this relationship but they are all tedious and involve considerable labor. However, in this research problem, the only stresses in which we are interested are along the boundaries of the cutout where the stress concentration is a maximum.

PART III

EQUIPMENT

A. POLARISCOPE

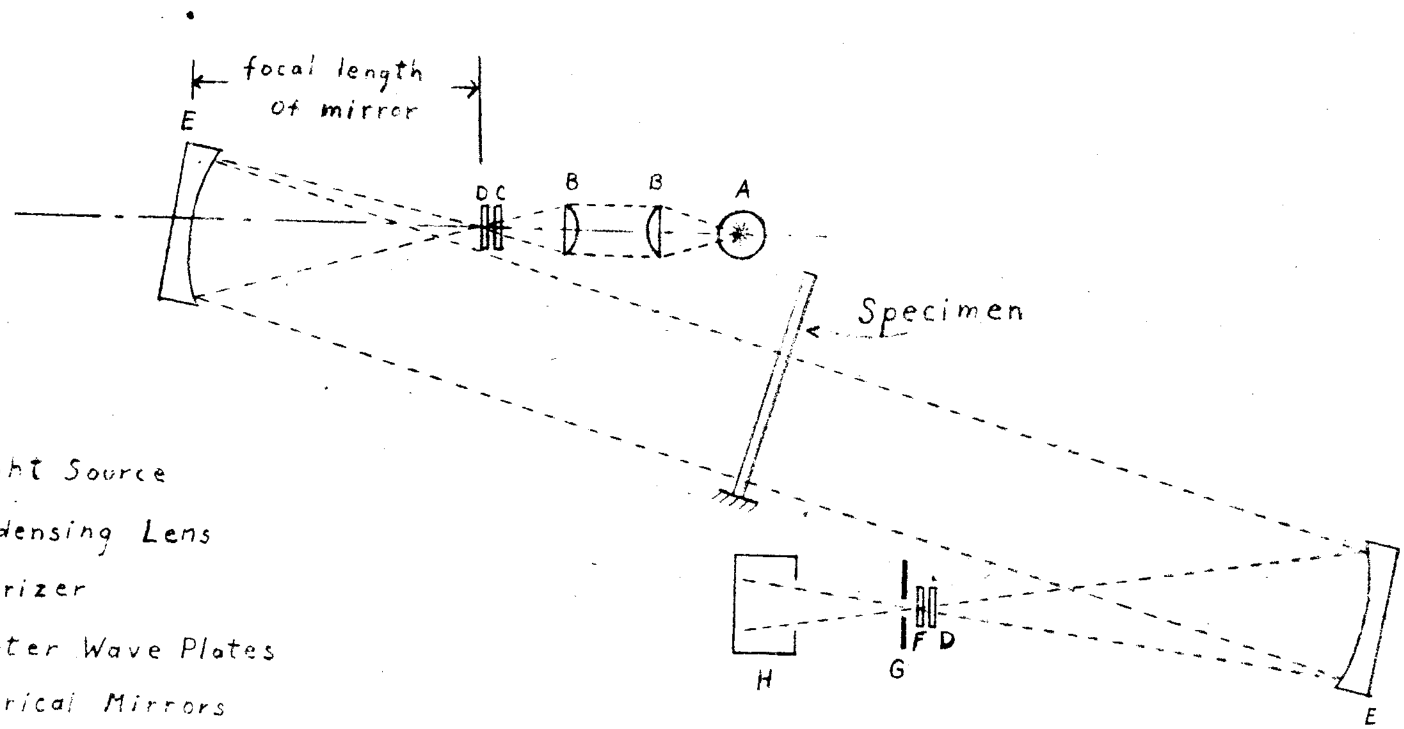
The polariscope consisted of a sodium vapor lamp, a set of condensing lenses, two polaroid lenses, two quarter wave plates, two spherical mirrors 20cm. in diameter and an aperture opening. The setup of the apparatus is illustrated in Fig. 4. This arrangement is a little unusual in that two spherical mirrors are used so that only about one third the usual room length for the polariscope is required. The image from this system is inverted and upside down.

The light source was a sixty watt sodium vapor lamp. Two plano-convex condensing lenses were placed between the source and the polarizer. The polarizer was placed at the focal point of the condensing lenses. The polarizer and analyzer were placed at the focal points of their respective mirrors (1 meter from the mirrors).

The polarizer and analyzer consisted of a polaroid lens 5 cm in diameter which was mounted in a stand. The quarter wave plate was mounted on the polaroid frame with a provision for relative movement between the two. Thus a light or dark field could be obtained.

It was found necessary to insert an aperture at the analyzer, (focal point of the mirror) in order to obtain clear images. Otherwise the distortion from the mirrors gave a fuzzy image.

The camera consisted of a box with a round opening in the front and the rear having a frame for receiving a 5" x 7" film holder. No lens was necessary in the camera because of the optical set up of the polariscope. A glass plate with a sheet of tracing paper attached was used to



A - Light Source
B - Condensing Lens
C - Polarizer
D - Quarter Wave Plates
E - Spherical Mirrors
F - Analyzer
G - Aperture
H - Camera

Fig. 4

MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	TOLERANCES ± .010 OR $\frac{1}{64}$ UNLESS OTHERWISE NOTED
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			POLARISCOPE (NAME)				DRAWING NO.

view the image and obtain the proper focus.

A stand for mounting the specimen was placed about halfway between the two mirrors. It consisted of a steel frame having bolt holes spaced 1/2" apart along a portion of one side to which the bakelite panel was bolted and rigidly held in place.

B. SPECIMEN

The specimen used was Bakelite BT-61-893. It was cut rectangular and measured 9 3/4" x 4" x 5/16". The original specimen was 12" long but was accidentally broken while determining the photoelastic constant. However, this shorter length proved to be sufficiently long so that the stress concentration fringes from the bolt holes did not distort the fringes around the cutout.

For mounting and loading purposes 3/16" holes spaced 1/2" apart and 1/2" from the edge were drilled on both ends of the specimen. One end of the specimen was mounted to the fixed stand and the loading beam was bolted to the other end. This gave a cantilever beam and by moving the loads along the loading beam, the ratio of bending to shear loads could be varied.

A sketch of the specimen with the first cutout which was elliptical is shown in Fig. 5. The ellipse measured 2" x 1" and the major axis was along the long dimension of the panel. Modifications to the ellipse are also shown in Fig. 5. The first consisted of modifying the ellipse to a rectangular cutout with 1/2" fillets, then to 1/4" fillets and finally to a purely rectangular opening.

The cutting operations were carried out by the machine shop which had had previous experience in working with this material. Holes were drilled in the specimen, then the rough shape of the cutout was obtained by using a filing machine. The final smooth finish was obtained

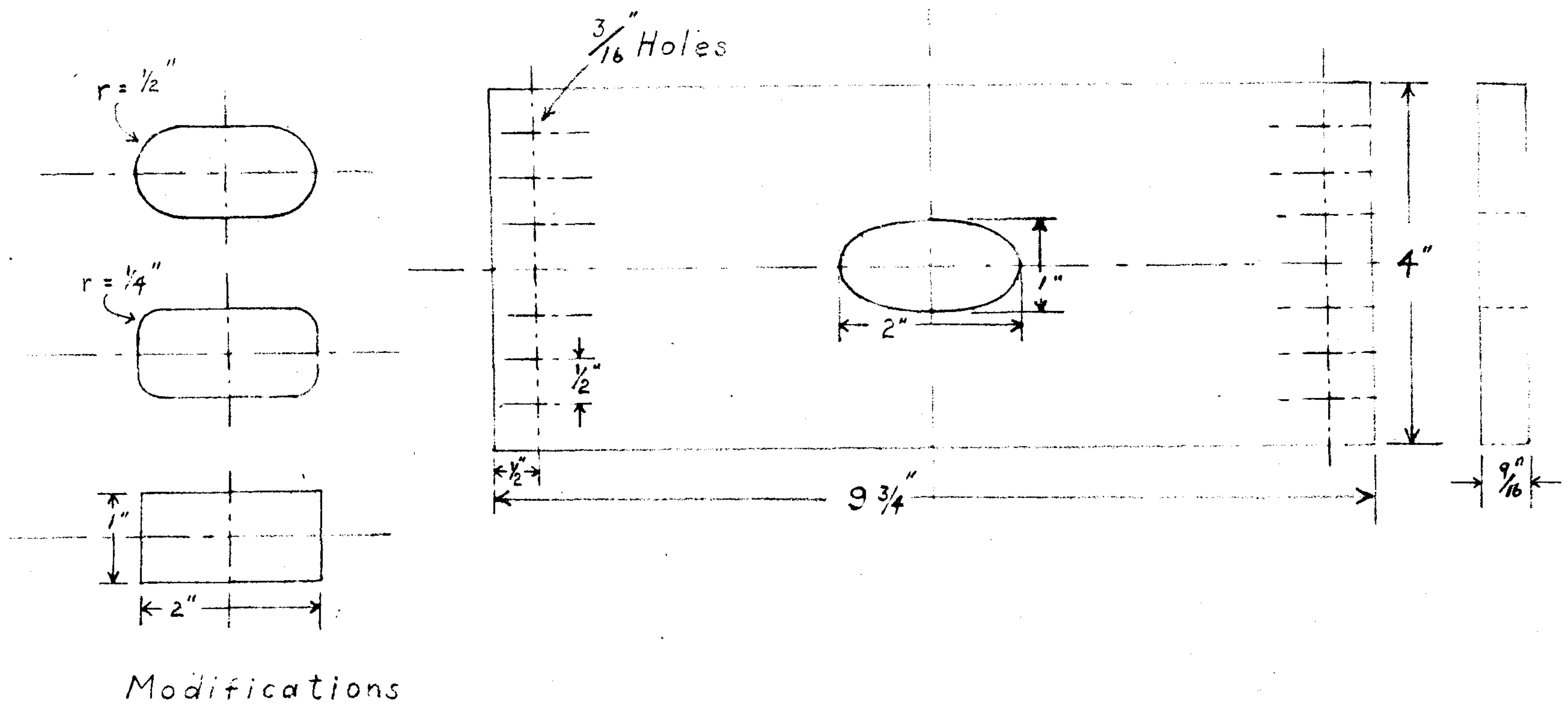


Fig. 5

MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	TOLERANCES ± .010 OR 1/64 UNLESS OTHERWISE NOTED
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			Specimen NAME				DRAWING NO.

by hand filing. The files used were sharp and only small amounts of material were removed at a time. In this way the Bakelite did not become heated and edge stresses were kept to a minimum. However as noted later, ageing stresses set in rather rapidly and the specimen should be used immediately after cutting. When cutting a photoelastic model great care should be taken that all cut surfaces are absolutely perpendicular to the face of the model.

PART IV

PROCEDURE

A. EQUIPMENT SETUP

For all of the work a dark field was used. There was no particular reason for favoring the dark field except that this procedure makes the dark lines whole numbers which was a little less confusing than the fractional fringe orders given by a light field. Frocht intimates that a dark field is standard unless special reasons dictate a preference for a light field. What Frocht calls a Standard or Crossed Circular Polariscope was used. The polarizer and analyzer were crossed and the axes of the quarter wave plate were at 90° with each other and at 45° to the polarizer axis.

The axes of the quarter wave plates were not marked, but were found by the following method: The polarizer and analyzer were crossed. A quarterwave plate was inserted between them and rotated until a dark field was obtained. Then the axes of the quarter wave plate were perpendicular and parallel to the polarizer axis.

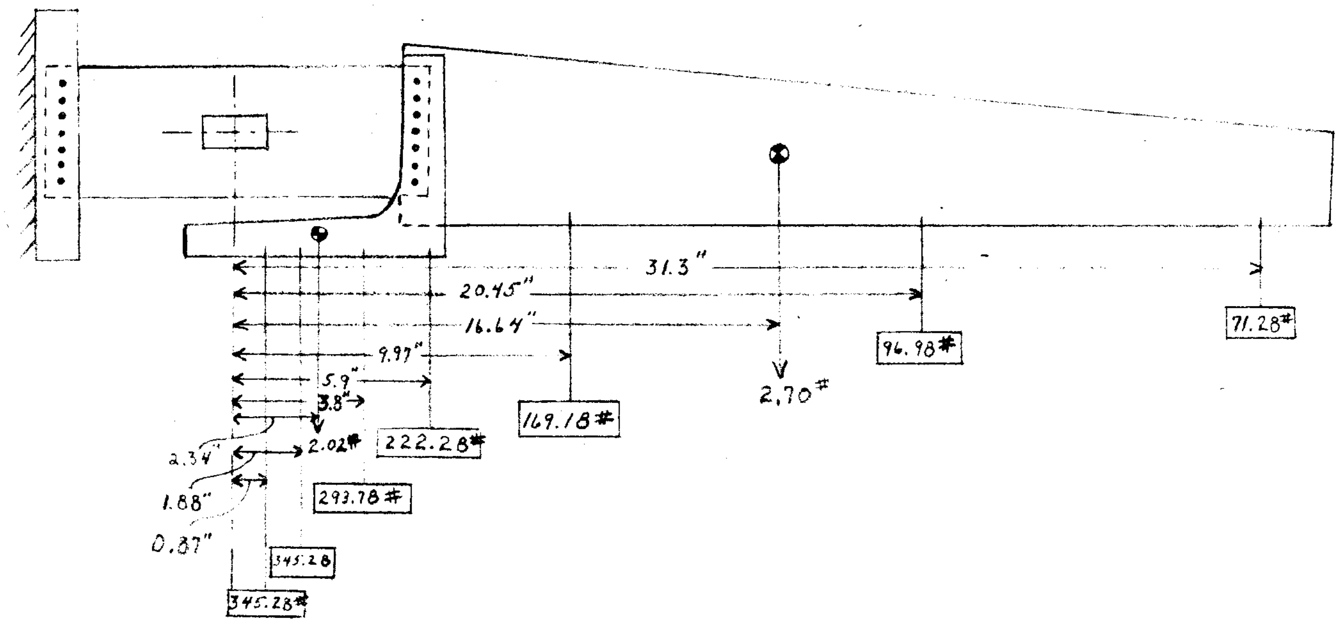
The spherical mirrors should be so arranged that the angle of reflection is as small as possible in order to minimize the distortion. In some of the work the horizontal unit length of the panel image was found to be 13% longer than the corresponding vertical unit length. This represents considerable stretching in the horizontal direction.

B. LOADING

The problem undertaken was to determine not only the effect of the various cutout shapes upon stress concentrations but also to determine the effect of different conditions of combined bending and shear loadings. For this reason the specimen was mounted as a cantilever beam, since this gave an easy means of varying the ratio between the bending and shear loads. For a cantilever beam the ratio of bending to shear stress is proportional to the distance out on the beam that the load is applied since the bending stress is a function of the load times the lever arm and the shear stress is a function of only the load.

A metal loading arm was bolted on to the free end of the specimen after it was attached to the fixed frame. A rather large bending moment could then be applied. Two auxiliary arms, one on either side of the loading arm and extending towards the center of the panel were used to obtain very short loading arms. See Fig. 6 for a sketch of the method of loading. Holes were drilled in these loading arms at the proper distances and the load suspended from a pin through the hole. Large lead and iron weights were used for the loads. By hanging the weights from a pin extending though the loading arm, the most uniform and torsion free loading was obtained. In using a photoelastic model great care should always be taken that the stresses are uniformly applied over the thickness of the model. Otherwise a two dimensional setup will not be obtained and the fringes will tend to become indistinct and discontinuous.

It was decided to use bending to shear load ratios of 1, 2, 4, 6, 10, 20, and 30 from a study of the results of the previous investigations. All of these ratios were measured at the center of the cutout. In order to obtain the maximum number of fringe orders, the loads were increased as the loading arm became shorter in such a manner as to keep the maximum permissible load on the specimen without breaking it. See the appendix for a detailed description of each loading position.



Note: Loads applied separately

							TOLERANCES ± .010 OR 1/64 UNLESS OTHERWISE NOTED
MATERIAL	FINISH	HEAT TREAT	DRAFTSMAN	CHECKED	APPROVED	ENGINEER	
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY			METHOD OF LOADING				
			NAME				DRAWING NO.

Fig. 6

C. CALIBRATION FOR PHOTOELASTIC CONSTANT

Before any cutouts were made in the panel, it was loaded and the photoelastic constant determined. Along the top and bottom edges of the panel the shearing stress is zero; also the normal stress perpendicular to the boundary must be zero. Therefore, the principal stress is the normal stress parallel to the edge. This stress, σ, may be calculated at any point knowing the bending moment and the dimensions of the specimen: $\sigma = \frac{My}{I}$.

M = bending moment

y = distance from neutral axis (center line of specimen)

I = moment of inertia of cross sectional panel area about the neutral axis

According to the photoelastic theory $(\sigma_1 - \sigma_2) = \frac{Nb}{C}$

N = fringe order

b = specimen thickness

C = photoelastic constant

$\sigma_2 = 0$, therefore: $\sigma = \frac{Nb}{C} = \frac{My}{I}$

$$C = \frac{MbI}{My}$$

Several points along the top and bottom were computed for each loading and a number of loadings were applied so that a large number of independent values were obtained and plotted. See the appendix for the detailed calculations. An average value of 86 was used for C. The individual values of C as computed are a little scattered. Frocht states that the above method of using a bending moment to calculate C is the least desirable and recommends that the specimen be loaded in tension for this computation. However this value of 86 corresponds rather closely to the values of C for Bakelite BT-61-893 given in most reports.

D. PHOTOGRAPHY

This was about the most important and difficult phase of the work. It was discovered that the photographs indicated more details than could be seen by the eye when the image was projected on a screen. Therefore, all work was done from photographs. A screen on which the image was projected was used to make rough sketches to obtain the number of the various fringe orders. The load was slowly applied and the fringes counted to obtain the number of each fringe as they show on the photographs.

As indicated previously, the spherical mirrors cause some distortion and an aperture was found necessary to obtain clear pictures. It was found that the smallest aperture possible which gave enough light to obtain a good photograph with a reasonable time exposure proved most successful. The size aperture used for photographing was 3/16". For viewing the image on a screen and on the focusing plate of the camera a slightly larger aperture was used in order to admit sufficient light. a 3/8" aperture was used for this purpose. While investigating the various fringe orders it was often found convenient to look directly into the polariscope at the analyzer and use a very small aperture about the size of a pin hole in front of the eye.

An Ortho type film is necessary for photographing sodium vapor light. The best results were obtained by using Super-Ortho-Press film. Because of the small aperture a very fast film is necessary. The time of exposure varied from about three minutes for the photographs showing the entire specimen to about four minutes for the enlarged views showing only the vicinity of the cutout. The films were developed immediately after each set of seven loadings was carried out. The developer used was Eastman D-19 and proved very successful.

The sodium vapor lamp is not entirely monochromatic and has a red and blue line in the spectrum. Filters are provided to take out these lines. However, the filters cut down on the light intensity to such a degree that an aperture larger than 3/16" was necessary. It was found that the smaller aperture without the filters in the lamp gave better pictures than the filtered light which required a larger aperture.

The pictures for the first two cutouts have stray light in them. This was caused by light from the second mirror which entered the camera around the outside of the analyzer frame. It was necessary to block off this light by installing a large shield around the analyzer.

The photographs of all cutouts except those for the elliptical hole were made immediately after the cutting operation had been carried out. This was found necessary in order to prevent the time edge stresses from setting in. The two photographs in Fig. 7 show the effect of time on the edge. The first picture indicates the condition of the boundary immediately after cutting and the second shows the same specimen under idential loading conditions but after the specimen had aged for a week. It can readily be seen that the boundary shows considerable difference around the corners. These pictures were taken using the same specimen. The cutouts were gradually made more extensive until a rectangle was the final result. In this procedure there is part of cutout which does not have material removed during successive modifications. Therefore there are parts of the cutout which exhibit ageing edge stresses even when the panel has been freshly cut. Photographs of the last two cutout modifications show this effect along the top and bottom middle edges. Therefore, if a good picture is desired it would be necessary to make entirely new cutouts in each case. T his would entail using a new panel for each modification unless the new cutout were so radically different from the old one that all of the old boundary material would be removed.

