A PHOTOELASTIC INVESTIGATION OF THE EFFECT OF CUTOUTS IN PANELS SUBJECT TO SHEAR AND BENDING

Thesis by

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PART I

INTRODUCTION

A. Statement of Thesis Problem

In the design of modern aircraft many structural problems have arisen which are not subject to a simple analysis. Although the usual formulas of elasticity apply, their application becomes too cumbersome for practical use. A problem of this type is the determination of stresses around openings or cutouts. Thus, in monocoque construction there are openings for doors, windows, hatches, etc. Frames and bulkheads are pierced for the passage of conduits, ducts, or other members. In many of these cases, no formal determinations of the stresses can be made. It is well known, however, that stress concentrations do occur at reentrant corners, openings, etc. It was the purpose of this investigation to determine by photoelastic means the amount of this stress concentration, and how it varied in intensity under different conditions of loading, and with various types of openings.

B. Previous Investigations

Numerous investigations have been made on flat plates with various size and shape holes.

Coker and Filon conducted extensive investigations on circular holes in tension bars. A. Schreyer, in Vol. 10 of Forschung, 1939, discusses "Photoelastic Tests on a Square Plate with a Central Square Opening." Dolan and Richards, in Vol. 7, Journal of Aeronautical Science, June 1940, investigated the stresses in wing ribs photoelastically. T. H. Frost of Massachusetts Institute of Technology, states in the 1930 Symposium on Aircraft Materials, that a recent examination has been made of the stresses in a side panel of the fuselage of a cabin passenger plane of monocoque construction. The perforations necessary for doors and windows in the fuselage of such aircraft constitute a real problem from the point of stress distribution. None of the above investigations were carried out under the same conditions as the one under consideration. The authors have knowledge of no investigation of the effect of cutouts in a flat panel subjected to combined bending and shear.

PART II

LABORATORY PROCEDURE

A. Equipment

The apparatus used consisted of the conventional circular polariscope, employing a sodium-vapor lamp for monochromatic light, polaroids, quarter-wave plates, and two 20 centimeter spherical mirrors. A diagram of the equipment is shown in Figure 1. Two condensing lenses were placed between the first polaroid and the light source with the polaroid placed at the focal point of this condensing system. The polarizer and analyser were placed at the focal points of their respective mirrors as shown , giving a light beam, in which the specimen was placed, of 20 centimeters in diameter while using 5 centimeter polaroids. The light beam is thus reversed in direction giving a long path with a wide field. No distortion of the image will result in this type of set-up as long as the angle, α , in Figure 1 is less than 8 degrees.

The monochromatic light source together with the quarter-wave plates produces circularly polarized light and allows the isochromatic lines of the model to be observed without the presence of the iso-clinic lines. The result is a pattern of definite light and dark fringes giving a clear picture.

The two polaroids are mounted in individual stands with the quarter-wave plates attached directly to the polaroid frames and inclined at 45 degrees to the polaroid axis.

The camera used consisted of a blackened box with a frame built to hold the plate holder of a 5 x 7, bellows type camera. The ground glass of this holder was used as a screen when tracing off the fringe orders for each picture. Ortho - x Eastman film was used, requiring a 1 minute exposure for clear pictures.

The loading method is shown in Figure 2. The combined bending and shear loading on the specimen was obtained by fixing the specimen at one end to a solid frame and attaching a loading beam at the other. The specimen was thus loaded as a cantilever beam. Shear and bending was varied by using different weights and different load positions on the beam.

B. Specimen

The specimen used throughout the investigation was a rectangular plate of bakelite, 10.5" x 3.3" x .3". The specimen was given a smooth finish on both faces, and after being cut to the size given above, showed no residual stresses. Difficulty in cutting the hole in the center was experienced at first due to the softness of the material and its tendency to plastic deformation unlessgreat care was exercised in cutting out the fillets.

The first hole cut was the 1 inch circle. Holding the specimen firmly on a wood base a hole was drilled with a vertical drill large enough to insert a fly-cutter. The circle was then cut out taking great care to use aslittle pressure as possible. A very sharp, well raked tool is necessary to avoid setting up permanent stresses.

Using the same specimen, the sides of the circle at the horizontal and vertical diameters were next flattened to a straight line and the 1 inch hole with 3/8 inch radius fillets was cut. Fillet radius was then reduced to 1/4, 1/8, 1/16 inch radius and the final specimen cut to a 1 inch square hole, by using a vertical planer.

For mounting the specimen seven 3/16 inch holes were cut in a vertical line 1/2 inch apart at each end of the piece. The loading beam was then secured to one end and two steel attaching plates at the other using 3/16 inch x 1 inch machine nuts and bolts with washers. These bolts were tightened securely and caused no induced stresses in the vicinity of the cutouts in the specimen. The fixed end of the specimen was attached to a solid frame in such a position as to be in the center of the light beam between the two spherical reflectors.

C. Procedure

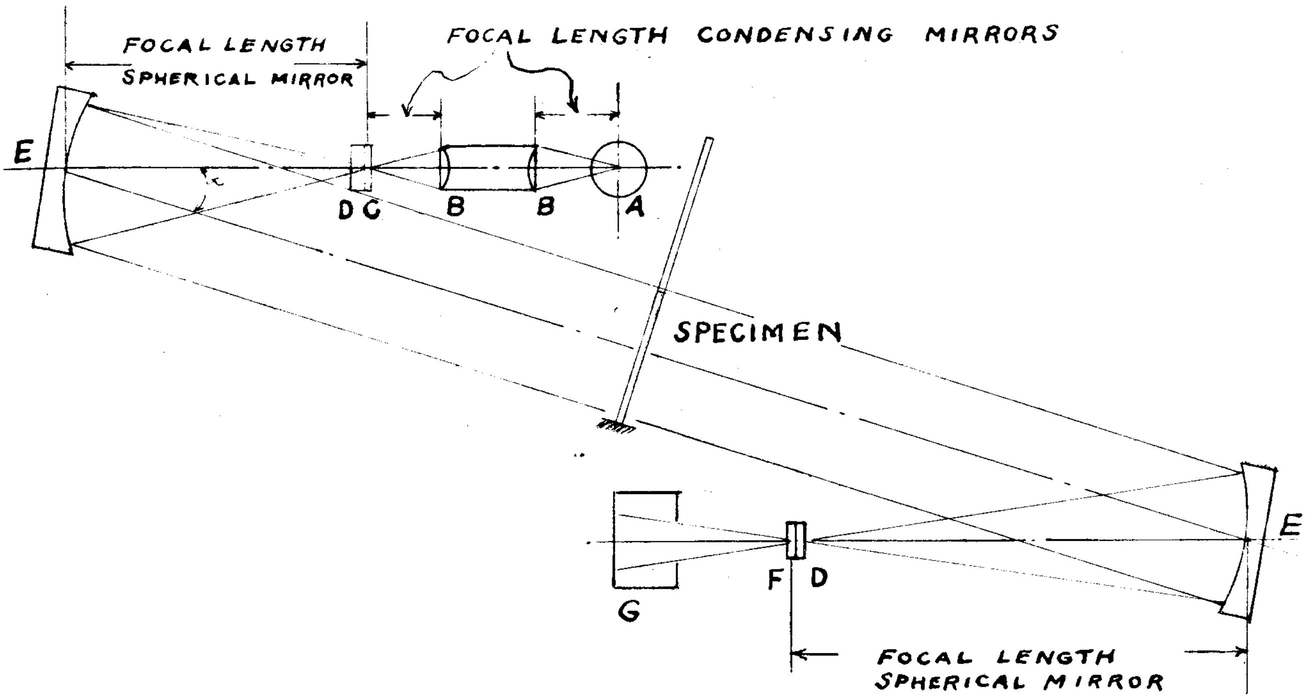
Basically, the problem is to determine the stress concentration factor resulting from a cutout in a panel under combined bending and shear loading. From this, an approximation of the stress ratio of the stresses resulting from cutouts in such a panel to the stresses which can be c alculated for a panel under similar loading without a cutout at corresponding points. To this end, the specimen with no hole was loaded first. Positions on the loading beam were numbered from outboard in, No's. 1, 2, 3, 4, 5. Maximum bending moment was obtained at position No. 1. Five loadings were investigated for each fillet size in the specimen.

A bare unlabelled photograph of a fringe pattern is always difficult and often impossible to analyse. It is necessary to know the order of each isochromatic. In addition, of course, it is necessary to know the load applied, the dimensions of the model and the stress-optical coefficient of the material of which the model ismade. To determine the fringe order of each isochromatic the image can be watched as load is gradually applied. Fringes will form first at points of high difference of principal stresses and travel toward points of low or zero stress.

Using the specimen without the cutout, a load of 41.53 lbs. was attached to the loading beam. A tracing was made of the resulting isochromatic pattern, numbering the fringe orders. The specimen was then photographed and the loading shifted to the other positions where the same procedure was repeated. It was necessary to reduce the total load applied as the lever arm was increased since stresses beyond the elastic limit of the material were reached with 41.53 lbs. at No. 1 position. However, since a variation in the ratio between shear load and bending moment was desired, this restriction did not complicate the procedure.

In the preliminary investigation an attempt was made to obtain fringe orders in the vicinity of eight or nine. At this high order of fringe, however, torsional instability of the loading beam and specimen resulted. Moreover, such a loading was found to leave a residual stress in the specimen. It was therefore necessary to use loadings which would not produce fringes of higher order than six. This gave a rather small variation of fringes at the fillets, varying from 1/2 order to 3 1/2. It was realized that higher loadings might result in more accurate results but due to the limited scope of this investigation the fringe orders obtained appeared satisfactory.

Even with the loadings used, with a maximum of 41.53 lbs. it was found necessary to use a specimen which was fairly thick and to utilize stabilizing wires, extending horizontally from the free end of the loading beam to fixed points, one on either side of the beam. In this manner the difficulties due to torsional instability of the specimen were eliminated.



DIAGRAMATIC PLAN VIEW OF POLARISCOPE

A = LIGHT SOURCE - 60 WATT SODIUM VAPOR LAMP

B = CONDENSING LENSES

C = POLARIZER

D = QUARTER-WAVE PLATES

E - SPHERICAL MIRRORS

F - ANALYSER

G = BOX CAMERA, APERTURE, GROUND GLASS PLATE

FIG. 1

PART III

THEORETICAL CONSIDERATIONS

The apparatus, as used, was set up as a circular polariscope. The polaroids and quarter-wave plate were adjusted so that with no load the screen was illuminated to maximum brightness, or in other words the "light field" condition was used. Then with the model in place, and defining the principal stresses at any point in the model as σ_1 and σ_2, the light intensity on the image of the point on the screen can be expressed as:

$$I = \frac{1}{2} I_m \left\{ 1 + \cos 2\pi \left[\frac{(\sigma_1 - \sigma_2) d}{C} \right] \right\}$$

where

I_m = light intensity at the image of the point if the model is unstressed.

σ_1, σ_2 = principal stress

d = thickness of the specimen parallel to the light beam

c = stress optical coefficient - a property of the material of which the model is made.

This equation can be shortened by writing:

$$N = \frac{(\sigma_1 - \sigma_2) d}{c}$$

where N is defined as the fringe order. Thus for light field:

$$I = \frac{1}{2} I_m (1 + \cos 2\pi N)$$

For light field whenever N is an integer, then $I = I_m$ and there is maximum brightness at the point. Whenever N is 1/2, 3/2, 5/2, etc., then $I = 0$, and there is total extinction of the light intensity at the point. For intermediate values of N the light intensity varies sinusoidally.

In the investigation made, the photograph of the specimen with no hole was used to determine the stress optical coefficient of the material. From this information the value of the fringe order which would have been obtained at the boundary of the hole, had no stress concentration taken place, was computed. When subsequent photographs were made, the values of the fringe order actually found were compared with the theoretical value of the fringe order computed for no concentration. In this manner the stress concentration or more accurately, a stress ratio was obtained.

It is to be noted that in the graphical presentation of the results that a true "stress concentration factor" was not plotted. Rather the value of a "stress ratio" is shown. Specifically, this "stress ratio" is the ratio between the stress obtained at any given location around the hole, to the stress occurring at the ends of the vertical center line diameter of the hole. This latter stress is computed using merely the equation $\sigma = \frac{My}{I}$ where " σ " is the tension or compression stress at the boundary of the hole at the center line station, "y" is the distance to the neutral axis, I is the moment of inertia of the section.

It should be remembered that the fringe pattern of the photographs represent a series of lines $\sigma_1 - \sigma_2$ = a constant. Without a very considerable increase in the amount of work, we do not have available a means of evaluating σ_1 and σ_2 separately at each point on the model. However, much valuable information can be obtained. The maximum shear stress can be written as

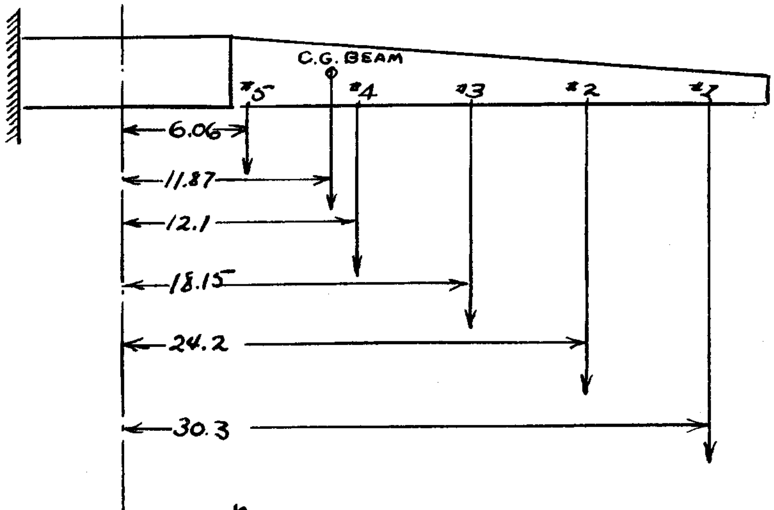
$$\tau_{maximum} = (\sigma_1 - \sigma_2)/2$$

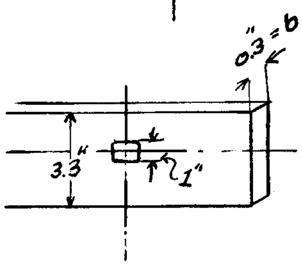
Hence the maximum shear stress can be found at all points. In many problems of design this is the criterion of failure. Hence the above information which can be obtained from the photographs for the conditions investigated, may have direct usefulness.

Also the principal stresses may be evaluated at a free boundary. The only principal stress that can exist at a boundary must be tangent to the boundary, and the other principal stress must be zero. In the case under analysis the largest values of the principal stresses occur at the free boundaries. It is this largest value of the principal stress which has been determined, and which is presented in the form of a dimensionless ratio in the graphs which have been developed.

PART IV

COMPUTATIONS





Load #1 = 20.74 lb.
" 2 = 20.74 lb.
" 3 = 31.0 lb.
" 4 = 41.53 lb.
" 5 = 41.53 lb.

M_s = Static moment due to loading arm

= 2.22 x 11.87" = 26.3 in. lbs.

I = Moment of inertia at of specimen with 1" hole

$$= \frac{bh^3}{12} = \frac{.3}{12}\left(\overline{3.3}^3 - \overline{1.0}^3\right)$$

$= 0.839 \text{ in.}^4$

A = Area of cross section with hole

$= .3\,(3.3 - 1.0) = 0.69 \text{ in.}^2$

For Load Position #1

$\sigma_{\mathfrak{C}}$ (for y = 0.5" = distance above neutral axis)

(ordinary calculation)

$$\sigma_{\mathfrak{C}} = \frac{My}{I}$$

M = (20.74 x 30.3) + 26.3 = 655 lb. in.

Y = 0.5"

I = 0.839

$$= \frac{655 \times .5}{0.839} = 390$$

S = Shear load = 20.74 + 2.22 = 22.96 lb.

A = 0.69 in.^{2}

$$\tau_{AVE} = \frac{S}{A} = \frac{22.96}{0.69} = 33.3$$

$$\frac{\sigma_{\mathfrak{C}}}{\tau_{ave}} \text{ (Position 1)} = \frac{390}{33.3} = 11.7$$

For Load Position #2

$\sigma_{\mathfrak{C}}$ = $\frac{My}{I}$ (y = 0.5" from neutral axis)

M = (20.74 x 24.2) + 26.3 = 552.3 in. lb.

y = 0.5"

I = 0.839 in.^{4}

$$\sigma_{\mathbb{C}} = \frac{552.3 \times 0.5}{.839} = 329 \text{ lb./sq. in.}$$

$$\tau_{Ave} = \frac{S}{A}$$

$S = 21.74 + 2.22 = 23.96$ lb.

$A = 0.69$ in.^{2}

$$\tau_{Ave} = \frac{23.96}{0.69} = 34.7 \text{ lb./sq. in.}$$

$$\frac{\sigma_{\mathbb{C}}}{\tau_{ave}} = \text{(Position \#2)} = \frac{329}{34.7} = 9.5$$

For Load Position #3

$$\sigma_{\mathbb{C}} = \frac{My}{I}$$

$M = (31.0 \times 18.15) + 26.3 = 589.3$ in. lb.

$y = 0.5''$ from neutral axis

$I = .839$

$$\sigma_{\mathbb{C}} = \frac{589.3 \times 0.5}{0.839} = 355 \text{ lb./sq. in.}$$

$$\tau_{ave} = \frac{S}{A}$$

$S = 31.0 + 2.22 = 33.22$ lb.

$A = 0.69$ sq. in.

$$\tau_{Ave} = \frac{33.22}{0.69} = 48.1 \text{ lb./sq. in.}$$

$$\frac{\sigma_{\mathbb{C}}}{\tau_{ave}} = \text{(Position \#3)} = \frac{355}{48.1} = 7.4$$

For Load Position #4

$$\sigma_{\mathcal{C}} = \frac{My}{I}$$

$M = (41.53 \times 12.1) + 26.3 = 529$ lb. in.

