A Method for the Representation and Manipulation of Uncertainties in Preliminary Engineering Design

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Abstract

Each stage of the engineering design process, and particularly the preliminary phase, includes imprecision, stochastic uncertainty, and possibilistic uncertainty. A technique is presented by which the various levels of imprecision (where imprecision is: *"uncertainty in choosing among alternatives"*) in the description of design elements may be represented and manipulated. The calculus of Fuzzy Sets provides the foundation of the approach. An analogous method to representing and manipulating imprecision using probability calculus is presented and compared with the fuzzy calculus technique. *Extended Hybrid Numbers* are then introduced to combine the effects of imprecision with stochastic and possibilistic uncertainty. Using the results, a preliminary set of metrics is proposed by which a designer can make decisions among alternative configurations in preliminary design.

In general, the hypothesis underlying the techniques described above is that making more information available than conventional approaches will enhance the decision-making capability of the designer in preliminary design. A number of elemental concepts toward this hypothesis have been formulated during the evolution of this work:

- Imprecision is a hallmark of preliminary engineering design. To carry out decisions based on the information available to the designer and on basic engineering principles, the imprecise descriptions of possible solution technologies must be formalized and quantified in some way. The application of the fuzzy calculus along with a fundamental interpretation provides a new and straight-forward means by which imprecision can be represented and manipulated.

- Besides imprecision, other uncertainties, categorized as stochastic and possi-

bilistic, are prevalent in design, even in the early stages of the design process. Providing a method by which these uncertainties can be represented in the context of the imprecision is an important and necessary step when considering the evaluation of a design's performance. Extended Hybrid Numbers have been introduced in this work in order to couple the stochastic and possibilistic components of uncertainty with imprecision such that no information is lost in the process.

- Because of the size, coupling, and complexity of the functional requirement space in any realistic design, it is difficult to make decisions with regard to the performance of a design, even with an Extended Hybrid Number representation. Defining and utilizing metrics (or figures of merit) in the evaluation of *how well* a design meets the functional requirements reduces the complexity of this process. Such metrics also have merit when we begin to think of languages of design and adding the necessary pragmatics of *"will a generated or proposed design satisfy the performance requirements with respect to the ever-present and unavoidable uncertainties?"*.

These concepts form the central focus of this work. The mathematical methods presented here were developed to support and formalize these ideas.

Contents

Acknowledgements ii

Abstract iii

List of Figures xvi

List of Tables xviii

1 Introduction 1

1.1 Summary and Hypothesis 1

1.2 Terminology and Mathematical Model 2

1.2.1 Design Definitions 2

1.2.2 Implementation Definitions 3

1.2.3 Mathematical Model 4

1.3 Problem Description and Instantiation 5

1.3.1 Imprecision: A Hallmark of Preliminary Design 5

1.3.2 Uncertainty in Engineering Design 6

1.4 A Delineation of Specific Goals 9

1.4.1 Design Problem Objectives 9

1.4.2 Computational Considerations 9

1.5 Organization of Thesis 10

2 The Imprecision Problem in Engineering Design **14**

2.1 Background 14

2.1.1 Representation and Interpretation of Imprecision 15

2.1.2 Existing Techniques 20

2.2 Approach 23

2.2.1 Fuzzy Arithmetic Operations 23

2.2.1.1 Analytical and Numerical Applications 24

2.2.2 Preference Function Shapes for Design Parameters 27

2.2.3 A Design Measure 29

2.2.3.1 Measure of Fuzziness 31

2.2.3.2 The γ-Level Measure 32

2.3 Example 36

2.3.1 Performance Specifications 38

2.3.2 Input Design Parameters 39

2.3.3 Output Performance Parameters 40

2.3.4 Applying the γ-Level Measure 49

2.3.5 Discussion 50

3 Discrete and Multiple Imprecision **52**

3.1 Introduction 52

3.2 Discrete Design Parameters 52

3.2.1 Two Representations for Discrete DPs 53

3.2.2 Discrete Frame Example 59

3.2.3 Combining Continuous and Discrete DPs; The γ Level Measure 63

3.3 Multiple-Source Imprecision 66

3.3.1 Combining Multiple Imprecision 66

3.4 Remarks on the Extensions 74

4 An Application of the "Semi-Automated" Approach to the Imprecision Problem 76

4.1 A Brake Design Example 76

4.1.1 Brake Example Nomenclature 76

4.1.2 The Problem Statement 78

4.1.3 Performance Specifications 84

4.1.4 Input Design Parameters 85

4.1.5 Output Performance Parameters 87

4.1.5.1 Drum Brake Output Performance Parameters 89

4.1.5.2 Disk Brake Output Performance Parameters 96

4.1.6 Applying the γ-Level Measure 105

4.1.6.1 Drum Brake γ-Level Measure 108

4.1.6.2 Disk Brake γ-Level Measure 108

4.1.7 Execution Times 112

4.1.8 Discussion 112

4.2 Conclusions 115

5 A Comparison of Fuzzy and Probability Calculus for Representing Imprecision 117

5.1 Introduction 117

5.2 The Probability Approach 118

5.2.1 Probability Function Operations 119

5.2.2 The Probability Interpretation 120

5.2.3 Analytical Application of the Probability Approach 123

5.2.4 Numerical Application of the Probability Approach 125

5.3 Examples 127

5.3.1 Example 5.1: Analytical Addition 127

5.3.2 Example 5.2: Numerical Addition 128

5.3.3 Example 5.3: Linear Equation 130

5.3.4 Example 5.4: Trigonometric Operations 132

5.3.4.1 Sine Operation 134

5.3.4.2 Cosine Operation 134

5.3.5 Example 5.5: Beam Shear Stress 136

5.3.6 Example 5.6: Torque for a Drum Brake 139

5.4 Discussion 144

5.5 Conclusions 147

6 Combining the Effects of Imprecision with Stochastic and Possibilistic Uncertainties 148

6.1 Introduction 148

6.2 Combining Imprecision With Other Uncertainties 148

6.2.1 Introducing Extended Hybrid Numbers 150

6.2.2 Computations with Extended Hybrid Numbers 152

6.2.2.1 The Stochastic Component of Uncertainty 153

6.2.2.2 A Stochastic Parameter Measure 155

6.2.2.3 Outline of Calculation Procedure 156

6.3 A Machine Design Example 162

6.3.1 Performance Expressions 162

6.3.2 Performance Specifications 167

6.3.3 Specifying Input Design Parameters 168

6.3.4 Output Performance Parameters 169
6.3.4.1 Spur Gear Output Performance Parameters 169
6.3.4.2 Helical Gear Output Performance Parameters 182
6.3.4.3 V-Belt Output Performance Parameters 183
6.3.5 Applying the γ-Level Measure 192
6.3.5.1 Spur Gear γ-Level Measure 196
6.3.5.2 Helical Gear γ-Level Measure 196
6.3.5.3 V-Belt γ-Level Measure 197
6.3.6 Measuring the Stochastic Contribution 197
6.3.7 Discussion 198
6.4 Conclusions 202

7 A Design Figure of Merit **204**
7.1 Introduction 204
7.2 A Design Figure of Merit 205
7.2.1 Example Use of the Design Figure of Merit 208
7.3 Functional Coupling: An Imprecision Approach 212
7.3.1 Functional Coupling 212
7.3.1.1 The Imprecision Extension 216
7.4 Discussion 217

8 Conclusions and Future Directions **218**
8.1 General Conclusions 218
8.2 Future Work 220
8.2.1 Current Research in the Design Community: Tendencies Toward a Mechanical Design Compiler 220

8.2.2 Transforming Continuum Design Descriptions 222

8.2.3 Representing Imprecise Functional Requirements and Constraints222

8.2.4 Developing a Formalism for the Imprecision Problem 223

8.2.5 Developing Utility Functions for the Imprecision Problem . . . 224

8.2.6 New Directions 224

A Appendix: Fuzzy Arithmetic 227

A.1 Fuzzy Set Concepts 227

A.1.1 Definitions 228

A.2 Fuzzy Number 229

A.2.1 Definitions 232

A.3 Operations for Fuzzy Numbers 233

A.3.1 Definitions 234

A.4 Miscellaneous Topics 239

A.4.1 Definitions 239

B Appendix: Derivation of the Probability Operations 241

B.1 Operations with Probability Density Functions 241

B.1.1 Addition 242

B.1.2 Subtraction 243

B.1.3 Multiplication 243

B.1.4 Division 243

B.1.5 Sine 244

B.1.6 Cosine 244

B.2 Operations with Cumulative Distribution 245

B.3 Powers of Uncertain Parameters 246

B.3.1 Case I: $x > 0$ 246

B.3.2 Case II: $x < 0$ 247

B.3.3 Alternative Derivation 248

C The Computational Model for Design Uncertainties **249**

C.1 Introduction 249

C.2 Computation of the Extension Principle 249

C.2.1 The FWA Algorithm 250

C.2.1.1 Implementation Scheme for FWA 254

C.2.1.2 Extending FWA to Numbers of Type 2 257

C.2.2 Revising the FWA Approach 258

C.2.2.1 Reducing the Complexity 258

C.2.2.1.1 Interval Analysis Definitions 258

C.2.2.1.2 Applying the Methods of Interval Analysis . 263

C.2.2.2 Extending FWA for Internal Extrema 266

C.3 Remarks on the Computational Model 267

D A Model of Engineering Design as a Process **268**

D.1 Summary 268

D.2 Introduction 268

D.3 Design Philosophy 270

D.4 A Systematic Design Approach 280

D.4.1 The Becker Model 281

D.4.1.1 Fuzzy Application 281

D.4.2 The Process 285

D.4.2.0.1 Level One 285

D.4.2.0.2 Level Two 285

D.4.2.0.3 Level Three 285

D.4.2.0.4 Level Four 285

D.4.2.0.5 Level Five 285

D.4.2.0.6 Level Six 286

D.4.2.0.7 Level Seven 286

D.4.2.0.8 Level Eight 286

D.4.2.0.9 Level Nine 286

D.4.2.0.10 Level Ten 286

D.5 Information and Decision 287

D.5.1 **Data-Flow Paradigm** 290

D.6 Future Directions 292

D.7 Conclusions 293

D.8 Acknowledgements 294

E Appendix: Calculating the Stochastic Component Output **295**

E.1 An Application of the Probability Operators 295

References **298**

List of Figures

1.1 Design Uncertainties: General Tendencies. 7

2.1 Preference Function Representation of "about 25 cm," with an α-cut at 0.5. 17

2.2 Preference Function Steel Alloy Data. 19

2.3 Locally Convex Function, Three Regions. 28

2.4 Measure of Fuzziness Example. 30

2.5 γ-Level Measure Application. 34

2.6 Design Problem: Frame Configuration. 37

2.7 Input Parameter: W. 41

2.8 Maximum Bending Stress σ. 43

2.9 Column Load F_B. 44

2.10 Euler-Johnson Condition C_{EJ}. 45

2.11 Critical Load P_{cr}. 46

3.1 Discrete Parameters, u_1 and u_2. 55

3.2 Discrete Parameter Representation A. 56

3.3 Discrete Parameter Representation B. 57

3.4 Output for $z = u_1 \odot u_2$. (Representation A (top) and B (bottom).) . 58

3.5 Discrete Parameter Representation A, More Choices. 60

3.6 Discrete Parameter Representation B, More Choices. 61

3.7 Output for $z = u_3 \odot u_4$. (Representation A (top) and B (bottom).) . 62

3.8 Design Parameters: l, t, and w_{AB}. 64

3.9 Discrete Case: Maximum Bending Stress σ. 65

3.10 Discrete/Continuous Case: Inputs u_1 and u_2. 68

3.11 Discrete/Continuous Case: $z = u_1 \odot u_2$. 69

3.12 Discrete/Continuous Case: z (Higher and Lower Resolutions). 70

3.13 Discrete/Continuous Case: Maximum Bending Stress σ. 71

4.1 Brake Problem. 79

4.2 Drum Brake System. 80

4.3 Disk Brake System. 81

4.4 Drum Brake: Actuating Force Output Set, $\tilde{F}_a$. 88

4.5 Drum Brake: Torque Output Set, $\tilde{\mathcal{T}}$. 90

4.6 Drum Brake: Temperature Rise Output Set, $\tilde{T}_r$. 92

4.7 Drum Brake: Temperature Decay Profile Output (t = 1sec), $\tilde{T}_d$. . . . 93

4.8 Drum Brake: Temperature Decay Profile Output (t = 60sec), $\tilde{T}_d$. . . . 95

4.9 Drum Brake: Temperature Decay Profile (t = 60sec, constant T_r), $\tilde{T}_d$. 97

4.10 Performance Specification, $\tilde{\mathcal{T}}^r$. 98

4.11 Disk Brake: Actuating Force Output Set, $\tilde{F}_w$. 100

4.12 Disk Brake: Actuating Force Output Set, $\tilde{F}_p$. 101

4.13 Disk Brake: Torque Output Set, $\tilde{\mathcal{T}}_w$. 103

4.14 Disk Brake: Torque Output Set, $\tilde{\mathcal{T}}_p$. 104

4.15 Disk Brake: Temperature Rise Output Set, $\tilde{T}_r$. 106

4.16 Disk Brake: Temperature Decay Profile Output (t = 1sec), $\tilde{T}_d$. 110

4.17 Disk Brake: Temperature Decay Profile Output (t = 60sec), $\tilde{T}_d$. . . . 111

4.18 Disk Brake: Temperature Decay Profile (t = 60sec, constant T_r), $\tilde{T}_d$. . 113

5.1 Example Input and Output Parameter. 121

5.2 Example 5.1: $z = u_1 + u_2$. 129

5.3 Example 5.2: $z = u_1 + u_2$. 131

5.4 Example 5.3: $z = u_1 \cdot u_2 + u_3$. 133

5.5 Example 5.4: Output Results for z_{sin} and z_{cos}. 135

5.6 Example 5.5: Output τ. 138

5.7 Example 5.6: Output $\mathcal{T}$. 142

6.1 Performance Parameter: p_j. 159

6.2 Cumulative Distribution for p_j^s. 161

6.3 Gear Drive Configuration. 163

6.4 V-belt Drive Configuration. 164

6.5 Shaft Length (L_s) *pdf*. 173

6.6 Brinell Hardness (H_B) *pdf*. 174

6.7 Face Width (w_F) *pdf*. 175

6.8 Failure Stress (S_f) *pdf*. 176

6.9 Tensile Strength (S_t) *pdf*. 177

6.10 Pulley Diameter (d_p) *pdf*. 178

6.11 Spur and Helical Gear: Fatigue Strength Factor of Safety n_f. 179

6.12 Spur and Helical Gear: Surface Durability Factor of Safety n_s. 181

6.13 Spur and Helical Gear: Shaft Diameter (Strength) d_s. 184

6.14 Spur and Helical Gear: Shaft Diameter (Deflection) d_s. 185

6.15 Spur and Helical Gear: Rated Bearing Load C. 186

6.16 V-Belt: Peak Force F_p. 187

6.17 V-Belt: Expected Belt Life L_e. 188

6.18 V-Belt: Shaft Diameter (Strength) d_s. 189

6.19 V-Belt: Shaft Diameter (Deflection) d_s. 190

6.20 V-Belt: Rated Bearing Load C. 191

A.1 Discrete Fuzzy Set fast. 230

A.2 Fuzzy Set fast. 231

A.3 Multiplication of $\tilde{E}$ and $\tilde{F}$. 236

C.1 Example Column Calculation: Exact and FWA. 252

C.2 FWA Pseudo-Code. 255

C.3 Example of d Array, where the "x" values are elements of "d." 256

C.4 Fuzzy Number of Type 2. 259

D.1 Randomness vs. Fuzziness – A Deterministic Relation. 271

D.2 French's Block Diagram of Design. 274

D.3 Asimow's Iterative Approach. 275

D.4 The Tree Structure of Decisions. 277

D.5 A Design Model. 279

E.1 Stochastic Component: V-Belt Shaft Diameter, $d^b_{s,str}$. 297

List of Tables

2.1 Example Problem: Fuzzy Design Parameter Data. 42

2.2 Design Example: "Constant" Data. 42

2.3 γ-*Level* Measure Results: Frame Configuration. 51

3.1 Possible z Values for Combinations of u_1 and u_2. 67

3.2 γ-*Level* Measure Results: Discrete/Continuous DPs for Frame. 67

4.1 Brake Example: Fuzzy Design Parameter Data. 86

4.2 γ-*level* Measure Results for Drum Brake. 107

4.3 γ-*level* Measure Results for Disk Brake. 109

4.4 Disk Brake: γ-*level* Measure Results for T_d. 109

5.1 Beam Example: Design Parameter Data. 140

5.2 Brake Example: Design Parameter Data. 140

6.1 Machine Design Example: Fuzzy Design Parameter Data. 170

6.2 Machine Design Example: Fuzzy Design Parameter Data (cont.). . . . 171

6.3 Machine Design Example: "Constant" Data. 172

6.4 γ-*level* measure results for Spur Gear (Imprecision). 193

6.5 γ-*level* measure results for Helical Gear (Imprecision). 194

6.6 γ-*level* measure results for V-Belt (Imprecision). 195

6.7 Relative Variance Contributions. 201

7.1 Frame Example: Figure of Merit and Weighting Coefficients. 209

7.2 Brake Example: Figure of Merit and Weighting Coefficients. 209

7.3 Transmission Example: Figure of Merit and Weighting Coefficients. . 209

C.1 Design Parameter Data for Column Equation. 253

Chapter 1

Introduction

1.1 Summary and Hypothesis

Engineering design, both in practice and research, is evolving rapidly, especially in the development of computer-based tools. Emphasis is moving from the later stages of design, to computational tools for preliminary design. In Appendix D, a general philosophical approach to computational tools in preliminary engineering design and a model of the design process is described. The global aim of this model is to provide a structure for the development of tools to assist the designer in: managing the large amount of information encountered in the design process; determining a design's functional requirements and constraints; evaluating the coupling between the design parameters; and carrying out the process of choosing between alternative design concepts.

A particular aim of this work is to develop tools to assist the designer in the *preliminary* phase of engineering design, by making more information available on the performance of design alternatives than is available using conventional design techniques. The most important design decisions (and potentially the most costly, if wrong) are made at the preliminary stage. Increased information, over what is available by traditional design methods, will enable these decisions to be made with greater confidence and reduced risk. The effect will be greater, the earlier in the design cycle additional information can be made available.

In order to contribute to a *theory of engineering design* [3] in the context of providing and representing design information, it is necessary to formulate a hypothesis that will lead to a better understanding of design practice by developing morphologies of design, or by improving current analytical or computational design procedures. This research primarily falls in the latter of these categories, where the main objective is to provide methods and tools for the decision-making aspect of *preliminary engineering design.*

1.2 Terminology and Mathematical Model

1.2.1 Design Definitions

Parameter: A variable or quantity used in the design process.

Design Parameter [DP]: Any free or independent parameter whose value is determined during the design process.
(synonyms: Design Variable, Input Parameter.)

Performance Parameter [PP]: Any parameter used in the design process that has a specified value, or range of values, [FR] determined independent of (and usually in advance of) the design process. The performance parameters [PPs] are usually dependent on the design parameters [DPs], and possibly some other PPs.

Output Parameter [OP]: Any parameter used in the design process that is dependent on the design parameters [DPs], and possibly some performance parameters

[PPs], but has no specified functional requirement [FR] value.

Functional Requirement [FR]: A value, or range of values, or fuzzy number that is the specified value for a Performance Parameter [PP].

This value is determined independent of (and usually in advance of) the design process. Each Performance Parameter has an FR.

(synonyms: Performance Specification, Constraint.)

(Note that this distinction between the Performance Parameter and its specified value [Functional Requirement] is to permit a Performance Parameter not to be identically equal to its specified Functional Requirement value at all times during the design process.)

Performance Parameter Expression [PPE]: An expression, relation, or equation relating some or all of the Design Parameters to a Performance Parameter. Can arise from exact, empirical, approximate, or qualitative engineering principles. Each PP has a PPE.

1.2.2 Implementation Definitions

Support: A crisp set of all values of a fuzzy set where the membership is greater than zero. Alternatively: The range of parameter values over which the fuzzy set membership is greater than zero.

Imprecision: The range (support) or spread of values about the peak [functional value of one (1)] of a parameter's preference function (in fuzzy form). The greater the imprecision, the greater the spread on the left or right (or both) sides of the fuzzy function. This is loosely analogous to variance in the stochastic sense.[1] The interpretation of imprecision, as used in design, was introduced earlier and will be discussed further in the next chapter.

1.2.3 Mathematical Model

The terminology presented in the previous section can be cast in the form of a mathematical model of the method proposed in this document. Such a model has the following advantages: it helps to enumerate a formalism for the method; consistency and understanding of the notation can be obtained.

The problem this document considers can be expressed formally as follows: Given a design universe $\mathcal{U}$ and a possible solution configuration (*i.e.*, design alternative) $\mathcal{A}$ in $\mathcal{U}$, there exists m design parameters $d_i, i = 1, 2, \ldots, m$ (where the d_i are sometimes referred to as u_i to underscore their uncertain nature) lying in an m-dimensional design parameter space $\mathcal{D}$ such that $\mathcal{A}$ is defined uniquely by specifying $\vec{d} = \{d_1, d_2, \ldots, d_m\} \in \mathcal{D}$. The associated performance of $\mathcal{A}$ can be specified by n performance parameters $p_j, j = 1, 2, \ldots, n$ (also denoted by z for the case of a single performance parameter and the deriving of certain mathematical operations), where $\vec{p} = \{p_1, p_2, \ldots, p_n\} \in \mathcal{P}$. $\mathcal{P}$ represents the n-dimensional performance parameter space. The p_j are related to the d_i through some relationship (a performance expression) $p_j = f_j(d_1, \ldots, d_n)$, *i.e.*, $\vec{p} = \vec{f}(\vec{d})$. To determine whether $\vec{d} \in \mathcal{D}$ gives a feasible design, a check must be made whether $\vec{p} = \vec{f}(\vec{d}) \in \mathcal{R} \subset \mathcal{P}$, where $\mathcal{R}$ is the functional requirement space.

[1] The mathematics of fuzzy sets are different from the mathematics of probability, and the fuzzy calculus is more well-suited to solving *imprecise* problems in the preliminary phase of design. Probability continues to be most appropriate for representing and manipulating the *stochastic* aspects of design problems. A comparison of probabilistic and fuzzy methods in design will be addressed in Chapter 5. Many design problems will require both methods.

1.3 Problem Description and Instantiation

1.3.1 Imprecision: A Hallmark of Preliminary Design

Engineering design, as a process, embodies many functions: analysis of requirements, concept generation, concept evaluation and refinement, evaluation of imprecise descriptions of simplified versions of the design, judgement of design feasibility, embodiment design, detail design, etc. [29, 30, 43, 44, 59, 67]. The concept generation and simplification processes will not be addressed by the research reported here; rather, the aim is to provide a technique for representing, manipulating, and evaluating the approximate, or imprecise, descriptions of the (preliminary) design artifact.

Once several concepts have been generated, the design process can be viewed as one of reducing the uncertainty with which each design alternative is described. At the preliminary stage, the designer is not certain what values will be used for each design parameter. Instead, the design parameters are usually given in terms of "approximate" (imprecise) values within a certain range. Uncertainty of this type may be referred to as *uncertainty in choosing among alternatives* (imprecision). Typical examples of imprecise descriptions in preliminary design include: an irregular cross-section structural member might be represented by a rectangular section for the purposes of initial evaluation; a gear set might be represented by a pair of circles rolling on each other (without slip), and an approximate speed ratio; a length of shaft might be represented as "about 25 cm"; etc. These are approximate, or *imprecise*, descriptions of the design artifact, not incomplete descriptions. The gear set, imprecisely represented above, has all of the *functional* attributes of a gear set, but none of the detail (gear teeth lengths, pitch, etc.).

As the design process proceeds from the preliminary stage to more-detailed design and analysis, the level of imprecision in the description of the design artifact is reduced. Naturally at the end of the design cycle, the level of imprecision is very small, although other uncertainties (*e.g.*, tolerances) usually remain. It is this spectrum of levels of precision (see Figure 1.1) that characterizes progress through the design process, from a description of a need, to a (precise) description of a device to fulfill that need.

While the description and quantification of imprecision used in this study pertains *directly* to the case of parameterizations of design alternatives (as introduced in the next section), the global discussion and representation of imprecision transcends the parametric view. It will be shown in future work that the general technique of representing imprecision can be expanded to higher level configuration design or other stages in the design process.

1.3.2 Uncertainty in Engineering Design

Figure 1.1 separates, in a new and evolutionary manner, the phenomenon of uncertainty in engineering design into a three-component structure. Uncertainty in choosing among alternatives (imprecision) is directly distinguished from stochastic and possibilistic uncertainty in the figure, where the ordinate represents a qualitative magnitude of uncertainty as a function of the time progression of the design cycle. (Note: the curves in Figure 1.1 describe the general trends of the magnitude of uncertainty for illustration only; they are not meant to precisely characterize uncertainty at each point in the design process.) As described above, an imprecise parameter in preliminary design is a parameter that may potentially assume any value within a possible range because the designer does not know, *apriori*, the final value that will emerge from the design process. The value of the coefficient of friction for a

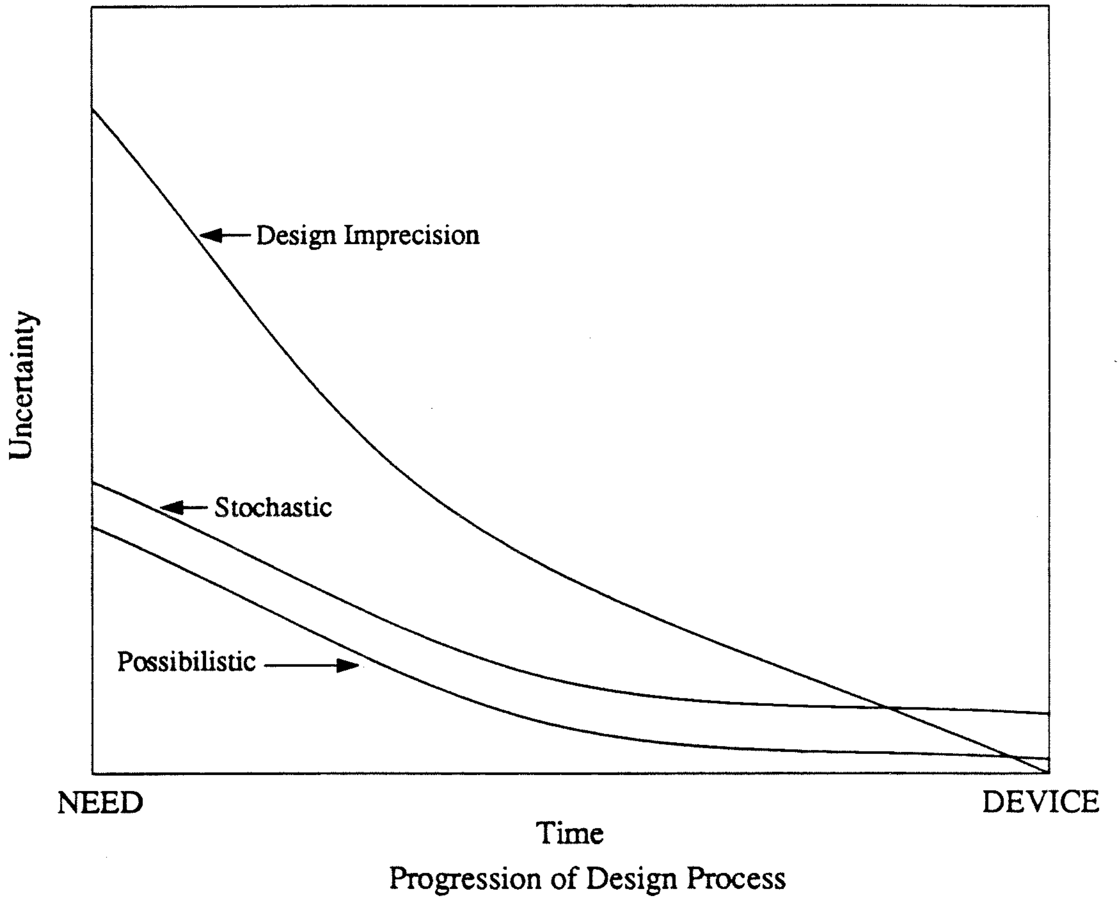

Figure 1.1: Design Uncertainties: General Tendencies.

vehicle's brake design is an example of an imprecise parameter. Even though the designer is uncertain about the final value of the coefficient of friction (depending on a set of possible materials), he or she usually has a preference or desire for choosing certain materials over others (in terms of cost, material properties, and so on). The additional (subjective) preference information can be exploited to quantify the imprecision with which the design parameters are known (in the preliminary phase of engineering design). The actual method of representing, interpreting, and manipulating this quantification of imprecision is referred to in this study as the *imprecision problem.*

Besides imprecision, there exist two other separable components of uncertainty in engineering design: stochastic and possibilistic. Engineers are usually most familiar with stochastic uncertainty. It consists of uncertainty in truth (plausibility), and arises from a lack of knowledge of a parameter due to some process the designer has no direct control or choice over. Using the coefficient of friction example once again, stochastic uncertainty enters because a particular material's coefficient of friction can be predicted (subjectively or objectively) only within certain limits, *i.e.*, a distribution of values. Uncertainties of this type are usually represented and manipulated with a quantification of plausibility using multi-valued probability logic.

Possibilistic uncertainty also consists of uncertainty of truth; however, in certain design situations, it is easier and more realistic to determine the possibility a parameter will assume a certain value, than the probability of this. The possible operating conditions (oil, weather, debris, etc.) for a brake design (coefficient of friction), for example, may be subjectively predicted and quantified. Chapter 6 will develop further the need to include possibilistic uncertainty in our model, instead of using multi-valued probability to quantify all uncertainties in truth.

Even though these two latter uncertainty components do not make up as large a magnitude of uncertainty as imprecision in preliminary design, they are of course very important when making decisions concerning the performance of design alternatives. Coupling the effects of stochastic and possibilistic uncertainties with imprecision represents a vital addition to the imprecision problem. This research addresses both the imprecision problem and this addition as they pertain to the hypothesis of enhancing the designer's decision-making capability.

1.4 A Delineation of Specific Goals

1.4.1 Design Problem Objectives

An approach for representing and manipulating the uncertainties encountered in design will be described. This method, developed to aid the designer in making decisions in the the preliminary phase of design, has the following objectives: (1) to determine the performance parameters for alternative designs including the designer's subjectivity concerning the choice of input design parameter values (the imprecision problem); (2) to determine the interaction of the input parameters with respect to the output (coupling and importance of inputs); (3) to rate the performance of each alternative design; (4) to compare the major differences among alternative designs; and (5) to include, without loss of information, the effects of other uncertainties in the context of the imprecision. Computational efficiency of the design technique is a final objective.

1.4.2 Computational Considerations

It has been difficult to provide computational tools for the preliminary phase of the design process, largely because of the relative paucity of algorithms and techniques that can operate on imprecise data. Solid modeling, optimization, mechanism analysis, and other CAD methods all require a highly precise representation of the objects being designed. This document presents a novel (to the engineering design process) application of a method for representing and manipulating imprecision.[2] A set of computational tools make up the foundation of this technique such that the designer is able to rate a design alternative according to its merit in relation to the others under consideration.

These computational tools are for use in the preliminary and conceptual synthesis stages of design, but do not attempt to supplant the designer. The idea is not to fully automate the design process, nor to automatically generate design alternatives, rather it is to make it easier for the designer to evaluate more alternatives in less time, and to provide more information on the performance of each of those alternatives. These developments form a *semi-automated* approach to design, where the emphasis is on computational efficiency, *i.e*, near real-time results for the design problem objectives.

1.5 Organization of Thesis

This thesis presents, chapter by chapter, a progression of the central ideas of the research in much the same way that the work was completed chronologically. Instead of attempting to describe a unified model for the three types of uncertainties from the beginning chapters, the imprecision problem, being the most significant in preliminary design, is considered first in Chapters 2 through 5. Stochastic and pos-

[2] Fuzzy sets have been applied to other domains including: seismic risk analysis [20, 18, 19, 21, 38], optimization [26, 60], reliability [50, 85], expert systems [52], logic and decision support [7, 8, 9, 10, 33, 37, 84, 91, 89], language and grammar [33, 40, 93], and others.

sibilistic uncertainties are added to the model in Chapter 6, and details related to decision making for complex designs, *i.e.*, designs with many design and performance parameters, follow in Chapter 7. A precise outline of this thesis is presented below:

- Chapter 2 introduces the imprecision problem, including background information on existing techniques for handling this type of uncertainty. A new and novel means of representing, interpreting, and manipulating imprecision is also presented, along with a design measure for determining the coupling and significance of design parameters. The chapter concludes with a simple frame example, illustrating the imprecision calculation technique.

- While Chapter 2 concentrates on a global means for modeling imprecision, only ideal, continuous, and single-source functions are used for modeling purposes. Because the activity of engineering design is by no means ideal, Chapter 3 addresses the problems of discrete design parameter data, multiple-source imprecision, and normalizing the preference information over the set of imprecise DPs. This chapter is not a central component in the understanding of the methodology; however, it does address relevant problems for any realistic design pursuit.

- Chapter 4 presents a real-world brake design problem using the approach of Chapter 2. Two primary alternatives are considered: an internally-actuated pivoted rim brake and a disk brake.

- Although Chapter 2 discusses possible existing techniques for representing and manipulating imprecision, no detailed comparison of other methods with the fuzzy calculus approach, adopted in this research, is given. Chapter 5 compares an analogous probability approach to the fuzzy technique, due to probability theory's wide use and applicability in related domains.

- Chapter 6 introduces *Extended Hybrid Numbers* to combine stochastic and possibilistic uncertainties with imprecision. Computational considerations for the stochastic component are discussed in this chapter, with an application in the form a of single-speed transmission design.

- The first six chapters combine to form a general semi-automated approach for making preliminary design decisions under uncertainty. For complex designs with varying degrees of coupling and conflicting performance parameter results, a metric (or figure of merit) is needed to provide an overall rating of design alternatives. Chapter 7 presents a preliminary metric for this purpose, and a design matrix for determining the coupling effects.

- Chapter 8 concludes the main body of this thesis with a discussion of the relevance of the research, how this work fits into the anatomy of current and existing design research, and possible future projects that exist as natural outgrowths of this research.

- Appendix A highlights certain fuzzy mathematical concepts used in this thesis.

- Appendix B derives the necessary probability density function operations used in Chapter 5. The derivation of the cumulative distribution form of these operations, as needed in Chapter 6, is also given.

- Appendix C presents, in detail, the algorithms and computation requirements of the techniques presented in Chapters 2 through 7.

- Appendix D contains a version of an earlier paper published by the author. It is included for completeness, as it is referenced in the thesis.
- Appendix E demonstrates the use of the cumulative distribution form of the probability operations, with an example from Chapter 6.

Chapter 2

The Imprecision Problem in Engineering Design

Chapter 1 discussed the various types of uncertainty encountered in engineering design. This chapter will discuss one type in particular: imprecision. Background information concerning the imprecision problem will be presented, including a representation and interpretation of imprecision, and how it relates to other possible techniques. An approach for modeling imprecision follows the background section, and the chapter concludes with a simple design example.

2.1 Background

Most of engineering, particularly design, can best be represented with some level of imprecision or approximation. According to Goguen [39]:

> *"Fuzziness is more than the exception in engineering design problems: usually there is no well-defined best solution or design."*

The imprecision that is being represented and manipulated by the technique reported here is meant to capture the approximations made during the early phases of engineering design. Chapter 1 introduced certain types of approximations, with particular emphasis on parametric design. (The general idea of representing imprecision is not only useful for this parametric view, but also has applications in configuration design and other stages of the design process.) Continuing in the theme of Chapter 1, it is common that geometry and other physical characteristics are described approximately. The length of a beam, for example, may be approximated as "about five meters," or an irregular three-dimensional object may be represented with a simplified geometry, such as a cube or sphere with varying dimension or diameter, for preliminary analysis. Imprecision is also common in the specification of material properties. The material used for a brake shoe lining may be described as "having a coefficient of friction (μ) of approximately 0.4," where there exists a number of choices of materials with μ values distributed about 0.4. Because these imprecise descriptions are a natural consequence of the simplifications and approximations made in the preliminary design, a method must be devised for representing and interpreting the varying degrees of imprecision.

2.1.1 Representation and Interpretation of Imprecision

A simple range might be used to represent the imprecision for a parameter. This is the technique used in interval analysis [57]. Instead of a range, the imprecise parameter will be represented by a range and a preference function to describe the desirability of using that particular value within the range. This preference function is similar to the notion of a *fuzzy set*, or more specifically a fuzzy number restricted to the set of real numbers.

A *fuzzy set* (as developed by Zadeh [87]) is a set with boundaries that are not sharply defined. Membership in the set is not the customary 0 or 1, but can be described by a continuum of grades of membership. In the approach described here, preference values are used, analogous to membership, to represent imprecision or approximation of engineering design parameters. For example, a designer may want to represent a dimension of "about 25 cm." He or she would do so by specifying a fuzzy preference function to represent that approximate parameter.

The first step is to decide the range of values that the parameter may assume. Values less than the low end of the range, and greater than the high end of the range will have a preference of zero (0) in the fuzzy representation. For example, there may be a restriction on the dimension to be being greater than 20 cm, and the designer may wish to keep it shorter than 30 cm. The value, or values, that the designer feels the greatest confidence in using, or desire to use, are assigned a preference of one (1). Certainly 25 cm will have a preference of 1 (one) in the preference function: "about 25 cm," and values away from 25 will have lower preference, as shown in Figure 2.1. Preference is assigned depending on the designer's desires to use those parameter values. The more confident, or the more the designer desires to use an input value, the higher its preference in the parameter's set. The resulting function of this process is a quantification of design preference, and not the usual notion of membership in a symbolically labeled fuzzy set, which usually denotes vagueness in meaning. In this way parameters whose values are not known precisely can be represented (and manipulated), and the designer's experience and judgement can be represented and incorporated into the design evaluation.

Therefore imprecision is interpreted as representing the designer's desire to use a particular value for a design parameter. Naturally these desires may change as the design proceeds, and this is easily accomplished using fuzzy preference functions. This evolution of knowledge, desire, and emphasis is a common element of the design process, and the technique reported here permits their representation and manipulation.

In the example given above, Figure 2.1, the input preference function depends

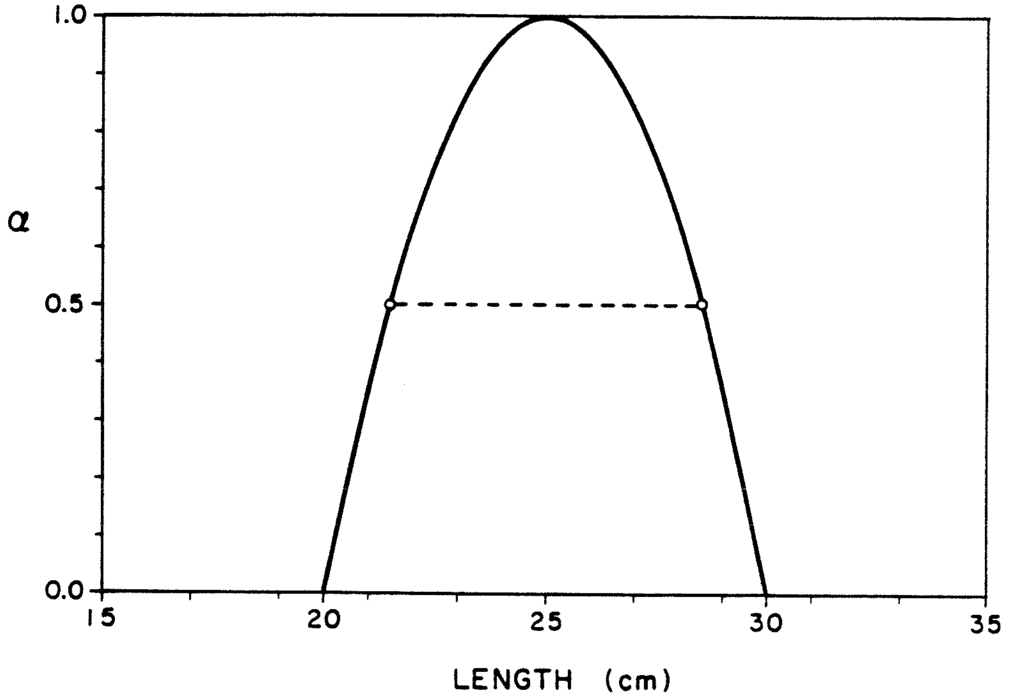

Figure 2.1: Preference Function Representation of "About 25 cm", with an α-cut at 0.5.

solely on the subjectivity of the designer. Preference functions need not always be dependent in this way; engineering data can also be used. For example, a variety of materials might be used, and the preference of the designer is to minimize cost, solubility, or some other measurable material property (or any combinations of these). If the cost or other material data are available, the preference function can be easily constructed by normalizing the data between zero and one, and interpolating a curve between the data points (a method for handling discrete data will be given in the following chapter). Figure 2.2 is an example preference function constructed from the cost data for certain steel alloys, where the designer has specified a preference of minimum cost.

The *desirability* interpretation, as discussed above, applies to input DPs (those parameters whose value the designer is free to choose). Target values for Performance Parameters are specified by Functional Requirements, not by the designer's desires (at least not in the same sense as the input DPs). Performance Parameters, resulting from calculations with imprecise input Design Parameters, will also be represented by fuzzy preference functions. These preference functions also represent the designer's desires, but in a slightly different way. The output parameter value with a preference of 1 (one) corresponds to the input values with preference of 1. This is a natural consequence of calculations with the Fuzzy Calculus [33, 46, 94]. This implies that if the designer's desires are met (inputs with preference of 1), then the performance will be the output value with preference of 1. Correspondingly, if the performance parameter output value with preference of 1 satisfies the Functional Requirement(s), then the designer can use the input Design Parameter values with preference of 1. If it is required to use an off-peak value for the performance (to satisfy a Functional Requirement), then either the designer's desires must be adjusted, or input values

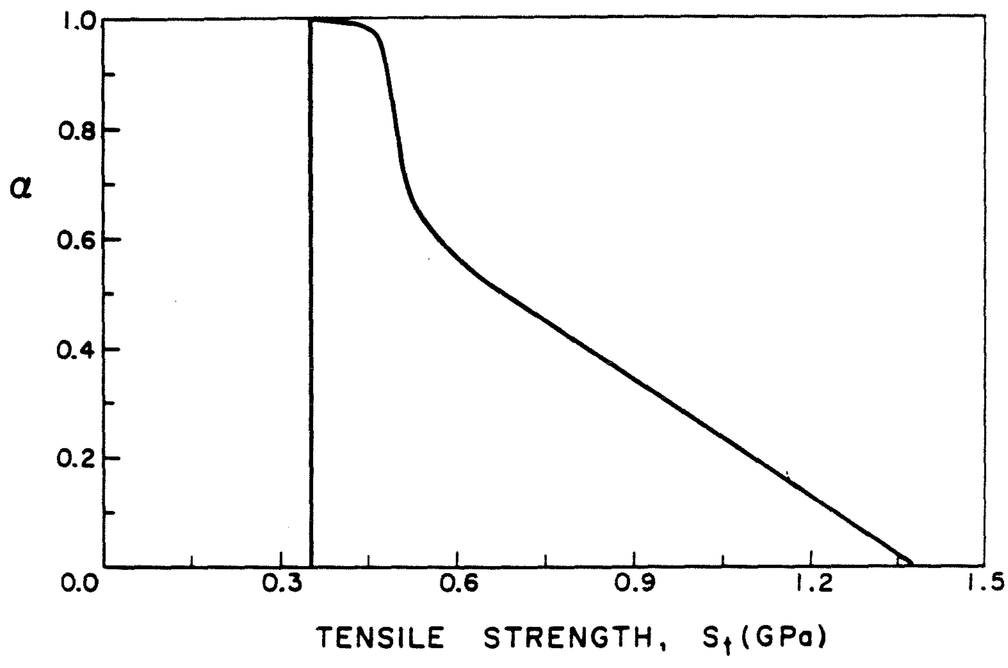

Figure 2.2: Preference Function Steel Alloy Data.

other than the most desirable must be used. This will be discussed in detail below.

2.1.2 Existing Techniques

There exists a variety of means by which imprecise parameters can be represented and manipulated in engineering design calculations. The most basic approach is to choose single (crisp, non-fuzzy) values for each of the parameters, substitute these into the governing equations, and record the crisp single-valued output. This method benefits from simplicity, but suffers from the time required to "explore" any real design space.

Optimization schemes potentially provide a means for handling imprecise parameters. These methods include direct search methods such as Simplex and three-point equal-interval searches, gradient methods such as Newton's and the Conjugate Gradient search [62]. However, conventional optimization methods require precise representations and analyses, and are therefore most useful in the latter stages of design. A. Diaz [26, 27, 28] is developing an optimization technique using imprecise (fuzzy) constraints. This method will be useful for solving imprecise optimization problems, but will not provide as much information on the performance of a design operating over a range of design parameters as the method reported here.

Interval analysis [57] is another method for carrying out computations with imprecise parameters. In this technique an interval (a range of numbers represented by its boundaries) is used to represent a DP in the design calculations. The output (PP) is similarly represented by the two numbers at the end points of an interval. This method has some similarity to the method developed by the authors in that it indicates ranges of possible values for inputs and outputs. Interval analysis, however, provides no information on the performance of a design *within* the interval. All that can be said, when interpreting a Performance Parameter output, is that the design will perform somewhere between the boundaries of the interval. Furthermore, the input values that contributed to any one particular value of the output cannot be directly determined (except at the boundaries). As the number of intervals used to represent DPs increases (*e.g.*, a succession of decreasing interval sizes may be used to cause a PP to approach a desired value), interval analysis could approach the method reported here.

G. Taguchi [23, 77] has developed a technique for evaluating the "quality" of a design based on his loss function. This function is essentially a preference function for a fuzzy representation.[1] Taguchi does not apply the mathematics of fuzzy sets to the evaluation or comparison of designs. Instead, his method uses the principles of "experimental design," which "explore" the design space one (or two) crisp design parameter value(s) at a time. Taguchi suggests that the Parameter Design phase will have the most impact on quality. In this phase the values for DPs can be selected to create a design that will be as insensitive as possible to manufacturing errors, environmental conditions, variability in use, etc. The design technique presented here will be a useful extension to Taguchi's method in the Parameter Design phase (by permitting a more thorough evaluation of the performance parameters over ranges of the design parameters), and performing its intended purpose in the preliminary design phase.

Sensitivity analysis permits the evaluation of the rate of change of an output PP as input DPs change. This relies on the evaluation of partial derivatives or Lagrange multipliers of system equations.[2] Sensitivity analysis is a powerful design tool, but provides information only at a single operating point each time it is evaluated,

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[1] See particularly the Quadratic Loss Function shown in Figure 3 of [23].

[2] Reference [62] pages 168 and 609.

and will provide no information when only discrete values of input design parameters are available. Furthermore, the change in desirability of inputs and outputs is not included in the calculation. For example: one input may have a narrow range of acceptable values, and a different input may have a much wider range of desirability. Even if the numerical sensitivity of one output is the same with respect to these two inputs, different design decisions should be reached regarding the effect of altering them. When a preference function is used instead of a range (to represent the designer's desires) even more information in the form of the rate of change of desirability of an output with respect to an input's desirability can be found. The γ-level measure will be introduced later to evaluate this effect. Sensitivity analysis, as it is usually applied, does not include the effects of imprecision, or the designer's desires.

If a multi-valued logic form of probability analysis is used (instead of the more common event-frequency form), imprecision of input DPs may be represented, and imprecise output PPs can be calculated [12, 24, 42, 69]. However, the probability calculus does not permit the relationships between inputs and outputs to be found. If, for example, a probability calculation shows that the desired performance has a low likelihood, determining which DPs to change, and how to change them is not possible from the probability calculations alone. Furthermore, some probability calculations (on imprecise parameters rather than uncertain parameters) can produce unexpected results.[3]

The method presented here, based on a fuzzy representation of imprecision,

[3]For example: $y = mx + b$ where m, x, and b have probabilistic representations centered at a value of 3.0, produces an output with a peak likelihood at $y = 11.6$ rather than the value 12. A detailed comparison of probability analysis and the author's technique will be addressed in Chapter 5.

extends the capabilities of the methods described above by permitting: representation of imprecise input Design Parameters; calculation of resulting Performance Parameters (with corresponding levels of imprecision); evaluation of Design Parameters to attain a desired Performance Parameter; and estimates the relationship between DPs and PPs over a wide range of values.

2.2 Approach

As described in the previous section, I have adopted the fuzzy calculus as a mathematical representation of imprecision in engineering design. The arithmetic and calculus of fuzzy sets and fuzzy numbers provides a method for manipulating the imprecise representations.

Fuzzy numbers and their associated arithmetic and calculus are the subject of many publications and several textbooks [33, 46, 94] and will not be presented here in detail. Instead, the necessary tools needed to understand the approach (and later compare with the probability calculus) will be described below and in Appendix A.

2.2.1 Fuzzy Arithmetic Operations

In his seminal paper [87], Zadeh puts forth the concept of a *fuzzy set* as "a class of objects with a continuum of grades of membership." From the many theoretical developments and applications that have appeared subsequent to this original work, a sub-area of research has been devoted to the concept of a *fuzzy number*, which exclusively concerns the universe of real numbers $\Re$. Kaufmann and Gupta [46] define a fuzzy number as "a fuzzy subset of $\Re$ that is convex and normal." Using this concept, and defining imprecise input design parameters as fuzzy numbers, the *extension principle* of Zadeh [91] may be directly applied to design computations, thus extending algebraic operations on real numbers to the fuzzy domain.

Specifically, let the imprecise input parameters $\tilde{u_1}, \tilde{u_2}, \ldots, \tilde{u_N}$ be defined in the universes $X_1, X_2, \ldots, X_N$, respectively. The mapping from $X_1 \times \cdots \times X_N$ to a universe Y is defined as a function f such that $y = f(x_1, \ldots, x_N)$. The extension principle then implies that with an imprecise performance parameter $\tilde{p}$ on Y, which is induced from $\tilde{u_1}, \ldots, \tilde{u_N}$ through f, the resulting membership function is

$$\mu_{\tilde{p}}^{(y)} = \sup_{\substack{x_1, \ldots, x_N \\ y = f(x_1, \ldots, x_N)}} \min(\mu_{\tilde{u}_1}^{(x_1)}, \ldots, \mu_{\tilde{u}_N}^{(x_N)}).$$

The ordinary binary operations and other function operations (extended trigonometric functions, etc.) can be derived from this principle in this max-min form. For example, the addition of two imprecise parameters $\tilde{u_1}$ and $\tilde{u_2}$ can be written as

$$\mu_{\tilde{u}_1 \oplus \tilde{u}_2}(y) = \bigvee_{y = u_1 + u_2} (\mu_{\tilde{u}_1} \wedge \mu_{\tilde{u}_2}), \tag{2.1}$$

where $\oplus$ denotes extended addition and $\vee$ and $\wedge$ denote max and min, respectively. Equivalently, extended addition may be expressed in the intervals of confidence, level of presumption, and α-cut form of Kaufmann and Gupta [46]:

$$\tilde{u}_{1,\alpha} \oplus \tilde{u}_{2,\alpha} = [u_{1,1}^{(\alpha)} + u_{2,1}^{(\alpha)}, u_{1,2}^{(\alpha)} + u_{2,2}^{(\alpha)}], \tag{2.2}$$

where $\tilde{u}_{i,\alpha} \in [u_{i,1}^{(\alpha)}, u_{i,2}^{(\alpha)}]$. The method for applying the extended addition and the other extended operations in the design domain are in Appendix A and the example section of this chapter. Note the change in nomenclature from Equation 2.1 (μ) to Equation 2.2 (α), reflecting the change from the general extension principle to the α-cut method.

2.2.1.1 Analytical and Numerical Applications

For a given design performance expression $z = f(u_i)$, where z is the performance parameter and the u_i are the design parameters, the fuzzy output $\tilde{z}$ can be determined analytically by applying the α-cut form of the extended operations (Kaufmann and Gupta [46]) as presented earlier, Equation 2.2. Such an application requires that the input parameters be represented algebraically, where the result after algebraic manipulation is $\tilde{z}_\alpha$, which is a function of the preference values α. Substituting a value for α will give an output corresponding to that level of desirability. (The analytical application for an example multiplication are in Appendix A, and the general approach for the analytical application of fuzzy arithmetic is in [33, 46].)

Even for a modest number of design parameters, the analytical fuzzy calculus method for calculating imprecise performance parameters (*e.g.*, application of combinations of equations of the form of Equation 2.2) is impractical for computer-assisted design applications, due to algebraic complexity. A discrete numerical method, such as the Fuzzy Weighted Average (FWA) algorithm [31] and its extensions (presented in Appendix C), is necessary to meet this need for computational efficiency. FWA approximates the analytical approach by discretizing the functions of the input fuzzy numbers into a prescribed number of α-cuts. Figure 2.1 shows an α-cut at preference 0.5. The discrete FWA algorithm treats each α-cut as an interval, and performs combinatorial interval analysis to calculate each output preference interval [31]. The important addition to interval analysis, however, is the preference value associated with each value in the fuzzy number. It can be seen that as successively smaller intervals are used in a calculation, interval analysis approaches the fuzzy set mathematics technique. A condensed version of the algorithm from [31] is in Appendix C (where the terminology has been changed to reflect the application to design calculations).

For N fuzzy design inputs and M discrete preference points, the algorithmic complexity of the FWA implementation is of order

$$H \sim M \cdot 2^{N-1} \cdot \kappa \tag{2.3}$$

where H equals the number of operations and κ equals the number of multiplications and divisions in $f(u_i)$. The extended version discussed in Appendix C further reduces this complexity to

$$H \sim M \cdot 2^{N-p-1} \cdot v \tag{2.4}$$

where $N - p$ is the number of repeated design parameters in the performance expression and v equals the number of interval operations in the expression $f(u_i)$.

One important ramification of fuzzy mathematics is that once a forward calculation is made (operating on inputs to determine an output fuzzy function), then *backward* calculations can be obtained with no further computation. The peak of a fuzzy output corresponds to the peak value for each of the inputs, and off-peak output values correspond to off-peak inputs with the same preference value. For example, if a designer performed a fuzzy calculation, and the output parameter's peak value (preference of one (1)) was not acceptable, then he or she could select a different output value and determine its preference value. The designer then knows that the inputs required to produce that output have the same preference or less. If the designer wishes to use an output parameter value with preference of 0.7, then he or she knows that at least one input must also have a preference of 0.7 or less, the other inputs having preference distributed about 0.7. In this way the relationship between inputs and outputs is readily observed. The backward path through the calculations is a natural consequence of the fuzzy arithmetic implementation developed by the author, and requires no further calculations once the forward path has been calculated.

In general, the author's computationally efficient implementation of FWA, when combined with an appropriate user interface, provides a means for carrying out real-time imprecise calculations, where the output results can be easily evaluated with respect to the inputs, and where a *backward path* can be utilized without further computation.

2.2.2 Preference Function Shapes for Design Parameters

A simple form of the preference functions described above is triangular (single most desired/confident value with linear interpolation to the zero confidence values) or trapezoidal (interval of most desired/confident values at preference of one (1)). For preliminary design, the experiments conducted to date indicate that these two classes of preference function shapes will adequately approximate many of the input DPs imprecise representations. These types of functions also satisfy the normality and convexity conditions required of fuzzy numbers. If it becomes necessary to use higher-order functions, they can be included without modification to the technique or implementation described here. For example, to bias a preference around the most preferred input, a quadratic function can be used. Likewise, to bias the preference in the opposite sense, an inverse quadratic function, which approaches a Dirac delta function in the extreme case, may be applicable. Furthermore, if multiple peaks are found to be required, then the convexity condition can be relaxed slightly such that the preference functions are treated as multiple locally convex functions (see Figure 2.3).

In addition to triangular and trapezoidal functions, preference functions may be constructed from engineering data (Figure 2.2), if the data and interpretation are available. For an incomplete set of data, a preference function can be approximated by curve fitting (somewhat analogous to the construction of subjective probability density functions) to certain known points of preference in a design parameter's input range.

For triangular inputs, the outputs of design performance analysis functions

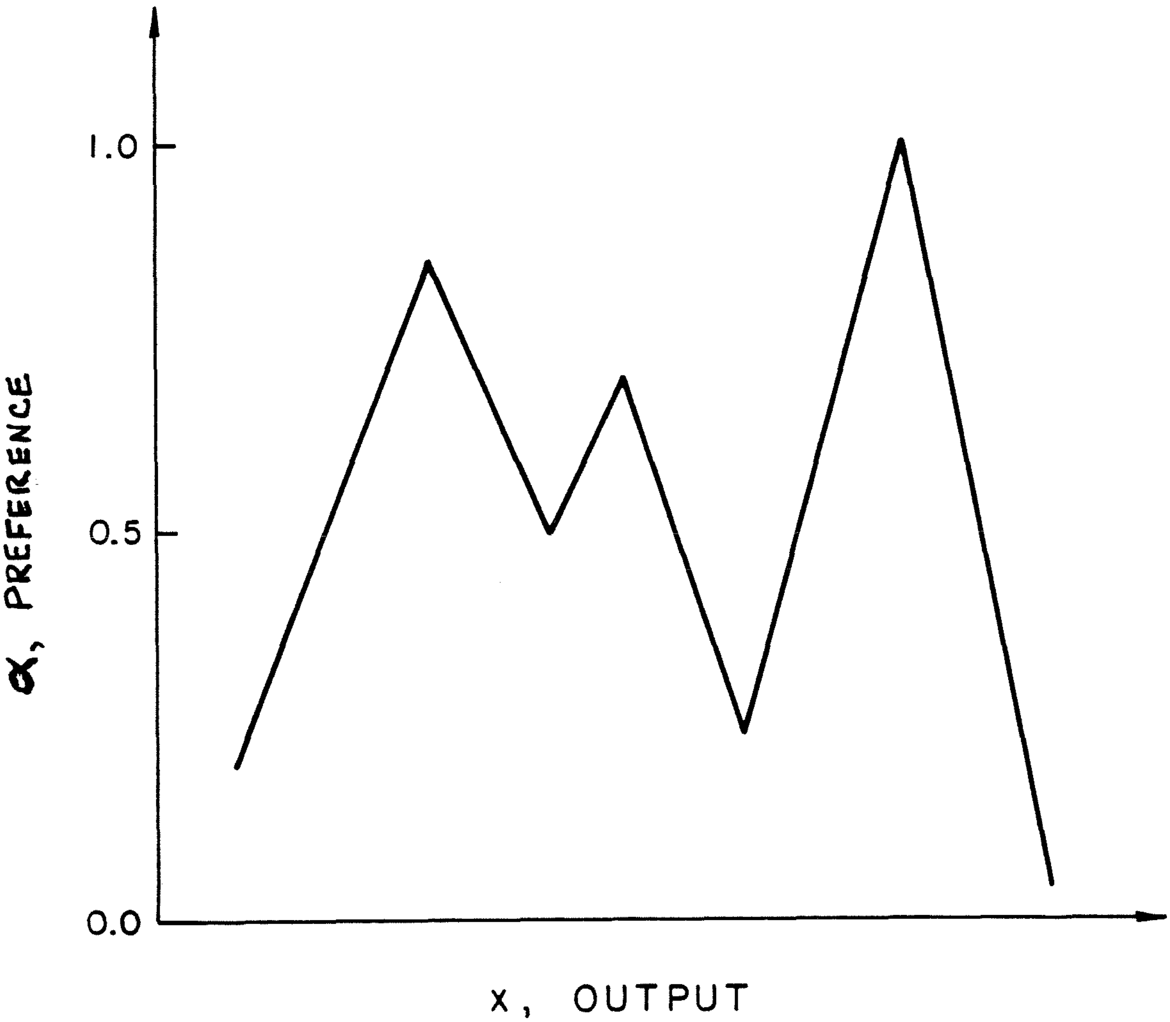

Figure 2.3: Locally Convex Function, Three Regions.

may not always be linear functions, as shown by the example in Appendix A. A fuzzy multiplication with triangular input functions does not result in a triangular output function, but instead two combined functions raised to the one-half power. Addition and subtraction will preserve the shape of the input function, but the multiplication and division operators both produce nonlinear results. In general, curves of different shape than the input can be expected for the results of fuzzy engineering design computations; however, the result of a fuzzy calculation can be interpreted as previously discussed, whatever its shape.

The intent of this section has not been to provide an exhaustive presentation of all possible preference function shapes, but, instead, to introduce certain classes of functions that can be applied easily in preliminary design. Future work will expand on these classes of functions, and present formal methods for selecting DP preference shapes.

2.2.3 A Design Measure

In any design calculation, some input parameters are very strongly coupled to the outputs, and others are nearly independent. A means of determining the relative coupling between imprecise (fuzzy) inputs and outputs can be used to determine which parameters the designer can change and produce little effect on the performance, and which parameters will have the most profound effect on the output. A new measure developed for this purpose, called the γ-level measure, is presented below, along with a well-known Measure of Fuzziness.

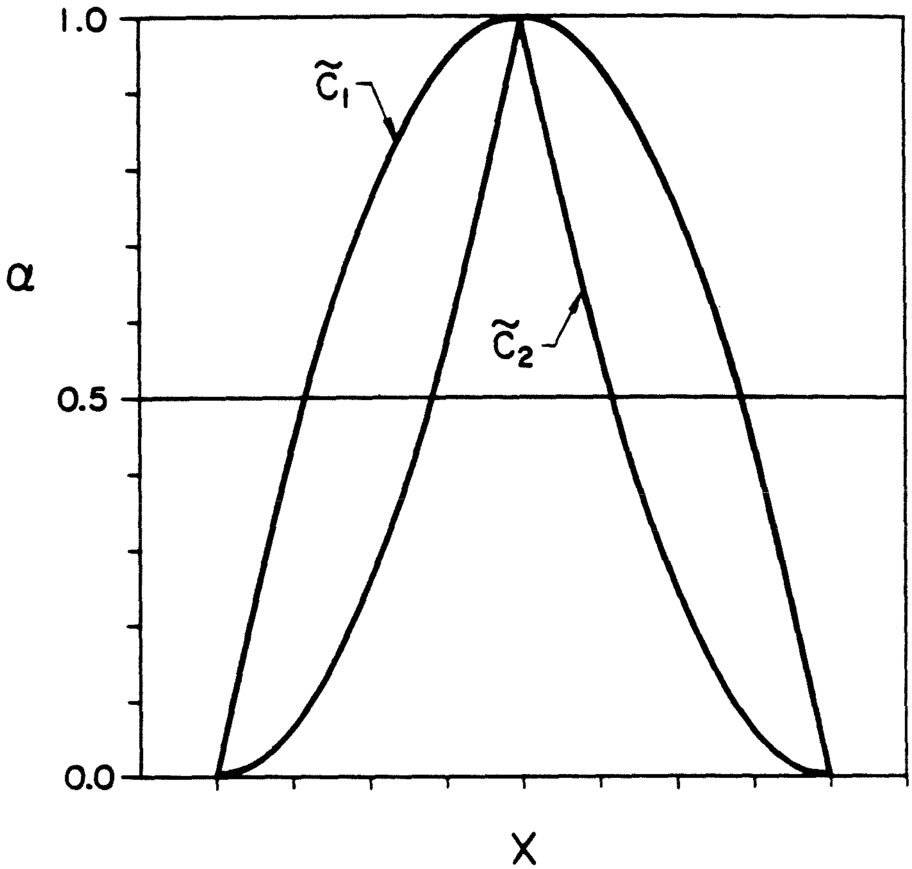

Figure 2.4: Measure of Fuzziness Example.

2.2.3.1 Measure of Fuzziness

The *Measure of Fuzziness* expresses "the difficulty of deciding which elements belong and which do not belong to a given fuzzy set" [33]. Using $d(\tilde{C})$ to denote the measure of fuzziness and using the definition for the *complement* of a fuzzy set (Appendix A), $d(\tilde{C})$ must satisfy the following conditions: [53]

1. $d(\tilde{C}) = 0$ iff $\tilde{C}$ is a crisp set in X, *i.e.*, the membership function takes on only values of zero and one.

2. $d(\tilde{C})$ must assume a maximum iff $\alpha_{\tilde{C}}(x) = \frac{1}{2}\ \forall x \in X$.

3. $d(\tilde{C}) \geq d(\tilde{C}^*)$ if $\tilde{C}^*$ is any "sharpened" version of $\tilde{C}$, *i.e.*, a crisper version. We can express this more precisely as $\alpha_{\tilde{C}} \leq \alpha_{\tilde{C}^*}$ for $\alpha_{\tilde{C}} \geq \frac{1}{2}$ and the converse for $\alpha_{\tilde{C}} \leq \frac{1}{2}$.

4. $d(\tilde{C}) = d(\hat{\tilde{C}})$ where $d(\hat{\tilde{C}})$ is the complement of $d(\tilde{C})$. This simply says that $d(\hat{\tilde{C}})$ is as fuzzy as $d(\tilde{C})$.

where $\alpha_{\tilde{C}}$ is the membership function of the fuzzy set (or number) $d(\tilde{C})$ defined $\forall x \in X$. The following entropy function satisfies the conditions required of a measure of fuzziness [53]:

$$d(\tilde{C}) = K \sum_{i=1}^{|X|} \Psi(\alpha_{\tilde{C}}(x_i)), \tag{2.5}$$

where:

$$\Psi(y) = -y\ln(y) - (1-y)\ln(1-y),$$

$\alpha_{\tilde{C}}$ is the membership function of the fuzzy set $\tilde{C}$, $|X|$ is the length of the discretized support (region of non-zero membership) of $\tilde{C}$, and K is an integer.

Unfortunately the entropy function as defined in Equation 2.5 measures values centered on $\alpha_{\tilde{C}} = \frac{1}{2}$. A membership value of one-half has the highest degree of "difficulty of deciding" whether it is a member of the set or not. Memberships close to one (1) are closer to being in the set, memberships close to zero (0) are closer to being out of the set. Thus this measure indicates how much of the membership function is close to one-half. In design, the engineer needs a measure of the values centered on $\alpha_{\tilde{C}} = 1$, indicating the "spread" of the preference function (near 1), not the steepness of the bounding curves (for membership functions). Figure 2.4 illustrates the difference. The *Measure of Fuzziness* will have the same value for membership functions $\tilde{C}_1$ and $\tilde{C}_2$ since these two curves have the same amount of x near $\alpha = 0.5$, however, $\tilde{C}_1$ has much greater imprecision (in the preference function interpretation) than $\tilde{C}_2$ (a much larger amount of x near $\alpha = 1.0$). To avoid this difficulty, a new measure has been developed here.

2.2.3.2 The γ-Level Measure

A new measure, which will be referred to as the *γ-level measure*, has been developed. This measure is defined in the following manner:

$$D(\tilde{C}) = \sum_{i=1}^{|X|}(e^{\beta(x_i)} - 1)^m, \tag{2.6}$$

where

$$\beta(x_i) = \begin{cases} \frac{\alpha_{\tilde{C}}(x_i)}{\gamma} & \text{if } \alpha_{\tilde{C}} \leq \gamma \\ \frac{2\gamma - \alpha_{\tilde{C}}(x_i)}{\gamma} & \text{if } \alpha_{\tilde{C}} \geq \gamma, \end{cases}$$

$$0 < \gamma \leq 1,$$

and m is an integer such that as m increases, the measure becomes more concentrated for values about $\alpha_{\tilde{C}} = \gamma$. The value of γ may be set so that $D(\tilde{C})$ measures values in the support centered about it. For $\gamma = \frac{1}{2}$ the *γ-level* measure satisfies the conditions for the *Measure of Fuzziness* [53], listed earlier. For the purpose of this study, $\gamma = 1.0$ and $m = 1.0$ will be used in Equation 2.6. (Note: a relevant property of the γ-level measure is that it does not depend on the unit-system used for the design parameters, i.e., non-dimensional parameter groups are not required in order to apply the γ-level measure.)

An outline of the process by which this γ-level measure can be used as a qualitative measure of the relationship between input design parameters and output performance parameters is shown below. Let $\tilde{u}_1, \ldots, \tilde{u}_N$ be N imprecise inputs (Design Parameters), and let $\tilde{P}$ be the output (Performance Parameter) of the computation $y = f(u_1, \ldots, u_N)$.

1. Determine $\tilde{P}$ using the FWA algorithm [31].

2. Let λ_1 and λ_2 be equal to the two x values for which $\alpha_{\tilde{P}} = minimum$ on both the left and right extremes of $\tilde{P}$. $\Lambda = [\lambda_1, \lambda_2]$ makes up an interval of the support of $\tilde{P}$.

3. Discretize the interval Λ into n equally spaced steps, such that $| X |= n$ in Equation 2.6.

4. For each input parameter, $\tilde{u}_i$, $i = 1, \ldots, N$, set all other $\tilde{u}_j, i \neq j$, to their nominal crisp value (where $\alpha_{\tilde{u}} = 1$). For $i = 1, \ldots, N$, use the FWA to calculate the output, Θ_i, where the ith fuzzy input remains fuzzy in the calculation, and all others are made crisp as above.

5. Calculate the γ-*level* measure ($\gamma = 1$) for $\tilde{P}$ and all Θ_i.

6. Normalize the $D(\Theta_i)$s with respect to $D(\tilde{P})$. The result is an ordering of the inputs according to importance (relative measure), giving a qualitative relationship of inputs to the output.

