HIGGS SCALARS AND THE NONLEPTONIC WEAK INTERACTIONS

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Christopher Thaddeus Hill

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ABSTRACT

We study the contributions to the nonleptonic weak Hamiltonian of Higgs bosons within the context of two typical gauge models of the flavor interactions and the SU(3) of color strong interactions (QCD). Processes mediated by charged bosons lead to $|\Delta I| = \frac{1}{2}$, $|\Delta S| = 1$ transitions and $|\Delta I| = 1$, $|\Delta S| = 0$ parity violations.

The effects of QCD upon the general intermediate spin-zero boson exchange Hamiltonian are investigated using the methods of short distance expansions and the renormalization group. It is found that induced gluonic processes, such as the "gluo-magnetic" moment transition $s \to d$ + gluon, are dramatically suppressed. On the other hand, operators containing lighter quarks, such as $\bar{s}u\bar{u}d + \bar{s}d\bar{d}d$, are found to mix with operators involving heavy quarks, such as $\bar{s}c\bar{c}d$, which can occur in the Hamiltonian with large coefficients. We argue that the purely light quark-involving operators have large matrix elements compared to their current-current Hamiltonian counterparts and, hence, Higgsons can account for a large part of the $|\Delta I| = \frac{1}{2}$ rule nonleptonic decays and nuclear parity violation. We found that this contribution is limited by the $K_L K_S$ mass difference, which may be due in part to second order weak Higgson exchange.

We also investigate the properties of several multi-Higgson Lagrangians with respect to their symmetry breaking properties, as are relevant to the models considered here.

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Chapter I

I.1 INTRODUCTION

The past few years have been exciting ones for particle physicists with the discovery of several new and important phenomena. These include the observation of neutral weak interaction current effects at various laboratories [1], the discovery of the psi and related new mesons and resonances [2] and the observation of scaling in deep inelastic electroproduction experiments [3]. The importance of these new phenomena lies in the fact that they were either anticipated or are readily explained within the context of a new class of theoretical models which embody a fundamental physical principle, namely, non-abelian local gauge invariance.

The idea of local gauge invariance is not new and forms a salient feature of Maxwell's equations of electrodynamics and a cornerstone of Einstein's theory of gravitation. The non-abelian nature of the new models is a manifestation of internal, apparently non-geometrical, symmetries which describe approximately the laws of elementary particles and their interactions. Examples of such symmetries are the ordinary isospin of nuclear physics and the approximate SU(3) invariance of the ordinary hadronic spectrum. When these non-abelian internal symmetries are combined with the notion of local gauge invariance one arrives at a theory first proposed by Yang and Mills [4] with the possibility of describing the interactions of elementary particles.

Today we recognize the existence of two distinct gauge theories of this kind which may someday be unified into a grand synthesis of all elementary particle interactions. First, we believe in the existence of a "color" theory of strong interactions based upon the gauge group SU(3), which is presumably responsible for the confinement of quarks within the hadrons [5]. The color theory, also known as Quantum Chromodynamics or QCD, accounts for the apparent scaling behavior of the electroproduction experiments [6] as well as providing a rationale for confinement of quarks by means of the infrared divergences associated with the on mass-shell amplitudes [7]. The important point we wish to emphasize here is that the SU(3) of color symmetry is an unbroken symmetry. Presumably only the color singlet states are observable.

Secondly, we presume the existence of a "flavor" symmetry which, when combined with the idea of local gauge invariance, leads to a theory of the weak and electromagnetic interactions of quarks and leptons. The flavor symmetry is built of a gauge group which is essentially undetermined but which would appear to contain the group SU(2)xU(1). This leads to scores of models of which the Weinberg-Salam model is the original and best contender [8]. Such a model predicts the existence of neutral currents and charmed quarks, the constituents of the psi and related new particles [9]. We emphasize here that the flavor symmetry, unlike SU(3) of color, cannot be an exact symmetry and must be broken in such a way as to preserve the masslessness of the photon while giving mass to the intermediate vector bosons and lifting the degeneracies in the masses of the different elementary fermions. Symmetry breaking is precisely why the exact determination of the complete flavor group is very difficult, requiring extremely high energies to unfold the complete particle spectrum. At some future date we may find that the color and flavor groups are not distinct but merely subgroups of of a larger unifying gauge group [10]. Clearly, these speculations emphasize the importance of understanding the nature of the symmetry breaking mechanisms.

Such a mechanism, which is commonly invoked to construct realistic models of flavor interactions, is the so-called Higgs mechanism [11]. The Higgs mechanism requires the existence of a new family of elementary particles which are spin zero scalars or pseudoscalars. One is encouraged by the successes of the gauge theories thus far to consider the possibility that such elementary scalars, or "Higgsons", may at some point be discovered in high energy experiments. As we shall see below, a natural mass scale for Higgs scalars is roughly $m_H \simeq \sqrt{\alpha}\, M_W$, where M_W is the mass of the W-boson. This result for m_H is presumably 5 to 15 Gev using the expected values of M_W and is not so large as to make the Higgsons inaccessible in the near future.

It should be mentioned that the Higgs mechanism is not the only possibility for breaking the flavor symmetries. Many authors believe in a dynamical symmetry breaking mechanism which, in analogy with superconductivity, provides symmetry breaking without the existence of <u>elementary</u> scalar fields.† Such a mechanism suffers from the drawbacks that a) it has not yet been demonstrated convincingly in four-dimensional model field theories b) it is doubtful that one could readily perform calculations with such a mechanism [12]. Should such a dynamical mechanism be operant it is possible, if not very likely, that it would mimic the simpler explicit Higgs mechanism and may, indeed, display quasi-

†Both mechanisms may be operant at some level, as has been considered by some authors [13].

elementary spin-zero excitations.

In this dissertation we have addressed the question what, if any, are the potential effects in the ordinary interactions of elementary particles due to Higgsons? Generally the effects involving leptons are small since Higgson couplings to the fermions are proportional to the fermion masses and leptonic masses are small. Nonetheless, if the existence of heavy leptons is established [14], Higgson mediated processes may become important in their decays; induced processes involving Higgsons such as $\mu \to e\gamma$ may also arise at some level as a result of Higgson-fermion interactions [15].

We have primarily focused upon the nonleptonic weak interactions where Higgson effects may already be operant. This is a rich problem in that it brings to bear the full apparatus of the renormalization group, short distance expansions, and QCD's unique property of being asymptotically free [6]. We employ these well known techniques in estimating the strong interaction corrections which we find in some cases to be important. The primary effects leading to an enhancement of Higgson amplitudes are due to a) the involvement of the charmed quark or other heavy quarks in the intermediate states which can lead to factors of m_c b) mixing angle effects as described below which lead to factors of (tan Θ) and may be large c) QCD enhancement effects.

We have considered two generic models: the standard Weinberg-Salam model with four quarks and four leptons and multiple Higgs doublets and a six-quark vectorlike scheme with minimally required numbers of Higgs multiplets [16]. Both models have the feature of leading to charged residual Higgsons and a wider class of Higgson mediated interactions than one obtains with only neutral Higgs bosons. We limited ourselves to models in which the neutral scalars couple only diagonally to the quarks, i.e. no $|\Delta S| = 1$ and $|\Delta C| = 1$ neutral scalar vertices, so as to avoid $|\Delta S| = 2$ and $|\Delta C| = 2$ processes in lowest order of the Higgson exchange. This becomes an equally important constraint in second order, or double Higgs exchange and Higgson-W exchange processes.

In considering these models we devote considerable attention to the structure and properties of the scalar self interactions, or Higgs potentials, which lead to the development of vacuum expectation values by different scalar fields and the spontaneous breakdown of the symmetry. This is a novel area of research and we are led to conjecture a number of interesting general features about symmetry breaking based upon our analysis of these potentials, as discussed in Appendix A.

The effects of the strong interactions lead to a standard but lengthy problem in short distance expansions. In the language of the renormalization group we find that we must construct anomalous dimensions of various basis operators occuring in the short distance expansions which mix under renormalization. We must then determine the linear combinations of operators that are multiplicatively renormalized in order to solve the renormalization group equations and thus determine the corrections to the Hamiltonian given in the tree approximation. Certain of the tree approximation operators will be enhanced and others suppressed by this process. Our treatment of the QCD effects is sufficiently general that it can be readily applied to any intermediate scalar exchange process. In the Appendices we discuss the technical apparatus that is required in the evaluation of the anomalous dimensions of composite operators.

In this work we have not reported on the purely leptonic and semi-leptonic Higgson induced processes. We have found that these effects are consistently small owing to the smallness of leptonic masses. Higher order radiative correction effects have been estimated, including mixing angle enhancements, for several of these processes. The potential sources of concern are the muon anomalous magnetic moment and the $K \to \mu\bar{\mu}$, which could receive large Higgson contributions. We have found that these are generally arbitrary in our models and can be separately controlled, with respect to the nonleptonic processes.

In summary, we find that Higgson processes can be competitive with the ordinary current-current weak interactions. By saturating the constraints imposed by the $K_L K_S$ mass difference, we may be able to account for at least a part of the nonleptonic $\Delta I = \frac{1}{2}$ rule in strangeness changing processes, as well as obtaining a significant contribution to $\Delta S = 0$ parity violating processes.

I.2 REVIEW OF HIGGSISM†

The prototype example of a model displaying the Higgs mechanism is based upon the U(1) gauge group and is described by the Lagrangian [11] of the form:

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2} m^2|\phi|^2 - \frac{1}{4!}|\phi|^4 \quad (I.1)$$

This is simply scalar electrodynamics with the wrong sign appearing in front of the m^2 term.

This Lagrangian is readily generalized to all Yang-Mills theories by introducing appropriate multiplets of the scalar fields and allowing the vector potentials to be members of the adjoint representations of the various gauge groups. The result of the unstable classical potential will be to force the scalar field to develop a vacuum expectation value which effectively breaks the gauge symmetry while preserving Lorentz invariance. The Higgs mechanism gives a mass to the vector fields corresponding to the charges in the group that are violated.

We wish to point out that, if we had started without the quartic self interaction, it would have been forced upon us when we computed scalar-scalar scattering amplitudes. The double seagull exchange of Fig.(I.1) is of order e^4 and requires a quartic counterterm for renormalization. Once one includes the counterterm, the theory is fully renormalizable. This is the basis of the standard assertion that λ is to be of order e^4. This is essentially a plausi-

† This section is purely pedagogical and is summarized in Table I.1

bility argument and cannot be justified rigorously since we are essentially free to specify the renormalized values of coupling constants arbitrarily.

To zeroth order in α the potential energy density of the vacuum is given by the expression:

$$V(\phi) = -\frac{1}{2} m^2 |\phi|^2 + \frac{\lambda}{4!} |\phi|^4 \qquad (I.2)$$

where ϕ is shorthand notation for the vacuum expectation value of the field ϕ, i.e. $\langle 0|\phi|0\rangle$.

A well defined procedure exists for the computation of the higher order corrections to the potential energy density of eq.(I.2) in a perturbation series in powers of Planck's constant. We describe this procedure here and refer the reader to the major references for details [17].

One considers the generating functional of single particle irreducible Green's functions, $\Gamma(J)$, where $J(x)$ is an arbitrary external source for the field ϕ. For scalar fields $J(x)$ is also a scalar, but in general one includes one source per field in $\Gamma(J)$ and each source has the same Lorentz property as the field represented. We are only interested in scalars here since a vector or spinor cannot develop a vacuum expectation value without spoiling Lorentz invariance.

One performs a Legendre transform upon $\Gamma(J)$ of the form:

$$\Gamma(J) - J\frac{\partial\Gamma}{\partial J} = \Gamma(\phi) \qquad (I.3)$$

where we use:

$$\frac{\partial \Gamma}{\partial J} = \phi \tag{I.4}$$

to eliminate J(x) in eq.(I.3). $\Gamma(\phi)$ has the following interpretation.

We may choose an ensemble of vacua all of which have the property that $<0|\phi|0> = \phi$. These are to be regarded as a set of trial vacuum wave functionals parameterized by ϕ and an infinite number of additional parameters. We vary the remaining parameters until we find the vacuum state of lowest energy for fixed value of ϕ. The energy density of this state is the value of $\Gamma(\phi)$. Therefore to find the absolute ground state we must simply minimize $\Gamma(\phi)$ with respect to ϕ. Generally, for values of ϕ which do not correspond to the lowest state, $\Gamma(\phi)$ will have an imaginary part corresponding to the decay rate (per unit volume) of the unstable vacuum.

$\Gamma(\phi)$ is known as the effective potential and is usually expanded in the loop expansion, or as a power series in Planck's constant. The lowest order term corresponding to $\hbar = 0$ is the classical energy density of eq.(I.2).

Coleman and Weinberg have investigated the Lagrangian of eq.(I.1) in the limit $m^2 = 0$ [18] using the one loop corrections to the effective potential. Their perturbative calculation is meaningful only when λ is of order α^2. In this case it is found the massless scalar electrodynamics is dynamically unstable and the scalar field will automatically develop a vacuum expectation value. We have mentioned this phenomenon in that it allows an alternative to the incorporation of Higgs scalars in a model explicitly. Many authors have considered the possibility that a composite operator such as $\bar{\psi}\psi$, for example, might play the role of the field ϕ and a mechanism such as the Coleman-Weinberg phenomenon might be operant [12]. We have briefly mentioned this possibility in section I.1 and we reiterate that it is difficult at present to see how to put dynamical symmetry breaking on a sound theoretical basis.

If $m^2 \neq 0$ we readily work out the properties of the potential in eq.(I.2). We find that $V(\phi)$ has a minimum when:

$$\phi = \sqrt{\frac{6m^2}{\lambda}}. \tag{I.5}$$

Returning to eq.(I.1) and shifting the field ϕ by the amount eq.(I.5), the vector boson is easily seen to acquire a mass. Essentially, the gradient of the phase of the complex field ϕ becomes the longitudinal component of the vector boson. This is most easily seen explicitly by setting $\phi = \rho e^{i\theta}$ and letting $\langle 0|\phi|0\rangle = \rho$ in (I.5). We find:

$$M^2_{V.B.} = e^2(\langle 0|\phi|0\rangle)^2. \tag{I.6}$$

There is also seen to be a leftover real scalar field corresponding to the shift $\rho = \langle 0|\phi|0\rangle + \eta$. We call the field η the "Higgson" and find that it has a mass:

$$\frac{1}{2} m_H{}^2 = -\frac{1}{2} m^2 + \frac{1}{4} (\langle 0|\phi|0\rangle)^2 = \frac{1}{2} m^2. \tag{I.7}$$

It is also seen that a Higgson can couple to a pair of vector bosons with the effective interaction:

$$\frac{1}{2} e^2\langle 0|\phi|0\rangle A_\mu A^\mu \eta = \frac{1}{2} eM_{V.B.} A_\mu A^\mu \eta. \tag{I.8}$$

In unified gauge theories we will have $M_{V.B.} \simeq M_W$, the mass of the intermediate vector boson. Of course, there will be numerical factors and mixing angles involved, but in what follows we will think of $M_{V.B.}$ as being always of order $\sqrt{\alpha/G_F}$ for the sake of discussion.

