THE LONGITUDINAL STABILITY OF A FLYING-BOAT IN THE PLANING CONDITION AS COMPUTED FROM TANK TEST DATA OF A HULL MODEL

Thesis by

Lieut. George A. Hatton, U.S.N.

In partial fulfillment of the requirements for the degree

of Master of Science in Aeronautical Engineering

California Institute of Technology
Pasadena, California
1940

ACKNOWLEDGMENT

The author wishes to express his gratitude to Dr. Theodore von Karman, Director of the Guggenheim Aeronautical Laboratory for suggesting the research, and to Dr. Clark B. Millikan, under whose direction the research was carried out, and who derived the formulae used throughout the problem. In addition the author wishes to thank Dr. Millikan for his advice and criticism.

INTRODUCTION

The question of "porpoising" of flying-boats has been the subject of investigation both abroad and in the United States. In Germany and England recourse has been had to the use of dynamically similar models, duplicating in the model as closely as possible all the features of the full scale flying-boat. In the towing tanks of the United States, usually a bare hull is tested, where measurements are made of resistance, load, trimming moment, and trim angle at various speeds. The results are usually furnished in the form of curves of trimming moment and draft against speed at various loadings and trim angles. Conclusions regarding "porpoising" of the full scale flying-boat cannot be drawn from the behaviour of the hull alone, but it is considered possible to evaluate certain hydrodynamic stability derivatives, which, in conjunction with aerodynamic derivatives obtained from wind-tunnel tests, may be used in the stability equation to determine the behaviour of the flying-boat in the planing condition.

In the following discussion the aerodynamic and hydrodynamic derivatives are deduced. The aerodynamic derivatives are similar to those normally used for airplanes, but they are evaluated in terms of beam, trim angle, and other hydrodynamic terms; then the hydrodynamic derivatives are deduced, also in terms of hull dimensions and attitudes. This permits direct addition of the hydrodynamic and aerodynamic derivatives for use in the longitudinal stability equation. The criteria of stability then are applied. In addition, a factorization of the stability quartic, formulated by Dr. Millikan, is applied to determine periods of the oscillations, as well as damping factors.

An example following the procedure above outlined and devised by Dr. Millikan is presented, using tank test data of a model 36 hull for hydrodynamic quantities. The aerodynamic quantities are based on an average of many modern flying-boats. The model 36 hull is selected as being fairly representative of present day flying boats.

SINGLE-STEP FLYING BOAT

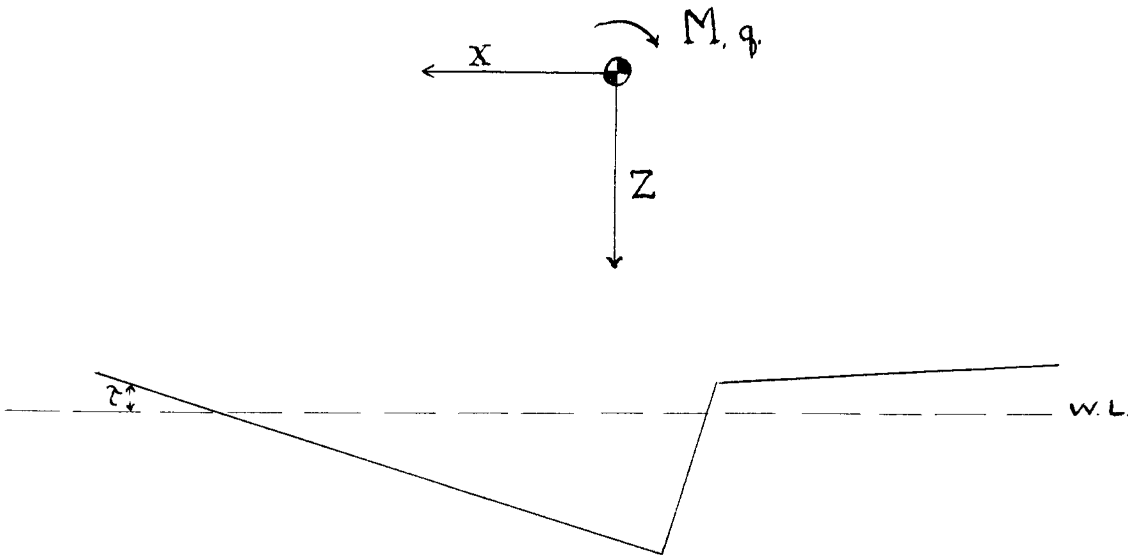


Fig. a

We make the following assumptions in deducing the expressions for the forces and moments:

1 - There is no variation in thrust and forward velocity.

2 - Only two degrees of freedom are considered, vertical movement and pitching motion, z and q.

3 - Only small oscillations are considered.

The following are the two equations for the vertical and pitching motions:

$$
(1) \quad \begin{cases} m \dfrac{d^2 z}{dt^2} = Z_z \cdot z + Z_w \cdot w + Z_\theta \cdot \theta + Z_q \cdot q \\ m K_\theta^2 \dfrac{d^2 \theta}{dt^2} = M_z \cdot z + M_w \cdot w + M_\theta \cdot \theta + M_q \cdot q \end{cases}
$$

where Z_z is the rate of change of vertical force with change of position $= \dfrac{dZ}{dz}$

M_z is the rate of change of moment with change of vertical position $= \dfrac{dM}{dz}$

The determinant for the stability equation then becomes:

$$\begin{cases} (m D^2 - Z_w D - Z_z)z & - (Z_q D + Z_\theta)\,\theta = 0 \\ -(M_w D + M_z)\,z & - (m K_B^2 D^2 - M_q D - M_\theta)\theta = 0 \end{cases}$$

Instead of using take-off speed to define the dimensionless derivatives we use hydrodynamic quantities, particularly the hull beam "b". Replacing V_0 (Glauert) by $\frac{V}{\sqrt{gb}}$, similar to the N.A.C.A., we define the hydrodynamic derivatives as follows:

$$(2)\quad \begin{cases} Z_z = -m\dfrac{V^2}{b^2} z_z & M_z = -\dfrac{m K_B^2 V^2}{b^3} m_z \\ Z_w = -m\dfrac{V}{b} z_w & M_w = -\dfrac{m K_B^2 V}{b^2} m_w \\ Z_\theta = -\dfrac{mV^2}{b} z_\theta & M_\theta = -\dfrac{m K_B^2 V^2}{b^2} m_\theta \\ Z_q = -mV z_q & M_q = -\dfrac{m K_B^2 V}{b} m_q \end{cases}$$

The stability equation for (1) becomes:

$$F(\lambda) = \begin{vmatrix} (m\lambda^2 - Z_w\lambda - Z_z) & (-Z_q\lambda - Z_\theta) \\ (-M_w\lambda - M_z) & (m K_B^2 \lambda^2 - M_q\lambda - M_\theta) \end{vmatrix} = 0$$

and following (2) this becomes:

$$F(\lambda) = \begin{vmatrix} \left(m\lambda^2 + \dfrac{mV}{b}\lambda z_w + \dfrac{mV^2}{b^2} z_z\right) & \left(mV\lambda z_q + m\dfrac{V^2}{b} z_\theta\right) \\ \left(\dfrac{m K_B^2 V}{b^2}\lambda m_w + \dfrac{m K_B^2}{b^3} V^2 m_z\right) & \left(m K_B^2 \lambda^2 + \dfrac{m K_B^2 V}{b}\lambda m_q + \dfrac{m K_B^2 V^2}{b^2} m_\theta\right) \end{vmatrix} = 0$$

This reduces (divide all terms by m, last line by K_B^2, multiply top line by $\frac{b^2}{V^2}$, bottom line by $\frac{b^3}{V^2}$ and divide second line by b) to:

$$F(\lambda) = \begin{vmatrix} \left(\left(\frac{b}{V}\right)^2 \lambda^2 + \frac{b}{V}\lambda z_w + z_z\right) & \left(\frac{b}{V}\lambda z_q + z_\theta\right) \\ \left(\frac{b}{V}\lambda m_w + m_z\right) & \left(\left(\frac{b}{V}\right)^2 \lambda^2 + \frac{b}{V}\lambda m_q + m_\theta\right) \end{vmatrix} = 0$$

(3) Let $\mu = \frac{b}{V}\lambda$

The stability equation now is:

(4) $$F(\mu) = \begin{vmatrix} (\mu^2 + \mu z_w + z_z) & (\mu z_q + z_\theta) \\ (\mu m_w + m_z) & (\mu^2 + \mu m_q + m_\theta) \end{vmatrix} = 0$$

(5) Write $F(\mu) = \mu^4 + B\mu^3 + C\mu^2 + D\mu + E$

$$B = z_w + m_q$$

$$C = z_z + m_\theta + z_w m_q + z_q m_w$$

$$D = z_z m_q - z_q m_z + z_w m_\theta - z_\theta m_w$$

$$E = z_z m_\theta - z_\theta m_z$$

$$R = BCD - D^2 - B^2 E$$

For stability: $B, C, D, E, R > 0$

Obviously $Z_z = M_z = 0$ and as in airplane stability, we put $Z_q = 0$.

$$\mathbf{Z}_w = -\frac{dL}{dw} = -\frac{dL}{V d\alpha} = -\frac{1}{V}\frac{dL}{d\alpha} = -\frac{1}{V}\frac{\rho}{2}V^2 S \frac{dC_L}{d\alpha}$$

$$\mathbf{Z}_\theta = -\frac{dL}{d\alpha} = -\frac{\rho}{2}V^2 S \frac{dC_L}{d\alpha}$$

$$\therefore \quad z_w = \frac{-\mathbf{Z}_w b}{mV} = \frac{\rho S b}{2m}\frac{dC_L}{d\alpha} = z_\theta$$

$$m_w = -\frac{1}{V}\frac{dM}{d\alpha} \times \frac{b^2}{m K_B^2 V} = \frac{\rho}{2}V^2 S t \frac{b^2}{K_B^2 m V}\frac{dC_M}{dC_L}\frac{dC_L}{d\alpha} = m_\theta$$

$$m_q = -\frac{b}{V}(M'_q) = +\eta_t K \frac{b}{V}\frac{\rho S V}{2m}\frac{S_t}{S}\frac{l^2}{K_B^2}\frac{dC_{L_t}}{d\alpha_t}$$

or

$$m_q = K\eta_t \frac{\rho S b}{2m}\frac{S_t}{S}\frac{l^2}{K_B^2}\frac{dC_{L_t}}{d\alpha_t}$$

Now if we let: $f = \frac{\rho S b}{2m}\frac{dC_L}{d\alpha}$; $h = -\frac{\rho S b}{2m}\frac{t b}{K_B^2}\frac{dC_L}{d\alpha}\frac{dC_M}{dC_L}$

and $j = \frac{\rho S b}{2m} K \eta_t \frac{S_t}{S}\frac{l^2}{K_B^2}\frac{dC_{L_t}}{d\alpha_t}$

then :

$$(6) \quad \left\{ \begin{array}{ll} z_z = 0 & m_z = 0 \\ z_w = f & m_w = h \\ z_\theta = f & m_\theta = h \\ z_q = 0 & m_q = j \end{array} \right\} \text{ dimensionless aerodynamic derivatives}$$

HYDRODYNAMIC DERIVATIVES

Single-step hull.