$y = 0.5''$

$I = 0.839$

$$\sigma_{\mathcal{C}} = \frac{529 \times 0.5}{0.839} = 315 \text{ lb/sq. in.}$$

$$\tau_{ave} = \frac{S}{A}$$

$S = 41.53 + 2.22 = 43.75$ lb.

$A = 0.69$ sq. in.

$$\tau_{ave} = \frac{43.75}{0.69} = 63.5 \text{ lb./sq. in.}$$

$$\frac{\sigma_{\mathcal{C}}}{\tau_{ave}} \text{ (Position \#4)} = \frac{315}{63.5} = 4.96$$

For Load Position #5

(for $y = 0.5''$ = distance above neutral axis)

$$\sigma_{\mathcal{C}} = \frac{My}{I}$$

$M = 41.53 \times 6.06 + M_s$

$= 252.0 + 26.3 = 278.3$ lb. in.

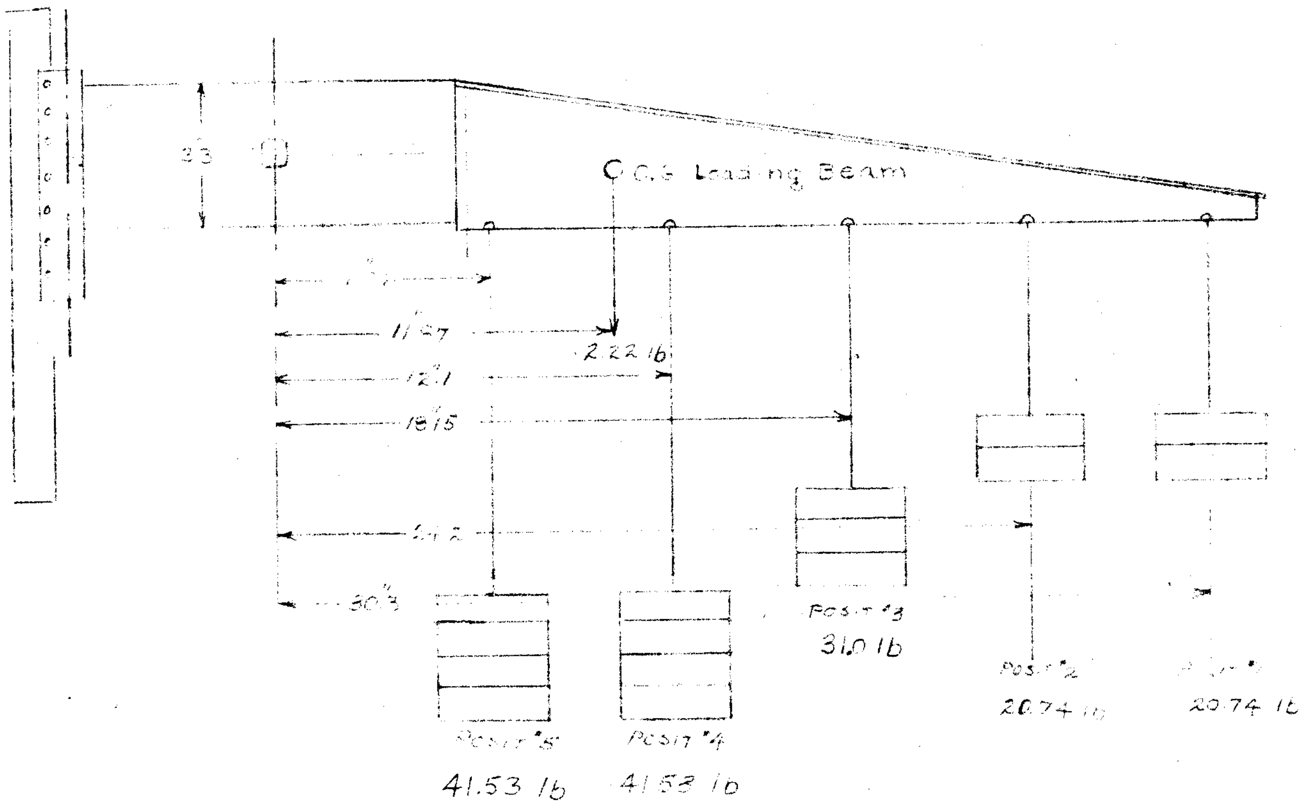
$$\sigma_{\text{¢}} = \frac{278.3 \times 0.5}{.839} = 166 \text{ lb/sq. in.}$$

$$\tau_{ave} = \frac{S}{A}$$

$$S = \text{Shear load} = 41.53 + 2.22 = 43.75$$

$$\tau_{ave} = \frac{43.75}{0.69} = 63.5$$

$$\frac{\sigma_{\text{¢}}}{\tau_{ave}} \text{ (Position \#5)} = \frac{166}{63.5} = 2.62$$



SPECIMEN
and
METHOD OF LOADING

Note - Loads applied as shown, separately.

FIG. 2

PART V

PHOTOGRAPHS

This section contains the photographs showing the isochromatic lines as obtained for the various headings. The photographs are arranged in several series, in order as follows:

Series A	No hole in the specimen
Series B	Hole 1 inch in diameter Fillet radius equals 1/2 inch
Series C	Hole 1 inch across Fillet radius equals 3/8 inch
Series D	Hole 1 inch across Fillet radius equals 1/4 inch
Series E	Hole 1 inch across Fillet radius equals 1/8 inch
Series F	Hole 1 inch across Fillet radius equals 1/16 inch
Series G	Hole 1 inch square. No Fillet

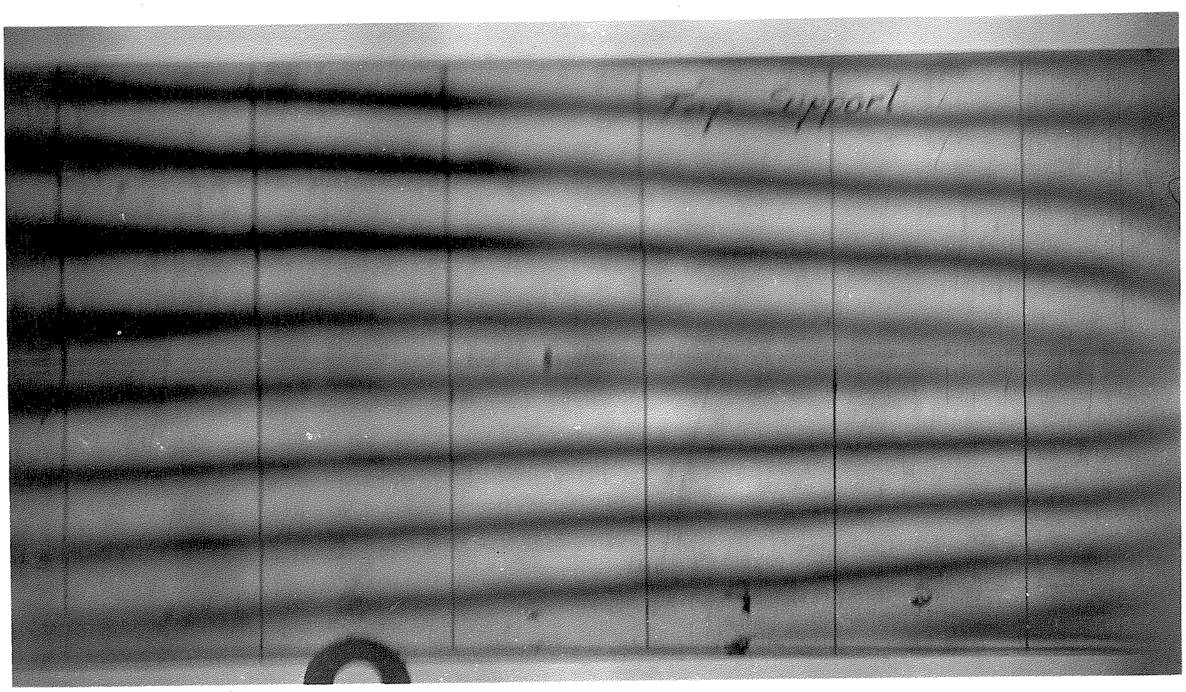
It will be noticed that on the lower edge of each specimen there are a series of numbers from one to five, and that at one number on each photograph a small circle appears. The position of this circle indicates the relative position of the load on the loading arm. All of the photographs are so mounted that the side corresponding to the fixed support of the specimen is at the right hand side of the picture. Correspondingly, the loading arm extended out to the left. If then, the circle is below the number 1 on the photograph it means that the load for that picture was at the number 1 position on the loading arm. This position is also the farthest to the left, and hence represents the greatest ratio of Bending Stress to Shear Stress. Conversely, if the circle is at the number 5 position on the photograph, it indicates that the load was at the number 5 position of the loading arm. That is, the load was close to the specimen and the value of Bending Stress to Shear Stress is a minimum.

The photographs of each series are mounted in order, starting with the maximum ratio of Bending Stress to Shear Stress, and then progressively decreasing this ratio in each subsequent picture. The actual values of this ratio for the different load position are given in the section under computations.

Series A

No Hole in the Specimen


Top Support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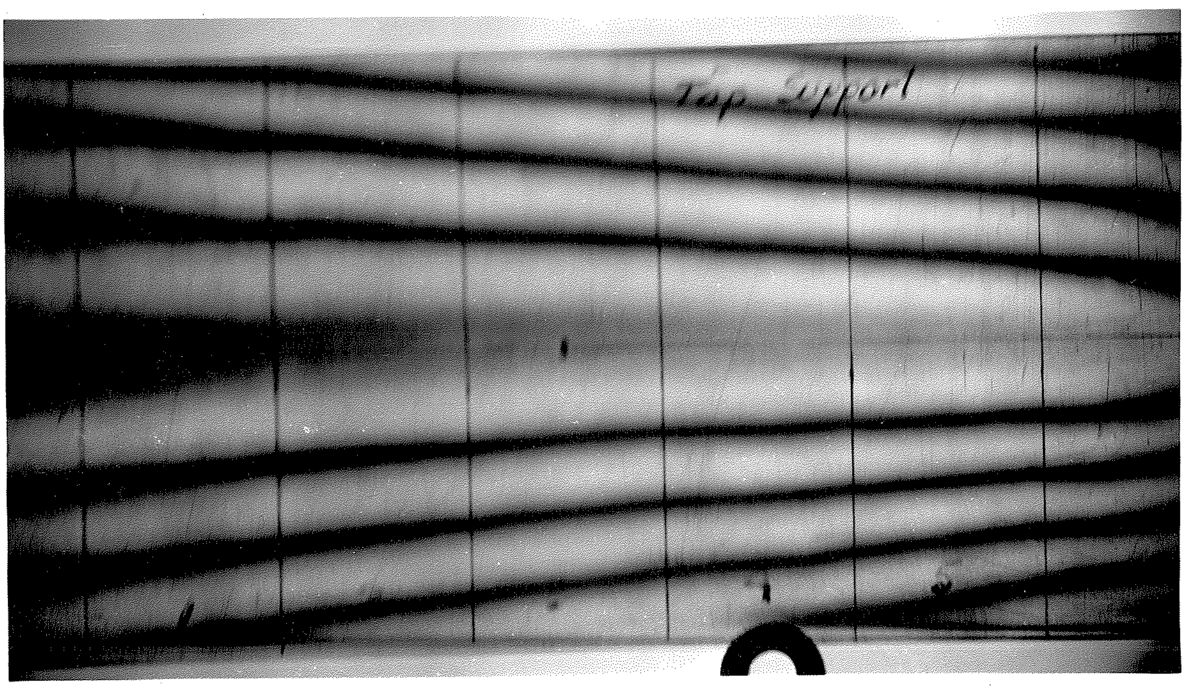
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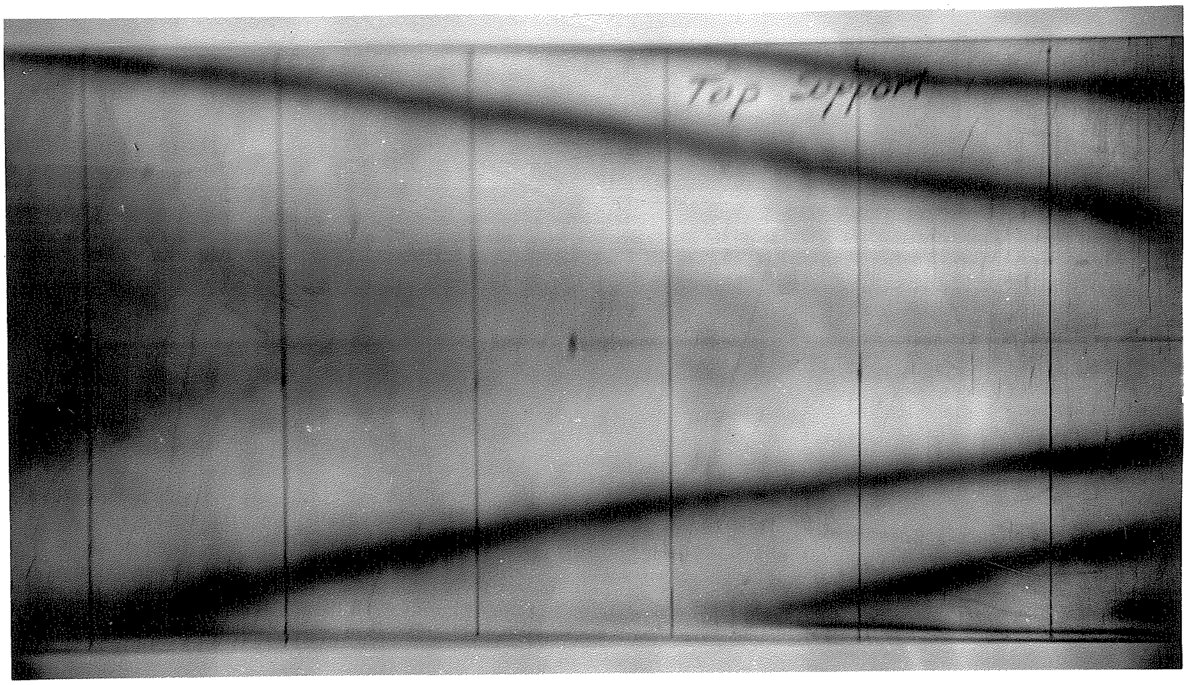
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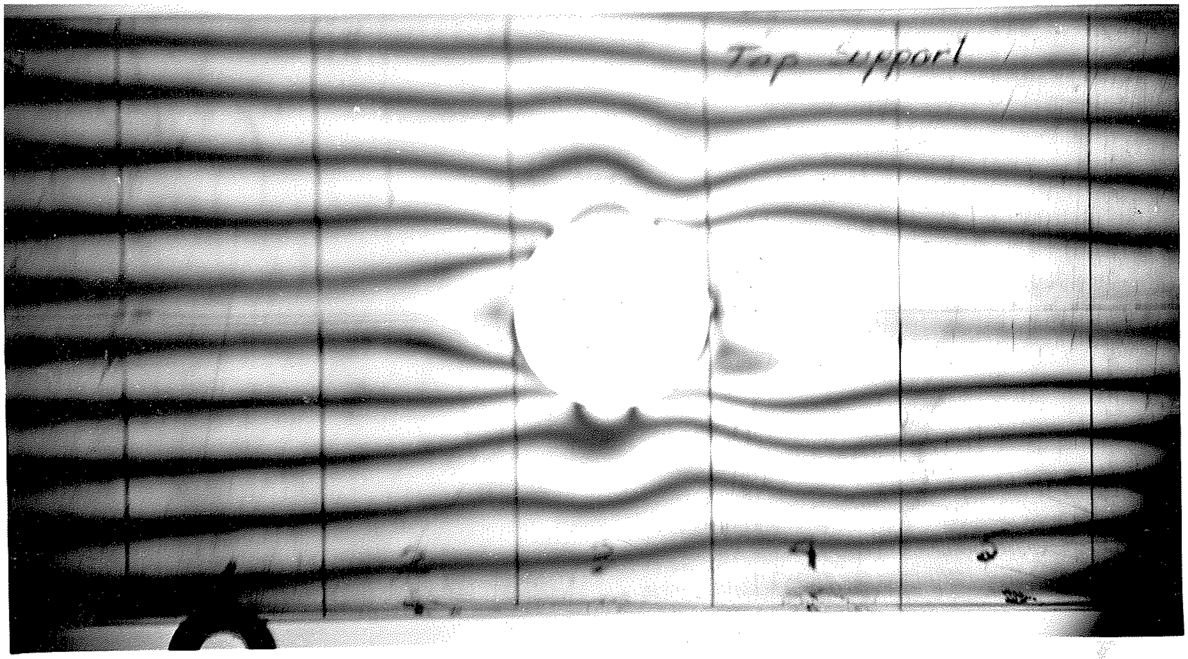
Series B

