ON THE MECHANICS OF SEDIMENTATION
IN ARTIFICIAL BASINS

Thesis by

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ABSTRACT

A theoretical and experimental study was made of the quiescent settling of the particles of a suspension in order to determine what governs the removal of particles when the suspension passes through a sedimentation basin. Settling column experiments were performed in tubes ranging from one foot to four feet in depth. Glass spheres in water, alum and clay in water, and Pasadena sewage were used as suspensions. While the suspensions settled in the columns, samples were withdrawn at time intervals at several depths. These were analyzed for concentration of suspended particles.

It was found that for free, quiescent settling of discrete particles, the removal of particles can be calculated in advance if the frequency distribution of particle settling velocities and the particle concentration are known as functions of position in the suspension at the beginning of settling.

For a suspension of flocculent particles it was almost impossible to predict removal without measuring the settling properties of each individual suspension. There is no single universal property of a settling tank with which removal is correlated. It was found that while for some flocculent suspensions removal may be strongly dependent upon detention time, no strong dependence on overflow rate was indicated.

Since so many suspensions encountered in settling tanks have flocculent particles, the kinetics of flocculation was studied qualitatively by theory and experiment.

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CHAPTER 1

INTRODUCTION

1-1 Sedimentation and Settling Tanks

The separation of solid particles from a liquid that carries them is a problem of frequent occurrence in engineering practice. Often the ability of the liquid to carry particles in suspension can be decreased by stilling the motion of the liquid. The term "sedimentation" usually refers to an operation by which the carrying power of the liquid is reduced in magnitude until the particles settle out under the force of gravity.

The equipment in which sedimentation is made to take place is often called a sedimentation basin or a settling tank. One of the primary functions of a settling tank is to permit the solid particles to settle to the bottom of the tank. The desired end result is either solid-free liquid or solids with a minimum of liquid. In either case, the amount of solid settled in the tank is an important quantity.

The amount of solid particles that settle in a tank is often called the total removal of suspended solids. The removal ratio, on the other hand, is the ratio of the total removal to the total amount of particles entering the tank with the liquid. The simplest study of the performance of a settling tank is to determine the removal ratio. For a given tank already in existence, the study of the removal ratio for various operating conditions may be very helpful. One may be able to find an operating procedure which produces maximum removal. For the study of sedimentation in general, however, this simple approach is not adequate.

The next level of study is to determine what should settle out in a tank. This problem has led to a great deal of theoretical and experimental work, which can be called the study of the fundamental aspects of sedimentation and settling tanks. With a few notable exceptions, the bulk of the experimental work has consisted of the study of a particular sedimentation operation.

1-2 Settling of Particles in a Suspension

A thorough comprehension of sedimentation is dependent on an adequate understanding of the settling of particles in a suspension. Of course the problem is not confined to a suspension of solid particles in a liquid with gravity as the activating force. One may encounter solid, liquid or gaseous particles in a liquid; and solid or liquid particles in a gas. As for the field of force to move the particles, it may be gravitational, centrifugal, electrostatic or magnetic. Whenever possible, concepts should be developed in a general enough form to include any system. The successful concepts have usually had this generality.

One such concept is the so-called "ideal" removal. It has been applied in principle to the operations of sedimentation (1,2), flotation (2), and cleaning gases (3). Since the main points of ideal removal will be discussed in chapter 2, only the most significant aspect need be mentioned here. The ideal removal depends on the characteristics of the suspension as well as the tank. This makes it a basic quantity.

Another important step in the basic study of settling was the theoretical and experimental analyses of settling in a turbulent fluid. Hurst (4) and Rouse (5) solved a differential equation for the equilibrium between particle settling and turbulent diffusion. Dobbins (6) considered the case where these two factors are not in equilibrium. Hurst, Rouse and Dobbins verified their results experimentally in similar apparatus.

In attacking the turbulent settling problem, Rouse and Dobbins considered the settling phenomenon at a general point in the suspension. Kynch (7) has done the same thing for settling where the concentration of particles is great and the fluid is essentially motionless. The study of settling at a point appears to be the avenue to a better understanding of the settling of particles in suspension.

The experimental study of this local settling behavior requires that data be taken at many points in the suspension and at many times during settling. In addition to Dobbins and Rouse, mentioned above, Comings (8,9) and Fitch (10) have actually measured concentration at various points in a suspension during settling. The great bulk of experimental work in the past has not involved such measurements. Most of the tests have been made on concentrated suspensions, and the subsidence rate has been the primary measurement. The subsidence rate refers to the motion of a sharp interface which often forms between the clear liquid above and the concentrated suspension below.

The scarcity of experimental data on settling at a point is a definite hinderance to progress in the study of sedimentation. In reading the literature on the subject one will observe that most of the data are of limited use. Except for the equipment or apparatus with which they were obtained, the data are of qualitative value only. A person wishing to study some aspect of sedimentation will seldom be able to use published data except in a qualitative manner.

How can experimental data be made more general? This can be done by relating the settling phenomenon at a point to conditions at the point. Experiments should be conducted with an eye to reducing the data until they are independent of the experiment. Then each investigation will add to the body of basic data. The need for such experiments was the incentive for this investigation.

1-3 Purpose of this Investigation

(a) One purpose of this investigation is to study the settling process at a general point in a suspension and to relate this local process to removal during settling. The approach has been both experimental and theoretical, with interrelation of the theory and experimental results.

(b) A second purpose is to study the effect of flocculation on settling.

1-4 Scope and Limitations

For the experimental part of the investigation, suspensions of solid particles in water were allowed to stand in vertical tubes. The particles settled under the influence of gravity. Both discrete and flocculent particles were used. Most of the tests were conducted with the water essentially motionless. While some work was done with the water in turbulent motion, it did not progress beyond the preliminary stage.

Two types of measurements were made. In the first, the concentration of particles at points in the suspension was determined as a function of space and time. The second measurement was the amount of material settled at the bottom of the suspension as a function of time.

To supplement the experimental investigations, theoretical analyses were employed for the following reasons:

(a) To give background for the interpretation of experimental results.

(b) To assist in the reduction of the measurements to data concerning settling at a point.

(c) To indicate what measurements should be made to improve the experiments and thus point the way for future work.

The symbols used are defined where they first occur and are listed in appendix 2.

CHAPTER 2

QUIESCENT, UNHINDERED SETTLING OF DISCRETE PARTICLES

The purpose of this chapter is two-fold. The first aim is to discuss certain idealizations of settling tanks while the second is to relate these tanks to the quiescent settling column.

The first section contains a review of the definitions related to a suspension of settling particles. The next three sections begin with the concept of an "ideal" tank and proceed to the case where the particles are not uniformly distributed at the inlet end. The aim of these four sections is to introduce the concentration at a point and the concentration profile as means of calculating removal.

Fitch (11) has presented an analysis which permits the "ideal" tank to be extended to all settling tanks with two-dimensional potential flow. The extension is given in section 2-5. The analysis itself has been modified and presented in appendix 1.

Finally, the settling column is introduced in section 2-6 which serves to relate the column to settling tanks. The details of column studies are reserved for later chapters.

2-1 Suspensions of Settling Particles

The term suspension refers to a mixture of fluid and particles. The fluid forms a continuous phase around the particles. An essential feature of a suspension is that the particles can be separated from the fluid by mechanical means. One of these means involves the settling of the particles through the fluid.

If the fluid through which the particle moves is motionless, the settling is said to be quiescent. Of course the moving particle will displace fluid and cause it to move; otherwise, however, the fluid is motionless.

The term "quiescent settling" is also used in reference to settling tanks where the fluid is moving. It is assumed that the motion of the particle through the body of the fluid is not affected by the motion of the fluid (i.e., no turbulence in the fluid).

A particle settling quiescently in a fluid of infinite extent reaches a constant velocity at which the frictional resistance of the fluid equals the force causing settling. Such settling is called "free" settling and the constant velocity is called the free settling velocity of the particle. When the particle settles near a boundary of the fluid, or near the other settling particles, its settling velocity is changed. Such settling is called hindered settling.

Although the determination of free and hindered settling velocities has been the subject of extensive research (12,13, 14,15,16), it is not an important part of this research.

Instead, a particle's settling velocity is assumed to be known or measurable and is taken as a starting point. The particle will be classified by this settling velocity, not by its size, shape or density. A suspension, on the other hand, may contain particles of many different settling velocities. To classify the suspension it is necessary to know the concentration of particles of each settling velocity, or a distribution of particle settling velocities.

The function F(w) will be called the cumulative frequency distribution of particle settling velocities. It is defined as the concentration of particles in suspension which have a settling velocity equal to or less than w. This function can be expressed in two ways; mass per unit volume, and per cent of total concentration of particles. In theoretical analysis it is usually better to consider it as mass per unit volume. On the other hand, it is often desirable to reduce the measured F(w) to the per cent basis.

Camp (17,18) introduced the use of F(w) in the analysis of sedimentation in ideal tanks. Ingersoll, McKee and Brooks (2) found it more convenient to use the derivative of F(w). This derivative, designated f(w) is related to F(w) by the equation

$$f(w) = \frac{dF(w)}{dw} \qquad (2\text{-}1)$$

In figure 2-1 the curves of F(w) and f(w) for a suspension of glass spheres are presented. The functions are

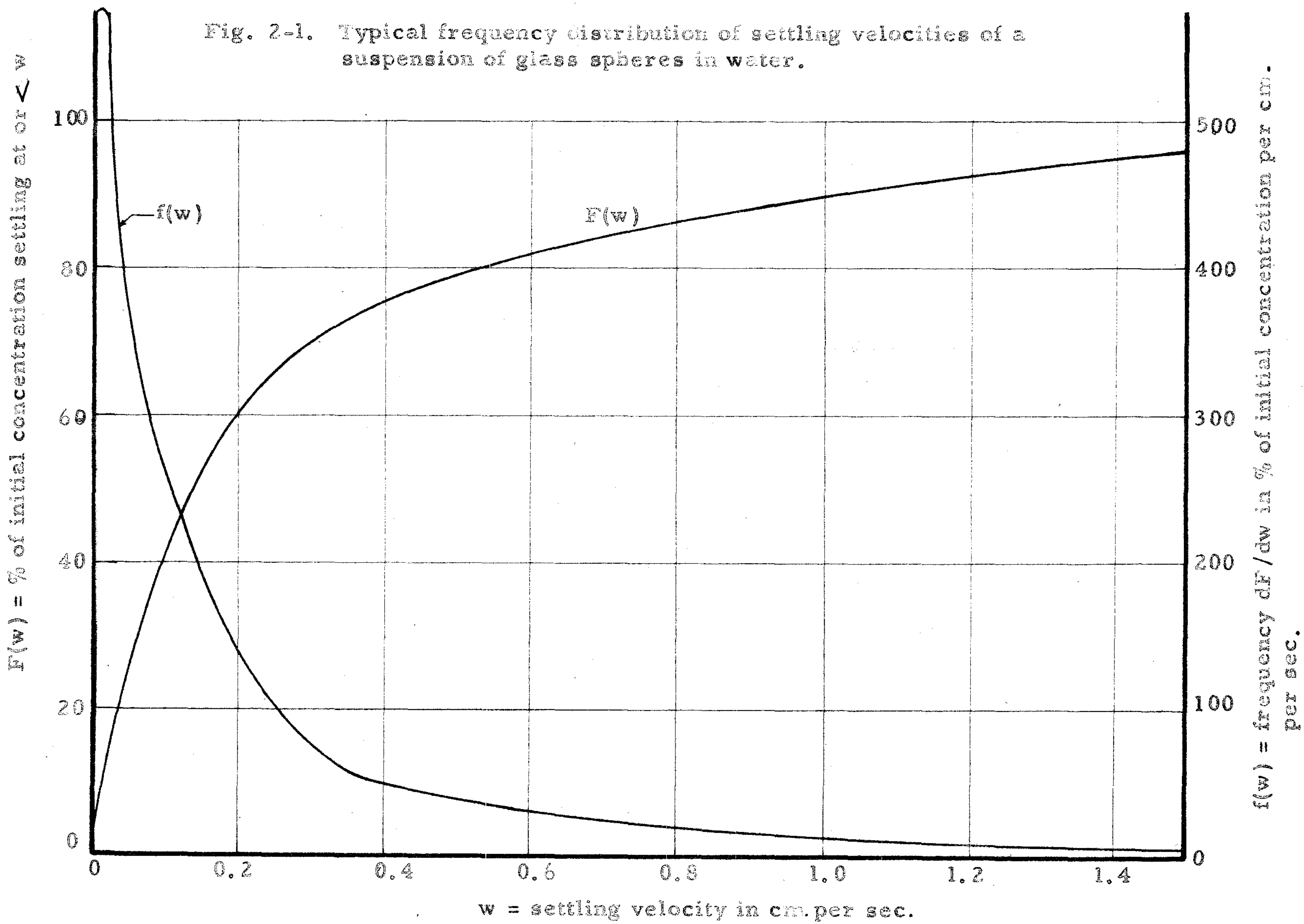

Fig. 2-1. Typical frequency distribution of settling velocities of a suspension of glass spheres in water.

plotted with w as the independent variable. In figure 2-2 the functions are plotted with the logarithm of w as the independent variable.

The function f(w) is the frequency distribution of particle settling velocities. Usually, it is simply called the settling velocity distribution. In figure 2-2, curve 2 shows f(w) as obtained by equation 2-1, but plotted against log w.* Curve 3 shows f(w) as obtained by differentiating F(w) plotted in curve 1. The relationship between curve 1 and curve 3 is

$$f_\ell(w) = \frac{dF(w)}{d(\log w)} \qquad (2\text{-}2)$$

In the theory of settling it is the f(w) defined by equation 2-1 that has the basic significance. Often, however, $f_\ell(w)$ is the function obtained experimentally. The two are related as follows:

$$f(w) = \frac{dF(w)}{dw} = \frac{dF(w)}{d(\log w)} \quad \frac{d(\log w)}{dw}$$

$$= \frac{0.434}{w} f_\ell(w) \qquad (2\text{-}3)$$

The quantity f(w)dw is the concentration of particles with settling velocity between $w + \frac{dw}{2}$ and $w - \frac{dw}{2}$. It is

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* log refers to common logarithm.

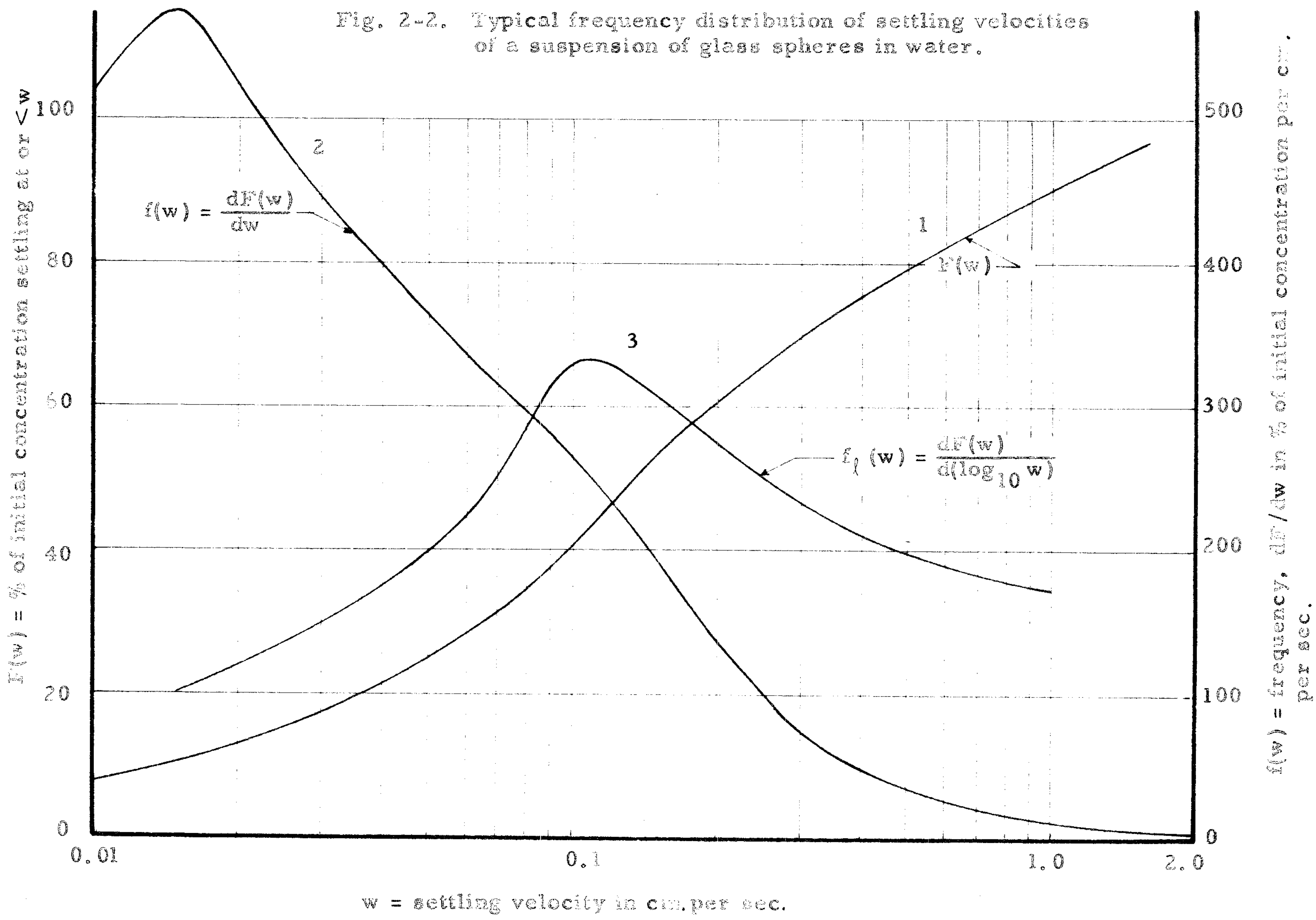


Fig. 2-2. Typical frequency distribution of settling velocities of a suspension of glass spheres in water.

a fixed property of a suspension only if the particle settling velocities do not change. Strictly speaking, therefore, it is not reasonable to speak of the f(w) of a suspension unless the settling is free or the concentration remains constant.

The function f(w) also has no significance if the particles can change character or become attached to other particles. A particle which does not change its shape, size or weight as it moves in a suspension is called a discrete particle. Hence, it may be said that a suspension has a unique settling distribution, f(w), only if the particles are discrete and the settling is unhindered.

The total concentration of all particles in suspension is related to the settling velocity distribution. If ϕ is defined as the total concentration, it is given by

$$\phi = \int f(w)dw = \int dF(w) \qquad (2\text{-}4)$$

The limits of integration should include all settling velocities occurring in suspension. For a suspension of rising and falling particles in gravitational sedimentation the integration may go from negative infinity to positive infinity.

2-2 Discrete Particles in the "Ideal" Tank

In order to analyze the behavior of a suspension in a settling tank, assumptions are usually made about the tank and the suspension. The problem is thus idealized until it can be handled analytically. A classical simplification is the "ideal" basin introduced by Camp (17,18). The "ideal" basin is a continuous flow rectangular settling basin with four zones. These zones, inlet, outlet, sludge and settling are shown in figure 2-3.

In the settling zone discrete particles undergo unhindered quiescent settling. In figure 2-3 the dimensions L, B, and D refer, respectively, to the length, breadth (or width), and depth of that zone.

The "ideal" basin has the following assumed characteristics:

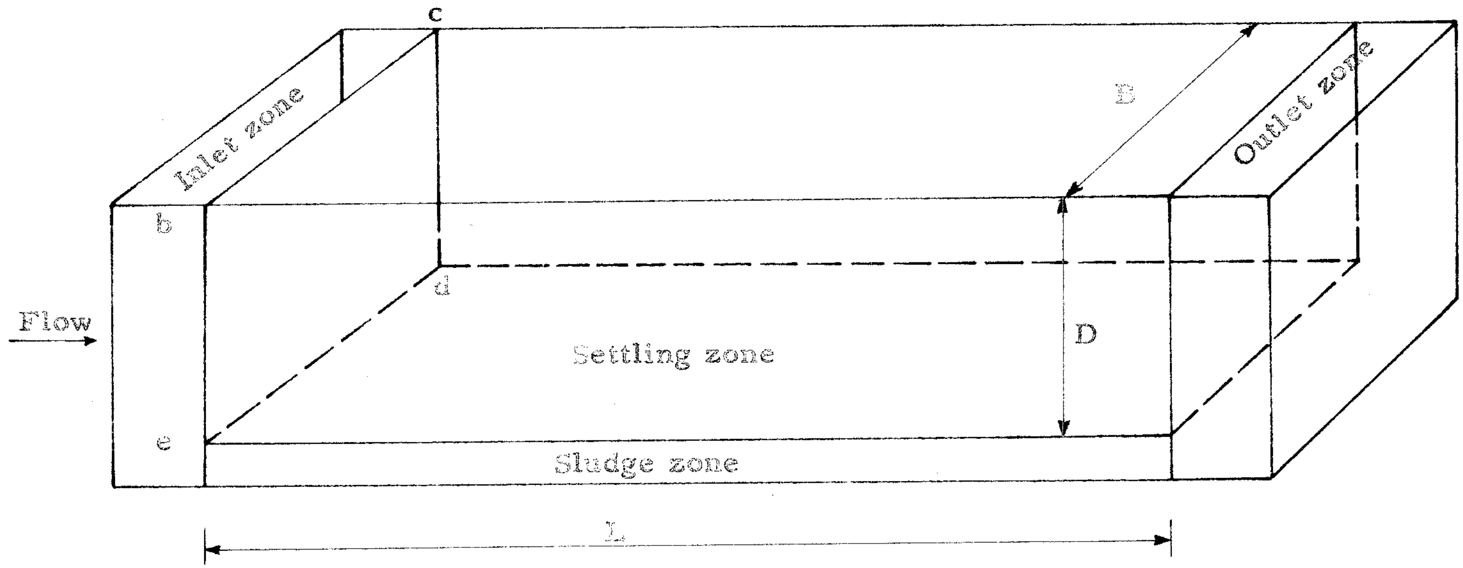
(a) In the settling zone the velocity of the fluid is horizontal, constant with respect to time, and uniform throughout the zone. The particles settle "quiescently."

(b) The concentration of suspended particles of each settling velocity is uniform over the vertical cross section at the inlet end of the settling zone.

(c) A particle is permanently removed from suspension when it reaches the bottom of the settling zone.

The first characteristic indicates that each portion of the suspending fluid will remain in the settling zone for the same length of time. This time is called the detention period, T. If Q represents the volume of suspension passing through the tank per unit of time, the detention time is

$$T = \frac{LBD}{Q} \tag{2-5}$$



L = length of settling zone
B = width of settling zone
D = depth of settling zone

Fig. 2-3. Zones in an "ideal" tank.

The detention time is also the time available for all particles to settle. Let the settling velocity of a particle that settles depth D exactly in time T be called w_0. Using equation 2-5, w_0 can be obtained as follows:

$$w_0 = \frac{D}{T} = \frac{QD}{LBD} = \frac{Q}{LB} \qquad (2\text{-}6)$$

The velocity w_0 is called the "overflow rate" of the tank, a somewhat misleading term. It derives its name from the fact that it is equal to Q divided by the horizontal area of the settling zone.

Camp (1) has shown that for a given influent suspension the fraction of suspended matter removed by ideal settling depends upon the overflow rate alone. For particles with $w > w_0$, the removal is complete; on the other hand, for $w < w_0$ the removal ratio may easily be shown to be w/w_0. For a constant overflow rate the removal is not changed by deepening the tank to increase detention time.

This removal expressed as a removal ratio R is defined by the equation

$$R = \frac{\text{Ø of influent} - \text{Ø of effluent}}{\text{Ø of influent}} \qquad (2\text{-}7)$$

Camp expressed R in terms of

$$R = 1 - F(w_0) + \frac{1}{w_0}\int_0^{F(w_0)} w\,dF(w) \qquad (2\text{-}8)$$

whereas Ingersoll, McKee and Brooks (2) have presented the same quantity in terms of f(w) as

$$R = \int_0^{w_0} \frac{w}{w_0} f(w)dw + \int_{w_0}^{\infty} f(w)dw \qquad (2\text{-}9)$$

Both equations 2-8 and 2-9 have the advantage that they can be calculated directly from the curves of figure 2-1. The equations fail, however, when the second condition for an ideal tank does not hold. That is, they apply only when the particles of each settling velocity are uniformly distributed over the inlet end of the settling zone. In the next section a method that does not suffer from this restriction is introduced.

2-3 Spatial Distribution of Particles in the "Ideal" Tank

The essence of the concept of the "ideal" tank is that particle paths can be calculated. The conditions which make the calculation simple are the following:

(a) The fluid velocity is constant throughout the tank.

(b) The particles are discrete.

(c) Settling is quiescent and unhindered.

(d) Removed particles never reenter the settling zone.

Because of conditions (b) and (c), particles of a settling velocity w settle as if no other particles are present. Their movement and concentration can be considered separately. Since it is known how fast these particles settle and where they are carried by the fluid, their position in the tank can be calculated as a function of time. The concentration of these particles at any point in the tank can be calculated if the concentration is known at the inlet end of the tank. At the inlet end, this concentration need not be uniform over the depth.

The spatial distribution of particles refers to the variation in concentration from point to point in space. The word "spatial" has been added to distinguish the term from the settling velocity distribution. The word "distribution," used alone, will refer to a function of both space and settling velocity.

In an ideal tank the spatial distribution of particles of each velocity is uniform at the inlet end of the settling zone. The settling in the tank could be analyzed just as well if this spatial distribution were not uniform. Let this spatial distribution be called the initial spatial distribution. It need not be the same for all particles.

Consider an initial spatial distribution which varies only with depth. The concentration of particles with settling velocity w at the inlet end of the tank will be a function of w and of depth below the top of the suspension. This distribution function will be called $f_0(z,w)$, where z is the depth. Therefore, the quantity $f_0(z,w)dw$ is defined as the concentration of particles with settling velocity between $w + \frac{dw}{2}$ and $w - \frac{dw}{2}$ at a depth z below the top of the suspension. Both z and w will be taken to be positive.

A hypothetical spatial distribution for particles of a settling velocity w is shown in figure 2-4. The plane of the figure is a vertical section through the settling zone parallel to the direction of flow. The distance in the direction of flow is represented by x as shown. At the inlet end $x = 0$.

The particles represented by the curve $f_0(z,w)$ are all on the same vertical line 0,z at the inlet end of the tank. As the suspension moves through the tank these particles remain in a single vertical line which moves through the tank with velocity, U, of the flowing fluid. This velocity is given by

$$U = \frac{Q}{BD} \tag{2-10}$$

In time t this line of particles moves a horizontal distance given by the equation

$$x = Ut \tag{2-11}$$

Each particle of velocity w settles a distance wt_1, in the time t_1. Therefore, the concentration of these particles at depth z_1, and distance $x_1 = Ut_1$ must be the same as the concentration at depth $z - wt_1$ at $x = 0$ and $t = 0$. Let $f(z,w,t)dw$ represent the concentration of particles of velocity w at depth z and at time t. According to the preceding

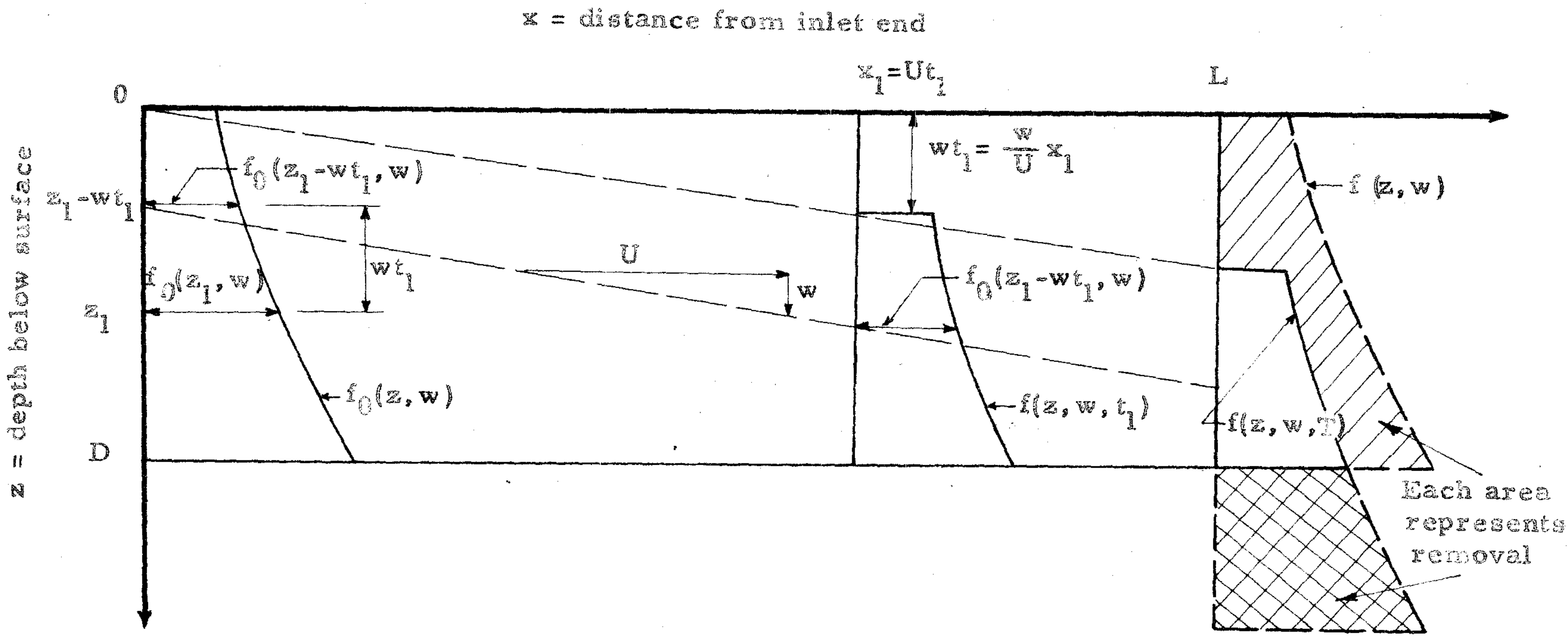


Fig. 2-4. Non-uniform spatial distribution at inlet in the "ideal" tank (w = constant).

argument and equation 2-11

$$f(z,w,t)dw = f(z,w,\tfrac{x}{U})dw$$

$$= f_0(z - wt,w)dw$$

$$= f_0(z - \tfrac{w}{U}x,w)dw$$

Hence the distribution function $f(z,w,t)$ is given by

$$f(z,w,t) = f_0(z - wt,w) \qquad (2\text{-}12a)$$

$$f(z,w,\tfrac{x}{U}) = f_0(z - \tfrac{w}{U}x,w) \qquad (2\text{-}12b)$$

In figure 2-4 the spatial distribution for $x = x_1$ and $t = t_1$ is represented by the curve marked $f(z,w,t_1)$. The value $f(z_1,w,t_1)$ is shown to be the same as the value $f_0(z_1 - wt_1,w)$. The spatial distribution f_0 is seen to move as a unit in the direction of w as the fluid moves in the direction of U. At the outlet end of the settling zone the spatial distribution has moved downward a distance wT and formed a new distribution $f(z,w,T)$ shown as a solid curve in figure 2-4.

The original inlet distribution is shown as a dashed curve at the outlet end. The crosshatched area between the dashed curve and the solid curve $f(z,w,T)$ represents the suspended matter removed in the settling zone. This area also equals the double crosshatched area under the dashed extension of the curve $f(z,w,T)$.

The reader is reminded that the foregoing analysis applies only to discrete particles with one velocity w. A suspension containing particles with many settling velocities is discussed in the next section.

2-4 Concentration Profiles and Removal in the Ideal Tank

The total concentration of particles at a point in the ideal tank is related to the distribution function just as the total concentration of particles in a suspension is related to the settling velocity distribution. If $\phi(z,t)$ is the total concentration of particles at depth z after a time t in the settling zone, it is given by the following equations

$$\phi(z,t) = \int f(z,w,t)dw \qquad (2\text{-}13a)$$

$$\phi(z,\tfrac{X}{U}) = \int f(z,w,\tfrac{X}{U})dw \qquad (2\text{-}13b)$$

Equation 2-13b gives concentration as a function of longitudinal position instead of time.

In equations 2-13 the limits of integration must include all settling velocities which exist. For the sake of simplicity, only particles settling downward from the surface will be considered (w positive). Since any particle with a velocity greater than z/t will have settled from the surface past the depth z in the time t, there are no particles faster than z/t at point (z,Ut). The upper limit of integration is thus fixed at z/t. Equations 2-12 and 2-13 can now be combined to give

$$\phi(z,t) = \int_0^{z/t} f(z,w,t)dw$$

$$= \int_0^{z/t} f_0(z - wt,w)dw \qquad (2\text{-}14a)$$

$$\phi(z,\tfrac{x}{U}) = \int_0^{z/t} f(z,w,\tfrac{x}{U})dw$$

$$= \int_0^{z/t} f_0(z - \tfrac{w}{U}x,w)dw \qquad (2\text{-}14b)$$

There is a simple graphical interpretation of equations 2-14. If a drawing similar to figure 2-4 is made for particles of each settling velocity, all the drawings can be superposed. The concentration at a point is obtained by adding the values of fdw at a point (z,x).

Having calculated $\phi(z,\frac{x}{U})$, one can plot ϕ as a function of z for any x. This curve will be called the concentration profile at distance x. It can also be called the concentration profile at the end of settling time t.

Hypothetical profiles are shown in figure 2-5. The plane of the figure is a vertical plane through the settling zone and parallel to the flow. The concentration profile at the

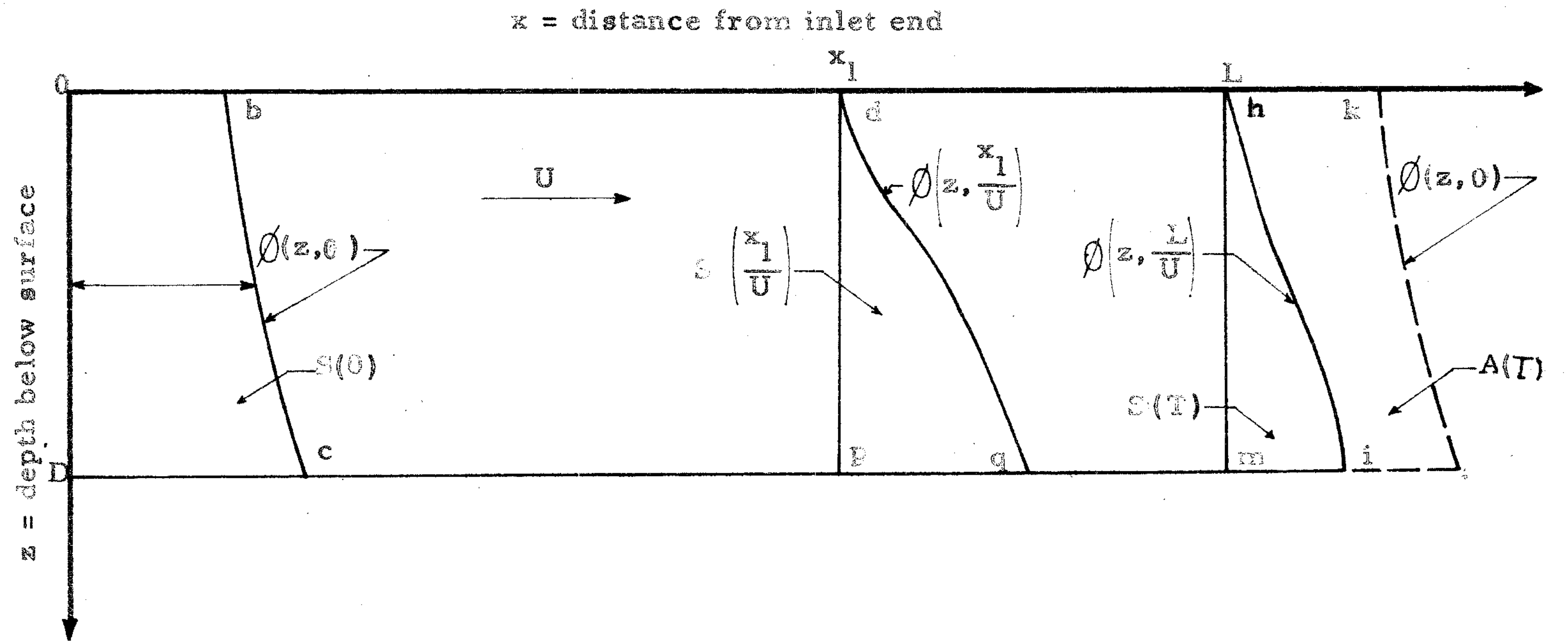


Fig. 2-5. Hypothetical concentration profiles in a settling tank.

inlet end is drawn along the line $x = 0$ and is shown by the curve marked $\phi(z,0)$. The area enclosed by this curve, the z and x axes, and the line $z = D$ represents the total amount of suspended matter above the depth D. Similarly at $x = x_1$ the area enclosed by lines dp, pq and dq represents the total amount of suspended matter above D. This area, which is a function of x, will be called $S(\frac{x}{U})$ or $S(t)$. It is defined by the equations

$$S(\tfrac{x}{U}) = \int_0^D \phi(z,\tfrac{x}{U})dz \qquad (2\text{-}15a)$$

$$S(t) = \int_0^D \phi(z,t)dz \qquad (2\text{-}15b)$$

Since ϕ has the units of mass per unit volume of suspension, S has the units of mass per unit horizontal area.

The values of S at $x = 0$ and $x = L$ are of special interest. The latter is represented in figure 2-5 by the area between outlet concentration profile $\phi(z,\frac{L}{U})$ and the line $x = L$. The inlet profile $\phi(z,0)$ has been redrawn along the line $x = L$ and is shown as a dashed line. The area between the dashed curve kj and the solid curve hi represents the material settled out of suspension in the time T. This area will be called $A(T)$ and is defined by the equation

$$A(T) = S(0) - S(T) \qquad (2\text{-}16)$$

The quantity A is the total amount of material removed from the suspension above D. This quantity is related to the removal ratio R by

$$R(T) = \frac{A(T)}{S(0)} = \frac{S(0) - S(T)}{S(0)} \qquad (2\text{-}17)$$

By combining equation 2-17 and 2-15, R can be expressed in terms of $\emptyset$

$$R(T) = 1 - \frac{\int_0^D \emptyset(z,T)dz}{\int_0^D \emptyset(z,0)dz} \qquad (2\text{-}18)$$

It should be noted that equations 2-15, 2-16, 2-17, and 2-18, are definitions. They are not limited to any tank. However, for the case of an "ideal" tank these equations must give the same results as the equations of Camp* and Ingersoll, McKee and Brooks.**

For an "ideal" tank with non-uniform initial spatial distribution, equation 2-14a can be used with equation 2-18 to obtain the result

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* Equation 2-8.

** Equation 2-9.

$$R(T) = 1 - \frac{\int_0^D \int_0^{z/T} f_0(z - wT, w)\,dw\,dz}{\int_0^D \int_0^{\infty} f_0(z,w)\,dw\,dz} \qquad (2\text{-}19)$$

For the "ideal" tank the distribution function at $t = 0$ is simply

$$f_0(z,w) = f(w)$$

The latter is the settling velocity distribution of the influent suspension. Equation 2-19 may then be written

$$R(T) = 1 - \frac{\int_0^D \int_0^{z/T} f(w)\,dw\,dz}{D\phi_0} \qquad (2\text{-}20)$$

where ϕ_0 is the average concentration of particles in the influent. It can be shown that the double integral in equation 2-20 can be changed to give

$$R(T) = 1 - \frac{\int_0^{D/T}\int_{wT}^{D} f(w)dzdw}{D\phi_0}$$

$$= 1 - \frac{\int_0^{w_0} (D-wT)f(w)dw}{D\phi_0}$$

$$= 1 - \frac{\int_0^{w_0} f(w)dw}{\phi_0} + \frac{\int_0^{w_0} \frac{w}{w_0} f(w)dw}{\phi_0} \qquad (2\text{-}21)$$

If ϕ_0 is taken as unity equations 2-21 and 2-8 are identical.

2-5 Idealized Settling Tanks with Two-Dimensional Potential Flow

The fluid flow in the settling zone of an ideal tank is a particular case of potential flow. One wonders if some of the characteristics of the ideal tank are common to any tank in which the fluid flow is potential. It is not difficult to answer this question when the potential flow is two-dimensional.

Fitch (11) discussed the settling of discrete particles through a fluid that is in two-dimensional flow. He showed, analytically, that in a tank with this type of flow, the removal of particles is the same as in an "ideal" tank. Of course both tanks must have the same overflow rate. Fitch's analysis is presented in appendix 1 in a somewhat modified form, but the idea is best explained by means of the following example.

Figure 2-6 shows sections through three continuous flow settling basins. The sections are in vertical planes parallel to the axes of the basins. In all three basins the flow is potential and two-dimensional. The streamlines are parallel to the sections shown.

Basin 2 has a variable depth in the plane of the section shown, but the depth is constant in the transverse direction. The other two basins are "ideal" tanks having the same constant width and same overflow rate as basin 2. To cause an even greater difference in the flow patterns of the basins, the effluent from number 2 is made to flow over a rectangular weir at the outlet end.

The flow in each basin can be described by flow net. Using the standard stream function ψ, the stream lines are labelled ψ_a, ψ_1, ψ_2, ψ_3, and ψ_b. Between the stream lines ψ_1 and ψ_2 flow of fluid per unit width is $\psi_2 - \psi_1$. The values of ψ increase in the direction of positive z, that is, down from the surface.