Weinberg has deduced a lower limit for the mass of the neutral Higgson in the standard Weinberg-Salam model with one scalar doublet [19]. The argument is essentially as follows.

Suppose we wanted to take m_H (= m) very small. Then, to hold $M_{V.B.}$ fixed so as not to spoil the strength of the weak interactions, it is necessary to take λ correspondingly small by eq. (I.6). However, λ is bounded by the renormalizability of the theory never to be smaller than α^2. Therefore, there must be a lower limit on m. We mentioned above that the renormalized value of λ is really arbitrary. Nonetheless, if one considers the one-loop corrections to $V(\phi)$,it may be seen that if one tries to take m too small, the $\langle 0|\phi|0\rangle = 0$ extremum becomes the physical minimum. This makes rigorous the renormalizability argument. The lower bound obtained for one doublet is a mass of 3.72 Gev without specifying the value of the Weinberg angle. Taking $\Theta_W = 35^o$ gives a lower bound of M_H = 4.9 Gev. Again, this result is suggestive of a natural mass scale of order 10 Gev for Higgsons, though no theoretical upper limit can be placed at present.

It is interesting to note that the Higgson-V.B.-V.B. coupling of eq. (I.8), which leads to diagrams such as Fig.(I.2), is of the same order as the tree diagrams if both divergences are present and if $m_H \simeq M_{V.B.}$. Again, this is basically the statement that perturbation theory is not sensible if m_H is too small.

We have been cavalier in this discussion about large numerical factors such as $1/16\pi^2$, etc. We are simply trying here to convey a qualitative feeling for the elements of Higgs theory.

The Higgs scalars are used to give masses in the Weinberg-Salam model to the elementary fermions. This will also occur in the vector-like theories, though singlet mass terms are also needed which may or may not arise from scalars (section II.2). A typical Yukawa interaction will be of the form (using the above developed U(1) model, we ignore the fact that ϕ is actually complex for the sake of illustration):

$$L_{Yukawa} = g\phi\,\bar{\psi}\psi\ . \tag{I.9}$$

Hence, the fermion mass will be of order $m_f = g\langle 0|\phi|0\rangle$, and the coupling to the leftover Higgson, η, is written:

$$L = \frac{m_f}{\langle 0|\phi|0\rangle}\,\eta\bar{\psi}\psi \simeq \sqrt{G_F}\,m_f\eta\bar{\psi}\psi . \tag{I.10}$$

This is the basis of the standard remark that the Higgson couplings to fermions are proportional to the fermion masses.

It is to be noted here that if additional Higgsons contribute vacuum expectation values to the vector boson masses, we will have:

$$M_{V.B.}{}^2 = e^2(\langle\phi\rangle_1{}^2 + \langle\phi\rangle_2{}^2 + \ldots + \langle\phi\rangle_n{}^2). \tag{I.11}$$

Hence:

$$\langle\phi\rangle_n = \frac{M_{V.B.}}{\sqrt{\alpha}}\sin\Theta \tag{I.12}$$

and if only ϕ_n couples to $\overline{\psi}_n\psi_n$, we will get a Yukawa interaction of strength:

$$L_{Yukawa} = \sqrt{G_F}\, m_f(\sin\,\Theta)^{-1}\eta\overline{\psi}\psi^{\dagger} \qquad (I.13)$$

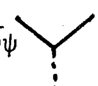
which can easily be substantially enhanced for small values of the mixing angle Θ. This indicates the possibility that Higgson exchange effects can be important when a) m_f is large, e.g. $m_f = m_c$ b) $\sin\,\Theta$ is small, e.g., $\Theta \simeq \Theta_{Cabbibo}$.

†Actually, it will be seen that when we rewrite the Higgs fields in terms of the physical mass matrix eigenfields, the factor of $\sin\theta$ in (I.13) becomes a $\tan\theta$.

TABLE I.1

SUMMARY OF HIGGSISM

$$V(\phi) = -\frac{m^2}{2}|\phi|^2 + \frac{\lambda}{4!}|\phi|^4$$

Vacuum Expectation Value	$\langle 0\|\phi\|0\rangle \approx \sqrt{\frac{6m^2}{\lambda}}$	
Vector Boson Mass	$M_W = e\langle 0\|\phi\|0\rangle \approx em/\sqrt{\lambda}$	
Higgson Mass	$M_H \approx m$	
Natural Higgson Mass Scale	$M_H \approx \alpha\sqrt{M_W}$	
V.B.-V.B.-Higgson Vertex	$L' = eM_W\phi W_\mu W^\mu$	
Fermion Mass	$m_f \approx g\langle 0\|\phi\|0\rangle \approx gM_W/e$	
(in theories with many Higgsons	$\approx g(\tan\Theta)M_W/e$)	
Higgson-Fermion-Fermion Vertex	$g\phi\bar{\psi}\psi \approx \sqrt{G_F}m_f\phi\bar{\psi}\psi$ $(\approx \sqrt{G_F}m_f\cot\Theta)$	

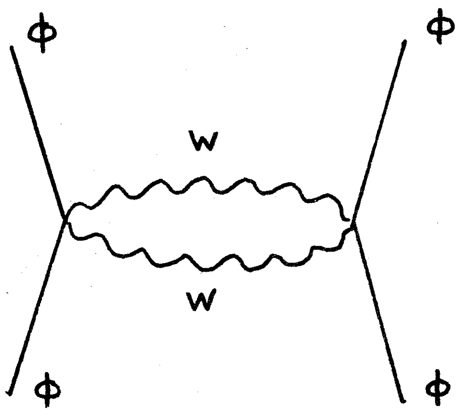


Fig. I.1

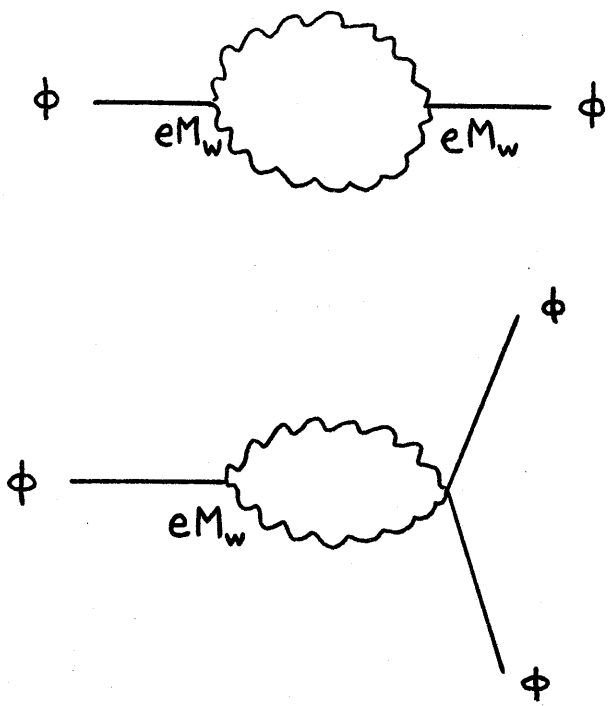


Fig. I.2

I.3 THE $|\Delta I| = \frac{1}{2}$ RULE

In the present section we review modern attempts to explain the puzzles of the $\Delta S = 1$ weak nonleptonic processes within the context of the unified gauge theories.

Lee and Gaillard and Altarelli and Maiani [20] offered an explanation of the apparent enhancement of the $|\Delta I| = \frac{1}{2}$ operators (and/or suppression of the $|\Delta I| = \frac{3}{2}$ operators) occurring in the weak nonleptonic current-current Hamiltonian within the context of the SU(3) color strong interaction picture. The nonleptonic Hamiltonian is constructed in the tree approximation as a short distance product of weak currents. The distance scale is set by $(M_W)^{-1}$ and is small compared to a conventional hadronic length scale, e.g. $(1 \text{ Gev})^{-1}$. The authors of ref.[20], using the techniques of short distance expansions and the renormalization group, as discussed in chapter III, computed the QCD radiative corrections to the operators appearing in the weak Hamiltonian. It was found that the octet components are enhanced relative to the $\underline{27}$ pieces that contain the $|\Delta I| = \frac{3}{2}$ operators by a factor of approximately 5.

We can readily enumerate a series of difficulties and objections to this result. These are as follows:

a) The desired enhancement factor is about 25 based upon the fact that $K^o \to 2\pi / K^+ \to 2\pi$ (rate) is = 625. The K^o decay is both isospin $\frac{1}{2}$ and $\frac{3}{2}$, whereas the K^+ decay is pure isospin $\frac{3}{2}$.

b) In the limit of exact SU(3) symmetry the arguments of

Gell-Mann [21] indicate that the K decay amplitudes must vanish due to the charge conjugation properties of the Hamiltonian.

c) The Lee-Sugawara relations [22]†, which appear to be experimentally sound for both the p-wave and s-wave Hyperon decays, are only obtained for the s-wave amplitudes in the current-current Hamiltonian (this is no doubt related to (b)). (Actually, one may note that both results are obtained with PCAC and current algebra [39]).

We should point out that the analysis of Lee, et. al.[20] is still somewhat incomplete in that the charge radius corrections have not been fully accomodated. It turns out that these corrections are probably not important due to the GIM cancellation, but the technical details are very intricate [23].

In attempt to remedy some of the alleged shortcomings of the previous analysis, Fritzsch and Minkowski [24] proposed an alternative explanation to the problem (also DeRujula, Georgi and Glashow) within the context of the vectorlike models of the flavor interactions. These authors call for a new operator of the form $\bar{s}\sigma_{\mu\nu}G^{A\mu\nu}\chi^{A}d_{L}$ (or $\bar{s}\not{D}\not{D}d$) which describes processes of the form $s \rightarrow d$ + gluon and, it was hoped, may be present with sufficient strength to account for the experimental situation. Such an operator is apparently nice from the point of view of item (b) having the correct charge conjugation to give the K decay in the continued SU(3) symmetry limit. Such an operator could occur in the vectorlike theory if there was an $\bar{s}\gamma_{\mu}c_{R}$ cur .

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†Only the s-wave "triangle" relation can be derived from the rent- current Hamiltonian [21]. The p-wave relation is observed exp ment- ally, though the original "derivation" of ref.[22] incorpora extra assumptions.

One may criticize the vectorlike explanation of the $|\Delta I| = \frac{1}{2}$ rule from the following point of view.

First, one hopes to start out with the attractive picture of the flavor currents of the form:

$$\begin{Bmatrix} u & c & t \ldots \\ d' & s' & b \ldots \end{Bmatrix}_L \begin{Bmatrix} u & c & t \ldots \\ b'' & s'' & d'' \ldots \end{Bmatrix}_R .$$

However, the arguments of Golowich & Holstein [25] indicate that the mixing angle between s_R and d_R must be less than .07 so as to maintain the purely left-handed chiral structure of the isospin transformation of the weak Hamiltonian. Furthermore, the observation of parity violation in the neutral current processes involving nucleons indicates that either d_R or u_R must be taken to have singlet components (otherwise the neutral current is purely vectorial [26]).

Wilczek and Zee, using the techniques we employ in Section II.4, [29] have studied the QCD corrections to the operators such as $\bar{s}\not{D}\not{D}d$ and $\bar{s}\sigma_{\mu\nu}G^{A\mu\nu}\chi^{A}d$ as they appear in short distance expansions These authors find that these operators are suppressed by factors of order $(\log M_W^2/\mu^2)^{-1/2} \approx 1/3$. Furthermore, the matrix elements of these operators can not be argued convincingly to be large (the argument given by Fritzsch and Minkowski relies on the ratio of two body to three body phase space in comparing $s \to$ gluon + d to $s \to d\bar{u}u$. This is hardly a believable estimate). A better estimate would be to compare $s \to d\bar{u}u$ (via gluon) to $s \to d\bar{u}u$ (via weak currents). This is of order unity.

There are many other proposed mechanisms for the $|\Delta I| = \frac{1}{2}$ rule in the literature relying upon different ideas [30]. Recently C. Schmid has proposed an interesting idea which we mention only because it illustrates that the effect may not be a "short-distance" enhancement at all. Schmid relies upon the finite size of the hadron to determine the approximate peaking in the quark wavefunctions for zero separation. Of course, the value of the enhancement is sensitive to one's choice of wavefunction, for which Schmid employs "charmonium-like" amplitudes for the relative separations of quarks within baryons (this approach only deals with baryonic decays and evidently fails for meson processes). This same kind of argument, though attractive, does not seem to work in the "bag" models [43].

It is a common feature to all of the enhancement mechanisms known to this author that they are basically undecidable in that they lead to no alternative measurable quantities one may use as a check on the mechanism. Actually, we may find that parity violation in the $|\Delta S| = 0$ processes may afford such a "second handle" on the problem (Altarelli, et. al. [31]).

In this dissertation we take a completely different tack and propose that Higgs scalars may be responsible, through a combination of effects, for the observed enhancement in nonleptonic $|\Delta I| = \frac{1}{2}$ weak interactions. Our approach is decidable in the sense that the discovery of Higgsons and the subsequent analysis of their decay properties would reveal a host of independent checks upon the hypothesis. Higgson decay modes would directly yield information about such parameters in the theory as the ratio of vacuum expectation values, etc.

Our point of view will be that Higgsons contribute in part to the nonleptonic interactions, but we will find that this requires saturating limits, such as the $K_L K_S$ mass difference, which will effectively require taking certain mixing angles small. In the end, we would require the independent information that would determine the absolute values of these parameters.

In the next section we give a brief summary of our work.

I.4 SUMMARY

We find that charged Higgson exchange leads to an effective Hamiltonian that contains a $|\Delta I| = \frac{1}{2}$, $|\Delta S| = 1$, piece of the form:

$$H = \frac{G_F}{\sqrt{2}} \frac{m_c^2}{m_H^2} \sin\Theta_c \cos\Theta_c \tan^2\chi \, (\bar{s}_L c_R \bar{c}_R d_L + \text{h.c.}) \qquad (I.14)$$

where χ is a mixing angle associated with the scalar vacuum expectation values (more generally, we simply regard $\tan\chi$ as representing all of the mixing effects). With other heavy quarks of charge 2/3 there will be additional terms involving $\bar{s}q\bar{q}d$. The structure of the entire nonleptonic Hamiltonian is, with the exception of the specific values of the parameters and numbers of quarks, model independent.