Hole 1 Inch in Diameter

Fillet Radius Equals 1/2 Inch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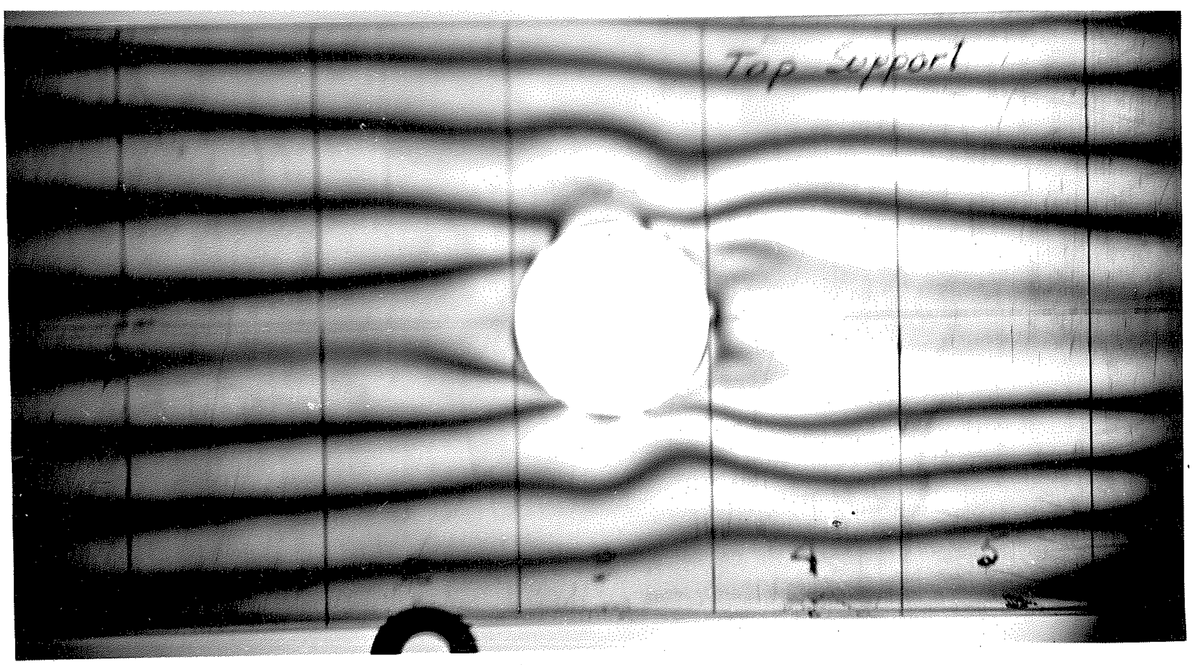
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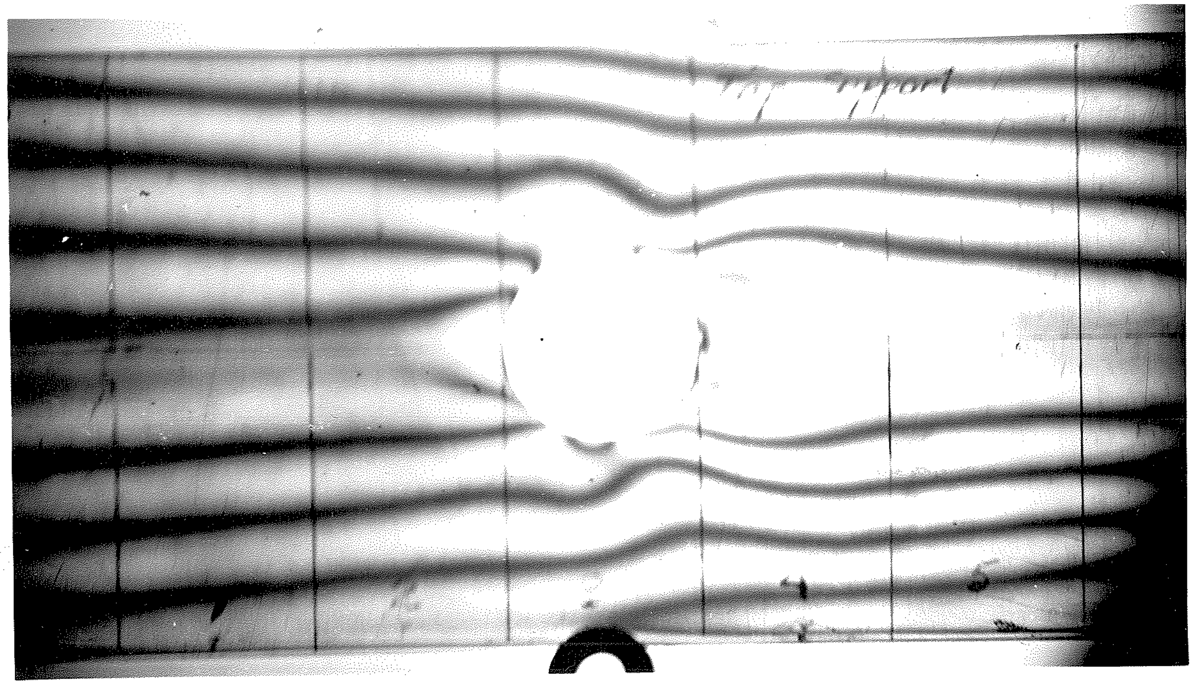
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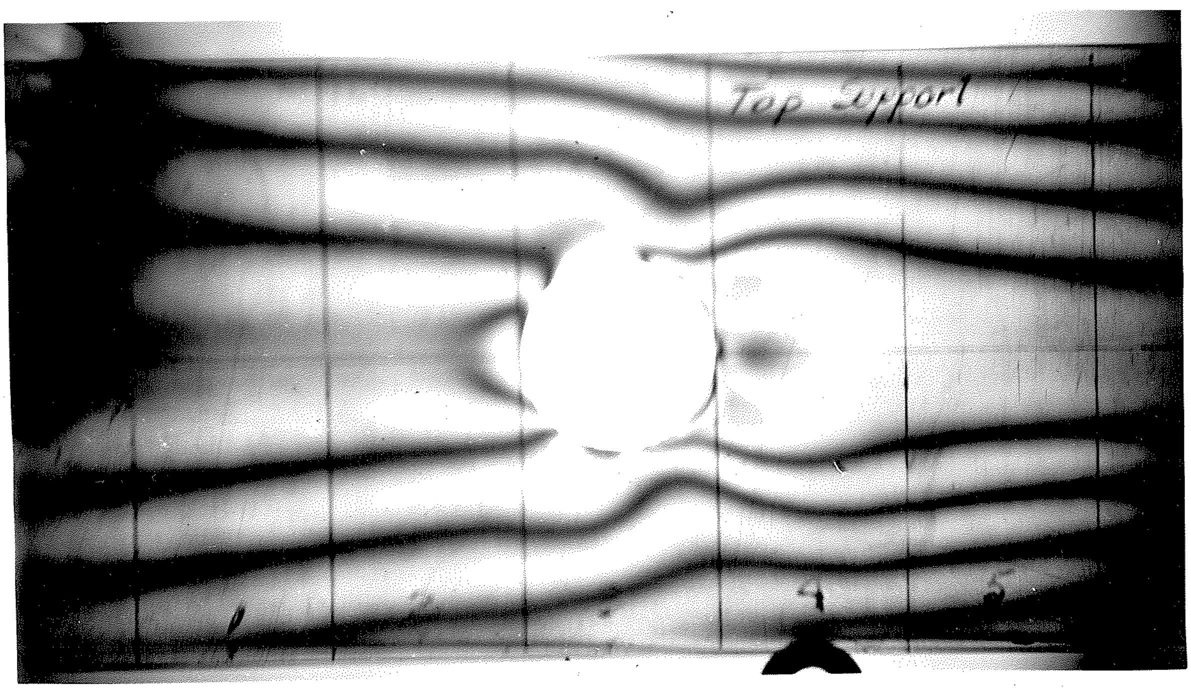
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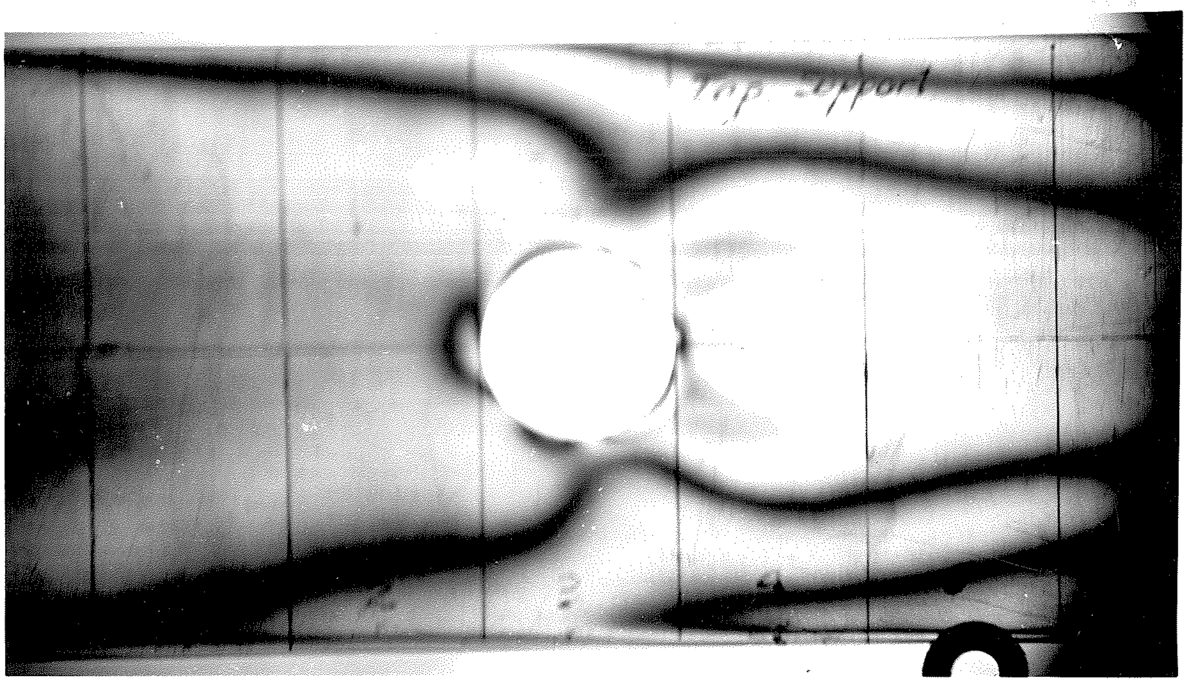
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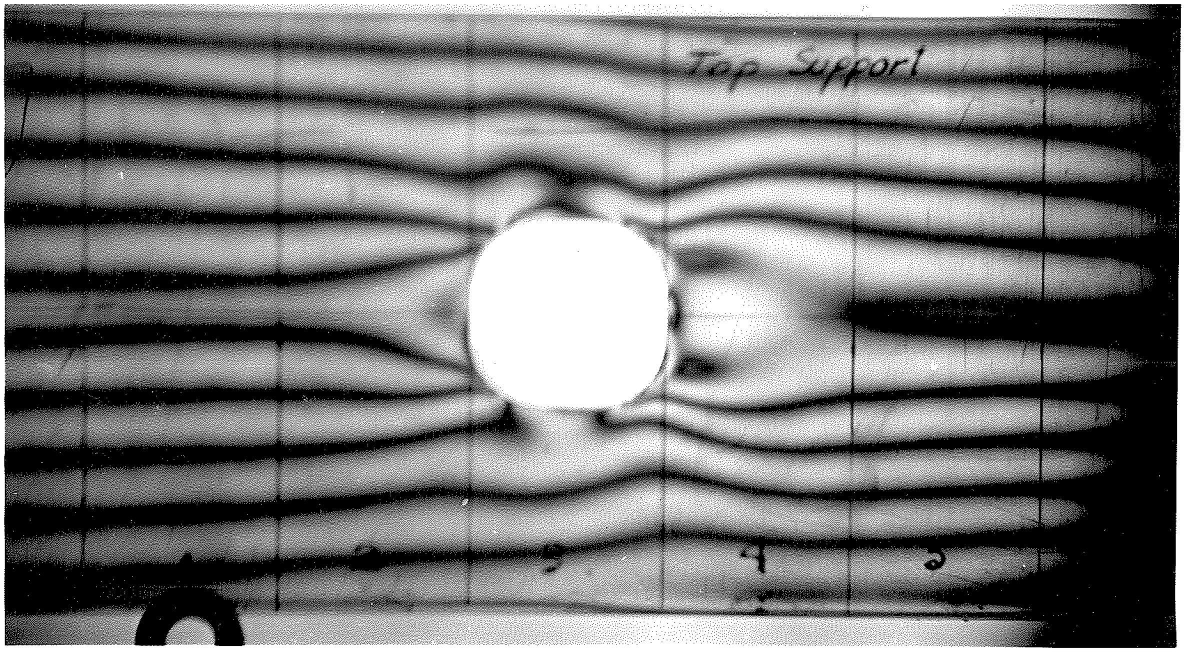
Series C

