THE DETERMINATION OF TWO PARAMETERS DEALING WITH POWER-ON STABILITY FOR A MODEL WITH RIGHT HAND PROPELLERS

THESIS

BY

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SUMMARY

This work presents expressions for determining the down-wash over the tail and the tail efficiency power-on and off. A method for the determination of empirical constants used by Millikan is presented. The method gives excellent agreement for the multiplicative factor A_P and good agreement for B_P . The theory checks experiment very well as A_P and B_P are nearly independent of T_c .

FOREWORD

This work was suggested by Dr. C. B. Millikan during the writing of his paper for the Third Wright Brothers Lecture. I have drawn freely from this work and have used the same notation except where I have felt that the work was hindered by it.

It is felt that when working from the experimental data, one should work with the quantities capable of measurement and not with the ratios suggested by Dr. Millikan.

It is to be noted in the thesis that I have left the final equations in the final camouflaged form because of the ease of writing. The notation introduced has been chosen to clarify rather than hinder.

This thesis has been written to be read by one who is familiar with Dr. Millikan's paper. It is to be considered as an appendix to that work.

TABLE I

Characteristics of the Model Used

(Dimensions are Full Scale)

WING

A = Aspect Ratio = 7.44

S = Area = 603.8 ft^2

c = Mac = 9.65 ft.

TAIL

A_t = Aspect Ratio = 3.58

S_t = Area = 122.7 ft^2

l = Length From To Elev. Hinge = 27.84 ft.

C_a = Wing Chord Behind Prop = 10.66 ft.

ν = No. of Engines Operating = 2

d = Propeller Diameter = 12.5 ft.

Q = Fraction of Tail Surfaces in Slipstream = 0.533

Aspect Ratio in Slipstream = 1.115

Giving $\lambda = 1.00$

The Determination of Two Parameters Dealing With Power-on Stability For a Model With Right Hand Propellers

In the paper by C. B. Millikan, "The Influence of Running Propellers on Airplane Characteristics," important effects were expressed in terms of empirical formulas whose justification rests on very meager experimental evidence. These are essentially the down-wash over the tail and the horizontal tail efficiency, power-on.

In considering these two factors it is convenient to first find expressions permitting their determination from power-on wind tunnel tests. To do this we must first find these two factors for the power-off case.

POWER-OFF ANALYSIS

In Ref. 1, Millikan gave the expression for the pitching moment of the tail assuming that the slope of the lift curve for infinite aspect ratio (a_0) was the same for the tail as for the wing. As the Davis wing sections have abnormally large values of a_0 , an analysis was made to discover if there was any change in the results for an airplane having Davis profiles for the wing and conventional sections for the tail.

Now if we consider EQ. 12, REF. 1*, $C_{L_t} = \zeta_t\, a_{ti}(\alpha - \varepsilon + i_t - i)$

and express $\varepsilon = m\,\varepsilon_0$

where $\varepsilon_0 = C_{L_W}/\pi A$

giving $\varepsilon = m\dfrac{C_{L_W}}{\pi A}$

* Notation used here is the same as in Ref. 1

putting $\alpha = \frac{C_L}{a}$

where "a" refers to the slope of the lift curve tail on;

we have

$$C_{L_t} = \zeta_t \, a_{ti} \left(\frac{C_L}{a} - \frac{m}{\pi A} C_{L_w} + i_t - i \right) \qquad 3$$

Now from $Eq. 11$ ref. 1

$$C_{L_w} = C_L - \frac{q_t}{q} \frac{S_t}{S} C_{L_t} \qquad 4$$

Inserting this into $Eq\ 3$ we have

$$C_{L_t} = \zeta_t \, a_{ti} \left(\frac{C_L}{a} - \frac{m}{\pi A} \left[C_L - \frac{q_t}{q} \frac{S_t}{S} C_{L_t} \right] + i_t - i \right)$$

or

$$C_{L_t} = \frac{\zeta_t \, a_{ti} \left(C_L \left[\frac{1}{a} - \frac{m}{\pi A} \right] + i_t - i \right)}{1 - \zeta_t \, a_{ti} \frac{q_t}{q} \frac{S_t}{S} \frac{m}{\pi A}} \; ; \qquad 5$$

letting $\zeta_t \frac{q_t}{q} = \eta_t$ we have

$$C_{L_t} = \frac{\zeta_t \, a_{ti} \left[\left(\frac{1}{a} - \frac{m}{\pi A} \right) C_L + i_t - i \right]}{1 - \eta_t \frac{S_t}{S} \frac{m}{\pi A} a_{ti}} \qquad 6$$

Now

$$\delta_t C_M = -\frac{q_t}{q} \frac{l}{c} \frac{S_t}{S} C_{L_t}$$

and letting $\eta_t = \zeta_t q_t / q$

we have

$$\delta_t C_M = \frac{-\eta_t \frac{l}{c} \frac{S_t}{S}}{1-\eta_t \frac{S_t}{S} \frac{m}{\pi A} a_{ti}} a_{ti}\left[\left(\frac{1}{a} - \frac{m}{\pi A}\right) C_L + i_t - i\right] \qquad 7$$

Where $a_{ki} = \dfrac{a_{ok}}{1+\dfrac{a_{ok}}{\pi A_k}}$

The subscript "k" refers to the surface in question

i.e. a_{ow} — a_o OF THE WING

a_{ot} — a_o " " TAIL

A_k = Aspect ratio of the surface in question

a_{ok} = Slope of the lift curve infinite aspect ratio

In the analysis we shall drop the subscript "k" when referring to the wing.

To determine the down-wash and tail efficiency from this equation we must find two equations with the two unknowns m and η_t. From wind tunnel tests we may measure directly the slope of the pitching moment curve and the stabilizer effectiveness. We shall therefore compute these factors:

The slope of the pitching moment curve

$$\frac{\partial}{\partial C_L} \delta_t C_M$$

and

The stabilizer effectiveness

$$\frac{\partial \, \delta_t C_M}{\partial i_t}$$

Performing the indicated operations on $Eq\ 7$ we have

$$\frac{\partial}{\partial C_L}\delta_t C_M = \frac{-\eta_t \frac{l}{c}\frac{S_t}{S}}{1-\eta_t \frac{S_t}{S}\frac{m}{\pi A} a_{ti}} a_{ti}\left(\frac{1}{a} - \frac{m}{\pi A}\right) \qquad 8a$$

$$\frac{\partial}{\partial i_t}\delta_t C_M = \frac{-\eta_t \frac{l}{c}\frac{S_t}{S}}{1-\eta_t \frac{S_t}{S}\frac{m}{\pi A} a_{ti}} a_{ti}\left(1 - \frac{\partial i}{\partial i_t}\right) \qquad 8b$$

Introducing the notation

$$\frac{\partial}{\partial C_L}\delta_t C_M = \gamma \quad ; \quad \frac{\partial}{\partial i_t}\delta_t C_M = \nu$$

$$1 - \frac{\partial i}{\partial i_t} = \beta$$

Equations 8 become

$$\gamma = \frac{-\eta_t \frac{l}{c}\frac{S_t}{S}}{1-\eta_t \frac{S_t}{S}\frac{m}{\pi A} a_{ti}} a_{ti}\left(\frac{1}{a} - \frac{m}{\pi A}\right) \qquad 9a$$

$$\nu = \frac{-\eta_t \frac{l}{c}\frac{S_t}{S}}{1-\eta_t \frac{S_t}{S}\frac{m}{\pi A} a_{ti}} a_{ti}\,\beta \qquad 9b$$

Divide 9a by 9b and get

$$\frac{\gamma}{v} = \frac{\frac{1}{a} - \frac{m}{\pi A}}{\beta} \qquad 10$$

or solving for m we have

$$m = \pi A\left[\frac{1}{a} - \beta\frac{\gamma}{v}\right]. \qquad 11$$

To solve for η_t substitute the value of m in Eq 9b

$$v = \frac{-\eta_t \frac{\ell}{c}\frac{S_t}{S}\, a_{ti}\,\beta}{1-\eta_t \frac{S_t}{S} a_{ti}\left(\frac{1}{a} - \beta\frac{\gamma}{v}\right)} \quad ; \qquad 12$$

now η_t is

$$\eta_t = \frac{v}{\frac{S_t}{S} a_{ti}\left(\frac{v}{a} - \beta\gamma\right) - \frac{\ell}{c}\frac{S_t}{S} a_{ti}\,\beta} \qquad 13$$

or rewriting in a more convenient form

$$\frac{1}{\eta_t} = \frac{S_t}{S}\, a_{ti}\left[\frac{1}{a} - \frac{\beta\left(\frac{\ell}{c} + \gamma\right)}{v}\right] \qquad 14$$

A summary of equations :

$$m = \pi A \left[\frac{1}{a} - \beta \frac{\gamma}{v} \right]$$

$$\frac{1}{\eta_t} = \frac{S_t}{S} a_{ti} \left[\frac{1}{a} - \beta \frac{\left(\frac{\ell}{c} + \gamma \right)}{v} \right] .$$

The factor $\beta = 1 - \frac{\partial i}{\partial i_t}$ takes into account the shift of zero lift due to the tail that has been neglected in many analyses.

The factor $\frac{\partial i}{\partial i_t}$ may be obtained by computation or by the measurement of a cross plot of C_L vs α curves for different i_t's . As an example see fig. 1.

The computation of m and η_t is most easily done by following certain well established steps. One finds that much time is spent in collecting the data and only a short while in reducing it to the appropriate form.

Now when making these calculations one therefore should collect the data first and draw the curves second.

This is the general idea given in the scheme below in Table II. These data were taken from Table I, which is a compilation of many of the pertinent model dimensions and is presented on page f .

After the scheme is included a calculation for the airplane in Ref. 1.

TABLE II

1. the wing area S
2. the wing aspect ratio A
3. the wing MAC c
4. the tail area S_t
5. the tail aspect ratio A_t
6. the tail length l
7. Slope of the lift curve $\frac{dC_L}{d\alpha} = a$
8. C_M vs C_L tail on for various stabilizer angles
   i.e. $i_t = a$, $i_t = b$, $i_t = c$, $i_t = d$, etc.
9. C_M vs C_L tail off
10. C_L vs α for the corresponding stabilizer angles
11. From 8, 9 + 10 plot $\delta_t C_M$ vs C_L for the corresponding i_t's
    $\delta_t C_M$ vs i_t for $C_L = 0, 0.5, 1.0$ AND i vs i_t (see figs 1, 2 + 3)
12. From 11, determine $\frac{\partial \delta_t C_M}{\partial C_L}$, $\frac{\partial \delta_t C_M}{\partial i_t}$ and $\frac{\partial i}{\partial i_t}$
13. Substitute in expressions for m and η_t

TABLE II a

DATA

1. $S = 603.8\ ft^2$
2. $A = 7.44$
3. $c = 9.65\ ft.$
4. $S_t = 122.7\ ft^2$
5. $A_t = 3.58$
6. $l = 27.84\ ft$
7. $a = 4.93$

$\gamma = -0.218$

$\nu = -1.605$

$\frac{\partial i}{\partial i_t} = 0.147$, $\beta = 0.853$

$$\frac{S_t}{S} = 0.203$$

$$\frac{l}{c} = 2.886$$

$$a_{ti} = \frac{5.7}{1 + \frac{5.7}{\pi\, 3.58}} = 3.79$$

CALCULATION

$$\frac{l}{c} + \gamma = 2.886 - .218 = 2.668$$

$$\beta\left(\frac{l}{c} + \gamma\right) = .853(2.668) = 2.280$$

$$\frac{1}{\eta_t} = .203\ (3.79) \left[\frac{2.280}{1.605} + .203\right] = 1.252$$

$$\eta_t = .799$$

$$m = \pi\ 7.44 \left[.203 - .853\ \frac{.218}{1.605}\right]$$

$$= \pi\ 7.44(.087)$$

$$= 2.035$$

$$m = 2.04$$

In practice the gap between the elevator and the stabilizer is not sealed off so this reduces the slope of the lift curve of the tail (a_t) . Since this reduction is not taken into consideration theoretically it is taken into account in the experimental values of η_t and m .

POWER-ON ANALYSIS

The method for determining the down-wash and tail efficiency power-on is essentially the same as for the power-off case with more complicated expressions.

The notation used is that of Ref. 1 unless otherwise stated.

From wind tunnel tests we may obtain, as in the power-off case, the slope of the pitching moment curve and the stabilizer effectiveness at a constant power. ($T_c = \text{Const.}$)

First, we must derive the expression for $\delta_t^P C_M$ for the general case as was done for Eq. 7

Consider now Eq. 25 of reference 1.

$$C_{L_t}^P = S_t^P a_{ti} \left(\alpha_p - \varepsilon_p + i_t - i_p \right) \qquad 15$$

and writing

$$\varepsilon_p = \frac{m}{\pi A} C_{L_W} + f_D \alpha_p + g_D \qquad 16$$

and substituting the expressions for $C_{L_t}^P$ and noting that

$\frac{C_L}{a_p} = \alpha_p$ we have

$$C_{L_t}^P = 5_t^P a_{ti} \left(\frac{C_L}{a_p} \{1 - f_D\} - \frac{m}{\pi A} C_{L_W} - g_D + i_t - i_p \right) \quad ; \qquad 17$$

now from Eq 21 and 22 of ref. 1. we have

$$C_{L_W} = (1 - f_L) C_L - \frac{q_t}{q} \frac{S_t}{S} C_{L_t}^P - g_L \qquad 18$$

Now substituting for C_{L_W} in Eq 17 and solving for $C_{L_t}^P$ we have

$$C_{L_t}^P = 5_t^P a_{ti} \left[\left\{ \frac{1}{a_p} - \frac{f_D}{a_p} - \frac{m}{\pi A}(1 - f_L) \right\} C_L + i_t - i_p + \frac{m}{\pi A} - g_D \right] \qquad 19$$

Now since $\eta_t^P = 5_t^P \frac{q_t}{q}$ and $\delta_t^P C_M = -\frac{l}{c} \frac{q_t}{q} \frac{S_t}{S} C_{L_t}^P$

we have

$$\delta_t^P C_M = \frac{-\eta_t^P \frac{l}{c} \frac{S_t}{S} a_{ti}}{1 - \eta_t^P \frac{S_t}{S} \frac{m}{\pi A} a_{ti}} \left[\left\{ \frac{1}{a_p} - \frac{f_D}{a_p} - \frac{m}{\pi A}(1 - f_L) \right\} C_L + i_t - i_p + \frac{m}{\pi A} g_L - g_D \right] \qquad 20$$

For the stabilizer effectiveness $\frac{\partial}{\partial i_t} \delta_t^P C_M$ we have

$$\frac{\partial}{\partial i_t} \delta_t^P C_M = \frac{-\eta_t^P \frac{l}{c} \frac{S_t}{S}}{1 - \eta_t^P \frac{S_t}{S} \frac{m}{\pi A} a_{ti}} a_{ti} \left[1 - \frac{\partial i_p}{\partial i_t} + \frac{m}{\pi A} \frac{\partial g_L}{\partial i_t} - \frac{\partial g_D}{\partial i_t} \right] \qquad 21$$

$\frac{\partial i_p}{\partial i_t}$

The quantity $\frac{\partial i_p}{\partial i_t}$ can be measured from wind tunnel tests in the same manner as $\frac{\partial i}{\partial i_t}$.