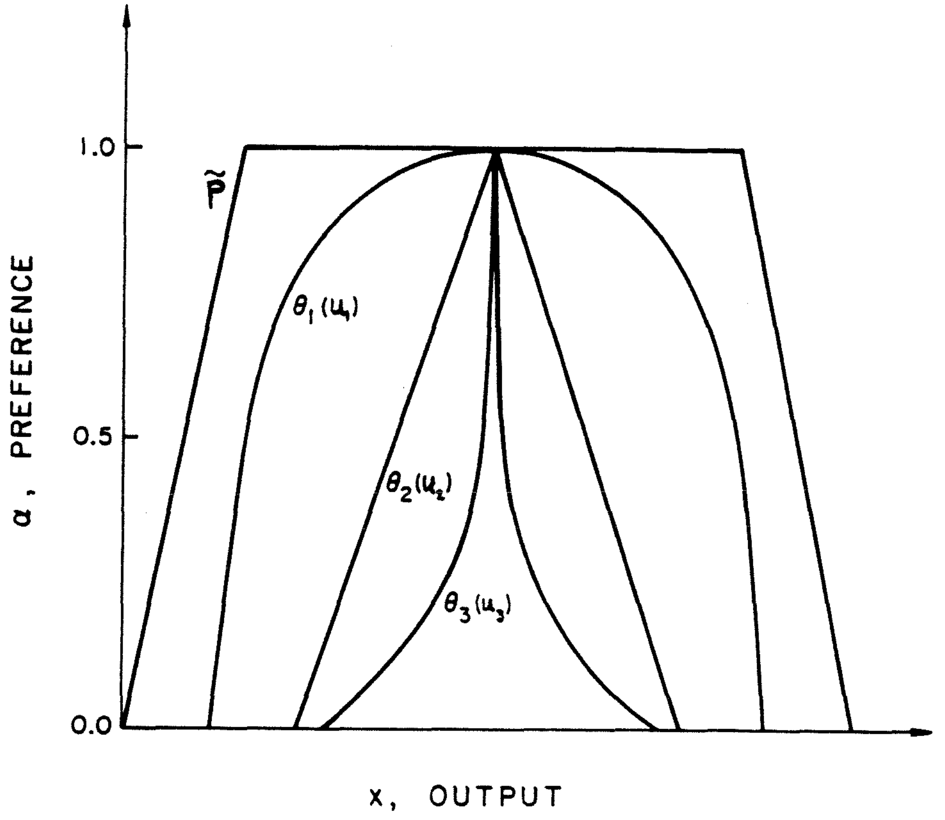

Figure 2.5: γ-Level Measure Application.

For the engineer who uses fuzzy preference functions in the description of design and performance parameters, this new measure provides the ability to determine some information on the coupling between the inputs and outputs of design calculations. The measure can also be used to determine which parameters the designer can change and produce little or no effect on the performance, and which parameters will alter the output the most. Those parameters with small influence may be fixed to the most-desired value by the engineer, resulting in a simplification of the design problem. The coupling information not only includes the rate of change of an output with respect to an input (over the range of acceptable values), but also includes the change in desirability of the parameters. If a small change of an input produces a large change in an output, but a small change in the desirability of the output, the γ-level measure will be small (even though the *sensitivity* of the output to that input is large). Similarly, if a large change of an input produces a small change in an output, but a large change in the desirability of the output, the γ-level measure will be large.

Figure 2.5 illustrates an example application of the γ*-level* measure. $\tilde{P}$ is the output fuzzy function of some performance parameter, functionally related through a PPE to three imprecise input parameters, u_1, u_2, and u_3. The Θ_i sets make up fuzzy outputs for only one fuzzy input parameter (and the other inputs held at their crisp value). After applying the γ-level measure to each of these output sets, the results may be ordered from largest to smallest. In this case, the ordering consists of the following: $D(\tilde{P})$, $D(\Theta_1)$, $D(\Theta_2)$, $D(\Theta_3)$. Normalizing the output measures $D(\Theta_i)$ with respect to $D(\tilde{P})$ shows that $D(\Theta_1)$ is much greater than for $D(\Theta_3)$. The parameter for Θ_3 (u_3) contributes very little to the preliminary design analysis when compared to the parameter for Θ_1 (u_1). Thus, the input parameter u_3 might be fixed to its crisp value (where its preference equals one (1)).

2.3 Example

A simple mechanical design example using the approach described in the previous section is presented here to illustrate the representation and manipulation of imprecise parameters in preliminary engineering design. The problem is to design a mechanical structure, attached to a wall at one end, which will support an overhanging vertical point load. Constraints on the problem include: the distance the load is from the wall; the total width of the supporting structure; and the materials used for the structural elements. One possible configuration, shown in Figure 2.6, consists of a two-member frame, where the compression member (AB) is attached to the wall at an angle of sixty degrees (60°) and both members have rectangular cross-sections. The global design objective is to avoid failure in either component of the frame. Performance expressions can be obtained for the two Functional Requirements by considering beam bending theory for the horizontal member (CD), and buckling for the compression member (AB).[4] The resulting Performance Parameters for the design are the maximum bending stress σ in CD and the column load F_B on AB:

$$\sigma = \frac{2l(W + \frac{W_{CD}}{6})}{w_{CD}t^2}, \tag{2.7}$$

$$F_B = \sqrt{\{\frac{9}{2\sqrt{3}}(W + \frac{W_{CD}}{2} + \frac{W_{AB}}{3})\}^2 + \{\frac{3}{2}(W + \frac{W_{CD}}{2})\}^2}. \tag{2.8}$$

The design parameters for this example are as follows: the applied load (W); the length of member CD (l); the width of the compression member (w_{AB}); and the thickness (t). If a different material is used, or a range of material properties are available, E and ρ may also be included as imprecise DPs. The relationships for the

[4]Shear stress in the horizontal member and elastic deformation of the entire frame do not contribute significantly to the problem.

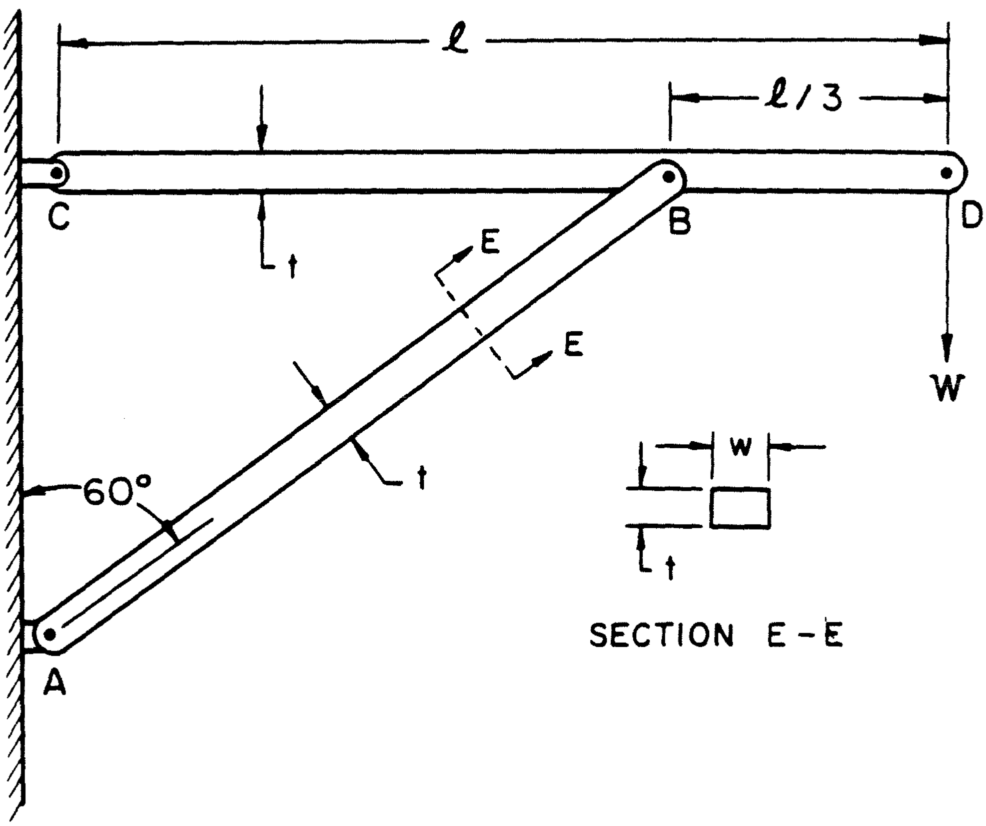

Figure 2.6: Design Problem: Frame Configuration.

weight of the two members, and a constraint on width (w) are:

$$W_{CD} = \rho g w_{CD} t l, \tag{2.9}$$

$$W_{AB} = \rho g w_{AB} t (\frac{4\sqrt{3}l}{9}), \tag{2.10}$$

$$w_{CD} = w_{AB} - 2.5 \text{ cm}. \tag{2.11}$$

2.3.1 Performance Specifications

In this design, σ must be less than the maximum bending stress before yield. This example assumes that the material has been specified to be steel. Thus, the Functional Requirement for maximum bending stress is:

$$\sigma \leq \sigma^r = 225 \quad \text{MPa},$$

where the superscript r denotes "requirement."

The performance expression for the column load F_B does not consist of a crisp (single value) inequality as in the case of σ^r. Instead, column-buckling theory is used to specify a requirement on the critical load of the compression member. Because the critical load depends on the dimensions of the member, the Euler-Johnson Condition must be calculated first to determine whether the Euler or Johnson equation should be applied. This condition along with the equations for the critical load can be given as:

$$C_{EJ} = \frac{\sqrt{2\pi^2 E}}{S_y} - (\frac{l}{0.289t})^2, \tag{2.12}$$

$$P_{cr} = \frac{w_{AB} t \pi^2 E}{n(\frac{4\sqrt{3}l}{2.6t})^2} \text{ (Euler)}, \tag{2.13}$$

$$P_{cr} = \frac{S_y w_{AB} t}{n} \left[1 - \frac{S_y (\frac{4\sqrt{3}l}{2.6t})^2}{4\pi^2 E} \right] \text{ (Johnson)}, \tag{2.14}$$

where S_y is the yield strength and n is the factor of safety (both assumed constant for simplicity). The resulting performance expression for the column load F_B becomes:

$$F_B \leq \; P_{cr}^r \; = \text{(Euler or Johnson Equation)} \; \text{ kN.}$$

Because Equations 2.13 and 2.14 depend on the design parameters of the problem, the performance specification P_{cr}^r will be a fuzzy requirement as a function of the DPs. Once the DPs have been determined, P_{cr}^r can be calculated and subsequently updated if any of the design parameters should change during the design process.

Overall, these two performance parameters, in the form of a crisp inequality and fuzzy inequality, make up the set of Functional Requirements for this simple design. This set can now be used to rate the frame configuration's performance parameters. For simplicity, the frame configuration shown in Figure 2.6 will not be compared with other alternative designs. The application presented in Chapter 4 will demonstrate the technique with a problem containing more realistic design complexities, and comparisons of design alternatives.

2.3.2 Input Design Parameters

The designer specifies the input parameters as preference functions according to the approach outlined previously. Here the parameters that need to be selected as part of the design process are: W, w_{AB}, l, and t. In this example, the subjective knowledge, experience, and desires of the engineer are used to imprecisely determine these input parameters. For example, the applied vertical load W is constrained by a maximum load that a proposed configuration is expected to withstand without failure. There also exists some latitude (due to other design considerations) by which this design load may be decreased such that the design is still *satisfactory*, but less *desirable* due to the decrease. Thus, the input parameter W is imprecisely defined in a range of possible values where the desirability decreases from the maximum value in the range to the minimum value shown in Figure 2.7. For this design problem, the maximum design load is 20 kN, which corresponds to the upper endpoint of the range. W may not be less than 15 kN, corresponding to the lower endpoint.

The remaining design parameters may be specified in a similar manner. Because each input set for this problem is in the form of a triangular function (naturally, more complex functions could have been used), the fuzzy DPs can be represented by three-values: left-extreme value for preference of zero, peak value for preference of one, and right-extreme value for preference of zero. Table 2.1 provides the necessary data for constructing the preference functions for the entire set of design parameters, and Table 2.2 lists other constant data used in this example design problem.

2.3.3 Output Performance Parameters

The fuzzy outputs for the performance parameters σ and F_B can be obtained by use of Equations 2.7 and 2.8 and the application of the FWA algorithm described earlier. The results are shown in Figures 2.8 through 2.11.

After the calculations have been performed to produce the outputs, the next step is to compare the output sets with the performance criteria. Figure 2.8 shows the imprecise performance parameter results for the maximum bending stress of member CD (Equation 2.7). The output at the peak of $\tilde{\sigma}_{(at\,\alpha=1)}$ is equal to 994 MPa. This peak output does not satisfy the Functional Requirement $\tilde{\sigma} < \sigma^r = 225$ MPa. To satisfy the requirement σ^r, the input parameters must deviate from the peak (most desired) values. At least one design parameter must decrease in preference, to the left of the peak, by between 0.5 and 0.6 ($\tilde{\sigma}_{(at\,\alpha=0.5)} = 259$ MPa and $\tilde{\sigma}_{(at\,\alpha=0.4)} = 206$ MPa), to meet the requirement on σ. (If a factor of safety is desired, a further decrease in

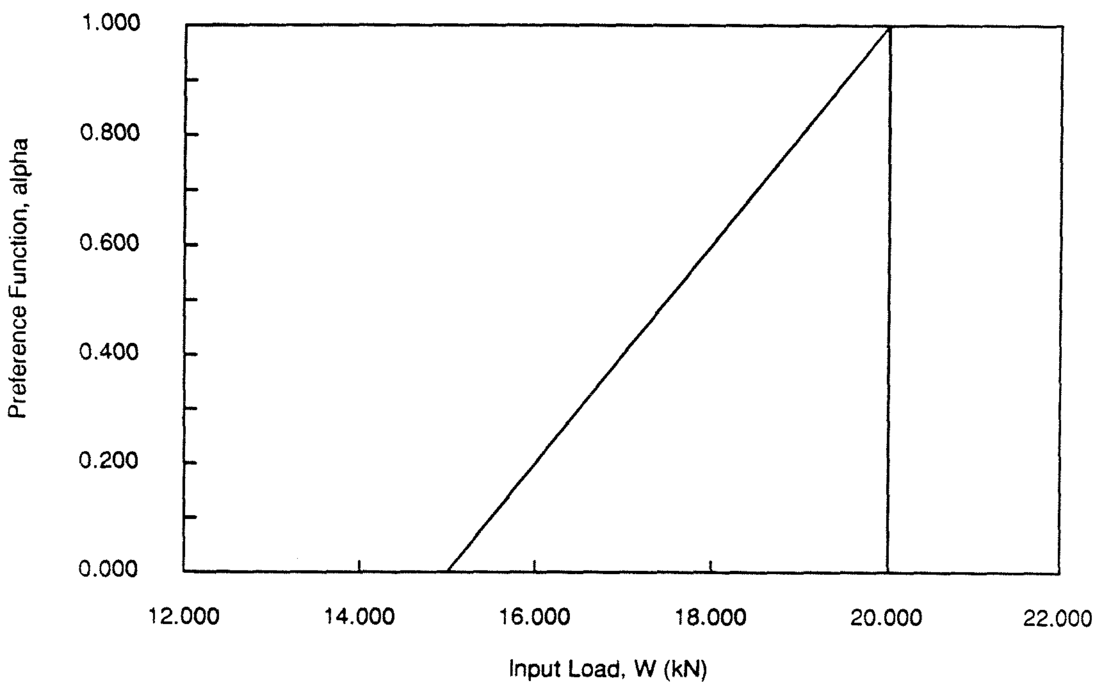

Figure 2.7: Input Parameter: W.

DPs (units)	$\alpha = 0$	$\alpha = 1$	$\alpha = 0$
W (kN)	15.0	20.0	20.0
w_{AB} (m)	0.04	0.07	0.13
l (m)	3.0	4.0	4.0
t (m)	0.04	0.06	0.10

Table 2.1: Example Problem: Fuzzy Design Parameter Data.

Constant (units)	Value
E (GPa)	207.0
ρ ($\frac{\mathrm{kg}}{\mathrm{m}^3}$)	7830.0
g (m/sec^2)	9.81
S_y (MPa)	225.0
n	5

Table 2.2: Design Example: "Constant" Data.

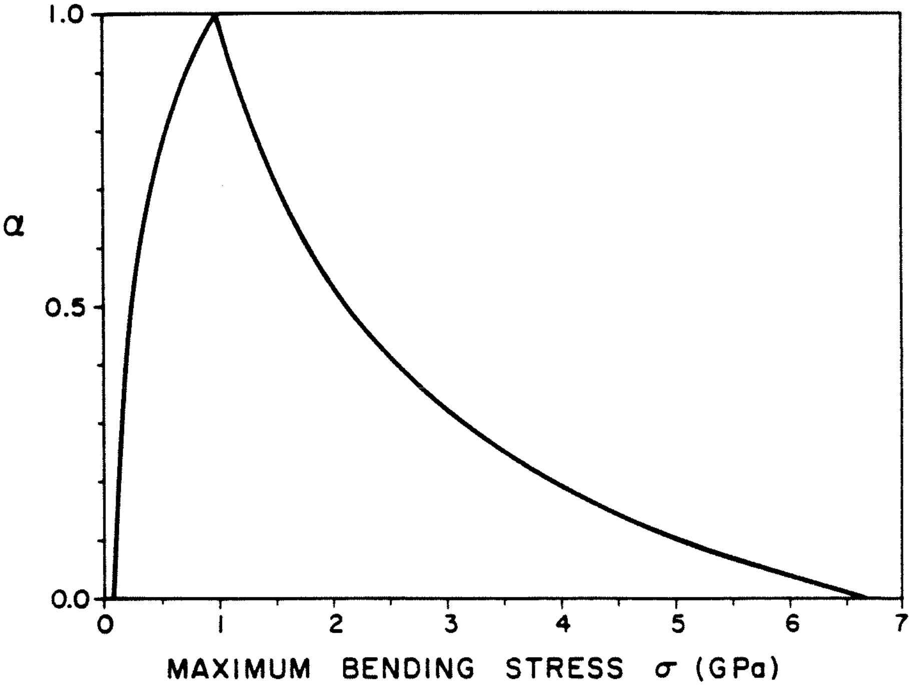

Figure 2.8: Maximum Bending Stress σ.

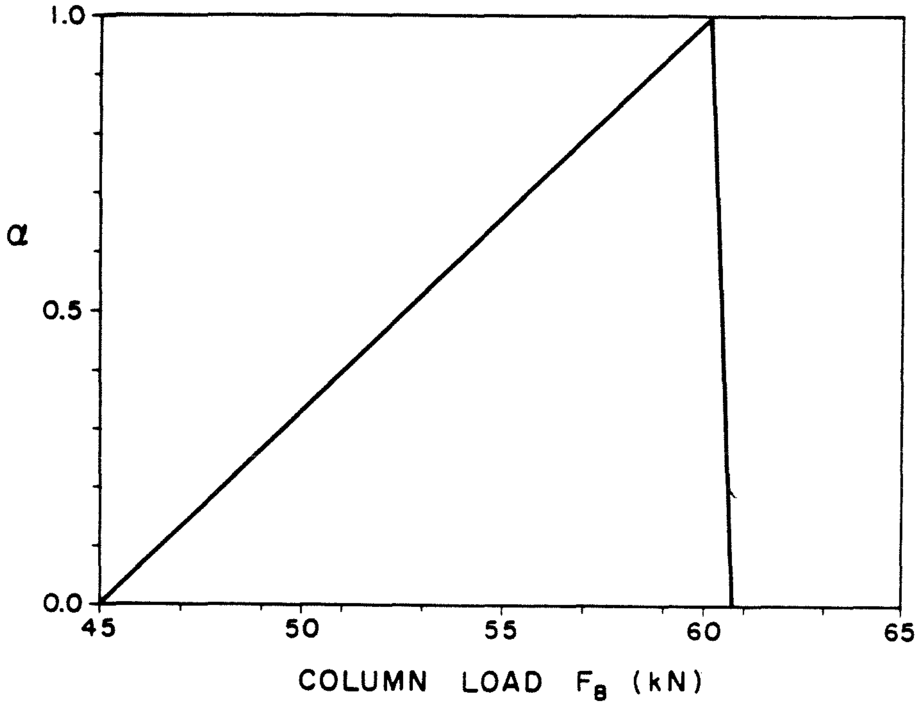

Figure 2.9: Column Load F_B.

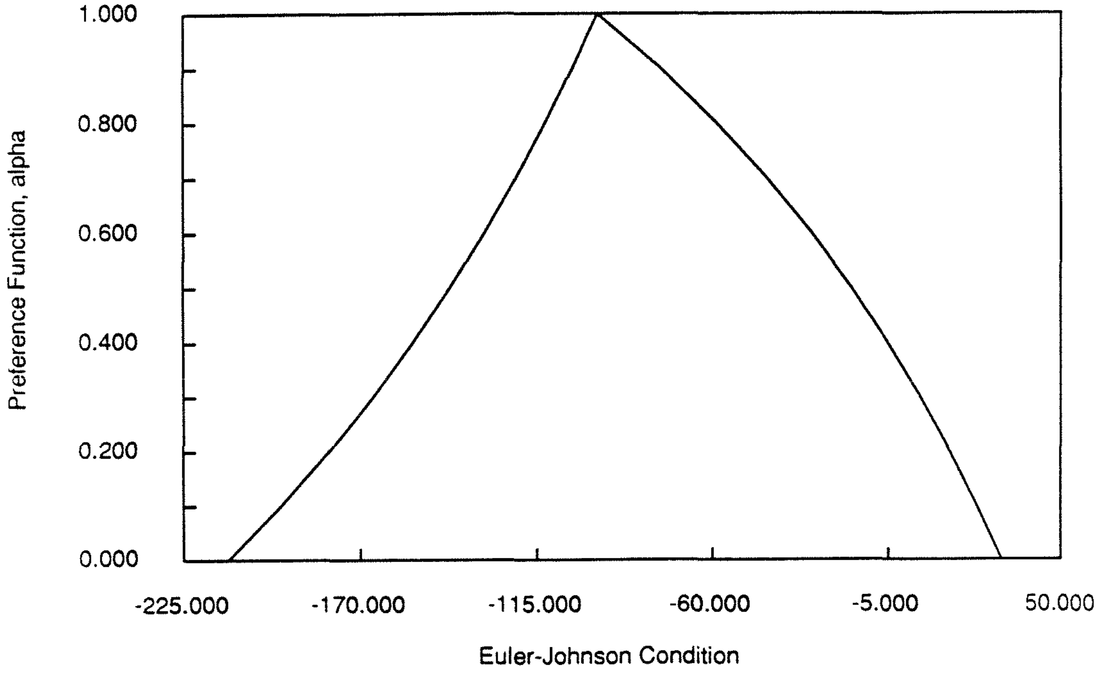

Figure 2.10: Euler-Johnson Condition C_{EJ}.

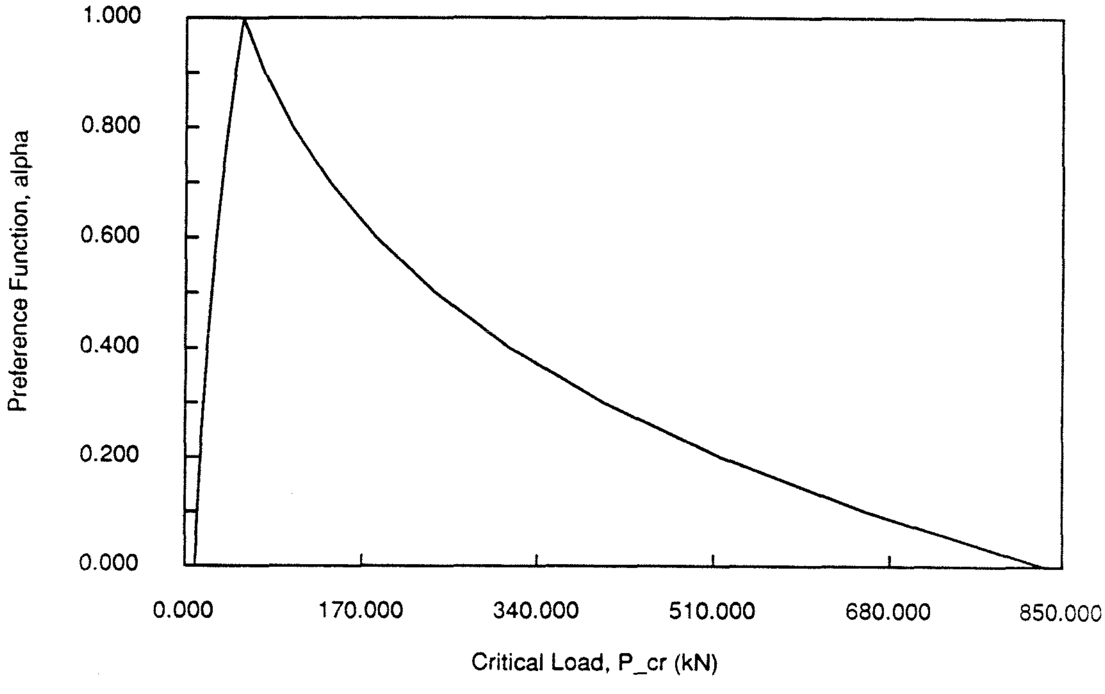

Figure 2.11: Critical Load P_{cr}.

preference will be required.)

The backward path of the imprecise calculation can be applied at this point to determine the effect of changing the preference of any one input design parameter. Data from the solution for $\tilde{\sigma}$ show that the input parameters of W and l could be decreased to the left of their peak values (at $\alpha = 1$) so that σ will meet its Functional Requirement, whereas the inputs w_{AB} and t must be decreased to the right of their peak values. This result cannot be obtained easily from inspection of the governing equation, because w_{AB} and t appear both in the denominator *and* the numerator of Equation 2.7, when combined with Equation 2.10. While this same result could be obtained through calculation of partial derivatives of the output with respect to each of the inputs, it was instead found by use of stored values calculated during the solution of the (imprecise) performance parameter by use of the author's implementation of the FWA algorithm. No additional calculations were required. These results show that σ^r may be satisfied by the frame configuration, but only with a large change in preference of the DPs from the most desired input peak values.

When considering other PPs as part of this design analysis (in addition to σ), care must be taken when adjusting the DPs (which are coupled to σ) to obtain acceptable performance values in those other PPs. A small adjustment of one DP to obtain a satisfactory performance value for one PP may adversely affect a different PP. The γ-level measure can be used to determine the magnitude of the coupling between parameters, and permit the designer to minimize the adverse effect of DP adjustment.

Figure 2.9 shows the output results for the column load performance parameter F_B. To compare $\tilde{F}_B$ with the performance criterion P_{cr}^r, the fuzzy requirement for the critical load must be determined. Figure 2.10 provides the output preference function for the Euler-Johnson Condition (Equation 2.12), where values less than zero correspond to an Euler column and values greater than zero to a Johnson column. Analyzing the figure, the majority of combinations of input parameters conform to an Euler column configuration. Although some combinations with large preference changes correspond to a Johnson column, it will be assumed at this preliminary design phase, that the Euler equation may be used as an approximation (over the entire input domain). This approximation can be verified later if the preference values of the design parameters deviate greatly from their peak values.

The critical-load results, using the Euler Equation 2.13, are in Figure 2.11. Comparing the peak values of $\tilde{F}_B$ (Figure 2.9) and $\tilde{P}_{cr}$, it is found that $\tilde{F}_{B(\,at\,\alpha=1)}$ is greater than $\tilde{P}_{cr(\,at\,\alpha=1)}$. Thus, the column load does not satisfy the buckling performance criterion P^r_{cr} for the most desired input parameter values. To satisfy the performance criteria, one of two avenues must be pursued: (1) adjust the input parameters with respect to F_B such that the critical load P^r_{cr} is satisfied, or (2) adjust the design parameters with respect to the Euler equation such that P^r_{cr} matches the output value for F_B. Because both F_B and P_{cr} depend on the same input parameters, coupling will be important. Using the γ-level measure results (presented later), the most important input parameter for F_B is W; t is the important parameter for P_{cr}; and both F_B and P_{cr} are uncoupled with respect to W and t. Thus, the preference of W may be decreased with respect to F_B without significantly affecting P_{cr}, whereas the converse is true for t with respect to P_{cr}. Considering the output curves $\tilde{F}_B$ and $\tilde{P}_{cr}$ once again, the preference of t need be changed only slightly (decreased by 0.1) to the right of the peak to satisfy the performance specification, while the preference of W would need a much greater decrease (approximately 0.4) to the left. These results demonstrate that P^r_{cr} may be satisfied with only a small sacrifice in desirability, depending on the design parameters that are changed.

2.3.4 Applying the γ-Level Measure

The γ-*level* measure, as described earlier, can be used to provide the engineer with qualitative information on the relationship between input parameters in the design. When a design parameter has the greatest qualitative importance for a given performance parameter, the numerical measure produces a normalized value of one (1). As the measure decreases in value, the corresponding input has little effect in determining the performance, meaning that even a large change in the design parameter (decrease in preference/desirability) produces a small change in output. The output of the γ-level measure is loosely analogous to sensitivity, but applies to the imprecise parameters, and represents the entire range of the parameters, not a single operating point. Moreover, this sensitivity is weighted by the designer's desires, as identified in the input parameters' fuzzy preference functions.

Table 2.3 lists the γ-level results for the frame configuration. While much information can be extracted from these data, only two important aspects will be discussed. First, analyzing the γ-level measures for σ, the input parameters t and w_{AB} are obviously the most important parameters that must be changed from their peak preference values to meet the FR. W and l contribute very little when compared with t and w_{AB}. Because l is relatively unimportant with respect to σ and the other performance parameters, it may be set to a representative or preferred value, resulting in a simplification of the frame design. Next, considering F_B and P_{cr}, W and t are by far the most important design parameters, respectively. In terms of F_B, this result verifies that the contributions of the weights of the frame members will be small. Further analysis also shows that due to the small γ-level measure (0.0135) for t with respect to F_B, F_B is nearly uncoupled from t. Similarly, P_{cr} is uncoupled from W.

2.3.5 Discussion

This example shows how imprecision in the design parameters can be handled, how the designer can move forward and backward through the design calculations to determine interactions of the DPs for the performance parameters, and how the γ-level measure can be used to determine information relative to the importance of the design parameters. Conclusions can be drawn from the results as to the ability of the configuration to satisfactorily meet the performance criteria (including consideration of the designer's desires), and if the configuration should be carried on to the next stage in the design process.

This design problem has been a simple example, with none of the complications that normally beset engineering designers, such as alternative configurations or technologies to compare; poor knowledge of the relationships between functional requirements and design parameters; and intangible requirements and specifications, such as aesthetics. The example does, however, demonstrate an enhanced capability for the designer to determine acceptable DP values, or ranges, simply and quickly by use of imprecise computations. Examples, which are considerably more complex in terms of comparing different design alternatives and in terms of including other uncertainty effects, in addition to imprecision, will be presented in later chapters.

	Performance Parameters		
DPs	σ	F_B	P_{cr}
W	0.130	1.0	0.0
w_{AB}	1.0	0.0234	0.342
l	0.130	0.00338	0.178
t	0.910	0.0135	1.0

Table 2.3: γ-*Level* Measure Results: Frame Configuration.

Chapter 3

Discrete and Multiple Imprecision

3.1 Introduction

Chapter 2 describes a method for representing and manipulating the design imprecision in the input design parameters. The intrinsic assumptions used in the method are: (1) the design parameters will be continuous over the full range of possible choices, and (2) the imprecision for a given DP will come from *one source.* While the use of these assumptions provides a straightforward means for presenting the method, any realistic design problem will violate at least one if not both of them. This chapter discusses an extension of the method to the domain of discrete design parameters and multiple-source imprecision for a given design parameter. It will be shown that each of these domains conforms to the interpretation and computational procedures outlined in Chapter 2.

3.2 Discrete Design Parameters

Discrete design parameters (*i.e.*, finite sets of mutually exclusive choices) are often present in engineering design disciplines; decision making with discrete data is a regular occurrence. Examples of discrete DPs range from variables associated with certain types of materials to parameters describing the geometric dimensions of mechanical components to the selection of electric motors from a catalog. As an example, there may exist only a finite number of alternatives for the choice of metals associated with a structure's tensile strength design parameter. Likewise, the choice for size, shape, and threading of the bolts for a subsystem bracket design may include only those bolts commercially available from one vendor. Having the ability to make decisions with such variables is a necessity in any design methodology, especially when considering the large dimensionality of discrete choices in preliminary design. (Many researchers have worked on discrete parameters, especially in optimization [4].)

This section will build on the information contained in Chapter 2, and extend the method for representing and manipulating imprecision to the discrete domain. As shown in Appendix A and in [33], the fuzzy calculus for discrete parameters is well developed. In fact, the extended operations discussed in Section 2.2.1 are essentially equivalent for both the continuous and discrete functions. However, because of the interpretation imposed on the imprecise design parameters, and because of the computational scheme (the extended FWA algorithm) used, the application of the mathematics is not the same for the discrete and continuous case. The method for representing a discrete parameter versus its continuous counterpart is the key to the problem.

3.2.1 Two Representations for Discrete DPs

Two distinct cases exist for representing, interpreting, and manipulating discrete-imprecise design parameters. Consider the case, for example, of two discrete design parameters, u_1 and u_2, which have values in the input set of real numbers shown in Figure 3.1. Because the computational technique requires a continuous representation at all values of preference, two possible representations can be constructed from "step" type functions, illustrated in Figures 3.2 and 3.3. The first discrete representation, Figure 3.2, is constructed, from the left boundary of the input range to the most preferred input, by first *stepping* vertically to the next level of preference where there exists a value of the design parameter. A horizontal line is then extended to the input with the higher preference level. The process is reversed for input design parameter values ranging from the most preferred value to the right boundary.

The second possible representation scheme, Figure 3.3, is constructed in a similar fashion to Representation A. However, in this case, the step function starts with the horizontal move, followed by the vertical step to the next level of preference (from the left boundary to the most preferred input). Once again, the process is reversed to the right of the most preferred design parameter input.

To determine which representation (A or B) will conform to the interpretation outlined in Section 2.1.1 and Section 2.2.1.1, consider an example calculation $z = f(u_1, u_2)$, where u_1 and u_2 are given by Figure 3.1, and

$$z = u_1 \odot u_2.$$

Figure 3.4 shows the results of this calculation. Representation A for u_1 and u_2 was used to determine the output curve shown on the top of Figure 3.4. Representation B was used to arrive at the bottom curve in the figure. Notice that no distinct levels in the output range are realizable for the case of Representation A, except for the end points. This implies that no information as to which input parameters combined to give a value within the output range can be obtained.

When considering Representation B, on the other hand, many distinct levels are present on the output curve for z. In fact, given all possible values of z as listed in Table 3.1, the curve clearly conforms to the interpretation outlined in Section 2.2.1.1. That is, for simplicity, a given output value z may be obtained by decreasing at least

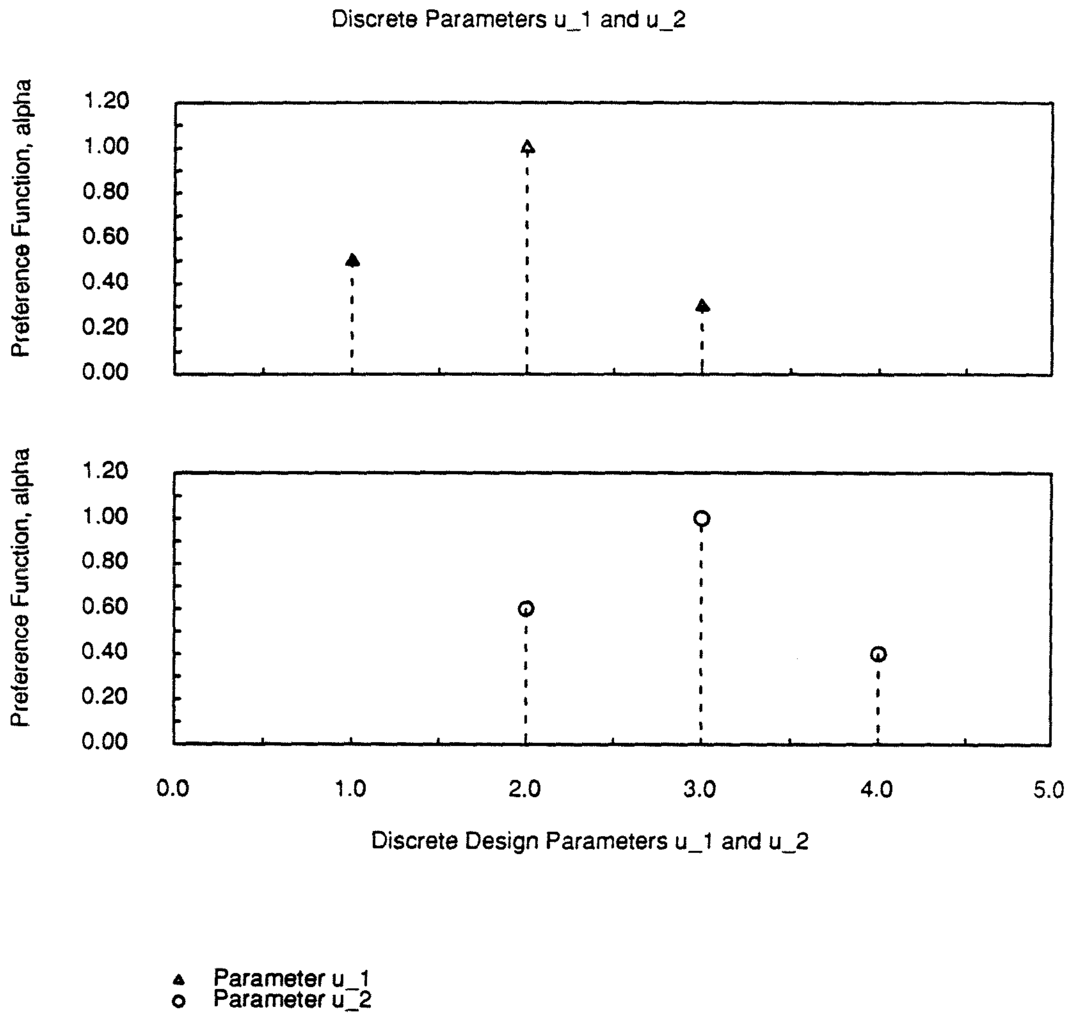

Figure 3.1: Discrete Parameters, u_1 and u_2.

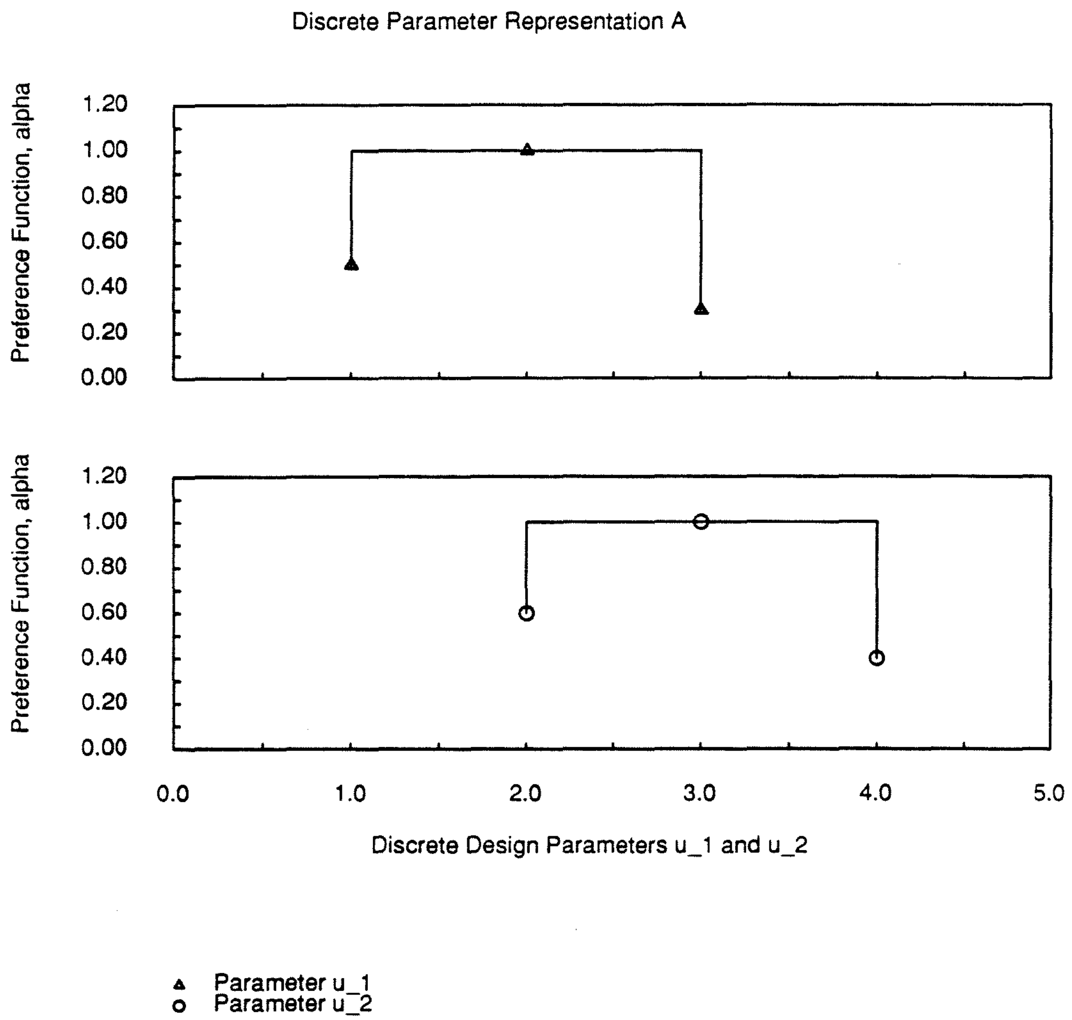

Figure 3.2: Discrete Parameter Representation A.

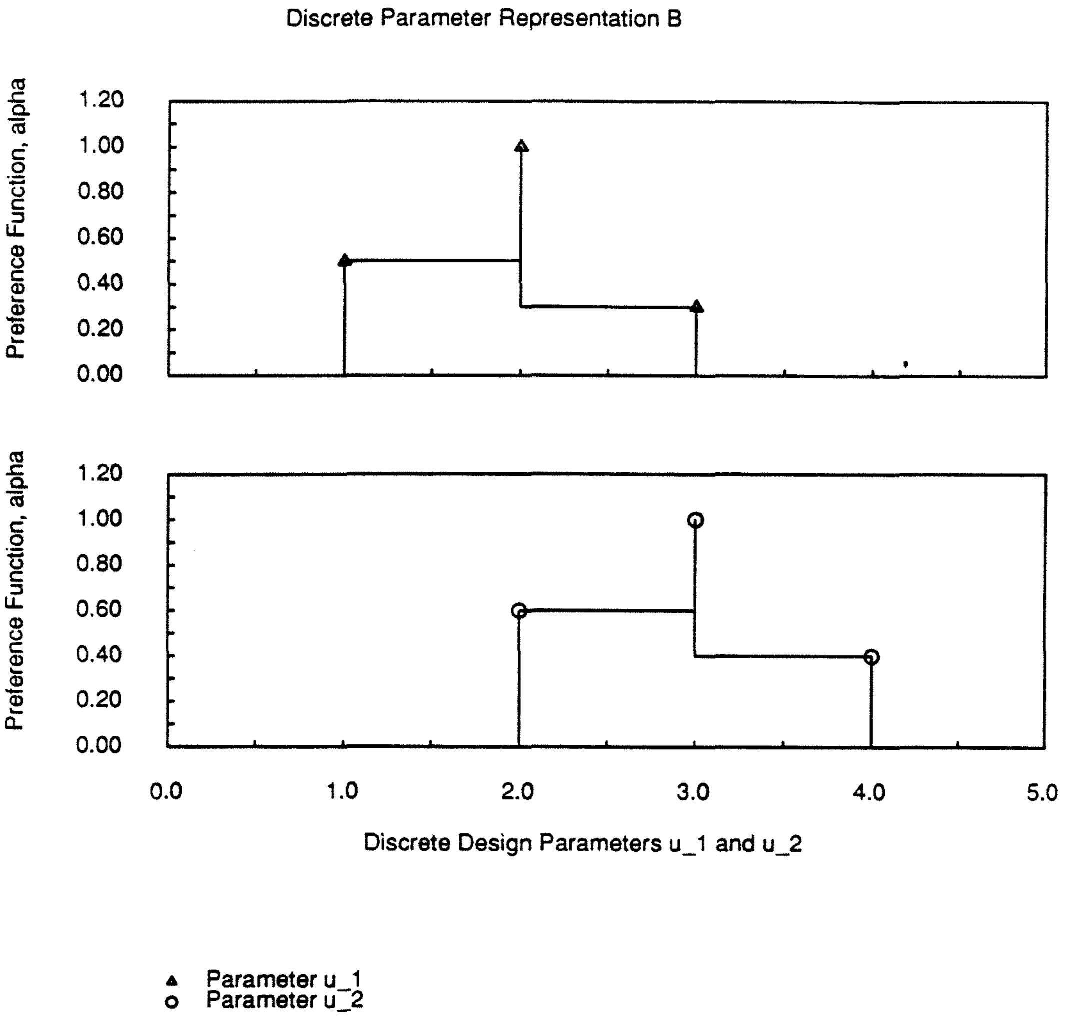

Figure 3.3: Discrete Parameter Representation B.

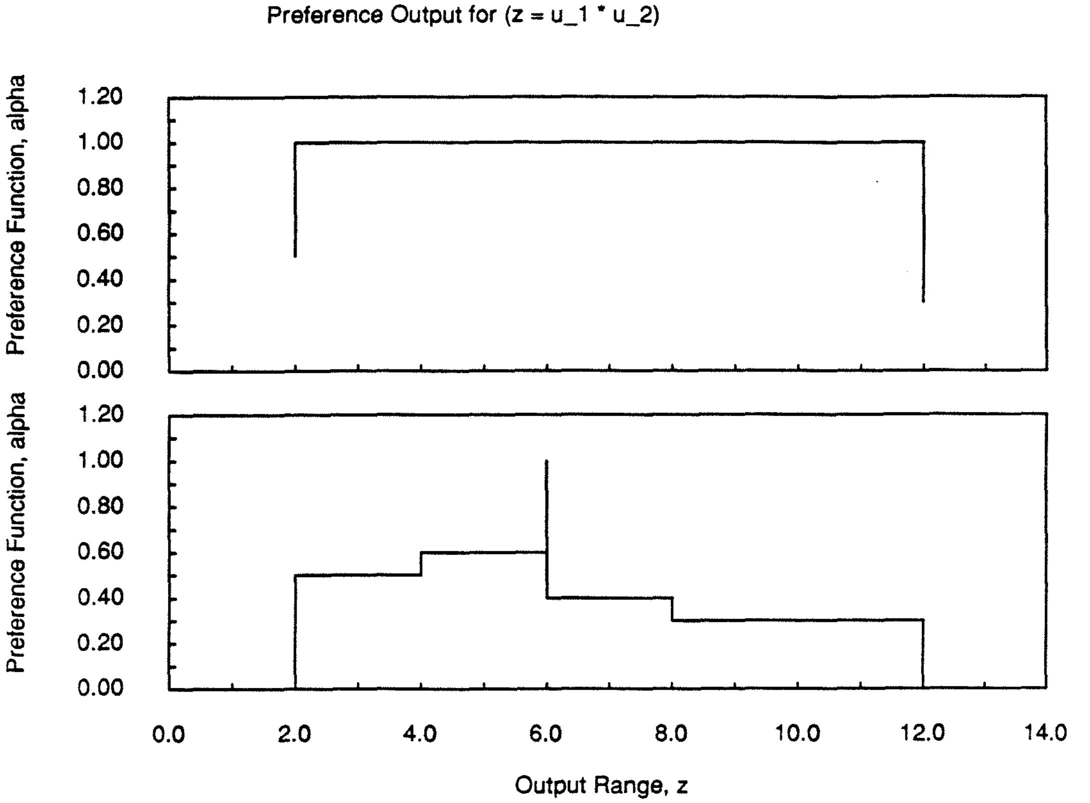

Figure 3.4: Output for $z = u_1 \odot u_2$. (Representation A (top) and B (bottom).)

one input parameter to the corresponding preference value on the output curve. For example, in Figure 3.4, the preference value for $z = 4$ is $\alpha_z = 0.6$. Table 3.1 confirms that at least one design parameter must decrease to $\alpha_{u_i} = 0.6$ (or lower) to achieve this value of z.

From these results, Representation B is the choice for representing discrete design parameters. To demonstrate this further, the same simple multiplication operation as above will be used for z. However, in this case, more discrete choices for each parameter u_i are available. Refer to the new design parameters as u_3 and u_4. Figures 3.5 and 3.6 show the input preference functions; Figure 3.7 shows the corresponding outputs for each representation. Notice that in the output range $z = 6$ to $z = 20$ for Representation A, the values of the inputs contributing to a particular z value cannot be discerned. This is not the case for Representation B, confirming once again, by induction, that Representation B conforms better to the interpretation used for imprecision.

3.2.2 Discrete Frame Example

The frame example presented in Chapter 2 will be used to demonstrate the application of the discrete representation shown above. Continuous preference functions were used in Chapter 2 to represent the design parameters: l, w_{AB}, t, and W. Assume, for this case, that the design parameters l, w_{AB}, and t have the same possible input ranges and the same general preference functions, but with only discrete points available. Assume also that the design parameter W is a constant for this case, $W = 20$kN. Figure 3.8 shows the discrete preference functions for the three design parameters. The result of the bending stress calculation (Equation 2.7 using the extended FWA algorithm) is provided in Figure 3.9. Notice from Figure 3.9 that the results are very

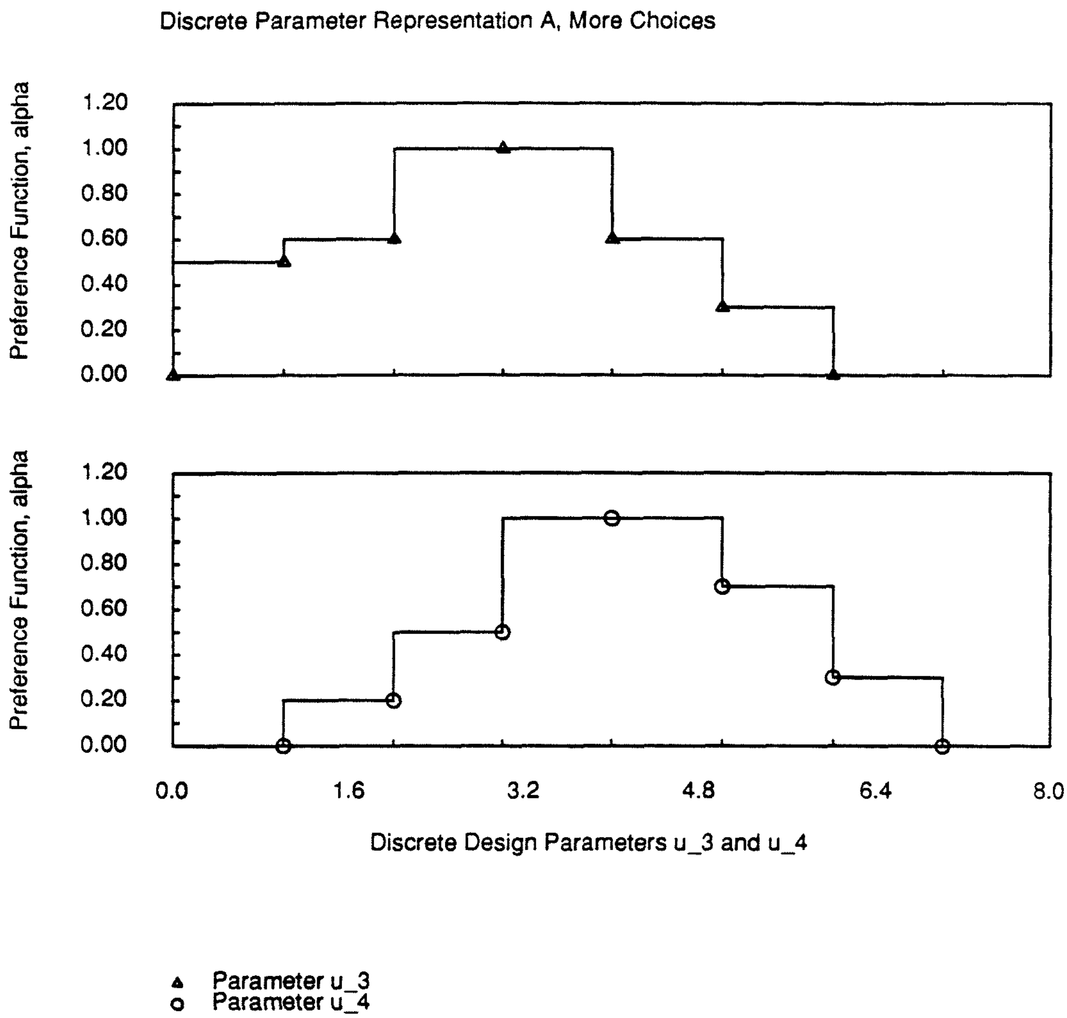

Figure 3.5: Discrete Parameter Representation A, More Choices.

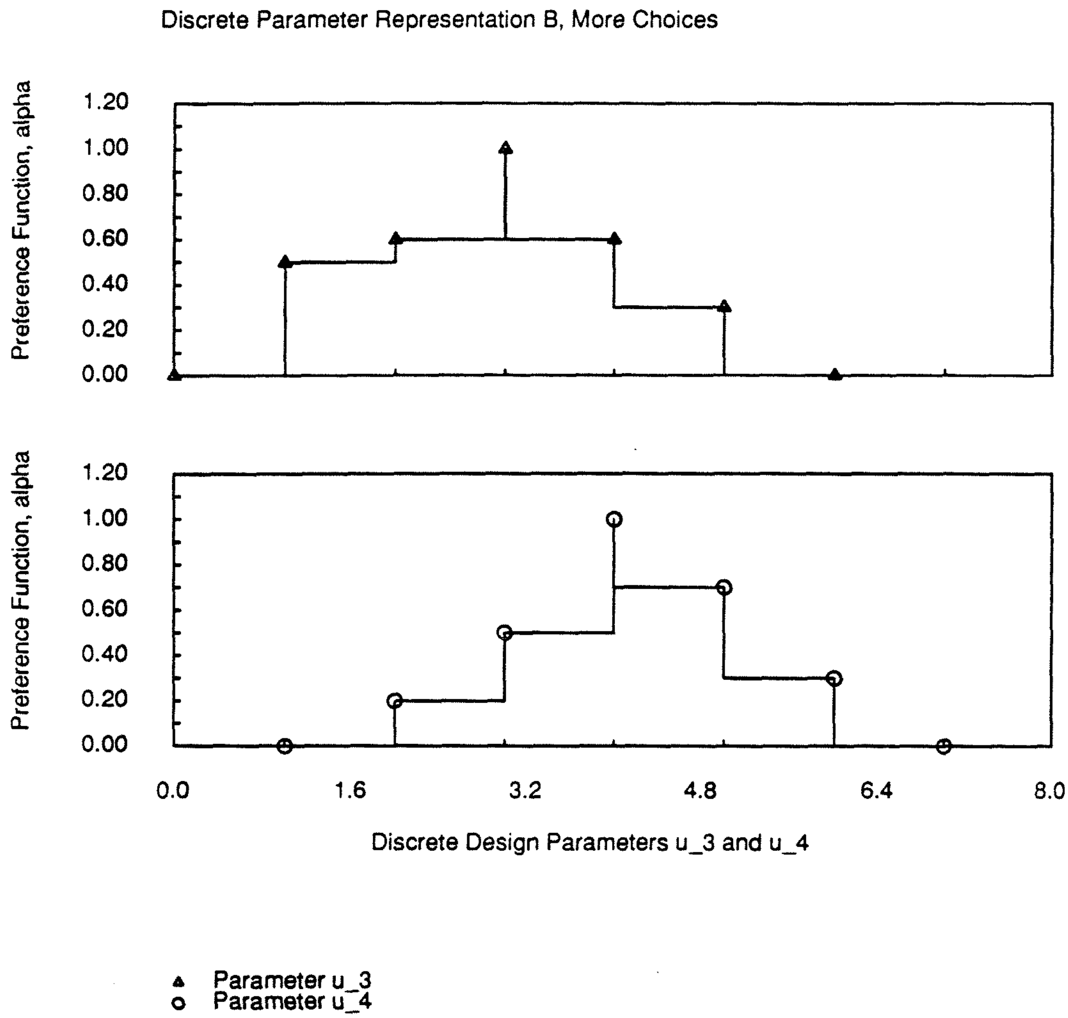

Figure 3.6: Discrete Parameter Representation B, More Choices.

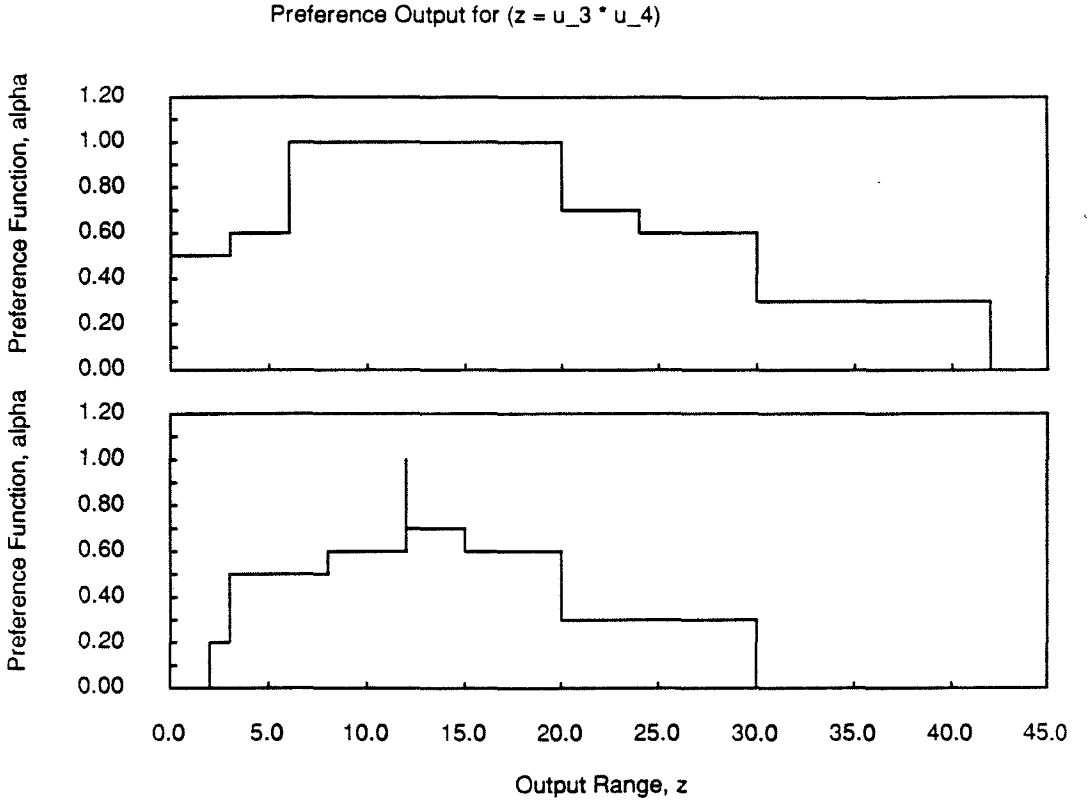

Figure 3.7: Output for $z = u_3 \odot u_4$. (Representation A (top) and B (bottom).)

similar to those in Chapter 2, with the exception that W is a constant.

3.2.3 Combining Continuous and Discrete DPs; The γ Level Measure

In the general case for a design alternative, there will exist both discrete and continuous design parameters. A method is therefore required to combine these two input types using the computation technique described in Chapter 2. The multiplication example ($z = u_1 \odot u_2$) discussed in the previous section will illustrate such a method.

Figure 3.10 shows the preference functions, one discrete and one continuous, for u_1 and u_2. A modified form of the extended FWA algorithm can be applied to the expression for z to determine the output function. Figure 3.11 shows the results at a 0.1 resolution of preference. Figure 3.12 gives the same results for a higher resolution (0.05) and lower resolution (0.2). Notice in the figures that discontinuity exists, as expected, due to the discrete design parameter. To capture the discontinuity effects, the application of the calculation technique must use α-cuts less than the α resolution desired. This means, for example, that if the preference for each output value is desired within 0.1, α-cuts every 0.05 must be applied within each discrete value of a design parameter, to achieve the desired resolution. Appendix C discusses the modification to the extended FWA needed to accomplish this task.

To further demonstrate the technique for combining continuous and discrete design parameters, consider the frame example once again. Instead of the input load W remaining as a constant, W will now be considered as having a continuous representation as shown in Figure 2.7. The other design parameters, l, t, and w_{AB}, will continue to be represented by the input functions of Figure 3.8. Applying the extended FWA algorithm, the resulting bending stress σ preference function is as shown in Figure 3.13. The results are very similar to those in Figure 2.8, a requirement

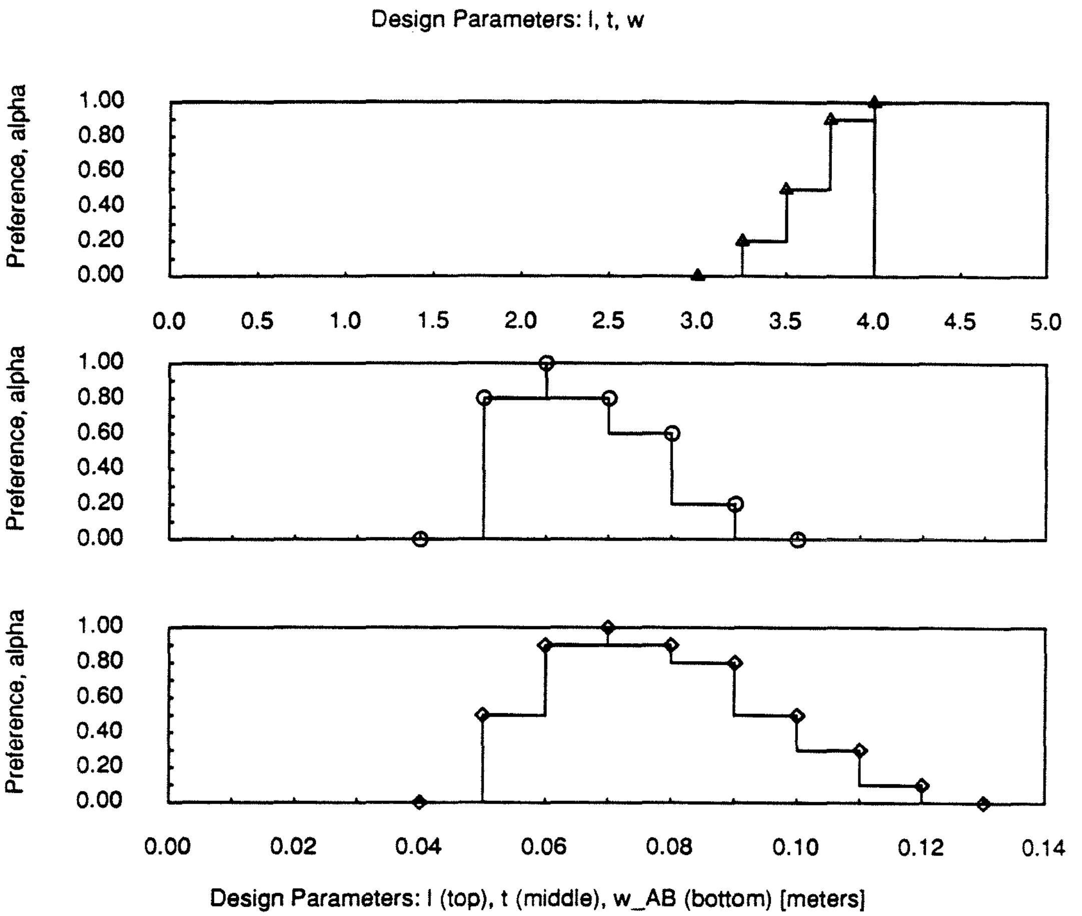

Figure 3.8: Design Parameters: l, t, and w_{AB}.

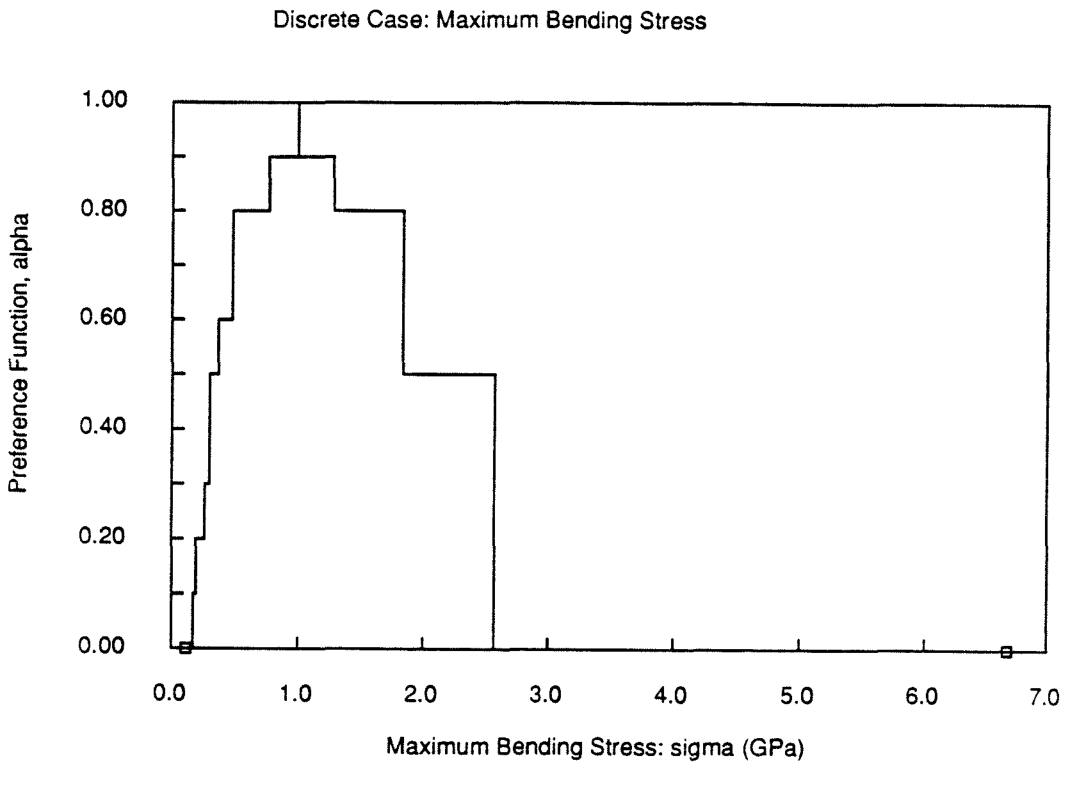

Figure 3.9: Discrete Case: Maximum Bending Stress σ.

of the technique. Moreover, the γ-level measures (Table 3.2) corresponding to this combined continuous and discrete example show little difference to those listed in Table 2.3.

3.3 Multiple-Source Imprecision

Chapter 2 discussed the interpretation of imprecision and design parameters in terms of single-source imprecision. That is, the preference function for a given design parameter depends only on subjective or objective data from one variable. For example, Figure 2.2 shows a preference function for tensile strength that depends only on cost for a given material. Single-source imprecision of this type will not always be the case. The imprecision for a DP may instead be associated with two or more sources. In the case of the tensile strength design parameter, the designer might not only wish to capture the imprecision due to cost, but also due to the various corrosive characteristics of the materials being used, for example. This section will present a straightforward method for combining multiple-source imprecision.

3.3.1 Combining Multiple Imprecision

It is unrealistic to use, simultaneously, multiple preference functions for one design variable. Because the performance parameter expression is a one-to-one mapping of design parameters (in a set fashion), the outcome of a calculation with multiple preference curves would, in general, not be unique. Thus, the preference functions should be combined in some fashion, such that the designer's information is maintained.

Many operations come to mind when considering the combination of preference functions. The most fundamental include the union and intersection set theoretic

Output z	α_{u_1}	α_{u_2}
2	0.5	0.6
3	0.5	1.0
4	1.0	0.6
4	0.5	0.4
6	1.0	1.0
6	0.3	0.6
8	1.0	0.4
9	0.3	1.0
12	0.3	0.4

Table 3.1: Possible z Values for Combinations of u_1 and u_2.

	Performance Parameter
DPs	σ
W	0.140
w_{AB}	1.0
l	0.124
t	0.821

Table 3.2: γ*-Level* Measure Results: Discrete/Continuous DPs for Frame.

