DESIGN STUDY OF AUXILIARY JET PROPULSION INSTALLATION ON THE YO-55 (ERCOUPE) AIRPLANE WITH AN ANALYSIS OF PERFORMANCE AND FLIGHT CHARACTERISTICS

Thesis

by

Carl F. Damberg
Captain, Air Corps
U. S. Army

and

Paul H. Dane
Captain, Air Corps
U. S. Army

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Nomenclature

A = Initial accelerating thrust for the normal airplane

B = Thrust decrement coefficient for normal airplane

F = Thrust from jet propulsion

h = Altitude

S_o = Normal take-off run

S_o' = Take-off run with jet propulsion

S_c = Distance to take-off over a 50 ft. obstacle

t_o = Time for normal take-off run

t_c = Time to climb 50 ft. with normal airplane

V = Velocity

V_H = Hovering velocity

V_{TO} = Take-off velocity for normal airplane

V_{max} = Maximum velocity at sea level

W = Normal gross weight

β = Ratio of overloaded weight to normal weight

ΔW = Permissible overload under conditions specified

$(\ )'$ = Refers to condition using jet propulsion

$(\ )_c$ = Refers to climb after take-off

$(\ )_\omega$ = Refers to overload condition

N = Propeller speed in r.p.m.

$\frac{dh}{dt}$ = Rate of climb

K = Conversion factor (550 in ft.-lb.-sec. system)

T_a = Ratio of power at altitude to power at sea level

T_v = Ratio of power at velocity V to power at V_{max}
ℓ_t = Thrust loading of airplane
ℓ_s = Span loading of airplane
ℓ_p = Parasite loading of airplane
η_m = Propulsive efficiency at V_{max}
e = Airplane efficiency factor
ρ_o = Standard sea level density of air
σ = Ratio of actual density of air to standard density
R_v = Ratio of actual velocity to maximum velocity
A_R = Area of rudder (each)
A_F = Area of fin (each)
A_E = Area of elevator
A_S = Area of stabilizer
AR_V = Aspect ratio of the vertical tail surfaces
AR_H = Aspect ratio of the horizontal tail surfaces
ℓ = Tail length
d = Perpendicular distance from jet thrust axis to center of gravity
D = Propeller diameter in feet
b = Wing span
η_r = Rudder efficiency
η_e = Elevator efficiency
a_o = Slope of lift curve for infinate aspect ratio
β_r = Rudder angle
β_e = Elevator angle
α = Angle of attack

q = Dynamic pressure

M_y = Yawing moment

M_p = Pitching moment

I. Introduction and Summary

In recent years the use of high wing loadings on large high performance airplanes has emphasized the problem of take-off. In many cases the allowable gross weight is limited primarily by take-off considerations.

The most effective way to decrease the take-off distance without decreasing wing loading is to increase the available thrust during take-off run and initial climb. The use of auxiliary jet propulsion has been proposed as a means of accomplishing this and considerable interest has been aroused in its development. A program of development and research on jet reaction motors using various types of fuels has been undertaken at the California Institute of Technology during the past year. The development of a jet motor using solid fuel (powder) has reached the stage where it is practical to begin actual flight test experiments with assisted take-offs. For this purpose the U. S. Army Air Corps has made available a YO-55 (Ercoupe) Airplane for installation of auxiliary jet propulsion units and for flight testing. This report is concerned with a study of the various problems involved in the use of jet propulsion on an airplane, and the design of a practical installation on the YO-55 airplane.

After a study of the various factors involved, it was decided to use two jet propulsion assemblies each containing three separate jet motors of approximately 25 lbs. thrust. The assemblies are mounted under the center section of the wing, one on each side just inboard of the main landing wheel fairing. The motors are so designed that in case of an excessive pressure build-up, the nozzle will fly off to the rear and the rest of the motor will be propelled forward clear of the airplane. Particular emphasis was placed on safety of the pilot and the airplane structure.

An estimate of the take-off performance of the YO-55 with auxiliary jet propulsion has also been made. The results showing the relative improvement in performance are listed below.

Summary of Take-off Performance of YO-55
Airplane with Auxiliary Jet Propulsion

Item	F=0 (Normal)	F=100 lbs.	F=150 lbs.
Take-off run	350 ft.	223 ft.	190 ft.
Percent saving on take-off run	0	36%	46%
Time for take-off run	9.84 sec.	6.65 sec.	5.73 sec.
Distance to take-off over a 50 ft. obstacle	766 ft.	466 ft.	393 ft.
Percent saving		39%	49%
Time to take off over a 50 ft. obstacle	16.77 sec.	10.71 sec.	9.12 sec.
Overload permissible with normal take-off run	0	223 lbs.	329 lbs.