$\frac{\partial g_L}{\partial i_t}$

g_L is given in Ref. 1, Eq 52. This gives us

$$\frac{\partial g_L}{\partial i_t} = \left(\eta_t^P a_{ti} \frac{S_t}{S}\right) g_{L_2} \frac{\partial i_p}{\partial i_t},$$

The quantity $\eta_t^P a_{ti} \frac{S_t}{S} g_{L_2}$ is independent of η_t^P or f_D and equals

$$-\frac{\nu d^2}{S}\left[2T_c + K_1(\lambda a_w - K_2)\right] \qquad 22$$

where ν = number of engines

d = propeller dia

S = wing area

For convenience we introduce the notation

$$\omega = \frac{\nu d^2}{S}\left[2T_c + K_1(\lambda a_w - K_2)\right] \qquad 23$$

Then

$$\frac{\partial g_L}{\partial i_t} = -\omega \frac{\partial i_p}{\partial i_t} \qquad 24$$

$\frac{\partial g_D}{\partial i_t}$

From *Eq 34, Ref. 1* we have

$$g_D = f_D (i_w - i_p) \qquad 25$$

giving

$$\frac{\partial g_D}{\partial i_t} = -f_D \frac{\partial i_p}{\partial i_t} \qquad 26$$

Substituting equations 24 and 26 in Eq. 21 we have

$$\frac{\partial}{\partial i_t} \delta_t^P C_M = \frac{-\eta_t^P \frac{l}{c} \frac{S_t}{S}}{1-\eta_t^P \frac{S_t}{S} \frac{m}{\pi A} a_{ti}} a_{ti} \left[1 - \frac{\partial i_P}{\partial i_t} \left(1 - \frac{m}{\pi A} w - f_D\right)\right] \qquad 27$$

For the slope of the tail pitching moment curve we have

$$\frac{\partial}{\partial C_L} \delta_t^P C_M = \frac{-\eta_t^P \frac{S_t}{S} \frac{l}{c}}{1-\eta_t^P \frac{S_t}{S} \frac{m}{\pi A} a_{ti}} a_{ti} \left[\frac{1}{a_P} - \frac{f_D}{a_P} - \frac{m}{\pi A}(1-f_L)\right] \qquad 28$$

Introducing the notation

$$\frac{\partial}{\partial C_L} \delta_t^P C_M = \gamma \ ; \quad \frac{\partial}{\partial i_t} \delta_t^P C_M = \nu \qquad \frac{\partial i_P}{\partial i_t} = \beta$$

f_D

We have, dividing ν by γ,

$$\frac{\nu}{\gamma} = \frac{1 - \beta\left(1 + \frac{m w}{\pi A} - f_D\right)}{\frac{1}{a_P} - \frac{f_D}{a_P} - \frac{m}{\pi A}(1-f_L)} \qquad 29$$

Solving for f_D we have

$$f_D = \frac{\frac{\nu}{\gamma}\left[\frac{1}{a_p} - \frac{m}{\pi A}(1-f_L)\right] - \left[1-\beta\left(1+\frac{m}{\pi A}w\right)\right]}{\frac{\nu}{\gamma a_p} + \beta} \qquad 30$$

Where from *Eq. 32, Ref 1* we get

$$f_L = \frac{w}{a_p} \qquad 31$$

We shall leave f_L in the equations to show where it goes with each algebraic manipulation.

We must check this equation to see if it disappears when T_C goes to 0. For $T_C = 0$ we get upon inspection of the various quantaties containing T_C

$$w = 0$$
$$f_L = 0$$
$$a_p = a$$
$$\beta = \frac{\partial i}{\partial i_t}$$

Upon substitution we have

$$f_D = \frac{\frac{\nu}{\gamma}\left[\frac{1}{a} - \frac{m}{\pi A}\right] - \left[1-\frac{\partial i}{\partial i_t}\right]}{\frac{\nu}{\gamma a} + \frac{\partial i}{\partial i_t}} \qquad 32$$

If we inspect $\frac{\nu}{\gamma a} + \frac{\partial i}{\partial i_t}$ we find that it is finite so we will turn our attention to the numerator.

From Eq 11 we find that $\left(\frac{1}{a} - \frac{m}{\pi A}\right) = \frac{\gamma}{\nu}\left(1 - \frac{\partial i}{\partial i_t}\right)$ so upon substitution in Eq 32 we have

$$\frac{\nu}{\gamma}\left[\frac{\gamma}{\nu}\left(1 - \frac{\partial i}{\partial i_t}\right)\right] - \left[1 - \frac{\partial i}{\partial i_t}\right] \qquad 33$$

Therefore our downwash f_D is zero when $T_c = 0$

So the expression is correct in the limit.

η_t^P

Now to solve for the tail efficiency we substitute the value for f_D in Eq 27 and get first

$$\frac{1}{\eta_t^P} = \frac{S_t}{S} a_{ti}\left[\frac{m}{\pi A} - \frac{l}{c}\frac{1}{\nu}\left\{1 - \beta\left(1 - \frac{m\omega}{\pi A} - f_D\right)\right\}\right] \qquad 34$$

and putting in f_D we have

$$\frac{1}{\eta_t^P} = \frac{S_t}{S} a_{ti}\left[\frac{m}{\pi A} - \frac{\frac{l}{c}\frac{1}{\gamma}\left\{\frac{1}{a_p} - \beta\frac{m}{\pi A}\right\}}{\frac{\nu}{\gamma a_p} + \beta}\right] \qquad 35$$

Using the same method as employed above we insert $T_c = 0$ and get

$$\left(\frac{1}{\eta_t^P}\right)_{T_c = 0} \equiv \frac{1}{\eta_t}$$

SUMMARY

$$f_D = \frac{\frac{\nu}{\gamma}\left[\frac{1}{a_p} - \frac{m}{\pi A}(1-f_L)\right] - \left[1-\beta\left(1+\frac{m w}{\pi A}\right)\right]}{\frac{\nu}{\gamma a_p} + \beta}$$

$$\frac{1}{\eta_t^p} = \frac{S_t}{S}\, a_{ti}\left[\frac{m}{\pi A} - \frac{\ell}{c}\,\frac{\frac{1}{\gamma}\left\{\frac{1}{a_p} - \beta\frac{m}{\pi A}\right\}}{\frac{\nu}{\gamma a_p} + \beta}\right]$$

DISCUSSION

The experimental data were obtained in December 1939 upon a model furnished by the North American Aviation Co. and run as GALCIT Rep. [illegible]. From this series of tests the tail pitching moments power-on and off were constructed. These are given in Figs. 4--16. The values of the parameters given in the equations are given in Table III below.

TABLE III

Tabulation of Wind Tunnel Test Results

T_c	a_p	γ	ν	$\frac{\partial i_p}{\partial i_t}$ FAIRED	$\frac{\partial i_p}{\partial i_t}$ TEST
0	4.960	$^-$.218	$^-$1.605	.1470	.147
.0744	5.31	$^-$.207	$^-$1.805	.1486	.152
.140	5.565	$^-$.197	$^-$1.965	.1493	.146
.214	5.760	$^-$.185	$^-$2.130	.1498	.152
.303	5.960	$^-$.171	$^-$2.310	.1500	.149

The values for $\frac{\partial i_p}{\partial i_t}$ were taken from a curve faired through the experimental points.

In figs. 17--19 are given the values in Table III plotted against T_c. The power-on values fair into the power-off values smoothly showing that at low T_c's the effect of rotation is not great. In computing f_D and η_t^p the equations were put into a different form so as to make the computation simpler.

They are

$$\left.\begin{aligned} f_D &= \frac{\frac{\nu}{\gamma}\left[1-\frac{m}{\pi A}(a_p-\omega)\right] - a_p\left[1-\beta\left(1+\frac{m\omega}{\pi A}\right)\right]}{\frac{\nu}{\gamma}+a_p\beta} \\ \eta_t^p &= \frac{S_t}{S}\, a_{ti}\left[\frac{m}{\pi A} - \frac{l}{c}\,\frac{\frac{1}{\gamma}\left(1-\beta\frac{m}{\pi A}a_p\right)}{\frac{\nu}{\gamma}+a_p\beta}\right] \\ \beta &= \frac{\partial i_p}{\partial i_t} \end{aligned}\right\} \qquad 37$$

f_D

From the above expressions the down-wash factor f_D was computed and has been plotted in Fig. 20. Millikan used a formula for f_D (EQ 33, REF. 1)

$$f_D = B_p \frac{QR(R-1)}{1+Q(R-1)} \qquad 38$$

Where Q = fraction of tail in slipstream = 0.533 for the above case.

$R = 1 + \frac{8}{\pi} T_c$

Calling this function φ we have

$$f_D = B_p \varphi \qquad 39$$

The factor B_p was assumed by C. B. Millikan to be 0.3 and is plotted in fig. 21. A good value from these tests is :

$B_P = 0.37$. Tests should be made on other types of airplanes to determine if this factor is correct. The variation of φ with Q and T_c has been plotted in fig. 22. For $Q=1$ the function is a straight line and is $\varphi = \frac{8}{\pi} T_c$

η_t^p

η_t^p has been computed and plotted in fig. 23. The reason why η_t^p increases as T_c increases is that the tail efficiency is defined not as a direct function of the tail pitching moment slope but as an interference factor times the ratio of q_t to q or $\eta_t^p = \zeta_t^p q_t/q$. Now if we consider $\zeta_t^p \doteq \zeta_t \frac{q_t^p}{q}$ we may write $\eta_t^p \doteq \zeta_t \frac{q_t^p}{q_t} \frac{q_t}{q}$ or since $\eta_t = \zeta_t \frac{q_t}{q}$ we have $\eta_t^p = \frac{q_t^p}{q_t} \eta_t$ and writing $q_t^p/q = 1 + \frac{8}{\pi} Q T_c$ and including a corrective factor we have

$$\eta_t^p = A_p \left(1 + \frac{8}{\pi} Q T_c\right) \eta_t \qquad 40$$

This same equation is given in Eq. 29 Ref. 1.

With this expression the value of A_p was calculated and plotted on Fig. 23.

From the discussion above it may be said that the tail interference factor ζ_t is changed by the influence of power. This change is shown by a variation in A_p from 1.02 to 0.95. This is within the accuracy of the approximations so the assumption that $A_p = 1$ as the first guess appears satisfactory for this case.

This entire power-on analysis was made possible by the fact that the runs were made at a constant value of T_c. This setting of constant power eliminated the measurement of slopes of the T_c vs i_t and T_c vs C_L relations for the points in question and other complicated terms. The analysis could be extended to include power-on wind tunnel tests that were made by setting $T_c = T_c(C_L)$, but becomes very complicated. It is felt that in running routine power-on wind tunnel tests, $T_c = $ const. runs should be included to offset the added complication of the $T_c(C_L)$ analysis.

CONCLUSION

From the expressions given for the down-wash over the tail and the tail efficiency power-off, it is suggested that available experimental data be reduced to give an analysis similar to that of L. E. Root.

A recommendation should be made at this point to persons using this type of analysis to obtain theoretical values power-on. The stability as defined by $\frac{\partial}{\partial C_L} \delta_t^P C_M$ is noted to be approximately proportional, at $T_c = \text{CONST}$ to the tail area and directly proportional to the tail length. Although a change in the tail length affects the down-wash on the tail, it is felt that this change in down-wash is not important. So to change the stability by a given amount it is usually more successful to change the tail area rather than the tail length. Examine Eq. 28.

The only variables that are within the control of the designer in regard to stability are the tail length, the tail area, the position of the nacelles and the propeller rotation.

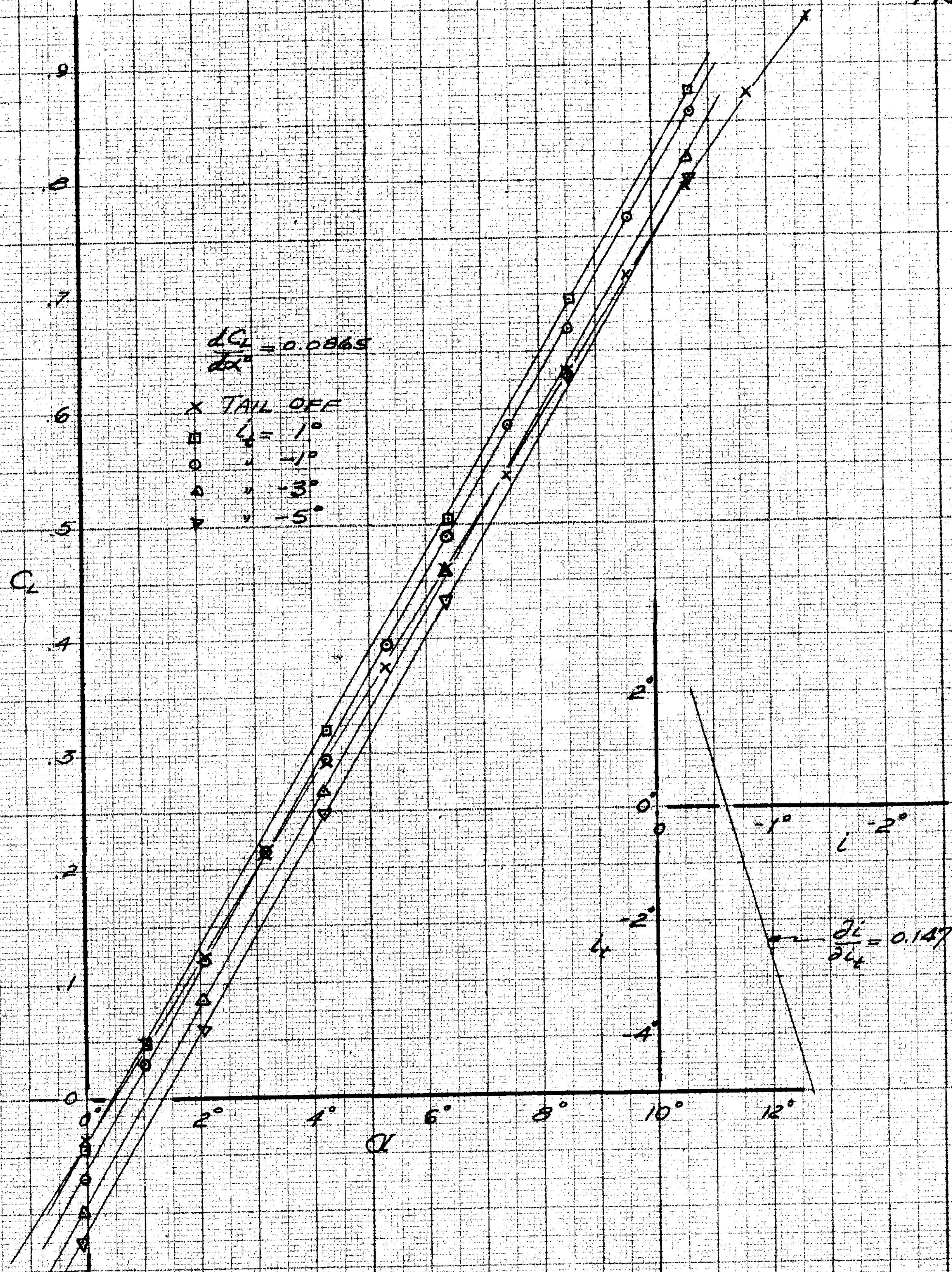
The effect of propeller rotation would seem to indicate different values of A_P and B_P for each case.