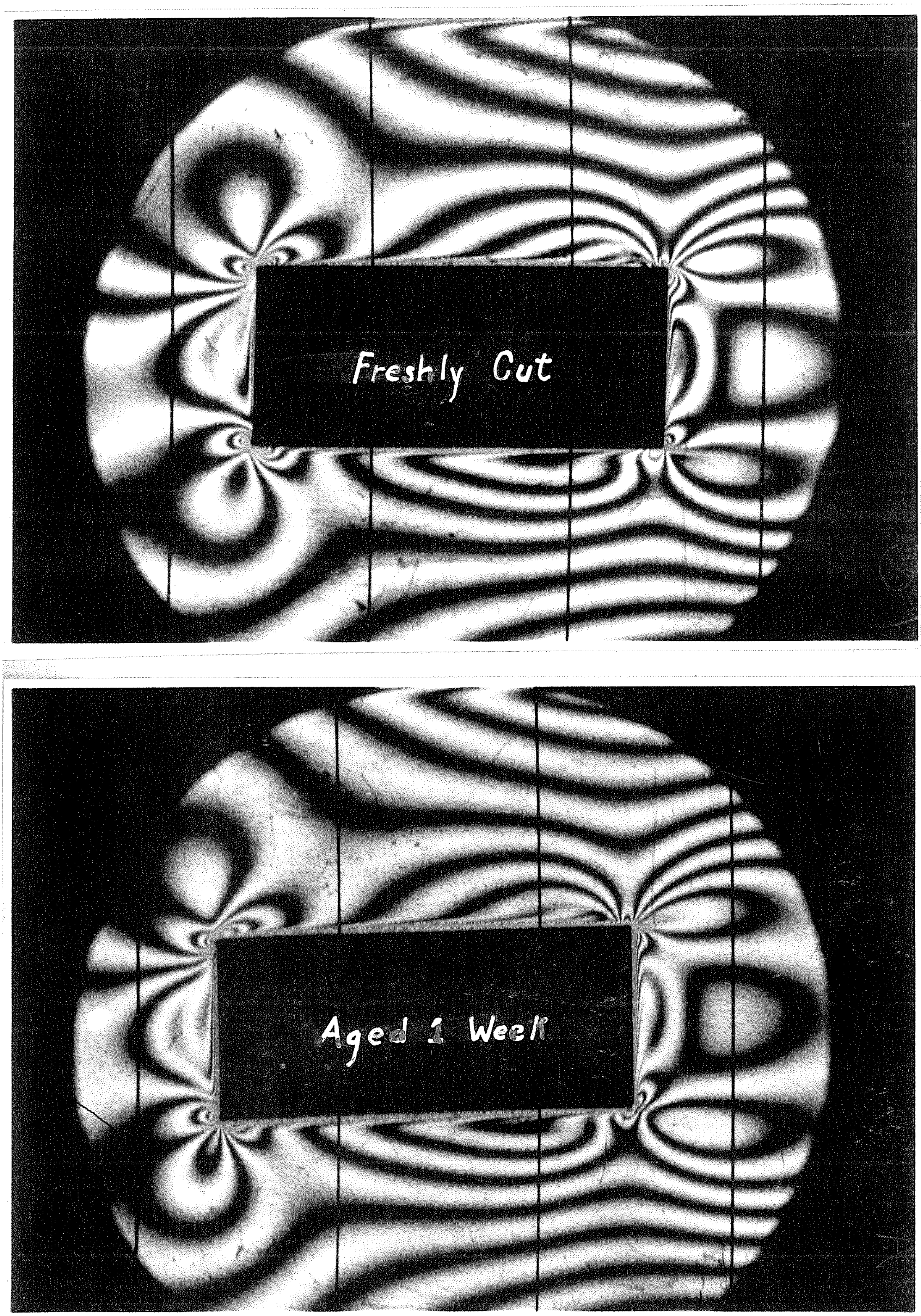


Fig. 7

PART V

CURVES

A. PREPARATION OF CURVES

As indicated previously, the value of the stress at the boundary of the cutout may be calculated knowing the fringe order at the point in question, the photoelastic constant and the thickness of the specimen. The fringe order numbers were ascertained and then the fringes on the photographs were numbered. However, the stress (which was normal to the cutout boundary) was not directly calculated. Instead a stress ratio at points around the boundary was determined. This ratio consists of the stress existing at any point along the cutout as determined by photoelastic methods divided by the maximum principal stress that would exist at some basic point in the panel assuming no cutout. This basic point of principal stress is at the edge of the cutout boundary on the vertical centerline. In this case it is 1/2" above the horizontal centerline and on the vertical centerline of the cutout where the bending to shear load ratio was computed. Details of these computations are in the appendix.

B. USE OF CURVES

The values of the stress ratio for the various loading conditions for each cutout are plotted in polar coordinates in Figs. 8, 9, 10 and 11. These curves may be used to determine the stress existing along cutout boundaries of similar shapped openings. First the ratio of bending to shear loads which will probably exist must be determined. Then with this value of $\frac{M}{S}$ the stress ratio at the point in question may be obtained from the proper cutout curve. This stress ratio multiplied by the principal stress existing at the intersection of the vertical centerline with the cutout boundary gives the stress at the chosen point. The principal stress can easily be computed by the use of the elementary formulae:

$$\tau = \frac{S}{Ib}\int_{y_1}^{h_2} b\,y\,dy$$

$$\sigma = \frac{My_1}{I}$$

$$\sigma_{pr} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

S = shear load

I = moment of inertia of panel cross section about neutral axis (equals $\frac{bh^3}{12}$)

b = panel thickness

h = panel height

y_1 = distance from centerline to proposed cutout boundary

M = bending moment at center of cutout

C. CURVES

Figs. 8 - 11 which follow are graphs of the stress ratio existing at each point around the boundary of the cutout. These curves are plotted in polar coordinates. Fig. 12 is a curve of maximum stress ratio vs. M/S for each of the cutouts.

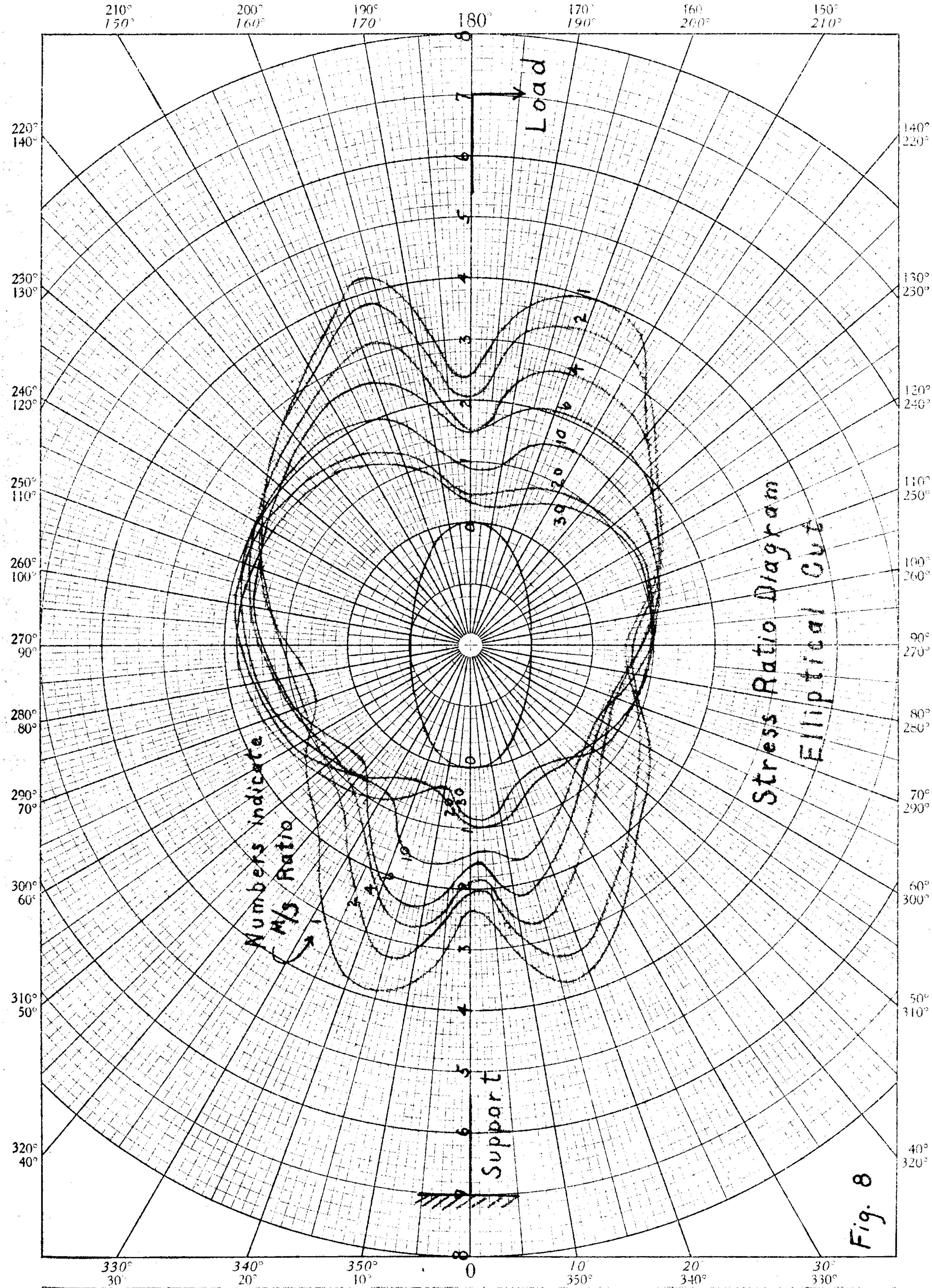

Load
Support
Stress Ratio Diagram
Elliptical Cut
Numbers indicate M/S Ratio
180°
0

Fig. 8

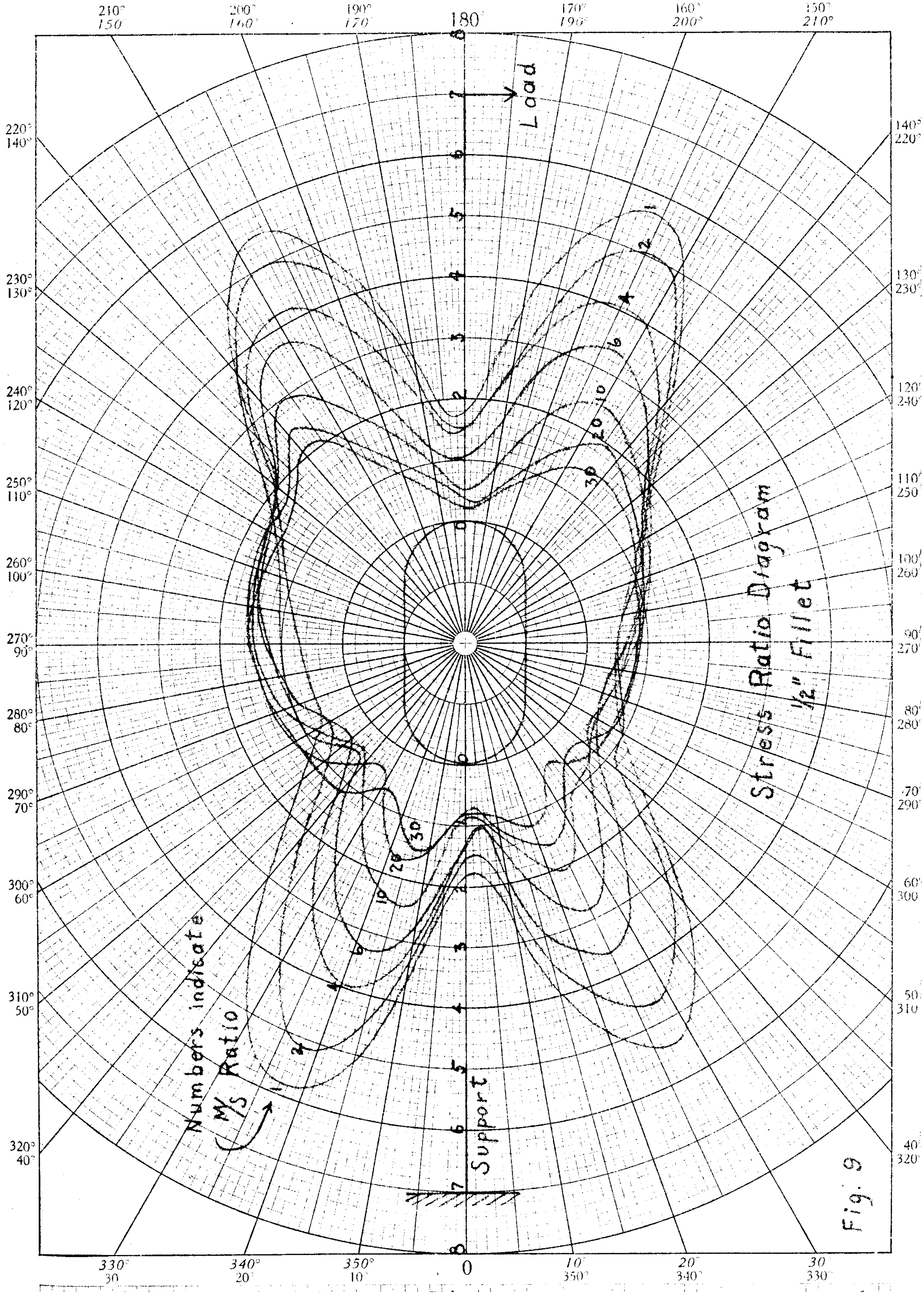
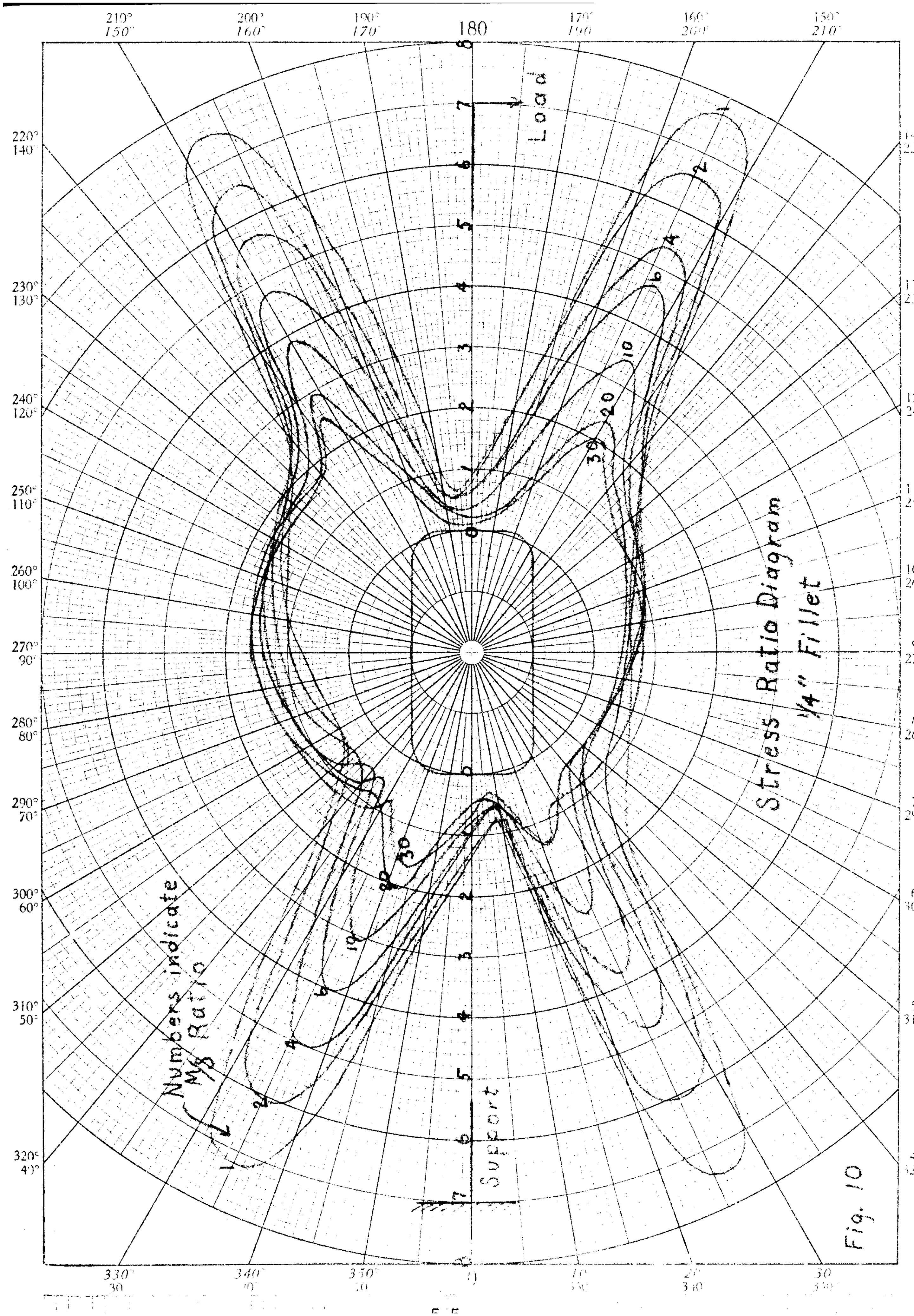
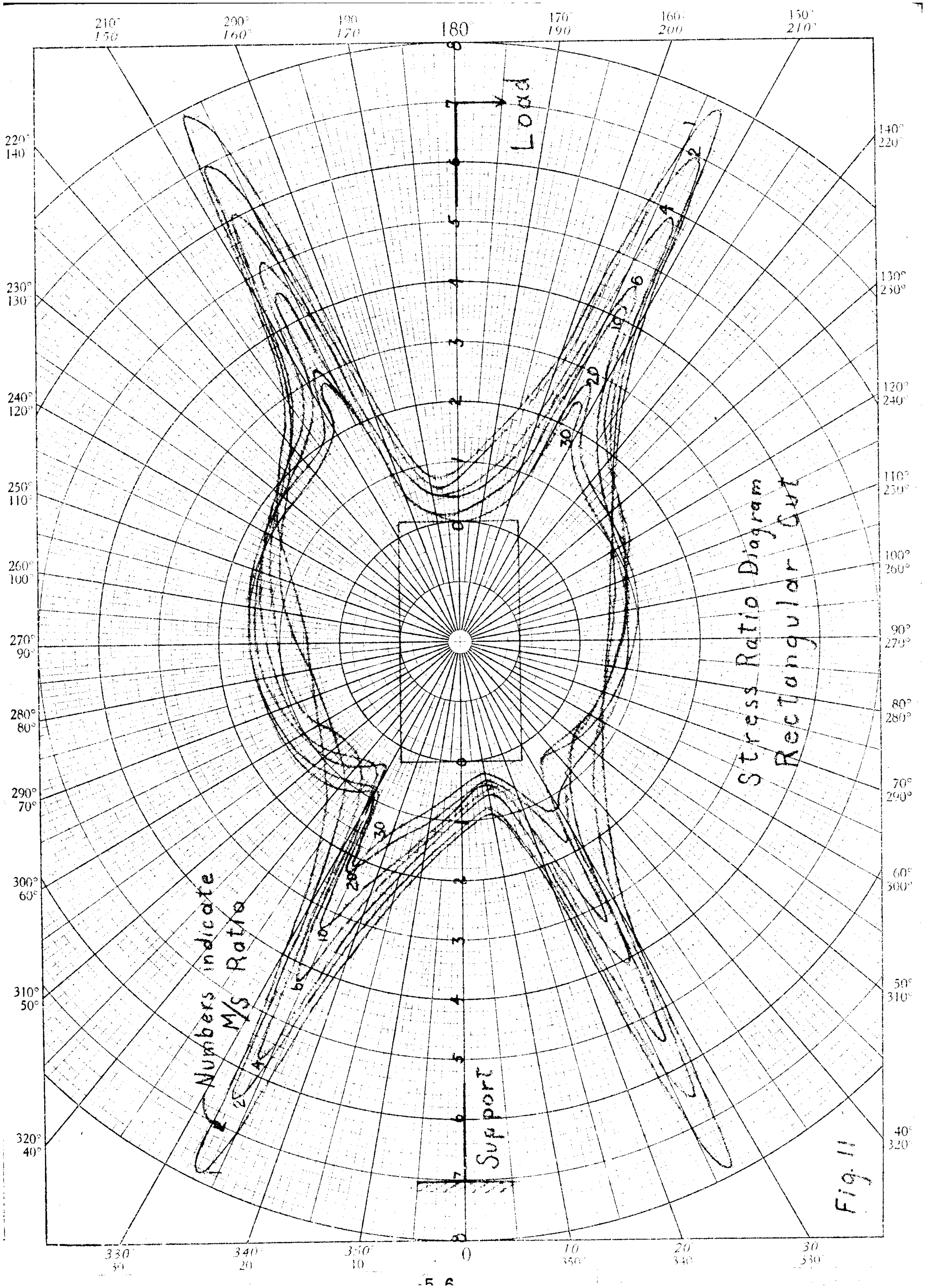
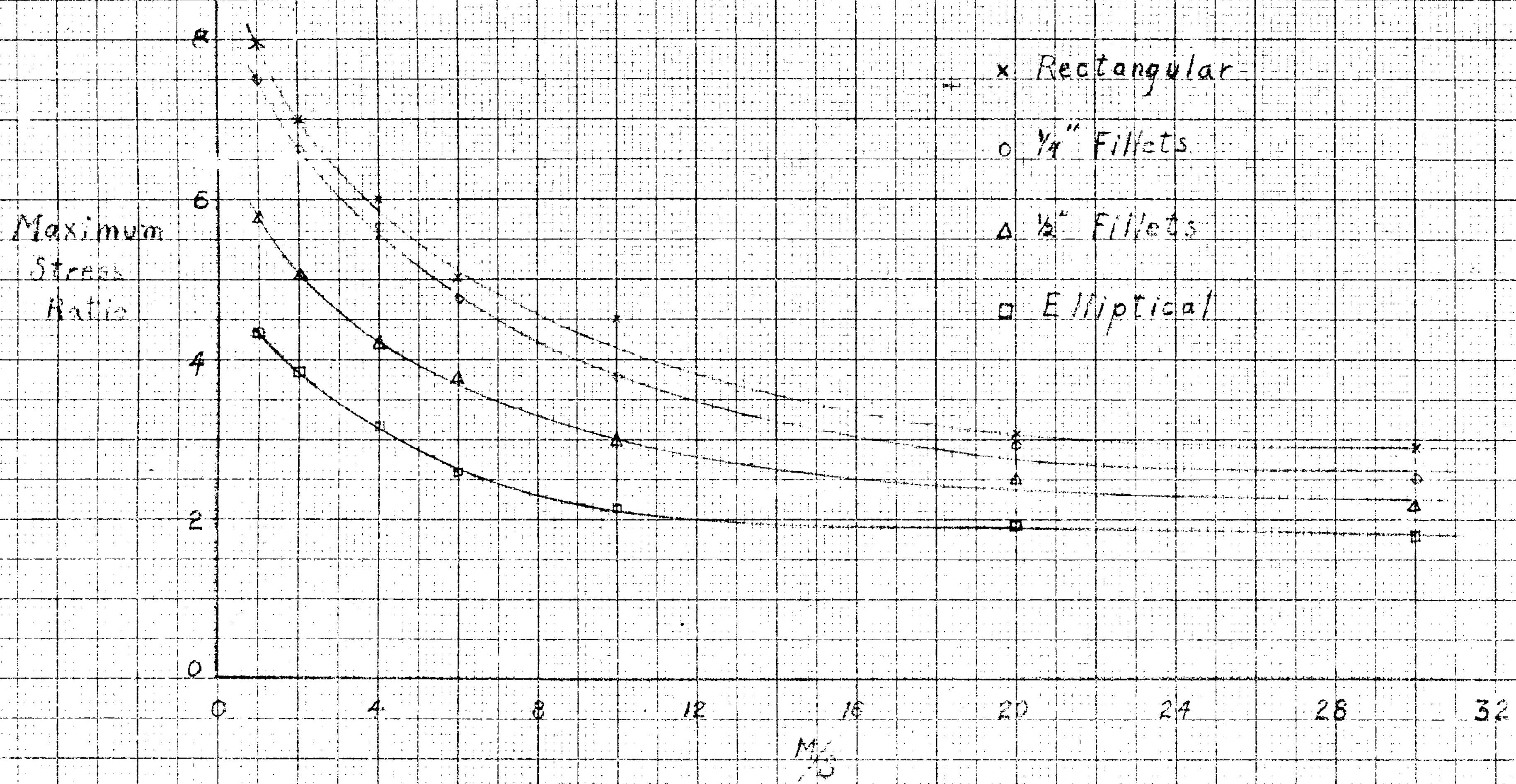

Stress Ratio Diagram
1/2" Fillet
Load
Support
Numbers indicate
M/S Ratio
180°
0

Fig. 9


Load
Support
Stress Ratio Diagram
1/4" Fillet
Numbers indicate M/S Ratio
1
2
4
6
10
20
30
180°
0
90
270
Fig. 10


180°
0
Load
Support
Stress Ratio Diagram
Rectangular Cut
Numbers indicate
M/S Ratio
Fig. 11



Max. Stress Ratio
vs
Loading Condition

Fig. 12

PART VI

CONCLUSIONS

As should be expected, the stress concentrations at the corners of the cutout were increased when the size of the fillets was reduced. However, the expected large increase in the stress at the sharp corners of the rectangle did not occur. Theoretically the stress at the corners should be infinite. The observations showed only a very moderate increase in stress for the sharp corners over that for the 1/4" fillets. This would indicate that the stress increases rapidly as the fillet radius is reduced until a moderately small fillet is obtained. After reaching this size, the effect on stress concentration of further reducing the fillet radius is not great. However, the calculated stress ratio curves for the rectangular cutout are a little doubtful and could indicate values which are too low. The fringes were quite close together and counting them accurately proved to be very difficult. Note that the stress was not a maximum at the corners, but the maximum occurred on the top and bottom edges of the cutouts a short distance towards the center from the corners.