When a particle enters the tank it begins to settle and to move forward with the fluid. As it settles it crosses stream lines. The number of stream lines crossed by a particle of settling velocity w as it moves a horizontal distance dx is given by the equation

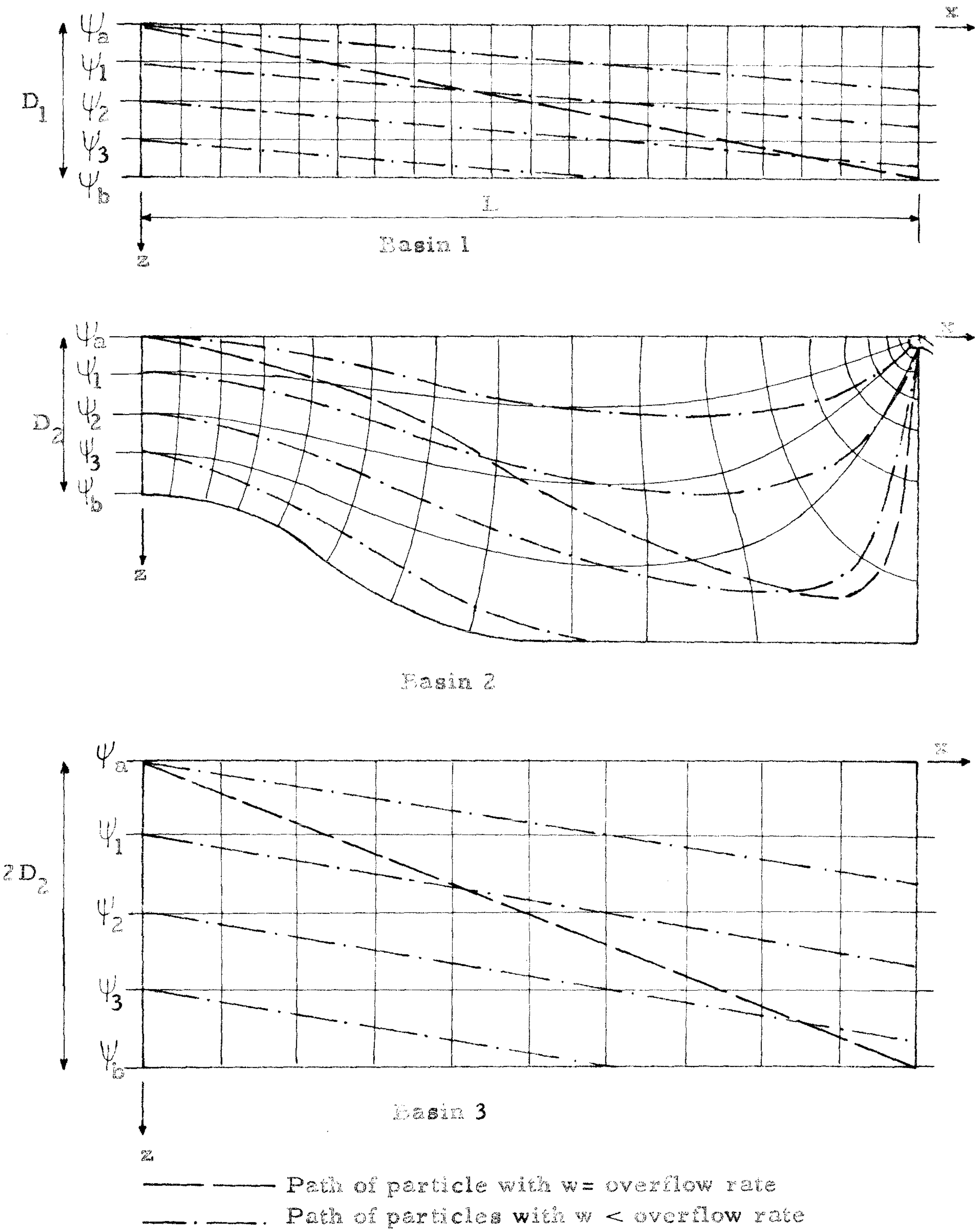


Fig. 2-6. Discrete particles in tanks with two-dimensional potential flow.

$$d\psi = w dx \qquad (2\text{-}22)$$

where w is positive in the positive z direction. The proof of equation 2-22 is given in appendix 1.

Integrating both sides of the equation 2-22 yields

$$\Delta\psi = wx \qquad (2\text{-}23)$$

where $\Delta\psi$ is the number of stream lines crossed when a particle moves a horizontal distance x.* A particle which settles from streamline ψ_a to stream line ψ_b in the distance L has a settling velocity

$$w = \frac{\psi_b - \psi_a}{L} \qquad (2\text{-}24)$$

However, the flow through the tank is $[\psi_b - \psi_a]B$. Hence,

$$w = \frac{(\psi_b - \psi_a)B}{LB} = w_0$$

A particle of settling velocity $w < w_0$ settles from stream line ψ_a to stream line ψ in the distance L. From

---

* This equation does not hold when the fluid velocity is exactly in the opposite direction to the settling velocity. In that case the particle crosses no stream lines. As an example, consider downward settling in upward flow.

equation 2-23, ψ is given by

$$\psi - \psi_a = wL \qquad (2\text{-}25)$$

If the inlet spatial distribution of these particles is uniform, and the stream lines are equally spaced at the inlet, the fraction removed is equal to

$$\frac{\psi - \psi_a}{\psi_b - \psi_a} = \frac{wL}{w_0 L} = \frac{w}{w_0} \qquad (2\text{-}26)$$

This is the same fraction removed as in an ideal tank. Hence R is the same for basin 2 as for basins 1 and 3.

The most valuable application of equation 2-22 is in construction of particle paths through the tank. A few of these paths are shown in figure 2-5. When particle paths can be constructed, it is possible to calculate the concentration at any point. Such a method is of great value when the inlet spatial distribution is not uniform.

In locating the particle path, the logical coordinates are ψ and x. Since the flow net is based on fluid flow the stream lines ψ = constant are laid out before the particle paths. It may be useful to write the inlet distribution in terms of ψ and w instead of z and w. An equation for $\phi(\psi, x)$ could be constructed in a form similar to that of equation 2-13.

From the preceding paragraphs it is possible to draw the following general conclusion about tanks in which the fluid flow is potential and two-dimensional. The removal, in such tanks, is completely dependent on the initial distribution $f_0(z,w)$ and the distribution of stream lines at the inlet end of the tank. If this stream line distribution is uniform the removal is exactly the same as in an ideal tank.

2-6 Unhindered Quiescent Settling of Discrete Particles in a Settling Column

In this thesis the term settling column refers to a suspension of particles and fluid held in some vertical container such as a graduated cylinder or glass tube. The particles settle vertically, and the container must be large enough to prevent the walls from affecting the settling. In the quiescent column the particles undergo quiescent settling.

Throughout the column the properties of the suspension are assumed to be functions only of depth below the surface. Since the properties do not change in any horizontal direction they can be written as functions of z and t. In the settling column, t is the time during which settling has been taking place.

Some of the equations and functions developed in previous sections can be applied directly to quiescent settling columns. The distribution function $f(z,w,t)$ is basic to the settling column. In order that its significance may be reviewed, a graphical representation of it is shown in figure 2-7. The

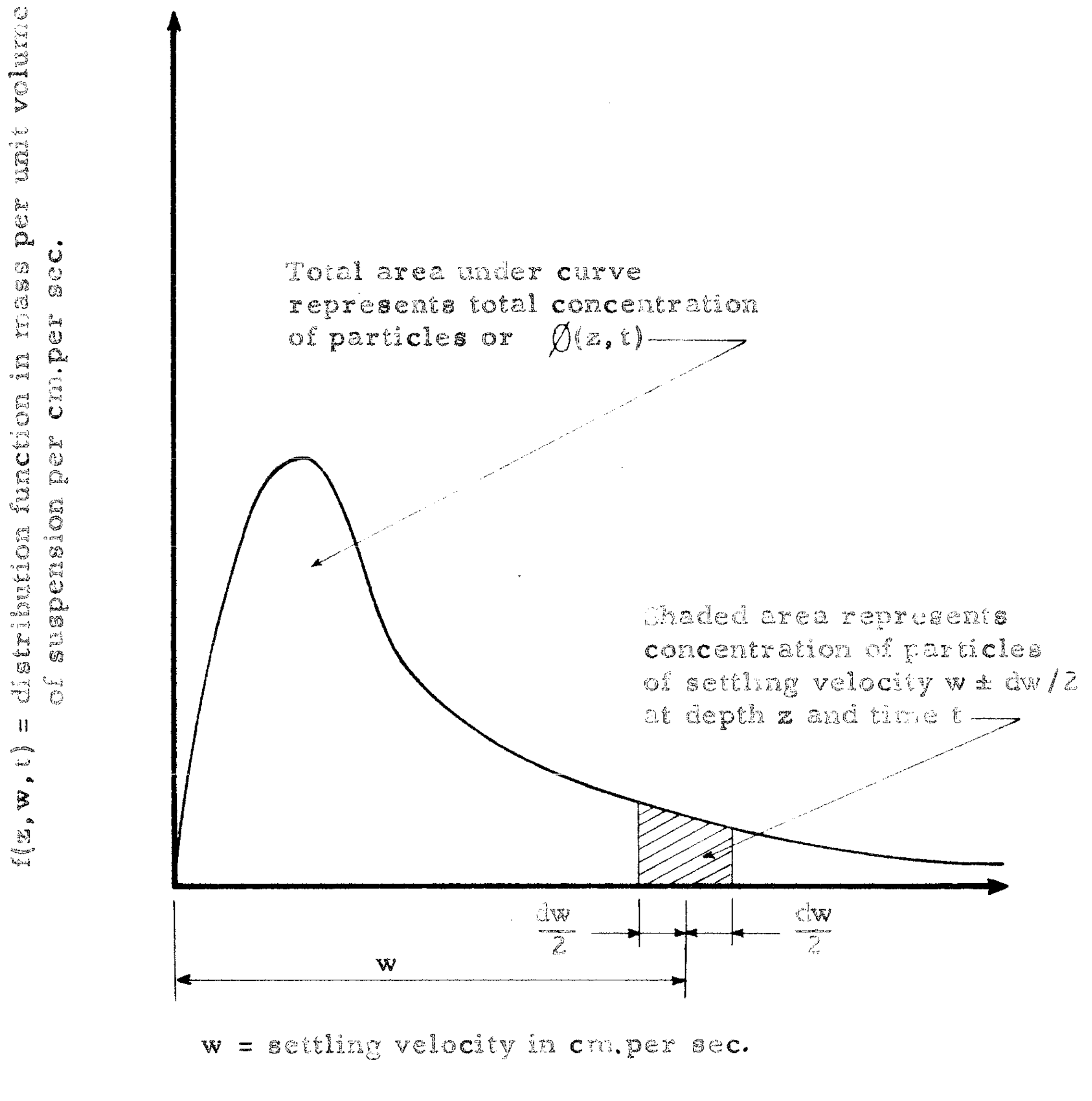


Fig. 2-7. Hypothetical distribution function.

quantity $f(z,w,t)$ is the concentration at depth z and time t of particles with settling velocity between $w + \frac{dw}{2}$ and $w - \frac{dw}{2}$. These particles are referred to as the particles of settling velocity w. For unhindered settling of discrete particles $f(z,w,t)$ can be written in terms of the initial distribution $f_0(z,w)$. This has been expressed in equation 2-12a as

$$f(z,w,t) = f_0(z - wt,w) \tag{2-12a}$$

In the settling column the total concentration of particles at depth z and time t is called $\phi(z,t)$. The quantities ϕ and f have already been related by the equations

$$\phi(z,t) = \int f(z,w,t)dw \tag{2-13a}$$

and

$$\phi(z,t) = \int_0^{z/t} f_0(z - wt,w)dw \tag{2-14a}$$

In the settling column, concentration profiles are curves of ϕ as a function of z for constant t. These can be plotted on the same graph to give a field of ϕ as a function of z with t as a parameter. Such a diagram will be called the ϕ,z diagram. A hypothetical ϕ,z diagram is shown in figure 2-8.

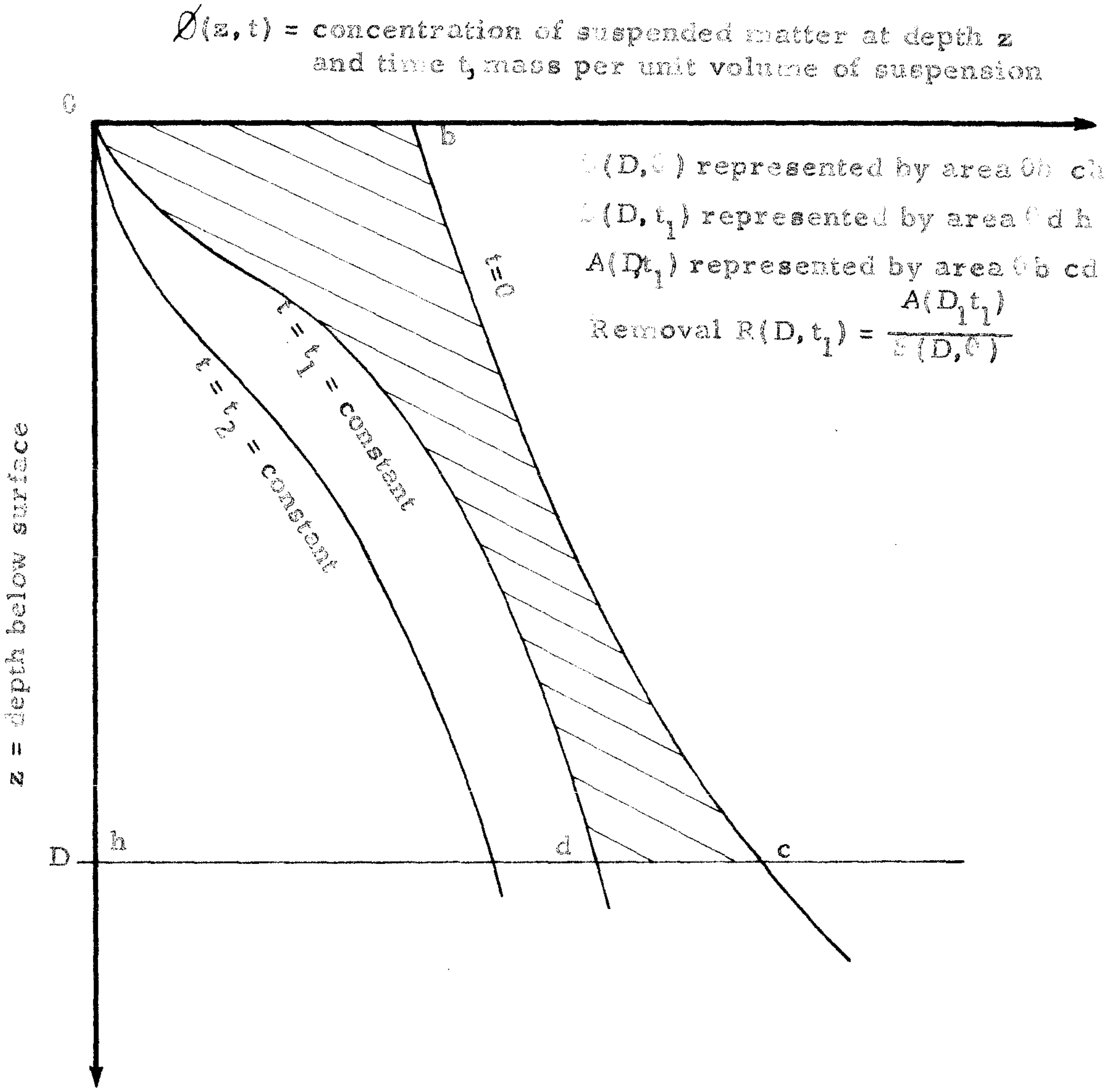


Fig. 2-8. Ø, z diagram for a settling column. Concentration as a function of depth with time as a parameter.

Intimately related to the concentration profiles and, hence, to the ϕz diagram are the quantities S,A and R. They have the same meaning as they had in section 2-4, except that it is now convenient to allow for a variable depth D. Thus, S(D,t) represents the total amount of material in suspension at time t above a unit horizontal area at depth z = D. In figure 2-8, S(D,0) is shown as the area Obch, while $S(D,t_1)$ is shown as the area Odh. As with the settling tank, A(D,t) is defined by

$$A(D,t) = S(D,0) - S(D,t) \tag{2-27}$$

while the removal ratio R(D,t) is defined by

$$R(D,t) = \frac{A(D,t)}{S(D,0)} = \frac{S(D,0) - S(D,t)}{S(D,0)} \tag{2-28}$$

This short discussion serves as an introduction to the analysis of settling in the settling column. The equations and sketches indicate the relationship between concentration profiles in a column and in a settling tank. However, in order to use the settling column as a research tool a more detailed analysis is necessary. This analysis is presented in the following chapter.

CHAPTER 3

CONTINUITY EQUATIONS FOR THE QUIESCENT SETTLING COLUMN

The purpose of this chapter is to develop and discuss certain continuity relationships for particles in the quiescent settling column. Like any other continuity relationships, the equations of this chapter involve only the concentration and motion of particles. On the other hand, equations relating the motion and the forces causing the motion are momentum relationships. Stokes' law for settling spheres is an example of the latter. Except for the effect of concentration on particle settling velocity, momentum equations are not considered in this chapter.

The chapter begins with a review of the simple continuity equations for settling particles. The equation is then used to obtain relationships between removal of suspended particles and flux of particles at a point. This whole section is important because the results are not limited to free settling or to discrete particles.

The second section of the chapter presents some continuity relationships for free settling of discrete particles. Free settling is a very simple process. It is usual for the most complicated equations to be used for the simplest processes, and free settling is no exception. Several equations have been derived to relate settling behavior at a point to conditions at the beginning of settling. Some of these will be of value in interpreting tests on flocculent suspensions.

The third section is an introduction to the analysis of hindered settling. The main reason for including this subject is completeness. While no tests on hindered settling were undertaken for this thesis, an analysis of local settling should at least mention the effect of concentration on velocity. Moreover, hindered settling can occur accidently in settling columns. It is wise therefore to be aware of some of its effects.

3-1 The Continuity Equation

(a) Flux of Particles. Before developing the continuity equation, it is necessary to discuss the flux of particles through a unit horizontal area in the settling column. This flux will be called $a(z,t)$. To determine $a(z,t)$, one must first consider the flux of particles of a single settling velocity, w.

At depth z below the top of the suspension and time t after settling has begun, the concentration of particles of velocity w is $f(z,w,t)dw$. The rate at which these particles cross a unit horizontal area at depth z is $wf(z,w,t)dw$. The flux is the sum of these rates over all possible settling velocities.

$$a(z,t) = \int_{-\infty}^{\infty} wf(z,w,t)\,dw \qquad (3\text{-}1)$$

From equation 3-1 it is clear that the flux at a point is related to the instantaneous mean settling velocity at that point. According to normal usage, the arithmetic mean settling velocity at z and t is defined as

$$\bar{w}(z,t) = \frac{1}{\phi(z,t)} \int_{-\infty}^{\infty} wf(z,w,t)\,dw \qquad (3\text{-}2)$$

Combining equations 3-1 and 3-2 gives

$$a(z,t) = \bar{w}(z,t)\phi(z,t) \qquad (3\text{-}3)$$

Equation 3-3 states that the instantaneous, local flux is the product of the instantaneous total local concentration and the instantaneous, local, average settling velocity.

(b) Conservation of Particles at a Point. Consider an elemental volume of suspension with its center at depth z; let the horizontal area of the volume be one unit and the vertical dimension be dz. The rate at which particles leave this volume is given by

$$a(z + \frac{dz}{2}, t)$$

while the rate at which they enter it is

$$a(z - \frac{dz}{2}, t)$$

The difference between the rates equals the rate of change of concentration in the volume.

$$a(z - \frac{dz}{2}, t) - a(z + \frac{dz}{2}, t) = dz\frac{\partial \phi}{\partial t}$$

This can be written simply as

$$-\frac{\partial a}{\partial z}dz = \frac{\partial \phi}{\partial t}dz$$

and the final continuity equation is

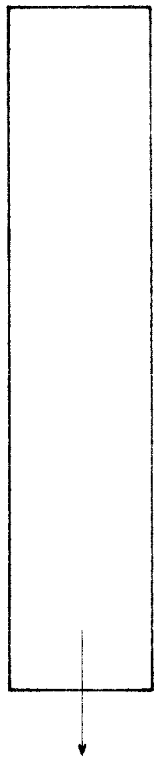
$$\frac{\partial a}{\partial z} + \frac{\partial \phi}{\partial t} = 0 \qquad (3\text{-}4)$$

Equation 3-4 states that the local derivative of concentration with respect to time plus the local derivative of flux with respect to distance is zero. This equation is general; it includes all forms of quiescent settling. In vector form the equation can account for motion of particles in all directions. This form is

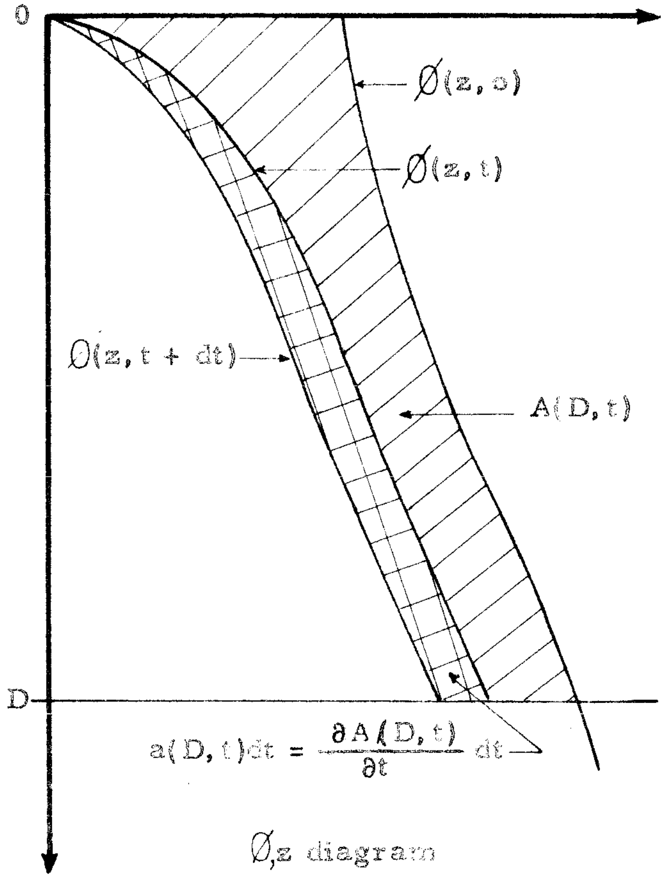
$$\frac{\partial \phi}{\partial t} + \nabla \cdot \underline{a} = 0 \qquad (3\text{-}5)$$

where the flux of particles must be considered as a vector $\underline{a}$.

(c) <u>Relationship Between Flux and Removal</u>. Consider the settling column and the related ϕ,z diagram shown in figure 3-1. The section of settling column shown there might



Settling column
above depth D



Ø,z diagram

Fig. 3-1. Relationship between flux and removal.

be taken as a free body diagram of the part of the settling column above D. No particles enter the free body through the top or the sides; hence the only change in the amount of suspended matter above D is caused by the flux a(D,t).

The ϕ,z diagram is shown for the portion of the column above D. In the diagram the area between the curves for $\phi(z,t)$ and $\phi(z,0)$ represents the function A(D,t). Moreover, the area between the curves for $\phi(z,t)$ and $\phi(z,t+dt)$ represents the change in A during the time dt, and it must be equal to adt. The flux can thus be given by the following equations.

$$a(D,t)dt = \frac{\partial A(D,t)}{\partial t}dt \tag{3-6}$$

$$a(D,t) = \frac{\partial A(D,t)}{\partial t} \tag{3-7}$$

Since equation 3-7 holds for any D, the variable z can be used in place of D. Therefore

$$a(z,t) = \frac{\partial A(z,t)}{\partial t} \tag{3-8}$$

Since equation 3-8 is quite general, it is not limited to any type of settling. It states, simply, that the flux of particles at depth z is equal to the rate of removal of particles from all of the suspension above z.

(d) Experimental Determination of a(z,t). Equation 3-8 is of considerable value in studying what is taking place at a point in the settling column. It is used in conjunction with a ϕ,z diagram obtained from settling column experiments.

If the diagram shown in figure 3-1 were obtained from an experiment, one would determine the flux in the following manner. First the concentration profiles would be measured in the column and plotted as lines of constant t in the ϕ,z diagram. Next, a value of D would be selected. The value of A(D,t) is then determined by graphical integration of the area between the curves for $\phi(z,0)$ and $\phi(z,t)$. The process is repeated for each t. These A(D,t) values are then plotted against time.

For any value of t the slope of the A(D,t) curve will give a(D,t). Plotting these values of a(D,t) will give a curve of a(D,t) as a function of time with D as a parameter. The whole process can be repeated for other values of D. Actual calculations following this method are to be found in chapter 6.

Perhaps even more interesting than the flux a(z,t) is the mean settling velocity $\bar{w}(z,t)$. In the study of flocculent suspensions, the function $\bar{w}(z,t)$ shows how much flocculation is affecting the settling at a point in the suspension. Since $\phi(z,t)$ and a(z,t) can both be determined, $\bar{w}(z,t)$ can be calculated using equation 3-3.

(e) The Suspension as a Continuum. In the preceding parts of this section the suspension of particles has been treated as a continuum of particles. Such an approach can be justified only by the examination of particle spacing in the suspension. This spacing determines whether or not the use of derivatives is justified.

In setting up the differential equations one tacitly assumes that such functions as ϕ, $\overline{w}$, a and A are continuous functions and possess derivatives. For space derivatives this assumption requires an element of length dz to be many times the distance between individual particles. For time derivatives the assumption requires an element of time dt to be long enough for many particles to pass a point under consideration.

The element of length, moreover, must be small with respect to the dimensions of the suspension; hence dz must be small with respect to z. The implication, then, is twofold. First, the particle spacing must be negligible compared to dimensions of the suspension. Secondly, any sample used to determine concentration must contain a large number of particles. The volume of such a sample may be the volume of a sample actually withdrawn from the suspension. On the other hand, it may be the volume included in the beam of light in a photo extinction technique. (19) In any case, the volume must contain a large number of particles. In all settling column experiments this condition was satisfied except as noted in chapter 7 for raw sewage.

3-2 Continuity Relationships for the Free Settling of Discrete Particles

(a) A Suspension of Uniform Particles. During free settling in a constant field of force,* a discrete particle maintains a constant settling velocity until it is removed from suspension. When this is the case and when the particles in suspension are all of one velocity w, the solution of the continuity equation is quite simple. Equation 3-4 then becomes

$$\frac{\partial \phi}{\partial t} + \frac{\partial (w\phi)}{\partial z} = 0 \qquad (3\text{-}9)$$

Since w is a constant one has

$$\frac{\partial \phi}{\partial t} + w\frac{\partial \phi}{\partial z} = 0 \qquad (3\text{-}10)$$

Equation 3-10 is a first order linear partial differential equation with constant coefficients. The solution is given in many standard texts as

$$\phi(z,t) = \phi_1(z - wt) \qquad (3\text{-}11)$$

where

$$\phi_1(z) = \phi(z,0) \qquad (3\text{-}12)$$

Equation 3-11 is exactly the same as equation 2-12a derived in chapter 2. The latter is based on physical considerations. The physical significance of equation 2-12a has already been explained in section 2-3.

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* In a centrifuge, for example, the force on a particle is a function of speed of rotation and distance from the center of rotation. The equations of this section would have to be modified for such a case. Gravity, on the other hand, creates a constant field of force.

(b) A Polydisperse Suspension. When a suspension contains particles of many sizes it is often called polydisperse. In this thesis polydisperse will also refer to suspensions with particles of many settling velocities. For such suspensions, equation 3-4 can be written

$$\frac{\partial \phi}{\partial t} + \overline{w}\frac{\partial \phi}{\partial z} + \frac{\partial \overline{w}}{\partial z}\phi = 0 \qquad (3\text{-}13)$$

If equation 3-13 is considered as a differential equation in ϕ, then the coefficients are unknown variables. It is much more direct to proceed as in section 2-4. In that section the analysis is based on the fact that in free settling, discrete particles of each settling velocity settle as if there are no other particles in suspension. For each settling velocity there is a solution in the form of equation 3-11. Since the concentration of particles of velocity w was given the form f(z,w,t)dw the following equation was obtained

$$\phi(z,t) = \int_0^{z/t} f_0(z - wt, w)\,dw \qquad (2\text{-}14a)$$

where $$f_0(z,w) = f(z,w,0)$$

The limits of integration are for a case where all the particles in suspension have positive settling velocities.

(c) Flux Distribution. Equation 2-14a gives $\phi(z,t)$ entirely in terms of the initial distribution function $f_0(z,w)$. The same method can be used to determine a(z,t) in terms of $f_0(z,w)$. The result follows from the definition* of $a(z,t)$ and is

$$a(z,t) = \int_0^{z/t} w f_0(z - wt, w)\,dw \qquad (3\text{-}14)$$

Again the limits are for a case where all particles have positive settling velocities.

Perhaps of greater interest than the flux itself is the rate of change of flux with time. In the settling of flocculent suspensions, $a(z,t)$ changes with time not only because of settling but also because of flocculation.** If the total change in $a(z,t)$ is measured, it would be very fortunate if the changes due to settling, and flocculation could be separated.

For very dilute flocculent suspensions, the settling between particle contacts is probably unhindered. If so, the function $\frac{\partial a(z,t)}{\partial t}$ for free settling is of some value. To obtain it, one first writes the continuity equation 3-10 for particles of velocity w as follows:

$$\frac{\partial}{\partial t}\Big[f(z,w,t)\,dw\Big] + w\frac{\partial}{\partial z}\Big[f(z,w,t)\,dw\Big] = 0 \qquad (3\text{-}15)$$

---

* See equation 3-1.

** See chapter 5.

Multiplying both sides by w gives

$$w\frac{\partial(fdw)}{\partial t} + w^2\frac{\partial(fdw)}{\partial z} = 0 \qquad (3\text{-}16)$$

Integrating with respect to w gives

$$\int_0^\infty \frac{(wfdw)}{\partial t} + \int_0^\infty \frac{(w^2fdw)}{\partial z} = 0 \qquad (3\text{-}17)$$

for cases where all $w > 0$.

It follows that

$$\frac{\partial}{\partial t}\int_0^\infty wf(z,w,t)dw + \frac{\partial}{\partial z}\int_0^\infty w^2f(z,w,t)dw = 0 \qquad (3\text{-}18)$$

The first term of equation 3-18 is simply $\frac{\partial a(z,t)}{\partial t}$. To interpret the second term it is first necessary to realize that the distribution function f(z,w,t) can be considered as a local instantaneous settling velocity distribution. It is shown in this form in figure 2-7. Consequently, the term

$$\int_0^\infty w^2f(z,w,t)dw$$

is the second moment of this velocity distribution about the axis w = 0. If $\sigma(z,t)$ is defined as the standard deviation of the settling velocity distribution, then

$$\int_0^{z/t} w^2f(z,w,t)dw = (\overline{w}^2 + \sigma^2)\phi \qquad (3\text{-}19)$$

and equation 3-18 can be written

$$\frac{\partial a}{\partial t} + \frac{\partial}{\partial z}\left[(\overline{w}^2 + \sigma^2)\phi\right] = 0 \qquad (3\text{-}20)$$

If in an experiment, one wishes to know the net change in a(z,t) due solely to free settling, one must measure the standard deviation of settling velocities at a point. This measurement is extremely difficult. Nevertheless, it is an advantage to know what should be measured in an ideal experiment.

(d) <u>Constant z/t in the ϕ,z Diagram</u>. Before leaving the subject of free settling of discrete particles, one additional relationship must be established. This relationship describes the lines of constant z/t in the ϕ,z diagram. In working with the ϕ,z diagram these lines are important.

For the present, it is sufficient to give these lines an elementary interpretation. Imagine that an observer is moving downward through the suspension at a constant velocity. When the observer reaches depth z, what concentration does he find there? The plot of this concentration as a function of z is a line of constant z/t in the ϕ,z diagram.* An example is shown in figure 3-2.

The slope of a line of constant z/t will be the derivative of ϕ with respect to z along the path z = Kt, where K is a constant. It is easiest to begin with the derivative of

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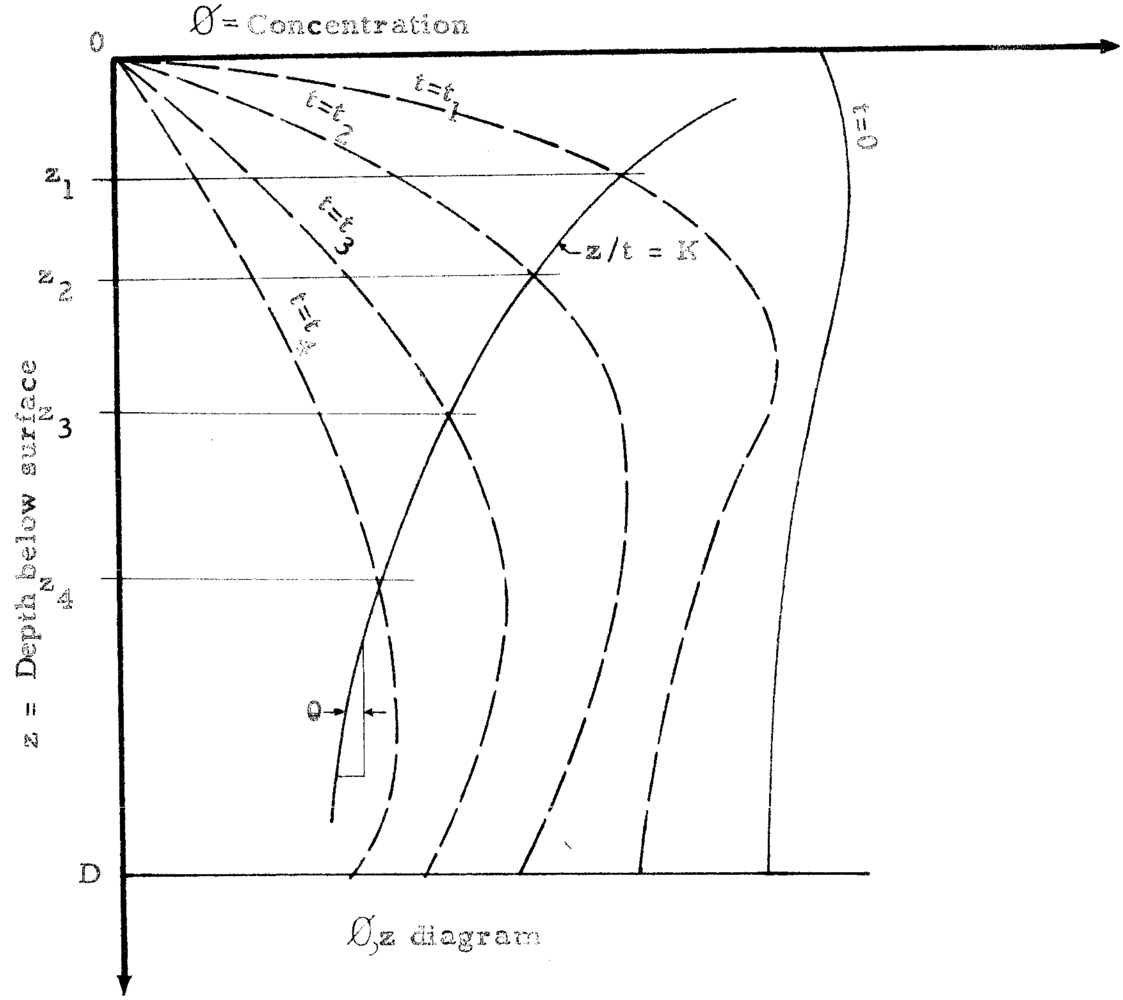
* To be precise, the observer must start at a depth z and time Kt if the value of z/t is to be a constant.



$$\frac{z_1}{t_1} = \frac{z_2}{t_2} = \frac{z_3}{t_3} = \frac{z_4}{t_4} = K \qquad \tan\theta = \left(\frac{d\emptyset}{dz}\right)_K$$

At z_1, observer experiences a concentration at one point in the Ø profile for $t = t_1$. At z_2, observer experiences concentration at one point in the Ø profile for $t = t_2$.

Fig. 3-2. Lines of constant z/t.

concentration of particles of a single velocity. Hence,

$$d\Big[f(z,w,t)dw\Big] = \frac{\partial}{\partial z}\Big[f(z,w,t)dw\Big]dz + \frac{\partial}{\partial t}\Big[f(z,w,t)dw\Big]dt$$

$$\left\{\frac{d}{dz}\Big[f(z,w,t)dw\Big]\right\}_K = \frac{\partial}{\partial z}\Big[f(z,w,t)dw\Big] + \frac{1}{K}\frac{\partial}{\partial t}\Big[f(z,w,t)dw\Big] \qquad (3\text{-}21)$$

Substituting the continuity equation 3-15 into equation 3-21 eliminates the time derivative and gives *

$$\left\{\frac{d}{dz}\Big[f(z,w,t)dw\Big]\right\}_K = (1 - \frac{w}{K})\frac{\partial}{\partial z}\Big[f(z,w,t)dw\Big] \qquad (3\text{-}22)$$

Both sides of this equation can be integrated with respect to w with the result that

$$\int\left[\frac{df(z,w,t)}{dz}\right]_K dw = \int(1 - \frac{w}{K})\left[\frac{f(z,w,t)}{z}\right]dw \qquad (3\text{-}23)$$

For a suspension with all settling velocities positive, there will be no particles with w K remaining at the depth z = Kt at time t. Therefore, the limits of integration are zero and K. Since both of these are constant, the derivatives may be taken outside of the integral sign. However, it is more convenient to leave the right hand side of equation 3-23

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* The subscript K on the left hand side of equation 3-21 and 3-22 indicates a total derivative in the direction z = Kt.

as it is. The final result is

$$\left(\frac{d\phi}{dz}\right)_K = \int_0^K (1 - \frac{w}{K})\left[\frac{f(z,w,t)}{z}\right] dw \qquad (3\text{-}24)$$

Substituting equation 3-11 or 2-12 into equation 3-24 gives

$$\left(\frac{d\phi}{dz}\right)_K = \int_0^K (1 - \frac{w}{K})\frac{\partial f_0}{\partial z'}\left[z(1 - \frac{w}{K}),\ w\right] dw \qquad (3\text{-}25)$$

where the differentiation is with respect to the basic variable $z' = z(1 - \frac{w}{K})$.

Equation 3-25 gives the slope of the lines of constant z/t. While the slope could be integrated to give the actual equation for the line, the slope is more interesting than the line itself. For example, if the initial spatial distribution $f_0(z',w)$ is independent of z', then

$$\frac{\partial f_0}{\partial z'}\left[z(1 - \frac{w}{K}),w\right] = 0 \qquad (3\text{-}26)$$

for each w. The lines of constant z/t are all straight lines parallel to the z axis. This is what occurs in the ideal tank and in the pipette analysis.

If the initial spatial distribution is not constant for all w, equation 3-26 is not true. What will be the slope of the lines z/t = K? It will depend on $f_0(z,w)$; if $f_0(z,w)$ decreases with depth then

$$\frac{\partial f_0}{\partial z'}\left[z(1 - \frac{W}{K}), w\right] < 0$$

Therefore, if the majority of particles have concentration decreasing with depth, the integral of equation 3-25 will be negative. The line z/t = K will have a negative slope like the one shown in figure 3-2. As a matter of fact, in an experiment without flocculation a negative slope of the z/t lines is a sure sign of one thing. At the beginning of the test there are more particles near the top of the column than near the bottom.* A similar statement can be made concerning z/t lines with positive slopes. If they occur in experimental results the initial distribution of particles must have increased from the top of the column to the bottom. Even for flocculent suspensions a positive slope will be evident for the early stages of the settling. During these stages the effect of flocculation is less than the effect of initial distribution.** Eventually the effect of flocculation predominates and the lines of constant z/t have a negative slope.

Several other uses for lines of constant z/t occur in other chapters. They will be discussed as they occur.

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* See the experiment discussed in section 4-d.

** See the ϕ,z diagram analysis of data by Fitch (9) in section 6-2.

3-3 Hindered Settling of a Suspension of Uniform Particles--Kynch's Assumption

(a) Relationship between Concentration and Particle Settling Velocity. Many workers have investigated the effect of particle concentration on the settling velocity of particles in a suspension. In this section, the results of one such investigation will be used. It is typical and adequate for the purposes of this section. For a more comprehensive presentation of the subject, the reader is referred to the text by Coulson and Richardson (12).

The results to be used are credited to Steinhour (20). He conducted experiments with suspensions of small uniform spheres settling in the Stokes' range. His results are expressed by the semi-empirical equation

$$\frac{w}{w_s} = (1 - \alpha\phi)^2 10^{-1.82\alpha\phi} \qquad (3\text{-}27)$$

The symbols have the following meaning:

w = particle settling velocity at concentration ϕ.

w_s = particle settling velocity when particle settles alone in a fluid of infinite extent.

α = reciprocal of specific gravity of the particles. Therefore, $\alpha\phi$ is the fraction of the volume of the suspension occupied by the particles.

In the experiments, the values of $\alpha\phi$ range from 0 to 0.55.