Experiment has usually shown that with the propellers rotating against the wing tip vortices the best stability is produced.

It is to be recommended that this sort of analysis be made using a similar model with running left and right hand propellers.

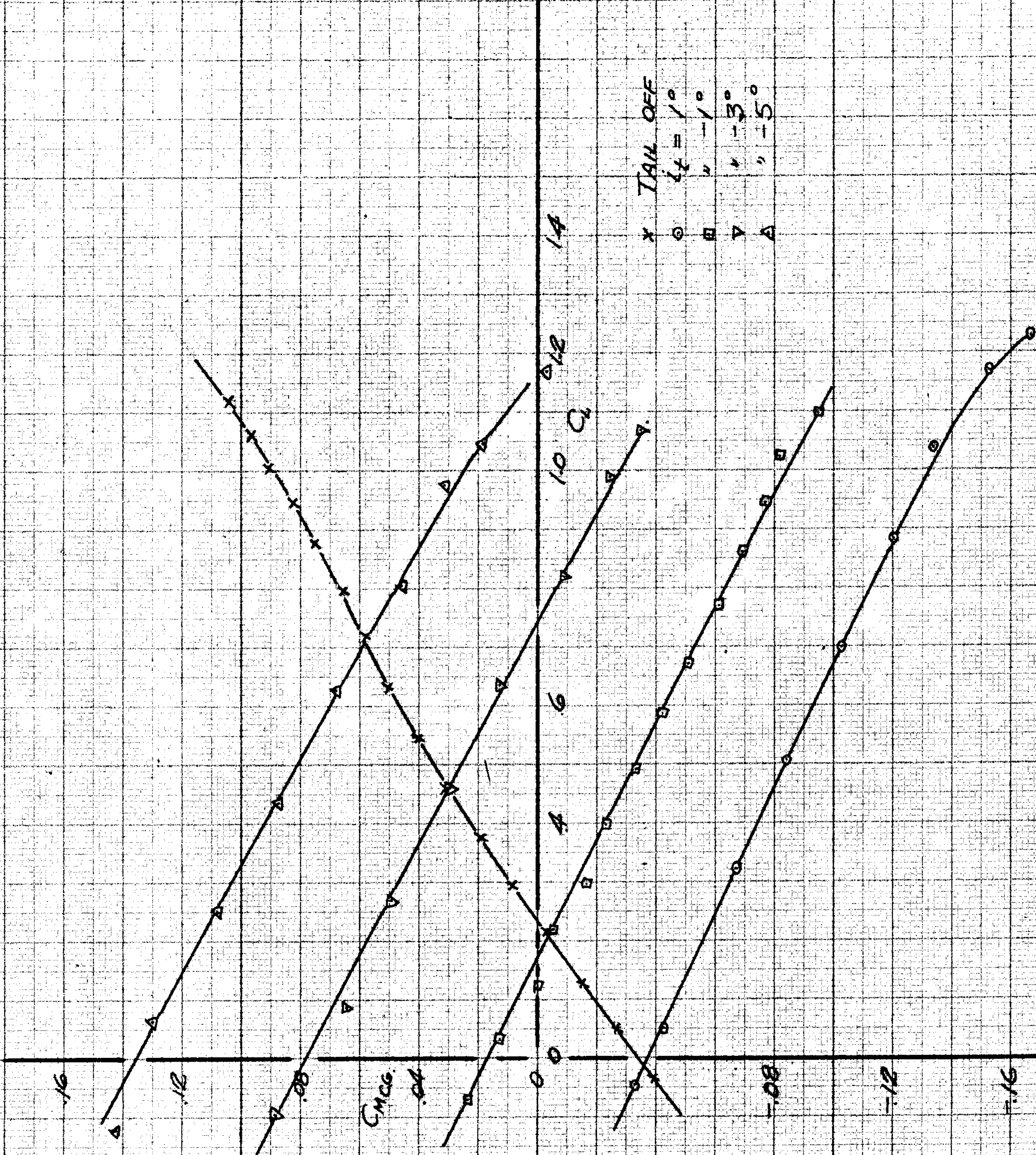
REFERENCES

1. Millikan, C. B. The Influence of Running Propellers on Airplane Characteristics, J. Ae. S., Vol. 7, No. 3, p. 85, Jan. 1940.



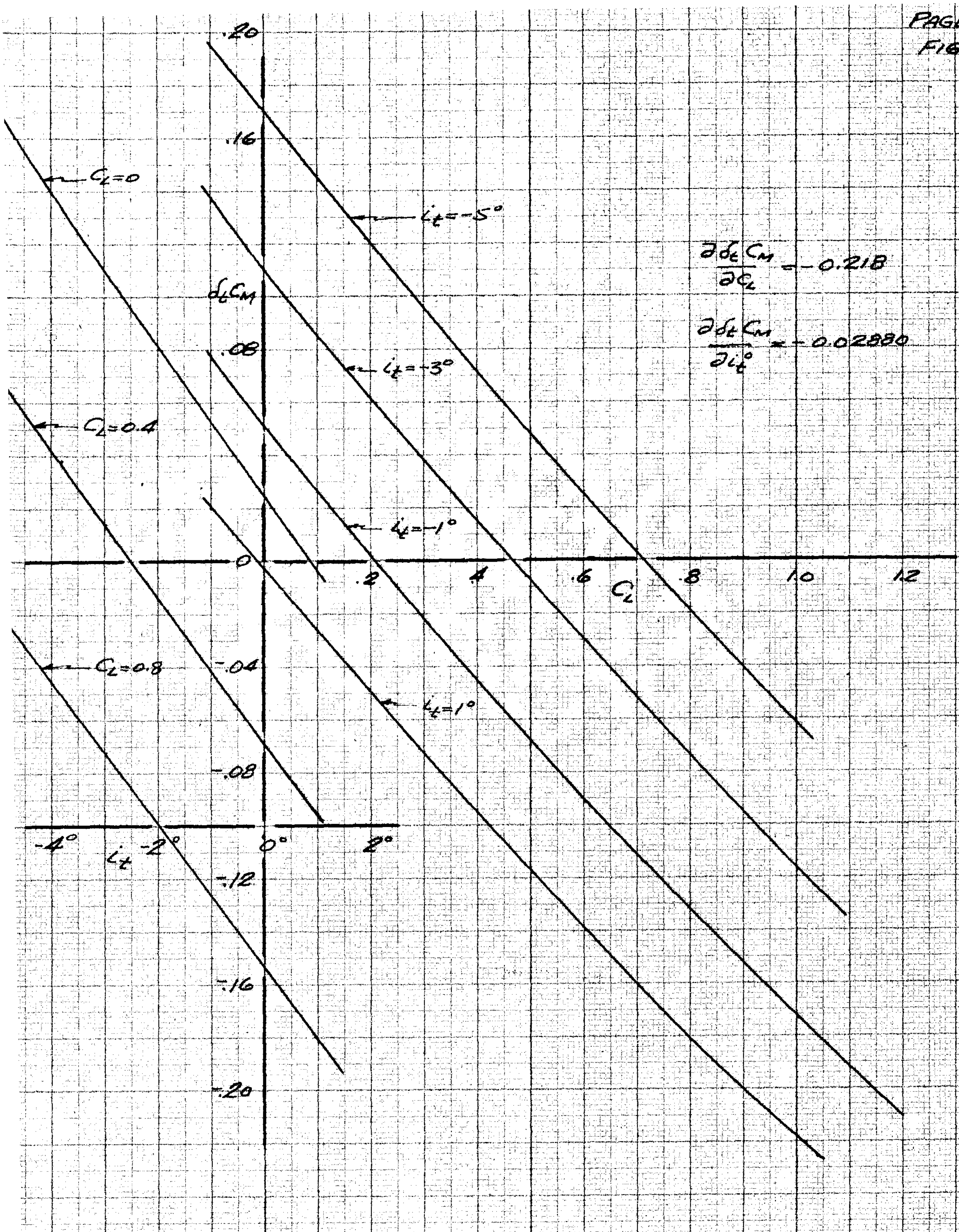
LIFT VS α AND DETERMINATION OF $\frac{\partial i}{\partial i_t}$;