Induced gluon processes of the form $s \to d$ + gluon, as described by the operators $\bar{s}\sigma_{\mu\nu}G^{A\mu\nu}\chi^A d$, $\bar{s}\not{D}\not{D}d$, etc., are found to be dramatically suppressed by the effects of QCD by factors of order 10^{-1}, as well as by the coefficients of their parent operators, $\bar{s}_R c_L \bar{c}_R d_L$, which are found to be of order m_c^2/m_H^2.

We find that the mixing angle contribution, $\tan^2\chi$, can assume large values without violating the smallness of the $K_L K_S$ mass difference, e.g., we can tolerate factors of order $\tan^2\chi \approx 10^1 m_H/m_c$ (or perhaps even larger).

The effects of QCD are striking in this example as they lead to a mixing of the operators in the Hamiltonian of eq.(I.14) with operators of the form $\bar{s}_L u_R \bar{u}_R d_L + \bar{s}_L d_R \bar{d}_R d_L$. These new operators have several desirable features a) they are of the correct chirality so as not violate the constraints of Golowich and Holstein [25] b) they enter the Hamiltonian with reasonable coefficients (of order 10^{-1}c)they are seen to have large matrix elements between ordinary hadron states, e.g., we show in section IV.2 that the matrix element of this operator between K and π states is of order $m_\pi^2/m_d m_s$ · (ordinary current-current matrix elements).

If we use this estimate for these matrix elements and saturate the bounds imposed upon our Hamiltonian by the $K_L K_S$ mass difference, we find that we can almost accommodate the observed enhancement of the isospin-$\frac{1}{2}$ processes in the nonleptonic weak interactions.

Chapter II

II.1 WEINBERG-SALAM MODEL WITH MULTIPLE HIGGS DOUBLETS

The Weinberg-Salam model [8] employs the standard left-handed doublets of quarks and leptons with the right-handed fields as singlets under the gauge group SU(2)xU(1). All quark fields are SU(3) color triplets and leptons are singlets. With four quarks and four leptons the model is free of axial vector current anomalies.

We will make use of the following notation:

$$\begin{array}{llll}
\psi_1 = \begin{Bmatrix} u \\ d \end{Bmatrix}_L & \psi_2 = \begin{Bmatrix} c \\ s \end{Bmatrix}_L & \chi_1 = \begin{Bmatrix} e \\ \nu_e \end{Bmatrix}_L & \chi_2 = \begin{Bmatrix} \mu \\ \nu_\mu \end{Bmatrix}_L \\
p^{(1)}_{L,R} = u_{L,R} & p^{(2)}_{L,R} = c_{L,R} & \ell^{(1)}_{L,R} = e_{L,R} & \ell^{(2)}_{L,R} = \mu_{L,R} \\
n^{(1)}_{L,R} = d_{L,R} & n^{(1)}_{L,R} = s_{L,R} & \nu^{(1)} = \nu_e & \nu^{(2)} = \nu_\mu .
\end{array} \qquad \text{(II.1)}$$

One may introduce a single Higgs doublet which is sufficient to break SU(2)xU(1) invariance, giving mass to the $W^\pm$ and Z bosons and arbitrary masses to the quarks and leptons. There is then a single left-over neutral Higgs scalar which couples diagonally to all flavors with coupling strengths proportional to the masses of the individual fermions.

This model is in good agreement with present experimental evidence regarding the weak and electromagnetic interactions, with the following few exceptions: a) it is impossible to introduce CP violating phases into the quark and vector boson sector b) the $|\Delta I| = \frac{1}{2}$ rule has not yet been satisfactorily explained c) we may require additional leptons and additional quarks to account for experimental observation [14]d) there may exist difficulties with parity violation in $|\Delta S| = 0$ nonleptonic processes such as $p + n \rightarrow D + \gamma$ [32].

These difficulties are not necessarily unrelated. For example, by including additional leptons and quarks (this must be done symmetrically to maintain the cancellation of anomalies) one remedies (c) as well as allowing the possibility of introducing complex phase angles for the purpose of violating CP as in (a). For the sake of the study of possible Higgson interaction effects in the weak Hamiltonian we will not further consider the question of more heavy fermions in the context of the Weinberg-Salam model.

It has been suggested by Weinberg [33] that by including additional Higgs multiplets one can account nicely for the problem of CP violation. After incorporating two or more doublets there remains an arbitrariness as to how they will couple to the ordinary fermion fields since there are more coupling constants than constraints which determine the values of the coupling constants. There is also the possibility of inadvertently introducing strangeness-changing and charm-changing neutral Higgson - fermion vertices which could lead to $|\Delta S| = |\Delta C| = 2$ processes in lowest order (single Higgson exchange). Such unwanted processes could be suppressed by taking the neutral Higgson mass heavy, e.g. of order 30 Gev, ignoring the possibility that such processes may also receive enhancement factors, which would require m_H even heavier.

Weinberg and Glashow [34] have suggested a general principle that "naturally" eliminates the off-diagonal neutral vertices and which completely determines the values of all the coupling constants in terms of the quark and lepton masses. Essentially, one requires that there be at most one Higgs doublet for each right-handed charge group in the model, i.e. one doublet for both s_R and d_R; one doublet for u_R and c_R. This would force the neutral Higgson corresponding to the pair s_R and d_R to couple to the diagonal operator $m_d\bar{d}d + m_s\bar{s}s$. This principle is implemented by building a discrete symmetry of the form $s_R \to -s_R$; $d_R \to -d_R$; $\phi \to -\phi$ (where ϕ is the Higgs doublet corresponding to s_R and d_R) into the model. With the principle described here we can accommodate no more than four Higgs doublets into the standard model with four fermionic doublets. One should not, however, take the naturalness assumption too seriously. For one, it rules out vectorlike theories for which one cannot incorporate discrete symmetries of this sort. Secondly, it is easy to construct a model in which the strangeness-changing and charm-changing neutral Higgson exchange processes are absent in lowest order and well within the bounds of tolerability that does not embody the suggestion of Weinberg and Glashow. The appearance of discrete symmetries does, however, lead to the absence of the strangeness and charm-changing processes to all orders of perturbation theory which may be of interest theoretically.

The most important consequence of incorporating additional Higgs doublets into the Weinberg-Salam model is the appearance of left-over charged, as well as neutral, Higgsons. These can easily connect light quarks to heavy quarks and can have large coupling strengths.

For simplicity, consider just the quark sector of the model which, invoking the natural elimination of the neutral strangeness and charm changing couplings, can have only two Higgs doublets. It is not possible to introduce CP violating phases with only two doublets and the discrete symmetries described above [33]. One can incorporate additional doublets for the leptons subsequently at which time it is possible to introduce the CP violation, or alternatively, one could abandon the discrete symmetries and employ a mechanism for eliminating the $|\Delta S| = |\Delta C| = 2$ processes in lowest order only, but which does afford the possibility of introducing CP phases.

We define the two doublets as:

$$\phi_1 = \begin{pmatrix} \phi_1^o \\ \phi_1^- \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_2^o \\ \phi_2^- \end{pmatrix} \qquad \phi^c = \begin{pmatrix} \phi^+ \\ -\phi^o \end{pmatrix} \tag{II.2}$$

$$\langle\phi_1^o\rangle = \eta \qquad \langle\phi_2^o\rangle = \rho \tag{II.3}$$

where we have also introduced the charge conjugated scalar field and defined the vacuum expectation values. The vacuum expectation values are determined by the Higgs potential which is discussed in Appendix A. We have assumed that the neutral fields will have non-zero vacuum expectation values so as to maintain the conservation of electric charge. This is the the "ferromagnetic" solution to the minimization of the potential and can be guaranteed only if it happens to be the solution with lowest energy. There will be three left-over real neutral fields which are linear combinations of Re ϕ_1, Im ϕ_1, Re ϕ_2, and Im ϕ_2. There will also be a charged Higgson which is a linear combination of ϕ_1^+ and ϕ_2^+. The other charged linear combination and neutral linear combination are the goldstone bosons which are eaten by the $W^\pm$ and Z bosons to give them mass. The precise form of the linear combinations and masses thereof can be found in the Appendix for the most general Higgs potential involving two doublets.

The most general Yukawa couplings that we can construct with the quark fields and the two doublets, disregarding the discrete symmetries, is of the form:

$$\mathcal{L} = \sum_{aij}\left(\Gamma^a_{ij}\bar{\psi}_i\phi^a p_{jR} + \Lambda^a_{ij}\bar{\psi}\phi^{ac} n_{jR}\right) \tag{II.4}$$

where Γ^a_{ij} and Λ^a_{ij} are arbitrary complex numbers.

If we invoke the naturalness assumption of Weinberg and Glashow we must require that $\Gamma^{(1)}_{ij} \neq 0$; $\Gamma^{(2)}_{ij} = 0$; and $\Lambda^{(1)}_{ij} = 0$; $\Lambda^{(2)}_{ij} \neq 0$. It is then verified that $\mathcal{L}$ is invariant under the discrete symmetry: $p \to -p$, $\phi^{(1)} \to \phi^{(1)}$; $n \to -n, \phi^{(2)} \to -\phi^{(2)}$. It should be noted that the Higgs potential must also be invariant under these reflection symmetries if the absence of neutral strangeness and charm changing processes is to be maintained to all orders. Of course, these effects do occur to second order in the weak interactions and, as we shall see in section II.5, the Higgson contributions can be comparable to the second order W-boson exchange processes.

An alternative to the naturalness assumption is to choose:

$$\Gamma_{ij}^{(2)} = f_\Gamma \Gamma_{ij}^{(1)} \qquad \Lambda_{ij}^{(2)} = f_\Lambda \Lambda_{ij}^{(2)} \tag{II.5}$$

which will also lead in lowest order to the absence of direct $|\Delta S| = |\Delta C| = 2$ processes from neutral Higgson exchange. The factors f_Γ and f_Λ are arbitrary.

After the scalar fields develop vacuum expectation values as in eq.(II.3) the fermions acquire masses. We may solve for the values of the numbers Γ_{ij}^a and Λ_{ij}^a necessary to give the correct quark mass spectrum. In this way the Yukawa interaction may be rewritten in terms of the physical quarks, and by using the masses and physical linear combinations of the Higgson fields, in terms of the physical massive Higgsons. We also use the fact that the W-boson mass is given in terms of the vacuum expectation values by:

$$M_W = \left(\frac{1}{2}\right)\frac{e}{\sin\theta_W}\sqrt{\eta^2 + \rho^2} \tag{II.6}$$

where θ_W is the Weinberg angle. We also introduce the vacuum expectation value mixing angle, χ, by:

$$\tan\chi = \eta/\rho. \tag{II.7}$$

This angle is important for the charged Higgsons since the charged Goldstone boson and the charged massive Higgson are always of the form:

$$\phi_{GB} = \phi_1 \cos\chi + \phi_2 \sin\chi \qquad M^2 = 0 \tag{II.8}$$

$$\phi_{Higgs} = -\phi_1 \sin\chi + \phi_2 \cos\chi \qquad M^2 = M_H^2. \tag{II.9}$$

Incorporating the discrete symmetries into the Yukawa interaction eq.(II.4) and using eqs. (II.6 - II.8) we obtain the following Higgson-fermion interaction Hamiltonian:

$$
\begin{aligned}
\mathcal{L} = \sqrt{\frac{G_F}{\sqrt{2}}}\ \frac{\cot\chi}{2}\ \phi_H^- \Big\{ & m_d \cos\theta_c\ \bar{d}_R u_L - m_d \sin\theta_c\ \bar{d}_R c_L \\
& + m_s \sin\theta_c\ \bar{s}_R u_L + m_s \cos\theta_c\ \bar{s}_R c_L \Big\} \\
+ \sqrt{\frac{G_F}{\sqrt{2}}}\ \frac{\tan\chi}{2}\ \phi_H^- \Big\{ & m_u \cos\theta_c\ \bar{d}_L u_R + m_u \sin\theta_c\ \bar{s}_L u_R \\
& - m_c \sin\theta_c\ \bar{d}_L c_R + m_c \cos\theta_c\ \bar{s}_L c_R \Big\} \\
+ H_{neutral} + \text{h.c.} &
\end{aligned}
\tag{II.10}
$$

We have not written the coupling of the neutral Higgsons since they conserve parity and couple to the quarks diagonally and therefore cannot be discerned in nonleptonic processes. The ϕ_H field is the physical charged Higgson. If one included additional Higgs multiplets it would be necessary to include the interactions of additional charged scalars which would be linear combinations of the various charged members of the multiplets.

II.2 VECTORLIKE MODEL

We will consider a six quark, six lepton vectorlike model which will exhibit the general Higgson interactions in this class of models. The quark sector of this model is similar to that of a model proposed by Ramond [35]based upon the exceptional group E_7. Although vectorlike models are intrinsically free of axial vector current anomalies, we will, nonetheless, maintain a lepton hadron symmetry and define the quark fields to be:

$$\psi_1 = \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \psi_2 = \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \quad \chi_1 = \begin{pmatrix} u \\ b_\chi \end{pmatrix}_R \quad \chi_2 = \begin{pmatrix} c \\ h_\chi \end{pmatrix}_R \tag{II.11}$$

$$\psi_3 = b_L \quad \psi_4 = h_L \quad \chi_3 = d_R \quad \chi_4 = s_R . \tag{II.12}$$

The most general SU(2)xU(1) invariant mass and Higgson coupling employing one real triplet and one complex doublet may be written:

$$\sum_{ij} m_{ij}\bar{\psi}_i\chi_j + \sum_{\substack{i,j= \\ (1,2)}} g_{ij}\bar{\psi}_i\tau^a\chi_j\phi^a + \sum_{\substack{i=(1,2) \\ j=(3,4)}} h_{ij}\bar{\psi}_i\tilde{\phi}^c\chi_j + \sum_{\substack{i=(3,4) \\ j=(1,2)}} h_{ij}\bar{\psi}_i\tilde{\phi}\chi_j + \text{h.c.}. \tag{II.13}$$

Note that there is an ordinary Cabbibo rotation in the s_L and d_L

fields, as well as a rotation in the b_R and h_R fields by the angle χ. ϕ^a is a real triplet and $\tilde{\phi}$ is a complex doublet.

The most general Higgs potential that is a function of a real triplet and a complex doublet is studied in Appendix A. We derive the physical mass spectrum and the mass matrix eigenfields in terms of the parameters appearing in the potential. It should be noted that this is the minimal Higgs scheme required in a vectorlike flavor theory based upon the gauge group SU(2)xU(1). We cannot replace the singlet mass term of eq.(II.13) by a Higgs field with a U(1) charge of the SU(2)xU(1) gauge group, because such a singlet would be electrically charged and cannot develop a vacuum expectation value. We can, however, enlarge the gauge group and treat the singlet term in eq.(II.13) as arising from a Higgson which breaks a part of the larger symmetry as well.

