EXPERIMENTAL INVESTIGATION OF ULTIMATE LOADS
CARRIED BY FLAT, UNSTIFFENED PANELS
UNDER COMBINED SHEAR AND COMPRESSION

Thesis
by
Henry T. Nagamatsu

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I. ACKNOWLEDGMENT

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He also wishes to thank the staff of the Guggenheim Aeronautical Laboratory for their constructive criticism and suggestions, and Mr. G. Tsubota for his helpful advice about the experimental technique.
II. SUMMARY

The behavior of flat, unstiffened panels was investigated for two length over width ratios and five thicknesses. In each case the panels were loaded until they failed. An empirical relation was developed for the variation of the ultimate combined shear and compression stress of the form

\[
\left( \frac{\sigma_c}{\sigma_{c0}} \right)^m + \left( \frac{\tau}{\tau_c} \right)^n = 1
\]

The values of \( m \) and \( n \) were found to be: \( m = 1.65 \) and \( n = 0.9 \) for panels with length over width ratio equal to 2, and \( m = 1.4 \) and \( n = 1.3 \) for panels with length over width ratio equal to 1 1/3. The above equations agree very closely with the experimental results.

There was a variation in the apparent shear modulus under different combined loading conditions, but no definite result was obtained.
III. INTRODUCTION

For the modern types of airplanes which use stressed skin construction to carry the external loads in the most efficient manner, the designers are often confronted by the problem of panels loaded with combined shear and compression forces, such as panels in the wing, fuselage, rudder, etc. The method of determining the ultimate failure stress for panels under combined shear and compression forces is lacking.

With these problems in mind, the present research project was begun two years ago at the California Institute of Technology by Mr. Butterworth, and last year Mr. Tsubota redesigned the loading mechanism, so as to load a panel simultaneously by shear and compression.

Throughout the test, 17ST duralumin sheet was used for the test panels, since it was considered to be a representative material for structural elements in modern airplanes. All specimens were cut from the sheet with grain parallel to the lengthwise direction. This was done in order to have consistent test results, because duralumin sheet is not an isotropic material; the physical properties parallel to the grain are different from those perpendicular to the grain. Five nominal thicknesses, .010", .020", .032", .040" and .050",
considered to be used widely for stressed skin construction, were used for the panel thicknesses. Previously, Mr. Tsubota tested only three nominal thicknesses, .020", .032", and .040", but these were inadequate to obtain the relationship between the ultimate failure stress and thickness; hence, nominal thicknesses of .010" and .050" were included to obtain more adequate experimental data.

Two different panel dimensions were used for the specimens. Both were 12 inches in length, but the widths were 6 inches and 9 inches, and the ratios of length to width were 2 and 1 1/3. Inasmuch as a considerable difference was found in the strength properties of these two series of panels, further tests should be made with different lengths and widths to compare the results with those obtained for this research project.

There were seven combinations of loading used in the experimental investigation: shear over compression ratios of tan 15\(^\circ\), tan 30\(^\circ\), tan 45\(^\circ\), tan 60\(^\circ\), tan 75\(^\circ\), compression alone, and shear alone. The ratios of shear over compression were made as indicated, in order to get enough experimental data correctly spaced to plot the interaction curve, \(\frac{\sigma_s}{\sigma_c}\) vs. \(\frac{\tau}{\tau_c}\).

Enough data was taken for each specimen during the application of the loads to determine the vari-
ation, if any, of shear modulus, $G$, for various loading conditions. For simplicity, the edges of the panels were simply supported by steel tubes with slots, making a sliding contact support.
IV. EXPERIMENTAL PROCEDURE

(a) Description of the Testing Machine

The apparatus which was used in this research project was modified by Mr. Tsubota, so as to get both shear and compression forces acting simultaneously on the panel. A very detailed description is given in Reference 2; hence, only the basic parts will be discussed. The only modification which was made, in order to prevent the bottom edge of the panel to rotate, was to put another roller on the left hand side of the panel, indicated by s in the drawing (Fig. 1).

The testing machine was constructed in a manner to permit various combinations of forces on the panel. For only shear load, one end of the tension strap gage was connected to the frame, and the load applied to the specimen, by tightening the screw on the tension strap. In order to get various ratios of shear to compression load, one end of the loading mechanism was connected to the beam, and by means of a lever system the reaction of the shear force was transmitted as a compression load to the panel. There were five slots on the bottom I-beam, and by placing the knife edge in these slots, it was possible to obtain the following ratios of shear to compression: \( \frac{F}{S} \), \( \tan 15^\circ \), \( \tan 30^\circ \), \( \tan 45^\circ \), \( \tan 60^\circ \), and \( \tan 75^\circ \). Pure compression
load was obtained by placing the end of the loading
cable, which was connected to the angles, to the frame
and the other end to the loading beam; then by tight-ning the loading screw, the load was transmitted by the
lever system to the panel as pure compression.

Three steel balls were used to get approximately
a point contact for the transmission of the force.
Knife edges were used as pivot points for the appli-
cation of the loads. Two Ames dial gages were used
to measure the vertical deflection of the panel, and
one for the horizontal deflection. The smallest
division on the dial gages was one-hundredth of a
millimeter.

The tension strap gage was calibrated in a
standard tension testing machine, and a curve of
deflection vs. load was plotted, as shown by Fig. 59.
Since the shear failure load for a nine inch panel
with a thickness of .040" was large, a heavier tension
strap gage was made and calibrated.

Frictional forces in the testing machine was kept
to a minimum by using roller bearings and rollers.

(b) **Description of the Test**

All panels were carefully cut out from 17ST
duralumin sheet with the grain parallel to the length-
wise direction. The thickness of the panels was
measured at three points on each side, and the average
value of the measured thicknesses was used to calculate the stress.