POWER-OFF



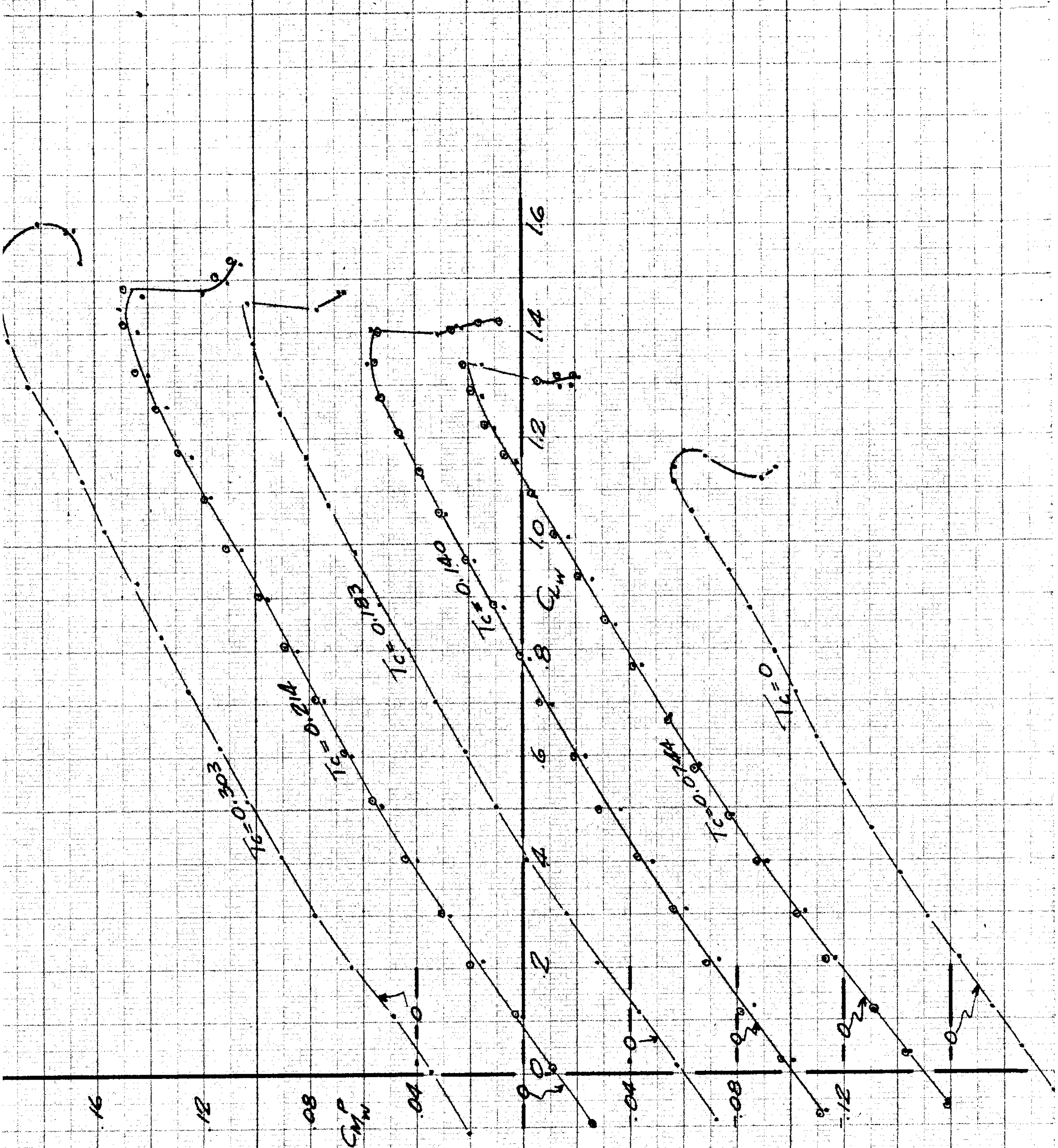
PITCHING MOMENTS, POWER-OFF

FIG. 3



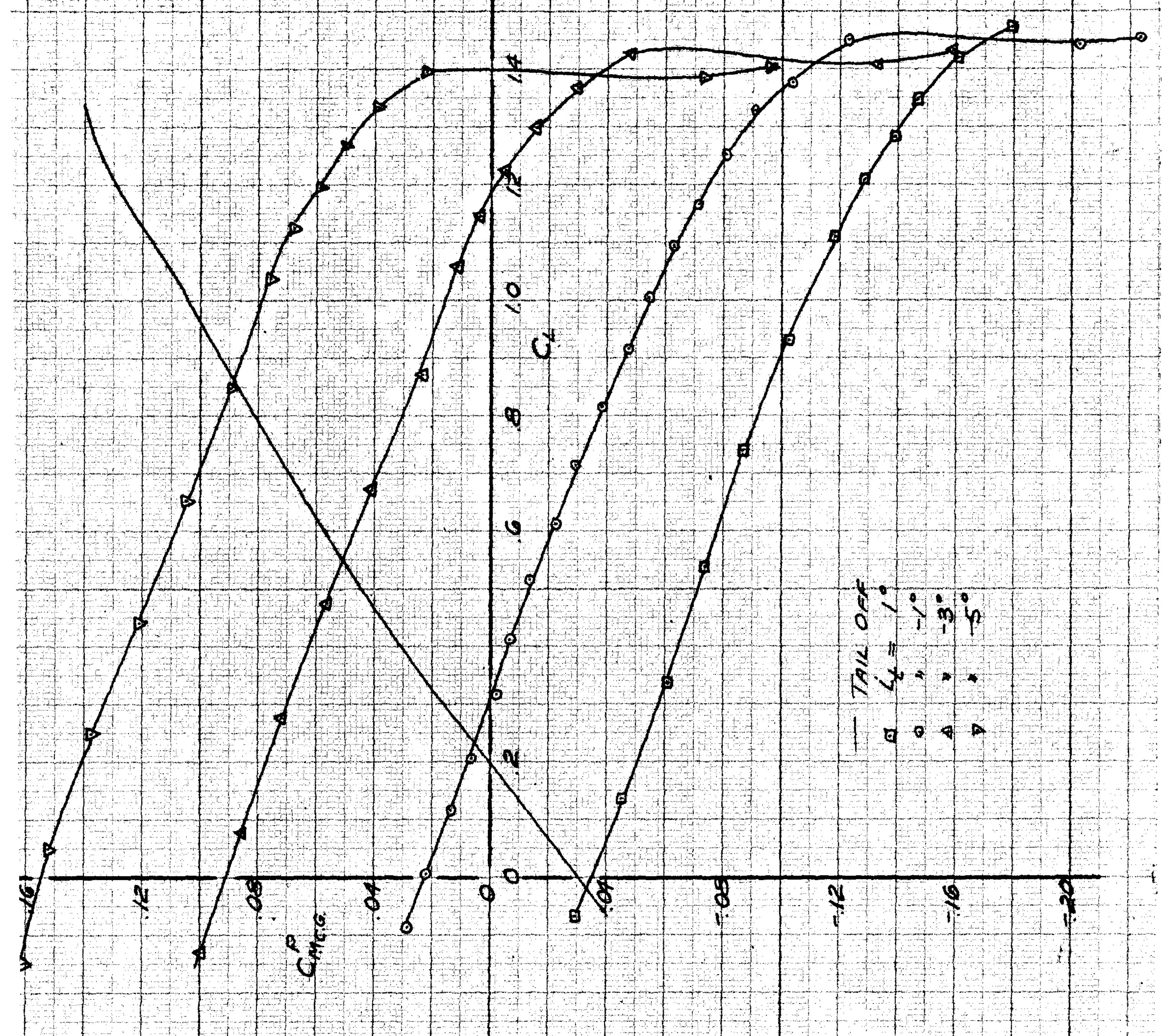
TAIL PITCHING MOMENTS AND STABILIZER EFFECTIVENESS