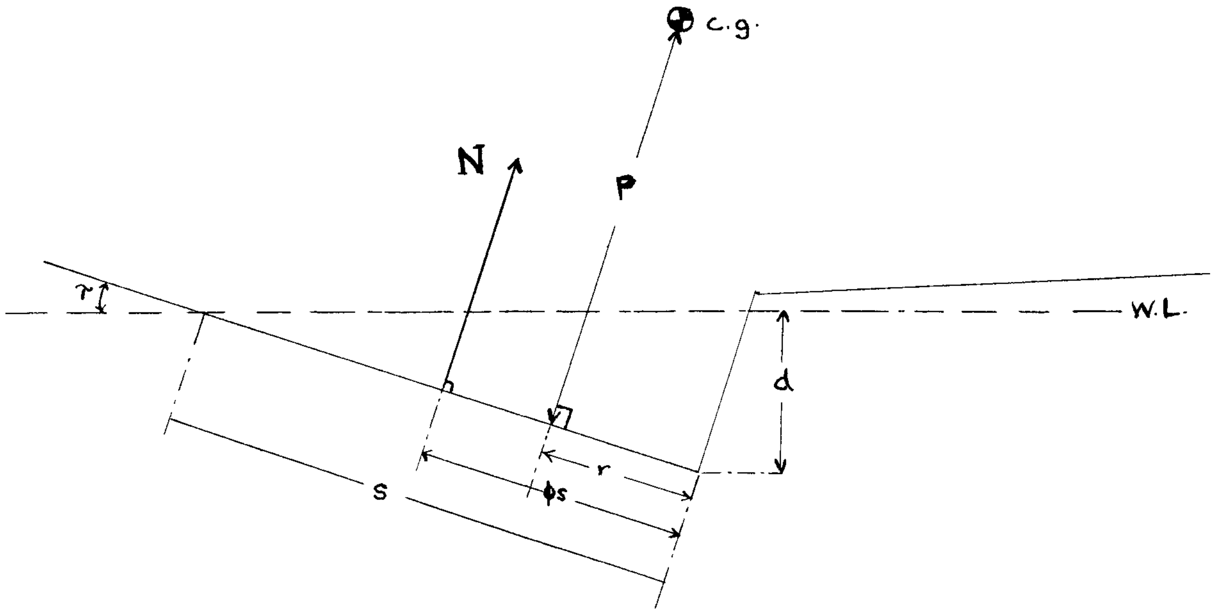


Fig. b

The geometric dimensions are shown in the sketch:

P = distance of c.g. above keel

d = draft

r = distance of c.g. forward of step

s = wetted length of keel from step forward

τ = trim angle

ϕs = distance forward of step to center of pressure of the planing surface

The expressions deduced by Glauert are reproduced here. Then they are compared with tank test data of N.A.C.A., T.N. 464, and modified as necessary to agree with the test results.

We assume that the hydrodynamic force N acts normal to the keel, as shown in the sketch, in a vertical plane, and that $Z = -N$ (τ is small). Also $M = N(\phi s - r)$.

Let $N = A\tau b s \rho_w V^2$ where A is a coefficient corresponding to the lift coefficient for airfoils.

Then
$$Z = -A\rho_w V^2 b s \tau$$
$$(10) \quad M = A\rho_w V^2 b s \tau (\phi s - r) \quad \text{(Glauert)}$$

A comparison of Glauert's expressions with tank tests results N.A.C.A. T.N. 464 leads us to adopt the following expressions:

$$(11) \quad \begin{aligned} Z &= -A\rho_w V^2 b^2 \left(d/b - \delta\right) \\ M &= A\rho_w V^2 b^3 \left(d/b - \delta\right)\left(\beta/\tau \frac{d}{b} + \sigma - \frac{r}{b}\right) \end{aligned}$$

It remains to determine the constants.

Tank tests furnish data as follows:

Curves of C_M vs C_v at various trim angles and various loadings.
C_d vs C_v at various trim angles and loadings.

We wish to determine: A and B which are constants. $(B = \beta/\tau)$
δ and σ as functions of τ .

$$(16) \quad \begin{aligned} A &= \frac{-Z}{\rho_w V^2 b^2 (d/b - \delta)} = \frac{\Delta}{\rho_w V^2 b^2 (d/b - \delta)} = \frac{C_\Delta}{C_v^2 (C_d - \delta)} \\ B\frac{d}{b} + \sigma &= \frac{M}{\Delta b} + \frac{r}{b} \ ; \quad B C_d + \sigma = \frac{C_M}{C_\Delta} + r. \end{aligned}$$

Then $$A(C_d - \delta) = \frac{C_\Delta}{C_v^2}$$

and $$B C_d + \sigma = \frac{C_M}{C_\Delta} + r.$$

Plotting data from the tank tests in the form of

$\frac{C_\Delta}{C_V^2}$ vs C_d and $\frac{C_M}{C_\Delta}$ vs C_d, we can determine the constants A, B and the values of δ and σ at each trim angle for which tests were made. Actually the curves plot as a family of straight lines with constant slope A, and in the case of C_M/C_Δ vs C_d, with a slope $\frac{\beta}{\tau} = B$; (β is constant). The intercepts are equal to δ, and to $r - \sigma$. Thus it is possible to determine δ and σ as functions of τ, the trim angle.

Now for the expressions of lift and moment we have:

$$(17.)\quad \begin{cases} Z = -A\rho_w V^2 b^2 \left(\frac{d}{b} - \delta\right) \\ M = A\rho_w V^2 b^3 \left(\frac{d}{b} - \delta\right)\left(\frac{\beta}{\tau}\frac{d}{b} + \sigma - \frac{r}{b}\right) \end{cases}$$

which are of the form of Glauert's (10).

Derivation of the hydrodynamic derivatives from (17)

$$\frac{\partial(\ )}{\partial z} = \frac{\partial(\ )}{\partial d}$$

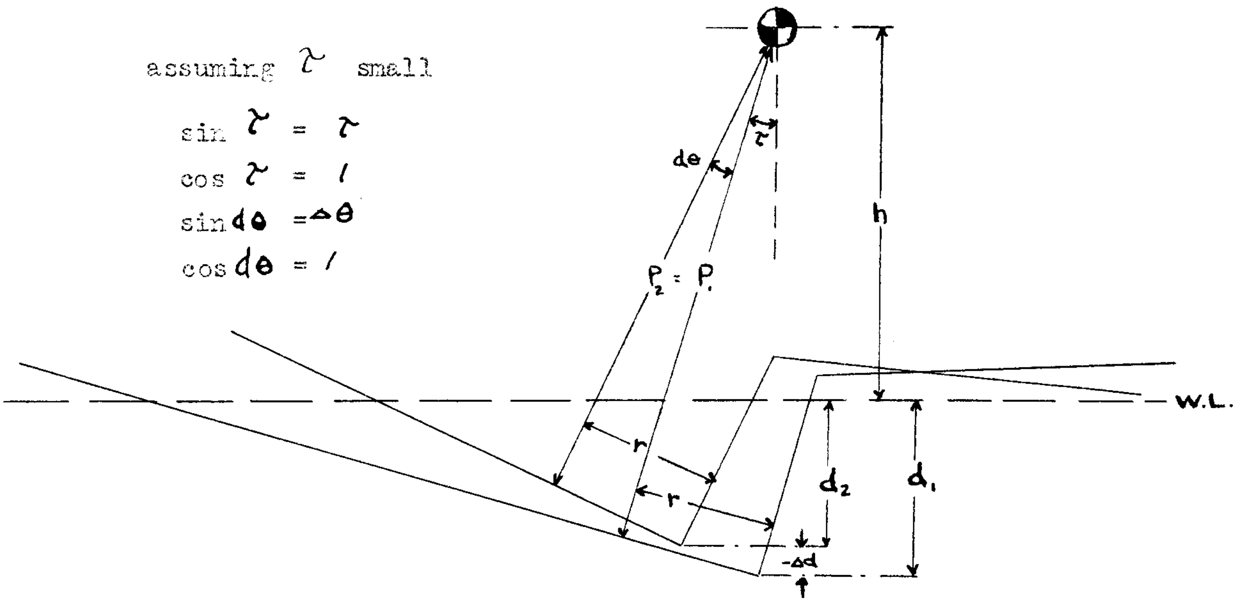
assuming τ small

$\sin \tau = \tau$

$\cos \tau = 1$

$\sin d\theta = \Delta\theta$

$\cos d\theta = 1$



$$d_1 = p \cos \tau + r \sin \tau - h$$

$$d_2 = p \cos(\tau + \Delta\theta) + r \sin(\tau + \Delta\theta) - h$$

$$-\Delta d = d_1 - d_2 = p\tau \,\Delta\theta - r \,\Delta\theta$$

$$= (p\tau - r)\,\Delta\theta$$

$$\frac{\partial d}{\partial \theta} = r - p\tau$$

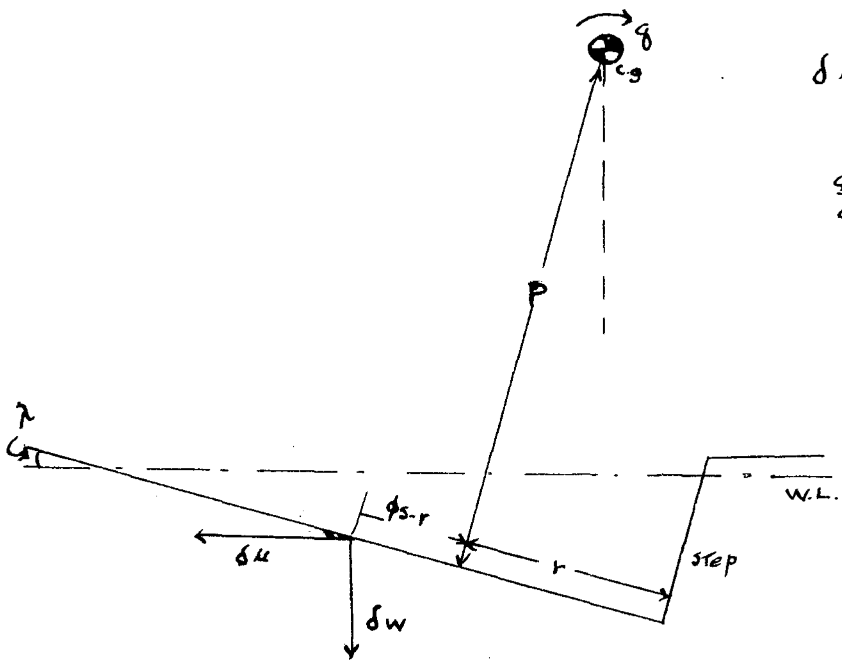
Now:

A change in θ means a change in both τ and d. So

$$\frac{\partial(\ )}{\partial \theta} = \frac{\partial(\ )}{\partial \tau} + (r - p\tau)\,\frac{\partial(\ )}{\partial d}$$

In considering the effect of q , the pitching velocity, Glauert neglects the effects of the rotation velocity itself, but considers the effects of the u and w velocity components of the center of pressure of the hull, produced by about the c.g.