The governing factor for the amount of stress concentration was the radius of the fillet. The larger was the radius, the smaller the stress ratio became. At first this would not seem to be true when examining Fig. 12 and noting that the maximum stress ratio for the ellipse was less than for the 1/2" fillet, whereas the smallest radius of the ellipse was less than the 1/2" fillet radius. However, the point of maximum stress occurs at a place where the radius of the ellipse is a maximum; i.e., at the ends of the minor axis. Instead of comparing the maximum stresses, if the stresses of the ellipse and 1/2" fillet are compared at the point where the ellipse radius is a minimum, the ellipse stresses are the greater.

Fig. 12 shows the effect of varying M/S on the maximum stress. A small M/S ratio gives the maximum stress around the cutout. As the ratio of M/S is increased the stress is reduced, but does not decrease appreciably for values of M/S greater than 25 or 30. It would, therefore, seem desirable to locate the cutouts at points in a structure where the bending stress is at least twenty times greater than the shearing stress.

A logical result which should be obtained from the two research projects on elliptical and modified cutouts is the condition of least stress concentration. As could be expected, the elliptical cutout with a horizontal major axis showed the minimum stress.

There were a number of interesting items encountered which are not readily explainable. The first is the fact that the maximum stress usually occurred on the side of the cutout towards the load. The normal expectation would be for the maximum stress to occur on the side towards the support where the bending moment is larger. This phenomena was also noticed in the two preceding investigatigations of panels with cutouts under combined loads by Gibbons and Dill, and Tyra and Hollister. It seems that there should be some physical explanation for such a result occurring in three independent investigations. There is a possibility that the explanation lies in an unsymetrical shear flow around the cutout.

Note that along the vertical side of the cutouts towards the loaded end the points of maximum stress move away from the horizontal centerline as the ratio of M/S increases. This could be expected since the bending stress becomes larger as M/S increases and also increases with distance from the neutral axis. However, on the side of the cutouts towards the support, these points of maximum stress shift towards the horizontal centerline as the ratio of M/S increases. This cannot be readily explained.

Other items which are not entirely understood are connected with the points of minimum stress which should occur where the horizontal centerline intersects the boundary of the cutout. The stress at this point might be expected to be zero since the bending stress is zero at the neutral axis and no shear should exist at a free boundary. However, the stress was not zero at the ends of the cutouts. This seeming inconsistency may be explained by Timoshenko's statement that a beam with a cutout along the neutral axis as indicated in Fig. 13 (a) may be replaced by an equivalent beam as shown in Fig. 13 (b). This equivalent beam would indicate a bending stress along the neutral axis at the end of the cutout due to the secondary bending of the material on both sides of the cutout. Note that the minimum stress becomes larger as the ratio of M/S becomes smaller. As the shear load becomes greater the secondary bending becomes more important.

However, what cannot be explained is the fact that these points of minimum stress were not on the horizontal centerline. They were symmetrically shifted on both ends of the cutout; i.e., shifted up on one end and down on the other. This can easily be seen on the photographs, especially of the rectangular cutout. The fringe lines are not symmetrical about the centerline, but are distorted towards one side. This is not a matter of residual stresses or an imperfection in the specimen because the panel was turned over so that the top and bottom were interchanged, and the exact picture as before was obtained. Therefore, this shifting of the fringes to one side of the horizontal centerline was not a property of the specimen but was due entirely to the loading.

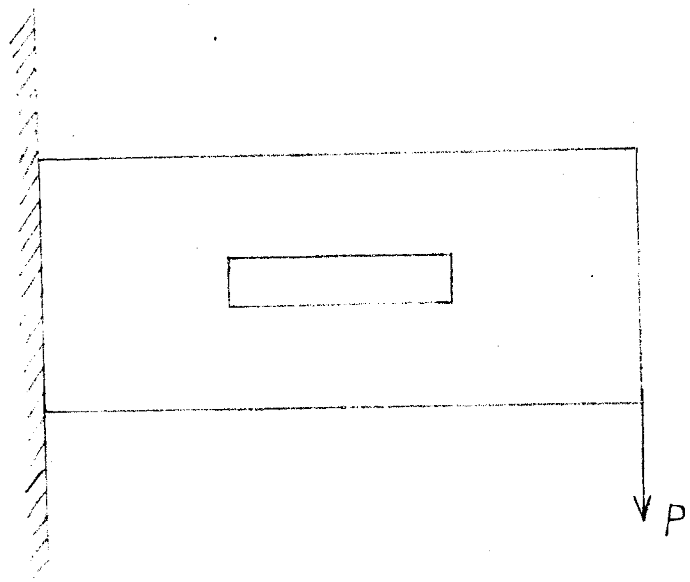


Fig. 13 (a)

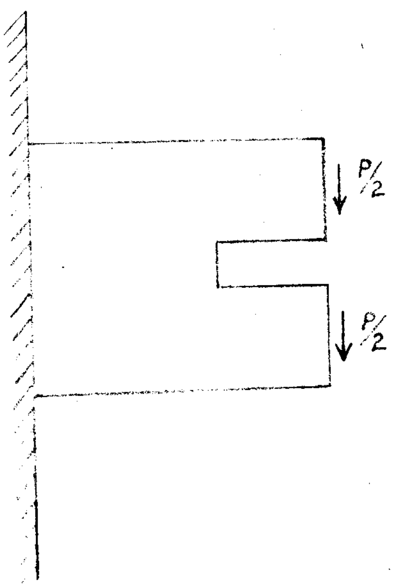


Fig. 13(b)

Another unsymmetrical result was the fact that the stresses were nearly always larger on the top of the cutout than on the bottom. This again was not due to the specimen itself as these same results occurred when the specimen was turned over.

All of these peculiar results indicate an unsuspected unsymmetrical condition. In order to form more definite ideas as to the actual causes of these results more photoelastic research should be carried out. It seems rather logical that the secondary bending of the material above and below the cutout as suggested in Fig. 13 (b) might explain some of the seeming inconsistencies. The effect of this secondary bending could be further investigated by testing panels in which the width of material between the cutout and outer edge of the panel is varied.

PART VII

APPENDIX

A. LOADING CALCULATIONS

(1) Weight of large loading beam = 2.70 lb.

Moment arm of large loading beam = 16.64 in.

Moment of auxiliary loading beam = 45 in. lb.

(2) Weight of auxiliary loading arms = 2.02 lb.

Moment arm of auxiliary loading arms = 2.34 in.

Moment of auxiliary loading arm = 4.7 in. lb.

(3) Static load $= S_s =$ 2.70 + 2.02 = 4.72 lb.

Static moment $= M_s =$ 45 + 4.7 = 49.7 in. lb.

(4)

$\frac{M(in.lb)}{S\ (lb)}$	S(lb)	M(in.lb)	Load $(S-S_s)$(lb)	Applied Moment $(M-M_s)$(in lb.)	Arm (in.)
1	350	350	345.28	300.3	0.87
2	350	700	345.28	650.3	1.88
4	298.5	1194	293.78	1144.3	3.83
6	227	1362	222.28	1312.3	5.90
10	173.9	1739	169.18	1698.3	9.97
20	101.7	2034	96.98	1984.3	20.45
30	76	2280	71.28	2230.3	31.30

B. PHOTOELASTIC CONSTANT CALIBRATION

$$\sigma = \frac{My}{I}$$

$$y = \frac{h}{2}$$

$$I = \frac{bh^3}{12}$$

$$\sigma = \frac{6M}{bh^2}$$

$$C = \frac{\sigma b}{N} = \frac{6M}{Nh^2}$$

h = 4 inches

$$C = \frac{3M}{8N}$$

The specimen was loaded and the intersection of the fringe orders with the outer boundary of the panel was noted. Knowing the load and the distance from the fringe to the load, the bending moment at the spot where the fringe intersected the boundary was known. This was done for a number of conditions of loading and the corresponding values of N and M were tabulated. Instead of calculating C for each value obtained, points of 3M vs 8N were plotted for each observation. Then an average straight line was drawn through the points. The slope of the line is $\frac{3M}{8N}$ which is also the value of C. In this case the average value of C was found to be 86. This curve is shown in Fig. 14.

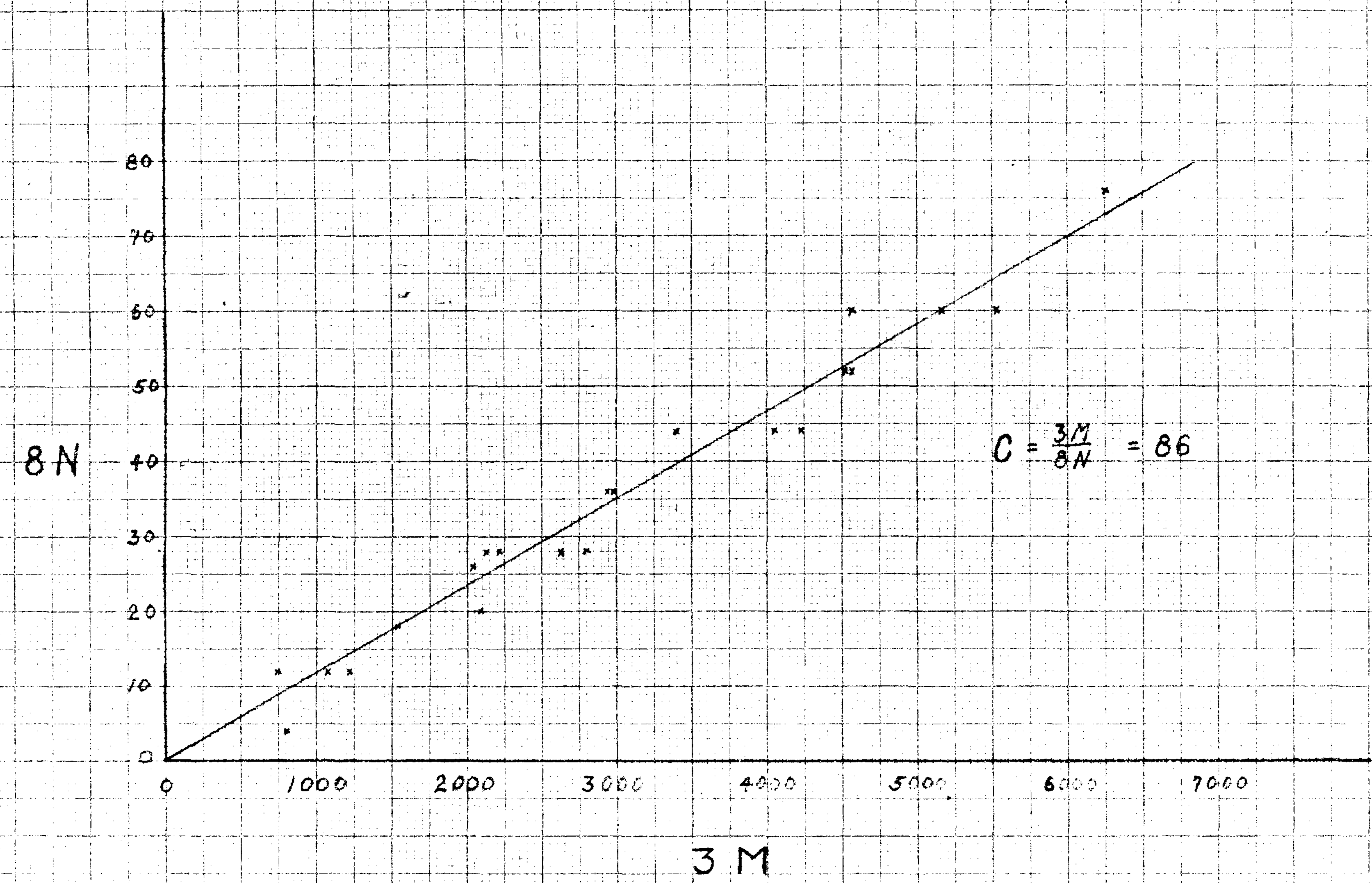


Fig 14 Calibration for Photoelastic Constant

C. <u>CALCULATION OF STRESS RATIO</u>

The basic stress is calculated at a point $\frac{1}{2}$" above the neutral axis at the vertical centerline of the proposed cutout.

$$\sigma_x = \frac{My_1}{I}$$

$$y_1 = \frac{1}{2}"$$

$$b = 9/16"$$

$$h = 4"$$

$$I = \frac{bh^3}{12} = 3 \text{ in.}^4$$

$$\sigma_x = \frac{M}{6}$$

$$\tau_{xy} = \frac{S}{Ib} \int_{y_1}^{h/2} b\, y\, dy = \frac{S}{2I} \left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau_{xy} = \frac{5S}{8}$$

Basic stress $= \sigma_{pr} = \sigma_x/2 + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$

. . . .

$$\sigma = \frac{CN}{b} \quad \text{(at boundary of cutout)}$$

$$C = 86$$

$$b = 9/16"$$

$\sigma = 153\ N$ (actual stress at cutout boundary)

M/S $\frac{(in.lb)}{(lb)}$	M(in.lb)	S (lb.)	σ_x(lb/in^2)	τ_{xy}(lb/in^2)	$\sigma_{pr.}$ (lb/in^2)	$\frac{\sigma}{\sigma_{pr.}}$
1	350	350	58.3	218.6	240.7	0.613N
2	700	350	116.7	218.6	284.7	0.537N
4	1194	298.5	119.0	186.8	311.3	0.491N
6	1362	227	227.0	142.0	259.5	0.517N
10	1739	173.9	289.5	108.7	325.8	0.469N
20	2034	101.7	339.0	63.6	350.5	0.436N
30	2280	76	380.0	47.5	385.9	0.396N

At each point on the cutout if the fringe order is multiplied by its value of $\frac{\sigma}{\sigma_{pr.}}$ as tabulated above, the correct stress ratio for that point is obtained. This was done and the results are plotted as Figs. 8-11.

The distortion of the image was in the order of nearly 13% in some cases. It was known by actual measurement that the ratio of the major axis to the minor axis of the cutout was 2. However by measuring the photograph this ratio came out as much as 2.26. This shows considerable elongation of the image. To compensate for this error a corrective grid was constructed and the points around the cutout on the photograph were transformed so that the cutout image was reconstructed to its proper shape before plotting.

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4. A Photoelastic Investigation of the Effect of Elliptical and Modified Cutouts in Flat Panels Subjected to Combined Bending and Shear. Thesis by R. M. Gibbons and Douglas Dill, California Institute of Technology, 1942.

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PART VIII

PHOTOGRAPHS

The following photographs are prints of the negatives from which the stress ratios were calculated. In all cases the photographs are mounted so that the loaded end is on the left side and the top of the photograph corresponds to the top of the specimen as mounted in the stand.

Each photograph has a few of the fringes numbered.

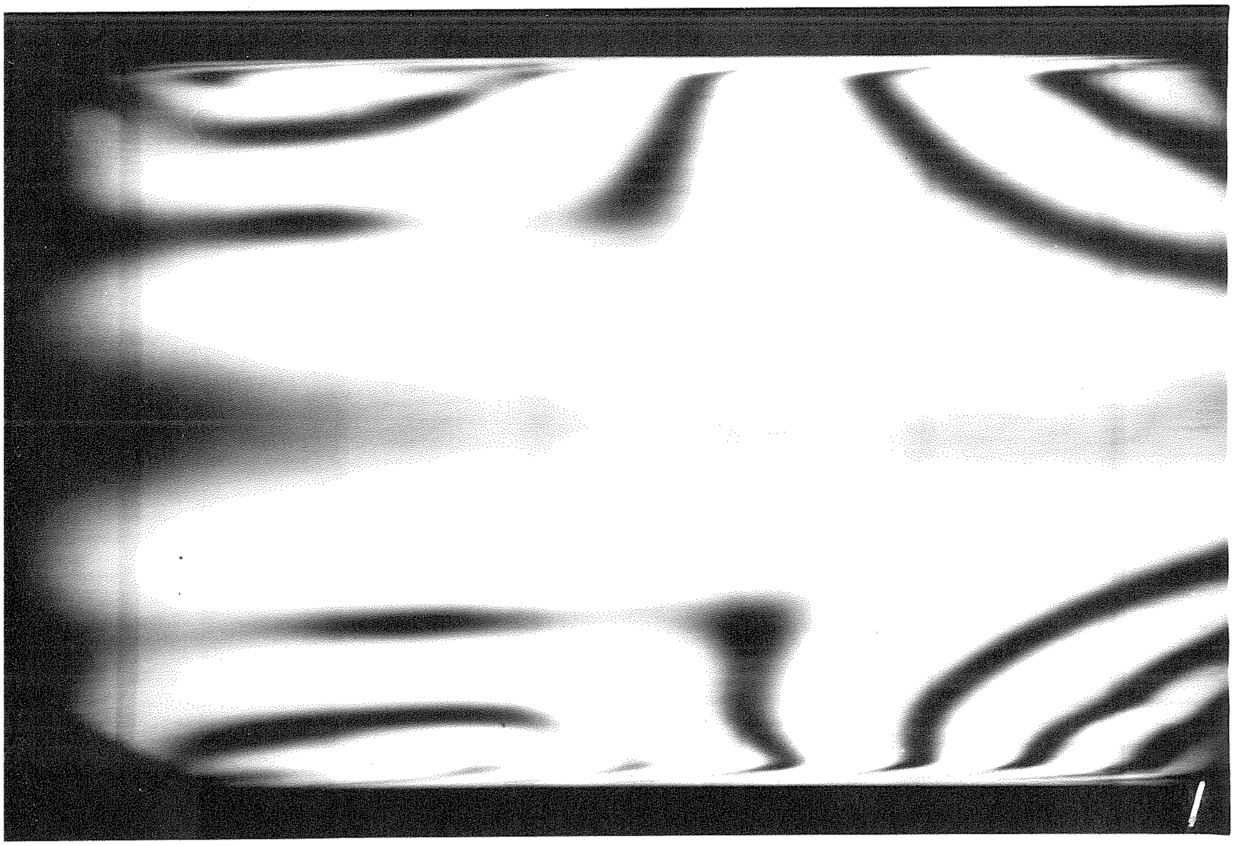
They are arranged according to types of cutouts as follows:

Group 1	No cutout
Group 2	Elliptical cutout
Group 3	Rectangular cutout with 1/2" fillets
Group 4	Rectangular cutout with 1/4" fillets
Group 5	Rectangular cutout

Each of the five groups of photographs as listed above contains pictures of the seven different loading conditions. These loading conditions are defined by the ratio of bending load to shear load (M/S). Each picture has the M/S ratio marked in the lower right hand corner.