POWER-OFF



FAIRED TAIL-OFF MOMENT CURVES, POWER-ON AND OFF

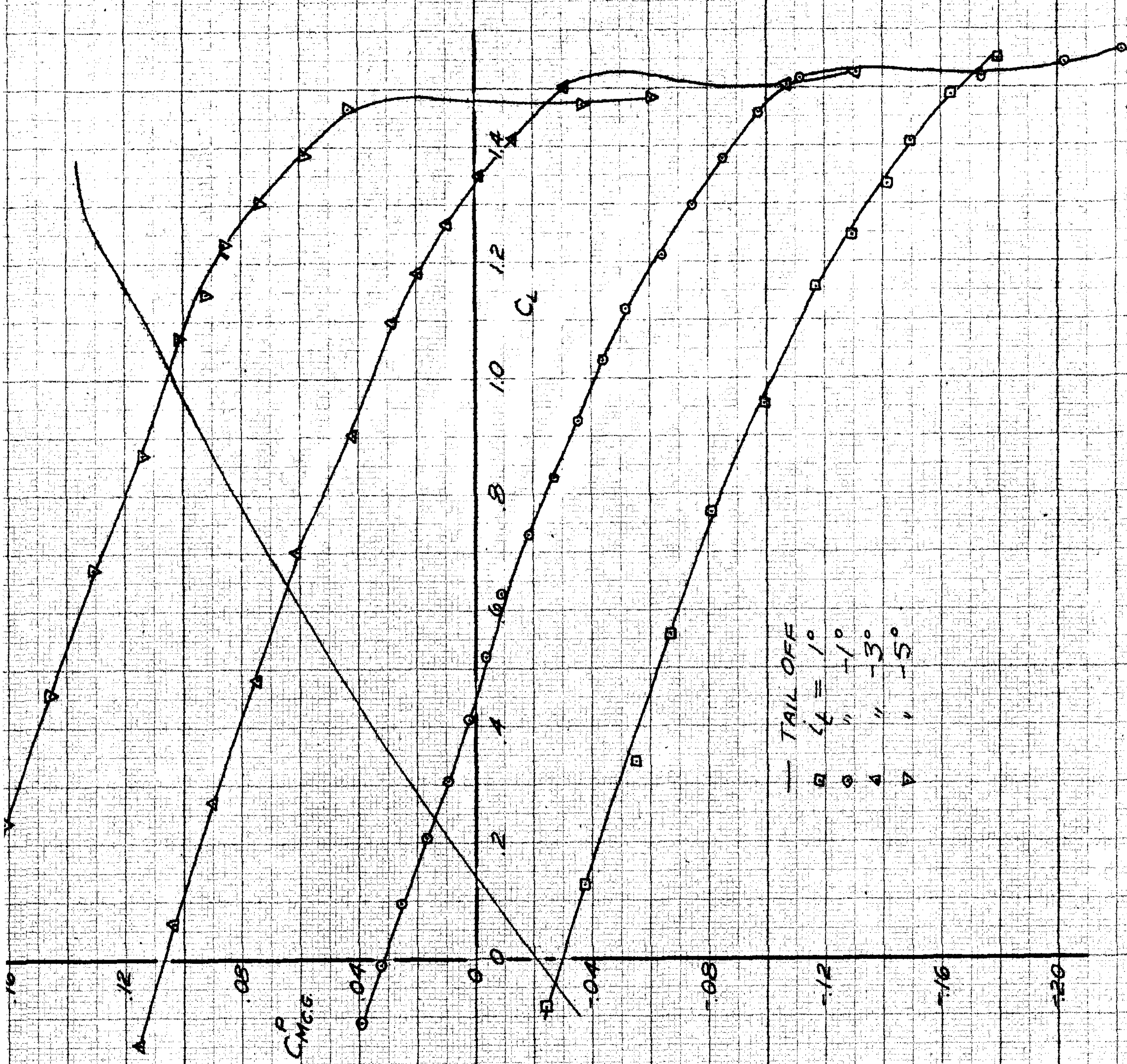
FIG. 5



PITCHING MOMENTS POWER-ON

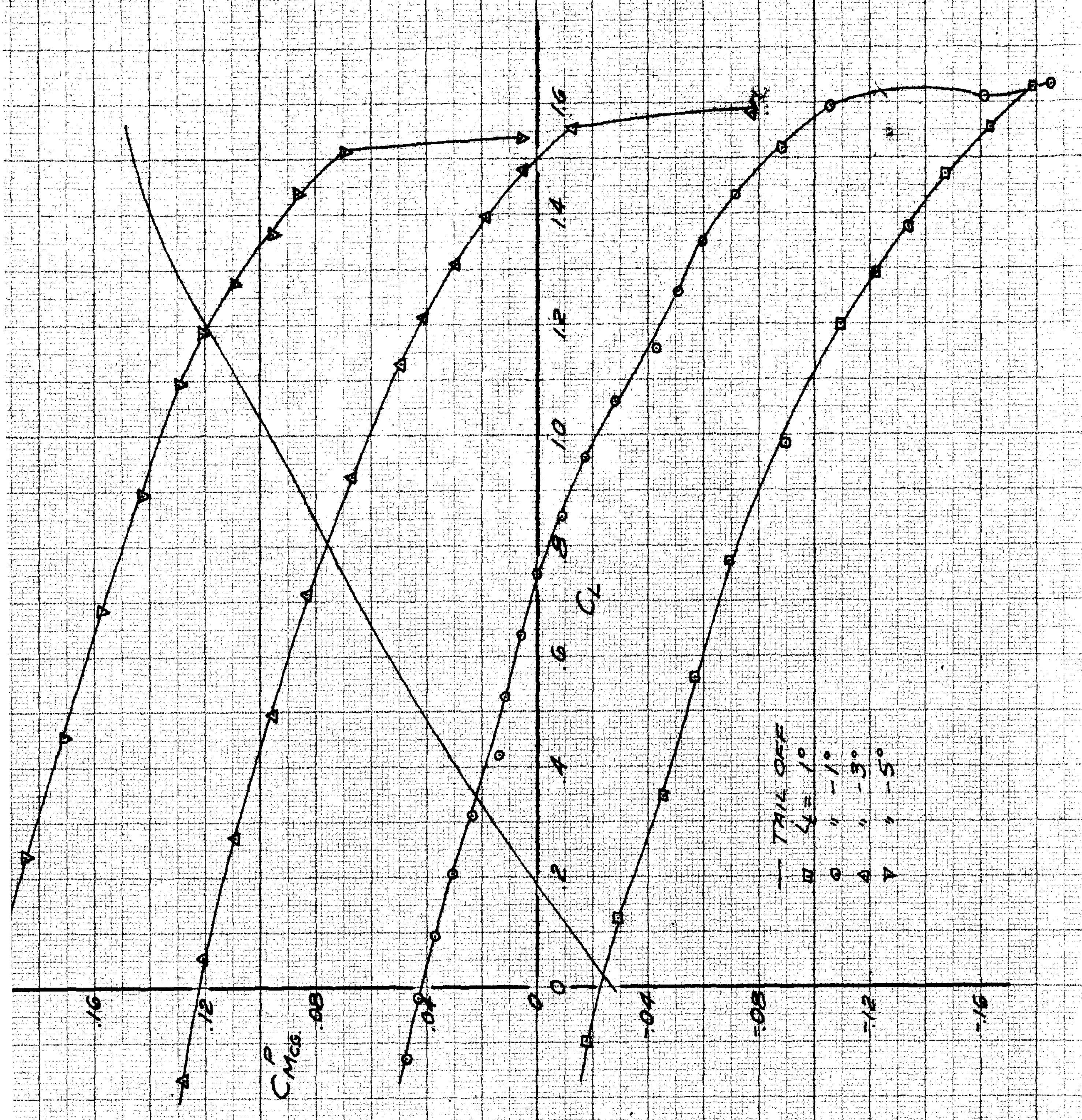
$T_C = 0.0744$

FIG. 6



PITCHING-MOMENTS POWER-ON ; $T_c = 0.140$

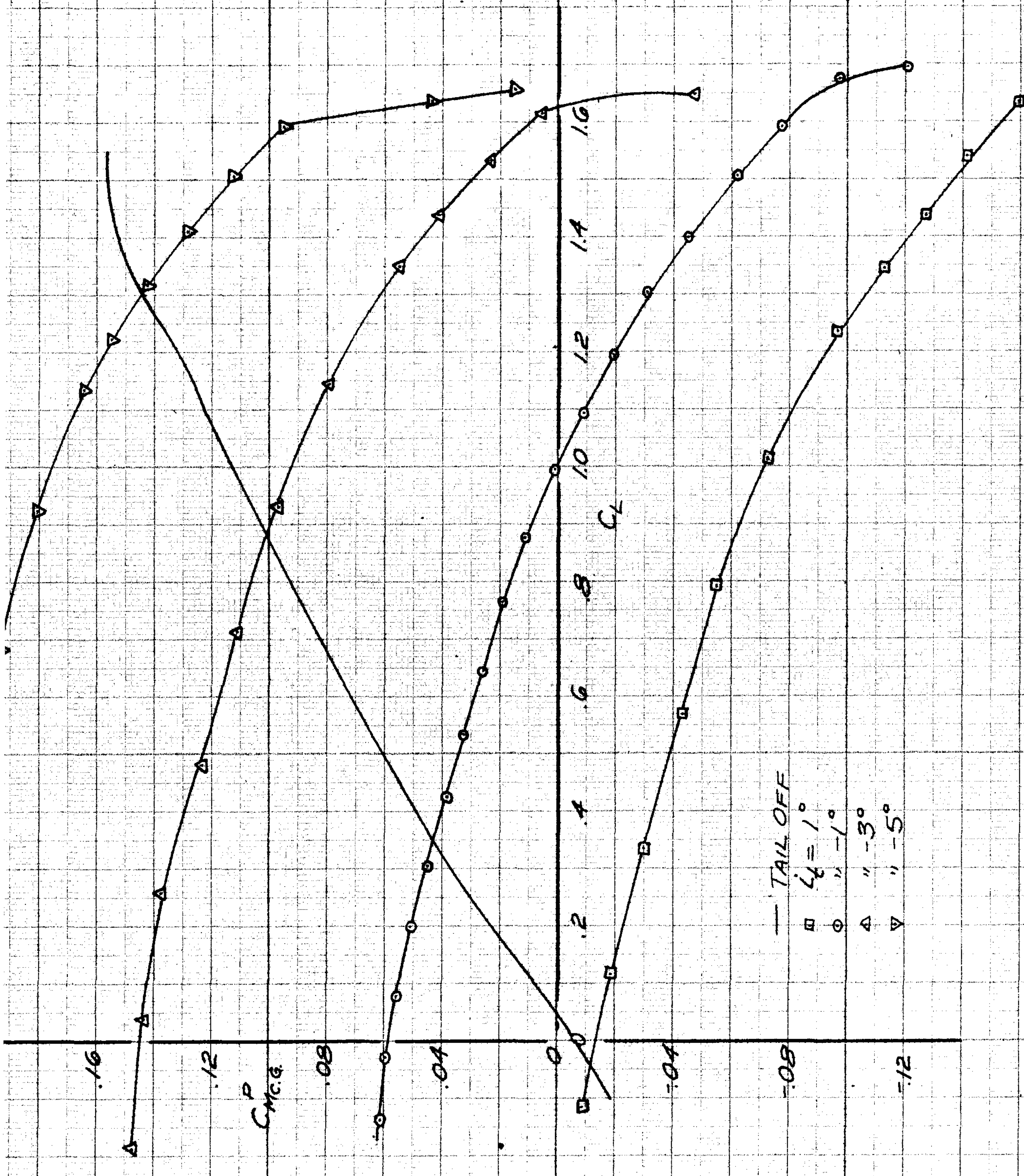
FIG. 7



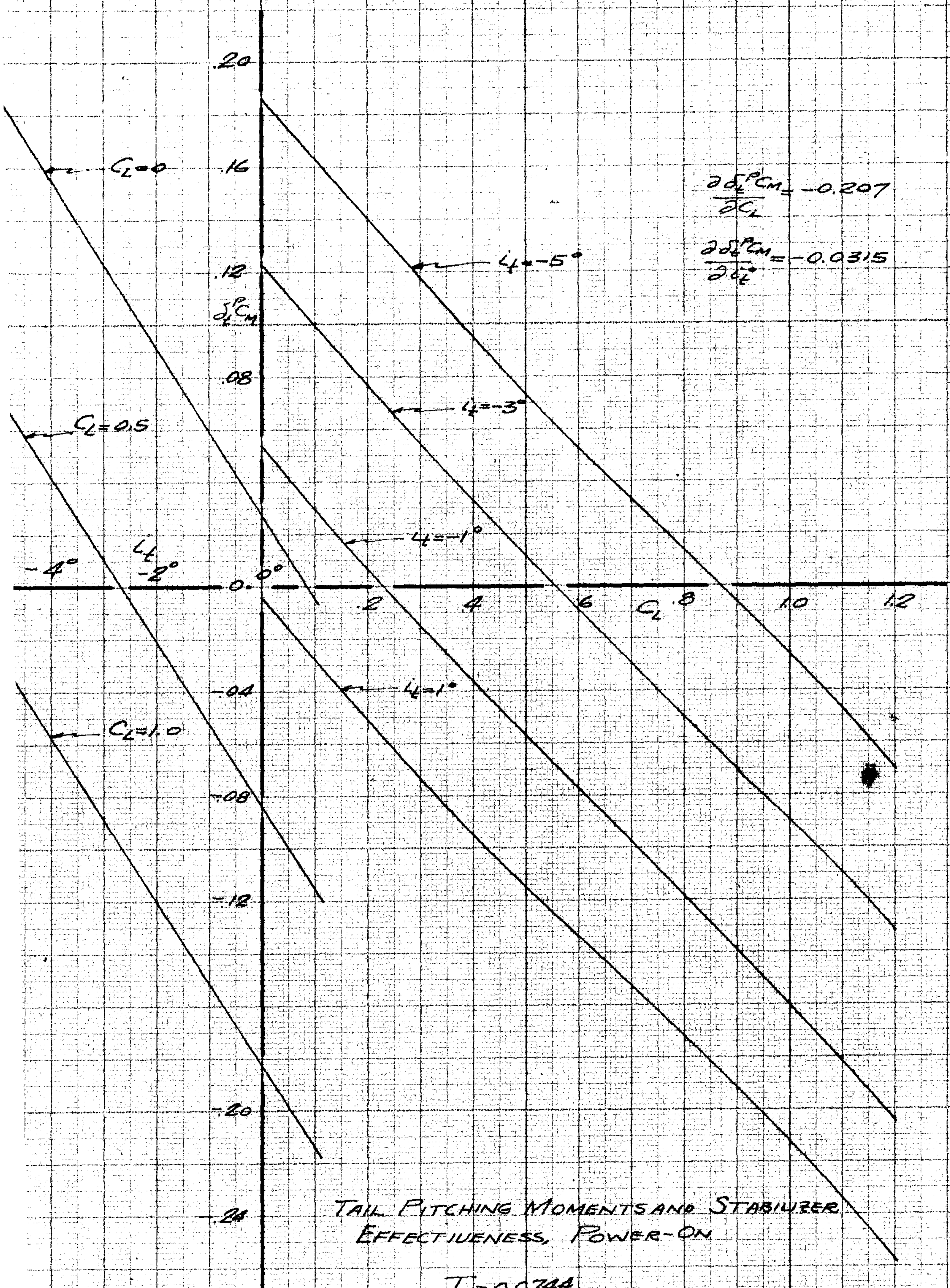
PITCHING MOMENTS, POWER-ON

$T_C = 0.214$

FIG. 8

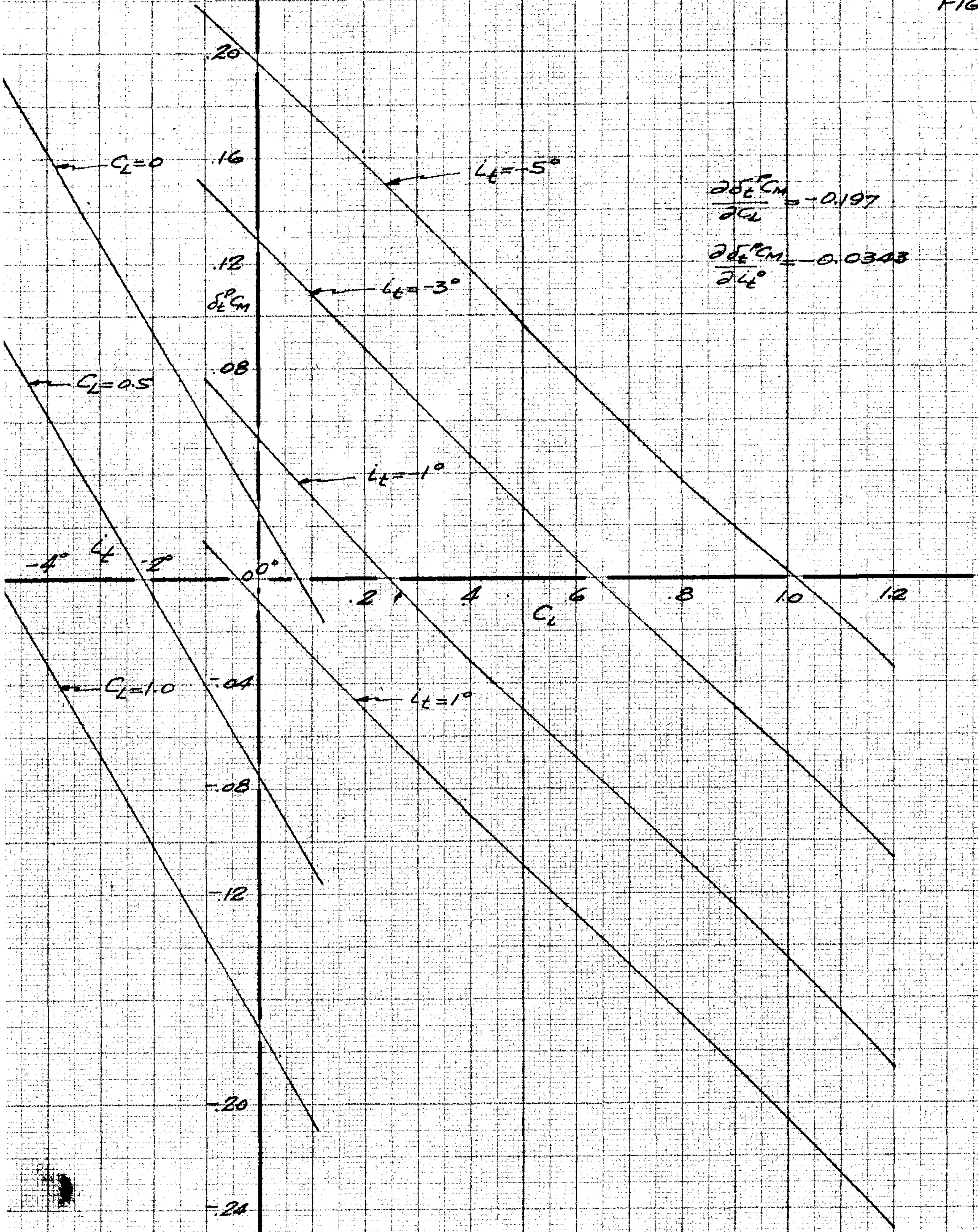


PITCHING MOMENTS, POWER-ON ; $T_c = 0.303$



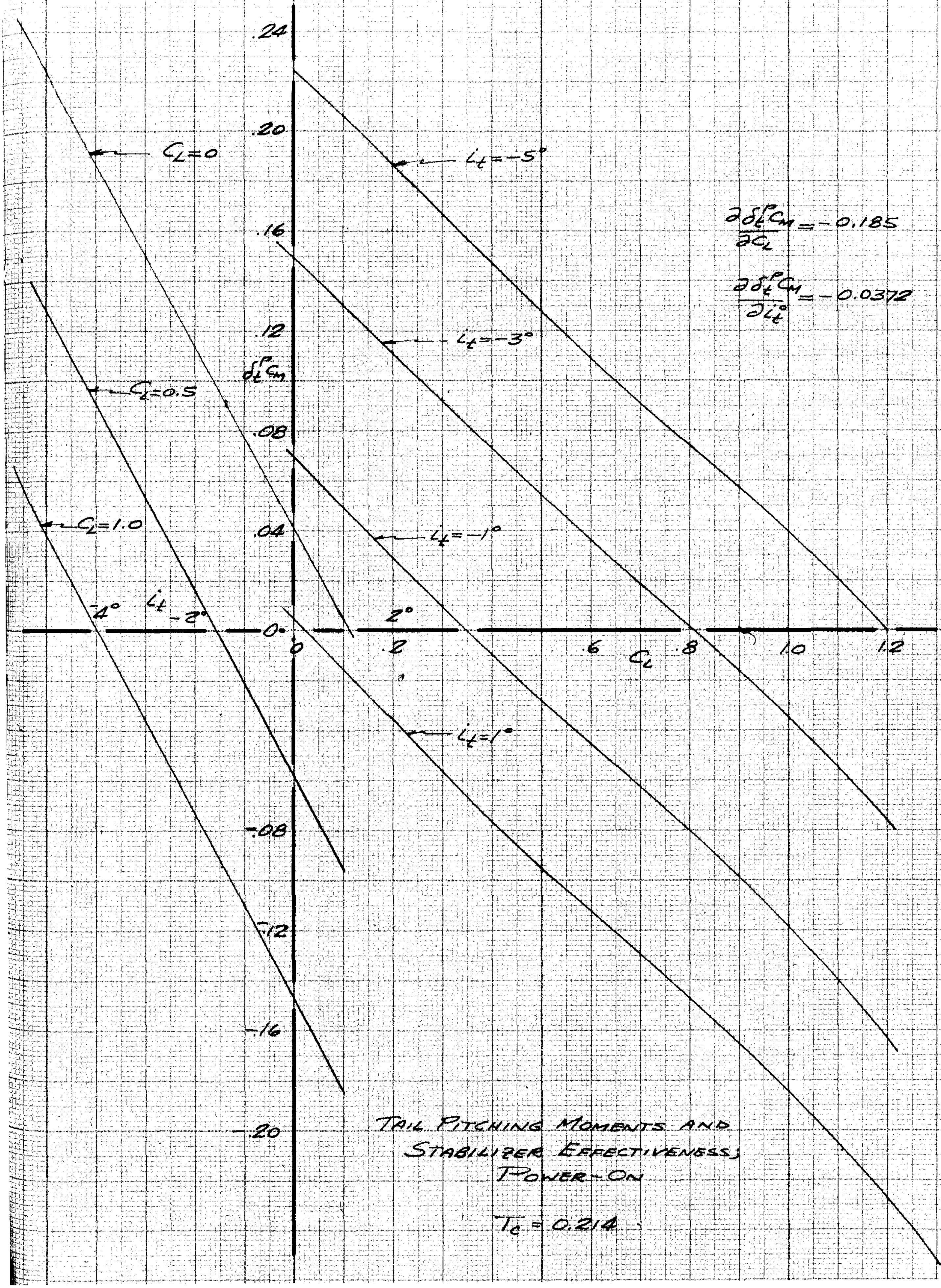
TAIL PITCHING MOMENTS AND STABILIZER EFFECTIVENESS, POWER-ON