In figure 3-3 equation 3-27 is plotted to show how the function $\frac{w}{w_s}$ varies with the volume concentration $\alpha\phi$. In the

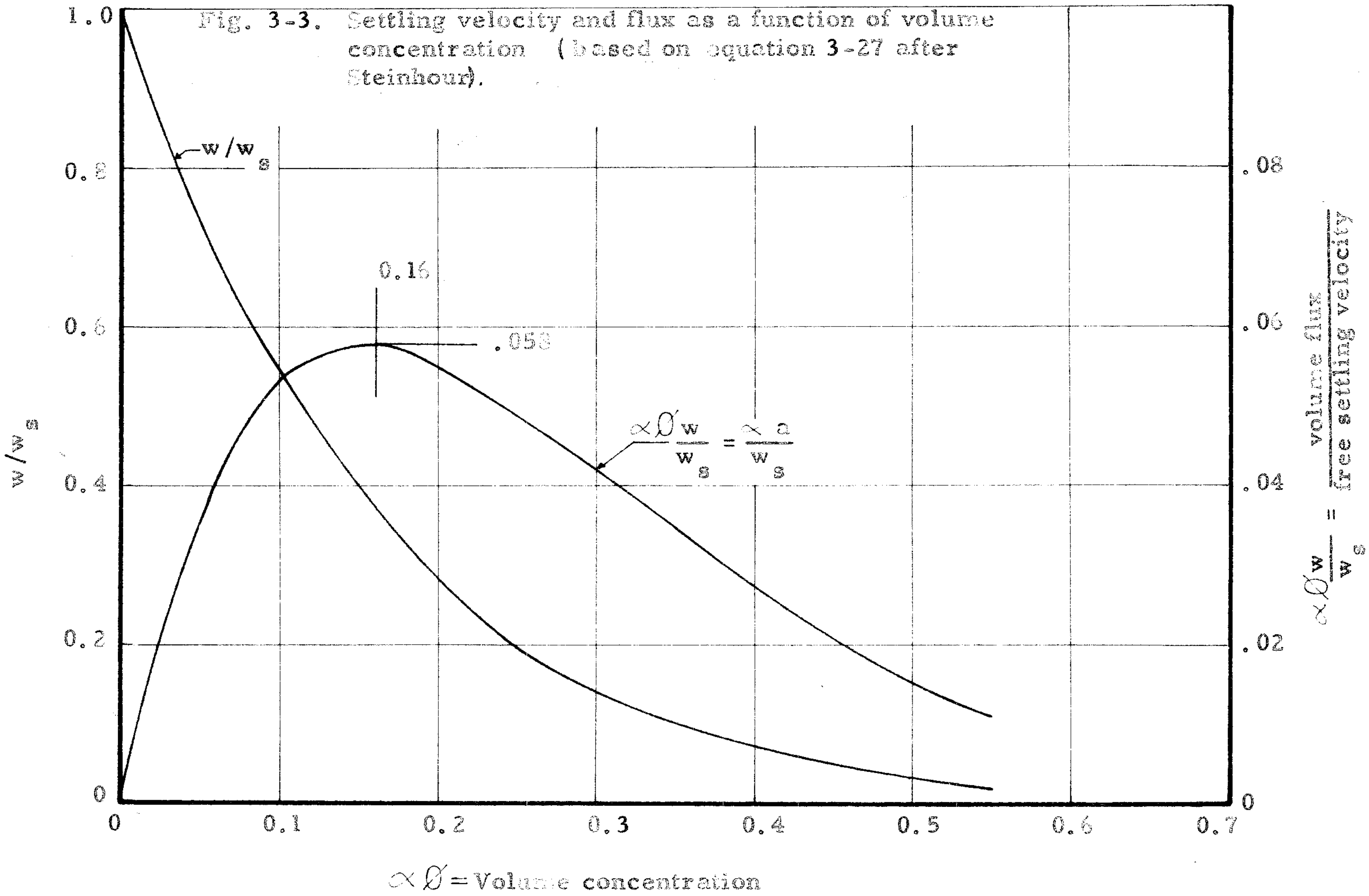

Fig. 3-3. Settling velocity and flux as a function of volume concentration (based on equation 3-27 after Steinhour).

same figure a curve for the product $\alpha \phi w/w_s$ shows how the flux varies with volume concentration.

Many workers have obtained results relating settling velocity and volume concentration. For example, McNown and Lin (14) have shown that $\frac{w}{w_s}$ is a function of Reynolds number and the ratio of particle diameter to mean particle spacing. This ratio is simply a constant times $(\alpha\phi)^{1/3}$.

In the light of the experimental evidence, Kynch (6) assumed that for a suspension of uniform particles w is a function of ϕ alone. Actually, his assumption is more explicit. It states that the settling velocity at any point in a suspension is a function only of the local concentration of particles. This assumption is not strictly correct since it neglects any effect of Reynolds number. However, Kynch's analysis is still valuable in interpreting results of settling column experiments.

(b) <u>Kynch's Solution of the Continuity Equation</u>. According to Kynch's assumption, w is a function of ϕ alone. Since the flux of particles of velocity w is equal to $w\phi$, this flux must also be a function of ϕ alone. It follows that the continuity equation 3-4, can be written

$$\frac{\partial \phi}{\partial t} + \frac{da}{d\phi}\frac{\partial \phi}{\partial z} = 0 \qquad (3\text{-}28)$$

where $\frac{da}{d\phi}$ is a function of ϕ.

Kynch solved equation 3-28 by the method of characteristics.* The solution has the form

$$\phi(z,t) = P(z - \frac{da}{d\phi}t) \qquad (3\text{-}29)$$

where P is a function of the term $z - \frac{da}{d\phi}t$. P is determined from the initial distribution of particles while $\frac{da}{d\phi}$ is determined from a relationship like equation 3-27.

The reader will recall that for free settling of uniform particles the ϕ,z diagram is very simple. Each profile is merely the initial distribution displaced along the z axis by distance wt. For hindered settling, this is not the general case. An example ϕ,z diagram is shown in figure 3-4. It was drawn for a suspension of uniform particles with a free settling velocity, w_s, of 0.01 cm. per sec.

The initial distribution of particles is given by the profile labeled t = 0. The subsequent profiles were calculated using equation 3-29 and figure 3-3. Each profile has a horizontal section that represents a sharp interface in the suspensions. Above the interface, the fluid is free of suspended particles. The calculation of the position of the interface requires additional techniques which are described in Kynch's original work.

(c) Particles Uniformly Distributed at t = 0. If at t = 0 the concentration ϕ is uniform, the initial profile of figure 3-4 is a straight line parallel to the z axis. If w depends solely on ϕ which is uniform, then w is uniform.

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* See reference 21, p. 373.

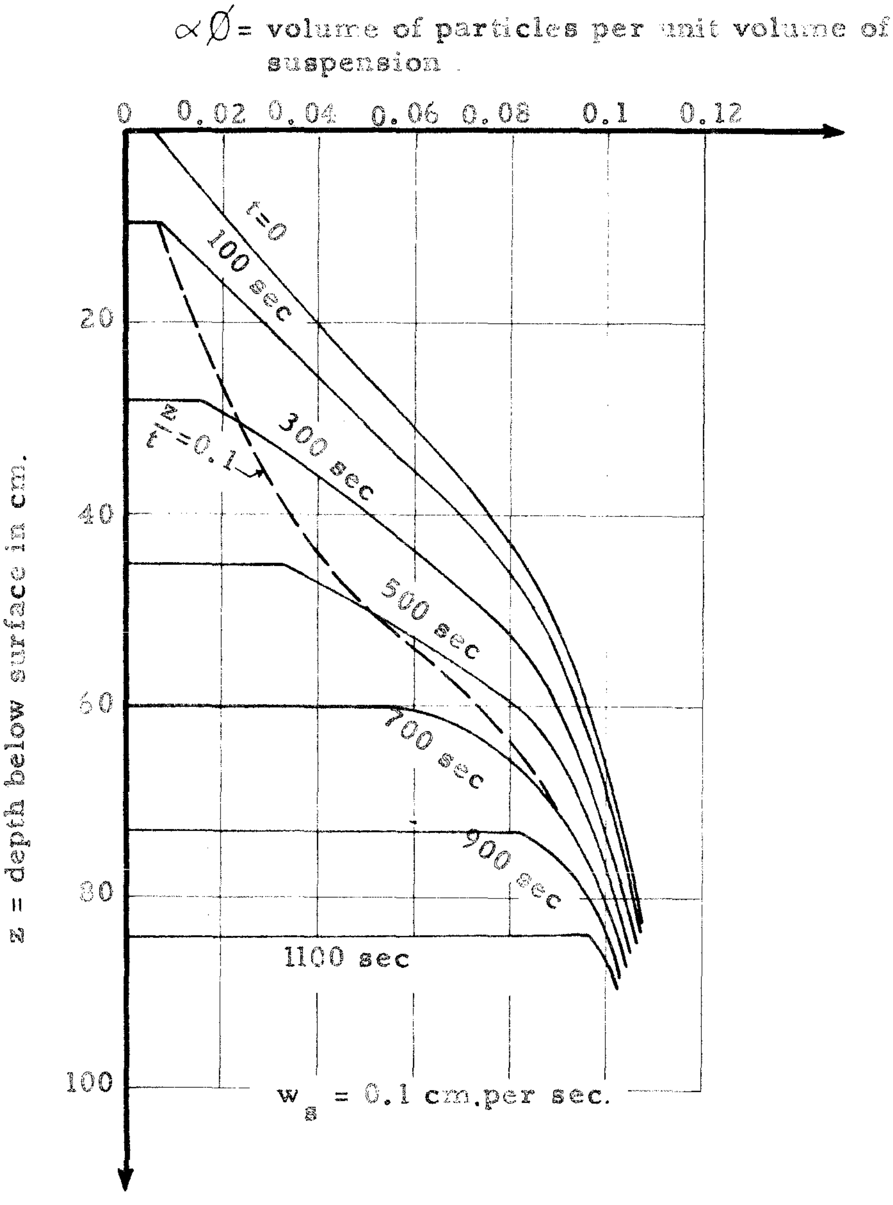

Fig. 3-4. ∅,z diagram for hindered settling of uniform particles (see text).

No force exists to change ϕ, and the initial uniform distribution settles as a unit.

(d) Lines of Constant z/t. A sample line of constant z/t is shown in figure 3-4 as a dashed curve labeled z/t = 0.1. It is seen to have positive slopes. As for the case of free settling the positive slope indicates non-uniform initial spatial distribution of particles.

If however, a suspension has particles of more than one settling velocity, it is difficult to interpret the z/t lines. Even when particles of each velocity are uniformly distributed at t = 0, the faster moving ahead of the slower cause a concentration gradient. This variation in ϕ causes particles originally of one velocity to have a velocity which varies with depth due to hindered settling.

Since the concentration increases with depth, particle velocities will decrease with depth. For each w the particles will "pile up" in suspension just as they are shown to do in figure 3-4. Thus, the lines of constant z/t have a positive slope. That is, the observer moving at the velocity $\frac{z}{t} = K$ moves into greater concentrations as the particles in front of him decrease in settling velocity. Therefore, unfortunately, a positive slope of a line of constant z/t is a symptom with two possible causes. It can be caused by non-uniform initial spatial distribution, or by hindered settling of a polydisperse suspension. Nevertheless, the symptom can be of value in diagnosing what takes place in a settling column.

CHAPTER 4

APPARATUS AND PROCEDURE FOR SETTLING ANALYSIS

The purpose of this chapter is to describe the apparatus used in the experiments of the research. The chapter also describes experiments performed for the purpose of testing the apparatus.

Section 4-1 describes the concept of settling analysis and some of the experimental problems involved therein. The following sections give detailed descriptions of the apparatus. Each section is written in the format of a recipe to be followed in using the apparatus. This is especially true of the subsection entitled "method." Unless stated otherwise, this subsection refers to the general method of using the apparatus.

In future chapters experiments will be mentioned by their names and only results will be given. Any deviations from the methods or apparatus described in this chapter will be described when applicable.

4-1 Settling Analysis

In this thesis the term settling analysis refers to experiments in which suspensions are allowed to settle by gravity in settling columns. In such an experiment, the particles may be discrete or flocculent; the settling may be hindered or free; the fluid may be quiescent or turbulent.

The name, settling analysis, must be credited to Camp (1) who suggested that such experiments be conducted in connection with the study of settling tanks.

On the other hand, the term settling velocity analysis refers to one special case of settling analysis. The particles must be discrete, and the settling must be free and quiescent. In addition, the particles of each settling velocity must be uniformly distributed at the beginning of settling. The object of settling velocity analysis is to determine the settling velocity distribution f(w) for the suspension. In most literature such experiments are called the sedimentation methods of particle size analysis.

The methods of settling velocity analysis have been summarized in many works (19,23). Since the methods of settling velocity analysis are similar to the methods of settling analysis, one will naturally review the former to find techniques for the latter. The choice of a settling velocity analysis technique for use in settling analysis is based on the nature of the suspension to be analyzed.

The essence of settling analysis is the testing of a suspension in its natural form. In general, the particles cannot be removed from the suspension for separate examination because such removal may change the particles from their natural form. Furthermore, the suspension cannot be diluted or concentrated unless it is certain that such process does not alter the natural settling behavior of the suspension. Finally, it would be very inconvenient if the apparatus for settling analysis had to be calibrated for each suspension. The suspensions to be tested may have a wide range of concentration, settling velocities and colors. These factors should affect the calibration as little as possible.

As an illustration of the problem, consider a suspension of colored turbid water which is to be treated with alum and activated silica. The treated water is to be allowed to settle in a tank, but first the water is to be tested in a settling column. From the point of view of settling analysis, the turbid water with the chemicals is the suspension. Even the mixing necessary to distribute the suspended particles in the column may be altering the suspension before settling begins. Therefore, this mixing must resemble the mixing that takes place when the suspension enters the settling tank.

In conducting a settling analysis only two types of measurements are primary.* One of these is the concentration $\phi(z,t)$ at various depths and various times. The other is a direct measurement of the removal, $A(D,t)$, of particles from the suspension above depth D. Other measurements such as temperature are necessary but can be considered as auxiliary to those for ϕ and A.

The methods of settling velocity analysis are usually divided into two groups, depending upon whether A or ϕ is

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* In many settling analyses of concentrated suspensions, the subsidence rate is the primary measurement. Behn (22) has presented a summary of such work.

measured. The concentration, ϕ, is measured in the pipette analysis, the photoextinction techniques and the plummet-hydrometer family of analyses (19,23). The total removal, A is measured in the Oden balance method, the manometer method, the bottom withdrawal method and decantation (19,24)

For the research described in this thesis, one method was selected from each of two groups of settling velocity analyses. From the first group, the pipette method was selected. The hydrometer method would not be suitable for measurements at several depths, and the photoextinction technique would be affected too greatly by the wide range of color and particle size encountered.*

From the second group, the bottom withdrawal tube was selected as a means of measuring the total removal A(D,t). This type of apparatus has been tested extensively by the Federal Inter-Agency River Basin (FIARB) Sub-committee on Sedimentation (26,24). In the testing, the apparatus exhibited consistency and accuracy as a technique for particle size analysis.

The selection of these two methods has been influenced by a consideration not yet mentioned. This consideration is the scale of the analysis. Most settling velocity analyses are conducted with samples ranging in volume from a fraction of a liter to a few liters. On the other hand, settling

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* Setter, Price and Grossman (25) found that they had to homogenize sewage in a Waring Blender in order to obtain satisfactory correlation between light transmission and the suspended solids as measured by Gooch crucible.

analyses are conducted with volumes of several gallons or greater. In its full stage of development settling analysis may be conducted in tubes as deep as settling tanks. Such tubes may be 15 feet deep and one foot in diameter. The two methods selected should operate quite satisfactorily on that scale.

4-2 Two-Depth Settling Apparatus

a. General. Many preliminary experiments were conducted using simple pipette analysis. Two methods were tried. In one method, the suspension settled in a liter graduate. The samples were withdrawn by means of an ordinary pipette, the tip of which had been broken to give a slightly larger opening. In the other method a fixed multiple intake pipette was used. The apparatus was similar to that developed by Jennings, Thomas, and Gardiner (27,23).

These methods are quite satisfactory for free settling of discrete particles uniformly distributed at the beginning of the test. Under these conditions the result is an experimental curve for the cumulative settling velocity distribution, F(w). On the other hand, with no information other than the experimental curve, how does one know that the necessary conditions have been met? One is much more certain if samples have been taken from two depths in the suspension and all points fall on a single curve.

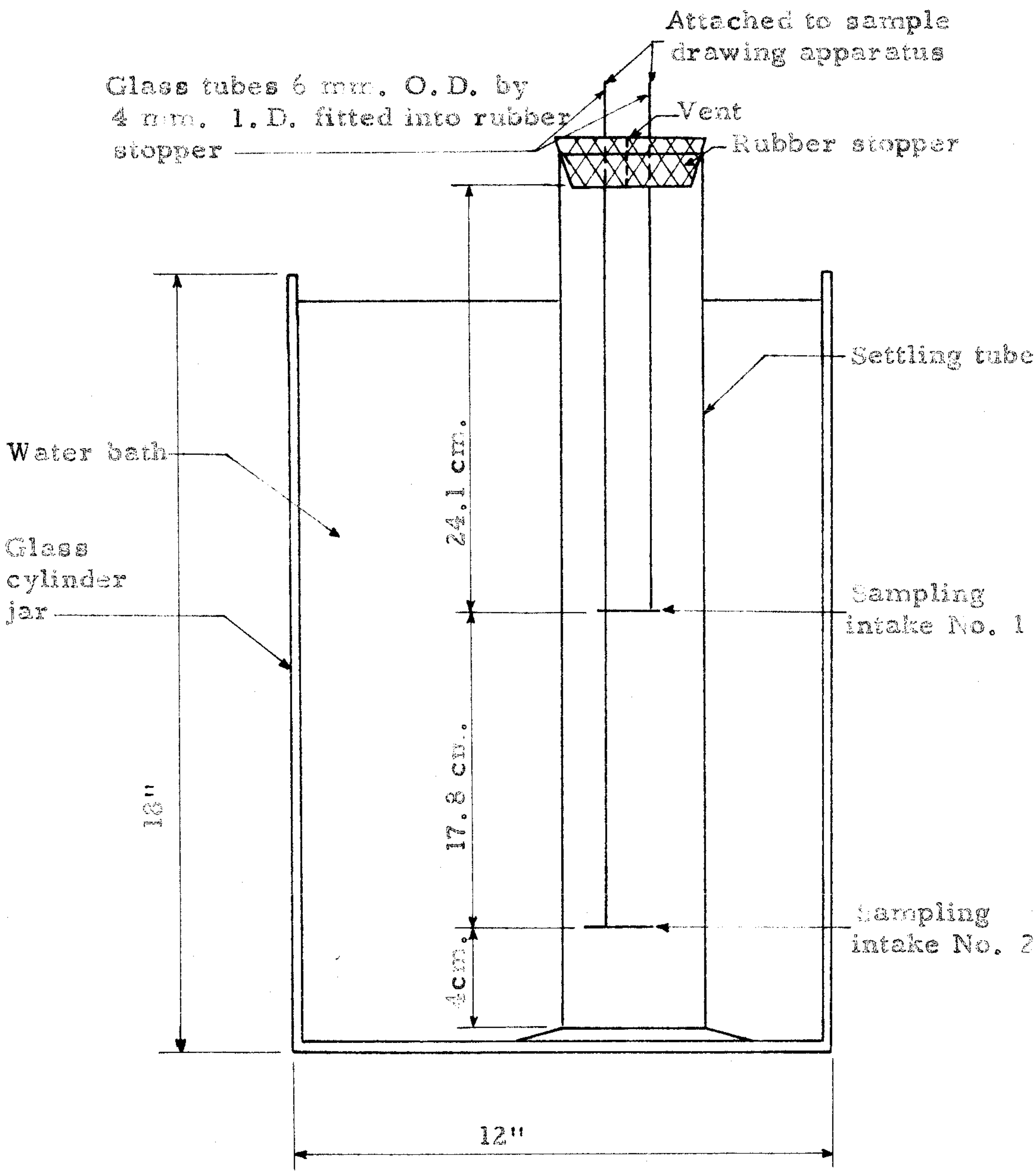
While two-depth tests are not infallible, their results can indicate errors in settling velocity analysis. These tests have a second value as a preliminary study for flocculent suspensions.

b. Apparatus. The apparatus used for taking samples at two depths is shown in figure 4-1. The settling suspension is contained in a settling tube made by flattening the pouring tip of a two-liter graduated cylinder. The graduations were converted to elevation readings and used for surface elevation readings. For temperature control, the settling tube sits in a water bath.

During the experiment, samples are withdrawn through sampling intakes similar to the one shown in figure 4-2. The intakes are attached to glass tubes which are, in turn, attached to a rubber stopper. The tube is assembled by pressing the rubber stopper tightly into the position shown.

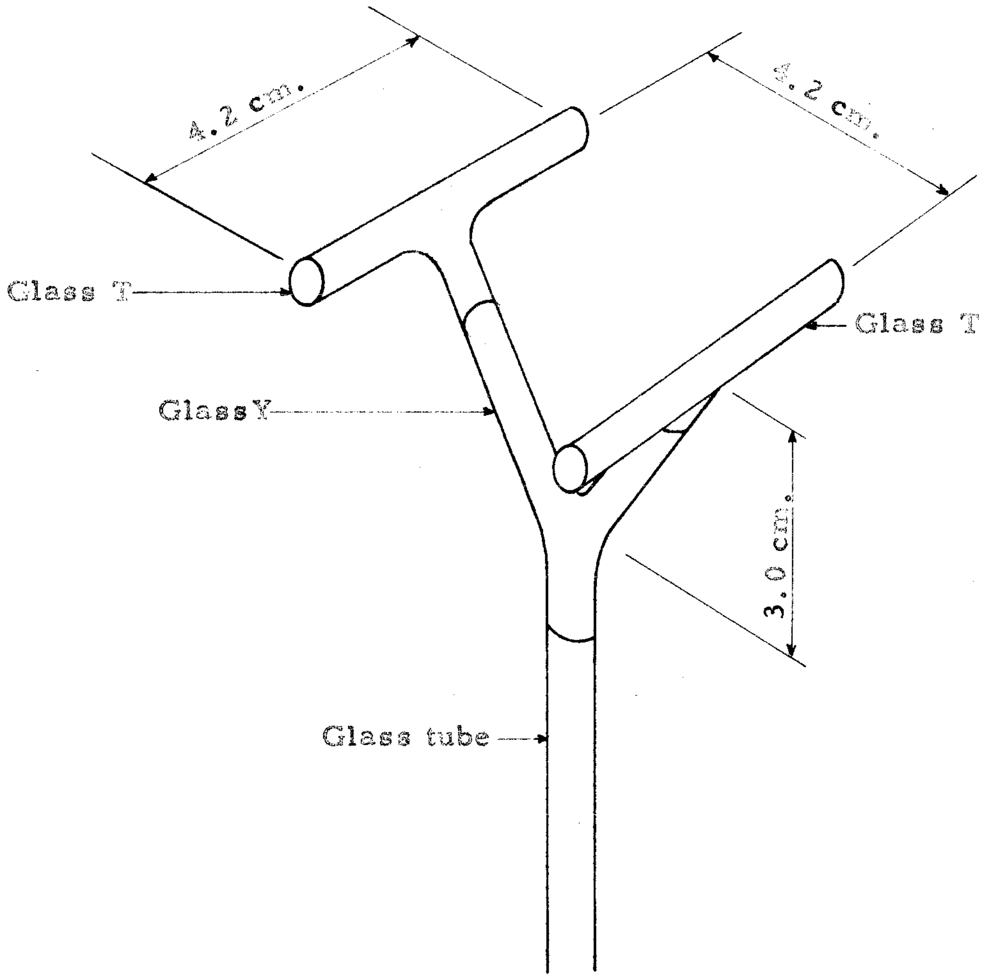
The actual withdrawing of samples is performed with the sample-drawing apparatus shown in figure 4-3. By means of the three-way stop cock, the suspension is made to flow up into the waste tube. This flow frees the intakes and tubes of any suspension remaining from the previous samples. The suspension is then made to flow immediately into the 50-ml. graduated tube where the sample is collected. Between samples the suspension is drained from the waste tubes in preparation for the next sampling operation.

c. Method. Two liters of the suspension to be tested are placed in the settling tube. The tube and contents are placed in the water bath until the suspension reaches the



Note: (1) For typical intake see Fig. 4-2
(2) For sample-drawing apparatus see Fig. 4-3

Fig. 4-1. Two-depth settling tube in water bath.



All glass sections
6 mm. O. D.
4 mm. I. D.

Fig. 4-2. Sampling intake (typical).

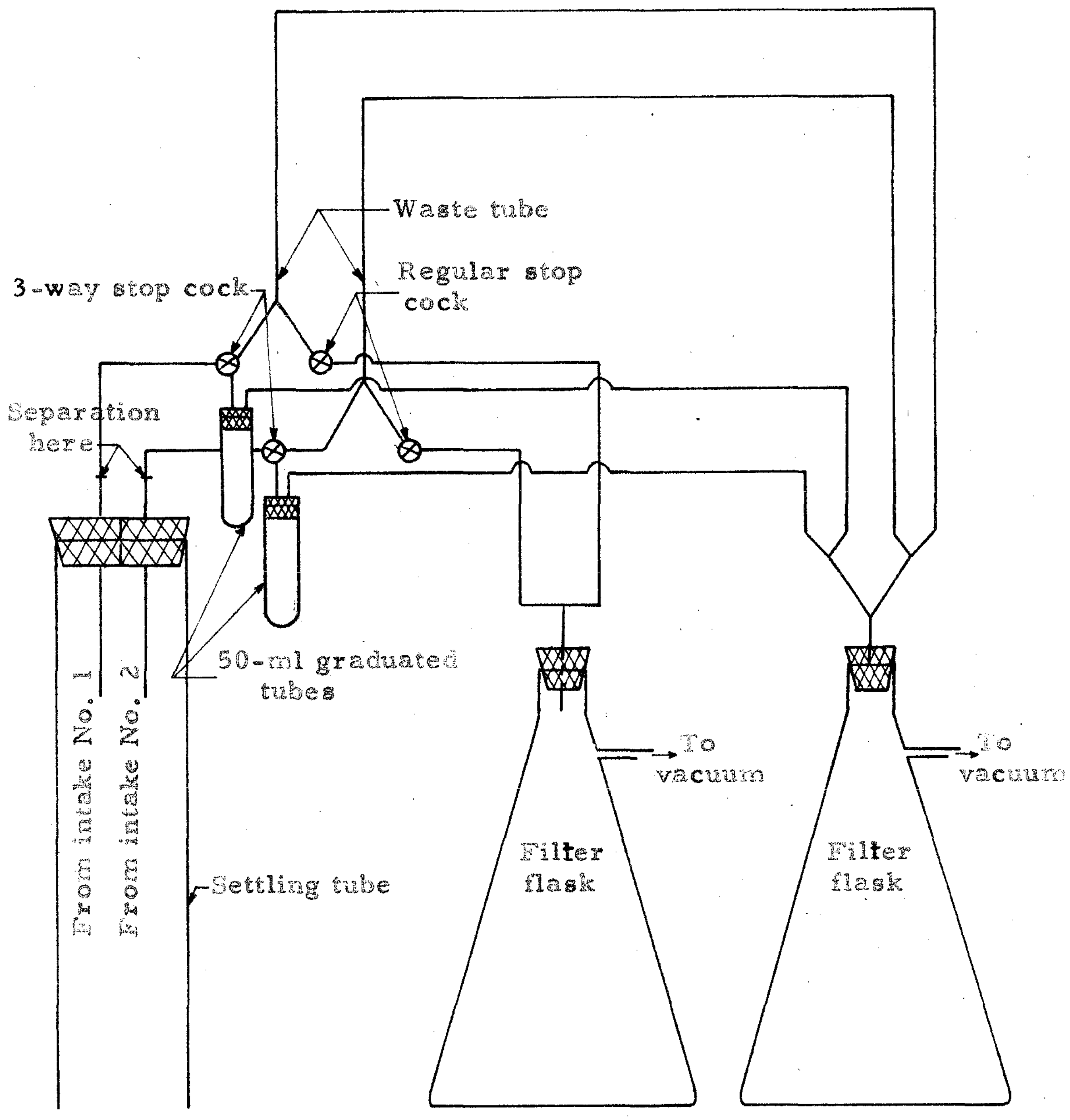


Fig. 4-3. Sample-drawing apparatus.

temperature of the bath.

After assembling the tube, the elevations of the intakes and of the surface of the suspension are read and recorded. The assembled tube is inverted in cycles and shaken for two minutes. At the end of the mixing, the tube is quickly placed in the water bath and the sample-drawing apparatus is attached.

The end of mixing has been arbitrarily selected as the beginning of settling. Therefore, the time of settling, t, for any sample is the time from the end of mixing to the taking of the sample. Samples are taken according to a predetermined time schedule, and after each sample the elevation of the surface of the suspension is read.

The volume of each sample is measured in the graduated tube in which it was collected. Normally the volume is between 30 and 40 ml. The suspended solids in each sample is then determined by filtration through Gooch crucibles according to standard methods (28).

d. Calibration of Apparatus. In order to test the apparatus, experiments were conducted with a suspension of glass spheres* in distilled water. The spheres were obtained in sieved fractions from the manufacturer, and suspension could be made up by weighing out quantities from individual fractions.

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* "Superbrite Glass Beads" manufactured by Minnesota Mining and Manufacturing Co., St. Paul, Minnesota.

According to the research by FIARB (26), the pipette method is not accurate for sand grains larger than 0.06 mm. In water at 25°C these grains would have a settling velocity of approximately 0.4 cm. per sec. The FIARB also found that for particle concentrations of less than 3000 mg. per liter, the experimental results are not consistent enough to be of value.

Preliminary experiments indicated that the FIARB findings might not be correct. It was decided, therefore, to use the apparatus for analysis of a suspension having a total concentration of 1000 mg. per liter and settling velocities from zero to greater than 1 cm. per sec. Such a suspension was made up and tested. The results are shown in figure 4-4. At the high velocities ($w > 0.6$ cm. per sec.) the test is too rushed to permit careful sampling with both intakes. As a result no data were obtained for the larger settling velocities. However, the data for the lower velocities and lower concentrations are not far from a fitted curve.

The test was repeated using one intake at a time. For this purpose two identical suspensions were prepared. One was analyzed using only the upper intake, the other using only the lower. The results are plotted in figure 4-5. Although two points have a significant deviation from any possible smooth curve, the others remain close to a single curve. The two points with large deviation represent samples taken 20 seconds after the beginning of settling. During this time the turbulence from the mixing had a large effect on the

Fig. 4-4. Two-depth settling analysis for glass spheres in distilled water.

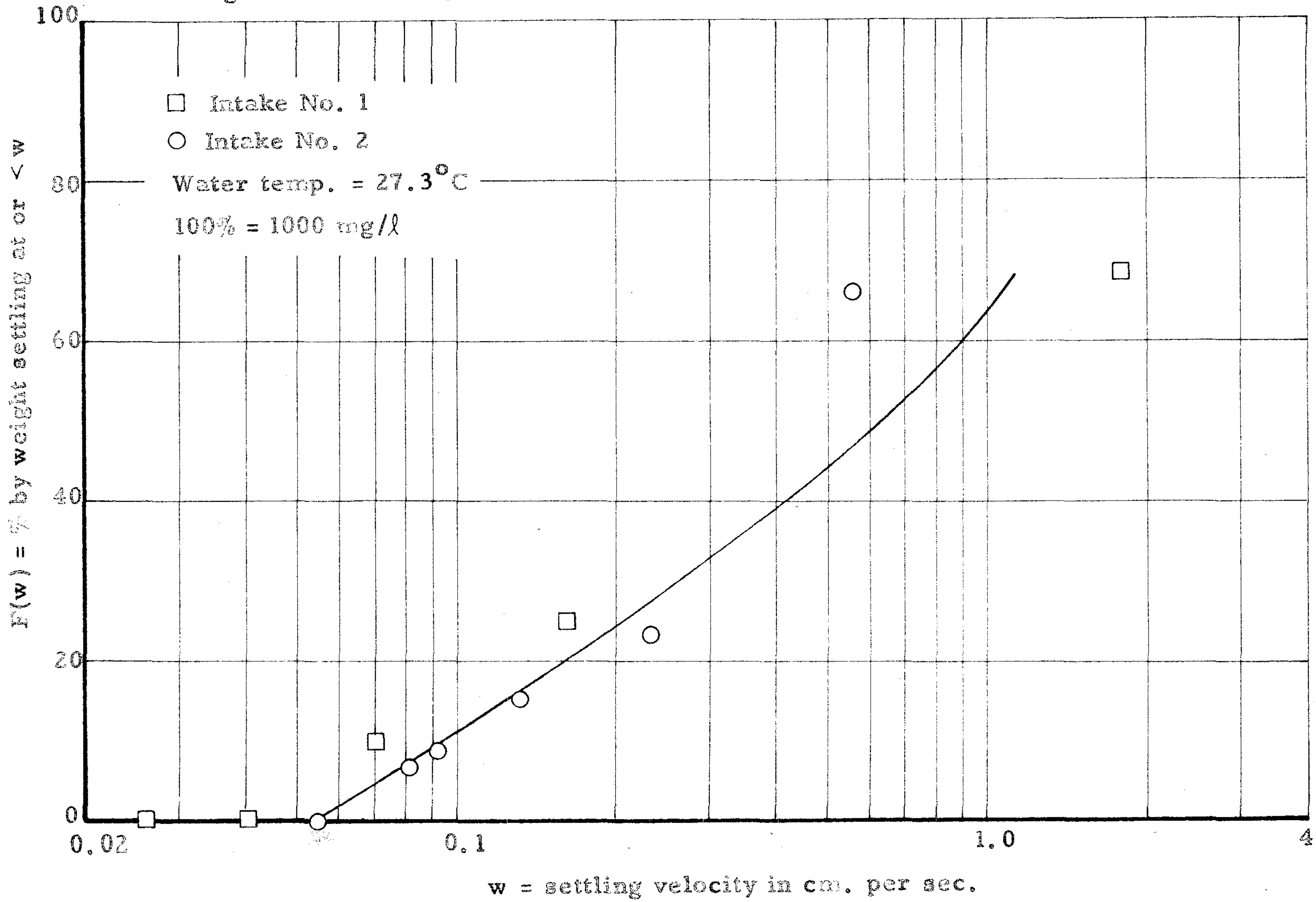
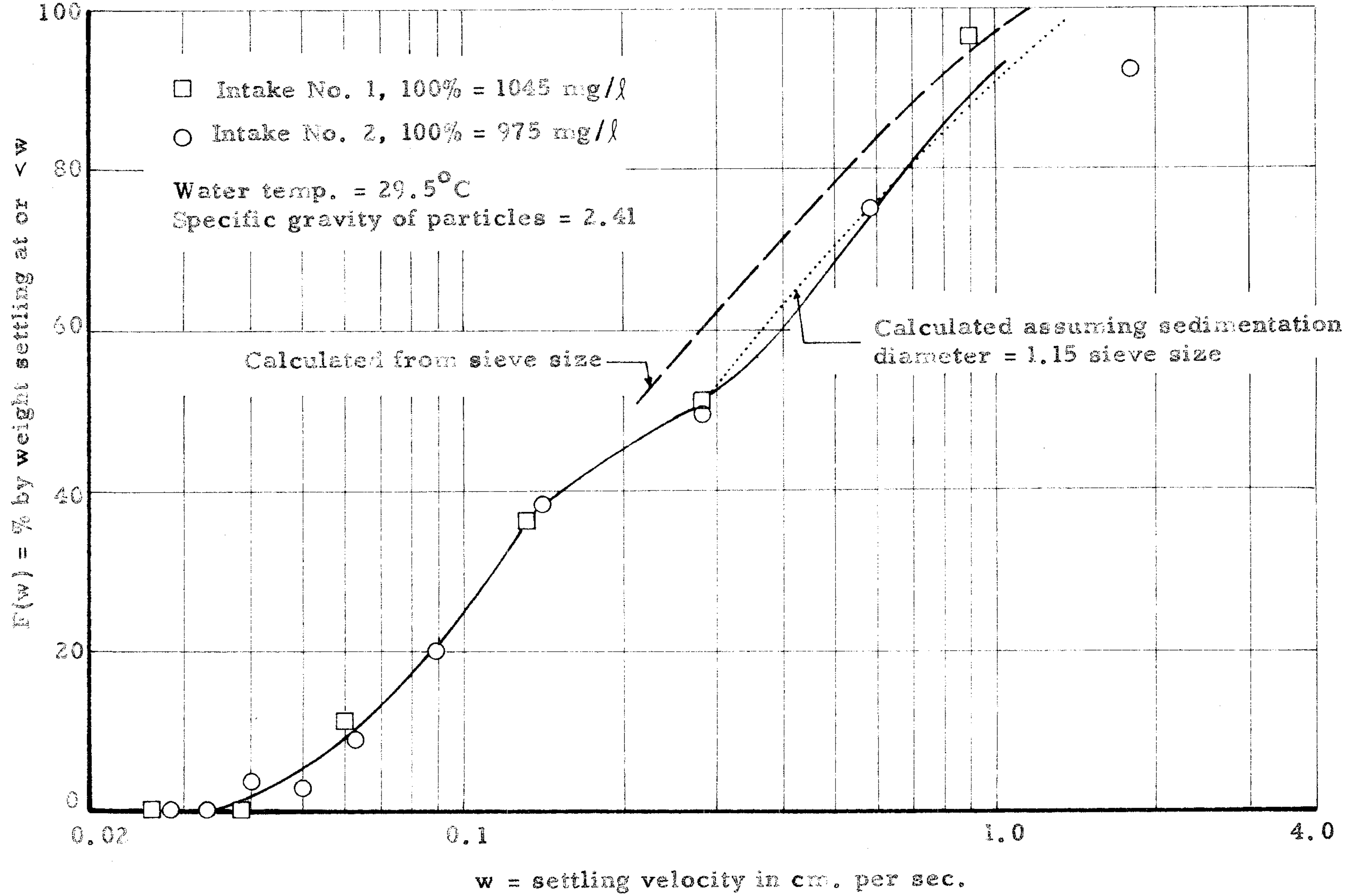


Fig. 4-5. Single depth analysis at each of two intakes for glass spheres in distilled water (see text).



settling of the particles.

A cumulative settling velocity distribution was calculated from the weight of each sieve fraction used in the suspension and the nominal sieve size for each fraction. This distribution was plotted in figure 4-5 as a dashed curve. The calculated and measured distribution do not agree. The only absolute method of determining which is correct consists of dropping individual particles in water and measuring the settling velocities.

Vanoni (29) has measured the settling velocities of individual sand grains. First a sieve analysis of the sample was made with a fourth root of two series of standard Tyler sieves which were shaken for 10 minutes in a Tyler Rotap Machine. For each sieve fraction, Vanoni measured the settling velocities of a number of randomly selected grains. From the measured velocities he calculated a mean sedimentation diameter for each fraction. For sieves finer than 150 mesh, the mean sedimentation diameter of a sieve fraction was found to be approximately 15 per cent larger than the mean sieve opening of that fraction.

The glass spheres used in the two-depth tests were small enough to pass a 150-mesh sieve. Assume, therefore, that the sedimentation diameter is 15 per cent larger than the nominal sieve size. As a result, the calculated settling velocities shown in figure 4-5 would have to be increased by approximately 30 per cent. The velocities shown by the dotted curve in figure 4-5 are based on a sedimentation diameter which is 15 per cent larger than the nominal sieve size. The dotted curve and solid curve agree well. Therefore, the difference between velocities measured in the settling analysis and the velocities calculated from sieve size is consistent with the findings of Vanoni.

There is another reason for believing that the data shown in figure 4-5 are reliable. It is the fact that the results from both intakes fall on one curve for $w \leq 0.7$ cm. per sec. Apparently, these results are independent of depth. This is equivalent to saying that the lines of constant z/t in a $\emptyset$,z diagram would be parallel to the z axis. Consequently, there is little likelihood of flocculation, hindered settling or non-uniform initial spatial distribution of particles in the experiment. This observation and the relatively good fit of the experimental points indicate that the distribution is correct for $w < 0.7$ cm. per sec.

If the apparatus is successful for the suspension tested, it should be more successful for suspensions with slower particles.

<u>e. Consistency of Results.</u> The above mentioned research of the FIARB definitely indicated that the pipette analysis will give very erratic results for particle concentrations of less than 3000 mg. per liter of suspension. Consequently the apparatus had to be tested for consistency.

These tests were performed in the following manner. First, four samples of glass spheres were taken from a single sieve fraction. Two of them were used to make up suspensions while the others were used for sieve analysis. The results of the sieve analysis are shown in figure 4-6. For each run, 10 grams of spheres were sieved for 10 minutes using standard Tyler sieves in a "rotap" machine. The main object of the analysis was to show that successive samples of spheres taken from a single sieve fraction would be similar. Any inconsistency of results in settling analysis would then be the fault of the apparatus.