Hole 1 Inch Across

Fillet Radius Equals 3/8 Inch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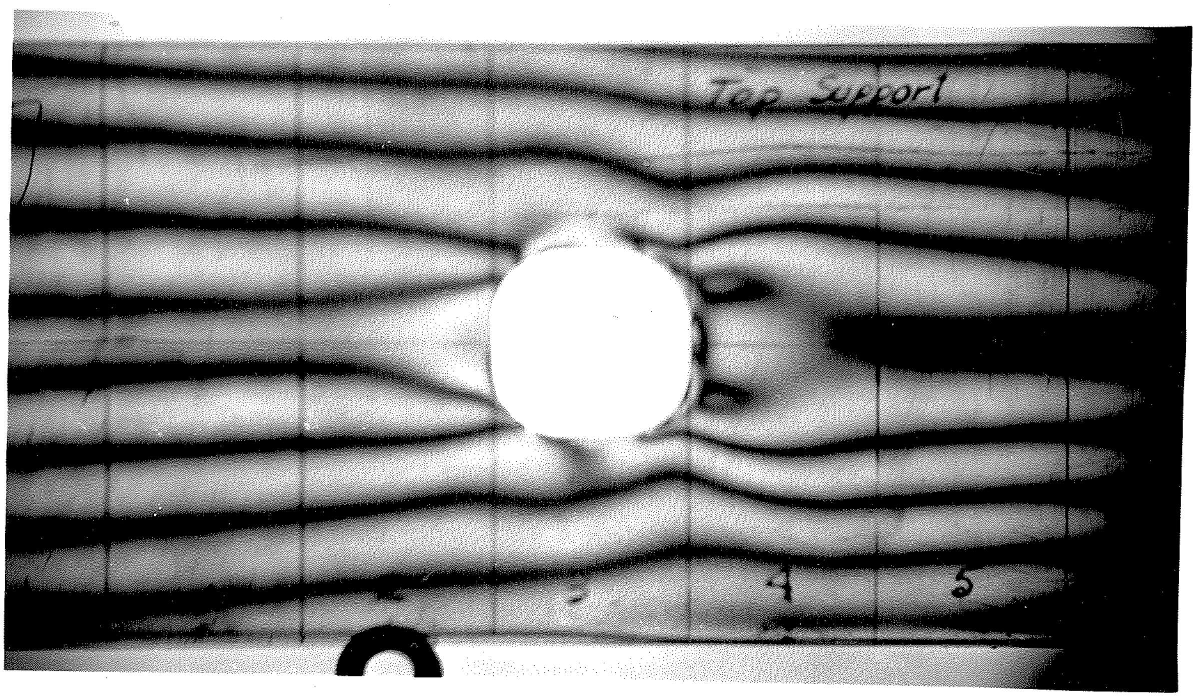
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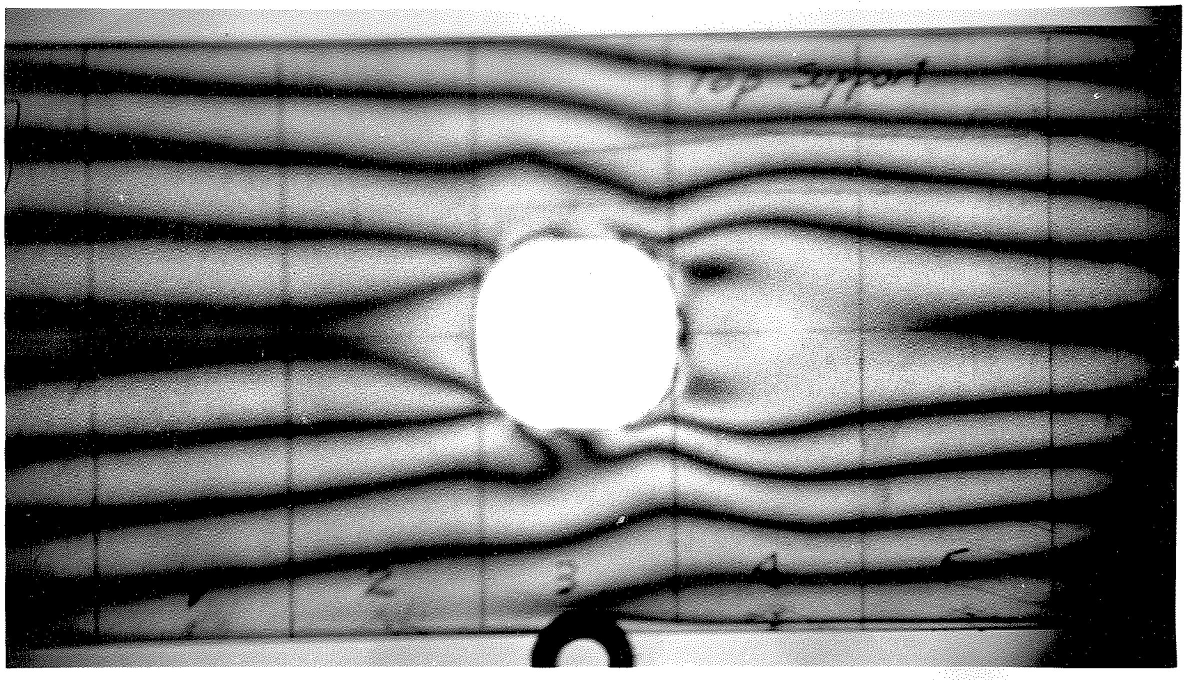
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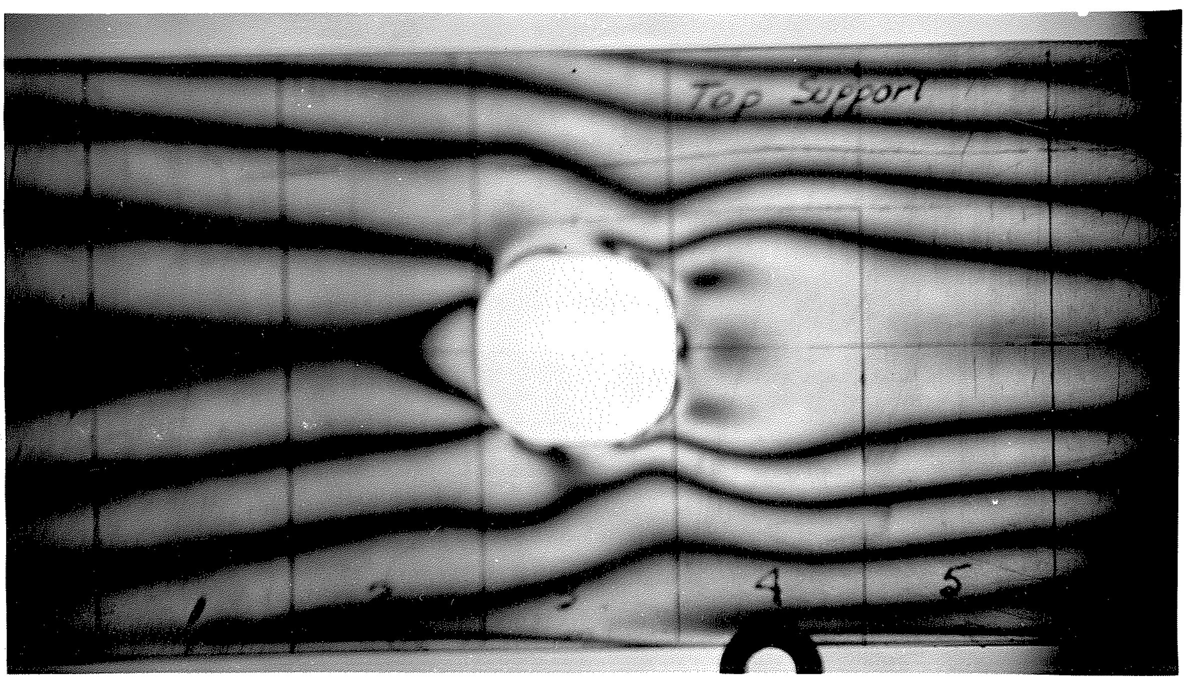
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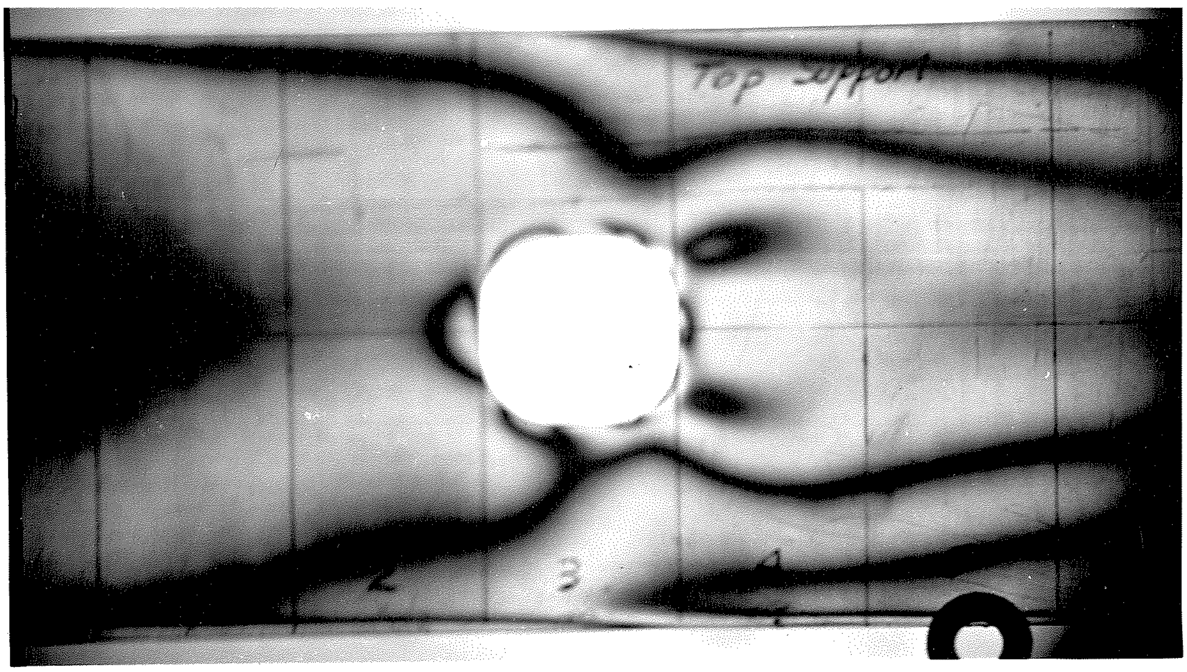
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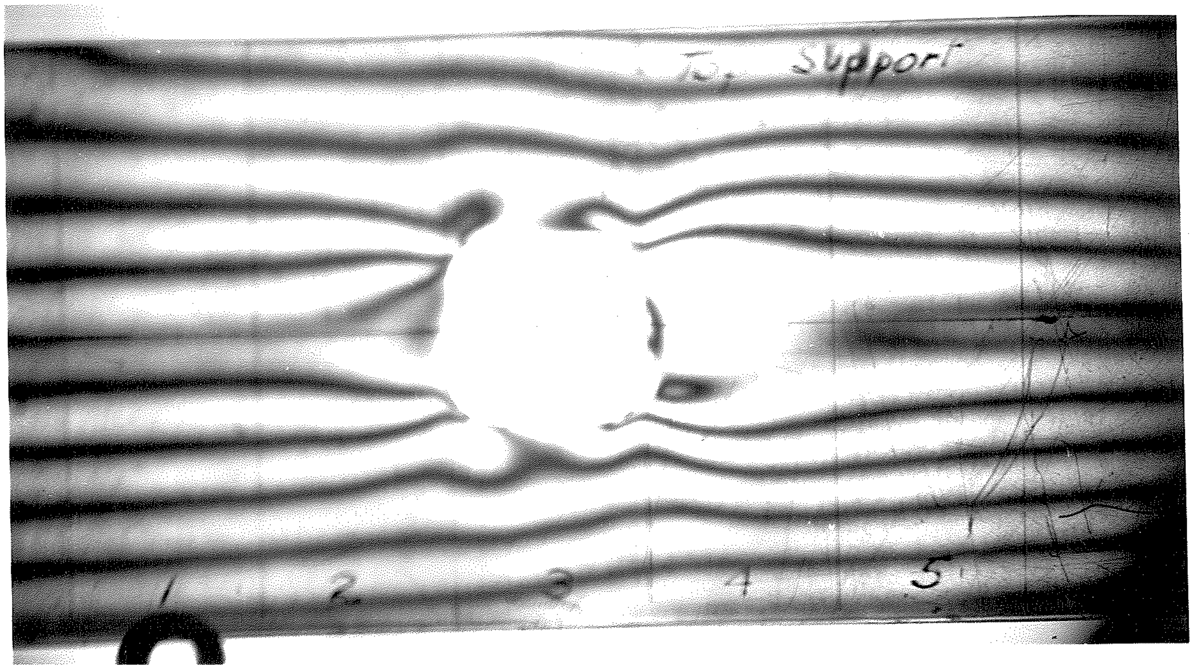
Series D

Hole 1 Inch Across

Fillet Radius Equals 1/4 Inch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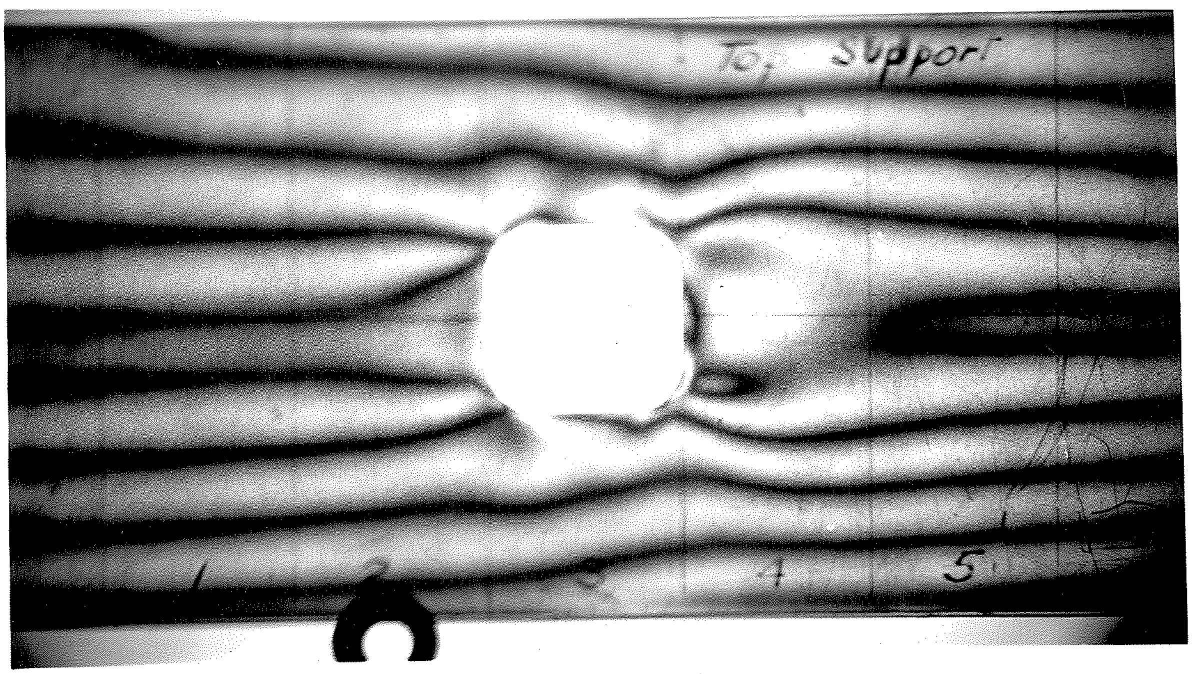
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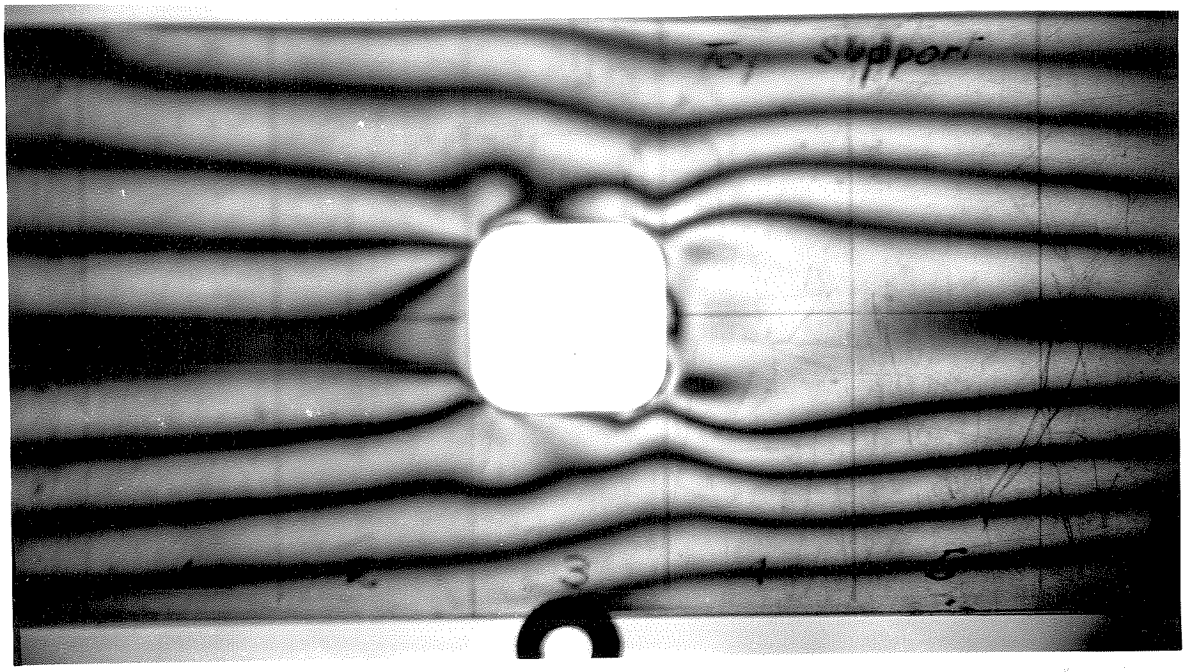
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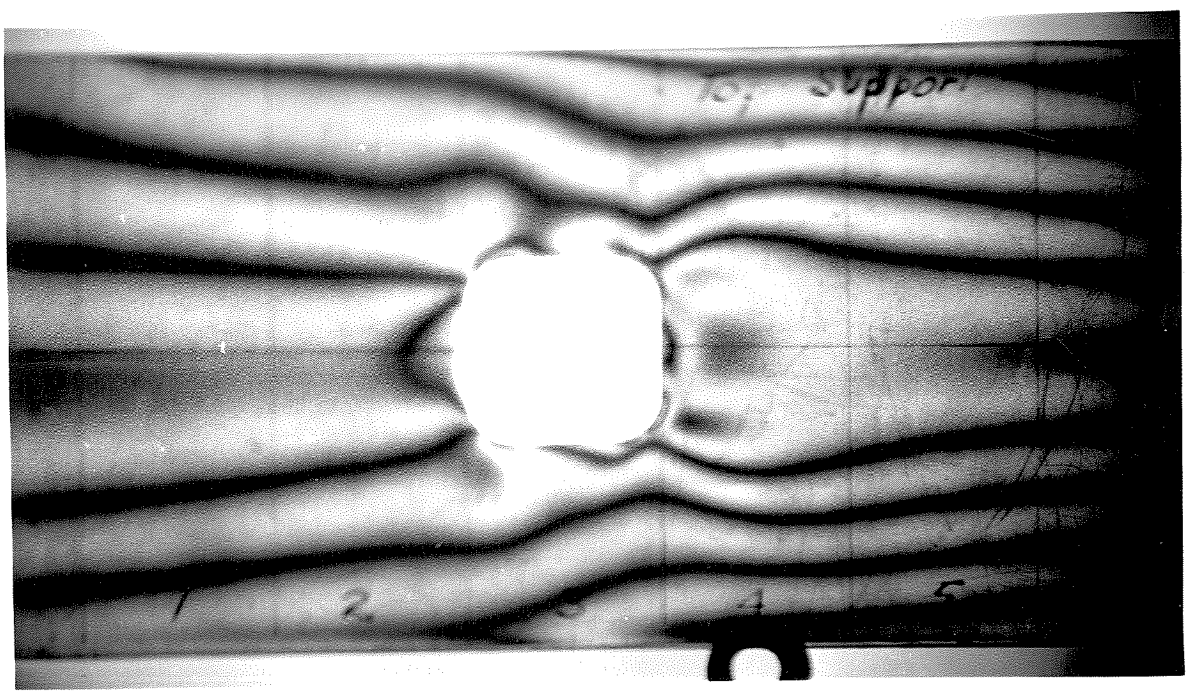
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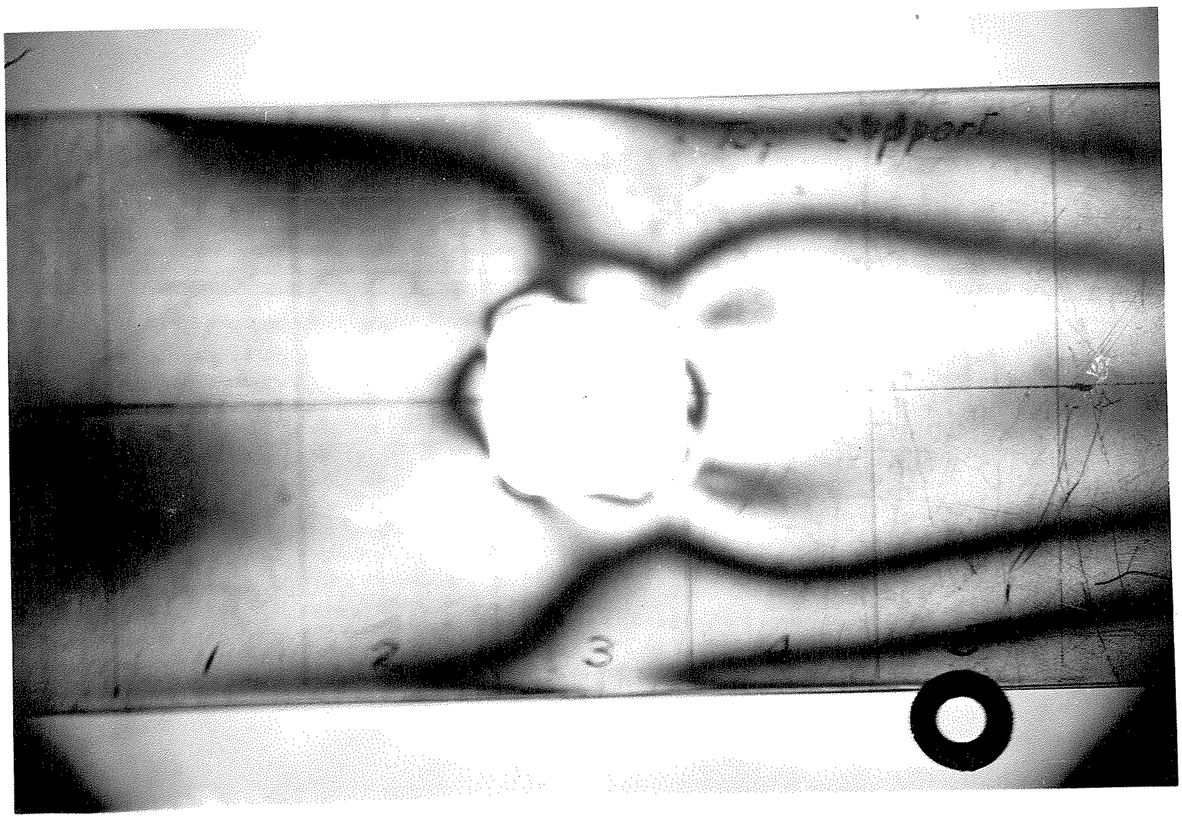
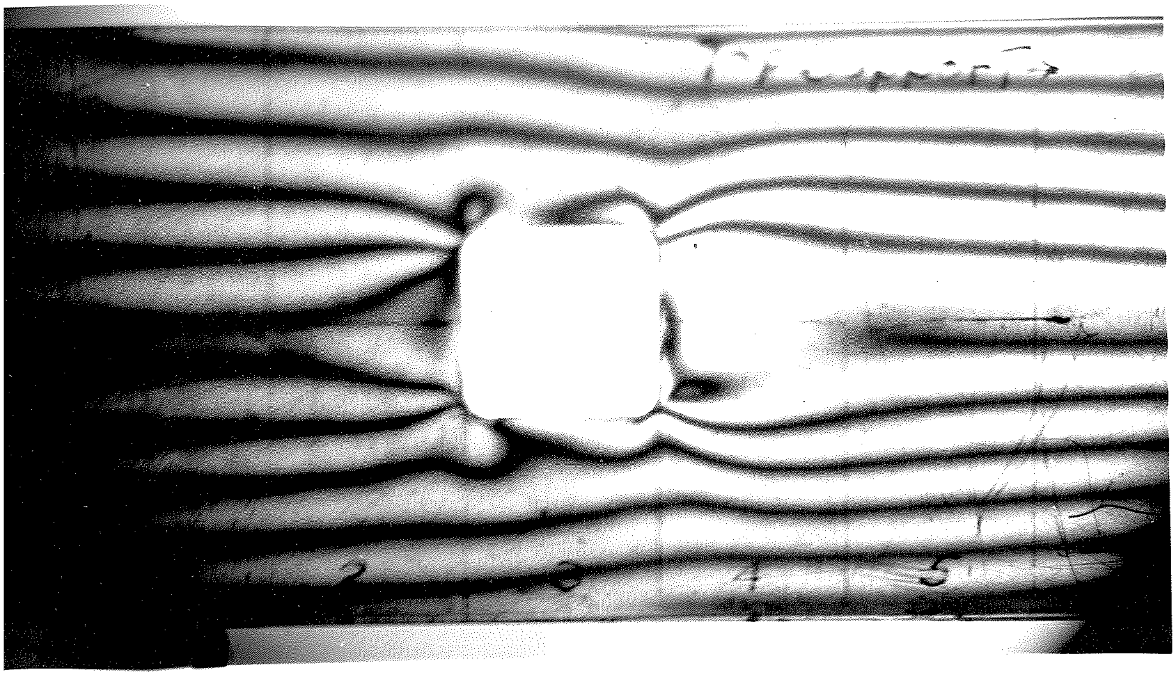