$T_C = 0.0744$



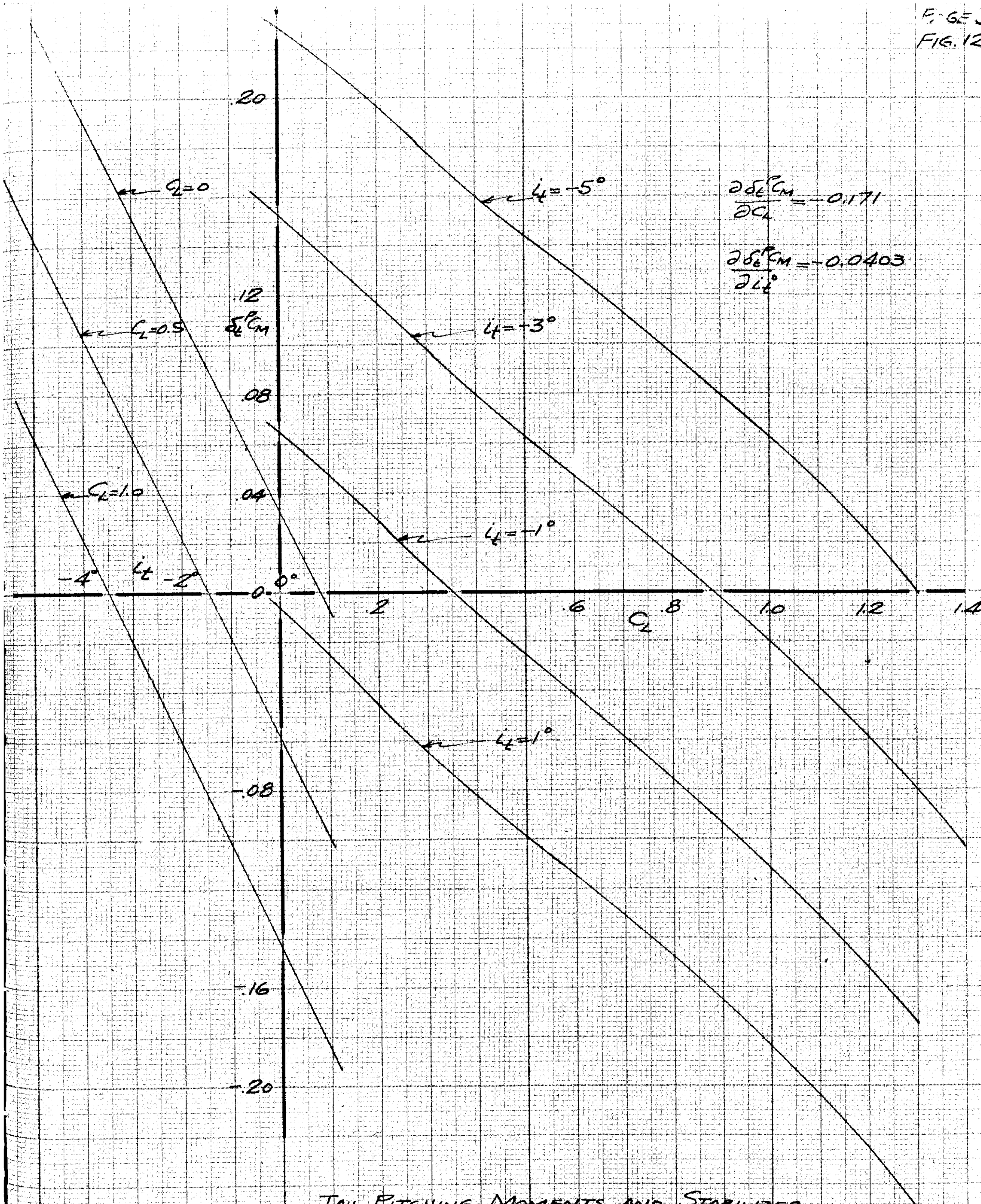
TAIL PITCHING MOMENTS AND STABILIZER EFFECTIVENESS, POWER-ON

$T_c = 0.140$



TAIL PITCHING MOMENTS AND
STABILIZER EFFECTIVENESS;
POWER-ON

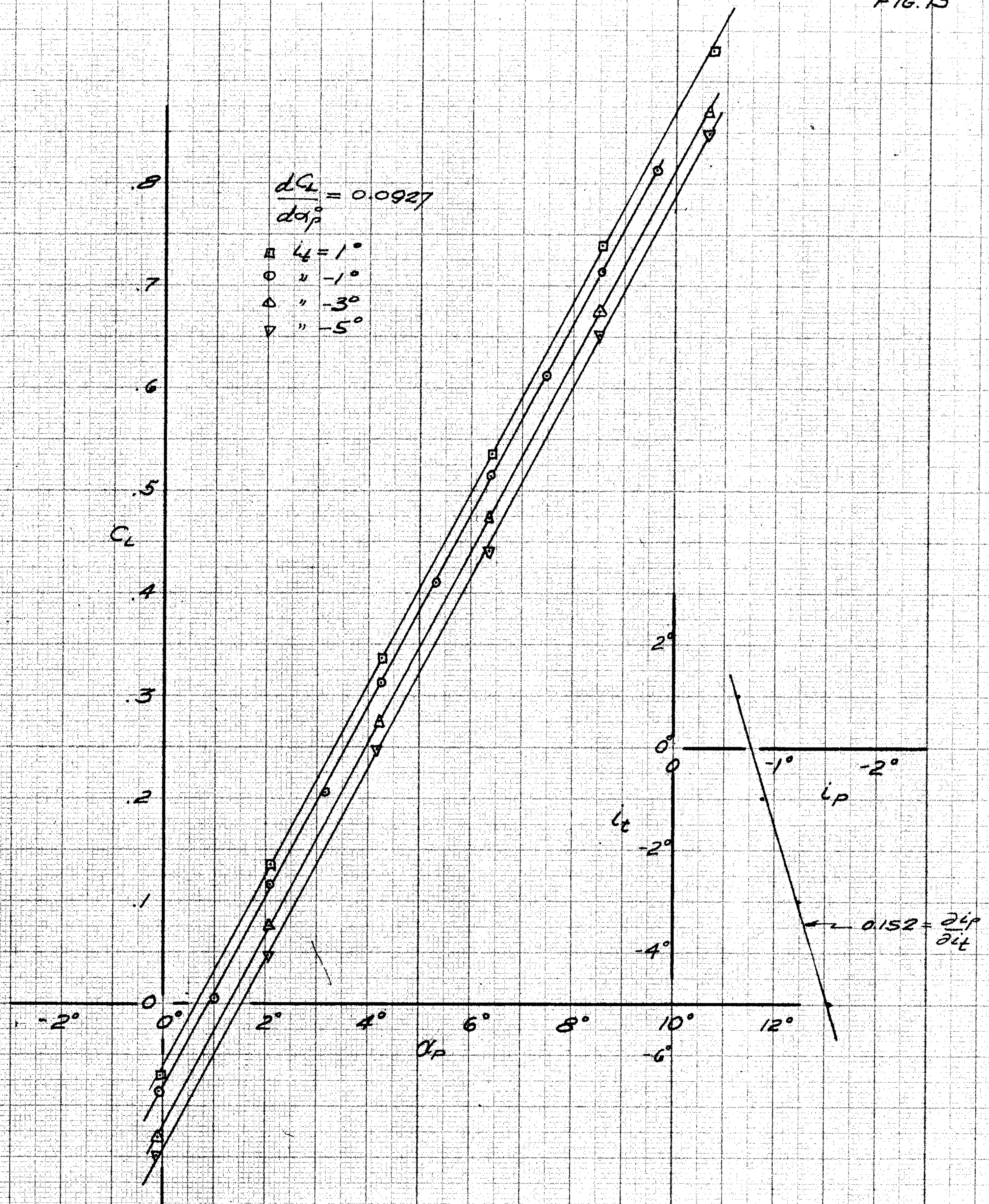
$T_c = 0.214$



TAIL PITCHING MOMENTS AND STABILIZER EFFECTIVENESS, POWER-ON

$T_c = 0.314$

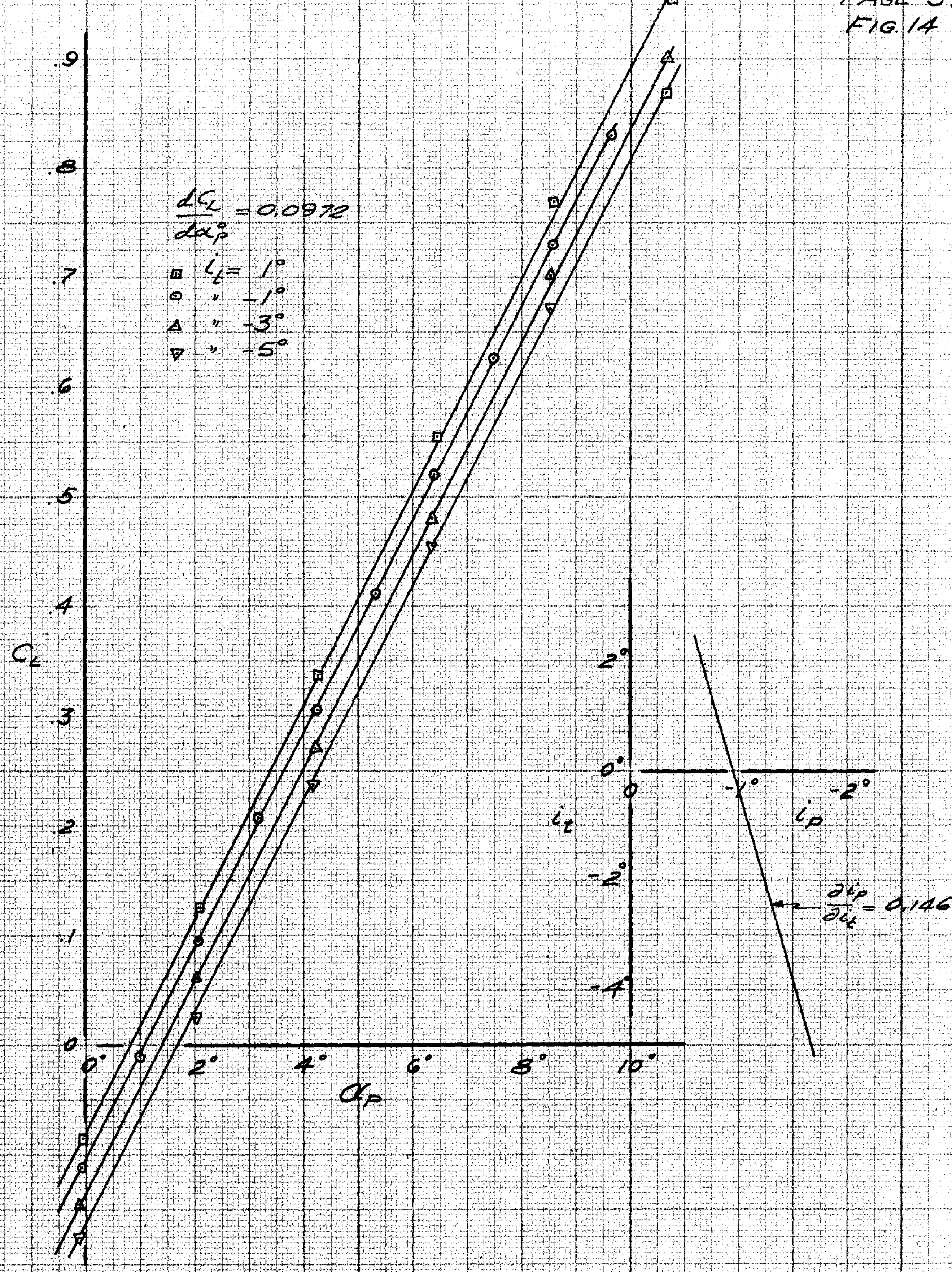
FIG. 13



LIFT VS α_p AND DETERMINATION OF $\frac{\partial i_p}{\partial i_t}$; POWER-ON

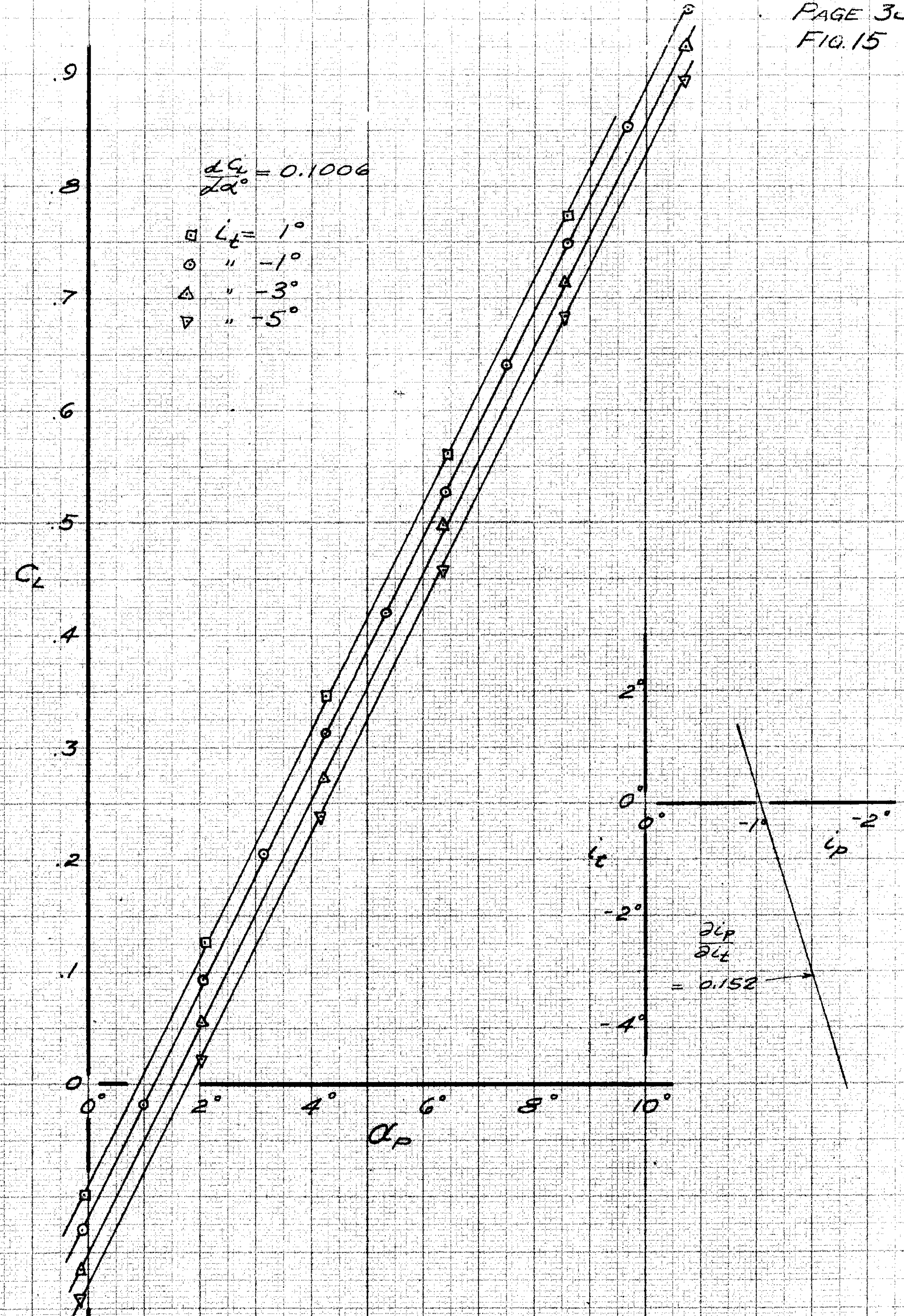
$T_c = 0.0744$

FIG. 14