An interesting feature of the most general Higgs potential depending upon a real triplet and a complex scalar is that the conservation of electric charge is "natural". As discussed in Appendix A, we cannot find solutions for the Higgs vacuum expectation values such that:

$$\langle 0|\phi^a|0\rangle \neq \langle 0|\tilde{\phi}^\dagger|0\rangle\tau^a\langle 0|\tilde{\phi}|0\rangle. \qquad \text{(II.14)}$$

We therefore choose:

$$\langle 0|\phi^z|0\rangle = \rho \quad \langle 0|\tilde{\phi}^o|0\rangle = \eta \quad \text{and} \quad \tan\chi = \eta/\rho. \quad \text{(II.15)}$$

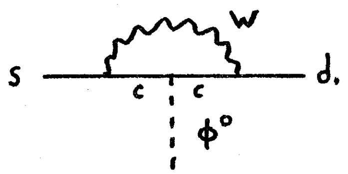
Since the z-component of the triplet is neutral, this corresponds to a conserved electric charge.

We may completely determine the Yukawa couplings in terms of the quark masses and G_F, as in the case with the Weinberg-Salam model. We find the following interaction Lagrangian for the charged Higgsons:

$$\begin{aligned}
\mathcal{L} = & \sqrt{\frac{G_F}{\sqrt{2}}}\,\frac{(-\tan\beta)}{4}\,\phi^+ \Big\{ m_u \bar{u} b_R \cos\chi + m_u \bar{u} h_R \sin\chi + \\
& m_c \bar{c} h_R \cos\chi - m_c \bar{c} b_R \sin\chi + m_u \bar{u} d_L \cos\theta_c + \\
& m_u \bar{u} s_L \sin\theta_c + m_c \bar{c} s_L \cos\theta_c - m_c \bar{c} d_L \sin\theta_c \Big\} \\
& \sqrt{\frac{G_F}{\sqrt{2}}}\,\frac{(\cot\beta)}{2}\,\phi^+ \Big\{ m_d \bar{u} d_R \cos\theta_c - m_d \bar{c} d_R \sin\theta_c + \\
& m_s \bar{u} s_R \sin\theta_c + m_s \bar{c} s_R \cos\theta_c - m_b \bar{c} b_L \sin\chi + \\
& m_b \bar{u} b_L \cos\chi + m_h \bar{c} h_L \cos\chi + m_h \bar{u} h_L \sin\chi \Big\} \\
& + \text{h.c.}.
\end{aligned} \tag{II.16}$$

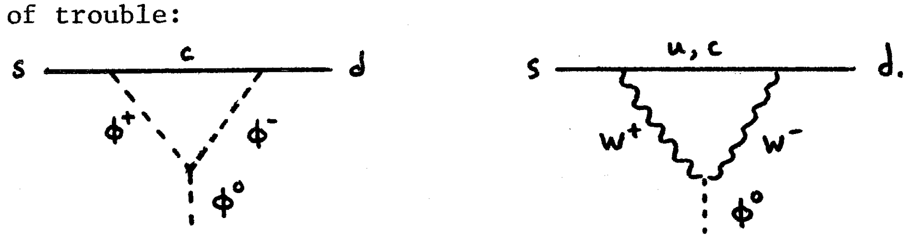
Though we are not explicitly considering the neutral Higgsons here, it deserves mention that when we depart from a model with the natural discrete symmetries forbidding the off-diagonal neutral vertices from occuring, such as the present vectorlike model, we run the risk of developing such terms in higher orders of perturbation theory. We wish to briefly comment upon this problem.

One might consider that the worst offending diagrams would be the W-boson radiative corrections to the neutral Higgson vertex:



Clearly, these diagrams always involve a chirality change if there are no right-handed currents and are proportional to $G_F^{3/2} m_c^2 m_s$, which is too small to be important in $|\Delta S| = 2$ processes that lead to $K_L K_S$ mass differences.

If the theory involves a right-handed current, e.g. $\bar{s}\gamma_\mu c_R$, then this will contain a logarithmic divergence and would appear to be a problem. But, note that the physical particle mass also receives a new logarithmic divergence which, to second order in perturbation theory, is equal to the vertex correction (this is not a Ward identity, but rather just an obvious result). Hence, the mass renormalization will also renormalize the vertex and maintain the absence of off-diagonals. Diagrams such as these are a potential source of trouble:



The first is automatically small $(G_F^{3/2} m_q^3)$ and the second is small by the GIM cancellation [9] between u and c quarks! Therefore, induced off-diagonal effects in Higgson exchange (of neutrals) is not a real problem for mass scales greater than 1 Gev.

II.3 NONLEPTONIC HIGGSON EXCHANGE HAMILTONIAN

In the Higgsified Weinberg-Salam model and the simple vectorlike model considered above we may summarize the largest effects due to the direct exchange of charged Higgsons. The result is an effective nonleptonic weak Hamiltonian which, aside from very few parameters, appears to be almost universal. We will ignore in this discussion the heaviest quarks, b and h considered in the vectorlike scheme, and we will also assume $m_u = m_d = m_s \sin^2\theta_c = 0$.

We then obtain the following effective Hamiltonian:

$$H = \frac{G_F}{\sqrt{2}} \frac{1}{m_H^2} \Big[(A^2 m_s^2 \cos^2\theta_c)\, \bar{s}_R c_L \bar{c}_L s_R \; + $$

$$(B^2 m_c^2) \Big\{ \bar{s}_L c_R \bar{c}_R s_L \cos^2\theta_c + \left(\bar{s}_L c_R \bar{c}_R d_L + \text{h.c.} \right) \frac{\sin 2\theta}{2}$$

$$\sin^2\theta_c\, \bar{d}_L c_R \bar{c}_R d_L \Big\} + \left(AB\, m_c m_s \frac{\sin 2\theta}{2}\right) \left\{ \bar{s}_R c_L \bar{c}_R d_L + \text{h.c.} \right\} +$$

$$(AB\, m_s m_c \cos^2\theta_c) \left\{ \bar{s}_R c_L \bar{c}_R s_L + \text{h.c.} \right\} \Big]. \qquad \text{(II.17)}$$

Several interesting features regarding the above expression require comment. First, we see from eqs. (II.9) and (II.10) that the parameters A and B take on the values:

Weinberg-Salam -

$$\begin{cases} A = (\cot\chi)/2 \\ B = (\tan\chi)/2 \end{cases}$$

Vectorlike -

$$\begin{cases} A = (\cot\beta)/2 \\ B = (-\tan\beta)/4. \end{cases} \qquad \text{(II.18)}$$

Apart from these variations in parameters, however, this structure would seem to be common to all models with the usual light quark pattern and weak currents.

Secondly, we note that, depending upon the particular values of the parameters A and B, certain terms in this Hamiltonian can, in principle, be quite large. For example, if $B^2(m_c^2/m_H^2) \gg 0$, then the term $\bar{s}_L c_R c_R d_L$ would be enhanced and could contribute to the $|\Delta I| = 1/2$; $|\Delta S| = 1$ decays. Likewise, for small B and large† A, we could obtain $AB(m_c m_s/m_H^2) \gg 0$ and the term of the form $\bar{s}_R c_L \bar{c}_R d_L$ becomes important in contributing to the same processes.

Note the interesting fact that no term in this expression contains the d_R quark. In fact, if we had written down the full Hamiltonian with $m_d \neq 0$, we would find terms that are $|\Delta I| = 1/2$ and $|\Delta S| = 1$ occuring with a strength of m_d/m_s compared to the terms with purely d_L. It is important that this be so as Golowich and Holstein have pointed out that the effective weak Hamiltonian must involve purely d_L in order to obtain the results of current algebra for the kaon and hyperon decays [25].

One might argue that operators of the form $\bar{s}c\bar{c}d$ cannot contribute substantially to the weak decays of hyperons and kaons due to the suppression of charmed quarks inside of the low-lying states. In fact, this is not the case as has been argued by Witten, Wilczek, Zee and others [29]. These operators have matrix elements between low-lying states such as $s \rightarrow d$ + (gluon) that arise by Higgson radiative corrections to gluon vertices with intermediate charmed quarks. The appearance of additional operators involving

† This does not occur for our models since $AB < 1$, but it may occur in certain "unnatural" models.

gluon fields is discussed in the next section and is based upon the suggestion originally due to Fritzsch and Minkowski [24] that effective anomalous gluo-magnetic moments may occur in the effective Hamiltonian for the nonleptonic weak interactions.

Finally, we wish to observe that the Hamiltonian of eq. (II.17) is derived in the tree approximation by considering the effects of single Higgson exchange. In section II.5 we consider the potentially harmful $|\Delta S| = 2$ processes occuring by way of W-boson and Higgson exchange as well as double Higgs exchange. Furthermore, in Chapter III we must deal with the technical problem of evaluating the QCD corrections to eq.(II.17) in the one-loop approximation making use of the renormalization group and short distance expansion techniques.

II.4 INDUCED GLUONIC PROCESSES

In addition to the Hamiltonian involving only quark fields of eq.(II.17) we must consider processes that are induced by radiative Higgson corrections of the form:

$$q_1 \to q_2 \; ; \quad q_1 \to q_2 + \text{gluon} \; ; \quad q_1 \to q_2 + 2 \text{ gluons}. \qquad \text{(II.19)}$$

These processes are represented by the Feynman diagrams appearing in Fig II.1 and can be described by the gauge invariant dimension five and six operators [29]:

$$O_1 = \bar{s}_R D_\mu D^\mu d_L \; ; \quad O_2 = \bar{s}_R \sigma_{\mu\nu} G^{\mu\nu A} \frac{\chi^A}{2} d_L \; ; \quad O_3 = \bar{s}_R \not{D}\not{D} d_L \qquad \text{(II.20)}$$

where D_μ is the covariant derivative, $D_\mu = \partial_\mu - igA^A\chi^A/2$. We have specifically written down the $|\Delta I| = 1/2$, $|\Delta S| = 2$ operators for the sake of example. In the Weinberg-Salam model with only four quarks these will be the most important induced gluonic operators. In the vectorlike theories we have the possibility of processes such as $c \to u$ + gluon, etc. as may be seen by studying the Higgson coupling Hamiltonian of eq.(II.16).

Operators of the form $\bar{s}_R d_L$, for example, may be ignored since they may be absorbed by a redefinition of the fields and a corresponding mass renormalization. Furthermore, processes such as $q_1 \to q_2$ + N gluons, where N > 2, are problematic and are generally ignored on the grounds that they involve higher powers of the gluon coupling.†

† There is a common fallacy in the literature [39] that these higher dimensional operators are suppressed by extra factors of $(m_H)^{-1}$, when in fact, they involve usually factors of $(m_c)^{-1}$. We have verified that these contributions are unimportant in our case, generally of order m_o/m_c for the largest effects (m_o is of order 500 Gev).

The operators of eq.(II.20) are not independent and are related by the algebraic relationship:

$$O_3 = O_1 - (\tfrac{1}{2})O_2. \qquad \text{(II.21)}$$

We have evaluated the diagrams of Fig. II.1 in the limit of vanishing external quark and gluon momenta (external momenta are taken small compared to m_c). The diagrams are all proportional to m_c. We obtain:

$$\left.\begin{aligned}
(1) &= i f_1 f_2 \frac{G_F}{\sqrt{2}} m_c p^2 \frac{\bar{s}_R d_L}{16\pi^2 m_H{}^2}[\delta] \\
(2) &= i f_1 f_2 \frac{G_F}{\sqrt{2}} g m_c q^\mu \varepsilon^\nu \frac{\bar{s}_R \sigma_{\mu\nu} \chi^A d_L}{16\pi^2 m_H{}^2}[\delta] \\
(3) &= i f_1 f_2 \frac{G_F}{\sqrt{2}} g^2 m_c \varepsilon\cdot\varepsilon \frac{\bar{s}_R d_L}{16\pi^2 m_H{}^2}\{\tfrac{\chi^A}{2}, \tfrac{\chi^B}{2}\}[\delta]
\end{aligned}\right\} \qquad \text{(II.22)}$$

where:

$$[\delta] = -(1-\alpha)^{-1} - (1-\alpha)^{-2}\ln\alpha \quad ; \alpha = m_c{}^2/m_H{}^2. \qquad \text{(II.23)}$$

Here f_1 is the $\bar{s}_R c_L$ Higgson fermion coupling constant; f_2 is the $\bar{c}_R d_L$ coupling. We have, referring to eqs.(II.18,II.16 & II.9):

Weinberg-Salam - Vectorlike-

$$\left.\begin{aligned}
f_1 &= \frac{\cot\chi}{2} m_s \cos\theta_c & f_1 &= \frac{-\tan\beta}{4} m_s \cos\theta_c \\
f_2 &= -\frac{\tan\chi}{2} m_c \sin\theta_c & f_2 &= \frac{-\cot\beta}{2} m_c \sin\theta_c.
\end{aligned}\right\} \qquad \text{(II.24)}$$

Hence, in general, $f_1 = Am_s\cos\theta_c$, and $f_2 = -Bm_c\sin\theta_c$. In the specific models considered, we always have $|AB| \leq 1$, and hence we

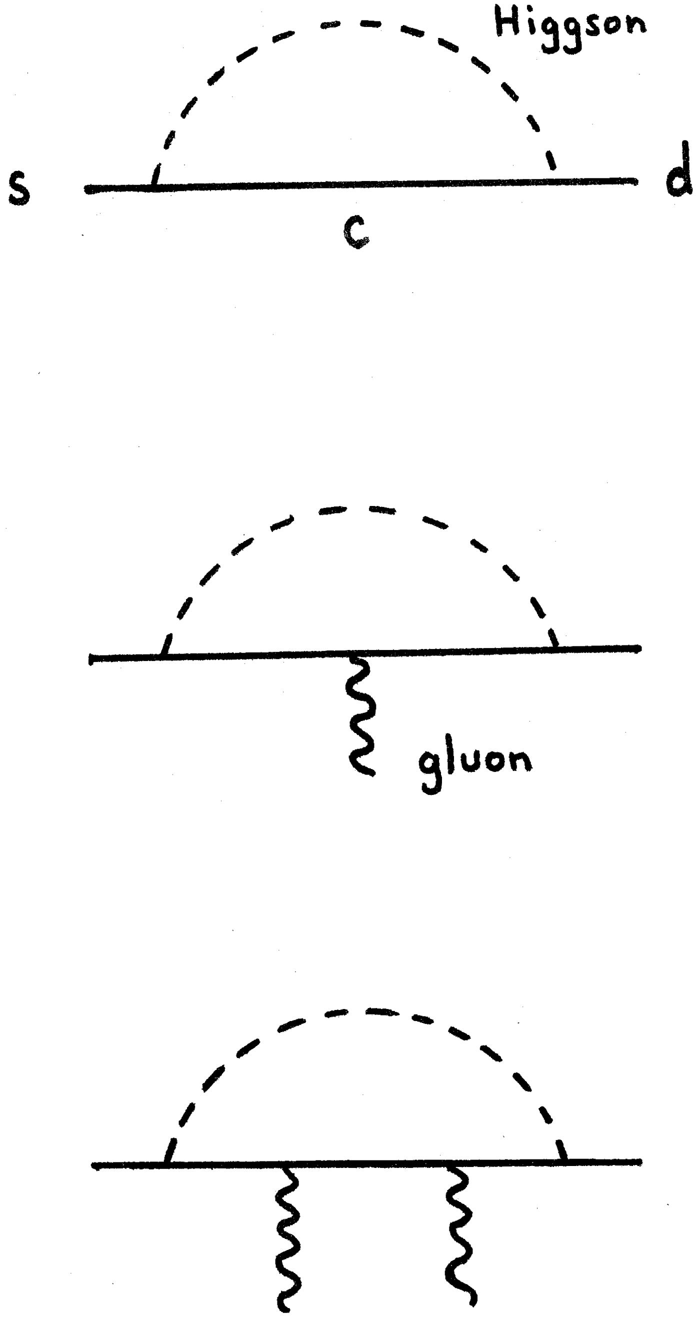


Fig. II.1

see that these operators do not contribute very strongly in the Hamiltonian. More general models would allow AB to be arbitrary (such as the parameter C in eq.II.17) and these operators could appear with arbitrary strength.