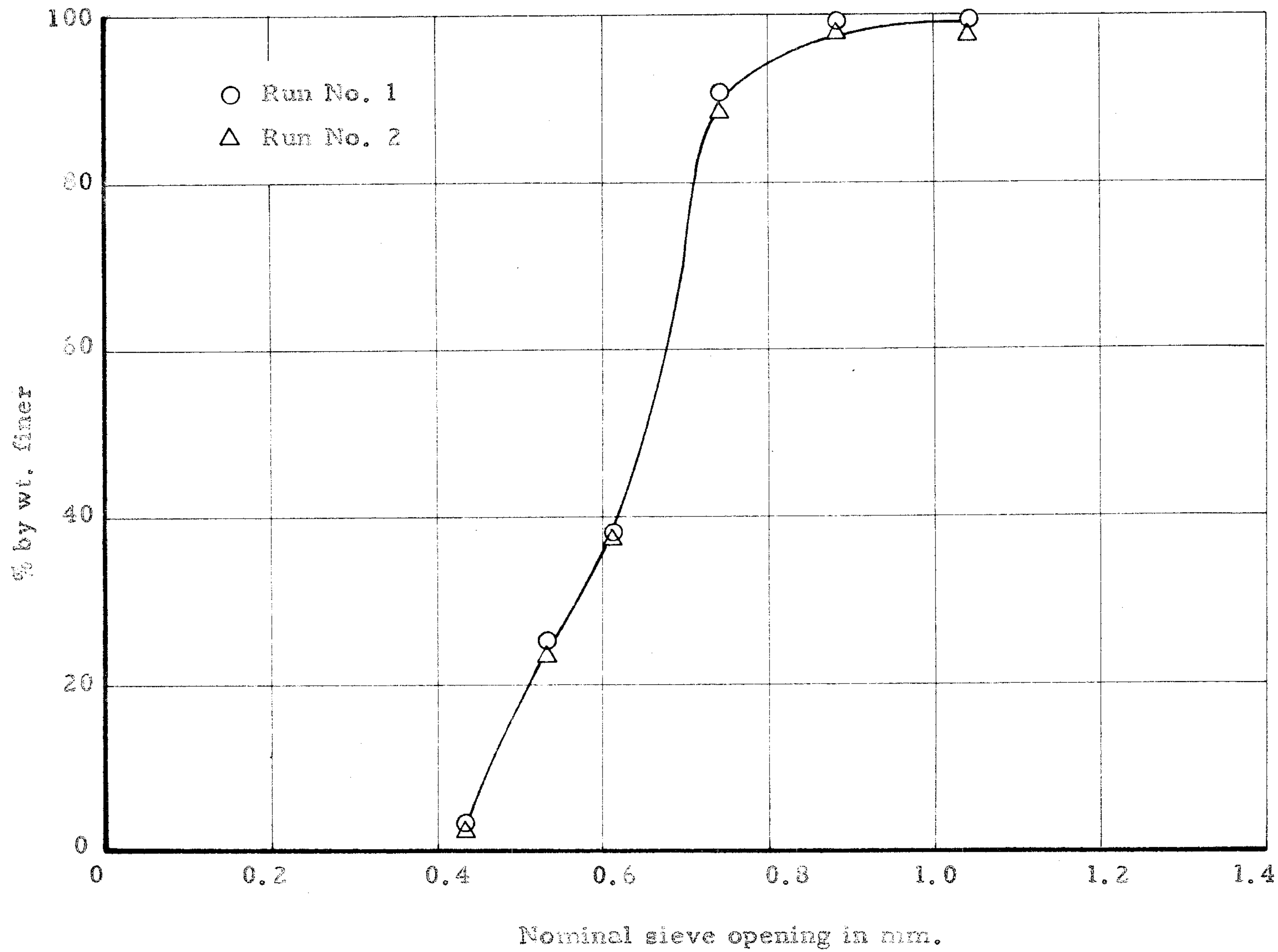
The suspensions used for settling analysis were made up with spheres and distilled water, each contained 1000 mg. of spheres per liter. Each suspension was analyzed in the two depth apparatus shown in figure 4-1, but only the lower intake was used. Consequently, the test was a simple pipette analysis. The consistency of results from two runs can be seen in figure 4-7.

4-3 Multiple Depth Settling Apparatus

a. General. In order to measure concentration profiles in a settling column, it is necessary to have a deep settling tube with sampling intakes at several depths. The settling tube described in this section has a depth of 120 cm. and has intakes at three depths. Obviously, this is not adequate for the study of settling in a deep tank. On the other hand, it is large enough to present many of the problems which must be solved for large scale tests.

By far the most important problem in the use of large

Fig. 4-6. Sieve analysis of a mixture of glass spheres.



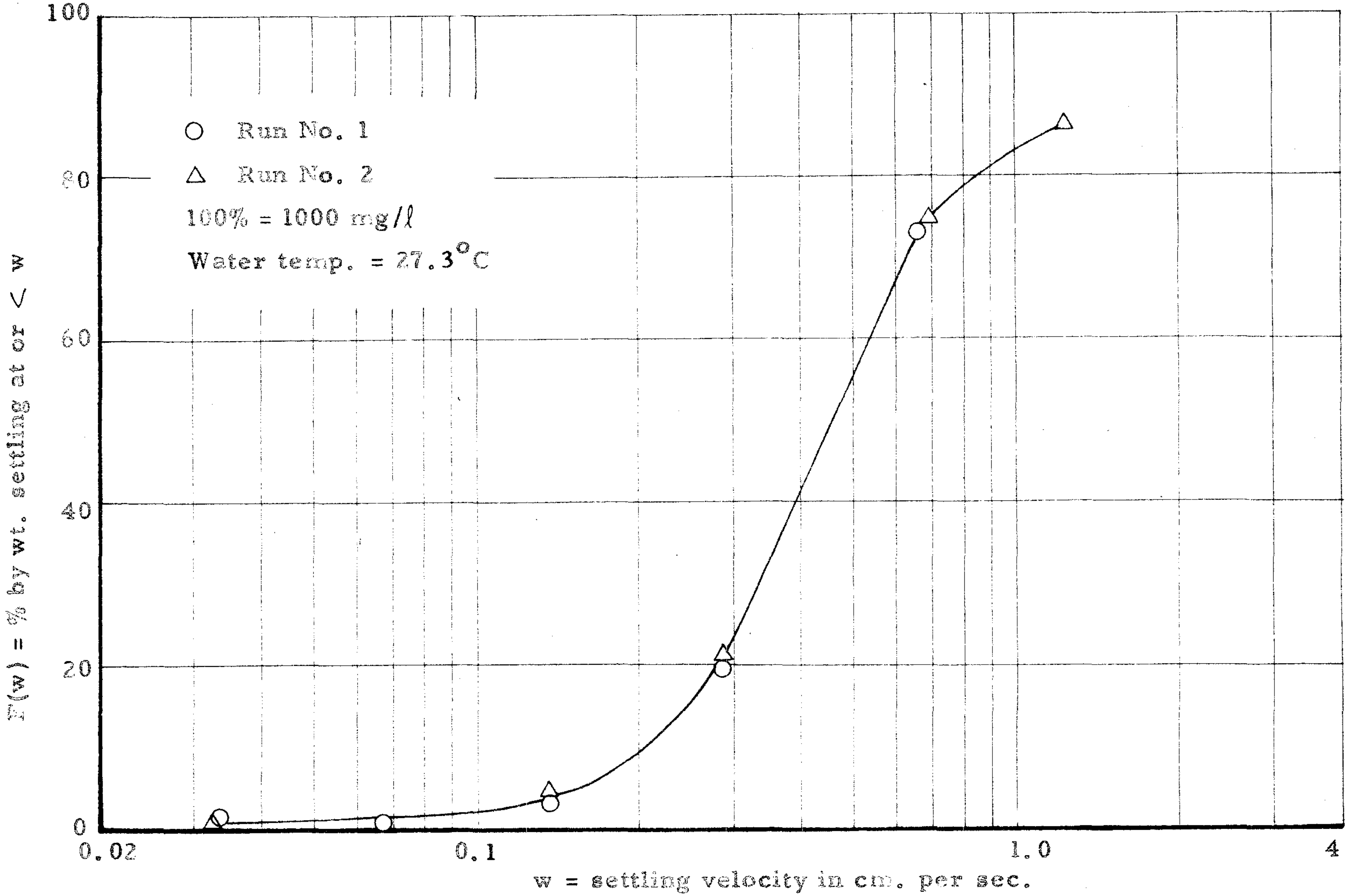


Fig. 4-7. Settling velocity analysis using lower intake of the two-depth settling tube. Glass spheres in water.

deep tubes, is the distribution of particles at the beginning of settling. For all the tests conducted in this research, the aim was to have particles of all settling velocities uniformly distributed. After many fruitless attempts, the problem was solved rather simply. The suspension is well mixed in a separate container and then poured quickly into the settling tube. The separate container is the mixing hopper described below.

Another problem occurring with deep tubes is the control of temperature. In this research the temperature was measured throughout the experiments, but it was not controlled. In addition to the obvious effects of temperature on particle settling velocities, there is the more subtle effect of convection currents. Any change of temperature outside of the tube will cause convection currents inside the tube. No effort was made to determine the effect of these currents.

One must also mention that deep settling tubes present a problem in the design of sampling intakes. If the intakes are attached to tubes which extend from the top or bottom of the suspension, these tubes will be long. They will contain a large volume of suspension. Before each sample this volume of suspension must be flushed out. The greater the volume, the more complicated the sampling.

Fitch (9) has used a settling tube in which the sampling intakes pass through the tube walls. The apparatus appeared to be quite satisfactory, and it has the advantage that all intakes operate by gravity flow.

b. Apparatus. The multiple-depth settling tube is shown in figure 4-8. It is essentially the same as the two depth tube, but there are two differences. At the bottom, there is a third sampling intake which operates by gravity flow. At the top, there is a platform which rests on the walls of the tube. The upper sampling intakes are attached to this platform, in order that they can be quickly positioned at the beginning of the settling.

Another major piece of apparatus is the mixing hopper shown in figure 4-9. It is simply a cylindrical container with a conical bottom. In the center of the bottom is a large opening closed by a rubber stopper. The size of the opening is an essential feature of the hopper. When the stopper is removed the contents of the hopper must fall into the tube before any appreciable settling can occur. The hopper holds seven liters of water which can be released in five seconds or less.

c. Method. The suspension to be tested is poured into the mixing hopper where it is mixed by the mechanical stirrer. The length of mixing time is usually two minutes, during which the temperature of the suspension is measured.

The settling tube is positioned directly under the hopper as shown in figure 4-9. The platform and upper intakes are removed from the tube leaving the top open. At the end of the mixing the stopper is removed and the contents of the hopper fall into the tube. The platform and upper intakes are replaced and the sample-drawing apparatus is attached.

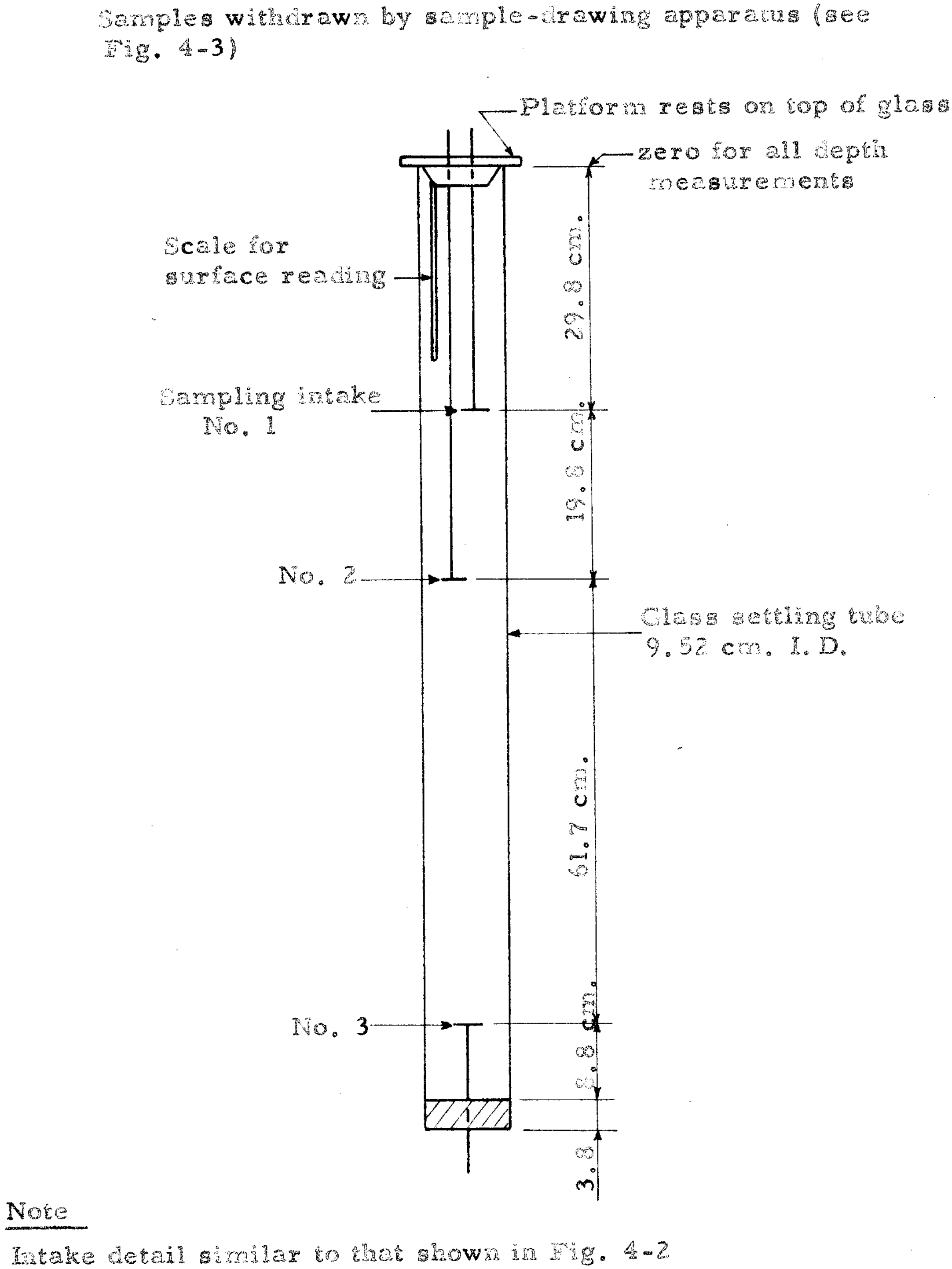


Fig. 4-8. Multiple-depth settling tube.

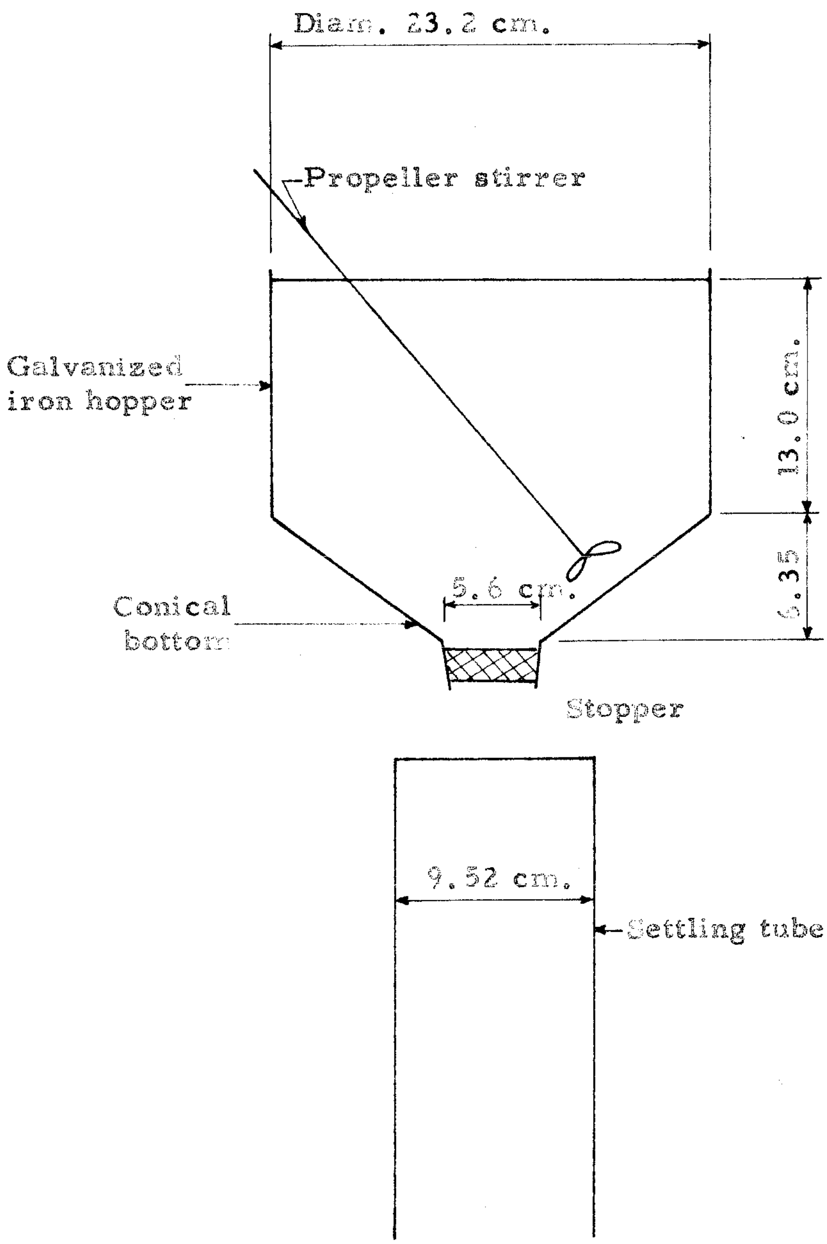


Fig. 4-9. Mixing hopper.

From this point, the rest of the experiment is similar to that for the two-depth tube. The one exception is that extra samples are taken through the third sampling intake. The time of sampling has been arbitrarily measured from the end of pouring. This is not precise but the results indicate that it is satisfactory.

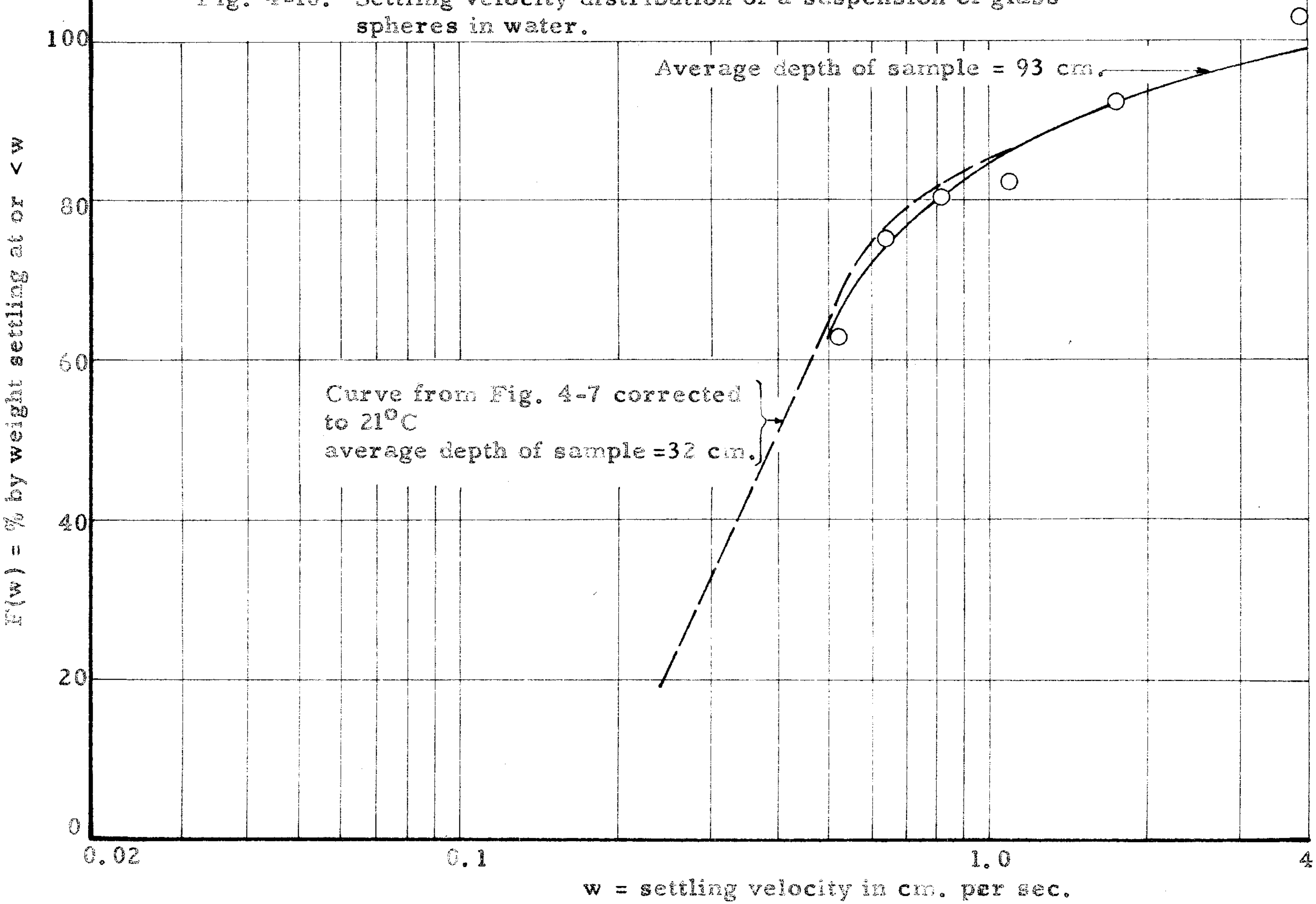
Throughout the experiment the temperature of the samples is measured immediately after they are taken. These measurements give the temperature inside the tube at the time of sampling. Additional temperature measurements are made at the surface of the suspension and in the air around the tube.

d. Comparison with the Two-Depth Tube. The two-depth tube was tested for consistency, and the results are shown in figure 4-7. The suspensions for this test contained glass spheres from a single sieve fraction.

A suspension of spheres from the same sieve fraction was analyzed in the multiple-depth tube. Samples were taken through the bottom intake and analyzed for suspended solids. The results are plotted in figure 4-10 as a solid curve.

The dashed curve of figure 4-10 represents results obtained from a similar analysis of an identical suspension in the two depth tube (figure 4-7). Of course, the data have been corrected so that both curves of figure 4-10 correspond to settling at 21°C. These two curves agree reasonably over the range of common velocities. Such agreement is an indication that both curves are reasonably correct and that the two-depth and multiple-depth tubes give consistent results.

Fig. 4-10. Settling velocity distribution of a suspension of glass spheres in water.



4-4 Multiple-Depth Analysis of a Suspension of Glass Spheres

a. General. A multiple-depth analysis was performed on a suspension of glass spheres in tap water. The purpose was to calculate the ideal removal of particles during quiescent settling. This ideal removal was to be compared with direct measurements of removal in the bottom withdrawal tube discussed in section 4-5.

b. Method. It has already been mentioned that the glass spheres were supplied in separate sieve fractions. By weighing out a definite quantity from each fraction any required mixture can be made. Four similar mixtures were made up in this manner.

Two of the mixtures were used for sieve analyses. They were sieved for 10 minutes through standard Tyler sieves. The sieve analysis curve is shown in figure 4-11.

Another one of the mixtures was mixed with tap water. The resulting suspension was analyzed in the multiple depth settling tube.

c. Observations. The data from the analysis are given in table 4-1 and the results are plotted in figure 4-12. No attempt has been made to find a single cumulative settling velocity distribution. Instead, the figure is considered merely as a plot having concentration as a function of z/t for each intake. As a result, there is a definite regular curve for each intake. Moreover, there is a definite trend; the lower concentrations occur at the lower intakes.

Fig. 4-11. Sieve analysis of a mixture of glass spheres.

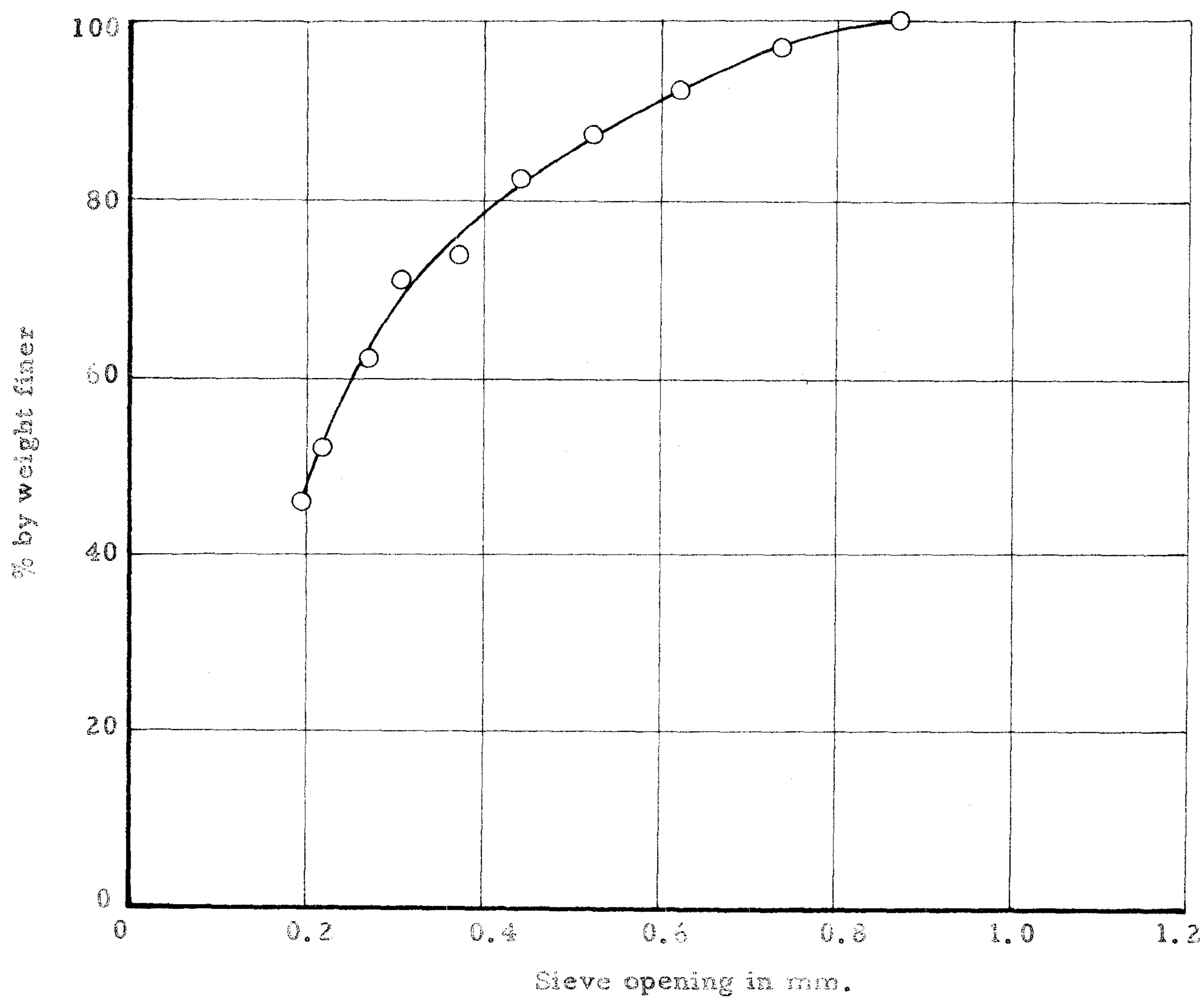


Table 4-1

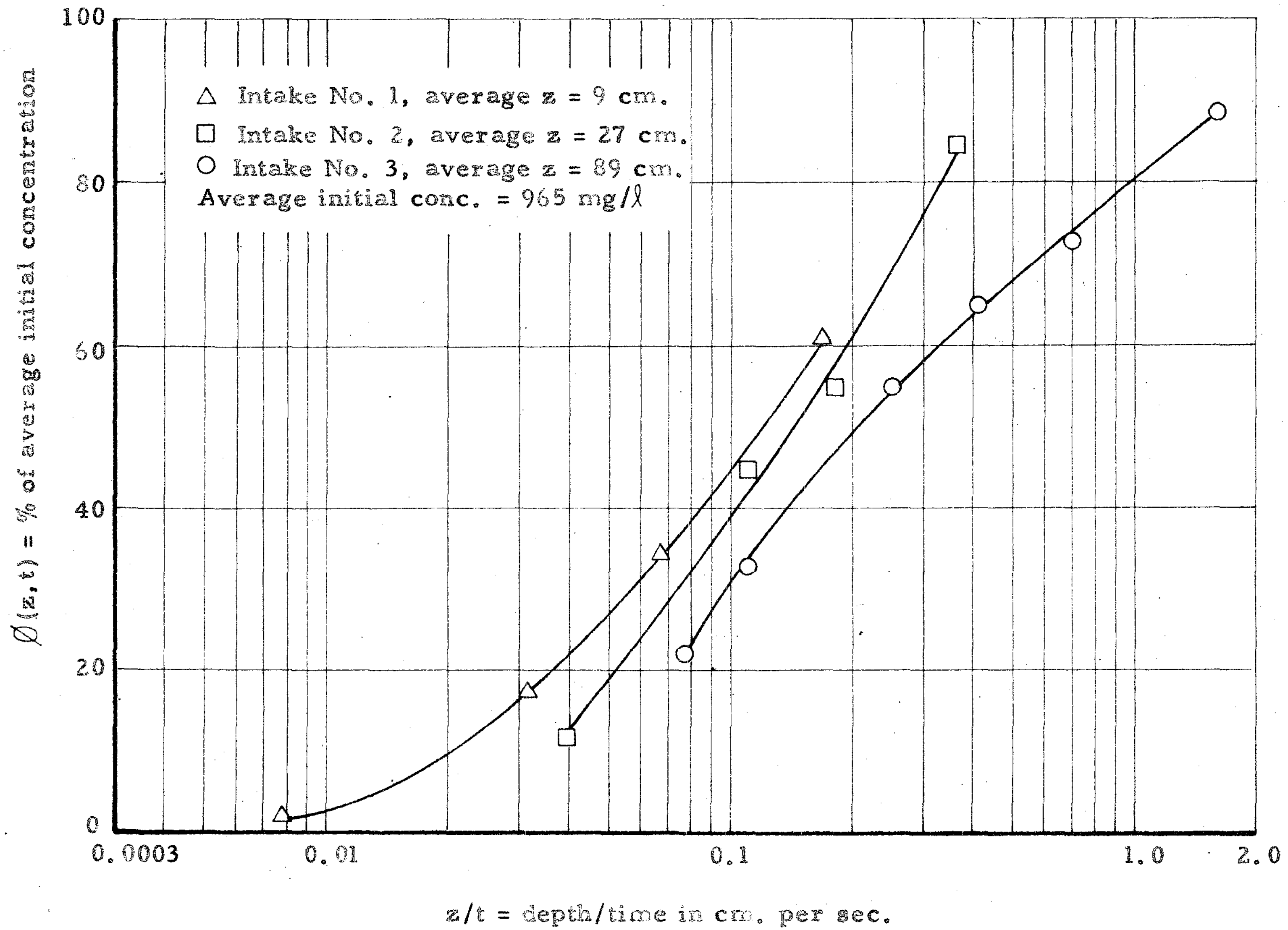
Multiple-Depth Analysis of a Suspension of

Glass Spheres in Water

Time After Start of Settling	Intake	Depth of Intake z	z/t	Volume of Sample	Temp. of Sample	Weight of Spheres in Sample	Conc. of Spheres in Sample Ø(z,t)	% of Average Initial Conc.
sec.	No.	cm.	cm./sec.	ml.	°C.	mg.	mg./ℓ	
0					18.5*			
60	3	94.9	1.6	36		30.8	855	88.5
75	1	12.5	0.17	32		18.7	583	60.5
90	2	32.3	0.36	33		26.9	815	84.5
135	3	94.0	0.70	38		26.6	700	72.5
150	1	10.0	0.067	30		10.0	328	34.0
165	2	29.8	0.18	31		16.6	525	54.4
225	3	91.5	0.41	42		26.0	627	65.0
240	1	7.4	0.031	39		6.6	169	17.5
255	2	27.2	0.11	45		19.4	430	44.5
365	3	88.9	0.25	42	18.5	22.4	533	55.2
480	1	3.8	0.0079	32		0.4	13	1.4
600	2	23.6	0.039	30	19.0	3.3	112	11.6
795	3	85.3	0.11	42	19.0	13.0	313	32.4
1083	3	82.6	0.076	44	19.0	9.3	214	22.2

* Measured in hopper during mixing.
Room temperature during test = 20.5 °C.

Fig. 4-12. Multiple-depth analysis of a suspension of glass spheres in water.



d. The ϕ,z Diagram. The curves of figure 4-12 show that for constant z/t the concentration decreases with depth. That is,

$$\left(\frac{\partial \phi}{\partial z}\right)_K < 0 \qquad (4\text{-}1)$$

Furthermore, the particles are discrete and the theory of section 3-2 is applicable. From this theory it must be concluded the initial distribution was non-uniform. Specifically, the equation 4-1 means that the initial concentration, $\phi(z,0)$ decreased with depth.

In order to investigate this conclusion more closely a diagram was drawn as shown in figure 4-13. First the data were plotted as ϕ against z for each intake, as shown by the dotted lines of the diagram. These lines correspond to the curves in figure 4-12. Next, the lines of constant z/t were plotted (dashed curves of figure 4-13).

By using the two sets of broken curves, approximate concentration profiles were sketched on the diagram. The profile for t = 0 is strictly hypothetical, but it is actually not needed in the analysis which follows. The other profiles have been calculated. Once they were obtained, the removal ratio R(D,t) was calculated as a function of overflow rate D/t for D = 90 cm. In figure 4-14, this removal is shown as a solid curve.

Figure 4-14 also contains three dashed curves. For each

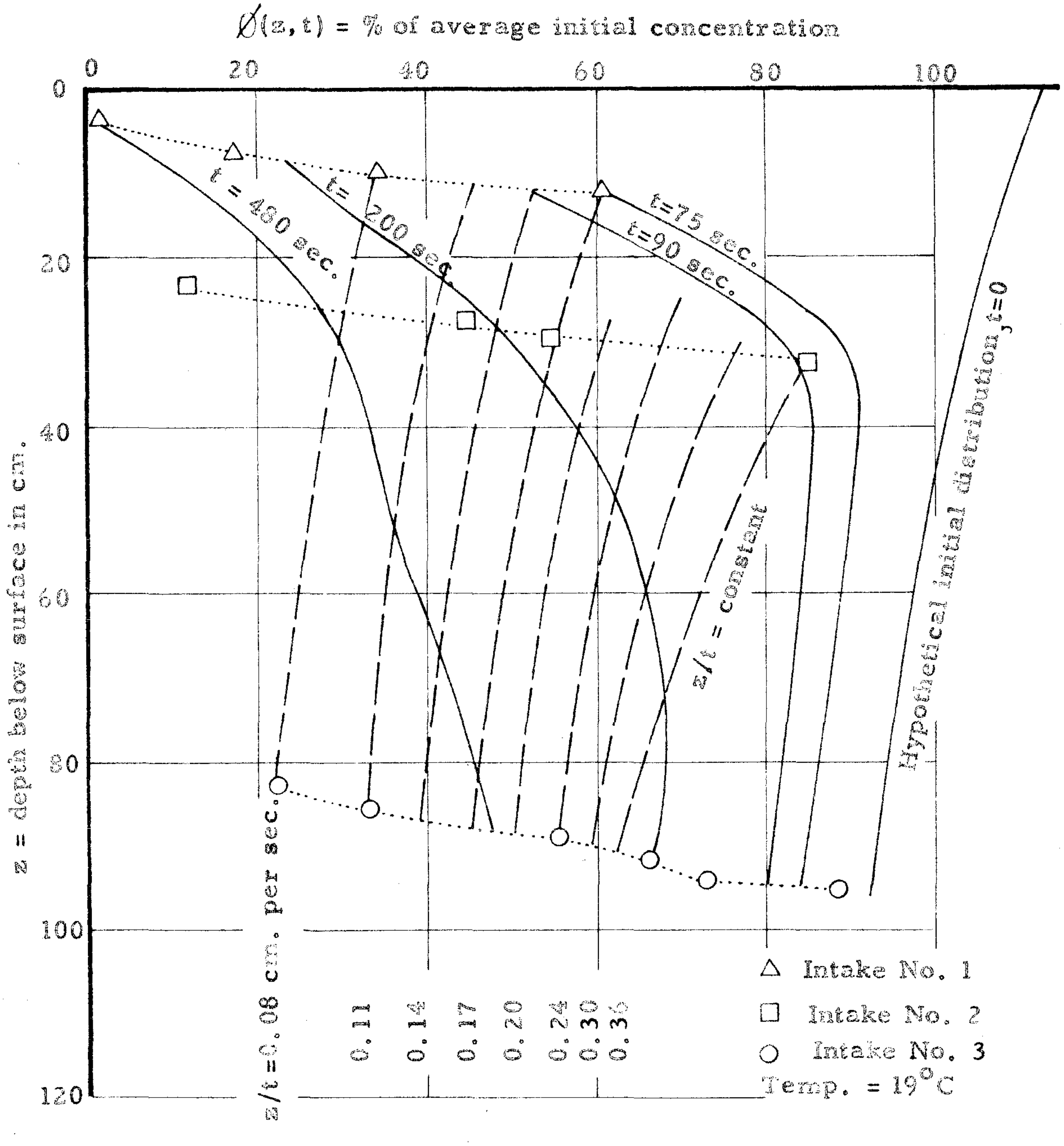


Fig. 4-13. Ø, z diagram for a suspension of glass spheres in tap water.

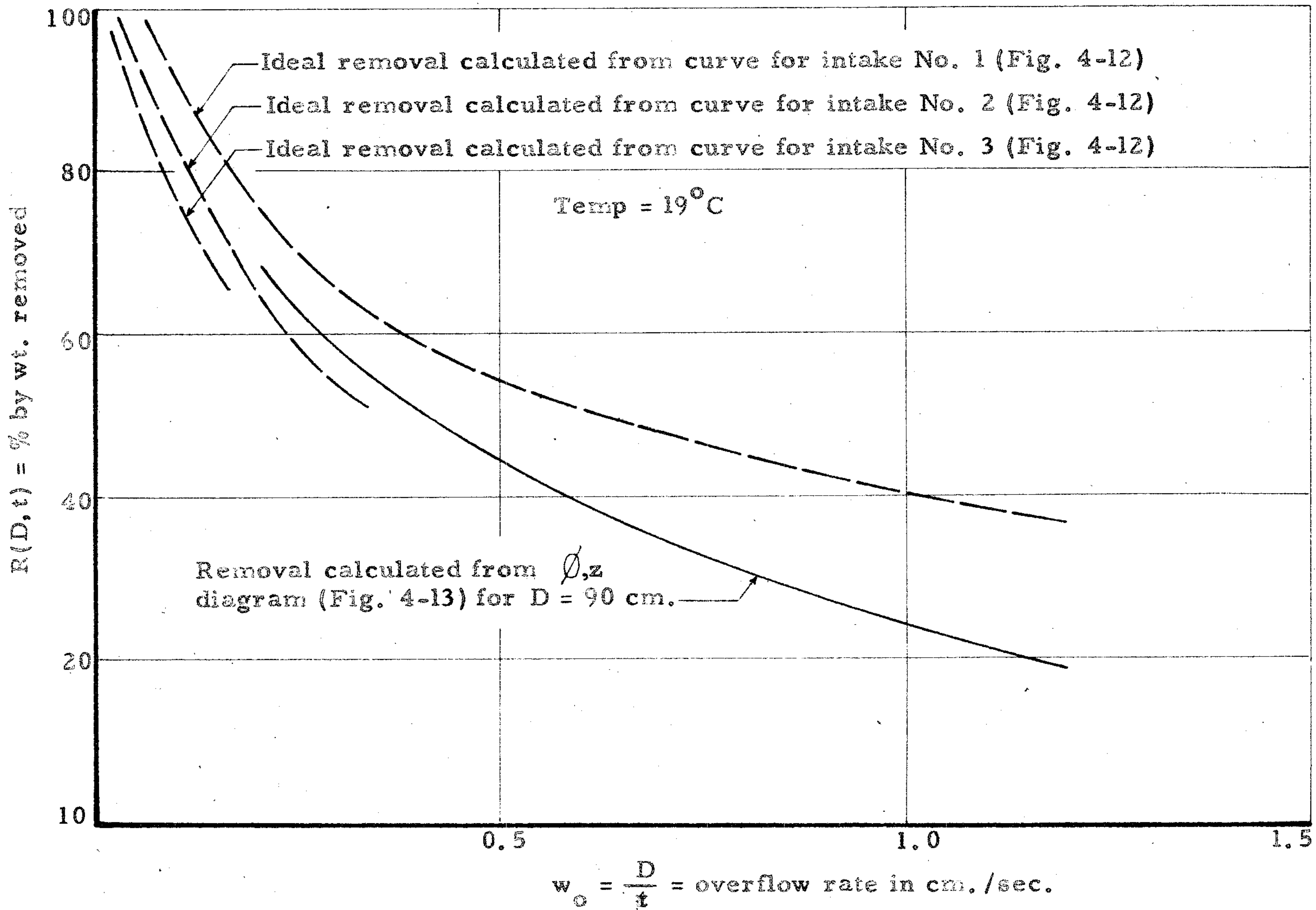


Fig. 4-14. % removal as a function of overflow rate.

curve, the data from one intake (figure 4-13) were used for a calculation of ideal removal (equation 2-8). It can be seen that the "ideal" removal based on data from only one intake is incorrect.

This error is due to the fact that the initial distribution of concentration $\phi(z,0)$ was not uniform. The non-uniformity was probably the fault of mixing hopper operation. Since the procedure was to turn off the stirrer as the suspension began to leave the hopper, it is possible that some particles settled in the hopper. As the last of the suspension left the hopper, it could have washed the settled particles out causing a high initial concentration at the top of the settling column.

The operation of the mixing hopper was modified for subsequent tests. However, it was not modified for the bottom withdrawal experiment described in section 4-5. Thus, the operation was similar for the multiple-depth analysis just described and the bottom withdrawal experiment described below.

4-5. The Bottom Withdrawal Tube

a. General. A logical way to measure the total removal in a column is to collect the settled matter at the bottom. In this type of experiment the suspension is allowed to settle in a container. Periodically during the settling, samples are withdrawn from the bottom of the container. Each sample is assumed to possess all of the particles which have reached the bottom since the previous sample. Consequently, the sum of the residues from the samples taken during time t is equal to the total removal during that time.

Preliminary experiments using this principle have been unsuccessful. The removal measured by bottom withdrawal has always been less than the removal predicted by other methods. It was found that the measured removal was greatly affected by the shape of the bottom of container in which settling takes place. Experiments with transparent suspensions have shown that particles which settle on the bottom are not easily removed by withdrawing a sample.