Before the specimen was placed into the testing machine, lead counter weights were placed on the loading beam to balance the loading system for each knife edge setting. After the system had been balanced, the panel was clamped between the angle sections. Slotted steel tubes were next attached to the panel to support the edges, and these tubes were clamped until sliding fit was made, in order to keep the load carried by the tubes, due to frictional forces, to a minimum. The resulting edge support can be assumed to be nearly simply supported.

There were four turnbuckles attached to the tubes by piano wire, which were used to prevent the tubes from spreading apart when the load was applied to the specimen. Care was taken when tightening the turnbuckles so that no edge moment was applied to the plate by the edge supporting tubes. At the corners of the specimen, between the tubes and the angle sections, additional supports were required in order to prevent a local failure from taking place.

After the plate was securely fastened to the apparatus, rollers at both sides of the panel were tightened to prevent the end of the panel from rotating, keeping the fixed and the movable ends
parallel throughout the test; but actually there was a slight rotation of the end.

The bottom I-beam with the knife edges was made horizontal by adjusting the jack. As the load was applied, the panel became shorter; hence, to keep the I-beam level, the jack had to be extended. Also, the loading cable attached to the angles was kept horizontal throughout the application of load by turning the screw $x$, attached to the pulley in Fig. 1.

The load was applied with an increment of about eight or ten readings, before the ultimate failure occurred. These readings were taken to determine whether there was a variation in shear modulus under various loading conditions; that is, various ratios of shear to compression. Near the maximum load the panel deflected without any increase in the load.
V. RESULTS

(a) Types of Failure

All panels that were tested failed by the formation of waves as shown by the photographs, Figs. 47-58. The panels of nominal thickness of .010" for both a/b ratios failed, by forming a number of secondary waves at the supporting edges as indicated by Fig. 47 and Fig. 53; thicker panels did not have many secondary waves at the edges.

Panels under compression failed at the center of the two edges. Before failure, one complete wave formed with a half wave length equal to half of the length for both a/b ratios. The wave became deeper and deeper until local failure took place at the center of the panel at both edges, illustrated by Figs. 52 and 58. When the panels of .010" were loaded by compression force, one complete wave formed at low stress; as the load was increased, secondary waves formed at the supports, the panels failing at many places along the edges, shown by Figs. 51 and 57 for both a/b ratios, similar to an Euler failure of a column.

The panels, when subjected to shear only, formed a diagonal wave, the angle of which was different for the two panel dimensions. For a/b = 2, the diagonal wave ran from one edge to the opposite edge at the
bottom of the panel; therefore, the angle with the horizontal for the wave was greater than 45 degrees. For panels with $a/b = 1 1/3$, the waves were nearly 45 degrees when subjected under shear and combined forces. The diagonal wave formed at low stress value, and it became deeper as the load was increased; secondary waves, induced by the main diagonal wave, formed at the edge supports. The panels failed locally at the supports, as shown by Figs. 48 and 54. The panels with thickness greater than .010" had only one secondary wave at the supports besides the main diagonal wave, but the panels of .010" thickness had many secondary waves at the supports, and when they failed, there were three or four local failures on both edges.

The wave formation for panels loaded with compression and shear was very similar to those subjected to shear alone.

(b) Variation of Ultimate Failure Stress with Thickness, $a/b$ ratios, and width, $b$

For $a/b = 2$, $b = 6$ inches, the curve of $\xi$ vs. $t$, thickness, for various ratios of $\frac{\xi}{\varepsilon}$ is a straight line. The curves for shear alone and for $\frac{\xi}{\varepsilon} = \tan 75^\circ$ are very close together for thicknesses of .010 and .020 inches. The increment between curves do not follow any logical sequence. The slope of the curves changes for different
loading conditions, being greatest for shear alone and smallest for \( \frac{\tau}{\sigma_c} = \tan 15^\circ \). The experimental scatter as shown by Fig. 17 is not very great, for the loading condition \( \frac{\tau}{\sigma_c} = \tan 15^\circ \) the scatter is very small.

The curves of \( \tau \) vs. \( t \) for \( a/b = 1 1/3 \) are nearly a straight line similar for the panels with \( a/b = 2 \). As the ratio of \( \frac{\tau}{\sigma_c} \) becomes larger, the curves of \( \tau \) vs. \( t \) deviate from a straight line. The slopes of the curves become greater as the amount of compression acting on the panel compared to the shear force becomes smaller. The increments between curves become smaller as the ratio \( \frac{\tau}{\sigma_c} \) increases. For the loading condition in which compression is the highest, the scatter of experimental points is very small, and the ultimate shear stress is very low.

For \( a/b = 2 \) and \( b = 6 \) inches, the curves of \( \tau \) vs. \( t \) for various loading conditions are again a straight line. The slopes of these curves decrease with the \( \frac{\tau}{\sigma_c} \) ratios, and the increments between curves are nearly constant, except for the case of pure compression, Fig. 18.

The curves of ultimate compression stress, \( \sigma_c \), vs. \( t \) for \( a/b = 1 1/3 \), \( b = 9 \) inches are straight lines. The slopes of these curves for various ratios of \( \frac{\tau}{\sigma_c} \) decrease with the increasing \( \frac{\tau}{\sigma_c} \) ratios. For \( a/b = 2 \) the increments between the different loading conditions
of compression to shear were nearly constant, but for this case the increment increases with the increase in the $\frac{\tau}{C_e}$ ratios.

(c) **Variation of the Ultimate Combined Stresses**

(1) **Variation of Ultimate Combined Stress with Thickness, $\sigma_c$ vs. $t$.**

The ultimate failure stresses were plotted as $\sigma_c$ vs. $t$ for various thicknesses and for two $a/b$ ratios.

For the smaller panels with $a/b = 2$, and $b = 6$ inches, the family of curves for the thickness are shown by Fig. 15 and are quite similar. By applying a small compression, there is a slight tendency for the thinner panels to increase the failure shear stress. This effect is very marked for a panel with nominal thickness of .010 inches. For thickness of .050 inches the effect is not very pronounced. When only compression force is acting on the panel, the ultimate failure compression stress is greatly reduced by the application of slight shear force.