II Description of YO-55 (Ercoupe) Airplane

The YO-55 airplane is a low wing monoplane with tricycle landing gear and a twin tail. The entire airplane is duralumin clad with the exception of the outer wing panels which are covered with doped fabric. The fuselage is of the monocoque type structure.

Power Plant

Continental Engine Model A-65-8 rated at 65 Horsepower at 2300 r.p.m.

Performance

Speed at rated r.p.m. (2300 r.p.m.)	111 m.p.h.
Hovering speed	41 m.p.h.
Take-off roll	350 ft.
Landing roll	300 ft.
Fuel capacity	14 gals.
Fuel consumption (at 73 mph at 1875 rpm)	2.9 gals./hr.
Range (4.83 hrs. at 73 r.p.m.)	352.6 miles

Weight

Weight empty	753 lbs.
Useful load	422 lbs.
Gross weight	1175 lbs.

Airfoil Data

Root section of wing	NACA 43013
Tip section of wing	NACA 43013
Wing span	30 ft.

Wing chord	5 ft.
Areas	
Wing --	
each	71.3 sq. ft.
total	142.6 sq. ft.
Aileron --	
each	8.4 sq. ft.
total	16.8 sq. ft.
Horizontal tail --	
stabilizer	10.2 sq. ft.
elevator	9.4 sq. ft.
total	19.6 sq. ft.
Vertical tail --	
each rudder	3.0 sq. ft.
each fin	1.65 sq. ft.
total	9.3 sq. ft.

Weight and Balance (Datum is Firewall)

Item	Weight		Arm	Horizontal Moment
	Special	Standard		
Weight empty		753	28.5	21460
Running lights and wiring	4.5		53	239
Battery installation	19.5		55	1072
Shielding harness	1.0		25	-25
Total special equipment	25.0			
Weight empty less special equipment		728		

Most Forward C. G. Position

Item	Weight (lbs.)	Arm (in.)	Horiz.Moment(in.lbs.)
Weight empty	753	28.5	21460
Pilot	170	36.5	6200
Fuselage tank	30	6.75	202
Wing tank	54	24.7	1332
Oil (1 gal.)	8	-13.87	111
TOTAL	1015	28.6	27083

Horizontal C.G. is 28.6 inches aft of datum.

Full Load and Rearward C.G. Position

Item	Weight (lbs.)	Arm (in.)	Horiz.Moment(in.lbs.)
Weight empty	753	28.5	21460
Pilot	170	36.5	6200
Fuselage tank	30	6.75	202
Wing tank	30	24.7	741
Oil (1 gal.)	8	-13.87	-111
Passengers	170	36.5	6200
Baggage	14	57.0	798
TOTAL	1175	30.2	35490

Horizontal C.G. is 30.2 inches aft of datum.

Approved C.G. Limits

Forward C.G. = 26.4" aft of datum
Aft C.G. = 30.3" aft of datum

III Location of Jet Propulsion Equipment

The ideal way to apply auxiliary jet thrust to an airplane would be to have a single jet propulsion assembly whose thrust line passes through the center of gravity of the airplane. Then, theoretically, the effects on the trim characteristics of the airplane would be zero. To attain this in the YC-55 airplane would mean locating the jet propulsion equipment so far to the rear of the center of gravity that balance would be exceedingly difficult if not impossible. Consequently, other locations had to be considered.

After careful consideration of the many factors involved, it was decided to use two jet propulsion assemblies, each of 75 lbs. thrust capacity. It was decided to locate these assemblies just inboard of the landing gear struts on the under surface of the wing center section. In reaching this decision, the following factors were taken into account:

(1) Safety for the pilot and airplane.
(2) Effect on airplane weight and balance.
(3) Effect on airplane flight characteristics.
(4) Structure considerations including ease of installation.
(5) Effect of blast from motor jets.

A detailed discussion of each of these factors follows:

1. Safety of the Pilot and Airplane

Realizing the importance of safety, the jet propulsion motors and auxiliary equipment were designed to afford maximum protection to both the pilot and the airplane structure. The jet motor nozzles are so designed that in case of an excessive pressure build-up, the bolts holding it to the main body of the motor will fail and the nozzle will fly off. For this reason, it is necessary to so locate the jet assemblies so that there is a minimum danger of the nozzle striking any portion of the fuselage or tail structure. It was also decided to extend the mounting frame considerably to the rear of the nozzles in order to provide protection for the main wing structure.

Each jet unit is so oriented that the main body of each jet motor will clear the propeller disk in case it flies forward.

