MANEUVERABILITY

Thesis

by

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SUMMARY

A method has been developed which makes possible the determination of the time to make a horizontal turn of any number of degrees, and also the control deflections necessary to make the turn. Accelerations encountered during the maneuver are easily obtained from the calculations which are used in obtaining the maneuvering time.

The method worked out is based on the six fundamental stability equations. The major assumptions are angle of bank, ϕ, and no sideslip during turn i.e. $V = 0$. Only the use of simple wind tunnel data is necessary.

Several cases are worked out for different bank assumptions for a twin engine attack bomber. The bank assumption which gives the most reasonable control deflections and accelerations during the maneuver is one which produced a helix angle of 0.090 and maximum bank of 75°. For this particular bank assumption calculations were carried out for sea level and 15,000 ft. altitude.

Calculations were also carried through for sea level and 15,000 ft. for a modified single engine airplane of same weight and horsepower as the twin engine one, using the bank assumption which gave ϕ max. of 75°.

A method is presented for the direct calculation of the stability derivative, M'_u, from wind tunnel data.

INTRODUCTION

The improvement of the flying qualities of modern aircraft is impeded by the absence of quantitative maneuverability and control criteria. As a result of several conferences held with members of the Aerodynamics Department of both the El Segundo and Santa Monica plants of the Douglas Aircraft, it seems that a concise definition of maneuverability is not yet available. As we know present day, modern aircraft, they are best conceived of as some "flying hunk", and to rate one airplane better than another is a difficult problem. The effect of wing loading, span loading, power loading, the effect of altitude and speed are all factors influencing the behaviour of the aircraft. To get a concise maneuverability criteria which takes into account all these variables is extremely difficult and no attempt has been made to do so in this report.

As a result of practical suggestions offered by Mr. Benny Howard of Douglas Aircraft it was decided to arbitrarily assign a flight path to the airplane and calculate the time necessary to execute the maneuver as well as the control motions. This may be considered a special definition of "maneuverability criteria".

The flight path chosen was simply a horizontal turn with no gain or loss in altitude. For necessary simplification constant power was considered throughout the turn. This phase of maneuverability is of great importance in the case of a night fighter plane where it is in pursuit only by instrument, and a rapid horizontal turn at full power is a necessity. The basic assumption for this designated flight path was that of angle of bank as a function of time. This was taken as an exponential function.

Calculations are worked out for a typical modern, twin engine, attack bomber. Methods presented in this paper are based on the interpretation of wind-tunnel data. Fortunately, such data will always be available at least in preliminary form during the design stages of the large aircraft on which rational analysis are important.

Several angles of bank and rolling velocities are assumed by varying the various parameters in the exponential equation, and calculations carried through for each assumption. Also the accelerations encountered for each maneuver are presented along with the time to execute the maneuver and the control deflections necessary as a function of time. For the assumed bank which seems to give the most logical results, calculations are made considering an altitude of 15,000 feet. All calculations come from the stability equation, the three determining lateral motions being used for all results except those pertaining to elevator deflections. For the first angle of bank assumption, two complete calculations are made, the first based on the simplifying assumption that if ϕ is the angle of bank, then $\sin \phi = \phi$, and the second considering the actual value of $\sin \phi$.

The last part of the paper considers the effect on maneuverability of engines mounted in the wings. A second airplane is considered, identical to the first except that it is a single motored one, the motor being mounted in the nose of the fusilage and weighing and developing the same amount of horse power as the two engines installed in the wing of the initial airplane.

All curves have been calculated for a conventional airplane of about 20,000 pounds gross weight. Although the above magnitudes are of no particular significance, the general trends and relationships should hold for airplanes of twice and half this size. Since outside of general specifications and speed conditions, the only factors necessary are:

Y_{o_r} - Change in side force per unit mass due to unit rudder deflection.

L_{o_r} - Change in rolling moment per unit moment of inertia due to unit aileron deflection.

N_{o_r} - Change in yawing moment per unit moment of inertia due to unit rudder deflection.

N_{o_a} - Change in yawing moment per unit moment of inertia due to unit aileron deflection.

and

L_r - Change in rolling moment due to yawing velocity.

L_p - Change in rolling moment due to rolling velocity.

N_r - " " yawing " " " yawing "

N_r - " " " " " " rolling "

The methods developed here should be of broad applicability.

DIMENSIONS AND CHARACTERISTICS OF TYPICAL TWIN MOTORED ATTACK BOMBER CONSIDERED

Wing area	= 465 sq. ft.
Wing span	= 61.33 ft.
Aspect ratio of wing	= 8.09
Weight	= 19,300 lbs.
Root chord	= 132.4 in.
Tip Chord	= 49.8 in.
M. A. C.	= 100.2 in.
Vertical surface area, S_V	= 63.28 sq. ft.
Aspect ratio vertical surface	= 1.38
Span of vertical surface	= 9.33 ft.
Distance of C.G. to rudder hinge line, l_V	= 307.6 in.
Distance of C.G. to elevator hinge line, l_H	= 316.5 in.
Wing loading, l_W	= 41.5 lbs. / sq. ft.
Horizontal surface efficiency, η_H	= .60 (Power on) = .72 (Power off)
$\frac{dC_{M_t}}{dC_L}$	= -0.170 (Power on) = -0.271 (Power off)
Horizontal surface area, S_H	= 101 sq. ft.
Overall length	= 568 in.
K_A^2	= 49.2 ft.^{2}
K_B^2	= 59.8 ft.^{2}
K_C^2	= 104.1 ft.^{2}

FOR SEA LEVEL

U_o	= 280 M.P.H
	= 410 F.P.S.
q'	= 199.5
C_L	= 0.208
B.H.P.	= 2400 (N.R.P.)

FOR 15,000 FT.

U_o	= 300 M.P.H.
	= 440 F.P.S.
q'	= 144.7

CALCULATION OF STABILITY DERIVATIVES AND ACCELERATIONS FOR UNIT CONTROL MOVEMENT FOR AIRPLANE CONSIDERED

FOR SEA LEVEL

A. RESISTANCE AND ROTARY DERIVATIVES FOR LONGITUDINAL MOTIONS

$X_u = -(2/mU_0)\, q'S\, C_D$

$= 0$ (For power on)

$Z_u = -(2/mU_o)\, q'S\, C_L$

$= -.151$

$$M_q = -\, q'S/mU_o \times t \times S_H/S \times l^2/K_B^2 \times \frac{a_o}{1 - \frac{a_0}{AR_t}} \times \eta_t$$

$$= -\, .377\ (8.44)\ 101/465\ (26.35 \times 26.35/59.8) \times 3.94 \times .60$$

$= -18.71$

The calculation of M_u presents some difficulty.

From British Confidential 3744, S. & C. 998,

$$T_C'\, V^3 = \text{Const.} = T_{C_o}'\, V_o^3$$

Therefore,

$$V = V_o\, (T_{C_o}' / T_C')^{1/3}$$

where ( )$_o$ indicates equilibrium trim

For Normal Rated Power,

$$T_{C_o}' = \frac{BHP \times \eta}{q'\, S}$$

BHP = 2400

η = .80 (app.)

q' = 199.5

S = 465

$T_{C_o}' = .0207$

For equilibrium trim,

$V_o = 410$ ft. / sec.

$T_{C_o}' = .0207$

Plotting C_M vs. C_L for $T_{C_o}' = .0207$ and shifting the C_L axis to intercept the curve at C_{L_o}, the values of C_M for various values of T_C' at trim C_L may be tabulated as C_M'. Since V is a function of T_C', it is also possible to tabulate various values of V for corresponding values of C_M'.

C_M'	T_C'	V
-.010	0	
0	.021	407
.020	.070	273
.046	.135	220
.072	.206	191
.096	.266	175

We know,

$$u\,M_u' = C_M' \times q' \times S \times t \times \frac{g}{W}$$

$$M_u' = q'\,S\,t\,\frac{g}{W} \times \frac{C_M'}{u}$$

$$M_u' = \frac{1}{C_{L_o}} \times \frac{g \times t}{l} \times \frac{dC_M'}{dV}$$

Plotting C_M' vs. V and taking the slope at V = 410 ft./sec we find,

$$\frac{dC_M'}{dV} = -0.0000635$$

(see Figure 6)

Therefore,

$$M_u' = -199.5 \times 465 \times 835 \times 0.0000635 \times 0.001667$$

$$= -49.1 \times 0.001667$$

$$= -0.0820$$

and dividing by K_B^2 gives the desired value of M_u

$M_u = -0.00137$

B. RESISTANCE DERIVATIVES FOR LATERAL MOTION

$$L_p = \frac{1}{\tau} l_p \qquad (\tau = m/eSV)$$

$$= \frac{1}{\tau} \times 1/4 \left(\frac{b}{K_A}\right)^2 \frac{dC_l}{d(pb/2V)}$$

$$\frac{dC_L}{d(pb/2V)} = .485 \quad \text{(NACA Report 635)}$$

$$= \frac{1}{1.323}(-9.25) = -7.00$$

$$N_p = \frac{1}{\tau} N_p$$

$$= \frac{1}{\tau} \times 1/4 \left(\frac{b}{K_C}\right)^2 \frac{dC_n}{d(pb/2V)}$$

$$\frac{dC_n}{d(pb/2V)} = .01542 \quad \text{(NACA Report 635)}$$

$$= \frac{1}{1.323}(-0.1391) = -0.1051$$

$$L_r = \frac{1}{\tau} \times \frac{1}{4} \left(\frac{b}{K_A}\right)^2 \frac{dC_l}{d(rb/2V)}$$

$$\frac{dC_l}{d(rb/2V)} = .0560 \quad \text{(MACA Report 635}$$

$$= \frac{1}{1.323}(1.066) = 0.805$$

$$N_r = \frac{1}{\tau} \left(\frac{b}{K_C}\right)^2 \frac{dC_n}{d(rb/2V)}$$

$$\frac{dC_n}{d(rb/2V)} = -\frac{1}{3} C_{D_W} - 2\eta_V \frac{l^2}{b^2} \frac{S_V}{S} \frac{dC_{LV}}{d\beta}$$

$$= -.113$$

$$N_r = \frac{1}{1.323}(-1.020) = -0.771$$

C. ACCELERATIONS FOR UNIT CONTROL MOVEMENT. (FROM WIND TUNNEL DATA)

$Y_{o_r} = C_{Y_{o_r}}\ q\ S\ /m$

$= (.00586)\ (199.5)\ (465)\ /m = -0.905$

$Zo_e = C_{Z_{o_e}}\ q\ S\ /m$

$= (0.27)\ (199.5)\ (465)\ /m = -41.8$

$M_{o_e} = C_{M_{o_e}}\ q\ S\ t\ /m\ K_B^2$

$= (.022)\ (199.5)\ (465)\ (8.35)\ /mK_B^2 = -0.467$

$Lo_a = C_{l_{o_a}}\ q\ S\ b\ /m\ K_A^2$

$= (.0040)\ (199.5)\ (465)\ (61.33)\ /m\ K_A^2 = 0.771$

$N_{o_r} = C_{n_{o_r}}\ q\ S\ b\ /m\ K_C^2$

$= (.00109)\ (199.5)\ (465)\ (61.33)\ /m\ K_C^2 = 0.0993$

$N_{o_a} = C_{n_{o_a}}\ q\ S\ b\ /m\ K_C^2$

$= (.00005)\ (199.5)\ (465)\ (61.33)\ /mK_C^2 = 0.00449$

For 15,000 ft.

A. RESISTANCE AND ROTARY DERIVATIVES FOR LONGITUDINAL MOTIONS

$X_u = 0$

$Z_u = -.109$

$M_q = -13.57$

$M_u = 1.041$

B. RESISTANCE DERIVATIVES FOR LATERAL MOTION

$L_p = -4.71$

$N_p = -0.0710$

$L_r = .544$

$N_r = -0.521$

C. ACCELERATIONS FOR UNIT CONTROL MOVEMENT

$Y_{o_r} = -0.656$

$Z_{o_e} = -30.3$

$M_{o_e} = -0.339$

$L_{o_a} = 0.559$

$N_{o_r} = 0.0720$

$N_{o_a} = 0.003254$

SUMMARY OF STABILITY DERIVATIVES AND
ACCELERATIONS FOR UNIT CONTROL MOVEMENT
FOR TWIN MOTORED AIRPLANE

TABLE A

ALTITUDE	X_u	Z_u	M_q	M_u	L_p	N_p	L_r	N_r	$Y_{\delta r}$	$Z_{\delta e}$	$M_{\delta e}$	$L_{\delta a}$	$N_{\delta r}$	$N_{\delta a}$
SEA LEVEL	0	-.151	-18.71	.000137	-7.00	-0.1051	0.805	-0.771	-0.905	-41.8	+0.467	0.771	0.0993	0.00449
15,000 Ft.	0	-.109	-13.57	.000099	-4.71	-0.0710	0.544	-0.521	-0.656	-30.3	+0.339	0.559	0.0720	0.00325

BASIC EQUATIONS AND ASSUMPTIONS

On account of the bilateral symmetry of the airplane it is customary to divide the notions into two independent groups, the lateral and the longitudinal, each consisting of three degrees of freedom:

1)	Lateral motions	Rolling Yawing Side slipping
2)	Longitudinal motions	Pitching Vertical translation Forward translation

Presumably the reactions to small increments of longitudinal speed or displacement do not sensibly influence the lateral motions and the two groups may be treated independently.