GROUP 1

No Cutouts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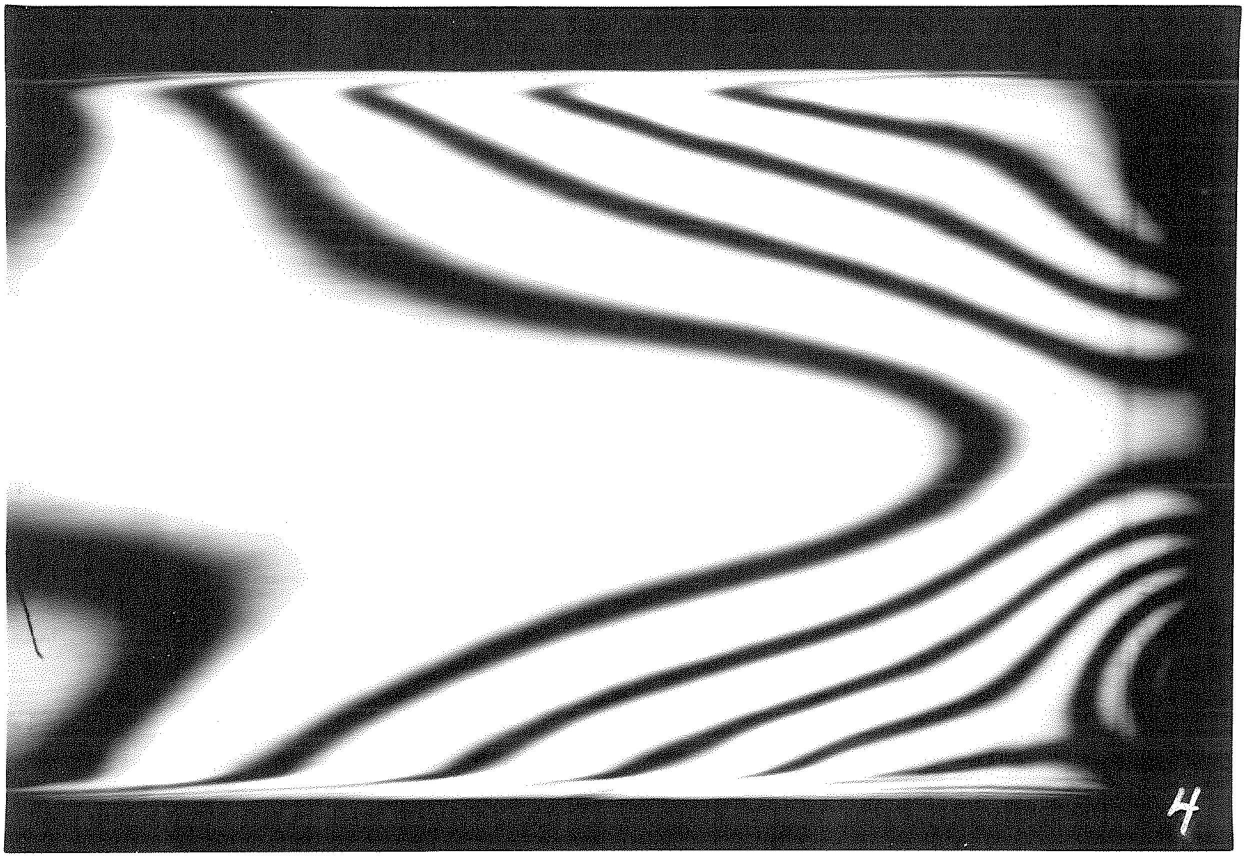
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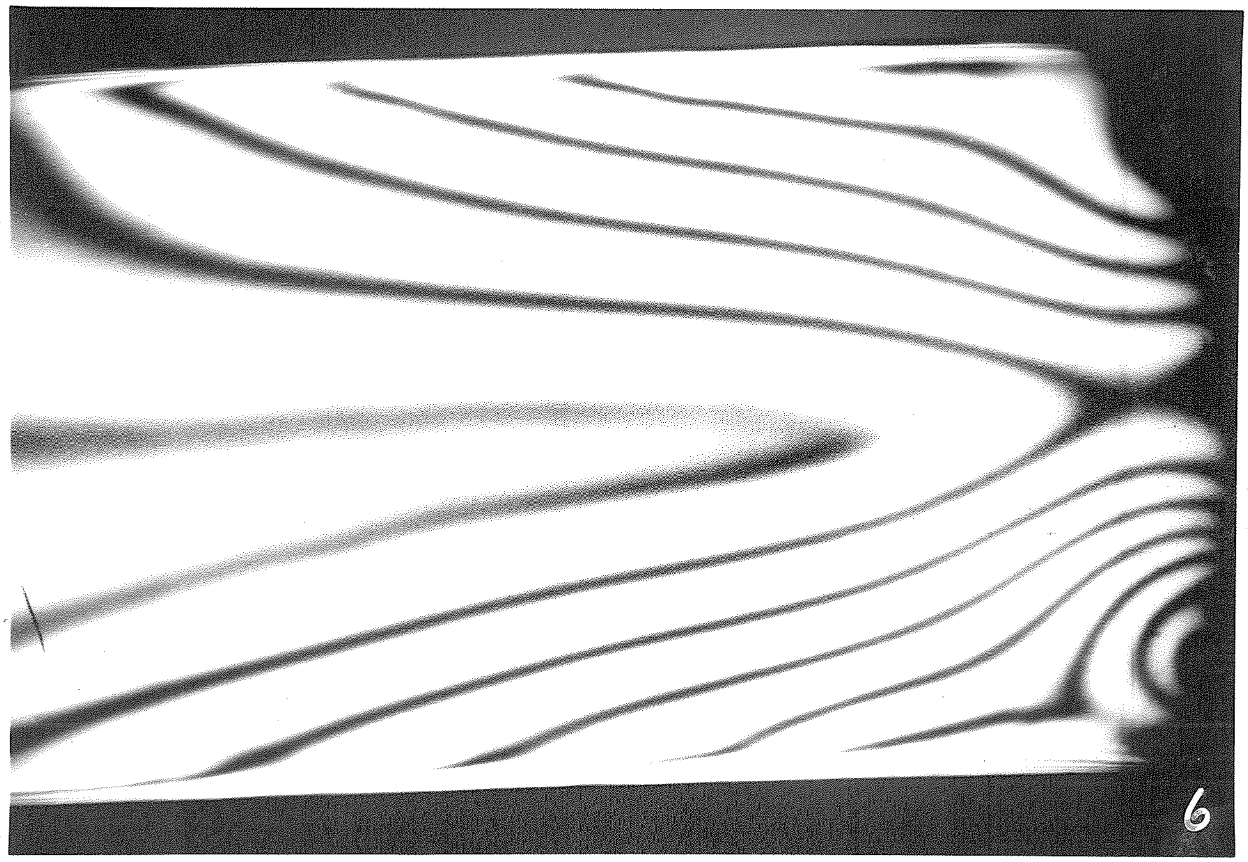
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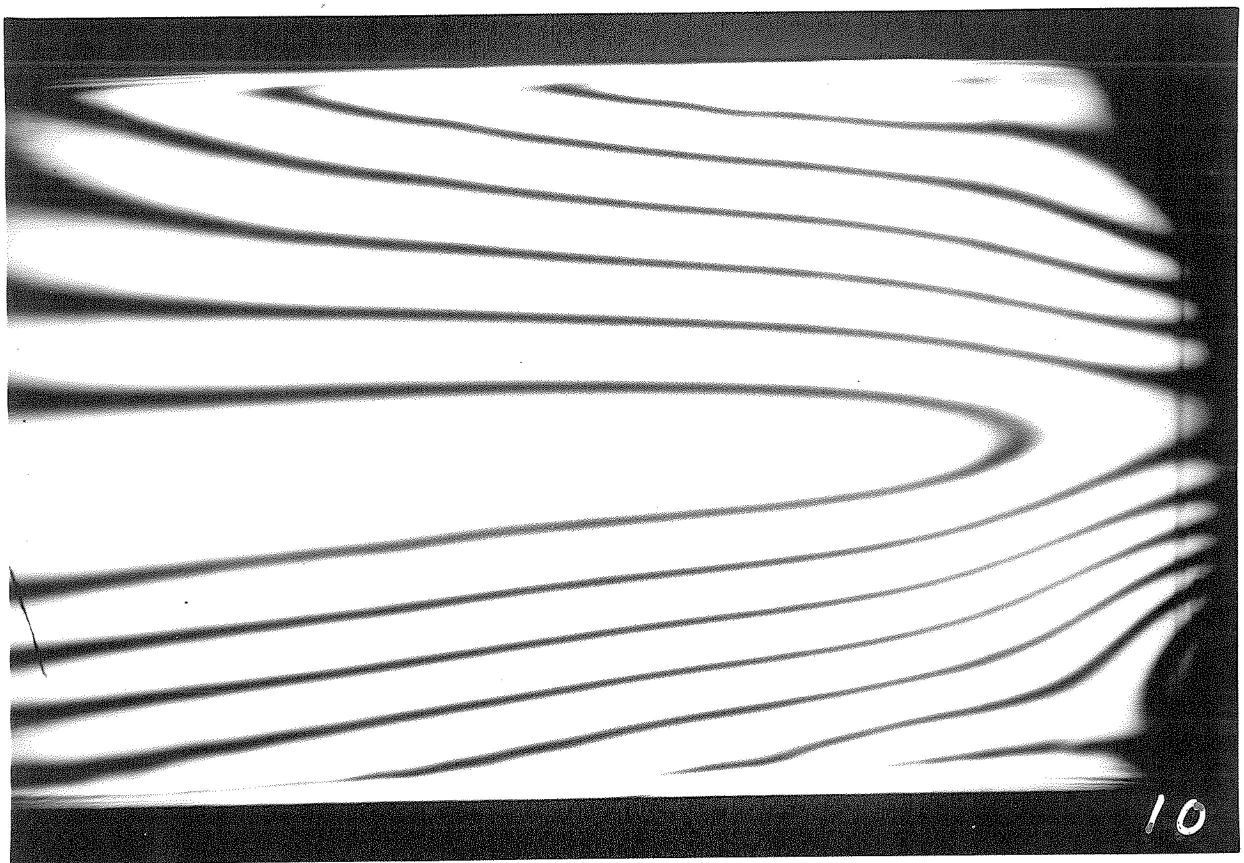
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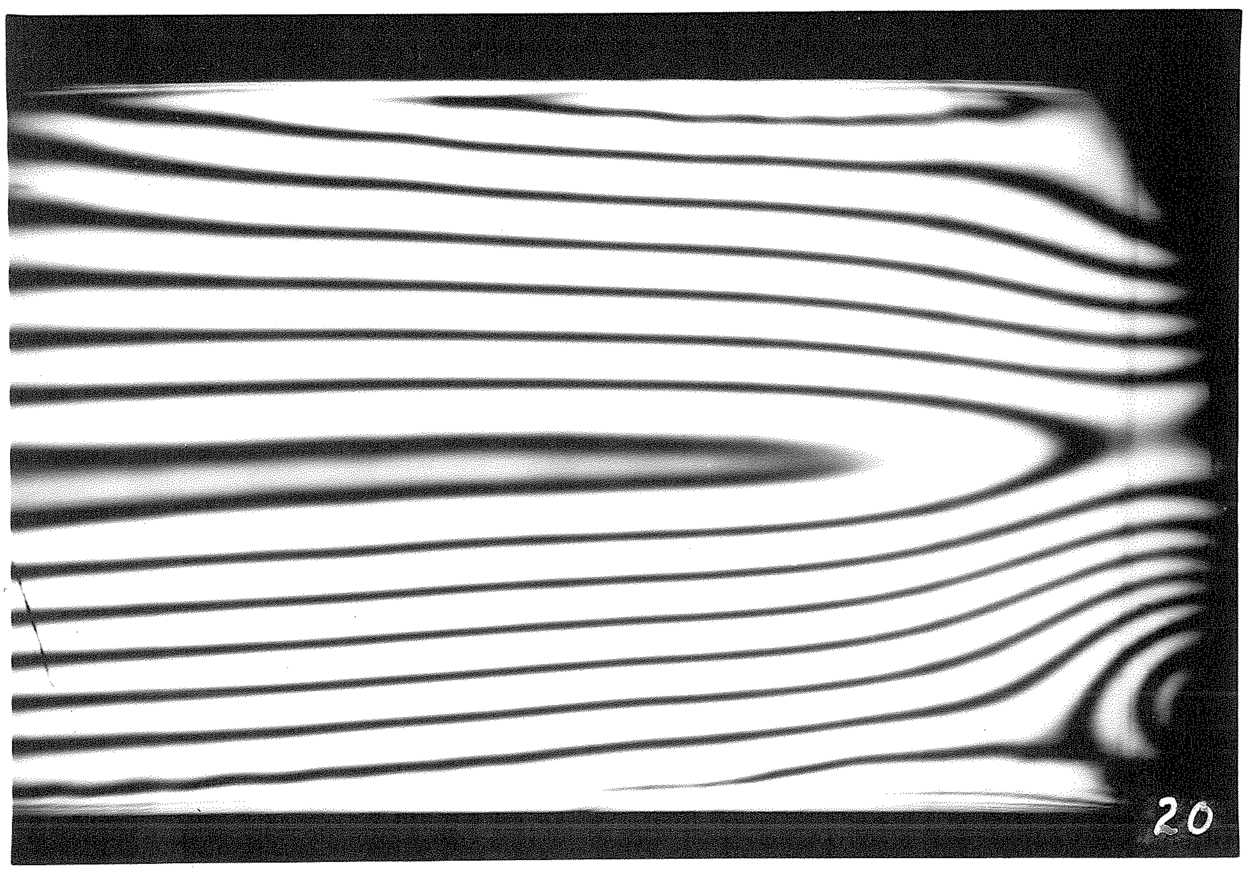
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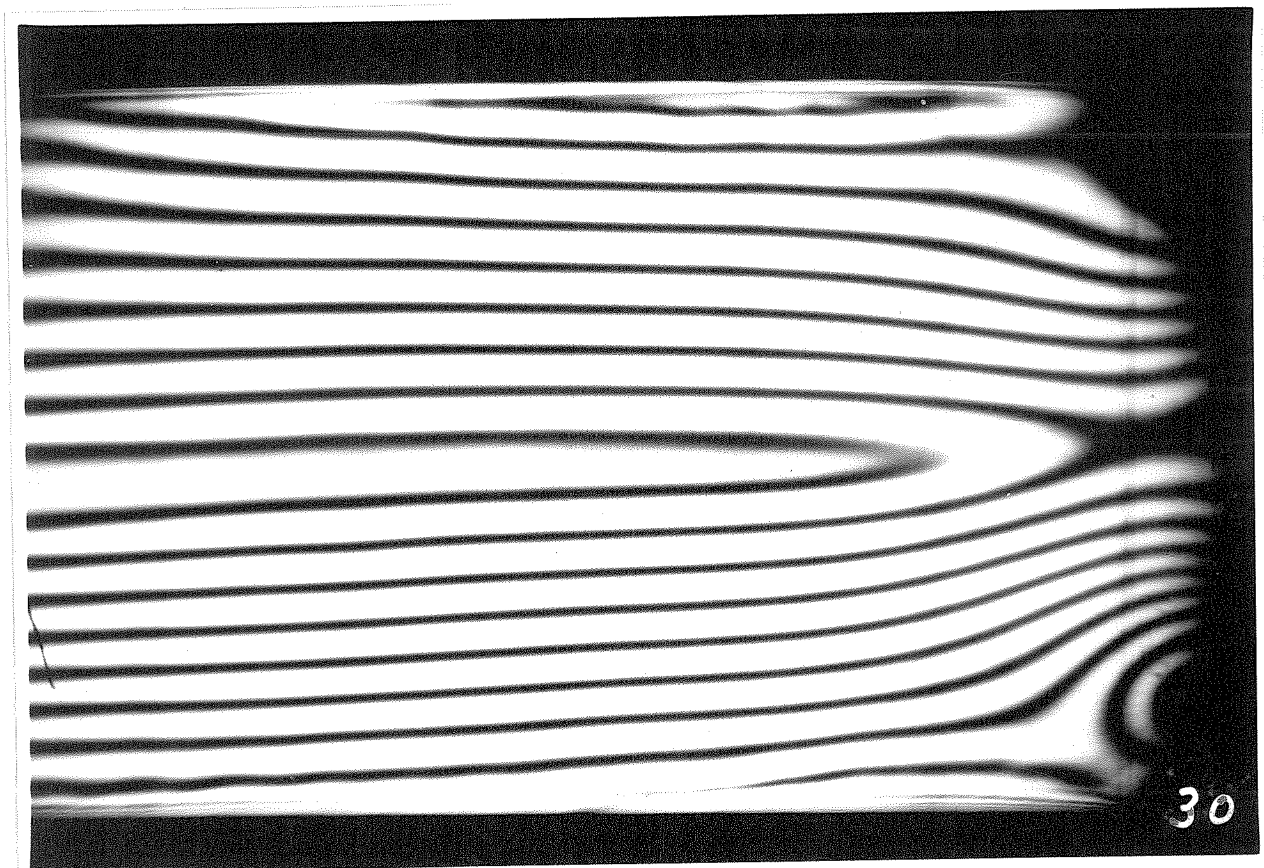
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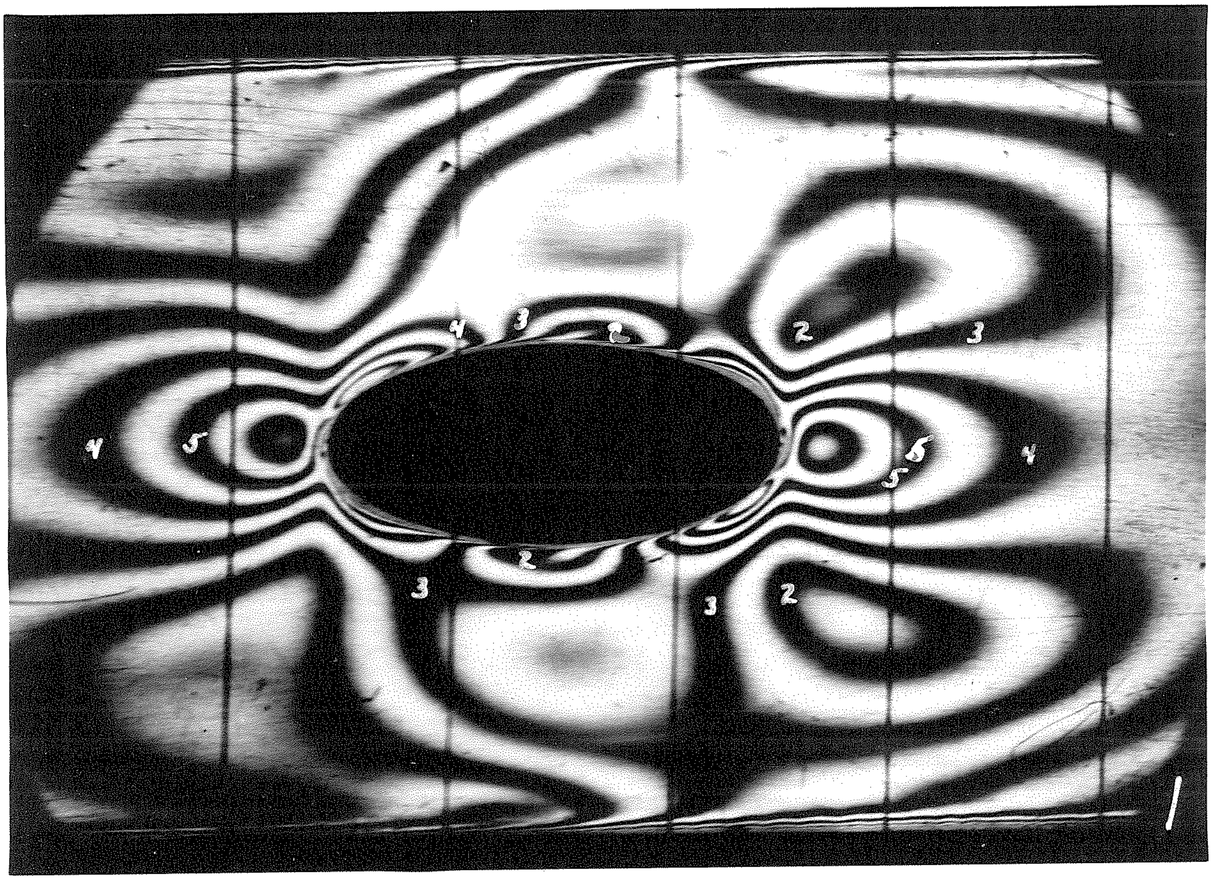
Elliptical Cutouts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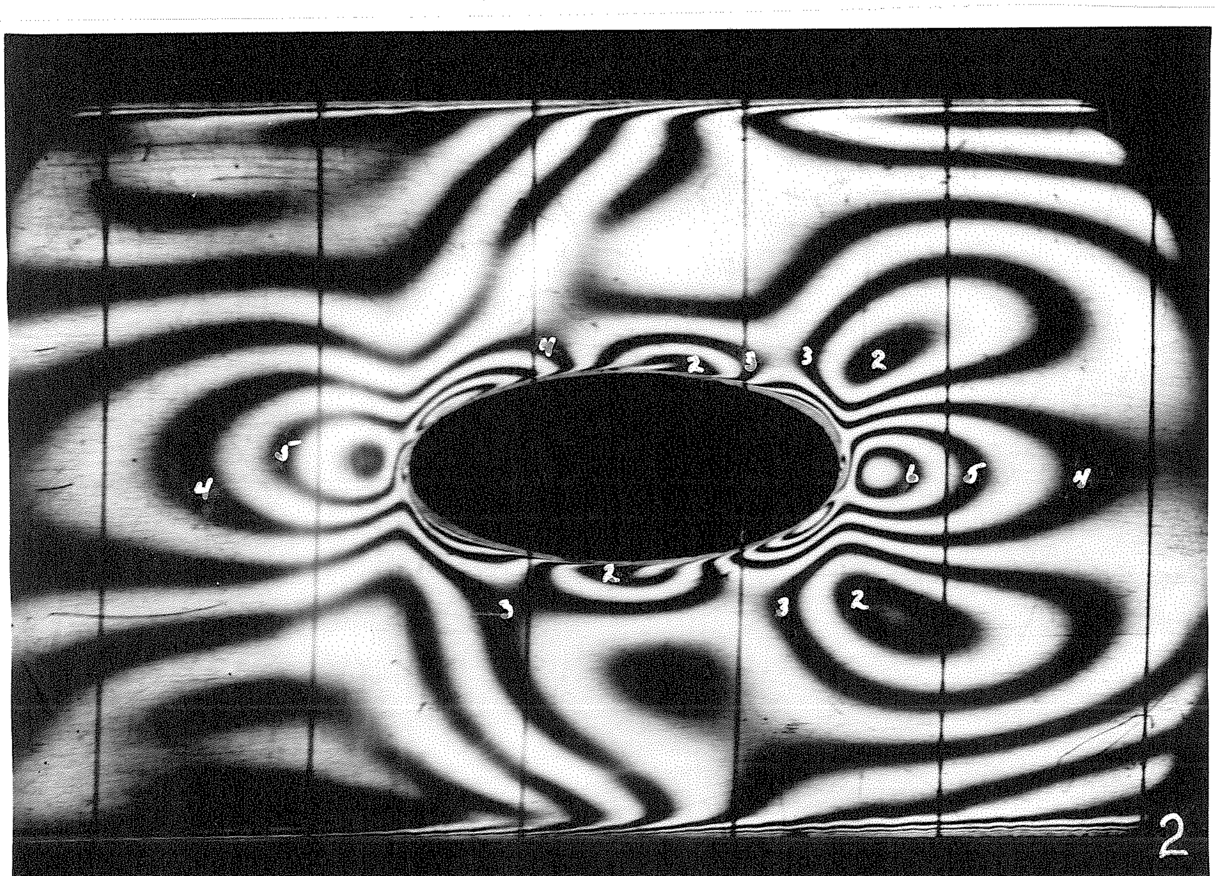
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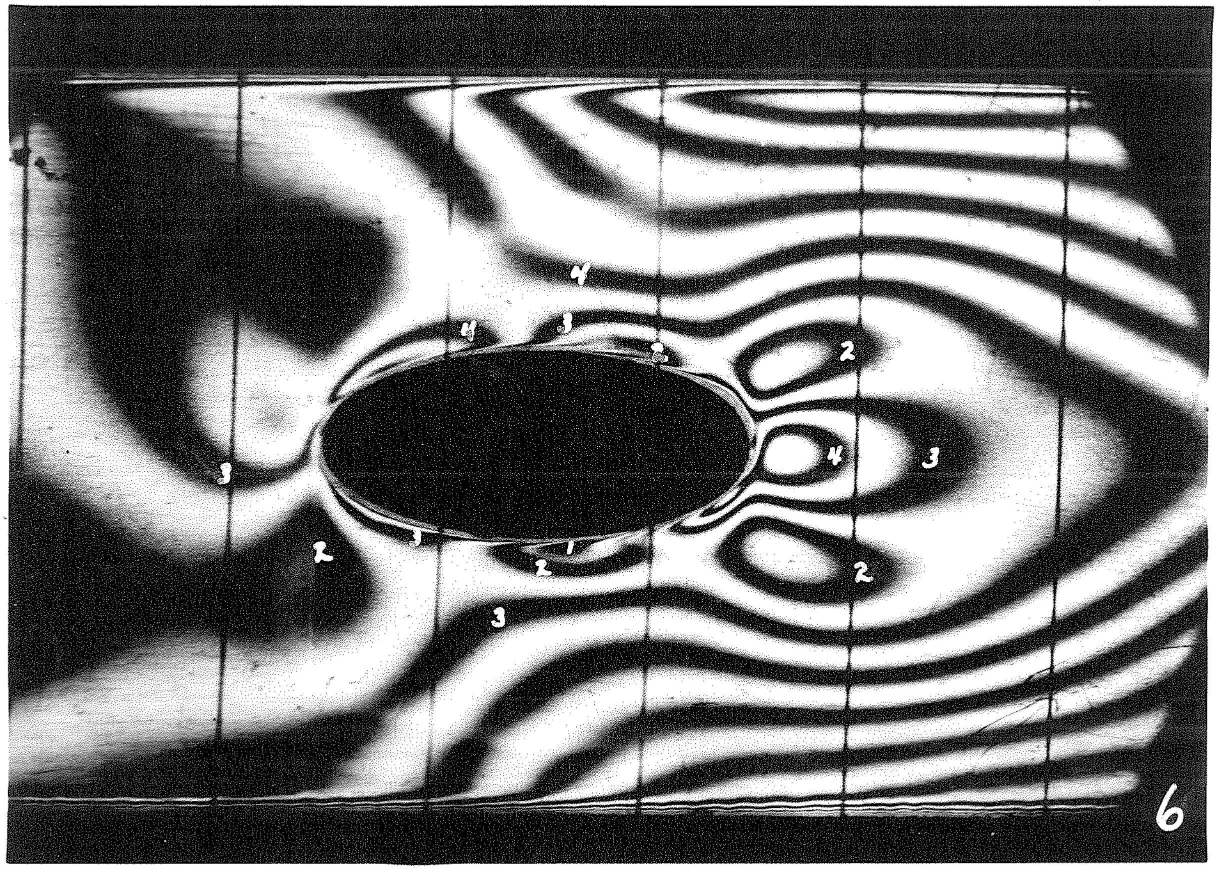
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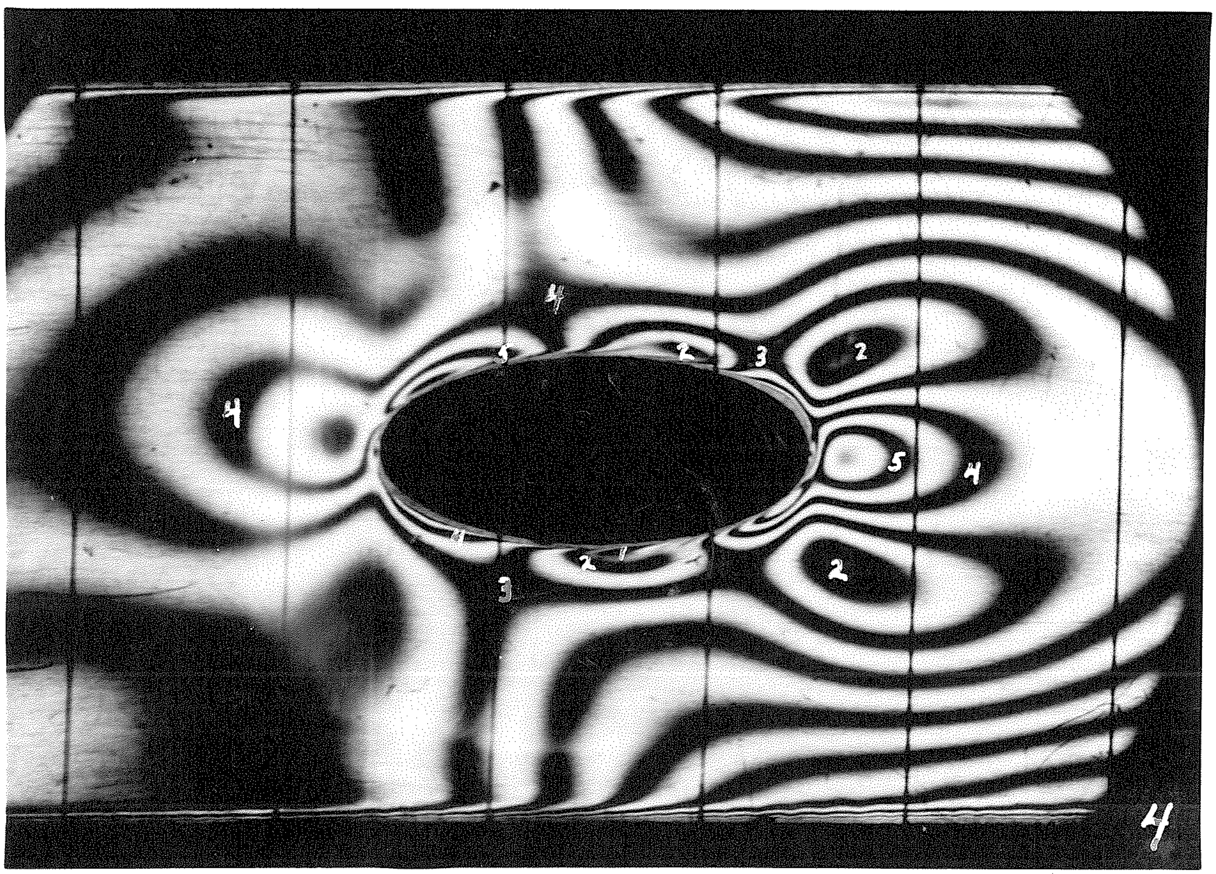
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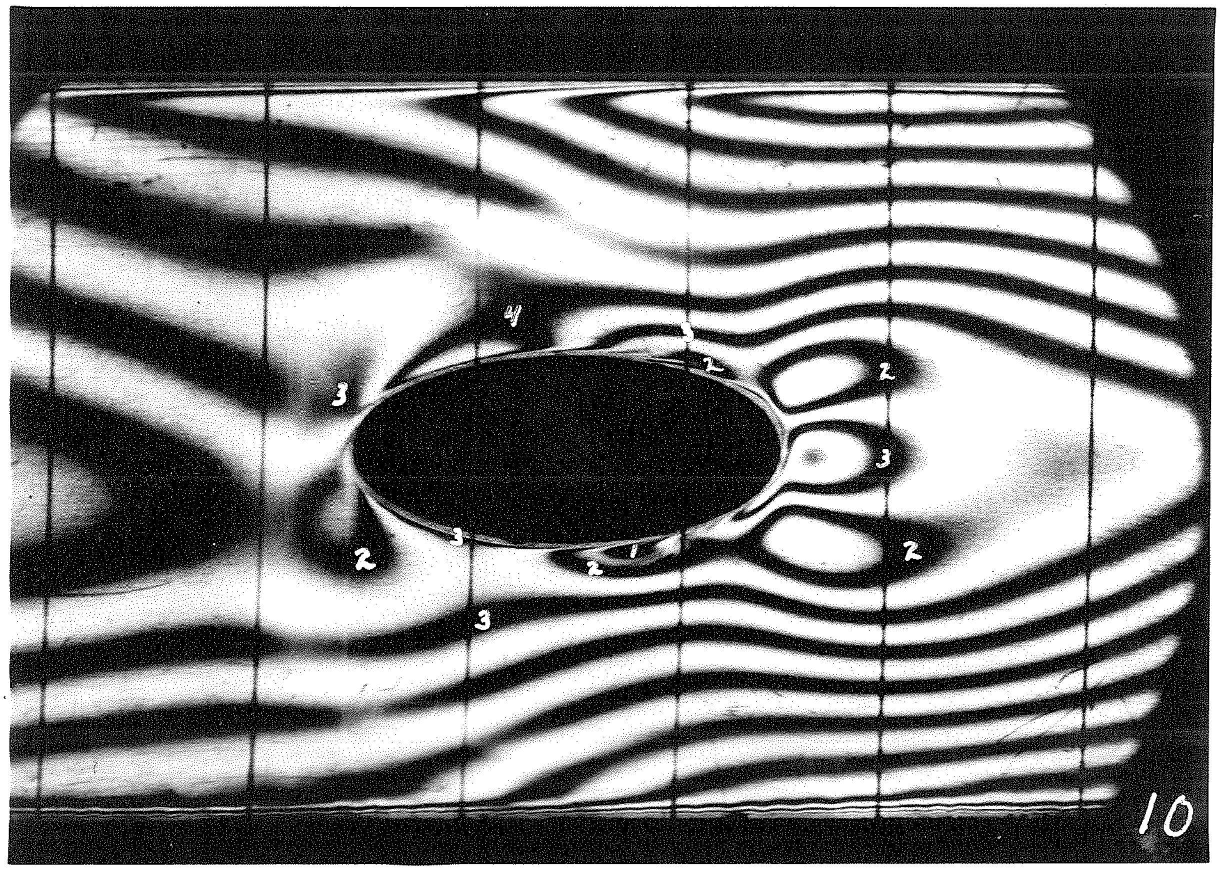
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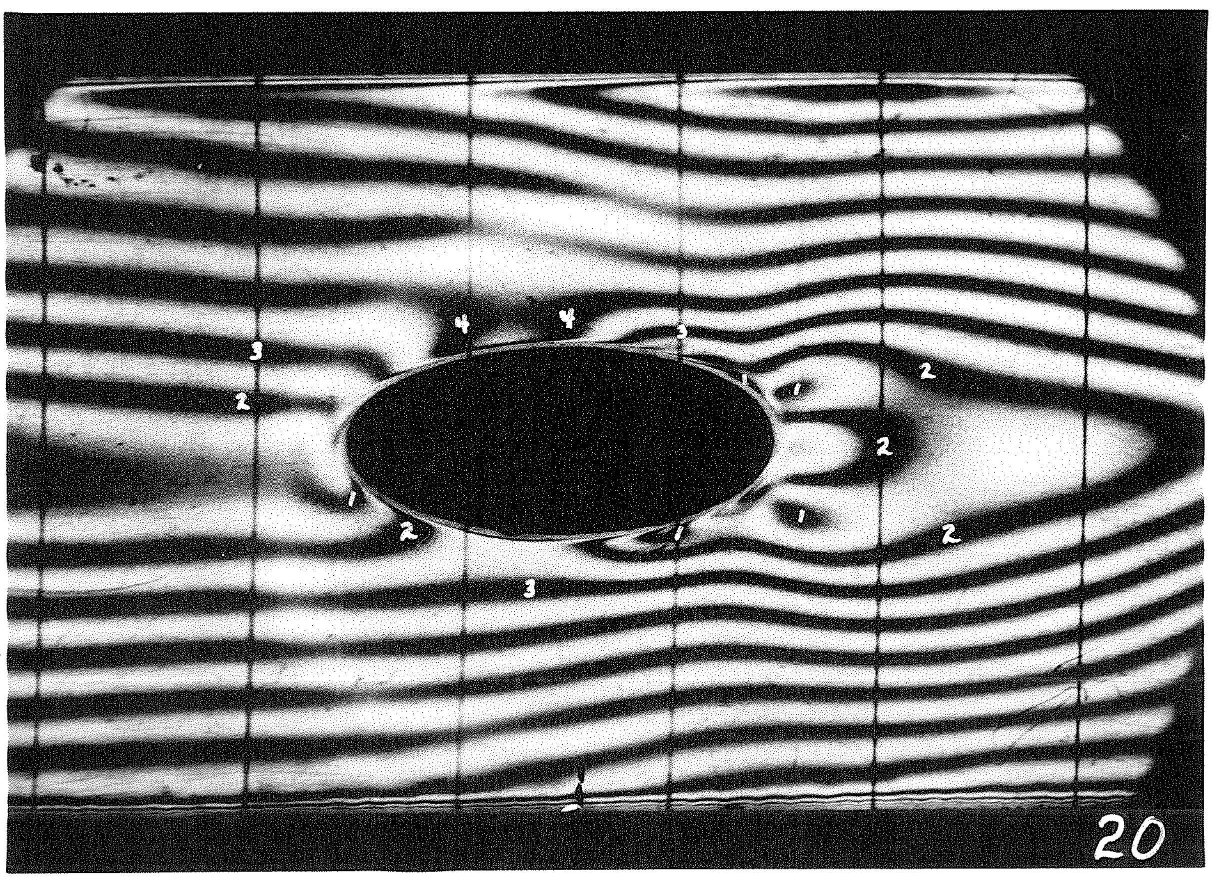
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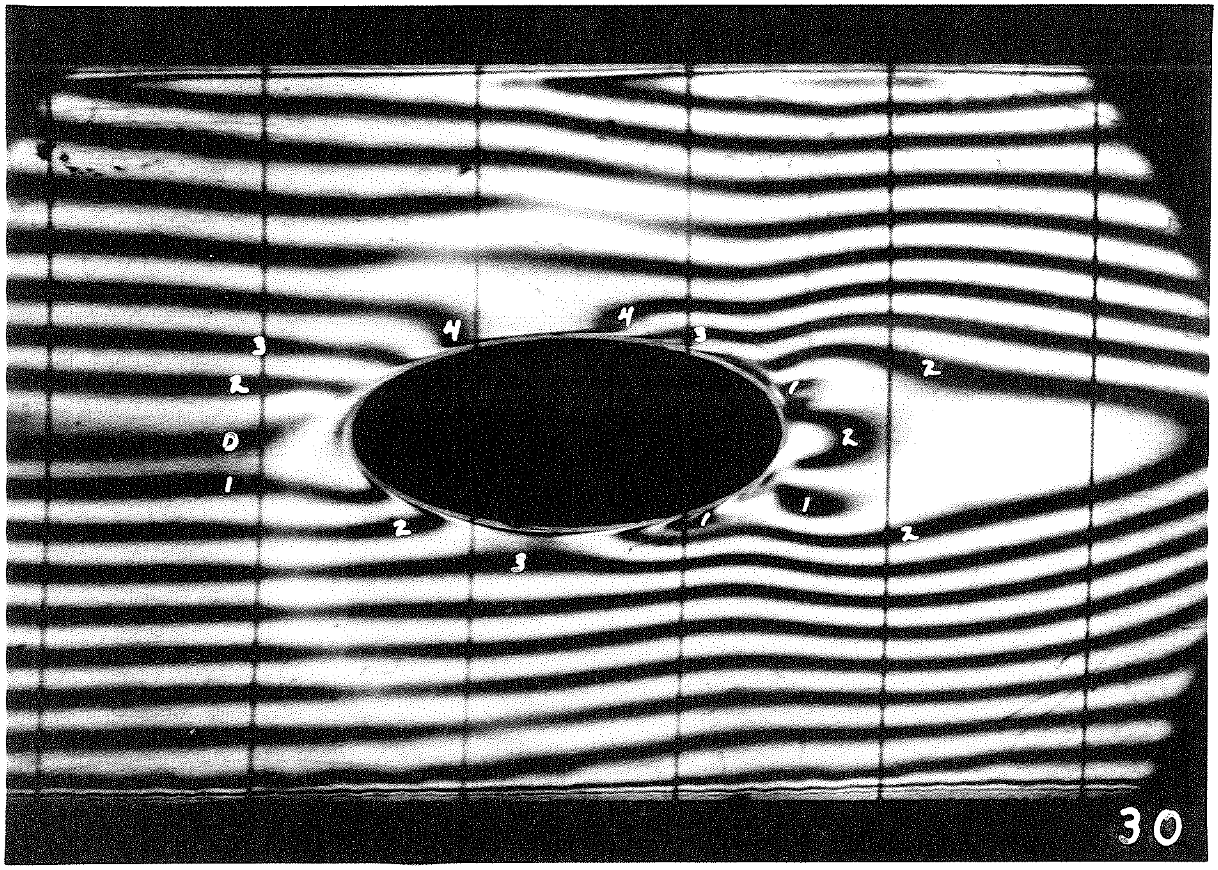
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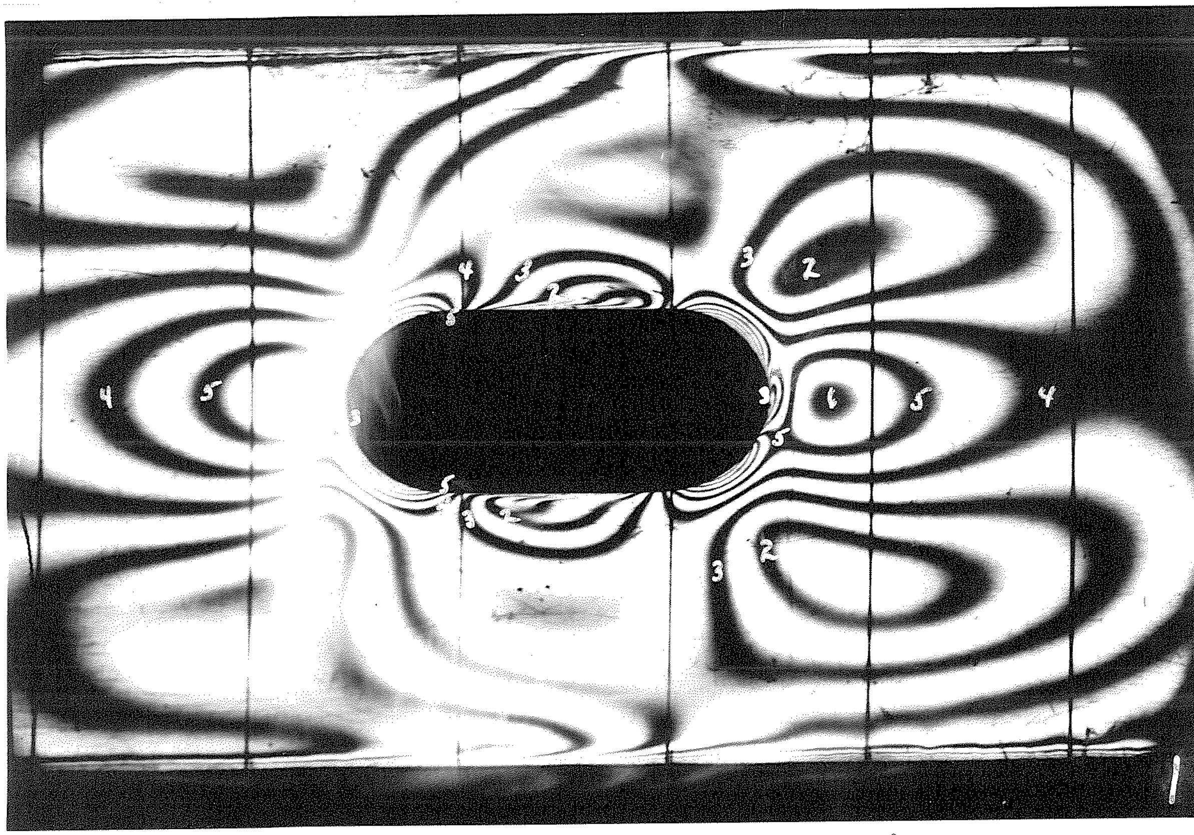
GROUP 3