Series E

Hole 1 Inch Across

Fillet Radius Equals 1/8 Inch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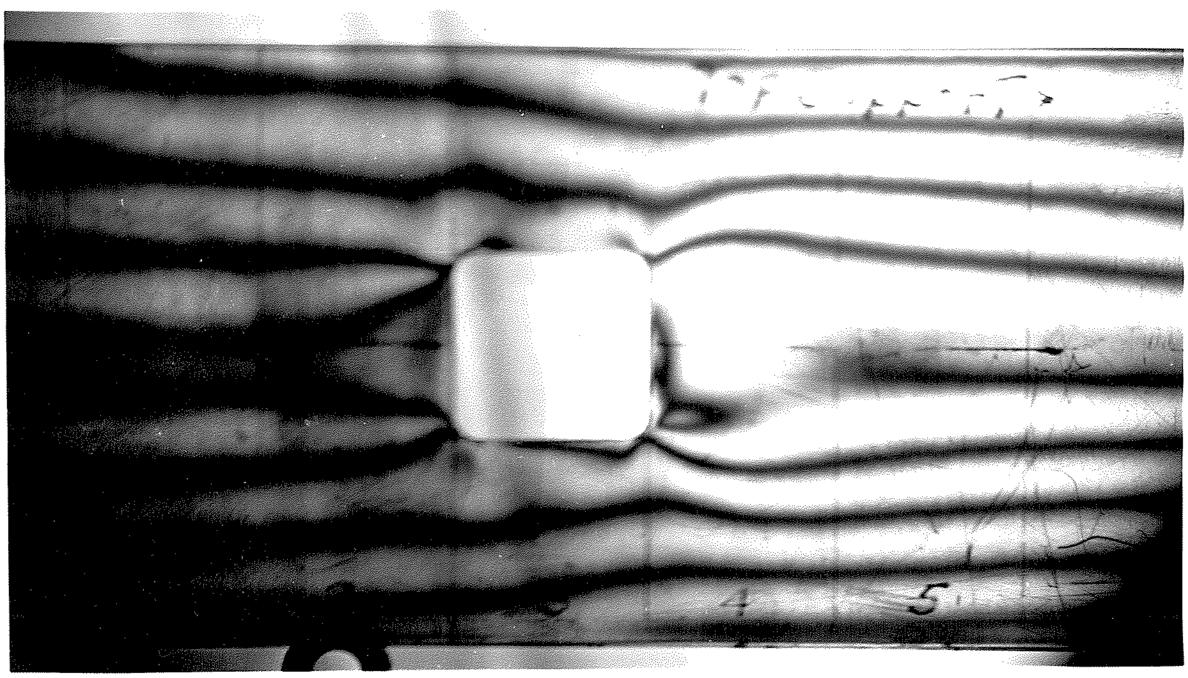
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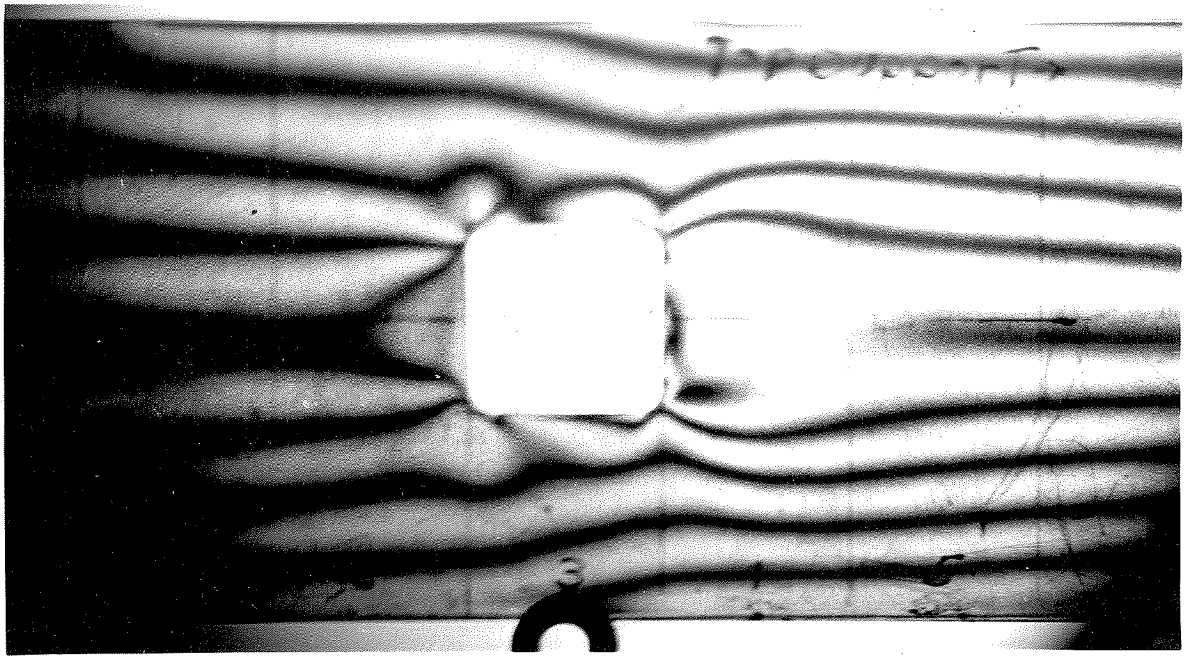

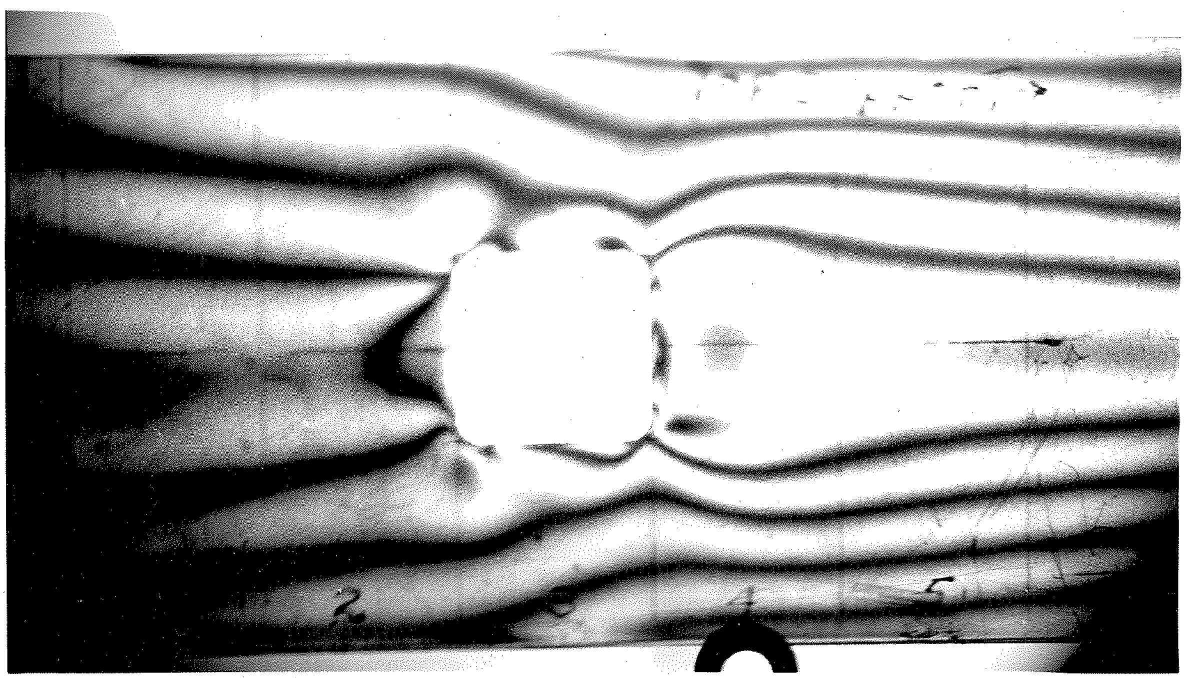


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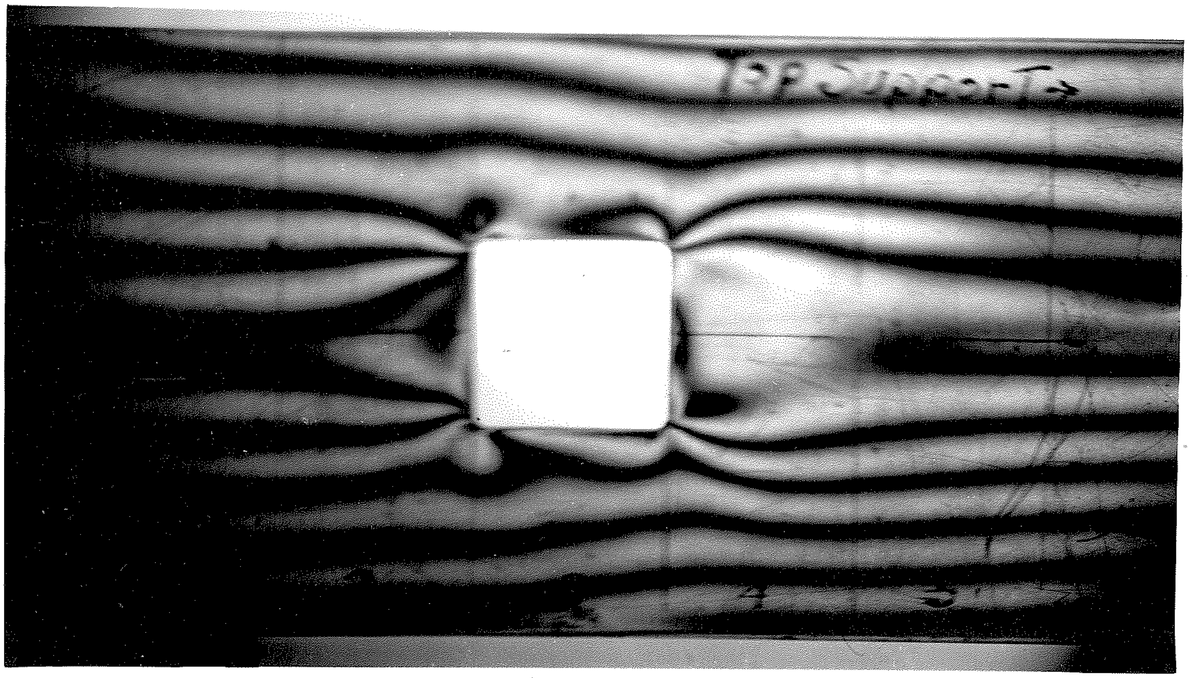
Series F

Hole 1 Inch Across

Fillet Radius Equals 1/16 Inch



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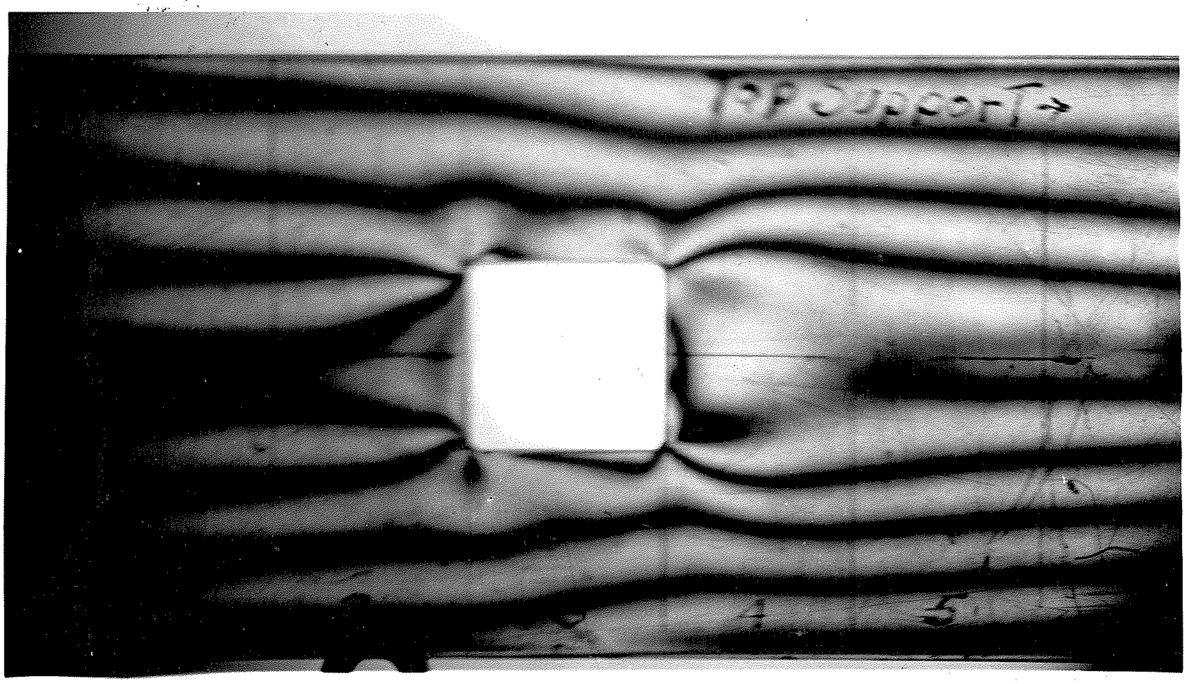