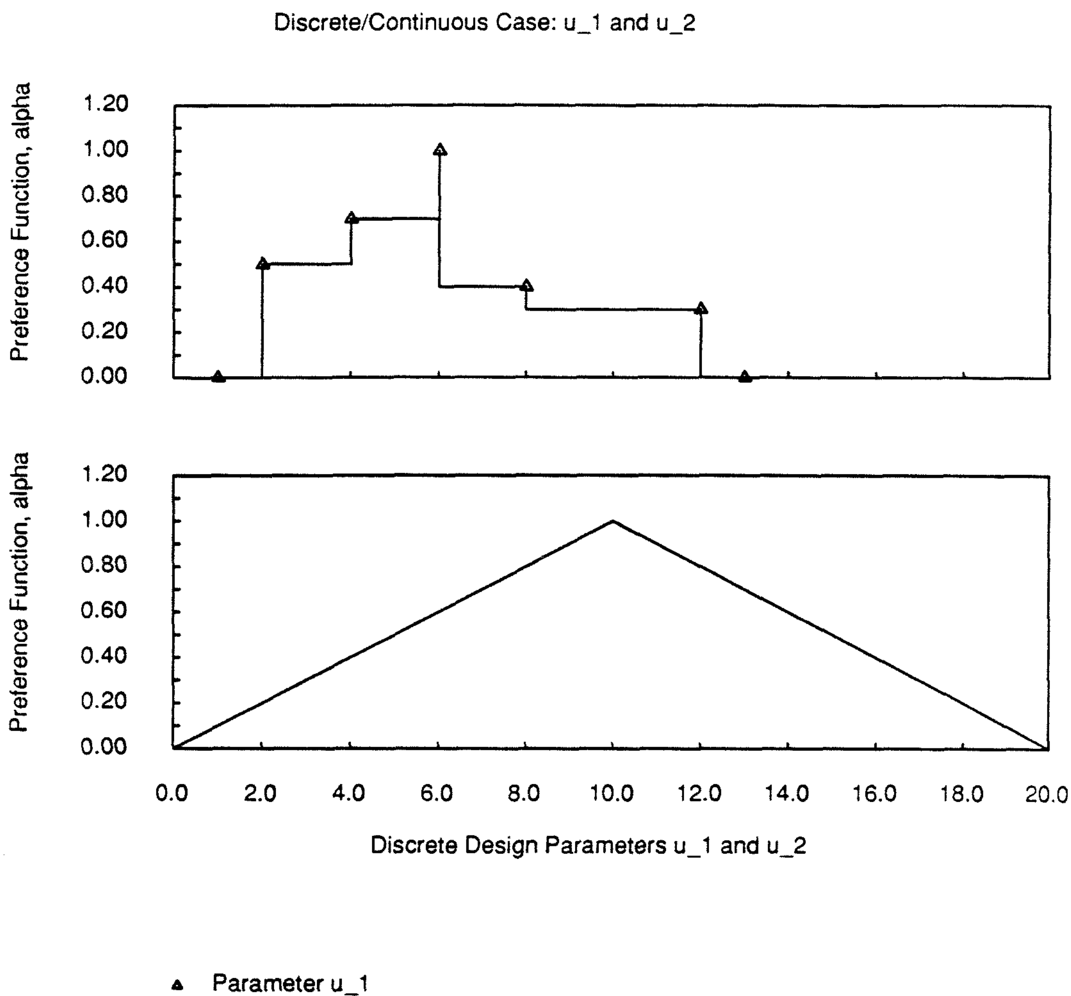

Figure 3.10: Discrete/Continuous Case: Inputs u_1 and u_2.

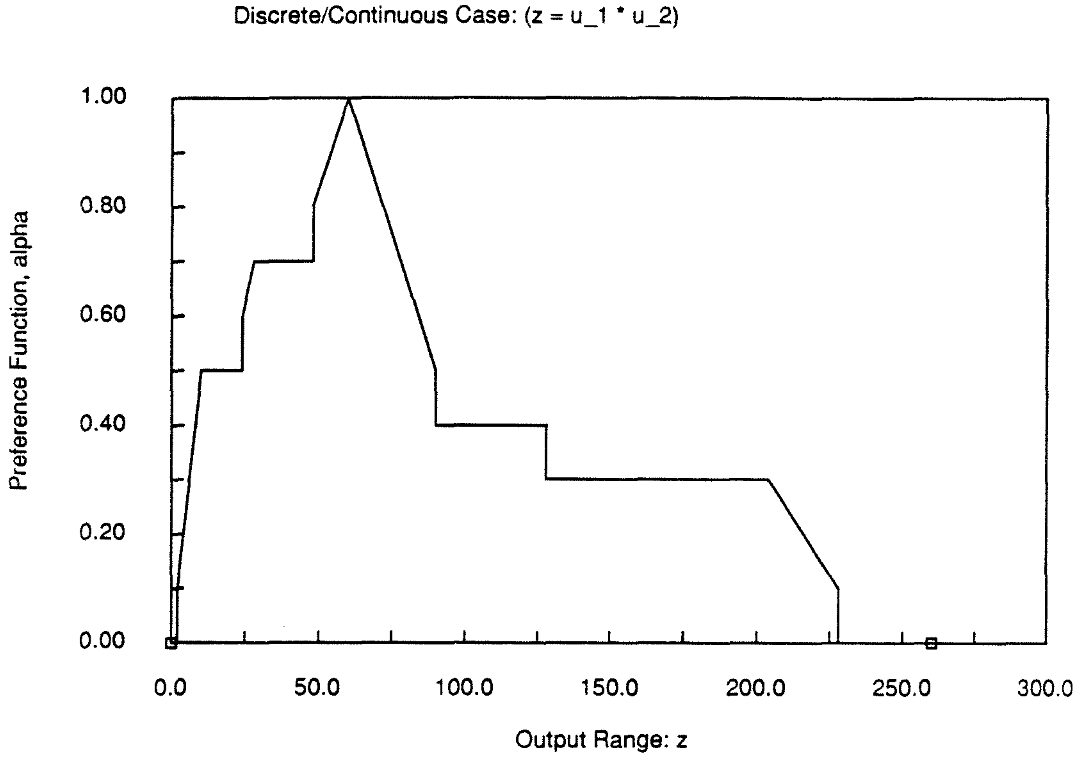

Figure 3.11: Discrete/Continuous Case: $z = u_1 \odot u_2$.

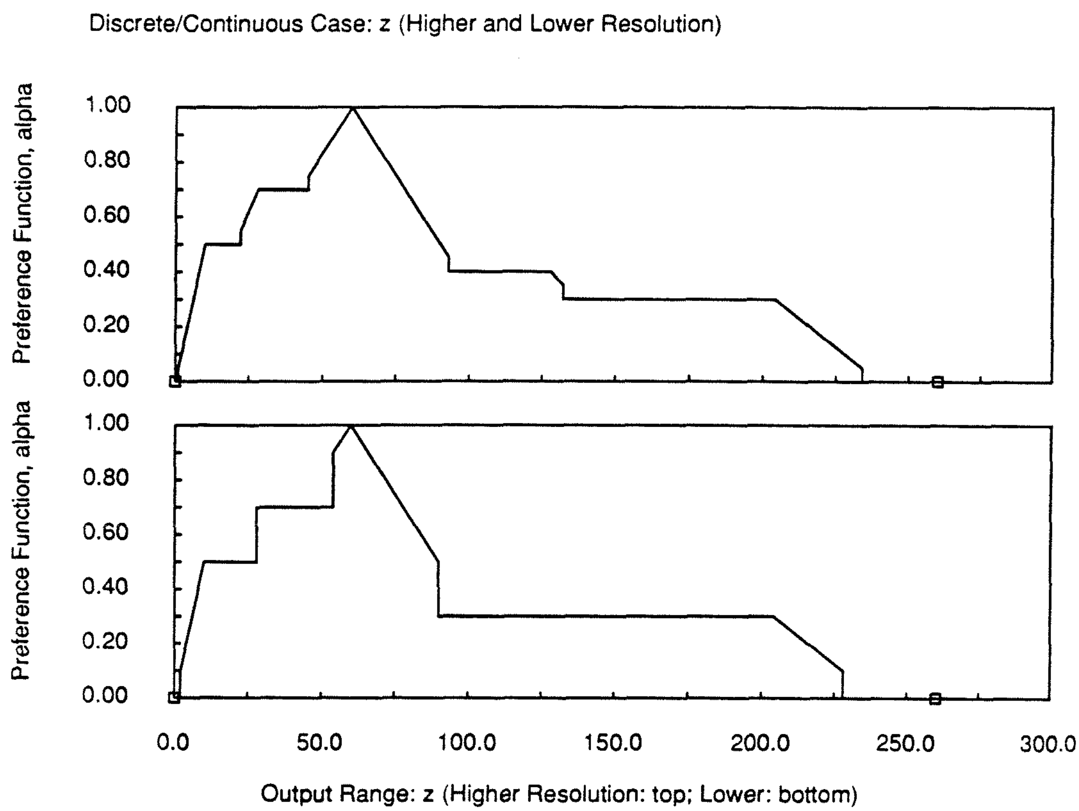

Figure 3.12: Discrete/Continuous Case: z (Higher and Lower Resolutions).

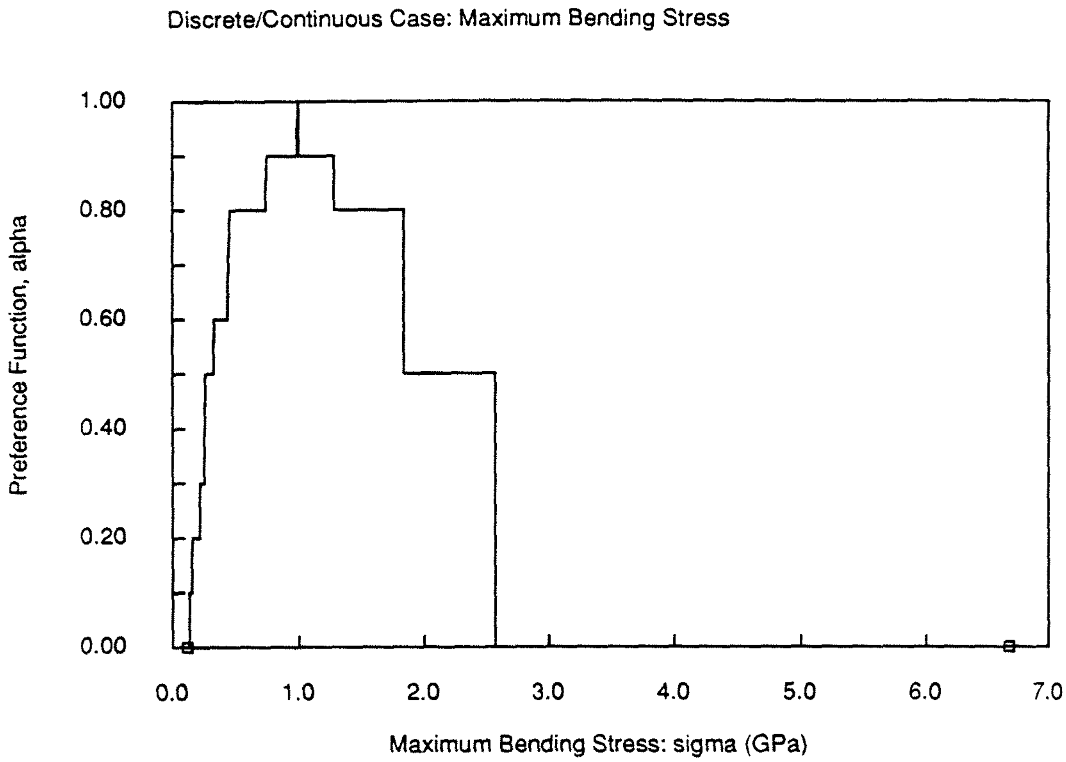

Figure 3.13: Discrete/Continuous Case: Maximum Bending Stress σ.

operations:

$$\alpha_{u_1 \bigcap u_2}(x) = \alpha_{u_1} \bigcap \alpha_{u_2}, \tag{3.1}$$

$$\alpha_{u_1 \bigcup u_2}(x) = \alpha_{u_1} \bigcup \alpha_{u_2}, \tag{3.2}$$

$\forall x \in U$, where U is universe of real numbers possible for u_1 and u_2. Intersection and union operations are context and situation dependent. The most common intersection operations are:

$$\alpha_{u_1 \bigcap u_2}(x) = \alpha_{u_1} \bigwedge \alpha_{u_2}, \text{ or} \tag{3.3}$$

$$\alpha_{u_1 \bigcap u_2}(x) = \alpha_{u_1} \cdot \alpha_{u_2}, \tag{3.4}$$

where $\bigwedge$ denotes the min operator, and Equation 3.4 is the algebraic product. Likewise, the most common union operators are:

$$\alpha_{u_1 \bigcup u_2}(x) = \alpha_{u_1} \bigvee \alpha_{u_2}, \text{ or} \tag{3.5}$$

$$\alpha_{u_1 \bigcup u_2}(x) = \alpha_{u_1} + \alpha_{u_2} - \alpha_{u_1} \cdot \alpha_{u_2}, \tag{3.6}$$

where $\bigvee$ denotes the max operator and Equation 3.6 is the algebraic sum.

An evaluation of these operations in the context of the interpretation of imprecision presented in Chapter 2 shows that the union operation can be discarded immediately. When the union operation is applied to two preference functions for the same design parameter, the resulting *support* will envelope each and every value of the design parameter having a preference greater than zero in *either* of the original preference functions. This violates the interpretation of preference. If a value having preference of zero in one preference function does not correspond to a zero preference in another preference function, the union operator (in either the form of Equation 3.5 or of Equation 3.6) will include the design value in the resulting function. But a design value with zero preference in any of the preference sources implies that the designer does not choose to use the value in any way. An operation used to combine multiple preferences should capture this principle.

The intersection operation, when applied to two preference functions, includes only design values in the resulting support that have preferences greater than zero in *both* of the original preference functions. Given that the intersection satisfies this principle, Equation 3.3 and Equation 3.4 must be compared to determine which is appropriate for the case at hand. The algebraic product (Equation 3.4) combines the preferences for two design preference functions such that the output preference for a given value is less than both of the originals. Essentially, this operation collapses the preference functions toward zero. For many preference functions for the same design variable and subsequent combinations, the collapsing effect may be dangerous because the preference values of a design parameter will not be distinguishable, *i.e.*, $\alpha_{u_i}(x) \approx 0 \ \forall x$.

The intersection operation given by Equation 3.3, on the other hand, does not result in a collapsing of the combined preference function. The min operator simply "selects" the minimum preference for each value of the design parameter. Such a combination makes sense when considering the imprecision interpretation of Chapter 2. The preference function associated with a design parameter denotes a range of acceptability of a design parameter value from completely unacceptable ($\alpha = 0.0$) to fully acceptable ($\alpha = 1.0$). By operating with the least preference from multiple-source imprecision, the acceptability of a value for a design parameter may always be justified in terms of the *worst case* scenario. Furthermore, the preference values resulting from a combination of the form of Equation 3.3 may always be traced back to the original preference curve contributing the value. This is useful to determine the effect of making changes to the preference functions during different stages of the design process.

Thus, the intersection operation given by Equation 3.3 will be used in this methodology to combine multiple-source imprecision. Note that a combined preference for a design parameter u_i must be extended vertically to a preference of one (1.0) because the operation may, in general, cause a non-normal curve. This extension, or normality condition, is required because the computation technique outlined in Chapter 2 requires a value (or interval) for a design parameter at each α-cut. The interpretation of this extension is straightforward. Because the combined preference curve for a design parameter u_i may have a peak less than one (1.0), no value in the input range is fully acceptable. By extending the peak of the combination vertically to a preference of one (1.0), the *most acceptable* value in the input range is used for calculations at higher α-cut levels. The *backward path* may be used to determine the preference values of the design parameters contributing to any value on the output performance curve.

3.4 Remarks on the Extensions

The problems addressed in this chapter, *i.e.*, discreteness and multiple-source imprecision, are often a source of high dimensionality in engineering design, especially when contemplating exhaustive design techniques that explore the entire design space. Such problems can often be intractable (combinatorially) for even the most basic analyses. By extending the imprecision problem approach (Chapter 2) to the discrete and multiple-source domains, the designer is not provided with all possible information to evaluate the alternatives; however, worst-case information and evaluations of sets of possible choices are easily presented and computed. The extensions discussed in this chapter require very little modification to the approach described in Chapter 2, and are in fact not fundamental in the understanding of the model of representing and manipulating imprecision. But when dealing with the pragmatics of practical design, the ability to reasonably represent discreteness and multiple imprecision is a necessity.

Chapter 4

An Application of the "Semi-Automated" Approach to the Imprecision Problem

4.1 A Brake Design Example

A brake design will illustrate the computational approach of Chapter 2 for a problem with realistic design complexities, *i.e.*, geometric and algebraic non-linearities in the performance parameter expressions, along with a need to simultaneously evaluate multiple (alternative) configurations.

4.1.1 Brake Example Nomenclature

a = *distance from center of drum brake to pivot*

b = *face width of frictional material*

c = *perpendicular distance, hinge to actuating force*

h = *convection coefficient*

k = *conductivity*

p_a = *maximum operating pressure of material*

p_{max} = *maximum pressure for disk brake, uniform wear*

p_p = *pressure for disk brake, uniform pressure*

r = *inner drum brake radius*

r_o = *outer radius of disk brake*

r_i = *inner radius of disk brake*

t = *time*

A = *factored common expression to simplify formulas*

B = *factored common expression to simplify formulas*

C = *specific heat*

F = *actuating force*

F_a = *actuating force for drum brake*

F_p = *actuating force for disk brake, uniform pressure*

F_w = *actuating force for disk brake, uniform wear*

F_{wheel} = *force on one wheel*

F_x = *x-component of actuating force*

F_y = *y-component of actuating force*

I_d = *mass moment of inertia of the drum system*

L_w = *width dimension of drum brake*

L_{w1} = *width dimension of disk brake*

M_b = *mass of brake parts*

M_μ = *moment due to frictional forces*

M_N = *moment due to normal forces*

M_v = *mass of vehicle*

R_1 = *outer radius of drum brake*

R_{wheel} = *radius of wheel*

T = *torque for drum brake*

$T_{ambient}$ = *ambient temperature*

T_r = *temperature rise*

T_d = *temperature decay profile*

$\mathcal{T}_p$ = *torque for disk brake, uniform pressure*

$\mathcal{T}_w$ = *torque for disk brake, uniform wear*

V = *velocity of vehicle*

μ = *coefficient of friction*

ω = *angular velocity of the drum brake*

ρ = *density of brake drum material*

θ_a = *angle of max pressure measured from pivot*

θ_1 = *angle to beginning of frictional material*

θ_2 = *angle to end of frictional material*

$\tilde{}$ $\Rightarrow$ *(tilde) denotes a preference function*

4.1.2 The Problem Statement

Brakes perform the action of transforming the energy of a moving vehicle into heat (usually by use of friction) over an interval of time. Many factors must be considered when designing such devices. Of primary importance is the ability of the device to avoid destructive temperature rises while still dissipating wide ranges of output power. Figure 4.1 schematically represents a brake system, where the drum has a rotational speed of ω and the ground symbol on the brake indicates no rotation.

The problem here is to design a braking system for a vehicle that will adequately stop the vehicle for a certain range of speeds and that will not degrade appreciably ('fade' due to temperature rise) over time. Two possible design configurations for the problem (Figures 4.2 and 4.3) will be considered. The first is an internally actuated, pivoted rim brake (a drum brake), where only one shoe has been shown in the figure. The other shoe would be placed in a symmetrical arrangement with the

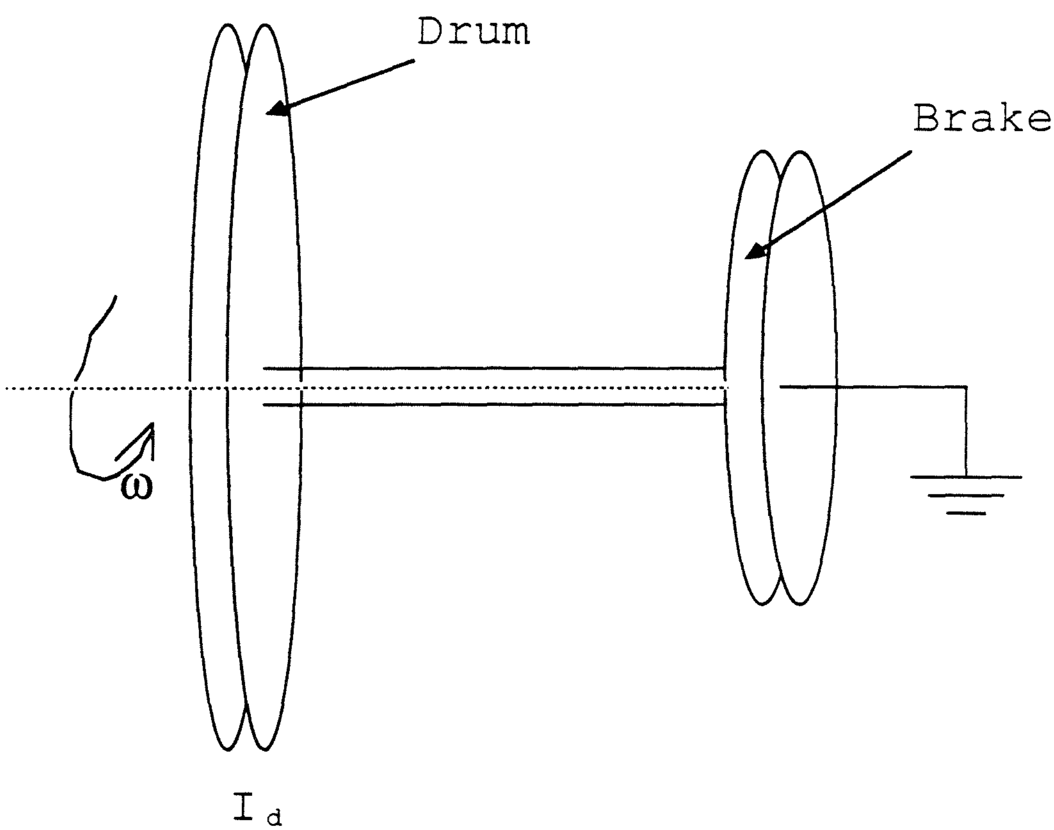

Figure 4.1: Brake Problem.

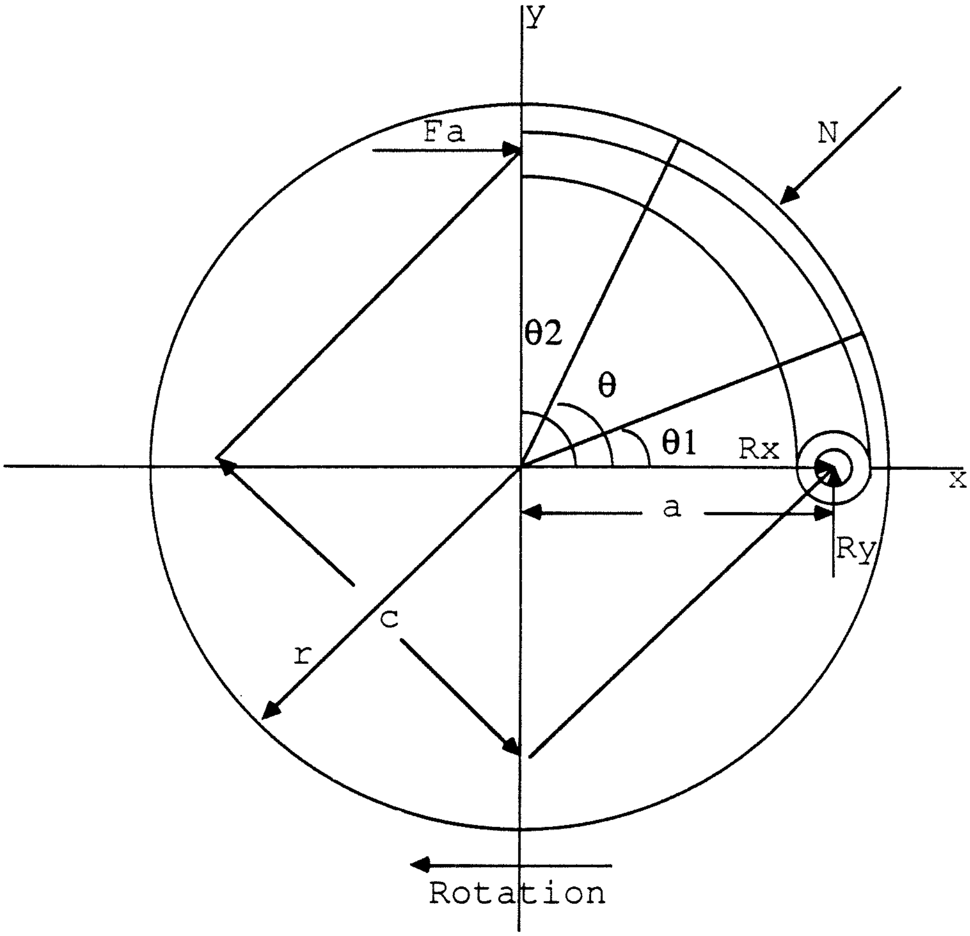

Figure 4.2: Drum Brake System.

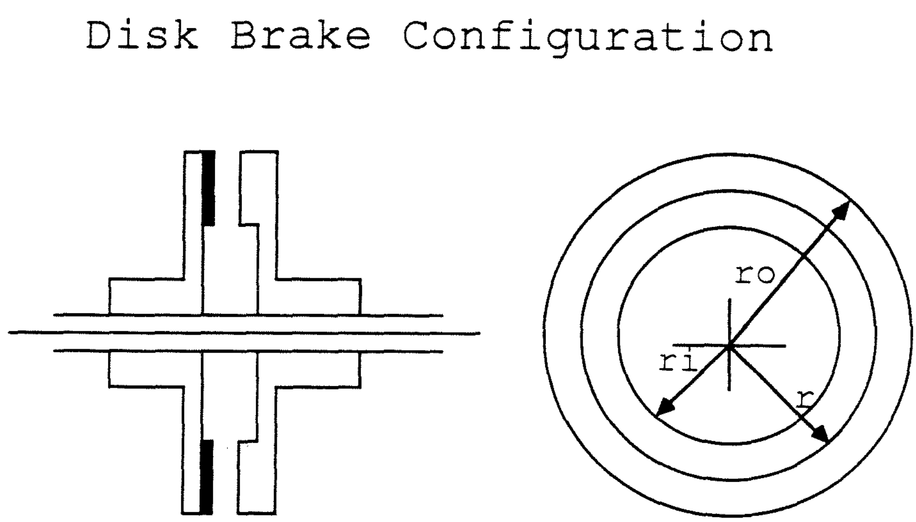

Figure 4.3: Disk Brake System.

one shown. Using, in part, the mathematical formulation of Shigley and Mitchell [67], four primary performance parameters can be chosen for the drum brake: actuating force F_a, transmitted torque $\mathcal{T}$, temperature rise T_r and temperature decay profile (proportional to heat rejection) T_d. In equation form, these PPs can be expressed as:

$$F_a = \frac{M_N - M_\mu}{c}, \tag{4.1}$$

$$\mathcal{T} = \frac{\mu\, p_a\, b\, r^2}{\sin\theta_a}(\cos\theta_1 - \cos\theta_2), \tag{4.2}$$

$$T_r = \frac{E}{C\, M_b}, \tag{4.3}$$

$$T_d = T_r - \left(T_r e^{-\frac{2\pi L_w t}{[(\frac{\ln(R_1/r)}{k} + \frac{1}{rh})M_b C]}} \right). \tag{4.4}$$

In order to completely describe the brake drum problem, the expressions for total system energy E, A, B, M_N, M_μ, I_d, and M_b must be given:

$$E = \frac{1}{2} I_d\, \omega^2 + \frac{1}{2} M_v\, V^2, \tag{4.5}$$

$$A = \frac{1}{2}(\sin^2\theta_2 - \sin^2\theta_1), \tag{4.6}$$

$$B = (\frac{\theta_2}{2} - \frac{\sin 2\theta_2}{4}) - (\frac{\theta_1}{2} - \frac{\sin 2\theta_1}{4}), \tag{4.7}$$

$$M_\mu = \frac{\mu\, p_a\, b\, r}{\sin\theta_a}[r(\cos\theta_1 - \cos\theta_2) - \frac{a}{2}(\sin^2\theta_2 - \sin^2\theta_1)], \tag{4.8}$$

$$M_N = \frac{p_a\, b\, r\, a}{\sin\theta_a} B, \tag{4.9}$$

$$I_d = \frac{1}{2} M_b\, (R_1^2 + r^2), \tag{4.10}$$

$$M_b = \pi\, \rho\, L_w\, (R_1^2 - r^2). \tag{4.11}$$

The design parameters for the problem are as follows: the pivot distance a; the radius r; the material angles θ_1 and θ_2; the coefficient of friction μ; the material pressure p_a; the actuating force distance c; the face width b; the angular velocity ω; the specific heat C; the vehicle mass M_v; the outer drum radius R_1; the drum width L_w; the drum material conductivity k; and the coefficient of convection h. Note that the velocity of the vehicle V is not an independent DP because it depends directly on ω. The imprecision due to the "constant" ρ is viewed as negligible for this design. If desired, ρ can be included as a DP (instead of a constant) to verify this assumption.

The second configuration to be considered is a disk brake system, comprising two simultaneously actuated brake pads that close on both sides of the rotating disk. The same PPs exist for the disk configuration as for the drum configuration. However, in this case, the actuating force and torque for the system depend on an assumption: uniform wear or uniform pressure. Denoting the actuating force and torque for uniform wear as F_w and T_w, and similarly denoting these properties for uniform pressure as F_p and T_p, the governing PPs can be expressed as:

$$F_w = 2\pi p_{max} r_i (r_o - r_i), \tag{4.12}$$

$$F_p = \pi p_p (r_o^2 - r_i^2), \tag{4.13}$$

$$\mathcal{T}_w = \pi \mu p_{max} r_i (r_o^2 - r_i^2), \tag{4.14}$$

$$\mathcal{T}_p = \frac{2}{3}\pi \mu p_p (r_o^3 - r_i^3), \tag{4.15}$$

$$T_d = T_r - \left(T_r e^{-\frac{h 2\pi r_o^2}{M_b C}} \right). \tag{4.16}$$

The energy expression and temperature rise performance parameter are the same as the drum brake configuration, except the expression for the mass of the brake parts M_b differs:

$$M_b = \pi \rho r_o^2 L_{w1}. \tag{4.17}$$

The corresponding DPs for the disk brake problem are: maximum pressure for uniform wear p_{max}; outer radius of disk r_o; inner point of contact of brake shoe r_i;

coefficient of friction μ; pressure for uniform pressure p_p; specific heat C; convection coefficient h; width of disk L_{w1}; angular velocity ω; and vehicle mass M_v.

4.1.3 Performance Specifications

Given the four PPs for each brake configuration as above, preliminary performance criteria can be specified. For example, the designer might associate a limiting value (functional requirement) F^r for the actuating force F_a, F_w, or F_p. Such a limiting value represents a maximum actuating force that can be used in the design, *i.e.*, F must be less than or equal to F^r where:

$$F \leq F^r = 25.0 \text{ kN}.$$

Consider the next PP, torque ($\mathcal{T}$, $\mathcal{T}_w$, or $\mathcal{T}_p$). Instead of a maximum as with the actuating force, the torque must meet a minimum limit for the design. Specifically, the frictional torque developed by the brake system must be matched by the frictional torque developed between the road and the tires. A common value to use for the torque on the brake is three-fourths (3/4) of the average force (weight) on one of the wheels multiplied by the radius of the wheel. The resulting minimum performance torque for the brake design can be expressed as:

$$\mathcal{T} \geq \mathcal{T}^r = \tfrac{3}{4} F_{wheel} \times R_{wheel} \text{ kN-m},$$

where F_{wheel} equals the force on the wheel, R_{wheel} equals the wheel radius. Notice that a single numerical FR value does not result for $\mathcal{T}^r$. Because $\mathcal{T}^r$ depends on the force on one wheel which in turn depends on the mass of the vehicle M_v, the functional requirement $\mathcal{T}^r$ is a preference curve as a function of the vehicle mass. (The radius of the wheel is assumed to be specified as a crisp constant for the design.) After the input design parameter M_v has been determined, T^r can be calculated and updated if M_v subsequently changes.

A specification for the maximum temperature rise can be obtained from empirical data:

$$T_r \;\leq\; T_r^r \;=\; (260 - T_{ambient}) \;\; {}^\circ\mathrm{C}.$$

The performance parameter T_d does not have a single FR value or a preference function representation. A qualitative specification (denoted by T_d^r) must therefore be used: *maximize heat dissipation to minimize the possibility of destructive temperatures for the frictional material.*

These four performance specifications, comprised of two single values, one preference function, and one qualitative statement, form the set of FRs for the design or functional requirement space (FRS). This FRS will be used to (1) rate each design configuration individually according to performance, (2) compare the alternatives to determine major differences, and (3) evaluate whether each configuration should be carried to the next stage in the design process.

4.1.4 Input Design Parameters

Triangular functions have been used for the input fuzzy parameters in both the drum brake and disk brake configurations. Table 4.1 lists the necessary data for constructing the brake design's input parameters, where the three data values for each DP have the following meaning: left-extreme value for preference of zero (0), peak value for preference of one (1), and right-extreme value for preference of zero (0). Various interpretations exist by which the engineer can assign the preference functions, *i.e.*, the preference functions can be constructed from objective or subjective data as discussed in Chapter 2. The input parameter ω is a special case. The preference

DPs (units)	$\alpha = 0$	$\alpha = 1$	$\alpha = 0$
a (m)	0.010	0.020	0.030
b (m)	0.025	0.050	0.075
μ	0.10	0.30	0.50
p_a (kPa)	300.0	1100.0	1900.0
r (m)	0.10	0.15	0.20
R_1 (m)	0.005	0.015	0.030
θ_1 (deg.)	0.0	0.0	30.0
θ_2 (deg.)	90.0	120.0	150.0
k (W/m °C)	50.0	65.0	80.0
h (W/m^2 °C)	120.0	150.0	180.0
M_v (kg)	200.0	900.0	1600.0
L_w (m)	0.025	0.060	0.100
C (J/kg °C)	400.0	500.0	900.0
p_{max} (kPa)	300.0	1100.0	1900.0
p_p (kPa)	300.0	1100.0	1900.0
r_o (m)	0.10	0.15	0.20
r_i (m)	0.000	0.025	0.040
L_{w1} (m)	0.0125	0.0250	0.0375

Table 4.1: Brake Example: Fuzzy Design Parameter Data.

value associated with each design parameter value of the support of $\tilde{\omega}$ (the range of inputs making up the set 0.0 to 30.0 rps) represents the complete range of speeds the brake is being designed to meet. The corresponding preference may be explained as follows. The peak of the preference function with preference of one (1) represents the speed that the brake design should adequately handle (design speed: $\omega = 15.0$ rps). The need for the design to meet values of speed greater than 15 rps is less important, hence for support values to the right of the input preference function's peak, the preference decreases. To the left of the peak, the preference takes on smaller values as the design speed ω decreases from $\omega = 15$ rps, indicating a lower desirability of a brake system designed to handle only those speeds.

Other preference functions with slightly different interpretations could have been used for the input ω parameter. For example, instead of a single peak at $\omega = 15$ rps, an interval of adequate or required speeds could have been specified (with preference of one (1)), resulting in a trapezoidal function.

The other input parameters have similar interpretations to that presented in Chapter 2.

4.1.5 Output Performance Parameters

With the input parameters specified, the outputs of the PPs can be determined using the performance parameter equations listed earlier, and the FWA implementation procedure discussed in Chapter 2. The schematic representations of the fuzzy outputs for both the drum and disk brake alternatives are in Figures 4.4 through 4.18.

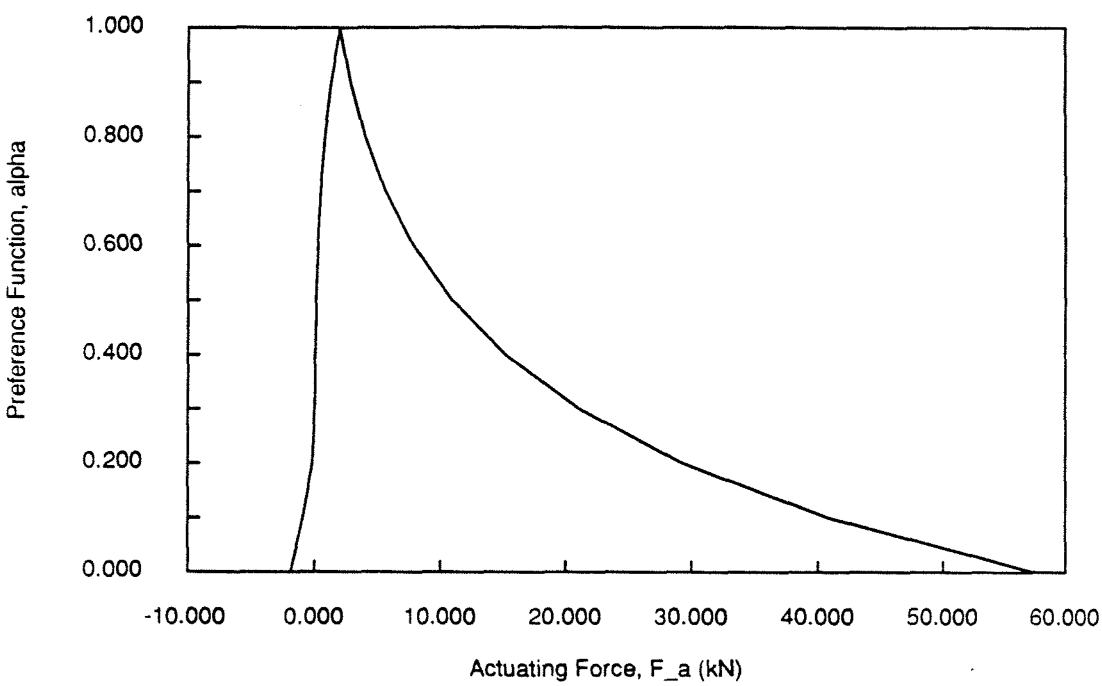

Figure 4.4: Drum Brake: Actuating Force Output Set, $\tilde{F}_a$.

4.1.5.1 Drum Brake Output Performance Parameters

Using the fuzzy outputs for the drum brake configuration (Figures 4.4 through 4.9), the performance parameters can be rated against the performance specifications, F^r, $\mathcal{T}^r$, T_r^r, and T_d^r. Figure 4.4 represents the fuzzy output $\tilde{F}_a$ for the drum brake actuating force performance parameter, from Equation 4.1. The value of $\tilde{F}_a$ for preference of one ($\alpha_{\tilde{F}_a} = 1$) is $F_a = 1.94$ kN. Comparing this result with F^r, $\tilde{F}_{a(\,at\,\alpha_{\tilde{F}_a}=1)}$ is less than F^r by at least a factor of ten. The imprecision in the actuating force output (*i.e.*, the change of $\tilde{F}_a$ with respect to preference $\alpha_{\tilde{F}_a}$) is very small when compared to the difference of F^r and $\tilde{F}_{a(\,at\,\alpha_{\tilde{F}_a}=1)}$. In fact, the actuating force for the drum brake does not exceed F^r until $\alpha_{\tilde{F}_a} = 0.2$, implying that a change in any, some, or all of the input parameters with preferences greater than or equal to 0.2 for one side of the preference function will still satisfy F^r. Thus, the drum brake configuration satisfies the actuating force functional requirement for the most confident (or desired) values of the input DPs, and input values with preferences far off the peaks.

The negative force results found in Figure 4.4 must also be considered. In this case, a geometrical dependency inherent in the problem, *i.e.*, the relationship between the input parameter a and the radius r, can produce negative actuating forces. Although such negative force values are perfectly correct physically, control problems arise of lifting the shoe from the drum with F_a instead of applying the shoe to the drum with this force. These output results for $\tilde{F}_a$ show that care must be taken when using one or more input parameters with preference less than 0.3.

The fuzzy output for the torque performance parameter $\mathcal{T}$, from Equation 4.2, is shown in Figure 4.5. Note the large range (typical in preliminary engineering design) for the performance parameter $\mathcal{T}$ ($> 10^3$) that results from reasonable ranges for the five design parameters (Table 4.1). This demonstrates the need for a computationally

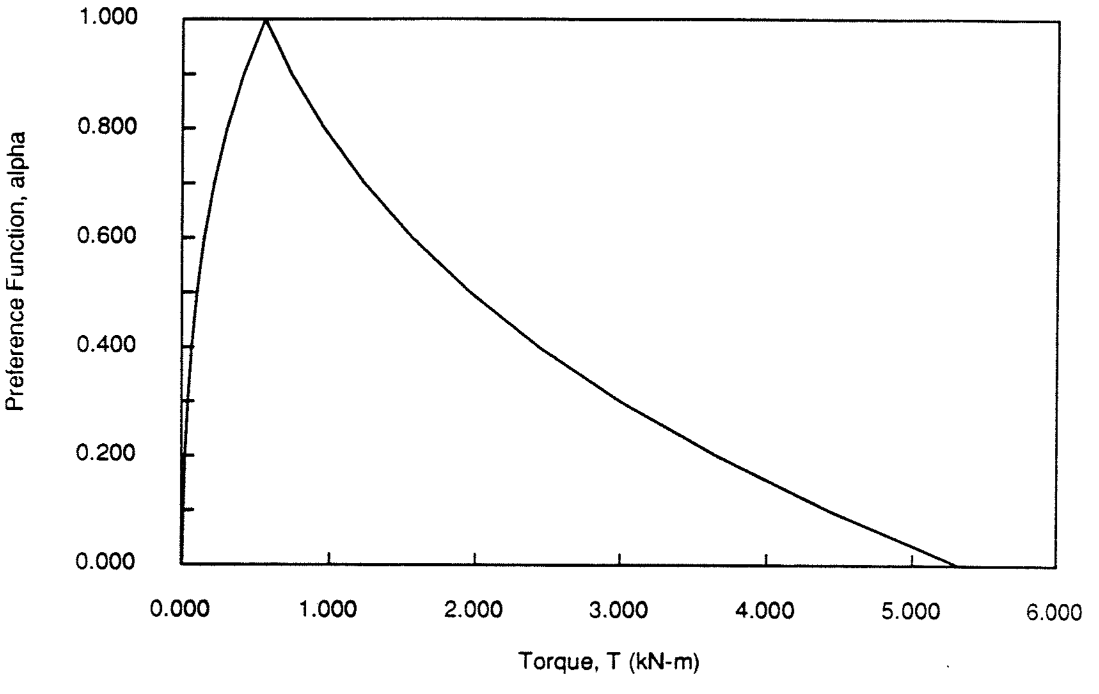

Figure 4.5: Drum Brake: Torque Output Set, $\tilde{\mathcal{T}}$.

efficient method for this stage of the design process.

Before this output can be compared with the performance specification, $\mathcal{T}^r$ must be calculated using the fuzzy input parameter M_v. Figure 4.10 shows the result. (The right side of the curve in Figure 4.10 corresponds to increasing values of M_v, while the left side corresponds to the converse.) The value of $\tilde{\mathcal{T}}^r$ for preference of one is $\mathcal{T}^r = 0.579$ kN-m. Comparing this value with the fuzzy output for $\mathcal{T}$, $\tilde{\mathcal{T}}_{(at\, \alpha_{\tilde{\mathcal{T}}}=1)}$ is less than $\mathcal{T}^r$; thus the torque output for the most confident inputs *does not* satisfy the FR. Values to the right of the peak in Figure 4.5 are found to satisfy $\mathcal{T}^r$; in fact, for $\alpha_{\tilde{\mathcal{T}}} = 0.9$, $\mathcal{T} = 0.736$ kN-m. Checking the fuzzy performance specification for $\mathcal{T}^r$ in Figure 4.10, it is found that if M_v is the parameter that changes, the corresponding $\mathcal{T}^r$ to the right of the peak is satisfied. The output torque results for the drum brake configuration demonstrate that further analysis must be performed to satisfy the functional requirements. Specifically, for the most confident input parameter values, and values with preference off the peaks, the torque requirement might be barely satisfied or not at all.

The temperature rise output set for the drum brake configuration is in Figure 4.6. Using a constant ambient temperature of $T_{ambient} = 24°$C, the corresponding performance specification for temperature rise is $T_r^r = 236°$C. As expected for any braking system, a complete stop from the design speed of $\omega = 15$ rps (or approximately 35 m/s) produces a temperature rise, $\tilde{T}_{r\,(at\, \alpha_{\tilde{T}_r}=1)} = 35°$C, which falls well within the functional requirement T_r^r. It can be inferred that a few stops in succession at this speed will also result in an accumulative temperature rise that meets T_r^r. However, for many multiple braking actions in succession, a nominal value of $T_r = 35°$C for each action will accumulate to temperatures approaching the performance requirement. Moreover, considering the output values for $\tilde{T}_r$, a small change

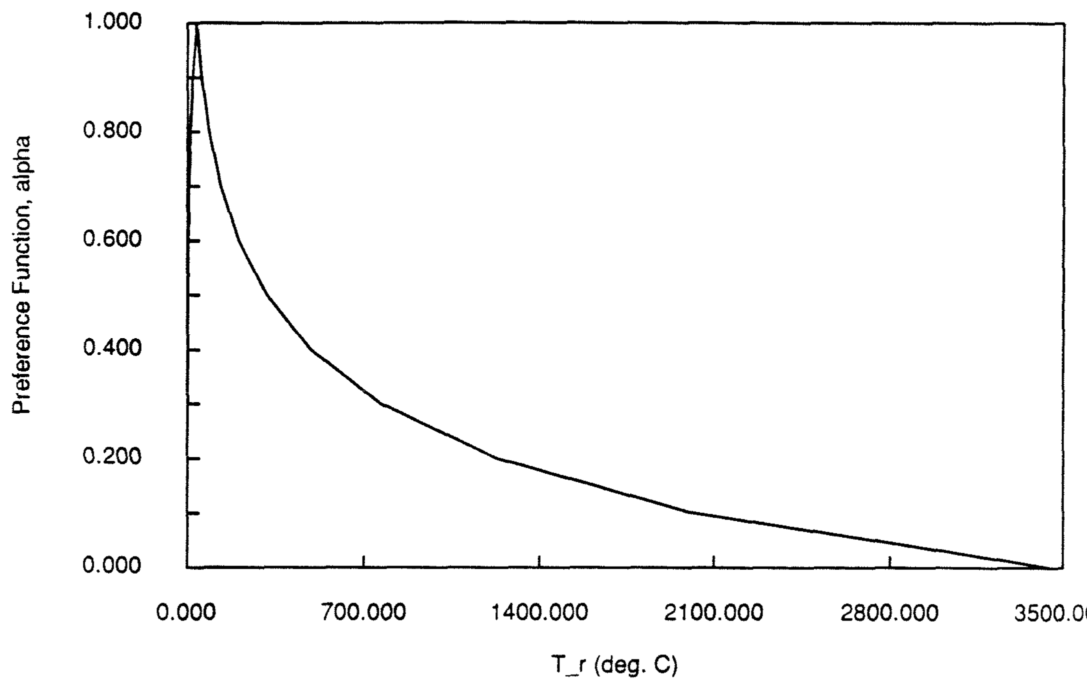

Figure 4.6: Drum Brake: Temperature Rise Output Set, $\tilde{T}_r$.

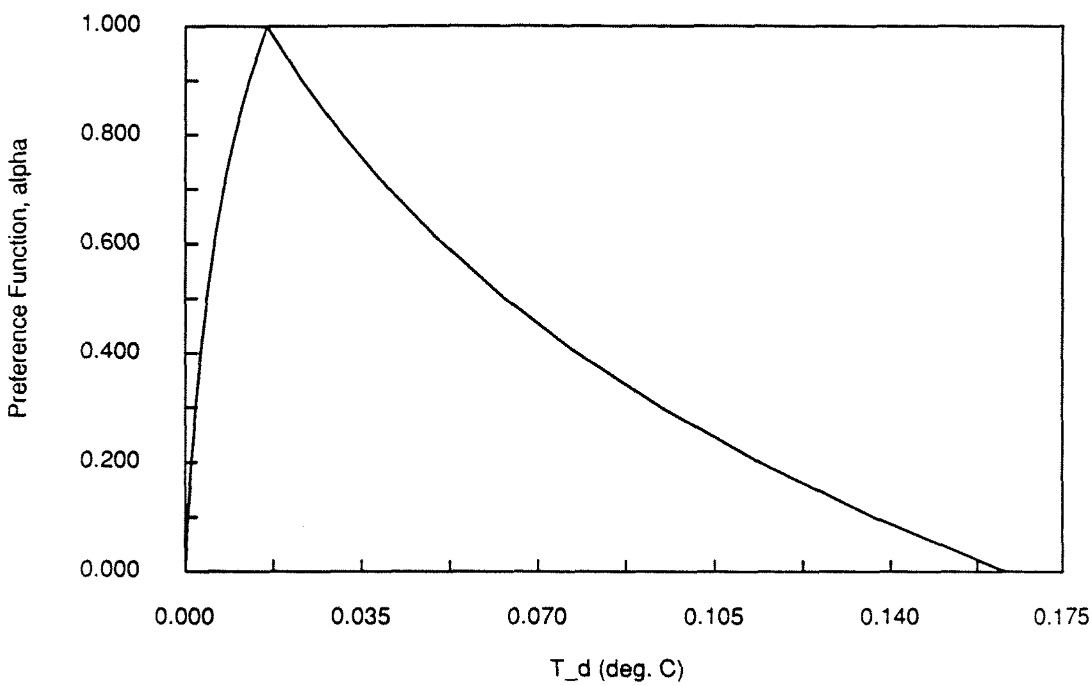

Figure 4.7: Drum Brake: Temperature Decay Profile Output ($t = 1\text{sec}$), $\tilde{T}_d$.

in $\alpha_{\tilde{T}_r}$ on the right portion of the curve (say $\alpha_{\tilde{T}_r} = 0.7$) results in temperature rises that quickly approach the FR in a few stops. This means that the imprecision of $\tilde{T}_r$ is very dramatic for increasing values of T_r to the right of the peak. While all values to the left of the peak of $\tilde{T}_r$ satisfy T_r^r, the peak of $\tilde{T}_r$ and values to the right demonstrate that a small decrease in preference of any or all of the inputs might produce undesired design results. Temperature rise is therefore a critical performance parameter, requiring special attention in later stages of the design with respect to the input parameters.

Equation 4.4 represents the PPE for the drum brake's temperature decay profile PP. Before analyzing the output results for T_d with respect to T_d^r, the form of Equation 4.4 warrants some explanation. The units of T_d are temperature (°C), where the rate of change is proportional to the heat rejection or dissipation of the drum brake. An alternative expression for heat dissipation is the time constant associated with the exponential term in Equation 4.4. Such an expression becomes difficult to evaluate, however, with respect to the performance specification T_d^r, which is dependent on the temperature rise T_r of the brake components. Equation 4.4, on the other hand, takes into account the imprecision inherent in T_r as an output parameter of the design, and provides a convenient comparison to T_d^r.

Figure 4.7 contains the output set for the temperature decay profile $\tilde{T}_d$ for a time of $t = 1$ sec and for a fuzzy input $\tilde{T}_r$ with peak at $T_r = 60$°C and right and left boundaries of $T_r = 175$°C and $T_r = 0$°C, respectively. At $\alpha_{\tilde{T}_d} = 1.0$, the output parameter value is $T_d = 0.016$°C. Figure 4.8 contains the output $\tilde{T}_d$ after $t = 60$ sec, where $\tilde{T}_{d\,(at\,\alpha_{\tilde{T}_d}=1)} = 0.96$°C. These results demonstrate that the temperature decay is approximately linear for the time interval being considered (within 60 seconds). The resultant values for $\tilde{T}_d$ where $\alpha_{\tilde{T}_d} = 1$ also show that the temperature decay is

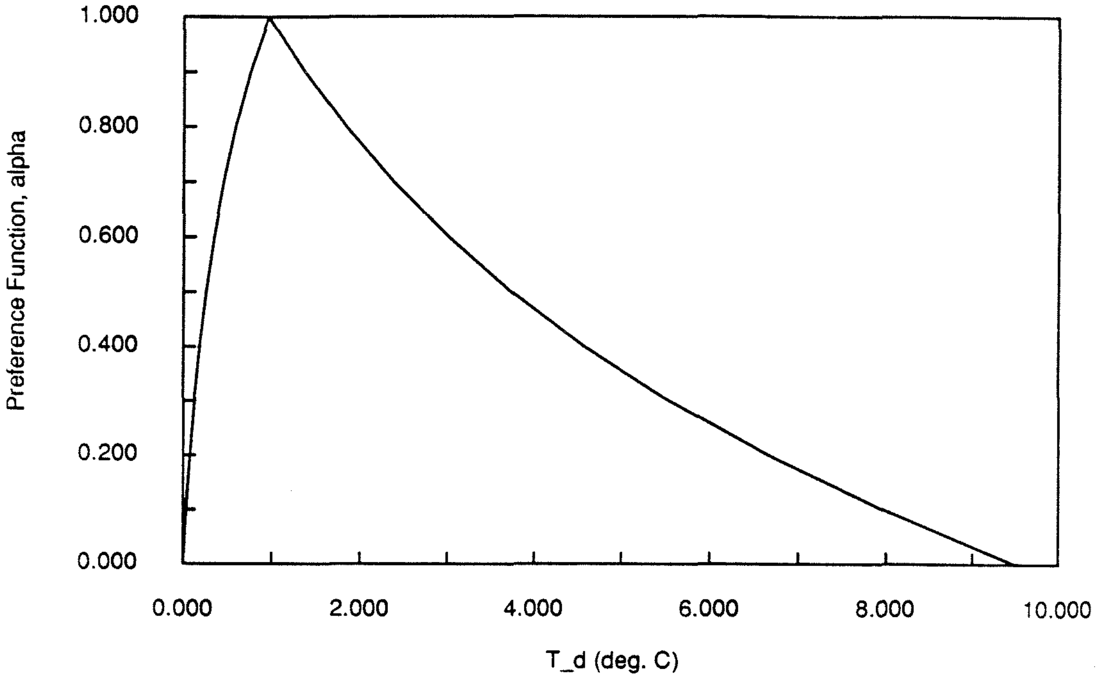

Figure 4.8: Drum Brake: Temperature Decay Profile Output (t = 60sec), $\tilde{T}_d$.

very slow for the most confident inputs. Analyzing Figures 4.7 and 4.8, it appears that much greater (by an order of magnitude) temperature rejection can be obtained by choosing inputs with small preference; however, these output values are not only dependent on the input design parameters under consideration (L_w, R_1, r, k, h, and C) but also on the output performance parameter T_r. Only the DPs can be altered to obtain required performance, not PPs such as T_r. Thus, to determine how the $\tilde{T}_d$ output set varies for different DPs, T_r must be set to a representative crisp value (*e.g.*, $T_r = 60°\text{C}$), and the output set T_d must be recalculated using this value. Figure 4.9 shows the result of such a calculation for time $t = 60$ sec. Notice that when compared with Figure 4.8, greater temperature decay occurs as T_r increases. Notice further that the imprecision due to T_r was the major contributor to the imprecision in $\tilde{T}_d$ in Figure 4.8, implying that any change of any or all of the DPs will not significantly change the amount of heat dissipation.

Relating this discussion to T_d^{τ}, the heat rejection of the drum brake does not compare well to the possible temperature rise values obtained in the system. Because of the small imprecision produced by changing one or many of the input design parameters, very little can be done to maximize the heat dissipation beyond that given by the peak of the $\tilde{T}_d$ output curve. The γ-*level* measure (discussed in the next section) will provide further evaluation of the T_d^{τ} performance specification.

4.1.5.2 Disk Brake Output Performance Parameters

The fuzzy output results for the disk brake configuration are in Figures 4.11 through 4.18. Our approach will show the ease by which the different assumptions of uniform wear or uniform pressure can be simultaneously evaluated for the disk configuration. An alternative means of performing this evaluation would be to classify each assumption

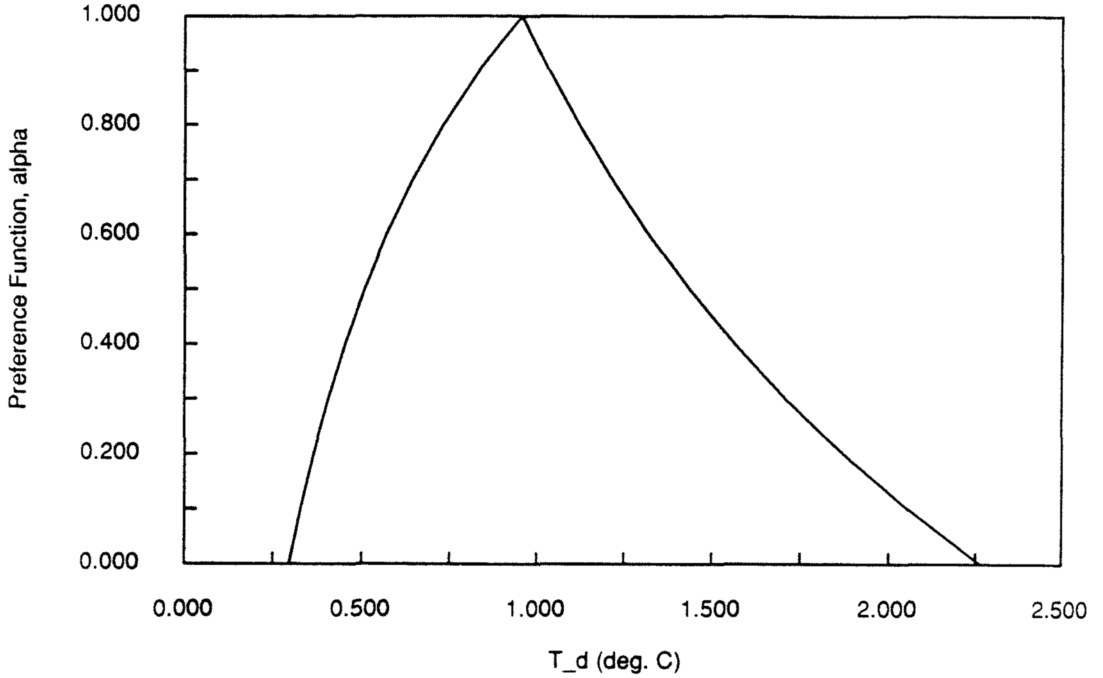

Figure 4.9: Drum Brake: Temperature Decay Profile ($t = 60$sec, constant T_r), $\tilde{T}_d$.

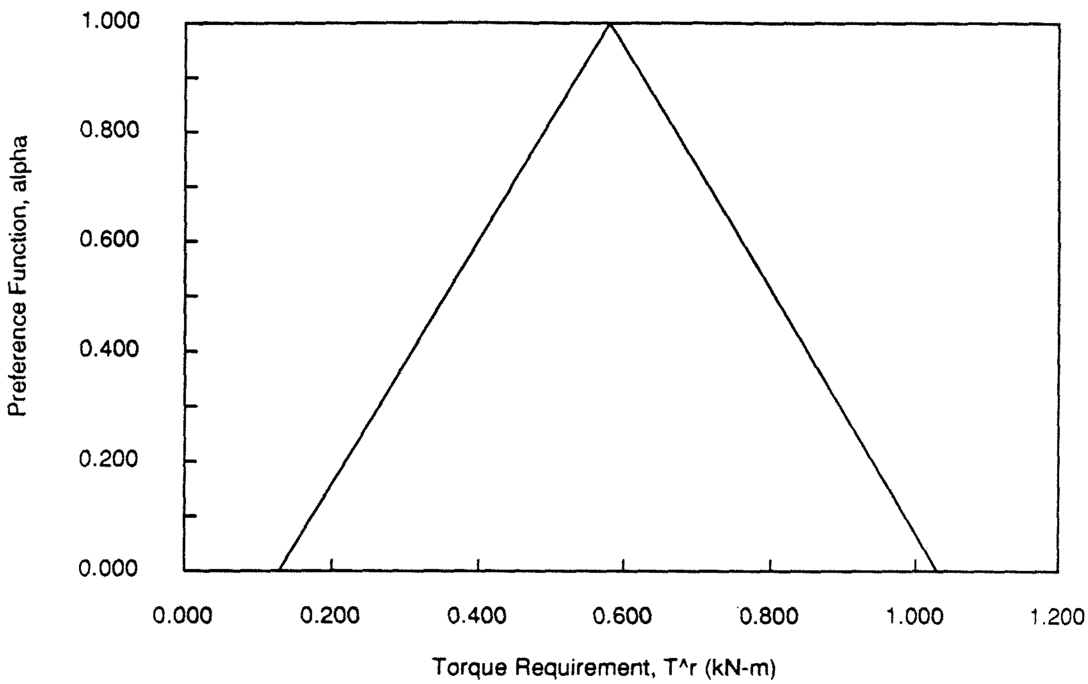

Figure 4.10: Performance Specification, $\tilde{T}^r$.

case as a separate configuration.

The actuating force parameters, with given PPEs (Equations 4.12 and 4.13), have output sets shown in Figures 4.11 and 4.12. The peaks of the curves are $\tilde{F}_{w\,(at\,\alpha_{\tilde{F}_w}=1)} = 21.6$ kN and $\tilde{F}_{p\,(at\,\alpha_{\tilde{F}_p}=1)} = 23.8$ kN, respectively. When compared to $F^r = 25$ kN, each actuating force value satisfies the FR. The uniform wear case performs slightly better than uniform pressure for small preference change to the right of the preference function's peaks. However, both cases have an imprecision in the output on the order of the difference between F^r and the peak actuating force value. Thus the actuating force performance parameter for either uniform wear or uniform pressure nominally satisfy the performance specification (at, and to the left of, the peak); yet, a small decrease in preference of one or more of the inputs may result in inadmissible output values.

The output sets $\tilde{\mathcal{T}}_w$ and $\tilde{\mathcal{T}}_p$ are shown in Figures 4.13 and 4.14. As in the drum brake configuration, Figure 4.10 represents the fuzzy performance specification for torque. For preference equal to one (1), $\mathcal{T}^r = 0.579$ kN-m, $\mathcal{T}_w = 0.890$ kN-m, and $\mathcal{T}_p = 0.983$ kN-m. A comparison of $\mathcal{T}_w$ and $\mathcal{T}_p$ to $\mathcal{T}^r$ shows that both uniform wear and uniform pressure for the disk brake meet the torque design requirement. For small changes of the input parameters (except M_v) to the left of the output torques' peaks (*e.g.*, a decrease in preference to 0.8), the values for $\mathcal{T}_w$ and $\mathcal{T}_p$ still satisfy the performance criteria. If the design parameter M_v changes preference proportionally to any other input parameter, the output torque values for uniform wear and uniform pressure will always meet the performance specification. Overall, the uniform pressure case performs better than uniform wear for torque considerations. Even though both cases have possible combinations of the input parameters not meeting the performance specification, satisfactory solutions do exist. Special care must be taken in adjusting

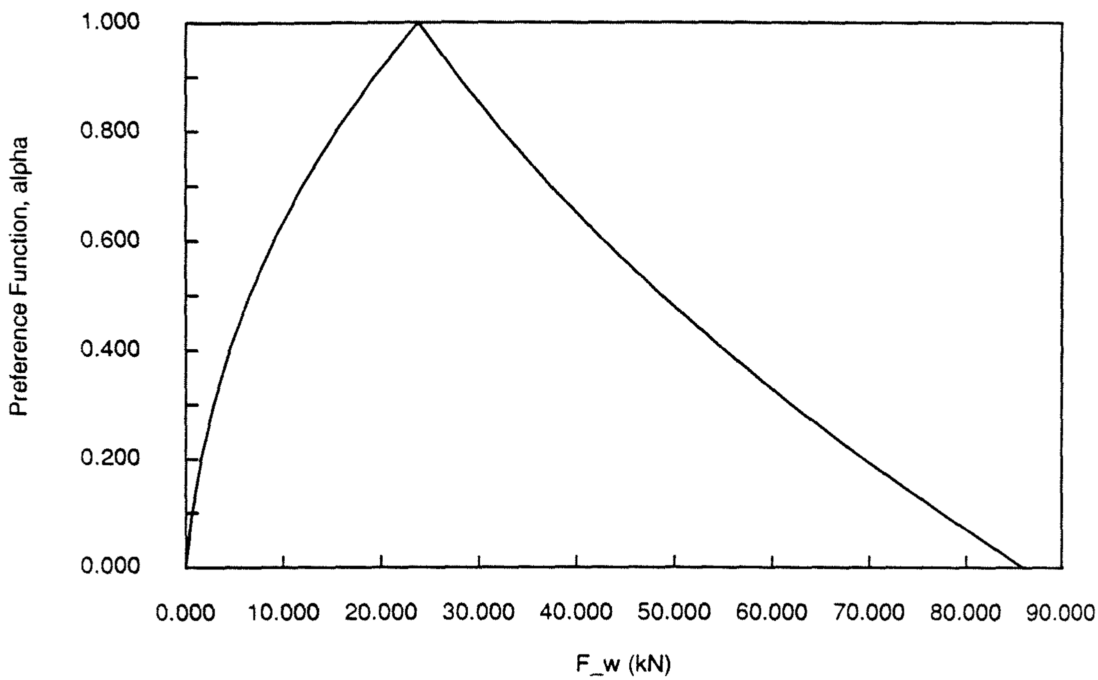

Figure 4.11: Disk Brake: Actuating Force Output Set, $\tilde{F}_w$.

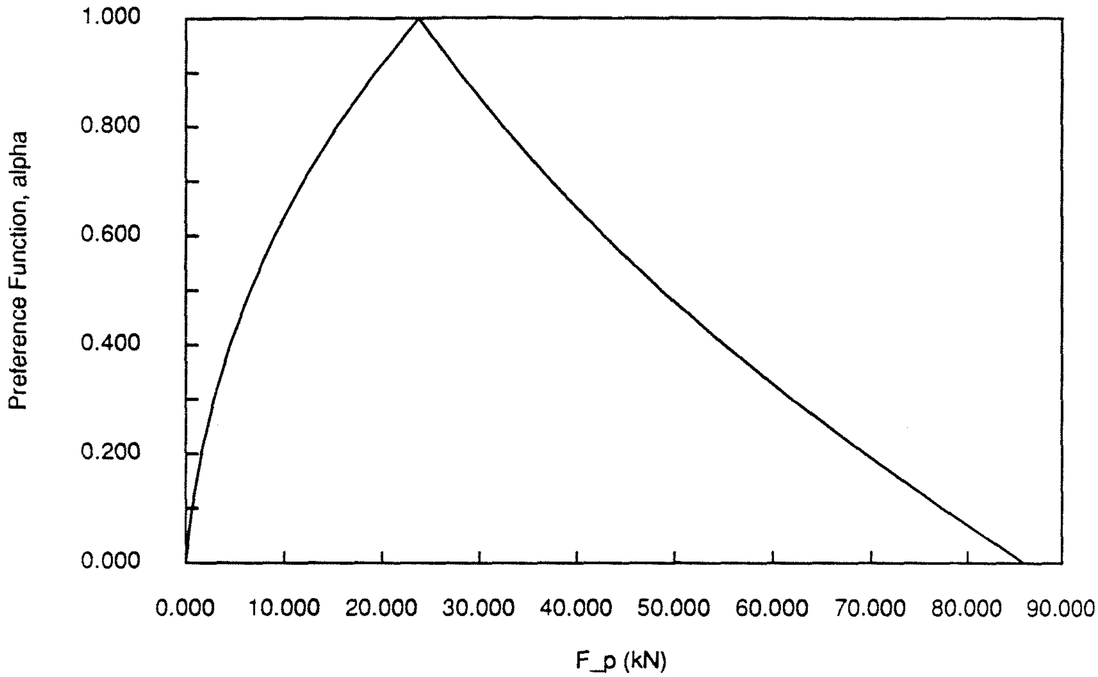

Figure 4.12: Disk Brake: Actuating Force Output Set, $\tilde{F}_p$.

M_v.

Using Figure 4.15, the disk brake configuration satisfies the performance specification for temperature rise at the peak value ($\tilde{T}_{r\,(at\,\alpha_{\tilde{T}_r}=1)} = 17.7°C$). For decreasing preference to the right of the peak, large changes in preference (*e.g.*, $\alpha_{\tilde{T}_r} = 0.5$) for any input parameter still satisfy T_r^r. For repeated braking episodes, the results are similar to the drum brake system. Due to the large imprecision in $\tilde{T}_r$, a decrease in preference of any of the inputs to 0.8 or below may result in T_r values that may accumulate for successive braking actions, such that they approach the performance criteria, T_r^r. Therefore, T_r for the disk brake configuration is once again a critical performance parameter, but not as critical as in the case of the drum brake because the values of $T_{r,disk}$ about the peak are smaller by a factor of two than $T_{r,drum}$ (*e.g.*, $T_{r,disk} = 17.7°C$ compared with $T_{r,drum} = 35.3°C$).

Figures 4.16 and 4.17 contain the output set for the temperature decay profile $\tilde{T}_d$ for times of $t = 1$ sec and $t = 60$ sec. The peak value for Figure 4.16 corresponds to $\tilde{T}_{d\,(at\,\alpha_{\tilde{F}_d}=1)} = 0.11°C$, whereas the peak value for Figure 4.3 occurs at $\tilde{T}_{d\,(at\,\alpha_{\tilde{F}_d}=1)} = 6.0°C$. These results illustrate that the slow rate of temperature decay is generally similar to the drum brake configuration. Yet, for output values about the peak of $\tilde{T}_d$, the magnitude of the temperature decay is significantly higher than the corresponding values for the drum brake.