2. Effect on Airplane Weight and Balance

The estimated weight of each jet propulsion assembly is 76 lbs. Since there are two of these, the total weight of the jet propulsion equipment will be 152 lbs. plus the weight of any necessary wing reinforcement. From the available airplane data, it is seen that the weight of one passenger and baggage, totaling 184 lbs., is included in the listed gross weight of the airplane. As it is necessary to have only the pilot for these experimental flights, it is possible to carry all the jet propulsion equipment without exceeding the normal gross weight of the airplane.

In order to maintain proper balance of the airplane, the jet propulsion equipment should be so located as to have a minimum effect on the horizontal position of the center of gravity. This has been accomplished by mounting the assembly almost directly below the normal horizontal center of gravity location.

3. Effect on Airplane Flight Characteristics

The optimum position for the jet assemblies insofar as effect on longitudinal trim is concerned would be such that the resultant thrust line passes through the center of gravity. However, other considerations made this impractical so the assemblies were located so that their resultant thrust line passes below the center of gravity of the airplane. The effect of this is discussed in Chapter VI where it is shown that there is adequate elevator control to compensate for the stalling moment.

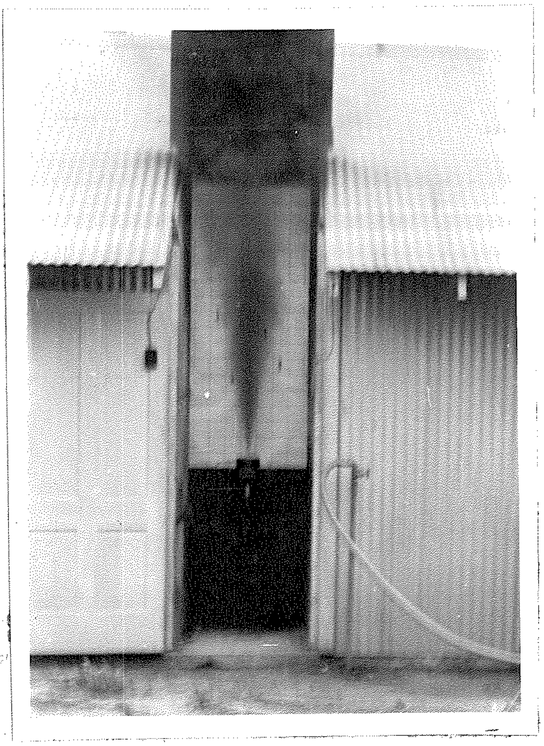
The use of two jet propulsion assemblies introduces the possibility of unfavorable yawing moments in case of failure of one assembly while the other jet assembly continues to function. This effect is discussed further in Chapter VI where it is shown that, with slight modifications, there is adequate rudder control to counteract this yawing effect.

4. Structural Considerations Including Ease of Installation

Since each jet propulsion assembly is located just inboard of a main landing gear strut and is fastened to the under surface of the center section of the wing, the installation of the assemblies should be relatively easy. The main wing spar can be used to advantage to support the major portion of the loads. Since the outer wing panels are detachable near the point of attachment of the jet assemblies, the interior of the wing should be readily accessible for the necessary work.

5. Effect of Blast from Motor Jets

It has been determined experimentally that the gases emerge from the nozzle of the jet motor in a cone whose angle is approximately 20° to 25°. Below is a photograph of a jet during one of these experiments.



Considering this fact, the jets are so located that no part of the wing, fuselage or tail structure is within a cone of 30° from the jet. As the heat from such a jet diffuses rapidly, no heat effects on the metal skin are anticipated.

IV Description of Jet Propulsion Motor

A jet propulsion motor consists essentially of a combustion chamber for burning either gaseous, liquid, or solid fuels and a nozzle through which the gaseous products of combustion escape at high velocity. The jet motors to be used in the YO-55 airplane installation will burn solid fuel (powder).

For the installation proposed in this report, the jet propulsion motor unit consists of a cylindrical length of Shelby tubing (12 7/8" x 2 3/8" x 3/16") with a cap screwed on one end and a nozzle bolted to the other end. The nozzle is fastened to the cylinder by tension bolts designed to fail at 6000 lbs. so that the nozzle will separate from the motor and relieve the pressure in case of an excessive pressure build-up.

Two fins running the length of the cylinder and located diametrically opposite to each other are welded to the outside of the cylinder. These fins are designed to fit into the guiding tracks of the track frame.

Light shear pins designed to shear at 50 lbs. hold each jet propulsion motor to the track frame. In case there is an excessive pressure build-up in the combustion chamber, the nozzle will fly to the rear. The reaction of the rest of the jet motor will fail the shear pins and the motor will be projected forward guided by the fins to clear the airplane.

The weight of a 25 lb. thrust motor loaded with 2 pounds of powder totals approximately 13.3 lbs.

From considerable experimental data available, this size jet motor will give an average thrust of 25 lbs. for a period of about 13 seconds.