The curves of ultimate compression stresses vs. ultimate shear stresses for $a/b = 1 1/3$ are different from those for $a/b = 2$. The curve for panels with $t = .010$ inches is different from the rest of the curves; it is similar to the curves for panels with $a/b = 2$. For the other thickness there is no tendency
for the ultimate shear stress to increase with the application of small compression force. The effect of shear load on the ultimate compression stress is not as great as for the panels with $a/b = 2$. The curves of $c_\tau$ vs. $\tau$ approach a circular shape with the increase in thickness of the specimen. Experimental scatter is small for both of the panels.

(2) Variation of Ultimate Combined Stresses

To indicate the variation of the ultimate combined stresses, the ratio of ultimate failure compression stress to ultimate compression stress without any shear load was plotted against the ratio of ultimate shear stress to ultimate shear stress without any compression force, $c_\tau$ vs. $\frac{X}{C_\tau}$.

When the values of $c_\tau$ and $\frac{X}{C_\tau}$ were obtained for panels with $a/b = 2$, for the five thicknesses, the resulting curves were practically coincident. The resulting curve has a vertical tangent with respect to the $\frac{X}{C_\tau}$ axis, and the curvature is greatest for low $c_\tau$ values. It has a shape similar to a cubic parabola. The scatter of experimental curves about the mean curve is not very great, especially for specimens with nominal thickness of .040 inches as shown by Fig. 7.

For panels with $a/b = 1 1/3$ and $b = 9$ inches,
the curves of $\frac{\sigma}{c_0}$ vs. $\frac{c}{c_0}$ for the various thicknesses did not fall on one common curve as for the panels with $a/b = 2$. The curve for a nominal thickness of .010 inches deviated from the others and is similar to the results for a panel with $a/b = 2$. Other curves are more or less symmetrical about a 45° line. These curves do not have nearly constant slope for low $\tau$ as for the panel with $a/b = 2$ as shown by Fig. 9. The experimental values are plotted with the mean curve in Figs. 4-14.

(d) **Stress Strain Diagrams**

Shear stress vs. shear strain curves are shown in Figs. 21-26. Shear stress, $\tau$, was calculated from the following equation:

$$\tau = \frac{S}{A}$$

$S$ = shear load

$A = b t =$ width times thickness

Unit shear strain, $\gamma$, was defined as

$$\gamma = \frac{\varepsilon}{\ell}$$

where $\varepsilon$ = deflection in the shear load direction

$\ell$ = length of the panel = $a$

For thin panels the load did not drop off sharply as for the thicker panels. Near the maximum stress the curve was poorly defined, because it was difficult
to obtain data, since the specimens deflected without any increase in the applied load. For low shear and compression stresses the curves are nearly a straight line. The stress at which the panels buckled was low; therefore, these curves show how the panels behave after they have entered the buckled state.

Compression stress vs. compression strain are shown by Figs. 27-32. The compression strain was calculated from the following equation:

\[ \sigma_c = \frac{P}{A} \]

where \( P \) = compression load

\( A = bt \) = width times thickness

The compression strain was obtained by dividing the deflection in the direction of the compression load, \( P \), by the length of the panel.

\[ \varepsilon = \frac{\Delta_x}{\ell} \]

\( \Delta_x \) = deflection in the direction of \( P \)

\( \ell \) = \( a \) = length of the panel

\( \varepsilon \) = compression strain, in./in.

These stress strain curves are similar to the shear stress strain curves, with a gradual drop off of compression load beyond the maximum load for the thin panel.

(e) **Variation of Apparent Shear Modulus**

The apparent shear modulus, \( G'_s \), was obtained from
the shear stress strain curves by measuring the slope of the curve at shear stress equal to zero. In order to get the slope with a reasonable degree of accuracy, the stress strain curves were plotted to a large scale from which the slope was determined.

For all thicknesses the apparent shear modulus reaches a maximum value when there is a slight compression on the panel. The rate of increase in apparent shear modulus with compression is greater than the rate of decrease in $G_e$ after the maximum value has been reached. The apparent shear modulus is greater for thicker panels, but the value of modulus $G_e'$ is much lower than the shear modulus obtained from the following equation:

$$G = \frac{E}{2(1+\nu)}$$

where: $E = \text{modulus of elasticity} = 10.3 \times 10^6 \text{ lbs./sq. in.}$

$\nu = \text{Poisson's ratio} = 0.3$

$$G = \frac{10.3 \times 10^6}{2(1+.3)} = 3.96 \text{ lbs./sq. in.}$$

For both panel dimensions, the variation of $G_e'$ vs. $\zeta$ is shown by Figs. 45-46.

Due to the fact that there are many possible sources of errors, the scatter of experimental points is great in the above two figures.
V. DISCUSSION OF RESULTS

(a) Ultimate Combined Stress

The curve faired through the experimental values for the $\frac{\sigma_x}{\sigma_e}$ vs. $\frac{x}{t_e}$ has the form of

$$(\frac{\sigma_x}{\sigma_e})^m + (\frac{x}{t_e})^n = 1.$$  \hspace{1cm} (1)

When the variation of thickness was eliminated by using the faired values to obtain $\frac{\sigma_x}{\sigma_e}$ and $\frac{x}{t_e}$ from the curve of $\sigma_x$ vs. $t$ and $x$ vs. $t$, Fig. 4 and 42, the curves for various thicknesses were coincident for the panels with $a/b = 2$. Such success was not obtained for the panels with $a/b = 1.1/3$; the .010 inch panel being very different from the other thicknesses, as shown by Fig. 9.