The apparatus described below was designed with special regard for the problem of removing particles at the bottom. In principle the apparatus is similar to the bottom withdrawal tube developed and tested by the Federal Inter-Agency River Basin (FIARB) Subcommittee on Sedimentation (24,26). In fact, the tube described below is so similar that it has been given the same name.

b. Apparatus. The bottom withdrawal tube used in this research is shown in figure 4-15. It consists of a vertical glass cylinder with a glass funnel attached to the bottom. Flexible tubing is attached to the bottom of the funnel. When samples are not being withdrawn, the tubing is closed by a pinch cock.

Inside the funnel is a small scraper made of thin plywood and cut to fit sides of the funnel. The scraper is attached to a thin rod which extends above the top of the

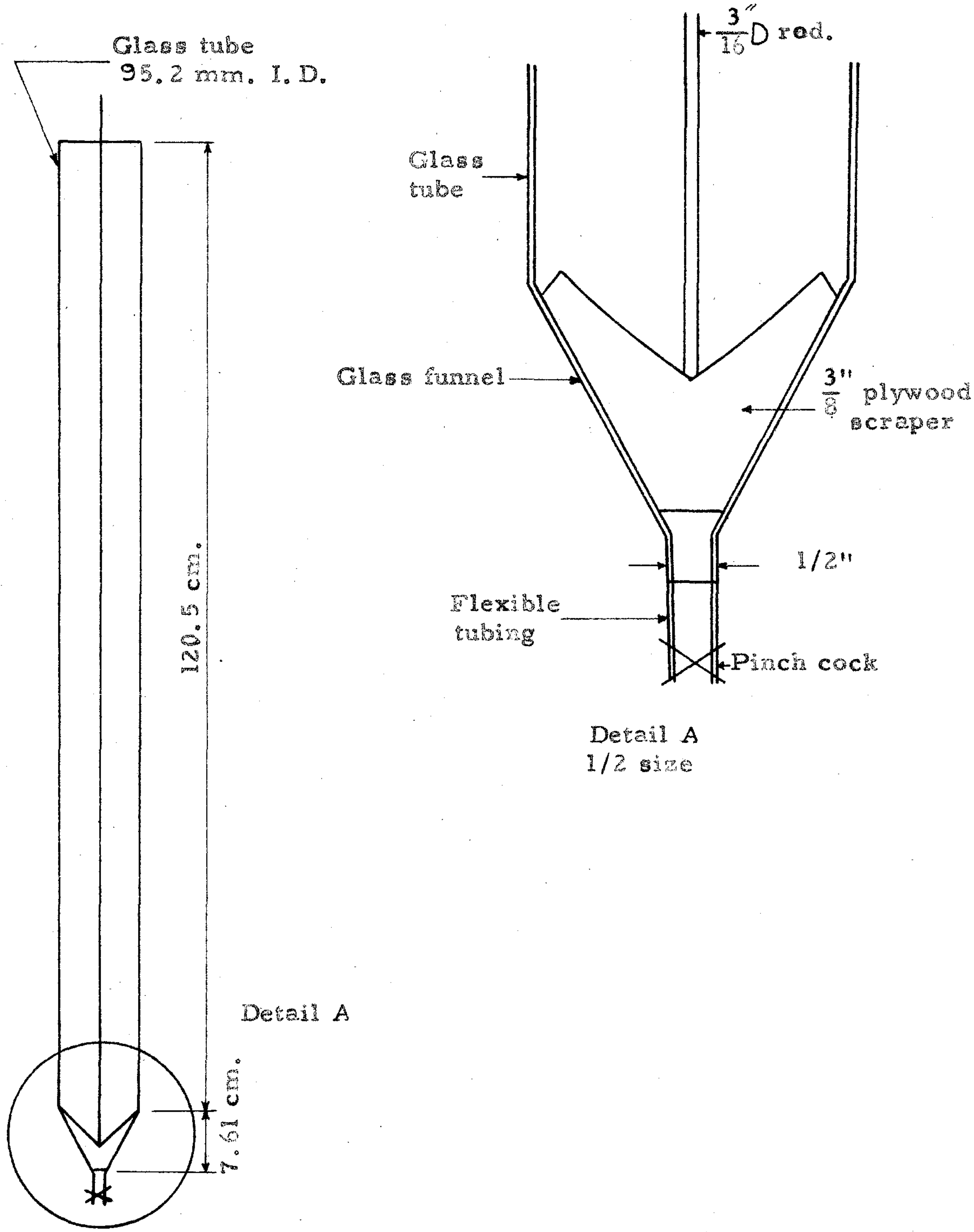


Fig. 4-15. Bottom withdrawal tube.

cylinder. Before each sample the rod is slowly twisted for a few revolutions so that particles adhering to the sides of the funnel will be forced to the bottom. It has been observed that the action of the scraper causes negligible disturbance to the suspension above the funnel. Since the volume of each sample is larger than the volume of the funnel, all of the disturbed suspension is removed in the sample.

c. Method. The suspension to be tested is mixed in the mixing hopper from which it falls into the bottom withdrawal tube. Periodically the scraper is turned and samples are withdrawn from the bottom of the tube. The samples are analyzed for suspended solids.

d. Analysis of the Suspension Described in Section 4-4. Four similar mixtures of glass beads were made up. Three of them were used as described in section 4-4. The fourth was made into a suspension and tested in the bottom withdrawal tube.

The data from the test are given in table 4-2 and plotted in figure 4-16. The plotted results have been joined by the smooth curve 1 . The curve 2 which is reproduced from figure 4-14 for convenience, gives the removal calculated by means of the ϕ,z diagram. It intersects curve 1 at an overflow rate of 1.2 cm. per sec. The relationship between curves 1 and 2 can be explained by means of the following visual observations made during the bottom withdrawal test.

Table 4-2

Bottom Withdrawal Analysis of a Suspension

of Glass Spheres in Water

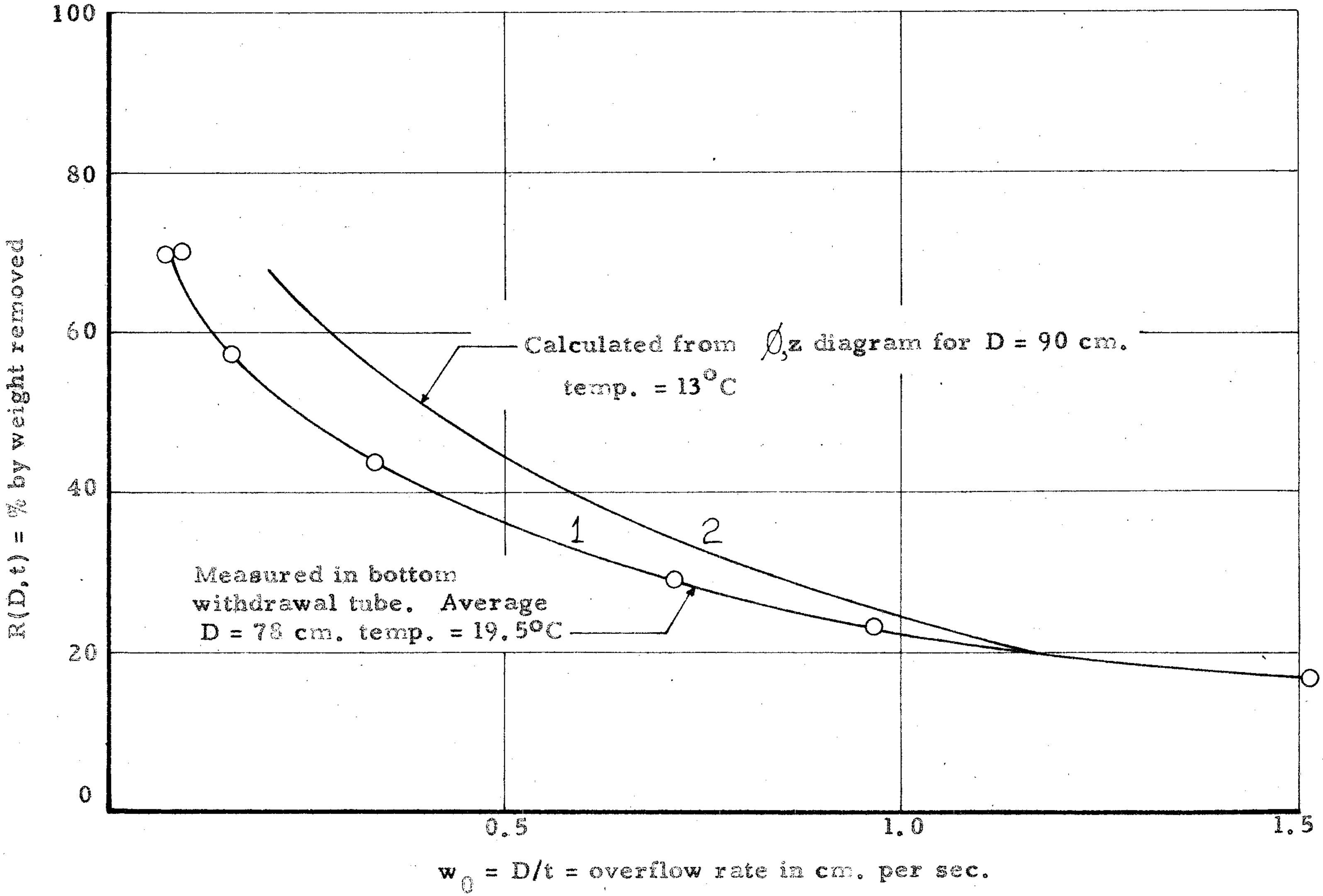
Time After Start of Settling t	Depth to Top of Sample D	Overflow Rate D/t	Volume of Sample	Temp. of Sample	Weight of Spheres in Sample	Total Weight of Spheres Removed	Removal Ratio R(Ø,t)
sec.	cm.	cm./sec.	ml.	°C.	gm.	gm.	%
0							
62	93.5	1.52	330		1.4087	1.41	16
92	88.5	0.97	335		0.6626	2.07	22
120	83.5	0.71	370	19.5	0.5642	2.64	28
240	79.0	0.33	405	19.5	1.1030	2.74	0.42
482	72.0	0.15	400	19.5	0.8655	4.60	0.55
780	67.0	0.085	435	19.5	0.7507	5.35	0.67
1020	60.5	0.063	470	19.5	0.1546	5.51	0.66

Room temperature during test = 20.0°C.

At t = 0 suspension contained 6.76 gm. of spheres and 7 liters of water.

Average initial concentration = 965 mg./ℓ.

Fig. 4-16. % removal as a function of overflow rate (see text).



At the beginning of the settling, only a few particles were seen adhering to the sides of the funnel. All of the settled particles must have been collected by the early samples which correspond to the large overflow rates for which the curves converge.

As the test proceeded, the scraper became only partially effective in keeping particles from adhering to the sides of the funnel. The increasing length of time between samples seemed to give the particles a better opportunity to adhere to the sides of the funnel. Consequently, the amount of settled particles remaining in the tube increased with time.

4-6 Discussion

Each of the three settling tubes described in this chapter has a specific place in settling analysis. All three might be used in the complete investigation of the settling properties of a suspension.

The two-depth settling analysis is valuable in preliminary studies of a suspension about which nothing is known. It was shown in section 4-2 that this analysis gives accurate and consistent results for a suspension with a particle concentration of 1000 mg. per liter and particle settling velocities up to 0.7 cm. per sec. Research by the FIARB indicates that this type of experiment will be more accurate for higher concentrations and smaller velocities.

The data from a two-depth analysis should be plotted showing concentration $\phi(z,t)$ as a function of z/t. If the data from both intakes fall on a single curve there is little likelihood of flocculation, hindered settling or non-uniform initial distribution of concentration.

Because of the calibration tests with glass spheres, one can rely on the uniformity of initial concentration $\phi(z,0)$. Therefore, if the data from each intake define a separate curve, flocculation or hindered settling occurred during the experiment. The difference between the curves is a measure of the effect of flocculation or hindered settling.

The two-depth analysis is useful in testing the consistency of settling properties of a suspension. Suppose that the results of analysis were consistent for a series of samples from a given suspension. Suppose, in addition that there was a consistent difference between the curves for the two intakes. Not only would the suspension be similar in all samples but the effect of flocculation or hindered settling would also be similar in all samples.*

The multiple-depth analysis is necessary for a comprehensive study of a suspension. The results are plotted in a ϕ,z diagram. Once the diagram is obtained, all of the theory of chapters 2 and 3 can be applied to the suspension. In particular, the removal can be determined as a function of overflow rate or detention time. The multiple-depth analysis is also necessary for a calculation of the effect of flocculation on settling at a point.

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* In section 6-1c the two-depth analysis is used to show the reproducibility of flocculation in a suspension of clay and alum in water. In section 7-3d the same test is used to show the variation of flocculation for raw Pasadena sewage.

For direct measurement of removal in routine analysis, the bottom withdrawal tube is most appropriate. For example, in the investigation of the removal in an existing settling tank of fixed depth, one would employ a bottom withdrawal tube as deep as the tank. One would then determine the removal as a function of overflow rate.

While the two-depth tubes seems to be adequate for its purposes, the multiple-depth tube and the bottom withdrawal tube both require some modification. The multiple-depth tube should be equipped with two more intakes, one with a depth of 65 cm. and the other near the surface of the suspension. The bottom withdrawal tube, on the other hand, should be equipped with a more efficient scraper. Such a scraper should fit snugly against the walls of the funnel and displace a small amount of fluid as it moves. In addition, it should rotate continuously at a small angular velocity.

These improvements are only a part development which should eventually take place. Both the multiple-depth tube and the bottom withdrawal tube should be increased in size (e.g. 15 feet deep and 1 foot in diameter). Both tubes should be equipped with means of controlling temperature and introducing turbulence into the suspension. In order to fill such tubes an adequate mixing hopper will have to be designed.

CHAPTER 5

QUIESCENT SETTLING OF FLOCCULENT SUSPENSIONS

This chapter contains an analytical discussion of the flocculation that occurs during quiescent settling. The purpose of the analysis is to determine the precise relationship between the flocculation and the removal of suspended particles.

In section 5-1, the effect of flocculation is introduced into the continuity equation. This analysis gives the effect of flocculation on a continuum of particles. In contrast, the succeeding section presents an analysis based on the motion of individual particles, that is, a kinetic theory of flocculation. The kinetic theory is carried far enough to show how experimental data can be used to build up an empirical description of flocculation at a point.

In the literature there is one example of an experimental study of the kinetic theory of flocculation during settling. The work is described and discussed in section 5-3 which concludes the chapter.

This chapter is closely related to chapter 6 in which the experiments with flocculent suspensions are described and analyzed.

5-1 The Effect of Flocculation upon the Settling of Particles

In spite of its frequent use, the word "flocculation" seems to have no universal definition. Therefore, it is necessary to define the term arbitrarily for the purpose of discussion. Throughout this thesis, flocculation refers to the actual agglomeration of suspended particles. Thus, when two or more suspended particles collide, become connected, and behave as a single particle thereafter, the phenomenon is called flocculation. Suspensions in which the particles have a tendency to flocculate are called flocculent suspensions, and the particles are called flocculent particles.

When two particles collide and settle as a single particle, the resultant settling velocity is different from the two original settling velocities. The resultant mass, on the other hand, is the sum of the masses of the original particles. Consequently, the product of mass and velocity changes while the mass remains constant.

This argument can be extended to include all the particles in a unit volume of suspension. Any flocculation involving particles in the volume does not change the mass per unit volume. Therefore, flocculation does not directly contribute to the local removal $\frac{\partial \phi(z,t)}{\partial t}$. The direct affect of flocculation is to change the local mean settling velocity $\overline{w}(z,t)$. However, $\overline{w}(z,t)$ also changes during the settling of discrete particles without flocculation. If some faster particles settle out of suspension at z the remaining particles have a smaller mean settling velocity. The change in $\overline{w}$ caused by settling (as if the particles were discrete) can be given the subscript s and can be written

$$\left(\frac{\partial \overline{w}}{\partial t}\right)_s$$

The change due to flocculation can be given the subscript F and can be written

$$\left(\frac{\partial \overline{w}}{\partial t}\right)_F$$

The local mean settling velocity is related to the local removal by the continuity equation 3-4

$$\frac{\partial \phi}{\partial t} = \frac{-\partial a}{\partial z} = \frac{-\partial (\overline{w}\phi)}{\partial z} \qquad (3\text{-}4)$$

Differentiating both sides of equation 3-4 with respect to t gives the following result

$$\frac{\partial^2 \phi}{\partial t^2} = -\frac{\partial^2 (\overline{w}\phi)}{\partial t \partial z} \qquad (5\text{-}1)$$

The quantity $\frac{\partial \overline{w}\phi}{\partial t}$ in equation 5-1 can be divided into the change due to flocculation

$$\left(\frac{\partial \overline{w}\phi}{\partial t}\right)_F = \phi\left(\frac{\partial \overline{w}}{\partial t}\right)_F = \left(\frac{\partial a}{\partial t}\right)_F$$

and the change due to settling

$$\left(\frac{\partial \overline{w}\phi}{\partial t}\right)_s = \left(\frac{\partial a}{\partial t}\right)_s$$

Equation 5-1 then becomes

$$\frac{\partial^2 \phi}{\partial t^2} = -\frac{\partial}{\partial z}\left[\phi\left(\frac{\partial \overline{w}}{\partial t}\right)_F + \left(\frac{\partial \overline{w}\phi}{\partial t}\right)_s\right]$$

$$= -\frac{\partial}{\partial t}\left[\left(\frac{\partial a}{\partial t}\right)_F + \left(\frac{\partial a}{\partial t}\right)_s\right] \qquad (5\text{-}2)$$

Equation 5-2 shows that the immediate effect of flocculation is a contribution to the rate of change of local removal. Therefore, to measure the local rate of flocculation directly, it is necessary to measure the second derivative with respect to time of the concentration. However, this second derivative will also include the effect of settling alone. The problem is simplified if the second derivative can be obtained at a point where settling is not causing any change. The theory behind this simplification is described in section 5-3. When the simplification is not possible, additional measurements are required.

5-2 A Kinetic Theory of Flocculation

In section 5-1, flocculation was discussed from the point of view of a continuum of particles. Most of the existing theories of flocculation are based on the kinetics of individual particles, and, hence, are kinetic theories of flocculation. A successful kinetic theory will give more detailed information about flocculation than will a continuum approach. However, the problem is so complicated that no successful theory has been developed. One feels called upon, therefore, to investigate the kinetic theory in order to see where the difficulties occur.

The classical mathematical theory of flocculation was derived by Smoluckowski (30). His purpose was to describe the agglomeration of uniform spherical particles in a colloidal suspension. Using a statistical description of Brownian motion, he was able to calculate the rate of inter-particle collisions. He then calculated the rate of inter-particle unions and the rate of decrease in total number of particles. The theory was verified by observing flocculation under a microscope and counting particles. Many excellent summaries of Smoluchowski's work are available in English (31,32,33).

The fact that Smoluchowski's theory was verified by experiment coupled with the fact it is a mathematical nicety has made it very popular. It has been extended and improved by other workers. However, neither the theory nor the extensions are adequate for the settling process. Certain assumptions that are valid for the Brownian motions in a colloid suspension are not valid for settling particles. Furthermore, for the colloid it is only necessary to predict the number of particles; for the settling particles it is necessary to predict settling velocities.

The settling particles in a suspension are best characterized by the settling velocities. Size and density are then taken as secondary properties. Consider a suspension having a set of distinct particle settling velocities $w_1, w_2 \ldots w_i \ldots w_\infty$. Let the number of particles per unit volume with settling velocity w_i be n_i. While a kinetic theory of flocculation caused by settling will have to be formulated in terms of w and n, it will be found that other information is necessary.

The first problem is to calculate the rate of interparticle contacts. A particle of velocity w_i will contact all slower particles that lie in its path. Assume, temporarily, that all of the particles are spheres and that the particles of velocity w_i have a radius r_i. It is now possible to calculate the rate of particle contacts between i-particles and slower particles.

In a unit time an i-particle will contact all of the l-particles with center located in a volume $\pi(r_i + r_l)^2(w_i - w_l)$. This term has units of volume per unit time, and is the product of an area and a relative velocity. Let the area be called G_{il}. Then

$$\pi(r_i + r_l)^2 = G_{il}$$

and the volume per unit time is

$$G_{il}(w_i - w_l).$$

The assumption of spherical particles is now dropped, and the area G_{il} is defined as the horizontal area under which the l-particles must be located to contact the i-particle. This area will not be circular in general. The total number of contacts per unit volume between i and l-particles is

$$n_i n_l G_{il}(w_i - w_l) \tag{5-3}$$

Since all of the contacts may not result in particle unions, assume that a constant fraction b_{il} result in unions.

When an i-particle becomes connected to any other particle and changes its velocity from w_i, it is considered to disappear from the group with velocity w_i. The resulting particle is considered to appear in another group. Therefore, the rate of disappearance of i-particles due to union with other particles can then be written as

$$\sum_{j}^{\infty} n_i n_j G_{ij} \left|(w_i - w_j)\right| b_{ij} \tag{5-4}$$

The summation will include all values of j, and the absolute value of $(w_i - w_j)$ is used to allow for i-particles overtaking slower particles and being overtaken by faster particles.

There must be some i-particles formed by the collision of other particles. Assume that a collision of a j-particle and a k-particle forms an i-particle. Then the rate of formation of i-particles is given by

$$\frac{1}{2} \sum_{j} \sum_{+ k \rightarrow i} G_{jk} \left|(w_j - w_k)\right| n_j n_k b_{ik} \qquad (5\text{-}5)$$

The summation includes only those combinations of j and k which result in an i-particle.

The total rate of change of the number of i-particles per unit volume is given by the sum of 5-4 and 5-5

$$\frac{dn_i}{dt} = \frac{1}{2} \sum_{j} \sum_{+ k \rightarrow i} G_{jk} \left|(w_j - w_k)\right| n_j n_k b_{jk}$$

$$- \sum_{j=1}^{\infty} G_{ij} \left|(w_i - w_j)\right| n_i n_j b_{ij} \qquad (5\text{-}6)$$

It is at this point in all kinetic theories of flocculation that a crucial assumption must be made. What value of j and k are used in the first summation? In other words, what is the resulting velocity when a particle of velocity w_j joins with a particle of velocity w_k? Assumptions concerning G_{ij} and b_{ij} are also necessary, but these assumptions are not nearly as difficult to make.

For a colloidal suspension, relatively simple assumptions are possible. On the one hand, the velocity of a particle in Brownian motion is inversely proportional to its size. On the other hand, the area G_{jk} increases with particle size.

Consequently, Smoluchowski was able to assume that the product $G_{jk}(w_j - w_k)$ was constant for all j and k without describing how either of the individual terms behaves. Furthermore, Smoluchowski characterized a particle by the number of original uniform spherical particles which it contains. In this case, a particle which results from union of j original particles contacts a particle which contains k original particles and form a particle containing j + k original particles. Therefore, i = j + k in equation 5-6.

Such fortunate circumstances do not occur with settling particles. On the contrary, experience indicates that when two particles join, both the size and velocity increase. For example, a large particle settling through a field of small particles increases in size and settling velocity. As a result, the term G_{jk} and the relative velocity $w_j - w_k$ both increase. The product $G_{jk}(w_j - w_k)$ certainly is not constant.

Nevertheless, the kinetic theory of flocculation has one great value. It indicates which experimental measurements have greatest significance. In settling, the important effect of flocculation is the change of flux rather than the change in particle size or numbers. To investigate this effect, equation 5-6 is multiplied by the settling velocity w_i.

$$w_i\left(\frac{dn_i}{dt}\right) = \frac{1}{2}\, w_i \sum_{j} \sum_{+\, k \to i} G_{jk} \left|w_j - w_k\right| n_j n_k b_{jk}$$

$$- w_i \sum_{j = 1}^{\infty} G_{ij} \left|w_i - w_j\right| n_i n_j b_{ij} \qquad (5\text{-}7)$$

The velocity w_i is the velocity of the class of particles and does not change with time. Thus, the left hand side of equation 5-7 can be written

$$\frac{dw_i n_i}{dt}$$

which is the rate of change of flux of i-particles. Summing over all values of i gives

$$\sum_{i=1}^{\infty} \frac{dw_i n_i}{dt} = \frac{d}{dt} \sum_{i=1}^{\infty} w_i n_i$$

$$= \frac{1}{2} \sum_{i=1}^{\infty} \left[w_i \sum_{j} \sum_{+ k \to i} G_{jk} \left| (w_j - w_k) \right| n_{jk} b_{jk} \right]$$

$$- \sum_{i=1}^{\infty} w_i \left[\sum_{j=1}^{\infty} G_{ij} \left| (w_i - w_j) \right| n_j b_{ij} \, n_i \right] \quad (5\text{-}8)$$

The left hand side of equation 5-8 is the rate of change of particle flux caused by flocculation. It corresponds directly to the term $\left(\frac{\partial a}{\partial t}\right)_F$ which occurs in equation 5-2. The right hand side of equation 5-8 contains terms of the form

$$\sum_{i=1}^{\infty} w_i \left[\sum_{j=1}^{\infty} G_{ij} \left| (w_i - w_j) \right| n_j b_{ij} \right] n_i$$

which can be written

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} G_{ij} \left|(w_i^2 - w_i w_j)\right| (n_i n_j) b_{ij} \qquad (5\text{-}9)$$

The problem is reduced to investigating experimentally how $\left(\frac{\partial a}{\partial t}\right)_F$ is related to a term of the form of 5-9. It is possible to be even more specific. The area G_{ij} depends on the size of the i and j-particles, while the numbers n_i and n_j are the numbers of i and j-particles respectively. Therefore, the sum of products $G_{ij}(n_i n_j)$ is a function of the volume of particles per unit volume of suspension.

The term 5-9 also contains the squares and cross products of the settling velocities. The sum of these squares and cross products is function of the mean and standard deviation of the settling velocities. Therefore, the term 5-9 is a function of the following quantities:

$\alpha\phi(z,t)$ = the instantaneous local volume concentration of particles, i.e. volume of particles per unit volume of suspension.

$\sigma(z,t)$ = local instantaneous standard deviation of settling velocities

$\bar{w}(z,t)$ = local instantaneous mean settling velocity

b = the fraction of particle contacts that result in union of particles

The object of an experimental study, then, is to find the relationship between $\left(\frac{\partial a}{\partial t}\right)_F$ and $\alpha\phi$, $\bar{w}$, and b. This relationship will be a property of the suspension alone. It should be reproducible to the same degree that the suspension is reproducible.

The quantities $\bar{w}$ and σ have an additional significance in the study of flocculation in very dilute suspensions. In such cases, the settling between particle contacts is probably unhindered. Consequently, the rate of change of flux due solely to settling $\left(\frac{\partial a}{\partial t}\right)_s$ is not very different from the rate of change of flux during free settling of discrete particles. However, the latter is given by equation 3-20. Substitution of $\left(\frac{\partial a}{\partial t}\right)_s$ in the equation gives

$$\left(\frac{\partial a}{\partial t}\right)_s + \frac{\partial(\bar{w}^2 + \sigma^2)\emptyset}{\partial z} = 0 \qquad (5\text{-}10)$$

Therefore, the measurement of $\bar{w}$ and σ will permit the separation of $\left(\frac{\partial a}{\partial t}\right)_F$ and $\left(\frac{\partial a}{\partial t}\right)_s$ when the total change $\frac{\partial a}{\partial t}$ is known.

5-3 Theory and Experiments of Tuorila

In one instance, flocculation during settling has been analyzed by kinetic theory and studied experimentally. The original work was done by Tuorila (34), but the following description is based on summaries by Krumbien (23) and Weigner (35).

In accordance with Tuorila's theory, consider a suspension containing spherical particles of two sizes. Let $n_1(t)$ be the number of smaller particles per unit volume of suspension. These particles have a mass m_1 and a radius r_1. Let $n_2(t)$ be the number of larger particles per unit volume of suspension. These will have mass m_2 and radius r_2.

The rate of change of particle concentration dn/dt has been given by equation 5-6. Normally, this equation would be written for both n_1 and n_2. However, Tuorila modified the approach somewhat in order to eliminate one equation. He considered the result of a union between a large and a small particle to be a large particle. Consequently, the number of large particles can be taken as a constant and can be written in the following manner

$$n_2(t) = n_2(0) = n_2$$

For the smaller particles, the rate of change dn_1/dt must be obtained from equation 5-6. Since no small particles are being formed, only the rate of disappearance must be calculated. Equation 5-6 becomes

$$\frac{dn_1(t)}{dt} = G_{12}(w_2 - w_1)n_1(t)n_2b_{12} \qquad (5\text{-}11)$$

In equation 5-11, the velocity w_2 is not constant as it was in equations 5-6, and 5-7. In the latter w_i was the velocity of a class of particles which moved to a new class as their velocity increased. In equation 5-11, on the other hand, w_2 is the varying velocity of the larger particles. Instead of considering the change in this velocity, Tuorila calculated the variation of the product $G_{12}(w_2 - w_1)$. Assuming spherical particles in the Stokes range he found that the product is nearly constant for following conditions:

$$0 < \frac{r_1}{r_2} < 0.4 \tag{5-12a}$$

$$0 < \frac{m_1 n_1}{m_2 n_2} < 0.4 \tag{5-12b}$$

Conditions 5-12b states that the ratio of mass concentration is less than 0.4. For condition 5-12, then, equation 5-11 can be integrated to give

$$n_1(t) = - n_1(0)\, e^{-n_2 d_{12} t} \tag{5-13}$$

where

$$d_{12} = G_{12}(w_2 - w_1) = \text{constant} \tag{5-14}$$

Tuorila conducted experiments to check equation 5-13. He prepared two fairly uniform fractions of washed and ignited silica flour. The two fractions were suspended in water and 0.5 N hydrochloric acid was added as a coagulant. The resulting suspension was mixed rapidly in a measuring cylinder and allowed to settle.

When the larger particles had settled to a depth z, all of the suspension above z was pipetted out of the cylinder. The average concentration of the removed suspension was then determined. Tuorila also derived an expression for this average concentration. Using equation 5-13 he obtained the following expression for the average n_1 above z.

$$\bar{n}_1 = \frac{n_1(0)\left[1 - e^{-n_2 d_{12} t}\right]}{n_2 d_{12} t} \tag{5-15}$$

Two sets of data from Tuorila's experiments are contained in table 5-1. The table also contains the average concentrations calculated from equation 5-15. For a more graphic comparison the calculated values have been plotted as a function of t in figure 5-1. The measured values have been plotted in the same figure.

The agreement between measured and calculated results is very good. However, in table 5-1a the term d_{12} is not constant. The reason for this inconsistency is explained in the discussion which follows.

5-4 Discussion of Tuorila's Experiment

In discussing the theory and experiment of Tuorila it is necessary to consider the spatial distribution of particles after a time t. In figure 5-2, spatial distributions for each size particles are shown for t = 0 and t = t_1. The mass concentration of each size particle is m n.

Table 5-1a
(After Tuorila)

Average Concentration of Small Particles

Above Depth z at Time t

(z/t = Velocity of Large Particles)

Large quartz grains 7-16 μ radius

Small quartz grains 0.5-2.0μ radius

7.35 gm. of large grains per liter

1.30 gm. of small grains per liter

Time t_1	$\bar{n}_1$		$n_2 d_{12}$
	Observed	Calculated	
sec.	gm.	gm.	liter per sec.
0	1.30	1.30	
330	1.04	1.01	0.0014
660	0.82	0.81	0.0015
990	0.64	0.66	0.0016
--	0.28	0.30	0.0018
			Mean 0.0016

Table 5-1b
(After Tuorila)

Average Concentration of Small Particles

Above Depth z at Time t

(z/t = Velocity of Large Particles)

Large quartz grains 7-16 μ radius

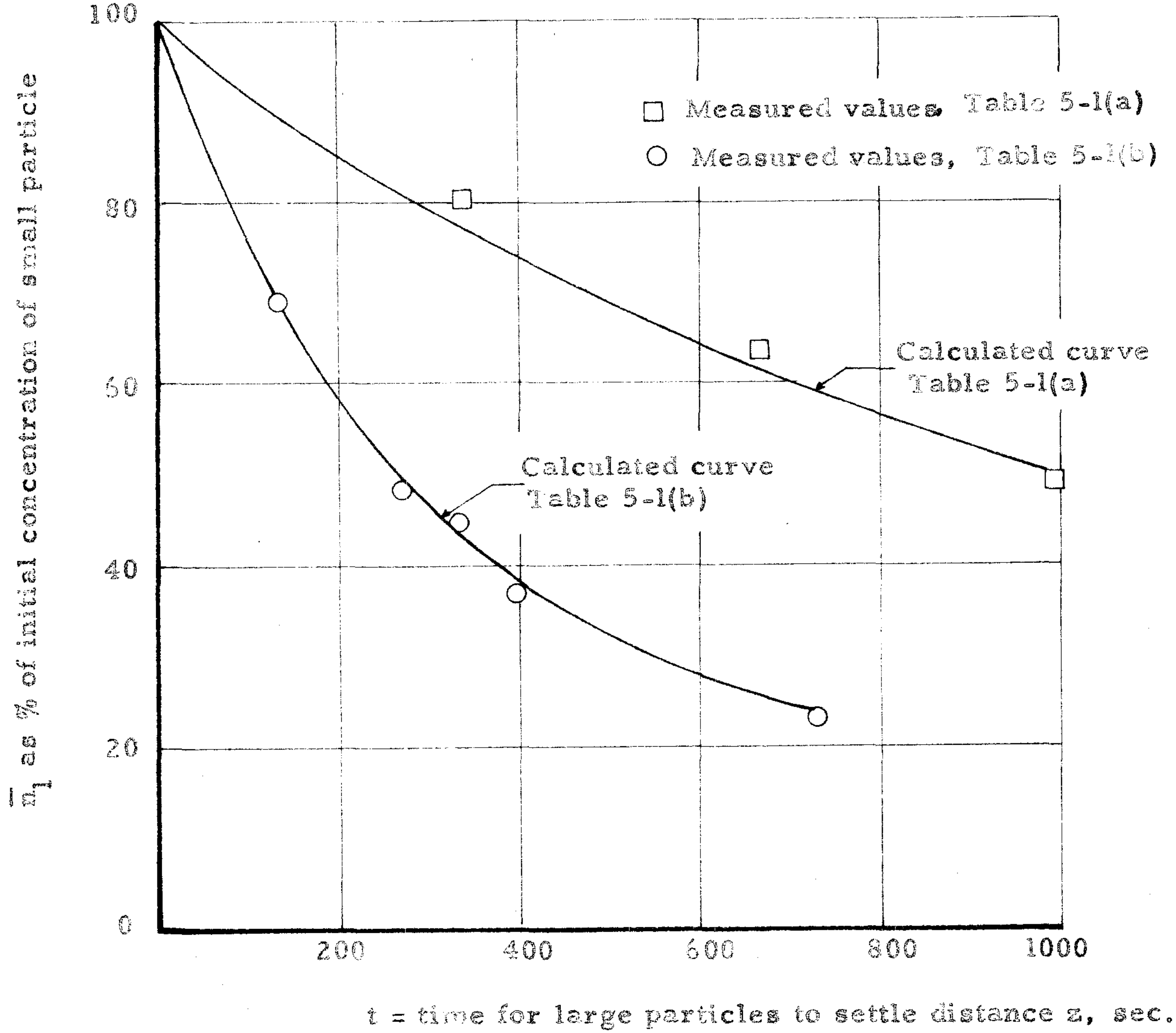
Small quartz grains 0.5-2.0μ radius

29.40 gm. of large grains per liter

1.30 gm. of small grains per liter

Time t_1 sec.	$\bar{n}_1$ Observed gm.	$\bar{n}_1$ Calculated gm.	$n_2 d_{12}$ liter per sec.
0	1.30	1.30	----
132	0.90	0.90	0.0058
264	0.63	0.65	0.0063
330	0.58	0.57	0.0058
396	0.48	0.50	0.0061
726	0.30	0.29	0.0058
			Mean 0.0060

Fig. 5-1. $\bar{n}_1$ as a function of time (after Tuorila).



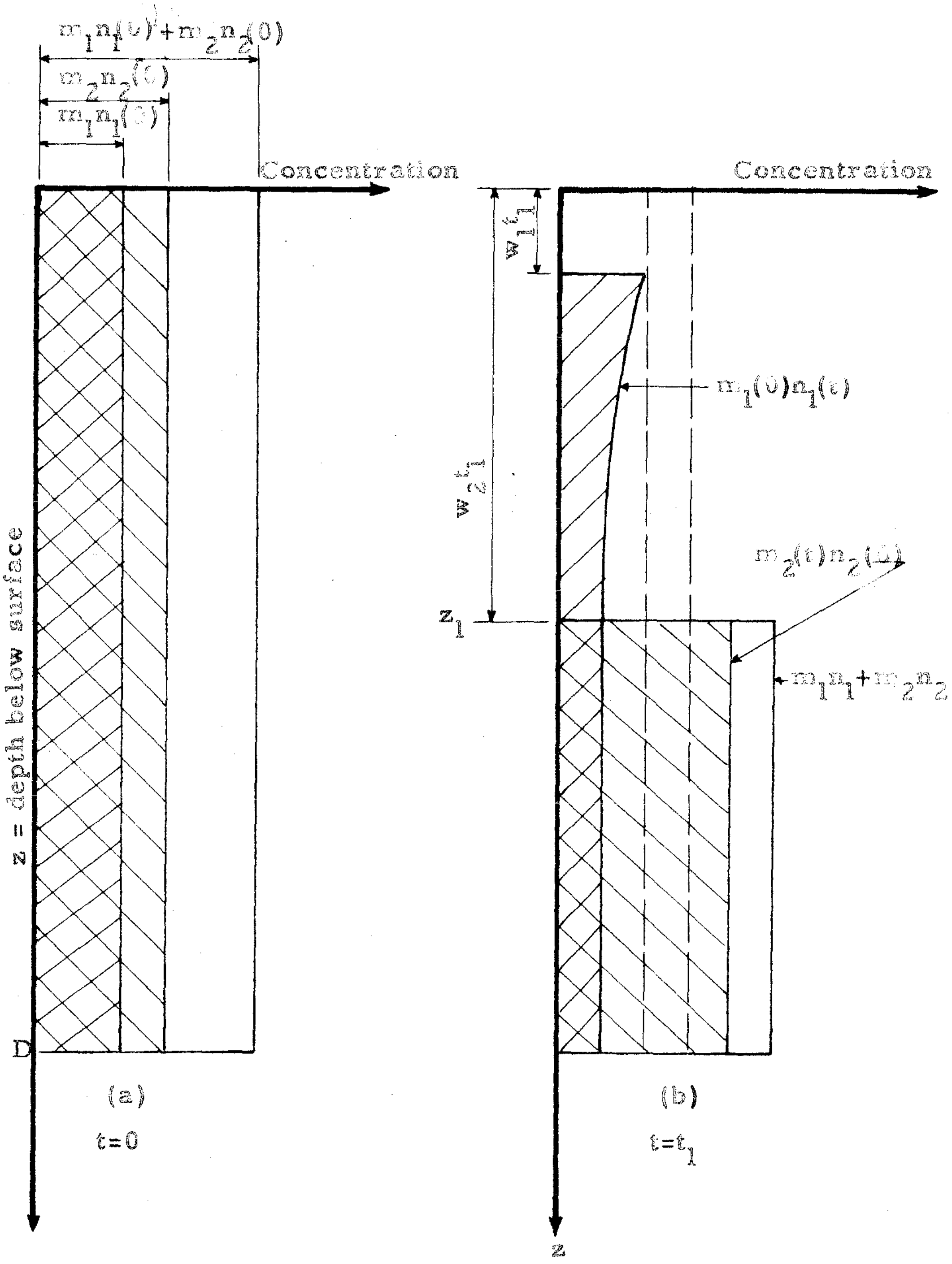


Fig. 5-2. Spatial distribution of particles in Tuorila's experiment.

In figure 5-2 (a) the initial distribution of concentration is uniform for each size and, hence, the total concentration is uniform. The concentration profile for each size is shown as a cross-hatched area.

At $t = 0$ the particle of both sizes begin to settle. The larger particles are moving with respect to the smaller ones. All of the large particles have the same settling velocity and settle at the same rate. The same holds true for the small particles. Since the particles were uniformly spaced at $t = 0$, each large particle overtakes the same number of small particles per unit of time. Consequently each large particle increases its size and velocity at the same rate.

This argument does not require Tuorila's assumption of constant size or velocity of the large particle. Furthermore, it does not require an assumption of free settling. The argument depends on the uniformity of conditions throughout the space in which the large particles are located. In that space, the rate of change of n_1 and of m_2 are functions of time alone.

On the other hand, if Tuorila's assumption is made, the term d_{12} will be constant. In the space where large particles are located, the rate of flocculation is given by equation 5-13. When the large particles have settled a distance $z_1 = w_2 t_1$, the flocculation at z_1 will stop. Therefore the concentration of small particles at $z_1 + w_1(t - t_1)$ will remain constant for $t > t_1$.

In figure 5-2 (b) the spatial distributions are shown for the time $t = t_1$. For $z > z_1$ flocculation is proceeding as a function of time, while for $z < z_1$ flocculation has stopped. For $z < z_1$ the curve $m_1 n_1$ decreases with depth because increasing depths correspond to a zone of flocculation for a longer time. The area under this part of the $m_1 n_1$ curve represented the particles removed and weighed in Tuorila's experiment. It also represents $t\bar{n}_1$ where $\bar{n}_1$ is given in equation 5-15.

At this point it is possible to explain why Tuorila's measured and calculated results agree when d_{12} is not constant. The quantity d_{12} affects the shape of the curve $m_1(0)n_1(t)$ in figure 5-2 (b). A small change in shape will not affect the area under the curve, and it is the area under the curve which is given by equation 5-15 and measured in the experiments.