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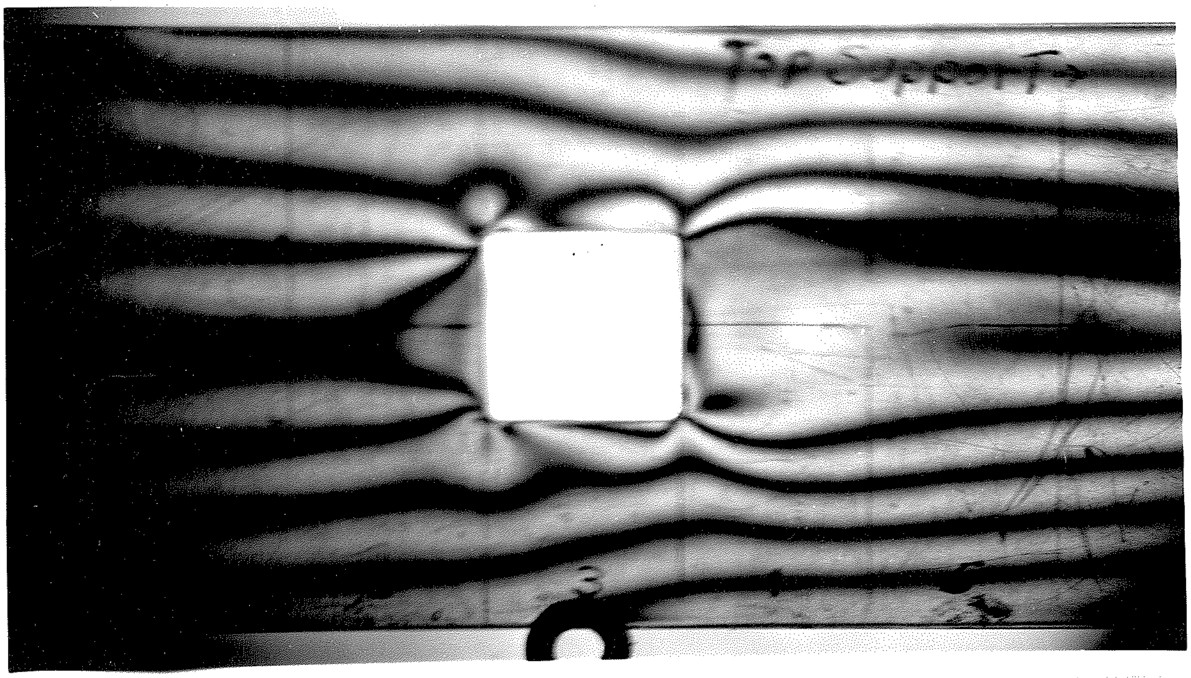
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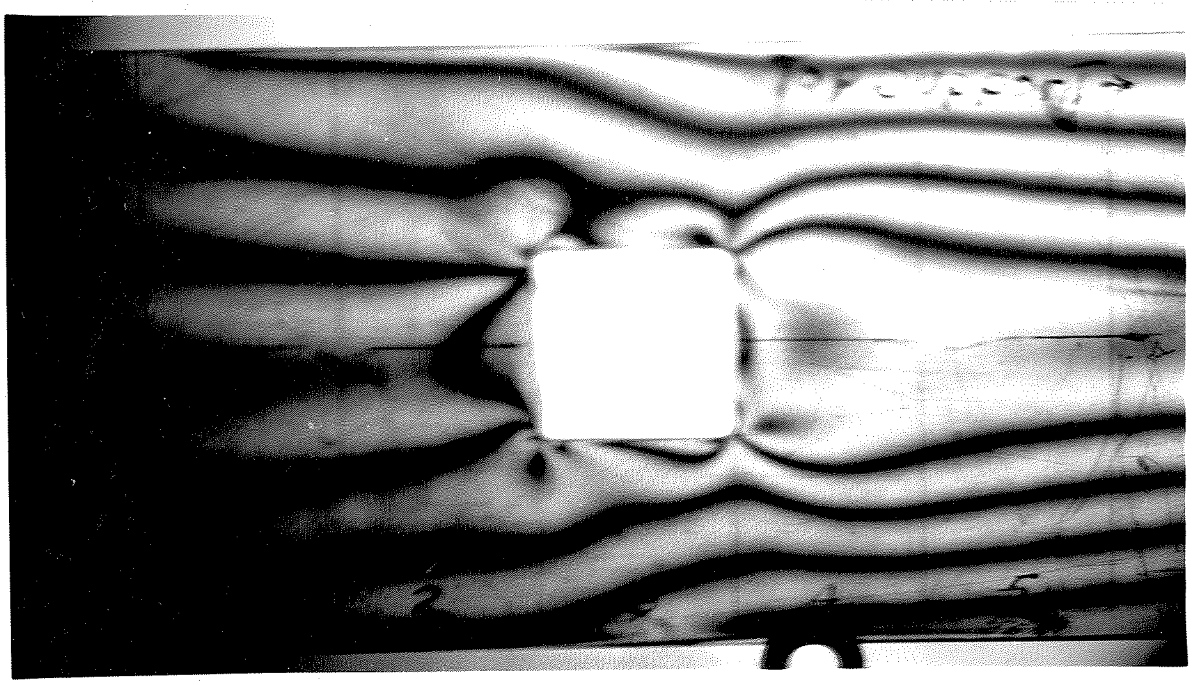
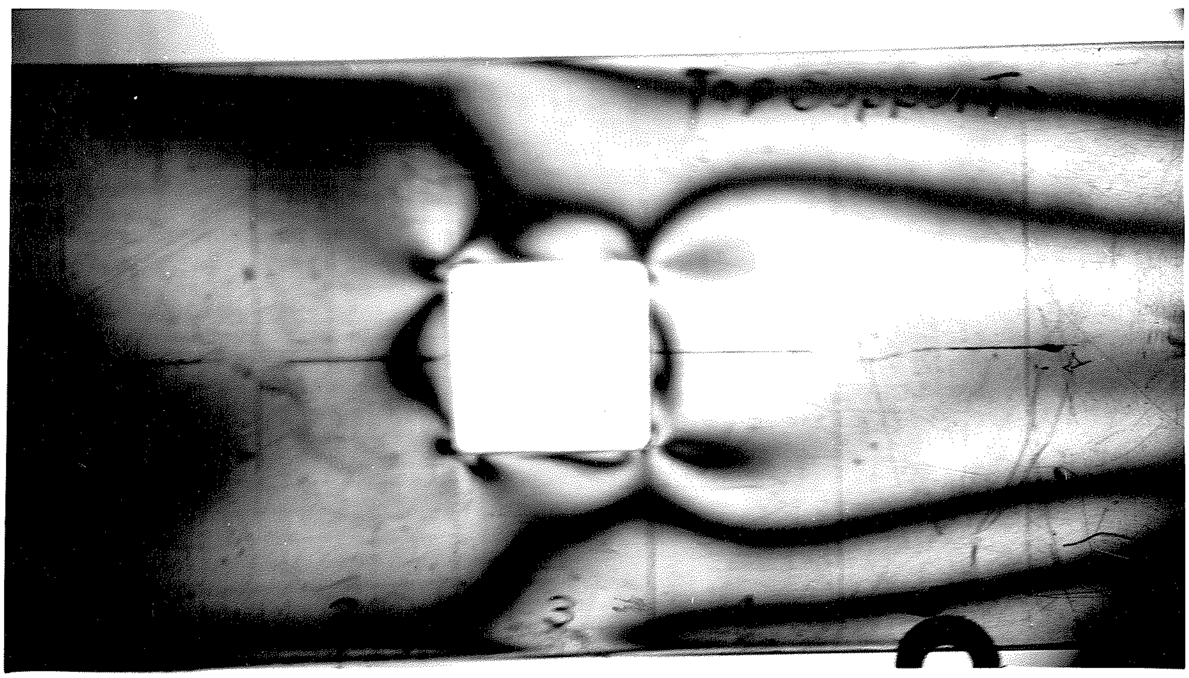
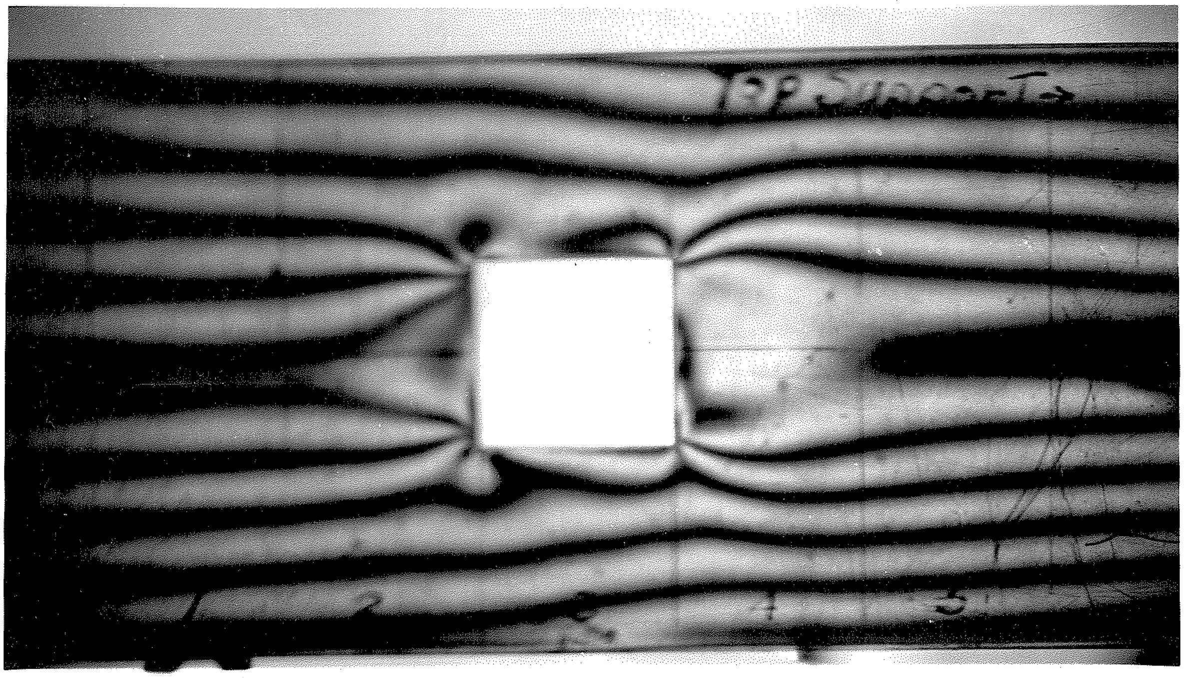


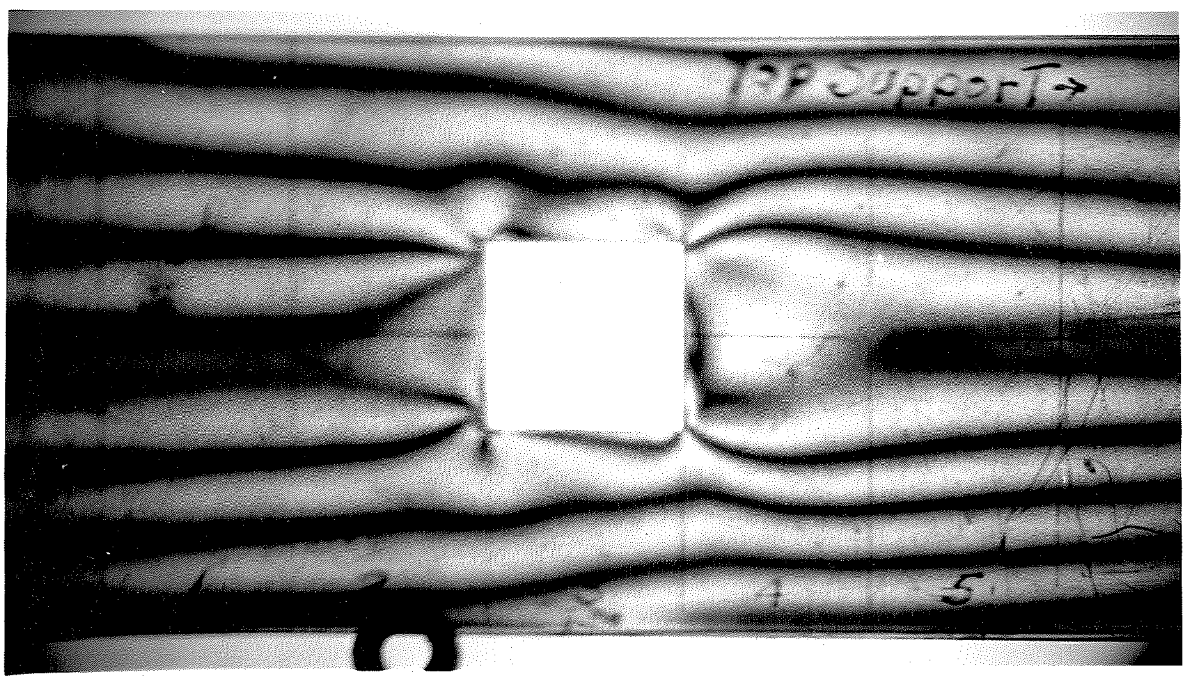


Series G

Hole 1 Inch Square

No Fillet


TOP Support →
1
2
3
4
5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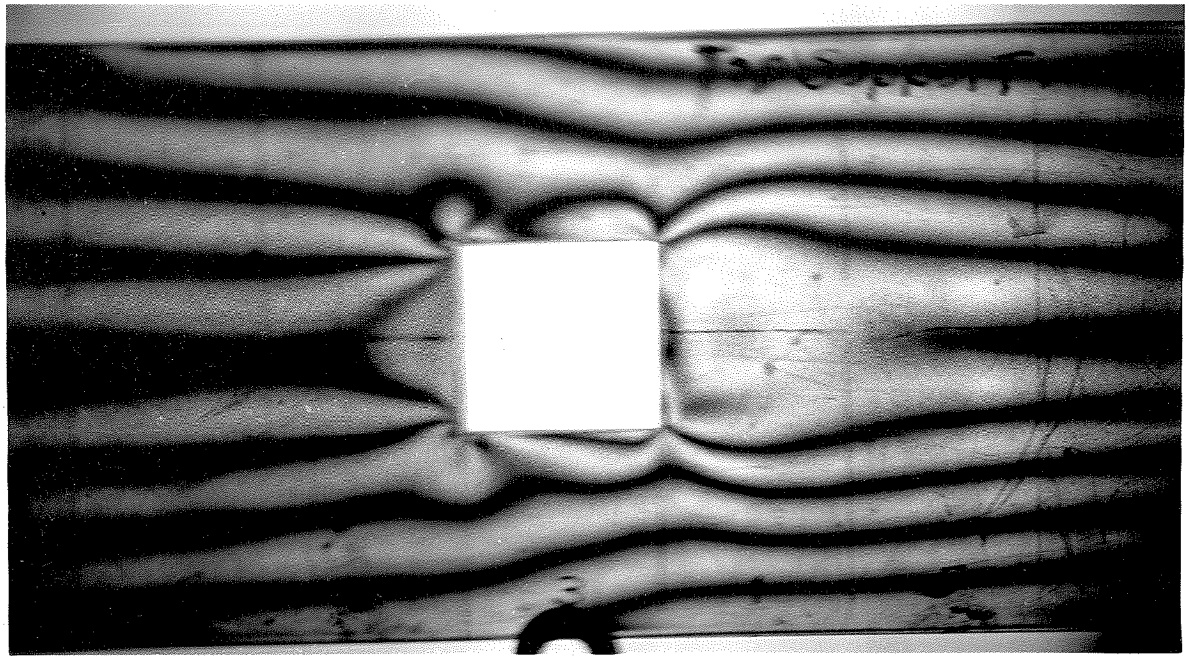
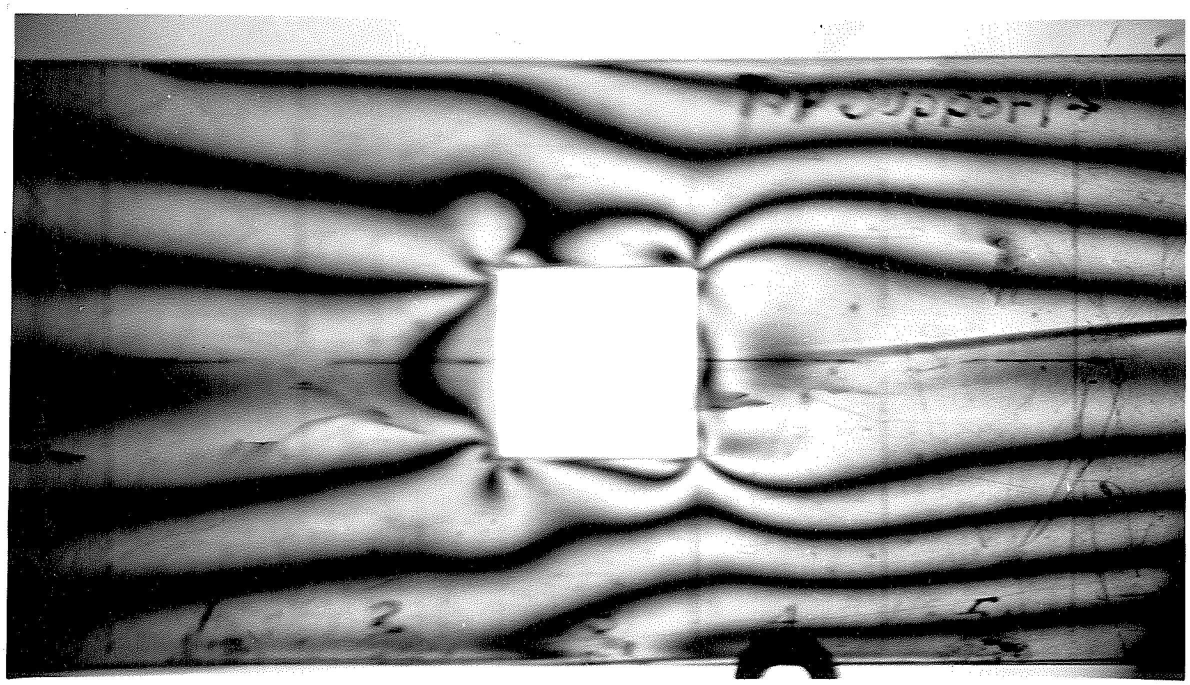
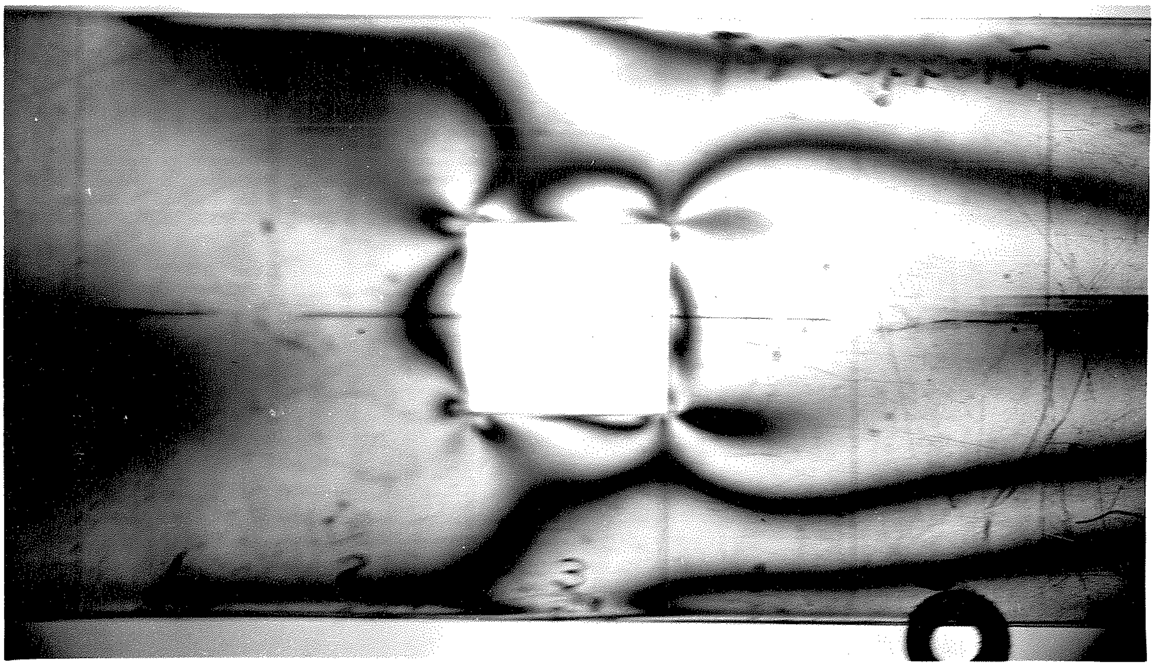