Other investigators have used Eq. 1 with different values for $m$ and $n$. Wagner found that $m = 1$ and $n = 2$ for buckling of flat strip under combined shear and compression (Ref. 4). Bridget used the formula with $m = 1$ and $n = 3$ for the critical stress in an unstiffened circular cylinder under combined shear and compression. Two investigators at the Guggenheim Aeronautical Laboratory, California Institute of Technology, have also used the above formula and obtained different values for $n$. Butterworth found that $m = 1$ and $n = 4$ for panels with $a/b = 2$ and 3
and for thickness of .020 and .032. His experimental results agree very closely for \( n = 4 \). Tsubota, with a different method of applying the combined shear and compression on the panel, found that it was safe to use \( m = 1 \) and \( n = 1 \) for design purposes.

The values of \( m \) and \( n \) obtained from the present experimental investigation were: for panels with \( a/b = 2, m = 1.4 \) and \( n = 1.3 \), and for panels with \( a/b = 1 \ 1/3, m = 1.65 \) and \( n = .9 \); hence, using \( m = 1 \) and \( n = 4 \) in Eq. 1 is unsafe and also using the values of \( m = 1 \) and \( n = 1 \) is conservative, especially when the compression load on the panel is small. Further tests should be made to determine whether varying the length and keeping \( a/b = 2 \) has any effect on the values of \( m \) and \( n \) in Eq. 1. To obtain an empirical equation for the ultimate combined stresses which can be used for design purposes, further investigations should be made for \( a/b = 3 \), and 1, so as to extend the results obtained by this investigation.

(b) **Ultimate Stress**

On Figs. 15-16 the curves of \( \phi \) vs. \( \tau \) for various thicknesses were plotted. For panels with \( a/b = 2 \) and for thinner thicknesses, the ultimate shear stress is slightly increased by applying a small compression load; therefore, after the above mentioned \( a/b \) ratios
have been investigated, it would be advantageous to investigate panels with slight tension, to see what effect it has on the ultimate shear stress.

The tendency to increase the ultimate shear stress by small compression load is not present for panels with $a/b = 1 1/3$. This may be due to the fact that the magnitude of ultimate shear stress for shear alone is nearly equal to the ultimate compression stress without any shear.

(c) **Apparent Shear Modulus**

The present investigation has shown that there is variation in the effective shear modulus with various combined loading conditions. An analysis was made to determine why the apparent shear modulus was low even in the unbuckled stage. This analysis was carried out for the case when the panel was loaded only with a shear force, and is given in the Appendix.

It was found that the horizontal deflection, i.e., the deflection in the direction of the applied shear force, was caused mainly by the bending deflection and the deflection due to shear was only about 17% of the total deflection in the unbuckled state.

Further deflection was caused by the rotation of the ends of the panel. Great care was taken to keep
the ends parallel throughout the application of the load, but actually there was a slight rotation of the loaded end. If the rotation of the end was taken into consideration, the calculated values for the apparent shear modulus were close to the experimental values obtained from the slope of the shear stress-strain curve. These results are indicated in the analysis given in the Appendix.

Another reason why the experimental values for \( G' \) were low for thin panels was due to the fact that the thin panels buckled at very low stresses.

Further theoretical investigations should be made to find the explanation for the increase in the apparent shear modulus with slight compression force on the panel. The data obtained for this research project can be used to check the analysis.

(d) Conclusion

The results obtained by this investigation are not conclusive enough to formulate any general empirical formula for the combined ultimate stresses which can be used by the designers. The present research project should be continued until such a general formula is obtained, which can be used with confidence in designing airplane structures.
VII. REFERENCES


VIII. GRAPHICAL RESULTS
VARIATION OF ULTIMATE COMBINED STRESS

\[
\left( \frac{\sigma}{\sigma_0} \right)^4 + \left( \frac{\varepsilon}{\varepsilon_0} \right)^3 = 1
\]

\[
\frac{\sigma}{\sigma_0} = 1.3
\]

\[
\left( \frac{\varepsilon}{\varepsilon_0} \right)^{0.65} + \left( \frac{\varepsilon}{\varepsilon_0} \right)^3 = 1
\]

\[
\frac{\sigma}{\sigma_0} = 2.3
\]
Variation of Ultimate Combined Stress
For \( b = 2, b = 6 \) in.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>o</td>
<td>0.10</td>
</tr>
<tr>
<td>x</td>
<td>0.20</td>
</tr>
<tr>
<td>Δ</td>
<td>0.30</td>
</tr>
<tr>
<td>*</td>
<td>0.40</td>
</tr>
<tr>
<td>o</td>
<td>0.50</td>
</tr>
</tbody>
</table>

FIG. 5
VARIATION OF ULTIMATE COMBINED STRESS
FOR $\theta = 90^\circ$, $A = 6$ IN.
THICKNESS = 0.10 IN.
VARIATION OF ULTIMATE COMBINED STRESS
FOR $\theta/\rho = 2$, $b = 6$ IN.
THICKNESS = 0.00 IN.
VARIATION OF ULTIMATE COMBINED STRESS
FOR \( q/\theta = 2 \), \( b = 6 \) IN.
THICKNESS = 0.32 IN.
Variation of Ultimate Combined Stress

For $a/b = 2$, $b = 6$ in.

Thickness = 0.40 in.
Variation of Ultimate Combined Stress
for \( \alpha = 2 \), \( b = 6 \) in.
Thickness = 0.50 in.
VARIATION OF ULTIMATE COMBINED STRESS
FOR $b = 1/2$, $b = 9$ IN.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>THICKNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.040</td>
</tr>
<tr>
<td></td>
<td>.050</td>
</tr>
<tr>
<td></td>
<td>.080</td>
</tr>
<tr>
<td></td>
<td>.092</td>
</tr>
<tr>
<td></td>
<td>.64 IN.</td>
</tr>
</tbody>
</table>

$\frac{C}{C_0}$

Fig. 9
VARIATION OF ULTIMATE COMBINED STRESS
FOR $a/b = 1/2$, $b = 9$ IN
THICKNESS = 0.11 IN
Variation of Ultimate Combined Stress
For \( \phi_0 = 1.5 \), \( D = 9 \text{ in.} \)
\( t = 0.020 \text{ in.} \)
Variation of Ultimate Combined Stress
for $\theta = 15^\circ$, $a = 9$ in.