The method of starting combustion in the jet motor chamber is electrical. At the nozzle end of the powder charge, a fine high resistance wire is located in the powder. When current passes through this wire, the heat generated starts combustion of the powder. An electric switch button located on the airplane control stick can be used to start the jet propulsion motors.

V. Description of Jet Propulsion Assembly

After carefully considering the various problems of installing jet propulsion motors on the YO-55 (Ercoups) airplane, it was decided to use two jet propulsion assemblies. Each assembly incorporates three twenty-five pound thrust jet motors for a total of 75 pounds thrust per assembly.

The complete jet propulsion assembly includes three jet motors, a track frame and mounting bracket.

The track frame is of welded steel construction and is designed with tracks that support the three jet motors by their fins. The jet motors mounted in the track frame are side by side and as close together as practical to achieve a compact unit.

The mounting bracket performs a dual function in that it supports the track frame and also provides protection for the pilot and airplane in case an excessive pressure build-up causes a jet motor nozzle to fly off. The bracket is constructed of steel plate formed into a U-shaped channel. The track frame is fastened to the mounting bracket by four bolts, two on each side. There are additional holes for the two rear bolts so that the angle of the jet motors can be varied in a vertical plane. Bolt holes are located in the top of the mounting bracket so that the entire assembly can be attached to the under surface of the wing.

A calculation of the weights of the components of the jet propulsion assembly gives the following results:

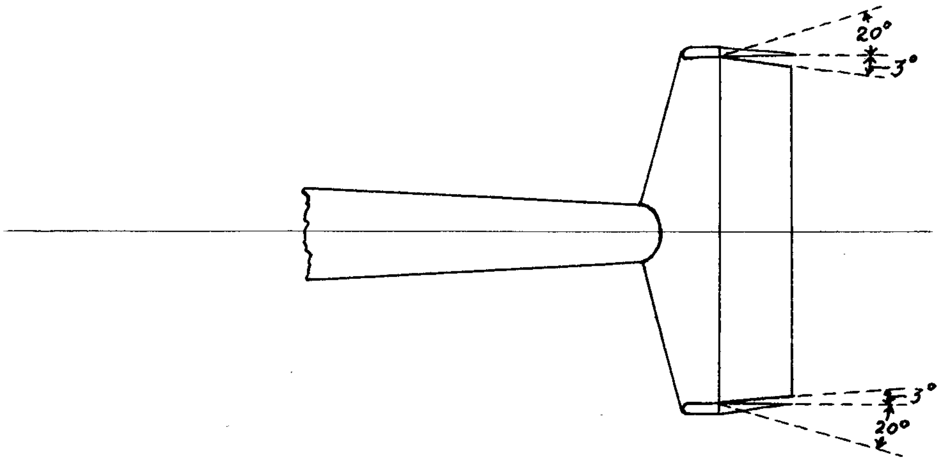
3 jet motors	39.9 lbs.
1 mounting bracket (channel)	27.0 lbs.
1 track frame	9.0 lbs.
TOTAL	75.9 lbs.

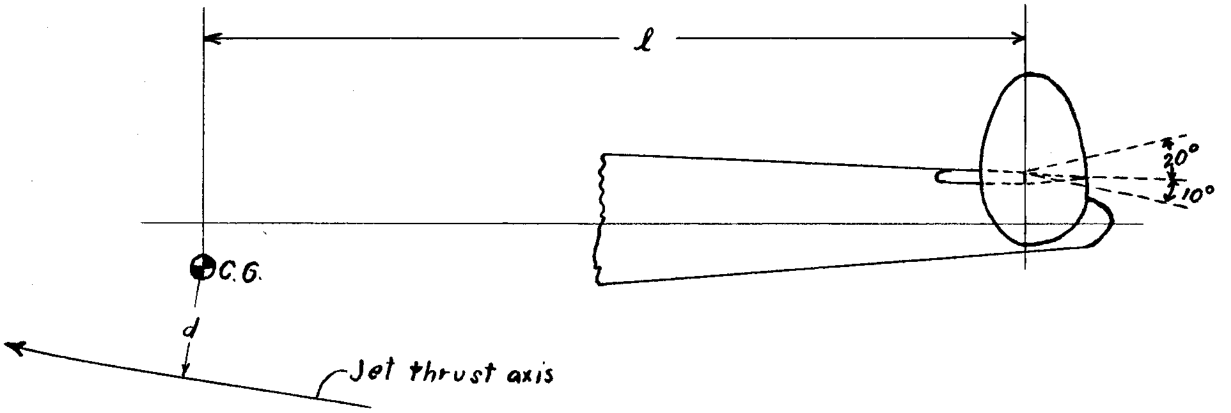
VI Flight Characteristics Analysis

The following data are necessary for this analysis:

Fin Area (each)	1.65 sq. ft.
Rudder Area (each)	3.00 sq. ft.
Stabilizer Area	10.2 sq. ft.
Elevator Area	9.4 sq. ft.
Tail Length (C.G. to surface hinge lines) 1 =	13.1 ft.