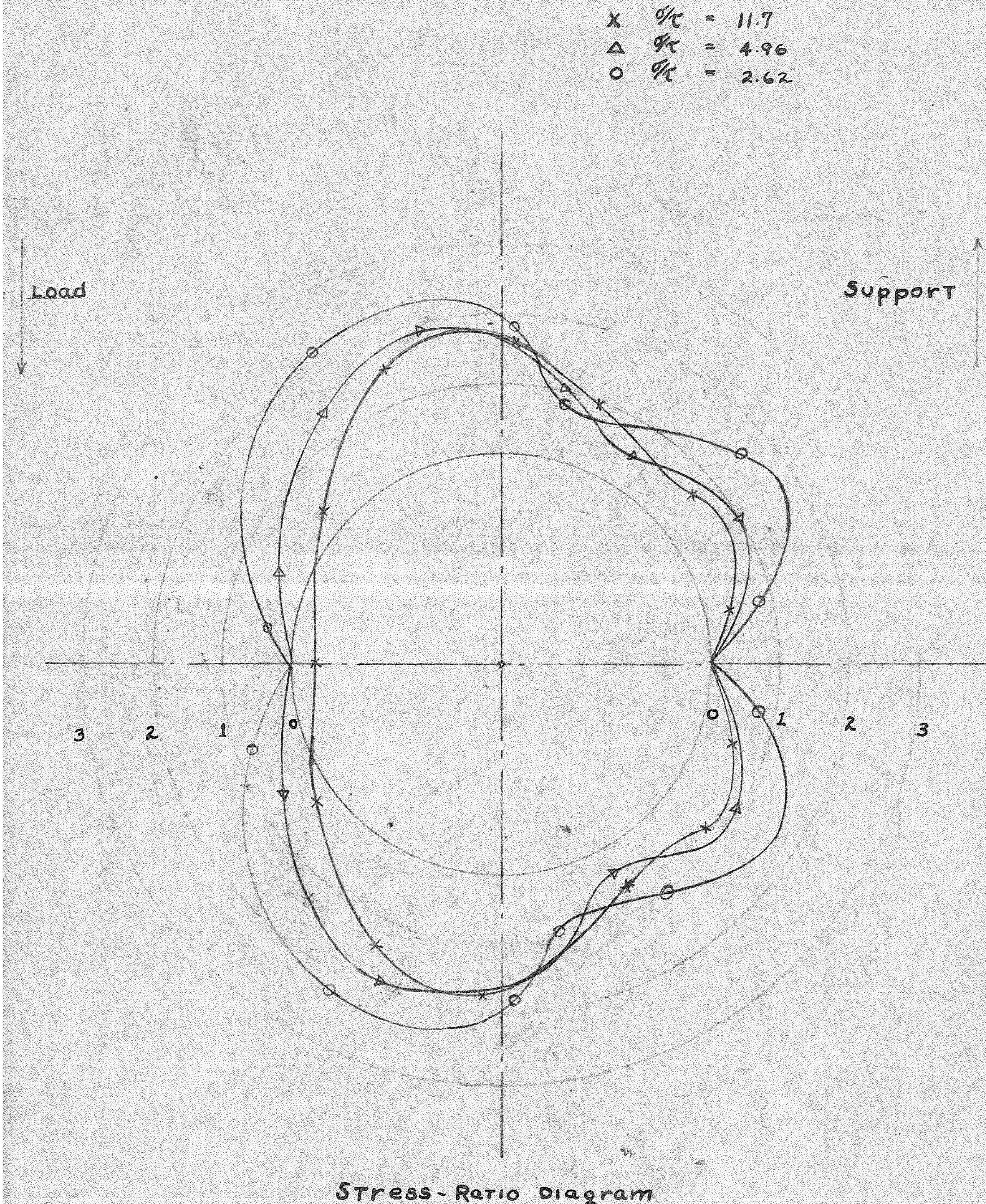
PART VI

METHOD OF USING THE CURVES

It is considered that in the design of a panel, the size and shape will be known , or else may be assumed as the first step in a trial and error solution. Also, it is considered that the size and type of the cutouts to be made will be known; and further, that the amount of the load and its point of application will be a given quantity.

With this information the following factors should be calculated: First, the moment of inertia of the cross section through the panel or member at the center line of the hole or cutout; Second, the bending moment at this station due to the load. Third, the average shear stress at the section in question using merely the value of load divided by the area of the section. With the first two quantities obtained it is possible to compute a tensile or compression stress which would be expected at the boundary of the hole provided no stress concentration took place. To do this the usual formula of "Stress equals the product of Bending Moment times distance from the neutral axis, divided by the moment of inertia of the section" is used. The ratio between stress due to bending, as determined for the edge of the hole, and the average shear stress is then determined. Also the ratio of twice fillet radius to hole diameter should be determined. With this information the graphs may be entered, and a value of the stress ratio found. This stress ratio is then the ratio of the maximum stress to be encountered at the boundary of the hole divided by the stress which would be computed for the station at the center line of the hole assuming that no stress concentration took place. The figures from 3 to 8 show the stress concentrations and their locations around the openings. Figures 9 and 10 show in a more convenient form the maximum value of the stress ratio.

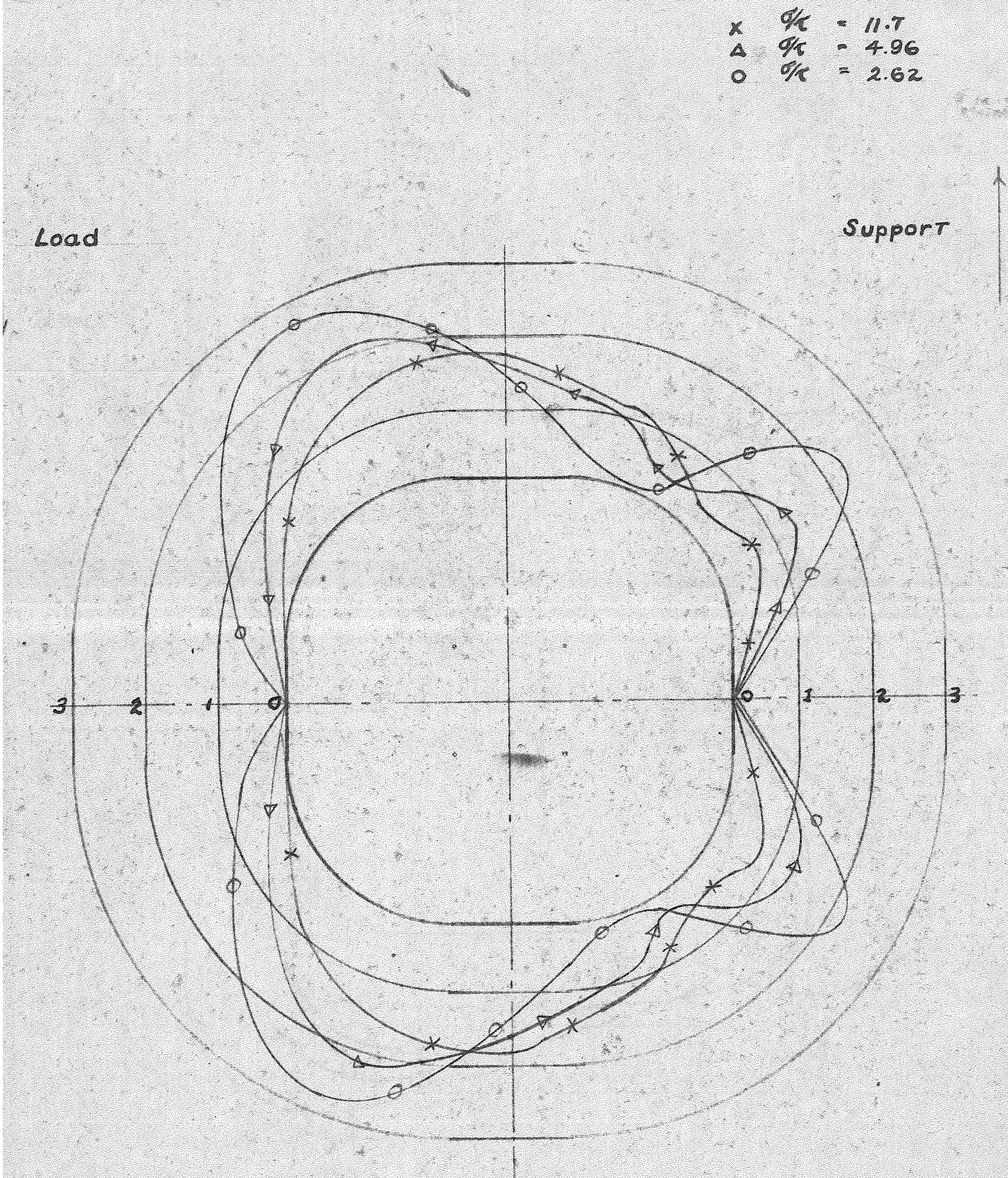
Having the stress ratio as determined from the curves, and knowing the computed value of the stress at the hole boundary the actual stress at the hole is readily obtained by multiplying the two together.



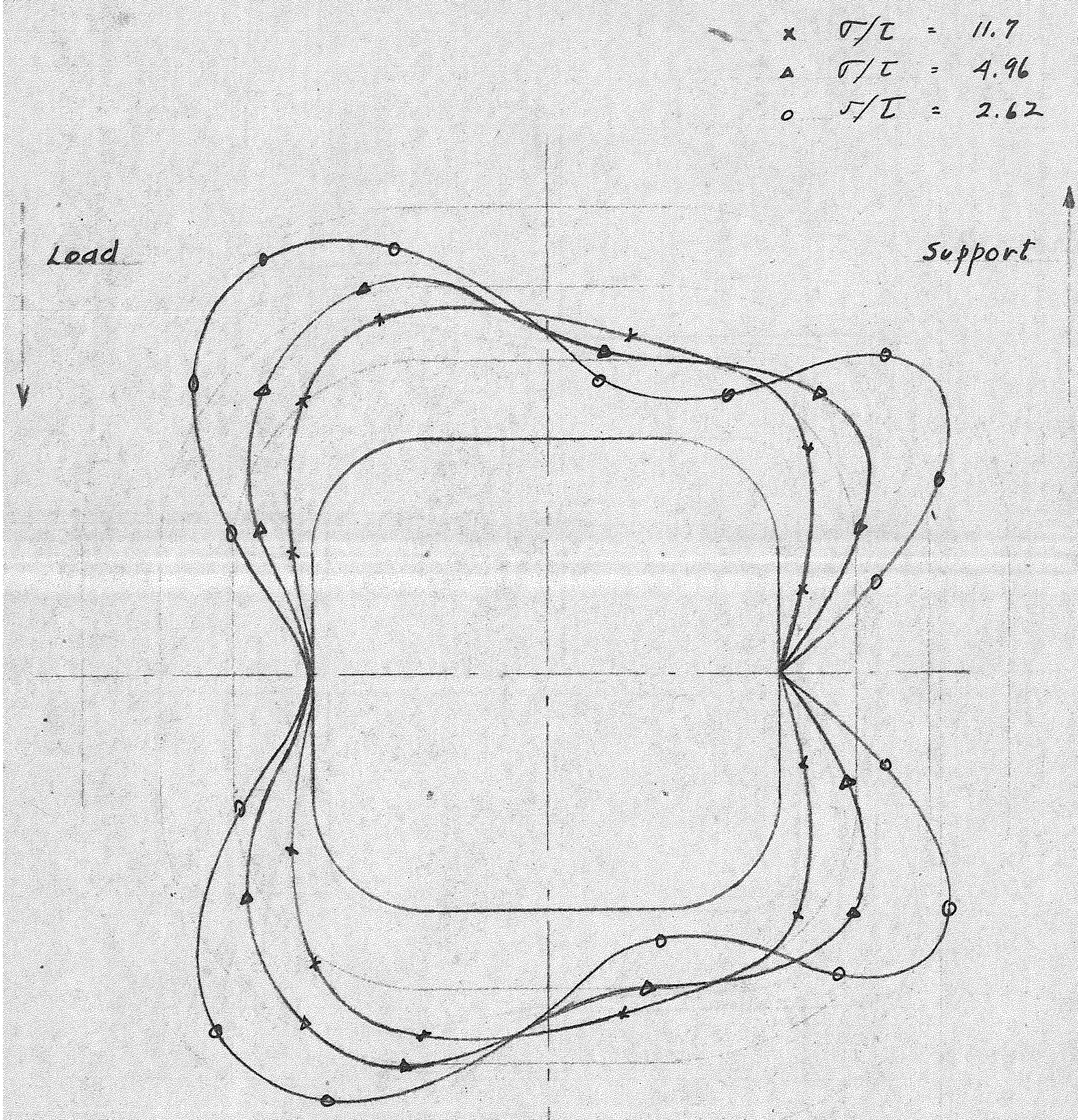
Stress-Ratio Diagram

1" Circular Hole.

Fig 3



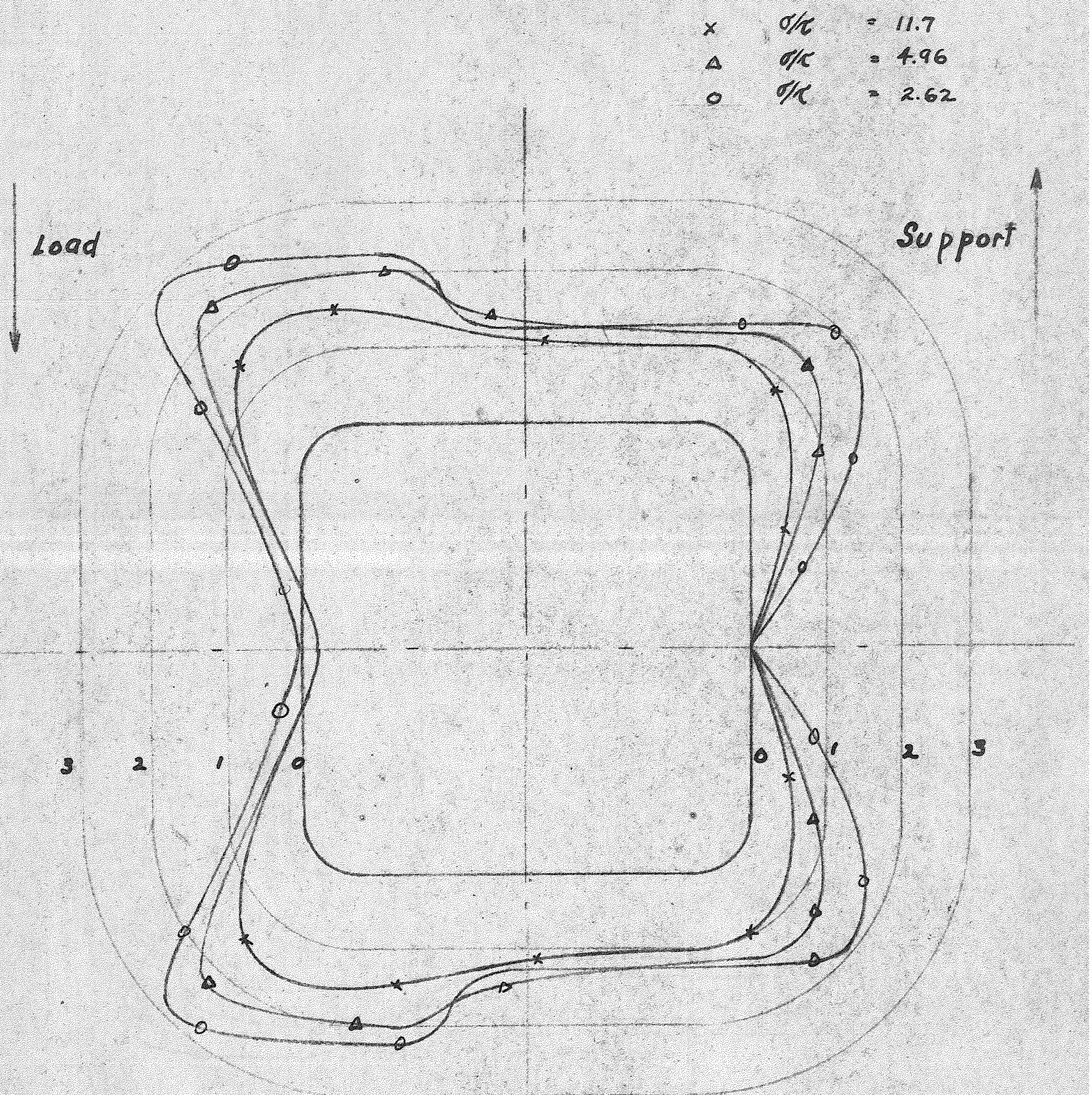
Stress-Ratio Diagram
1" Hole 3/8 inch fillet
Fig. 4