The area under the curve above $z = z_1$ has a third interpretation. It is equal to the quantity

$$1 - R(z_1, t)$$

where $R(z_1, t)$ is the removal from above z_1. Therefore, Tuorila was actually measuring the removal above z_1 at t_1. However, from assumption that the large particles are settling at a fairly uniform rate, z_1/t_1 is constant. One concludes that Tuorila was measuring the removal at constant overflow rate as a function of depth. Obviously, this removal is not constant.

Tuorila's experiment and theory involved a very simple suspension of flocculent particles settling in a settling column. However, some of the conclusions reached in the above discussion have a bearing on more complicated cases. Consider a suspension of particles in a settling column. Assume that particles of each settling velocity are uniformly distributed at $t = 0$, and let flocculation occur during settling. If an observer moves downward through the column at a constant velocity $z/t = K$, what concentration will be experienced?

At $t = 0$ there are some particles with settling velocity $w \geq K$. These particles move ahead of the observer. As they move ahead, they capture or sweep up some particles with settling velocity $w < K$. Therefore, the total number of particles with $w < K$ is affected like the number of small particles in Tuorila's experiment. Consequently the observer will experience decreasing concentration as he moves at the velocity $z/t = K$. Moreover, the lines of constant z/t in the $\emptyset$,z diagram will look like the curve $m_1 n_1$ for z z_1 in figure 5-2 (b). Therefore, for flocculent suspensions one expects that

$$\left(\frac{d\emptyset}{dz}\right)_K < 0$$

If the observer is moving faster than any particle in suspension he will be in location corresponding to $z > z_1$ in figure 5-2b. That is, for z/t greater than the velocity of fastest particle, flocculation is a function of time only. No particles are settling out of suspension. If the rate of change of flux is measured for these values of z/t, this change will be due to flocculation alone and will be a direct measure of $\left(\frac{\partial a}{\partial t}\right)_F$. Therefore, with suspensions of slow enough particles or with columns of sufficient depth, the change of flux due to flocculation alone can be measured near the bottom of the column. This measurement is possible only until the fastest particles have settled from the surface to the bottom of the column.

CHAPTER 6

EXPERIMENTAL MEASUREMENT OF LOCAL MEAN SETTLING VELOCITY

This chapter describes the experimental determinations of the local mean settling velocity, $\bar{\bar{w}}(z,t)$, in a flocculent suspension. Section 6-1 describes the experiments and the analysis of the experimental data. In section 6-2 some data from the literature are analyzed in a similar fashion, and $\bar{\bar{w}}(z,t)$, is calculated.

The determination of the local mean settling velocity is the first step in studying the effect of flocculation on settling. The next step is to determine the relationship between the rate of change of $\bar{\bar{w}}(z,t)$ and other properties of the suspension. The kinetic theory in section 5-2 predicts which properties will be most significant. The experimental study of the effect of these properties on $\bar{\bar{w}}(z,t)$ is a subject for future research.

6-1 Settling Analysis of a Clay-Water-Alum Suspension

a. General. The study of flocculation has two parts. One is based on the rate of aggregation of particles and is discussed in the previous chapter. The other is based on the chemical mechanism which allows particles to aggregate and is not discussed in this thesis.

Owing to time limitations it was decided to study the effects of particle aggregation using a suspension for which the chemical mechanism has already been studied. A suspension of clay in water meets this requirement. It has been studied extensively as a class of colloidal suspensions (36). Furthermore, Langlier, Ludwig and Ludwig have studied the action of coagulants in causing flocculation in clay suspensions (37).

The information from these studies is available if it should be needed. Therefore a suspension of clay in water was used in an experimental study of flocculation during settling. Experiments were also performed with domestic sewage. However, these experiments were not as successful as those with clay and water.

Bentonite clay was used in all of the experiments. A gross sample was obtained from a commercial supplier and was divided into small portions by means of a sample splitter. These portions were mixed with Pasadena tap water. The resultant suspensions ranged in concentration from 655 to 1290 mg. per liter.

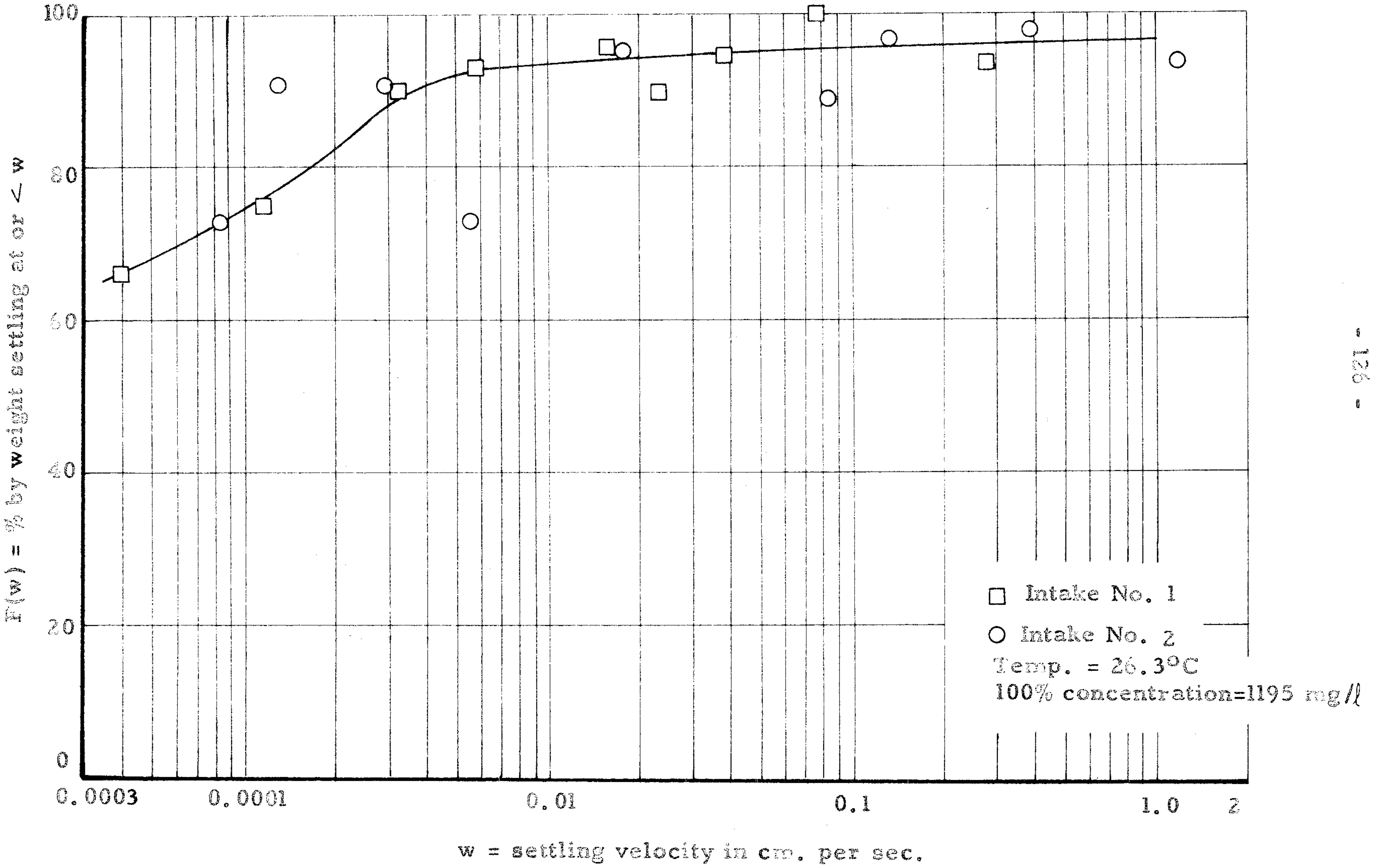
b. Settling Velocity Analyses of Deflocculated Clay. A suspension of clay, water and deflocculating agent was prepared according to the method used in the hydrometer analysis of fine grained soils (38). Sodium silicate was used as the deflocculating agent. The resultant suspension was analyzed in the two-depth settling tube and samples were taken at both intakes as described in section 4-2. The object of the experiment was to obtain an approximation of the settling velocity distribution for the original clay particles.

The results of the analysis have been plotted in figure 6-1. Since flocculation is absent, the results can be plotted as a cumulative settling velocity distribution. While some of the experimental points lie a considerable distance from a single smooth curve, there is no consistent difference between the upper and the lower intake. Therefore, the deviations must be caused by experimental error rather than by flocculation or non-uniform initial distribution.

There is a possibility that some hindered settling occurred during the test. The initial concentration of particles was 1195 mg. per liter and the particles had a specific gravity of approximately 2.65. It follows that the volume concentration of particles was approximately 0.45 ml. per liter. The effect of this concentration on particle settling velocity can be estimated by Steinhour's equation, 3-27; the reduction turns out to be negligible. The effect can also be calculated using the equation of McNown and Lin (14). According to this equation the settling velocity of the particles was reduced to 94 per cent of their free settling velocity. On the logarithmic scale of figure 6-1 a 6 per cent change in velocity is negligible.

Admittedly, the data in figure 6-1 do not represent a precise measurement of the settling velocity distribution. However, such a measurement was not the aim of the experiment. The aim was to obtain a distribution which could be compared

Fig. 6-1. Settling velocity analysis of a suspension of bentonite clay in water.



with the results obtained when flocculation occurs. Figure 6-1 is sufficiently accurate for this purpose.

<u>e. Two-Depth Analyses of Clay and Alum in Water.</u> A suspension of clay in water was mixed with a solution of alum and analyzed in the two-depth tube. The experiment was run in duplicate. (See section 4-2 for method.)

This experiment had two purposes. The first was to determine the order of magnitude of the effect of flocculation on settling. The second was to show that the effect of flocculation is reproducible.

Before the settling analysis was performed, a suspension of clay and tap water without alum was tested for pH and alkalinity. It was found that for 1200 mg. of clay per liter of tap water the pH was 7.8 and the total alkalinity* was 5.9 milliequivalents per liter.

The two suspensions used in the settling analyses had concentrations of 1194 and 1290 mg. per liter. Consequently, the pH and alkalinity before addition of alum must have been approximately the same as the values just given. In both runs alum was added until the resultant concentration was 50 mg. per liter of suspension. Since pH and alkalinity affect the flocculation caused by alum, the chemical aspects of flocculation were similar in both runs.

The flocculation during the settling analysis is also

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* The total alkalinity was determined by potentiometric titration to pH 4.3 (see reference 28).

affected by the mixing at the beginning of the analysis. In both runs the alum was added just before mixing and the mixing was performed in the same manner for the same length of time. Therefore, at the beginning of settling, the suspensions should have been in the same condition for both runs.

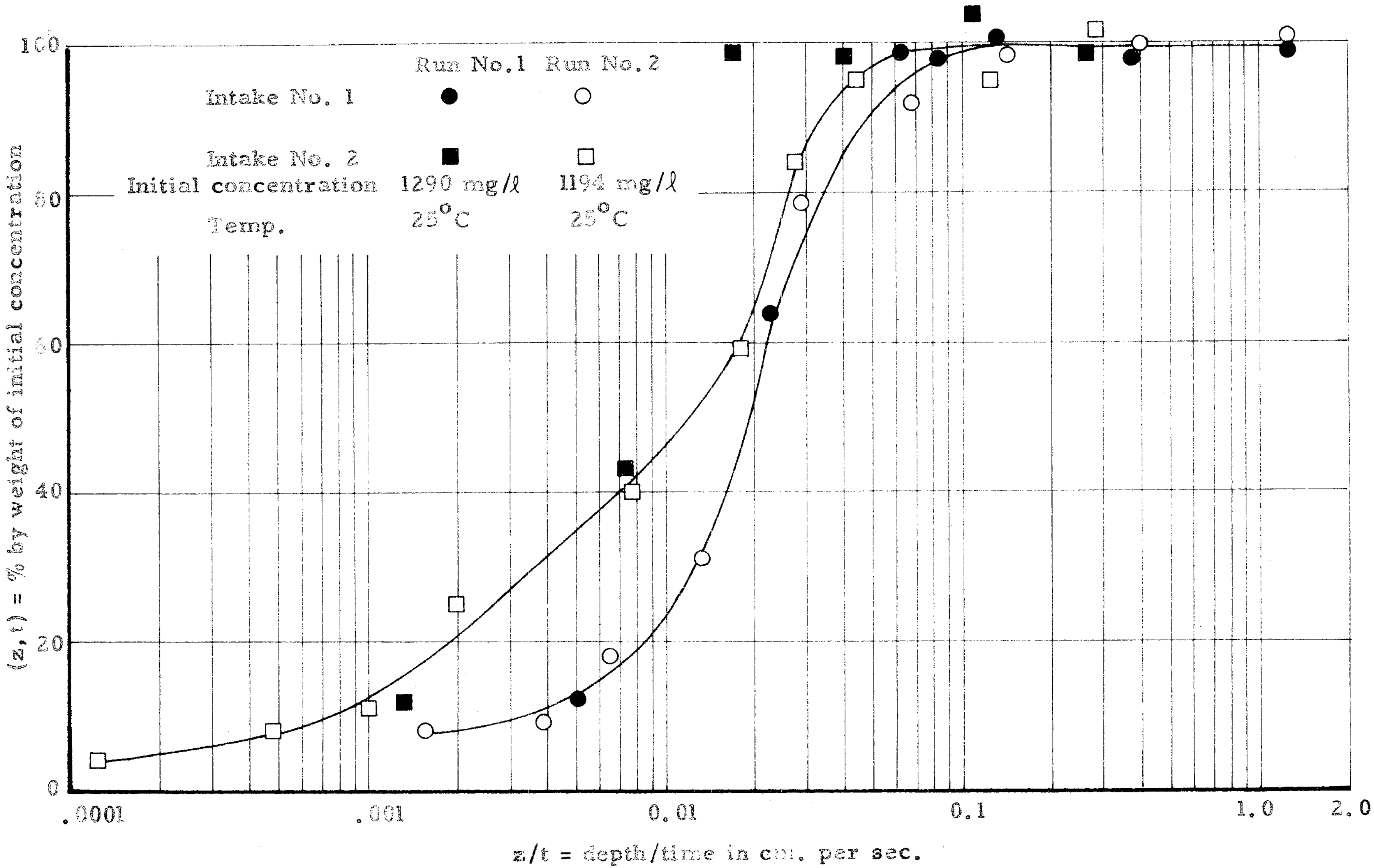
The results of both analyses are plotted in figure 6-2. Since the data are quite similar, only one curve need be drawn for each intake. With the exception of one stray point, the maximum vertical deviation of a plotted point from its smooth curve is 5% of the initial concentration.

The data have been plotted using per cent concentration and z/t as coordinates. The term z/t is not a settling velocity, and the curves are not settling velocity curves. In this form of plot, however, the data can be compared with the cumulative settling distribution of figure 6-1. Comparison shows the marked effect of flocculation.

The difference between results for upper and lower intake is a measure of the flocculation during settling. From figure 6-2 it is seen that the difference was the same for both runs. Consequently, the effect of flocculation was the same for both runs. This result leads to the conclusions that for a given suspension, subjected to given conditions, the effect of flocculation on settling is reproducible.

d. Multiple-Depth Analysis of Clay and Alum in Water. A suspension of clay and water was mixed with a solution of alum and analyzed in the multiple-depth tube described in section 4-3. The object of the experiment was to study the

Fig. 6-2. Two-depth analysis of a suspension of bentonite clay and alum.



local mean settling velocity $\bar{w}(z,t)$ during settling and flocculation.

The suspension contained 655 mg. of clay per liter, and the final alum concentration was 25 mg. per liter. Immediately after the addition of the alum, the experiment was started.

At the time of the experiment the mixing hopper had not been developed, but the mixing was similar to that done in the hopper. The suspension was mixed in a separate container and poured into the settling tube through a funnel.

The results of the experiment are given in table 6-1 and plotted in figure 6-3. The concentration has been plotted as a function of z/t in order to show the effect of flocculation. Except at the higher z/t values, the points define a separate curve for each intake. The deviation of the points from the curves is less than the separation of the curves. The few scattered points show no trend in their location. Therefore, they are random deviations and not signs of non-uniform initial distribution.

The same data are plotted in the ϕ,z diagram shown in figure 6-4. The dotted lines show ϕ as a function of z for each intake. These curves show the lowering of the surface during the test and the resulting change in intake depth. Moreover, they correspond to the curves shown in figure 6-3.

Since samples were taken at only three depths, drawing the concentration profiles required some interpolation between these depths. Instead of trying to draw the profiles directly,

Table 6-1

Multiple-Depth Analysis of a Suspension of

Clay and Alum in Water

Time After Start of Settling	Intake	Depth of Intake z	z/t	Volume of Sample	Temp. of Sample	Weight of Particles in Sample	Conc. of Particles in Sample ø(z,t)	% of Initial Conc.
sec.	No.	cm.	cm./sec.	ml.	°C.	mg.	mg./l	
0					29.5*			
60	2	42.6	0.71	28		17.7	622	99
90	3	104.3	1.2	27		17.3	654	104
120	1	21.7	0.18	32½		20.3	625	99
180	3	103.2	0.57	33		21.6	655	104
245	2	39.7	0.16	28		18.1	647	103
300	1	18.4	0.062	37		23.6	637	102
360	3	101.4	0.28	31½		20.0	635	101
480	1	17.1	0.036	28		16.7	595	95
600	2	37.5	0.063	31½	30.3	21.4	677	108
720	3	99.2	0.14	32	30.3	22.9	716	114
960	1	15.2	0.016	22		8.1	369	59
1200	2	35.6	0.030	25		11.2	448	71
1440	3	97.3	0.068	36		12.9	368	59
1920	1	13.2	0.069	27		7.7	285	45
2400	2	33.7	0.014	31		9.0	290	46
2880	3	95.4	0.033	29	31.5	7.2	248	40
3840	1	11.1	0.0029	31	32.0	5.1	164	26
4860	2	31.6	0.0065	32	32.3	5.0	156	25
5760	3	93.3	0.016	27	32.8	4.6	170	27
7740	1	8.9	0.0012	31	33.4	2.7	87	14
10100	2	29.4	0.0029	40	34.0	4.1	100	16
14100	3	91.1	0.0065	26	34.8	1.2	46	7

* Measured in mixer

100% = 655 mg./l   Alum Concentration = 25 mg./l

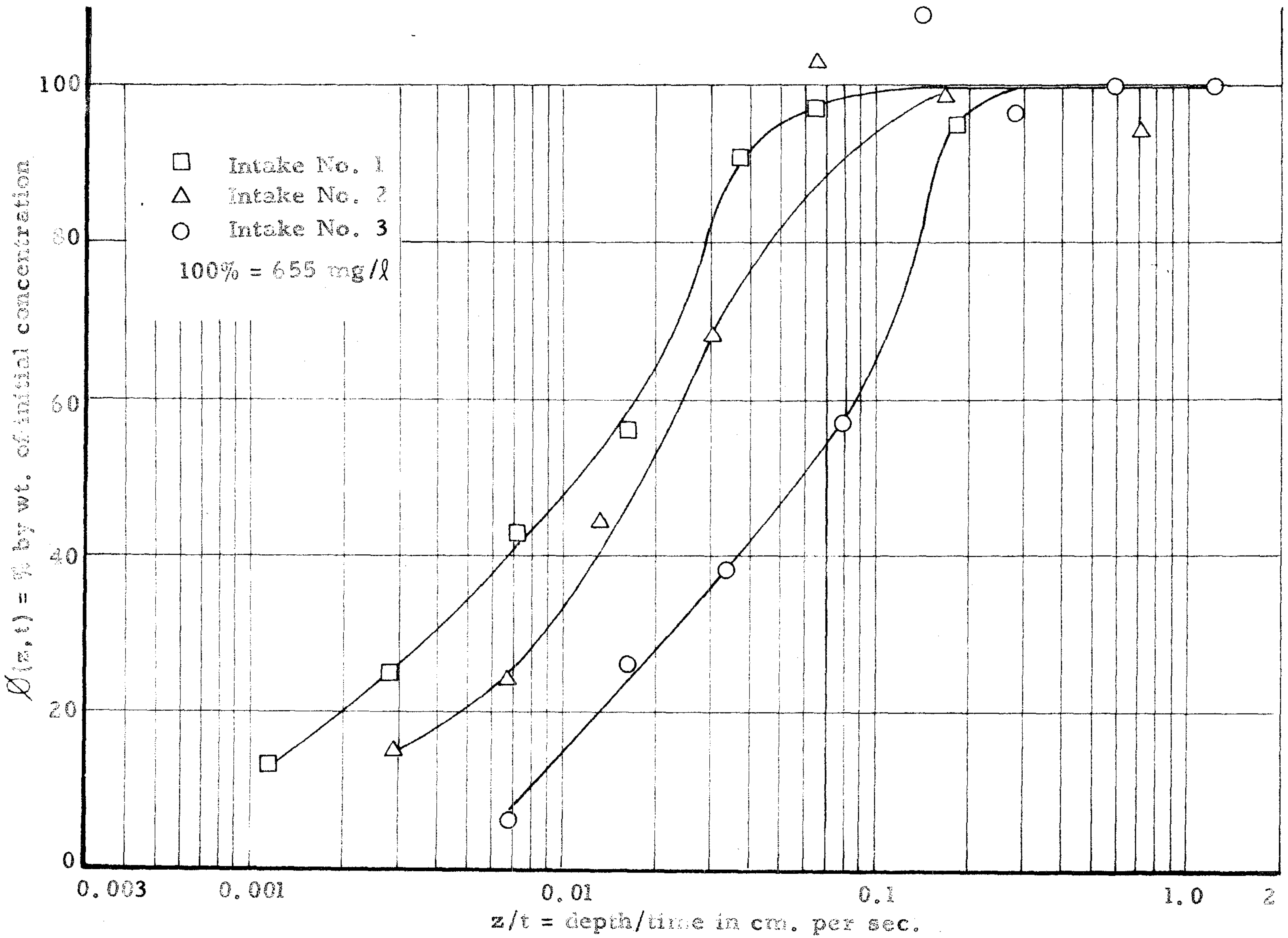


Fig. 6-3. Multiple depth analysis of a suspension of bentonite clay and alum in water (see Table 6-1).

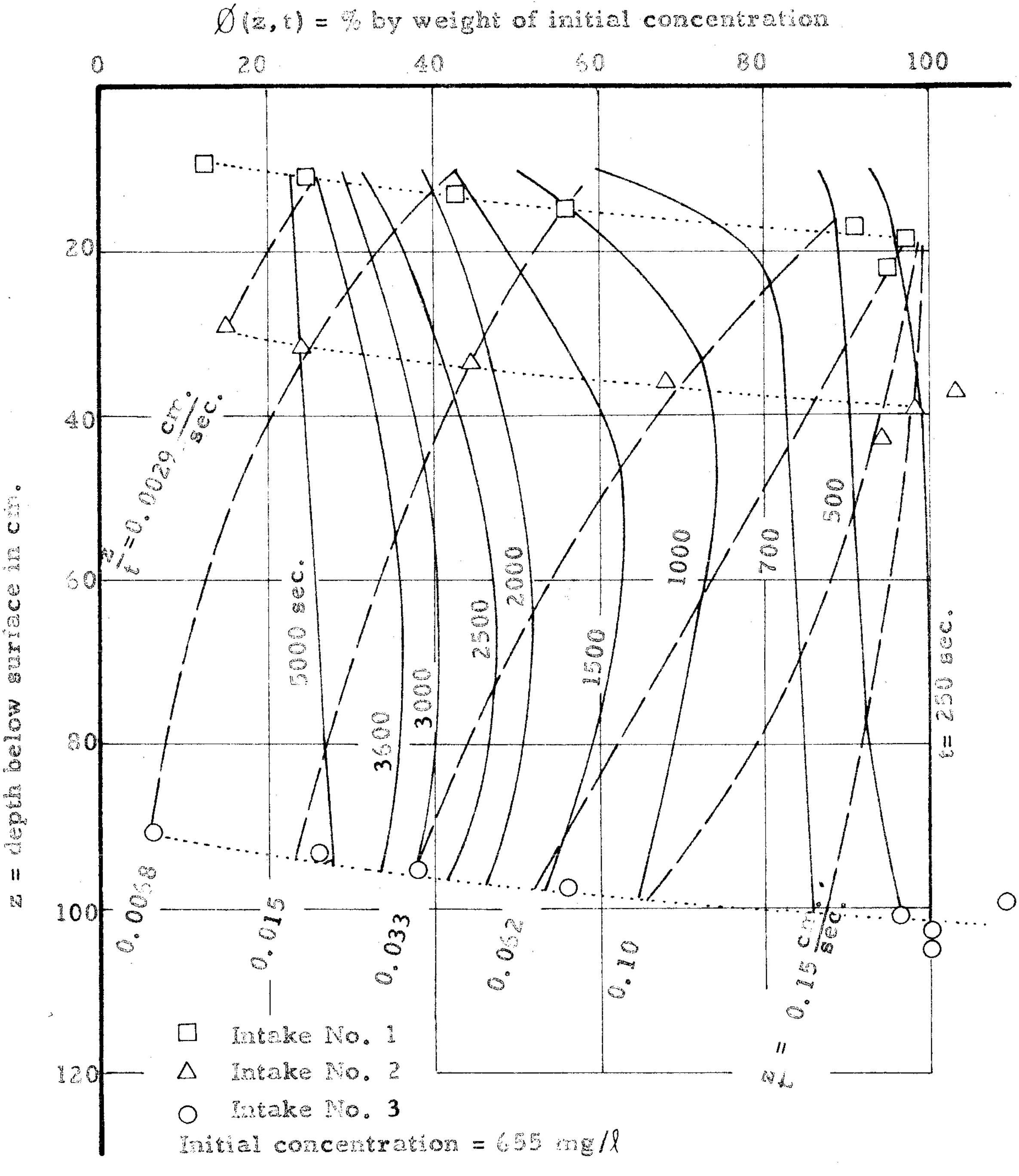


Fig. 6-4. Ø, z diagram for a suspension of bentonite clay and alum in water.

the lines of constant z/t were drawn first. These are the dashed lines in the ϕ,z diagram. Once the two sets of broken lines had been drawn in figure 6-4, the concentration profiles were drawn.

e. Local Mean Settling Velocity. The total removal was calculated using the ϕ,z diagram in the manner outlined in section 3-1. In order to perform the required graphical integrations, it was necessary to assume a shape of the profiles for $z < 10$ cm. It was assumed that the profiles were straight lines drawn from the value $\phi(10,t)$ to origin $\phi = z = 0$.

The curves of A(D,t) are shown in figure 6-5. The slopes of these curves give the local flux as a function of time, and the local flux divided by the local concentration gives the local mean settling velocity shown in figure 6-6.

At any depth the mean settling velocity shown in figure 6-6 increases after t = 200 seconds. At z = 90 cm., this increase continues until t = 700 seconds. From the ϕ,z diagram it is evident that at t = 700 seconds, $\phi(z,700)$ is less than $\phi(z,0)$ for z = 90 cm. Consequently, the mean settling velocity continued to increase after the concentration had begun to decrease.

The first particles to settle out of suspension at depth z are the first particles to settle from the surface to this depth. These are the fastest particles in suspension. Therefore, when the concentration at a point decreases, the fastest particles are being lost. The change in mean settling velocity due to settling $(\partial\overline{w}/\partial t)_s$ must be negative. On the

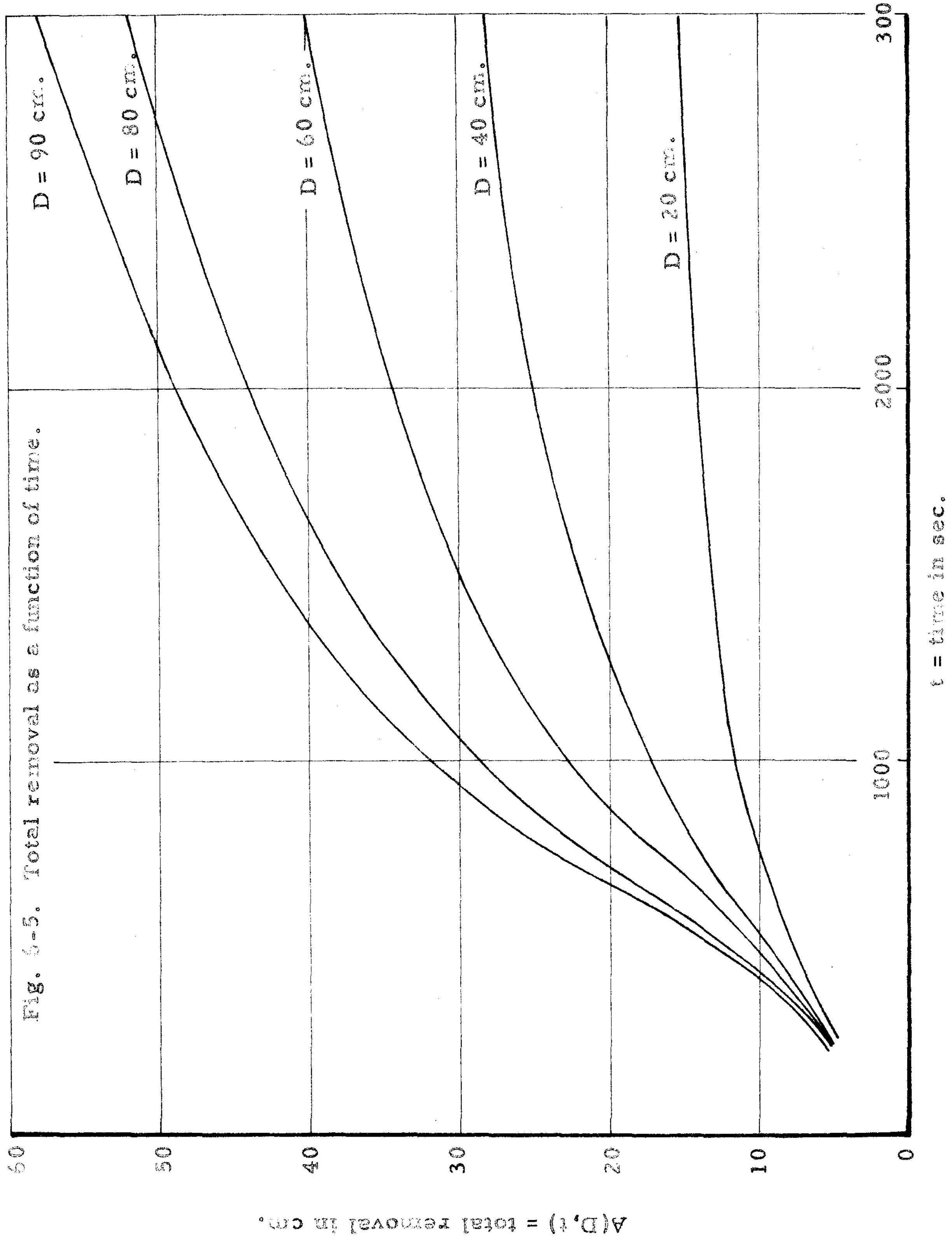
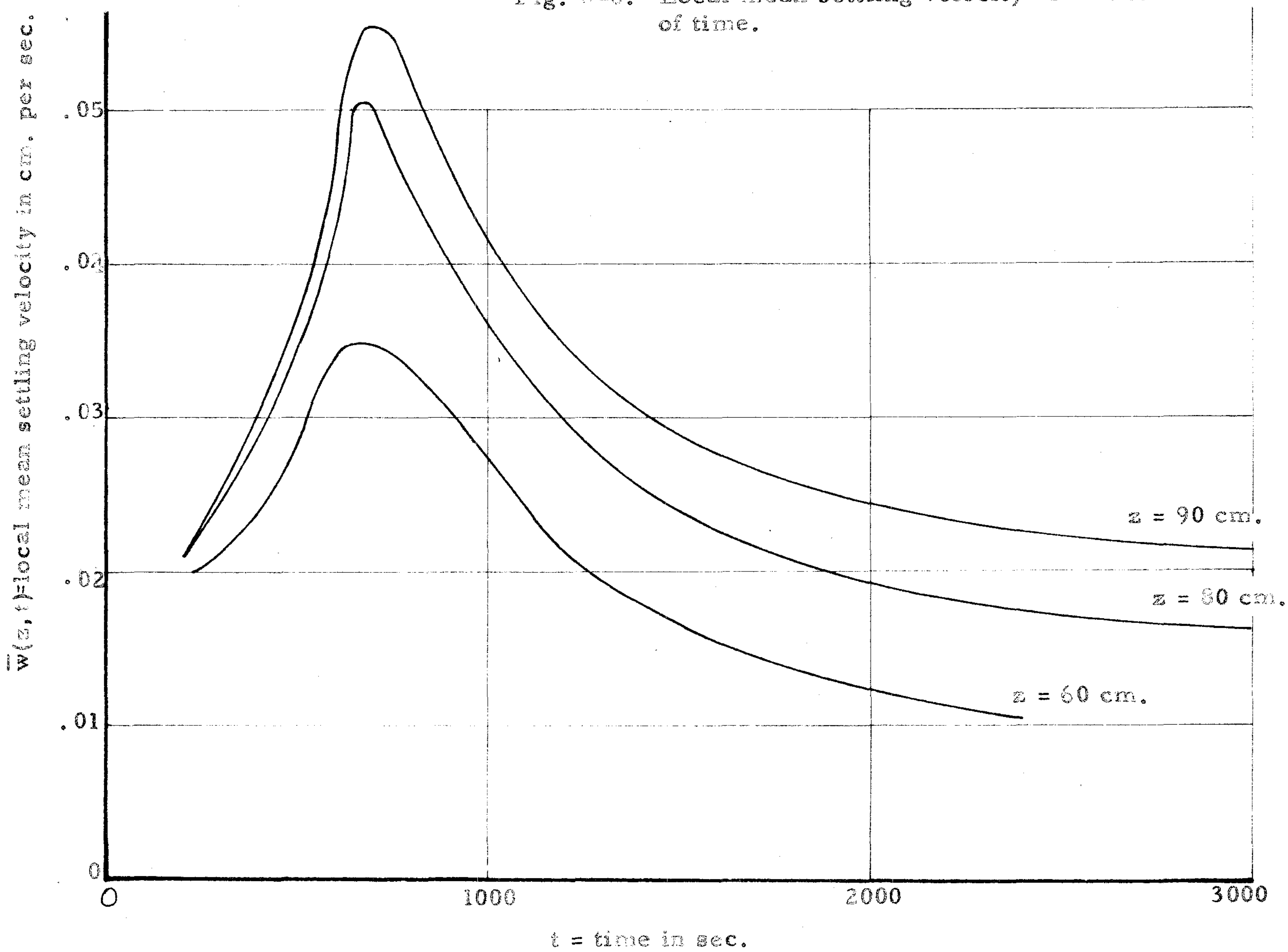


Fig. 6-5. Total removal as a function of time.

Fig. 6-6. Local mean settling velocity as a function of time.



other hand, the total change in $\overline{w}$ is positive in the early stages of settling. Consequently, the increase due to flocculation $(\frac{\partial \overline{w}}{\partial t})_s$ is greater than the total increase shown in figure 6-6. That is

$$(\frac{\partial \overline{w}}{\partial t})_F = \frac{\partial \overline{w}}{\partial t} - (\frac{\partial \overline{w}}{\partial t})_s = \frac{\partial \overline{w}}{\partial t} + \left|(\frac{\partial \overline{w}}{\partial t})\right|_s \qquad (6\text{-}1)$$

Equation 6-1 states that the rate of change of $\overline{w}$ caused by flocculation is equal to the total rate of change shown in figure 6-6 plus the absolute value of the decrease caused by faster particles settling out of suspension.

It is possible to calculate the order of magnitude of flocculation effects on local settling. At a depth of 90 cm. the mean settling velocity increases from 0.021 cm. per second at t = 200 seconds to 0.037 at t = 500 seconds. This change represents an increase of 75 per cent and an average rate of increase of 0.000053 cm. per sec. per sec. Furthermore, the change in concentration during the same time was only 3 per cent. The effect of particles settling out of suspension must have been small. Therefore, the change and rate of change are primarily due to flocculation.

The maximum mean settling velocity for z = 90 cm. occurs at 700 seconds. Its value of 0.056 cm. per second is 2.8 times the mean settling velocity at t = 200 seconds. The average rate of change between t = 200 and t = 700 seconds was 0.00007 cm. per sec. per sec.

A maximum value of $\overline{w}$ occurs at all depths in the settling tube. After this value, the mean settling velocity decreases. Particles are settling out of suspension, and the loss of faster particles offsets the effect of flocculation.

f. Conclusions. The results of the multiple-depth analyses have given the order of magnitude of the changes in mean settling velocity due to flocculation during settling. These results would be much more accurate if two additional sampling intakes were added to the apparatus. One should be located at a depth of about 65 cm. while the other should be capable of taking samples near the surface. These additional intakes would make the profiles in the ϕ,z diagram more accurate. This improvement would be carried through the calculations to the values of $\overline{w}(z,t)$.

6-2 Experiments by Fitch

a. General. Fitch (9) and his associates performed multiple-depth analyses on a suspension of whiting ($CaCO_3$) and ferrisul ($Fe_2(SO_4)_3 \cdot 9H_2O$) in water. The concentration of $CaCO_3$ was 400 parts per million (ppm) while the concentration of ferrisul was 15 ppm.

Fitch's experiments were similar to those described in section 6-1. Immediately after adding the ferrisul he allowed the suspension to settle in a settling tube. The first experiments were conducted in settling tubes five feet in depth. Samples were withdrawn at one, three and five feet depths. By determining the amount of $CaCo_3$ in each sample, the particle concentration $\phi(z,t)$ was obtained.

Later experiments were more elaborate. The settling tube was seven feet deep and had an internal diameter of 5 1/2 inches. For temperature control, the outside of the tube was covered with insulation one inch thick. Samples were taken at seven depths by means of veterinary hypodermic needles which passed through the walls of the tube.

Fitch used his results to demonstrate the effect of detention time on removal for flucculent suspensions. This aspect of the problem will be discussed in chapter 7. For the present, his data will be used in a calculation of the effect of flocculation at a point.

<u>b. Calculation of Local Mean Settling Velocity</u>. The diagram in figure 6-7 is based upon the data from Fitch's experiment with the 7 foot tube. The solid curves are concentration profiles or lines of constant time t, and the dashed curves are curves of constant z/t. For $t < 0.25$ hours and $z < 2$ feet, the constant z/t lines have positive slopes. Since the initial concentration is too small for hindered settling, the positive slopes are a sign of non-uniform initial distribution. To be exact, the initial concentration $\phi(z,0)$ increased with depth.

Following the method described in section 6-1 the local mean settling velocity was calculated. The mean velocities for $z = 3$ feet and $z = 6$ feet are shown in figure 6-8.