Rectangular Cutout with 1/2" Fillets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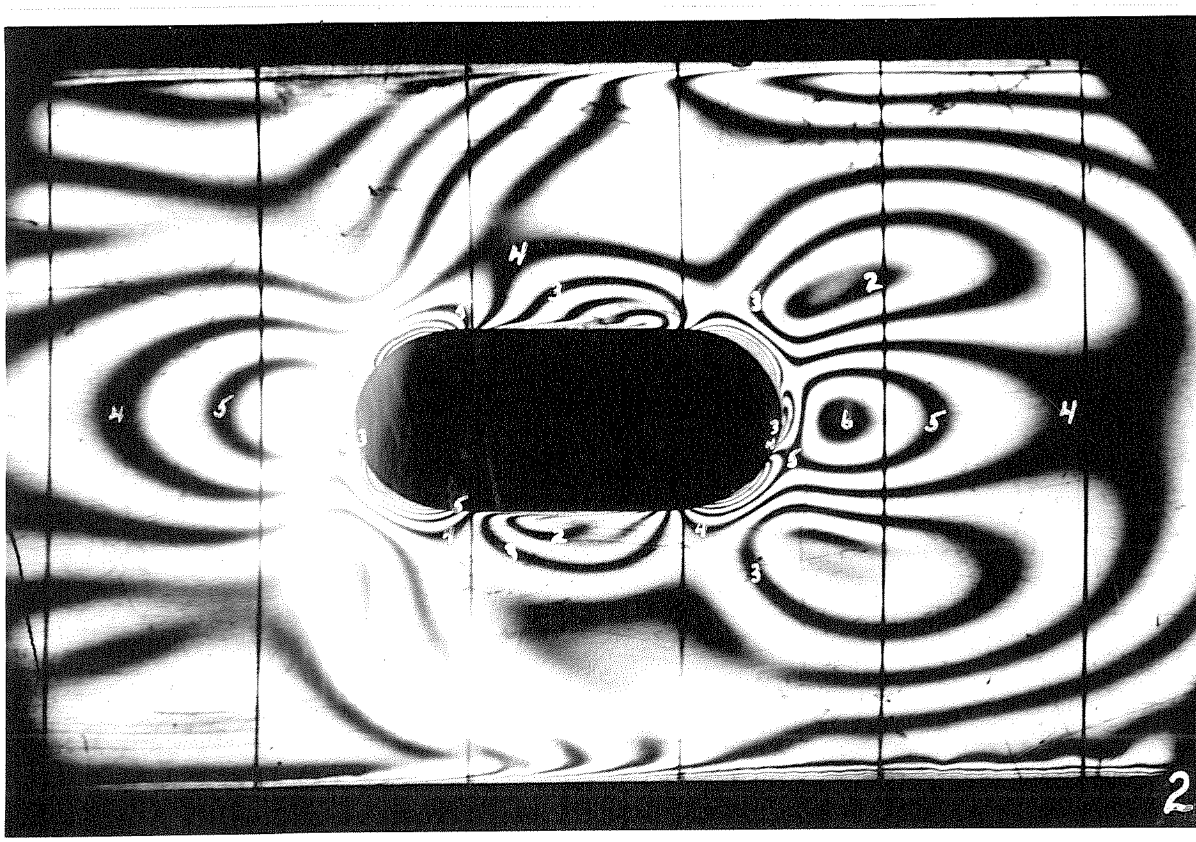
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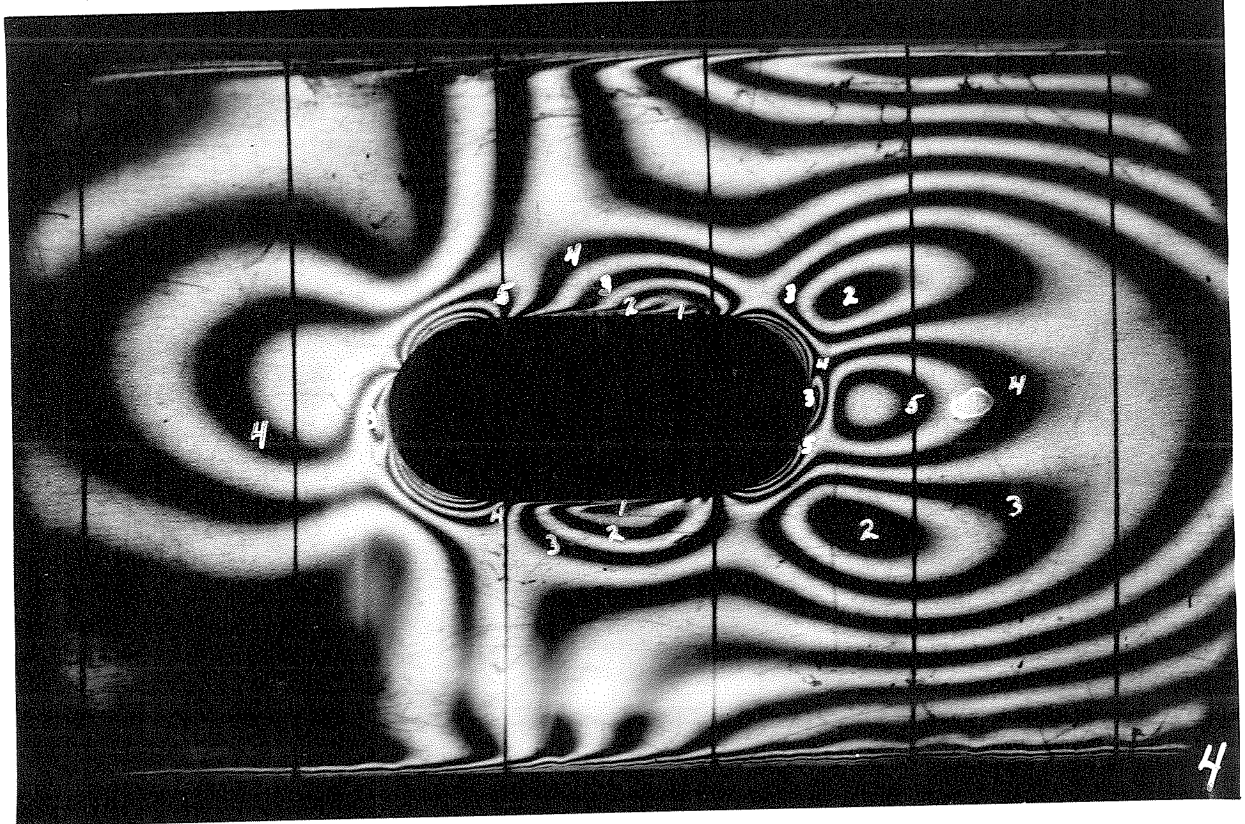
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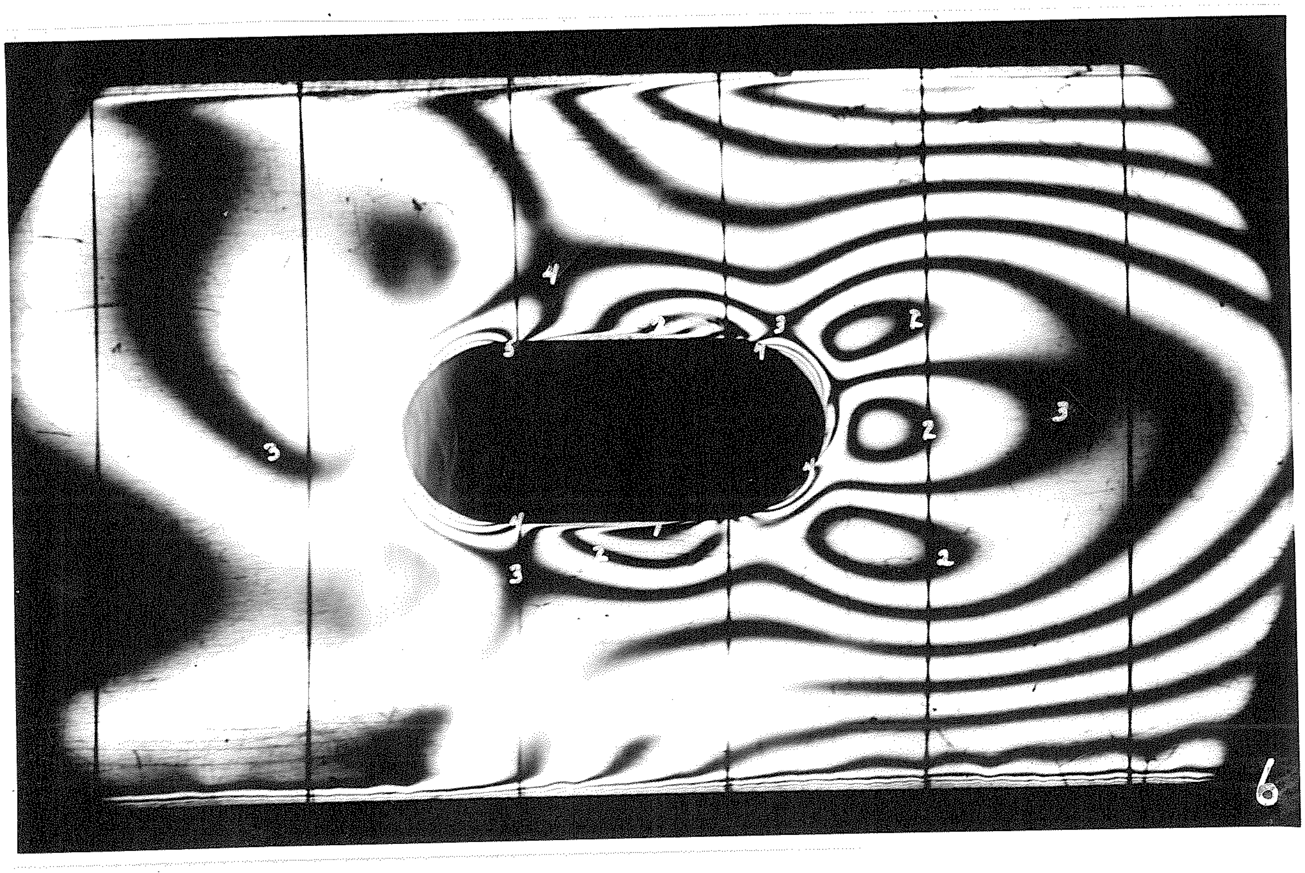
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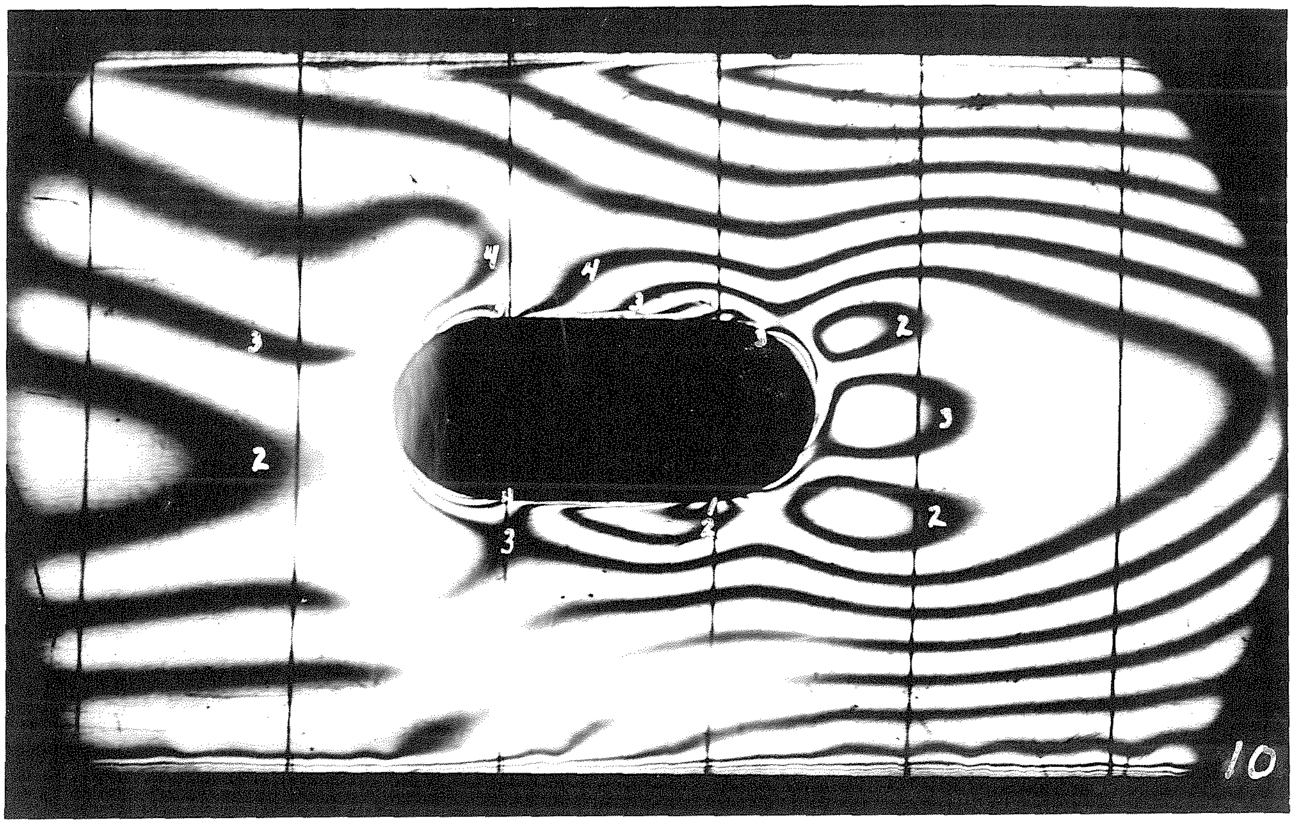