It has been emphasized by Witten, Wilczek and Zee [29] that the operators of II.20 need not appear in the Hamiltonian to have effects that are essentially of the same form. Witten has shown that the operator $\bar{s}_R c_L \bar{c}_R d_L$ has matrix elements between single quark and single quark, single gluon states that are proportional to m_c and which are essentially of the form of the matrix elements directly associated with the operators O_1, O_2 and O_3. In fact, it is shown by Wilczek and Zee that the operator O_2, suggested by Fritzsch and Minkowski [24], does not even occur in short distance operator products of currents, but that the matrix elements of the current-current operator $\bar{s}_R \gamma_\mu c_R \bar{c}_L \gamma^\mu d_L$ between s and d + gluon states mimic the effect of O_2 between on shell quark states.

We shall see that the operators, O_i, are suppressed further by the effects of gluon radiative corrections.

II.5 K_LK_S MASS DIFFERENCE

A classic problem of weak interaction physics comes into play if we attempt to enhance the coupling strength of charged Higgsons to operators such as $\bar{s}_Rc_L$, and $\bar{c}_Rd_L$. The effects of double Higgson exchange and Higgson-W boson exchange threaten to give, to second order in the weak coupling constant, too large a mass difference to the K_L and K_S CP eigenstates of the $K^0,\bar{K}^0$ complex. We may use the constraint that the new Higgson exchange diagrams of Fig.II.2b&c be of the same order of magnitude as the usual W-boson diagrams of Fig.II.2a to place constraints on the parameters A, B and C, of the effective Hamiltonian eq.(II.17).

The Higgson box diagrams are automatically second order weak because of the four vertices, each carrying a factor of $\sqrt{G_F}$. Higgson W-boson box diagrams, as in Fig.II.2b are second order weak because the two Higgson vertices supply a factor of G_F, and the loop integral is of order M_W^{-2}. Recall that the Glashow, Iliopoulos,Maiani [9] mechanism is responsible for the double W-boson diagrams being second order weak as opposed to the superficially, $G_F\alpha$ one would obtain without the u-c quark cancellation. Of course, there can be no GIM cancellation in the Higgson diagrams because the vertices are proportional to m_c (or m_u in the case of u-quark intermediate states).

We will use only the dominant terms and the lightest four quarks in evaluating the box diagrams (heavy quarks such as b,h in the vectorlike model can contribute to the D_LD_S mass difference, which is not sufficiently well established to place limits on second order weak processes, but which would appear to be light [28]).

We have carefully evaluated the diagrams of Figs.(II.2a, b, & c) where it is extremely important to keep track of signs as well as magnitudes. Since we are interested in putting constraints on large values of B (see II.18), it is possible to ignore the Higgson-W boson exchange processes which are proportional to AB in our standard models and we see from eq.(II.18) that AB $\ll$ 1. Hence, we need consider only the Higgson-Higgson and W-W box diagrams of Figs(II.2a,c). As is essential for the success of the model we find that the Higgson-Higgson diagrams have the same sign as the W-W diagrams!

The resulting effective Hamiltonian describing the $K_L K_S$ transition is:

$$H_{eff} = -\left(1 + \frac{m_c^2}{4m_H^2} B^4\right) \frac{G_F^2 m_c^2}{16\pi^2} \sin^2\Theta_c \cos^2\Theta_c \; \bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma^\mu d_L. \quad \text{(II.25)}$$

We estimate the $\langle\bar{K}^o|$, $|K^o\rangle$ matrix elements of the above Hamiltonian in the usual way:

$$\begin{aligned} \langle\bar{K}^o|\bar{s}_L\gamma_\mu d_L \bar{s}_L \gamma^\mu d_L|K^o\rangle &= \left(\frac{1}{2}\right)^2 \langle\bar{K}^o|\bar{s}\gamma_5\gamma_\mu d|0\rangle\langle 0|\bar{s}\gamma_5\gamma^\mu d|K^o\rangle \\ &= \frac{1}{4} f_K^2 \, m_K^2. \qquad \text{(II.26)} \end{aligned}$$

Following Gaillard and Lee [36], we find that:

$$m_{K_L} - m_{K_S} = -\frac{1}{m_K} \langle\bar{K}^o|H_{eff}|K^o\rangle. \qquad \text{(II.27)}$$

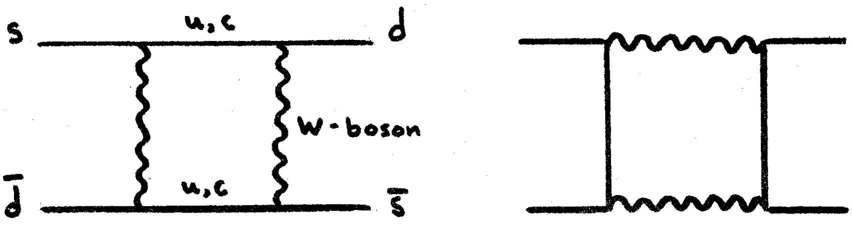


Fig. II.2a

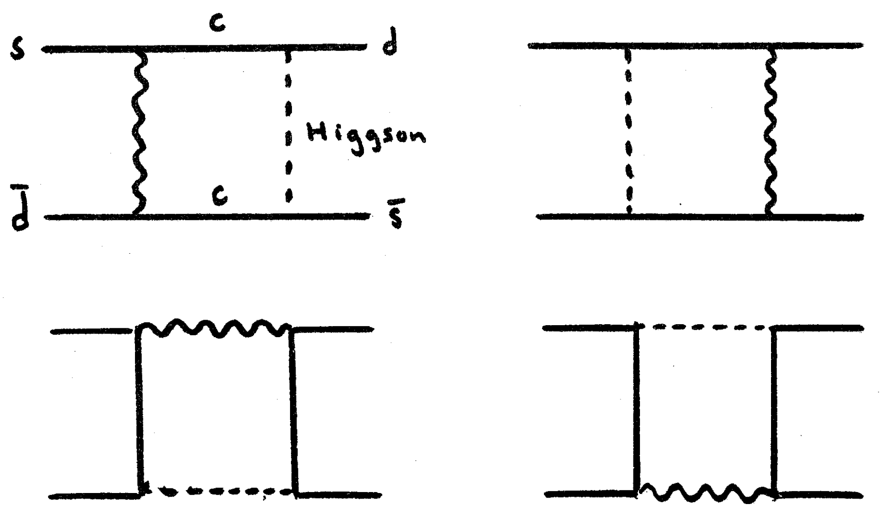


Fig. II.2b

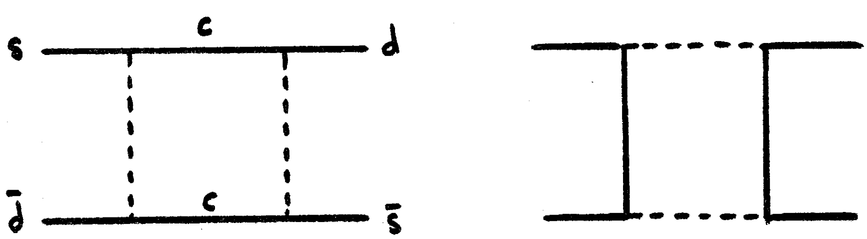


Fig. II.2c

Combining (II.26) and (II.27) yields:

$$\frac{m_L - m_S}{m_K} = \left(1 + \frac{1}{4}\frac{m_c^2}{m_H^2} B^4\right)\left\{\frac{f_K^2 G_F^2 \, m_c^2 \sin^2\Theta_c \cos^2\Theta_c}{16\pi^2}\right\}$$

$$= 2 \times 10^{-16}\left(1 + \frac{1}{4}\frac{m_c^2}{m_H^2} B^4\right). \qquad \text{(II.28)}$$

Experimentally, the left hand side of (II.28) is equal to $.7 \times 10^{-14}$ and we find, therefore, that we can tolerate:

$$B^2 = \left(\frac{\tan^2\chi}{4}\right)_{WS} \approx 10\left(\frac{m_H}{m_c}\right). \qquad \text{(II.29)}$$

Actually, we can tolerate even a larger value of B^2 than this since the QCD effects upon this effective Hamiltonian are to suppress the Higgs contribution substantially (similarly, we may well have underestimated the W-W contribution).

Our estimates have been performed in the Feynman-t'Hooft gauge. We have actually normalized our results by the Lee-Gaillard calculation for simplicity. It should also be mentioned that the operators of the form $\bar{s}d\bar{s}d$ (as opposed to $\bar{s}\gamma_\mu d\bar{s}\gamma^\mu d$) do not lead to enhanced matrix elements as in our estimates of section IV.2. These operators arise in the Higgson-W boson graphs of Fig.II.2b.

Chapter III

III.1 ANOMALOUS DIMENSION EVALUATIONS

Higgson exchange processes involve the short distance product of scalar and pseudoscalar densities. By "short" distance we are always referring to lengths that are small compared to the length scale defined by a renormalization group invariant mass, m_o, typifying the quark momentum in a hadron. Hence, our unit of length is effectively, m_o^{-1}.

This mass scale cannot be given precisely since short distance effects depend, generally, only logarithmically upon it. It should be regarded as a simple parameter in the theory with which we hope to describe the matrix elements of operators. It is generally taken to be of order:

$$.3 \text{ Gev} < m_o < 1 \text{ Gev}. \qquad \text{(III.1)}$$

With Higgson masses of order five to ten Gev we are justified in considering short distance expansions.

The short distance effects are calculable due to the remarkable property of the SU(3) color gauge theory of strong interactions being asymptotically free [1]. In addition to the radiative gluon corrections of operators, such as appear in the Higgson exchange Hamiltonian of eq.(II.17), there are also Higgson radiative corrections to gluonic vertices, as discussed in section II.4, which must be included in the analysis of short distance products.

Higgson radiative corrections lead to two different effects.

First, in the operator products of the form $(\bar{q}_R q_L)(\bar{q}_R q_L)$, there will appear the gluon "anomalous magnetic moment" term, or operator O_2 of eq.(II.20). Of course, we will also find the operators O_1 and O_3 since none of the diagrams of eq.(II.22) vanished. This is in contrast to the case in vectorlike theories where one encounters operator products of the form $(\bar{q}_R \gamma_\mu q_R)(\bar{q}_L \gamma^\mu q_L)$ and the gluon "anomalous magnetic moment", O_2, does not occur. We carry out the analysis of the short distance products including these operators in analogy to ref.[20]. We find a substantial suppression of the operators O_i.

Secondly, there will occur "charge radius" contributions in the operator products of the form $(\bar{q}_L q_R)(\bar{q}_L q_R)$, which are operators such as $\bar{q}\gamma_\mu \chi^A q \partial_\nu G^{\mu\nu A}$. We will use the equations of motion to relate these to the four quark operators $\bar{q}\gamma_\mu \chi^A q \bar{q}\gamma^\mu \chi^A q$. The appearance of these operators leads to substantial complications in the computation of the anomalous dimensions. This sort of complexity does not occur in the current-current products in the limit $m_c = m_u$ because of the GIM mechanism [9], but interestingly enough, the $m_c \neq m_u$ corrections to the usual current-current analysis may be important [23].

The methodology is, by now, well known for treating the strong interaction corrections to short distance operator products [20]. To the best of our knowledge, this is the first treatment of scalar exchange processes. For that reason, we make every effort to keep the results model independent so that they may be taken over to any fundamental scalar mediated interaction.

In Appendix B we present a systematic review of the methods that directly apply to our analysis as well as several example calculations. We shall proceed in this chapter to consider the two distinct classes of operator products. We shall refer to these as follows:

$$\text{(class I)} \quad (\bar{q}_L q_R)(\bar{q}_L q_R) \qquad\qquad \text{(class II)} \quad (\bar{q}_R q_L)(\bar{q}_L q_R) \qquad \text{(III.2)}$$

Note that the orders of the chirality indices of the form RLRL and LRRL are easily obtained by hermitian conjugation of these two classes. The quarks, q, may be of any flavor and we will only be considering the products of color singlet bilinears.

Class I is essentially trivial, though the appearance of the gluonic operators of section II.4 must be dealt with. The analysis and computation of the anomalous dimensions of the operators O_i of II.4 is distinct from the analysis of the dimension 6 four quark operators of class I and has been carried out for the vector-like theories with right-handed weak currents by Wilczek and Zee [29]. We will be able to borrow their results presently.

The operators of class II are considerably more complicated. In computing the anomalous dimensions we have found a trick that enables us to simplify the problem. We resolve any four quark operator with the class II chiral structure, e.g. $\bar{s}_L c_R \bar{c}_R d_L$, into a sum of four quark operators that transform as irreducible flavor representations. For example, for SU(4) we may write $\bar{s}c\bar{c}d$ as a singlet plus an adjoint ($\underline{15}$) plus an operator that is a mixture of higher representations ($\underline{20} + \underline{84}$). We then compute separately the contributions to the anomalous dimensions from each of these terms. The essential simplification that we discover is that the mixed representation operator is quite trivial to treat, and it will receive no net enhancement in QCD. The $\underline{1}$ and the $\underline{15}$ operators in SU(4) receive different amounts of enhancement in QCD, i.e. they have different anomalous dimensions.

The fact that the different irreducible representations are enhanced by different factors leads to an interesting consequence. An operator such as $\bar{s}c\bar{c}d$ of class II will be mixed with operators such as $\bar{s}u\bar{u}d + \bar{s}d\bar{d}d$, etc. Hence, although our Hamiltonian of eq. (II.17) involves heavy quarks in the tree approximation, the corrected Hamiltonian will contain terms that involve only light quarks. In Chapter IV we argue that these light quark operators have large matrix elements.