$\theta = 0.338$ in.

Fig. 12
VARIATION OF ULTIMATE
COMBINED STRESS
FOR $0.625/4$, $L = 8$ in
$f = 0.090$

FIG. 13
VARIATION OF ULTIMATE COMBINED STRESS
FOR $q_1 = 18$, $L = 9$ IN.

$\tau = 0.50$

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>THICKNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.011 IN.</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>0.050</td>
</tr>
</tbody>
</table>
Ultimate Failure Stress

$\sigma_e$ vs $C$

For $gb = 2$, $A = 6$ in.

Symbol | Thick in
--- | ---
O | 0.011
x | 0.020
A | 0.032
+ | 0.040
* | 0.050

Ultimate Compression Stress, $\sigma_e$, lbs/sq.in

Ultimate Shear Stress, $\tau$, lbs/sq.in

Fig. 13
Ultimate Failure Stress

G_c vs. \tau

For a/b = 1/8, b = 9 in.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>THICK</th>
<th>in</th>
</tr>
</thead>
<tbody>
<tr>
<td>⬤</td>
<td>.011</td>
<td></td>
</tr>
<tr>
<td>⬤</td>
<td>.020</td>
<td></td>
</tr>
<tr>
<td>⬤</td>
<td>.030</td>
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</tr>
<tr>
<td>⬤</td>
<td>.040</td>
<td></td>
</tr>
<tr>
<td>⬤</td>
<td>.050</td>
<td></td>
</tr>
</tbody>
</table>

Ultimate Compressive Stress, \sigma_c \text{ ksi/ln}

Ultimate Shear Stress, \tau \text{ lb/sq in.}

Fig. 16
ULTIMATE SHEAR STRESS vs THICKNESS
FOR $\beta = 8, \delta = 6$ in

SHEAR STRESS ATTAINABLE in Eqs./ptm

THICKNESS $t$ in INCHES

Fig. 17
Ultimate Shear Stress vs. Thickness

For \( \frac{b}{t} = 1 \frac{5}{6} \), \( b = 9 \text{ in} \)

Shear Stress at Failure in KSI

Thickness, \( t \) in inches

Fig. 10
ULTIMATE COMPRESSIVE STRESS vs THICKNESS
FOR $d/h = 1/8$, $d = 8$ in

Thicknes, $t$, in inches

Fig. 20
SHR LOAD - DEFLECTION FOR $\frac{\theta}{\theta_c} = \cos 15^\circ$

$\theta = 12^\circ$, $\theta_c = 6^\circ$

- SYMBOL: THICKNESS
  - O10
  - 0.17
  - 0.24
  - 0.09
  - 0.09

Shear Unit Deflection: in/in
SHEAR LOAD - DEFLECTION FOR $T_e = 10$ at $15^\circ$

$a = 12^\circ$, $b = 6^\circ$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.503</td>
<td></td>
</tr>
<tr>
<td>0.406</td>
<td></td>
</tr>
<tr>
<td>0.320</td>
<td></td>
</tr>
<tr>
<td>0.201</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

Shear Load, lb/ft

Shear Unit Deflection, in/in
COMPRESSIVE LOAD DEFLECTION
for Pure Compression
2-12", 6"-6"

Symbol | Thickness
--- | ---
O | 0.04
x | 0.05
A | 0.06
+ | 0.07
G | 0.08

Compressive Stress, kip/in²

Compressive Unit Deflection, in/in

Graph showing the relationship between compressive stress and compressive unit deflection for various thicknesses.
COMPRESSION LOAD DEFLECTION for θ = 45°

q = 12°, A = 6

SYMBOL | THICKNESS
--- | ---
0 | 0.0409
1 | 0.0195
2 | 0.0313
3 | 0.0414
4 | 0.0495

COMPRESSION UNIT DEFLECTION in in

Fig. 26
Shear Load Deflection for \( \frac{H}{L} = 100, 60 \)

Shear Unit Deflection in/in
Shear Load-Deflection for $\frac{c}{c} = 100.15^\circ$
$q/\delta = 15$, $b = 9''$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>0.0102 in</td>
</tr>
<tr>
<td>*</td>
<td>0.0202</td>
</tr>
<tr>
<td>*</td>
<td>0.0316</td>
</tr>
<tr>
<td>*</td>
<td>0.048</td>
</tr>
<tr>
<td>*</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

Shear Stress, lbs/in $^2$

Shear Unit Deflection, in/in

Fig. 33
COMPRESSION LOAD-DEFLECTION FOR $\frac{L}{C} = 70\%$, $\theta = 15^\circ$

$\theta = 12^\circ$, $b = 9''$

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>THICKNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.002 IN.</td>
</tr>
<tr>
<td></td>
<td>0.0202</td>
</tr>
<tr>
<td></td>
<td>0.0316</td>
</tr>
<tr>
<td></td>
<td>0.0412</td>
</tr>
<tr>
<td></td>
<td>0.0496</td>
</tr>
</tbody>
</table>
COMPRESSION LOAD-DEFLECTION for \( \frac{L}{L_2} \) tan 30° 
\( \alpha = 12^\circ \), \( b = 2'' \)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>THICKNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>0.198</td>
</tr>
<tr>
<td></td>
<td>0.320</td>
</tr>
<tr>
<td></td>
<td>0.400</td>
</tr>
<tr>
<td></td>
<td>0.494</td>
</tr>
</tbody>
</table>