Following is a sketch of the tail assembly of the YO-55 showing the general shape and the maximum elevator and rudder deflections:





1. Analysis of Rudder Effectiveness

Since two jet propulsion assemblies are to be used on this installation, it is necessary to make certain that the rudder will have sufficient power to overcome the adverse yawing moment caused by one unit failing while the other continues to act. This problem is similar to the problem of single engine operation of a bimotored airplane.

Since the airplane has a tricycle type landing gear, the natural stability of this type of gear should prevent the possibility of ground-loop during the take-off run due to failure of one jet assembly. The critical period will be just after take-off before the airplane has gained speed and when the control effectiveness is at a minimum. We will therefore investigate the rudder effectiveness at the take-off speed.

Since the maximum deflection of one rudder is 3° inward when the other is 20° outward, we will make our analysis on the assumption that only one rudder is used and that its maximum deflection is the sum of the maximum deflections of both rudders.

The yawing moment due to a rudder angle β_r is given by the formula

$$M_y = -\eta_r q S_v \ell \frac{a_o}{1+\frac{a_o}{\pi A\!\!R_v}} \frac{\partial \alpha_r}{\partial \beta_r} \beta_r$$

The aspect ratio of each vertical surface is

$$A\!\!R_v = \frac{(3.25)^2}{4.65} = 2.27$$

From Figure 5, Reference 4, the rudder efficiency

$$\eta_r \doteq .80$$

$$\frac{A_R}{A_R + A_F} = .645$$

From Figure 11, Reference 5

$$\frac{\partial \alpha_r}{\partial \beta} = 0.9$$

Assume $a_0 = 5.7$

$$\frac{a_0}{1 + \frac{a_0}{\pi AR_V}} = 3.17$$

$$S_V = 4.65 \text{ sq.ft.}$$

$$l = 13.1 \text{ ft.}$$

$$q = \frac{\rho_0}{2}(V_{T0})^2 = 4.3$$

The maximum rudder deflection in one direction is 20° on one rudder and 3° on the other, making a total deflection of 23°.

$$\therefore \beta = \frac{23}{57.3} = .401 \text{ radians}$$

$$\therefore M_y = 0.80 \times 4.3 \times 4.65 \times 13.1 \times 3.17 \times .9 \times .401 = 240 \text{ ft.lbs.}$$

Assuming a total jet force of 150 lbs., each unit would have 75 lbs. thrust.

Distance from centerline of ship to center of each jet assembly is 2.8 ft.

Maximum yawing moment from one jet assembly:

$$= 2.8 \times 75 = 210 \text{ ft. lbs.}$$

Although this shows that the rudder effectiveness is sufficient to overcome the yawing moment due to a single jet assembly, the margin is rather small. It is therefore thought advisable to increase the rudder area to provide more control.

Assume that the area of each rudder is increased by 1 sq. ft.

Then $$R\!\!\!\!/\,A_V = \frac{(3.25)^2}{5.65} = 1.87$$

$$\eta_V = .75$$

$$\frac{a_0}{1 + \frac{a_0}{\pi R\!\!\!\!/\,A_V}} = 2.89$$

$$\frac{\partial \alpha_r}{\partial \beta} = 0.92$$

$$S_V = 5.65$$

$$\therefore\ M_y = .75 \times 4.3 \times 5.65 \times 13.1 \times 2.89 \times .92 \times .401 = 255 \text{ ft. lbs.}$$

It is considered that this is a sufficient margin of control. It should also be noted that the rudder effectiveness was calculated at zero angle of yaw. If the ship is yawed, the rudder effectiveness is increased, since the angle of attack on the rudder is greater.

2. Analysis of Elevator Effectiveness

Since the resultant thrust line of the jet assemblies lies below the center of gravity, addition of the jet thrust will introduce a stalling moment to the airplane. It is therefore desirable to investigate the elevator effectiveness at take-off speed, to be certain there is sufficient elevator control to overcome this moment.