Consider first the lateral motions. If the flight path is assumed to be horizontal (or nearly so) and the main forward velocity U_o to be substantially constant, the equations of motion in a lateral disturbance may be written:

(In side slipping) $$\frac{dv}{dt} = g \sin \phi - rU_o + vY_v + Y_o$$

(In rolling) $$\frac{dp}{dt} = vL_v + pL_p + rL_r + L_o$$

(In yawing) $$\frac{dr}{dt} = vN_v + pN_p + rN_r + N_o$$

The quantities that arise in the consideration of the lateral motions are defined in the following table:

Velocities and displacements of airplane axes:

U_o, equilibrium flight velocity along X-axis.

v, component of flight velocity along Y - axis (side slipping) p, component of angular velocity about X - axis (rolling)

r, component of angular velocity about Z - axis (yawing)

Ø, angle of bank

Forces and moments resolved along airplane axes:

Y, component of force along Y - axis

L, component of moment about X - axis (rolling moment)

N, component of moment about Z - axis (yawing moment)

Accelerations of airplane:

$Y_o = Y/m$ (control force per unit mass)

$L_o = L/m\,K_A^2$ (control moment per unit moment of inertia)

$N_o = N/m\,K_C^2$ (control moment per unit moment of inertia)

Airplane characteristics used as parometers:

Y_v
Y_r
L_p
L_v
L_r
N_v
N_p
N_r

Stability derivatives in terms of accelerations of airplane, thus:

$$Y_v = \frac{\partial Y}{\partial v} \Big/ m$$

$$L_r = \frac{\partial L}{\partial r} \Big/ mK_A^2$$

$$N_p = \frac{\partial N}{\partial p} \Big/ mK_C^2$$

It has been found convenient to transform all stability derivatives and disturbing effects into terms of accelerations of the airplane rather than retaining them as moments and forces. This transformation is accomplished by dividing out the appropriate moments of inertia and the mass of the airplane. For example, $\frac{\partial L}{\partial p} \Big/ mK_A^2$ may be written simply as L_p;

similarly $\frac{\partial N}{\partial v} \Big/ m\, K_C{}^2 = N_v$ and $\frac{\partial Y}{\partial r} \Big/ m = Y_r$.

On the basis of these definitions, the above equations for sideslipping, rolling and yawing were written. Since the axes chosen will ordinarily lie near the axes of the principle moments of inertia of the airplane, terms involving the products of inertia have been neglected.

FOR SOLUTION OF THE LATERAL EQUATIONS

From the three lateral equations, the unknowns are:

ϕ, r, v, Y_o, L_o, N_o

Therefore we have three equations and six unknowns. However one of the unknowns can be eliminated if we let:

$Y_o = \delta_r\, Yo_r$

$L_o = \delta_a\, Lo_a$

$N_o = \delta_r\, No_r + \delta_a\, No_a$

For the two other unknowns, we can

1) Specify ϕ ,

2) Specify v

We shall take v = 0 (as suggested by Jones in NACA Report 560)

For ϕ, let $\phi = K\left[\frac{1 - e^{-nt}}{n} - \frac{1 - e^{-(n+m)t}}{n+m}\right]$

The three equations of lateral motion may now be written:

$$r\,U_o - \delta r\, Yo_r = g \sin \phi \qquad (1)$$

$$r\,L_r + \delta_a\, Lo_a = \frac{dp}{dt} - pLp \qquad (2)$$

$$\frac{dr}{dt} - r\,N_r - \delta_r\, No_r - \delta_a\, No_a = pNp \qquad (3)$$

where,

Y_{o_r} = acceleration sideways due to unit rudder deflection

L_{o_r} = rolling moment due to unit aileron deflection

N_{o_r} = Yawing moment due to unit rudder deflection

N_{o_a} = Yawing moment due to unit aileron deflection

The three unknowns in equations (1), (2), and (3) are:

r, δ_r, δ_a

Multiplying (1) by $\frac{N_{o_r}}{Y_{o_r}}$, and (2) by $\frac{N_{o_a}}{L_{o_a}}$

$$r \frac{N_{o_r}}{Y_{o_r}} U_o - \delta_r N_{o_r} = g \frac{N_{o_r}}{Y_{o_r}} \sin \phi \tag{4}$$

$$r \frac{N_{o_a}}{L_{o_a}} L_r + \delta_a N_{o_a} = \frac{N_{o_a}}{L_{o_a}} \frac{dp}{dt} - p \frac{N_{o_a}}{L_{o_a}} L_p \tag{5}$$

and rewriting (3)

$$\frac{dr}{dt} - r N_r = p N_p + \delta_r N_{o_r} + \delta_a N_{o_a} \tag{6}$$

From (5)

$$\delta_r N_{o_r} = -g \frac{N_{o_r}}{Y_{o_r}} \sin \phi + r \frac{N_{o_r}}{Y_{o_r}} U_o \tag{7}$$

From (6)

$$\delta_a N_{o_a} = \frac{N_{o_a}}{L_{o_a}} \frac{dp}{dt} - p \frac{N_{o_a}}{L_{o_a}} L_p - r \frac{N_{o_a}}{L_{o_a}} L_r \tag{8}$$

substituting $\delta_r N_{o_r}$ and $\delta_a N_{o_a}$ in (6)

$$\frac{dr}{dt} - r N_r = pN_p + r \frac{N_{o_r}}{Y_{o_r}} U_o - g \frac{N_{o_r}}{Y_{o_r}} \sin \phi$$

$$+ \frac{N_{o_a}}{L_{o_a}} \frac{dp}{dt} - p \frac{N_{o_a}}{L_{o_a}} L_p - r \frac{N_{o_a}}{L_{o_a}} L_r$$

or, rewriting

$$\frac{dr}{dt} + \left(\frac{N_{o_a}}{L_{o_a}} L_r - \frac{N_{o_r}}{Y_{o_r}} U_o - N_r\right) r = pN_p$$

$$-g \frac{N_{o_r}}{Y_{o_r}} \sin \phi + \frac{N_{o_a}}{L_{o_a}} \frac{dp}{dt} - p \frac{N_{o_a}}{L_{o_a}} L_p \qquad (9)$$

From equation (7), the rudder deflection may be written,

$$\delta_r = -g \frac{1}{Y_{o_r}} \sin \phi + r \frac{1}{Y_{o_r}} U_o \qquad (10)$$

From equation (8), the aileron deflection may be written,

$$\delta_a = \frac{1}{L_{o_a}} \frac{dp}{dt} - p \frac{1}{L_{o_a}} L_p - r \frac{1}{L_{o_a}} L_r \qquad (11)$$

Since ϕ is known, and $p = \frac{d\phi}{dt}$, equation (9) is a first degree linear differential equation in r and t.

Recalling $\phi = k\left(\frac{1-e^{-nt}}{n} - \frac{1-e^{-(n+m)t}}{n+m}\right)$

The solution of (9) is greatly simplified by letting $\sin \phi = \phi$

Putting $\frac{N_{o_a}}{L_{o_a}} L_r - \frac{N_{o_r}}{Y_{o_r}} U_o - N_r = A$

Equation (9) may be written,

$$\frac{dr}{dt} + A_r = N_p\left[ke^{-nt} - ke^{-(n+m)t}\right]$$

$$-g \frac{N_{o_r}}{Y_{o_r}} \left[k\left(\frac{1-e^{-nt}}{n} - \frac{1-e^{-(n+m)t}}{n+m}\right)\right]$$

$$+ \frac{N_{o_a}}{L_{o_a}} \left[-kne^{-nt} + k(n+m)\, e^{-(n+m)t}\right] - \frac{N_{o_a}}{L_{o_a}} L_p \left[ke^{-nt} - ke^{-(n+m)t}\right]$$

Collecting terms on right hand side,

$$\frac{dp}{dt} + A_r = C_1 + C_2\, e^{-nt} + C_3\, e^{-(n+m)t} \qquad (12)$$

Where,

$$C_1 = -g \frac{N_{o_r}}{Y_{o_r}} \frac{k}{n} + g \frac{N_{o_r}}{Y_{o_r}} \frac{k}{(n+m)}$$

$$C_2 = N_p\, k + g\,\frac{N_{o_r}}{Y_{o_r}}\,\frac{k}{n} - \frac{N_{o_a}}{L_{o_a}}\,kn - \frac{N_{o_a}}{L_{o_a}}\,L_p k$$

$$C_3 = -N_p k - g\,\frac{N_{o_r}}{Y_{o_r}}\,\frac{k}{n+m} + \frac{N_{o_a}}{L_{o_a}}\,k(n+m) + \frac{N_{o_a}}{L_{o_a}}\,L_p k$$

Solving equation (12)

$$r = Be^{-At} + \frac{C_1}{A} + \frac{C_2}{A-n}\,e^{-nt} + \frac{C_3}{A-(n+m)}\,e^{-(n+m)t}$$

B can be determined from the boundary conditions which are obviously:

$$r = 0 \text{ at } t = 0$$

$$B = -\frac{C_1}{A} - \frac{C_2}{A-n} - \frac{C_3}{A-(n+m)}$$

$$r = \frac{C_1}{A}\left(1-e^{-At}\right) + \frac{C_2}{A-n}\left(e^{-nt} - e^{-At}\right) + \frac{C_3}{A-(n+m)}\left(e^{-(n+m)t} - e^{-At}\right) \qquad (13)$$

If the actual value of sin ϕ is used, rather than the simplified case where sin $\phi = \phi$, equation (9) may be written,*

$$\frac{dr}{dt} + Ar = N_p\left[ke^{-nt} - ke^{-(n+m)t}\right] - g\,\frac{N_{o_r}}{Y_{o_r}}\,\sin\left[k\left(\frac{1-e^{-nt}}{n} - \frac{1-e^{-(n+m)t}}{n+m}\right)\right] + \frac{N_{o_a}}{L_{o_a}}\left[-kne^{-nt} + k(n+m)\,e^{-(n+m)t}\right] - \frac{N_{o_a}}{L_{o_a}}\left[L_p\;ke^{-nt} - ke^{-(n+m)t}\right]$$

$$\frac{dp}{dt} + Ar = C_1\,e^{-nt} + C_2\,e^{-(n+m)t} - g\,\frac{N_{or}}{Y_{or}}\,\sin\phi \qquad (14)$$

where

$$C_1 = N_p\,k - \frac{N_{o_a}}{L_{o_a}}\,kn - \frac{N_{o_a}}{L_{o_a}}\,L_p\,k$$

* As a matter of interest, equation (9) was solved by the approximate method and also by the exact method for a reasonably small angle of maximum bank of only 67°, and the time to make a given turn calculated. The results are shown in figure IV; and it is obvious that the error involved in the simplified (approximate) solution of r necessitates the exact method of solution whenever accurate results are desired.

$$C_2 = -N_p \, k + \frac{N_{o_a}}{L_{o_a}} \, k \, (n+m) + \frac{N_{o_a}}{L_{o_a}} \, L_p \, \dot{k}$$

Solving equation (14)

$$r = Be^{-At} + \frac{C_1}{A-n} e^{-nt} + \frac{C_2}{A-(n+m)} e^{-(n+m)t}$$

$$-g \frac{N_{o_r}}{Y_{o_r}} e^{-At} \int_o^t e^{At} \sin \phi \, dt$$

As before, the boundary conditions are:

$$r = 0 \text{ at } t = 0$$

$$B = -\frac{C_1}{A-n} - \frac{C_2}{A-(n+m)}$$

$$r = \frac{C_1}{A-n} e^{-nt} - e^{-At} + \frac{C_2}{A-(n+m)} e^{-(n+m)t} - e^{-At}$$

$$-g \frac{N_{o_r}}{Y_{o_r}} e^{-At} \int_o^t e^{At} \sin \phi \, dt \qquad (15)$$

The integral term in this equation must be evaluated.

$$\text{Let true integral} = \int_o^t e^{Ax} \sin \phi \, (x) \, dx$$

$$\text{Let approximate integral} = \sin \phi \, (t) \int_o^t e^{Ax} \, dx$$

$$R = \frac{\text{True}}{\text{Approximate}}$$

$$R = \frac{\int_o^t e^{Ax} \sin \phi \, (x) \, dx}{\sin \phi \, (t) \int_o^t e^{-Ax} \, dx}$$

$$R = \frac{A \int_o^t e^{A(x-t)} \sin \phi \, (x) \, dx}{\sin \phi \, (t) \left(1-e^{-At}\right)}$$

For t small,

$$\sin \phi(t) = \phi(t) + \frac{[\phi(t)]^3}{3!} + \text{---------}$$

$\phi(t)$ less than $\frac{1}{2}$

t " " $\frac{1}{2}$

we can say,

$$I_1 = A\int_0^t e^{A(x-t)} \sin \phi(x)\, dx = A\int_0^t e^{A(x-t)} \phi\, dx$$

Integrating by parts

$$I_1 = \phi(t) - k\int_0^t e^{A(x-t)}\left(e^{-nx} - e^{-(n+m)x}\right) dx$$

$$\text{since } \phi(x) = k\left(\frac{1-e^{-nx}}{n} - \frac{1-e^{-(n+m)x}}{n+m}\right)$$

$$\frac{d\phi}{dx} = k\left(e^{-nx} - e^{-(n+m)x}\right)$$

$$I_1 = \phi(t) - ke^{-At}\left(\frac{e^{(A-n)t}-1}{A-n} - \frac{e^{(A-n-m)t}-1}{A-(n+m)}\right)$$

$$= \phi(t) - k\left(\frac{e^{-nt}-e^{-At}}{A-n} - \frac{e^{-(n+m)t}-e^{-At}}{A-(n+m)}\right)$$

$$R = \frac{\phi(t) - k\left(\frac{e^{-nt}-e^{-At}}{A-n} - \frac{e^{-(n+m)t}-e^{-At}}{A-(n+m)}\right)}{\phi(t)\left(1-e^{-At}\right)}$$

$$R = \frac{1-\frac{k}{\phi(t)}\left(\frac{e^{-nt}-e^{-At}}{A-n} - \frac{e^{-(n+m)t}-e^{-At}}{A-(n+m)}\right)}{1-e^{-At}} \qquad (16)$$

Therefore for values of t up to approximately $\frac{1}{2}$ second,

$$\int_0^t e^{At} \sin \phi(t)\, dt = R \sin \phi(t)\int_0^t e^{At}\, dt$$

Where R is given above in equation (16).