III.2 QCD ENHANCEMENTS AND MASSES

The different operators which we will be studying receive different enhancement factors from the effects of QCD. These factors are always of the form:

$$X^{k_i/2b_o} \quad \text{where} \quad X = \left(1 + \frac{\kappa_{\mathfrak{m}} b_o}{2\pi} \ln \frac{m_H}{\mathfrak{m}}\right) \qquad \text{(III.3)}$$

where $\mathfrak{m}$ is the operator renormalization mass and $\kappa(\mathfrak{m})$ is the corresponding value of $g^2/4\pi$ at $\mathfrak{m}$. b_o is the g^3 coefficient in $\beta(g)$ and is given by $b_o = (11 - \frac{2}{3} n_f)$ where n_f is the number of flavors. We discuss k_i below.

Ideally, we would like to choose $\mathfrak{m}$ equal to the value m_o of a typical quark momentum in a hadron, since then we would expect the matrix elements between hadrons to be of order unity. Unfortunately, with $\mathfrak{m} \sim m_o \sim 500$ Mev, the strong coupling constant $\kappa(\mathfrak{m})$ is becoming infinite. Therefore, we must compromise, for the sake of perturbation theory, and choose $\mathfrak{m}$ of order 1 to 2 Gev where $\kappa(\mathfrak{m})$ is between 1 and 1/3, but which is hopefully sufficiently close to m_o so that matrix elements can still be roughly estimated.

Note that we have been speaking of several mass scales here. There is a renormalization group invariant mass, μ, as defined in Table III.1. μ is the characteristic of the trajectory $\kappa(M)$ and represents the mass at which $\kappa(M)$ diverges in perturbation theory (.45 Gev is equivalent to a value of $\kappa = \frac{1}{2}$ at M = 2 Gev). Secondly, there is a mass m_o typifying the hadronic world which may be thought of as the characteristic quark momentum, or hadronic mass (.3 - 1 Gev). There is also the mass scale $\mathfrak{m}$ at which we choose to normalize the operators (the "compromise" mass $\approx$1 to 2 Gev). We also use a sliding scale mass M which can take any value. Quark masses are functions of M and any other renormalization group invariant (RGI) mass, and are therefore not physically measurable (see IV.2). Finally there are other RGI masses that are physically measurable, e.g. m_H, m_p, f_π, etc. These are summarized in Table III.1.

The quantity X is derived by solving renormalization group equations for the coefficient functions $C_i(x,\mathfrak{m})$ of a given operator O_i. The Hamiltonian density is constructed by integrating the operator product over x, multiplied by $D_H(x)$, the Higgson propagator. $D_H(x)$ is peaked strongly about $|x| < m_H^{-1}$, though it has a tail for larger $|x|$. Hence:

$$X^{k_i/2b_o} = \int d^4x\, D_H(x)\, C_i(x,\mathfrak{m}). \qquad \text{(III.4)}$$

The quantity k_i emerges as the coefficient of $g^2/16\pi^2$ in the anomalous dimension $\gamma_{O_1} + \gamma_{O_2} - \gamma_{O_i}$ (where, for example, $O_1 = \bar{s}c$, $k_1 = -8$; $O_2 = \bar{c}d$ $k_2 = -8$; and $O_i = \bar{s}c\bar{c}d$, k_i). The problem of computing k_i is the subject of the remainder of this chapter.

Some typical values of X are, for $\mathfrak{m} = 1$ Gev and $\kappa = 1$: X = 3.37 for $m_H = 6$ Gev, X = 4.05 for $m_H = 10$ Gev, and X = 4.59 for $m_H = 15$ Gev.

TABLE III.1

μ	RGI mass defined by: lim $M\exp-(\frac{2\pi}{b_o\kappa(M)})$ as $M \to \infty$	(450 Mev)
m_o	Typical quark momentum (RGI)	(.3 to 1 Gev)
$\mathfrak{m}$	RGI "compromise" operator norm. mass	(~1 to 2 Gev)
M	sliding scale mass (non-RGI)	-
$m_q(M)$	quark mass (non-RGI)	-
m_H,m_p	examples of other RGI masses (Higgson, proton; ~10, ≈1 Gev).	

III.3 OPERATORS OF THE FORM $\bar{q}_L q_R \bar{q}_L q_R$

III.3A We have computed the anomalous dimensions of the operators $A = \bar{q}_L q_R \bar{q}_L q_R$ ($A^\dagger = \bar{q}_R q_L \bar{q}_R q_L$), and $B = \bar{q}_L \sigma_{\mu\nu} q_R \bar{q}_L \sigma^{\mu\nu} q_R$ ($B^\dagger = \bar{q}_L \sigma_{\mu\nu} q_R \bar{q}_L \sigma^{\mu\nu} q_R$) as an exercise in Appendix B. Presently, the symbol q stands for a quark field of any flavor, e.g., included in this class of operators are the following; $\bar{s}_L c_R \bar{c}_L d_R$, $\bar{s}_R u_L \bar{c}_R d_L$, $\bar{s}_R s_L \bar{s}_R d_L$ etc. For example, included in this class of operators is $\bar{s}_R c_L \bar{c}_R d_L$ which occurs in the Higgs exchange Hamiltonian of eq.(II.17), and therefore this class is of direct physical interest.

The result of the evaluation of the diagrams in Fig.(III.1) is the anomalous dimension matrix:

$$\gamma \begin{pmatrix} A \\ B \end{pmatrix} = \frac{g^2}{16\pi^2} \begin{bmatrix} -16 & 0 \\ 1/3 & 16/3 \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix}. \tag{III.5}$$

The eigenvectors and eigenvalues of this matrix are:

$$\left.\begin{aligned} V_1 &= A + .015\,B && = \frac{g^2}{16\pi^2}(-16) \\ V_2 &= B && = \frac{g^2}{16\pi^2}(16/3) \end{aligned}\right\}. \tag{III.6}$$

These linear combinations of operators, V_1 and V_2, are multiplicatively renormalized to second order in QCD. In practice one could always assume that $V_1 = A$, since the effects of mixing are so small, but as a simple illustration of the methods to be employed here, consider a Hamiltonian that in the tree approximation involves only the operator A, e.g.,

$$H_{tree} = G_F \frac{m_c^2}{m_H^2} f \left[V_1 - .015\, V_2 \right]. \tag{III.7}$$

The renormalization group-scaling equation when applied to the short distance expansion used to construct this Hamiltonian instructs us to multiply each of the operators by a factor of $\{1 + \frac{\kappa b_o}{2\pi}\ln\lambda\}^{k_i/2b_o}$ where $\lambda = m_H/m$, i.e., we multiply by a factor of $(X)^{k_i/2b_o}$. The parameter k_i is given by inspection of eq.(III.5) to be:

$$k_i = k_1 + k_2 - k_{0_i}. \tag{III.8}$$

We have already computed the anomalous dimensions for the scalar and pseudoscalar operators in Appendix B and we obtain from eq.(III.18), $k_1 = k_2 = -8$. Likewise, from eq.(III.43) we see that $k_{V_1} = -16$, and $k_{V_2} = \frac{16}{3}$. Hence, our QCD corrected version of the example Hamiltonian of eq.(III.44) is:

$$\begin{aligned} H_{\text{corrected}} &= G_F \frac{m_c^2}{m_H^2} f \left[V_1 - .015\, V_2 (X)^{-.76} \right] \\ &\cong \quad '' \quad \left[V_1 - .032\, V_2 \right] \\ &\cong \quad '' \quad \left[\bar{q}_L q_R \bar{q}_L q_R - .019\, \bar{q}_L \sigma_{\mu\nu} q_R \bar{q}_L \sigma^{\mu\nu} q_R \right]. \end{aligned} \tag{III.9}$$

This is not a substantial modification of the original Hamiltonian that involved only the operator A. We see that the effects of QCD are to introduce a 2% mixture of the operator B. [In eq.(III.9) we have used the value X = 3.]

III.3B In section II.4 we found the contribution to the non-leptonic Hamiltonian from operators of the form $O_1 = \bar{q}_R D_\mu D^\mu q_L$, $O_2 = \bar{q}_R \sigma_{\mu\nu} G^{\mu\nu A} \chi^A q_L/2$, and $\bar{q}_R \not{D}\not{D} q_L = O_3$. These operators will arise in the operator product expansion of scalars and pseudoscalars and are associated with the operators we have just discussed by the similar chirality. We may work with O_1 and O_2, since O_3 is given by eq.(II.21) in terms of these.

Wilczek and Zee [29] have discussed the anomalous dimensions of these operators in the current-current operator product expansion. We may readily take their work over to our case. The anomalous dimension matrix is found to be:

$$\gamma \begin{pmatrix} O_1 \\ O_2 \end{pmatrix} = \frac{g^2}{16\pi^2} \begin{bmatrix} +71/24 & +7/16 \\ -23/4 & 161/24 \end{bmatrix} \begin{pmatrix} O_1 \\ O_2 \end{pmatrix} \tag{III.10}$$

and the multiplicatively renormalized operators are found to be given by:

$$\left.\begin{array}{lll} V_3 = O_1 - .500\, O_2 & \gamma(g) = \dfrac{35}{3}\dfrac{g^2}{16\pi^2} \\[2ex] V_4 = O_1 - .152\, O_2 & \gamma(g) = \dfrac{23}{3}\dfrac{g^2}{16\pi^2} \end{array}\right\}. \tag{III.11}$$

Therefore, in constructing the Hamiltonian, one should supply the operator V_3 as it appears in the tree approximation with an additional factor of $(X)^{-3.32} = .027^\dagger$ $(+3.32 = (\frac{35}{3} + 16)/(25/3))$. Likewise, V_4 will receive an extra factor of $(X)^{-2.84} = .043.^\dagger$

There will also be an extra suppression which we have not discussed due to the fact that these operators are proportional to

---

†We have taken X = 3 in these examples for definiteness.

m_c as they enter the Hamiltonian. Color invariant mass terms are seen to be suppressed, and treated as coupling constants, they tend to zero at short distances in analogy with the effective coupling constant. We need not include this effect here since the principal source of suppression is simply the one described above.

Hence, we may summarize: Gluonic operators are not important in Higgson exchange processes as they are severely suppressed by QCD.

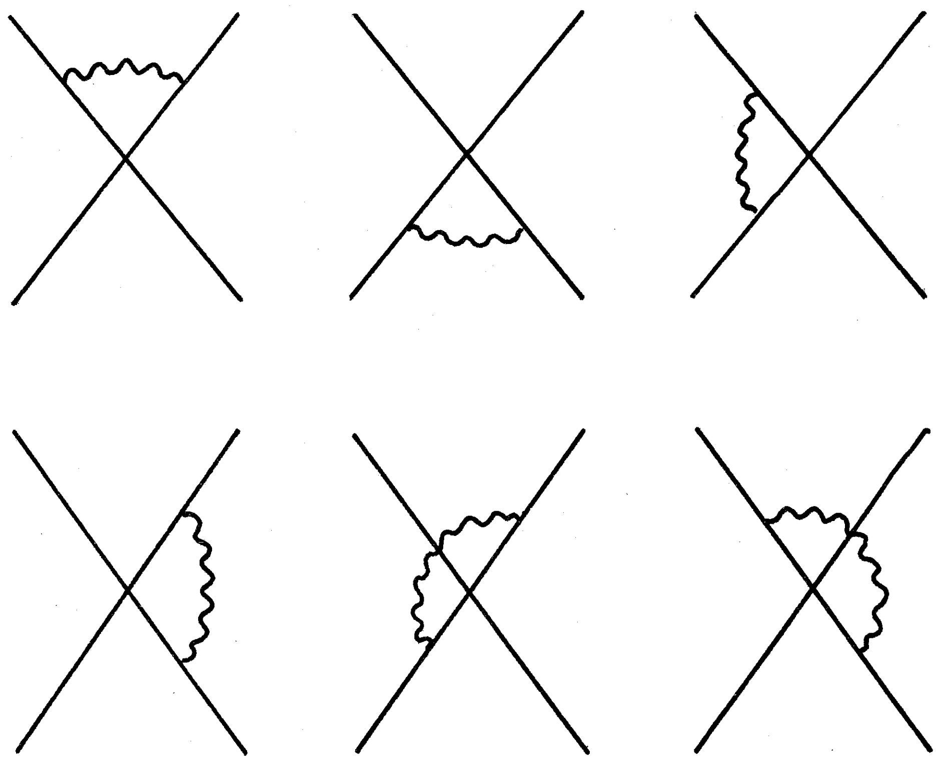


Fig. III.1

III.4 OPERATORS OF THE FORM $\bar{q}_L q_R \bar{q}_R q_L$

III.4A We now turn to the more difficult analysis of the short distance products of the form $(\bar{q}_L q_R)(\bar{q}_R q_L)$. This analysis is complicated by the appearance of the operators $\bar{q}_L \gamma_\mu \chi^A q_L \partial_\nu G^{\mu\nu A}$ and $\bar{q}_R \gamma_\mu \chi^A q_R \partial_\nu G^{\mu\nu A}$, which were not permitted previously by chirality. Actually, a more convenient expression of these operators is obtained by making use of the equations of motion to write:

$$\bar{q}\gamma_\mu \chi^A q \partial_\nu G^{\mu\nu A} = \frac{g}{2} \bar{q}\gamma_\mu \chi^A q \bar{q}\gamma^\mu \chi^A q. \qquad \text{(III.12)}$$

We prefer to use strictly the four quark operators and the above equation confirms that this is always possible.

The diagrams that lead to the appearance of this new operator are the quark-antiquark loops of Figs.(III.2 & III.3). We will make use of the Fierz identities to rewrite all four quark operators without the appearance of the color generators χ^A. The required formulae are stated in Appendix B.

Though we will make our analysis general enough to be taken over to any scalar-pseudoscalar operator product, it will be convenient to describe certain procedures in terms of SU(4), e.g., we will often think in terms of the four quark Weinberg model in describing the representation content of certain terms. When a result that is easily generalized to SU(N) is encountered, we will specify N instead of 4 in the formula.

Of course, the value chosen for N in any given application requires some care. We are considering the strong interaction corrections to short distance products with separations of the product operators of order $(m_H)^{-1}$. Therefore, in computing $\beta(g)$

and the anomalous dimensions, $\gamma(g)$, of the various operators, we must include the effects of only the quarks with masses lighter than m_H. Heavy quarks do not enter the renormalization group or scaling equations until the distances involved are small compared to the heavy particle's Compton wavelength. This is the content of the Appelquist-Carrazone theorem [38].