From the sketch:



$$\delta u = [\rho \cos\tau - (\phi s - r)\sin\tau]\,\delta q$$

$$\delta u = \delta q(\rho - \tau(\phi s - r)]$$

$$\frac{du}{dq} = [\rho - \tau\ (\phi s - r)]$$

$$\delta w = \delta q[\rho\tau + (\phi s - r)]$$

$$\frac{dw}{dq} = [\rho\tau + (\phi s - r)]$$

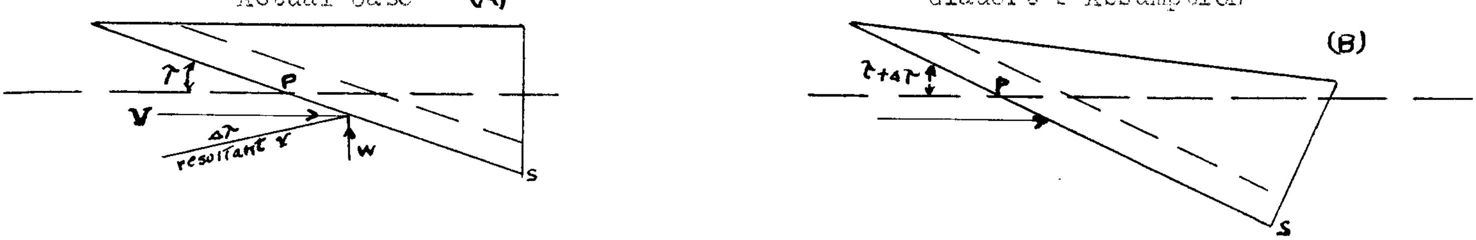
$$\therefore (\ )_q = \frac{\partial(\ )}{\partial q} = \frac{\partial(\ )}{\partial V}\frac{du}{dq} + \frac{\partial(\ )}{\partial w}\frac{dw}{\partial q}$$

We have to find $\dfrac{\partial(\ \ )}{\partial w}$

To determine $\frac{\partial(\ )}{\partial w}$

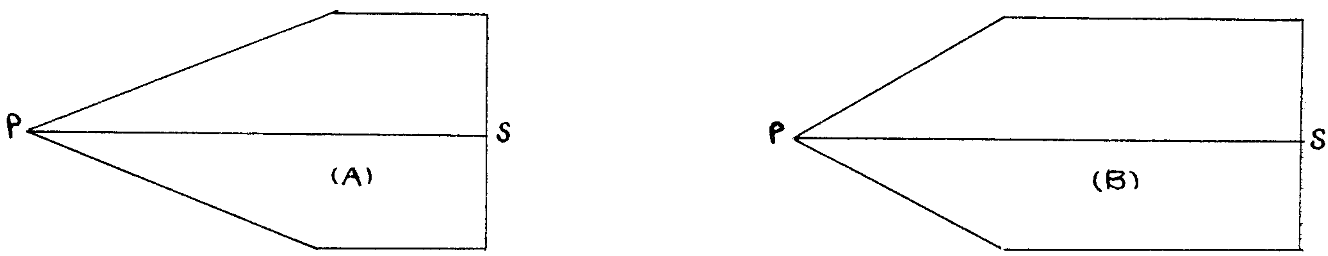
Glauert assumes the effect of w is identical to an increase of τ; ($\Delta\tau = \frac{w}{V}$) where length s is held unchanged.

Actual Case (A) Glauert's Assumption



(a) The displacement in (B) is larger than in (A).

(b) In a vee-bottom hull more of the bottom is wetted in case (B) as the sketches below, of the bottom surface wetted, show.



Both factors cause Glauert's Z_w to be higher than the correct value. Factor (a) is probably small at high speed where displacement is slight, but the effect of (b) must increase as V increases.

We are considering τ and d as basic variables.

Hence $\frac{\partial(\ )}{\partial w} = \frac{1}{V}\left(\frac{\partial}{\partial \tau}\right)_{s=const.} \neq \frac{1}{V}\left(\frac{d}{d\tau}\right)_{d=const}$ but

$$\frac{\partial}{\partial w} = \frac{1}{V}\left(\frac{\partial}{\partial \tau}\right)_{d=\text{const}} + \left(\frac{\partial d}{\partial w}\right)_s \left(\frac{\partial}{\partial d}\right)_{\tau=\text{const}}.$$

But $\left(\frac{\partial d}{\partial w}\right)_{s=\text{const}} = \frac{1}{V}\left(\frac{\partial}{\partial \tau}\right. = \frac{s}{V} = \frac{d}{\tau V}$

$$(19).\quad \frac{\partial}{\partial w} = \frac{1}{V}\left(\frac{\partial}{\partial \tau}\right)_{d=\text{const}} + \frac{d}{\tau V}\left(\frac{\partial}{\partial d}\right)_{\tau=\text{const}}$$

$\frac{\partial(\;)}{\partial q}$ from page

$$= \left[p - \tau(\phi s - r)\right]\frac{\partial(\;)}{\partial V} - \left[p\tau + (\phi s - r)\right]\frac{1}{V}\left[\left(\frac{\partial}{\partial \tau}\right)_d + \frac{d}{\tau}\left(\frac{\partial}{\partial d}\right)\right]$$

but we have replaced ϕs by $\frac{\beta}{\tau}d + \sigma b$ after comparing test data with Glauert's expressions.

$$(20.)\ \therefore\ \frac{\partial}{\partial q} = \left[p - \tau\left(\frac{\beta d}{\tau} + \sigma b - r\right)\right]\frac{\partial}{\partial V} - \left[p\tau + \left(\frac{\beta}{\tau}d + \sigma b - r\right)\right]\frac{1}{V}\left[\left(\frac{\partial}{\partial \tau}\right)_d + \frac{d}{\tau}\left(\frac{\partial}{\partial d}\right)\right]$$

From: (17) $\begin{cases} Z = -A\rho_w V^2 b^2\left(\frac{d}{b} - \delta\right) \\ M = A\rho_w V^2 b^3\left(\frac{d}{b} - \delta\right)\left(\frac{\beta}{\tau}\frac{d}{b} + \sigma - \frac{r}{b}\right) \end{cases}$

and (18) $\quad Z_z = -A\rho_w V^2 b$

(17) & (19) $\quad Z_w = -A\rho_w V b^2\left(\frac{1}{\tau}\frac{d}{b} - \delta'\right)$

(17) & (18) $\quad Z_\theta = A\rho_w V^2 b^2\left(\frac{p\tau - r}{b} + \delta'\right)$

(17) & (20) $\quad Z_q = A\rho_w V b^3 \frac{d}{b\tau}\left[\left(\frac{\beta}{\tau}\frac{d}{b} + \sigma - \frac{r}{b}\right)\left(1 - \delta\tau\frac{b}{d}\right) - \frac{p\tau}{b}\left(1 - 2\frac{b}{d}\delta\right)\right]$

The M derivatives

$$M_z = A\rho_w V^2 b^2 \left[2\frac{\beta}{\tau}\frac{d}{b} + \sigma - \frac{r}{b} - \beta\frac{\delta}{\tau} \right]$$

$$M_w = A\rho_w V b^3 \frac{d}{b\tau}\left[\left(\frac{\beta d}{\tau b} + \sigma - \frac{r}{b}\right)\left(1 - \delta'\tau\frac{b}{d}\right) + \left(\frac{d}{b} - \delta\right)\sigma'\tau\frac{b}{d} \right]$$

$$M_\theta = A\rho_w V^2 b^3 \left[\left(\frac{\beta d}{\tau b} + \sigma - \frac{r}{b}\right)\left(\frac{r - p\tau}{b} - \delta'\right) + \left(\frac{d}{b} - \delta\right)\left\{ \frac{\beta}{\tau}\left(\frac{r - p\tau - d/\tau}{b}\right) + \sigma' \right\} \right]$$

$$M_q = -A\rho_w V b^4 \frac{d}{b\tau}\left(\frac{\beta d}{\tau b} + \sigma - \frac{r}{b}\right)\left[\left(\frac{\beta d}{b\tau} + \sigma - \frac{r}{b}\right)\left(1 - \delta'\tau\frac{b}{d}\right) - \frac{p\tau}{b}\left(1 - 2\delta\frac{b}{d}\right) + \sigma'\tau\left(1 - \frac{b}{d}\delta\right) \right]$$

(Terms containing τ^2 have been neglected)

Combining expressions for Z_c, and M_c, with (2) on page we obtain, corresponding to (15).

Write $G = \frac{\rho_w b^3}{m}$ $\quad H = G\frac{b^2}{K_B^2}$

$r_1 = r/b$; $p_1 = p/b$; $d_1 = \frac{d}{b}$

$$F = \left(\frac{\beta}{\tau}\frac{d}{b} + \sigma - \frac{r}{b}\right)$$

$$(21.)\quad z_z = -\frac{b^2}{mV^2} Z_z = \frac{A\rho_w b^3}{m} = AG$$

$$z_w = -\frac{b}{mV} Z_w = \frac{A\rho_w b^3}{m}\left(\frac{d_1}{\tau} - \delta\right) = AG\left(\frac{d_1}{\tau} - \delta\right)$$

$$z_\theta = -\frac{b}{mV^2} Z_\theta = \frac{A\rho_w b^3}{m}\left(r_1 - p_1\tau - \delta'\right) = AG\left(r_1 - p_1\tau - \delta'\right)$$

$$z_q = -\frac{1}{mV} Z_q = -AG\,\frac{d_1}{\tau}\left[F\left(1 - \delta'\frac{\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right)\right]$$

$$m_z = \frac{-b^3}{mK_B^2V^2} M_z = -AH\left[F + \beta/\tau\,(d_1 - \delta)\right]$$

$$m_w = \frac{-b^2}{mK_B^2 V} M_w = -AH\frac{d_1}{\tau}\left[F\left(1 - \frac{\delta'\tau}{d_1}\right) + (d_1 - \delta)\frac{\sigma'\tau}{d_1}\right]$$

$$m_\theta = \frac{-b^2}{mK_B^2V^2} M_\theta = -AH\left[F\left(r_1 - p_1\tau - \delta'\right) + (d_1 - \delta)\left\{\frac{\beta}{\tau}\left(r_1 - p_1\tau - \frac{d_1}{\tau}\right) + \sigma'\right\}\right]$$

$$m_q = \frac{-b}{mK_B^2 V} M_q = +AH\frac{d_1}{\tau}F\left[F\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right) + \sigma'\tau\left(1 - \frac{\delta}{d_1}\right)\right]$$