<u>For t large</u>

For values of t greater than ½ second, the above value of R does not hold accurately for $\sin \phi(t) \neq \phi(t)$

However we can evaluate the integral by an assymptotic evaluation.

We can say,

$$I_2 = e^{-At} \int_0^t e^{Ax} \sin \phi(x)\, dx$$

Expressing $\sin \phi(x)$ in a Taylor's series

$$\sin \phi(x) = \sin \phi(t) + \cos \phi(t) \frac{d\phi}{dt} \frac{(x-t)}{1!} + \text{------}$$

$$\int_0^t e^{Ax} \sin \phi(x)\, dx = \sin \phi(t) \int_0^t e^{Ax}\, dx + \cos \phi(t) \times \frac{d\phi}{dt} \int_0^t e^{Ax} (x-t)\, dx$$

$$= \sin \phi(t) \frac{e^{At}-1}{A} + \cos \phi(t) \frac{d\phi}{dt} \left(\frac{x-t}{A} e^{A-x} - \frac{1}{A^2} e^{A-x} \right|_0^t$$

$$= \sin \phi(t) \frac{e^{At}-1}{A} + \cos \phi(t) \frac{d\phi}{dt} \left(\frac{t}{A} - \frac{e^{At}-1}{A^2} \right)$$

Therefore

$$I_2 = \frac{\sin \phi(t)}{A} \left(1-e^{-At}\right) + \frac{\cos \phi(t)}{A^2} \frac{d\phi}{dt} \left(-1+e^{-At} + Ate^{-At}\right)$$

$$I_2 = \frac{\sin \phi(t)}{A} \left(1-e^{-At}\right) \left(1 - \frac{\cot \phi(t)}{A} \frac{d\phi}{dt} \frac{1-e^{-At}-Ate^{-At}}{1-e^{-At}}\right)$$

For evaluation of the integral in equation (15) for various values of t, one method must be used for t less than ½ second and another method used for t greater than

$\frac{1}{2}$ second.

Summarzing,

For t less than $\frac{1}{2}$

$$e^{-At}\int_0^t e^{At} \sin \phi (t)\, dt = R \frac{\sin \phi (t)}{A} \left(1-e^{-At}\right) \qquad (17)$$

where R is given in equation (16)

For t greater than $\frac{1}{2}$

$$e^{-At}\int_0^t e^{At} \sin \phi (t)\, dt = \frac{\sin \phi (t)}{A} \left(1-e^{-At}\right) \left[1 - \frac{\operatorname{Cot} \phi(t)}{A} \times \frac{d\phi}{dt} \frac{1-e^{-At} -Ate^{-AT}}{1-e^{-At}}\right] \qquad (18)$$

Therefore by solution of the lateral equations of motion through assuming a bank ϕ in a horizontal turn in which there is no sideslip, the rudder deflection, δ_r , the aileron deflection δ_a, and the angular velocity of yaw, r, can be found in terms of the airplane accelerations for unit control movement, Y_{Or}, L_{Oa}, N_{Or}, N_{Oa} and damping derivatives L_r, L_p, N_p, N_r.

Consider now the longitudinal motions.

If the flight path is assumed to be horizontal and the main forward velocity U_o to be substantially constant, the equations of motion in a longitudinal disturbance may be written,

$$\frac{du}{dt} = u\,X_u + w\,X_w - g\,\theta + X_o$$

$$\frac{dw}{dt} = u\,Z_u + w\,Z_w + q\,U_o + Z_o$$

$$\frac{dq}{dt} = u\,M_u + w\,M_w + q\,M_q + M_o$$

The quantities which arise in the consideration of the longitudinal motions are defined in the following table.

Velocities and displacements of airplane axes:

U_o, equilibrium flight velocity along X-axis (same as for lateral motion)

u, Component of flight velocity along X-axis.

w, Component of flight velocity along Z-axis.

q, Component of angular velocity about Y-axis (pitching)

Forces and moments resolved along airplane axes:

X, Component of force along X-axis.

Z, Component of force along Z-axis.

M, Component of moment about Y-axis (pitching moment)

Accelerations of airplane:

$X_o = X/m$ (control force per unit mass)

$Z_o = Z/m$ (control force per unit mass)

$M_o = M/m\,K_B^2$ (control moment per unit moment of inertia)

Airplane characteristics used as parameters:

X_u

X_w $\qquad X_u = \frac{\partial X}{\partial u} / m$

Z_u

Z_w $\qquad M_q = \frac{\partial M}{\partial q} / m\ K_B^2$

M_u

M_w

M_q

On the basis of these definitions, the above equations were written.

FOR SOLUTION OF THE LONGITUDINAL EQUATIONS

From the three longitudinal equations, the unknowns are:

$u,\ w,\ \theta,\ X_o,\ Z_o,\ M_o$

As in the case concerning the lateral equations, we have three equations and six unknowns. One of the unknowns can be eliminated if we let:

$$M_o = \delta_e\ M_{o_e}$$

$$Z_o = \delta_e\ Z_{o_e}$$

And for the other two unknowns, we can assume:

1) $X_o = 0$ i.e. no change in thrust

2) $\theta = \frac{w}{U_o}$ i.e. horizontal motion (Eulerian Axes)

3) Steady state i.e. $\frac{du}{dt} = 0$, $\frac{dw}{dt} = 0$.*

* Only the steady state condition of the turn will be considered, for that will determine maximum elevator deflection.

For some relation to give q (pitching velocity), let us consider the motion of the airplane as it executes a horizontal turn.

We can say,

$$\bar{w} = p\,\bar{i} + q\,\bar{j} + r\,\bar{k}$$

where,

$\bar{w}$ = angular velocity of coördinate

If β = change of direction in horizontal plane from original flight path (degree of turn executed), then,

$$\bar{w} = \frac{d\phi}{dt}\,\bar{i} + \frac{d\beta}{dt}\sin\phi\,\bar{j} + \frac{d\beta}{dt}\cos\phi\,\bar{k}$$

Therefore,

$$p = \frac{d\phi}{dt}$$

$$r = \frac{d\beta}{dt}\cos\phi$$

$$q = \frac{d\beta}{dt}\sin\phi$$

Eliminating $\frac{d\beta}{dt}$,

$$q = r\tan\phi \qquad (18)$$

Rewriting $r = \frac{d\beta}{dt}\cos\theta$,

$$\frac{d\beta}{dt} = r\sec\phi \qquad (19)$$

Using assumptions 1, 2, and 3, and $q = r\tan\phi$, the longitudinal equations may be written,

$$u\,X_u + w\,X_w - g\,\frac{w}{U_o} = 0$$

$$u\, Z_u + w\, Z_w + \delta_e\, Z_{\delta e} = -U_o\, q$$

$$u\, M_u + w\, M_w + \delta_e\, M_{\delta e} = (D - M_q)\, q$$

$X_u = 0$*, therefore $w = 0$ (see calculations of stability derivatives)

$$u\, Z_u + Z_{\delta e}\, \delta_e = -U_o\, q$$

$$u\, M_u + M_{\delta e}\, \delta_e = (D - M_q)\, q$$

Solving for δ_e:

$$\delta_e = - \frac{U_o\, M_u\, q + Z_u\, (D - M_q)\, q}{Z_{\delta e}\, M_u - M_{\delta e}\, Z_u} \tag{20}$$

Integrating equation (19) gives the turn as a function of time

$$\beta = \int_o^t r \sec \phi \tag{21}$$

Knowing β, the acceleration produced during the turn can be easily calculated.

Normal acceleration = $U_o \omega$ ft./sec.^{2}

where,

$$\omega = \frac{d\beta}{dt}$$

Gravitational acceleration = 32.2 ft./sec.^{2}

Therefore,

$$a = \sqrt{(a_n)^2 + (a_g)^2} \tag{22}$$

where a_n and a_g are normal and gravitational acceleration respectively.

* The assumptions X = 0 and horizontal flight with $X_u = 0$ are not consistent except the case for horizontal flight in a straight line at the speed U_o. The error produced by this inconsistency is small.

CALCULATIONS FOR TIME TO EXECUTE A HORIZONTAL TURN AND THE CONTROL DEFLECTIONS NECESSARY, CONSIDERING SEVERAL BANK CONDITIONS

Recalling $\phi = k\left(\frac{1-e^{-nt}}{n} - \frac{1-e^{-(n+m)t}}{n+m}\right)$

An examination of this exponential expressing the angle of bank as a function of time will reveal that the parameter, k, determines the maximum angle of bank, whereas the other two parameters, n and m, determine the rolling velocity, and therefore the helix angle pb/2V.

A helix angle of at least .070 is necessary, and one of 0.100 is desired for maximum aileron deflection as is pointed out in NACA Report 715.

I For a maximum bank of 80° and a helix angle of approximately 0.100, the parameters should be:

$k = 3.1$

$m = 3.0$

$n = 1.5$

Therefore,

$$\phi = 3.1\left(\frac{1-e^{-1.5t}}{1.5} - \frac{1-e^{-4.5t}}{4.5}\right)$$

Considering the exact solution of r as expressed in equation (15), and using the values of the stability derivatives, and accelerations for unit control deflection for sea level given in table A, page (13), we have:

$A = 45.8$

$C_1 = -.2268$

$C_2 = .2818$

Equation (15) may be written:

$$r = -0.00513 \left(e^{-1.5t} - e^{-45.8t}\right) + 0.00682 \left(e^{-4.5t} - e^{-45.8t}\right) + 3.53\, e^{-45.8t} \int_{o}^{t} e^{45.8t} \sin \phi \, dt$$

Solving this equation by the exact method previously outlined and graphically integrating equation (21) in order to determine β, it is convenient to tabalize the results. Several of the items used in the solution of r are included in table I (a).

Using equations (10) and (11) and values (for sea level) from table (A), we can obtain the elevator and rudder deflections necessary to make the turn given in table I (a). Since $\frac{d\beta}{dt} = r \sec \phi$, the acceleration produced during the turn may be calculated.

$$\delta r = 35.6 \sin \phi - 453r$$

$$\delta a = 1.298 \frac{dp}{dt} + 9.07\, p - 1.042\, r$$

$$a = \sqrt{(a_n)^2 + (a_g)^2}$$

where,

$$a_n : U_o \frac{d\beta}{dt}$$

Table I (b) shows aileron and rudder deflections as well as accelerations. See page (43)

Considering only the maximum elevator deflection, that is, $\frac{dq}{dt} = o$, where $q = r \tan \phi$ as given in equation (18).

δ_e from equation (20) becomes

$$\delta_e = - \frac{U_o M_u q + Z_u M_q q}{Z_{oe} M_u - M_{oe} Z_u} = - D \times q$$

where,

$$D = \frac{U_o M_u - Z_u M_q}{Z_{oe} M_u - M_{oe} Z_u}$$

Refefring to table (A) for the values to determine δ_e in the above mentioned equation , we have,

$$\delta_e = -Dq = -\frac{410 \times -0.00137 + 0.151 \times 18.71}{-41.8 \times -0.00137 + .467 \times .151} \quad (.384)$$

$$= -17.70 \times .384$$

$$= -6.81^{\circ}$$

Figure 1 (a) shows Ø and p plotted against time, t; figure 1 (b) shows turn, elevator and rudder deflections plotted against time; and figure 1 (c) shows acceleration produced in turn as a function of time, t.

II. The control deflections necessary to execute the turn as worked out for a maximum bank of 80° are of reasonable magnitude. However, the maximum acceleration of 5.1 g is extremely high and could be maintained for only a few seconds by the average pilot. Considering a turn in which the maximum angle of bank is 75°, an acceleration more nearly equal to what the pilot may voluntarily develop will arise. Since the parameter k determines the maximum bank, and m and n determine the helix angle, it is only necessary to change k for the aileron control is sufficient to maintain the helix angle of approximately 0.100. The values of k,m,n will be

$k = 2.95$

$m = 3.0$

$n = 1.5$

Therefore,

$$\phi = 2.95\left(\frac{1-e^{-1.5t}}{1.5} - \frac{1-e^{-4.5t}}{4.5}\right)$$

For this case, as the one previously discussed, only the exact solution of r as given by equation (15) will be considered. Using the values of the stability derivatives and accelerations for unit control deflection for sea level given in table (A), page (13), we have:

$A = 45.8$

$C_1 = -.2160$

$C_2 = .2680$

And so equation (15) may be written:

$$r = -0.00488\left(e^{-1.5t} - e^{-45.8t}\right) + 0.00650\left(e^{-4.5t} - e^{-45.8t}\right) + 3.53\, e^{-45.8t}\int_0^t e^{45.8t} \sin\phi \, dt$$

Putting in tabular form various items necessary in the exact solution of the above equation, and graphically integrating equation (21) in order to determine β, we have table II (a), page (44).

The expressions for δ_r and δ_a obtained from equations (10) and (11) are unchanged since sea level conditions are those under consideration. Obtaining $\frac{d\beta}{dt}$ from table II (a), the acceleration produced during the maneuver may be readily calculated. Table II (b) shows the aileron and rudder deflections as well as the accelerations for various values of t.

Since $\delta_e = -D \times q$ for maximum deflection

where $q = r \tan \phi$

$= .282$

We have,

$\delta_e = -5.0^\circ$

It is seen that for a reduction of maximum bank from 80° to 75°, the acceleration is reduced from 5.1 g to 3.80 g. There is little change in control deflections, the initial aileron position being reduced only one half of one degree, and the final rudder position being only two degrees less. However, the time to execute a turn of a given number of degrees is much greater for the latter case as can be readily seen from a comparison of tables I (a) and II (a).