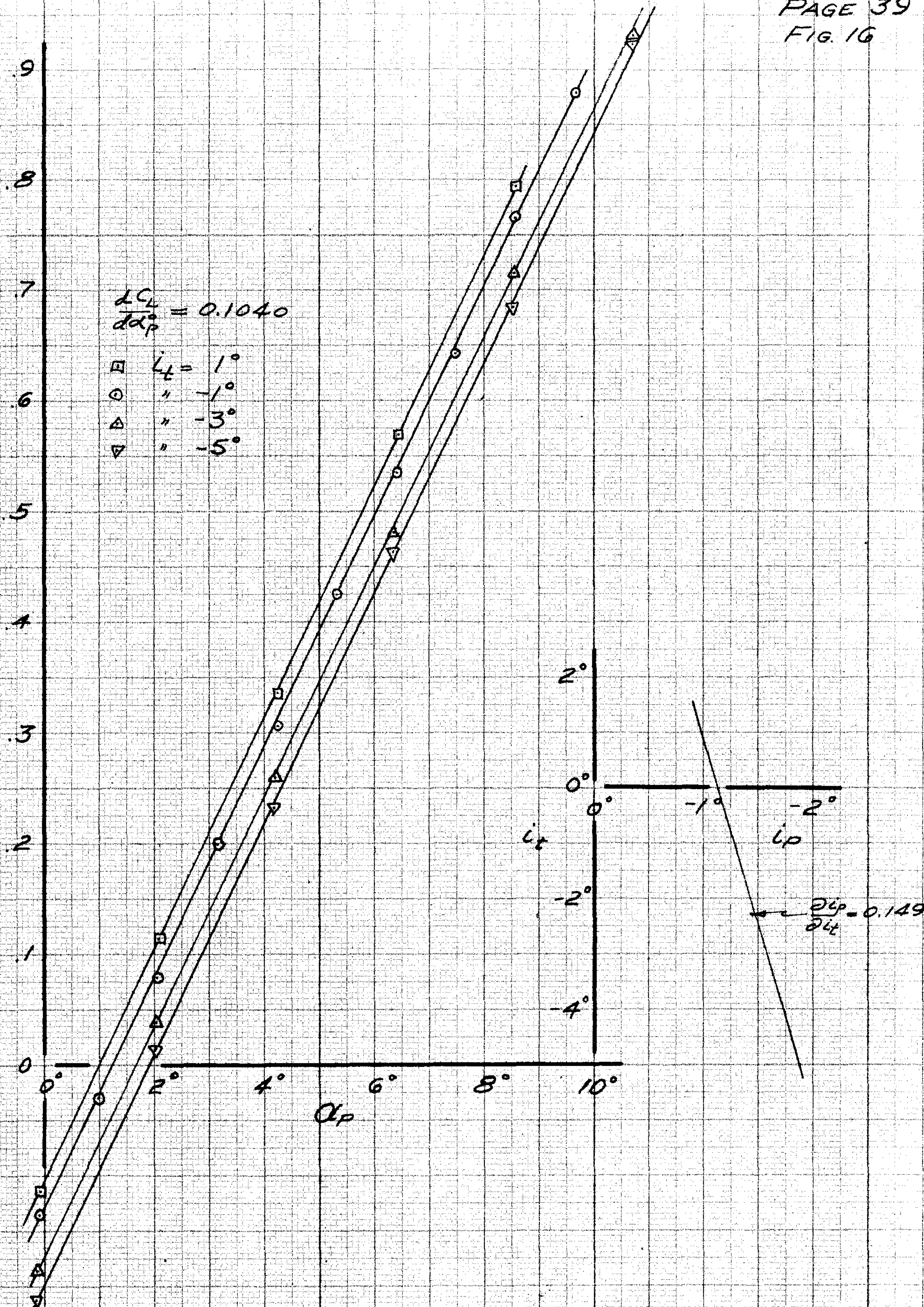
LIFT VS α_p AND DETERMINATION OF $\frac{\partial i_p}{\partial i_t}$; POWER-ON

$T_c = 0.140$



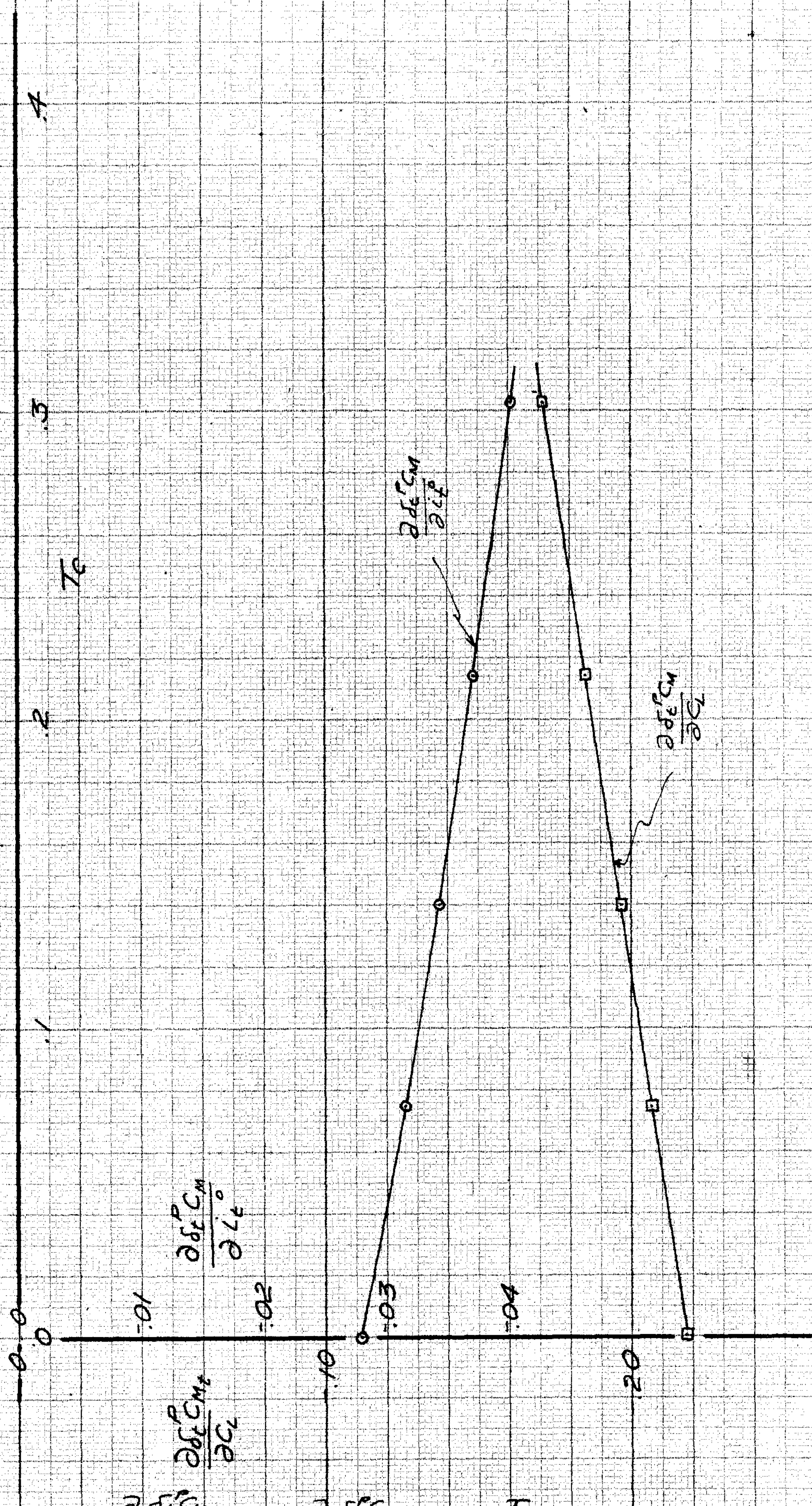
LIFT VS α_p AND DETERMINATION OF $\frac{\partial i_p}{\partial i_t}$, POWER-ON

$T_c = 0.214$



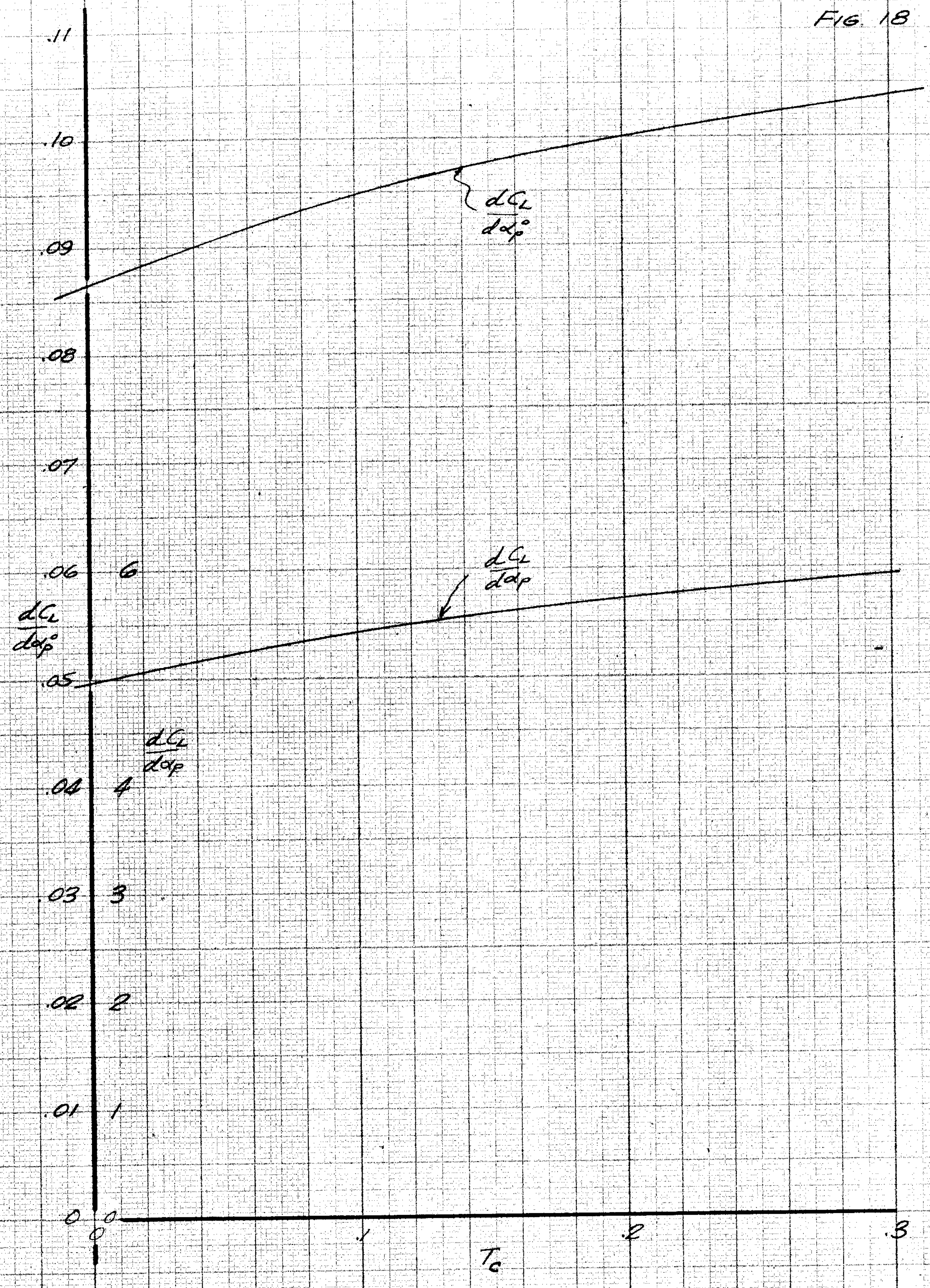
LIFT VS α_p AND DETERMINATION OF $\frac{\partial i_p}{\partial i_t}$, POWER-ON

$T_c = 0.303$



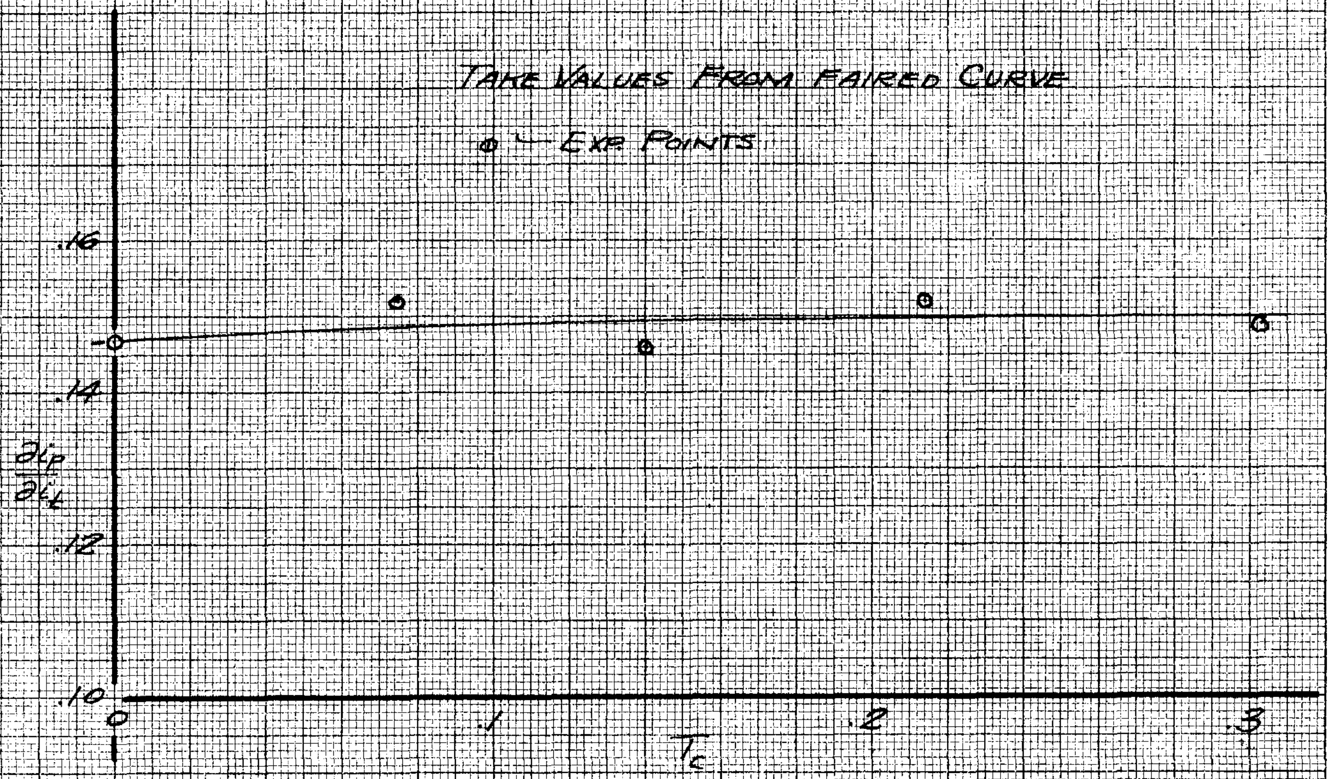
VARIATION OF $\frac{\partial \delta_t^P C_M}{\partial C_L}$ AND $\frac{\partial \delta_t^P C_M}{\partial i_t^\circ}$ WITH T_c