In practice we always treat m_c perturbatively. The leading effects of mass insertions are to introduce the operators such as O_1 and O_2 of section III.3B. Treating the charmed quark as a heavy quark in this analysis would, presumably, only slightly modify our results, but it drastically complicates the technical problems. The primary complication in treating m_c as large is to make the anomalous dimension matrices depend upon m, the sliding scale renormalization mass. This makes the relatively simple problem of diagonalizing the anomalous dimensions to find the multiplicatively renormalized operators impossible, since the multiplicatively renormalized combination now depends upon m, and we cannot write simple solutions to the renormalization group equations. We will assume in practice, then, that there are only four quarks whose masses are significantly less than the Higgson mass and that only these should be counted in the computation of the various renormalization group functions.

Examples of different flavor representations that will occur in our analysis are operators of the form:

$$\text{(I)}\ \bar{u}_L d_R \bar{s}_R c_L \quad \text{(II)}\ \bar{s}_L c_R \bar{c}_R d_L,\ \bar{u}_L c_R \bar{c}_R c_L \text{(III)}\ \bar{s}_R s_L \bar{s}_L s_R,\ \bar{s}_L c_R \bar{c}_R s_L.$$

We have listed these categories because each leads to a distinct mixing problem. Category (I), corresponding to an SU(4) 84 representation, is essentially trivial because only the diagrams of Fig.(III.1) occur, and there are no quark-antiquark loops. Category (II) will involve at least one fermion loop, and category (III) involves two such loops. These radiative corrections are depicted in Figs.(III.2 & III.3) respectively.

We can organize our treatment of the different categories of operators by noting that any four quark operator with arbitrary flavors, a,b,c, & d, can be written in the form:

$$\begin{aligned}\bar{q}^a q^b \bar{q}^c q^d &= \bar{q}^i q^j \bar{q}^k q^l \,(\alpha\delta^{jk} + \sum_A c_A \lambda^{kjA})(\beta\delta^{il} + \sum_B c'_B \lambda^{ilB}) \\ &= \Big(f_1\, \bar{q}^i q^j \bar{q}^j q^i + f_2 \sum_A c_A \bar{q}^i q^j \bar{q}^k q^i \lambda^{kjA} \\ &\quad + f_3 \sum_A c'_A \bar{q}^i q^j \bar{q}^j q^k \lambda^{ijA} + f_4 \sum_{AB} c_A c'_B \bar{q}^i q^j \bar{q}^k q^l \lambda^{Akj} \lambda^{Bil} \Big)\end{aligned} \tag{III.13}$$

where λ^{ijA} are the generators of the flavor group, SU(4) (SU(N)) and the c_A are coefficients which, for our purposes, will always lie in the root space of the group, i.e., only c_3, c_8, c_{15}, ... will be nonzero. This is the advantage of choosing the above linear combinations as a complete set of basis operators with respect to flavor, as opposed to operators such as $\bar{q}\lambda q\bar{q}\lambda q$, etc.

The first operator on the second line is a singlet with respect to SU(N). The mixing of the singlet flavor representations is the first non-trivial problem we will discuss. It will be necessary to choose a complete set of basis operators of the form $\bar{q}\Gamma q\bar{q}\Gamma q$ which also transform as singlets and which will mix only amongst themselves. This problem will present a 7 x 7 anomalous dimension matrix, which will be diagonalized with the assistance of a computer to find the multiplicatively renormalized operators.

The operators of the form $\bar{q}^i q^j \bar{q}^k q^i \lambda^{kjA}$ and $\bar{q}^i q^j \bar{q}^j q^k \lambda^{ikA}$ form two sets of operators which transform as <u>15</u> (the adjoint) representations under SU(4) (SU(N)). We will find by choosing the linear combinations:

$$\bar{q}^i q^j \bar{q}^k q^i \lambda^{kjA} \pm \bar{q}^i q^j q^j q^k \lambda^{ijA} \tag{III.14}$$

that our problem will be reduced to two inequivalent 5 x 5 anomalous dimension matrices which are also easily diagonalized with the aid of a computer.

The operators $\bar{q}^i q^j \bar{q}^k q^l \lambda^{ilA} \lambda^{kjB}$ would superficially appear to pose the most difficult mixing problem, since this operator is actually a mixture itself of the <u>15</u>, <u>20</u>, and <u>84</u> representations of SU(4) (or a similarly complicated mixture in the case of SU(N)). In fact, however, this turns out to be as trivial a mixing problem as the case studied in III.3A. This is due to the fact that the quark-antiquark loops would lead to a $\mathrm{tr}\lambda^A$ or a $\mathrm{tr}\lambda^B$, which is zero, and therefore we need only consider the diagrams of Fig.(III.1).

III.4B We define the following basis operators for the treatment of the SU(4) (SU(N)) singlet mixing problem. The indices a, b, etc. refer to flavors and repeated indices are, as usual, summed.

$$
\begin{array}{lll@{\qquad}lll}
A_1 & = & \bar{q}_L^a q_R^b \bar{q}_R^b q_L^a & A_1^C & = & \bar{q}_L^a \chi^A q_R^b \bar{q}_R^b \chi^A q_L^a \\
A_2 & = & \bar{q}_L^a \sigma_{\mu\nu} q_R^b \bar{q}_R^b \sigma^{\mu\nu} q_L^a & A_2^C & = & \bar{q}_L^a \sigma_{\mu\nu} \chi^A q_R^b \bar{q}_R^b \sigma^{\mu\nu} \chi^A q_L^a \\
A_3 & = & \bar{q}_L^a \gamma_\mu q_L^a \bar{q}_R^b \gamma^\mu q_R^b & A_3^C & = & \bar{q}_L^a \gamma_\mu \chi^A q_L^a \bar{q}_R^b \gamma^\mu \chi^A q_R^b \\
A_4 & = & \bar{q}_L^a \gamma_\mu q_L^a \bar{q}_L^b \gamma^\mu q_L^b & A_4^C & = & \bar{q}_L^a \gamma_\mu \chi^A q_L^a \bar{q}_L^b \gamma^\mu \chi^A q_L^b \\
A_5 & = & \bar{q}_R^a \gamma_\mu q_R^a \bar{q}_R^b \gamma^\mu q_R^b & A_5^C & = & \bar{q}_R^a \gamma_\mu \chi^A q_R^a \bar{q}_R^b \gamma^\mu \chi^A q_R^b \\
A_6 & = & \bar{q}_L^a \gamma_\mu q_L^b \bar{q}_L^b \gamma^\mu q_L^a & A_6^C & = & \bar{q}_L^a \gamma_\mu \chi^A q_L^b \bar{q}_L^b \gamma^\mu \chi^A q_L^a \\
A_7 & = & \bar{q}_R^a \gamma_\mu q_R^b \bar{q}_R^b \gamma^\mu q_R^a & A_7^C & = & \bar{q}_R^a \gamma_\mu \chi^A q_R^b \bar{q}_R^b \gamma^\mu \chi^A q_R^a
\end{array}
\qquad \text{(III.15)}
$$

We may use the Fierz transformations and the identities amongst the color matrices to rewrite the A_i^C in terms of the A_i. These identities are tabulated in Appendix B. We obtain:

$$
\left.
\begin{array}{lll@{\qquad}lll}
A_1^C & = & -\frac{2}{3} A_1 - A_3 & A_4^C & = & -\frac{2}{3} A_4 + 2A_6 \\[2ex]
A_2^C & = & -\frac{2}{3} A_2 & A_5^C & = & -\frac{2}{3} A_5 + 2A_7 \\[2ex]
A_3^C & = & -\frac{2}{3} A_3 - 4A_1 & A_6^C & = & -\frac{2}{3} A_6 + 2A_4 \\[2ex]
 & & A_7^C = -\frac{2}{3} A_7 + 2A_5 & & &
\end{array}
\right\} . \qquad \text{(III.16)}
$$

(It is useful to observe that the color octet current products may be expressed conveniently in terms of these operators as follows:

$$\left.\begin{aligned} E_1 &= j^A_{\mu L}\, j^A_{\mu L} = -\tfrac{2}{3} A_4 + 2A_6 \\ E_2 &= j^A_{\mu L}\, j^A_{\mu R} = -\tfrac{2}{3} A_3 - 4A_1 \\ E_3 &= j^A_{\mu R}\, j^A_{\mu R} = -\tfrac{2}{3} A_5 + 2A_7 \end{aligned}\right\} . \qquad \text{(III.17)}$$

where $j^A_{\mu L,R}$ are the left or right handed color octet vector currents ($j^A_\mu/2$ couples to the gluon field).

To compute the anomalous dimension matrix we may successively insert the operators, A_i, into the diagrams of Fig.III.2. We follow the standard procedure of extracting the coefficient of the $\frac{1}{\varepsilon}$ singularity. The result of this tedious but straightforward calculation is:†

$$\gamma(g) = \frac{g^2}{16\pi^2} \begin{bmatrix} -20 & 0 & 12 & 2 & 2 & 8 & 8 \\ \frac{1}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 & 0 \\ -\frac{2}{3} & 0 & 2 & \frac{1}{3} & \frac{1}{3} & \frac{4}{3} & \frac{4}{3} \\ -\frac{1}{3} & 0 & 0 & \frac{7}{3} & 0 & -\frac{14}{3} & 0 \\ -\frac{1}{3} & 0 & 0 & 0 & \frac{7}{3} & 0 & -\frac{14}{3} \\ 1 & 0 & 0 & -7 & 0 & -2 & 0 \\ 1 & 0 & 0 & 0 & -7 & 0 & -2 \end{bmatrix} \left\{\begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix}\right\} . \qquad \text{(III.18)}$$

It is possible to reduce the problem to a 5x5 anomalous dimension matrix and a 2x2 matrix by choosing linear combinations $A_6 \pm A_7$ and $A_4 \pm A_5$.

† These results are true for SU(4) in this and the other matrices described in this section.

We find that the multiplicatively renormalized operators are given by (we include the anomalous dimension eigenvalues):

$$\gamma_i(g) = \frac{g^2}{16\pi^2} k_i$$

$$\left.\begin{array}{lll}
k_1 = -20.400 & V_1 = & .997\, A_1 - .013\, A_2 + .036\, A_3 + .004\, \bar{A}_4 - .053\, \bar{A}_6 \\
k_2 = -\ 5.475 & V_2 = & .574\, A_1 - .018\, A_2 - .184\, A_3 + .364\, \bar{A}_4 + .568\, \bar{A}_6 \\
k_3 = \quad 1.625 & V_3 = & -.560\, A_1 + .050\, A_2 - 1.012\, A_3 - .093\, \bar{A}_4 + .025\, \bar{A}_6 \\
k_4 = \quad 5.333 & V_4 = & A_2 \\
k_5 = \quad 6.583 & V_5 = & -.267\, A_1 - .071\, A_2 - .152\, A_3 + .527\, \bar{A}_4 - .461\, \bar{A}_6
\end{array}\right\}$$

$$\left.\begin{array}{lll}
k_6 = -\ 5.946 & V_6 = & -.347\, \bar{A}_5 - .615\, \bar{A}_7 \\
k_7 = \quad 6.279 & V_7 = & .550\, \bar{A}_5 - .465\, \bar{A}_7
\end{array}\right\} \qquad \text{(III.19)}$$

where we employ the definitions:

$$\bar{A}_4 = A_4 + A_5;\ \bar{A}_5 = A_4 - A_5;\ \bar{A}_6 = A_6 + A_7;\ \bar{A}_7 = A_6 - A_7.$$

It should be noted that these are not normalized eigenvectors. We have tabulated the groups in ascending anomalous dimension which corresponds to descending contribution at short distances.

Our next task for the computation of the corrections to the Hamiltonian would be to rewrite the operators in terms of the V_i. However, we are only interested in the corrections to the operator A_1, which is the only object that contributes to the tree approximation to the Hamiltonian. Notice, however, that the operator V_1 is equal to the operator A_1 up to 5% accuracy. It is certainly sufficient for our purposes to choose $V_1 = A_1$ with the indicated eigenvalue -20.4. This is justified in part by the fact that the solution to the renormalization group equations requires we must compute the coefficient function for an effective coupling constant evaluated at $\sim$1 Gev, which to lowest order can be no more accurate than 5% anyway. We see that with the eigenvalue of -20.4, this operator will actually be very slightly enhanced in the short distance product by a factor of $(X)^{.24}$ where the exponent is $k/2b_o$ (k = 20.4 - 16). (See Table IV.1).

Fig. III.1 +

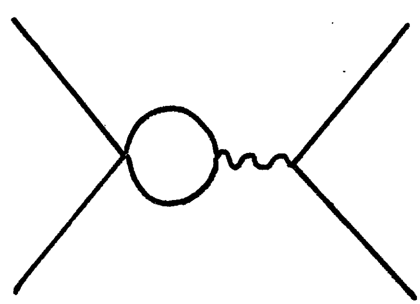


Fig. III.2

III.3C To treat the adjoint representation of SU(N), corresponding to the $\underline{15}$ of SU(4), we find that we must introduce the following basis operators in analogy with the preceding case:

$$\left.\begin{aligned}
B_1^{A\pm} &= \bar{q}_L^a q_R^b \bar{q}_R^c q_L^d (\delta^{ad}\lambda^{cbA} \pm \delta^{bc}\lambda^{adA}) \\
B_2^{A\pm} &= \bar{q}_L^a \sigma_{\mu\nu} q_R^b \bar{q}_R^c \sigma^{\mu\nu} q_L^d (\delta^{ad}\lambda^{cbA} \pm \delta^{bc}\lambda^{adA}) \\
B_3^{A\pm} &= \bar{q}_L \gamma_\mu q_L \bar{q}_R \gamma_\mu \lambda^A q_R \pm \bar{q}_L \gamma_\mu \lambda^A q_L \bar{q}_R \gamma_\mu q_R \\
B_4^{A\pm} &= \bar{q}_L \gamma_\mu q_L \bar{q}_L \gamma_\mu \lambda^A q_L \pm \bar{q}_R \gamma_\mu q_R \bar{q}_R \gamma_\mu \lambda^A q_R \\
B_5^{A\pm} &= \bar{q}_L^a \gamma_\mu q_L^b \bar{q}_L^c \gamma_\mu q_L^d (\lambda^{cbA}\delta^{ad}) \pm \bar{q}_R^a \gamma_\mu q_R^b \bar{q}_R^c \gamma_\mu q_R^d (\lambda^{cbA}\delta^{ad})
\end{aligned}\right\} \quad \text{(III.20)}$$

where we have already chosen linear combinations that simplify the problem to two distinct 5x5 anomalous dimension matrices.