Figure II (a) shows ϕ and p plotted against time, t; figure II (b) shows turn, elevator and rudder deflections plotted against time; and figure II (c) shows accelerations produced in turn as a function of the time, t.

Since 75° seems to be the most logical value of ϕ max., it is of interest to investigate for this same value of ϕ the maneuverability of the airplane at some altitude rather than sea level as previously done. Let us arbitrarily assume 15,000 ft.

Again considering only the exact solution of r, and using the values of the stability derivatives and accelerations for unit control deflections for 15,000 ft. given in table (A), page (13), we have:

$$A = 48.77$$

$$C_1 = -.1544$$

$$C_2 = .2059$$

Therefore r may be written,

$$r = -0.00327\left(e^{-1.5t} - e^{-48.77t}\right) + 0.00465 \times \left(e^{-4.5t} - e^{-48.77t}\right) + 3.53\, e^{-48.77t}\int_0^t e^{48.77t}\sin\phi\, dt$$

As before, putting in tabular form various items necessary in the exact solution of the above equation, and graphically integrating equation (21) in order to determine β, we have table II (c) as given on page (46).

Using equations (10) and (11) and values from table (A), we can obtain, as before, the elevator and rudder deflections necessary to make the turn given in table II (c). The acceleration can be obtained as previously described.

$$\delta_r = 49.1 \sin\phi - 670r$$

$$\delta a = 1.822\frac{dp}{dt} + 8.44\, p - .947\, r$$

Table II (d) page (47) shows aileron and rudder deflections. Also accelerations produced.

Since $\delta_e = -D\, xq$ for maximum deflection

where $q = r \tan\phi$

$= .261$

We have,

$$\delta_e = -3.94^0$$

From examination of tables II (a) and II (c), it is apparent that for the same bank assumption, there is little difference in the time required to make a specified turn. As would be expected, however, from tables II (b) and II (d) it is shown that the control deflections must be considerably increased for the high altitude case. Since the deflections developed in making the turn at sea level are about the maximum the pilot could exert, it is obvious that the altitude maneuver will be considerably slower since not enough aileron control will be available to produce the same rolling velocity as in the previous case. The accelerations produced are unchanged as regards to sea level.

Figure II (b) shows turn, elevator and rudder deflections plotted against time; and figure II (c) shows accelerations produced in turn as a function of the time.

III. So far we have considered a twin motored attack bomber of late design. For this particular plane the time to make a specified turn and the control deflections necessary to execute this turn have been worked out, both for sea level and for 15,000 ft. As previously seen, the only major assumption made in the calculations was the angle of bank as a function of time, which was represented as an exponential curve.

As a matter of interest, let us investigate the maneuvering characteristics of this same plane considering it a single engine airplane rather than a twin engine one. The aerodynamic qualities necessary for the calculations remain unchanged, and only the radii of gyration come in as influencing factors.

We shall assume the "new" single engine airplane to have the one motor mounted in the fuselage and this motor to be of the same weight as the two motors on the previous plane. The top speed remains unchanged. Also, this single motor will be considered mounted approximately six feet forward of the two motors on the previously discussed airplane.

We have known:

1) Radii of gyration on twin motored airplane.

$K_A^2 = 49.2$ ft.^{2}

$K_B^2 = 59.8$ ft.^{2}

$K_C^2 = 104.1$ ft.^{2}

2) Weight of engine on twin motored airplane.

$W = 2690$ lbs.

3) Weight of engine on single motored airplane.

$2W = 5380$ lbs.

4) Distance of motor from fuselage on twin motored airplane.

$y = 8.75$ ft.

5) Distance moved forward of motor on single motored airplane.

$x = 6.00$ ft.

By denoting I_s and I_T as moment of inertia of single and twin motored airplane respectively:

I_s = mass of airplane $x K_A^2$ - inertia due to motors

$$= 600 \times 49.2 - 167 \times (8.75)^2 = 16720$$

$$K_A^2 = 16720/600 = 27.9 \text{ ft.}^2$$

In a similar manner K_B^2 and K_C^2 can be approximated for the single motored airplane.

Finally, we have,

$$K_A^2 = 27.9 \text{ ft.}^2$$

$$K_B^2 = 69.8 \text{ ft.}^2$$

$$K_C^2 = 92.9 \text{ ft.}^2$$

Using these new radii of gyration, the stability derivatives and accelerations for unit control deflection for sea level come out:

$L_r = 1.421$

$L_p = -12.37$

$N_r = -0.865$

$N_p = -0.1180$

$Y_{o_r} = -0.905$

$L_{o_a} = 1.360$

$N_{o_r} = 0.1115$

$N_{o_a} = 0.00669$

Assuming the angle of maximum bank as 75°, and considering only the exact solution of r, we have, using the above values of the stability derivatives and unit control accelerations:

$A = 51.37$

$C_1 = -.1907$

$C_2 = .2342$

Therefore,

$$r = -0.00383\left(e^{-1.5t} - e^{-51.37t}\right) + 0.00500\left(e^{-4.5} + e^{-51.37t}\right)$$

$$+ 3.97\, e^{-51.37t}\int_{0}^{t} e^{51.37t} \sin \phi \, dt$$

Putting in tabular form necessary items in the exact solution of the above equation and in the determination of β, we have; Table III (a) page (48).

The elevator and rudder deflections necessary to make the turn are:

$\delta_r = 35.6 \sin \phi - 453$ (unchanged since Y_{o_r} is independent of moment of inertia)

$$\delta_a = 0.736 \frac{dp}{dt} + 9.09\, p - 1.045$$

Table III (b) gives aileron and rudder deflections and also accelerations produced.

$\delta_e = D \times q$

where, as before,

$q = r \tan \phi$

$= .282$

$\delta_e = -5.0^{\circ}$

It is of interest to note that for this "new" airplane consisting of one engine the maneuverability is practically the same as for the twin engine one, for the time to execute the turn remains substantially the same. The initial aileron deflection for this single engine airplane is fairly low, but from table III (b) it is seen that the aileron position must reach in short time almost that required for the first airplane considered. There is only a slight change in maximum rudder deflection, and the accelerations remain unchanged.

Figure III (a) shows turn, elevator and rudder deflections plotted against time; figure III (b) shows accelerations produced in turn as a function of time.

Following the usual procedure, the effect of altitude may be seen for the single motored airplane at 15,000 ft. from the tables III (c) and III (d)

Using the correct stability derivatives and unit control accelerations:

$$C_1 = -0.1356$$

$$C_2 = 0.1791$$

$$A = 54.73$$

$$r = -0.00255\left(e^{-1.5t} - e^{-53.19t}\right) + 0.00357\left(e^{-4.5t} - e^{-53.19t}\right)$$

$$+ 3.97\, e^{-53.19t} \int_0^t e^{53.19t} \sin \phi \, dt$$

The aileron and rudder deflections necessary are:

$$\delta_r = 49.1 \sin \phi - 671\, r$$

$$\delta_a = 1.016 \frac{dp}{dt} + 8.49\, p - 0.975\, r$$

Table III (d) shows aileron and rudder deflections. Also accelerations.

$$\delta_e = -4.07^\circ$$

The effect of altitude in this case appears very interesting. The time to execute the maneuver is only slightly less at 15,000 ft., but the initial aileron deflection is increased several degrees. However the maximum deflection of the the aileron necessary to make the turn at this altitude is exactly that required at sea level, and therefore the aileron control necessary for these two cases is unchanged. From an examination of tables III (c) and (d) and Figure III (a), it is seen that the steady state position of the rudder for the airplane at 15,000 ft. is practically the same as for sea-level. The elevator angle is increased, but remains small in comparison with other control movements.

As a result, it seems in order to conclude that for sea-level maneuverability there is little difference between the twin engine airplane and the single engine one, but that the effect of altitude is greater on the former airplane than on the latter.

The results of tables III (c) and III (d) are plotted in figure III (a) and III (b).

CONCLUSION

In calculating the time to make an arbitrary turn with no change in propeller thrust or gain or loss in altitude and the control deflections necessary to execute this maneuver, it appears that the fundamental assumption of angle of bank as a function of time is of great importance. In order to keep the acceleration during the turn below 4 "g", a maximum bank of 75° obtained in approximately four seconds was used.

The effect of altitude (15,000 ft.) on maneuvering characteristics of a twin motored airplane is very apparent. There appears to be little difference in the time to make a specified turn, but the aileron deflection must be considerably increased for the high altitude case.

Investigation of the maneuvering characteristics at sea level of this same airplane considering it powered by a single engine, but otherwise unchanged, showed that for this modified condition the time to make a given turn and the rudder and aileron deflections necessary to execute the maneuver differed little from that required for the twin motored airplane. The initial aileron deflection required for the modified airplane is less than that necessary for the twin engine one, but quickly builds up to nearly the same maximum. Therefore, for this altitude, it appears that there is little difference in the maneuverability of the two airplanes.

The effect of altitude on this single engine airplane is very slight. The initial aileron deflection is increased, but the maximum required remains the same. The rudder deflections are only slightly affected. As before, the time to make a specified turn is unchanged by altitude. Since altitude has such a small effect on this modified single engine airplane, it certainly should be superior in maneuvering qualities at 15,000 ft. to the twin engine one.

Accelerations remain the same for all cases considered.

TABLE I (a) (SEA LEVEL CONDITIONS)

t	Ø rad.	p	$\frac{dp}{dt}$	sin Ø	Cot Ø	R	r	SecØ	r Sec Ø	β rad.	.β deg.
0	0	0	9.300	0			0	1	0	0	0
.125	.059	.803	4.095	.059	16.96	.742	.0030	1.002	.0030	.0002	.02
.250	.177	1.124	1.333	.176	5.57	.872	.0105	1.017	.0107	.0010	.06
.375	.326	1.192	- .071	.320	2.96	.920	.0210	1.056	.0222	.0031	.18
.500	.471	1.141	- .739	.454	1.96	-	.0316	1.121	.0354	.0067	.38
.750	.729	.902	-1.038	.666	1.11	-	.0488	1.341	.0655	.0193	1.11
1.000	.923	.657	-1.192	.797	.76	-	.0596	1.660	.0990	.0399	2.28
1.500	1.159	.322	- .474	.917	.44	-	.0701	2.498	.1750	.1084	6.21
2.000	1.273	.155	- .232	.956	.31	-	.0734	3.366	.2470	.2140	12.28
2.500	1.326	.073	- .109	.970	.25	-	.0747	4.133	.3085	.3529	20.23
3.000	1.354	.034	- .052	.977	.22	-	.0754	4.660	.3517	.5180	29.70
4.000	1.373	.008	- .012	.981	.20	-	.0757	5.094	.3858	.8868	50.90
∞	1.376	0	0	.981	.20	-	.0757	5.164	.3918		

TABLE I (b) (SEA LEVEL CONDITIONS)

t	δ_r	δ_a	$\frac{d\beta}{dt}$	a_n	a_g	"g"	
0	0	12.07	-	-	32.2	32.2	1.00
.125	.74	12.59	-	-	"	32.2	1.00
.250	1.50	11.91	-	-	"	32.2	1.00
.375	1.88	10.70	.0222	9.1	"	33.2	1.03
.500	1.85	9.38	.0354	14.5	"	35.4	1.10
.750	1.60	6.79	.0655	27.8	"	43.5	1.35
1.000	1.37	4.35	.0990	40.6	"	53.3	1.65
1.500	1.01	2.23	.1750	71.7	"	81.9	2.54
2.000	.80	1.03	.2470	101.2	"	102.9	3.21
2.500	.72	.44	.3085	126.4	"	131.8	4.09
3.000	.64	.16	.3517	144.0	"	143.7	4.46
4.000	.62	-.03	.3858	158.0	"	163.9	5.00
	.59	-.04	.3918	160.7	"	166.9	5.18

TABLE II (a) (SEA LEVEL CONDITIONS)

t	Ø rad.	p	$\frac{dp}{dt}$	Sin Ø	Cot Ø	R	r	Sec Ø	r Sec Ø	β rad.	β deg.
0	0	0	8.850	0	-	-	0	1	0	0	0
.125	.056	.764	3.895	.056	-	.742	.0029	1.001	.0029	.0002	.01
.250	.168	1.070	1.269	.167	-	.867	.0091	1.014	.0093	.0009	.05
.375	.310	1.134	-.068	.305	-	.920	.0200	1.050	.0210	.0028	.16
.500	.449	1.082	-.703	.434	2.070	-	.0302	1.110	.0335	.0062	.36
.750	.694	.859	-.988	.640	1.201	-	.0468	1.302	.0610	.0180	1.03
1.000	.878	.626	-1.134	.769	.830	-	.0574	1.568	.0900	.0369	2.12
1.500	1.102	.306	-.451	.892	.506	-	.0679	2.211	.1500	.0969	5.55
2.000	1.212	.148	-.221	.936	.375	-	.0719	2.850	.2048	.1857	10.73
2.500	1.260	.070	-.104	.952	.322	-	.0734	3.271	.2400	.2969	17.01
3.000	1.289	.032	-.049	.961	.290	-	.0741	3.599	.2666	.4236	24.23
4.000	1.308	.008	-.011	.966	.269	-	.0745	3.861	.2879	.7009	40.08
∞	1.313	0	0	.967	.210	-	.0746	3.922	.2929	-	-