FINAL DIMENSIONLESS DERIVATIVES

1. Hydrodynamic

$$z_z = AG$$

$$z_w = AG\left(\frac{d_1}{\tau} - \delta'\right)$$

$$z_\theta = AG\left(r_1 - p_1\lambda - \delta'\right)$$

(21) $$z_q = -AG\,\frac{d_1}{\tau}\left(F\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right)\right]$$

$$m_z = -AH\left[F + \beta/\tau\,(d_1 - \delta)\right]$$

$$m_w = -AH\,\frac{d_1}{\tau}\left[F\left(1 - \frac{\delta'\tau}{d_1}\right) + (d_1 - \delta)\frac{\sigma'\tau}{d_1}\right]$$

$$m_\theta = -AH\left[F(r_1 - p_1\tau - \delta') + (d_1 - \delta)\left\{\beta/\tau\left(r_1 - p_1\lambda - \frac{d_1}{\tau}\right) + \sigma'\right\}\right]$$

$$m_q = +AH\,\frac{d_1}{\tau}\left[F\left\{F\left(1 - \frac{\delta'\tau}{d_1}\right) - p_1\tau\left(1 - 2\frac{\delta}{d_1}\right) + \sigma'\tau\left(1 - \frac{\delta}{d_1}\right)\right\}\right]$$

$$F = \beta/\tau\,\frac{d}{b} + \sigma - \frac{\kappa}{b}$$

$$G = \frac{\rho_w b^3}{m} = \frac{1}{C_{\Delta r}}$$

$$H = G\,\frac{b^2}{K_B^2}$$

$$(\ )_1 = \frac{(\ )}{b}$$

$$(\ )' = \frac{\partial(\ )}{\partial\tau}$$

2. Aerodynamic

(6)

$$z_z = 0 \qquad m_z = 0$$

$$z_w = f \qquad m_w = h$$

$$z_\theta = f \qquad m_\theta = h$$

$$z_q = 0 \qquad m_q = j$$

$$f = J\,\frac{dC_L}{d\alpha} \qquad J = \frac{\rho\,(b_a/b)^2}{\rho_w\ 2\,AR}\,G$$

$$h = -J\,\frac{b^2}{K_B^2}\,\frac{t}{b}\,\frac{dC_L}{d\alpha}\,\frac{dC_M}{dC_L}$$

$$j = J\,\frac{b^2}{K_B^2}\,K\,\eta_t\,\frac{l^2}{b^2}\,\frac{S_t}{S}\,\frac{dC_{L_t}}{d\alpha_t}$$

We wish to investigate porpoising stability as function of velocity and shall assume that the elevator is operated so as to give a definite equilibrium attitude τ as function of V. (We shall at first assume the "best trim" relation between τ and V). Using subscripts $(\ )_a$ and $(\ )_h$ for "aerodynamic" and "hydrodynamic" respectively, we have two equilibrium conditions:

$$(22) \quad \begin{cases} L_a + L_h = W = mg \\ M_a + M_h = 0 \end{cases}$$

Let us express M_a in the form

$$(23) \qquad M_a = \frac{\rho}{2} V^2 S t \left[C_{M_0} + \frac{dC_M}{dC_L} C_L + C_{Me} \right]$$

where C_{M_0} and $\frac{dC_M}{dC_L}$ correspond to the complete airplane with $e = 0°$ and C_{Me} is the moment coefficient corresponding to elevator deflection. Similarly we write

$$(24) \qquad L_a = \frac{\rho}{2} V^2 S \frac{dC_L}{d\alpha} (\tau - \tau_0) = \frac{\rho}{2} V^2 S\, a (\tau - \tau_0)$$

$$a = \frac{dC_L}{d\alpha}$$

where τ_0 = trim angle for $L_a = 0$, which for accuracy may have to be taken as a function of elevator angle, i.e., of C_{Me}.

Notice that in considering porpoising stability at any speed we take elevator angle, i.e., C_{Me} as constant. However C_{Me} is a parameter which in general will vary from one equilibrium V to another.

Using (10) for the hydrodynamic derivatives we get from (22):

$$mg = \frac{\rho}{2} V^2 S\, a (\tau - \tau_0) + A \rho_w V^2 b^2 (d_1 - \delta)$$

$$1 = V^2 \left[\frac{\rho S a}{2mg} (\tau - \tau_0) + \frac{A \rho_w b^2}{mg} (d_1 - \delta) \right]$$

$$= \frac{V^2}{gb} \left[\frac{\rho S b}{2m} a (\tau - \tau_0) + \frac{\rho_w b^3}{m} A (d_1 - \delta) \right]$$

Write (with Glauert)

$$(25) \qquad V = \frac{V}{\sqrt{gb}} = C_v \quad (N.A.C.A.)$$

Then $\left[f(\tau - \tau_0) + AG(d_1 - \delta) \right] C_v^2 = 1$

which corresponds with Glauert's Eq. (19)

From the moment equation:

$$\rho/2\, V^2 S t \left(C_{M_0} + \frac{dC_M}{dC_L} C_L + C_{Me} \right) + A \rho_w V^2 b^3 (d_1 - \delta) F = 0$$

But $C_L = a(\tau - \tau_0)$

$$\frac{\rho S t}{2 \rho_w b^3} \left[C_{M_0} - a \sum (\tau - \tau_0) + C_{Me} \right] + AF(d_1 - \delta) = 0$$

(26) where $\sum = -\frac{dC_M}{dC_L}$ = static stability

$$\therefore \left(\frac{\rho S b}{2m} \Big/ \frac{\rho_w b^3}{m} \right) \left[C_{M_0} - a \sum (\tau - \tau_0) + C_{Me} \right] \frac{t}{b} + AF(d_1 - \delta) = 0$$

$$\therefore \left[C_{M_0} - a \sum (\tau - \tau_0) + C_{Me} \right] t_1 + \frac{G}{J} AF(d_1 - \delta) = 0$$

where $t_1 = t/b$

Hence the two equilibrium relations are

$$(27) \quad \begin{cases} f(\tau - \tau_0) + AG(d_1 - \delta) = \dfrac{1}{C_v^2} \\ C_{M_0} - a \sum (\tau - \tau_0) + C_{Me} = -\dfrac{AG}{J t_1} F(d_1 - \delta) \end{cases}$$

These correspond to Glauert Eq (19) and (20) and for a particular seaplane give $d_1(C_v)$ and $\tau(C_v)$ for any assumed $C_{Me}(C_v)$. Or assuming $\tau(C_v)$ they give $d_1(C_v)$ and $C_{Me}(C_v)$ directly without solving simultaneous equation.

GENERAL PROCEDURE FOR A DEFINITE SEAPLANE

1) Assume the basic characteristics of the seaplane
2) Assume $\tau(C_v)$
3) Calculate $d_1(C_v)$ and $C_{Me}(C_v)$ from (27)
4) Calculate hydro- and aerodynamic derivatives as functions of C_v from (15) and (6)
5) Investigate stability as function of C_v.

GENERAL EQUILIBRIUM RELATIONS

We can transform (27) into more useful forms using

$$C_\Delta = A C_v^2 (d_1 - \delta)$$

$$(28) \qquad C_\Delta = \frac{1 - f C_v^2 (\tau - \tau_0)}{G}$$

$$C_{Me} = \sum a(\tau - \tau_0) - C_{M_0} - \frac{GA}{\sqrt{t_1}} F(d_1 - \delta)$$

APPENDIX: SUMMARY OF HYDRODYNAMIC DERIVATIVES

We start with the expressions for hydrodynamic derivatives in the planing region, in the form found empirically from T.N. 464, 638.

$$Z = -A\rho_w V^2 b^2 \left(\frac{d}{b} - \delta\right)$$

$$M = A\rho_w V^2 b^3 \left(\frac{d}{b} - \delta\right)\left(\frac{\beta}{\tau}\frac{d}{b} + \sigma - \frac{r}{b}\right)$$

where for a given hull shape A, β are constants and δ, σ are empirical functions of τ.

The references below are to pages and equations

We have as variables V, d, τ or $V, d/b, \tau$

$$\frac{d}{dz} = \frac{b\,\partial}{\partial\, d/b} = \frac{\partial}{\partial d} \qquad \text{Ref.}(\text{p. } 8.)$$

$$\frac{d}{d\theta} = \frac{d}{d\tau} + \left(\frac{r - p\tau}{b}\right)\frac{\partial}{\partial\, d/b} \qquad (\text{p. } 8)$$

$$\frac{\partial}{\partial w} = \frac{1}{V}\left(\frac{d}{d\tau} + \frac{d}{b\tau}\,\frac{\partial}{\partial\, d/b}\right) \qquad \text{Ref.}((19) - \text{p } 11.)$$

$$\frac{d}{dq} = \left[\frac{p}{b} - \tau\left(\frac{\beta d}{\tau b} - \frac{r}{b} + \sigma\right)\right] b\,\frac{\partial}{\partial V} - \left[\frac{p}{b}\tau + \left(\frac{\beta}{\cancel{b}}\frac{d}{\tau} + \sigma - \frac{r}{b}\right)\right] b\,\frac{\partial}{\partial w}$$

Ref. (20.) p.11

where $\frac{\beta}{t}\frac{d}{b} = \phi\,\frac{s}{b}$ i.e. $\beta \equiv \phi$ (Glauert)

For the dimensional derivatives we have:

Writing: $F = F(d, \tau) = \left(\frac{\beta}{\tau} \frac{d}{b} + \sigma - \frac{r}{b} \right)$

$$Z_z = -A\rho_w V^2 b$$

$$Z_w = -A\rho_w V b^2 \frac{d}{b\tau} \left[1 - \delta' \tau \frac{b}{d} \right]$$

$$Z_\theta = -A\rho_w V^2 b^2 \left[\frac{r - p\tau}{b} - \delta' \right]$$

$$Z_q = A\rho_w V b^3 \frac{d}{b\tau} \left[F\left(1 - \delta'\tau \frac{b}{d}\right) - \frac{p}{b}\tau\left(1 - 2\delta \frac{b}{d}\right) \right]$$

$$M_z = A\rho_w V^2 b^2 \left[2\frac{\beta}{\tau}\frac{d}{b} - \frac{r}{b} + \sigma - \frac{\beta}{\tau}\delta \right]$$

$$M_w = A\rho_w V b^3 \frac{d}{b\tau} \left[F\left(1 - \delta'\tau\frac{b}{d}\right) + \left(\frac{d}{b} - \delta\right)\sigma'\tau\frac{b}{d} \right]$$

$$M_\theta = A\rho_w V^2 b^3 \left[F\left(\frac{r - p\tau}{b} - \delta'\right) + \left(\frac{d}{b} - \delta\right)\left(\frac{\beta}{\tau}\, \frac{r - p\tau - d/\tau}{b} + \sigma'\right) \right]$$

$$M_q = -A\rho_w V b^4 \frac{d}{b\tau} F \left[F\left(1 - \delta'\tau\frac{b}{d}\right) - \frac{p}{b}\tau\left(1 - 2\delta\frac{b}{d}\right) + \sigma'\tau\left(1 - \delta\frac{b}{d}\right) \right]$$

We get the final dimensionless derivatives (**21**), p.14 as indicated on p.**13** .