To obtain a measure of the imprecision of the temperature decay in terms of the DPs (*i.e.*, excluding T_r), T_r must be set to a crisp value as shown earlier for the drum brake. This is because T_r is a performance parameter, not a design parameter, and the designer cannot select values for T_r. The resultant calculation, using constant T_r and fuzzy design parameters, is in Figure 4.18. Once again, T_r was the major contributor to the overall imprecision in $\tilde{T}_d$ (comparing Figures 4.17 and 4.18), but the output

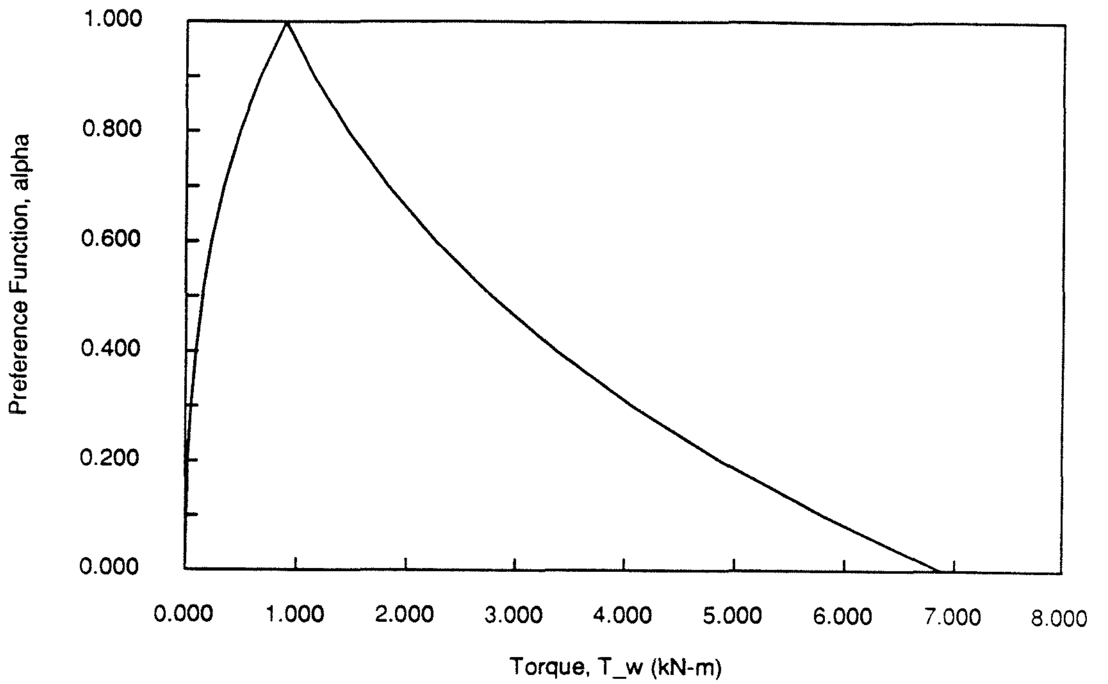

Figure 4.13: Disk Brake: Torque Output Set, $\tilde{\mathcal{T}}_w$.

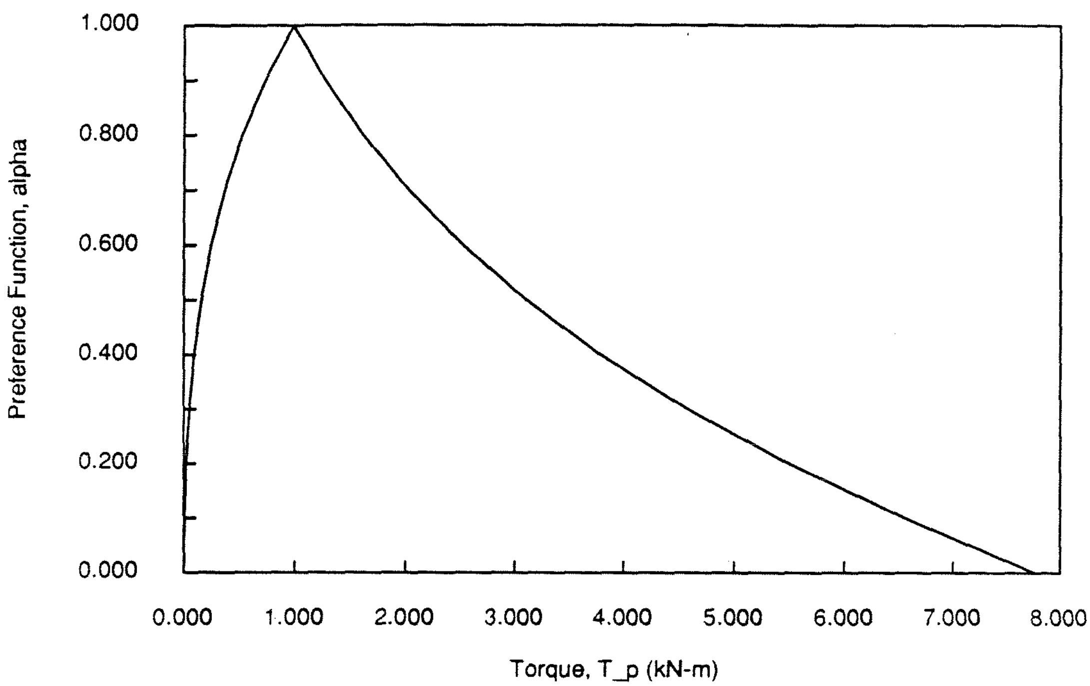

Figure 4.14: Disk Brake: Torque Output Set, $\tilde{T}_p$.

imprecision for the case of constant T_r is still significant, implying that a change in one or more of the DPs will result in a large change of temperature decay. In terms of the performance criteria T_d^r, the disk brake system dissipates heat to a greater extent than the results found for the drum brake system. Due to the possibility of large imprecision in the temperature decay profile for a corresponding change in the disk brake DPs, heat dissipation may be maximized beyond the values centered about the peak of the $\tilde{T}_d$ output curve. The following section (γ-*level* measure) will provide information on discerning which of the input parameters will bias such a maximization.

4.1.6 Applying the γ-Level Measure

The γ-*level* measure can be used to provide the engineer with qualitative information concerning the role of the input parameters in the brake design. The process of using and algorithmically implementing the γ-*level* measure is in Chapter 2. Tables 4.2, 4.3, and 4.4 list normalized γ-*level* measures for the brake design.

When an input is not related to a performance parameter, the numerical measure produces a zero (0.0). As the normalized numerical measure increases in value, the corresponding input parameter has a greater qualitative importance in determining the particular performance parameter in question, meaning that a comparatively small change in the input parameter produces a large change in the output. The γ-*level* measure can be used to order the input parameters according to importance. Further, if one input parameter has a significantly higher γ-*level* measure for an output set, when compared to another input parameter (where the second parameter's measure is not zero), the designer can fix the value of the second parameter without affecting performance.

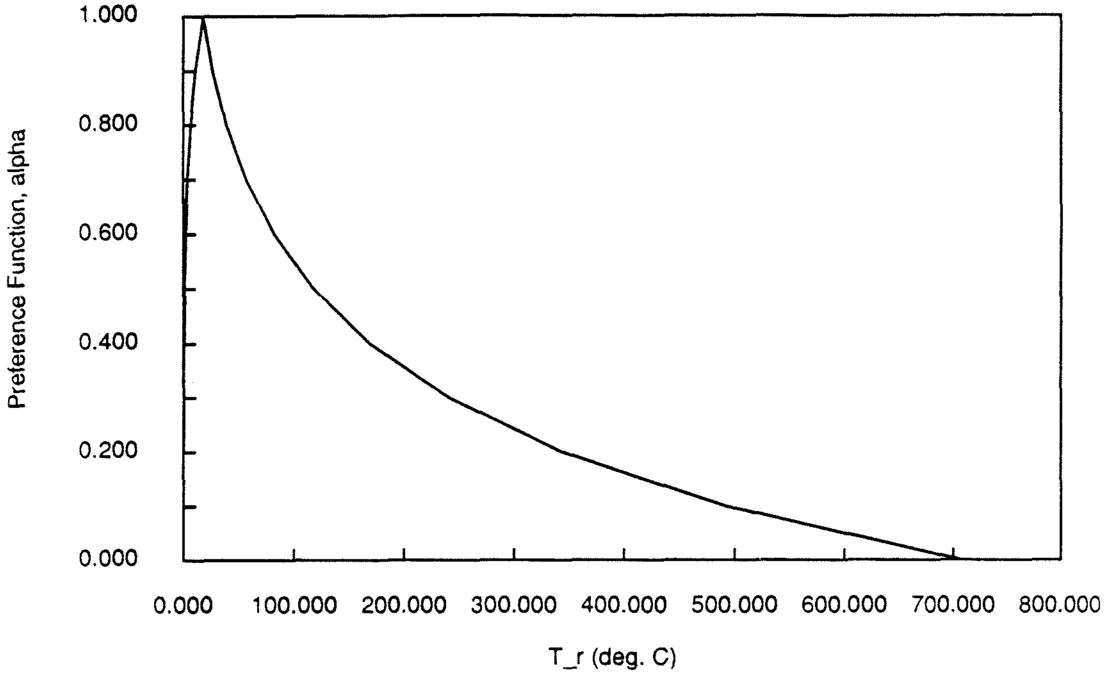

Figure 4.15: Disk Brake: Temperature Rise Output Set, $\tilde{T}_r$.

	Performance Parameters				
DPs	F_a	$\mathcal{T}$	E	T_r	T_d
a	0.21	0.0	0.0	0.0	0.0
r	1.0	0.92	0.0	0.44	0.0
θ_1	0.30	0.04	0.0	0.0	0.0
θ_2	0.43	0.41	0.0	0.0	0.0
μ	0.77	0.92	0.0	0.0	0.0
p_a	0.94	1.0	0.0	0.0	0.0
c	0.93	0.0	0.0	0.0	0.0
b	0.64	0.0	0.0	0.0	0.0
ω	0.0	0.0	1.0	1.0	0.0
C	0.0	0.0	0.0	0.19	0.19
M_v	0.0	0.0	0.39	0.39	0.0
R_1	0.0	0.0	0.001	0.21	0.19
L_w	0.0	0.0	0.0	0.36	0.0
k	0.0	0.0	0.0	0.0	0.004
h	0.0	0.0	0.0	0.0	0.09
T_r	0.0	0.0	0.0	1.0	1.0

Table 4.2: γ-*level* Measure Results for Drum Brake.

4.1.6.1 Drum Brake γ-Level Measure

Table 4.2 lists the results of the *γ-level* measure for the drum brake configuration as applied to each input parameter with respect to each output set.

In the case of the energy expression E, the *γ-level* measure indicates that the only significant input parameters for energy considerations are the mass of the vehicle and the angular velocity ω. This makes perfect sense when examining the terms of Equation 4.5. When this equation was derived, all contributing factors to the energy were taken into account. The left portion of the equation contains the energy term due to the brake parts, while the right portion contains the contribution due to the kinetic energy of the vehicle. The energy of the brake parts will be insignificant when compared to the vehicle's kinetic energy. The *γ-level* measure indicates this result.

Table 4.2 lists the *γ-level* measure results for the torque performance parameter $\mathcal{T}$ in the second column. The design parameters r, μ, and p_a are shown to have nearly the same importance with respect to $\mathcal{T}$. Notice, however, that the input parameter θ_1 contributes very little to the output set. This indicates that θ_1 is essentially orthogonal to $\mathcal{T}$.

The conductivity input parameter k has little contribution to T_d (a *γ-level* measure result of 0.004) and can be fixed to a representative material value. The *γ-level* measure is especially useful for determining the interactions between input DPs and output PPs in this way.

4.1.6.2 Disk Brake γ-Level Measure

Table 4.4 lists the *γ-level* results for the temperature decay performance parameter where T_r takes on a representative crisp value. Because the temperature rise T_r can take on a wide range of values, depending on vehicle speed, etc., it has been fixed to

	Performance Parameters					
DPs	F_w	F_p	$\mathcal{T}_w$	$\mathcal{T}_p$	T_r	T_d
p_{max}	1.0	0.0	0.95	0.0	0.0	0.0
r_o	0.55	0.49	1.0	0.99	0.37	0.0
r_i	0.93	1.0	0.82	0.96	0.0	0.0
μ	0.0	0.0	0.82	0.96	0.0	0.0
p_p	0.0	0.98	0.0	1.0	0.0	0.0
ω	0.0	0.0	0.0	0.0	1.0	0.0
C	0.0	0.0	0.0	0.0	0.19	0.18
h	0.0	0.0	0.0	0.0	0.0	0.10
L_{w1}	0.0	0.0	0.0	0.0	0.28	0.27
M_v	0.0	0.0	0.0	0.0	0.39	0.0
T_r	0.0	0.0	0.0	0.0	1.0	1.0

Table 4.3: γ-*level* Measure Results for Disk Brake.

	Design Parameters				
PP (constant T_r)	r_o	C	h	L_{w1}	T_r
T_d	0.0	0.69	0.35	1.0	0.0

Table 4.4: Disk Brake: γ-*level* Measure Results for T_d.

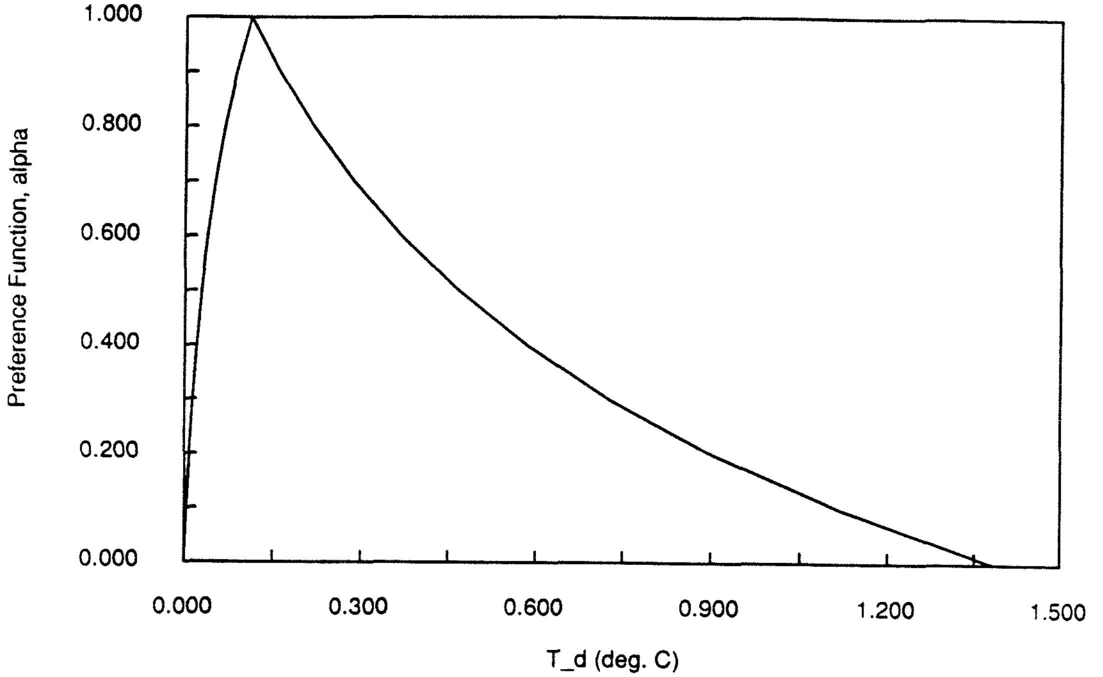

Figure 4.16: Disk Brake: Temperature Decay Profile Output (t = 1sec), $\tilde{T}_d$.

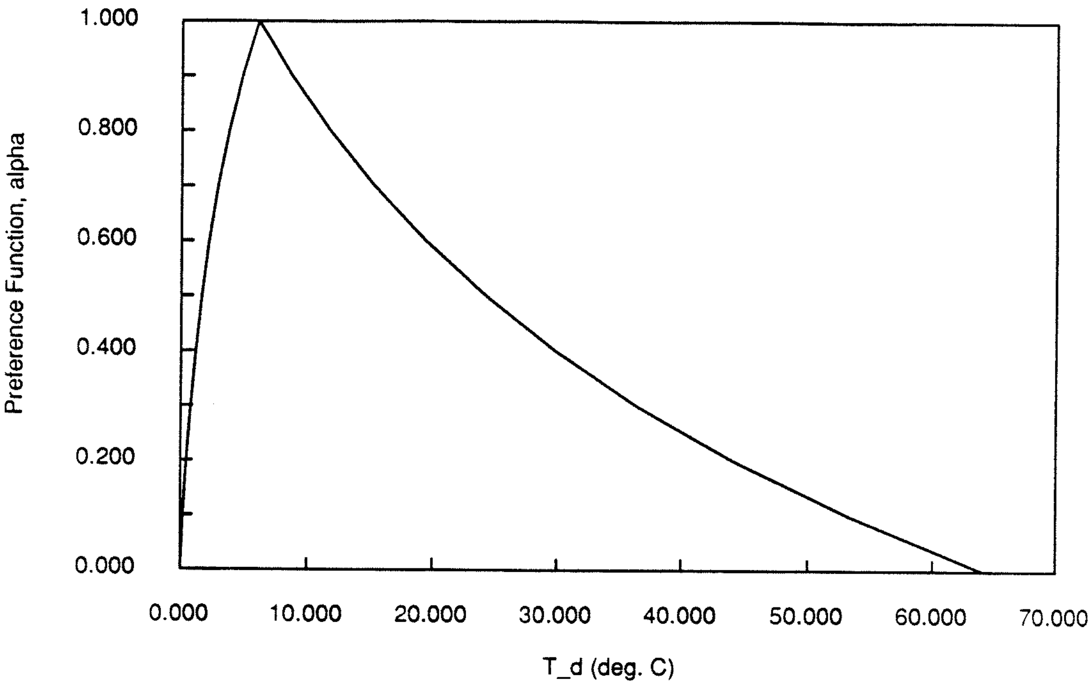

Figure 4.17: Disk Brake: Temperature Decay Profile Output ($t = 60$sec), $\tilde{T}_d$.

a nominal value here for comparison of the two alternative design configurations.

Because the output results of $\tilde{T}_d$ for the disk brake (discussed above) indicated that a change in the input parameters will have a large effect on the temperature decay, the most important design parameters must be determined to focus the design efforts on those that will contribute the greatest to maximizing heat dissipation (*i.e.*, satisfying T_d^r). Table 4.4 shows that the design efforts should first focus on L_{w1} and subsequently C to produce better heat dissipation.

4.1.7 Execution Times

Timing results for the brake example illustrate that one calculation of a PP preference function takes 2.5 seconds on average (Sun Microsystems 3/260 workstation without a floating point accelerator).

4.1.8 Discussion

The brake design example presented in this section demonstrates how imprecisely described configurations can be evaluated with respect to a functional requirement space. The alternative configurations can be compared for each dimension in the Functional Requirement Space (FRS) to determine the major differences. It is seen (above) that both of the brake configurations satisfy the *actuating force* requirement where the output performance parameter preference $(\alpha) = 1$. Although a small change in a DP associated with the disk brake can make a large change in the actuating force PP so that it no longer satisfies its FR, the DPs associated with the actuating force PP are not likely to change. A discussion of the results above illustrates that the primary input parameters that can require change from the peak values are L_{w1} and C (determined by the γ-*level measure*, and the influence of those parameters on the

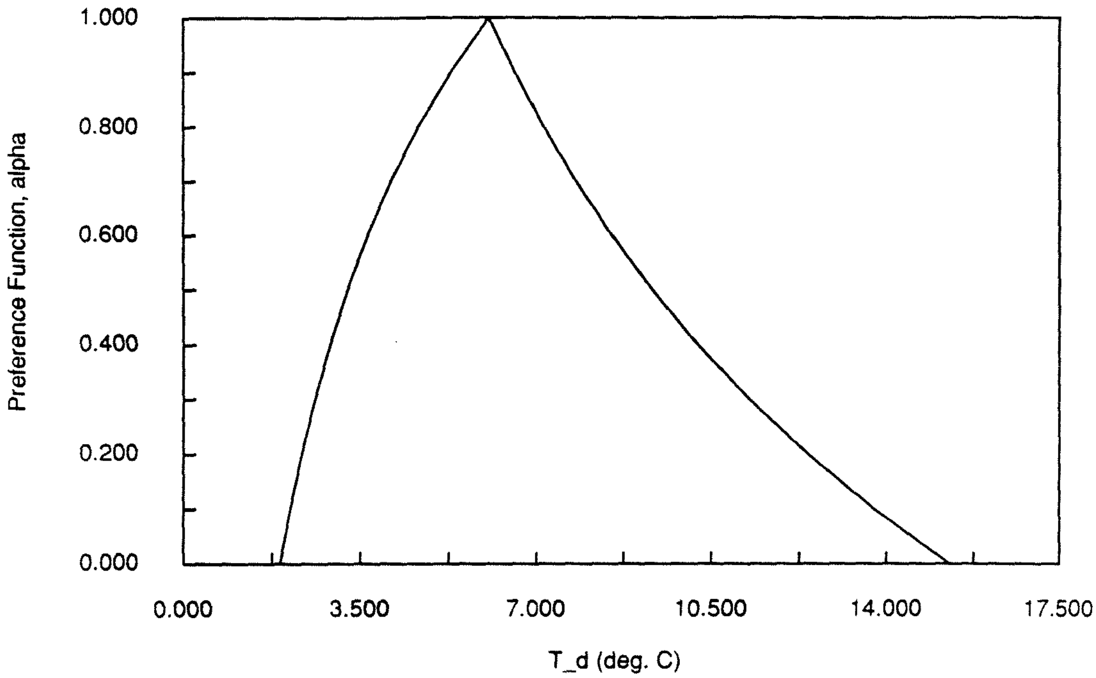

Figure 4.18: Disk Brake: Temperature Decay Profile ($t = 60$sec, constant T_r), $\tilde{T}_d$.

temperature rise). These DPs are independent of the actuating force PPs for the disk brake, and will thus have no effect on the actuating force output set.

The torque results for the disk brake configuration (also discussed earlier) are superior to those for the drum brake. Both cases have combinations of the input parameters where the performance specification will not be satisfied. However, the disk brake is less sensitive to changes in the DPs when compared to the drum brake case.

The temperature rise PP was found to be a critical parameter for both brake alternatives. Small changes in the DPs result in large changes in T_r, often beyond the acceptable range. In comparing the two alternatives, the disk brake configuration out-performed the drum brake by a factor of two in the region of interest. Care will need to be taken as the input parameters are adjusted in the later design stages for either configuration because of the large degree of coupling between the temperature rise PP and the temperature decay profile PP.

The final dimension of the FRS to be considered is the temperature decay profile. The disk brake far out-performed the drum brake design. Not only do the results illustrate that heat is rejected faster (by a factor of six) for the disk brake, but they also show that better results can be obtained by adjusting the DPs relevant to the disk brake's T_d performance parameter. This is not the case for the drum brake.

With this data, the engineer is equipped to choose between the disk and drum brakes. The only advantage of the drum brake (in this example) is the small change in actuating force for comparatively large changes in DPs. As shown, the more important performance parameters of temperature rise and temperature decay profile are handled much better by the disk brake design.

4.2 Conclusions

The example presented above shows how the theory of computations with imprecise parameters developed in Chapter 2 are applied to modest engineering design problems. The solutions presented here were reached with a minimum of application-specific programming, and required only a few seconds of CPU time (to compute PP preference functions). Imprecise design parameters are easily handled, even in relatively complex design equations (PPEs). The imprecise performance parameter results provide powerful insight into these particular design examples, especially with regard to the interaction of parameters and the effect of varying parameter values.

The brake design example illustrates the semi-automated approach to preliminary design. Alternatives can be evaluated easily and efficiently by providing the designer with performance information and by reducing the usual complexities of working with the imprecise descriptions of the alternatives. Although this approach was constructed for use in the preliminary stages of design, it can also be useful later in the design process, especially when combined with other more traditional design tools such as optimization techniques.

The method demonstrated here for performing computations with imprecise parameters is only one small, and separable, part of an overall methodology for engineering design, as generally described in Appendix D. However, this technique is central to the methodology, both from theoretical and implementational points of view. Coping with imprecision in engineering design is vital to the development of comprehensive approaches to augmenting designers' abilities. Finding techniques for *efficiently* computing with those imprecise representations is similarly necessary.

The brake example contained one imprecise (fuzzy) functional requirement (FR) (along with other crisp inequality FRs), as did the frame example shown in

Chapter 2. Later in this document, this technique will be extended further into the areas of imprecise specifications and requirements, and the development of a design methodology, terminology, and environment will continue.

Chapter 5

A Comparison of Fuzzy and Probability Calculus for Representing Imprecision

5.1 Introduction

Chapter 2 through Chapter 4 describe the first steps toward the realization of the central hypothesis of this work. Those chapters present a method for modeling the *uncertainty in choosing among alternatives* found in preliminary design (which is defined as *imprecision*), using the fuzzy calculus. They also develop analytical and computational procedures for carrying out calculations with this type of uncertain (imprecise) design parameter. This chapter will develop a similar technique using probability calculus to represent and manipulate imprecise parameters, and will compare it with the fuzzy calculus method.

Other methods (*e.g.*, probability logic [42, 69], interval analysis [56], sensitivity analysis [16, 17, 51], the Taguchi method [23, 77], etc.) have previously been applied to representing and manipulating the subjective or uncertain aspects of engineering problems. Probability calculus, in particular, is known to be appropriate for manipulating one type of uncertainty (referred to here as *stochastic uncertainty*). As a comparison with these methods, an approach is presented here using the *probability* calculus to manipulate the *imprecision* aspect of uncertainty (not the stochastic uncertainty) in a way analogous to the fuzzy calculus method. The next section will develop, in full, a probabilistic alternative to the theory of imprecise calculations. Examples using both methods are then presented, followed by a discussion of the major differences. It will be shown that the probability method is applicable for calculations with *stochastically uncertain* parameters, and the fuzzy calculus is well-suited for representing and manipulating imprecise design descriptions. It will also be shown that the probability method is not as well-suited as the fuzzy calculus for modeling imprecise design descriptions.

5.2 The Probability Approach

The first portion of this section will present the necessary tools to construct the probability approach. Operation rules for imprecise calculations are then derived, followed by the interpretation scheme for carrying out these calculations. Analytical and numerical applications of the method conclude this section.

The probability approach can be developed on either of two interpretations: the "classical" relative frequency of occurrences, or ("Bayesian") probability logic as a measure of plausibility of propositions [24, 45]. However, the calculus is the same for both interpretations. Here we use the calculus of probability and introduce another interpretation of its meaning: that of *preference.* For convenience, this chapter will continue to use the term "probability" although it is emphasized once again that the term does not denote its usual meaning, but only its associated calculus. This is done to compare the use of the probability calculus with the fuzzy calculus for representing and manipulating imprecision. Naturally the usual notion of probability would continue to be used to represent stochastic uncertainty.

Only three axioms are required in the process of constructing the calculus for probability logic [24]:

$$P(a \mid b) \geq 0, \text{ and } P(a \mid a) = 1, \tag{5.1}$$

$$P(a \mid b) + P(\tilde{a} \mid b) = 1, \tag{5.2}$$

$$P(a, b \mid c) = P(a \mid b, c) \cdot P(b \mid c). \tag{5.3}$$

The first of these axioms simply states the conventions that the probability of a given information b must be nonnegative, and the probability of a given itself is unity. The second axiom represents a statement giving the probability of the negation of a in terms of the probability of a, under the same hypothesis b, *i.e.*, the probability of a given b summed with the probability of the contradiction of a on the same information must equal unity. Finally, axiom 5.3 is the product rule giving the probability of a and b under the hypothesis c in terms of more elementary probabilities, or equivalently [45] "*the probability of the joint assertion of two propositions on any data c is the product of the probability of one of them on data c and that of the other on the first and c.*" The form of the axioms given in Equations 5.2 and 5.3 is partly conventional and partly a requirement for internal consistency of the calculus [24]. Axioms 5.2 and 5.3 imply that

$$P(a \textbf{ or } b \mid c) = P(a \mid c) + P(b \mid c) - P(a, b \mid c), \tag{5.4}$$

which is referred to here as the addition rule.

5.2.1 Probability Function Operations

To construct the analogous form of the fuzzy calculus method for imprecise calculations, the rules for calculation operations must be developed first. For the sake of brevity, only the rules for the binary operations of addition, subtraction, multiplication, and division, along with the unitary operations for the sine and cosine functions, will be presented. Appendix B presents the detailed derivations of these operations given two independent "input" parameters $\hat{x}$ and $\hat{y}$ and "output" parameter $\hat{z}$:

$$\hat{z} = f(\hat{x}, \hat{y}),$$

where the function f is made up of any combination of the six possible operations listed. The resulting probability density functions (*pdfs*) for the output parameter $\hat{z}$ are summarized below:

$$p_{add}(z \mid I) = \int_{-\infty}^{\infty} p_{\hat{x}}(z - y)\, p_{\hat{y}}(y)\, dy. \tag{5.5}$$

$$p_{sub}(z \mid I) = \int_{-\infty}^{\infty} p_{\hat{x}}(z + y)\, p_{\hat{y}}(y)\, dy, \tag{5.6}$$

$$p_{mul}(z \mid I) = \int_{-\infty}^{\infty} \frac{1}{y} p_{\hat{x}}(\frac{z}{y})\, p_{\hat{y}}(y)\, dy, \tag{5.7}$$

$$p_{div}(z \mid I) = \int_{-\infty}^{\infty} y \cdot p_{\hat{x}}(z \cdot y)\, p_{\hat{y}}(y)\, dy, \tag{5.8}$$

$$p_{sin}(z \mid I) = \frac{1}{\sqrt{1 - z^2}} p_{\hat{x}}(\sin^{-1}(z)),\ (\frac{-\pi}{2} \leq \sin^{-1}(z) \leq \frac{\pi}{2}), \tag{5.9}$$

$$p_{cos}(z \mid I) = \frac{1}{\sqrt{1 - z^2}} p_{\hat{x}}(\cos^{-1}(z)),\ (0 \leq \cos^{-1}(z) \leq \pi). \tag{5.10}$$

5.2.2 The Probability Interpretation

As discussed earlier, the fuzzy calculus technique represents imprecision by a range and a function defined in that range to describe the desirability or preference of using one particular over another. The more confident, or the more designer desires to use an input value, the higher its preference value in the parameter's normalized (between zero and one) preference function. A similar interpretation for the *probability* approach to representing imprecision is developed below.

The input design parameters can be described by probability density functions (*pdf*) $p_i()$ with each $p_i()$ having unit area. Figure 5.1 shows an example input parameter u_i. The vertical axis values of the *pdf* capture the subjective imprecision or

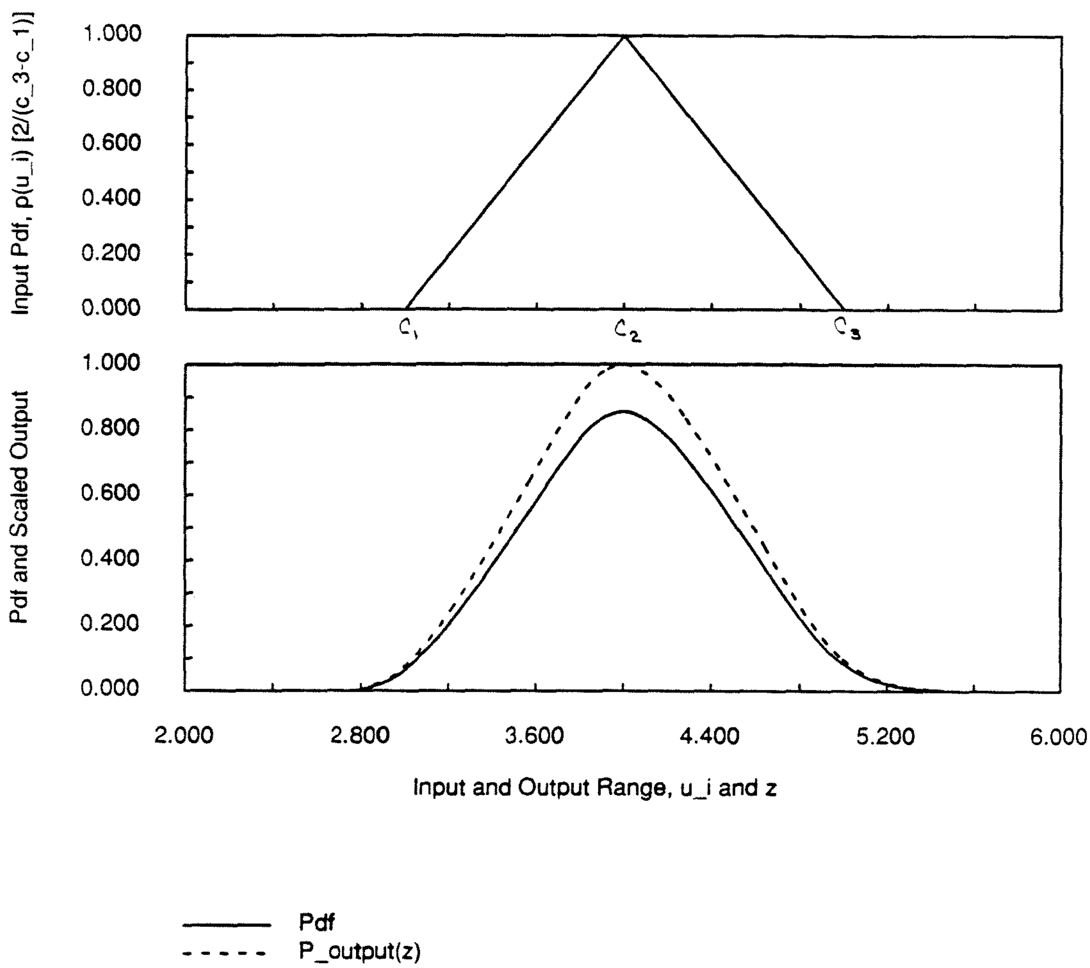

Figure 5.1: Example Input and Output Parameter.

approximate character of u_i, in a way analogous to the membership for the fuzzy calculus approach. $p_i(u_i)$ value(s) of maximum height correspond to the value(s) of u_i for which the designer rates as highest in terms of confidence or desirability. Conversely, u_i values with $p_i(u_i)$ equal to zero are those the designer rates as least preferred or unacceptable (the endpoints of the interval of confidence for u_i). A fuzzy number captures the desires of the designer by use of a function ranging from zero to one, whereas a corresponding *pdf* for some input parameters may vary in height depending on the need to meet the unit-area condition. These two methods of representing the subjective nature of an *input* parameter's imprecision are equivalent in what follows, because the ratio of any value in the input range to the most desired input(s) is kept the same for the fuzzy and probability approaches.

When using Equations 5.5 through 5.10, the result of a calculation $\hat{z} = f(\hat{x}, \hat{y})$ will be a *pdf*, with unit area. This *output* form does not have a direct analogous interpretation to the fuzzy approach. Figure 5.1 provides an example output *pdf* from some $f(x, y)$, along with a uniformly scaled version of the output, ranging from (0.0) to (1.0) on the vertical axis. The output *pdf* is normalized ($P(z)$ with respect to $\max[P(z)]$) to compare it directly with the fuzzy result. The normalization is mathematically equivalent to the following procedure:

1. Consider a finite interval, δ, centered at the peak of the output *pdf* curve.

2. Calculate the area under the curve for this interval. The result is the probability of obtaining the most probable output, plus or minus $\frac{\delta}{2}$. Call the result Θ_1.

3. For each interval, of width δ, in the range of the output off the peak, calculate the area under the curve, Θ_i, $i = 2, \ldots, n$.

4. Normalize the results, Θ_i, $i = 1, \ldots, n$ with respect to Θ_1.

The result of this process is the scaled curve P_{output} given in Figure 5.1. This curve is called the *relative desirability of* $\hat{z}$ as it represents the ratio of the *pdf* for a given value of $\hat{z}$ to the maximum *pdf* value.

5.2.3 Analytical Application of the Probability Approach

Calculations with fuzzy numbers (defined as an interval of confidence and level of presumption) are performed using operation rules based on α-cuts [46, 33]. Using the interpretation scheme and operation rules defined previously, an analytical method for calculating outputs for the probability approach can be likewise developed.

Consider a general input design parameter, u_i, graphically represented by the *pdf* shown in the top of Figure 5.1. Here only triangular *pdfs* will be considered such that $p_{u_i}(u_i)$ may then be defined as follows:

$$p_{u_i}(u_i) = \begin{cases} 0 & u_i < c_1 \\ \frac{2}{(c_3-c_1)(c_2-c_1)}(u_i - c_1) & c_1 \leq u_i \leq c_2 \\ \frac{2}{(c_3-c_1)(c_2-c_3)}(u_i - c_3) & c_2 \leq u_i \leq c_3 \\ 0 & u_i > c_3 \end{cases} \tag{5.11}$$

where

$$(c_3 - c_1) = \text{input range} \in \Re.$$

For a given functional requirement expression $\hat{z} = f(u_i)$, $i = 1, \ldots, N$ and known *pdfs* for $p_{u_1}(u_1), \ldots, p_{u_N}(u_N)$, the end-product $p(z)$ is equal to a multiple integral expression of order $(N-1)$. To deal with the discontinuity in the triangular input and the intervals for which the integrals will be applied, a Heaviside function will be defined for convenience:

$$\mathcal{H}(x) = \begin{cases} 0 & x \le 0, \\ 1 & x > 0. \end{cases} \tag{5.12}$$

$\mathcal{H}(x)$ is defined to be zero for $x = 0$ to properly handle the limits of integration on the integral terms for $p(z)$. The *pdf* in Figure 5.1 and Equation 5.11 may now be redefined in terms of the Heaviside function:

$$\begin{aligned} p_{u_i}(u_i) &= [\mathcal{H}(u_i - c_1) - \mathcal{H}(u_i - c_2)] \cdot D_1 \cdot (u_i - c_1) \\ &\quad +[\mathcal{H}(u_i - c_2) - \mathcal{H}(u_i - c_3)] \cdot D_2 \cdot (u_i - c_3), \end{aligned} \tag{5.13}$$

where

$$\begin{aligned} D_1 &= \frac{2}{(c_3 - c_1)(c_2 - c_1)} \qquad \text{and} \\ D_2 &= \frac{2}{(c_3 - c_1)(c_2 - c_3)}. \end{aligned}$$

The general form of the analytical imprecise calculation for the probability approach is given by:

$$p(z) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} (\ldots)\, du_{N-1} \cdots du_1, \tag{5.14}$$

where the term $(\ldots)$ is made up of the product of *pdf*s in the form of Equation 5.13. Substitution of Equation 5.13 for the *pdf*s in the term $(\ldots)$ will reduce Equation 5.14 to

$$p(z) = \sum_j [G_k(e_j(c_l, z)) - G_k(f_j(c_l, z))] \cdot \mathcal{H}(e_j(c_l, z) - f_j(c_l, z)), \tag{5.15}$$

where

$$\begin{aligned} G_k(x) &= \int^x g_k(\xi)\, d\xi, \\ g_k(x) &= \text{a product of } D_1(u_i - c_1) \text{ with itself, with } D_2(u_i - c_3), \\ &\quad \text{or } D_2(u_i - c_3) \text{ with itself,} \end{aligned}$$

$$
\begin{aligned}
e_j(c_l, z), & \\
f_j(c_l, z) &= \text{functions of } c_l \text{, and } z \text{ depending on the } j^{th} \text{ operation} \\
& \quad \text{and limits of integration,} \\
l &= 1, \ldots, 3, \\
e_j() &\neq f_j().
\end{aligned}
$$

5.2.4 Numerical Application of the Probability Approach

As shown earlier, the fuzzy calculus method for *analytically* calculating imprecise performance parameters is impractical for computer-assisted design applications, due to algebraic complexity. A discrete numerical method thus becomes necessary to satisfy computational requirements. A form of the Fuzzy Weighted Average algorithm [31] and its extensions was presented in Chapter 2 (detailed algorithm in Appendix C) to meet this need.

When considering the analytical application of the *probability* approach, a similar combinatorial problem arises for an increasing number of design parameters due to the resulting multiple integrations. A numerical scheme for calculating the imprecise output as expressed by Equation 5.14 must therefore be developed. Because the calculation rule given by Equation 5.14 depends on the output variable of interest (z in this case), the output range must be combinatorially determined through interval analysis and then discretized for a numerical approximation. Implementing such a discretized output range, a numerical algorithm for the probability approach can be used as follows:

1. Assuming the general case $\hat{z} = f(u_i)$, determine the upper and lower bound on $\hat{z}$, denoted by z_u and z_l, through interval analysis.

2. Discretize the output parameter $\hat{z}$: $\hat{z} = m\Delta z + z_l$, where $m = 0, 1, \ldots, \frac{(z_u - z_l)}{\Delta z}$.

3. For each integration variable u_i with corresponding triangular input function $p_{u_i}(u_i)$ (Figure 5.1), discretize u_i: $u_i = n_i \Delta u_i + c_1$.

4. Replace the integral(s) in Equation 5.14 with discretized summation(s) such that

$$p(z) = p(m\Delta z + z_l) = \Delta u_1 \cdots \Delta u_{N-1} \sum_{u_1} \cdots \sum_{u_{N-1}} (\ldots) \tag{5.16}$$

For N design parameters and corresponding discretizations, the complexity of the numerical algorithm is of order

$$H \sim 2^{N-1} \cdot \kappa_1 \cdot \frac{(c_3 - c1)^{N-1}}{\Delta u_1 \cdots \Delta u_{N-1}} \cdot \frac{(z_u - z_l)}{\Delta z} \cdot (N-1) \cdot \kappa_2$$

where H is the number of operations, κ_1 equals the number of multiplications and divisions in $f(u_i)$, and κ_2 equals the number of multiplications and divisions in the $(\ldots)$ term of Equation 5.16. If all input parameter ranges as well as the output range are discretized into M intervals, the complexity becomes:

$$H \sim (N-1) \cdot M^N \cdot \kappa_1 \cdot \kappa_2, \tag{5.17}$$

which can be compared to the complexity of the extended fuzzy calculus FWA algorithm (either Equation 2.3 or Equation 2.4) shown earlier.

5.3 Examples

Using the probability approach along with its analytical and numerical applications, as described in Section 5.2, this section will present a number of examples of imprecise calculations. The complexity of the examples will be a progression from simple addition to calculations involving real-world design equations. The results of both the probability approach and fuzzy calculus method are shown graphically; however, only the mathematical development for the probability method is shown due to the thorough treatment of the fuzzy calculus' case earlier. In all fuzzy calculations, the input parameters are represented with an interval of confidence and a preference function, and the extended FWA algorithm is applied to obtain the output, where, for these examples, M (the number of discretized preference function points) is equal to 11.

5.3.1 Example 5.1: Analytical Addition

Given two input parameters u_1 and u_2 both in the form of Figure 5.1, the problem is to calculate the output

$$\hat{z} = u_1 + u_2 \tag{5.18}$$

using the analytical application. From Equation 5.5, $p(z)$ is given by

$$p(z) = \int_{u_2} p_{u_1}(z - u_2)\, p_{u_2}(u_2)\, du_2.$$

Substituting p_{u_1} and p_{u_2} into Equation 5.13:

$$\begin{aligned} p(z) \;=\; & \int_{-\infty}^{\infty} \{[\mathcal{H}((z-u_2)-c_1) - \mathcal{H}((z-u_2)-c_2)] \cdot D_1 \cdot ((z-u_2)-c_1) \\ & +[\mathcal{H}((z-u_2)-c_2) - \mathcal{H}((z-u_2)-c_3)] \cdot D_2 \cdot ((z-u_2)-c_3)\} \cdot \\ & \{[\mathcal{H}(u_2-c_1) - \mathcal{H}(u_2-c_2)] \cdot D_1 \cdot (u_2-c_1) \\ & +[\mathcal{H}(u_2-c_2) - \mathcal{H}(u_2-c_3)] \cdot D_2 \cdot (c_2-u_2)\}\, du_2. \end{aligned} \tag{5.19}$$

Let $c_1 = 2$, $c_2 = 3$, and $c_3 = 4$. Multiplying the terms of Equation 5.19 and applying Equation 5.15, the result (after considerable algebraic manipulation) is

$$\begin{aligned} p(z) &= \mathcal{H}(z-4)\cdot\{\frac{z^3}{6} - 2z^2 + 8z - \frac{32}{3}\} \\ &\quad -2\mathcal{H}(z-5)\cdot\{\frac{z^3}{3} - 5z^2 + 25z - \frac{125}{3}\} \\ &\quad +6\mathcal{H}(z-6)\cdot\{\frac{z^3}{6} - 3z^2 + 18z - \frac{108}{3}\} \\ &\quad -2\mathcal{H}(z-7)\cdot\{\frac{z^3}{3} - 7z^2 + 49z - \frac{343}{3}\} \\ &\quad +\mathcal{H}(z-8)\cdot\{\frac{z^3}{6} - 4z^2 + 32z - \frac{256}{3}\}. \end{aligned} \tag{5.20}$$

Considering z in the output interval $[4, 8]$, the *pdf* $p(z)$ can be constructed as shown in Figure 5.2. The corresponding $P_{output}(z)$ curve and the fuzzy result for z can also be determined, as in Figure 5.2.

Comparing the results, it is found that the probability approach output approaches a normal distribution (recall the *central limit theorem*), while the fuzzy approach results in a triangular function. The peaks of $P_{output}(z)$ and the fuzzy approach results both fall at the sum of the most preferred inputs $z_{peak} = 6.0$ (Figure 5.2). The probability and fuzzy output curves are not of identical form; however, very similar results occur for the output ranges where the vertical axis values (Figure 5.2) fall between (0.7) and (1.0).

5.3.2 Example 5.2: Numerical Addition

Given the same problem as Example 5.1 (Equation 5.18), this example will illustrate the numerical application of the probability approach. The upper and lower bound on z are:

$$z_l = 4.0,$$

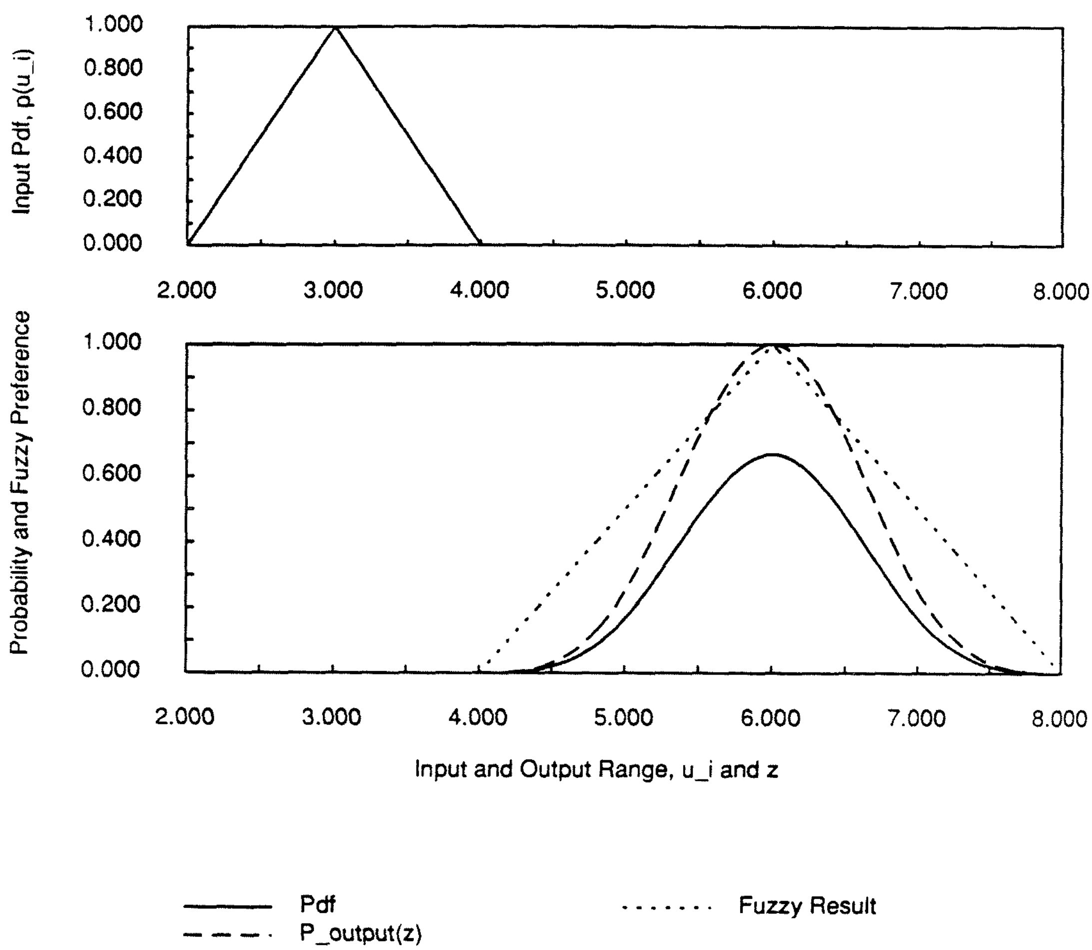

Figure 5.2: Example 5.1: $z = u_1 + u_2$.

$$z_u = 8.0.$$

The discretizations for $\hat{z}$ and u_2 can be specified as

$$\begin{aligned} z &= m\Delta z + 4.0 \\ u_2 &= n_2 \Delta u_2 + 4.0. \end{aligned}$$

Substituting into Equation 5.16, the result becomes

$$p(z) = \Delta u_2 \sum_{n_2} p_{u_1}(z - u_2)\, p_{u_2}(u_2). \tag{5.21}$$

Letting $m, n_2 = 0, 1, \ldots, 1000$ such that $\Delta z = 0.004$ and $\Delta u_2 = 0.004$, the calculated result of Equation 5.21 can be determined as in Figure 5.3.

When comparing the fuzzy result of Figure 5.3 with the probability approach, the same statements can be made as for Example 5.1, where $z_{peak} = 6.0$. Notice from the figure that the numerical scheme, when compared with the analytical results, is accurate within the small numerical error.

5.3.3 Example 5.3: Linear Equation

In this case, the governing equation is given by

$$\hat{z} = u_1 \cdot u_2 + u_3 \tag{5.22}$$

where the top of Figure 5.1 represents u_i, $i = 1, 2, 3$ for $c_1 = 2$, $c_2 = 3$, and $c_3 = 4$. The numerical approach will be used here. The bounds on z are then:

$$\begin{aligned} z_l &= 6.0, \\ z_u &= 20.0. \end{aligned}$$

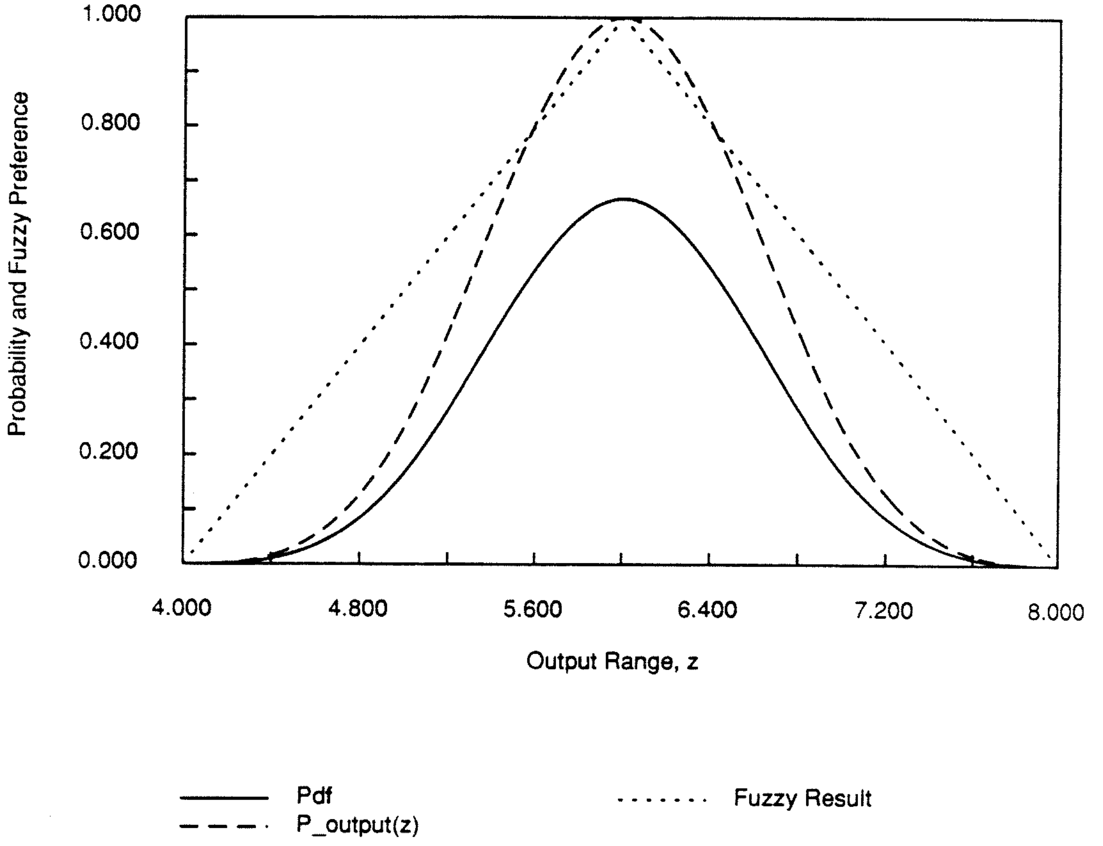

Figure 5.3: Example 5.2: $z = u_1 + u_2$.

Discretizations for z, u_2, and u_3 are expressed as

$$\begin{aligned} z &= m\Delta z + 6.0, \\ u_2 &= n_2\Delta u_2 + 2.0, \\ u_3 &= n_3\Delta u_3 + 2.0. \end{aligned}$$

Equation 5.16 becomes

$$p(z) = \Delta u_2 \cdot \Delta u_3 \sum_{n_3} \sum_{n_2} \frac{1}{u_2} p_{u_1}\left(\frac{z - u_3}{u_2}\right) p_{u_2}(u_2) p_{u_3}(u_3). \tag{5.23}$$

For discretization $n_2, n_3 = 0, 1, \ldots, 100$ and $m = 0, 1, \ldots, 1000$, $\Delta u_2 = 0.02$, $\Delta u_3 = 0.02$ and $\Delta z = 0.014$. Substituting these values into Equation 5.23, the calculated result of Equation 5.22 are as in Figure 5.4, along with the fuzzy calculus result.

In contrast to the addition operation examples (Example 5.1 and 5.2), the probability and fuzzy output curves are not symmetric about the peak. Furthermore, due to the nonlinear operation of multiplication, the peak of $P_{output}(z)$ does not fall at the linear combination of the most preferred inputs, but instead at $z_{peak} = 11.6$. The fuzzy result's peak, on the other hand, does correspond to such a combination, $z_{peak} = 12.0$. In general, the curves have very little similarity because of the shifted peaks as well as the collapsed range of the $P_{output}(z)$ curve versus the fuzzy output.

5.3.4 Example 5.4: Trigonometric Operations

This example will consider simple trigonometric operations on a single parameter u_1. Only the output parameter, z, requires discretization due to the closed form of Equations 5.9 and 5.10.

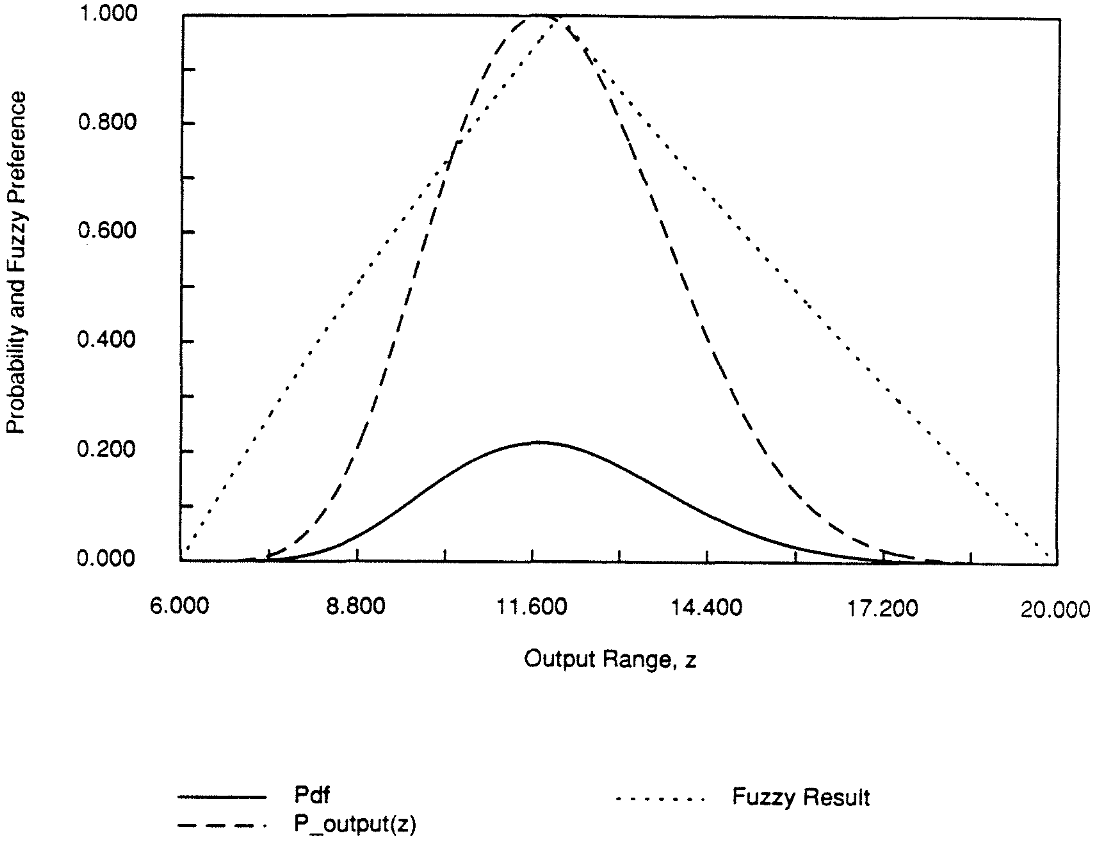

Figure 5.4: Example 5.3: $z = u_1 \cdot u_2 + u_3$.

5.3.4.1 Sine Operation

The equation of interest in this case is

$$\hat{z}_{sin} = \sin u_1, \tag{5.24}$$

where the input parameter u_1 is a triangular function with the peak at $u_{1,peak} = \frac{\pi}{6}$ and interval range of $u_1 \in [0.0, \frac{\pi}{3}]$. From Equation 5.9, the output parameter's *pdf* can be calculated such that

$$p(z_{sin}) = \frac{1}{\sqrt{1 - z_{sin}^2}} p_{u_1}(\sin^{-1}(z_{sin})). \tag{5.25}$$

Considering the output range $z_{sin} \in [0.0, \frac{\sqrt{3}}{2}]$, the result of the calculation $p(z_{sin})$ along with $P_{output}(z_{sin})$ are shown in Figure 5.5.

5.3.4.2 Cosine Operation

Using the same input parameter u_1 as for the sine operation, this case will calculate the cosine output:

$$z_{cos} = \cos u_1, \tag{5.26}$$

where the resulting *pdf* can be determined from

$$p(z_{cos}) = \frac{1}{\sqrt{1 - z_{cos}^2}} p_{u_1}(\cos^{-1}(z_{cos})). \tag{5.27}$$

Figure 5.5 shows the output curves $p(z_{cos})$ and $P_{output}(z_{cos})$, where $z_{cos} \in [0.5, 1.0]$.

The limits on the ranges of the input angles for the trigonometric operations used in this example were chosen to produce monotonic outputs. The sine output functions are very similar for the probability approach and the fuzzy approach ($u_1 \in [0.0, \frac{\pi}{3}]$). The cosine function for the same input, however, produces dissimilar results as the input parameter approaches a value for which the cosine function has zero slope.

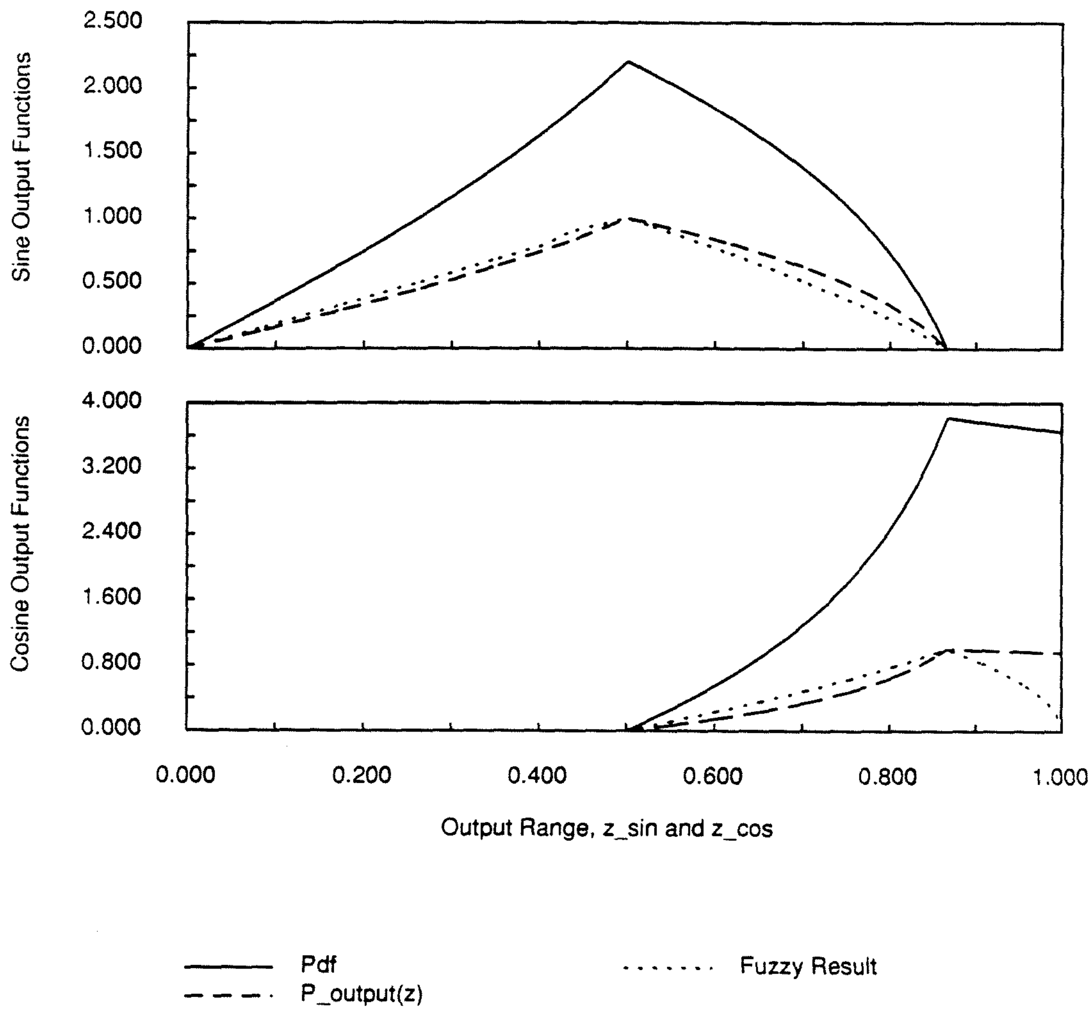

Figure 5.5: Example 5.4: Output Results for z_{sin} and z_{cos}.

Instead of $P_{output}(z)$ approaching zero for the right extreme input (Figure 5.5), as in the other examples, the output values are very high relative to the peak. Notice that both trigonometric operations resulted in peak values corresponding to the expected input, *i.e.*, for $u_1 = \frac{\pi}{6}$, $z_{sin,peak} = 0.5$ and $z_{cos,peak} = \frac{\sqrt{3}}{2}$.

5.3.5 Example 5.5: Beam Shear Stress

The problem is to design a horizontal beam that will not fail when subjected to a vertical load distributed along its length. The configuration under consideration for this example is a simply-supported beam with a pin connection on the left, a roller connection on the right, a rectangular cross-section and a uniformly distributed vertical load. Given design parameters of beam length L, width b, height h, and applied load w, one important performance parameter is the maximum shear stress, τ:

$$\tau = \frac{3\,w\,L}{4\,b\,h}. \tag{5.28}$$

Other performance parameters of interest, which are not considered here, might be: mid-point deflection, maximum bending stress, etc. Table 5.1 lists the representative data (left-extreme, right-extreme, and peak values) for the triangular design inputs, where we define $\hat{w} = \frac{3}{4}$ so that Equation 5.28 becomes

$$\tau = \frac{\hat{w}\,L}{b\,h}. \tag{5.29}$$

Applying Equations 5.7 and 5.8, $p(\tau)$ is given by

$$p(\tau) = \int_h \int_b \int_L \left(\frac{h \cdot b}{L}\right) p_{\hat{w}}\left(\frac{\tau \cdot h \cdot b}{L}\right) p_L(L) p_b(b) p_h(h)\; dL\, db\, dh. \tag{5.30}$$

The numerical solution to Equation 5.30 can be formulated as follows:

1. Combinatorially determine upper and lower bounds for τ.

$$\tau_l = 0.9 \text{ MPa},$$
$$\tau_u = 202.5 \text{ MPa}.$$

   Note the large range (typical in preliminary engineering design) for the performance parameter τ ($> 10^2$) that results from reasonable ranges for the five design parameters (Table 5.1). This demonstrates the need for a computationally efficient method for this stage of the design process.

2. Discretize the output range for τ and the input parameters L, b, and h:

$$\tau = m\Delta\tau + 0.9,$$
$$L = n_1\Delta L + 3.0,$$
$$b = n_2\Delta b + 0.1,$$
$$h = n_3\Delta h + 0.1.$$

3. Express Equation 5.30 in the numerical application form:

$$p(\tau) = \Delta L \cdot \Delta b \cdot \Delta h \sum_{n_3}\sum_{n_2}\sum_{n_1} \left(\frac{h \cdot b}{L}\right) p_{\hat{w}} \left(\frac{\tau \cdot h \cdot b}{L}\right) p_L(L)\, p_b(b)\, p_h(h). \quad (5.31)$$

4. Letting $n_i = 0, 1, \ldots, 50$ and $m = 0, 1, \ldots, 1000$ such that $\Delta L = 0.12$, $\Delta b = \Delta h = 0.008$, and $\Delta\tau = 0.2016$, calculate $p(\tau)$ from Equation 5.31. The result of such a calculation is shown in Figure 5.6.

Again we see that the peak of the curve produced by the probability calculus does not occur at the shear stress corresponding to the most preferred design parameter values.

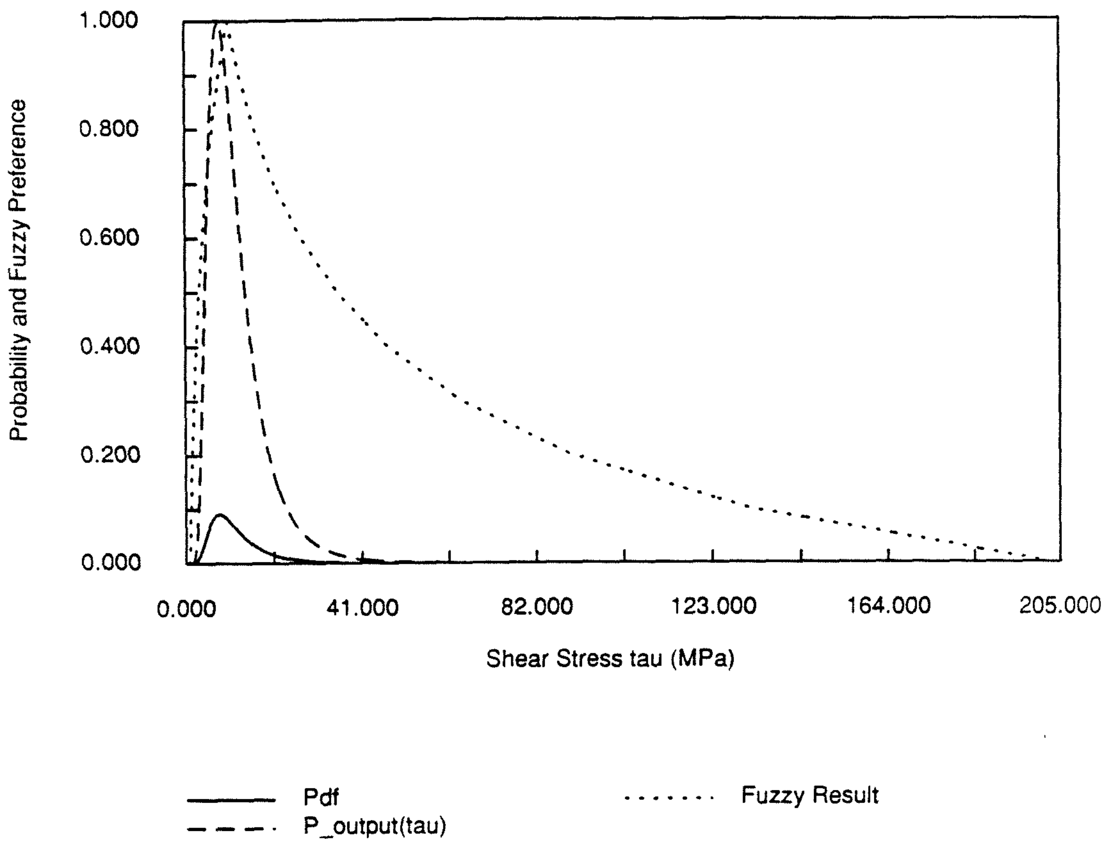

Figure 5.6: Example 5.5: Output τ.