TABLE II (b) (SEA LEVEL CONDITIONS)

t	δ_r	δ_a	$\frac{d\beta}{dt}$	a_n	a_g	a	"g"
0	0	11.49	-	-	32.2	32.2	1.00
.125	.68	11.99	-	-	"	32.2	1.00
.250	1.82	11.35	-	-	"	32.2	1.00
.375	1.80	10.18	.0210	8.6	"	33.5	1.04
.500	1.74	8.91	.0335	13.7	"	36.0	1.12
.750	1.59	6.46	.0610	25.0	"	41.5	1.29
1.000	1.35	4.15	.0900	36.9	"	48.9	1.52
1.500	.99	2.12	.1500	61.5	"	66.6	2.07
2.000	.74	.98	.2048	84.0	"	86.5	2.69
2.500	.64	.42	.2400	98.4	"	103.8	3.22
3.000	.62	.15	.2666	109.2	"	114.2	3.55
4.000	.62	-.02	.2879	118.1	"	122.2	3.80
	.62	-.03	.2929	120.0	"	124.2	3.85

TABLE II (c) 15,000 Ft.

t	ø rad.	p	$\frac{dp}{dt}$	Sin ø	Cot ø	R	r	sec ø	r sec ø	β rad.	β deg.
0	0	0	8.850	0	-	-	0	1	0	0	0
.125	.056	.764	3.895	.056	-	.756	.0030	1.001	.0030	.0002	.01
.250	.168	1.070	1.269	.167	-	.874	.0098	1.014	.0100	.0010	.06
.375	.310	1.134	-.068	.305	-	.925	.0195	1.050	.0202	.0029	.17
.500	.449	1.086	-.703	.434	2.070	-	.0289	1.110	.0321	.0062	.35
.750	.694	.859	-.988	.640	1.201	-	.0446	1.302	.0581	.0174	1.00
1.000	.878	.626	-1.134	.769	.830	-	.0544	1.568	.0854	.0354	2.03
1.500	1.102	.306	-.451	.892	.560	-	.0642	2.211	.1420	.0923	5.29
2.000	1.212	.148	-.221	.936	.375	-	.0677	2,850	.1930	.1762	10.09
2.500	1.260	.070	-.104	.952	.322	-	.0689	3.271	.2252	.2805	16.06
3.000	1.289	.032	-.049	.961	.290	-	.0696	3.599	.2504	.3995	22.85
4.000	1.308	.008	-.011	.966	.269	-	.0700	3.861	.2701	.6595	37.74
∞	1.313	0	0	.967	.210	-	.0702	3.922	.2750		

TABLE II (d) 15,000 Ft.

t	δ_r	δ_a	$\frac{d\beta}{dt}$	a_n	a_g	a	"g"
0	0	16.12	-		32.2	32.2	1.00
.125	.74	13.55	-		"	32.2	1.00
.250	1.64	11.33	-		"	32.2	1.00
.375	1.92	9.43	.0202	9.8	"	33.6	1.05
.500	1.94	7.84	.0321	17.3	"	36.5	1.14
.750	1.56	5.41	.0581	28.1	"	42.8	1.33
1.000	1.33	3.16	.0854	39.5	"	51.8	1.58
1.500	.85	1.70	.1420	60.0	"	68.1	2.11
2.000	.58	.78	.1930	77.5	"	84.0	2.61
2.500	.65	.33	.2252	97.1	"	102.3	3.18
3.000	.55	.11	.2504	108.6	"	113.4	3.52
4.000	.45	-.02	.2701	118.9	"	123.0	3.82
	.42	-.02	.2750	121.0	"	126.0	3.92

TABLE III (a) (SEA LEVEL CONDITIONS)

t	Ø rad.	p	$\frac{dp}{dt}$	sin Ø	Cot Ø	R	r	sec Ø	r sec Ø	β rad.	β deg.
0	0	0	8.850	0	-	-	0	1	0	0	0
.125	.056	.764	3.895	.056	-	.768	.0030	1.001	.0030	.0002	.01
.250	.168	1.070	1.269	.167	-	.880	.0104	1.014	.0105	.0010	.06
.375	.310	1.134	-.068	.305	-	.929	.0206	1.050	.0216	.0031	.18
.500	.449	1.086	-.703	.434	2.070	-	.0308	1.110	.0342	.0065	.38
.750	.694	.859	-.988	.640	1.201	-	.0473	1.302	.0616	.0185	1.06
1.000	.878	.626	-1.134	.769	.830	-	.0581	1.568	.0911	.0376	2.15
1.500	1.102	.306	-.451	.892	.506	-	.0683	2.211	.1510	.0981	5.62
2.000	1.212	.148	-.221	.936	.375	-	.0723	2.850	.2060	.1875	10.73
2.500	1.260	.070	-.104	.952	.322	-	.0735	3.271	.2404	.2992	17.12
3.000	1.289	.032	-.049	.961	.290	-	.0743	3.599	.2674	.4261	24.42
4.000	1.308	.008	-.011	.966	.269	-	.0747	3.861	.2885	.7040	40.30
∞	1.313	0	0	.967	.210	-	.0748	3.922	.2934		

TABLE III (b) (SEA LEVEL CONDITIONS)

t	δ_r	δ_a	$\frac{d\beta}{dt}$	a_n	a_g	a	"g"
0	0	6.51	-	-	32.2	32.2	1.00
.125	.63	9.81	-	-	"	32.2	1.00
.250	1.23	10.64	-	-	"	32.2	1.00
.375	1.52	10.24	.0216	8.3	"	33.3	1.04
.500	1.47	9.31	.0342	12.0	"	24.3	1.07
.750	1.36	7.02	.0616	26.2	"	41.6	1.29
1.000	1.05	4.79	.0911	35.8	"	48.1	1.50
1.500	.84	2.38	.1510	62.4	"	70.2	2.18
2.000	.65	1.11	.2060	83.5	"	89.3	2.77
2.500	.62	.48	.2404	98.0	"	103.2	3.21
3.000	.59	.18	.2674	107.9	"	112.6	3.49
4.000	.58	-.01	.2885	118.2	"	122.8	3.82
	.58	-.01	.2934	120.0	"	124.0	3.85

TABLE III (c) 15,000 Ft.

t	∅ rad.	p	$\frac{d\phi}{dt}$	sin ∅	Cot ∅	R	r	sec ∅	r sec ∅	β rad.	β deg.
0	0	0	8.850	0	-	-	0	1	0	0	0
.125	.056	.764	3.895	.056	-	.778	.0031	1.001	.0031	.0002	.01
.250	.168	1.070	1.269	.167	-	.887	.0102	1.014	.0104	.0011	.06
.375	.310	1.134	-.068	.305	-	.933	.0202	1.050	.0212	.0031	.18
.500	.449	1.086	-.703	.434	2.070		.0294	1.110	.0356	.0065	.37
.750	.694	.859	-.988	.640	1.202		.0448	1.302	.0585	.0179	1.02
1.000	.878	.626	-1.134	.769	.830		.0547	1.568	.0860	.0360	2.06
1.500	1.102	.306	-.451	.892	.506		.0644	2.211	.1423	.0931	5.33
2.000	1.212	.148	-.221	.936	.375		.0678	2.850	.1931	.1769	10.11
2.500	1.260	.070	-.104	.952	.322		.0691	3.271	.2261	.2817	16.21
3.000	1.289	.032	-.049	.961	.290		.0697	3.599	.2444	.3995	22.86
4.000	1.308	.008	-.011	.966	.269		.0700	3.861	.2702	.6565	37.64
∞	1.313	0	0	.967	.210		.0710	3.922	.2705		

TABLE III (d) 15,000 FT.

t	δ_r	δ_a	$\frac{d\beta}{dt}$	a_n	a_g	a	"g"
0	0	8.99	-	-	32.2	32.2	1.00
.125	.67	10.45	-	-	"	32.2	1.00
.250	1.36	10.36	-	-	"	32.2	1.00
.375	1.45	9.54	.0212	8.8	"	33.4	1.04
.500	1.61	8.47	.0356	12.8	"	34.7	1.08
.750	1.42	6.25	.0585	28.1	"	42.8	1.33
1.000	1.10	4.11	.0860	38.4	"	50.1	1.56
1.500	.75	2.08	.1423	58.4	"	66.6	2.07
2.000	.58	.96	.1931	84.3	"	90.3	2.80
2.500	.50	-.11	.2261	97.7	"	102.8	3.19
3.000	.48	-.09	.2444	106.7	"	111.7	3.47
4.000	.48	-.07	.2702	119.0	"	123.2	3.83
	.48	-.07	.2705	119.0	"	123.2	3.83

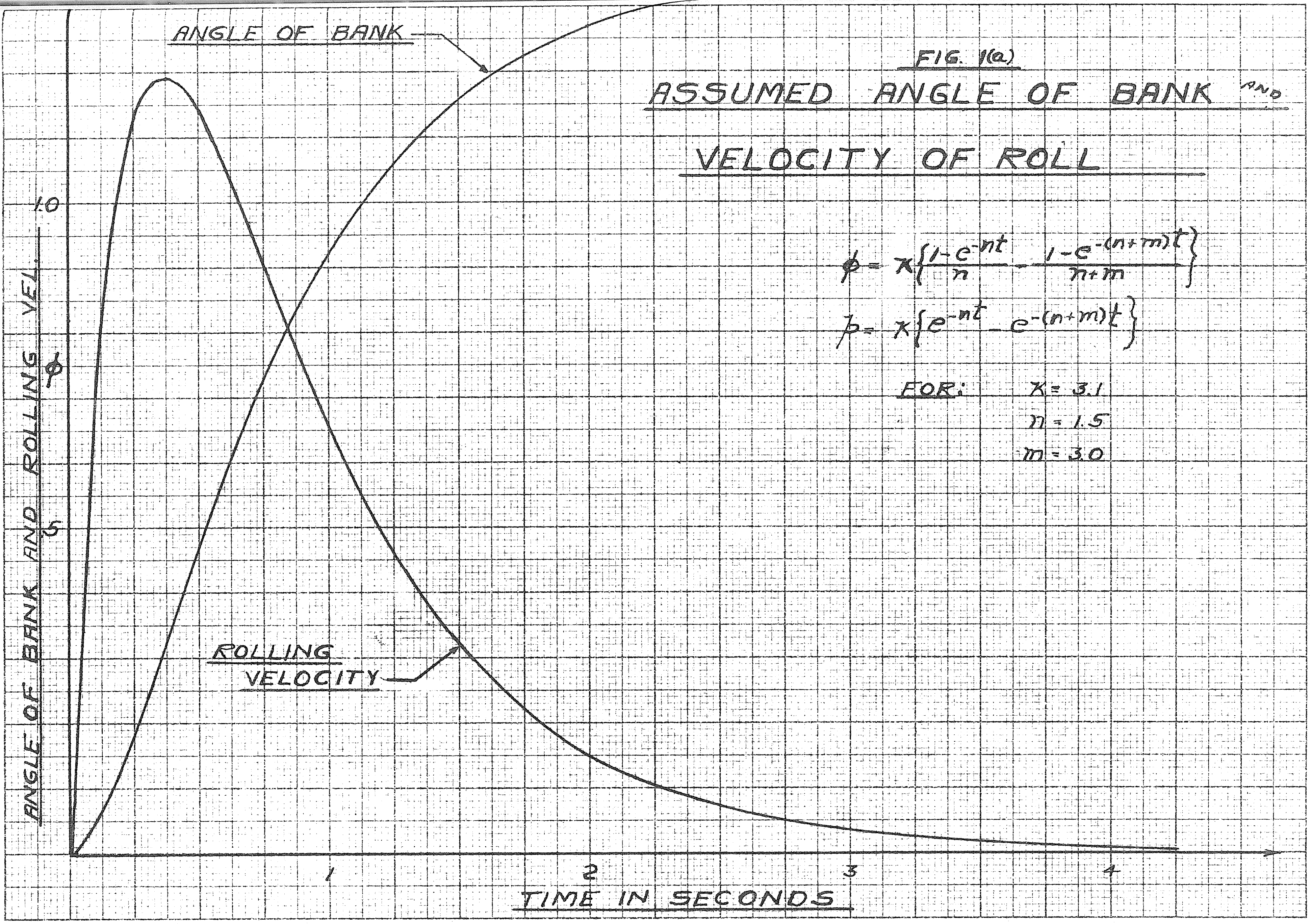


FIG. 1(a)