The formula for the change in pitching moment due to elevator angle β_e is

$$\Delta M_p = -\eta_H\, q\, l\, S_H \frac{a_0}{1 + \frac{a_0}{\pi R\!\!\!\!/\,A_H}} \frac{\partial \alpha_H}{\partial \beta_e} \beta_e$$

The aspect ratio of the horizontal tail surfaces is

$$AR_H = \frac{(8.21)^2}{19.6} = 3.44$$

From Figure 6, Reference 4, the horizontal tail efficiency

$$\eta_e = .79$$

Assume $a_o = 5.7$

$$\frac{a_o}{1 + \frac{a_o}{\pi AR_H}} = 3.73$$

$$\frac{A_E}{A_E + A_S} = .48$$

From Figure 11, Reference 5

$$\frac{\partial \alpha_H}{\partial \beta_e} = .81$$

$$\ell = 13.1 \text{ ft.}$$

$$q = 4.3$$

$$S_H = 19.6 \text{ sq.ft.}$$

The maximum down elevator angle

$$\beta_e = 10^\circ = .0175 \text{ radians}.$$

The change in pitching moment due to full down elevator deflection

$$\Delta M_p = .79 \times 4.3 \times 13.1 \times 19.6 \times 3.73 \times .81 \times .0175 = 462 \text{ ft. lbs.}$$

The perpendicular distance from the resultant jet thrust axis to the C.G. position

$$b = 1.26 \text{ ft.}$$

The pitching moment due to auxiliary jet thrust of 150 lbs

$$= 150 \times 1.26 = 188 \text{ ft. lbs.}$$

Therefore there is sufficient elevator control to offset the stalling moment from the jet force.

It should be noted that the propeller thrust axis is 1.30 ft. above the C.G. position. Since flight tests show that there is not an excessive change in trim from full power on to power off, it is felt that the effect of jet thrust on trim will not be excessive.

Another condition which should be investigated is the possibility of the nose wheel leaving the ground at the start of the take-off run due to the stalling moment from the jet thrust. This is balanced by the moment due to the reactions of the main landing wheels to the airplane weight.

The horizontal distance from the C.G. to vertical line through point of contact of main landing wheels is

$$d = 14.4''$$

The moment about the C.G. due to vertical reactions from main landing gear (assuming main gear to be carrying full weight of airplane)

$$M_1 = -W \times d = -1175 \times 14.4 = -16950 \text{ in. lbs.}$$

The maximum stalling moment due to jet thrust

$$M_2 = 2260 \text{ in. lbs.}$$

Therefore, there is no danger of the airplane nose wheel leaving the ground at the start of the take-off run.

VII Calculation of Take-off Performance with Auxiliary Jet Propulsion

In this section the improvement in take-off performance of the YO-55 with auxiliary jet propulsion is calculated. This analysis is based partially on the formulas derived by Dr. C. B. Millikan and Dr. H. J. Stewart, Reference 1, and partially on the method of airplane performance calculation developed by Dr. W. B. Oswald, Reference 2. Two conditions of jet thrust are assumed for the basis of these calculations:

1. A constant jet force of 100 lbs. applied during entire take-off period.
2. A constant jet force of 150 lbs. applied during entire take-off period.

The take-off run and distance to clear a 50 ft. obstacle with the normal gross weight are calculated for both conditions. Also the overload permissible with normal take-off run and the time of take-off for each case are calculated.

The following is a summary of the known data pertaining to the airplane which will be necessary in these calculations:

Gross Weight $W = 1175$ lbs.
Maximum Speed (Sea Level) $V_{max} = 111$ m.p.h.
Maximum Power (Sea Level) $B.H.P. = 65$ h.p.
RPM (At Rated Speed) $= 2300$ r.p.m.
Propeller Diameter $D = 6$ ft.
Wing Span $b = 30$ ft.
Hovering Speed $V_h = 41$ m.p.h.
Normal Take-off Run $S_0 = 350$ ft.

1. Take-off Run with Jet Propulsion and Normal Gross Weight

Calculation of static thrust

The method of Diehl for calculation of static thrust, Reference 3, is used here.

$$\frac{V}{ND} = \frac{88 \times 111}{2300 \times 6} = .708$$

From Figure 5, Reference 3, the static thrust coefficient for best performance propeller

$$K_{TO} = 67500$$

Static Thrust

$$A = \frac{K_{TO} \times bhp}{rpm \times D} = \frac{67500 \times 65}{2300 \times 6} = 318 \text{ lbs.}$$

The formula for take-off run for the normal airplane without auxiliary jet propulsion as given by Millikan and Stewart, Reference 1, is:

$$S_o = -\frac{W}{2gB} \log\left(1 - \frac{B}{A} V_{TO}^2\right)$$

In this analysis it will be assumed that the take-off speed V_{TO} is equal to the hovering speed V_h given in the airplane data, since V_h indicates the minimum speed at which adequate control can be maintained.

$$\therefore V_{TO} = V_h = 41 \text{ m.p.h.} = 60 \text{ ft. per sec.}$$

Since S_o, W, A, and V_{TO} are known, equation (1) can be solved for B

$$B = .0608$$

The formula for take-off run of the normal airplane with auxiliary jet propulsion, Reference 1, is:

$$S_o' = -\frac{W}{2gB} \log\left(1 - \frac{B}{A+F} V_{TO}^2\right)$$

Assuming the jet force $F = 100\ lbs.$

$$S_o' = 223\ ft.$$

$$\frac{S_o - S_o'}{S_o} = .36$$

This means that an auxiliary jet force of 100 lbs. will cause a saving of 36% in take-off distance.