At $z = 6$ feet the mean settling velocity increased from 0.013 cm. per sec. at $t = 0.25$ hours to 0.035 cm. per sec. at

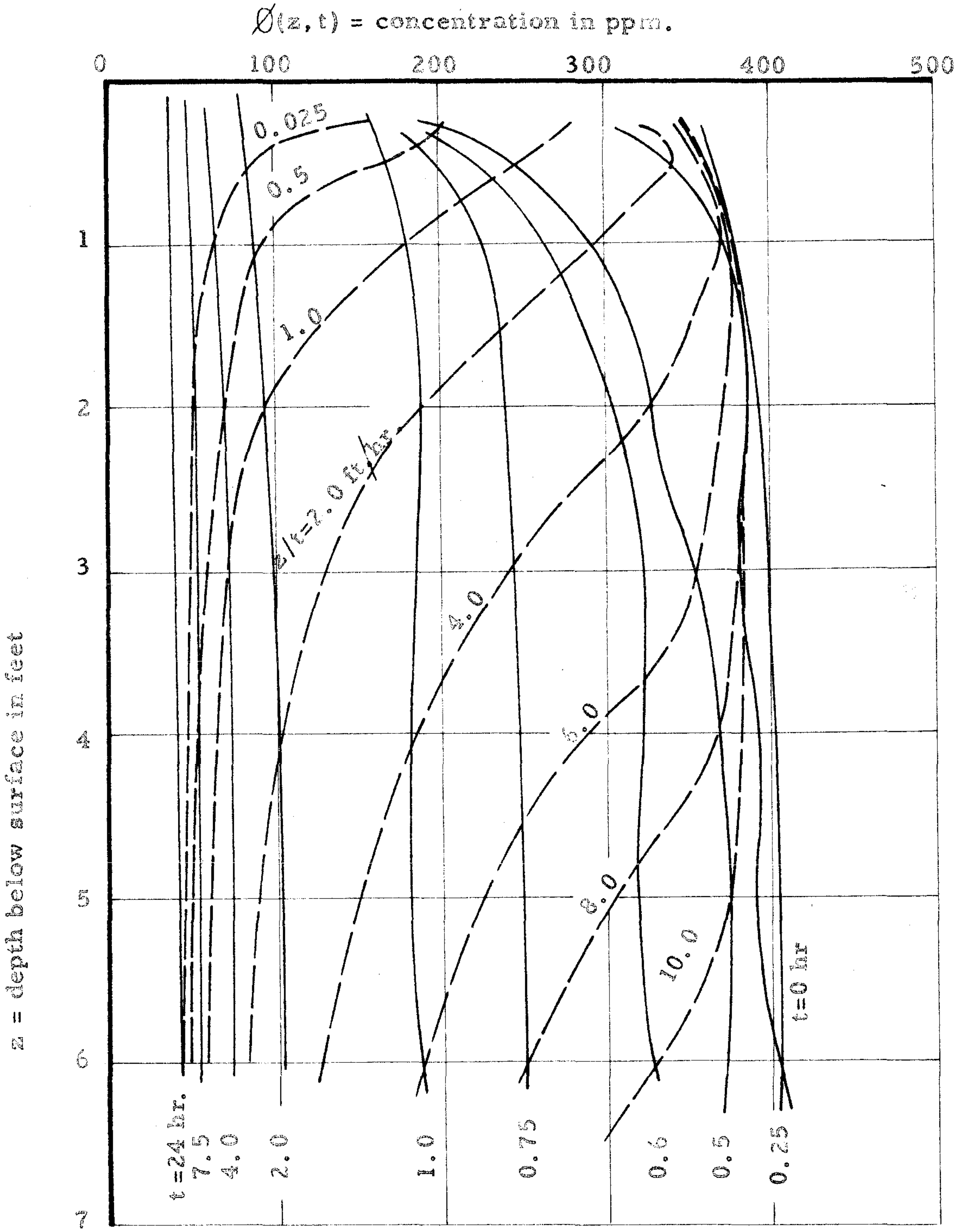
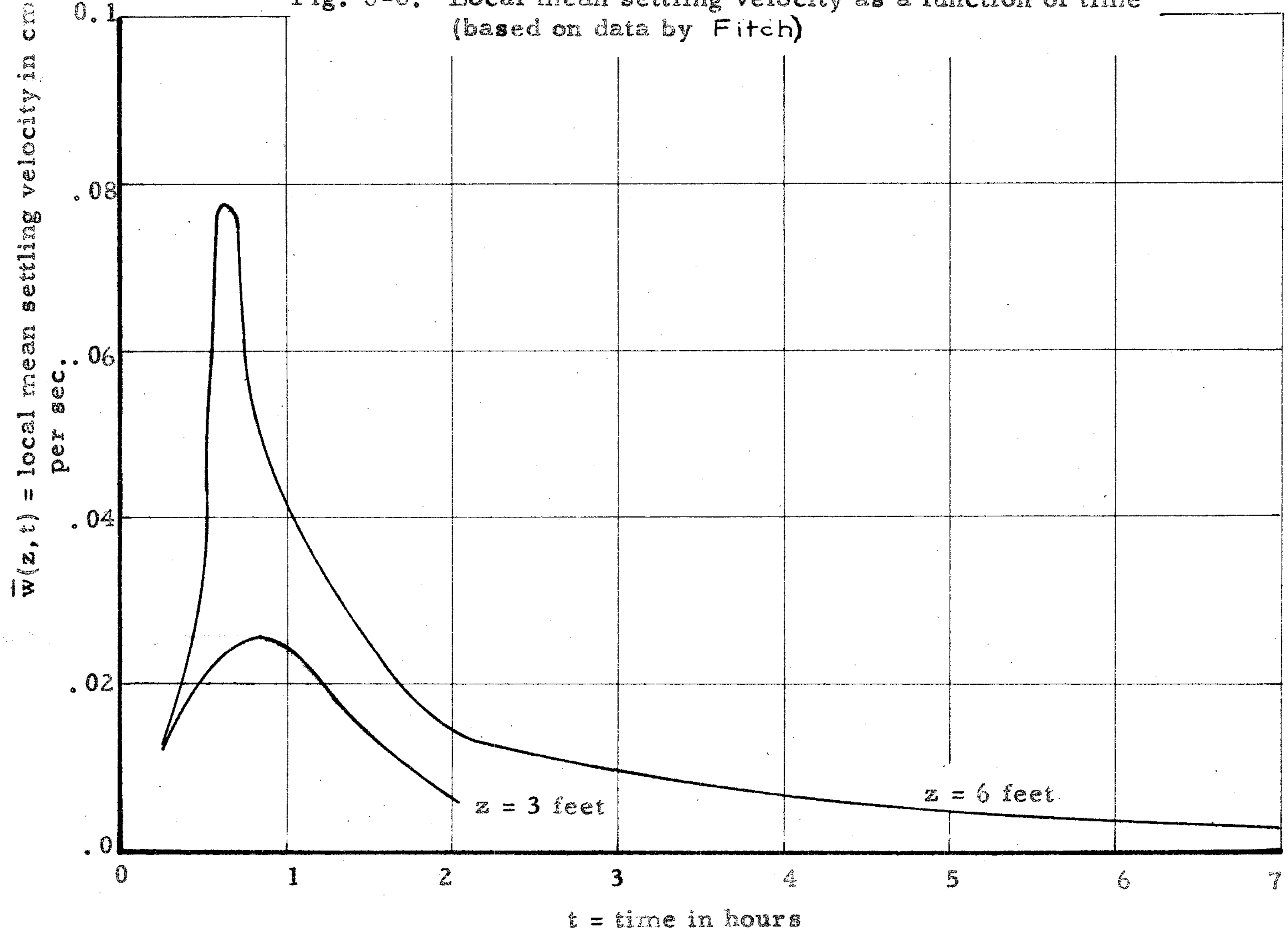


Fig. 6-7. Ø,z diagram for a suspension of Ca CO_3 and $Fe_2(SO_4)_3 \cdot 9H_2O$ in water (after Fitch).

Fig. 6-8. Local mean settling velocity as a function of time (based on data by Fitch)



t = 0.5 hours. The average rate of change during this time was 0.000025 cm. per sec. per sec. At t = 0.5 hours the concentration at z = 6 feet is 94 per cent of the initial concentration. Consequently, up to this time the change in $\overline{w}$ is caused primarily by flocculation alone.

It is interesting to compare the change in $\overline{w}$ for whiting and ferrisul with the change for clay and alum. The comparison is valid only for those stages of settling for which the concentration has not decreased significantly. For whiting and ferrisul the rate of change during this stage of settling has been calculated in the previous paragraph and was 0.000025 cm. per sec. per sec. For clay and alum the corresponding rate of change was 0.000053 cm. per sec. per sec. The values differ by a factor of two. Without auxiliary experiments the difference cannot be attributed to specific causes.

CHAPTER 7

SETTLING ANALYSIS IN THE STUDY OF SETTLING TANKS

The purpose of this chapter is to describe the use of settling analysis in the investigation of removal in settling tanks. The discussion is limited to tanks which are similar in shape and flow pattern to the "ideal" tank.

The use of the multiple-depth analysis is described in some detail. This experiment is particularly helpful in determining the relative effects of detention time and overflow rate on removal. The effects are calculated for the clay-alum and $CaCO_3$-Ferrisul suspensions.

This chapter also presents the results of some simple pipette analyses of Pasadena sewage. These results show the need for large scale experiments, using samples of large volume.

A brief discussion of bottom withdrawal tests and turbulent settling columns is presented. The former is the most logical choice for routine analysis. The latter is necessary for the determination of maximum possible removal.

7-1 Øz Diagram Analysis for Quiescent Settling in a Rectangular Tank

a. General. The discussion in this section is limited to tanks which are similar to the ideal tank described in section 2-2. These tanks have the same zones as the ideal tank, and the flow of the suspension is identical to that in the ideal tank. At the inlet end of the settling zone, the particles of each settling velocity are uniformly distributed over the depth D. In the settling zone, however, flocculation and hindered settling may occur, but turbulence is assumed to be absent.

For such a tank, the removal is the same as the removal in an equivalent quiescent settling column. The ϕ,z diagram for the settling column can be used to calculate this removal. Furthermore, if the column is deep enough the ϕ,z diagram can be used to study the effect of overflow rate and detention time on removal.

In order to use the ϕ,z diagram for this more general study, it is necessary to give additional interpretation to the lines of the diagram. A hypothetical diagram is shown in figure 7-1. It represents a multiple-depth analysis of an unknown suspension. The lines of this diagram show how the suspension responds to change in depth and overflow rate.

b. Lines of Constant Detention Time. The use of the concentration profile at the outlet end of a tank is shown in figure 2-5. The profile corresponds to a settling time equal to the detention time.

Assume that the suspension described by figure 7-1 passes through a tank of depth D_1 and detention time T_1. The concentration profile for the outlet end is given by the curve $t = T_1$ for the range $0 \leq z \leq D_1$. If the tank has a depth D_2 and the same detention time, the profile is given

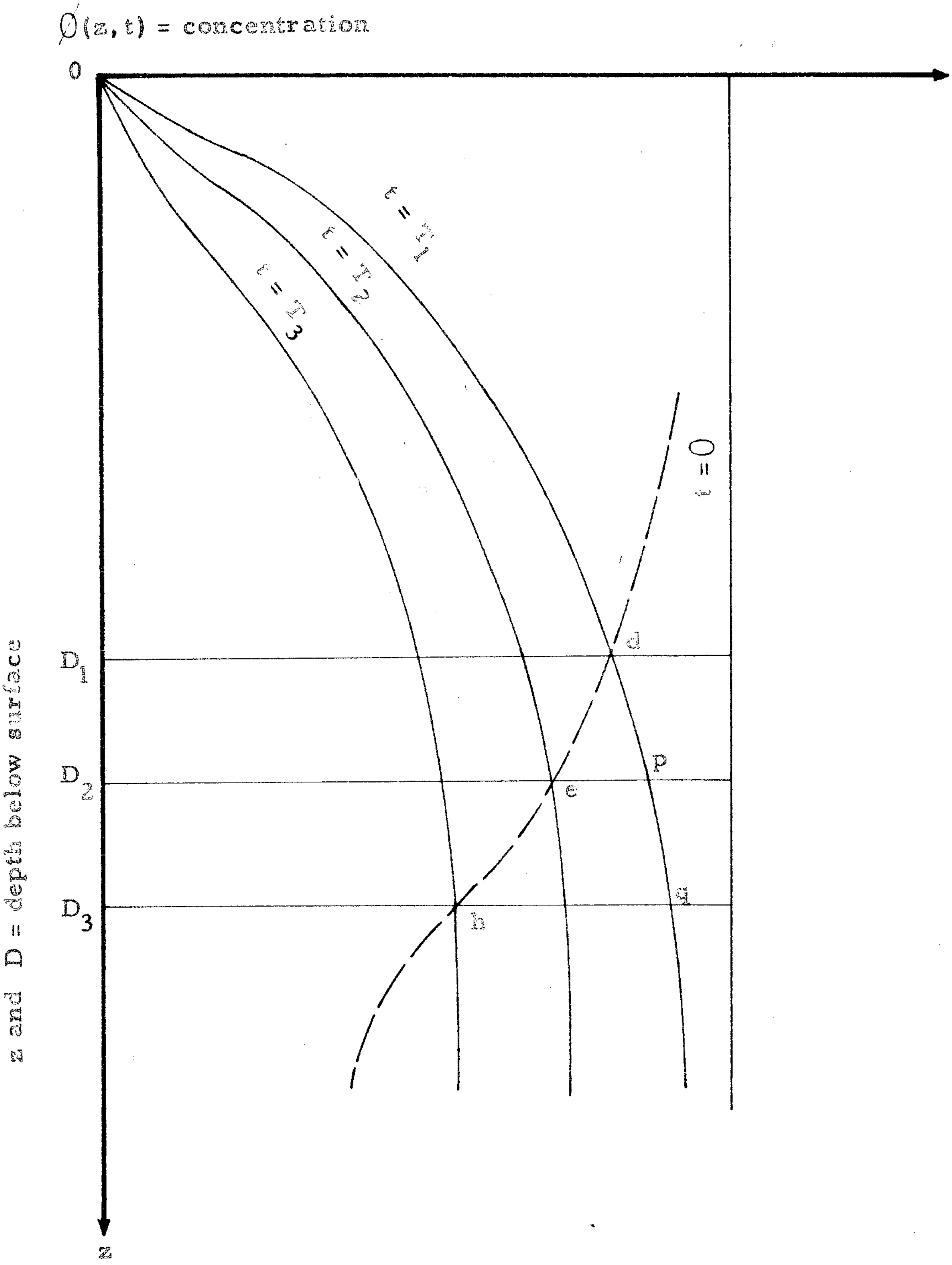


Fig. 7-1. Lines in the Ø, z diagram.

by the same curve $t = T_1$ for the range $0 \leq z \leq D_2$. Consequently, the points dpq are on a line of constant detention time. Therefore, a profile in the ϕ,z diagram corresponds to the outlet profiles of a series of tanks of different depths but with the same detention time.

c. Lines of Constant Overflow Rate. If the suspension of figure 7-1 passes through a tank of depth D_2 in time T_2, the outlet profile will be the line $t = T_2$ for the range $0 \leq z \leq D_2$. A similar argument holds for depth D_3 and time T_3. If all three tanks have the same overflow rate w_0, the values of D/T must be the same. As a result

$$\frac{D_1}{T_1} = \frac{D_2}{T_2} = \frac{D_3}{T_3} = w_0 \qquad (7\text{-}1)$$

Consequently, the points deh lie on a line of constant overflow rate. Since constant D/T corresponds to constant z/t, the lines of constant z/t are lines of constant overflow rate.

d. Effect of Depth on Removal when T = Constant. The rate of change of removal R(D,T) with depth D of tank for constant T is merely the partial derivative

$$\left(\frac{\partial R}{\partial D}\right)_T$$

The removal has already been defined by equation 2-28.

$$R(D,T) = \frac{A(D,T)}{S(D,0)} \qquad (2\text{-}28)$$

From this equation, the partial derivative with respect to D is

$$\left(\frac{\partial R}{\partial D}\right)_T = \frac{1}{[S(D,0)]^2}\left[S(D,0)\frac{A}{D} - A\frac{\partial S(D,0)}{\partial D}\right] \qquad (7\text{-}2)$$

The following derivatives are obtained from the definitions of S and A in equations 2-15 and 2-27:

$$\frac{\partial A}{\partial D} = \phi(D,0) - \phi(D,T) \qquad (7\text{-}3)$$

$$\frac{\partial S(D,0)}{\partial D} = \phi(D,0) \qquad (7\text{-}4)$$

Substituting these derivatives in equation 7-2 gives

$$\left(\frac{\partial R}{\partial D}\right)_T = \frac{1}{S(D,0)}\left[\phi(D,0) - \phi(D,T) + \frac{A}{S(D,0)}\phi(D,0)\right]$$

$$= \frac{1}{S(D,0)}[(1 - R)\phi(D,0) - \phi(D,T)] \qquad (7\text{-}5)$$

When removal depends solely on detention time, the partial derivative with respect to depth is zero. Consequently

$$(1 - R)\phi(D,0) = \phi(D,T) \qquad (7\text{-}6)$$

However, R is constant for this case; hence, $\phi(D,T)$ is a constant ratio of the initial concentration $\phi(D,0)$. Since the initial concentration is uniform the following conclusion can be drawn. In regions of the ϕ,z diagram where detention time governs the removal, the profiles are straight lines parallel to the z axis.

e. Effect of Depth on Removal when w_0 = Constant. The relationship between depth and removal for constant overflow rate can be derived from the geometry of the ϕ,z diagram. One begins by considering the special type of ϕ,z diagram shown in figure 7-2(a). In this figure the lines $\frac{z}{t} = K$ are all straight and parallel to the z axis. Along these lines, the concentration is constant. Hence, the concentration at $z = KT_1$ is equal to that at $z = KT_2$ for any value of K. For this reason, the profiles $t = T_1$ and $t = T_2$ are geometrically similar.* In fact, all of the profiles in this ϕ,z diagram must be geometrically similar.

In figure 7-2(a) let one value of K be w_0, the constant overflow rate to be considered. A tank with detention time T_1 and overflow rate w_0 will have a depth $D_1 = w_0 T_1$. The outlet profile for this tank will be $t = T_1$ from the origin to point 4. Similarly, a second tank with the same overflow rate and with detention time T_2 will have a depth $D_2 = w_0 T_2$, and the outlet profile will be the curve $t = T_2$ from the origin to point 7.

For the first tank the removal ratio can be given as the

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* If ϕ were plotted against z/t instead of z, the two profiles would fall on a single curve. Hence, ϕ is a function of z/t alone.

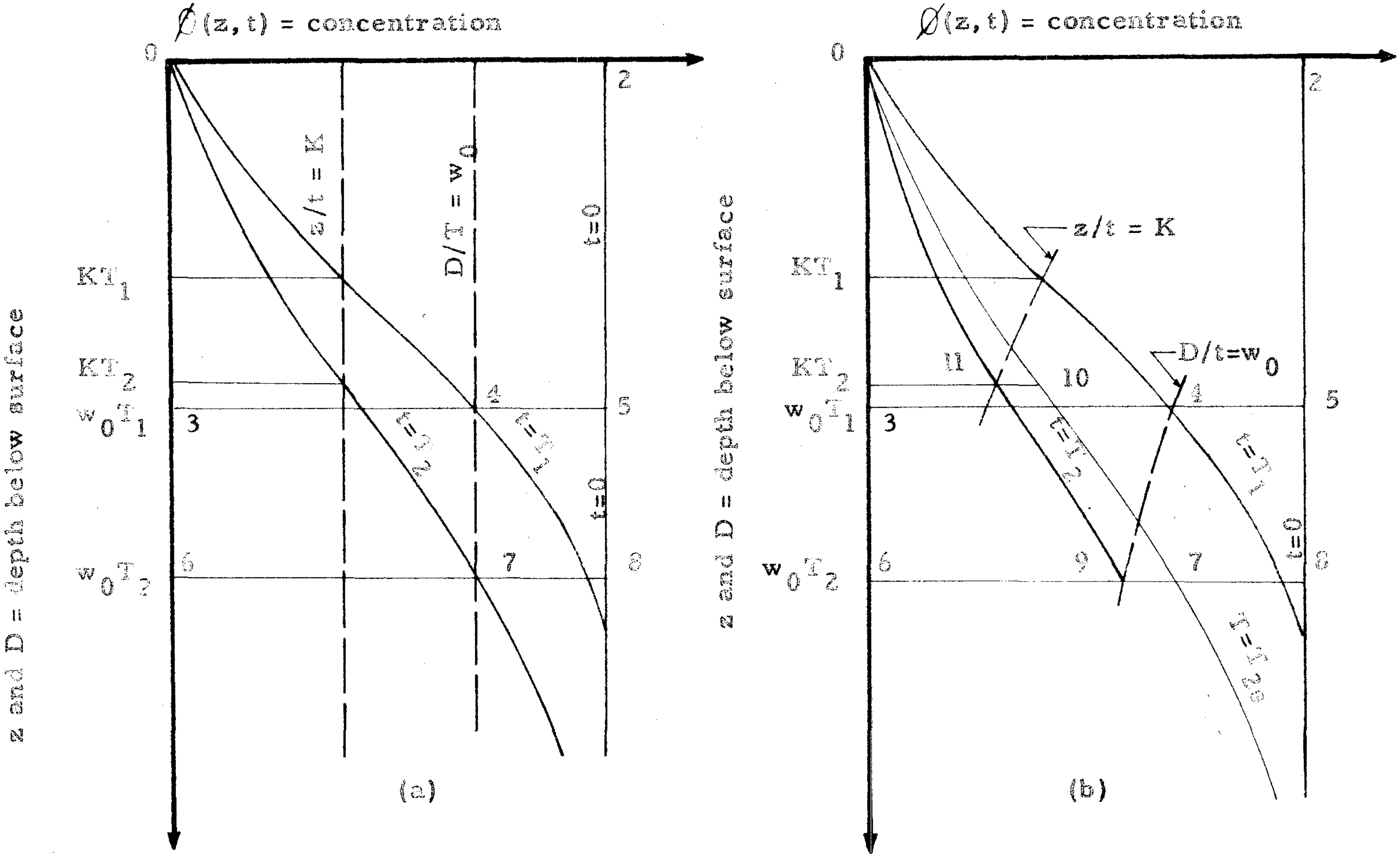


Fig. 7-2. Similar profiles in the Ø, z diagram.

ratio of two areas as follows:

$$R(D_1,T_1) = \frac{\text{area } 0,2,5,4}{\text{area } 0,2,5,3} \tag{7-7}$$

Similarly, for the second tank

$$R(D_2,T_2) = \frac{\text{area } 0,2,8,7}{\text{area } 0,2,8,6} \tag{7-8}$$

From the similarity of the profiles

$$\frac{\text{area } 0,2,5,4}{\text{area } 0,2,8,7} = \frac{T_1}{T_2}$$

while

$$\frac{\text{area } 0,2,5,3}{\text{area } 0,2,8,6} = \frac{T_1}{T_2}$$

Therefore

$$R(D_2,T_2) = R(D_1,T_1) \tag{7-9}$$

Equation 7-9 holds for any D_2 and T_2 as long as $\frac{D_2}{T_2} = w_0$. Consequently when the lines of $\frac{z}{t} = K$ are all straight lines parallel to the z axis, one has a sufficient condition for $R(D,T)$ to depend solely upon w_0.

It remains now to prove that the condition is also necessary. To do this, one shows that no other pattern of lines gives constant R(D,T) for constant w_0.

In figure 7-2(b), the profile $t = T_1$ is identical to the T_1 profile in figure 7-2(b). However, in figure 7-2(b) one assumes that the lines of constant z/t are not vertical but have the shapes shown. At time T_2, these lines must reach the depths $z = KT_2$ and $D_2 = w_0T_2$ just as they do in figure 7-2(a). Therefore, the profile $t = T_2$ in figure 7-2(b) must pass through points 9 and 11.

For depth D_1 and time T_1 the removal $R(D_1,T_1)$ is still given by equation 7-7, but for depth D_2 and time T_2 the removal ratio is

$$R(D_2,T_2) = \frac{\text{area } 0,2,8,9}{\text{area } 0,2,8,6} \qquad (7\text{-}10)$$

If removal ratio depends solely on overflow rate

$$R(D_2,T_2) = R(D_1,T_1)$$

as in equation 7-9.

In figure 7-2(b) a profile $t = T_{2s}$ is shown. This profile is identical to the line $t = T_2$ in figure 7-2(a) and is, therefore, geometrically similar to $t = T_1$. It has already been shown that

$$R(D_2,T_{2s}) = R(D_1,T_1) \qquad (7\text{-}11)$$

where $R(D_2,T_{2s})$ is given by equation 7-8. Therefore, in figure 7-2(b)

$$R(D_2,T_2) = R(D_2,T_{2s}) \tag{7-12}$$

and the net area between profiles $t = T_2$ and $t = T_{2s}$ (area 0,7,9) must be zero.

It follows that these two profiles must coincide or cross. However, if they cross there will be some w_0 for which the area 0,7,9 is not zero. If it is to be zero for arbitrary w_0 the two profiles must coincide. As a result the lines of $z/t = K$ must parallel to the z axis.

In the preceding paragraphs the following statement has been proven: The removal ratio R(D,T) is a function of overflow rate alone, if and only if the lines of constant z/t are all straight and parallel to the z axis. This statement is based strictly on the geometry of the ϕ,z diagram and the fact that $\phi(z,0)$ = constant. No assumptions have been made about the type of settling that produces the diagram.

7-2 Examples of ϕ,z Diagram Analysis

a. Clay and Alum in Water. The ϕ,z diagram in figure 6-4 is based on the multiple depth analysis of a suspension of clay and alum in water. The diagram can be used to study the effect of depth and detention time upon this suspension. Of course, the range is limited to tanks with $D < 3.0$ feet and $T < 2$ hours.

In figure 6-4, the lines of constant z/t or w_0 show that removal is not dependent solely on overflow rate. None of the lines is parallel to the z axis.

In the same diagram, the profiles show that the detention time is not the sole governing criterion for removal. None of the profiles are straight lines parallel to the z axis. The shape of the profiles is explained in part (c) below.

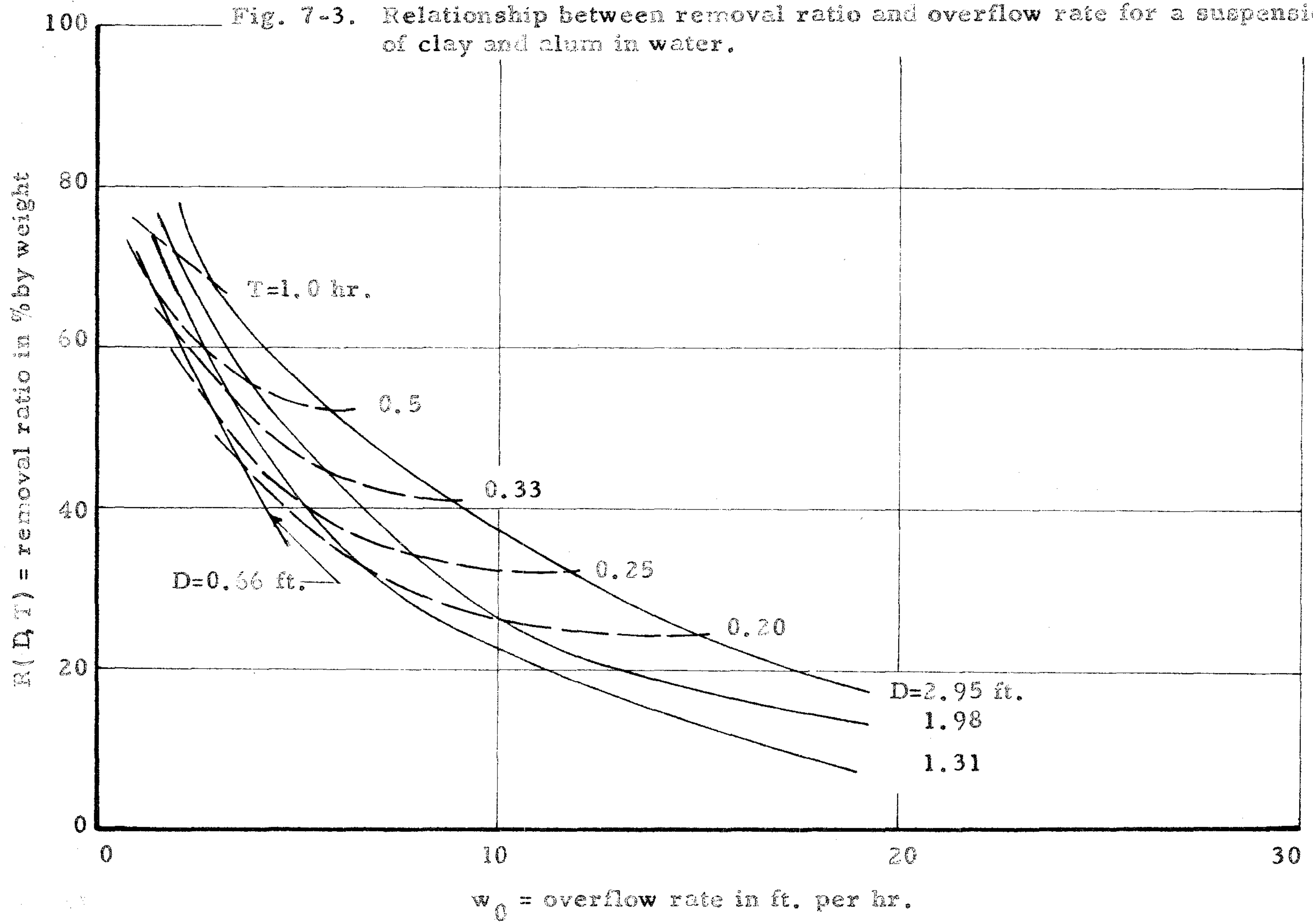
In figure 7-3 the removal is plotted against overflow rate for the clay-alum suspension. Depth and time are used as parameters. The curves substantiate the conclusions drawn in the previous paragraphs. Neither overflow rate nor detention time alone governs the removal. For a given overflow rate, the removal for D = 2.95 feet (90 cm.) is approximately 1.5 times the removal for D = 1.31 feet (40 cm.).

b. Fitch's Experiment. Fitch's multiple-depth analysis has been discussed in section 6-2. The results have been plotted as a ϕ,z diagram in figure 6-7. This diagram can be used to determine the effect of depth on the removal.

The lines of constant z/t or w_0 show that overflow rate does not govern removal until the removal is almost complete. Therefore, during the major part of the settling, removal is not governed by overflow rate.

On the other hand, removal appears to be governed by detention time. For $T \geq 0.25$ hours, the profiles are lines reasonably parallel to the initial distribution. Fitch actually calculated removal and found that it was more

Fig. 7-3. Relationship between removal ratio and overflow rate for a suspension of clay and alum in water.



closely related to detention time than to overflow rate.

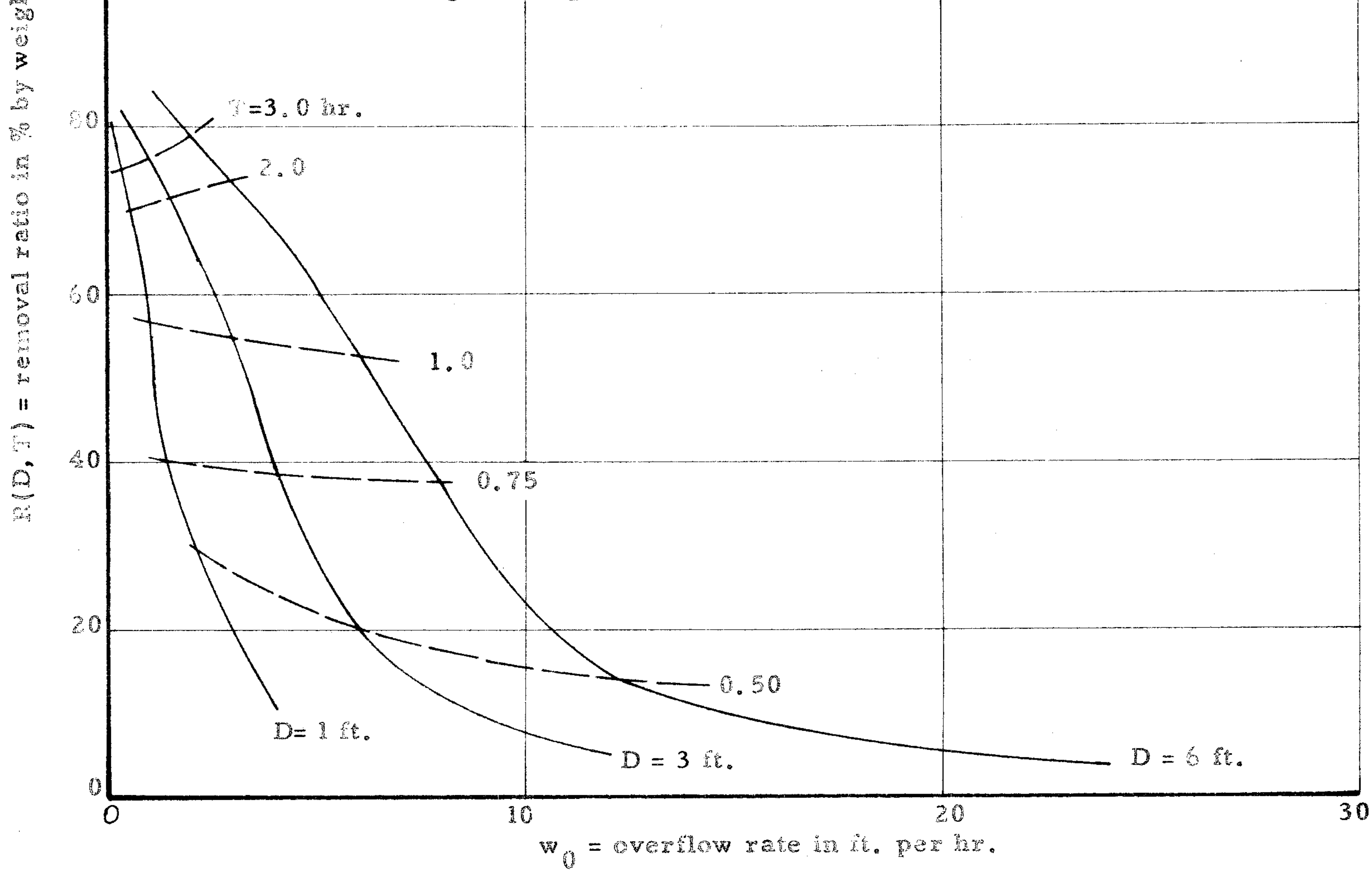
In figure 7-4 the removal is plotted against overflow rate for the suspension of whiting and Ferrisul. Depth and time are used as parameters. For a few values of T, the removal is almost independent of overflow rate.

c. Effect of Flocculation on the Shape of the Profiles. The shape of the profiles in the ϕ,z diagram determines whether or not detention time governs removal. If these profiles are straight lines parallel to the z axis, detention time is more important than overflow rate.

In order to relate flocculation to the shape of the profiles it is best to return to the simple suspension discussed in section 5-3. This suspension has particles of only two sizes. Figure 5-2(b) gives the spatial distribution of each size at time $t = t_1$. It is obvious that after the larger particles have settled out of suspension, the concentration of small particles will decrease with depth. When there is only one size of small particle, this shape of profile will exist.

A more complicated suspension has particles of many velocities. Assume that a group of large particles has settled out leaving a concentration of small particles which decreases with depth. Let the small particle be a collection of particles with varying velocities. These particles will not maintain the profile shown in figure 5-2(b). On the contrary, differential settling among the small particles will tend to make the remaining concentration increase with

Fig. 7-4. Relationship between removal ratio and overflow rate for a suspension of $CaCO_3$ and $Fe_2(SO_4)_3 \cdot 9H_2O$ in water (based on data by Fitch).



depth.

It follows that the profiles in the ϕ,z diagram are the result of the combination of the two effects, and the profile shapes depend upon the relative magnitude of the two effects. Figure 6-4 shows the shapes of the profiles for the suspension of clay and alum. For each profile the concentration first increases with depth and then decreases. The faster particles have settled out leaving the decreasing profile. At the same time, settling of the remaining particles has begun to form an increasing profile. The result is a maximum concentration near mid-depth.

If the velocities of the remaining slow particles had been nearer the velocities of the faster particles the shape of the profile would have been different. No maximum concentration would have occurred at mid-depth. The resulting profile would be more like a straight line. In fact, a certain distribution of large and small velocities could produce profiles which are nearly straight lines parallel to the z axis.

It appears that such a combination of settling velocities occurred in Fitch's experiment. The profiles have been shown in figure 6-7. They are, indeed, almost parallel to the z axis. One must admit, therefore, that flocculation can produce a settling regime in which removal is determined by detention time.

d. Effect of Flocculation on the Shape of Lines of Constant Overflow Rate. This effect has already been discussed in section 5-3. The conclusion was that flocculation causes lines of constant z/t to have negative slopes. However, if overflow rate is to govern removal, and if the particles were uniformly distributed at $t = 0$, the lines of constant overflow rate must be parallel to the z axis. Consequently, flocculation will work against the possibility of removal being governed by overflow rate.

7-3 Experiments with Sewage

a. General. Many settling tanks are required to handle suspensions which vary in character from hour to hour. In addition, there may be a longer range variation from day to day or season to season. If settling analyses are to be used in a study of the suspension, data must be obtained for the whole range of variation.

Variation in the character of the suspension may be obscured by the variation in samples. The settling properties of samples of a suspension may vary from sample to sample, if the samples are too small. If the samples are used in settling analysis, the results of the analysis will vary accordingly.

During the research, many pipette analyses were performed with sewage as the suspension. The sewage presented many of the difficulties encountered in trying to predict settling behavior from settling analysis. The concentration of particles was below 500 mg. per liter; the settling velocities were as large as 1 cm. per sec.; and the suspension varied from sample to sample.

The sewage was obtained from a trunk sewer of the Los Angeles County Sanitation Districts where it passes through the new golf course in Alhambra, near the site of the abandoned Tri-Cities Sewage Treatment Plant. This trunk sewer serves most of Pasadena, San Marino, South Pasadena, and parts of contiguous communities, with a total sewered population of about 200,000. Owing to the sloping topography, velocities in the trunk and lateral sewers are high. In addition, the sampling point was very near a confluence and drop in the sewer. As a result, the sewage was well mixed.

The samples obtained from the sewer were called gross samples. They were taken in the morning and tested in the afternoon of the same day. Before the experiment, the gross sample was poured into a large ceramic crock. When a smaller sample was required for testing, the gross sample was stirred, and while stirring continued, the smaller sample was taken with a dipper.

Occasionally several similar small samples were required. They were obtained in the following manner. The containers for the smaller samples were placed in a row. While the gross sample was being stirred, a dipperful of sewage was placed in each container. The process was repeated until each container held a sample of the desired size.

b. Variation in the Settling Properties of Gross Samples. The gross samples, obtained from the trunk sewer, were usually five gallons in volume. It was assumed that each gross sample was representative of the sewage from which it was taken.

From each of several gross samples, a liter sample was taken in the manner described above. The liter sample was used for a pipette analysis. General information about the analyses is given in table 7-1, and the results are plotted in figure 7-5. The curves of figure 7-5 differ considerably. The difference in ordinates at $\frac{z}{t}$ = 0.03 cm. per sec. is 13 per cent of the initial concentration. This difference is greater than the maximum deviation of any individual point from its respective curve. Only two individual points have a deviation of 8 per cent of initial concentration. All other points have less.

At least some of the difference between the curves in figure 7-5 is due to difference in the liter samples tested. However, the liter samples may not be truly representative of the 5-gallon gross samples. The variation of small samples from a single gross sample will be discussed next.

c. Duplicate Pipette Analysis for a Single Gross Sample. Two similar samples of about 3.8 liters were taken from a single gross sample. The method of obtaining similar samples has already been described in part a above. These samples were used for duplicate pipette analysis.

For the purpose of duplicate tests, two identical settling were constructed. In design, they were similar to the lower

Table 7-1

Pipette Analyses of Pasadena Sewage

Settling tube - 1 liter graduate

Volume used - 1 liter

Temperature control - none

Size of samples withdrawn - 25 ml.

Method of withdrawing samples - 25 ml. broken tip pipette lowered into suspension by hand for each sample.

Depth of samples - 22 cm. below surface of sewage.

Date	Gross Sample Obtained	Pipette Analysis Began	Initial Concentration	Temp. During Test
Thur. Oct. 21/54	8:30 a.m.	2:35 p.m.	304 mg./l	
Thur. Oct. 28/54	8:45 a.m.	11:40 a.m.	320	22-27°C.
Mon. Nov. 15/54	9:00 a.m.	1:45 p.m.	374	21-23.5°C.
Mon. Nov. 22/54	9:00 a.m.	1:00 p.m.	380	23-30°C.

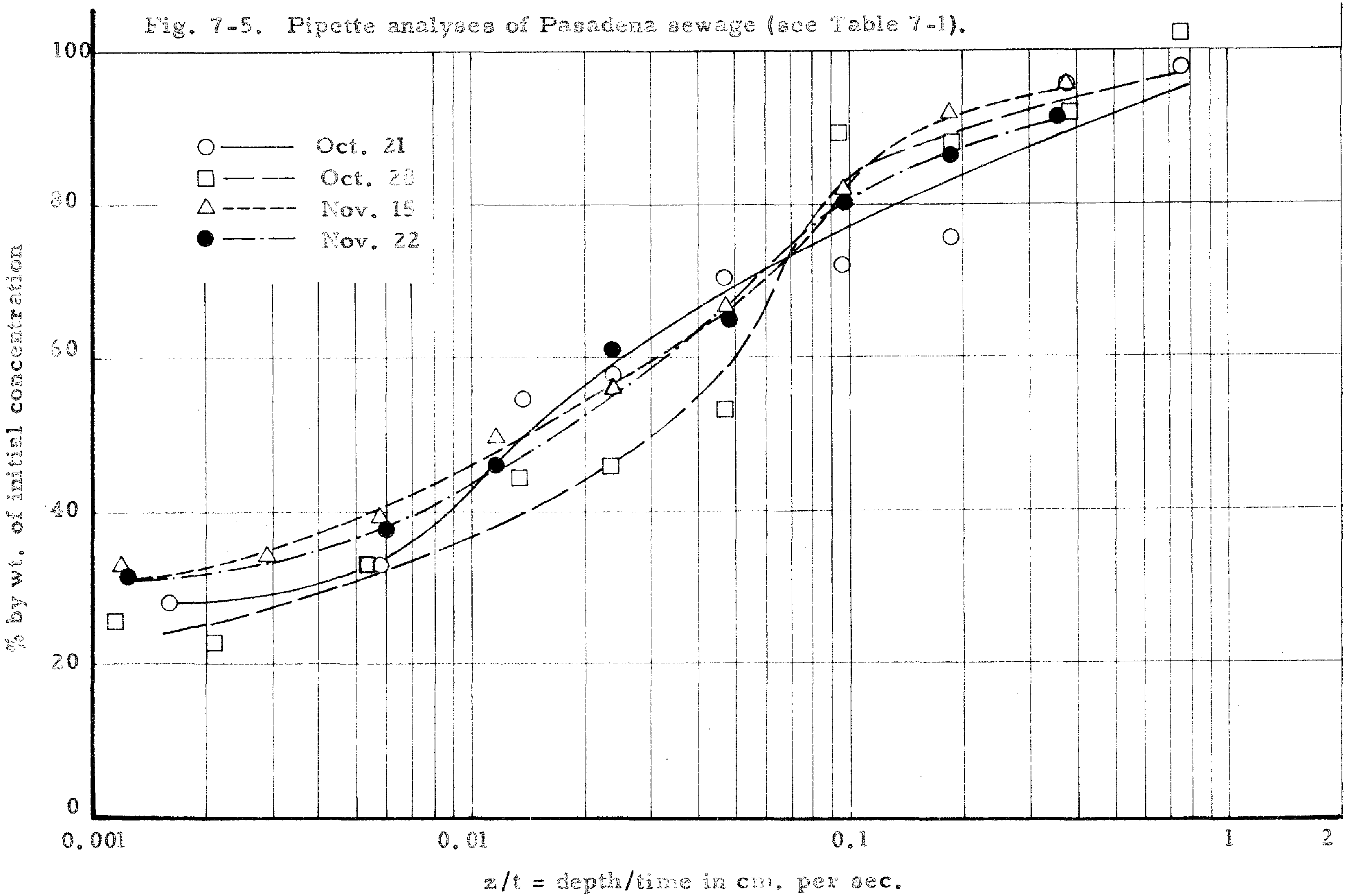


Fig. 7-5. Pipette analyses of Pasadena sewage (see Table 7-1).

half of the multiple-depth tube shown in figure 4-8. Each tube had only one intake. In order to control the temperature, both tubes were placed in the same water bath. For the sake of differentiation, these tubes were called tube A and tube B.