5.3.6 Example 5.6: Torque for a Drum Brake

The motivation of this example is the design of a braking system for a vehicle that will adequately stop the vehicle for a certain range of speeds. This example was explored thoroughly in Chapter 4 with the fuzzy calculus to represent design imprecision. A portion of the same example is repeated here to compare results with the probability calculus. Given that a drum brake configuration is under consideration for this design, one important performance parameter is the torque $\mathcal{T}$ [67]:

$$\mathcal{T} = \mu\, p_a\, b\, r^2 \,(\cos\theta_1 \;-\; \cos\theta_2), \qquad (5.32)$$

where

$$\begin{aligned}
b &= \textit{face width of frictional material},\\
p_a &= \textit{maximum operating pressure of material},\\
r &= \textit{inner drum brake radius},\\
\mu &= \textit{coefficient of friction},\\
\theta_1 &= \textit{angle to beginning of frictional material},\\
\theta_2 &= \textit{angle to end of frictional material.}
\end{aligned}$$

The representative data for the torque input parameters (b, p_a, r, μ, θ_1, and θ_2) are in Table 5.2. Applying Equations 5.6, 5.7, and 5.10, the numerical scheme for determining $\mathcal{T}$ is as follows:

1. Determine the bounds for $\mathcal{T}$:

$$\begin{aligned}
\mathcal{T}_l &= 0.00375\,(\text{kN-m}),\\
\mathcal{T}_u &= 5.3181\,(\text{kN-m}).
\end{aligned}$$

Note the large range for the performance parameter $\mathcal{T}$ ($> 10^3$) that results from reasonable ranges for the six design parameters (Table 4.1).

DPs u_i (units)	$p_{u_i}(u_i)_{left} = 0$	$p_{u_i}(u_i) = 1$	$p_{u_i}(u_i)_{right} = 0$
L (m)	3.0	6.0	9.0
b (m)	0.10	0.30	0.50
h (m)	0.10	0.30	0.50
$\hat{w}$ (kN/m)	75.0	150.0	225.0

Table 5.1: Beam Example: Design Parameter Data.

DPs u_i (units)	$p_{u_i}(u_i)_{left} = 0$	$p_{u_i}(u_i) = 1$	$p_{u_i}(u_i)_{right} = 0$
b (m)	0.025	0.050	0.075
μ	0.10	0.30	0.50
p_a (kPa)	300.0	1100.0	1900.0
r (m)	0.10	0.15	0.20
θ_1 (deg.)	0.0	30.0	60.0
θ_2 (deg.)	90.0	120.0	150.0

Table 5.2: Brake Example: Design Parameter Data.

2. Write the analytical form of the expression for $p(\mathcal{T})$:

$$\begin{aligned} p(\mathcal{T}) = & \int_{\mu}\int_{p_a}\int_{b}\int_{u_3}\int_{u_2} \left(\frac{1}{u_3 \cdot \mu \cdot p_a \cdot b}\right) p_{\mu}(\mu) p_{p_a}(p_a) p_b(b) \\ & p_{u_4}\left(\frac{\mathcal{T}}{u_3 \cdot \mu \cdot p_a \cdot b}\right)\left(\frac{1}{\sqrt{1-(u_3+u_2)^2}}\right) p_{\theta_1}(\cos^{-1}(u_3+u_2)) \\ & \left(\frac{1}{\sqrt{1-u_2^2}}\right) p_{\theta_2}(\cos^{-1}(u_2))\, du_2\, du_3\, db\, dp_a\, d\mu, \end{aligned} \tag{5.33}$$

where

$$\begin{aligned} u_1 &= \cos(\theta_1), \\ u_2 &= \cos(\theta_2), \\ u_3 &= u_1 - u_2, \\ u_4 &= r^2, \text{ and} \\ p_{u_4} &= \frac{p_r(r)}{2 \cdot r}. \end{aligned}$$

3. Discretize the input and output ranges:

$$\begin{aligned} \mathcal{T} &= m\Delta\mathcal{T} + \mathcal{T}_l, \\ u_3 &= n_1\Delta u_3 + 0.5, \\ \mu &= n_2\Delta\mu + 0.1, \\ p_a &= n_3\Delta p_a + 300.0, \\ b &= n_4\Delta b + 0.025, \\ u_2 &= n_6\Delta u_2 - \frac{\sqrt{3}}{2}. \end{aligned}$$

4. Transform Equation 5.33 into the form of Equation 5.16.

5. Letting $n_i = 0, 1, \ldots, 10$ and $m = 0, 1, \ldots, 100$, the result of the numerical calculation can be determined as shown in Figure 5.7.

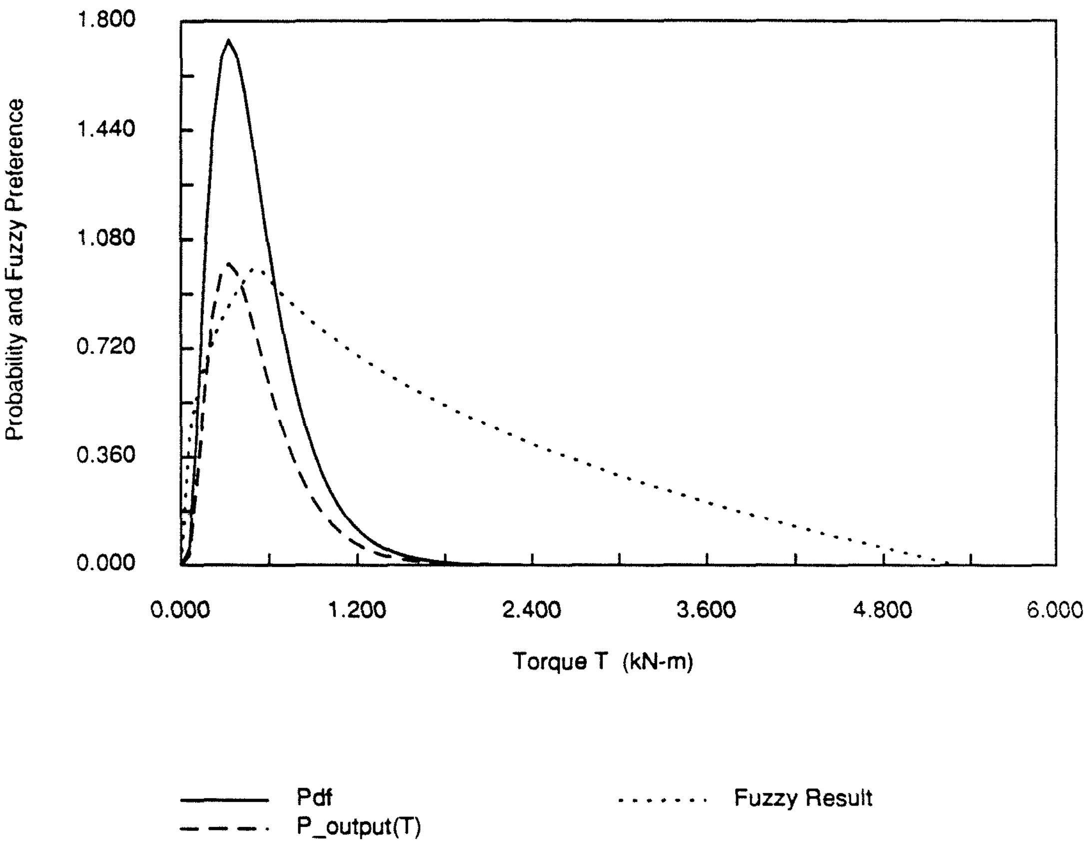

Figure 5.7: Example 5.6: Output $\mathcal{T}$.

Examples 5.5 and 5.6 consist of calculations with real-world design equations, building from the operations carried out in the previous examples. Figures 5.6 and 5.7 show that the nonlinear operations shift the peak of P_{output} when compared with the fuzzy result. The height of the probability function versus the fuzzy preference function also varies greatly in the output horizontal range. When compared with the previous results for Examples 5.1 through 5.4, this effect is much more dramatic for the shear stress and brake torque due to the increased number of operations.

5.4 Discussion

Section 5.3 presents several examples of the probability calculus approach applied to equation calculations with imprecise input parameters. This section will compare the results of these examples with the fuzzy calculus approach.

The output parameters from the example calculations in Section 5.3 are made up of continuous *pdf*s, which must have the following two properties:

1. $p(z) \geq 0$,

2. $\int_{-\infty}^{\infty} p(z)dz = 1$.

Figures 5.2 through 5.7 possess both these properties. Furthermore, in the case of Example 1 and Example 2, the *central limit theorem* applies, *i.e.*, for a large number of uncertain parameters $u_1, \ldots, u_N$, the sum of the parameters will result in an approximate Normal (Gaussian) distribution. Figures 5.2 and 5.3 illustrate that for triangular input parameters, a single addition operation produces an output similar to a Normal distribution.

The discussion of the fuzzy calculus approach, compared to the probability approach for representing and manipulating imprecision, will be three-fold: an assessment of the general character of the output (performance parameter) preference functions in the two cases, the differences in the interpretations of these curves, and the usefulness of applying either method in the design domain. It is found that two primary differences occurred in the outputs of the probability approach calculations compared to the fuzzy set method: the output peak is shifted, and the output height over the range of the performance parameter is different. The differences in the axiomatic development of the two calculi, of course, are the basis for these output dissimilarities.

The fuzzy approach relies on max-min operations using the *extension principle* [91], which assures that the peak of a calculation with triangular inputs will occur at the calculated combination of the most desirable values (peaks) of the input parameters. The peak of the output from the probability approach, on the other hand, will generally be shifted, and will have similar results to the fuzzy calculus only when the calculation involves linear operations, but not for nonlinear operations.

The differences in the output height for the two methods can be explained similarly. The fuzzy approach once again relies on the max-min solution, which tends to broaden the output (*i.e.*, greater preference values for points that approach the extremes of the output range interval). The probability approach, however, relies on the addition rule. For an increasing number of operations in $f(u_i)$, combinations of input parameters with values far off the input curve's peak will contribute very little to the output curve, resulting in a collapsing of the output range (*i.e.*, small output height for the extreme output range values).

The interpretation of the output results for the two methods is also different. The result of the fuzzy approach to imprecise calculations is in general a curve that peaks (preference of one (1)) at the output value corresponding to the most preferred input values, and has an associated interval range of all possible combinations of the input parameters. The value at any point on the output preference function can be directly traced back to the combination of input parameters that resulted in the corresponding output value. Alternatively, the result of the probability approach is a curve that peaks $P_{output}(z) = 1$ at a point shifted, in general, from the output value corresponding to the most preferred input values, and has the same output range as the fuzzy calculus method. (If the usual interpretation of probability was being used, the peak would occur at the most likely output value.) Because of the addition rule for probability, input values cannot be practically traced back from a given output on the curve $P_{output}(z)$. The reason for this is that a probability calculation for a point on $P_{output}(z)$ is a combination of *all* the probabilities from inputs corresponding to a specific output z value.

The use of probability to represent design imprecision has an additional difficulty. Probability calculations with large numbers of uncertain input parameters (*e.g.*, greater than 10) will be slow and computationally expensive (Equation 5.17) in comparison to fuzzy calculations. As shown in the next chapter, it will be necessary to use probability calculations for including the effects of stochastic uncertainty along with the fuzzy calculus approach to design imprecision, as long as the number of stochastically uncertain parameters is small.

5.5 Conclusions

Using the analysis of the differences in output curves and interpretation as discussed above, one can determine the applicability and usefulness of the fuzzy approach and probability approach for the representing and manipulating imprecise parameters in the design domain. The fuzzy approach is a technique that presents more information to a designer than conventional single-valued or interval analysis, by indicating the relative importance of input parameters and providing a method for comparing different solution alternatives. The probability approach can satisfy the first objective of this research (Chapter 1) in that the subjectivity of the designer can be represented and manipulated. However, the performance parameter results are in a form that makes the evaluation of the relative importance of inputs more difficult, due to the narrowing of the output peak common to probability calculations (as shown in the examples). The shifting of the peak of the output from the probability calculations, away from the combination of the most preferred inputs, also reduces their usefulness. Additionally, the ability to trace an output value back to a set of inputs that produced it is absent in probability calculations with imprecise parameters. Finally, probability calculations are far more computationally complex than fuzzy calculations. All of the above contribute to making fuzzy calculations on design imprecision more applicable and useful for preliminary engineering design.

Chapter 6

Combining the Effects of Imprecision with Stochastic and Possibilistic Uncertainties

6.1 Introduction

The fuzzy calculus has been shown to be useful for representing and manipulating design imprecision. This chapter introduces a technique for handling stochastic and possibilistic uncertainty in addition to imprecision. A single-speed transmission example, with all three types of uncertainty present, demonstrates the method.

6.2 Combining Imprecision With Other Uncertainties

In Chapter 1, the imprecision component of design uncertainty and the stochastic and possibilistic components were described. At least two of these effects are usually present simultaneously. For example, a dimension of a part might be only imprecisely known to the designer, *and* the manufacturing method might introduce an uncertainty (tolerance). Similarly for a coefficient used in the design process, such as the convection coefficient in heat-transfer, some contributions to this coefficient the designer can choose, such as geometry or surface finish; others, such as the conditions under which the device will operate, he or she has no control over. The first of these effects is represented as imprecision, determined by the designer's desires or by design specifications. The second of these is represented as either stochastic or possibilistic uncertainty, or both.

Stochastic and possibilistic uncertainty, in addition to imprecision, are present at all stages of the design process. The usual sources of these two types of uncertainties are tolerances in manufacturing processes resulting in uncertain dimensions and uncertainties in material properties. Uncertainties can also exist related to application. The coefficient of friction between a tire and the road can take on a wide range of values depending on road, tire, and weather conditions. Similarly, there might exist uncertainty in physical property models [49]. Sometimes these data will represent measured (objective) probability data; sometimes they will represent subjective possibility data. As will be seen later, these two types of design uncertainty are kept distinct.

This section presents a method for representing and manipulating imprecision in conjunction with other uncertainties. Calculations are performed with these variables according to the governing performance expressions of the system. Qualitative relations between the input and output performance parameters are determined such that the designer is able to rank the design parameters according to their impact on the performance results. The designer is also provided with the necessary information by which a design alternative may be rated according to its merit in relation to the design's functional requirements, and in relation to the other alternatives under consideration.

6.2.1 Introducing Extended Hybrid Numbers

Chapters 2 through 5 develop a method for representing and manipulating imprecision. To include stochastic and possibilistic uncertainty effects, an additional method must be employed. Several methods already exist for representing and manipulating these two complementary types of uncertainty in engineering [14, 36, 42, 50, 68, 69]. The need exists to combine the effects of stochastic and possibilistic uncertainties with imprecision. Because imprecision is modeled with preference functions and the fuzzy calculus, a logical way to include uncertainty effects is to also transform them into a fuzzy representation. For example, an uncertain parameter that is described by a probability density function can be normalized to have a peak of one (1), as shown by Kaufmann and Gupta [46, pages 79–82]. Likewise, if a stochastic parameter $\hat{u}$ is modeled by a Gaussian distribution, the following algorithm may be employed to transform the parameter to an approximate fuzzy representation. Calculate the area from the expected value to infinity, which corresponds to the probability of $\hat{u}$ being greater than the expected value. Denote this probability by P_E. If areas are then calculated from certain $\hat{u}$ values to infinity such that the results are increments from 0.5 to 0.0, and if these are normalized with respect to P_E, values in increments from 1.0 to 0.0 will be obtained. Symmetry of the Gaussian distribution automatically gives the same values for the left portion of the distribution for $\hat{u}$. An interpolated curve between the resultant incremental values may now be considered as an approximate membership function α_G if the fuzzy interpretation of the results is adopted.

With the uncertain parameter represented fuzzily, and assuming an expression exists relating the uncertain parameters to the imprecise parameters, the fuzzy calculus might be used to combine the scaled *pdf* representation or the α_G representation of the stochastic uncertainty with the imprecision of the design, resulting in a single output curve. The fuzzy output in this case has both stochastic uncertainty and imprecise contributions. Unfortunately, as pointed out by Kaufmann and Gupta [46, page 82], information is lost by this process.

> "Basically, we have transformed a measurement of an objective (measured) value to a valuation of a subjective (fuzzy) value, which results in the loss of information. Although this procedure is mathematically correct, it decreases the amount of information that is available in the original data, and we should avoid it."

There is no way to determine which portion of the result is due to stochastic uncertainty and which is due to imprecision. Because these two effects are independent, and the need exists to be able to determine their ramifications separately, a method for keeping stochastic uncertainty and imprecision distinct in design calculations must be developed.

Kaufmann and Gupta in [46] have proposed the notion of *hybrid numbers* as a method of representing stochastic uncertainty and fuzzy uncertainty without reducing the information content. This is somewhat similar to complex numbers, with a real component and an imaginary component. Here, instead, the resulting number includes a fuzzy component, and a stochastic one. In the model of the engineering design process proposed in this document, the fuzzy component of the hybrid numbers will be used to represent imprecision (or approximation); the other component will represent stochastic uncertainty.

In engineering design, the random-variable frequency-based model of probability won't adequately represent all uncertainties in truth (*i.e.*, uncertainty in predicting a value a parameter will assume at any stage of the design). Some uncertainties of this form are subjective, rather than measured. For example, the coefficient of friction of a brake shoe under a variety of possible operating conditions. These subjective uncertainties can be represented by *possibility*, introduced by Zadeh [92]. This representation of uncertainty (rather than incorporating all uncertainty into a multi-valued logic probability formulation) has been adopted for computational efficiency, and to adhere to the interpretation of these two distinct forms of uncertainty proposed by Kubic and Stein in [50] as well as others.

Because these two separate forms of truth uncertainty, in addition to imprecision, are a useful representation scheme in preliminary engineering design, Kaufmann and Gupta's hybrid numbers have been augmented to represent all three components. This new representation will be referred to as *Extended Hybrid Numbers*.

6.2.2 Computations with Extended Hybrid Numbers

The three distinct components that can comprise design parameters (imprecision, possibilistic uncertainty, and stochastic uncertainty) can be operated on in a design calculation, and then recombined into an extended hybrid representation of the result. Imprecision is represented and calculated, and the result is interpreted as discussed previously. The same fuzzy mathematics can be applied to the possibilistic uncertainties [92], but the interpretation of the inputs and outputs are different. The input interpretation corresponds to Kubic and Stein's, and the output represents the performance over the range of possible values of the input parameters. The designer's judgement can be incorporated at this stage by determining over what range of possible values the design should function; or a specification may require the design to operate in some range of possible conditions. For example, in extreme cases the design must operate over all possible conditions, and the range of performance would extend over the entire possible output range.

6.2.2.1 The Stochastic Component of Uncertainty

The construction and choice of input parameter *pdfs* for the stochastic component can be carried out in one of the following ways [11, 69]:

1. Subjectively determine the *pdf* $p(u_i)$ for an input parameter u_i based upon past experience or based upon the data from a similar former design scenario.

2. Subjectively construct $p(u_i)$ from a known range of plausible values of u_i. The shape and area of such a subjective *pdf* can be manipulated by curve fitting functions and a normalization routine, respectively, in order to satisfy the unit area condition.

3. Construct $p(u_i)$ from known data and by application of the maximum entropy function.

Examples of input *pdfs* using these methods of choice are given in the machine design problem at the end of the chapter.

Operation rules may be constructed from Cox's [24] formulation of the calculus for probability logic in order to carry out calculations with the stochastic component of the extended hybrid numbers. These operation rules, along with a general analytical and numerical approach, have been previously defined in Chapter 5. In contrast to Chapter 5, the definition of probability as a quantification of preference is no longer used; instead, it is necessary to return to the basic notion of probability as a quantification of plausibility.

Although the numerical application presented in Chapter 5 proved useful and adequate when comparing the fuzzy and probability calculus for the imprecision problem, the general approach has two short-comings: (1) the method is difficult to apply when dealing with a mathematically defined *pdf* $p(u_i)$ that has infinite tails; and (2) the complexity of the method (Equation 5.17) makes it costly for a large number of parameters. These short-comings may be overcome, at least in part, by discretizing the cumulative probability rather than the range of each parameter in the multiple integral expression (Equation 5.14). A change of this form, however, requires that the binary operations (Equations 5.5 through 5.8) be defined in terms of the cumulative distribution of the input parameters instead of the probability density function. Appendix B contains the derivation of the alternative forms for the binary operations, summarized below:

$$\begin{aligned} p_{add}(z \mid I) &= \frac{d}{dz} P(\hat{z} \leq z \mid I) \\ &= \int_0^1 p_{\hat{x}}(z-y)\,dP(y), \end{aligned} \tag{6.1}$$

$$p_{sub}(z \mid I) = \int_0^1 p_{\hat{x}}(z+y)\,dP(y), \tag{6.2}$$

$$p_{mul}(z \mid I) = \int_0^1 \frac{1}{y} p_{\hat{x}}(\frac{z}{y})\,dP(y), \tag{6.3}$$

$$p_{div}(z \mid I) = \int_0^1 y p_{\hat{x}}(y \cdot z)\,dP(y). \tag{6.4}$$

Applying these operation rules in the context of Equation 5.14, the new numerical application is formulated as an algorithm in the following manner:

1. Assuming the general case $\hat{z} = f(u_i)$, determine the upper and lower bound on $\hat{z}$, denoted by z_u and z_l, through combinatorial interval analysis.

2. Discretize the output parameter $\hat{z}$: $\hat{z} = m\Delta z + z_l$, where $m = 0, 1, \ldots, \frac{(z_u - z_l)}{\Delta z}$.

3. For each integration variable u_i with corresponding input *pdf* $p_{u_i}(u_i)$, convert each $p_{u_i}(u_i)$ to its cumulative distribution $P_{u_i}(u_i)$.

4. Discretize each $P_{u_i}(u_i)$ into L_i equally spaced intervals, where

$$P_{u_i}(u_i) = (j - \frac{1}{2})/L_i,$$

$$j = 1, 2, \ldots, L_i.$$

5. Replace the integral(s) in Equation 5.14 with discretized summation(s) such that

$$p(z) = p(m\Delta z + z_l) = \frac{1}{L_1} \cdots \frac{1}{L_{N-1}} \sum_1^{L_1} \cdots \sum_1^{L_{N-1}} (\ldots) \tag{6.5}$$

The usual likelihood interpretation of probability theory is applied to the normalized output results of computations with Equation 6.5. The usual convergence and stopping criteria apply for the algorithm.

6.2.2.2 A Stochastic Parameter Measure

The γ-level measure, defined in Chapter 2, has been developed on the premise of determining the importance and coupling of the imprecision component of the design parameters. Likewise, a measure is needed to determine the relative importance of the stochastic component of uncertainty such that parameters with little influence on the stochastic uncertainty can be assigned to their expected values. This results in a decrease in dimension of the parameter space, and a corresponding reduction in computation, especially in the context of the calculations performed over the entire design cycle from the highly imprecise preliminary stage to the reliability measures determined at a design's completion.

Variance analysis provides a useful means by which the relative contribution of the stochastic uncertainty of each design parameter may be determined with respect to the stochastic component of the output extended-hybrid-number representation. Consider, for example, that z_j $j = 1, \ldots, M$ are the performance parameters, where z_j^s

represents the stochastic portion of the j^{th} output and u_i^s, $i = 1, \ldots, N$ the stochastic inputs. The variance of z_j^s may be determined using the second central moment of the output *pdf* $p(z_j)$:

$$\nu_{z_j} = \int_{z_j^s} (z_j^s - \mu_{z_j^s})^2 \, p(z_j) \, dz_j^s, \tag{6.6}$$

where $\mu_{z_j^s}$ is the mean of z_j^s. Similarly, the variance of the input parameters may be found from the following:

$$\nu_{u_i} = \int_{u_i^s} (u_i^s - \mu_{u_i^s})^2 \, p_{u_i}(u_i) \, du_i^s. \tag{6.7}$$

The relative variance of input parameter u_i^s to the total output variance of the performance parameter z_j^s can be estimated using

$$r_{j,i} = \left(\frac{\partial z_j^s}{\partial u_i^s} \right)^2 |_{u_k^s} \cdot \left(\frac{\nu_{z_j}}{\nu_{u_i^s}} \right), \tag{6.8}$$

where the partial derivative of the performance parameter with respect to the input parameter may be estimated by numerical differentiation if necessary. (Note: instead of the variance approach, an application of the entropy measure of the marginal distributions could have been used.)

6.2.2.3 Outline of Calculation Procedure

The calculation method for extended hybrid numbers may be summarized as follows:

- Given performance parameter expressions for the design, relating the design parameters to the performance parameters, calculate the imprecision using the nominal values for the possibilistic and stochastic uncertainties and the FWA algorithm with extensions.

- Repeat with the nominal values for the imprecision and stochastic uncertainties to determine the possibilistic component.

- Determine the stochastic component using the numerical scheme presented above, along with the nominal values for the imprecision and possibilistic uncertainties, and normalize this result to get a relative plausibility function.

- Combine these resulting output functions.

The extended hybrid representation of the result can be shown on a single graph. All three output functions will have a (relatively) common peak (at the nominal value for the output). The three curves represent the imprecision, possibility, and probability respectively, and can be compared to the design's functional requirements. For example, consider the output performance for fatigue strength of a spur gear system as shown in Figure 6.11. Assuming that the desired performance (functional requirement) of the factor of safety is 2.0, the corresponding membership value (α) of the imprecision curve can be read directly from the ordinate. In this case α is approximately equal to 0.8. This implies that at least one design parameter must decrease in membership to 0.8 in order to achieve a factor of safety equal to 2.0 and satisfy the performance requirement.

The possibilistic uncertainty, however, must also be accounted for in the design. In Figure 6.11 the *possible* ranges of the fatigue strength factor of safety for the spur gear are indicated by the possibilistic uncertainty curve. Because a minimum value (2.0) must be achieved for this parameter, the left portion (from the peak) of the curve represents the possibilistic uncertainty that will be present. If a PP must meet a maximum constraint (instead of a minimum) then the right portion of the curve would be used. The effect of this uncertainty is to introduce a range of possible values for each output PP value. Because the uncertain range may include unsatisfactory performance values, the value of the PP must be chosen such that all *possible* values satisfy the required performance.

A particular range of possibilistic uncertainty is determined by the degree of uncertainty the designer chooses to include. For example, the designer may wish this device to operate over no less than 50% of the possible operating conditions. In this case the range of possible outputs would be determined by the interval between the left and right portions of the possibilistic uncertainty curve at a membership of 0.5. For this example PP, shown in Figure 6.11, the nominal value is 1.61 for the spur gear configuration. The value of the left portion of the curve at $\alpha = 0.5$ is 0.99, and 1.70 for the right. Thus the range of uncertainty in this instance is 0.71, however it is asymmetrically distributed, with the major portion (0.62) to the left of the PP value. If the nominal value for this PP is used (1.61), the possible range of performance would be from 0.99 to 1.70. Because the requirement on this PP is a minimum of 2.0, a value larger than the peak (nominal) output value must be used, such that the lower end of the possible range still meets the requirement. A satisfactory PP value can be obtained by adding the left portion of range to the required value. In this case the designer would have to use a value no less than $0.62 + 2.0 = 2.62$ to be sure that no less than 50% of the possible values of the output would meet or exceed the requirement.

A similar procedure can be carried out for the stochastic component of uncertainty shown on the output curve, where the chosen stochastic uncertainty is subsequently added to the functional requirement to obtain an equivalent uncertainty performance specification in terms of both a possibilistic and stochastic contribution. For example, assume that the imprecision and stochastic components of a performance parameter (p_j) have been calculated as shown in Figure 6.1, and that the designer wishes to achieve 99% reliability for p_j. Assume also that the functional requirement for this performance parameter is expressed as a crisp inequality in one

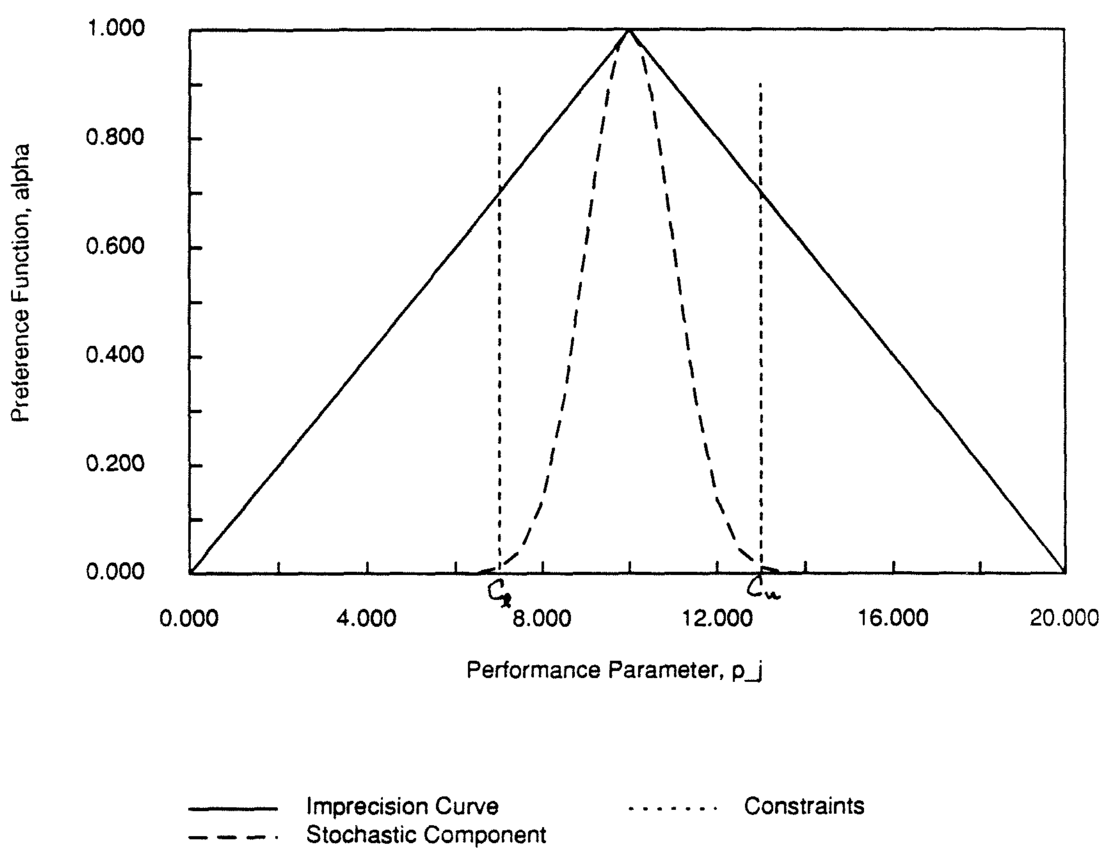

Figure 6.1: Performance Parameter: p_j.

of two possible forms: $p_j \leq C_u$ or $p_j \geq C_l$, where "C" denotes a constraint. Example functional requirement values are shown in the figure. According to the interpretation of the imprecision calculation, at least one design parameter must be decreased in preference to the value where the vertical line from either C_u or C_l intersects the imprecision curve. In order to include the stochastic component for 99% reliability, the value of the functional requirement must be shifted by an appropriate amount, either δ_u^s (to the right for C_u) or δ_l^s (to the left for C_l). The values for δ_u^s and/or δ_l^s can be determined from the equivalent cumulative distribution of the stochastic component p_j^s, as shown in Figure 6.2. $p_{j,u}^s$ represents the value for which $P(p_j^s \leq \hat{p}_j^s) = 0.99$. Likewise, $p_{j,l}^s$ represents the value for which $P(p_j^s \leq \hat{p}_j^s) = 0.01$. δ_u and δ_l can be calculated from these values according to the following relationships:

$$
\begin{aligned}
\delta_u &= p_{j,u}^s - p_{j,\alpha=1}^s \\
\delta_l &= p_{j,\alpha=1}^s - p_{j,l}^s.
\end{aligned}
$$

Notice here that the peak of the imprecision curve is not used in these calculations due to the possibility that the peak will be shifted for a non-linear probability computation. δ_u is now added to C_u or δ_l is subtracted from C_l to determine the resultant functional requirement for the stochastic component of uncertainty.

In this procedure, the contribution of the stochastic component has been handled by shifting the functional requirement. It must be noted that this is *equivalent* to shifting the imprecision curve by δ^s either to the right or left depending on the functional requirement. A shifting of the imprecision curve conforms to the mathematical model of the performance parameters, because functional requirements are really fixed constants for a design. No matter which method is used (either shifting the functional requirement or the imprecision curve by δ^s), however, the results will be the same.

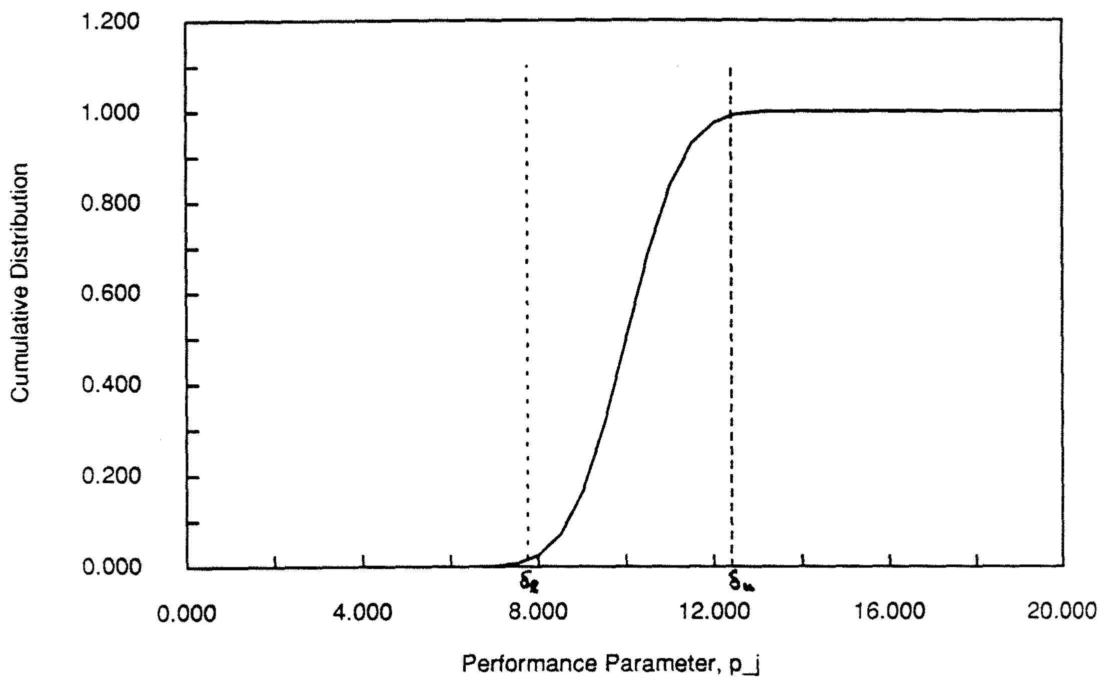

Figure 6.2: Cumulative Distribution for p_j^s.

6.3 A Machine Design Example

To demonstrate the approach described briefly above, an example preliminary design problem will be presented. The problem is to design a single speed power transmission for the "spin" cycle of a conventional domestic clothes washing machine. The details of changing modes from agitation to spin are omitted for clarity. The drive motor is a 1.5 kW (2 hp) electric motor with a nominal no-load speed of 1750 rpm. The desired top speed of the drum is approximately 350 rpm. Both the motor shaft and the drive shaft on the washer drum are vertical. As a design goal it is desirable to minimize cost, as well as to achieve satisfactory performance in terms of strength, durability, and belt life. For simplicity in this example, cost will be assumed to be directly related to the diameter of the shafts. Three different alternative drives will be compared: spur gears, helical gears, and a V-belt. The configuration of the gear drives is shown in Figure 6.3, and the V-belt is shown in Figure 6.4.

6.3.1 Performance Expressions

A variety of performance issues arise when designing a speed reduction system as described in the problem statement. For the spur and helical gear configurations, gear strength, surface durability, lubrication, and reliability should be considered in order to rate the design's performance. Similarly, a successful V-belt design should perform satisfactorily with respect to belt life, efficiency, reliability, etc.

A discussion of all performance aspects in this design example will not be provided. Instead, a typical set of performance characteristics has been chosen for each of the three proposed configurations. For the spur gear and helical gear configurations, the choice of performance parameters include: factor of safety for fatigue strength n_f; factor of safety for surface durability n_s; and bearing load rating for the resul-

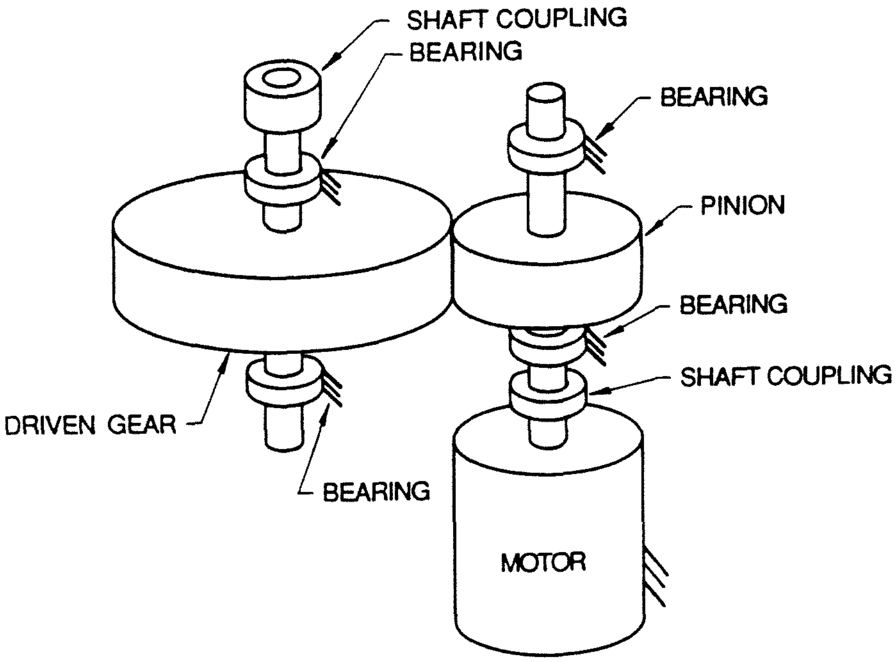

Figure 6.3: Gear Drive Configuration.

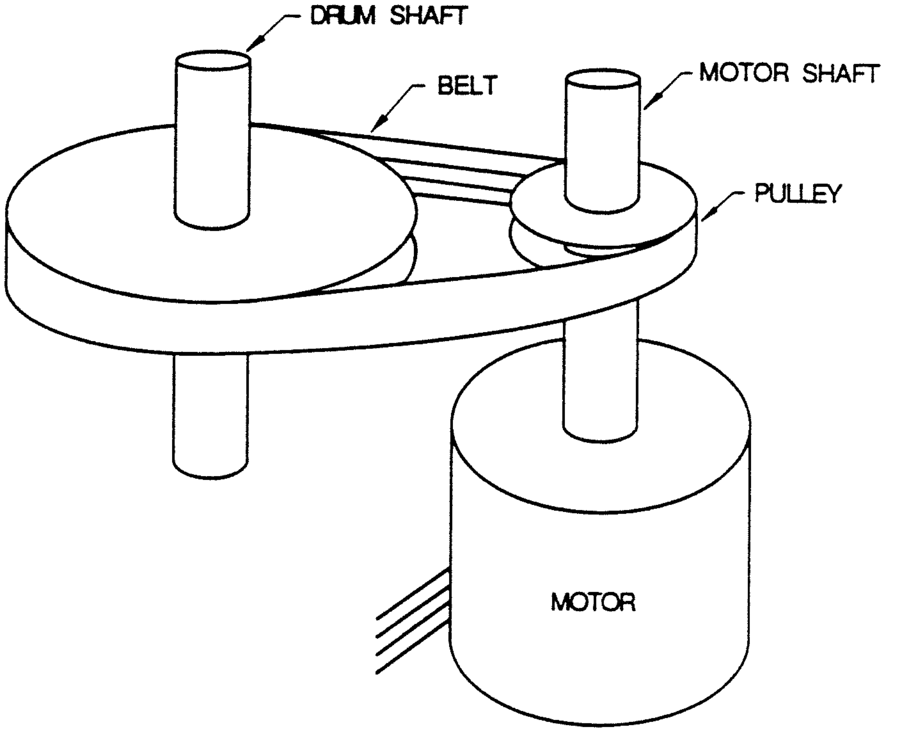

Figure 6.4: V-belt Drive Configuration.

tant gear forces C. In addition, two calculations relating to shaft diameter will be considered: shaft diameter for deflection $d_{s,def}$; shaft diameter for strength (bending) $d_{s,str}$. Using, in part, the mathematical formulations found in [25, 32, 67, 71], the corresponding performance expressions for these PPs may be specified as:[1]

$$n_{f,spur} = \frac{0.5\,\pi\, K_a K_s K_r K_t K_e K_J K_A^s S_t N m^2 n_{rpm} w_F}{60\,(10)^3\, K_o K_l P [K_A^s + (\frac{\pi\, N m n_{rpm} B}{10^3})]}, \tag{6.9}$$

$$n_{f,helical} = \frac{0.5\,\pi\, K_a K_s K_r K_t K_e K_J \sqrt{K_A^h} S_t N m^2 n_{rpm} w_F}{60\,(10)^3\, K_o K_l P \cos^2\psi \sqrt{[K_A^h + (\sqrt{\frac{\pi\, N m n_{rpm} B}{10^3}})]}}, \tag{6.10}$$

$$n_{s,spur} = \frac{(\frac{K_L}{K_T K_R K_{elas}}(2.76\, H_B - 70)(10)^6)^2 (\pi K_A^s w_F N^2 m^2 n_{rpm} \cos\phi \sin\phi m_G)}{2\,(60)\,(10)^3\, P K_o K_l [K_A^s + (\frac{\pi N m n_{rpm} B}{10^3})](m_G + 1)}, \tag{6.11}$$

$$n_{s,helical} = \frac{(\frac{K_L}{K_T K_R K_{elas}}(2.76\, H_B - 70)(10)^6)^2 \cdot (\Lambda_1 \cdot \Lambda_2)}{2\,(60)\,(10)^3\, P \cos^2\psi cos\phi_n K_o K_l \sqrt{[K_A^h + \sqrt{(\frac{\pi N m n_{rpm} B}{10^3})}](m_G + 1)}} \tag{6.12}$$

$where:$

$$\Lambda_1 = ((0.95)\sqrt{K_A^h} w_F N^2 m n_{rpm} \cos\phi_t \sin\phi_t m_G),$$

$$\Lambda_2 = [\sqrt{(\frac{Nm}{2\cos\psi} + m)^2 - (\frac{Nm\cos\phi_t}{2\cos\psi})^2} + \sqrt{(\frac{m_G N m}{2\cos\psi} + m)^2 - (\frac{m_G N m \cos\phi_t}{2\cos\psi})^2} - (\frac{3Nm\sin\phi_t}{\cos\psi})],$$

$$d_{s,str}^s = \left(\frac{32\,(60)\,(10)^3 P L_s K_f K_k K_{sc} K_{df} \sqrt{1 + \tan^2\phi}}{4\,(0.55)\,\pi^2\, N m n_{rpm} S_f}\right)^{\frac{1}{3}}, \tag{6.13}$$

$$d_{s,def}^s = \left(\frac{4\,(60)\,(10)^3 P L_s^3 \sqrt{1 + \tan^2\phi}}{3\,\pi^2\, N m n_{rpm} E y_s}\right)^{\frac{1}{4}}, \tag{6.14}$$

[1]The performance parameters will not be derived here. The equations shown for factor of safety, rated belt load, and belt life may be formulated from the material in the cited references. Likewise, the shaft diameter equations come directly from beam bending theory, and may be reproduced by considering maximal moments from the moment and loading diagrams.

$$d^h_{s,str} = \left(\frac{32\,(60)\,(10)^3 P \cos\psi K_f K_k K_{sc} K_{df} \sqrt{[\frac{\tan\phi_t L_s}{4} + \frac{\tan\psi N m}{4\cos\psi}]^2 + \frac{L_s^2}{16}}}{(0.55)\,\pi^2 N m n_{rpm} S_f} \right)^{\frac{1}{3}}, \quad (6.15)$$

$$d^h_{s,def} = \left(\frac{4\,(60)\,(10)^3 P \cos\psi L_s^3 \sqrt{1 + \tan^2\phi_t}}{3\,\pi^2 N m n_{rpm} E y_s} \right)^{\frac{1}{4}}, \quad (6.16)$$

$$C_{spur} = \left(\frac{60\,(10)^3 K_{sh} K_p K_{os} P \tan\phi}{\pi N m n_{rpm}} \right) \left[(\frac{L_D}{L_R})(\frac{n_{rpm}}{n_R})(\frac{1}{6.84}) \right]^{\frac{1}{3}} \left(\frac{1}{[\ln(\frac{1}{R})]^{\frac{1}{3.51}}} \right), \quad (6.17)$$

$$\begin{aligned} C_{helical} \;=\; & \left(\frac{60\,(10)^3 P}{\pi N m n_{rpm}} \right) \cdot (0.5\, K_{sh} K_p K_{os} \cos\psi \tan\phi_t + 1.4\,\sin\psi) \\ & \left[(\frac{L_D}{L_R})(\frac{n_{rpm}}{n_R})(\frac{1}{6.84}) \right]^{\frac{1}{3}} \left(\frac{1}{[\ln(\frac{1}{R})]^{\frac{1}{3.51}}} \right). \end{aligned} \quad (6.18)$$

From these performance expressions, the design parameters for the spur and helical gear configurations are as follows: module m; speed n_{rpm}; face width w_F; acceptable shaft deflection y_s; Brinell hardness H_B; surface finish factor K_a; design factor K_{df}; miscellaneous effects factor K_e; surface finish and environment factor K_f; service factor K_k; load-distribution K_l; overload factor K_o; elastic coefficient K_{elas}; preloading factor K_p; oscillation factor K_{os}; shock factor K_{sh}; reliability factor K_r; size factor K_s; temperature factor K_t; velocity factor K_A; life factor K_L; geometry factor K_J; reliability factor K_R; temperature factor K_T; shaft length L_s; failure stress S_f; and tensile strength S_t. The speed ratio m_G and the input power P are constants as specified in the problem statement. The pressure and helix angles, ϕ and ψ respectively, as well as the number of teeth N, rated bearing life L_R, rated bearing speed n_R, and modulus of elasticity E are also considered as "constants" for this design. Although these terms could be represented as imprecise parameters, their contribution is viewed as either negligible (*e.g.*, modulus of elasticity) or prescribed by requirements on the design problem (*e.g.*, the number of teeth is set at the minimum value as prescribed in design tables).

The final configuration to be considered is the V-belt system shown in Figure 6.4. The same performance parameters exist for the V-belt alternative as found for the gear systems, with one exception. Instead of safety factors for fatigue strength and surface durability, belt life is used as a measure of performance, where belt life is directly related to the *peak force* encountered per belt-pass. Denoting expected belt life by L_e and peak force by F_p, the governing expressions, that relate the V-belt design parameters to the performance parameters, may be written as:

$$F_p = \frac{60\,(10)^3\,PS_fT_r}{(T_r-1)\pi d_p n_{rpm}} + \frac{K_bC_b}{d_p} + \frac{K_cC_b(\pi d_p n_{rpm} C_c)^2}{10^6}, \tag{6.19}$$

$$L_e = \frac{N_T\left\{[3.57(d_p(m_G+1)-C_d m_G)] + \frac{[d_p(m_G-1)-C_d m_G]^2}{4(d_p(m_G+1)-C_d m_G)}\right\}}{60\,\pi d_p n_{rpm}}, \tag{6.20}$$

$$d^b_{s,str} = \left(\frac{32\,(60)\,(10)^3 K_{sf}PL_sK_fK_kK_{sc}K_{df}\sqrt{\{\sqrt{3}\frac{T_r+1}{T_r-1}\}^2+1}}{8\,(0.55)\,\pi^2\,d_p n_{rpm} S_f}\right)^{\frac{1}{3}}, \tag{6.21}$$

$$d^b_{s,def} = \left(\frac{4\,(60)\,(10)^3 K_{sf}PL_s^3\sqrt{T_r^2+T_r+1}}{3\,\pi^2\,(T_r-1)d_p n_{rpm} E y_s}\right)^{\frac{1}{4}}, \tag{6.22}$$

$$C_{belt} = \left(\frac{60\,(10)^3\,K_{sf}K_{sh}K_pK_{os}P}{\pi d_p n_{rpm}}\right)\left[(\frac{L_D}{L_R})(\frac{n_{rpm}}{n_R})(\frac{1}{6.84})\right]^{\frac{1}{3}}\left(\frac{1}{[\ln(\frac{1}{R})]^{\frac{1}{3.51}}}\right), \tag{6.23}$$

The additional design parameters for the V-belt configuration are: pulley diameter d_p; belt bending force factor K_b; belt centrifugal force factor K_c; service factor K_{sf}; total belt passes N_T; and tension ratio T_r.

6.3.2 Performance Specifications

In this design example it is assumed that preliminary performance criteria (functional requirements) have been specified for: the fatigue strength factor of safety; surface durability safety factor; shaft diameter; belt life; and bearing load rating. For the first two of these, the designer will usually use a "rule-of-thumb" to determine the factor of safety PPs. The result represents a minimum safety factor that must be achieved by the design. In other words, n_f and n_s must be greater than or equal to n_f^r and n_s^r, *i.e.*:

$$n_f \geq n_f^r = 2.0,$$
$$n_s > n_s^r = 1.0.$$

In a similar manner, a minimum limit may be specified for the expected belt life performance parameter. Assuming, under nominal conditions, that a washing machine will be used an average of ten hours per week, the resultant minimum expected life of the belt is:

$$L_e \geq L_e^r = 16.0 \text{ khr}.$$

A single functional requirement or specification is usually not given for the two remaining output parameters (d_s and C) in the preliminary design phase. For this example, it has been chosen to *minimize shaft diameter* to minimize material and manufacturing costs. This specification is denoted by d_s^r. A similar specification, C^r, will be used for the bearing load rating.

These five performance specifications make up the preliminary functional requirement set for this transmission example. When combined with the imprecise output and γ-level results (shown later), this set provides a means of rating each design alternative according to individual performance, and in comparison to other alternatives.

6.3.3 Specifying Input Design Parameters

As discussed earlier, each input parameter represents a triplet of information: imprecision modeled with preference functions; possibilistic uncertainty modeled with possibility functions; and stochastic uncertainty modeled with probability density functions. Triangular and linear (fuzzy) functions have been used to represent the imprecision and possibilistic uncertainty in the input parameters. Probability density functions are used to represent the last component of information. Tables 6.1 and 6.2 list the data for constructing the imprecise and possibilistic components of the input parameters, where the three data values for each DP represent the following: left-extreme value for preference equal to zero, peak value for preference of unity, and right-extreme value with zero preference. Table 6.3 lists the values used for the design "constants." Figures 6.5 through 6.10 show the probability density functions for those parameters that include a stochastic component of uncertainty. The interpretation and use of these data conforms to the explanations in Chapter 2 and Section 6.2.

6.3.4 Output Performance Parameters

Using the performance expressions (Equations 6.9 through 6.23) along with the design parameter data given in the previous section, the extended FWA procedure was applied to obtain the imprecise and possibilistic performance outputs. The cumulative-distribution form of the probability technique was also used to determine the stochastic components of the performance parameters. (An example calculation of the stochastic component for the shaft diameter $d^b_{s,str}$ performance parameter is derived in Appendix E.) Figures 6.11 through 6.20 show the performance outputs for the spur gear, helical gear, and V-belt alternatives.

6.3.4.1 Spur Gear Output Performance Parameters

The spur gear output results contained in Figures 6.11 through 6.15 will be compared to the performance criteria: n^r_f, n^r_s, d^r_s, and C^r. Figure 6.11 shows the extended-hybrid output for the fatigue strength factor of safety (Equation 6.9). The imprecision curve

	Imprecision(i)/Possibilistic Uncertainty(u)					
DPs (units)	$\alpha_i = 0$	$\alpha_i = 1$	$\alpha_i = 0$	$\alpha_u = 0$	$\alpha_u = 1$	$\alpha_u = 0$
K_a	0.63	0.70	0.85	0.69	0.70	0.71
K_s				0.8	1.0	1.0
K_r	0.702	0.814	0.897	0.780	0.814	0.830
S_t (MPa)	350.0	1065.0	1550.0			
m (mm)	1.2	1.4	1.6			
n_{rpm} (rpm)	1500.0	1750.0	2000.0			
w_F (mm)	6.0	9.5	13.0			
K_t				0.95	1.0	1.0
K_e				1.31	1.33	1.35
K_A^s (m/s)				800.0	1200.0	1300.0
K_A^h (m/s)				60.0	78.0	100.0
K_J				0.34	0.35	0.36
K_o				1.0	1.0	1.4
K_l				1.3	1.3	1.5
K_L	1.0	1.0	1.4	1.0	1.0	1.1
K_R	0.8	0.8	1.2	0.8	0.8	0.9
K_{elas} (k$\sqrt{\text{Pa}}$)				187.0	191.0	191.0
H_B	100.0	310.0	460.0			
K_T				0.95	1.0	1.0
K_f	1.0	1.1	1.3	1.05	1.1	1.15

Table 6.1: Machine Design Example: Fuzzy Design Parameter Data.

	Imprecision(i)/Possibilistic Uncertainty(u)					
DPs (units)	$\alpha_i = 0$	$\alpha_i = 1$	$\alpha_i = 0$	$\alpha_u = 0$	$\alpha_u = 1$	$\alpha_u = 0$
L_s (mm)	125.0	150.0	205.0			
S_f (MPa)	175.0	350.0	1050			
K_k				1.0	1.2	1.3
K_{sc}				1.4	1.6	1.8
K_{df}				1.2	1.5	1.7
y_s (mm)	1.0	2.0	3.0			
L_D (khr)	12.0	16.0	16.0			
R	0.95	0.99	0.99			
K_{sh}				1.4	1.5	1.6
K_p				1.0	1.0	1.1
K_{os}				0.95	1.0	1.05
T_r	3.3	3.6	3.8			
d_p (mm)	75.0	85.0	125.0			
K_c	0.1	0.6	2.0	0.4	0.6	0.9
K_b				125.0	175.0	225.0
N_T (Mpasses)	0.5	123.5	1400.0	75.0	123.5	175
K_{sf}				1.0	1.2	1.3

Table 6.2: Machine Design Example: Fuzzy Design Parameter Data (cont.).

"Constant" (units)	Value
N	18
P (kW)	1.5
B	3.2809
C_b	4.4482
C_c	0.00328
ϕ (deg.)	20.0
ψ (deg.)	15.0
ϕ_t (deg.)	20.65
ϕ_n (deg.)	20.0
E (GPa)	207.0
L_R (khr)	3.0
n_r (rpm)	500.0
m_G	5

Table 6.3: Machine Design Example: "Constant" Data.

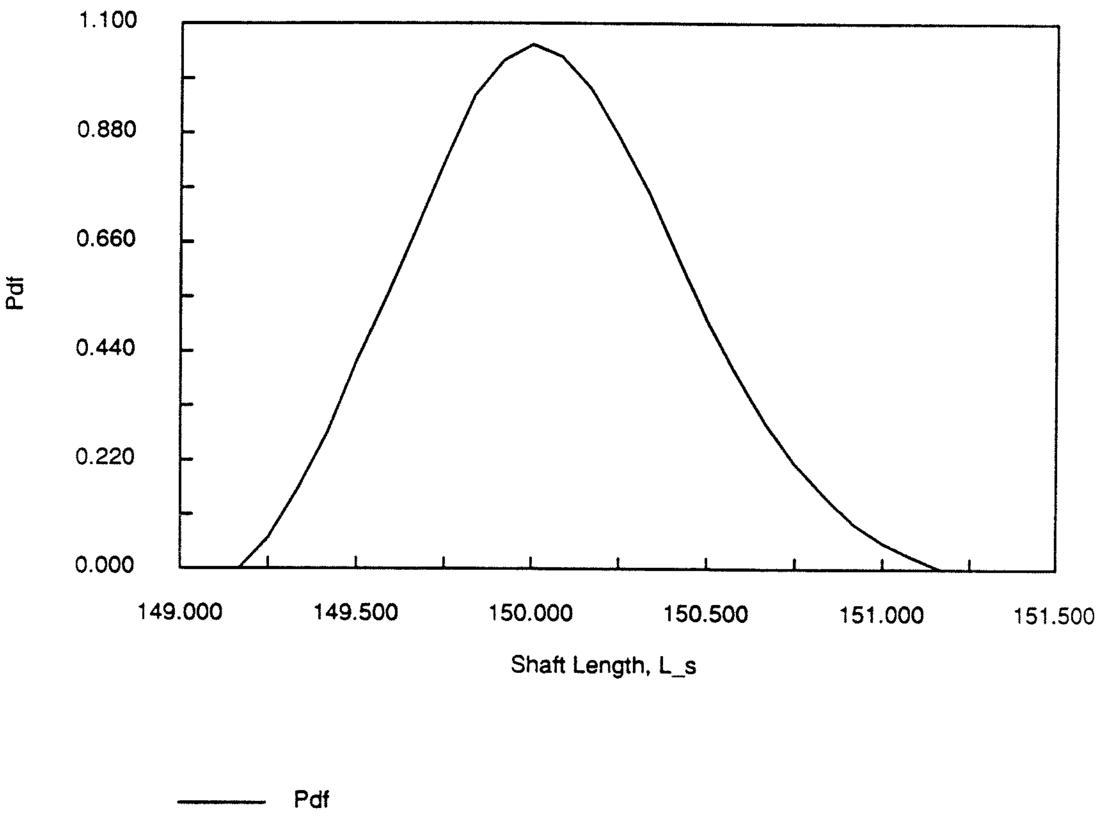

Figure 6.5: Shaft Length (L_s) *pdf.*

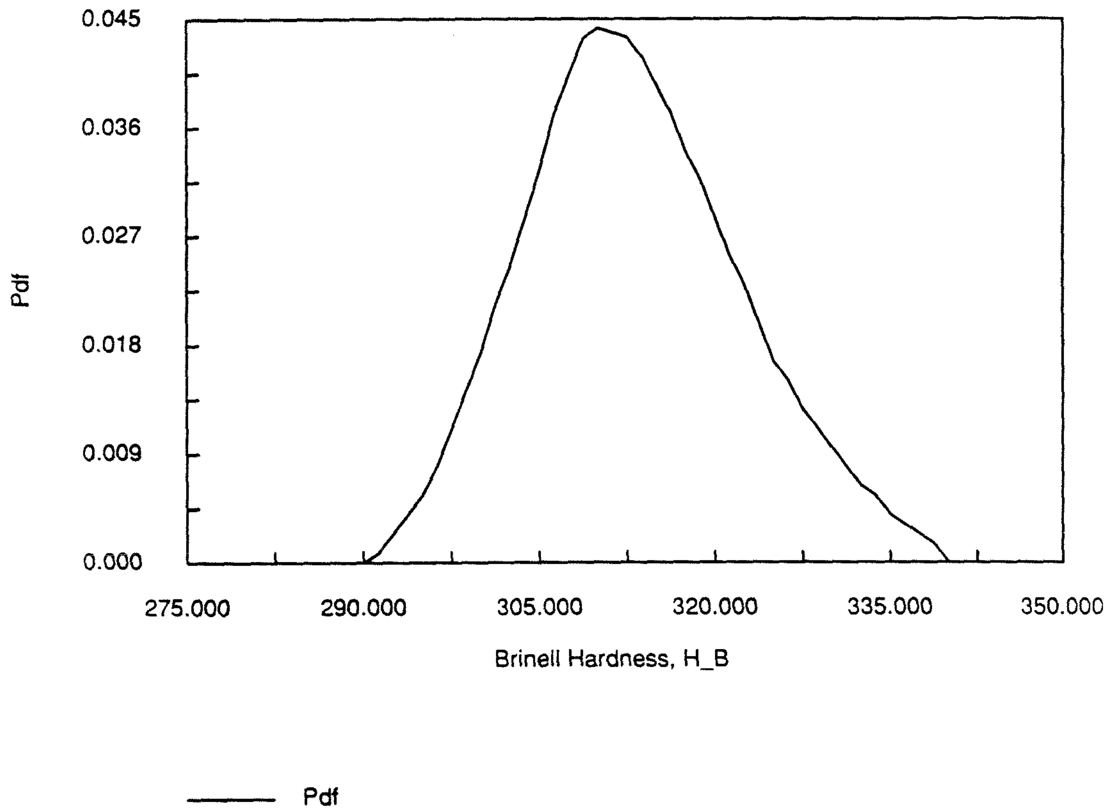

Figure 6.6: Brinell Hardness (H_B) *pdf.*

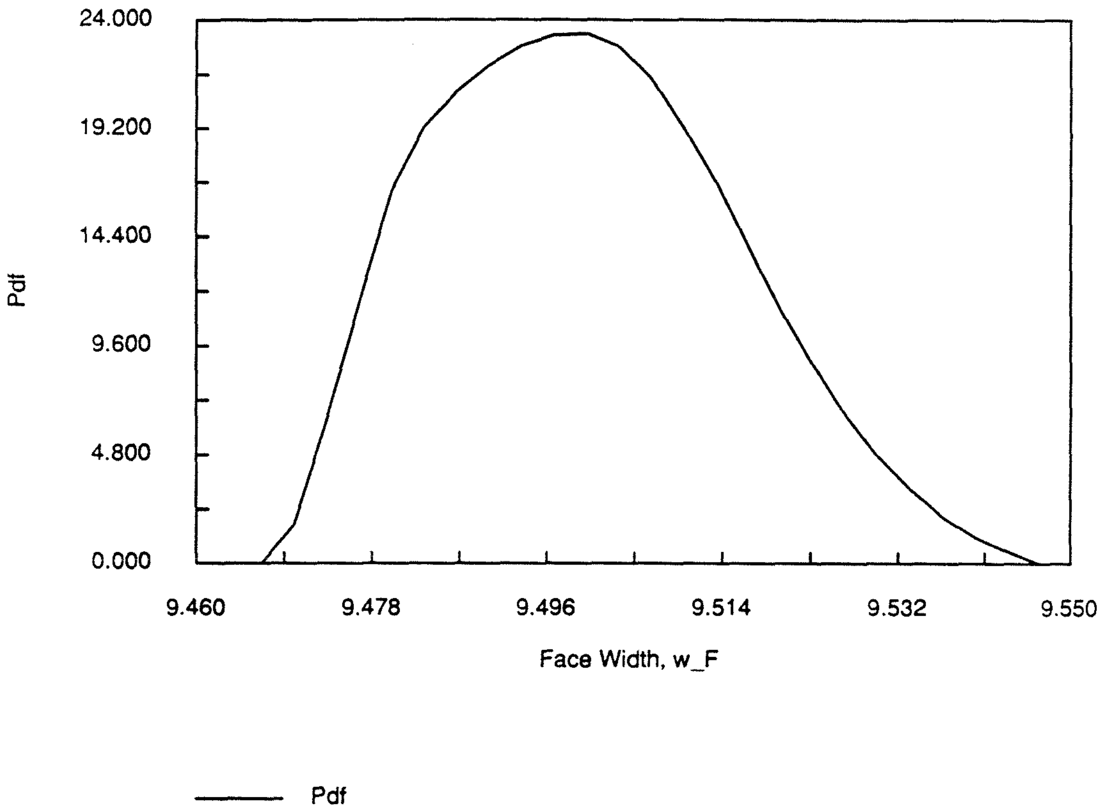

Figure 6.7: Face Width (w_F) *pdf*.

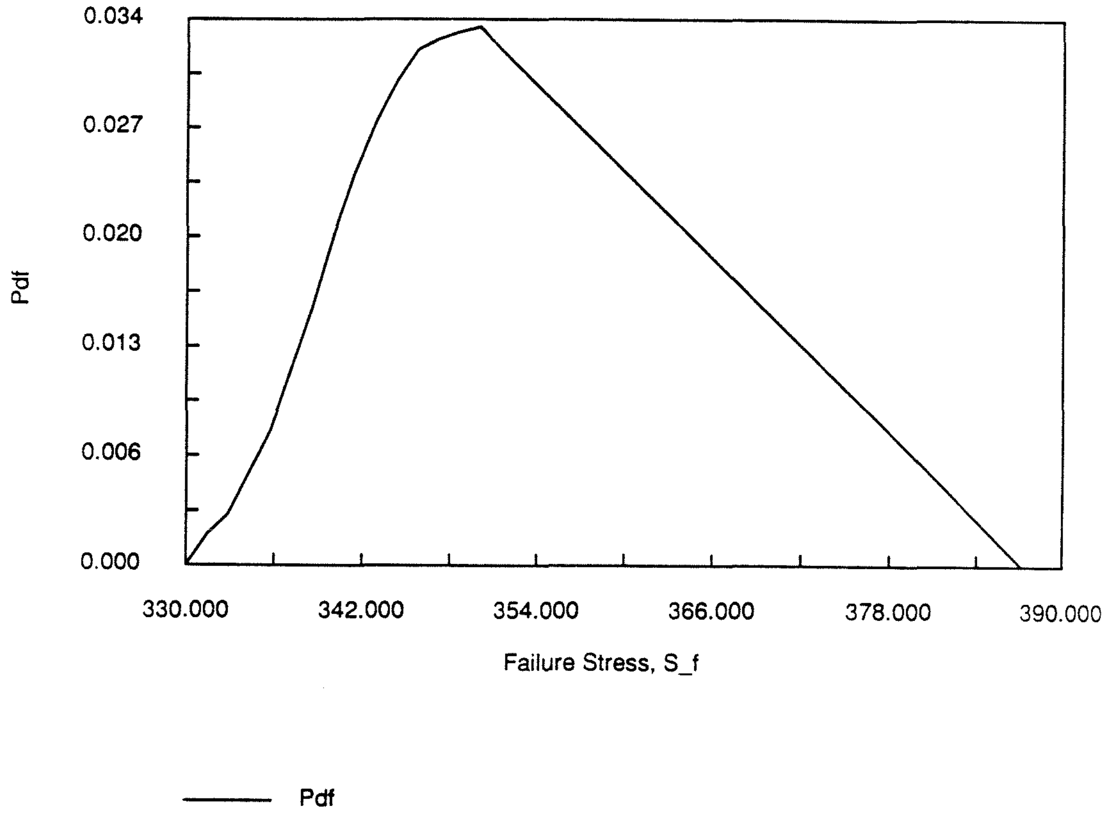

Figure 6.8: Failure Stress (S_f) *pdf*.

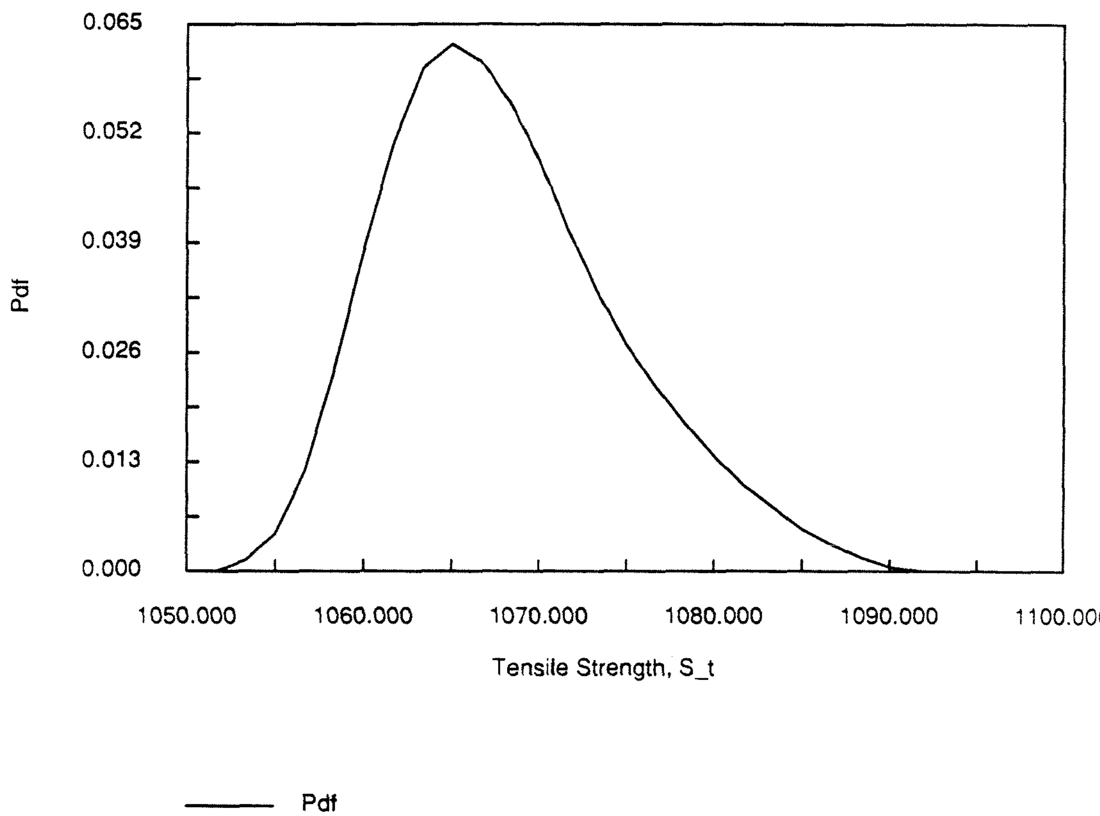

Figure 6.9: Tensile Strength (S_t) *pdf.*

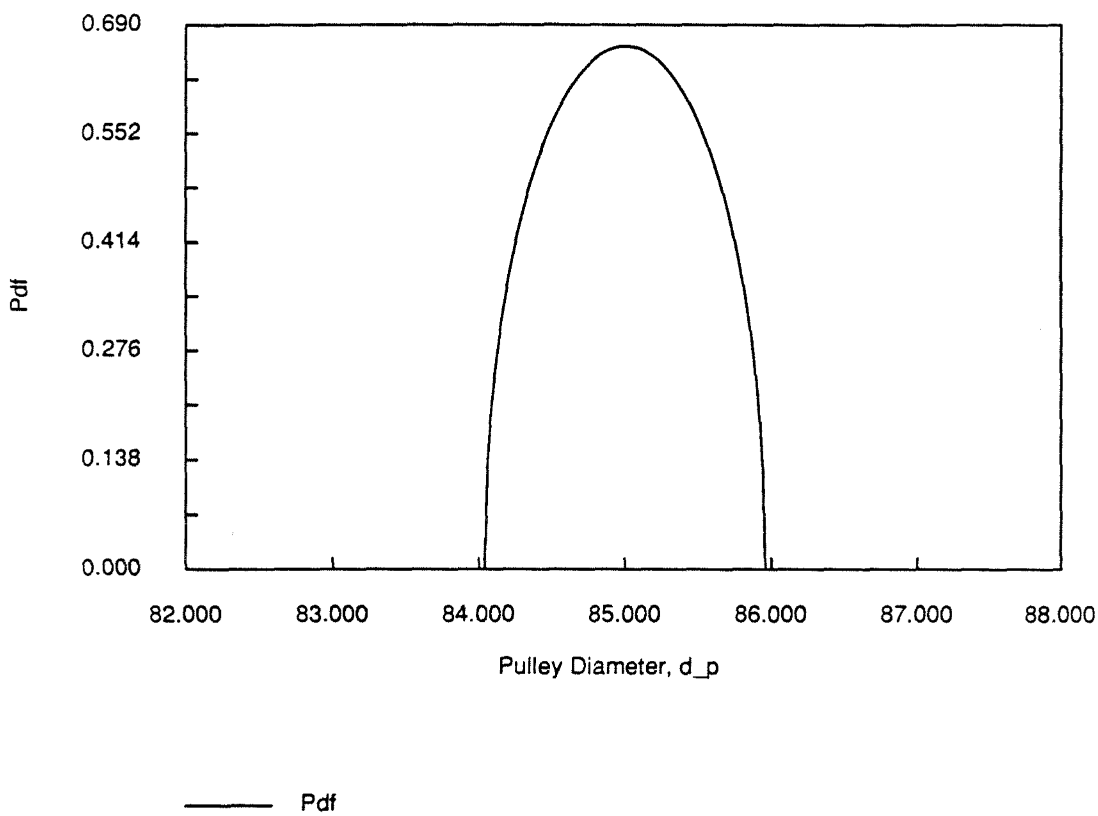

Figure 6.10: Pulley Diameter (d_p) *pdf*.