FACTORIZATION OF THE STABILITY QUARTIC

$$F(\lambda) = \lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

$$= (\lambda^2 + a\lambda + b)(\lambda^2 + \alpha\lambda + \beta)$$

Approx. $= (\lambda^2 + b_1)(\lambda^2 + \beta_1)$

This leads to the following expression for determining the actual roots:

1) $B = a + \alpha$

2) $C = b + \beta + a\alpha$

3) $D = a\beta + b\alpha$

4) $E = b\beta$

Result:

(1).

$$\lambda_{1,2} = -\frac{1}{4}\left(B + \frac{BC-2D}{\sqrt{C^2-4E}}\right) \pm i\sqrt{\frac{C}{2} + \frac{1}{2}\sqrt{C^2-4E} - \frac{1}{16}\left(B + \frac{BC-2D}{\sqrt{C^2-4E}}\right)^2}$$

$$\lambda_{3,4} = -\frac{1}{4}\left(B - \frac{BC-2D}{\sqrt{C^2-4E}}\right) \pm i\sqrt{\frac{C}{2} - \frac{1}{2}\sqrt{C^2-4E} - \frac{1}{16}\left(B - \frac{BC-2D}{\sqrt{C^2-4E}}\right)^2}$$

Glauert's Cases (Reference 4, Tables 4 - 27)

Here $B << C < D << E$

$B \backsim 20 \backsim 10$
$C \backsim 1500 \backsim 10^3$
$D \backsim 15{,}000 \backsim 10^4$
$E \backsim 500{,}000 \backsim 10^5$

Here we know only that B is very small and E very large, while C and D are intermediate. From page (equations 1 to 4) we see that these conditions are satisfied if $b, \beta >> a, \alpha$ in particular if $a, \alpha = O(1)$ while $b, \beta > O(1)$. The relative magnitudes of C and D depend then on the signs of a, α, b, β . The two equations for determining b and β are then 2 and 4 . This leads to exactly the same approximate expressions for $\alpha_1, \beta_1, a_1, b_1$ as we found before in Klemin's case, i.e. the assumption $C >> D$ was not required for this approximation.

We know that instability occurs through vanishing of the Routh's discriminant, i.e.

$$R = BCD - D^2 - B^2E = 0$$

It can be shown that this corresponds to a vanishing of the real part of roots $\lambda_{3,4} = f_{3,4}$ (say)

$$-f_{3,4} = \frac{1}{4}\left(B - \frac{BC - 2D}{\sqrt{C^2 - 4E}}\right)$$

Now

$$-4f_{1,2} = B + \frac{BC - 2D}{\sqrt{C^2 - 4E}}$$

$$\therefore -4f_{1,2} \cdot f_{3,4} = \frac{1}{4}\left[B^2 - \frac{B^2C^2 - 4BCD + 4D^2}{C^2 - 4E}\right]$$

$$= \frac{1}{4}\left[\frac{-4B^2E - 4D^2 + 4BCD}{C^2 - 4E}\right] = \frac{R}{C^2 - 4E}$$

$$(2.) \therefore f_{3,4} = \frac{R}{4(C^2-4E)\,f_{1,2}}$$

Now (C^2-4E) and $f_{1,2}$ are both non-vanishing in all normal cases. Hence $R=0 \backsim f_{3,4}=0$ and porpoising corresponds to the $(\ )_{3,4}$ motion changing from a damped to a divergent oscillation.

CONCLUSION

It is not claimed that the method outlined is conclusive. It is recommended that further calculations be made to indicate the effects of variation of trim angles, height of center of gravity, fore and aft position of center of gravity, and changes in static stability.

A comparison of flight test results with the calculated behaviour should indicate whether the method is reliable. There should be available a complete tank test of the actual hull used, and the aerodynamic characteristics and dimensions should be those of the actual flying-boat.

The example solution is based on data taken from Reference 1. Figs. 14 and 15 present the function C_Δ plotted against C_V for two runs up to a take-off speed. Points were selected of C_Δ and C_V.

. The best trim angles corresponding to these various points were selected from Fig. 11 (Ref 1) for use in the formulae. Values of δ, σ, δ', β and σ' were taken from the curves, Figs. 1 - 4, included herewith. These curves are obtained from the test data, as indicated on page .

The aerodynamic quantities are selected as the average of several present-day boats. The dimensionless hydrodynamic and aerodynamic derivatives are calculated and added. The coefficients of the stability quartic and Routh's discriminant are calculated. Finally the factorization, developed by Dr. Millikan, is applied to the equations.

Steps In Stability Problem

1. From tank test data, plot curves of:

C_M/C_Δ vs C_d at various values of
(Figs. 1 and 2
C_Δ/C_v^2 vs C_d at various values of

2. From these plots, determine A, B, δ, σ. (Figs. 1,2,3,4.
3. Plot δ and σ vs τ and differentiate (Figs. 1,2,3,4. graphically to obtain δ', σ'.
4. Determine A, G, H, F, f, j, h from wind tunnel data and plane dimensions.
5. Calculate derivatives, Page 14 (Table II)
6. Determine constants of stability quartic and R on Page 3, equation 5. (Table III)
7. Plot R vs C_v to find limit of stability (Fig. 6)
8. Factor quartic by equation 1, Page 20
9. Figure 5 is plot of τ_b, C_{Me}, C_d (or d,) C_Δ vs C_v.

STATISTICAL DATA ON SEVERAL FLYING BOATS

No.	#1	#2	#3	#4	#5	#6
W (lb)	60,500	50,000	100,000	21,100	45,500	28,500
P_0	4000	4000	4400	1700	3600	1750
S (ft^2)	1267	1048	2880	1402	1780	1296
b_a (ft.)	125	110	180	104	115	95
b "	10	9.16	13.9	10.21	10.5	9.17
l "	47.4	45	65.7	37.8	45.5	
t "	11.5	10.3	17.75	13.78	16.1	14.6
S_t	217.4	158.4	450	158.4	267	
AR_t	3.06	3.06	3.92	3.79	4.59	
Σ	.21	.19	.10	.05	.08	09
C_{Mo}	.10	.09	07	.01	.03	.03
C_{M_o}	.16	.15	.09			
γ_0	-4°	-3.5°	-6°	-7°	-4°	-10°
τ_0	-7°	-7°	-10°			
$\partial C_M/\partial e$	.020	.025	.016			
η_t	.70	.75	.64	.67	.65	
p (ft)			16.8			
r (ft.)			5.8			
AR	12.34	11.56	11.2	7.72	7.43	6.97
l_w	47.8	47.8	34.7	15.1	25.6	22
$C_{\Delta r}$	.945	1.01	.58	.31		.58
b_a/b	12.5	12.	12.95	10.2	10.95	10.4
r/b			.42			
i_w°	3°	3°	5°	6°	3°	7.35°
$l_w/W^{1/3}$	1.22	1.30	.75	.55	.8	.72
S_t/S	.171	.151	.156	.113	.150	
l/t	4.29	4.37	3.14	2.75	2.82	
P/b			1.19			
a	5.9	5.6	5.1	4.7	4.85	5.1

STATISTICAL DATA ON SEVERAL FLYING BOATS

No.	# 7	8	9	10		
W	20,013	10,400	65,000	84,000		
P_o	1650	900	4800	4800		
S (ft^2)	1296	720	1779	2867		
b_a(ft)	95	73	115	153		
b "	8.5	6	10.5	12.5		
l "	40.67		46.9	66.25		
t "	14.6		16.12	18.75		
S_t	233		323.3	518.4		
AR_t	3.86		4.47			
Σ	.09		.19			
C_{M_o}	.09		.12			
C_{M_o}	.07		.20			
r_o	-9°		-2.5°			
r_o	-14°		-6.0°			
$dC_M/de°$			.0175			
η_t			.85			
P(ft.)				18.75		
r(ft.)				7.47		
AR.	6.97	7.5	7.43	8.16		
l_w	15.4	14.4	36.5	29.3		
$C_{\Delta r}$	.51	.76	.89	.68		
b_a/b	11.2	12.2	10.95	12.2		
r/b	5.16					
i_w			3°			
$l_w/W^{1/3}$	.57	.66	.90	.67		
S^t/S	.18		.182	.18		
l/t	2.79		2.91	3.54		
P/b				1.5		
a	5.16		5.2			

ASSUMED NUMERICAL VALUES

From studies of a considerable number of modern planes we find, as generally used average values

$$\mathcal{R} = 10 \quad ; \quad b_a/b = 12 \quad ; \quad \frac{K_B}{b} = 1.5 \quad ; \quad \left.\begin{array}{l} a = 5.2 \ (\text{radian}^{-1}) \\ \ \ = .09 \ (\text{degree}^{-1}) \end{array}\right\} \frac{t}{b} = 1.2$$

$$\frac{S_t}{S} = .17 \quad ; \quad \frac{l}{t} = 3.5 \quad ; \quad \mathcal{R}_t = 4.0 \quad ; \quad a_t = \begin{cases} 4.0 \ (\text{radian}^{-1}) \\ .07 \ (\text{degree}^{-1}) \end{cases} ; \quad \eta_t = .70$$

$$K = 1.25 \quad ; \quad \tau_0 = \begin{array}{l} -5^\circ \ (\text{flaps } 0^\circ) \\ -8^\circ \ (\text{flaps } 20^\circ) \end{array} ; \quad \frac{\partial C_M}{\partial e^\circ} = -.02$$

$$\text{Power-off trim @ } C_L = .6$$

As an assumption giving the variation of design parameters with size we take $C_{\Delta r} = \dfrac{W}{\rho_w g b^3}$ = const. This quantity varies from about 0.5 on old boats to about 1.3 on the most modern boats (e.g. Consol.31). The old Rohrbach Romar had 2.4. As typical of current practice we take $C_{\Delta r} = \frac{1}{G} = .625$

Then:

$$l_w = \frac{W}{S} = \frac{W\mathcal{R}}{b_a^2} = \frac{W}{b^2}\left(\frac{b}{b_a}\right)^2 \mathcal{R} = \frac{W}{\rho_w g b^3}\left(\frac{b}{b_a}\right)^2 \mathcal{R}\,\rho_w g b$$

$$= \frac{\rho_w g\, C_{\Delta r}}{(\rho_w g\, C_{\Delta r})^{1/3}} \frac{\mathcal{R}}{(b_a/b)^2} W^{1/3} = (64\, C_{\Delta r})^{2/3} \frac{\mathcal{R}}{(b_a/b)^2} W^{1/3}$$

With our assumed numerical values this gives:

$$l_w = \frac{16 \times 10 \times .851}{144} W^{1/3} = .952\, W^{1/3}$$

W	1000	10,000	50,000	100,000
l_w	9.52	20.6	44	35.1

This gives the following values

which are reasonable for modern flying boats.