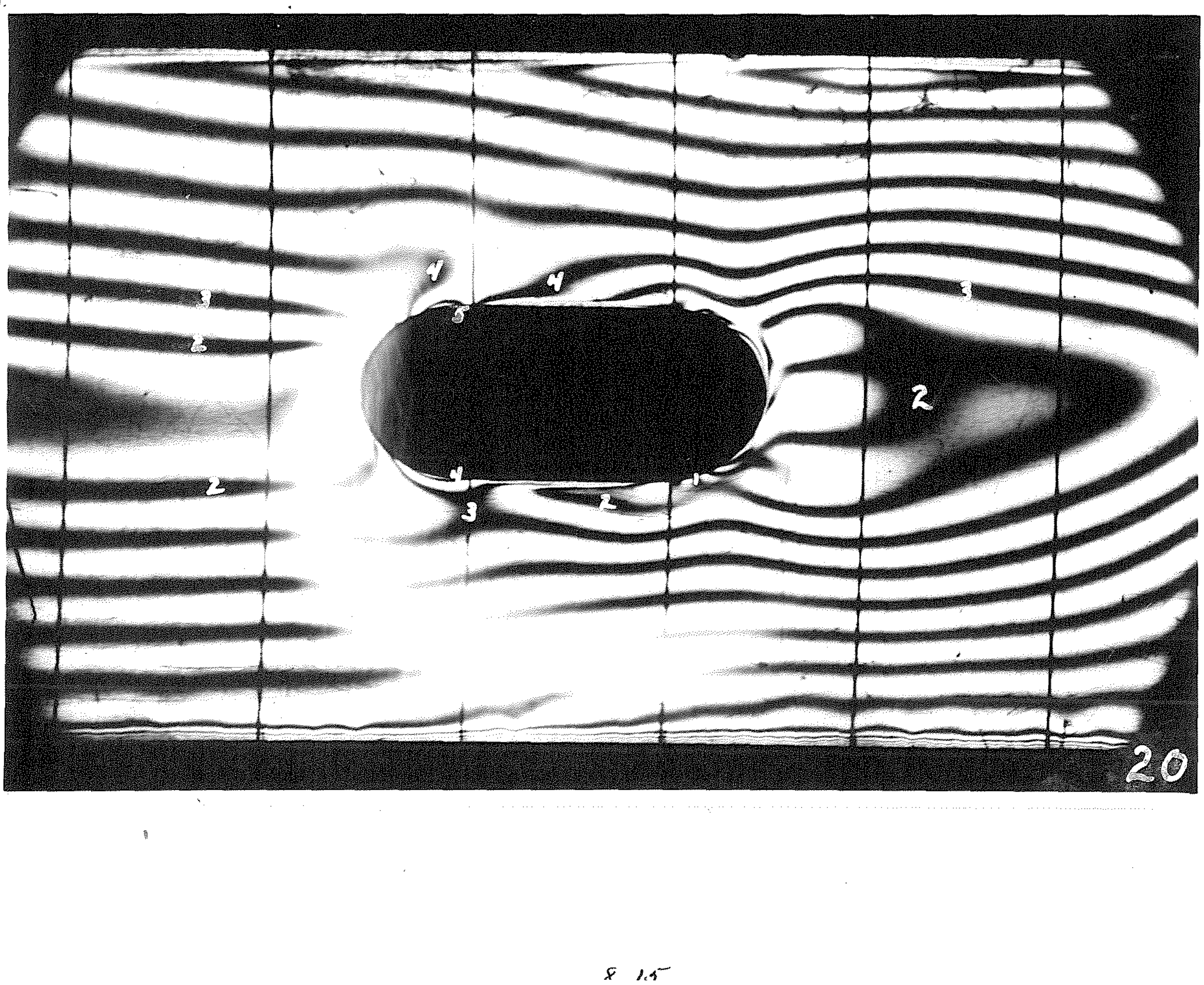

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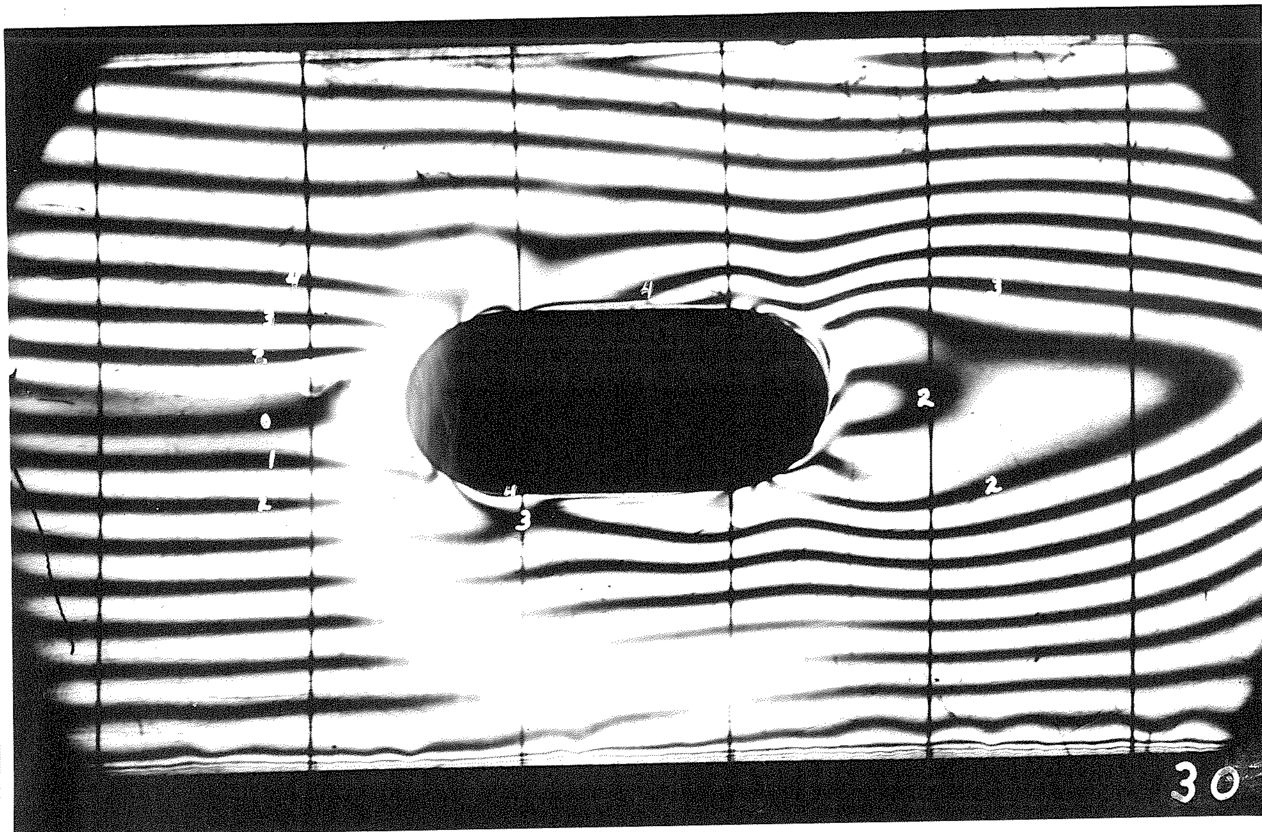
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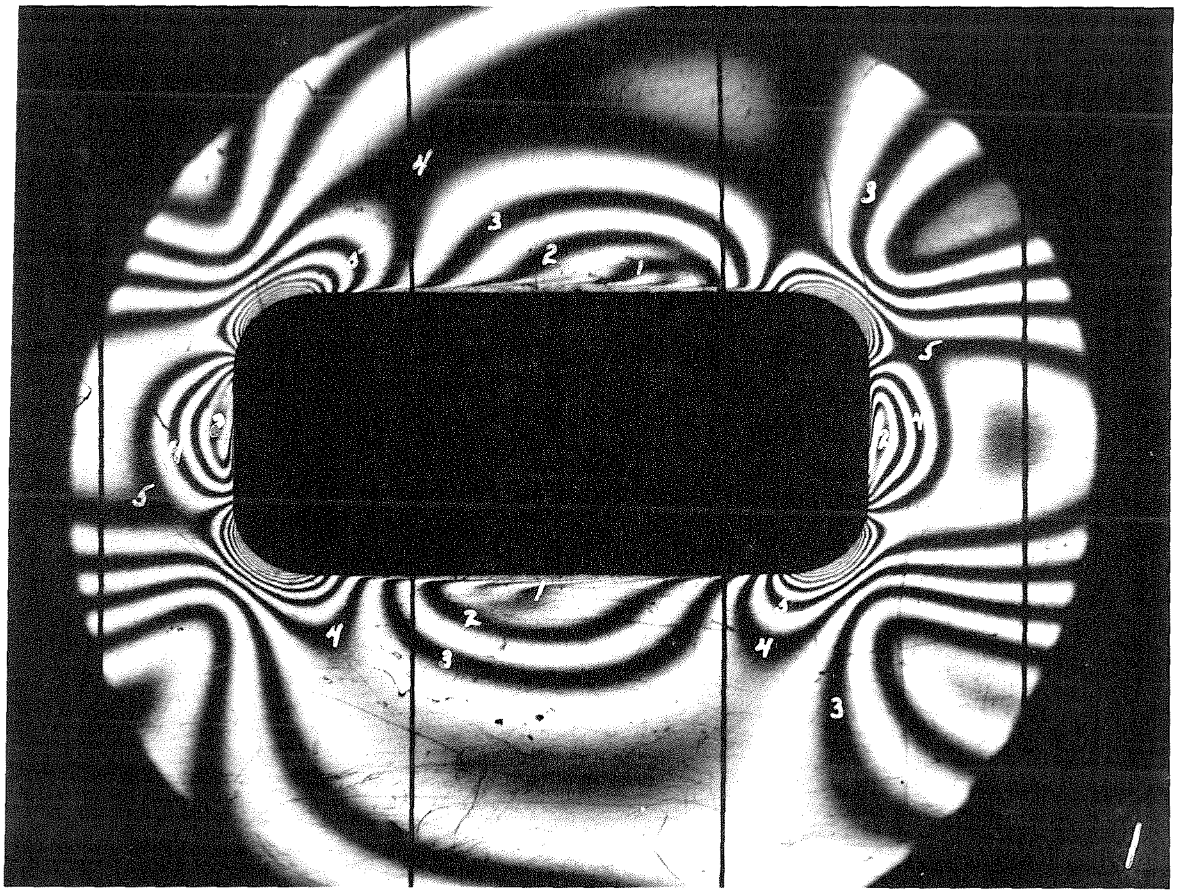
GROUP 4