FIG. 18



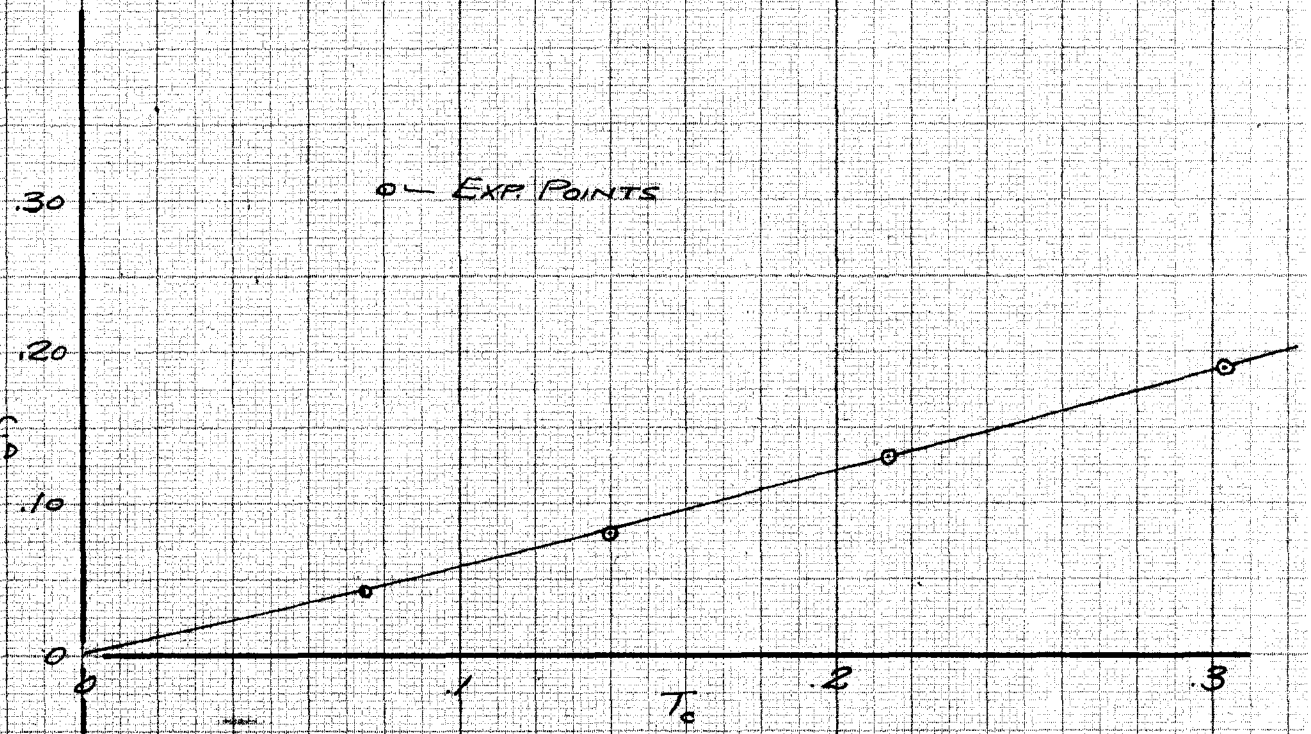
VARIATION OF $\frac{dC_L}{d\alpha_p^\circ}$, $\frac{dC_L}{d\alpha_p}$ WITH T_c

FIG. 19



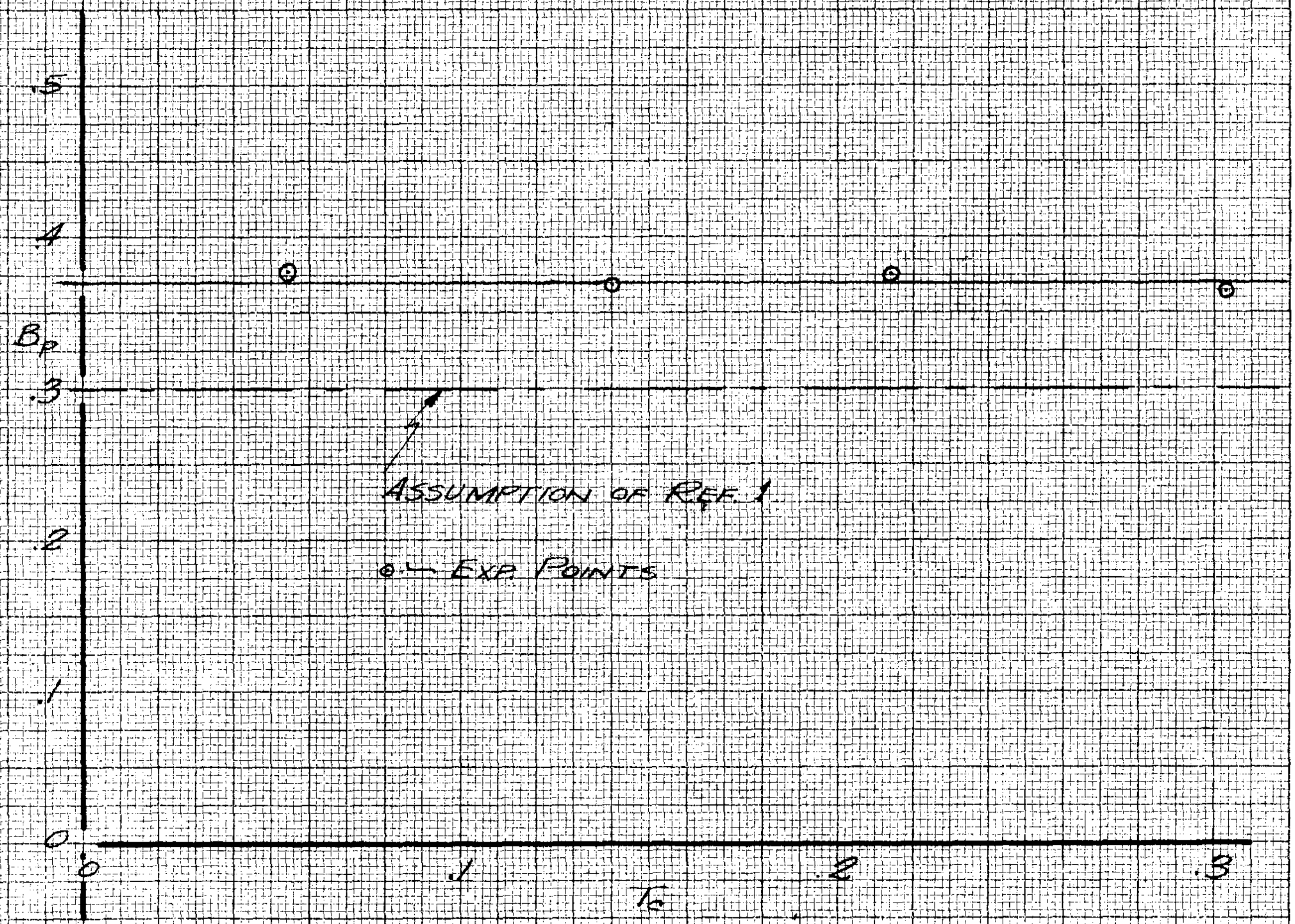
DETERMINATION OF $\frac{\partial i_P}{\partial i_L}$ TO BE USED IN COMPUTING f_D

FIG. 20



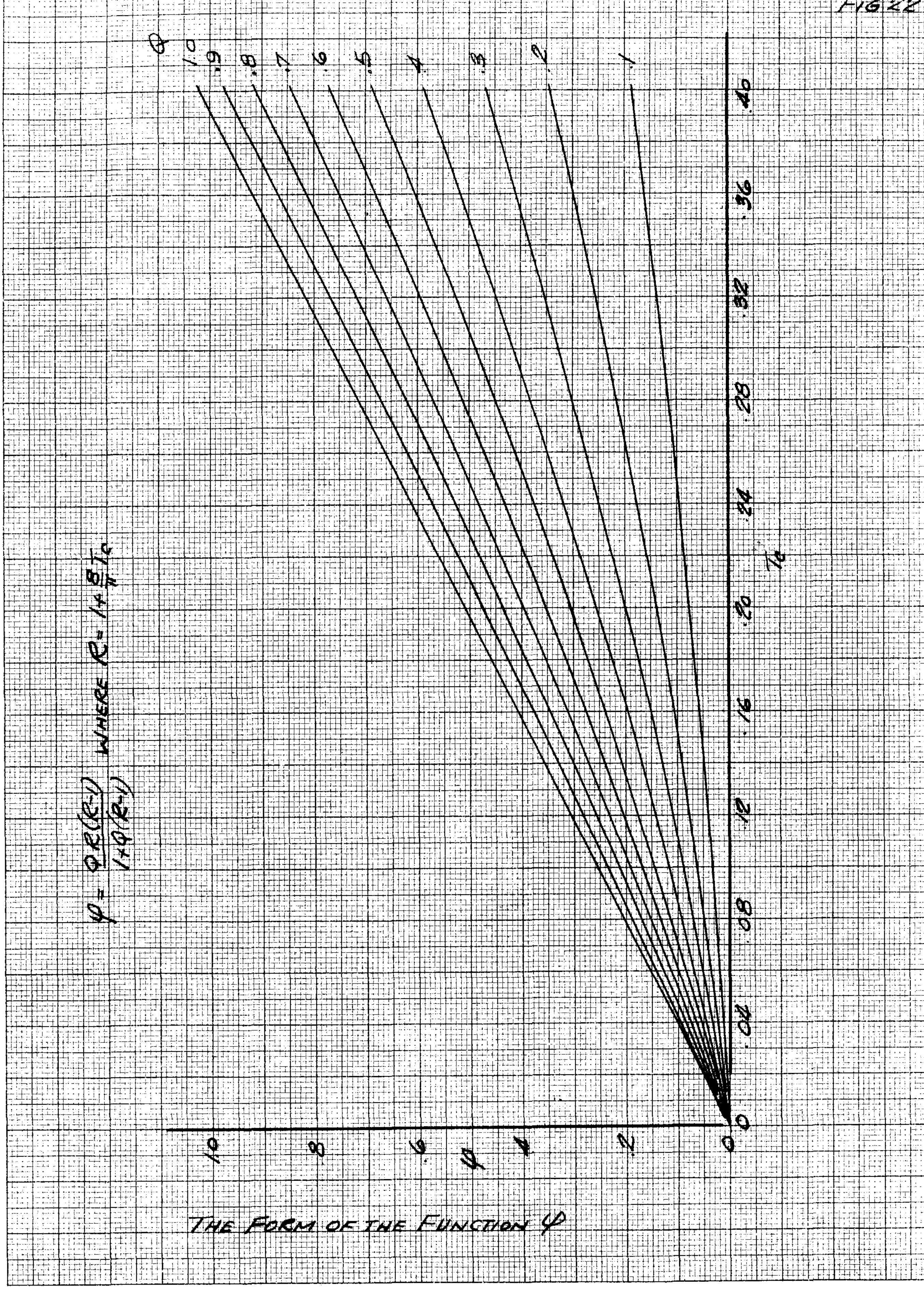
VARIATION OF THE DOWNWASH AT THE TAIL DUE TO POWER

FIG. 21



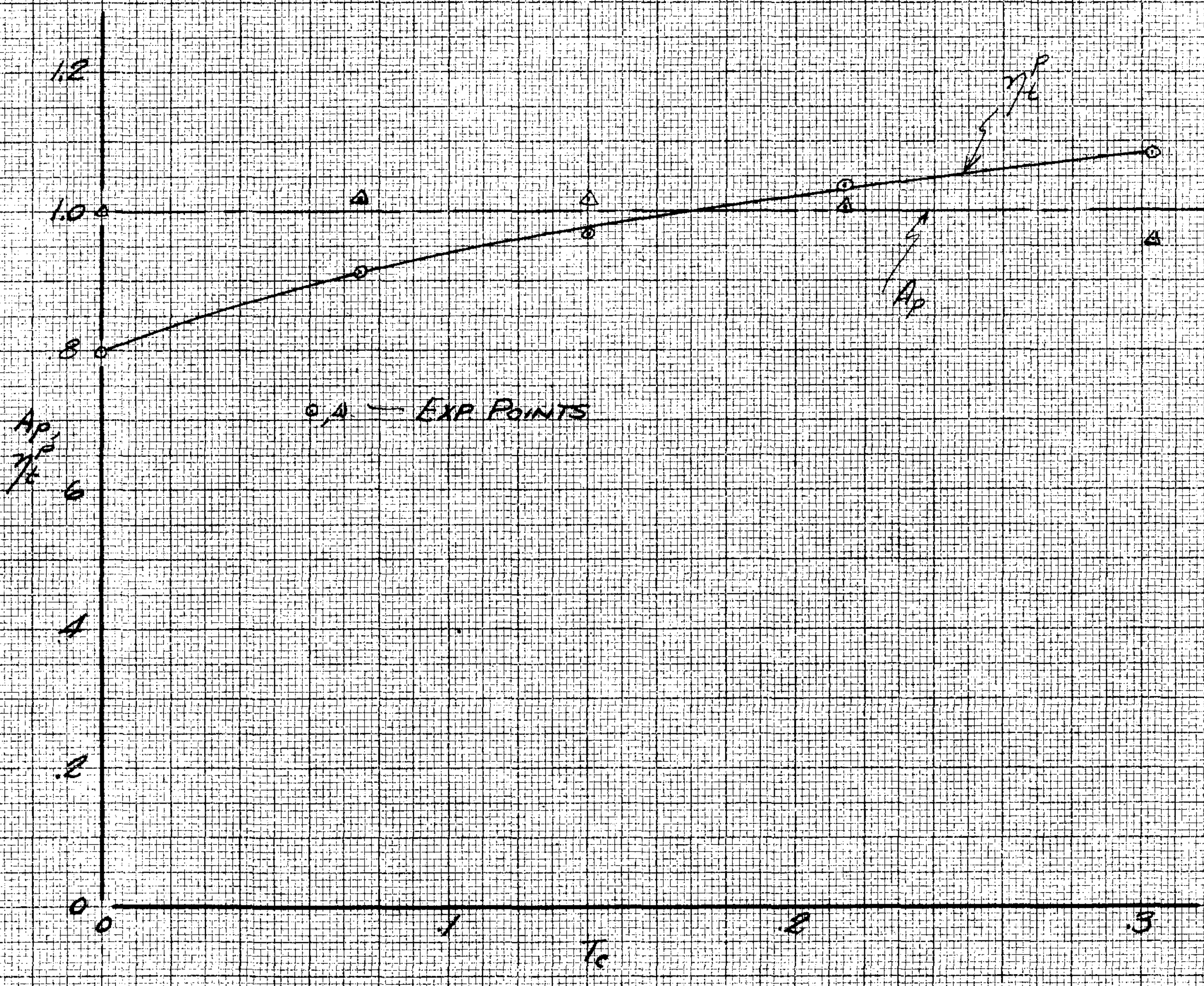
VARIATION OF B_P WITH T_c

FIG 22



THE FORM OF THE FUNCTION φ

FIG. 23



VARIATION OF η_t^p AND A_p WITH T_c