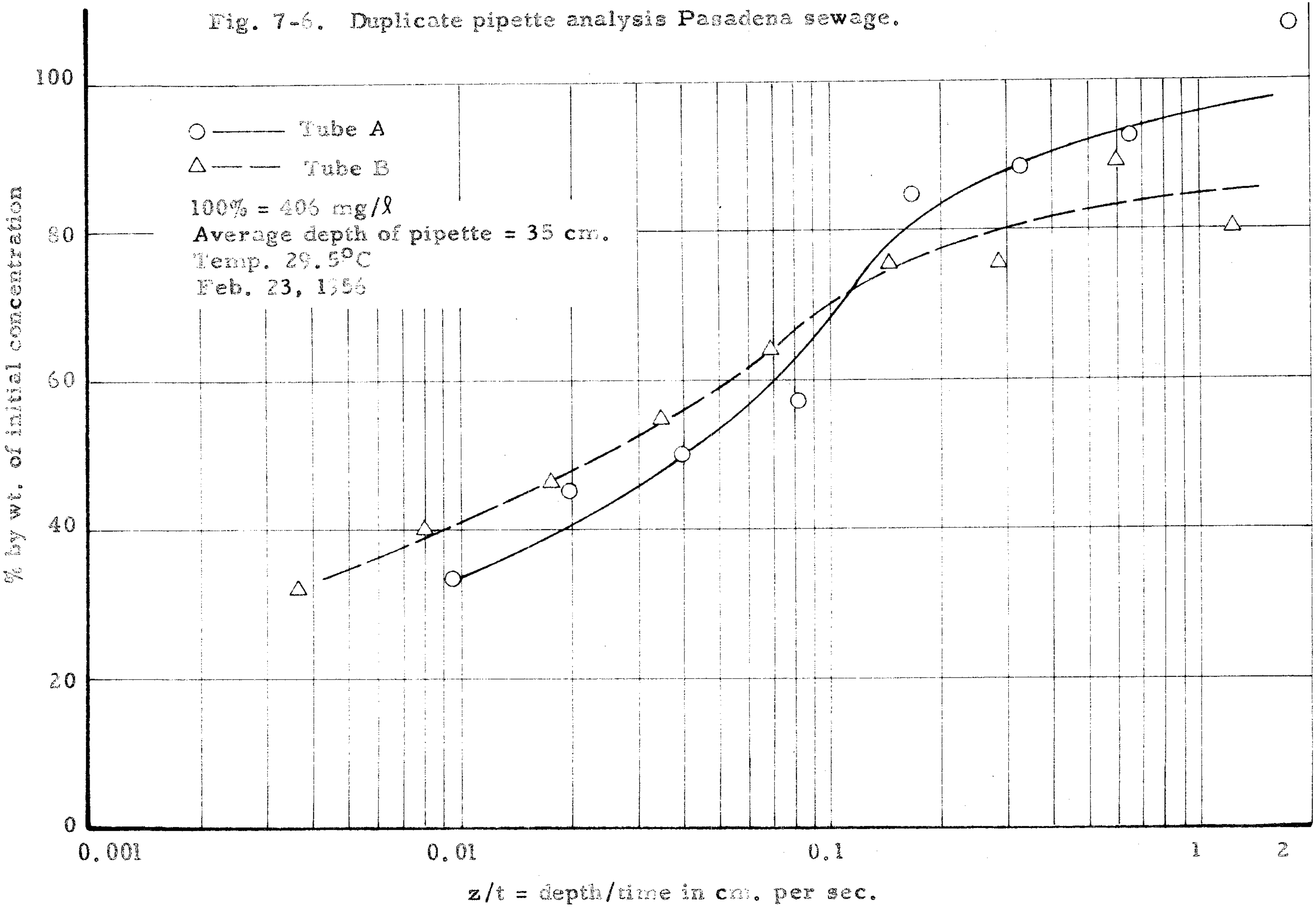
Figures 7-6 and 7-7 show the results for two gross samples. The difference between curves for the same gross sample is nearly as large as the difference between gross samples shown in figure 7-5. A third set of duplicate pipette analyses substantiated this conclusion.

d. Variation in the Effect of Flocculation. Since the results of a pipette analysis of sewage vary from experiment to experiment, one would expect the effect of flocculation to vary as well. The two-depth settling analysis is sufficient to determine whether or not the effect of flocculation is constant.

Figure 7-8 shows the results of a two-depth settling analysis of Pasadena sewage. Figure 7-9 shows the results of a similar test on a different day. In each figure, the difference between curves for lower and upper intakes is a measure of flocculation. In figure 7-8 the effect of flocculation appears to be negligible, while in figure 7-9 it appears to be relatively large. This variation was exhibited by other two-depth tests on sewage.

e. Sewage as a Continuum of Particles. Figures 7-5 to 7-9 inclusive, show that the individual plotted points have fairly large deviations from the fitted curves. The Federal

Fig. 7-6. Duplicate pipette analysis Pasadena sewage.



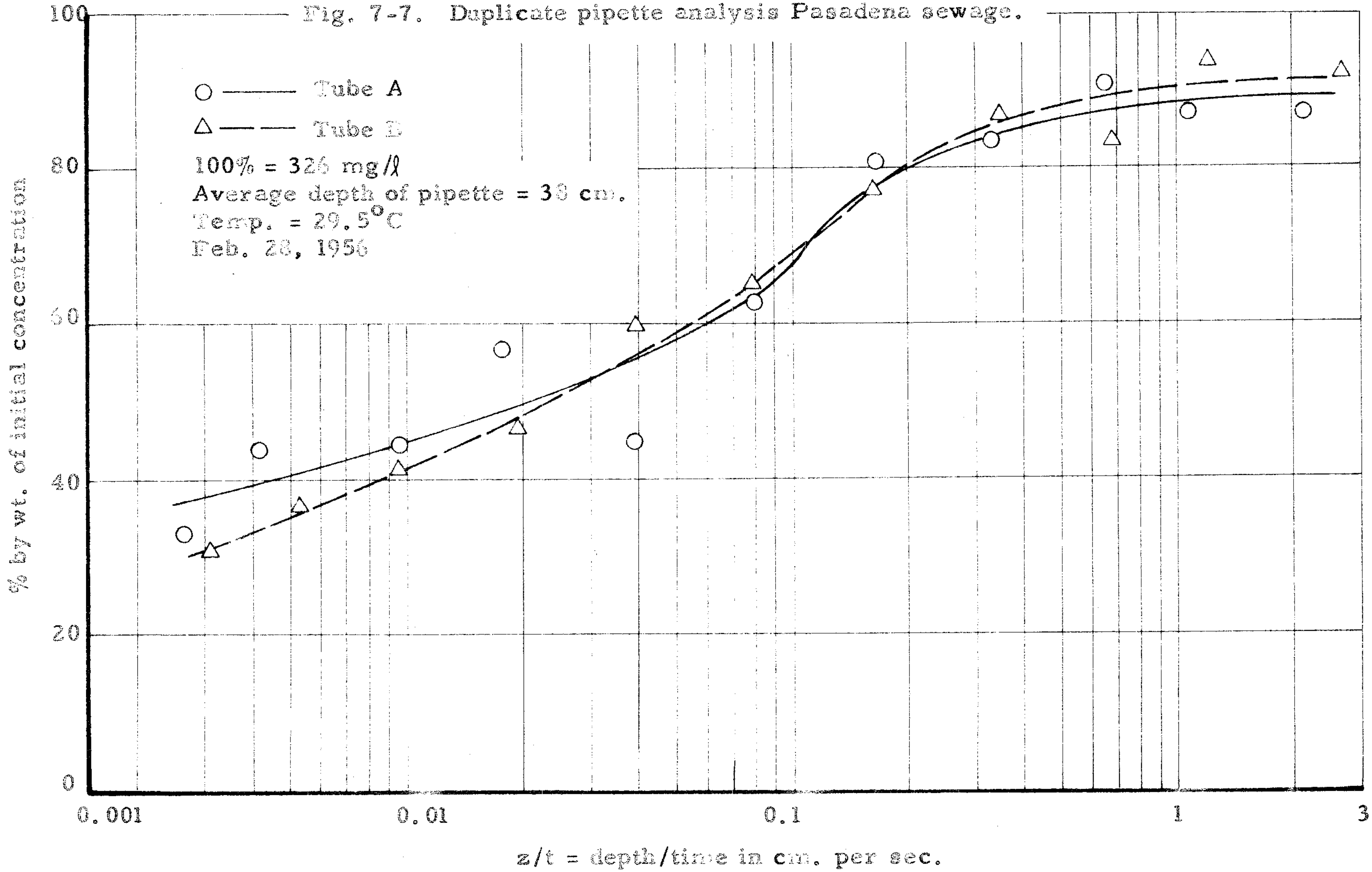


Fig. 7-7. Duplicate pipette analysis Pasadena sewage.

Fig. 7-8. Two-depth analysis of Pasadena sewage.

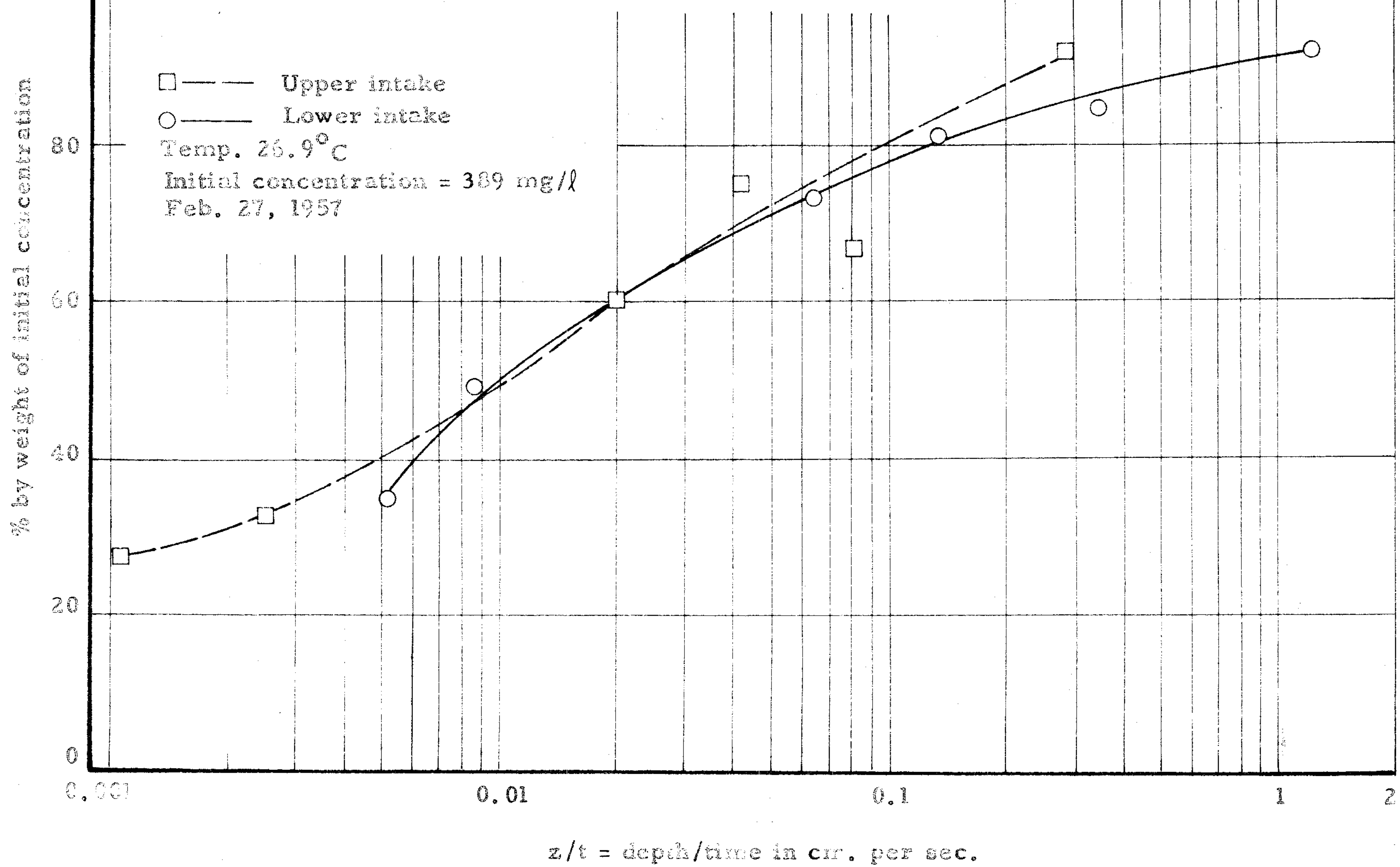
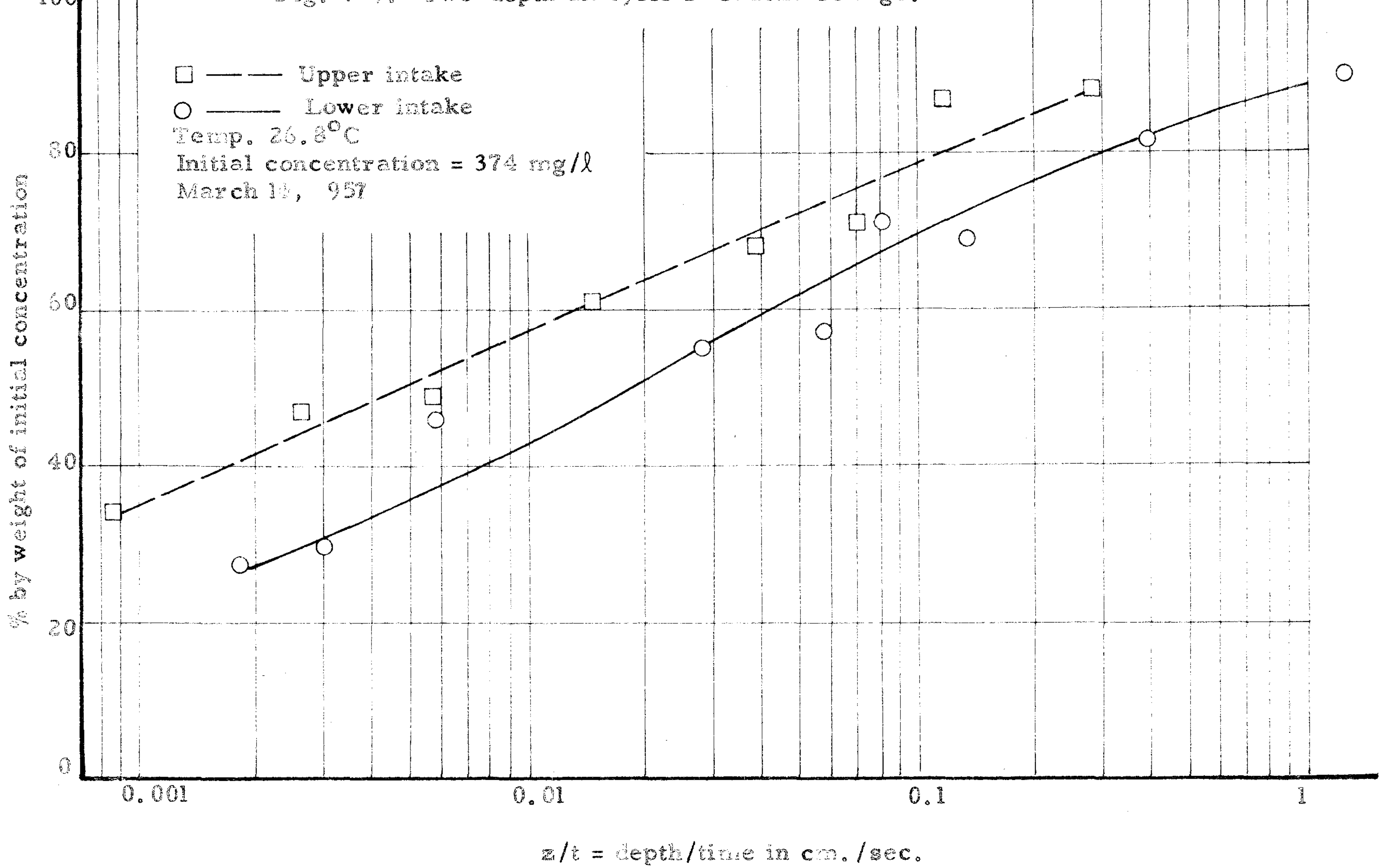


Fig. 7-9. Two-depth analysis Pasadena sewage.



Inter-Agency River Basin (FIARB) Subcommittee on Sedimentation found that such erratic data are common to the pipette analysis when the particle concentration is small (26). The implication is that the deviation is caused by the error in weighing small quantities. This simple explanation does not satisfactorily explain the pattern of deviation. A more general approach is to investigate the assumption that sewage is a continuum of particles.

For the pipette analysis of sewage, the error in a single point can be estimated. Each 25 ml. sample was filtered through a weighed membrane filter,* or a Gooch crucible. The filter and residue were dried, cooled and weighed. Both weighings were performed with an analytic balance sensitive to 0.1 mg. As a result, the weight of dry solids on the filter had an absolute error of 0.2 mg. The weight of solids varied from approximately 1 mg. to 10 mg.

As an example, consider the data in figure 7-5. The weight of solids on a filter varied from 1 mg. for small z/t to 9 mg. for large z/t. The relative error, therefore, ranges from 2.2 per cent to 20 per cent. The relative error in the volume of a sample was always less than 2 per cent. In general, then, the relative error in the concentration of an individual sample ranges from 4 per cent at large z/t to 22 per cent at small z/t.

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* The method of filtering for suspended particles was similar to the method of filtering for bacterial assay (28).

For the curves shown in figure 7-4, this error is approximately 8 per cent in the range $0.01 < z/t < 0.06$. In this range the concentration is approximately 50 per cent of the initial concentration. It follows that the error in a single point is (0.08)(50%) = 4 per cent. The actual deviations of the points from the curves agree with this figure.

At large values of z/t, the deviation of a point should be approximately (0.04)(95%) = 4 per cent. However, the deviations appear to be as large as 8 per cent. At first glance, one would say that the large values of z/t have errors due to the error in t. The latter is usually attributed to the fact that the turbulence from the initial mixing persists during the early stages of settling. Visual observation during the tests shows that the error in t may be 50 seconds. By the time t = 200 seconds, this error is 25 per cent. The error in z is relatively negligible; hence, in z/t the error is 25 per cent. On the logarithmic scale of figure 7-5, an error of 25 per cent causes a small horizontal deviation. Therefore, most of the deviation shown by the point must be in the vertical direction. That is, the deviation is in concentration.

If the experimental error cannot account for deviation at large z/t, the error must be in the assumption that the concentration changes continuously. This assumption is equivalent to the assumption that the suspension can be treated as a continuum of particles. The latter was discussed in section 3-1 and was shown to depend upon the particle spacing.

The spacing of particles in a suspension can be calculated from a settling velocity distribution, if the particle shape and density are known. In order to make such a calculation, the data of November 15 from figure 7-5 were taken as a settling velocity distribution. The particles of sewage were assumed to be spheres with a specific gravity of 1.03. With this assumption, the particle size and spacing was calculated for finite increments of settling velocity. The results of the calculation are shown in table 7-2.

From the table one would conclude that the slower particles can be taken as a continuum. On the other hand, the faster particles have an average spacing of the order of 1 cm. With such a spacing, there can only be one particle per ml. When this particle has a diameter of 0.36 mm., its weight is .025 mg. The weight of four such particles is as large as the sensitivity of the balance used for weighing. There are only 25 of these particles in a 25 ml. sample. Consequently, any irregularity in the particle spacing can affect the measured concentration of the sample.

Of course, theparticles in sewage are not spherical. They vary in size, shape and density. Nevertheless, the conclusion that single large particles affect results is still valid. During experiments, larger individual particles were observed in the settling tubes. Often their individual identity persisted through the filtering and drying operations.

Table 7-2

Estimate of Particle Spacing in

Pasadena Sewage

Settling Velocity w	Arith. Mean w_i	Concentration Slower F(w)	Fraction F_i	Particle Diameter $2r_i$	Ratio of Radius to Spacing $r_i/$ i	Particle Spacing i
cm./sec.	cm./sec.	mg./	mg./	mm.		mm.
	0.37		14		.015	11.
0.37		360				
	0.23		16	0.36	.016	11.4
0.18		344				
	0.14		40	0.28	.021	6.7
0.092		304				
	0.069		52	0.24	.024	5.0
0.046		252				
	0.0034		40	0.14	.021	3.3
0.023		212				
	0.0017		24	0.10	.018	2.8
0.012		188				
	0.0086		40	0.074	.021	1.8
0.0057		148				
	0.0043		20	0.05	.017	1.5
0.0029		128				
	0.0022		4	0.04	.010	2.0
0.0014		124				
	0.00085		44	0.02	.022	0.45
0.00028		80				
	Colloidal		80		.027	

Basis - Nov. 15 Data from figure 7-5.

- Particles assumed to be spheres with specific gravity of 1.03.

The faster particles in sewage cannot be considered as a continuum of particles, unless the smallest sample is relatively large; perhaps 100 or 200 ml. This size sample cannot be considered as a point sample unless the volume of the settling tube is large; perhaps 20 or 30 liters. For experiments conducted on this scale, the variation in results from experiment to experiment may be greatly reduced.

7-4 Bottom Withdrawal Tube Tests

For routine studies of removal it is desirable to have an experiment which is simpler than the multiple depth analysis. The bottom withdrawal tube is more suitable for these measurements. The removal measured with this apparatus is exactly the same as the removal in a quiescent tank of the same depth. When the depths are the same, the effect of hindered settling and flocculation are automatically included in the experimental results.

The experiment consists of placing the required volume of suspension in the mixing hopper and pouring it from the hopper to the tube. During the settling, five or six samples are withdrawn at the bottom of the tube and filtered. The result is a curve giving removal as a function of time for the depth used in the experiment. If desired, the results can be converted to a function of overflow rate.

In section 4-5, the preliminary design of the bottom withdrawal tube is described. In this design, the problem of removing the settled particles has not been completely solved. Consequently, the error in the measured removal is as high as 15 per cent. The improvements suggested in section 4-6 should greatly reduce the error.

7-5 Turbulence in Settling Analysis

The settling in continuous flow settling tanks is rarely quiescent. There is always some degree of turbulent motion in the fluid. The effect of the turbulence is two-fold; it increases the effect of flocculation while decreasing the rate of settling. If settling analysis is to be fully developed, it will be necessary to add turbulence to the settling column.

A settling column in which turbulent motion is produced can be called a turbulent settling column. During the course of the research, an attempt was made to perform multiple-depth analysis in a turbulent settling column. The experiments were performed in the turbulence apparatus used by Rouse (4). While this apparatus could not be suitably adapted to the needs of settling analysis, it was possible to obtain ϕ, z diagrams and to study the removal. The experiments were of limited value because of the resuspension of particles which had once settled. For completely successful experiments, it will be necessary to incorporate a device for preventing the resuspension. Dobbins (5) used such a device in his experiments on turbulent settling.

With a turbulent settling column it will be possible to determine the maximum removal for a particular suspension and an arbitrary overflow rate. For such a study, a multiple-depth analysis will be performed with a constant degree of turbulence. The experiment will be repeated several times, using a different degree of turbulence each time. For each degree, a ϕ,z diagram can be drawn, and the optimum turbulence level will be that which produces the most favorable ϕ,z diagram. The most favorable diagram can then be analyzed for optimum depth and detention time. The removal corresponding to optimum turbulence, depth and detention time can be used in studying the efficiency of settling tanks.

Ingersoll, McKee and Brooks have pointed out that analyses of the pipette type are tedious to perform and not suitable for routine studies of settling tank efficiency. Since multiple depth analysis is of the pipette type, it is subject to the same criticism.

If simplicity is an important factor, the most logical approach would be to produce turbulence in a bottom withdrawal tube. On the other hand, the bottom withdrawal tube would not give basic information about the settling.

7-6 Discussion

For many years the design of settling tanks has been based on the detention period (detention time). More recently, this approach has become somewhat unpopular. The so-called "rational" design based on overflow rate has been gaining popularity.

Actually neither overflow rate nor detention time can be used as a universal basis for tank design. A truly rational approach would consider the suspension as a material with its own set of settling properties. Naturally, then, one would make measurements to determine these properties. The multiple-depth analysis is well suited for this determination. The experimental data of such an analysis will show the relationship between removal, detention time and overflow rate.

It is interesting to note that concentration measurements were made in tanks and in settling columns before the "ideal" tank was conceived. (See discussion in reference 17). The investigators often observed that the concentration was substantially uniform throughout the depth, for a large portion of the time available for settling. This observation corresponds to vertical profiles in the ϕ, z diagram. The theory of this chapter shows that such profiles indicate a correlation between removal ratio and detention time. These investigators had experimental justification for using detention time in designing tanks for suspension with which they had to deal. Obviously, they did not recognize the relationship between their observation and their design criterion.

CHAPTER 8

SUMMARY

8-1 General

The investigation described in this thesis was limited to the settling aspect of sedimentation in artificial basins. Thus, the suspended particles were considered from the time that they enter the tank with the influent until they either touch the bottom or leave the tank with the effluent. No attempt was made to determine the behavior of the particles after they settle, and the possibility of their resuspension.

One purpose of the research was to study the characteristics of removal of suspended particles during settling. The relationships between the removal ratio and the following factors were analyzed theoretically and experimentally:

(1) The frequency distribution of the settling velocities of the particles suspended in the influent.

(2) The spatial distribution of particles at the inlet of the tank.

(3) The flow pattern of the suspension in the tank.

(4) Flocculation during settling.

(5) Overflow rate (i.e. ratio of depth to detention time).

(6) Detention time.

Another purpose of the research was to determine the effect of flocculation on settling at a general point in the suspension, and to relate this effect to the properties of the suspension at the same point. The order of magnitude of the effect was successfully determined using data from settling column experiments. In order to determine which properties of the suspension are most closely related to the effect of flocculation, a kinetic theory of flocculation was partially developed.

The findings of the research are summarized in the following sections.

8-2 Concentration Profiles and the ϕ,z Diagram

a. The concentration profile can be used in the calculation of removal of suspended particles. For a continuous flow rectangular basin, the concentration profile is a curve giving the local concentration ϕ in mg. per liter as a function of depth z along a vertical line. Examples of profiles are shown in figure 2-4.

b. For the purpose of analysis, it is assumed that the concentration profile does not vary with distance across the width of the tank. In this case the total removal of suspended particles can be expressed as the area under the profile at the inlet end of the tank less the area under the profile at the outlet end.

c. Concentration profiles are also used in the analysis of settling in a settling column. The measured profiles for the column are plotted in a single diagram showing concentration as a function of depth with the parameter being time (instead of distance along the tank). This diagram is called the ϕ,z diagram for the column.

d. Whenever settling in a settling column is equivalent to settling in a basin, the measured profiles in the column correspond to profiles in the tank. In particular, if settling time in the column corresponds to detention time in the tank, the column profiles correspond to those for the outlet end of the tank. The ϕ,z diagram for the column then gives a series of outlet profiles for tanks of various depths.

With this interpretation, the ϕ,z diagram for a very deep settling column can be used to determine the effects of detention time and overflow rate on removal. When the profiles in the diagram are approximately straight lines parallel to the z-axis ($\phi = 0$), the removal ratio is more closely correlated to detention time than to overflow rate. On the other hand, when the lines of constant z/t are parallel to the z-axis, the removal ratio is more closely correlated to overflow rate than to detention time.

8-3 Continuity Equations

a. A suspension is treated as a continuum of settling particles, and a simple continuity equation is developed for the conservation of particles at a point. The treatment is valid as long as the particle spacing is small compared to the dimensions of the suspension or to the size of any samples taken during an experiment. Some erratic results from experiments with sewage are explained by estimating the particle spacing and showing that for sewage, the assumption of a continuum is sometimes doubtful.

b. The continuity equation is solved for some particular cases of settling. With the aid of these solutions and the continuity equation itself, the ϕ,z diagram can be used to isolate possible difficulties in settling column experiments. A list of such diagnoses is given as follows:

(1) If the distribution of particles is known to be uniform at the beginning of settling, the ϕ,z diagram will detect the occurrence of flocculation during settling.

(2) If the initial distribution of particles is known to be uniform at the beginning of settling, the ϕ,z diagram will detect the occurrence of hindered settling during the experiment.

(3) If flocculation and hindered settling are known to be absent during settling, the ϕ,z diagram will detect a non-uniform distribution of particles at the beginning of settling. It will often be possible to tell whether the initial concentration increased or decreased with depth.

8-4 Effect of Flocculation

a. In settling tanks, the important effect of flocculation is its contribution to the time rate of change of mean settling velocity at a point in the suspension.

b. Flocculation is analyzed by considering the rate of inter-particle collisions and the resulting particle unions, caused by non-uniform settling velocities of particles. This kinetic theory of flocculation cannot be completed. It can, however, be carried far enough to show that the degree of flocculation depends primarily on the following properties of the suspension at a point:

(1) The volume concentration of particles.

(2) The mean settling velocity of particles.

(3) The standard deviation of the settling velocity of the particles.

(4) The fraction of inter-particle contacts that result in the union of particles.

c. The change in mean settling velocity at a point was determined for a suspension of bentonite clay in water with alum as a coagulant. In the early stages of the settling, the local mean settling velocity increased from 0.02 to 0.033 cm. per sec. in 250 sec. for an average rate of increase of 0.000052 cm. per sec. per sec. This change was due primarily to flocculation during settling. The increase in mean settling velocity even continued after the concentration at the point had started decreasing. Similar results were found by a reanalysis of data by Fitch (9).

8-5 Experimental

a. All the experiments of this research were conducted in settling columns. Each column was tested for accuracy with suspensions of glass spheres in water.

b. The removal of glass spheres from a suspension in a quiescent settling column was measured by means of multiple-depth pipette analysis. The removal from an identical suspension was measured directly in a bottom withdrawal tube. The bottom withdrawal tube was found to have an error of as much as 15 per cent. In the bottom withdrawal of samples at time intervals, all settled particles are not removed from the bottom of the tube as they should be. However, preliminary studies with a scraper at the bottom indicate that this source of error can be eliminated.

c. Multiple-depth analyses were performed using a suspension of clay in water with alum as a coagulant. It was found that the effect of flocculation is reproducible in repeated experiments.

The results were used in the calculation mentioned in section 8-4 c above. They were also used in studying the effect of flocculation on removal.

d. Single-depth pipette analyses were performed with Pasadena sewage. The consistency of the experimental results was not satisfactory and should be improved by using larger apparatus and larger samples.

Two-depth analyses of Pasadena sewage indicated that the effect of flocculation varied considerably between samples taken on different days.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

9-1 Conclusions

a. Except for routine analysis of previously tested suspensions, settling column experiments should include concentration measurements at several depths. These measurements can be conveniently organized in a concentration-depth-time diagram. From the pattern of this "$\emptyset$,z diagram", it is possible to detect errors or unexpected effects in the experiments. Furthermore, it is possible to determine whether the removal is more closely related to overflow rate or to detention time.

b. For flocculent suspensions in a settling tank, neither overflow rate nor detention time alone governs removal. In general, removal is related to both. The relationship between removal and these two variables can be determined by multiple-depth analysis.

Under certain conditions the removal is more closely related to detention time than to overflow rate. This occurence can be explained theoretically.

c. The bottom withdrawal tube can be developed for use in routine studies of removal in settling tanks. The tube must be as deep as the tank being studied. It is quite possible to combine the bottom withdrawal tube with a vibrating grid measurement of removal during turbulence.

d. A kinetic theory of the flocculation of settling particles cannot be completed with existing information. However, the theory can be carried far enough to show qualitatively the relationship between flocculation at a point in a quiescent suspension and properties of the suspension at the same point. The most important properties are:

(1) The volume concentration of particles.

(2) The mean settling velocity of particles.

(3) The standard deviation of the settling velocities of the particles.

(4) The fraction of inter-particle contacts that result in union of the particles.

Of these, only the mean settling velocity can be estimated from multiple-depth analysis.

9-2 Recommendations

a. A large scale settling tube should be developed for multiple depth pipette analysis. The tube should be as deep as existing settling tanks, and of sufficient volume to permit samples of several hundred milliliters to be taken through the pipettes. Furthermore, the apparatus should be designed for temperature control, visual observations, and introduction of turbulence.

b. The bottom withdrawal tube should be developed to operate on the same scale as the tube described in the first recommendation. Such an apparatus will be very valuable for routine analysis.

c. The large scale tube described in the first recommendation can be used in the investigation of the mechanics of flocculation. It has the advantage that individual samples withdrawn through the pipettes are large enough to be analyzed.

According to the theory in this thesis, the individual samples should be analyzed for volume concentration of particles and for the standard deviation of settling velocities. Satisfactory techniques for performing these analyses are not at hand. These techniques must be developed if the study of the mechanics of flocculation is to advance.

d. For an arbitrary suspension, the optimum removal is a function of overflow rate, detention time, and turbulence during settling. This optimum removal should be determined by multiple-depth analysis in a turbulent settling column. For this reason, the turbulent settling column should be perfected.

APPENDIX 1

SETTLING TANKS WITH TWO-DIMENSIONAL POTENTIAL FLOW

General

Fitch (10) has demonstrated that there are other simplifications of settling tanks for which the removal is the same as that in the "ideal" tank. With slight modifications, Fitch's analysis can be used to calculate the removal for any tank which meets the following requirements:

(1) Two-dimensional potential flow of suspension in the settling zone.

(2) Particles of each settling velocity uniformly distributed at the inlet to the settling zone.

(3) Unhindered settling of discrete particles in the settling zone.

This application of the analysis has already been discussed in section 2-5. In this appendix the modified analysis itself is presented using the stream function notation.

Removal in Rectangular Tank

The modified analysis will be applied to the example given by Ingersoll, McKee and Brooks (2). The tank in the example was a continuous flow rectangular basin with an overflow weir at the effluent end. The velocity distribution is uniform at the influent end.

A cross section is shown in figure A1-1 where the flow is shown in the z,x plane. For steady flow in the tank let

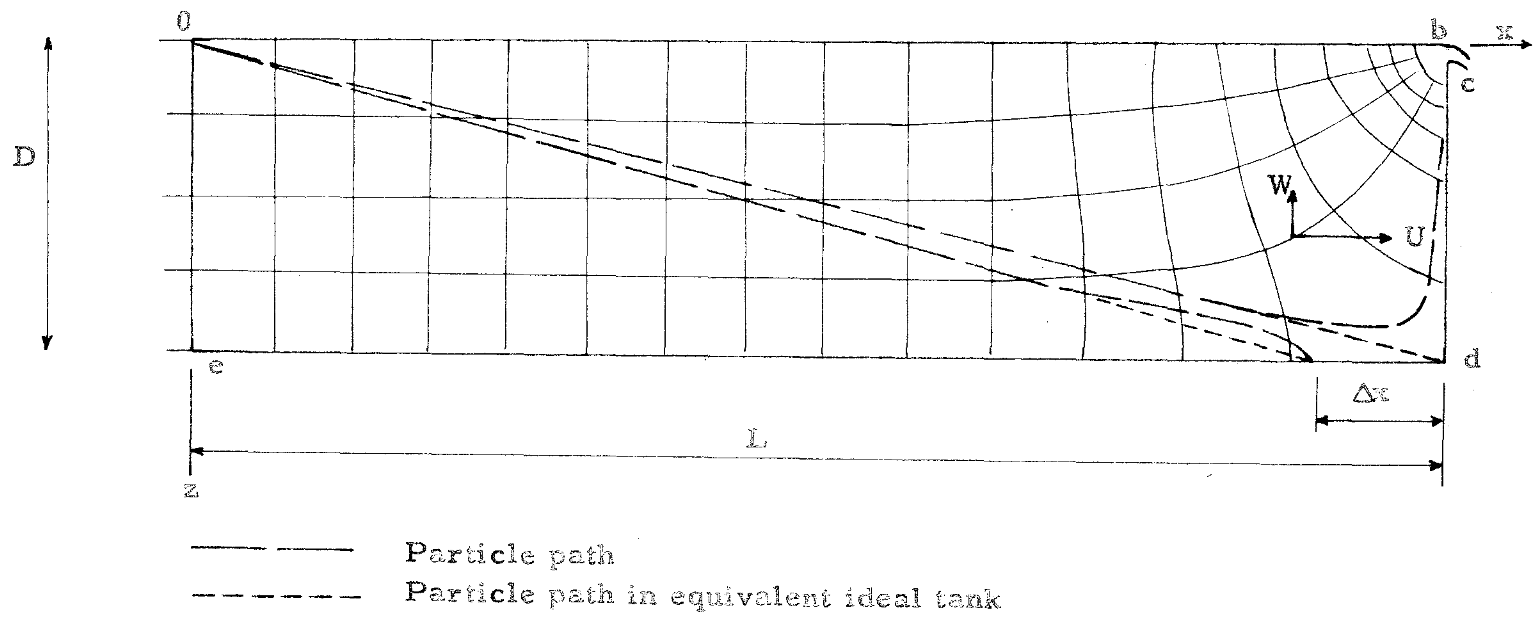

Fig. A1-1. Removal in a tank with two-dimensional potential flow.

(z,x) be the stream function. Assuming incompressible flow, the velocities in the z and x directions are given by

$$W = -\frac{\partial\psi}{\partial x} \qquad U = \frac{\partial\psi}{\partial z} \tag{A1-1}$$

respectively.

As a particle moves, it crosses stream lines. The total differential of ψ for this motion is given by

$$d\psi = \frac{\partial\psi}{\partial x}\, dx + \frac{\partial\psi}{\partial z}\, dz \tag{A1-2}$$

The relationship between dx and dz is given by*

$$\frac{dx}{U} = \frac{dz}{w + W} \tag{A1-3}$$

Substituting A1-1 in A1-3 gives

$$\left[w - \frac{\partial\psi}{\partial x}\right] dx = \frac{\partial\psi}{\partial z}\, dz \tag{A1-4}$$

Equation A1-2 can be written in terms of dx by using equation A1-4

$$d\psi = \frac{\partial\psi}{\partial x}\, dx + \left[w - \frac{\partial\psi}{\partial x}\right] dx \tag{A1-5}$$

*Equation A1-3 is not permissible when $U = 0$ (i.e. purely vertical flow).

Therefore

$$d\psi = w\,dx \qquad \text{(A1-6)}$$

What is the settling velocity of a particle which enters the tank on stream line ψ_a and reaches stream line ψ_b in the horizontal distance L? Integrating equation A1-6 gives the result

$$\psi_b - \psi_a = w_0 L$$

$$w_0 = \frac{\psi_b - \psi_a}{L}$$

$$= \frac{\text{Flow per unit width of tank}}{\text{Horizontal area per unit width of tank}}$$

$$= \text{Overflow rate} \qquad \text{(A1-7)}$$

The question arises as to where this particle actually touches stream line ψ_b. The point is shown in figure A1-1 as point c. This particle is essentially lost from the flow. However, a particle with a slightly larger velocity $w_0 + \Delta w$ would reach the stream line ψ_b in a horizontal distance $L - \Delta x$. The quantities Δx and Δw can be made arbitrarily small and the particle will still reach the line ed. Therefore, particles with settling velocity larger than but arbitrarily close to w_0 will settle from line Ob to line ed in the horizontal distance L. It should be noted that no particles touch the line dc.

When a particle with $w < w_0$ enters the tank on stream line ψ_a, it will reach a stream line ψ in the horizontal distance L. From equation A1-6, ψ is given by

$$\psi - \psi_a = wL \tag{A1-8}$$

$$\frac{\psi - \psi_a}{\psi_b - \psi_a} = \frac{w}{w_0} \tag{A1-9}$$

Therefore, if the stream lines are uniformly distributed at $x = 0$, the removal ratio for these particles is given by equation A1-9. But this removal ratio is the same as that for an ideal tank. Consequently the removal ratio for each settling velocity must be the same as that for the "ideal" tank.

APPENDIX 2

SUMMARY OF NOTATION

The following summary omits, for simplicity, definitions of some letters of secondary importance which appear only a few times in a single section, or which name points in a figure.

$a(z,t)$ = flux of particle at some arbitrary depth and time.

A = total removal of particles from above an arbitrary level in some arbitrary time.

b = fraction of inter-particle contacts that result in union.

B = breadth of settling zone in a rectangular settling tank.

$d_{ij} = G_{ij}(w_i - w_j)$

D = depth of settling zone in a rectangular settling tank.

$f_0(z,w)$ = initial distribution function.

$f(w)$ = frequency distribution of particle settling velocities.

$f(z,w,t)$ = distribution function.

$F(w)$ = cumulative frequency distribution of particle settling velocities.

G_{ij} = horizontal area beneath which the center of a j-particles must lie to be contacted by an i-particle.

i, j, k = summation variables.

K = constant value of z/t.

ℓ_i = average center to center spacing of i-particles.

L = length of settling zone in a rectangular settling tank.

m_i = mass of an i-particle.

n_i = number of i-particles per unit volume of suspension.

Q = volume rate of flow of a suspension through a rectangular settling tank.

r = particle radius.

R = removal ratio.

S = total amount of suspended particles in suspension above a unit horizontal area at some arbitrary depth.

t = time after beginning of settling.

T = detention period or detention time for a rectangular tank.

U = horizontal component of fluid velocity.

w = particle settling velocity.

$\bar{w}$ = arithmetic mean settling velocity.

w_0 = overflow rate.

W = vertical component of fluid velocity.

x = horizontal distance from the inlet end of a rectangular settling tank, taken along the axis of the tank.

z = depth below the surface of the suspension.

α = reciprocal of the specific gravity of suspended particles = (unit mass of water)/(unit mass of particles).

$\alpha\phi$ = volume of particles per unit volume of suspension.

= standard deviation of particle settling velocities.

$\phi(z,t)$ = total concentration of particles at some arbitrary depth and time.

ϕ_0 = average concentration of particles in the influent suspension.

ψ = stream function.

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