Rectangular Cutout with 1/4" Fillets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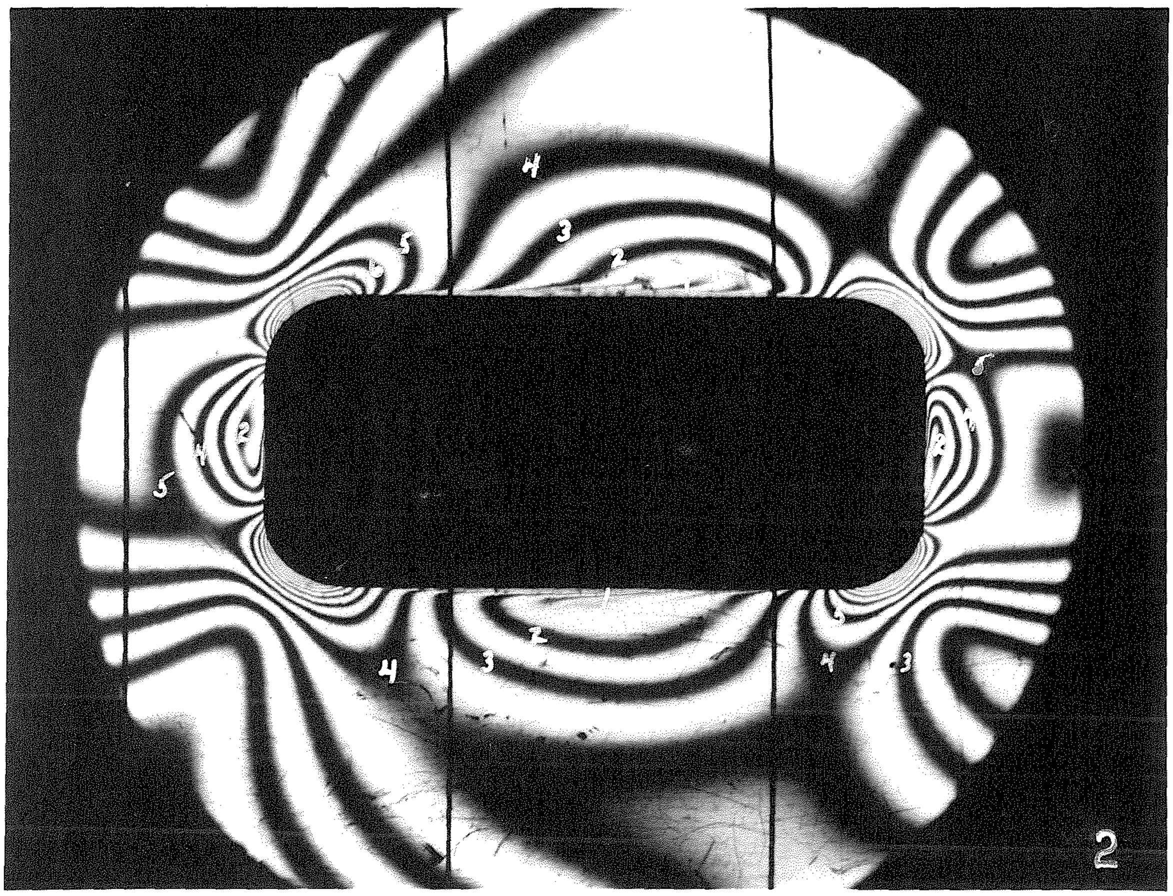
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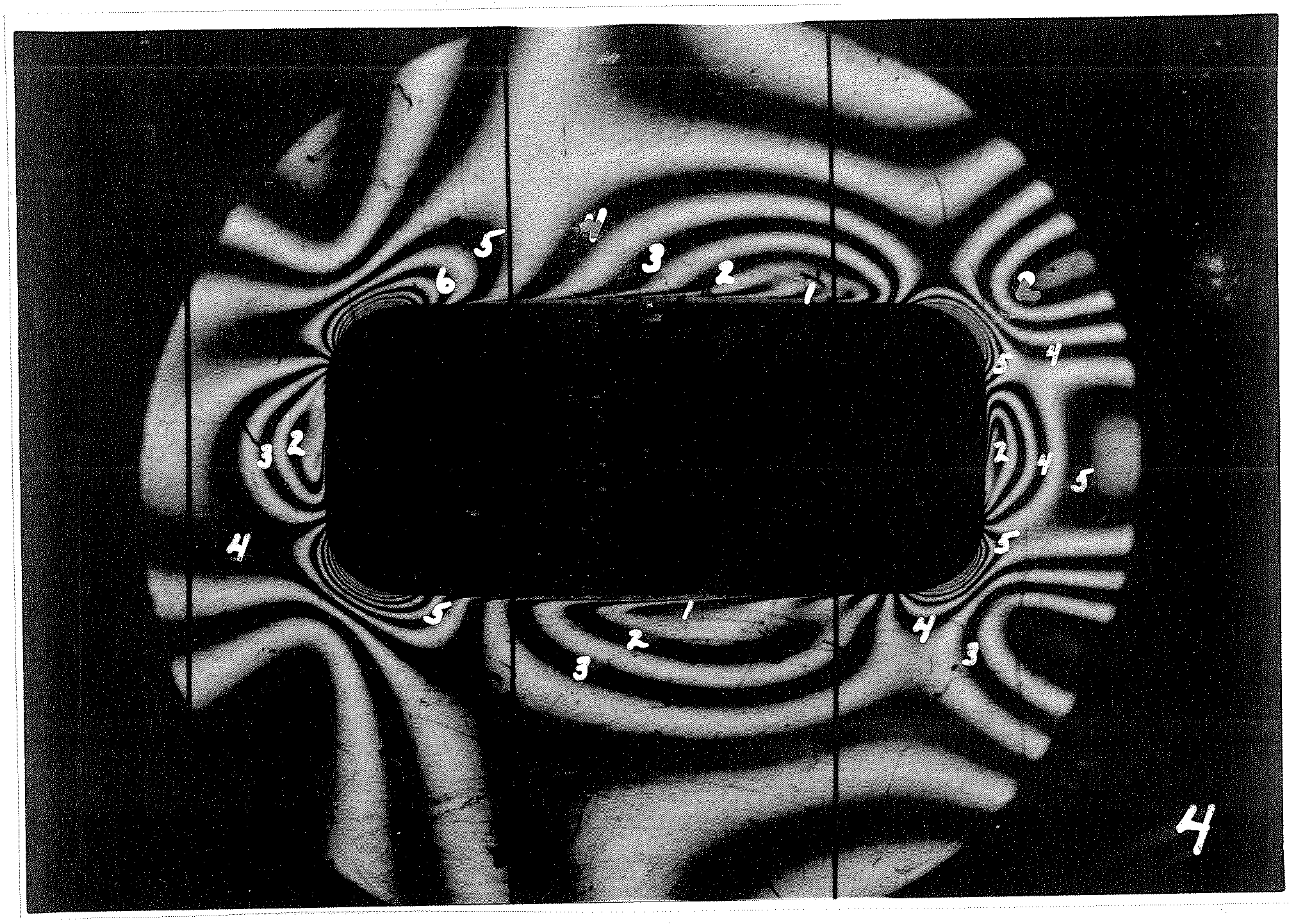
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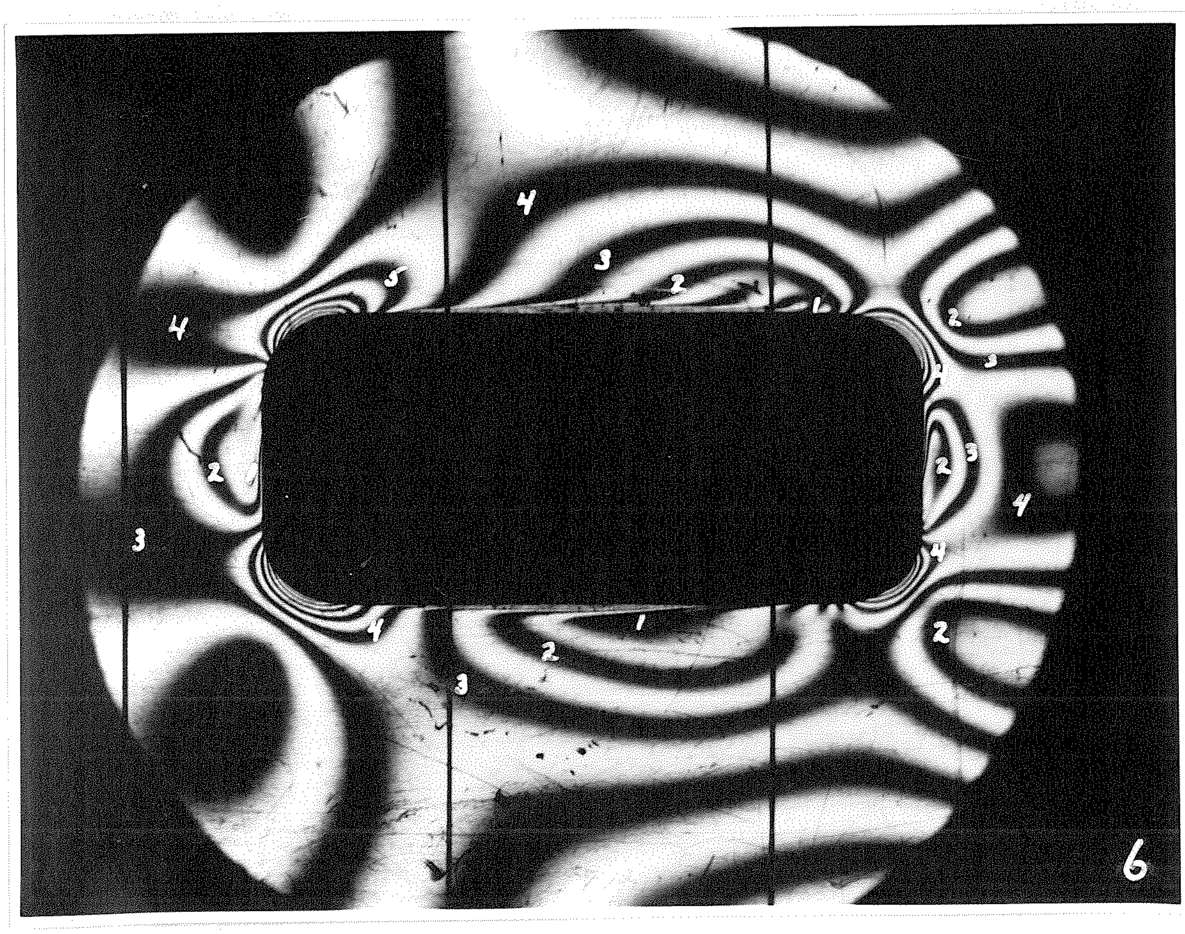
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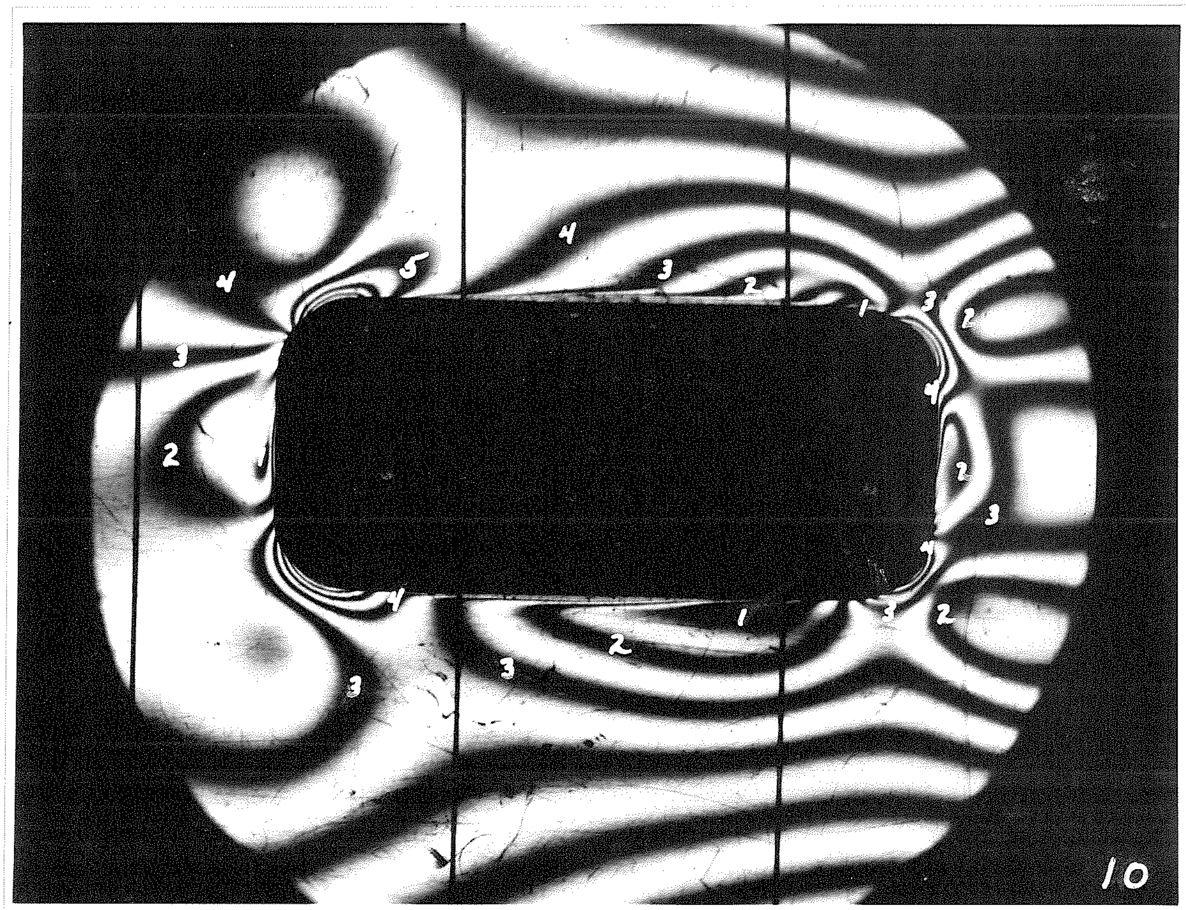
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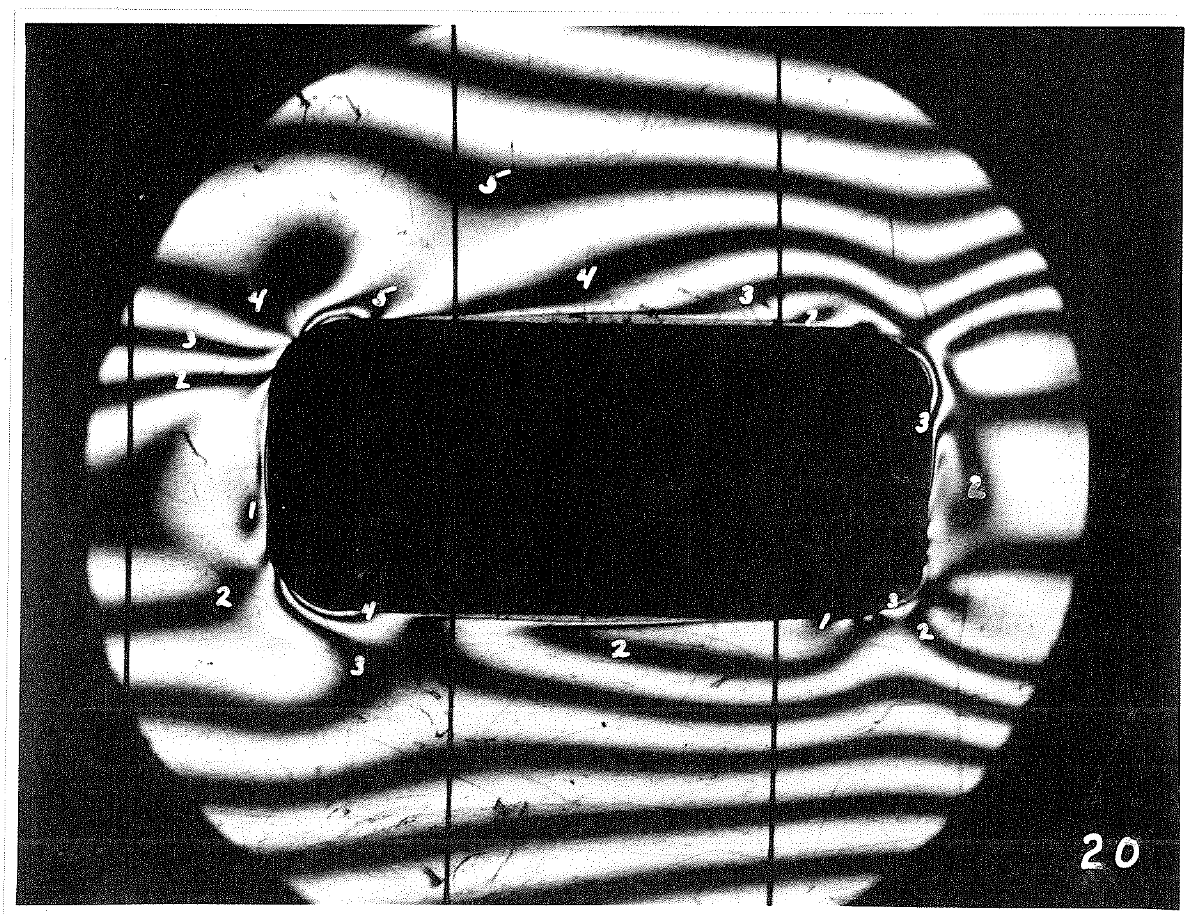
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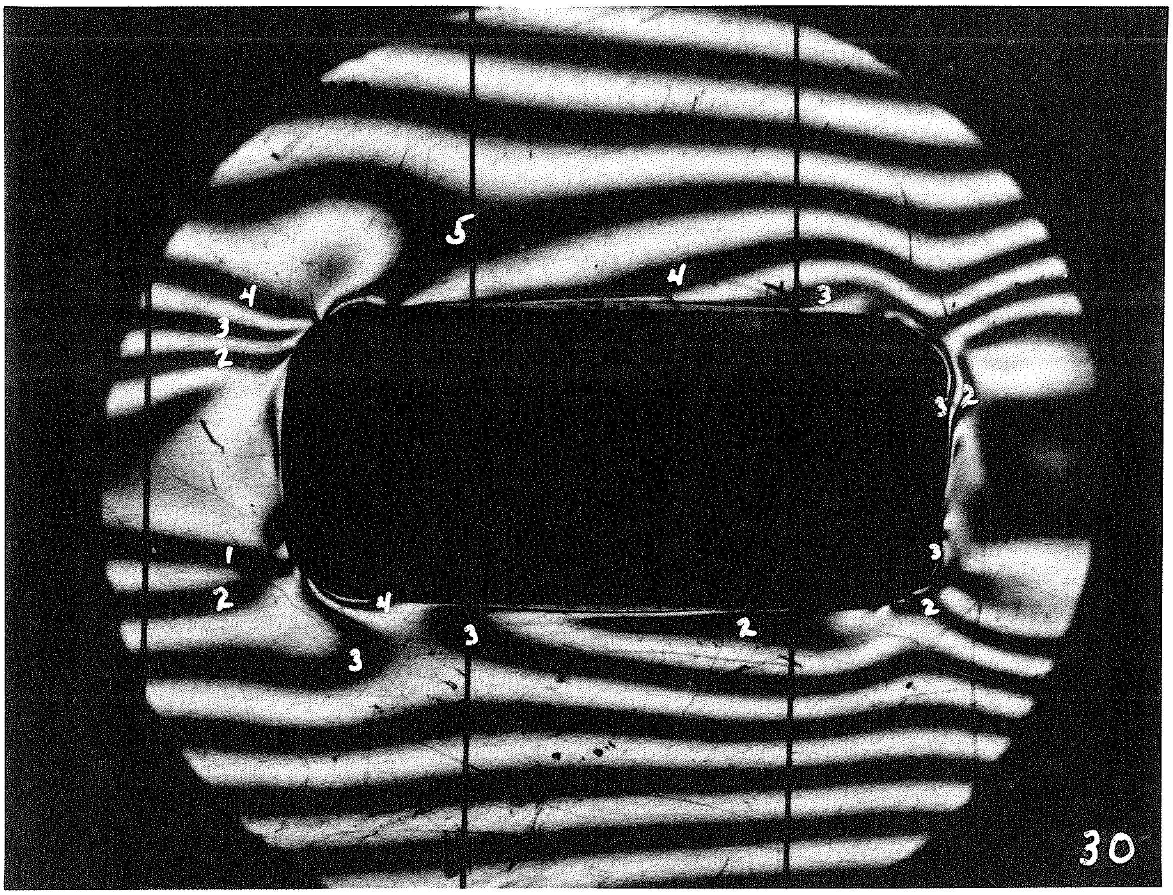
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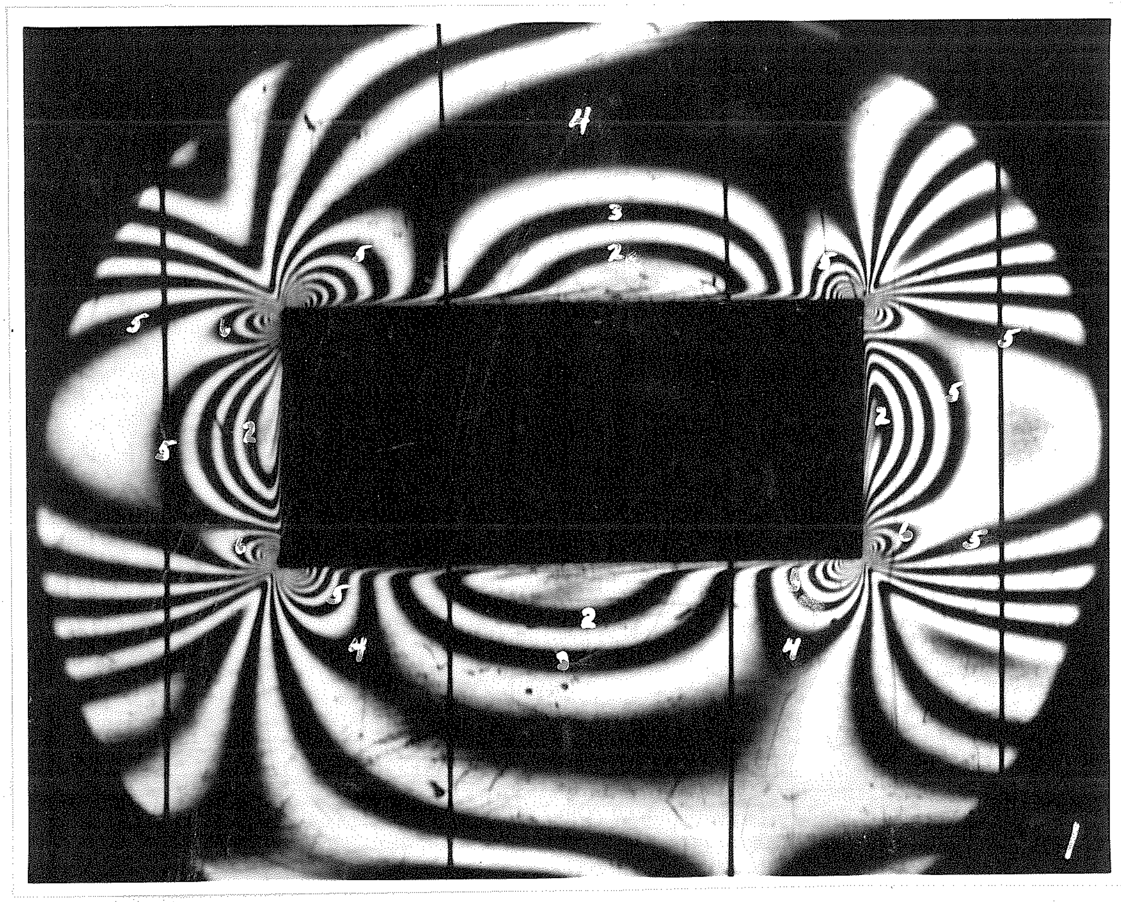
GROUP 5