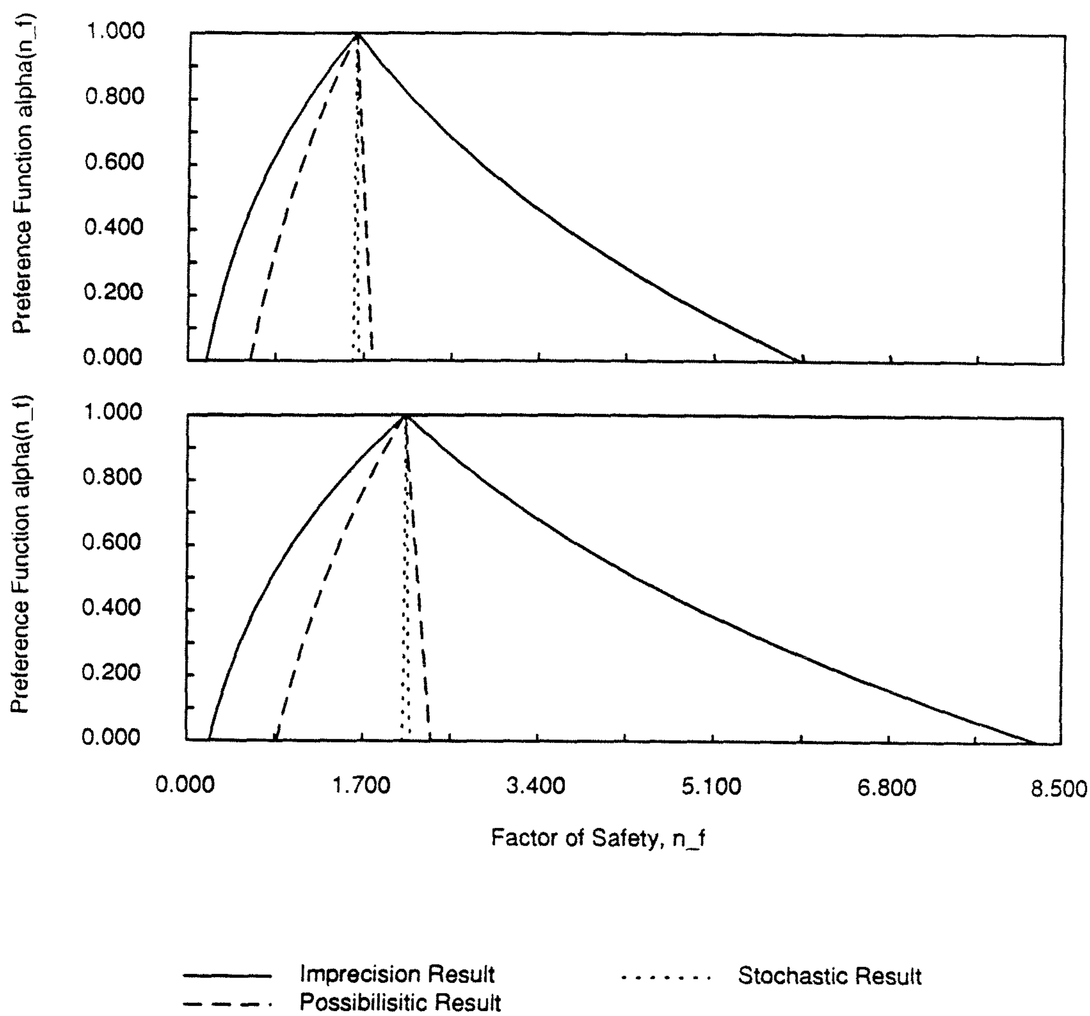

Figure 6.11: Spur and Helical Gear: Fatigue Strength Factor of Safety n_f.

is denoted by $\tilde{n}_f^s$. Checking the output at the peak of $\tilde{n}_f^s$ (preference of unity, $\alpha = 1$), it is found that $\tilde{n}_{f,(at\,\alpha=1)}^s$ equals 1.61. This output safety factor does not satisfy the performance specified for the design, $n_f^r \geq 2.0$. To reach this required value the input parameters must deviate from their peak values. To achieve the minimum desired performance at least one design parameter must decrease in preference of approximately 0.2, *i.e.*, $\tilde{n}_{f,(at\,\alpha=0.8)}^s = 2.15$. Only imprecision has been considered to arrive at this conclusion. If the other uncertainty effects shown in the figure are also taken into consideration, the preference of one or more input parameters must decrease by between 0.3 and 0.4 ($\tilde{n}_{f,(at\,\alpha=0.7)}^s = 2.47$, and $\tilde{n}_{f,(at\,\alpha=0.6)}^s = 2.83$) to satisfy the performance specification including both the functional requirement and uncertainty involved. These results demonstrate that the fatigue strength factor of safety may be satisfied by the spur gear configuration, but only with a large change in preference, and therefore also a large change in the choice of DP values. They also show that care must be taken when adjusting PPs that are coupled, because adjusting one DP may affect more than one PP.

A similar situation occurs in the case of the output surface durability safety factor for the spur gear, as shown in Figure 6.12. Once again, the performance criterion n_s^r is not satisfied at the peak (nominal value), $\tilde{n}_{s,(at\,\alpha=1)}^s = 0.73$. Considering output values to the right of the peak and taking into account the represented uncertainty, satisfactory performance will be *just* achieved at an output of $\tilde{n}_{s,(at\,\alpha=0.7)}^s = 1.63$. Of course, this means that a corresponding decrease in preference of the design parameters is required; however, the decrease is not as drastic when compared to $\tilde{n}_f^s$ due to the nearness of the peak output to the functional requirement, and due to the greater imprecision of the right portion of the curve ($\tilde{n}_s^s$ changes faster for a given change in preference than $\tilde{n}_f^s$). Even though $\tilde{n}_s^s$ is closer to meeting its FR than $\tilde{n}_f^s$, it may still

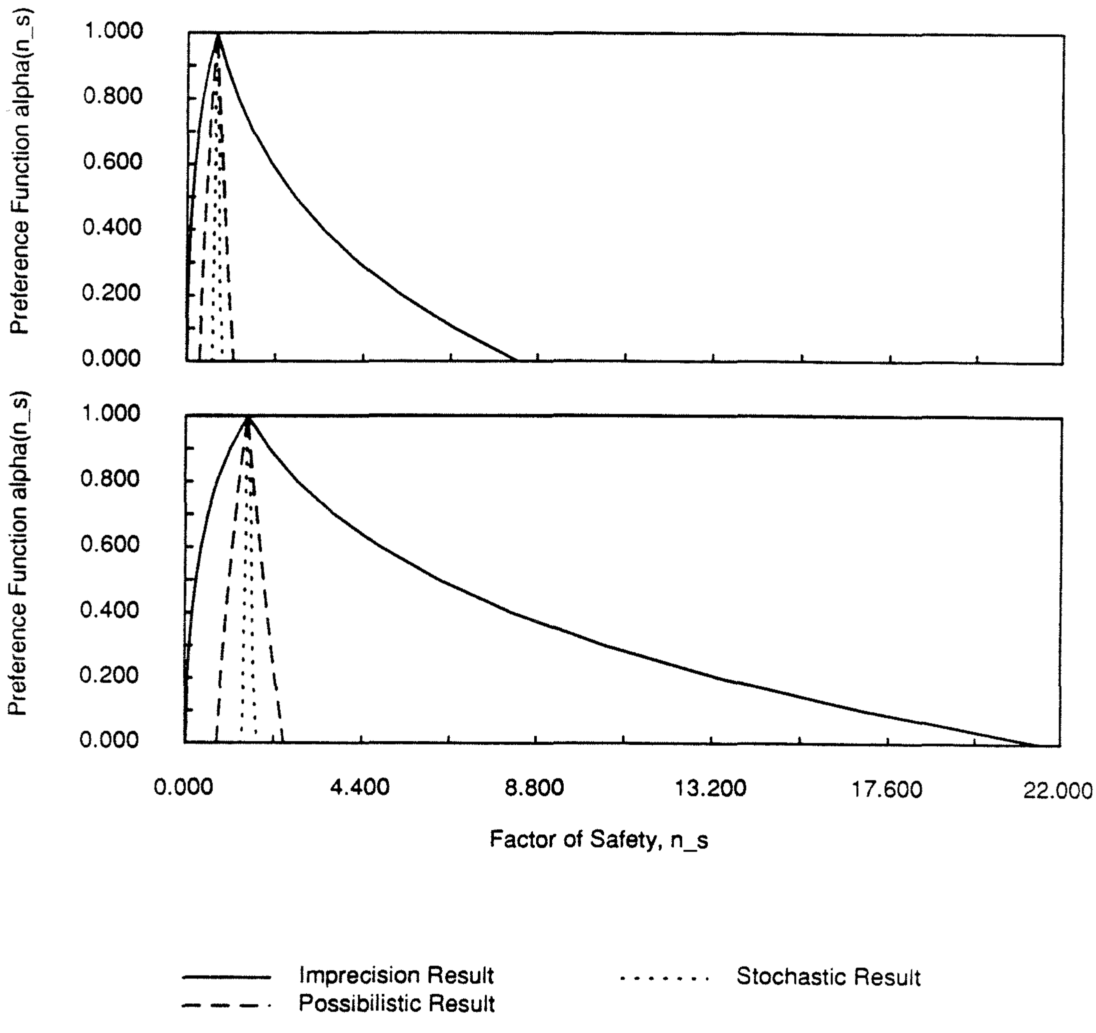

Figure 6.12: Spur and Helical Gear: Surface Durability Factor of Safety n_s.

be influenced by DP changes made to adjust other coupled PPs.

The remaining performance results for the shaft diameter calculations and rated bearing load are shown in Figures 6.13, 6.14 and 6.15. These will be discussed later, when the design alternatives are compared.

6.3.4.2 Helical Gear Output Performance Parameters

The imprecise and uncertainty output results for the helical gear configuration may be found in Figures 6.11 through 6.15. For the case of the fatigue strength factor of safety, Figure 6.11 shows the output sets as determined from Equation 6.10. The peak of the imprecision curve corresponds to: $\tilde{n}^h_{f,(at\,\alpha=1)} = 2.1$. When compared with the requirement $n^r_f \geq 2$, the factor of safety is satisfactory. Considering the additional uncertainty effect, the peak output does not meet n^r_f. To satisfy the functional requirement, an output preference value of 0.9 must be used ($\tilde{n}^h_{f,(at\,\alpha=0.9)} = 2.46$). Combining this result with the fact that the output curve for $\tilde{n}^h_f$ has imprecision on the order of the difference between n^r_f and the output peak, special care must be taken when adjusting other PPs that are coupled to n^h_f. Despite these potential difficulties the required performance is nearly satisfied with the nominal (peak) value for the helical gear fatigue strength safety factor.

Using Figure 6.12, the helical gear configuration performs satisfactorily for the surface durability factor of safety, $\tilde{n}^h_{s,(at\,\alpha=1)} = 1.53$. Even with regard to uncertainty considerations the nominal output meets the functional requirement n^r_s. The only concern involved with $\tilde{n}^h_s$ is the coupling with other performance parameters. Because of the relatively small imprecision on the left-hand side of $\tilde{n}^h_s$, small changes of a design parameter's preference to the left will have only a small influence on performance (this may be verified by the *backward path* FWA implementation as described in Section 2).

Thus, even though there exist combinations of input parameters that do not satisfy the performance specifications, n_s^h is a performance parameter that is closer to its FR than n_f^h or n_s^s, and is not influenced as strongly by other coupled PPs as shown by the small left-hand side imprecision.

6.3.4.3 V-Belt Output Performance Parameters

Figures 6.16 through 6.20 show the output performance curves for the V-belt configuration. Equation 6.19 represents the performance parameter for belt peak force. No performance is specified for peak force in this design. Instead, the performance focus is on the expected life of the belt, as represented in PPE form in Equation 6.20. Because the N_T design parameter implicitly depends on F_p, which in turn depends on other design parameters in the problem, a method is required for determining L_e given the imprecise output for belt peak force. The approach used in this design is to calculate the imprecise and uncertain peak force performance for both pulleys, as shown in Figure 6.16. Using design tables and these results, the input parameter N_T may be constructed (an approximation used for N_T may be found in Table 6.1) and used in Equation 6.20. The result is the output for expected life as given in Figure 6.17.

Analyzing Figure 6.17, the peak output value is: $\tilde{L}_{e,(\,at\,\alpha=1)} = 8.3$ khr. Comparing this result with $L_e^r \geq 16.0$ khr, the nominal design does not satisfy the specified performance, by a factor of two. However, the imprecision of the output is on the order of the difference between L_e^r and $\tilde{L}_{e,(\,at\,\alpha=1)}$, implying that only a small change in input parameter preference is required. In the context of the uncertainty shown in the figure, satisfactory performance is obtained between $\alpha_{\tilde{L}_e} = 0.8$ and $\alpha_{\tilde{L}_e} = 0.9$, *i.e.*, $\tilde{L}_{e,(\,at\,\alpha=0.8)} = 26.0$ khr and $\tilde{L}_{e,(\,at\,\alpha=0.9)} = 17.0$ khr.

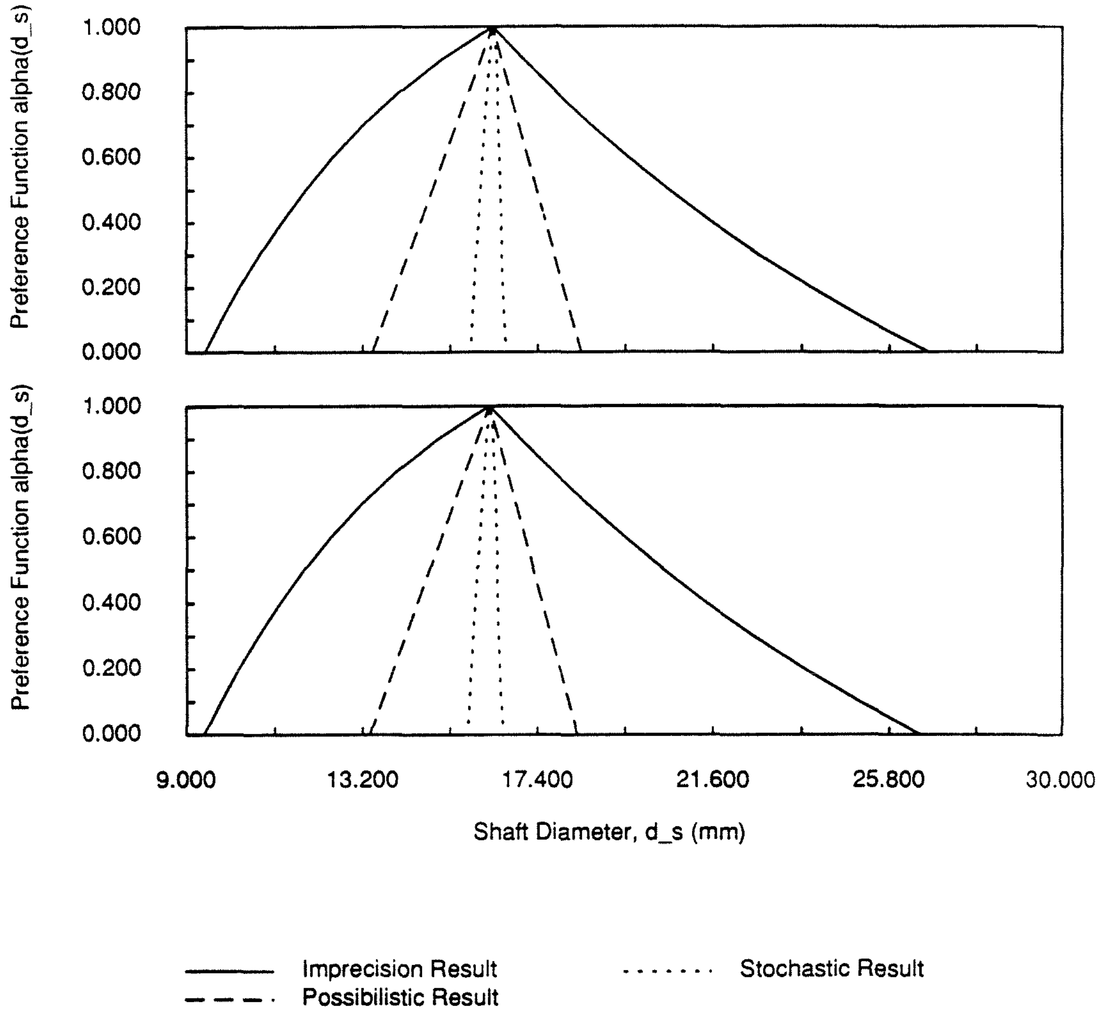

Figure 6.13: Spur and Helical Gear: Shaft Diameter (Strength) d_s.

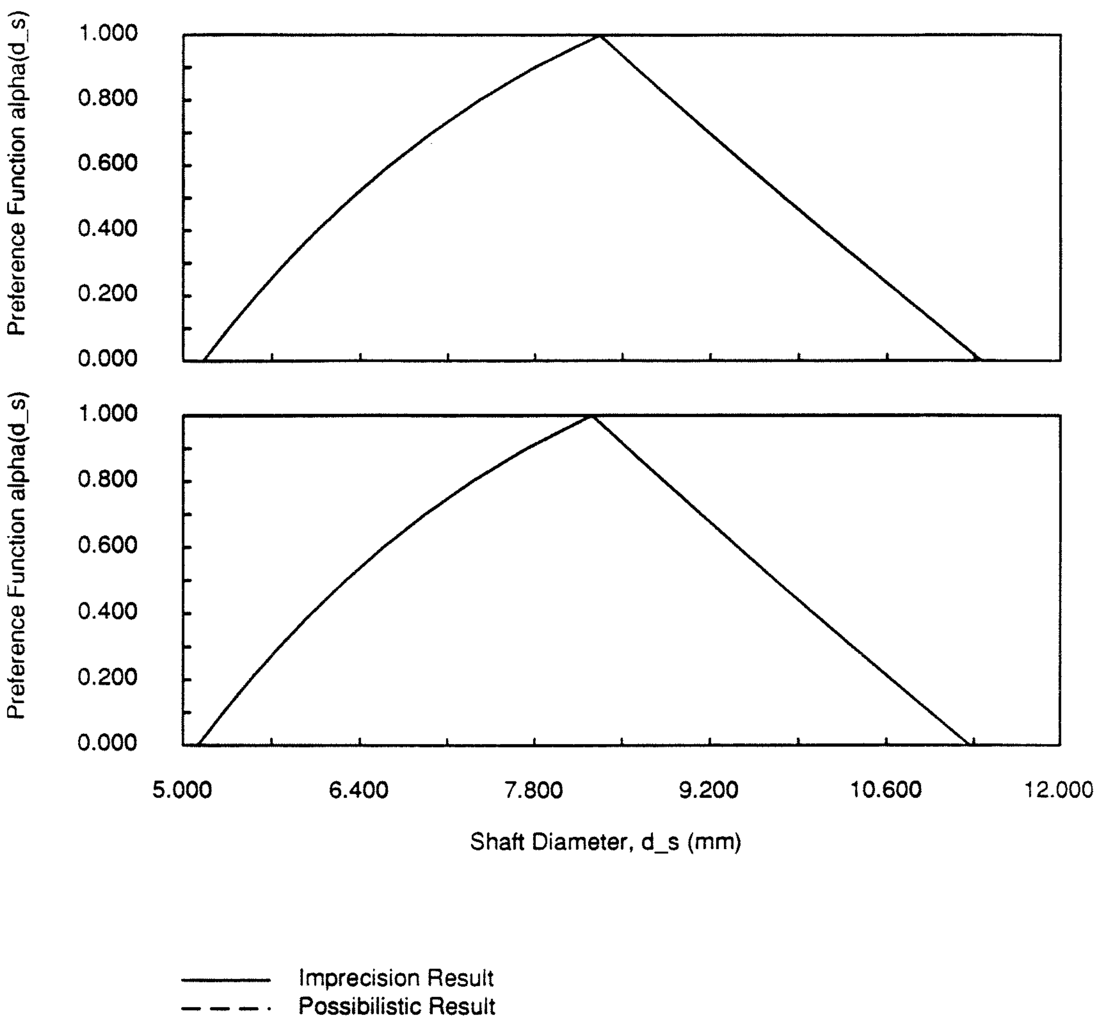

Figure 6.14: Spur and Helical Gear: Shaft Diameter (Deflection) d_s.

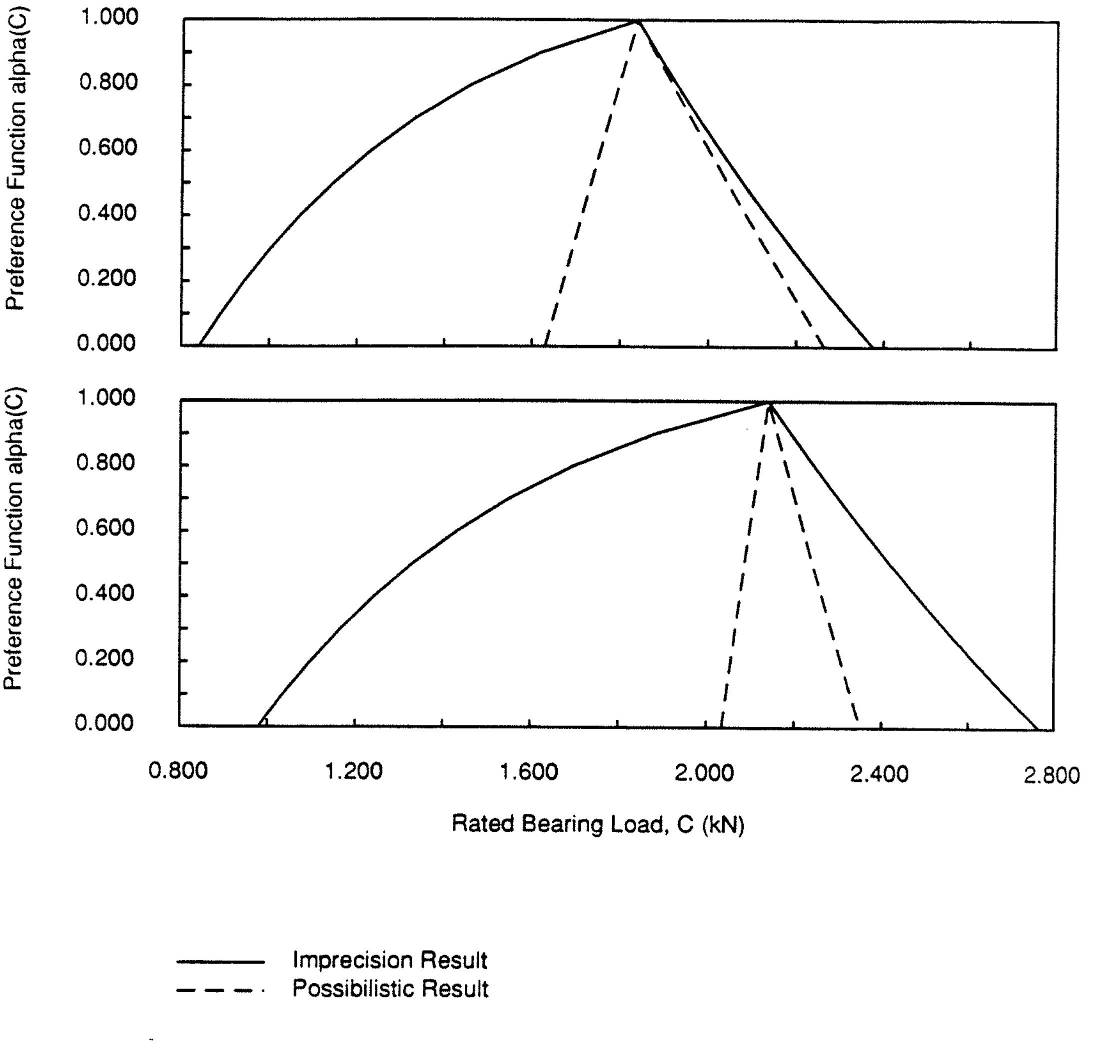

Figure 6.15: Spur and Helical Gear: Rated Bearing Load C.

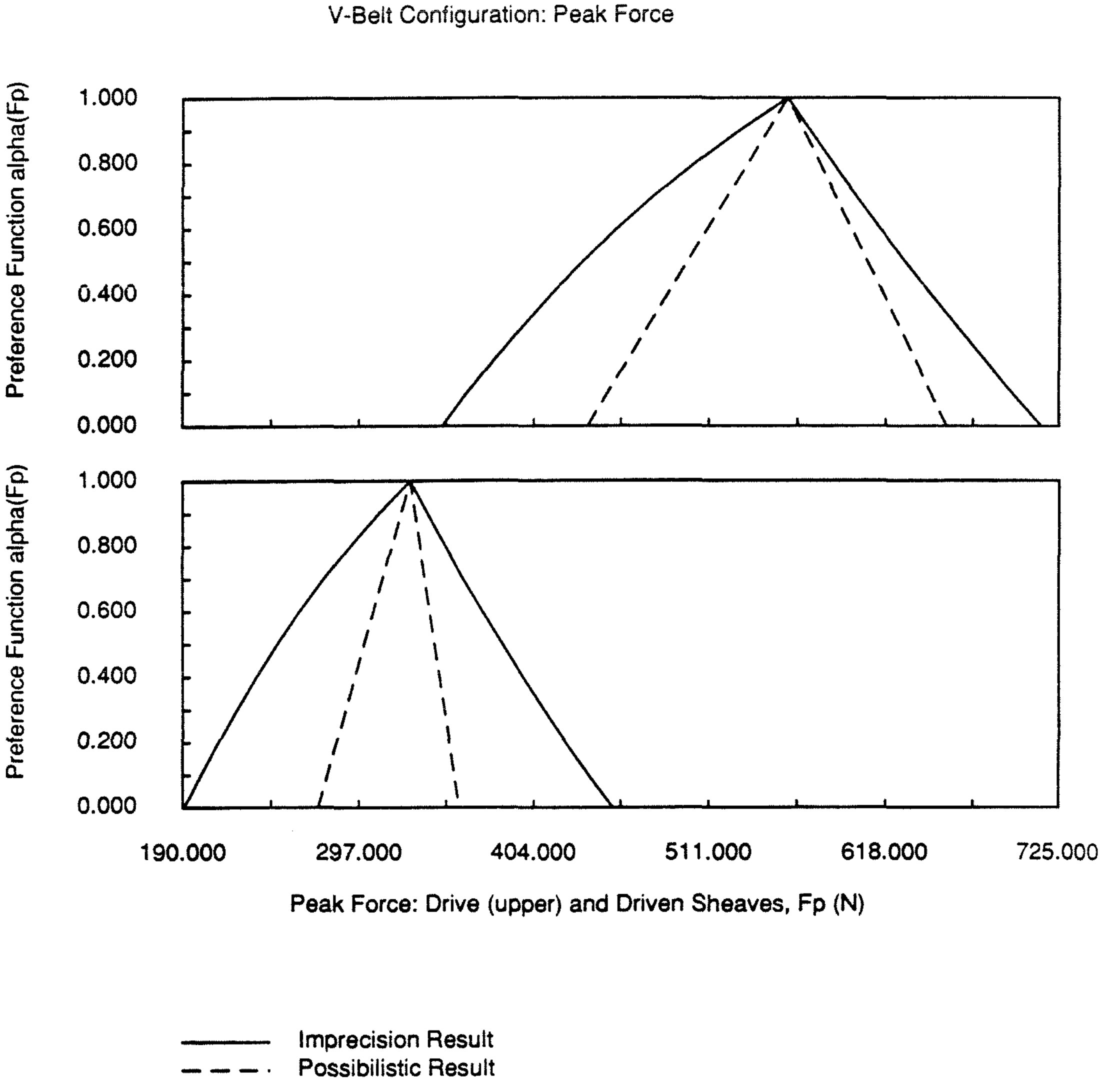

Figure 6.16: V-Belt: Peak Force F_p.

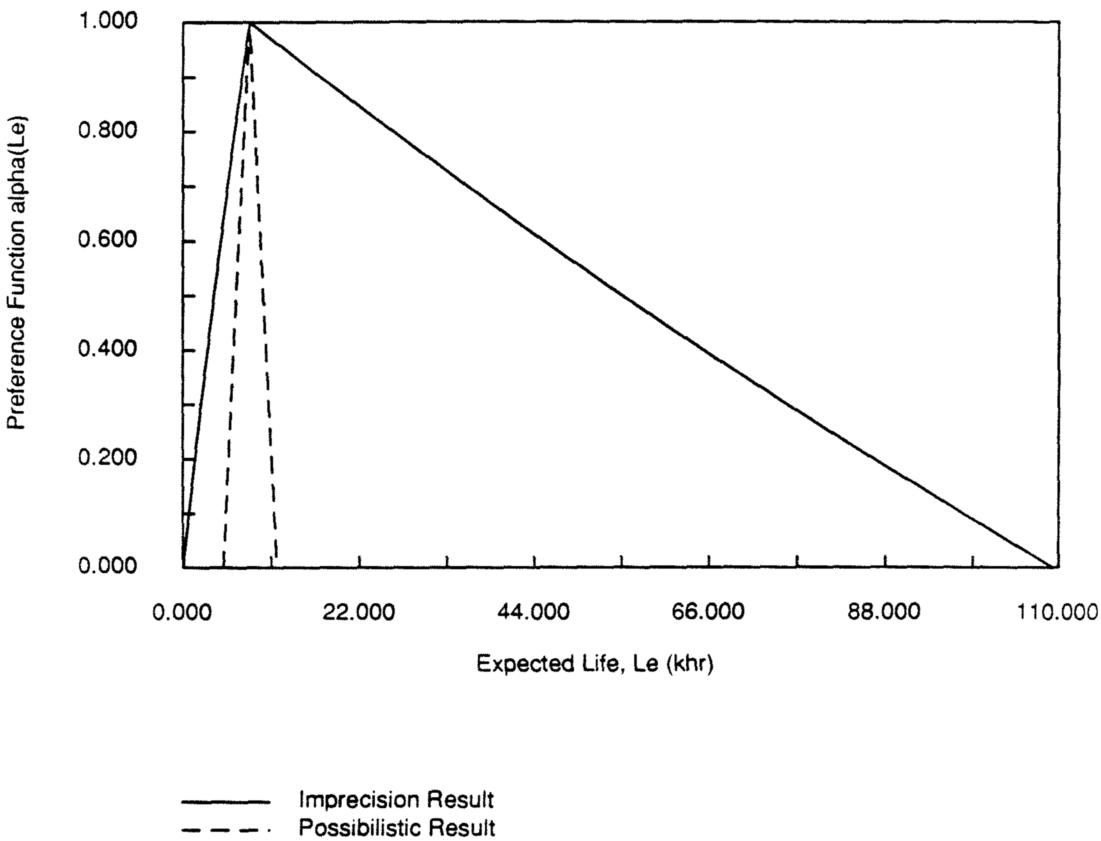

Figure 6.17: V-Belt: Expected Belt Life L_e.

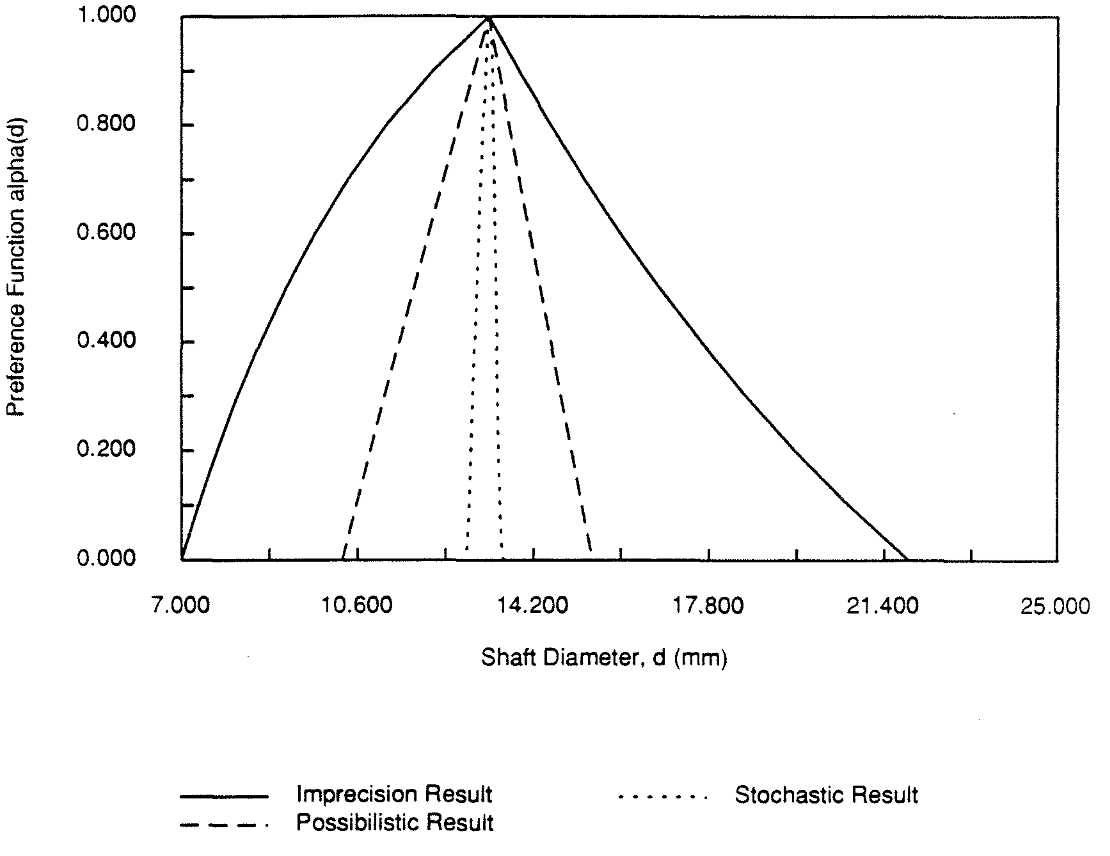

Figure 6.18: V-Belt: Shaft Diameter (Strength) d_s.

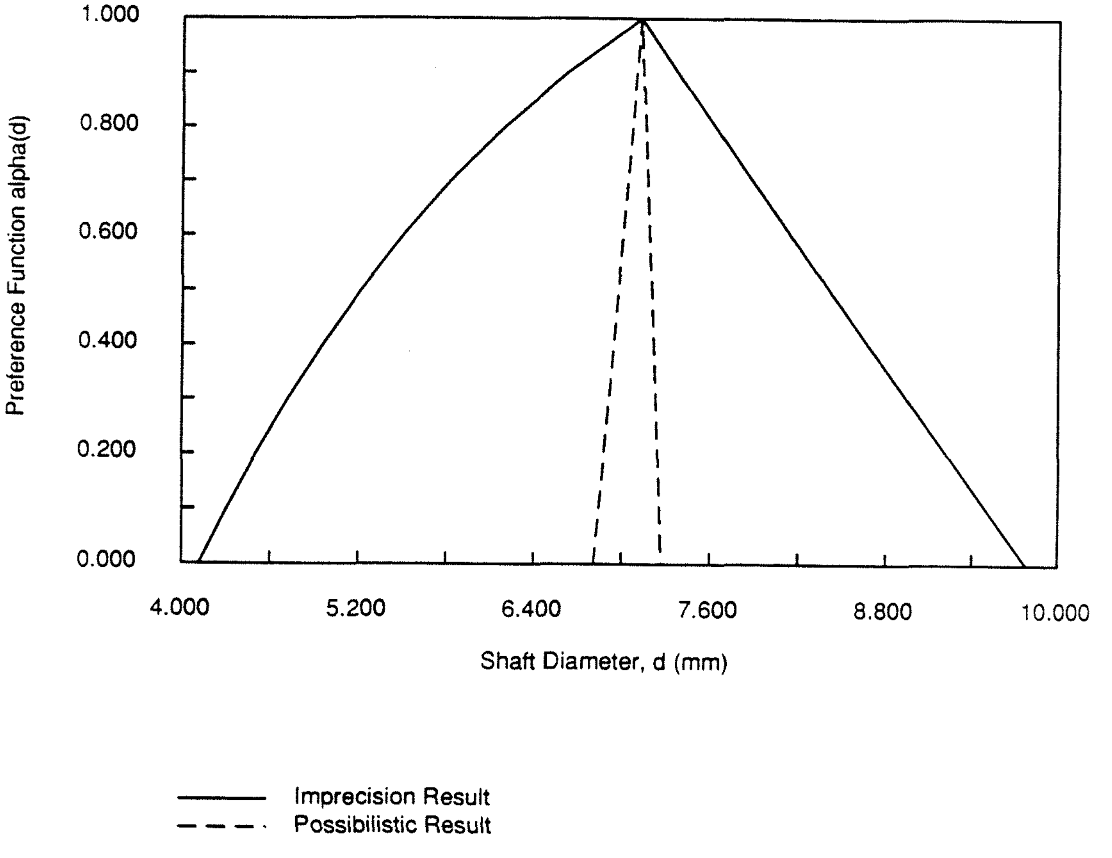

Figure 6.19: V-Belt: Shaft Diameter (Deflection) d_s.

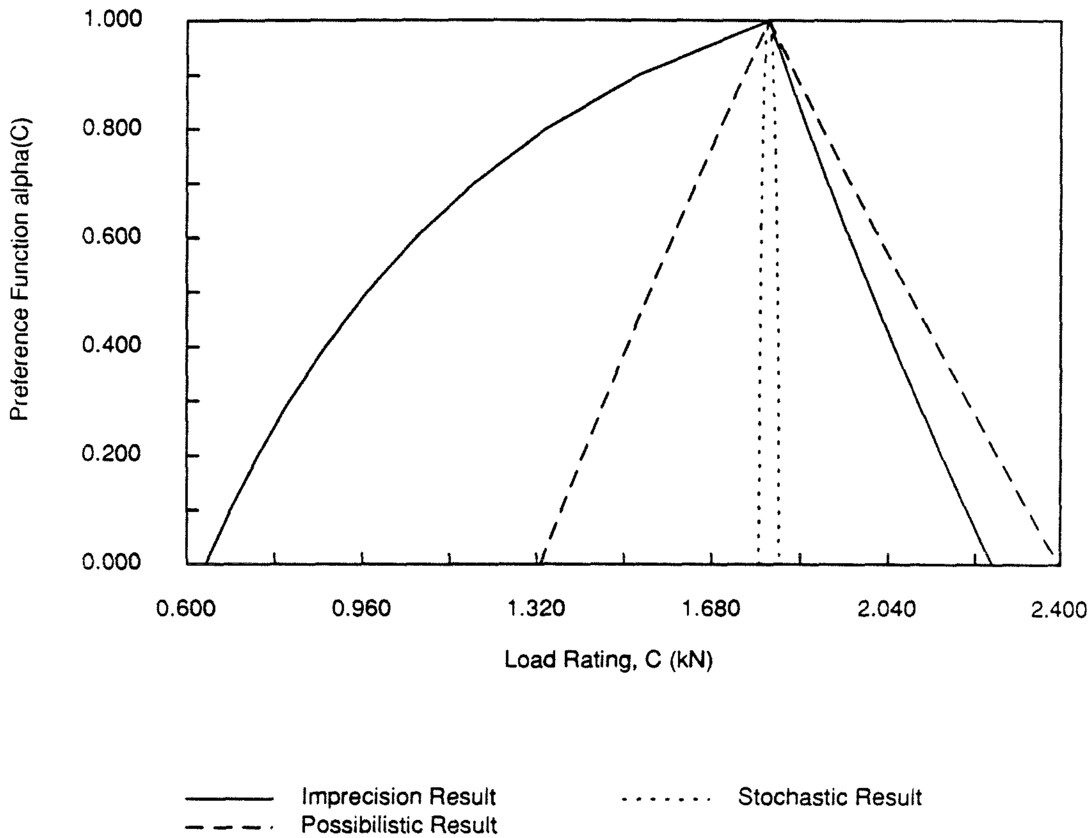

Figure 6.20: V-Belt: Rated Bearing Load C.

6.3.5 Applying the γ-Level Measure

The γ-level measure may be applied to the performance parameter outputs using the procedure described in Chapter 2. The results provide the designer with qualitative information concerning the importance and coupling of the input parameters for this transmission design. Tables 6.4, 6.5, and 6.6 list normalized γ-level measures for the three alternatives under consideration.

Reviewing the purpose and application of the γ-level measure: when a design parameter has the greatest qualitative importance for a given performance parameter, the numerical measure produces a normalized value of one (1). As the measure decreases in value, the corresponding input has little effect in determining the performance parameter, meaning that even a large change in the design parameter (decrease of preference) produces a small change in the output. The output is loosely analogous to sensitivity, but applies to imprecise parameters, and represents the entire range of the parameters, not a single operating point. Moreover, this sensitivity is weighted by the designer's desires, as identified in the input parameters' preference functions. The details of this weighting are discussed in Chapters 2 and 4.

The γ-level measure may be used to determine importance of inputs simply by comparing the normalized measure of one input relative to the others. This information suggests that parameters with small γ-level measures may be fixed, as changes in those parameters will have only a small effect on performance. Coupling information is also obtained. If a design parameter has a high measure with respect to one PP but a very small measure with respect to another, the performance parameters in question may be viewed as uncoupled with respect to the design parameter. Further, in terms of possibilistic uncertainty, the γ-level measure can also provide an indication of which parameters contribute the greatest to the uncertainty of the problem.

	Performance Parameters				
DPs	n_f	n_s	d_s (str.)	d_s (def.)	C
m	0.44	0.21	0.16	0.18	0.55
n_{rpm}	0.18	0.09	0.16	0.18	0.37
w_F	0.65	0.31	0.0	0.0	0.0
S_t	1.0	0.0	0.0	0.0	0.0
S_f	0.0	0.0	1.0	0.0	0.0
K_a	0.28	0.0	0.0	0.0	0.0
K_r	0.21	0.0	0.0	0.0	0.0
K_{elas}	0.0	0.13	0.0	0.0	0.0
K_L	0.0	0.38	0.0	0.0	0.0
K_R	0.0	0.29	0.0	0.0	0.0
H_B	0.0	1.0	0.0	0.0	0.0
K_f	0.0	0.0	0.15	0.0	0.0
L_s	0.0	0.0	0.28	1.0	0.0
y_s	0.0	0.0	0.0	0.77	0.0
R	0.0	0.0	0.0	0.0	1.0
L_D	0.0	0.0	0.0	0.0	0.17

Table 6.4: γ-*level* measure results for Spur Gear (Imprecision).

	Performance Parameters				
DPs	n_f	n_s	d_s (str.)	d_s (def.)	C
m	0.49	0.23	0.15	0.18	0.55
n_{rpm}	0.24	0.11	0.16	0.18	0.37
w_F	0.65	0.31	0.0	0.0	0.0
S_t	1.0	0.0	0.0	0.0	0.0
S_f	0.0	0.0	1.0	0.0	0.0
K_a	0.28	0.0	0.0	0.0	0.0
K_r	0.21	0.0	0.0	0.0	0.0
K_{elas}	0.0	0.13	0.0	0.0	0.0
K_L	0.0	0.49	0.0	0.0	0.0
K_R	0.0	0.29	0.0	0.0	0.0
H_B	0.0	1.0	0.0	0.0	0.0
K_f	0.0	0.0	0.15	0.0	0.0
L_s	0.0	0.0	0.28	1.0	0.0
y_s	0.0	0.0	0.0	0.77	0.0
R	0.0	0.0	0.0	0.0	1.0
L_D	0.0	0.0	0.0	0.0	0.17

Table 6.5: γ-*level* measure results for Helical Gear (Imprecision).

	Performance Parameters				
DPs	F_p (sm. pull.)	L_e	d_s (str.)	d_s (def.)	C
d_p	1.0	0.001	0.29	0.33	0.95
n_{rpm}	0.33	0.03	0.16	0.18	0.37
T_r	0.07	0.0	0.04	0.05	0.0
S_f	0.0	0.0	1.0	0.0	0.0
K_c	0.07	0.0	0.0	0.0	0.0
K_f	0.0	0.0	0.15	0.0	0.0
L_s	0.0	0.0	0.28	1.0	0.0
y_s	0.0	0.0	0.0	0.78	0.0
N	0.0	1.0	0.0	0.0	0.0
R	0.0	0.0	0.0	0.0	1.0
L_D	0.0	0.0	0.0	0.0	0.17

Table 6.6: γ-*level* measure results for V-Belt (Imprecision).

6.3.5.1 Spur Gear γ-Level Measure

Table 6.4 categorizes the γ-level results for the spur gear configuration. Two important input parameters for a spur gear reduction unit are the speed n_{rpm} and the module m. Analyzing the γ-level measures shows that n_{rpm} contributes very little to any of the performance parameters. This implies that the speed of the motor can be fixed with respect to the spur gear design and not affect performance. While the module design parameter also contributes very little to the shaft diameter results, the γ-level measure shows that m may not be fixed relative to the factors of safety, n_f and n_s, and the bearing load rating, C. As a result, it can be concluded that n_f, n_s, and C are coupled with respect to m, but uncoupled with respect to d_s.

6.3.5.2 Helical Gear γ-Level Measure

The γ-level measures for the helical gear alternative are shown in Table 6.5. Similar results occur for the speed n_{rpm} and the module m as found for the spur gear. Additional information that can be inferred from the γ-level results concerns the shaft diameter performance specification, d_s^r. The relevant design parameters for shaft diameter (strength calculation, Equation 6.15) include: S_f, L_s, n_{rpm}, m, and f. From Table 6.5, the material property, S_f, obviously dominates the output performance, with secondary effects from L_s. This tells us that when attempting to satisfy d_s^r, minimization efforts should be first (and foremost) placed on choosing a material property just to the right of the $d_{s,str}$ peak value (Figure 6.13), with subsequent effort placed in changing L_s.

6.3.5.3 V-Belt γ-Level Measure

Table 6.6 lists the γ-level measures for the V-belt configuration. A preliminary analysis of the table shows that the pulley diameter and speed contribute very little to belt life performance L_e, with N_T dominating the imprecision. However, because N_T implicitly depends on F_p, the peak force design parameters will bias the imprecision of expected belt life to the greatest extent. Thus, considering the γ-level results for F_p, it is found that the most important parameters are pulley diameter (a γ-level measure of 1.0) and speed (0.334).

Another interesting result with regard to the peak force is the contribution of the centrifugal force factor, K_c. When the equation for belt peak force was derived, all force contributions were considered, including tension force, bending force, and centrifugal force. Of course, for the given belt tensions and bending, the centrifugal component will contribute very little. The γ-level measure verifies this result.

Finally, as with the helical gear, the failure stress design parameter plays the most significant role in terms of imprecision of the shaft diameter output. In order to meet the performance criteria on shaft diameter, efforts should once again be focused on S_f, with secondary considerations of shaft length L_s and pulley diameter d_p, as these parameters have the largest γ-level measures.

6.3.6 Measuring the Stochastic Contribution

Equation 6.8 may be applied to those performance parameters that included at least one design parameter with a stochastic component of uncertainty. A numerical differentiation scheme was used to calculate the $r_{j,i}$. Table 6.7 shows the results. From the table, the only parameters that contribute greatly to the stochastic PP output are S_t, H_B, and S_f. The stochastic uncertainty (due to manufacturing tolerances) of the geometric dimensions impact the results very little. Thus, subsequent stochastic calculations with a refined model of the design alternatives need only include S_t, H_B, and S_f, resulting in a decrease in the computation and stochastic parameter space.

6.3.7 Discussion

This example has demonstrated the evaluation of the imprecise and uncertain output performance with respect to individual functional requirements. γ-level measure results have also provided information concerning the relative importance of certain design parameters of the problem. Major differences of the alternative configurations may now be determined. When comparing the spur gear and helical gear alternatives, both configurations do not satisfy the nominal fatigue strength performance n_f^r, especially when uncertainty is included. Even though both the spur and helical gears only require a small deviation in input preference from the most desired, the helical configuration slightly out-performs the spur gear configuration. Figure 6.11 illustrates this higher performance due to the closer proximity of the peak of n_f^h to n_f^r, and due to the slightly higher imprecision of $\tilde{n}_f^h$ to the right of the peak.

More drastic differences occur when comparing n_s^h and n_s^s. The helical system satisfies the performance specification without change of input parameters. A significant change, by comparison, is required for the spur gear alternative. The order of the imprecision is also significantly different. Figure 6.12 shows that a given change in preference to the right of the peak will produce a change in output of more than a factor of two for n_s^h compared to n_s^s. Thus, the helical gear alternative is less sensitive to variance in design parameters, especially in terms of coupling with any other performance parameters.

The performance criteria for shaft diameter and rated bearing load may be compared directly from Figures 6.13, 6.14, and 6.15. Both shaft diameter calculations (Equations 6.13 through 6.16) were carried out to determine the minimum requirements for satisfying both strength and deflection considerations. Because the strength calculation produces higher shaft diameters, Figure 6.13 will be used as a basis for comparison. Considering Figure 6.13, the output performance curves are essentially identical for both the spur and helical configurations. This implies that the requirement d_s^r will not be satisfied to any greater extent by choosing one gear system over the other. The γ-level measure suggests that efforts should concentrate on varying the material property S_f and subsequently L_s in order to decrease diameter.

Figure 6.15 shows the performance outputs for the rated bearing load. In this case, the spur gear outperforms the helical system with respect to the nominal output of the imprecision gear. Notice however that greater uncertainty exists for the spur gear output. Considering the greatest uncertainty to the right of the peak for both curves, the spur gear rated bearing load is less than the helical system, but not significantly less.

At this point, both gear configurations may be compared directly with the V-belt drive. However, the direct comparison of the gear systems showed that the *only* advantage of the spur gear in terms of the functional requirements is the smaller rated bearing load. Because the difference in bearing load was not significant, and because the helical system outperformed the spur gear alternative in terms of fatigue strength and surface durability, only the helical gear configuration will be compared with the V-belt.

Using Figure 6.17 and the results discussed earlier, the nominal V-belt alternative does not satisfy the expected life requirement nominally, but requires a change in design parameter preference. Although the expected life of the helical gear is not determined by a performance expression, the fatigue strength and surface durability implicitly depend upon life through the factors involved in the equations. Comparing L_e to n_f^h and n_s^h in this sense, it is found that the helical gear requires similar changes of design parameters to achieve the specified performance.

Similar results occur for the shaft diameter and rated bearing load. Even though the V-belt alternative has lower nominal values at the peak of the imprecision curve for both cases, the addition of uncertainty considerations shows little difference between the V-belt and helical system. Thus, very little advantage exists for choosing one configuration over the other to minimize either shaft diameter or rated bearing load. Because of these results, a cost function might be devised for the next design stage, in which material cost or volume might be used as a measure.

	Design Parameters					
PP	S_t	w_F	H_B	L_s	S_f	d_p
$n_{f,spur}$	0.93	0.06	0.0	0.0	0.0	0.0
$n_{f,helical}$	0.92	0.06	0.0	0.0	0.0	0.0
$n_{s,spur}$	0.0	0.0006	0.98	0.0	0.0	0.0
$n_{s,helical}$	0.0	0.0006	0.98	0.0	0.0	0.0
$d^s_{s,str}$	0.0	0.0	0.0	0.005	0.99	0.0
$d^h_{s,str}$	0.0	0.0	0.0	0.005	0.995	0.0
$d^b_{s,str}$	0.0	0.0	0.0	0.005	0.97	0.027
C_{belt}	0.0	0.0	0.0	0.0	0.0	1.0

Table 6.7: Relative Variance Contributions.

6.4 Conclusions

This power transmission design example demonstrates an application of the semi-automated approach to representing and manipulating both imprecision and other uncertainties in preliminary design. The designer is able to represent preliminary descriptions of design alternatives, even when they are very imprecisely described. In the design example shown above, none of the three alternatives was precisely described, yet conclusions could be drawn regarding the performance of each in this application, subject to the requirements specified here. For example, it was found that the helical gear drive is less sensitive to changes in design parameters, and appears to meet the functional requirements more easily than the spur gear configuration. Nearly the same performance was obtained for the helical gear and V-belt alternatives, when compared to the functional requirements used here.

Further, the introduction of other uncertain data (to complement the imprecise data in the design process) contributes additional information on the performance of each design alternative. The other uncertain data are comprised of two distinct components: a stochastic (objectively/subjectively measured) part, and a possibilistic (subjective) portion. These three data are combined by use of Extended Hybrid Numbers, which provide a consistent representation for input design parameters, and evaluation of results. In this example, the possibilistic data played an important role in indicating how the imprecise performance results might change over the possible range of input parameters. The stochastic component indicated how sensitive the design alternatives were to manufacturing and material processing uncertainties.

The γ-level measure also helps in the process of determining the relative importance of design parameters. Input parameters are seen to have a large effect on performance parameters in some cases, and a small effect in others, indicated by large or small (respectively) γ-level measures. This is an indication of the coupling between the DPs and the PPs. In some cases, parameters with small γ-level measures can be fixed at a value of the designer's choosing, and removed from further design consideration, thus reducing the number of parameters that need to be considered. In the design example used here the γ-level measure showed that the motor speed, in the case of the gears, affects the performance of the transmission very little.

Imprecision and other uncertainty data play an important role in engineering design. The approach described and demonstrated here comprises a method for representing and manipulating all uncertainty aspects simultaneously. This method provides more information to the designer at the preliminary stage than is available using conventional design tools, especially with respect to the subjectivity of the designer. In the next chapter, the approach will be extended by further developing the method of determining the coupling of the design parameters, and by introducing a preliminary metric by which designs with conflicting performance information can be evaluated.

Chapter 7

A Design Figure of Merit

7.1 Introduction

Chapters 2 through 6 discuss an approach for representing, interpreting, and manipulating uncertainties in preliminary engineering design. The focus of the approach is to improve a designer's decision-making capability by calculating with a two-dimensional parameter space instead of the physical parameter space, *i.e.*, the design parameters are a function of the physical range of parameter values as well as preference ($DP(d, \alpha)$). The method relies on the designer to generate the design alternatives ($\mathcal{A}_k$) for a given design problem, as well as the preference functions for the parameterizations of the $\mathcal{A}_k$. Once performance parameter information has been computed, the designer is then called on to analyze the results in the context of the functional requirements.

The example problems given in Chapters 2, 4, and 6 demonstrate the effectiveness of this semi-automated method for making decisions between alternative designs. However, as the dimensions of the design parameter and performance parameter spaces grow, two problems become apparent: (1) choosing between multiple, and possibly conflicting, functional requirements can be difficult; and (2) the influence of the coupling between performance parameters is not easily deciphered. This chapter introduces a preliminary *design metric* (or *figure of merit*) for dealing with multiple, conflicting goals. A *factorial* design scheme and a *design parameter matrix* are also introduced to aid in determining the influence of functional coupling.

7.2 A Design Figure of Merit

The global objective of this research is to enhance the decision-making capability of the designer concerning preliminary solution *alternatives* by presenting, in a comprehensive manner, the imprecise information that is available during the preliminary design stages. The technique, as described so far, relies on a semi-automated approach for representing and manipulating design uncertainties. When the performance parameter space is large and non-linear, and when the design alternatives under consideration contain conflicting information regarding "how well the functional requirements are satisfied," it may be difficult to rate one design alternative with respect to any other. When a given functional requirement or constraint is deemed *more important* than another, or when there exist imprecise descriptions of the FRs and the importance of FRs, no systematic means have been described to understand the differences between solution alternatives. A design metric (or figure of merit) is needed to obtain a quantified design measure or design rating of the alternatives.

In the semi-automated approach presented to this point, the performance parameter results are decoupled, and the evaluation of each functional requirement with respect to each PP is carried out on a one-to-one basis. In order to determine a viable figure of merit, a recoupling of the performance parameter space (PPS) is necessary. Many mathematical functions come to mind for recoupling the PPS. The set theoretic operations described in Chapter 3 are a possible option. As shown by Bellman and Zadeh [15], however, a weighted linear combination of performance parameter results has a very useful property beyond the operations described in Chapter 3. This property is the ability to evaluate results "in which some goals and perhaps some constraints are of greater importance than others." One function that exhibits this property is:

$$\mu_{FWA}(x) = \frac{\sum_{i=1}^{n} \alpha_i(x)\mu_{G_i}(x)}{\sum_{i=1}^{n} \alpha_i(x)} + \frac{\sum_{j=1}^{m} \beta_j(x)\mu_{C_j}(x)}{\sum_{j=1}^{m} \beta_j(x)} \tag{7.1}$$

where

$\mu_{FWA}(x)$ = membership function of decision set,

$\mu_{G_i}(x)$ = membership function of goal set i,

$\mu_{C_j}(x)$ = membership function of constraint set j,

n = number of goals,

m = number of constraints,

α_i = weighting coefficient for goal i,

β_j = weighting coefficient for constraint j.

This function is referred to as the Fuzzy Weighted Average (Schmucker [66]), and may be viewed mathematically as a union of intersections operation. Notice that the classical symbol for membership μ is used in the equations, denoting the general case of symbolically labeled sets. Notice further that the goal, constraint, and weighting sets are all considered as fuzzy, resulting in a fuzzy decision set on the output. In the context of the design imprecision problem, this is useful when the functional requirements are imprecisely specified, along with weighting for each goal and constraint.

For the purposes of this study, the design metric will not include imprecise weighting coefficients. The weighting will be a scalar over the entire output support of each performance parameter. Moreover, only inequality functional requirements will be considered, where the term "functional requirement" includes both goals and constraints. Given these simplifications, a scalar form of Equation 7.1 may be used as a preliminary figure of merit:

$$\zeta_{\mathcal{A}_k} = \frac{\sum_{j=1}^{m} \beta_j \cdot \alpha_{FR_j}(p_j)}{\sum_{j=1}^{m} \beta_j}, \tag{7.2}$$

where

$\zeta_{\mathcal{A}_k}$	=	the figure of merit for design alternative $\mathcal{A}_k$,
$\alpha_{FR_j}(p_j)$	=	the point of intersection of the FR on the PP output curve,
β_j	=	the weighting coefficient for the j^{th} PP, p_j,
m	=	the number of performance parameters.

Equation 7.2 has been chosen because it possesses the following properties:

1. Equation 7.2 recouples the performance parameter space such that each design alternative can be directly compared by a numerical design rating;

2. The importance of a functional requirement may be subjectively specified and differentiated through a simple weight, β;

3. The range of the figure of merit given by Equation 7.2 are unity and zero respectively, *i.e.*, $\max[\zeta_{\mathcal{A}_k}] = 1$ and $\min[\zeta_{\mathcal{A}_k}] = 0$. This means that a design alternative that satisfies all design specifications will have a figure of merit of unity. If any design specification is not satisfied, $\zeta_{\mathcal{A}_k}$ will be less than unity, and zero if an alternative meets none of the functional requirements;

4. Given that 0.5 is the neutral preference point between acceptable and unacceptable values in a design parameter's input set, Equation 7.2 will return a value of 0.5 when all of the $\alpha_{FR_j} = 0.5$, no matter what values the β_j assume.

7.2.1 Example Use of the Design Figure of Merit

The figure of merit described by Equation 7.2 can be applied to the results of the design calculations discussed and demonstrated in Chapters 2 through 6. For a given design alternative $\mathcal{A}_k$, the α_{FR_j} are equated to the preference value where each functional requirement FR_j intersects the $\tilde{p}_j$ output curve (as long as the peak of $\tilde{p}_j$ does not satisfy FR_j). If the peak of $\tilde{p}_j$ satisfies FR_j, $\alpha_{FR_j}(p_j) = 1$. The assigning of weights will not be discussed in detail here. Many other authors [15, 33, 48, 84, 94] have presented various approaches for assigning weights. A simple approach in the form of a normalized rating of functional requirements is used for example purposes in this section.

Tables 7.1, 7.2, and 7.3 show the results of the application of Equation 7.2 to the frame design (Chapter 2), the brake design (Chapter 4), and the single-speed transmission design (Chapter 6). The values for $\mathcal{A}$ and β mean the following in the tables:

- Table 7.1 (Frame Design):

$$\mathcal{A}_1 = \text{frame alternative shown in Figure 2.6;}$$

$$\beta_1 = \text{the weighting coefficient for the bending stress PP;}$$

$$\beta_2 = \text{the weighting coefficient for the column load PP.}$$

Design Alternative	Figure of Merit and Weighting		
$\mathcal{A}_k$	$\zeta_{\mathcal{A}_k}$	β_1	β_2
$\mathcal{A}_1$ (Frame)	0.725	0.5	0.5

Table 7.1: Frame Example: Figure of Merit and Weighting Coefficients.

Design Alternatives	Figure of Merit and Weighting				
$\mathcal{A}_k$	$\zeta_{\mathcal{A}_k}$	β_1	β_2	β_3	β_4
$\mathcal{A}_1$ (Drum)	0.400	0.1	0.2	0.2	0.5
$\mathcal{A}_2$ (Disk_w)	0.980	0.1	0.2	0.2	0.5
$\mathcal{A}_3$ (Disk_p)	0.980	0.1	0.2	0.2	0.5

Table 7.2: Brake Example: Figure of Merit and Weighting Coefficients.

Design Alternatives	Figure of Merit and Weighting					
$\mathcal{A}_k$	$\zeta_{\mathcal{A}_k}$	$\beta_{1,1-2}$	$\beta_{2,1-2}$	$\beta_{3,3}$	$\beta_{4,1-3}$	$\beta_{5,1-3}$
$\mathcal{A}_1$ (Spur Gear)	0.663	0.25	0.25	0.0	0.25	0.25
$\mathcal{A}_2$ (Helical Gear)	0.920	0.25	0.25	0.0	0.25	0.25
$\mathcal{A}_3$ (V-belt)	0.783	0.0	0.0	0.5	0.25	0.25

Table 7.3: Transmission Example: Figure of Merit and Weighting Coefficients.

- Table 7.2 (Brake Design):

$\mathcal{A}_1$ = brake design alternative shown in Figure 4.2;

$\mathcal{A}_2$ = disk brake alternative, Figure 4.3, uniform wear;

$\mathcal{A}_3$ = disk brake alternative, Figure 4.3, uniform pressure;

β_1 = the weighting coefficient for the actuating force PP;

β_2 = the weighting coefficient for the torque PP;

β_3 = the weighting coefficient for the temperature rise PP;

β_4 = the weighting coefficient for the temperature dissipation PP.

- Table 7.3 (Transmission Design):

$\mathcal{A}_1$ = spur gear alternative shown in Figure 6.3;

$\mathcal{A}_2$ = helical gear alternative, Figure 6.3;

$\mathcal{A}_3$ = V-belt alternative, Figure 6.4;

$\beta_{1,1-2}$ = weighting coefficient: fatigue factor of safety PP;

$\beta_{2,1-2}$ = weighting coefficient: surface durability PP;

$\beta_{3,3}$ = weighting coefficient for the V-belt life PP;

$\beta_{4,1-3}$ = weighting coefficient for the diameter strength PP;

$\beta_{5,1-3}$ = weighting coefficient for the rated bearing load PP.

For the brake design, all design alternatives are rated using the same performance parameters. In the case of the transmission design, however, the gear alternatives have one more performance parameter than the V-belt alternative. This is due to the fact that the global functional requirement of "life-cycle" or "expected-life" is evaluated by both the fatigue and surface durability factors of safety for the gear designs. In order to make $\zeta_{\mathcal{A}_1}$ and $\zeta_{\mathcal{A}_2}$ comparable with $\zeta_{\mathcal{A}_3}$, the following simple condition must be satisfied: $\beta_{1,1-2} + \beta_{2,1-2} \equiv \beta_{3,3}$. Table 7.3 shows this condition for the transmission design. Overall, the results in the tables confirm the decisions and discussions of the example designs in Chapters 2, 4, and 6. The values used for α_{FR_j} in the $\zeta_{\mathcal{A}_k}$ calculations can be obtained directly from the chapters and figures that describe the example design problems.

7.3 Functional Coupling: An Imprecision Approach

In Chapter 2, the γ-level measure was introduced to determine the coupling and importance of the design parameters with respect to the performance parameters. The mathematical and algorithmic approach described in the γ-level section of Chapter 2 can be extended using the concept of a *design (or coupling) matrix.* It will be shown that this extension will help to determine the degree of functional coupling in proposed design alternatives, as well as to facilitate the application of the figure of merit presented in the previous section.

7.3.1 Functional Coupling

The idea of functional coupling is especially important when considering an axiomatic approach to design, as proposed by Suh, Bell, and Gossard [75]. In an axiomatic approach, the belief is that there exists a set of fundamental principles that should direct design practices. These principles, in turn, can be used to evaluate design decisions at each point in the design process. Two basic axioms were proposed by Suh, Bell, and Gossard in [75]:

- **Axiom 1** *Maintain the independence of functional requirements.*
- **Axiom 2** *Minimize the information content of designs.*

The first axiom simply states that each facet of a product or process design should be satisfied independently by some feature or component within the design. The second axiom formalizes the concept that designs should approach a minimal state of complexity.

Many publications regarding applications of these axioms have appeared since their introduction. In particular, the work of Rinderle and Suh [65] is significant to the research reported in this document. In [65], they develop a mathematical analogy to the first of these axioms. Relating this analogy to the mathematical model described in Chapter 1, the basic idea is that the functional requirements (FRs) can be mapped into a physical vector field representation of the design problem such that there exists a function vector $\vec{F}$ given by:

$$\begin{aligned}\vec{F} \;=\;\; & f_1(d_1, d_2, \ldots, d_n)\vec{u}_1 \;+ \\ & f_2(d_1, d_2, \ldots, d_n)\vec{u}_2 \;+ \\ & \quad\vdots \qquad\qquad\quad + \\ & f_m(d_1, d_2, \ldots, d_n)\vec{u}_m,\end{aligned} \tag{7.3}$$

where

$$\begin{aligned} f_j &= \text{scalar functions of the design parameters, } d_i, \\ \vec{u}_j &= \text{unit vectors in the FR}_j \text{ direction.}\end{aligned}$$

If the FRs are known to be related linearly to the DPs (the more general, non-linear case is presented later), this representation of $\vec{F}$ (Equation 7.3) leads to the construction of a design matrix that relates the functional requirements (FRs) of a design to the design parameters (DPs):

$$\vec{F} = \begin{bmatrix} \text{FR}_1 \\ \text{FR}_2 \\ \vdots \\ \text{FR}_m \end{bmatrix} = \begin{bmatrix} a_{11}\, a_{12} \cdots a_{1n} \\ a_{21}\, a_{22} \cdots a_{2n} \\ \vdots \\ a_{m1}\, a_{m2} \cdots a_{mn} \end{bmatrix} \begin{bmatrix} \text{DP}_1 \\ \text{DP}_2 \\ \vdots \\ \text{DP}_n \end{bmatrix}. \tag{7.4}$$

The coefficients a_{ji} make up the *design matrix* in Equation 7.4. Denoting this matrix as $\boldsymbol{A}$, $\boldsymbol{A}$ provides the functional coupling information for the design vector $\vec{F}$. If the matrix $\boldsymbol{A}$ contains only diagonal members, *i.e.*, only the diagonal elements are nonzero (or nearly so), the design is considered to be *uncoupled*. An uncoupled design implies that a desired change in an FR can be accomplished by a change in only one

DP, without affecting the other FRs. Thus, the FRs are completely independent, satisfying Axiom 1.