Compressive Unit Deflection, in./in.
COMPRESSION LOAD DEFLECTION for \( \frac{h}{h_0} = 0.6 \) at 15°

\[ \alpha = 12°, \beta = 9° \]

**Symbol** | **Thickness**
--- | ---
0 | 0.013
1 | 0.010
2 | 0.008
3 | 0.006
4 | 0.005
5 | 0.004
6 | 0.003
7 | 0.002
8 | 0.001
9 | 0.000

**Compressive Unit Deflection in/ln**

**Fig. 42**
COMPRESSION LOAD DEFLECTION for $z$ = 60°,

$\theta = 0^\circ$, $\delta = 0^\circ$

**Symbol** | **Thickness**
--- | ---
0 | 0.06
1 | 0.20
2 | 0.30
3 | 0.40
4 | 0.60

**Compressive Unit Deflection, in/in**

*Fig. 45*
COMPRESSION LOAD DEFLECTION FOR $\theta = 120^\circ$

$\alpha = 12^\circ$, $b = 9^\circ$

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>THICKNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>0.0405</td>
</tr>
<tr>
<td></td>
<td>0.0403</td>
</tr>
</tbody>
</table>

COMPRESSIVE UNIT DEFLECTION IN./IN.
\text{Ge vs } t

\text{AT } T = 0 \quad \frac{\rho}{\lambda} = 0.2

\text{Apparent Shear Modulus, Ge (in GPa)}

\text{Thickmess, } t, \text{ (in)}

\text{Synax} \quad \frac{\rho}{\lambda}

- 0.268

- 0.78

- 1.06

- 1.79

- 3.73

Fig. 45
**Pure Shear**

\[ a/b = 2, \ b = 6 \]
\[ t = .0109 \]

---

**Fig. 47**
Pure Shear

$a/b = 2, b = 6$

t = 0.0320

Fig. 48
Combined Shear & Compression

\[ \frac{\tau}{\sigma_c} = 1.00 \]
\[ a/b = 2, \quad b = 6, \quad t = 0.0108 \]

\[ \tau = 479 \]
\[ \tau = 771 \]

\[ \tau = 1098 \]
Failed

\[ \tau_{\text{max}} = 1511 \]

Fig. 49
Combined Shear & Compression

\[ \frac{\gamma}{\gamma_c} = 1.00 \]

\[ \frac{a}{b} = 2, \ b = 6, \ t = 0.0310 \]

\[ \gamma = 0 \]

\[ \gamma = 1380 \]

\[ \gamma = 2080 \]

Failed

\[ \gamma_{\max} = 3440 \]

Fig. 50
Pure Compression

$a/b = 2, \ b = 6$
$t = .0119$

$\sigma_c = 373$

$\sigma_c = 616$

$\sigma_c = 2342$

Fig. 51
Pure Compression

\[ \frac{a}{b} = 2, \ b = 6 \]

\[ t = 0.0319 \]

\[ \sigma_c = 1452 \]

\[ \sigma_c = 1992 \]

\[ \sigma_c = 5065 \]

Failed

\[ \sigma_{\text{max}} = 7950 \]

Fig. 52
Pure Shear

$s/b = 1 1/3, b = 9$

$t = .0104$

$r = 52.4$

$r = 182$

$r = 1540$

Failed

$r_{max} = 203$

Fig. 53
Pure Shear

\[ a/b = 1 1/3, \quad b = 9 \]
\[ t = .0313 \]

\( \gamma = 771 \)  
\( \gamma = 1176 \)

Failed

\( \tau_{\text{max}} = 2730 \)

Fig. 54
Combined Shear & Compression

\[ \frac{\tau}{\sigma_c} = 1.00 \]

\[ a/b = 1 \frac{1}{3}, \quad b = 9, \quad t = .0111 \]

\[ \tau = 160 \]
\[ \tau = 260 \]

\[ \tau = 701 \]
\[ \tau = 1142 \]
Failed

\[ \tau_{max} = 1252 \]

Fig. 55
Combined Shear & Compression

\( \frac{\tau}{\tau_s} = 1.00 \)

\( a/b = 1 \; 1/3, \; b = 9, \; t = .0316 \)

\[ \tau = 369 \]

\[ \tau = 1290 \]

Failed

\[ \tau_{max} = 2920 \]

Fig. 56
Pure Compression

\[ a/b = 1 \frac{1}{3}, \quad b = 9 \]
\[ t = 0.0104 \]

\[ \sigma_c = 642 \]
\[ \sigma_c = 1091 \]

\[ \sigma_c = 2725 \]

Failed

\[ \sigma_{c,max} = 2725 \]

Fig. 57
Pure Compression

$s/b = 1\ 1/3$, $b = 9$

t = .0317

\[ \sigma_e = 1590 \]

\[ \sigma_e = 2475 \]

\[ \sigma_e = 4060 \]

Failed

$\sigma_{e\ max} = 5500$

Fig. 58
IX. APPENDIX
Apparent Shear Modulus, $G'$, for a Flat Panel with
Slight Rotation of the Loaded End

1. Panel loaded at the end with a load, $P$, while there is a slight rotation of the end through an angle $\phi$.

2. Assumptions:
   a. Plate is held rigidly at the fixed end.
   b. Shear deflection shall be considered.
   c. Section are $\perp$ to the neutral axis.
   d. Couple applied at the loaded end will hold the section $\parallel$ to the fixed edge.
   e. Plate is not in the wave stage.

3. Method of Solution
   a. Obtain the expression for $K$ in terms of $G'_e$ and $E$.

   $$K = \frac{\phi_{\text{actual}}}{\phi_{\text{theoretical}}}; \quad M = \frac{EI}{L}(K\phi)$$

   b. Find the relationship between $\phi$ and $z$.
   c. Determine $\phi_{\text{exp.}}$ from $\phi_{\text{theo.}}$ and $K$.

   $\phi_{\text{theo.}}$ = ends are held parallel.
   d. Determine the values of $K$ for various $G'_e$.