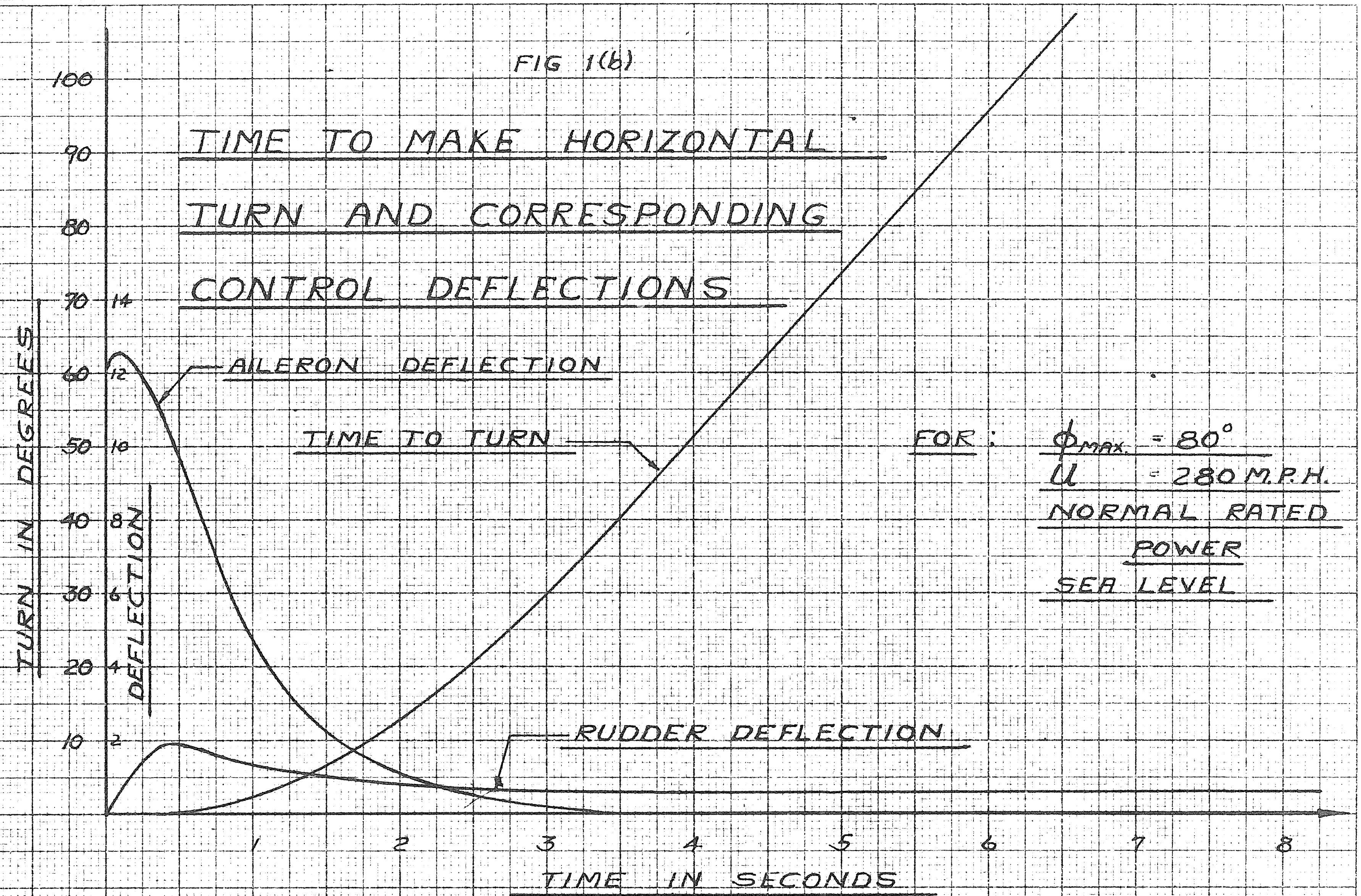
ASSUMED ANGLE OF BANK AND VELOCITY OF ROLL

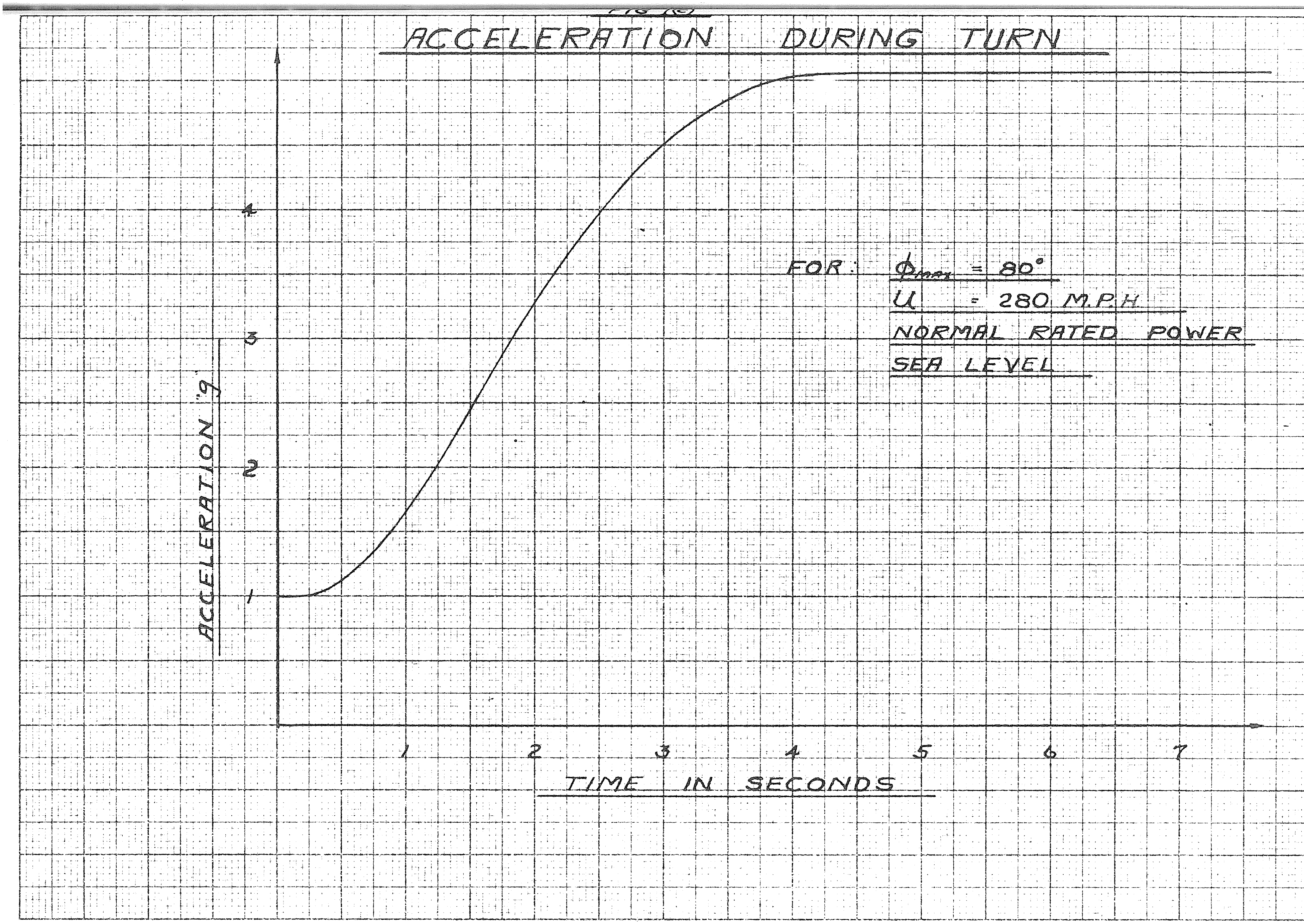
$$\phi = K\left\{\frac{1-e^{-nt}}{n} - \frac{1-e^{-(n+m)t}}{n+m}\right\}$$

$$p = K\left\{e^{-nt} - e^{-(n+m)t}\right\}$$

FOR: $K = 3.1$
$n = 1.5$
$m = 3.0$

FIG 1(b)


TIME TO MAKE HORIZONTAL
TURN AND CORRESPONDING
CONTROL DEFLECTIONS
AILERON DEFLECTION
TIME TO TURN
RUDDER DEFLECTION
FOR :
ΦMAX. = 80°
U = 280 M.P.H.
NORMAL RATED
POWER
SEA LEVEL
TURN IN DEGREES
DEFLECTION
100
90
80
70
60
50
40
30
20
10
14
12
10
8
6
4
2
1
2
3
4
5
6
7
8
TIME IN SECONDS


ACCELERATION DURING TURN
FOR: ϕ_{max} = 80°
U = 280 M.P.H.
NORMAL RATED POWER
SEA LEVEL
ACCELERATION "g"
1
2
3
4
1
2
3
4
5
6
7
TIME IN SECONDS

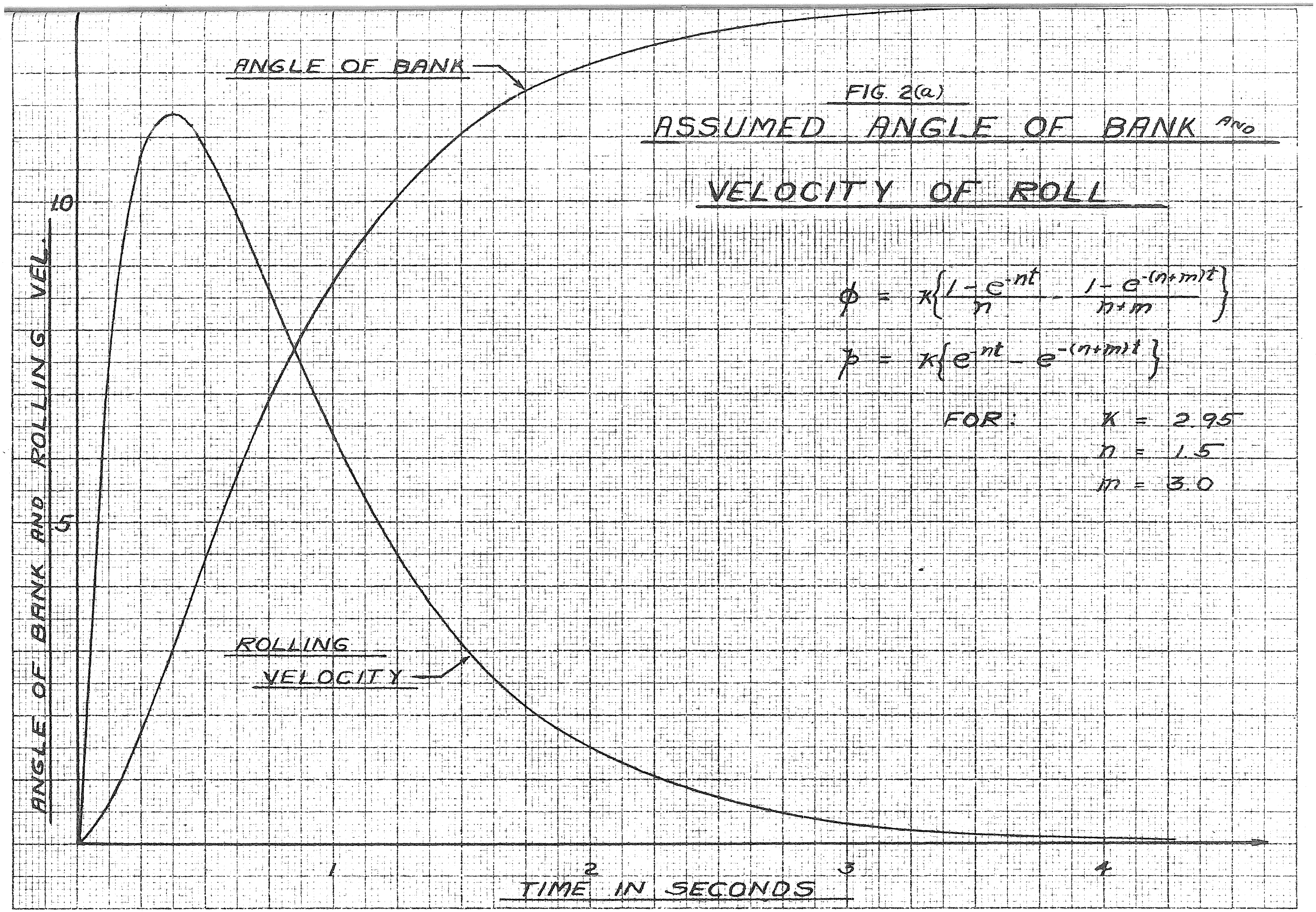


FIG. 2(a)
ASSUMED ANGLE OF BANK AND
VELOCITY OF ROLL
$\phi = K\left\{\frac{1-e^{-nt}}{n} - \frac{1-e^{-(n+m)t}}{n+m}\right\}$
$p = K\left\{e^{nt} - e^{-(n+m)t}\right\}$
FOR: K = 2.95
n = 1.5
m = 3.0
ANGLE OF BANK
ROLLING VELOCITY
ANGLE OF BANK AND ROLLING VEL.
10
5
1
2
3
4
TIME IN SECONDS

FIG. 2(b)