The above assumptions give for the constant coefficients in the dimensionless derivatives (cf. Eq. (6)(15)):

$G = 1.6 \quad H = .712 \quad J = .0137$

$f = .0712$

$j = .064$

$h = 0 = .038\,\Sigma$ (We assume $\Sigma = 0$)

Hence:

$G = 1.6 \qquad f = .0712$ f, g, h are all (radians^{-1})

$H = .712 \qquad j = .064$

$J = .0137 \qquad h = 0$

From T.N. 638 study we find $A = .3 \quad \beta = 33$

δ and σ (plotted in Figs. 1-2)

We assume as a first case: no flaps : $\tau_0 = -5^\circ$

$\Sigma = 0$

$C_{M_0} = 0$

$C_{L(max)} = 1.5$

EQUILIBRIUM CONDITIONS

From tank test data we have τ_b best trim angle as function of C_v and $C_\Delta = \frac{L_h}{\rho_w g b^3} = C_{\Delta_r} \frac{L_h}{w}$

But $\frac{L_h}{w} = \frac{A \rho_w b^2 v^2 (d_1 - \delta)}{w} = A \rho_w \left(\frac{V}{\sqrt{gb}}\right)^2 \frac{b^3 g}{w} (d_1 - \delta) = A \frac{C_v^2}{C_{\Delta R}} (d_1 - \delta)$

$$\therefore \; C_\Delta = A C_v^2 (d_1 - \delta)$$

The lift equilibrium equation (27) gives :

$$f(\tau - \tau_0) + AG(d_1 - \delta) = \frac{1}{C_v^2}$$

$$d_1 - \delta = \frac{1}{AG}\left[\frac{1}{C_v^2} - f(\tau - \tau_0)\right]$$

$$\therefore \; C_\Delta = A C_v^2 (d_1 - \delta) = \frac{1}{G}\left[1 - f(\tau - \tau_0) C_v^2\right]$$

To determine $\tau (C_v)$ $(d_1 - \delta)$

a) Assume $C_\Delta (C_v)$ and determine τ_b from tank data

b) Calculate $C_\Delta = \frac{1}{G}\left[1 - f(\tau - \tau_0) C_v^2\right]$

c) Repeat using the new C_Δ until assumed and calculated C_Δ agree.

d) Calculate $(d - \delta)$ vs. C_v from

$$d_1 - \delta = \frac{C_\Delta}{A C_v^2}$$

e) Having $\delta(\tau)$ from tank data this gives $d_1 (C_v)$

Now determine elevator moment to trim from:

$$C_{M_e} = a \sum (\tau - \tau_0) - C_{M_0} - \frac{GA}{\sqrt{t_1}} F(d_1 - \delta)$$

or with our numerical values

$$C_{M_e} = .09 \sum (\tau - \tau_0)^\circ - C_{M_0} - 29.2\ F(d_1 - \delta)$$

If this C_{M_e} is reasonable, calculate derivatives from (6) and (21). Then calculate coefficients of stability quartic from Eq. (5) p. 3. Here derivatives are the sum of hydro-and aerodynamic terms. Repeat for other values of C_v, and plot R vs. C_v to find stability limit.

TABLE I

TABLE OF CONSTANTS

C_v	C_Δ	$\tau_0°$	δ	$d_1-\delta$	d_1	σ	δ'	σ'
3	.47	5.7	.05	.174	.224	−.357	−.58	3.25
4	.355	5.4	.052	.074	.126	−.375	−.53	3.58
5	.235	5.25	.053	.0314	.0844	−.385	−.51	3.75
6.	.09	4.95	.056	.00633	.0643	−.405	−.455	4.1

$P/b = 1$ $r/b = .714$ $C_{\Delta r} = .625$ $G = 1.6$

$A = .3$ $B = \frac{33}{\tau°}$ $J = .0137$ $H = .712$ $f = .0712$

$j = .064$

$C_{v_{T.O.}} = 6.55$ $h = 0$

TABLE II. TABLE OF DERIVATIVES

	$C_v = 3$	$C_v = 4$	$C_v = 5$	$C_v = 6$
a	0	0	0	0
h				
z_z	.48	.48	.48	.48
a	.0712	.0712	.0712	.0712
h	1.31	.898	.687	.575
z_w	1.3812	.9692	.7582	.6462
a	.0712	.0712	.0712	.0712
h	.574	.551	.544	.52
z_θ	.6452	.6222	.6152	.5912
a	0	0	0	0
h				
z_q	−.252	.297	.381	.374
a	0	0	0	0
h				
m_z	−.265	.0285	.0795	.136
a	0	0	0	0
h				
m_w	−.26	.071	+.15	+.17
a	0	0	0	0
h				
m_θ	.172	.0915	.126	.154
a	.064	.064	.064	.064
h	.0533	.0242	.0822	.11
m_q	.1173	.0882	.1462	.174

TABLE III

CONSTANTS OF THE STABILITY QUARTIC

C_v	B	C	D	E	R
3	1.498	.734	.395	.254	-.292
4	1.057	.635	.0784	.026	.176
5	.904	.66	.044	.012	.145
6	.820	.672	.0325	-.006	.0208

FACTORIZATION OF THE QUARTIC

$C_v = 3 \quad \{$

Note : $4E > C^2$ @ $C_v = 3$.

$$C_v = 4 \quad \begin{cases} \lambda_{1,2} = -.499 \pm i\,.585 \\ \lambda_{3,4} = -.03 \pm i\,.205 \end{cases}$$

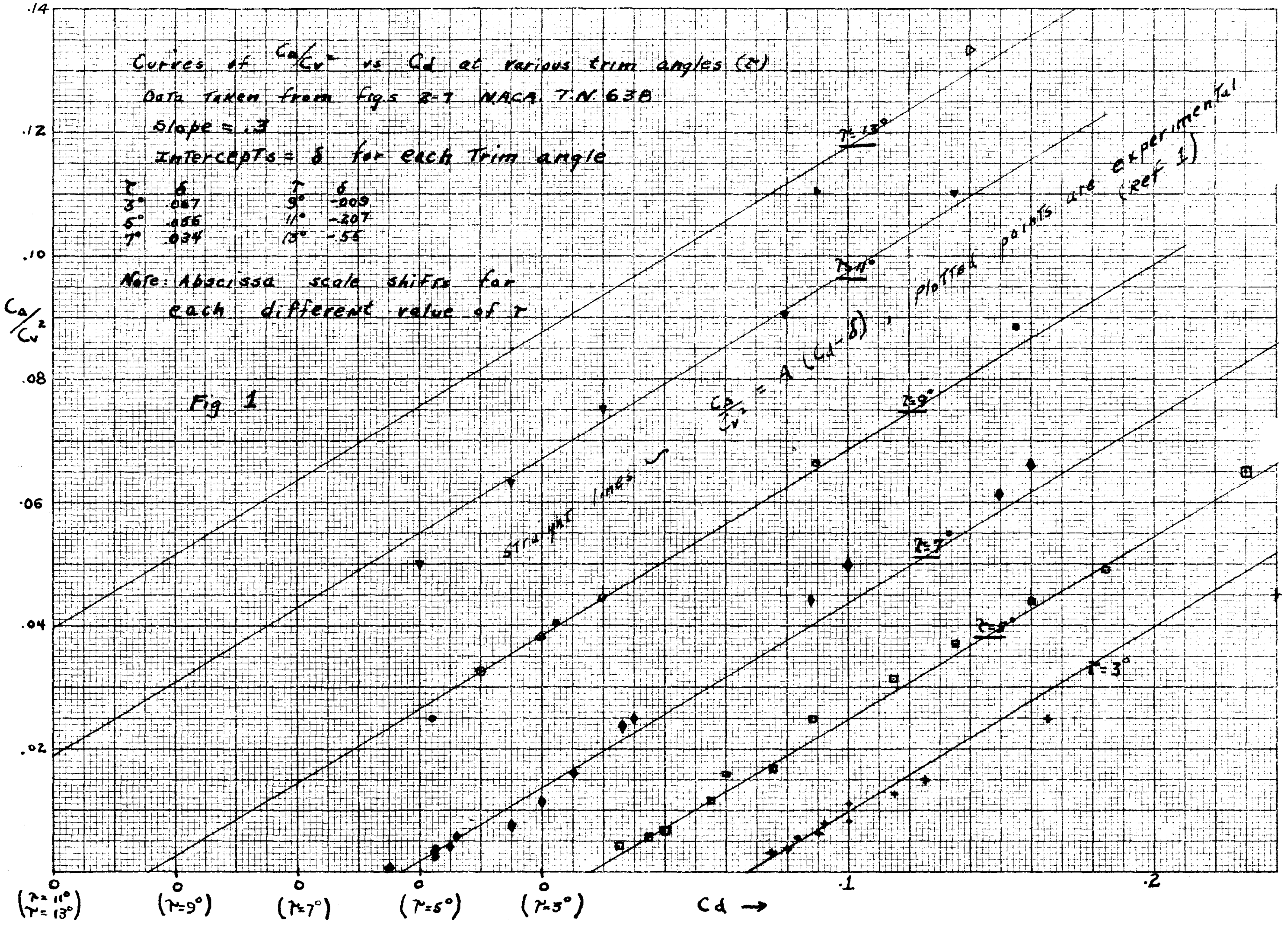
$$C_v = 5 \quad \begin{cases} \lambda_{1,2} = -.43 \pm i\,.674 \\ \lambda_{3,4} = -.022 \pm i\,.137 \end{cases}$$