      (1) Also determine $\phi_{\text{exp.}}$ for each value of $K$.  

     -84-
e. Plot the experimental results of $\gamma_{\text{exp}}$ vs. $x$.

(1) The value of $K$ can be determined from the slope of the curve.

f. Comparison between calculated $G_e'$ and actual $G_e'$.

h. Conclusions.

Solution

l. $G_e'$ as a function of $M = \frac{EI}{\ell} (K\varphi)$

\[ M = \frac{EI}{\ell} (K\varphi) \quad (1) \]

\begin{align*}
\gamma &= \frac{v}{\ell} = \frac{P\ell^3}{3EI} + \frac{\varphi}{5IG} - \frac{M\ell}{2EI} \\
&= \varphi_{\text{actual}} - \varphi_{\text{theo}}
\end{align*}

where $v = \text{total deflection at end due to the bending and shear deflection.}$

b. Expression for $M$.

\[ M = \frac{EI}{\ell} (K\varphi) \]

\[ K = \frac{\varphi_{\text{actual}}}{\varphi_{\text{theo}}} \quad \varphi_{\text{actual}} = \text{experimental value of } \varphi \]

\[ \varphi_{\text{theo}} = \text{the ends are held parallel} \]

c. Expression for $\varphi_{\text{theo}}$.

(1) Slope of the neutral axis without the end moment, $M$. 

-85-
The expression for $v$ at $y = 0$, considering the fixed end is rigidly fixed and also the shear deflection.

$$
(v)_{y=0} = \frac{px^3}{6EI} - \frac{pl^2}{2EI} + \frac{pl^3}{3EI} + \frac{2}{5} \frac{pc^r}{IG} (l-x)
$$

(3)

Slope at any point

$$
\frac{d(v)_{y=0}}{dx} = \phi = \frac{px^r}{3EI} - \frac{pl^r}{2EI} - \frac{2}{5} \frac{pc^r}{IG}
$$

(4)

Slope at $y = 0, x = 0$, (B).

$$
= -\frac{pl^r}{2EI} - \frac{2}{5} \frac{pc^r}{IG}
$$

(5)

but $dv = \tan \phi \frac{dx}{dx}$

hence

$$
\tan \phi = +\frac{pl^r}{2EI} + \frac{2}{5} \frac{pc^r}{IG}
$$

(6)

from which

$$
\phi = \tan^{-1} \left( \frac{pl^r}{2EI} + \frac{2}{5} \frac{pc^r}{IG} \right)
$$

(7)

$$
\Rightarrow \quad \phi = \left( \frac{pl^r}{2EI} + \frac{2}{5} \frac{pc^r}{IG} \right)
$$

(7a)
From (7a) \( \phi \) (actual) can be expressed in terms of \( \phi \) (theo.) by applying correction factor, \( K \).

\[
\phi \text{ (actual)} = K \left( \frac{PL}{2EI} + \frac{2}{5} \frac{Pc}{IG} \right) \tag{8}
\]

and

\[
M = \frac{EI}{l} \phi = \frac{EI}{l} K \left( \frac{PL}{2EI} + \frac{2}{5} \frac{Pc}{IG} \right) \tag{9}
\]

d. Finally the expression for \( \gamma \) becomes

\[
\gamma = \frac{PL}{3EI} + \frac{2}{5} \frac{Pc}{IG} - \frac{1}{2EI} \left( K \frac{EI}{l} \left( \frac{PL}{2EI} + \frac{2}{5} \frac{Pc}{IG} \right) \right)
\]

\[
= \frac{PL}{12EI} (4-3K) + \frac{Pc}{5IG} (2-K) \tag{10c}
\]

for \( c = \frac{b}{a} = \frac{1}{4}, \quad \frac{a}{b} = 2 \)

\[
G = \frac{E}{2.6}
\]

Substitute these values into (10)

\[
\gamma = \frac{PL}{12EI} (4-3K) + \frac{Pc}{5IG} (2-K)
\]

\[
= \frac{PL}{EI} \left[ (4-3K) + \frac{1}{8} (2-K) \right]
\]

\[
= \frac{PL}{EI} \left[ .3333 - .25K + .065 - .0325K \right]
\]

\[
= \frac{PL}{EI} \left[ .3983 - .2825K \right] \tag{11}
\]

but

\[
P = \tau bt = \frac{\tau l t}{2}
\]

and

\[
I = \frac{bh^3}{12} = \frac{tl^3}{4l}
\]

\[
G^{'I} = \frac{\tau}{\gamma} = \frac{\frac{2PL}{3EI} \left[ .3983 - .2825K \right]}{(4.8) \left[ .3983 - .2825K \right]} \tag{12}
\]

-87-
Solve for $K$ from eq. (12)

$$(49) (0.39 83 - 2.825 K) G_e' = E$$

hence

$$K = \frac{G_e' (49) (0.39 83) - E}{(49) (2.825) G_e'}$$  \hspace{1cm} (13)$$

Assume $E = 10.3 \times 10^6$ lbs./sq. in.

for which

$$K = \frac{19.15 G_e' - 10.3 \times 10^6}{13.59 G_e'}$$  \hspace{1cm} (13a)$$

2. Now express $\phi_K$ as a function of $r = \frac{P}{bt}$

a. $\phi_K = K \left( \frac{p l^3}{2EI} + \frac{3}{5} \frac{p c^3}{IG} \right)$ \hspace{1cm} (14)$$

but $c = \frac{l}{h}$, $I = \frac{t l^3}{96}$

$$G = \frac{E}{2.5}, \quad p = \frac{r l t}{2}$$

Substitute these values into eq. (14) so as to obtain a simple expression for $\phi_K$

$$\phi_K = K \left[ \frac{(\frac{r l t}{5}) l^3}{2EI} + \frac{3}{5} \frac{(\frac{r c^3}{h}) l^3}{IG} \right]$$

$$= K \left[ \frac{\frac{r l t}{5}}{E} + \left( \frac{c}{5} \right) \frac{r l}{E} \right]$$

$$= 27.12 K \frac{r}{E}$$  \hspace{1cm} (14a)$$

For $\phi_{\text{theo.}} = 1$

$$\therefore \phi_{\text{theo.}} = \frac{r}{E} (27.12) \quad \text{(i.e., no rotation of the ends; they are held parallel).}$$
b. \( \phi_{\text{exp.}} \) can be obtained from \( \phi_{\text{theo.}} \) and \( \phi_k \)

\[
\phi_{\text{exp.}} = \phi_{\text{theo.}} - \phi_k
\]

where:

\[
\phi_k = K \phi_{\text{theo.}}
\]