Rectangular Cutout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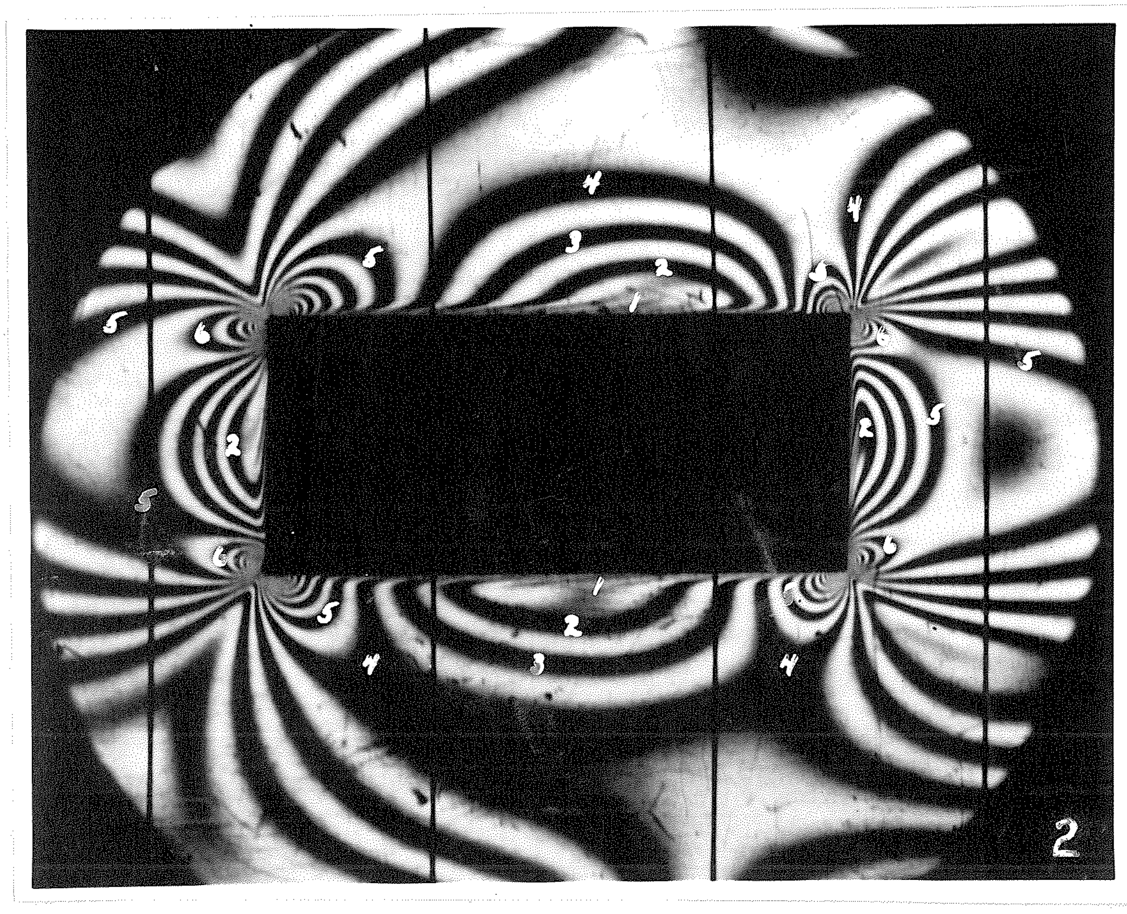
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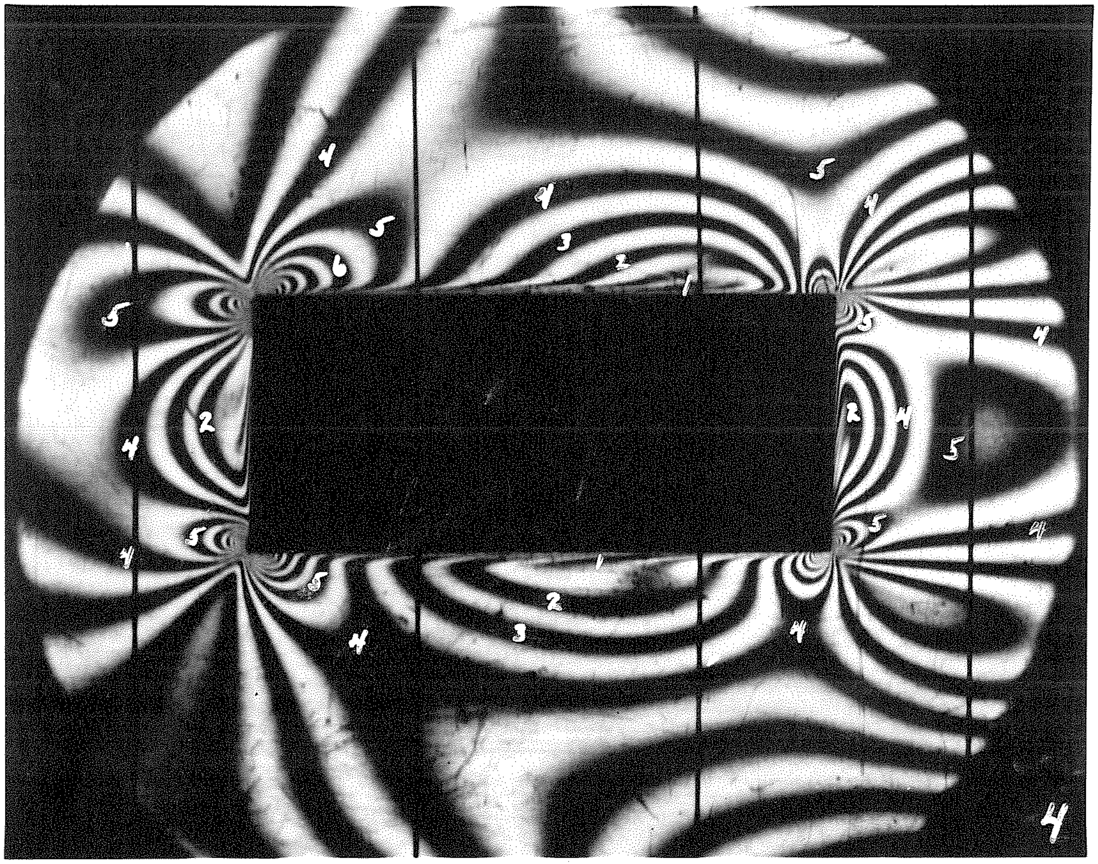
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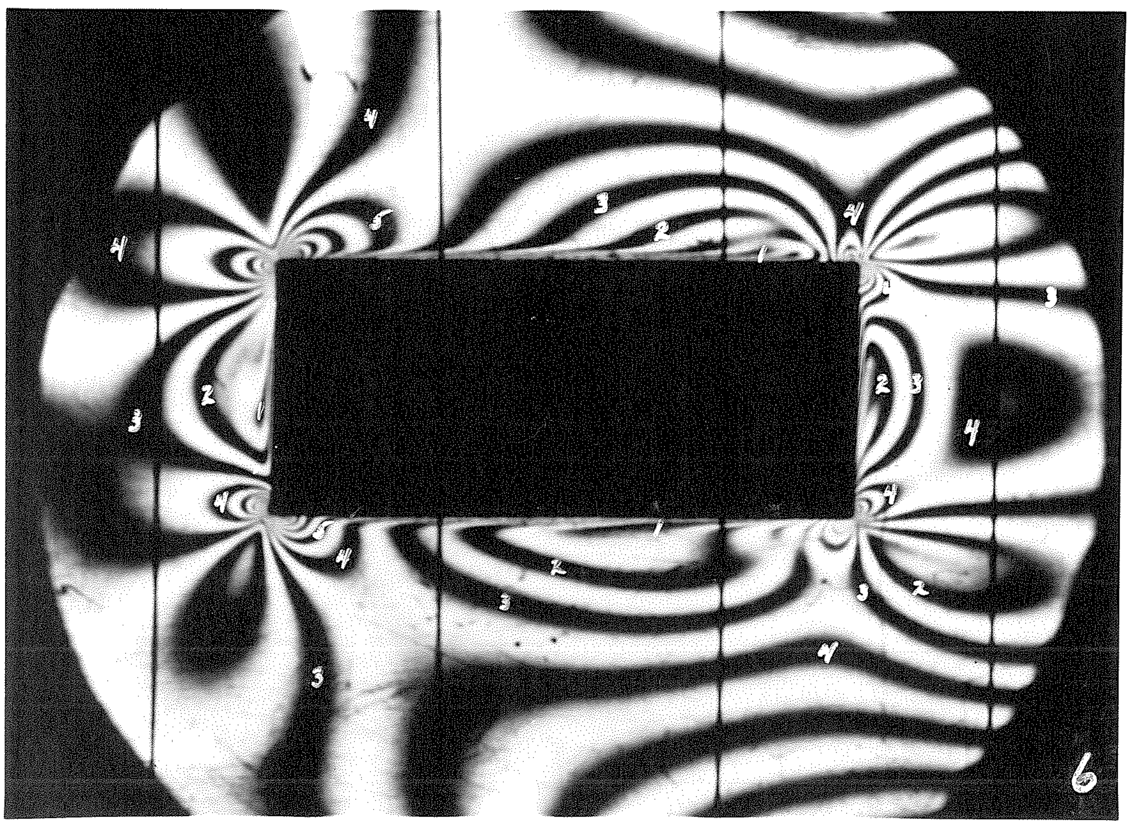
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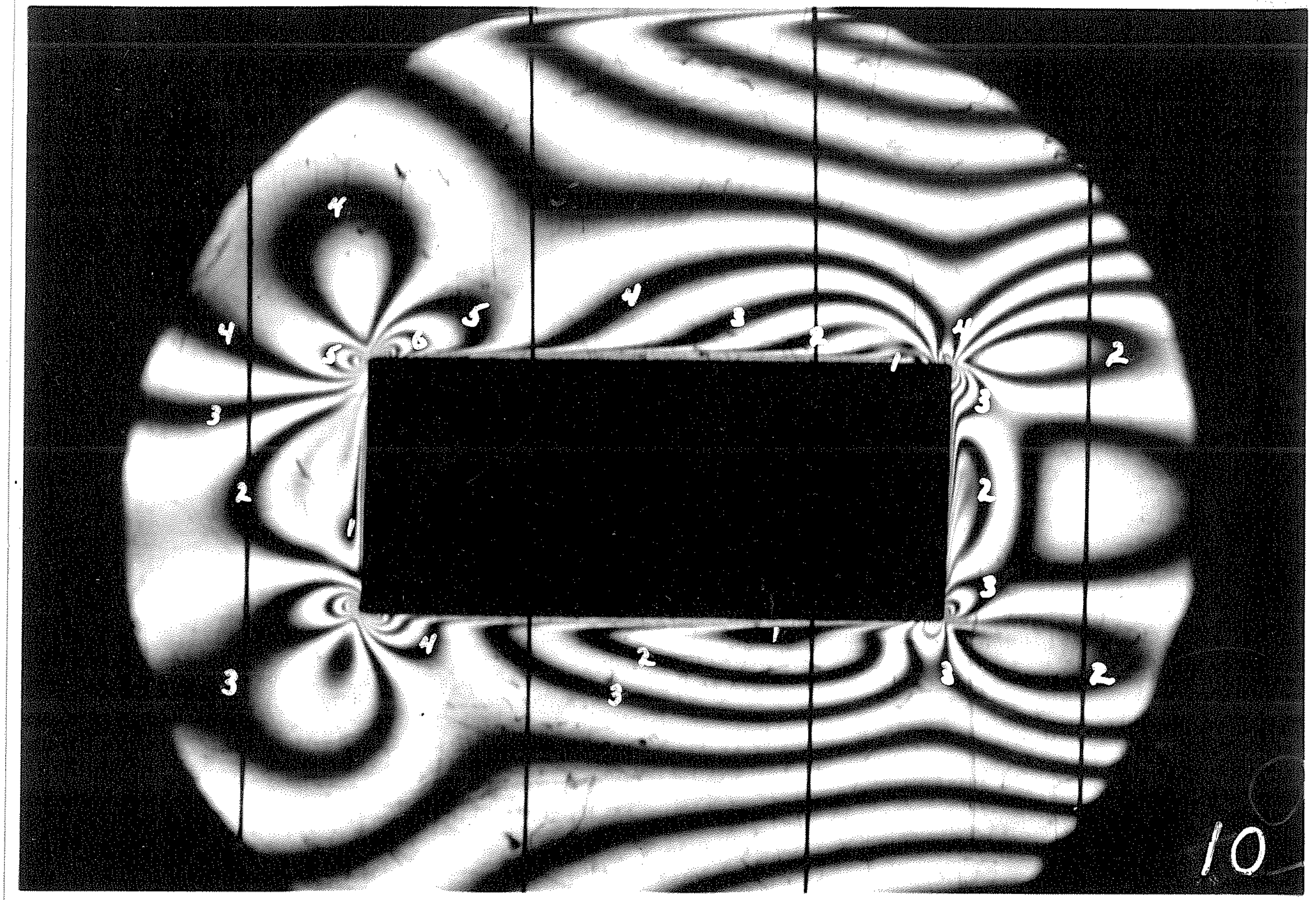

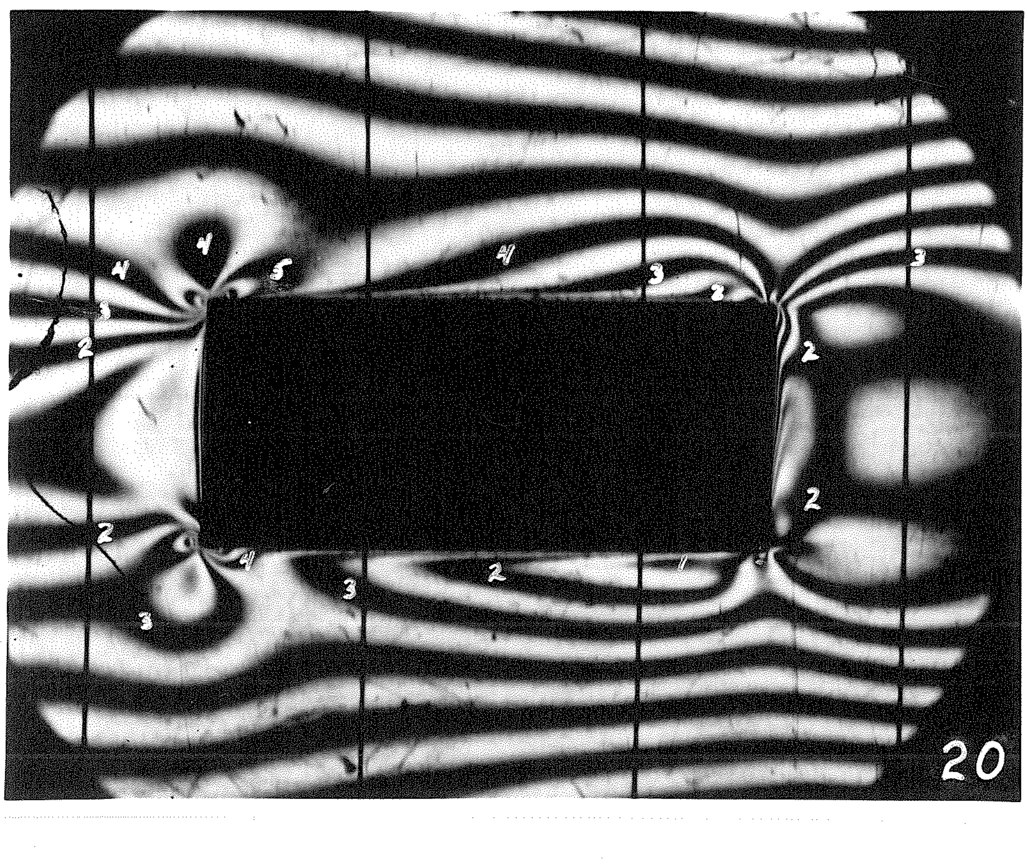

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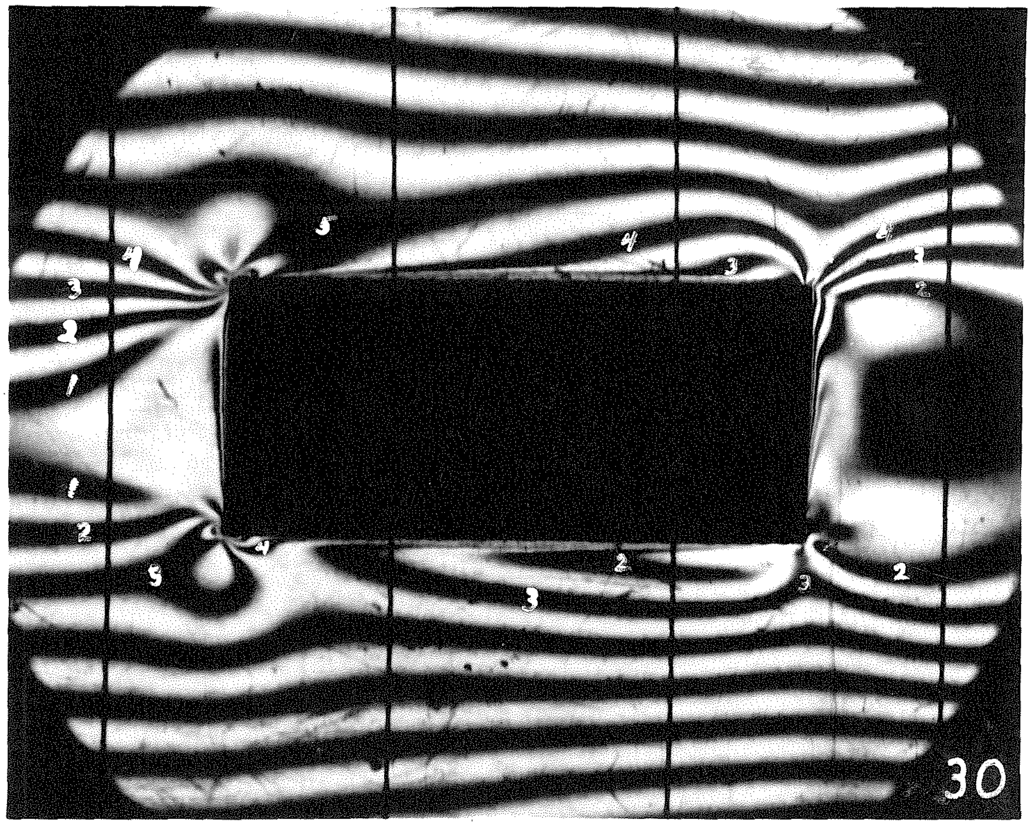
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