Assuming the jet force $F = 150\ lbs.$

$$S_o' = 190\ ft.$$

$$\frac{S_o - S_o'}{S_o} = .46$$

This means that an auxiliary jet force of 150 lbs. will cause a saving of 46% in take-off distance.

2. Take-off Run with Jet Propulsion and Overload

The formula for take-off run with auxiliary jet propulsion and overload, Reference 1, is

$$S_{o_\omega}' = -\frac{\beta W}{2gB} \log\left(1 - \frac{\beta B}{A+F} V_{To}^2\right)$$

Where $\beta = \frac{W_\omega}{W}$

We will assume that the take-off run with auxiliary jet propulsion and overload is to be the same as the normal airplane without jet propulsion.

i.e. $S_{o_\omega}' = S_o = 350\ ft.$

Assuming the jet force F = 100 lbs., we can solve equation (3) for β

$$\beta = 1.19$$

The permissible overload for normal take-off distance with jet propulsion is then

$$\Delta W_1 = (\beta - 1) W = (1.19 - 1) \times 1175 = 223 \text{ lbs.}$$

Assuming the jet force F = 150 lbs.

$$\beta = 1.28$$

The permissible overload for this condition is

$$\Delta W_2 = (\beta - 1) W = (1.28 - 1) \times 1175 = 329 \text{ lbs.}$$

3. Distance to Take-off over a 50 ft. Obstacle with Auxiliary Jet Propulsion and Normal Gross Weight

In this analysis it is assumed that the airplane makes a normal take-off and then continues to climb to 50 ft. holding the same speed as at take-off. The take-off in this condition is separated into two regimes - the take-off run and the climb after take-off. The take-off run with auxiliary jet propulsion has already been determined. It now remains to determine the rate of climb under the conditions specified. It is necessary to first determine the initial rate of climb of the normal airplane without jet propulsion at the specified speed. For this calculation the method of Oswald, Reference 2, will be used. The fundamental performance equation is

$$\frac{dh}{dt} = \frac{K\, T_a\, T_v}{l_t} - \frac{\sigma \rho_o V^3}{2 l_p} - \frac{2 l_s}{\pi \sigma \rho_o V}$$

If now we can determine the three airplane parameters $\ell_t, \ell_p,$ and ℓ_s , we can determine the rate of climb corresponding to any given velocity V .

Determination of ℓ_t (thrust loading)

given - $V_{max} = 111$ m.p.h.

$N = 2300$ r.p.m.

$B.H.P. = 65$

The propeller coefficient

$$C_{s_m} = \frac{\rho_0^{\frac{1}{5}} V_{max}}{(550 \times b.h.p.)^{\frac{1}{5}} \times N^{\frac{2}{5}}} = 1.42$$

From Figure 27, Reference 2, the propulsive efficiency at V_{max}

$$\eta_m \doteq .85$$

$$\ell_t = \frac{W}{(bhp)\,\eta_m} = \frac{1175}{65 \times .85} = 21.3$$

Determination of ℓ_s (span loading)

From consideration of the airplane type and its relative cleanness we shall assume the airplane efficiency factor to be

$$e = .80$$

for monoplanes $K = 1$

Then

$$\ell_s = \frac{W}{e(K b_1)^2} = \frac{1175}{.80 \times (30)^2} = 1.635$$

Determination of l_p (parasite loading)

$$l_s \times l_t = 34.8$$

$$V_{max} = 111 \text{ m.p.h.}$$

From Figure 29, Reference 2

$$\frac{l_p}{l_t} = 10.5$$

$$\therefore l_p = 10.5\, l_t = 224$$

Since we are considering sea level conditions

$$T_a = \sigma = 1$$

$$T_v = f(R_v)$$

where $R_v = \dfrac{\text{actual velocity}}{\text{max. velocity}}$

In the present case we are assuming that the speed during climb is the take-off speed.