(1) From the figure it can be shown that

\[
\phi_{\text{exp.}} = \phi_{\text{theo.}} - \phi_k
\]

\[
= (1 - K) \phi_{\text{theo.}}
\]

\[
= \frac{\tau}{E} \left( 27.12 \right) (1 - K)
\]

3. The values of K for various \( G'_e \) values

a. \( K = \frac{G'_e \left( 10.15 \right) - 10.3 \times 10^4}{13.59 \ G'_e} \)

In deriving this expression for K, the value of G was taken to be \( \frac{E}{2(1 + \nu)} = \frac{E}{2.6} \)

\( \nu = 0.3 \)

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<th>( \phi_{\text{exp.}} \times 10^{-9} )</th>
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\( \phi_{\text{exp.}} = a \tau \)

\[ 1 - K = \frac{a}{2.63 \times 10^{-6}} \]

\( a \) is the slope of the curve
Graph showing $C_p$ vs. $K$ and $t$ for $\alpha = 2$.

The graph includes:
- A line labeled "Experimental $C_p$".
- A line labeled "$C_p$ for top values of $K$".

The graph axes are labeled:
- Vertical axis: Temperature (°F) with values 0, 50, 100.
- Horizontal axis: $K$ (dimensionless) with values 0, 20, 40, 60, 80, 100.
- Additional axis labeled "THICKNESS, t (in.)" with values 0, .01, .02, .03, .04, .05, .06, .07, .08.
for \( t = 0.040 \)

\[
a = \frac{(15.6 - 10.5) \times 10^4}{(2200 - 1200)} = 4.9 \times 10^{-1}
\]

\[
K = 1 - \frac{4.3 \times 10^{-1}}{2.63 \times 10^4} = 0.817
\]

\( G' \) from the graph of \( G' \) vs. \( K \).

\[
= 1.28 \times 10^6
\]

4. Calculation of \( G'_{\exp} \) from Experimental Data

\[
\tan \phi_{\exp} = \frac{\delta n - (\delta n + \delta s)}{1 + \frac{K E I}{2}} (17)
\]

or \( \phi_{\exp} = \frac{\delta n - (\delta n + \delta s)}{c} \) (17a)

4. \( G' \) in terms of \( G \) and \( K \).

\[a. \ \gamma = \frac{PL}{EI} + \frac{2\mu}{5} \frac{PC}{IG} - \frac{L}{AEI} \left[ K \frac{E I}{2} \left( \frac{PL}{AEI} + \frac{2\mu}{5} \frac{PC}{IG} \right) \right]
\]

\[
= \frac{PL}{12EI} (4-3K) + \frac{PC}{5IG} (2-K)
\]

\[c = \frac{b}{2} = \frac{L}{4}, \quad \frac{a}{b} = 2
\]

\[
= \frac{PL}{I} \left[ \left( \frac{4-3K}{12E} \right) + \frac{(2-K)}{80G} \right]
\]

\[
\zeta = \frac{2P}{IK}, \quad I = \frac{tL^3}{4L}
\]

\[G' = \frac{c}{\gamma} = \frac{2P}{12L} \left[ \frac{PL}{tL^3} \left( \frac{4-3K}{12E} \right) + \frac{(2-K)}{80G} \right]
\]

\[
= \frac{1}{48} \left[ \frac{9L_0}{2L} \frac{EG}{3L_0 E - 2L_0 K G + 24E - 12KE} \right]
\]

\[G' = \frac{5EG}{20G(4-3K) + 3E(2-K)}
\]

-92-
b. \( G' \)

\[
G = \frac{-3 \, E G_e' \, (2-K)}{20 \, G_e' \, (4-3K) - 5E}
\]  

6. Comparison between the calculated values of \( G_e' \) and the experimental results.

a. \( G_e' \) was calculated by the following method:

(1) The value of \( K \) was determined from the curve of \( G_{exp} \) vs. \( \tau \) for each thickness.

(2) \( G_e' \) was determined from the equation

\[
G'_e = \frac{E}{(48)(.3383 - .2825K)}
\]

by using the experimental values of \( K \).

b. The calculated and experimental \( G'_e \) are plotted on page 92.

(1) The calculated values of \( G'_e \) are higher than the experimental results.

(2) \[
G'_e = \frac{5 \, E G}{20(4-3K) + 3E(2-K)}
\]

\[
= \frac{5 \, E}{20(4-3K) + 3 \, E \, (2-K)} \, \frac{c}{G}
\]

The calculated values of \( G'_e \) are larger than the experimental values because of the following reasons:

(a) In obtaining the value of \( G'_e \), the value of \( G \) was assumed to be \( \frac{E}{2(1+.3)} \) but if \( G \) is
actually less than the assumed value, the calculated $G'_c$ will be lower, if this correct $G$ is used.

(b) $G'_c$ will be lower if the value of $K$ is taken to be smaller than the experimental result.

7. Conclusion:

a. Further careful study should be made to determine why the calculated $G'_c$ is larger than the experimental $G'_c$; whether it is due to the variation of $G$ or $K$, or due to the buckling phenomena.
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