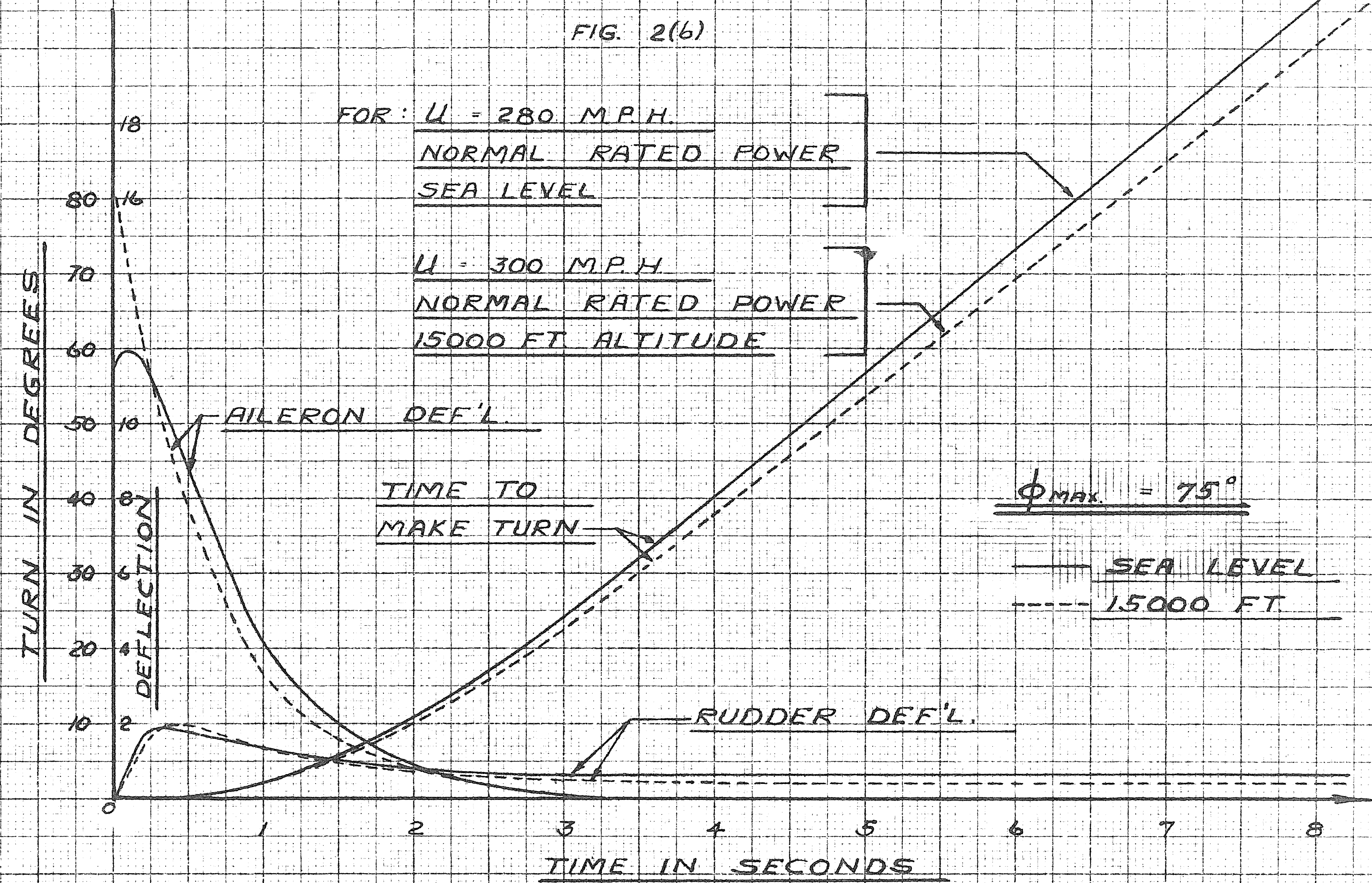

FOR: U = 280 M.P.H.
NORMAL RATED POWER
SEA LEVEL
U = 300 M.P.H.
NORMAL RATED POWER
15000 FT ALTITUDE
AILERON DEF'L.
TIME TO
MAKE TURN
$\phi_{MAX.}$ = 75°
SEA LEVEL
15000 FT.
RUDDER DEF'L.
TURN IN DEGREES
DEFLECTION
TIME IN SECONDS
0
1
2
3
4
5
6
7
8
10
20
30
40
50
60
70
80
2
4
6
8
10
16
18

FIG. 2(c)

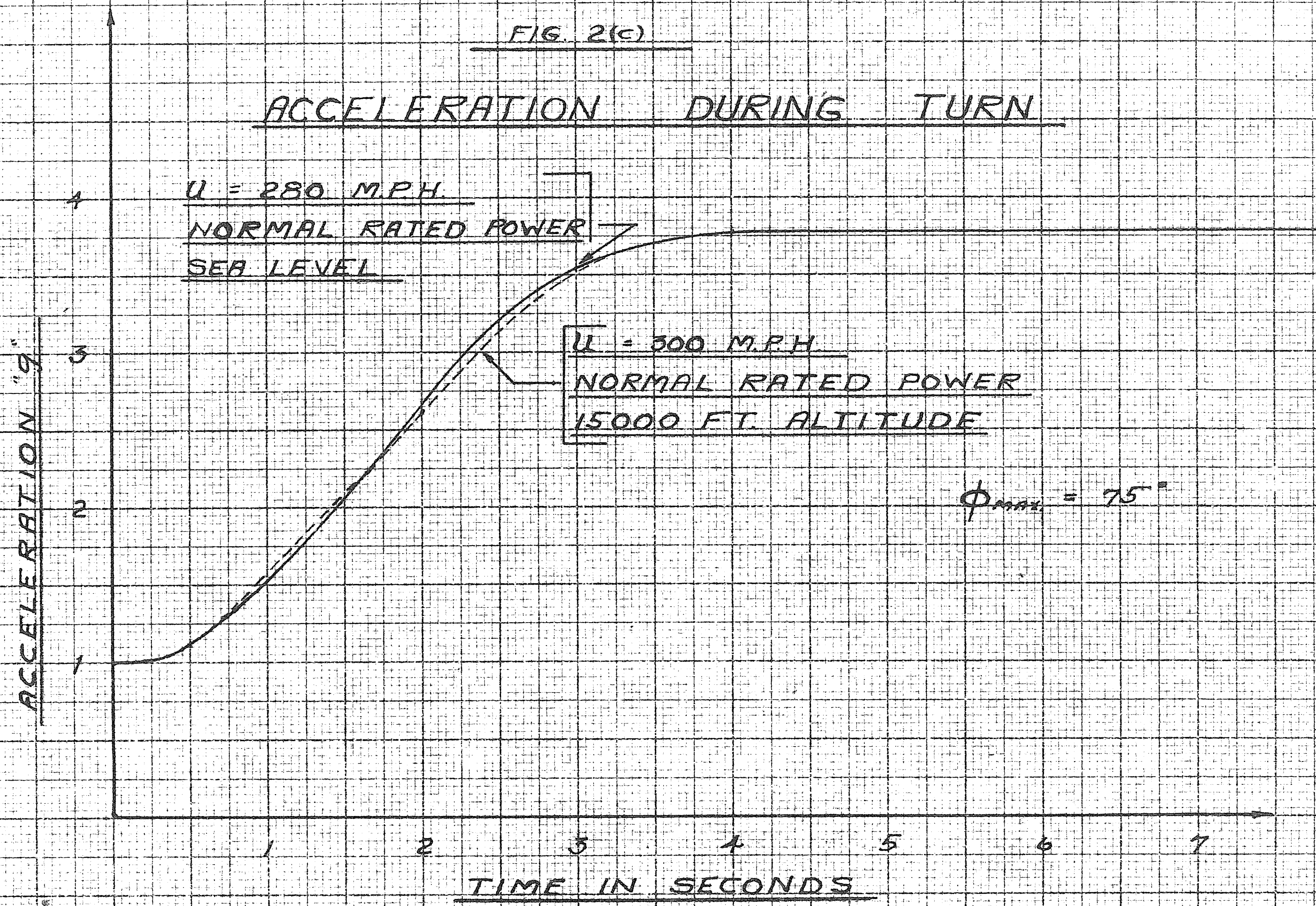

ACCELERATION DURING TURN
U = 280 M.P.H.
NORMAL RATED POWER
SEA LEVEL
U = 300 M.P.H.
NORMAL RATED POWER
15000 FT. ALTITUDE
ϕmax = 75°
ACCELERATION "g"
1
2
3
4
TIME IN SECONDS
1
2
3
4
5
6
7

FIG 3(a)

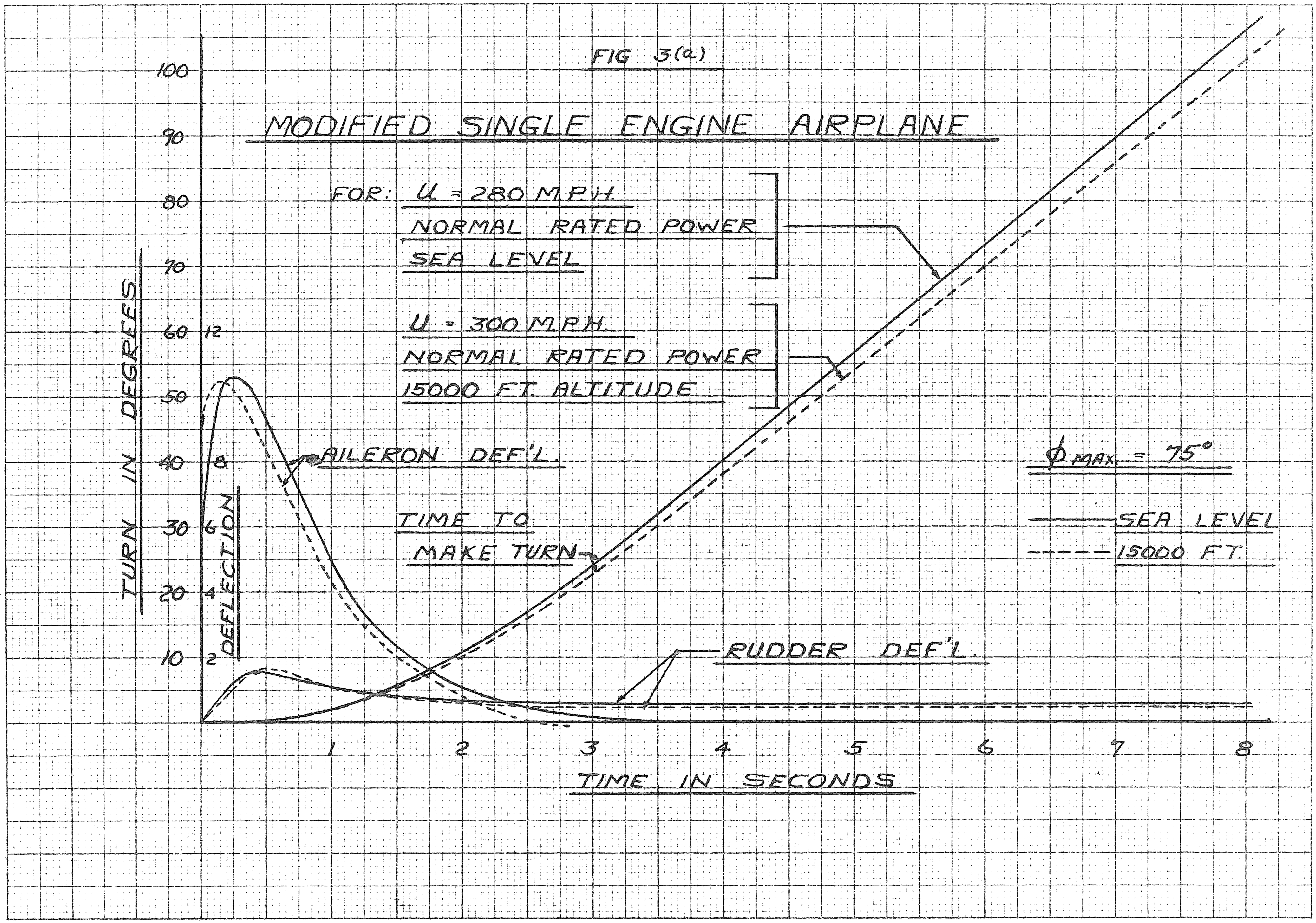
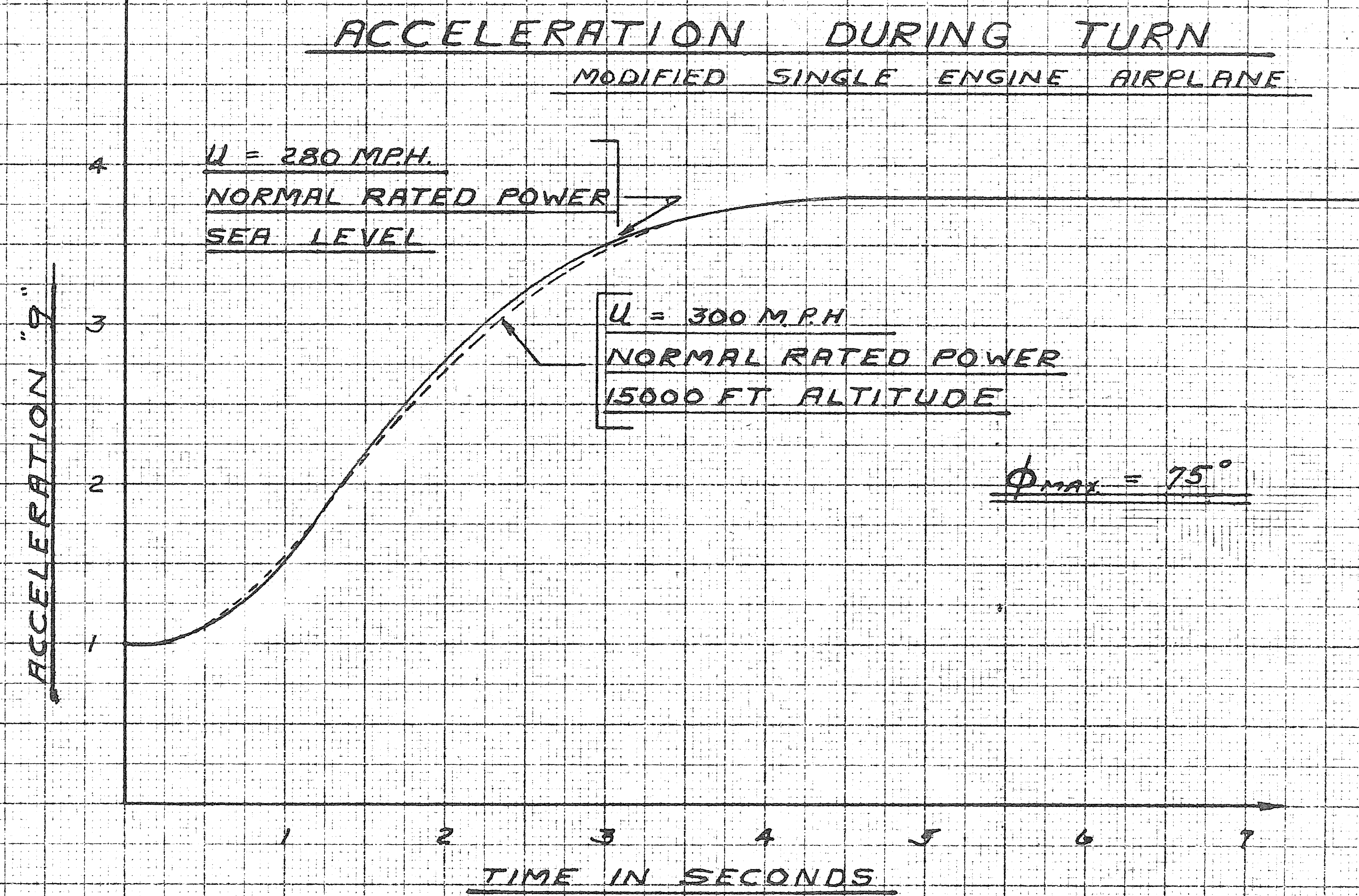