If the design matrix $\boldsymbol{A}$ is not diagonal but can be rearranged into an upper-triangular form, the design representation $\vec{F}$ is considered to be *quasi-coupled.* A quasi-coupled design has special properties. In particular, the design system can be rearranged into the following system:

$$\begin{aligned} \mathrm{FR}_1 &\simeq f_1(d_1) \\ \mathrm{FR}_2 &\simeq f_2(d_1, d_2) \\ &\vdots \\ \mathrm{FR}_{m-1} &\simeq f_{m-1}(d_1, d_2, \ldots, d_{n-1}) \\ \mathrm{FR}_m &\simeq f_m(d_1, d_2, \ldots, d_{n-1}, d_n), \end{aligned} \tag{7.5}$$

where the "$\simeq$" is used instead of a strict equality to denote that a FR_j is *approximately* or nearly independent of all other design parameters except those shown in the functional relationship. When a design can be placed in the form of Equation 7.5, *i.e.*, an upper-triangular $\boldsymbol{A}$, the functions can be adjusted in a proper sequence, resulting in the satisfying of a given FR_j without affecting the previous FR_{j-1} to FR_1. As stated by Rinderle and Suh, "a quasi-coupled system is in general superior to a coupled system because the detailed function interactions do not need to be known, but it is inferior to an uncoupled system since information is required about the order of adjustment."

When the design matrix $\boldsymbol{A}$ is neither diagonal nor upper-triangular (meaning that the design is neither uncoupled nor quasi-coupled), the design is said to be *coupled.* Two cases can be distinguished for a coupled design. If the design matrix is full, *i.e.*, all of the elements a_{ji} are nonzero, the design is completely coupled. This means that a change in any of the DPs will affect all of the FRs simultaneously. If, on the other hand, the design matrix is not full and cannot be rearranged into an upper-triangular form, the design is said to be dually: coupled and partially quasi-coupled. The reason for distinguishing this case from a fully coupled system is that under special circumstances certain design parameters can be assigned nominal (constant) values such that the system can be transformed into a quasi-coupled design. Of course, in general, this last case is rarely encountered in design systems, and is usually handled as a fully coupled system.

The ideas of functional coupling presented to this point relate to design systems in which the functional requirements are linear functions of the design parameters. Rinderle and Suh generalized these concepts to mappings between nonlinear spaces (where the functional requirements make up a curvilinear coordinate system in the physical space) by the introduction of a *coupling matrix*, $\boldsymbol{C}$, which has the following elements:

$$c_{ji} = \frac{\partial(FR_j)}{\partial(d_i)}. \tag{7.6}$$

The c_{ji} give the coupling between the FR_j and d_i at a given design point. Likewise, "the magnitudes of off-diagonal elements of the coupling matrix $\boldsymbol{C}$ determine the degree of coupling in a design." For the case of linear designs, the coupling matrix takes the form of Equation 7.4, as previously shown.

Using the definition of the coupling matrix (Equation 7.6) and the concepts of uncoupled, quasi-coupled, and coupled design systems, Rinderle and Suh derived two measures of functional coupling for the general design problem. These measures rely on constructing DP isograms in the physical space, with respect to FR coordinates superimposed in that space. The first measure, referred to as the *skew*, measures the cross sensitivity of the design parameters. In other words, the skew is the degree of orthogonality between DP isograms. The second measure is called the *alignment.* This measure determines a normalized correspondence between pairs of FRs and DPs. When the alignment is unity, for a given skew, the DP isograms "align" most closely with the FR coordinates, and the highest degree of uncoupling is achieved. The reader is referred to [65] for the mathematical definitions of these measures, as well as an example application of their use.

7.3.1.1 The Imprecision Extension

For the imprecision problem, where it is desired to determine the functional coupling of a number of design alternatives $\mathcal{A}_k$, the idea of a coupling matrix as described above can be extended to include preference information. Notice that the coupling matrix, defined by Equation 7.6, is simply a sensitivity of the functional requirements with respect to the design parameters. To extend this representation to the two-dimensional design parameter space of a preference function $(DP_i(d_i, \alpha_i))$, another coupling matrix, C^γ must be used in conjunction with Rinderle and Suh's, Equation 7.6. The elements of this new matrix are simply the γ-level measures for each design parameter (d_i) with respect to an alternative's performance parameters (p_j). The elements of the new coupling matrix, C^γ, are defined as follows:

$$c^\gamma_{ji} = D_{p_j}(\Theta(d_i)), \tag{7.7}$$

where $D_{p_j}(\Theta(d_i))$ is described in Chapter 2.

A coupling matrix of the form of Equation 7.7 can be determined for each alternative $\mathcal{A}_k$ in a preliminary design analysis. Like the coupling matrix defined by Equation 7.6, the off-diagonal elements of $C^\gamma_{\mathcal{A}_k}$ determine the degree of coupling in an alternative $\mathcal{A}_k$, where the coupling is with respect to the preference information of the design parameters. The concepts of uncoupled, quasi-coupled, and coupled systems apply equivalently to $C^{\gamma}_{\mathcal{A}_k}$. Thus, using this new coupling matrix in the context of the figure of merit presented previously, two alternatives with nearly equivalent figures of merit can be subsequently compared through the application of Equation 7.7. If one alternative, say $\mathcal{A}_1$, exhibits a lower degree of coupling than another alternative ($\mathcal{A}_2$) and the figures of merit of both $\mathcal{A}_1$ and $\mathcal{A}_2$ are nearly the same, then $\mathcal{A}_1$ is the superior design, as given by Axiom 1. This result occurs because in a coupled design, the satisfying of one functional requirement through the change of certain design parameters will in general adversely affect another functional requirement, especially with regard to preference. The higher the degree of coupling, the greater the adverse affect.

7.4 Discussion

For complex designs with varying degrees of coupling and conflicting performance parameter results, it can be difficult to make decisions between design alternatives, even with the representation of subjective information in the form preference functions. The addition of a figure of merit and a design coupling matrix to the methodology presented in Chapters 2 through 6 will aid a designer in making decisions in this highly complex and vague environment. The specific design metric and specific representation of the coupling matrix C^{γ} discussed in this chapter are not meant to be the only all-inclusive choices for handling conflicting design information and for determining functional coupling. They are preliminary choices which have proved useful in this study. Hopefully, the global pursuits of deriving applicable design metrics and methods for determining the degree of functional coupling of a very imprecise design system will motivate new approaches to these problems, especially in preliminary engineering design.

Chapter 8

Conclusions and Future Directions

8.1 General Conclusions

One of the central goals of this research is to increase the amount of information available to engineering designers regarding the performance of design alternatives, over that available with conventional design analyses. The effect will be greater when the information is made available earlier in the design process. Ultimately the most important (and costly) decisions in the design cycle are made in the very early stages. Engineering designs are typically represented imprecisely at the early, conceptual (preliminary) stage of design. Computational tools for this area of the design process are rare, largely because of the scarcity of techniques capable of handling imprecise data. One of the central hypotheses of the research reported here is that representing and manipulating imprecise descriptions of design artifacts during the preliminary phase (and hence increasing the information available to the designer) will enable design decisions to be made with greater confidence and reduced risk, and that this will ultimately result in better designs.

The technique and implementation reported here represent a new application (to the engineering design process) of a powerful approach to represent and manipulate imprecise engineering design data, as well as other uncertainties. The examples shown demonstrate that the method can be applied to engineering design problems, and provide the ability to perform design calculations on a variety of imprecise and uncertainty parameters (with the extended hybrid number representation). The (correspondingly) imprecise-calculation and extended-hybrid-number results provide more information to the designer than conventional single-valued design analyses. The fuzzy calculus is used here to represent the designer's desire to use particular values for imprecise parameters, and then operate on these representations. The probability calculus is used to represent and manipulate the stochastic component of uncertainty. The imprecise-numerical technique used is an extensively modified implementation of the Fuzzy Weighted Average operating on preference representations of design parameters. Similarly, a cumulative-distribution numerical technique is used to calculate with the stochastic parameters.

Additional useful information that this method can provide, through the use of the *γ-level measure*, is the coupling between imprecise representations of design parameters (inputs) and the performance parameter results. This coupling information can be used to focus the engineer's resources on those aspects of the design problem with the largest effect on the resulting performance. When a high level of coupling and conflicting performance parameter results exist, a simple but very useful figure of merit (or metric) can be applied. A figure of merit in the form presented in this document helps the designer to further manage and evaluate the information-intensive preliminary design alternatives, and represents the first steps towards a method for quantifying design decisions.

8.2 Future Work

This section discusses possible future projects that exist as natural outgrowths of the research presented in this document. A design research context for these projects is presented at the beginning of this section, followed by a description of individual research tasks.

8.2.1 Current Research in the Design Community: Tendencies Toward a Mechanical Design Compiler

Research in engineering design is evolving rapidly, especially with regard to computational models for the preliminary phase of design. In the last decade, a few researchers have presented interesting results in the areas of "pre-parametric mechanical design" and "design representations and languages," including Ulrich and Seering [78]; Rinderle [64]; Stiny [74]; Fitzhorn [34]; Dixon [80]; Ward and Seering [79]; and others. As with any basic research, these first efforts have helped to define the general scope of problems in the area, as well as to raise a number of fundamental questions. Overall, this work, along with recent discussions in the design community, has led to the concept of a *mechanical design compiler.* [1] In an idealistic view, a *mechanical compiler* consists of the basic method (*i.e.*, rules, grammar, and approach) for representing an initial design need (and corresponding information), for transforming this representation into a valid and appropriate set of possible designs that satisfy the need, and for presenting the alternative designs (in terms of

[1] Ward and Seering [79] discuss the idea of a "mechanical design compiler" in the context of quantitative inference for artifact sets. This idea has lead to a computer program that transforms a schematic description of a mechanical transmission to a component description in the form of catalog numbers of basic mechanical components.

fundamental mechanical artifacts) to the designer so that decisions may be carried out directly.

This idealistic view of a mechanical design compiler has merit in terms of a broad future goal for the work. However, a more realistic view is needed for present research efforts. Such a view entails relaxing the first component in the structure of a *mechanical design compiler.* Instead of seeking solely to synthesize designs from a representation of the initial need, the first component of a *mechanical design compiler* consists of representing initial *proposed* designs as well as representing basic mechanical components, scientific principles, and subjective data. By representing proposed alternatives rather than just the need, the engineer is still the overall concept generator; and the compiler is the machinery for computing possible configurations from the design representation as well as from previous designs in the same class of problems.

The work reported in this document may be integrated directly into the idea of a *mechanical design compiler.* The modeling of subjective and objective uncertainties is necessary in order to realistically evaluate the variable (in terms of uncertainties) functional requirement space of generated alternatives. The approach as described in Chapters 2 through 7 presents a useful method by which alternatives can be evaluated, one performance dimension at a time, and subsequently recoupled using a design metric. The general algorithmic and computational structure can be directly used within the framework of a compiler. A number of additional problems become apparent, however, when considering the use of the method in a *mechanical design compiler.* These research tasks will be referred to as "*Transforming Continuum Design Descriptions,*" "*Representing Imprecise Functional Requirements and Constraints,*" "*Developing a Formalism for the Imprecision Problem,*" "*Determining Utility Functions for the Imprecision Problem*" below.

8.2.2 Transforming Continuum Design Descriptions

The technique developed in this document relies on two assumptions: the existence of a preliminary parameterization of a design alternative and the existence of either approximate or exact performance expressions. A method is needed by which the initial (continuum) representation of a design alternative may be transformed into a preliminary parameterization and a preliminary set of performance expressions. The determination of these two factors is a complex problem due to their interrelated structure, as well as to the breadth of possible alternatives in mechanical design. Even though this complexity exists, fundamental results from research into this problem (based on engineering principles) are definitely foreseeable. For example, in the domain of mechanical structures, a complex differential equation representation of a system may be very difficult if not computationally infeasible to solve for the initial parameterization and performance relationships. A technique can be envisioned in which approximations are made to the differential equations (without solving them) in order to determine approximate performance expressions and the important design parameters of the problem. Such approximations will, in turn, influence the form of the imprecision and uncertainties in a design problem. Likewise, for a complex geometric representation of a mechanism (*e.g.*, an irregular cross-section for a structural member), geometric and mathematical relations may suggest possible simplifications (*e.g.*, preliminary parameterization and performance expressions based on a simplified rectangular, circular, or other common cross-section).

8.2.3 Representing Imprecise Functional Requirements and Constraints

As stated earlier, the uncertainty method allows the designer to evaluate preliminary alternative designs one performance dimension at a time. The information added in this process is a quantification of the imprecision, as well as other uncertainties, in the design parameters. The inherent imprecision in the *design specifications and requirements* is not *directly* represented in the current method, however. Because design specification and requirements are, in general, imprecise (not crisp) values in preliminary design (and, in fact, in all phases of engineering design), the uncertainty method must be extended to relate the trade-offs between imprecise design parameters and imprecise design specifications. The general problem of *what sources contribute to the imprecision in design specifications and requirements* must also be considered. One obvious source of imprecision for a design specification arises from implicit dependencies on the imprecise design parameters. For example, in a preliminary brake design for a vehicle, the torque performance specification is a function of two imprecise design parameters: the radius of the vehicle's wheel and the weight of the vehicle. Other sources of imprecision, besides implicit relationships with design parameters, must also be investigated, including the subjective preferences of the designer.

8.2.4 Developing a Formalism for the Imprecision Problem

A formalism of the imprecision problem represents a direct extension of the approach described in Chapters 1 through 7. Such a formalism should include the following facets:

- Refine the use and specification process of preference functions, including continuous and discrete design parameters as well as multiple source imprecision.

- Investigate more rigorous methods for interpreting imprecise performance parameter results, especially in the context of functional coupling and design measures such as the γ-level measure.

- Introduce methods for combining imprecise calculation results with imprecise design specifications, and relate the methods to the extended hybrid representation.

8.2.5 Developing Utility Functions for the Imprecision Problem

The objective of the method for representing and manipulating uncertainties is to provide the necessary information for making decisions between preliminary design alternatives. When a given alternative is carried forward to later stages in the design process, an approach is needed for determining the best possible choice of the design parameter vector. Such an approach should construct utility functions that encapsulate the trade off between meeting the performance specifications and seeking the highest degree of preference for the design parameters. Utility functions of this form represent an extension to the design metric discussed in Chapter 7.

8.2.6 New Directions

The previous sections relate the research reported in this document to the concept of a *mechanical design compiler*, and introduce needed research projects as direct extensions to the work. The following section will present: (1) indirect extensions of this research as they relate to recent developments in the field; and (2) a project that considers the application of parallel processing to design computations.

The introductory portion of this section briefly mentioned other researchers' work in the field. The uncertainty method described herein can be combined with other methods to produce interesting results. Such a combination will be referred to as indirect extensions to the research, because these extensions are not direct outgrowths from the analytical and numerical procedures of the uncertainty method.

An itemized list of the indirect extensions follows:

- Fitzhorn [34], Stiny [74], and others have introduced work on formal grammars (languages) in design. These works present a grammatical analogy to the design process, and represent the first steps toward the formal machinery of the representation and transformation components of a *general mechanical design compiler*. The addition of the necessary pragmatics of representing and manipulating uncertainties, however, has not been addressed. Combining the uncertainty method with a grammatical analogy to design should prove to be constructive, especially in the context of how a symbolic and computational representation of uncertainties influences the grammar.

- Ward and Seering [79] have developed "a theory of quantitative inference for artifact sets as applied to a mechanical design compiler," which returns component catalog data from a description of a mechanical design. This compiler uses a "labeled interval specification language" for the representation and transformation of a design's description. Because Ward and Seering's approach relies on interval operations, the mathematics may be extended to include the uncertainty method such that further subjective and objective data of the designer may be included.

Besides the indirect extensions to the uncertainty method discussed above, a *data-flow paradigm* to design represents a needed research project in terms of dealing with computational complexity in design automation. "Data-driven" programming, an environment vastly different from conventional languages, will have significant advantages for manipulating analyses functions and design representations. The programming will be much like assembling a three-dimensional block diagram, with the functional blocks consisting of symbolic representations of functions and analyses. In these transformed functional blocks, parallel paths of computations are directly realized, such that the computational complexity is ultimately reduced. A simple example of the use of "data-driven" programming is in the domain of mechanical systems. Because kinematic and dynamic systems of equations may be transformed into canonical form, functional blocks may be built directly from the canonical representation, leading to parallel computation with respect to each block.

Appendix A

Appendix: Fuzzy Arithmetic

The following appendix contains background information on fuzzy set theory and fuzzy arithmetic.

A.1 Fuzzy Set Concepts

The concepts involved with fuzzy sets will be described in detail on the basis of the presentations in references: [33], [46], [47], [87], [89], and [94]. Only those tools that are required to understand this document will be discussed. The reader is referred to the references for a more rigorous (axiomatic) approach of fuzzy set theory.

In the classical sense, a set of elements or objects makes up a *Universe*, Γ, such that an element ω of the subset Ω of Γ may be represented by the characteristic function μ_Ω, mapping $\Gamma \rightarrow \{0,1\}$

$$\mu_\Omega = \begin{cases} 0 & \text{iff } \omega \notin \Omega, \\ 1 & \text{iff } \omega \in \Omega. \end{cases} \tag{A.1}$$

The set $\{0,1\}$ is of course a binary set in the classical theory. If the characteristic function is allowed to vary over the real interval [0,1], the subset Ω would be termed a fuzzy subset of a universe of discourse Υ (Upsilon), where we use Υ instead of Γ in order to differentiate classical theory from its fuzzy counterpart (N.B.: neither universe is fuzzy in terms of their members). The characteristic function (now referred to as membership) may be represented as $\mu_\Omega : \Upsilon \rightarrow [0,1]$. The membership function may be stated as a qualitative operator representing the degree (vagueness or uncertainty, or belief) to which an element belongs to a subset, where 1 denotes full membership and 0 the converse. The greater the value of μ_Ω the greater the membership. Zadeh [89] describes the membership mapping clearly as

> [the membership function] associates with each element y of Υ a number $\mu_\Omega(y)$ in the interval [0,1] which represents the grade of membership of y in Υ.

A.1.1 Definitions

a. Discrete Domain If Ω contains a finite number of members, *i.e.*, a finite support $\{y_1, y_2, \dots, y_n\}$, then the subset may be represented by

$$\Omega = \sum_{i=1}^{n} \mu_i | y_i \tag{A.2}$$

or equivalently,

$$\Omega = \mu_1 | y_1 + \dots + \mu_n | y_n \tag{A.3}$$

where the summation convention represents the *union* operation and the vertical bar sign denotes a delimiter. (N.B.: The set symbols of { and } will be assumed.)

b. Example Using the label *fast* as in the speed of an automobile, the fuzzy subset for the universe $\Upsilon = \{40, 55, 70, 80, 100\}$ (mph) might be (see Figure A.1)

$$\text{fast} = 0.05/40 + 0.25/55 + 0.6/70$$
$$+0.8/80 + 1/100.$$

This is quite different from the crisp set ‘speeding,’

$$\text{speeding} = \{70, 80, 100\}.$$

c. Continuum The summation in Equation A.2 may be replaced in a continuous domain as follows

$$\Omega = \int_{\Upsilon} \mu_{\Omega}(y)|y, \tag{A.4}$$

where the integral sign denotes the union operation once again.

d. Example 2 Referring to the first example, *fast* might be given now as (see Figure A.2)

$$\text{fast} = \int_{40}^{100} (y^2/10000)|y.$$

A.2 Fuzzy Number

An alternative view that may clarify these ideas concerns the universe of real numbers $\Re$. For an uncertain value, say $C \in \Re$, we may express C as an interval value:

$$C = [c_l, c_r],$$

such that this may be termed the *interval of confidence* of C. Intervals of confidence help to represent uncertainty in parameters; yet, such an interval does not include the concept of membership values. Kaufmann and Gupta [46] have defined the idea of a *level of presumption* that directly combines the concept of the interval of confidence with a membership value (α) at each value in the interval. For instance, let the interval of confidence for the length input parameter l of a column stress equation be:

$$\tilde{l} = [20, 30].$$

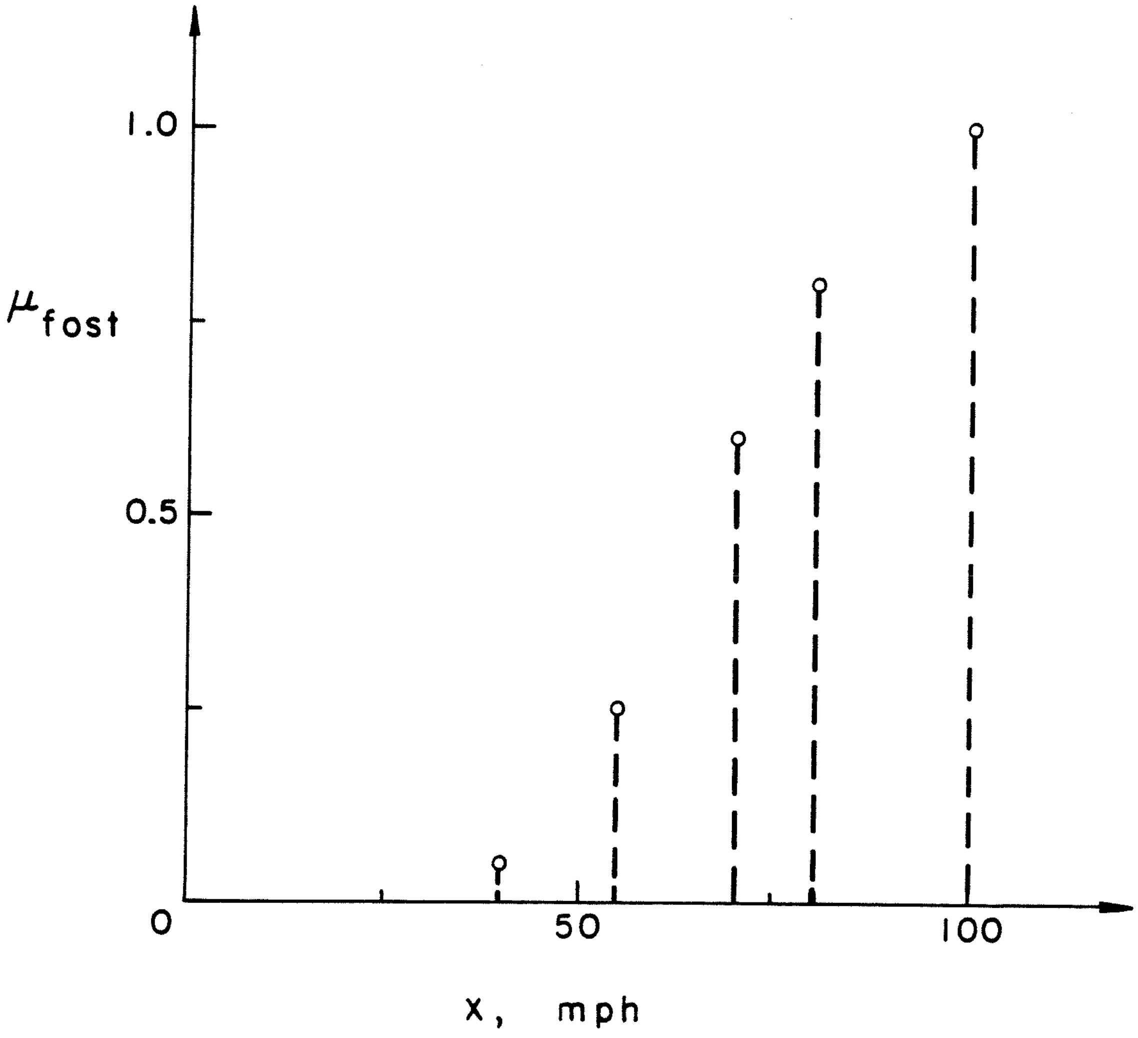

Figure A.1: Discrete Fuzzy Set fast.

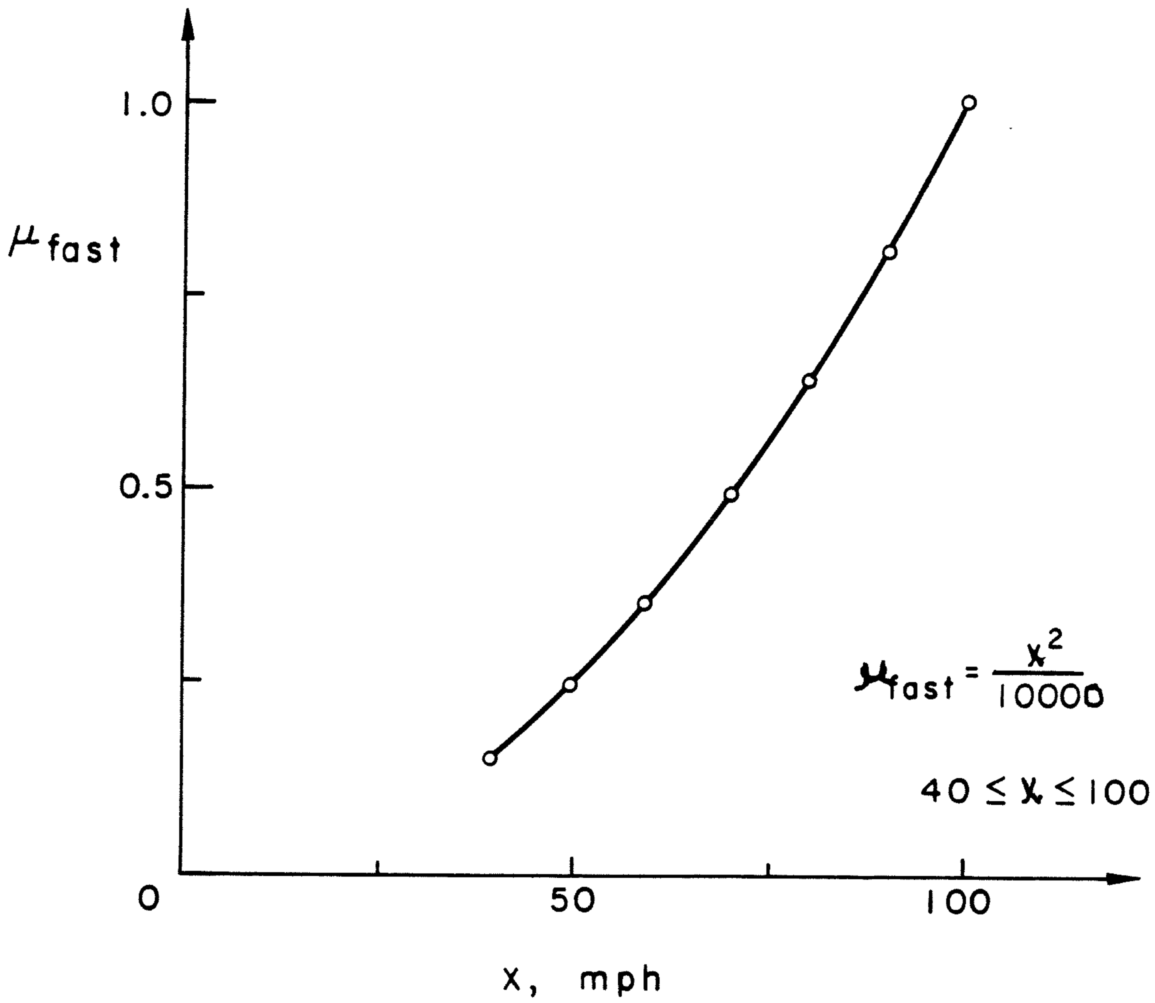

Figure A.2: Fuzzy Set fast.

A level of presumption (α_l) may be assigned by the designer for any value in $\tilde{l}$, *e.g.*, $\alpha_l = 1$ may be associated with $l = 25$. Subsequent levels of presumption (or membership values) may be given as in Figure 2.1. The interpretation here is straight forward, *i.e.*, the designer is the most confident in $l = 25$ when compared to any other value in $\tilde{l}$, and conversely, the designer has zero confidence in $l = 20$ and $l = 30$. We thus define $\tilde{l}$ or more generally $\tilde{C}$ as a *fuzzy number*, where the association between interval and membership values "corresponds to the natural, often implicit, mechanism of human thinking in the subjective estimation of a value for a dimension." [46] If we generalize this idea of a fuzzy number, we arrive back to the definition of a fuzzy set as given earlier.

A.2.1 Definitions

a. α **- Level-Set** The discussion of a fuzzy number with respect to its membership values leads directly to the idea of defining crisp sets (or intervals of confidence) for each level, α. Specifically, an α*-level-set* (or α-cut), C_α, is a crisp set taken from the fuzzy set $\tilde{C}$ such that

$$C_\alpha = \{x | \mu_{\tilde{C}}(x) \geq \alpha\}, \alpha \in [0, 1]. \tag{A.5}$$

Referring to Figure 2.1, we see that by example

$$\begin{aligned} l_{1.0} &= [25, 25], \\ l_{0.5} &= [21.5, 28.5], \textit{and} \\ l_{0.0} &= [20, 30]. \end{aligned}$$

b. Condition on a Fuzzy Number A condition follows from definition (a.), *i.e.*, a fuzzy number $\tilde{C}$ must satisfy

$$\forall \alpha_1, \alpha_2 \in [0,1],$$
$$(\alpha_2 > \alpha_1) \Longrightarrow ([c_l^{(\alpha_2)}, c_r^{(\alpha_2)}] \subseteq [c_l^{(\alpha_1)}, c_r^{(\alpha_1)}]).$$

c. Normality and Convexity Conditions A fuzzy set (or subset of $\Re$) $\tilde{C}$ is said to be *normal* if

$$sup_x \mu_{\tilde{C}} = 1,$$

which means that $\mu_{\tilde{C}max} = 1$. Likewise, $\tilde{C}$ is *convex* if

$$\mu_{\tilde{C}}(\lambda x_1 + (1-\lambda))x_2 \geq min(\mu_{\tilde{C}}(x_1), \mu_{\tilde{C}}(x_2)),$$
$$\forall x_1, x_2 \in [c_l, c_r] \, and \, \forall \lambda \in [0,1].$$

Convexity of a fuzzy set is also assured if all of the α-level sets are convex, C_α.

d. Precise Definition of a Fuzzy Number Definitions a.-c. provide the necessary conditions for a precise definition of a fuzzy number according to Kaufmann and Gupta: "A fuzzy number in $\Re$ is a fuzzy subset of $\Re$ that is convex and normal." As seen earlier in the article, we may relax the convexity condition slightly in order to allow for multiple peaks for an input design parameter represented as a fuzzy number.

A.3 Operations for Fuzzy Numbers

Zadeh [90] introduced the extension principle as one of the fundamental ideas of fuzzy set theory. Using this idea, classical mathematics may be extended to the fuzzy domain. Specifically, let the fuzzy sets (or fuzzy numbers) $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_N$ be defined in the universes $X_1, X_2, \ldots, X_N$, respectively. The mapping from $X_1 \times \cdots \times X_N$

to a universe Y may be defined as a function f such that $y = f(x_1, \ldots, x_N)$. The extension principle then gives that a fuzzy set (or number) $\tilde{D}$ on Y can be induced from $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_N$ through f such that the resulting membership function is

$$\mu_{\tilde{D}}(y) = sup_{x_1,\cdots,x_N} min(\mu_{\tilde{C}_1}, \cdots, \mu_{\tilde{C}_N})$$

where $y = f(x_1, \ldots, x_N)$. The ordinary binary operations then become extended operations in the fuzzy domain (extended addition, extended multiplication, etc.).

Even though the development of these extended operations may be completed rigorously using the extension principle approach, it is more useful to present operations for fuzzy numbers in a slightly different manner. Notably, interval operations for α-level sets become the natural means for performing calculations with fuzzy numbers.

A.3.1 Definitions

a. Addition and Subtraction Two fuzzy numbers, $\tilde{E}$ and $\tilde{F}$, may be summed or subtracted level by level ($\alpha \in [0, 1]$) according to the following formulas:[1]

$$\begin{aligned} E_\alpha \oplus F_\alpha &= [e_l^\alpha + f_l^\alpha, e_r^\alpha + f_r^\alpha], \\ E_\alpha \ominus F_\alpha &= [e_l^\alpha - f_r^\alpha, e_r^\alpha - f_l^\alpha] \end{aligned}$$

where

$$E_\alpha = [e_l^\alpha, e_r^\alpha], F_\alpha = [f_l^\alpha, f_r^\alpha].$$

b. Multiplication and Division Similarly, two fuzzy numbers, $\tilde{E}$ and $\tilde{F}$, may be multiplied or divided [1] (considering $\Re^+$ only here)

$$E_\alpha \odot F_\alpha = [e_l^\alpha \cdot f_l^\alpha, e_r^\alpha \cdot f_r^\alpha],$$
$$E_\alpha \oslash F_\alpha = [e_l^\alpha / f_r^\alpha, e_r^\alpha / f_l^\alpha].$$

c. Example of Fuzzy Multiplication For simplicity, consider the fuzzy numbers, $\tilde{E}$ and $\tilde{F}$ as shown in Figure A.3. The membership functions are given by

$$\begin{aligned} \mu_{\tilde{E}} &= \frac{1}{5}x,\ 0 \leq x \leq 5, \\ &= -\frac{1}{5}x + 2,\ 5 \leq x \leq 10, \\ &= 0,\ otherwise, \end{aligned} \tag{A.6}$$

$$\begin{aligned} \mu_{\tilde{F}} &= \frac{1}{5}x - 2,\ 10 \leq x \leq 15, \\ &= -\frac{1}{5}x + 4,\ 15 \leq x \leq 20, \\ &= 0,\ otherwise. \end{aligned} \tag{A.7}$$

In terms of the levels of presumption, α, Equation A.6 becomes

$$\alpha = \frac{e_l^\alpha}{5} \tag{A.8}$$

and

$$\alpha = -\frac{e_r^\alpha}{5} + 2. \tag{A.9}$$

Similarly, equation A.7 leads to

$$\alpha = \frac{f_l^\alpha}{5} - 2 \tag{A.10}$$

[1] Although nonfuzzy operations are easily extended to their fuzzy counterparts, it must be noted that certain properties of the classical binary operations are lost in the process. [46] Caution is therefore warranted in the use of fuzzy computations.

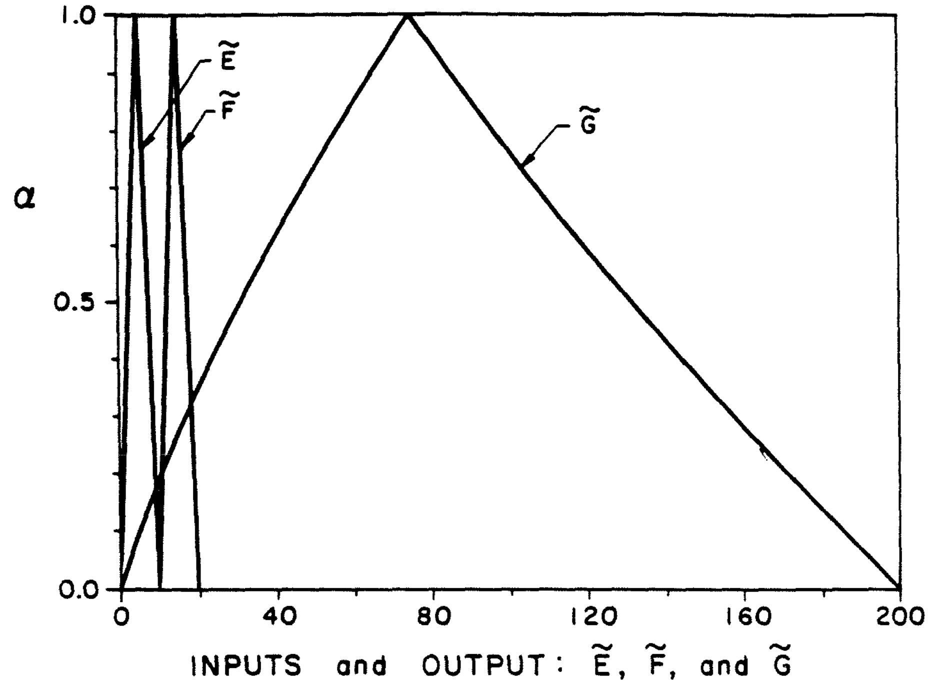

Figure A.3: Multiplication of $\tilde{E}$ and $\tilde{F}$.

and

$$\alpha = -\frac{f_r^\alpha}{5} + 4. \tag{A.11}$$

Combining the results, we end up with expressions for E_α and F_α:

$$E_\alpha = [5\alpha, -5\alpha + 10]$$

and

$$F_\alpha = [5\alpha + 10, -5\alpha + 20].$$

Multiplying leads to

$$\begin{aligned} G_\alpha &= [(5\alpha)(5\alpha + 10), \\ &\quad (-5\alpha + 10)(-5\alpha + 20)] \\ &= [25\alpha^2 + 50\alpha, 25\alpha^2 - 150\alpha + 200], \end{aligned} \tag{A.12}$$

where $G_\alpha = E_\alpha \odot F_\alpha$. Solving the quadratics on each end of the interval and retaining only two roots for α, we have

$$x_l = 25\alpha^2 + 50\alpha \implies \alpha_l = -1 + \sqrt{\frac{x_l}{25} + 1},$$
$$x_r = 25\alpha^2 - 150\alpha + 200 \implies \alpha_r = 3 - \sqrt{\frac{x_r}{25} + 1},$$

where

$$0 \leq x_l \leq 75,$$
$$75 \leq x_r \leq 200.$$

d. **Trigonometric Operations** Binary operations have been our focus previously; yet, in all disciplines of engineering, trigonometric operations are just as crucial. Thus, the sine, cosine, and tangent functions must be extended to the fuzzy domain as we did with the addition, subtraction, multiplication, and division operators. Consider first, then, a fuzzy angle defined in the following manner

$$\theta^{(\alpha)} = [\theta_l^{(\alpha)}, \theta_r^{(\alpha)}].$$

The trigonometric functions are defined below.

$$\begin{aligned} sin(\theta^{(\alpha)}) &= [sin(\theta_l^{(\alpha)}), sin(\theta_r^{(\alpha)})], \\ cos(\theta^{(\alpha)}) &= [cos(\theta_l^{(\alpha)}), cos(\theta_r^{(\alpha)})], \\ tan(\theta^{(\alpha)}) &= [tan(\theta_l^{(\alpha)}), tan(\theta_r^{(\alpha)})]. \end{aligned}$$

For the cosine membership functions, problems obviously occur at the points of the cosine curve where monotonicity is not true, *i.e.*, at $\theta = 0$ and $\theta = \pi$. To correct the problems for all three functions, the condition on fuzzy numbers presented in Appendix A.2, definition b must be employed. For the cosine function, this implies that when $(\alpha_2 > \alpha_1)$, the interval $cos(\theta^{\alpha_2})$ must be included in the interval $cos(\theta^{\alpha_1})$. Within $\frac{\pi}{2}$ of $\theta = \pi$, we then have

$$cos[\theta^{\alpha}] = [-1, max(cos(\theta_1^{\alpha}, cos(\theta_1^{\alpha}))].$$

Kaufmann and Gupta [46] list the interval expressions for the trigonometric functions for the full unit circle. Examples may be found below, where the following restriction has been imposed:

$$\forall \alpha \in [0,1], 0 \leq \mid \theta_2^{\alpha} - \theta_1^{\alpha} \mid \leq \frac{\pi}{2}.$$

$$(i)\ 0 \leq \theta_1^{\alpha} \leq \theta_2^{\alpha} \leq \frac{\pi}{2}:$$

$$\begin{aligned} sin(\theta^{(\alpha)}) &= [sin(\theta_l^{(\alpha)}), sin(\theta_r^{(\alpha)})], \\ cos(\theta^{(\alpha)}) &= [cos(\theta_r^{(\alpha)}), cos(\theta_l^{(\alpha)})], \\ tan(\theta^{(\alpha)}) &= [tan(\theta_l^{(\alpha)}), tan(\theta_r^{(\alpha)})]. \end{aligned}$$

$$(ii)\ 0 \le \theta_1^\alpha \le \frac{\pi}{2} \le \theta_2^\alpha \le \pi :$$

$$\begin{aligned} sin(\theta^{(\alpha)}) &= [min(sin(\theta_l^{(\alpha)}), sin(\theta_r^{(\alpha)})), 1], \\ cos(\theta^{(\alpha)}) &= [cos(\theta_r^{(\alpha)}), cos(\theta_l^{(\alpha)})], \\ tan(\theta^{(\alpha)}) &= (-\infty, tan(\theta_r^{(\alpha)})] \cup [tan(\theta_l^{(\alpha)}), \infty). \end{aligned}$$

$$(iii)\ \frac{\pi}{2} \le \theta_1^\alpha \le \theta_2^\alpha \le \pi :$$

$$\begin{aligned} sin(\theta^{(\alpha)}) &= [sin(\theta_r^{(\alpha)}), sin(\theta_l^{(\alpha)})], \\ cos(\theta^{(\alpha)}) &= [cos(\theta_r^{(\alpha)}), cos(\theta_l^{(\alpha)})], \\ tan(\theta^{(\alpha)}) &= [tan(\theta_l^{(\alpha)}), tan(\theta_r^{(\alpha)})]. \end{aligned}$$

$$(iv)\ \frac{\pi}{2} \le \theta_1^\alpha \le \pi \le \theta_2^\alpha \le \frac{3\pi}{2} :$$

$$\begin{aligned} sin(\theta^{(\alpha)}) &= [sin(\theta_r^{(\alpha)}), sin(\theta_l^{(\alpha)})], \\ cos(\theta^{(\alpha)}) &= [-1, max(cos(\theta_l^{(\alpha)}), cos(\theta_r^{(\alpha)}))], \\ tan(\theta^{(\alpha)}) &= [tan(\theta_l^{(\alpha)}), tan(\theta_r^{(\alpha)})]. \end{aligned}$$

A.4 Miscellaneous Topics

A.4.1 Definitions

a. Fuzzy Sets of Type-m A *fuzzy set of type-m* consists of a fuzzy set with membership values, which are themselves fuzzy sets of *type-$m-1$*, where m is an integer, $m > 1$. For example, a *type-$m = 2$* fuzzy set is a fuzzy set with membership values that are themselves fuzzy, *i.e.*, a fuzzy set of *type 1*.

b. Complement The *complement* of a fuzzy set $\tilde{C}$, denoted by $\hat{\tilde{C}}$, has a membership function defined by

$$\mu_{\hat{\tilde{C}}}(x) = 1 - \mu_{\tilde{C}}, \forall x \in X.$$

c. Support of a Fuzzy Set The *support* of a fuzzy set $\tilde{C}$ is the crisp set of all $x \in X$ such that $\mu_{\tilde{C}} > 0$.

Appendix B

Appendix: Derivation of the Probability Operations

B.1 Operations with Probability Density Functions

The following section contains the derivation of the probability approach operations summarized in Section 5.1.

In general, let $\hat{x}$ and $\hat{y}$ be two independent uncertain or imprecise "input" parameters, and let I be the proposition stating that their imprecision or uncertainty is assumed to be described by the subjectively assigned probability density functions (pdfs) $p_{\hat{x}}(x)$ and $p_{\hat{y}}(y)$, that is,

$$P(\hat{x} \leq x \mid I) = \int_{-\infty}^{x} p_{\hat{x}}(\xi) d\xi,$$

and

$$P(\hat{y} \leq y \mid I) = \int_{-\infty}^{y} p_{\hat{y}}(\eta) d\eta. \tag{B.1}$$

Note that for a finite range of possible values, $p_{\hat{x}}(x)$ and $p_{\hat{y}}(y)$ would be set to zero for x and y outside their respective input ranges.

Let $\hat{z}$ be the uncertain or imprecise "output" parameter, given by:

$$\hat{z} = f(\hat{x}, \hat{y}).$$

We wish to determine the resulting pdf for $\hat{z}$. From the axioms in Equations 5.1 to 5.3:

$$\begin{aligned}
P(\hat{z} \leq z \mid I) &= P(\hat{z} \leq z, \hat{y} \in (-\infty, \infty) \mid I) \\
&= \sum_{n=-\infty}^{\infty} P(\hat{z} \leq z, \hat{y} \in [y_n, y_n + \delta y] \mid I) \\
&= \sum_{n=-\infty}^{\infty} P(\hat{z} \leq z \mid \hat{y} \in [y_n, y_n + \delta y], I) \times P(\hat{y} \in [y_n, y_n + \delta y] \mid I) \\
&= \sum_{n=-\infty}^{\infty} P(\hat{z} \leq z \mid \hat{y} \in [y_n, y_n + \delta y], I) \cdot p_{\hat{y}}(y_n)\, \delta y,
\end{aligned}$$

where the real line is divided into equal intervals δy by the points y_n, $n = 0, \pm 1, \pm 2, \ldots, \pm\infty$. Taking the limit as δy tends to zero, we get:

$$P(\hat{z} \leq z \mid I) = \int_{-\infty}^{\infty} P(\hat{z} \leq z \mid \hat{y} = y, I) p_{\hat{y}}(y) dy. \tag{B.2}$$

We apply this result to each of the binary operations.

B.1.1 Addition

Here $\hat{z} = \hat{x} + \hat{y}$; so from Equation B.1:

$$\begin{aligned}
P(\hat{z} \leq z \mid \hat{y} = y, I) &= P(\hat{x} + \hat{y} \leq z \mid \hat{y} = y, I) \\
&= P(\hat{x} \leq z - y \mid I) \\
&= \int_{-\infty}^{z-y} p_{\hat{x}}(x) dx.
\end{aligned}$$

Therefore,

$$\frac{d}{dz} P(\hat{z} \leq z \mid \hat{y} = y, I) = p_{\hat{x}}(z - y),$$

and

$$\begin{aligned}
p_{add}(z \mid I) &= \frac{d}{dz} P(\hat{z} \leq z \mid I) \\
&= \int_{-\infty}^{\infty} p_{\hat{x}}(z - y) p_{\hat{y}}(y) dy. \qquad \text{(B.3)}
\end{aligned}$$

B.1.2 Subtraction

Here $\hat{z} = \hat{x} - \hat{y}$, and, as above, we get:

$$p_{sub}(z \mid I) = \int_{-\infty}^{\infty} p_{\hat{x}}(z+y)p_{\hat{y}}(y)dy. \tag{B.4}$$

B.1.3 Multiplication

Here $\hat{z} = \hat{x} \cdot \hat{y}$; so, assuming first that $y \neq 0$:

$$\begin{aligned} P(\hat{z} \leq z \mid \hat{y} = y, I) &= P(\hat{x} \cdot \hat{y} \leq z \mid \hat{y} = y, I) \\ &= P(\hat{x} \cdot y \leq z \mid I) \\ &= P(\hat{x} \leq \frac{z}{y} \mid I) \\ &= \int_{-\infty}^{\frac{z}{y}} p_{\hat{x}}(x)dx. \end{aligned}$$

Therefore,

$$\frac{d}{dz}P(\hat{z} \leq z \mid \hat{y} = y, I) = \frac{1}{y}p_{\hat{x}}(\frac{z}{y}).$$

If $y = 0$ is a possible value of $\hat{y}$, we note that this function approaches zero as y tends to zero because $p_{\hat{x}}(x)$ must be integrable over $(-\infty, \infty)$. Thus, regardless of whether $y = 0$ is possible or not:

$$p_{mul}(z \mid I) = \int_{-\infty}^{\infty} \frac{1}{y}p_{\hat{x}}(\frac{z}{y})p_{\hat{y}}(y)dy. \tag{B.5}$$

B.1.4 Division

Here $\hat{z} = \frac{\hat{x}}{\hat{y}}$.

$$\begin{aligned} P(\hat{z} \leq z \mid \hat{y} = y, I) &= P(\frac{\hat{x}}{\hat{y}} \leq z \mid \hat{y} = y, I) \\ &= P(\hat{x} \leq y \cdot z \mid I) \\ &= \int_{-\infty}^{y \cdot z} p_{\hat{x}}(x)dx. \end{aligned}$$

Therefore,

$$\frac{d}{dz}P(\hat{z} \leq z \mid \hat{y} = y, I) = yp_{\hat{x}}(y \cdot z).$$

Again, there is no complication if $y = 0$ is a possible value, and:

$$p_{div}(z \mid I) = \int_{-\infty}^{\infty} yp_{\hat{x}}(y \cdot z)p_{\hat{y}}(y)dy. \quad \text{(B.6)}$$

B.1.5 Sine

Here $\hat{z} = \sin \hat{x}$; so $\hat{z} \in [a, b] \subseteq [-1, 1]$, and:

$$\begin{aligned} P(\hat{z} \leq z \mid I) &= P(\sin \hat{x} \leq z \mid I) \\ &= P(\hat{x} \in \bigcup_{n=-\infty}^{\infty} [(2n-1)\pi - \sin^{-1} z, 2n\pi + \sin^{-1} z] \mid I) \\ &= \sum_{n=-\infty}^{\infty} P(\hat{x} \in [(2n-1)\pi - \sin^{-1} z, 2n\pi + \sin^{-1} z] \mid I), \end{aligned}$$

where $\sin^{-1} z$ is the unique value $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ such that $\sin x = z$. Therefore,

$$P(\hat{z} \leq z \mid I) = \sum_{-\infty}^{\infty}[P(\hat{x} \leq 2n\pi + \sin^{-1} z \mid I) - P(\hat{x} \leq (2n-1)\pi - \sin^{-1} z \mid I)],$$

which implies:

$$\begin{aligned} p_{sin}(z \mid I) &= \frac{d}{dz}P(\hat{z} \leq z \mid I) \\ &= \frac{1}{\sqrt{1-z^2}} \sum_{-\infty}^{\infty}[p_{\hat{x}}(2n\pi + \sin^{-1} z) + p_{\hat{x}}((2n-1)\pi - \sin^{-1} x)]. \quad \text{(B.7)} \end{aligned}$$

If the possible values of $\hat{x}$ lie in the range $(-\frac{\pi}{2}, \frac{\pi}{2})$, then $p_{\hat{x}}(x) = 0$ for x outside this range, and:

$$p_{sin}(z \mid I) = \frac{1}{\sqrt{1-z^2}} p_{\hat{x}}(\sin^{-1} z). \quad \text{(B.8)}$$

B.1.6 Cosine

Here $\hat{z} = \cos \hat{x}$, and:

$$P(\hat{z} \leq z \mid I) = P(\cos \hat{x} \leq z \mid I)$$

$$
\begin{aligned}
&= P(\hat{x} \in \bigcup_{n=-\infty}^{\infty} [2n\pi + \cos^{-1} z, 2(n+1)\pi - \cos^{-1} z] \mid I) \\
&= \sum_{n=-\infty}^{\infty} P(\hat{x} \in [2n\pi + \cos^{-1} z, 2(n+1)\pi - \cos^{-1} z] \mid I), \\
&= \sum_{-\infty}^{\infty} [P(\hat{x} \leq 2(n+1)\pi - \cos^{-1} z \mid I) - P(\hat{x} \leq 2n\pi + \cos^{-1} z \mid I)],
\end{aligned}
$$

where $\cos^{-1} z$ is the unique value $x \in [0, \pi]$ such that $\cos x = z$. Therefore,

$$p_{cos}(z \mid I) = \frac{1}{\sqrt{1-z^2}} \sum_{-\infty}^{\infty} [p_{\hat{x}}(2(n+1)\pi - \cos^{-1} z) + p_{\hat{x}}(2n\pi + \cos^{-1} x)]. \tag{B.9}$$

If the possible values of $\hat{x}$ lie in the range $(0, \pi)$, then $p_{\hat{x}}(x) = 0$ for x outside this range, and:

$$p_{cos}(z \mid I) = \frac{1}{\sqrt{1-z^2}} p_{\hat{x}}(\cos^{-1} z). \tag{B.10}$$

B.2 Operations with Cumulative Distribution

In the previous section, the operation rules for the probability calculus were derived. Because a numerical application of these operations is necessary to carry out design computations, an equivalent form of the binary operation rules (addition, subtraction, division, and multiplication) using the cumulative distributions of the input parameters is needed such that greater efficiency can be achieved.

Recall Equation B.2:

$$P(\hat{z} \leq z \mid I) = \int_{-\infty}^{\infty} P(\hat{z} \leq z \mid \hat{y} = y, I) p_{\hat{y}}(y) dy.$$

Using the definition of the probability density function $p_{\hat{y}}(y)$,

$$p_{\hat{y}}(y) = \frac{d}{dy} P(\hat{y} \leq y \mid I),$$

$p_{\hat{y}}(y)\, dy$ can be replaced in Equation B.2 along with a change of the limits of integration such that

$$P(\hat{z} \leq z \mid I) = \int_{0}^{1} P(\hat{z} \leq z \mid \hat{y} = y, I) dP(y), \tag{B.11}$$

where the values for y in Equation B.11 correspond to

$$P(y) = \int_{-\infty}^{y} p_{\hat{y}}(y)dy.$$

This result may be applied to each of the binary operations in a similar manner to that presented in the previous section. The results are listed below:

$$\begin{aligned} p_{add}(z \mid I) &= \frac{d}{dz}P(\hat{z} \leq z \mid I) \\ &= \int_0^1 p_{\hat{x}}(z-y)dP(y), \end{aligned} \tag{B.12}$$

$$p_{sub}(z \mid I) = \int_0^1 p_{\hat{x}}(z+y)dP(y), \tag{B.13}$$

$$p_{mul}(z \mid I) = \int_0^1 \frac{1}{y} p_{\hat{x}}(\frac{z}{y})dP(y), \tag{B.14}$$

$$p_{div}(z \mid I) = \int_0^1 y p_{\hat{x}}(y \cdot z)dP(y). \tag{B.15}$$

B.3 Powers of Uncertain Parameters

The brake torque example in Section 5.3.6 and the transmission design example in Chapter 6 include DPs that are raised to a power. The discussion below develops the operation rules for powers.

Consider the case of an uncertain parameter, $\hat{y}$, raised to a non-zero, crisp (certain) power x, where x can be either a fraction or an integer. We wish to determine the *pdf* $p_{\hat{z}}(z)$ for $\hat{z} = \hat{y}^x$. Assuming that the uncertainty for $\hat{y}$ is described by a subjectively assigned *pdf* $p_{\hat{y}}(y)$, and considering $\hat{y} \in \Re^+$ only (*i.e.*, $p_{\hat{y}}(y) \equiv 0$ for $-\infty \leq y \leq 0$), two cases become apparent when deriving $p_{\hat{z}}(z)$: $x > 0$ and $x < 0$.

B.3.1 Case I: $x > 0$

From the basic definition of the cumulative distribution,

$$P(\hat{z} \leq z \mid I) = P(\hat{y}^x \leq z \mid I)$$

$$
\begin{aligned}
&= P(\hat{y} \leq z^{\frac{1}{x}} \mid I) \\
&= \int_{-\infty}^{z^{\frac{1}{x}}} p_{\hat{y}}(y)\, dy.
\end{aligned}
$$

Taking the derivatives of both sides of this result and applying the chain rule:

$$
\begin{aligned}
\frac{d}{dz} P(\hat{z} \leq z \mid I) &= \frac{d}{dz} \int_{-\infty}^{z^{\frac{1}{x}}} p_{\hat{y}}(y)\, dy \\
&= p_{\hat{y}}(z^{\frac{1}{x}})\, \frac{d}{dz}(z^{\frac{1}{x}}) \\
&= p_{\hat{y}}(z^{\frac{1}{x}})\, \frac{1}{x}\, z^{(\frac{1}{x}-1)}.
\end{aligned}
$$

Using the basic definition for a *pdf* from Equation B.1, the resulting probability density function for $\hat{z}$ may be expressed as:

$$
p_{\hat{z}}(z) = \frac{p_{\hat{y}}(z^{\frac{1}{x}})}{x\, z^{(1-\frac{1}{x})}}. \tag{B.16}
$$

B.3.2 Case II: $x < 0$

Once again from the basic definition of the cumulative distribution,

$$
\begin{aligned}
P(\hat{z} \leq z \mid I) &= P(\hat{y}^x \leq z \mid I) \\
&= P(\hat{y} \geq z^{\frac{1}{x}} \mid I) \\
&= \int_{z^{\frac{1}{x}}}^{\infty} p_{\hat{y}}(y)\, dy.
\end{aligned}
$$

Taking the derivatives of both sides of this result and applying the chain rule:

$$
\begin{aligned}
\frac{d}{dz} P(\hat{z} \leq z \mid I) &= \frac{d}{dz} \int_{z^{\frac{1}{x}}}^{\infty} p_{\hat{y}}(y)\, dy \\
&= -p_{\hat{y}}(z^{\frac{1}{x}})\, \frac{d}{dz}(z^{\frac{1}{x}}) \\
&= p_{\hat{y}}(z^{\frac{1}{x}})\, \frac{1}{\mid x \mid}\, z^{(\frac{1}{x}-1)}.
\end{aligned}
$$

In a similar fashion to Case I, the *pdf* for $\hat{z}$ is given by:

$$
p_{\hat{z}}(z) = \frac{p_{\hat{y}}(z^{\frac{1}{x}})}{\mid x \mid\, z^{(1-\frac{1}{x})}}. \tag{B.17}
$$

B.3.3 Alternative Derivation

It should be noted that the derivation for a probability density function of an uncertain parameter raised to a power can be alternatively formulated using the fact that the probability of any value z of $\hat{z}$ must equal the corresponding probability of the value y of $\hat{y}$, *i.e.*,

$$p_{\hat{z}}(z)\,dz \;=\; p_{\hat{y}}(y)\,dy.$$

We can rearrange this result into the following form:

$$p_{\hat{z}}(z) = \frac{p_{\hat{y}}(y)}{dz/dy}. \tag{B.18}$$

Applying Equation B.18 to $\hat{z} = \hat{y}^x$, we get:

$$p_{\hat{z}}(z) = \frac{p_{\hat{y}}(z^{\frac{1}{x}})}{x\,z^{(1-\frac{1}{x})}}. \tag{B.19}$$

Two notes of caution must be enumerated when using this alternative derivation:

1. For a negative power x, an absolute value must be applied to the denominator of Equation B.19 in order to ensure a positive result for $p_{\hat{z}}(z)$.

2. Implicit in this derivation is that $\hat{y} \in \Re^+$. If this is not the case, the more general approach to the derivation of the power operation using the cumulative distribution (Equation B.1) must be used.

Appendix C

The Computational Model for Design Uncertainties

C.1 Introduction

The main body of this document presents a methodology by which uncertainties in preliminary engineering design can be represented and manipulated. This appendix discusses the component of the methodology that requires the computation of Zadeh's *extension principle*, *i.e.*, fuzzy arithmetic for the purpose of this study.

C.2 Computation of the Extension Principle

In Appendix A and Chapter 2, the analytical method of calculating a fuzzy output (application of Zadeh's *extension principle*) from imprecise inputs is shown. Although the method is straight-forward in its approach, the manipulation of symbols and the solution of expressions that include high order polynomials, both in the numerator and denominator (for extended division), make this method infeasible for computer-assisted design applications. This is compounded by the fact that the exact solution to the analytical application of the *extension principle* can be shown to be equivalent to a non-linear programming problem [6]. A discrete numerical approach is therefore necessary to meet the computational requirements for handling many design parameters. This section will discuss a useful numerical technique, the FWA algorithm, along with a number of extensions.

C.2.1 The FWA Algorithm

In reviewing the literature, many discrete and analytical methods exist for carrying out extended operations with fuzzy sets (or fuzzy numbers). The Fuzzy Weighted Average (FWA) algorithm as presented by W.M. Dong and F.S. Wong in [31] outlines a simple and efficient algorithm that the author finds to be useful for carrying out engineering design calculations. Comparing the algorithm to the analytical method outlined in Appendix A, FWA uses the interval analysis techniques as described; yet, FWA simplifies the process extensively by discretizing the membership functions of the input fuzzy numbers into a prescribed number of α-cuts (Appendix A.2.a). Performing interval analysis for each α-cut and combining the resultant intervals, the output is a discretized fuzzy set, the performance parameter output of input preference functions for the case of a design calculation. Dong and Wong also include a combinatorial interval analysis technique in order to avoid the problems of the multiple occurrence of variables for division and multiplication in the algebraic equation expression. A condensed version of the algorithm from [31] has been provided below (where the terminology has been changed to reflect the application to design calculations).

For N real imprecise design parameters, $\tilde{u}_1, \ldots, \tilde{u}_N$, let d_i ($i \in [1, N]$) be an element of $\tilde{u}_i$. Given a performance parameter represented by the mapping $p = f(d_1, \ldots, d_N) \forall d_i \in \tilde{u}_i$ respectively, let $\tilde{P}$ be the fuzzy output of the mapping. The following steps lead to the solution of $\tilde{P}$.

1. For each $\tilde{u}_i$, discretize the preference function into a number of α values, $\alpha_1, \ldots, \alpha_M$, where M is the number of steps in the discretization.

2. Determine the intervals for each parameter $\tilde{u}_i, i = 1, \ldots, N$ for each $\alpha-$cut, $\alpha_j, j = 1, \ldots, M$.

3. Using one end point from each of the N intervals for each α_j, combine the end points into an N-*ary* array such that 2^N distinct permutations exist for the array.

4. For each of the 2^N permutations, determine $p_k = f(d_1, \ldots, d_N), k = 1, \ldots, 2^N$. The resultant interval for the $\alpha-$cut, α_j, is then given by

$$P^{\alpha_j} = [min(p_k), max(p_k)].$$

The complexity of this algorithm is given in Chapter 2, Equation 2.3. Figure C.1 shows the results of the analytical application of the *extension principle* to the column equation (Equation C.1), along with the results using FWA.

$$\sigma_a = \frac{\pi^2 E}{n\left(\frac{Kl}{r}\right)^2}. \tag{C.1}$$

The input parameters for the maximum allowable stress σ_a (Equation C.1) are triangular preference functions as listed in Table C.1. No noticeable differences can be seen in the figure; however, minimal error is of course encountered due to discretization and the precision of the computer used.

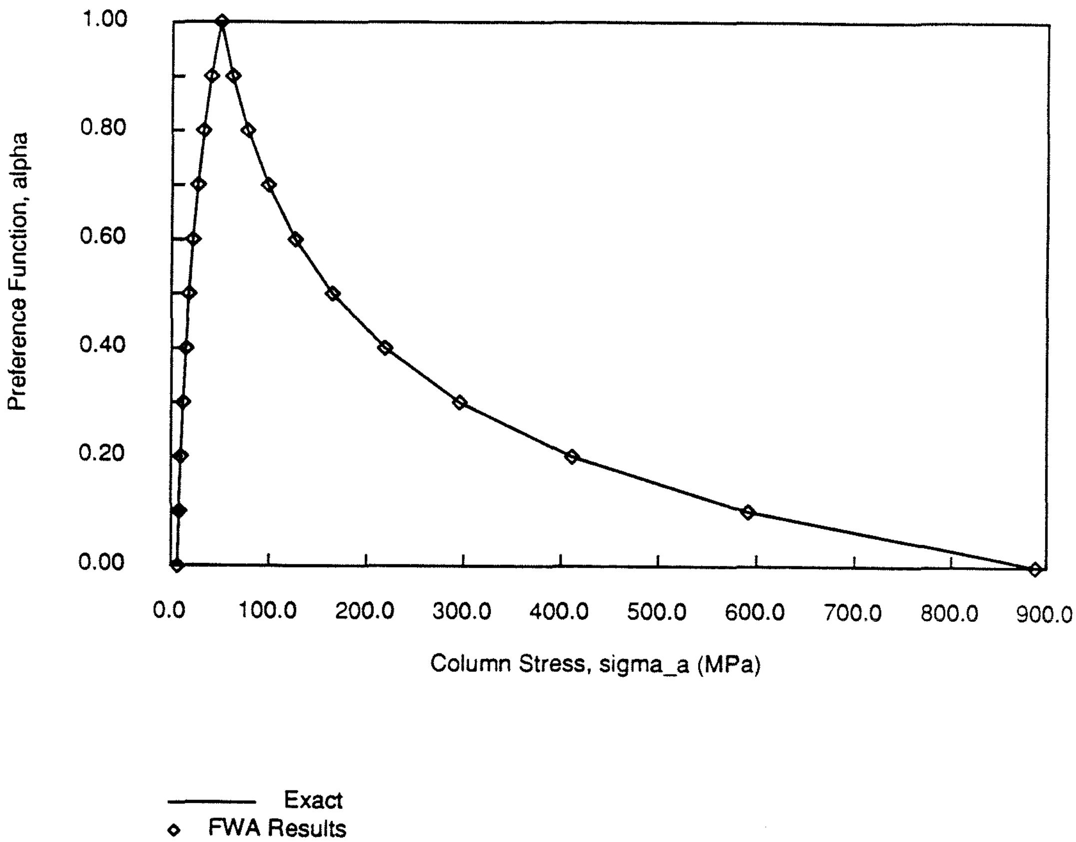

Figure C.1: Example Column Calculation: Exact and FWA

DPs (units)	$\alpha = 0$	$\alpha = 1$	$\alpha = 0$
n	1.0	3.0	5.0
l/r	60.0	100.0	160.0
E (GPa)	75.0	150.0	225.0
K (simply-supported)	1.0	1.0	1.0

Table C.1: Design Parameter Data for Column Equation.

C.2.1.1 Implementation Scheme for FWA

For the FWA to be an efficient tool within real-time computer-assisted design, the implementation scheme must be considered in detail. A new, highly computationally efficient implementation of Dong and Wong's algorithm [31] was developed by the author, and a current version, written in pseudo-code, is shown in Figure C.2.

The input array d contains the discretized elements for each input design parameter. For example, given two parameters, $\tilde{u}_1$ and $\tilde{u}_2$, and three α-cuts, $\alpha = 0.0$, $\alpha = 0.5$, and $\alpha = 1.0$, the d array is expressed as follows:

$$\begin{aligned} d = \; & [d_{11}, d_{12}, d_{13}, d_{14}, d_{15}, d_{16}, \\ & d_{21}, d_{22}, d_{23}, d_{24}, d_{25}, d_{26}] \end{aligned}$$

where the first index corresponds to the parameter number and the second to the elements in the parameter's support due to the α-cuts. Figure C.3 illustrates two possible examples (out of many, Chapter 2) of fuzzy parameters (preference functions) that can be expressed in the d array.

Looking further into the implementation scheme, another array (integer mask), is established in order to step through the d input array. The method consists of a bitwise system to determine the offset into the array. Employing the method conserves memory. Besides the masking system, the routine *func*() contains the actual algebraic expression to be solved to get the fuzzy output. In *func*() itself, standard software engineering and symbolic programming rules are applied to reduce the number of multiplications. Such rules have lead to the *consts*() function seen later in the code, which multiplies the result by any constants in the original expression that were not necessary in evaluating the variables *fmin* and *fmax*. The routine *consts*() may save at least 2^N multiplications and/or divisions, where N equals the number of fuzzy parameter inputs. Finally, the *fwa*() scheme calls another routine, *table_vals*().

```
Inputs:
(1) d, array of elements of input parameters.
(2) n, number of fuzzy inputs.
(3) M, number of discrete points.

Outputs:
(1) p, array containing fuzzy result.

fwa(d,n,M)

BEGIN
  integer ialpha_cuts, i, j,
      l, j_lim, icut, ioffset;
  integer mask[no_of_input parameters];
  real fvar[no_of_input parameters],
      f, fmin, fmax;

  ialpha_cuts = k / 2; /* No. of alpha cuts. */
  /* Create the bit masks to get a value in the d array.*/
  for i = 0 to (n-1)
    mask[i] = 2^i;
  for icut = 0 to (ialpha_cuts-1)
  BEGIN
    for i=0 to n-1
      fvar[i] = d[i*k+icut];
    fmin = func(fvar,n);
    fmax = fmin;
    j_lim = 2^n - 1;
    for j=1 to j_lim
    BEGIN
      for l=0 to (n-1)
      BEGIN
        if((mask[l] AND j) == mask[l])
          ioffset = k - icut - 1;
        else
          ioffset = icut;
        fvar[l] = d[l*k+ioffset];
      END;
      f = func(fvar,n);
      fmin = min(fmin,f);
      fmax = max(fmax,f);
      table_vals(f,fmin,fmax,fvar,n,M,icut);
    END;
    consts(fmin,fmax);
    p[icut] = fmin;
    p[k-icut-1] = fmax;
  END;
  return results in p[], with implied α values from ialpha_cuts;
END;
```

Figure C.2: FWA Pseudo-Code.

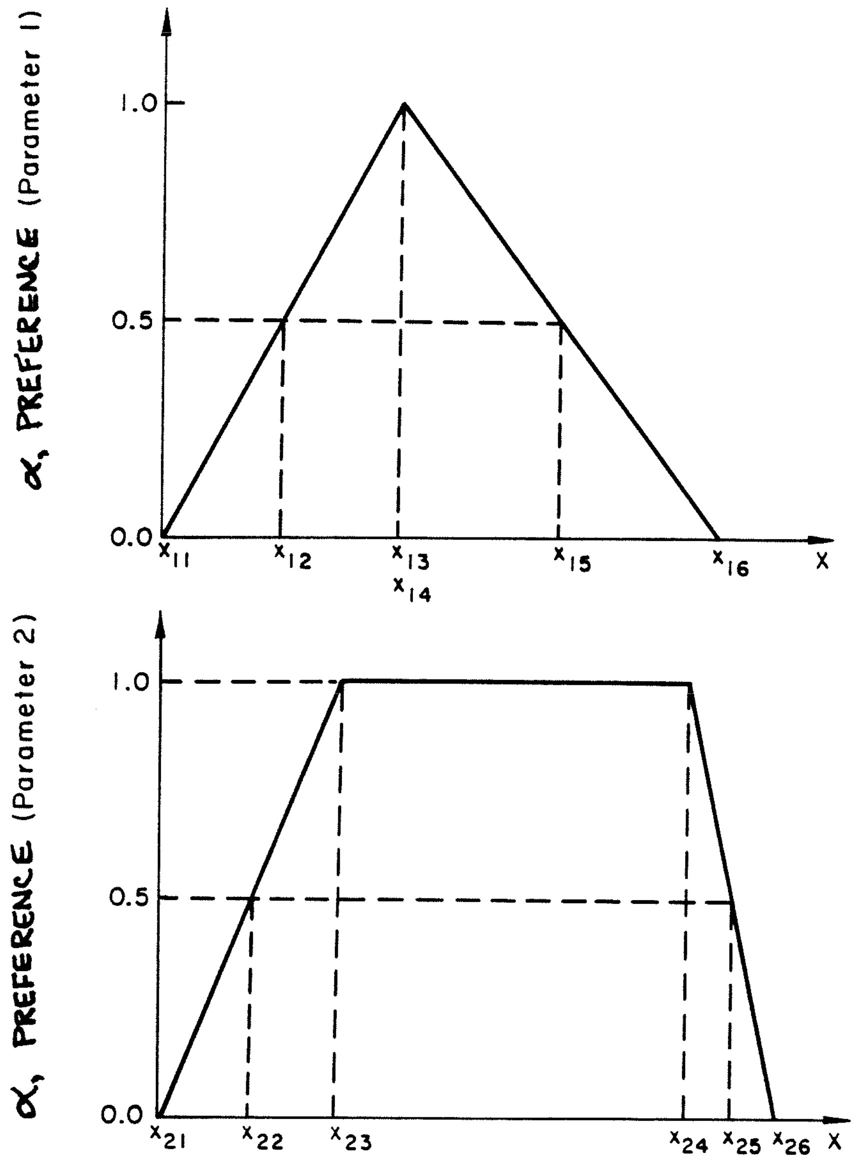

Figure C.3: Example of d Array

table_vals() constructs a look-up table of the input parameters that form the discrete outputs lying on the actual membership curve at the α-cut levels (used in the *backward path*). Interpolation may be employed to determine the values of the corresponding inputs for other points on the curve.