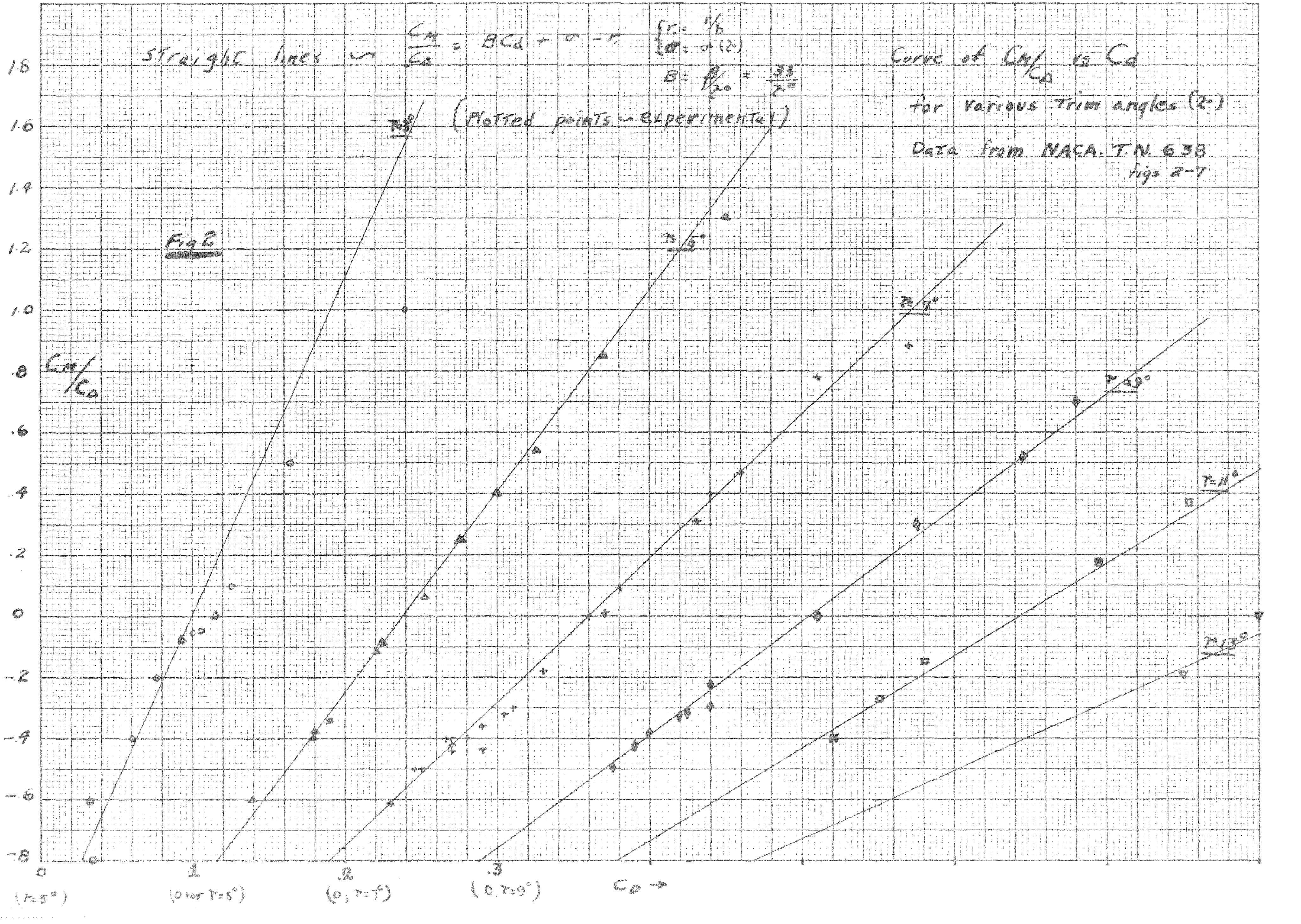
$$C_v = 6 \quad \begin{cases} \lambda_{1,2} = -.381 \pm i\,.73 \\ \lambda_{3,4} = -.029 \pm .1 \end{cases}$$

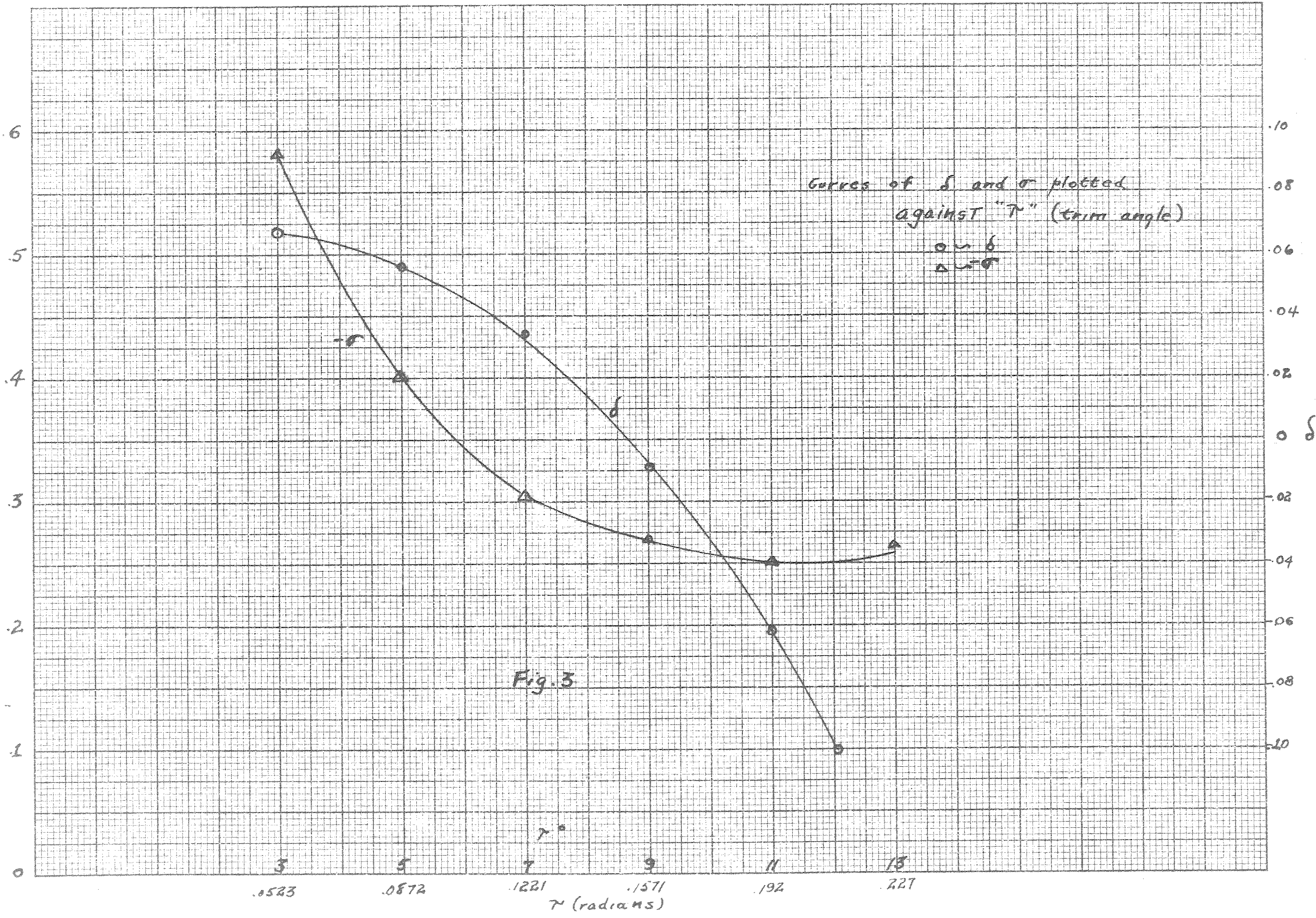
Note E is −

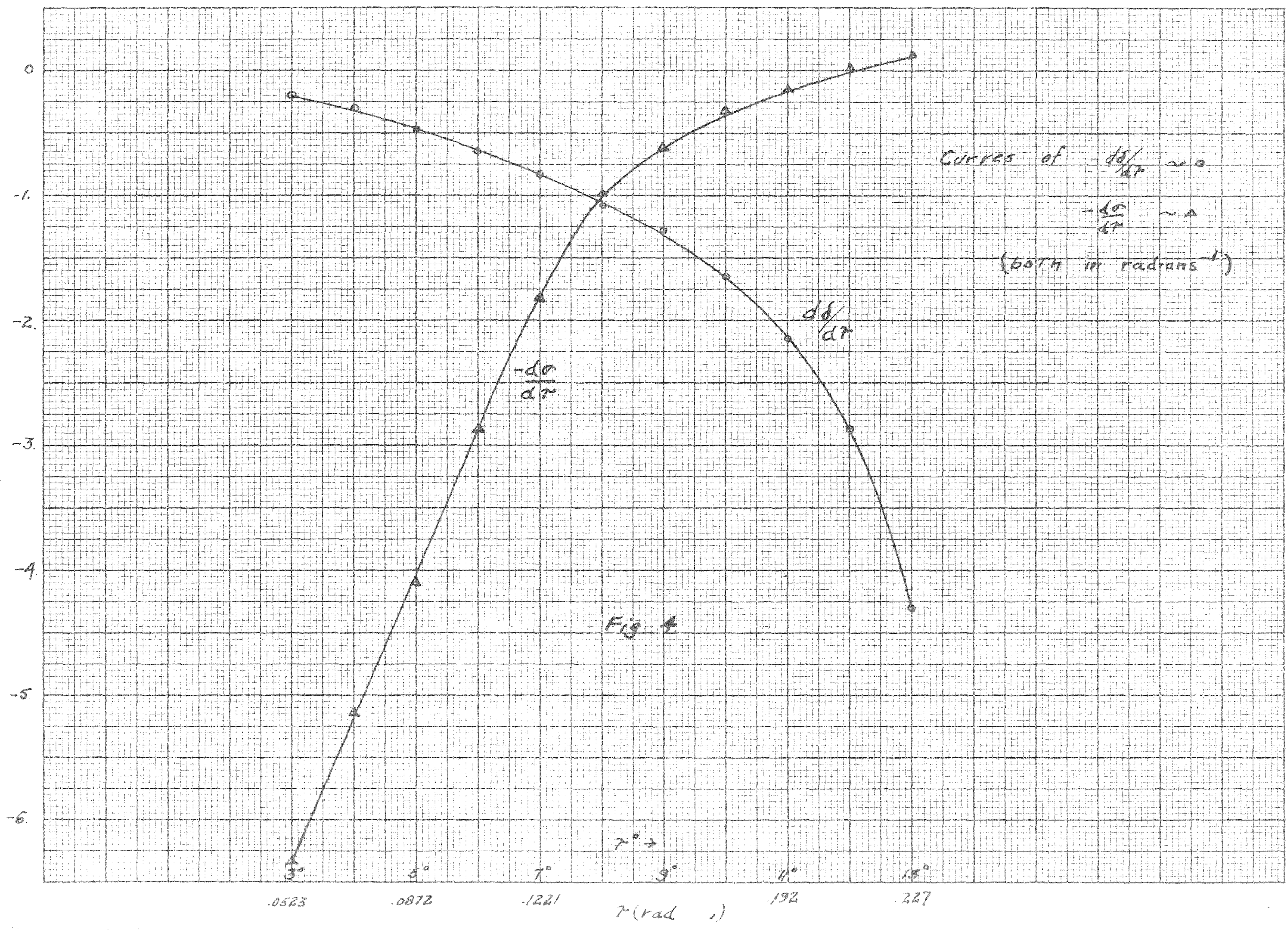
Check factorization by multiplying $(\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)(\lambda-\lambda_4)$

$$\mu^4 + .904\mu^3 + .697\mu^2 + .0446\mu + .0123 \qquad (C_v = 5)$$


Curves of C_Δ/C_v^2 vs C_d at various trim angles (τ)
Data taken from figs 2-7 NACA T.N. 638
Slope = .3
Intercepts = δ for each trim angle
τ δ
3° .067
5° .056
7° .034
τ δ
9° -.009
11° -.207
13° -.55
Note: Abscissa scale shifts for each different value of τ
Fig 1
$\frac{C_\Delta}{C_v^2} = A(C_d - \delta)$, straight lines
plotted points are experimental (Ref 1)
τ=13°
τ=11°
τ=9°
τ=7°
τ=5°
τ=3°
C_Δ/C_v^2
.14
.12
.10
.08
.06
.04
.02
0
(τ=11°, τ=13°)
0
(τ=9°)
0
(τ=7°)
0
(τ=5°)
0
(τ=3°)
.1
.2
Cd →