$$\therefore R_v = \frac{41}{111} = .37$$

From Figure 18, Reference 2

$$T_v = .52$$

We can now calculate the sea level rate of climb at V_{TO} from equation (4)

$$\left(\frac{dh}{dt}\right)_0 = \frac{K T_v}{l_t} - \frac{\rho_0 V^3}{2 l_p} - \frac{2 l_s}{\pi \rho_0 V} =$$

$$\frac{550 \times .52}{21.3} - \frac{.001689 (60)^3}{224} - \frac{1.45}{\pi \times .001689 \times 60} = 7.21 \text{ ft. per sec.}$$

time to climb 50 ft. with normal airplane

$$t_c = \frac{50}{7.21} = 6.93 \text{ sec.}$$

Distance to take-off over a 50 ft. obstacle with normal airplane without jet propulsion

$$S_c = S_o + t_c V_{To} = 350 + 6.93 \times 60 = 766 \text{ ft.}$$

Since V_{To} is the same with or without auxiliary jet propulsion, all energy obtained from the additional thrust of the jet goes into increasing the potential energy of the airplane. Therefore, the rate of climb at the velocity V_{To} with a constant jet force F is

$$\left(\frac{dh}{dt}\right)' = \left(\frac{dh}{dt}\right)_o + \frac{F\ V_{To}}{W}$$

Assuming jet force F = 100 lbs.

$$\left(\frac{dh}{dt}\right)' = 7.21 + \frac{100 \times 60}{1175} = 12.32 \text{ ft. per sec.}$$

The time to climb 50 ft.

$$t_c' = \frac{50}{12.32} = 4.06 \text{ sec.}$$

Total distance to clear a 50 ft. obstacle

$$S_c' = S_o' + t_c' V_{To} = 223 + 4.06 \times 60 = 466 \text{ ft.}$$

$$\frac{S_c - S_c'}{S_c} = .39$$

This means that a jet force of 100 lbs. will effect a saving of 39% in the take-off distance to clear a 50 ft. obstacle.

Assuming a jet force F = 150 lbs.

$$\left(\frac{dh}{dt}\right)' = 7.21 + \frac{150 \times 60}{1175} = 14.77 \text{ ft. per sec.}$$

The time to climb 50 ft.

$$t_c' = \frac{50}{14.77} = 3.39 \text{ sec.}$$

Total distance to clear a 50 ft. obstacle

$$S_c' = 190 + 3.39 \times 60 = 393 \text{ ft.}$$

$$\frac{S_c - S_c'}{S_c} = \frac{373}{766} = .49$$

This means that a jet force of 150 lbs. will effect a saving of 49% in the take-off distance to clear a 50 ft. obstacle.

4. Calculation of Time of Take-off

Calculation of the total time of take-off is important with auxiliary jet propulsion since this determines the length of time the jet units must deliver power.

The formula for time of take-off with auxiliary jet propulsion is given as, Reference 1

$$t_o' = \frac{W}{2g\sqrt{(A+F)B}} \log\left[\frac{1+\sqrt{\frac{B}{A+F}}\ V_{TO}}{1-\sqrt{\frac{B}{A+F}}\ V_{TO}}\right]$$

For the case where F = 100 lbs.

$$t_o' = 6.65 \text{ sec.}$$

For the case where F = 150 lbs.

$$t_o' = 5.73 \text{ sec.}$$

The total time for take-off over a 50 ft. obstacle can be obtained by adding the time of take-off and the time to climb 50 ft. for each condition.

Summary of Take-off Performance of YO-55
Airplane with Auxiliary Jet Propulsion

Item	F=0 (Normal)	F=100 lbs.	F=150 lbs.
Take-off run	350 ft.	223 ft.	190 ft.
Percent saving on take-off run	0	36%	46%
Time for take-off run	9.84 sec.	6.65 sec.	5.73 sec.
Distance to take-off over a 50 ft. obstacle	766 ft.	466 ft.	393 ft.
Percent saving	0	39%	49%
Time to take off over a 50 ft. obstacle	16.77 sec.	10.71 sec.	9.12 sec.
Overload permissible with normal take-off run	0	223 lbs.	329 lbs.

References

1. Millikan, C. B., and Stewart, H. J.: Aerodynamic Analysis of Take-off and Initial Climb as Affected by Auxiliary Jet Propulsion. Air Corps Jet Propulsion Research Report No. 5

2. Oswald, W. Bailey: General Formulas and Charts for the Calculation of Airplane Performance. N.A.C.A. Technical Report No. 408

3. Diehl, Walter S.: Static Thrust of Airplane Propellers. N.A.C.A. Technical Report No. 447

4. Root, L. E.: Empennage Design with Single and Multiple Vertical Surfaces. Journal of the Aeronautical Sciences, Vol. 6, No. 9, July, 1939

5. Wenzinger, Carl J.: Pressure Distribution over an Airfoil Section with a Flap and Tab. N.A.C.A. Technical Report No. 574

6. Glauert, H.: Theoretical Relationships for an Airfoil with Hinged Flap. Reports and Memoranda No. 1095