Stress-Ratio Diagram

1" Hole $\frac{1}{4}$" Fillet

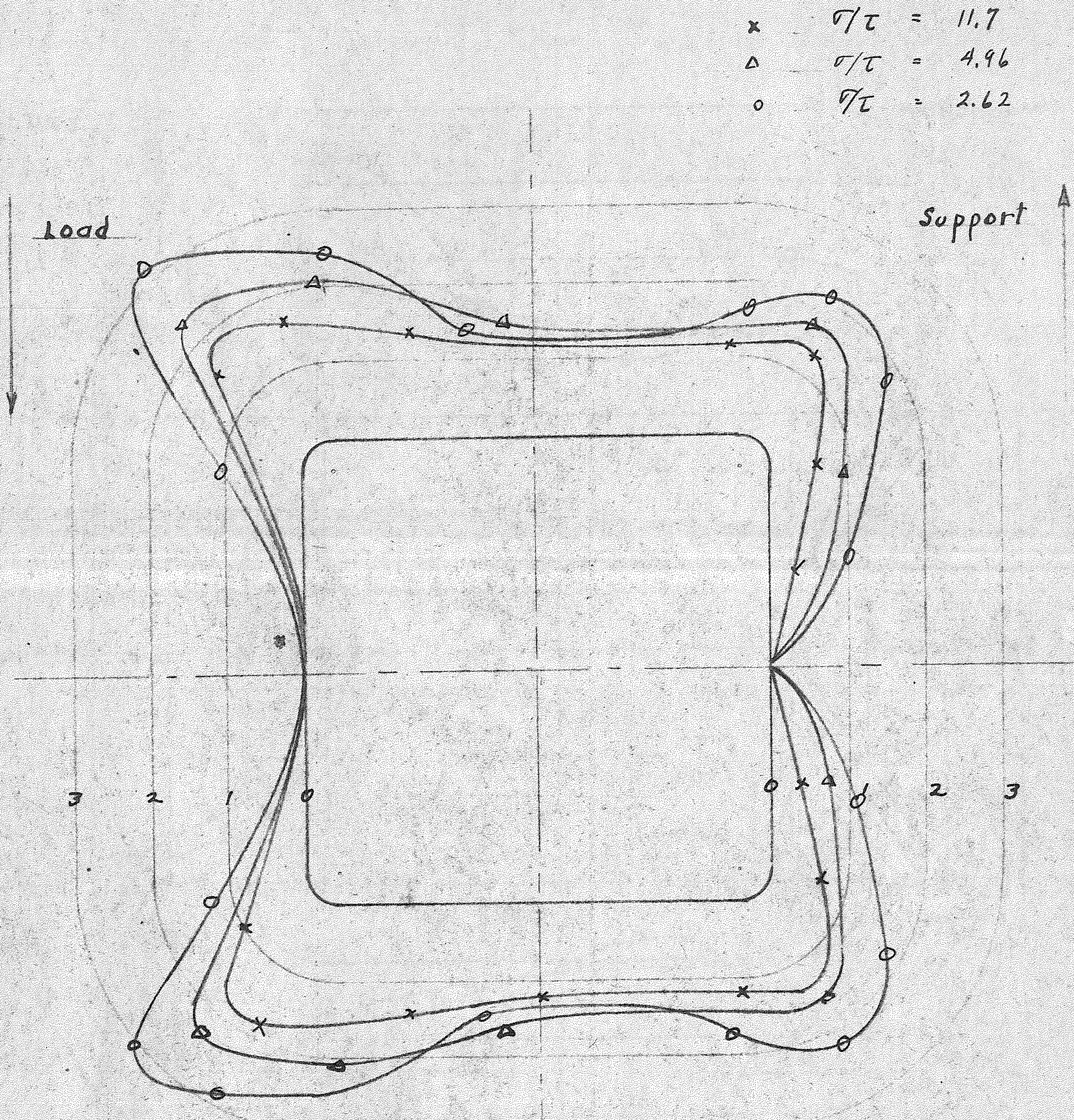
Fig. 5



Stress-Ratio Diagram

1" Hole $\frac{1}{8}$" Fillet

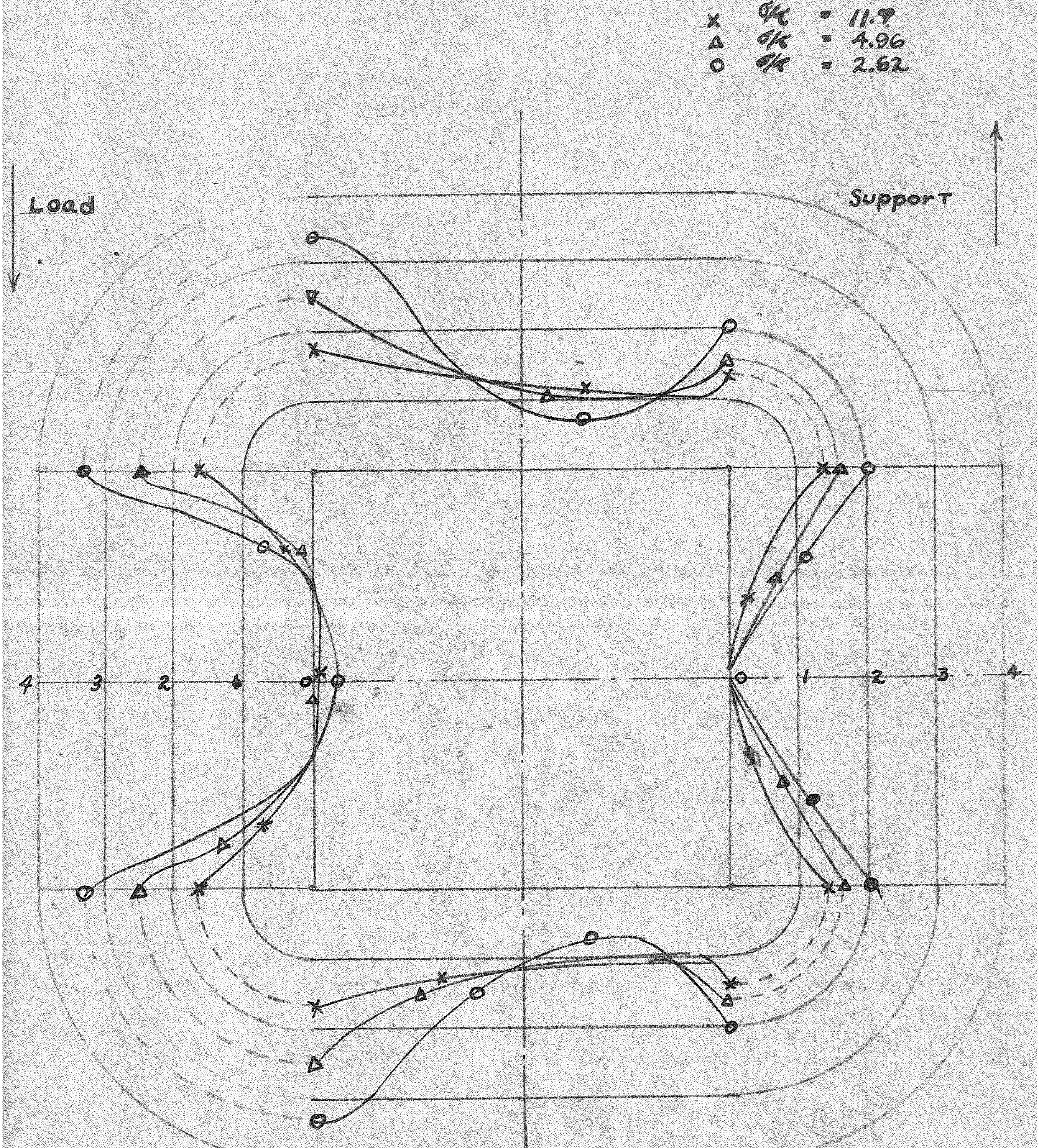
Fig. 6



Stress-Ratio Diagram

1" Hole $\frac{1}{16}$" Fillet

Fig. 7



Stress-Ratio Diagram
1" Square Hole

Fig. 8

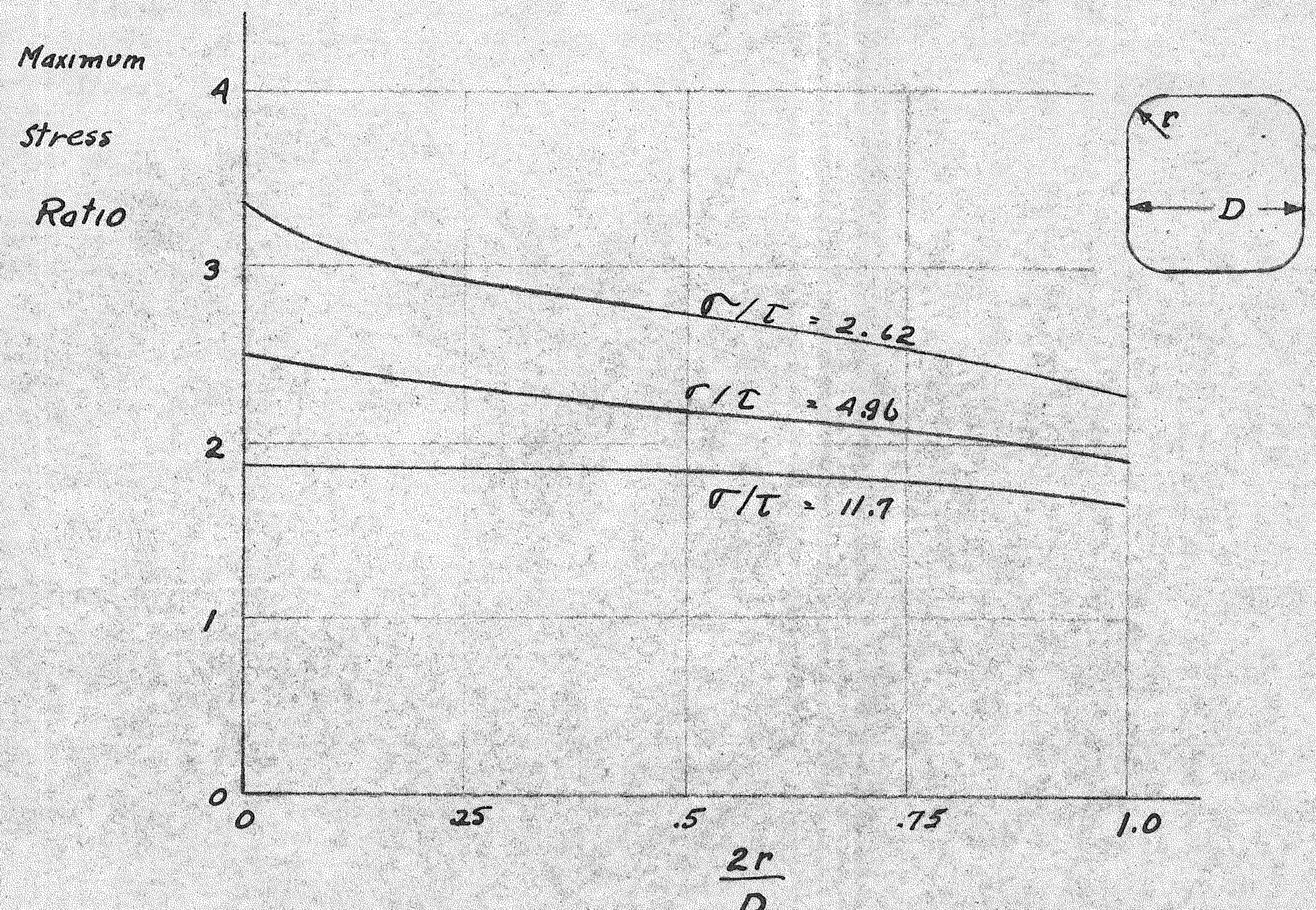


Fig. 9 Stress Ratio vs Fillet Radius

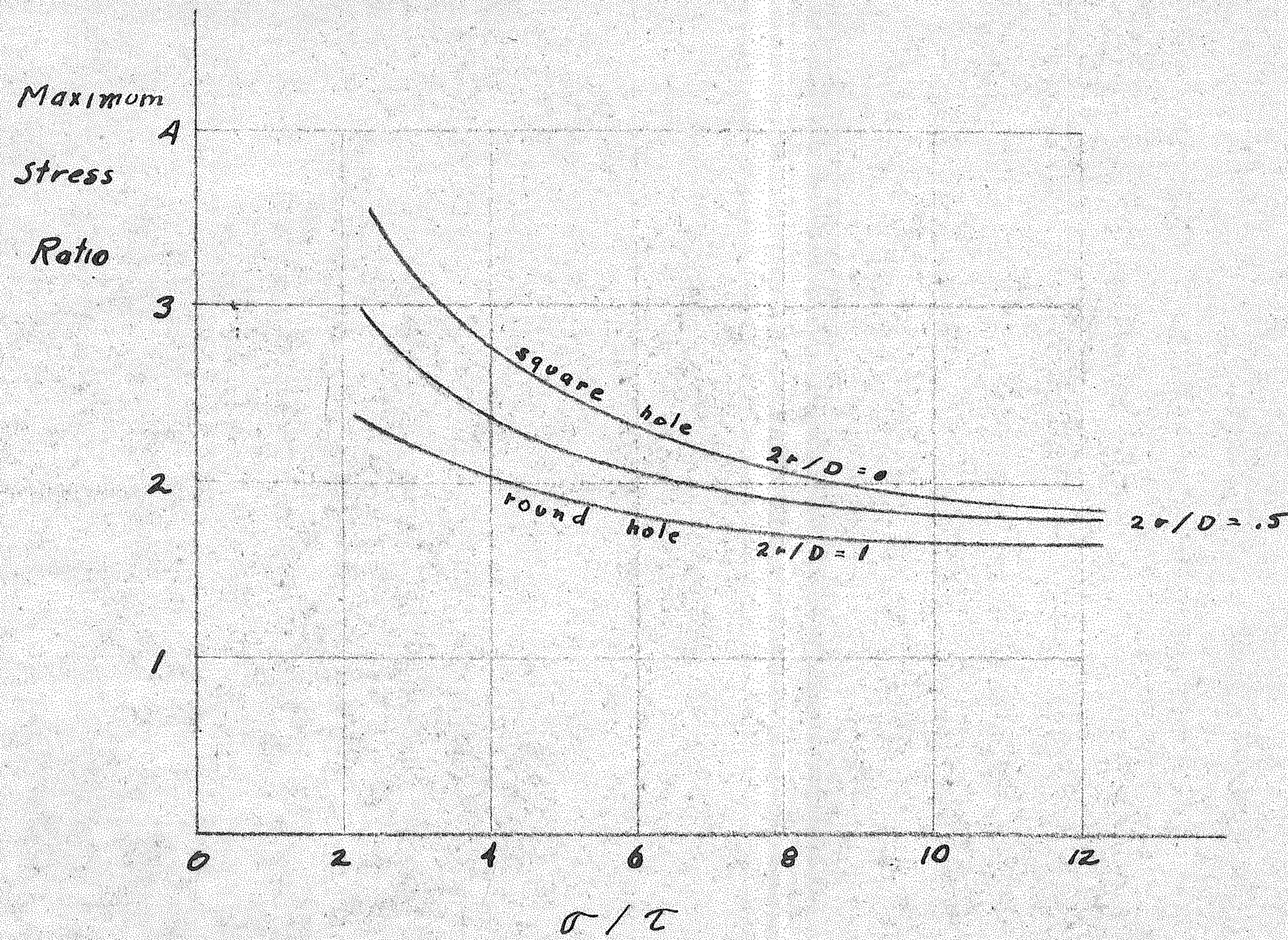


Fig. 10 Maximom Stress Ratio vs σ/τ

PART VII

CONCLUSIONS

A number of interesting results are apparent from an examination of the stress ratio charts. As was expected, the stress concentration at the corners increased as the fillet radius was made smaller. However the increase was not large as theoretical considerations alone might have lead one to anticipate. Thus, at the sharp corners of the square hole a mathematical analysis would indicate an infinite stress. Although such a stress could obviously not have been shown in the fringe pattern, it was thought that some rather high fringe orders might have appeared. However, fringe orders higher than four did not occur. A possible explanation of this phenomena is plastic deformation of the specimen. However, the specimen showed no apparent residual stresses or plastic deformation as a result of the application of the load.

Another interesting result is brought out in figure 10. Here, the manner in which the stress concentration increases as the proportion of Bending Stress to Shear Stress becomes smaller, is clearly demonstrated. Also evident, is the fact that at large values of stressdue to bending, the size or shape of the fillets has but little effect upon the stress concentration. However, as the proportion of shear stress increases, the stress concentration becomes progressively worse for the small filleted and square cornered holes.

Perhaps the most unexpected development indicated by the curves is that in all cases the maximum stress occured at that corner of the hole which was closest to the applied load. At this corner the bending moment was smaller than at any other part of the hole. It should also be noted that the maximum values of the stress decreased rapidly as distance from the point of maximum stress was increased.

In carrying out the investigation five different values of bending stress to shear stress were used. In plotting the stress-ratio curves for the various shape holes it was found that the ratios for positions #2 and #3, having values of Bending Stress to Shear Stress of 9.5 and 7.4 respectively, fell in between the curves for positions #1 and #4. For the sake of clarity curves for positions #2 and #3 were omitted from figures 3 to 8. Photographs of the isochromatics for these loadings are included, however, and an inspection will show that the variations in pattern are small from loading No. 1 to 3. Figure 10 also shows this as the curves tend to flatten out above a σ/τ ratio of about 6.

In general it isbelieved that the accuracy of the results is within the limits of ordinary engineering computation. In determining fringe orders for the various loadings duplicate tests were made on various specimens to check original data, since the determination of the fringe order was the basis of the investigation. It must be remembered that the engineering value of the curves obtained is restricted to the problem of panels under combined bending and shear only

Extrapolation of the data to conditions other than those investigated should be done with the utmost circumspection.

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