Straight lines ~ $\frac{C_M}{C_\Delta} = BC_d + \sigma - r$
$\{ r = r/b$
$\{ \sigma = \sigma(\tau)$
$B = \beta/\tau^\circ = \frac{33}{\tau^\circ}$
(Plotted points ~ experimental)
Curve of C_M/C_Δ vs C_d
for various Trim angles (τ)
Data from NACA. T.N. 638
figs 2-7
Fig 2
C_M/C_Δ
C_D →
$\tau = 3°$
$\tau = 5°$
$\tau = 7°$
$\tau = 9°$
$\tau = 11°$
$\tau = 13°$
($\tau = 3°$)
(0 for $\tau = 5°$)
(0; $\tau = 7°$)
(0, $\tau = 9°$)
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
0
.1
.2
.3


Curves of δ and σ plotted against "τ" (trim angle)
○ ~ δ
△ ~ σ
δ
-σ
τ°
τ (radians)
3
5
7
9
11
13
.0523
.0872
.1221
.1571
.192
.227
0
.1
.2
.3
.4
.5
.6
δ
.10
.08
.06
.04
.02
0
.02
.04
.06
.08
.10
Fig. 3



Curves of $-\frac{d\delta}{d\tau}$ ~ o
$-\frac{d\sigma}{d\tau}$ ~ Δ
(both in radians^{-1})
$\frac{d\delta}{d\tau}$
$\frac{-d\sigma}{d\tau}$
Fig. 4
0
-1.
-2.
-3.
-4.
-5.
-6.
3°
5°
7°
9°
11°
13°
$\tau° \rightarrow$
.0523
.0872
.1221
.192
.227
τ (rad ,)

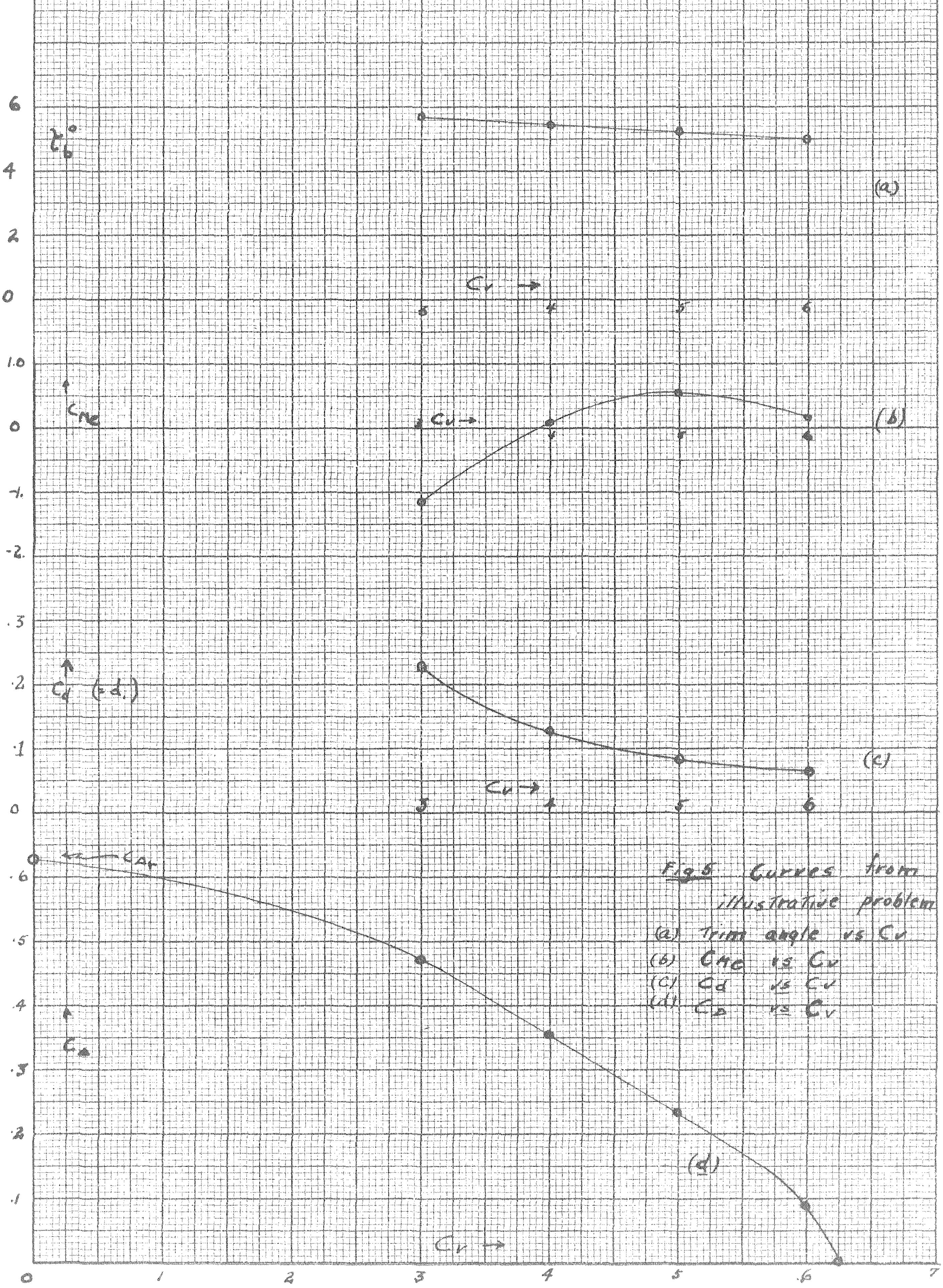
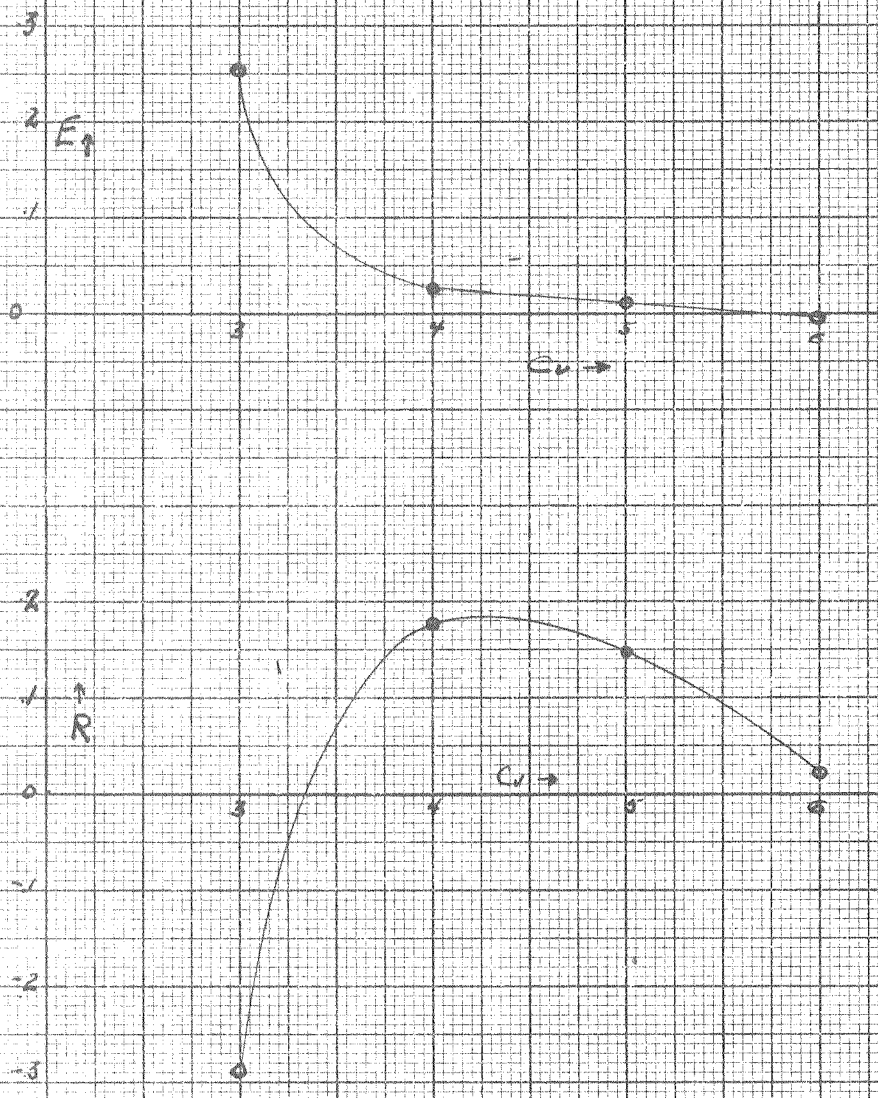


Fig 5 Curves from illustrative problem
(a) Trim angle vs C_v
(b) C_{MC} vs C_v
(c) C_d vs C_v
(d) C_D vs C_v

Fig 6

Curves of: E vs C_v

R vs C_v



REFERENCES

1. Allison, John M.: Tank Tests of Model 36 Flying-Boat Hull. N.A.C.A., T.N. 638, 1938.

2. Klemin, Alexander, Pierson, John D. and Storer, Edmund M.: An Introduction to Seaplane Porpoising. Jour. of Aero. Sci., June 1939.

3. Perring, W.G.A. and Johnston, L.: Hydrodynamic Forces and Moments on a Simple Planing Surface and on a Flying-Boat Hull. A.R.C. R. & M. 1646.

4. Perring, W.G.A. and Glauert, H.: Stability on the Water of a Seaplane in the Planing Condition. A.R.C. R. & M. 1493.

5. Dawson, John R.: A Complete Tank Test of the Hull of the Sikorsky S-40 Flying-Boat - American Clipper Class. N.A.C.A. T.N. 512, 1934.

6. Shoemaker, James M. and Parkinson, John B.: A Complete Tank Test of a Model of a Flying-Boat Hull. N.A.C.A. Model No. 11. N.A.C.A. T.N. 464, 1933.

7. Parkinson, John B., Olson, Roland E., and House, Rufus O.: Hydrodynamic and Aerodynamic Tests of a Family of Models of Seaplane Floats with Varying Angles of Dead Rise. N.A.C.A. models 57-A, 57-B, 57-C. N.A.C.A. T.N. 716, 1939.

8. Parkinson, John B. and Dawson, John R.: Tank Tests of N.A.C.A. model 40 Series of Hulls for Small Flying Boats and Amphibians. N.A.C.A. T.R. 543, 1936.