The algorithmic complexity of the implementation (repeated from Equation 2.3) may be determined. For $n = N$, the complexity is of order

$$H \sim M \cdot 2^{N-1} \cdot \kappa$$

where H equals the number of operations and κ equals the number of multiplications and divisions in the routine *func*(). From a practical standpoint, a minicomputer or workstation executes this routine (binary operations only in *func*()) essentially in real-time for at least fifteen fuzzy input parameters for one equation. When adding trigonometric functions to the expression in *func*(), a time delay of seconds becomes apparent for around ten inputs.[1] Overall, the complexity result demonstrates the efficiency of FWA and its implementation scheme.

C.2.1.2 Extending FWA to Numbers of Type 2

While the FWA algorithm along with its implementation scheme handle calculations with fuzzy numbers of *type-one* very efficiently, further work has shown that fuzzy numbers of *type-two* may be required to provide a means for dealing with and interpreting input parameters that have geometrical or other physical dependencies. By definition (Appendix A), a *fuzzy set of type-m* consists of a fuzzy set with membership values that are themselves fuzzy sets of *type*-$m-1$, where m is an integer, $m > 1$.

[1] These qualitative comments are generalizations of benchmarks taken from a Sun Microsystems 3/160 workstation without a floating point accelerator. Of course, the comments here will vary somewhat depending on the expression in *func*().

FWA must be extended to handle fuzzy numbers of *type-two*. The extension procedure can be executed in a straightforward manner by separating the fuzzy number of *type-two* into two functions: $\mathcal{A}$ and $\mathcal{B}$, as shown in Figure C.4. Extended operations can be performed with fuzzy numbers of *type-two* in the same manner as described previously for FWA. The only enhancement concerns the fact that the function $\mathcal{B}$ is not normalized (its maximum value is not one (1.0), Appendix A), resulting in the need to change the input format of the d array to deal with α ranges besides $\alpha \in [0, 1]$. Once the output intervals for each function $\mathcal{A}$ and $\mathcal{B}$ are determined, they can be combined to form the output fuzzy result of *type-two*.

C.2.2 Revising the FWA Approach

C.2.2.1 Reducing the Complexity

In the previous sections of this appendix, the FWA algorithm was presented in detail. Extensions to the algorithm in terms of implementation, reducing the complexity of the functional expression, and computations with fuzzy numbers of *type-two* have been shown. Although the extended algorithm, as it stands, is usable for *real-time* engineering design calculations, its complexity can be reduced further. An implementation of the methods of interval analysis [57] is the key to the reduction in complexity.

C.2.2.1.1 Interval Analysis Definitions Certain definitions and theorems as presented by Moore [57] are needed to construct and explain the reduction in complexity of the FWA algorithm (as developed in this document). These definitions and theorems are provided below, where Moore [57] can be referenced for the details and

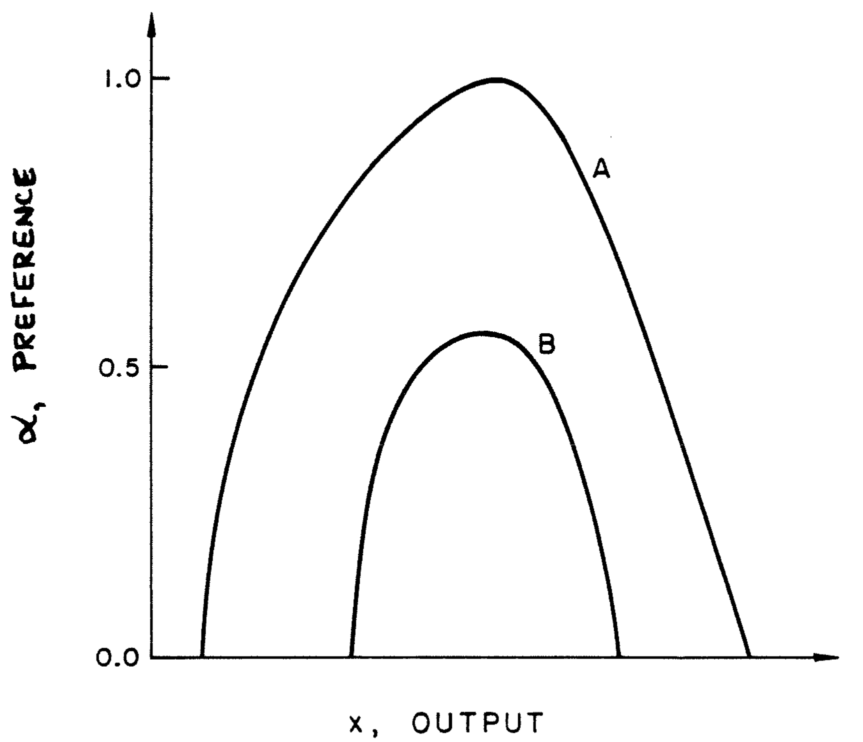

Figure C.4: Fuzzy Number of Type 2

proofs. [2]

Definition C.1 *An* **interval number** *($\mathcal{X}$) is a closed bounded set of real numbers such that*

$$\mathcal{X} = [\underline{x}, \overline{x}] = \{x : \underline{x} \leq x \leq \overline{x}\}.$$

Definition C.2 *The* **width** *of an interval number $\mathcal{X} = [\underline{x}, \overline{x}]$ is defined by $w(\mathcal{X}) = \overline{x} - \underline{x}$. Likewise, the* **center** *of an interval number $\mathcal{X}$ is determined from $c_{\mathcal{X}} = (\overline{x} + \underline{x})/2$.*

Definition C.3 *Let Λ and Υ be arbitrary sets, and let $g : \Lambda \to \Upsilon$ be an arbitrary mapping (function) from Λ into Υ. Denoting $S(\Lambda)$ and $S(\Upsilon)$ as the families of subsets of Λ and Υ respectively, the* **set-valued mapping,** *$\overline{g} : S(\Lambda) \to S(\Upsilon)$,*

$$\overline{g}(\mathcal{X}) = \{g(x) : x \in \mathcal{X}, \mathcal{X} \in S(\Lambda)\}$$

is the **united extension** *of g. This can also be written as*

$$\overline{g}(\mathcal{X}) = \bigcup_{x \in \mathcal{X}} \{g(x)\}.$$

Definition C.4 *A* **rational interval function** *is a function whose interval values are defined by a specified finite sequence of interval arithmetic operations.*

Definition C.5 *Using Definition C.3, and the assumptions therein, the united extension $\overline{g} : S(\Lambda) \to S(\Upsilon)$, has the* **subset property***:*

$$\mathcal{X}, \mathcal{Y} \in S(\Lambda) \quad with\ \mathcal{X} \subseteq \mathcal{Y} \implies \overline{g}(\mathcal{X}) \subseteq \overline{g}(\mathcal{Y}).$$

[2] Definitions C.1 through C.10 and Theorems C.1 through C.4 are originally presented in Moore [57], Chapters 1 through 4.

Definition C.6 *Let f be a real valued function of n real variables $x_1, x_2, \ldots, x_n$. An* **interval extension** *of f is an interval valued function F of n interval variables $\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n$ with the property*

$$F(x_1, x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n), \quad \textit{for real arguments.}$$

Definition C.7 *An interval valued function F of the interval variables $\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n$ is* **inclusion monotonic** *if*

$$\mathcal{Y}_i \subseteq, \quad i = 1, 2, \ldots, n,$$

implies

$$F(\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_n) \subseteq F(\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n).$$

United extensions, which all have the subset property, are inclusion monotonic. Interval arithmetic is inclusion monotonic, as are rational interval functions and the natural interval extensions of all the standard functions used in computing.

Definition C.8 *An interval extension F is* **Lipschitz** *in $\mathcal{X}_0$ if there is a constant L such that $w(F(\mathcal{X})) \leq L \cdot w(\mathcal{X})\ \forall \mathcal{X} \subseteq \mathcal{X}_0$.*

Lemma C.1 *If a real valued function $f(x)$ satisfies an ordinary Lipschitz condition in $\mathcal{X}_0$, i.e., $\mid f(x) - f(y) \mid \leq L \cdot \mid x - y \mid$ for x and y in $\mathcal{X}_0$, then the united extension of f is a Lipschitz interval extension in $\mathcal{X}_0$.*

Definition C.9 *A* **uniform subdivision** *of an interval vector $\mathcal{X} = (\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n)$ is defined as follows. Let N be a positive integer. Define $\mathcal{X}_{i,j} = [\underline{x}_i + (j-1) \cdot w(\mathcal{X}_i)/N, \underline{x}_i + j \cdot w(\mathcal{X}_i)/N]$, $j = 1, 2, ldots, N$. Then $\mathcal{X}_i = \bigcup_{j=1}^{N} \mathcal{X}_{i,j}$ and $w(\mathcal{X}_{i,j} = w(\mathcal{X}_i)/N$. Furthermore, $\mathcal{X} = \bigcup_{j_i=1}^{N} (\mathcal{X}_{1,j_1}, \mathcal{X}_{2,j_2}, \ldots, \mathcal{X}_{n,j_n})$ with $w(\mathcal{X}_{1,j_1}, \ldots, \mathcal{X}_{n,j_n}) = w(\mathcal{X})/N$.*

Definition C.10 *Let $f(x_1, x_2, \ldots, x_n)$ be a rational function. Rewriting f in the following manner,*

$$f(x_1, \ldots, x_n) = f(c_1, \ldots, c_n) + g(x_1 - c_1, \ldots, x_n - c_n),$$

an interval extension $F(\mathcal{X})$ of f derived from the righthand side of the above equation is said to be in **centered form**.

Theorem C.1 *If $F(\mathcal{X})$ is an inclusion monotonic, Lipschitz, interval extension for $\mathcal{X} \subseteq \mathcal{X}_0$, then the excess width of a refinement, $F_{(N)}(\mathcal{X})$, the union of interval values of F on the elements of a uniform subdivision of $\mathcal{X}$, is of order $1/N$. This gives*

$$F_{(N)}(\mathcal{X}) = \bigcup_{j_i=1}^{N} F(\mathcal{X}_{1,j_1}, \ldots, \mathcal{X}_{n,j_n}) = \overline{f}(\mathcal{X}_1, \ldots, \mathcal{X}_n) + \mathcal{E}_N$$

and there is a constant K such that

$$w(\mathcal{E}_N) \leq K \cdot w(\mathcal{X})/N,$$

where $\mathcal{E}_N$ is the error compared with the exact solution. If $F(\mathcal{X})$ is in centered form, the corresponding width of the error interval of the N^{th} refinement is

$$w(\mathcal{E}_N) \leq K_c \cdot w(\mathcal{X})/N^2,$$

for some K_c.

Theorem C.2 *If there does not exist a multiple occurrence of the real variables $x_1, x_2, \ldots, x_n$ in a given real valued function $f(x_1, x_2, \ldots, x_n)$, then the interval extension $F(\mathcal{X})$ corresponding to $f(x)$ is the united extension of f for all $\mathcal{X} \subseteq \mathcal{X}_0$.*

Theorem C.3 *Let $F(\mathcal{X}_1, \ldots, \mathcal{X}_j, \ldots, \mathcal{X}_n)$ be a rational interval function. If $\mathcal{X}_j$ occurs only once in F and*

$$\mathcal{X}_j = \bigcup_{i=1}^{N} \mathcal{X}_j^{(i)},$$

then

$$F(\mathcal{X}_1, \ldots, \mathcal{X}_j, \ldots, \mathcal{X}_n) = \bigcup_{i=1}^{N} F(\mathcal{X}_1, \ldots, \mathcal{X}_j^{(i)}, \ldots, \mathcal{X}_n)$$

for all $\mathcal{X} \in \mathcal{X}_0$.

Theorem C.4 *Let* $F(\mathcal{X})$ *be a rational interval function written in centered form. Each of the interval variables* $\mathcal{X}_{p+1}, \ldots, \mathcal{X}_n$ *occurs just once in* F. *Subdividing each of the interval variables* $\mathcal{X}_1, \ldots, \mathcal{X}_p$ *so that*

$$\mathcal{X}_i = \bigcup_{j=1}^{N} \mathcal{X}_i^{(j)} \text{ with } w(\mathcal{X}_i^{(j)}) = \frac{1}{N} w(\mathcal{X}_i),$$

there is a positive number K *such that*

$$\bigcup_{i_1=1}^{N} \cdots \bigcup_{i_p=1}^{N} F(\mathcal{X}_1^{(i_1)}, \ldots, \mathcal{X}_p^{(i_p)}, \mathcal{X}_{p+1}, \ldots, \mathcal{X}_n) = \overline{f}(\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n) + \mathcal{E}_N$$

where

$$w(\mathcal{E}_N) \leq \frac{K}{N^2} w(\mathcal{X}).$$

C.2.2.1.2 Applying the Methods of Interval Analysis The interval analysis definitions and theorems given above can be used to reduce the complexity of the FWA algorithm. Using design terminology and nomenclature, the problem is to compute the performance parameter results p_j, $j = 1, 2, \ldots, m$ at each α-cut for a class real valued design functions $DF(\mathcal{X}_0)$, where $p_j = f(d_1, d_2, \ldots, d_n)$. The $DF(\mathcal{X}_0)$ compose the arithmetic functions $\{+, -, /, \cdot\}$ and unary functions $\{\sin(\cdot), \cos(\cdot), \exp(\cdot), \ln(\cdot), \sqrt{\cdot}$, etc.$\}$, ***i.e.*****, all standard** computing functions. Assuming that a function f in the class of functions $DF(\mathcal{X}_0)$ is defined for all $\mathcal{D} \in \mathcal{X}_0$, and assuming that no division by zero or unary operation of zero for $\{\ln(\cdot) or \sqrt{\cdot}\}$ occur, the interval extensions $F(\mathcal{D})$ of f will be Lipschitz and inclusion monotonic.

Under these conditions, Theorem C.1 can be applied such that the performance parameter output interval is obtained for arbitrarily sharp bounds (the excess width) for each discretized α-cut. Denote the output interval as $P_j^\alpha(\vec{D}^\alpha)$. When the d_i, $i = 1, 2, \ldots, n$, occur only once in f, only one computation is required (Theorem C.2) to obtain exact bounds for $P_j^\alpha(\vec{D}^\alpha)$ (within the precision of the computation). For multiple occurrences of the d_i, a number of refinements can be applied to determine arbitrarily sharp bounds. Alternatively, as shown by Dong and Wong [31], a combinatorial interval analysis scheme can be used to determine exact bounds for a function f with multiple occurrences of variables. Section C.2.1 describes this scheme algorithmically.

To reduce the complexity of the FWA algorithm, two changes to Dong's and Wong's approach are required:

1. Apply Theorem C.2 and Theorem C.3 such that the permutations in the combinatorial step of the FWA algorithm are applied only to the variables that are repeated; and

2. Use interval arithmetic to determine the bounds instead of normal single-valued arithmetic.

These changes affect the FWA algorithm as follows. For N real imprecise design parameters, $\tilde{u}_1, \ldots, \tilde{u}_N$, let x_i ($i \in [1, N]$) be an element of $\tilde{u}_i$. Given a performance parameter represented by the mapping $p = f(d_1, \ldots, d_N) \forall d_i \in \tilde{u}_i$ respectively, let $\tilde{P}$ be the fuzzy output of the mapping. The following steps lead to the solution of $\tilde{P}$:

1. For each $\tilde{u}_i$, discretize the preference function into a number of α values, $\alpha_1, \ldots, \alpha_M$, where M is the number of steps in the discretization.

2. Determine the intervals for each parameter $\tilde{u}_i, i = 1, \ldots, N$ for each $\alpha-$cut, $\alpha_j, j = 1, \ldots, M$.

3. Given p unrepeated design parameters in f and $N-p$ repeated DPs, separate the intervals for the $\tilde{u}_i$ as follows $\vec{\tilde{u}} = \{\tilde{u}_1, \ldots, \tilde{u}_p, \tilde{u}_{p+1}, \ldots, \tilde{u}_N\}$, where the repeated variables are contained in $\vec{\tilde{u}}$ between $\tilde{u}_{p+1}$ and $\tilde{u}_N$.

4. Using one end point from each of the $N-p$ intervals for each α_j, combine the end points into an $(N-p)$-*ary* array such that 2^{N-p} distinct permutations exist for the array.

5. For each of the 2^{N-p} permutations, determine through interval computations $p_k = f(d_1, \ldots, d_N), k = 1, \ldots, 2^{N-p}$. The resultant interval for the α−cut, α_j, is then given by

$$P^{\alpha_j} = [\min(\underline{p}_k), \max(\overline{p}_k)].$$

The complexity of this modified algorithm is

$$H \sim M \cdot 2^{N-p-1} \cdot v$$

where $N-p$ is the number of repeated design parameters in the performance expression and v equals the number of interval operations in the expression $f(d_i)$. As p approaches N, the new algorithm is much more efficient than the original FWA; however, there does exist a trade-off in overall complexity because the modified version of FWA uses interval operations whereas the original does not. For implementations of the modified algorithm where the interval operations are carried out in assembly code, the effect will not be dramatic. But when the interval operations are implemented in subprogram calls, and when p is much less than N, the values of κ and v should be calculated to determine whether the modified FWA or the original will perform better.

C.2.2.2 Extending FWA for Internal Extrema

The FWA algorithm and extensions presented above are valid only for real valued functions f, and corresponding interval extensions $F(\mathcal{X}_0)$ that do not include internal *extrema* for the intervals in question, $\mathcal{X} \in \mathcal{X}_0$. This is because only the endpoints (at a given α-cut) of the input parameters d_i $i = 1, 1, \ldots, n$ are used in the computation. If internal extrema are present for a given α-cut α_j and calculation of $P^{\alpha_j}(\vec{D}^{\alpha_j})$, the FWA algorithm can be extended to determine the correct bounds of $P^{\alpha_j}(\vec{D}^{\alpha_j})$ by the implementing the following procedure:

1. For each α-cut α_j, determine if an internal extrema exists for the α-cut intervals of d_i $i = 1, 2, \ldots, n$, $p = f(d_1, d_2, \ldots, d_n)$. This may be accomplished by either analytically or numerically solving

$$\frac{\partial p}{\partial d_i} = 0$$

   for each d_i.

2. Denoting the extrema by ξ_l, and denoting the values of the d_i that make up a given ξ_l by ε_i, determine for each ξ_l if every ε_i lies in the α-cut intervals for α_j.

3. If the above condition is true, compare the calculated extrema $f(\xi_l)$ with P^{α_j} from the FWA algorithm such that

$$P^{\alpha_j} = [\min(\underline{p}^{\alpha_j}, f_l(\xi_l)), \max(\overline{p}^{\alpha_j}, f_l(\xi_l))]$$

   for all l.

Alternatively, Skelboe [70] developed a *general* algorithm for computing interval expressions with or without internal extrema. Skelboe's approach, which implements Theorem C.3 and Theorem C.4, relies on a subdivision of the argument intervals of an expression and a subsequent recomputation of the expression with the new intervals. His approach can compute refinements of an interval extension to arbitrarily sharp bounds. In the best case, the Skelboe algorithm can outperform the FWA algorithm, even with extensions, when internal extrema are present. However, the complexity of the Skelboe algorithm is dependent on the subdivision structure and the placement of the internal extrema with respect to the subdivisions. Such a dependency causes the upper bound on complexity to greatly exceed that of the *guaranteed* complexity of the extended FWA algorithm.

C.3 Remarks on the Computational Model

The FWA algorithm and extensions presented in this appendix provide a basis for computing with sets of *imprecise* parameters in preliminary engineering design. When combining the extended FWA with other tools, real-time representation and manipulation of design uncertainties are not only feasible but practical with current computing technology. The other tools, in addition to the extended FWA, that were used in completing the design examples in this study include: an efficient method for computing the stochastic component of the extended hybrid representation (briefly discussed in Chapter 6), a factorial functional coupling tool relying on both sensitivities and the γ-level measure, an interface for constructing preference curves and probability density functions for the imprecise and stochastic components of the design parameters, a language tool for syntactically representing and parsing a PPE description of a design alternative, and a symbolic computing approach for manipulating PPE design descriptions and for reducing computational complexity.

Appendix D

A Model of Engineering Design as a Process

D.1 Summary

The following appendix is a portion of a previous publication [81], and has been included for completeness. The primary thesis of this appendix is to present a general framework for the design process and for developing computational design tools.

D.2 Introduction

Computers have impacted much of the scientific and engineering work carried out today. Engineering design is no exception. However, at present, most of the computational efforts in engineering design are in the area of analysis. The facet of CAD that has received the most attention up to now (particularly in mechanical systems) has been the very last portion of the design process, that stage when very little of the design is subject to change, but the designer is interested in a sophisticated analysis of some of the parts. Thus contemporary CAD is usually single-part oriented, and in general is largely used as a tool for finite-element analysis and other analyses of a similar nature. The current status of CAD (Computer Aided [Engineering] Design) may really be viewed as *Computer Analysis in Design.*

Though computational analysis is obviously needed, this research emphasizes the synthesis and decision components of engineering design. The focus on analysis in contemporary CAD doesn't contribute to the underlying purpose of design work, i.e., "a creative and purposeful activity directed toward the goal of fulfilling human needs" [5]. Computation has the potential for far greater impact on the design process in the early, preliminary, conceptual stages of a design. The objective of this appendix is to outline an approach by which computational tools may be constructed for the synthetic part of the engineering design process, as well as the analytic.

At a basic level, engineering design combines art and applied science. These two characteristics contain, to a great extent, many *uncertain* (or imprecise) terms, for example the concepts of *safety* and *economy* are two contributing factors to such terms. Overall, the synthetic and analytic problems are complicated by uncertainty, both in the design requirements and in the proposed solutions (at the preliminary phase). The work of Yao [86], Blockley [20], Becker [13], and Rao [60] provide further insight into engineering design in this context. For example, uncertainty might be defined according to the guidelines of Yao and Furuta [86]:

> Generally, the term 'uncertainty' may be associated with ambiguity, fuzziness, randomness, vagueness, and imprecision of the events under consideration. These ... uncertainties may be delineated from one another as follows:
>
> - Randomness is due to factors in the complex phenomena which are random in nature.
> - Fuzziness results from the complexity of natural events, the knowledge of which is imprecise and/or incomplete, and/or subjective.
> - Ambiguity results from the use of natural languages which can be

meaningful but not clearly defined.

- Blur or vagueness is accompanied with inexact and/or ill-defined figures, pictures and scenes ...

Yao and Furuta combine the last three items into a more general expression of *fuzziness*, where the first item remains as the pure expression of randomness. Notice the lack of cohesiveness and clarity in the description of uncertainty above. In fact, there exists some degree of circular definitions. These difficulties demonstrate the problems of performing analysis in an uncertain domain. If we correlate the definition(s) of uncertainty into some representative relationship structure (as did Yao and Furuta – Figure D.1), we may conjecture that a large portion of engineering design problems fall into a highly complex, imprecise, and uncertain domain, while the minority exist in the classical deterministic space.

Many problems in engineering design (especially during the preliminary stages) can be better understood with an ability to manipulate imprecision and uncertainty. In this appendix, preliminary and proposed work in this direction is presented. Section D.3 presents a general philosophy of engineering design and the design process. A systematic approach to design is constructed in Section D.4. Section D.5 outlines more specific computational aides in this morphological structure. The final section is devoted to concluding remarks.

D.3 Design Philosophy

It will be useful to develop a model of the engineering design process and its relation to the work reported here. The task of developing a theory describing the engineering design process has begun only in the last century [61] [63]. Many engineers have

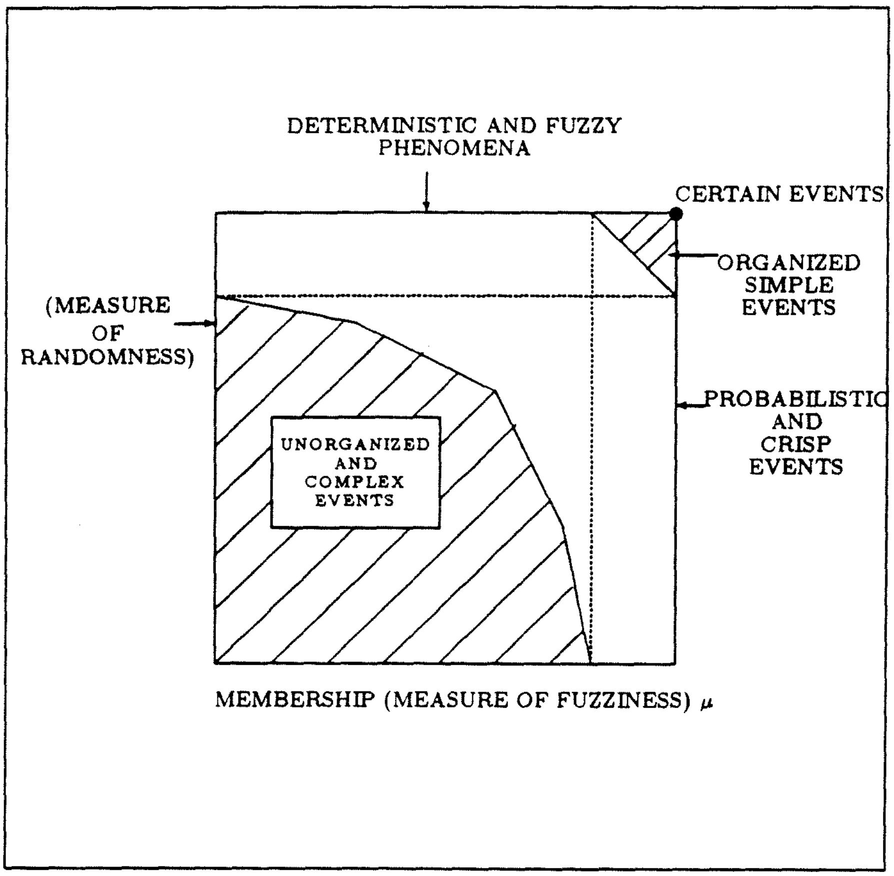

Figure D.1: Randomness vs. Fuzziness – A Deterministic Relation.

attempted to construct a model by first studying the work of psychologists in the area of human thought. While these efforts have contributed to the general knowledge, this paper will not focus at this psychological level; rather, an effort will be made to study the actions of the engineer in the process of a design, not necessarily the thoughts he has while at work.

During an analysis of previous work in the area of a design philosophy, certain terms have surfaced as being fundamental, such as the view that design is an *iterative* process. The term *problem solving* seems to be equally pervasive. Becker's work [13], as previously cited, describes in more detail engineering design as a problem solving. Of course, we have an intuitive feel for this in our everyday existence. For example, starting with a problem (or need), e.g., building a fence for our yard, we define the problem, apply creative thought and preliminary analysis (arrange and develop the structure of the fence), delineate options (type of fences: wood, chain-ink, etc.), perform analysis in more detail (price options, analyze life-expectancy, determine aesthetic value), and then select the solution (choose 'best' fence).

Along with these ideas, another basic trend in the literature is apparent, namely that design may be broken down into three primary phases [13]: *synthesis*, *evaluation*, and *decision*. Equivalently, according to Adams [1], the terms may be stated as synthesis, analysis and judgement. Even though these three areas may be designated as distinct entities within design, the boundaries separating them are always imprecise. When constructing a model for the process, it should be kept in mind that design is not a recipe, but a changing process within each problem case.

Let us now consider these three dominant characteristics of design as listed. *Analysis*, more than the others, has been developed extensively for all areas of engineering. In CAD systems today, most efforts have been devoted to the development of computational systems specially constructed for the later stages of design. While preliminary analysis obviously exists in the process, these CAD systems are oriented primarily toward refinement and evaluation of particular solutions previously developed (often by hand). Analysis, of course, exists at every stage in the design process.

In many ways, *decision theory* [72] has been developed as extensively as analysis. However, decision theory has been applied in only a rudimentary way to design. In fact, decision in the design process exists more as an 'arbitrary' judgement [1] by an engineer or a group of engineers. Similar to analysis, decision exists at all stages of a design.

Synthesis in preliminary or conceptual design represents the most creative of the three characteristics. Harris [41] provides a description of preliminary design:

> The designer ... collects and assimilates as much fact as he can relevant to his design, using the full gamut of analytical technique, if needed. He examines it, turns it over, changes it round, immerses himself in it, lets it sink into his subconsciousness, drags it out again, walks all round it, prods it. The hope is that, at some unsuspected moment, by who knows what mysterious process of imagination, intellect or inspiration, by influence of the genius, the daemon, the muse – the brilliant, the obvious, the definitive concept of the work will flash into the mind.

Figure D.2 and Figure D.3 show the five basic descriptors (iterative, problem solving, synthesis, analysis and decision) in models of the design process. Both French's and Asimow's models conform to the major ideas presented here. Three other characteristics of the design model to be included are: (1) the externalization of design, (2) the evaluation and re-evaluation of the overall need, and (3) a representation for the structure of decisions among alternative solutions [5], [35], [13] and

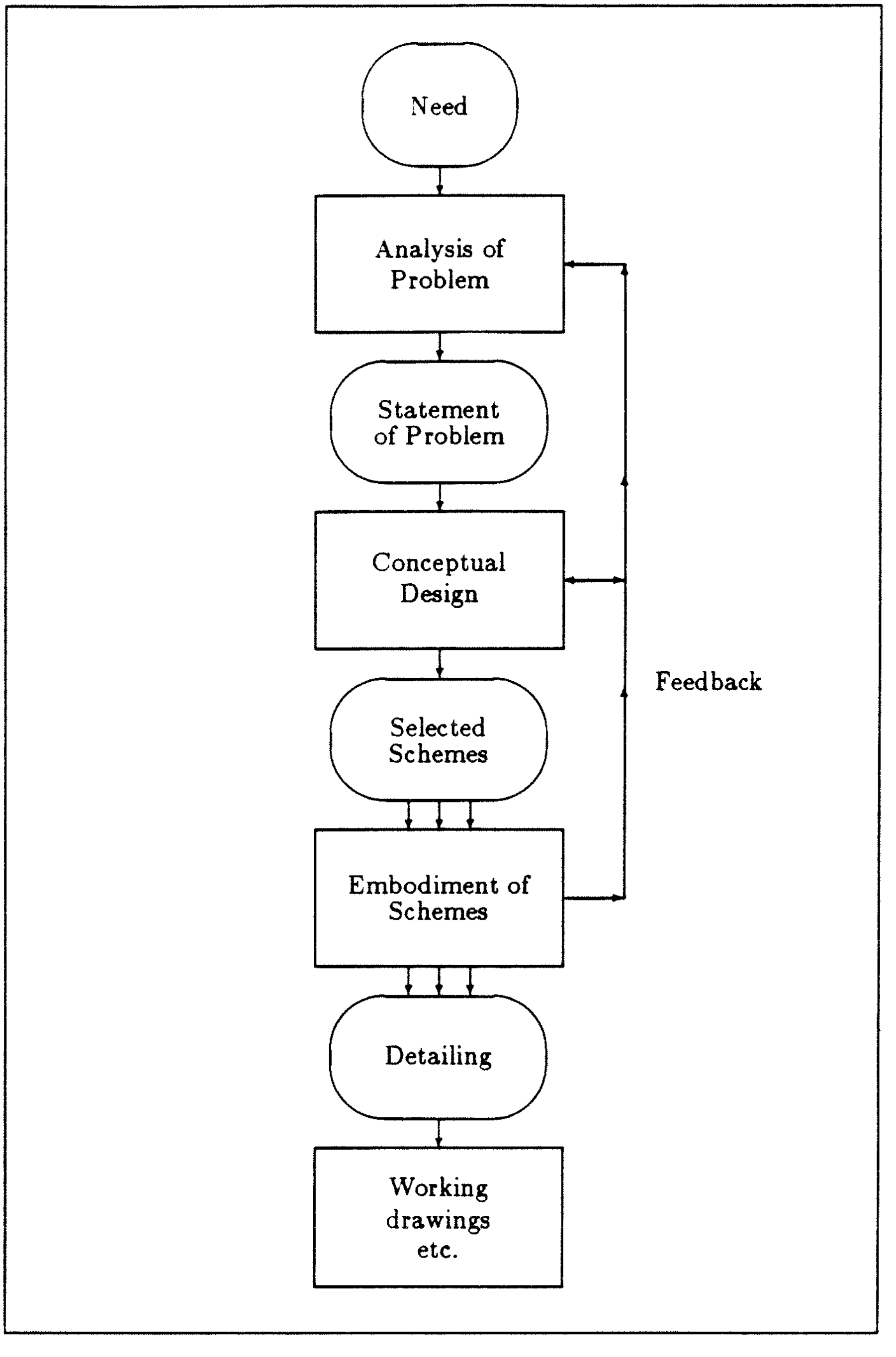

Figure D.2: French's Block Diagram of Design.

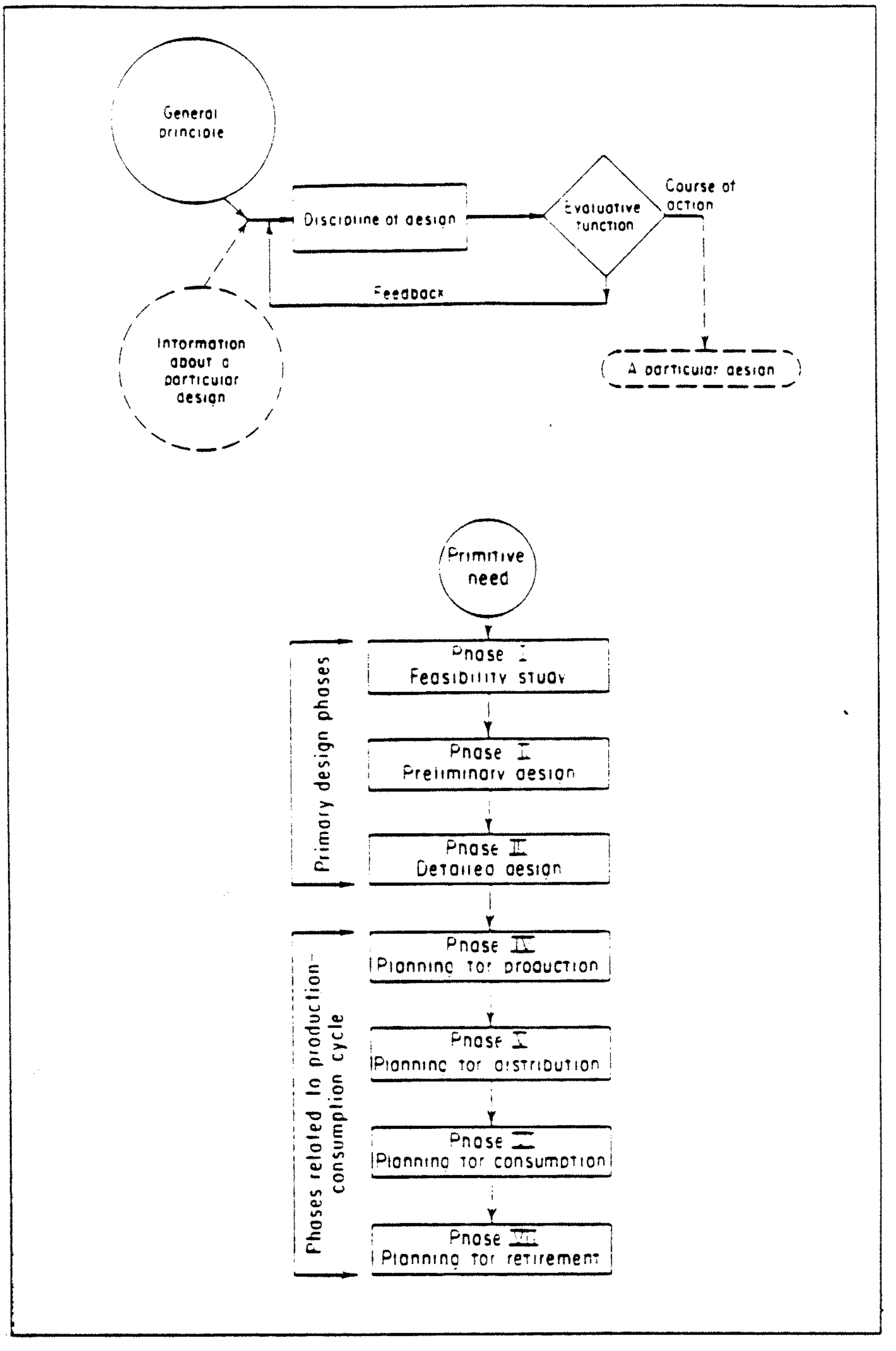

Figure D.3: Asimow's Iterative Approach.

[1].

Externalization is the process of delineating which information (from the universe of engineering information) to include and which to not include in the process of design. For example, when designing a bridge, certain codes and specifications must be used. Past experience is also a possible source of external information. Externalization as an important ingredient of a design model becomes clear in several ways: to avoid redesigning something that has already been designed, to avoid leaving an important design element or consideration out, to create a situation in which an engineer is conscious of his decisions among relevant and unrelevant information, and to emphasize the need to evaluate the information used in a given project.

Along with externalization, a re-evaluation of the original need should take place periodically in the design process. This evaluation should be in the form of feedback of the analysis results and knowledge obtained back to a consideration of the problem statement. For example, let an initial need be stated as: design a rover vehicle for sample collection on Mars. One design requirement is to collect samples over a surface area of 1000 square kilometers. Because of the phrasing used in the initial description of the need, a common 'rover' vehicle (e.g., wheeled, track, or legged) will likely be designed so that it meets the performance specification listed, along with other design considerations such as high fuel efficiency (low weight), long mission time requirements, etc. We observe in this design approach that due to the term 'rover', the design progressed forward as if the typical type of wheeled rover was an intrinsic component. No evaluation of the original need was carried out. If such an evaluation had occurred, the result may have been a ballistic-hopping vehicle [83] that can collect samples over a greater area of terrain with a higher probability of success than a conventional rover.

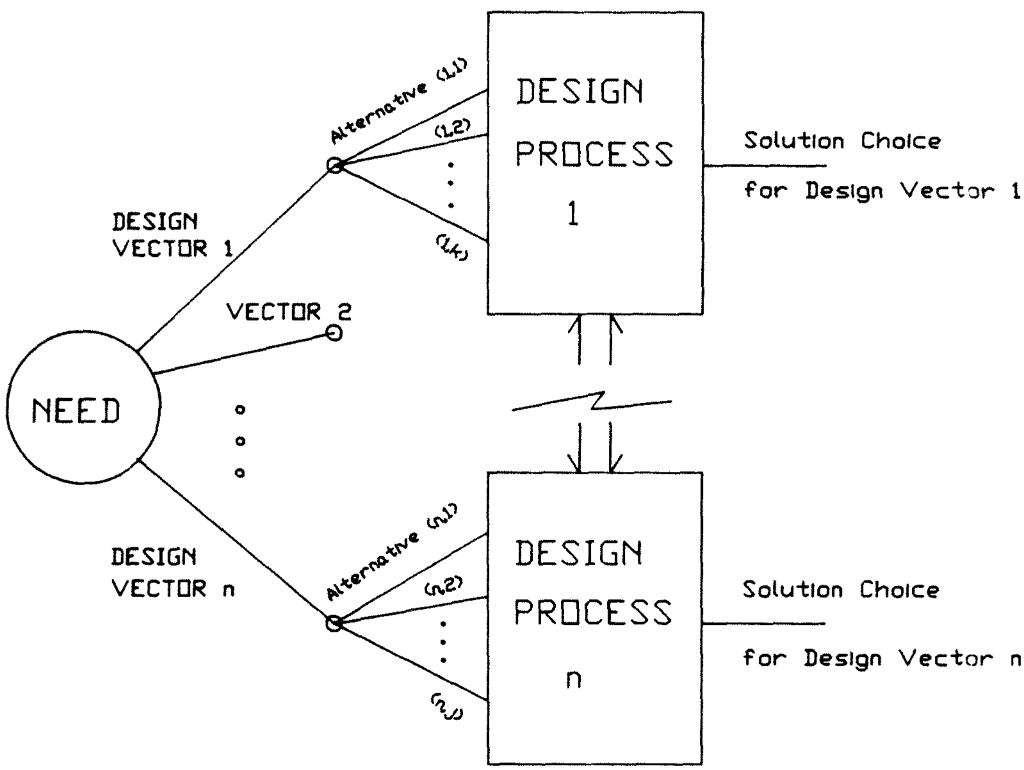

Figure D.4: The Tree Structure of Decisions.

The third of the three additional characteristics of design is to structure decisions among alternative solutions. One element of the earlier discussion was *decision*; however, that section dealt primarily with decisions during the creation of a single design alternative. No symbolic representation was presented describing the way in which decisions among alternatives are made. Indeed, considering most of the design literature, authors devote very little effort to the area, even though decision appears to be second only to the creative phase of design. To help develop a formalism we present a preliminary model of the structure of the decisions or alternative solutions within a design (see Figure D.4).

The structure resembles a tree, where the need (and its refinement) is the root, and overall design choices (or design vectors) make up the branches. Each design vector represents one functional requirement or constraint [75] [76], and each design vector has a number of different solutions, shown as leaves. For example, to use the Mars rover again, one vector might be the ability to maneuver among obstacles of a certain size and spacing. This vector might have four alternatives: legs, wheels, flight, and hopping. During the design process each vector's solution alternatives are evaluated, and a solution is suggested, which may depend upon the solution choice made for other vectors. Thus the leaves symbolize the various detailed choices as a design progresses such that certain leaves become pruned when a decision is made, while others are carried through to the last decision stage. Even though branches usually originate from the root, branches may form from other branches when a certain vector must be factored into two, three, or n *distinct and important* choices to be considered. Overall only one leaf exists on each branch at the end point when the final decision for each vector in the design takes place.

Using the philosophical characteristics described above, we propose a model

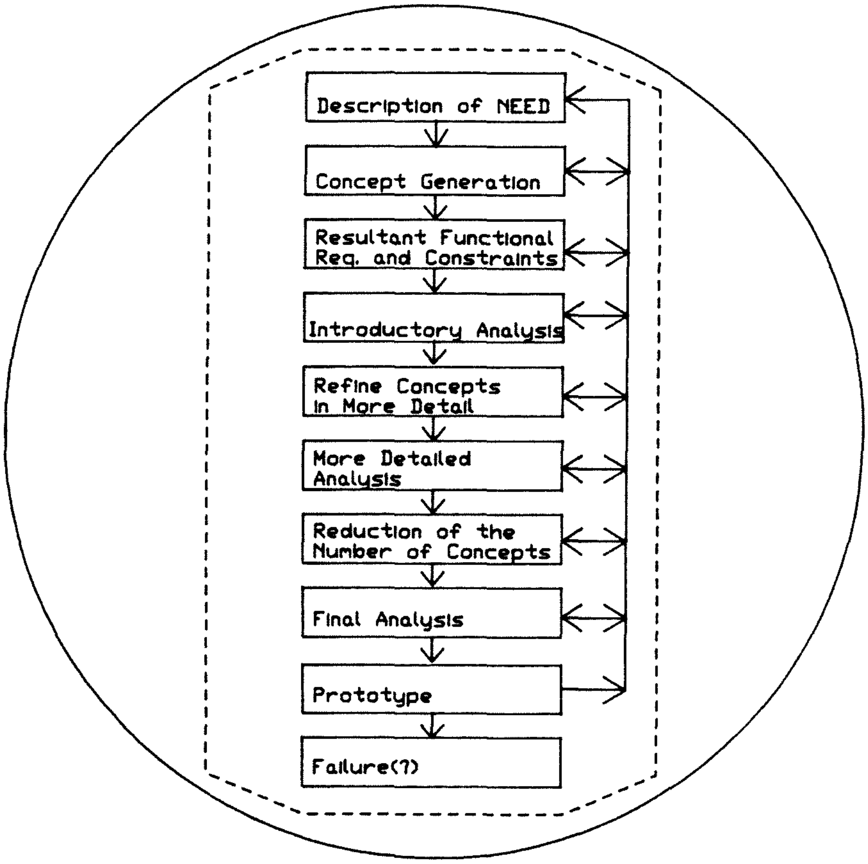

Figure D.5: A Design Model.

of the engineering design process as shown in Figure D.5 [2]. The overall elements shown will be briefly described. The circle acts as the externalization boundary in the process, delineating unnecessary information excluded from important information included. The solid line denoting feedback from the various blocks represents the iterative process in design. Note that feedback can occur from any stage in the process back to any previous stage for more work (as required) for the design. Iteration is concluded, and a solution is declared when time or other resources run out. The dotted line encapsulates the entire internal model to indicate that the iteration can include a re-evaluation of the original need and requirements. Alternative design solutions, their development and analysis are implicitly included throughout the process as shown.

The design model developed here is not to be taken as absolute. It is subject to modification according to the wishes and styles of individual designers, and each particular design's requirements. We have developed it as a basis for description of development of computational preliminary engineering design tools.

D.4 A Systematic Design Approach

In Section D.3, we constructed a philosophical model of design as a process. While the model forms a foundation, it doesn't help the engineer in everyday practice. We endeavor in this section, therefore, to build on the foundation, presenting preliminary work of a systematic approach to design with emphasis upon analytical, symbolic, and computational tools. References [87, 88, 39, 33, 7, 58] contain background material on fuzzy sets, fuzzy logic, probability and possibility, and fuzzy programming.

Every stage in the design process includes a corresponding precision, or level of detail. Starting at the conceptual phase with a coarse level of detail, and proceeding to more and more precision. The level of precision in the description of a design can be made available to the corresponding analysis, not only to influence the complexity of that analysis, but also to produce a corresponding error bound on the results. It appears that the mathematical research in fuzzy sets will find direct application to this problem.

> The concept of fuzzy sets offers an analytic formulation for dealing with ambiguities which arise in the course of engineering practice. [22]

It appears that fuzzy sets will not only permit imprecision (uncertainty in choosing among alternatives) in the description of a design, but also permit a quantification of that imprecision and a determination of its collateral effect on analysis.

D.4.1 The Becker Model

D.4.1.1 Fuzzy Application

Only a few years after Zadeh's first paper on fuzzy sets [87], James Becker constructed a model and methodology for the structural design process, where fuzzy sets played a large role in the theoretical development and application [13]. Even though Becker stressed structural design, many fundamental ideas of his work apply directly to the approach we have in mind, especially considering the definitions of fuzziness from Section D.2.

From Becker's model, we find that design may be visualized as having three basic components: an input set $\mathcal{I}$, an output set $\mathcal{O}$, and a universe of information $\mathcal{K}$, where $\mathcal{K}$ consists of two parts: the engineer's cognitive information and abilities (K) and the external information (E). In terms of the input, $\mathcal{I}$ consists of the information required for a given design that did not originally reside in K. The inclusion of this information into $\mathcal{K}$ usually occurs when the need $\mathcal{N}$of the design is delineated.

Considering the output set, $\mathcal{O}$ "is the set of information whose development is the goal of the ... process" [13]. In many respects, a set of this nature does not fulfill the need, but simply gives the information necessary to fulfill it. For example, given the need for a new interstate roadway, the set $\mathcal{O}$ might consist of the cross-section drawings needed to construct the roadway, not the physical interstate itself.

The design universe $\mathcal{U}$ may now be introduced as the informational (and cognitive) aggregate of a given design from input through output. During a multiphase design, many areas of engineering contribute to the effort so that we have in essence multiple $\mathcal{K}$ sets. $\mathcal{U}$ may be viewed as the union of these sets, i.e.,

$$\mathcal{U} = \bigcup_{i=1}^{t} \mathcal{K}_i. \tag{D.1}$$

The underlying meaning of $\mathcal{U}$ as the design universe is a continually updated and encompassing informational shell which results in the mapping of the need (N) into the input $\mathcal{I}$ by the designer.

$$\mathcal{U} : N \rightarrow \mathcal{I} \tag{D.2}$$

It follows,

$$\mathcal{I} : a \rightarrow \mathcal{A} \tag{D.3}$$

where 'a' is a requirement for an informational component, $\mathcal{A}$, at some point in the design process. Overall,

$$\mathcal{A} = [(x, \mu_{\mathcal{A}})] x \in \mathcal{U} \tag{D.4}$$

where $\mu_{\mathcal{A}}$ is the membership of x in $\mathcal{A}$, or equivalently the relevance of x to $N \cap \mathcal{A}$. According to Becker, "this mapping process, continual cognition, is generally a human function of information discrimination."

A design description, $\vec{d}$, as it evolves, represents a possible fulfillment of the need in the output set. $\vec{d}$ may be defined as a vector in the design space, $\mathcal{D}$. $\mathcal{D}$ is described as a space of 'fuzzily differentiated information,' represented by design dimensions, d_i, such that

$$\mathcal{D} = \bigcup_{i=1}^{n} d_i. \tag{D.5}$$

These design dimensions ultimately evolve into the final $\vec{d}$, implying that the d_i exist initially (as either null members or preliminary ideas) and become refined (decisions) in the temporal design process.

Parallel to the design space, $\mathcal{D}$, performance dimensions, p_j, make up a performance space, $\mathcal{P}$,

$$\mathcal{P} = \bigcup_{j=1}^{m} p_j. \tag{D.6}$$

The performance dimensions exist to assure the state and development of the system components under design.

From the performance space, an *acceptable design* may be expressed as a vector, $\vec{p_d}$, where $\vec{p_d} \in P$. In conjunction, a predicted performance vector of a design, $\vec{p_p}$, must be produced as a by-product of a design evaluation by the engineer in terms of the performance space. Yet, before such evaluations may take place in a design, the context (or environment) must be known. A given context includes such things as environmental conditions, climate, seismic conditions, consumer interests, cultural effects (legal, political, economic), etc. Used to predict or estimate conditions for the final design, each context factor is called c_k such that the entire context set is

$$\mathcal{C} = \bigcup_{k=1}^{l} c_k. \tag{D.7}$$

In summary to this point, Becker states

There are then three basic descriptive sets of information in the problem solving [design] cycle. Set $\mathcal{D}$ describes the solution. Set $\mathcal{P}$ contains the information necessary in judging the design. Set C simulates the context in which the design will eventually exist.

Several functions operate on these sets. The first of these represents the evaluation stage of a design by the engineer, i.e., the mapping of the current state of the design vector and context to the predicted performance vector

$$\mathcal{H}[\vec{d}, C] \rightarrow \vec{p_p}. \tag{D.8}$$

Similarly, a decision function may be defined as

$$\Delta[\vec{p_p}, \vec{p_d}] \rightarrow m \tag{D.9}$$

where m is the merit (acceptability) for a given design. Implicit to the Δ function is its iterative stature; a designer may stop or continue to develop the design depending on the decision value (merit).

The result of $\mathcal{H}$ and Δ, $\vec{p_p}$ and m respectively, may be integrated into the design for refinement purposes, constructing a new design term, d'. So, if the decision function does not result in a merit for completion, the design will continue such that

$$d' = \{\vec{d} \cup \vec{p_p} \cup m\}. \tag{D.10}$$

It follows that the design space will be modified

$$\mathcal{D}' = \mathcal{D} \cup \bigcup_{i=1}^{n} d'_i. \tag{D.11}$$

The actual refinement of the design occurs in the conceptual function, Λ,

$$\Lambda[\mathcal{D}', \mathcal{P}, \mathcal{C}] \rightarrow \vec{d}. \tag{D.12}$$

This function produces an improved design vector, $\vec{d}$, returning the process to stage one of the cycle.

These symbolic representations of various aspects of engineering design may be combined to describe the process.

D.4.2 The Process

Becker described the interaction of the different model components according to a ten step method. The general content of each step will be given.

D.4.2.0.1 Level One An expression of the problem need along with any relevant information occurs here. The information (in so-called 'library' form) is reviewed and rated on the basis of each components relevance to the design, performance, context, evaluation, decision, and conception set. Relevance, in this case, is a measure delimited by membership values for each set per information component.

D.4.2.0.2 Level Two Secondary information (information which further describes the primary members – the library) is introduced and assigned membership values.

D.4.2.0.3 Level Three Establishment of the performance space makes up the third level. The major factors that will form a basis for decision become known.

D.4.2.0.4 Level Four Here the establishment of the evaluative methods for the $\mathcal{H}$ function takes place.

D.4.2.0.5 Level Five Using the qualitative nature of the evaluative methods, level five consists of identifying the dimensions of the design space.

D.4.2.0.6 Level Six With the performance space and the design space now defined, the designer must decide on the relevant information for decision making. This level exists primarily to assure that a decision class of information exists for each performance dimension.

D.4.2.0.7 Level Seven Subjective values are assigned here delineating, in the designer's view, the 'potential' of a given design dimension to be useful in the resultant $\vec{d}$. These values are not considered as primary informational components; yet, they may help in reducing the design space to a manageable domain.

D.4.2.0.8 Level Eight The composite library of information for the design is reviewed here so that the designer will have a chance to modify relevant items.

D.4.2.0.9 Level Nine The process up to this point has integrated the information into a singular domain, termed the design space. This space will now be uncoupled, separating the relevant design alternatives for each subspace and removing incompatible portions of the design dimensions. The judgmental weighting of the performance dimensions and criteria are assigned as well.

D.4.2.0.10 Level Ten The weighting terms enter as input to level ten, resulting in the recoupling of compatible design subspaces so that alternatives may be rated for acceptability. If a design does not emerge here at a merit value greater than or equal to the expectations of the designer, the process may return to a previous level.

D.5 Information and Decision

Becker's model helps us to identify, mathematically, the important sets or spaces in a design. The ten stage methodology constructs a framework by which the design process may unfold. Although Becker's philosophical view differs slightly from the one outlined in Section D.3, let us consider his mathematical approach as very well defined for our purposes. Let us, however, dissect the methodology so that our systematic approach may be constructed.

In Becker's ten level taxonomy for the design process, two distinct areas become apparent: information and decision. Levels one through eight consist primarily of determining and assigning relevance to information for a given design. Such a process results in a so-called 'information base,' which steadily grows and evolves as the design is refined. Upon close inspection, an information base may be defined as a partially ordered collection of knowledge components (analytical tools, empirical data, descriptions, etc.) whose membership values map on to the real interval $(0, 1]$ with regard to the design dimensions, d_i. Notice that the interval does not contain values of zero, thus excluding information which has no relevance.

Breaking down an information base further, we see that the individual components come from a variety of sources. Examples include reference manuals such as Shigley's and Mitchell's *Mechanical Engineering Design* [67], the engineer's knowledge and experience, experiments, field testing, government regulations and codes, and *Past Designs*. The last of these has been highlighted to emphasize its importance as a contributor to the information base. As discussed in Section D.3, without past designs, the scenario of redesigning the wheel and making the same mistakes of previous designs may be fatally realized.

From a computational standpoint, the information base may be extracted from a more general/global database (perhaps on-line as an object-oriented system [73]), containing the reference manuals, experimental data, and so on. As a design progresses, designers enter items directly into the database such that the information may be referenced in later projects. They then assign weights (values or functions) to each component of relevance so that it may be automatically extracted as part of the individual design's information base.

A second area of major importance, besides information, in a systematic approach to design is decision. Becker attempts to handle the decision requirements in levels nine and ten of his methodology. Recall in level nine, the design dimensions are decoupled according to the performance criteria so that acceptable (partially based upon judgement) alternatives per dimension may be separated. The design space must be recoupled so that the overall alternatives may be distinguished. Becker does this using a method for calculating the membership grade of a decision set based on two sets called 'goals and constraints,' where both the goals and constraints originate from the design vector. These sets are equivalent to the terms 'functional requirements and constraints' [75] [76] as shown in Figure 5. In our definition, a functional requirement may be thought of as a performance criterion which has a membership value (in the interval $[0,1]$) between 0.5 and 1.0, while a constraint's membership value falls into the domain $(0, 0.5]$. This performance criterion membership value is a weight, indicating the importance of making the design meet that performance specification. The dividing line between functional requirements and constraints is 0.5. The significance of fuzzy sets is most apparent in the development and manipulation of these performance weights. The weight need not be a fixed value (as in conventional *crisp* sets), but can be a function, even a function dependent on the design's abilities to meet other performance criteria. For example: Cost may have a function that produces a weight close to 1.0 if some performance parameter is not met very well, indicating that reducing cost is of prime importance. On the other hand, the cost weight may be very small if other performance parameters are well met, resulting in a high performance and possibly high cost device.

Although Becker's decision function will not be described in detail, a brief description will be helpful. Given the functional requirements and constraints, there exist "some situations ... in which some goals and constraints are of greater importance than others." A function then arises which may be used to calculate the membership values over the decision set:

$$\begin{aligned} \mu_D(x) = \textstyle\sum_{i=1}^{n} \alpha_i(x)\mu_{G_i}(x) \\ + \textstyle\sum_{j=1}^{m} \beta_j(x)\mu_{C_j}(x) \end{aligned} \tag{D.13}$$

and

$$\sum_{i=1}^{n} \alpha_i(x) + \sum_{j=1}^{m} \beta_j(x) = 1 \tag{D.14}$$

where

$\mu_D(x) =$ *membership function of decision set,*

$\mu_{G_i}(x) =$ *membership function of goal set i,*

$\mu_{C_j}(x) =$ *membership function of constraint set j,*

$n =$ *number of goals,*

$m =$ *number of constraints,*

$\alpha_i =$ *weighting coefficient for goal i,*

$\beta_j =$ *weighting coefficient for constraint j.*

The function Becker uses to calculate a decision set of the design alternatives seems to be very useful, for it recouples the design vectors into a cohesive decision space. This application of fuzzy decision theory will not supplant the judgement nor decisions of the engineering designer. Rather it is to provide additional information to the designer during the preliminary or conceptual phase of the design process by developing a unified calculation paradigm for evaluation of performance parameters and their weights. The designer will be providing the creative input, and evaluate the design/analysis results in a similar manner to what is done now, but will be able to reach his design decisions more quickly with better information. The design decisions will not be made automatically, and hence our developments are neither AI nor Expert Systems. Engineers operate now in the imprecise preliminary design phase, and we feel that computational tools to augment the data they have available will permit them to design faster, or to evaluate more alternatives. We do not suggest that designers operate without clarity of thought, but rather that the complexity of engineering systems and phenomena require classification and analysis with techniques that permit imprecision in the descriptions of designs. Fuzzy sets promises to be a technique to provide these capabilities.

D.5.1 Data-Flow Paradigm

The mathematical model of Becker along with extensions to his process methodology has provided a basis for our design approach. Yet, this must be extended even further to encompass specific representations of synthesis, analysis, and complexity in design. We propose to do this by symbolically and graphically representing the design process according to a data-flow paradigm. The idea of data-flow arises from an approach to computing which differs greatly from the classical Von Neumann philosophy. In the Von Neumann sense, actions (or computer instructions) are carried out sequentially such that the flow of a program or algorithm may be said to be dependent upon the ordering of the instructions. In data-flow models, on the other hand, actions result as a consequence of the input data, not of any ordering.

To clarify the difference, data-flow models may be visualized in the same way as the layout of a circuit board. Each chip on the board represents a function in data-flow. Depending on the input signals, outputs (transformations) occur from the chips. Notice that the results of the layout do not depend on the ordering or placement of the chips, just the connections and subsequent data (signals) that flow. (The power of this analogy becomes even more relevant, for our data-flow model is an attempt to handle complexity in design in a similar way to that in VLSI systems [55],[54].)

The data-flow concept may be integrated into the design approach through the use of what we call *sheets*, where the analogy is a plane in three dimensional Cartesian space with the z-value held constant. Each sheet may be considered to be a design dimension, d_i, in the design space, $\mathcal{D}$. When viewing each sheet separately, we find that it is made of of functional blocks, where each block has inputs and outputs. An internal structure of a functional block (function for short) relates closely to a chip and results in a transformation of the inputs in some sense. Specifically, a function may be a governing equation for the design dimension, a drawing of its components, empirical data, a descriptive sentence, etc. In this sense, a function and its structure represent components of the information base of a given design.

Besides functions, a sheet may also contain sub-sheets, delineating the alternatives per design dimension. Sub-sheets contain functions in a similar and more detailed manner to sheets. However, sub-sheets do not intrinsically have one characteristic of sheets, i.e., a direct relationship to the performance space. For each performance dimension in the performance space, $\mathcal{P}$, there exists a three dimensional geometric space such that a sheet is mapped into this geometric space. The mapping only concerns the third (z) component in a space, where this component varies between zero and one. If a performance criteria is not relevant to a given sheet (design dimension), then the sheet maps to a z-value of zero. If there is complete relevance, z equals one. Partial relevance, in a parallel sense to the mathematical model, creates a mapping between zero and one. For n performance dimensions, there will exist n geometric spaces.

At this point, we could develop the paradigm further with regard to the complete mathematical model and process methodology; however, the details are not as important as the ideas behind the paradigm. The functions, as described briefly above, encapsulate the power for building design synthesis and analysis tools. For example, a function might be Newton's Second Law, with inputs mass and acceleration and outputs a force vector. Because the data-flow paradigm may be expressed quite easily in algorithmic form, it will handle the bookkeeping chores of the design approach (e.g., fuzzy calculations, database functions, etc.). The designer may concentrate on working with the functions on a given sheet to describe the design and performance spaces fully, where one of the primary advantages of this programming approach is that it permits rapid rearrangement of the interactions between the various computations and analyses in the preliminary design phase. Intrinsic functions in the paradigm of course include anything already in the information base.

D.6 Future Directions

The design perspective, philosophical model, and methodology presented above represent only preliminary work. Each of these will be refined and investigated further. Particular emphasis will be placed on the systematic approach described in Section D.3. The results of the data-flow paradigm, in conjunction with the mathematical development, should yield new insight into the engineering design process.

Overall the primary research goal of this work is to develop computational tools for preliminary design and to provide insight into the aggregate design process. The later of these should be realized during the process of refining the philosophical model, along with development of the data-flow paradigm. However, the specific design tools must be developed for a specific design problem, one that is limited enough in scope that some useful testing of design methodology can be performed. Such problems have a "well bounded design space", meaning that the range of possible solution technologies is small. Since the goal of this research is an increased understanding of preliminary design, and how to aid that process, the specific engineering design tools will simply be test-beds for the ideas and approaches developed, rather than ends of themselves.

D.7 Conclusions

The research outlined in this presentation is particularly aimed at addressing the problems of aiding preliminary engineering design. The preliminary phase is that portion of the design sequence where several different concepts (usually imprecisely defined) are simultaneously evaluated for feasibility, and relative merit. This work will develop methods and tools to aid the designer in developing designs, making analyses (of a variety of levels of sophistication), and in choosing between alternatives. The kinds of decisions that designers are called upon to make in the preliminary phases are often between competing technologies or solution schemes. A useful paradigm to describe this is when the designer needs to choose between gears and chains for a power transmission. Contemporary Computer Aided Design and Engineering (CAD) systems are of little or no help at that level now.

These tools are not "artificial intelligence", in that the designer is providing both the creative idea generation, and making the decisions. In an implementation of such a preliminary design system, the designer will provide the suggested alternative designs, and choose the analyses to perform, and interpret the results. The computer will perform the requested analyses of the imprecisely described designs. Both fuzzy sets and data flow programming appear at this stage to be very promising techniques for implementing computationally aids for preliminary design, in addition to contemporary three-dimensional solid modeling, manipulation, and graphical representation.

The main motivation for this work is to develop systems that will increase the information a designer can have available to him to make engineering design decisions and choices from the beginning to the end of the design process. Perhaps the central hypothesis in this research is that the greatest impact of computation on the design process will occur when tools to *aid* the designer during the concept generation and early analysis phase are available.

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Appendix E

Appendix: Calculating the Stochastic Component Output

In Chapter 6, *extended hybrid numbers* were introduced to represent the three-component structure of uncertainty information encountered in preliminary design. Equations 6.1 through 6.4 are used to perform arithmetic calculations with the stochastic component of the design parameters. This appendix will demonstrate the use of these equations with an example.

E.1 An Application of the Probability Operators

As an example use of the binary probability operators (cumulative distribution formulation), the stochastic component of uncertainty for the shaft diameter strength of the V-belt alternative (Chapter 6) will be calculated. Recalling Equation 6.21, the only stochastic parameters involved in the equation are: L_s, d_p, and S_f. Thus, using the nominal values of the imprecision and possibilistic parameters, the shaft diameter can be determined from

$$d^b_{s,str} = \left(\frac{CL_s}{d_pS_f}\right)^{\frac{1}{3}}, \tag{E.1}$$

where C is a composite of the nominal values of the other parameters. Let $x = \frac{S_f}{C}$ such that $p_x(x) = p_{S_f}(S_f) \cdot C$. Equation E.1 then becomes

$$d^b_{s,str} = \left(\frac{L_s}{d_p x}\right)^{\frac{1}{3}}, \tag{E.2}$$

Applying Equations 6.3, 6.4, and B.16, the resulting probability density from Equation E.2 is

$$p_{d^b_{s,str}}(d^b_{s,str}) = 3y^{\frac{2}{3}} \int_0^1 \int_0^1 S_f d_p p_x(y d_p S_f) dP_{S_f} dP_{d_p}, \tag{E.3}$$

where

$$y = (d^b_{s,str})^3.$$

The numerical scheme outlined in Section 6.2.2.1 can be used to approximate Equation E.3, giving

$$p_{d^b_{s,str}}(d^b_{s,str}) = 3y^{\frac{2}{3}} \frac{1}{L_1} \frac{1}{L_2} \sum_{i=1}^{L_1} \sum_{j=1}^{L_2} S^j_f d^i_p p_x(y d^i_p S^j_f), \tag{E.4}$$

where

$$P_{d_p}(d^j_p) = \frac{i - \frac{1}{2}}{L_1}, i = 1, 2, \ldots, L_2,$$

$$P_{S_f}(S^j_f) = \frac{j - \frac{1}{2}}{L_2}, j = 1, 2, \ldots, L_2.$$

Figure E.1 and Figure 6.18 show the results from Equation E.4.

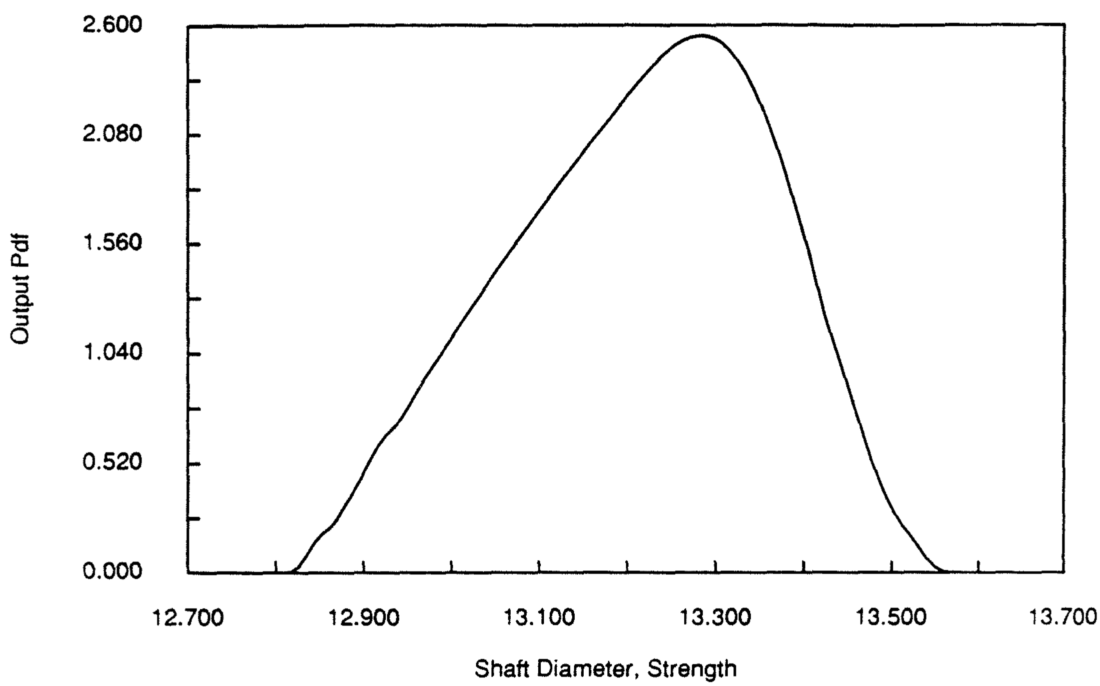

Figure E.1: Stochastic Component: V-Belt Shaft Diameter, $d^b_{s,str}$.

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