MODIFIED SINGLE ENGINE AIRPLANE
FOR: U = 280 M.P.H.
NORMAL RATED POWER
SEA LEVEL
U = 300 M.P.H.
NORMAL RATED POWER
15000 FT. ALTITUDE
AILERON DEF'L
TIME TO
MAKE TURN
RUDDER DEF'L.
ϕ MAX. = 75°
SEA LEVEL
15000 FT.
TURN IN DEGREES
DEFLECTION
100
90
80
70
60
50
40
30
20
10
12
8
6
4
2
1
2
3
4
5
6
7
8
TIME IN SECONDS

FIG 3(b)



ACCELERATION DURING TURN
MODIFIED SINGLE ENGINE AIRPLANE
U = 280 MPH.
NORMAL RATED POWER
SEA LEVEL
U = 300 M.P.H
NORMAL RATED POWER
15000 FT ALTITUDE
$\phi_{max} = 75°$
ACCELERATION "g"
1
2
3
4
TIME IN SECONDS
1
2
3
4
5
6
7

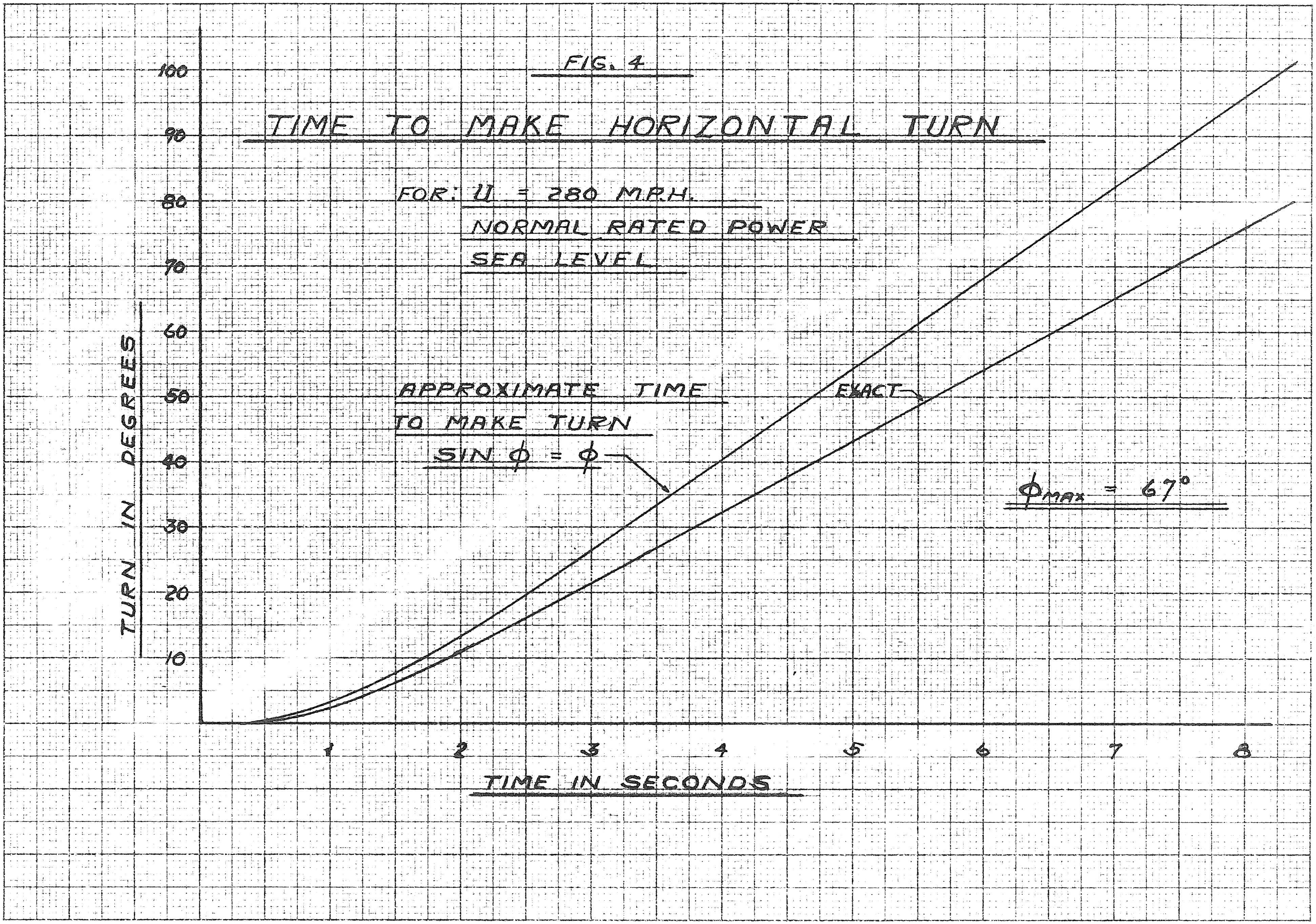
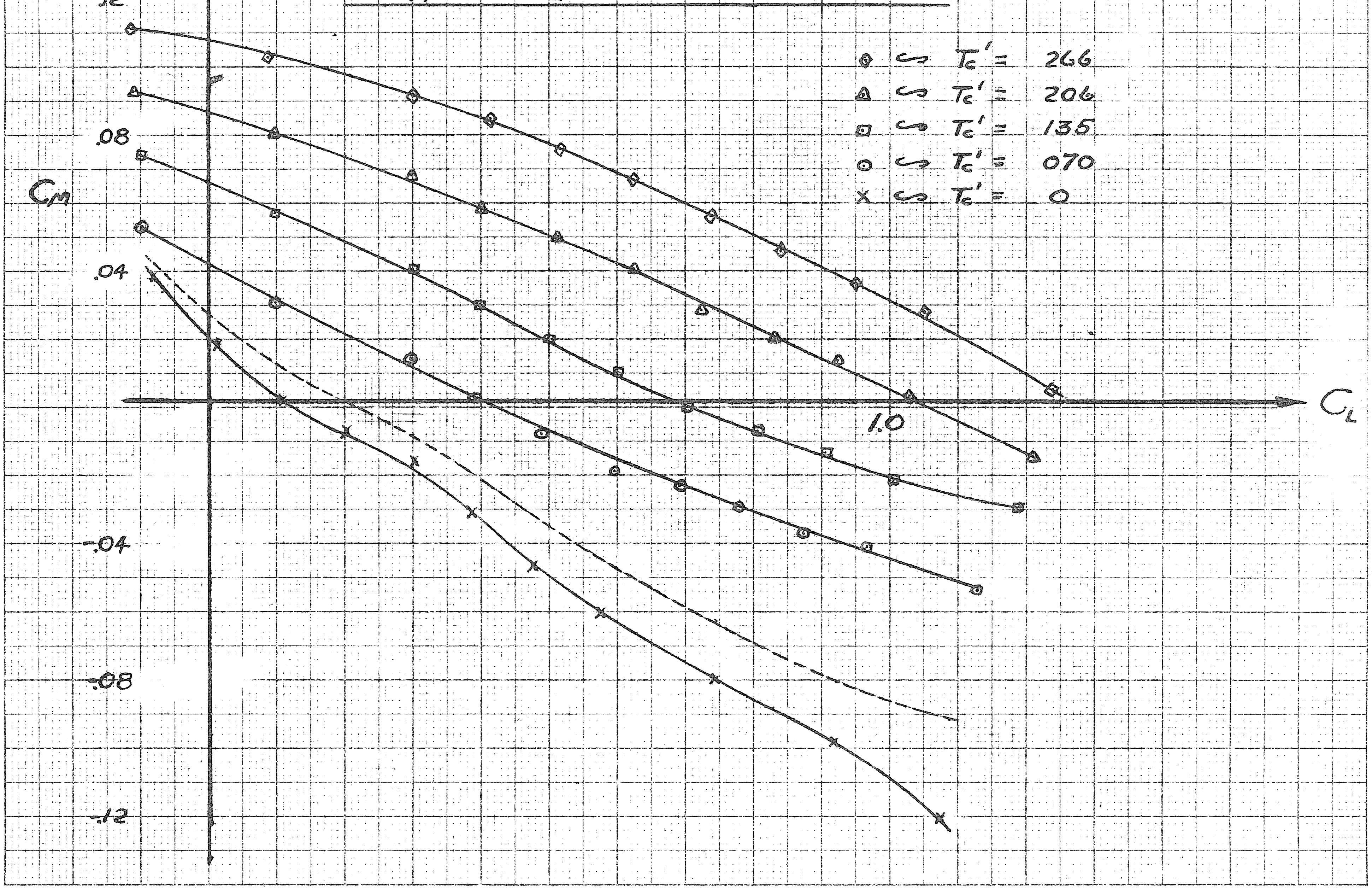


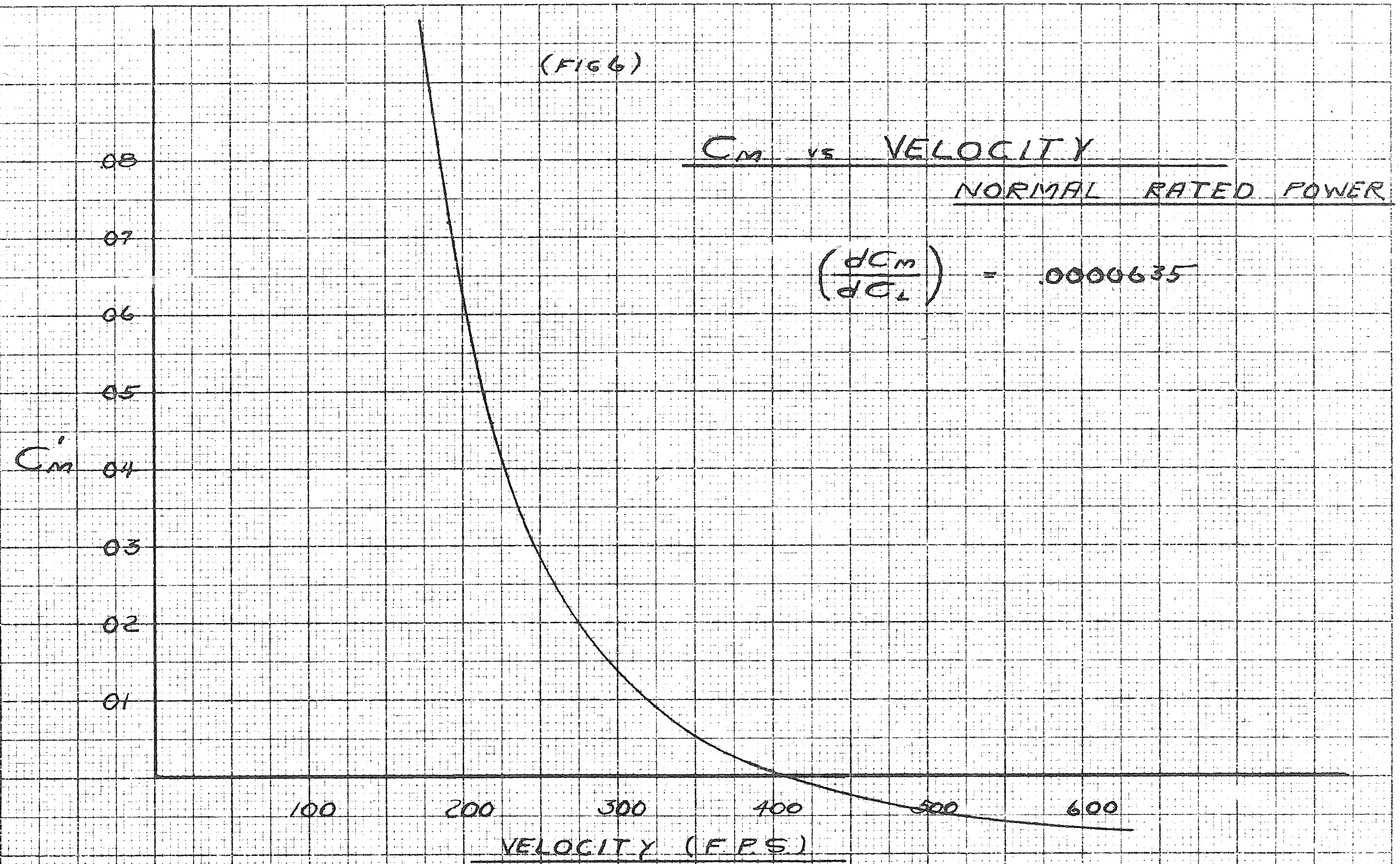
FIG. 4
TIME TO MAKE HORIZONTAL TURN
FOR: U = 280 M.P.H.
NORMAL RATED POWER
SEA LEVEL
APPROXIMATE TIME
TO MAKE TURN
SIN φ = φ
EXACT
φMAX = 67°
TURN IN DEGREES
100
90
80
70
60
50
40
30
20
10
1
2
3
4
5
6
7
8
TIME IN SECONDS

FIG. 5

C_M VS. C_L FOR VARIOUS T_c'



(FIG 6)


Cm vs VELOCITY
NORMAL RATED POWER
(dCm / dCL) = .0000635
C'm
.08
.07
.06
.05
.04
.03
.02
.01
100
200
300
400
500
600
VELOCITY (F.P.S.)