We find:

$$\begin{bmatrix} -18 & 0 & 12 & 2 & 4 \\ \frac{1}{3} & \frac{16}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 2 & \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & 0 & 0 & \frac{7}{3} & -\frac{16}{3} \\ 1 & 0 & 0 & -7 & 0 \end{bmatrix} \begin{Bmatrix} B_1^{A+} \\ B_2^{A+} \\ B_3^{A+} \\ B_4^{A+} \\ B_5^{A+} \end{Bmatrix} \qquad \begin{bmatrix} -18 & 0 & 12 & -2 & -4 \\ \frac{1}{3} & \frac{16}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 2 & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & 0 & 0 & \frac{7}{3} & -\frac{16}{3} \\ 1 & 0 & 0 & -7 & 0 \end{bmatrix} \begin{Bmatrix} B_1^{A-} \\ B_2^{A-} \\ B_3^{A-} \\ B_4^{A-} \\ B_5^{A-} \end{Bmatrix}. \quad \text{(III.21)}$$

Again, this has been computed in the usual way, but the diagrams of Fig.III.3 contain only one quark-antiquark loop as opposed to two in the preceding case.

The multiplicatively renormalized operators are:

$$
\begin{array}{lll}
k_1 = -17.993 & W_1^{\pm} = & .998B_1^{\pm} - .014B_2^{\pm} + .018B_3^{\pm} \pm .002B_4^{\pm} \mp .055B_5^{\pm} \\
k_2 = -\ 4.929 & W_2^{\pm} = & \mp .249B_1^{\pm} \pm .008B_2^{\pm} \pm .093B_3^{\pm} - .595B_4^{\pm} - .795B_5^{\pm} \\
k_3 = \quad 1.809 & W_3^{\pm} = & \pm .614B_1^{\pm} \mp .058B_2^{\pm} \pm 1.007B_3^{\pm} + .095B_4^{\pm} - .029B_5^{\pm} \\
k_4 = \quad 5.333 & W_4^{\pm} = & B_2^{\pm} \\
k_5 = \quad 7.446 & W_5^{\pm} = & .068B_1^{\pm} + .010B_2^{\pm} + .035B_3^{\pm} \mp .722B_4^{\pm} \pm .688B_5^{\pm}
\end{array}
\qquad \text{(III.22)}
$$

where we have suppressed the adjoint representation label$(^A)$.

As in the preceding example of the singlet operators it suffices to take $W_1 = B_1$ with the corresponding eigenvalue, -17.993. This leads to an enhancement of $(X)^{.12}$, or about 1.14, choosing $X = 3$, and about 1.21 with $X = 5$.

Fig. III.1 +

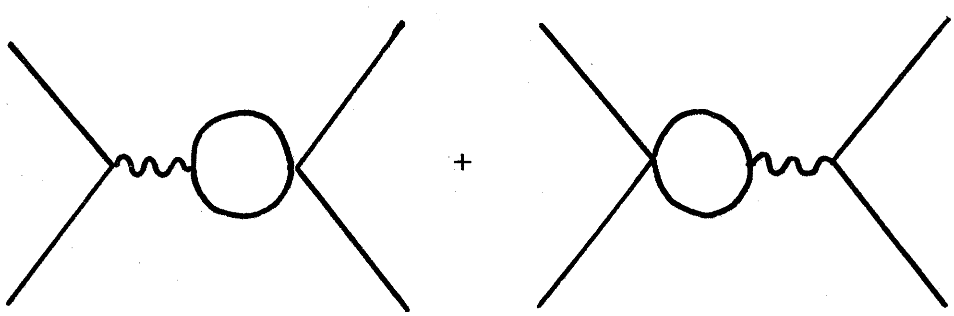


Fig. III.

III.4D The remaining operators are mixtures of $\underline{15}$, $\underline{20}$, and $\underline{84}$ for SU(4) and more general mixtures for SU(N). One might think it would be necessary to write out the specific representations and perform the same analysis as above. This would be a very tedious procedure, but it is not necessary and, in fact, this case is easy to treat.

Our basis operators are:

$$\left.\begin{aligned} D_1^{AB} &= \bar{q}_L^{a} q_R^{b} \bar{q}_R^{c} q_L^{d} \lambda^{cbA} \lambda^{adB} \\ D_2^{AB} &= \bar{q}_L^{a} \sigma_{\mu\nu} q_R^{b} \bar{q}_R^{c} \sigma^{\mu\nu} q_L^{d} \lambda^{cbA} \lambda^{adB} \end{aligned}\right\}. \tag{III.23}$$

It is easily seen that there are no quark-antiquark diagrams in the computation of this anomalous dimension matrix due to the tracelessness of the λ-matrices. Therefore, the relevant diagrams here are those of Fig.III.1. We readily obtain the anomalous dimension matrix as before:

$$\frac{g^2}{16\pi^2}\begin{bmatrix} -16 & 0 \\ \frac{1}{3} & \frac{16}{3} \end{bmatrix}\begin{Bmatrix} D_1^{AB} \\ D_2^{AB} \end{Bmatrix}. \tag{III.24}$$

The multiplicatively renormalized operators are therefore:

$$\left.\begin{aligned} k_1 &= -16 \qquad & U_1 &= D_1 - \frac{1}{64} D_2 \\ k_2 &= \frac{16}{3} \qquad & U_2 &= D_2 \end{aligned}\right\}. \tag{III.25}$$

To be consistent with the preceding analyses, we should approximate

$U_1 = D_1$, with the eigenvalue -16. Since this eigenvalue is just the opposite of the contribution of the scalar and pseudoscalar operators that make up the tree approximation, we see that this operator will not be enhanced at all in the short distance product, i.e. it receives a factor of $(X)^0 = 1$.

One interesting consequence of the fact that both the singlet and adjoint representation operators receive factors of about 1.3, whereas the mixed operator receives no enhancement, is that we would expect the flavors of the operators that appear in the tree approximation Hamiltonian to become mixed when we construct the corrected Hamiltonian. For example, we will see that the operator $\bar{s}c\bar{c}d$ will mix with the operator $\bar{s}u\bar{u}d + \bar{s}d\bar{d}d$.

Chapter IV

IV.1 THE QCD CORRECTED HAMILTONIAN

Previously we obtained a parameterization of the Hamiltonian due to Higgson exchange in two sample models, eq.(II.17). In the preceding chapter we have described the calculation of the corrections to eq.(II.17) due to the effects of the strong interactions in the SU(3) color picture. Now we wish to discuss the implications of this for the physical processes that are contained in the corrected version of the Hamiltonian.

The operators that occurred in the tree approximation were of the following form:

$$\left.\begin{array}{l} s_R c_L c_L s_R \\ s_L c_R c_R s_L \\ d_L c_R c_R d_L \\ s_R c_L c_L d_R \end{array}\right\} \text{(class I)} \qquad \left.\begin{array}{l} s_R c_L c_R d_L \\ s_R c_L c_R s_L \end{array}\right\} \text{(class II)} \tag{IV.1}$$

as well as the hermitian conjugates of these.

There were additional operators which we have ignored since they had coefficients that were of order m_u, m_d, or $m_s \sin\theta_c$. Of course, one might be concerned that these operators have larger matrix elements than the set we have included above, but the discussion to follow will alleviate this concern.

We found that the class II operators were not substantially enhanced by the effects of QCD in section III.3 and III.5. This then justifies simply ignoring these operators for the class of "typical" models which we have studied since an inspection of eq.(II.17) reveals that these operators occur with a coefficient of order $G_F AB m_c m_s/m_H^2$. (II.18) shows that $AB \lesssim 1$, and only models with unnatural conservation of charm and strangeness are expected to violate this constraint. Furthermore, there is no good reason to believe that these operators will have large matrix elements as, a) they involve charmed quarks and we expect that the charm quark content of low lying states is rather small, b) the induced $s \to d$ + gluon processes are all suppressed in QCD by small factors of order 10^{-1}. The primary source of suppression of these operators is the factor of $m_s/m_H \approx .015$, which would be hard to overcome in any model without violating the $K_L K_S$ mass difference. Henceforth we will only be concerned with the class I operators.

Naively, one might dismiss these operators on the grounds that the charmed quark content of the light mesons and baryons is small. However, as was noted in the previous analysis, the effects of QCD will be to introduce operators such as $\bar{s}_L u_R \bar{u}_R d_L +$ $\bar{s}_L d_R \bar{d}_R d_L$ by way of the charge radius diagrams involving quark-antiquark loops such as in Figs.(III.2 & 3). Technically, the way these new operators enter is by the fact that the singlet four quark and adjoint four quark operators receive different amounts of enhancement than the mixed operators. Therefore, the linear combinations of basis operators that initially represent the operator $\bar{s}_L c_R \bar{c}_R d_L$ is altered by the different enhancement factors of the different basis operators, and one ends up with new linear combinations, representing operators with terms like $\bar{s}u\bar{u}d + \bar{s}d\bar{d}d$. These new terms can have very big matrix elements, as we will see below.

Let us first see what we are dealing with by writing the class I operators in terms of the basis operators of Chapter III. We find by simple algebra (hereafter we suppress the chiral labels which are understood to be either LRRL or RLLR)

$$\bar{s}c\bar{c}s = \frac{1}{16}\left[A_1 - \sqrt{6}\,B_1^{15} - \frac{4}{\sqrt{3}}\,\bar{B}_2^{8} + 4\sqrt{2}\,D_1^{15,8} - 2\,D_1^{15,15}\right]$$

$$\bar{d}c\bar{c}d = \frac{1}{16}\left[A_1 - \sqrt{6}\,B_1^{15} - 2\,\bar{B}_2^{3} + \frac{2}{\sqrt{3}}\,\bar{B}_2^{8} + \frac{\sqrt{6}}{3}\,\bar{B}_2^{15} + 2\sqrt{6}\,D_1^{15,3} + 2\sqrt{6}\,D_1^{15,3} - 2\sqrt{2}\,D_1^{15,8} - 2\,D^{15,15}\right]$$

$$\bar{s}c\bar{c}d = \frac{1}{8}\left[\bar{B}_2^{4} - i\bar{B}_2^{5} - \sqrt{6}(D_1^{15,4} - iD_1^{15,5})\right]. \qquad \text{(IV.2)}$$

Of course, as we have seen, the last operator only occurs with d_L since the d_R is down by a factor of m_d/m_s in the Hamiltonian.

In eq.(IV.2) we have introduced the operators $\bar{B}_i^A$ and B_i^A which are defined in terms of $B_i^{A\pm}$ in section III.3C by:

$$B_i^A = \frac{1}{2}(B_i^{A+} + B_i^{A-})\,; \quad \bar{B}_i^A = \frac{1}{2}(B_i^{A+} - B_i^{A-}). \qquad \text{(IV.3)}$$

The different operators A and B_i^A are enhanced in QCD by different factors of the form X^p. These factors are reproduced in Table IV.1 for different values of X. Recall that the operators $D_i^{A,B}$ are not enhanced, i.e., p = 0.

The effect of this disparity in the enhancement factors of the two sets of basis operators is to bring into the nonleptonic Hamiltonian the following four operators:

$$\left.\begin{aligned} X_1 &= \frac{-1}{16}\left[\sqrt{6}\,B_1^{15} + \frac{4}{\sqrt{3}}\,\bar{B}_1^{8}\right] \\ X_2 &= \frac{-1}{16}\left[\sqrt{6}\,B_1^{15} + 2\,\bar{B}_1^{3} - \frac{2}{\sqrt{3}}\,\bar{B}_1^{8} - \frac{\sqrt{6}}{3}\,\bar{B}_1^{15}\right] \\ X_3 &= \frac{1}{8}\left(\bar{B}_1^{4} - i\bar{B}_1^{5}\right) \qquad X_4 = A_1 \end{aligned}\right\}^{\dagger} . \tag{IV.4}$$

X_1, and X_2 do not change strangeness and are mixtures of isospin 0,1 and 2. These operators may, in general, induce parity violation in the strong interactions and may be of interest for that reason. X_3 is isospin 1/2 and changes strangeness by one unit. These operators will occur with a numerical factor of $(X^P - 1)$ in the same location in the nonleptonic Hamiltonian as their respective parent operators, eq.(IV.1).

The new induced operator X_3 is easily seen to be of the form:

$$X_3 = \frac{1}{4}\left(\bar{s}_L u_R \bar{u}_R d_L + \bar{s}_L d_R \bar{d}_R d_L + \bar{s}_L s_R \bar{s}_R d_L + \bar{s}_L c_R \bar{c}_R d_L\right) \tag{IV.5}$$

where we have restored the chirality labels to emphasize that only d_L will occur in the Hamiltonian. Of course, the constraints imposed upon the structure of the nonleptonic Hamiltonian by current algebra, as emphasized recently by Golowich and Holstein [25], require that it transform as $I = \frac{1}{2}$ with respect to the left handed charges only, and one can readily verify that (IV.3) is $I_L = \frac{1}{2}$ and $I_R = 0$. We note the appearance of the $u\bar{u} + d\bar{d}$ structure.

The effective $\Delta I = \frac{1}{2}$ nonleptonic Hamiltonian becomes:

---

† Recall that the subscript (1) denotes the Dirac structure.

$$\frac{G_F}{\sqrt{2}} \sin\Theta_c \cos\Theta_c \; (\delta) \; \frac{B^2 m_c^2}{m_H^2} X_3 + \; \text{h.c.,} \qquad \text{(IV.6)}$$

where we take $\delta = X^p - 1$ and these values are seen in Table IV.1 for the various values of X (note that we will be talking exclusively of the operators B_i^A henceforth, and these have p = .26).

The operator X_2 also contributes to the effective Hamiltonian and is $\Delta I = 0$ and 1 and leads to parity violation in nuclear processes. The effective Hamiltonian is:

$$\frac{G_F}{\sqrt{2}} \sin^2\Theta_c (\delta) \; \frac{B^2 m_c^2}{m_H^2} X_2 \quad + \quad \text{h.c..} \qquad \text{(IV.7)}$$

We reemphasize that all of these operators have the chiral structure LRRL. The operator X_1 will also enter the Hamiltonian, but it leads to the term:

$$\frac{G_F}{\sqrt{2}} \cos^2\Theta_c (\delta) \; \frac{A^2 m_s^2}{m_H^2} X_1 \quad + \quad \text{h.c..} \qquad \text{(IV.8)}$$

We may write out the operators X_1 and X_2 as follows:

$$\left.\begin{aligned} 16X_1 &= -\sqrt{6}\; \bar{q}_i c \bar{c} q^i \; - \; \tfrac{4}{3}(\bar{u} q_i \bar{q}^i u + \bar{d} q_i \bar{q}^i d \; - \; 2\bar{s} q_i \bar{q}^i s) \\ 16X_2 &= -\sqrt{6}\; \bar{q}_i c \bar{c} q^i \; + \; 4(\bar{d} q_i \bar{q}^i d \; - \; A_1). \end{aligned}\right\} \qquad \text{(IV.9)}$$

If we ignore the heavy quarks, s and c, we find that we may write

$$\left.\begin{aligned} X_1 &= -\frac{1}{12}\,(\bar{u}u\bar{u}u + \bar{u}d\bar{d}u + \bar{d}u\bar{u}d + \bar{d}d\bar{d}d) \\ X_2 &= \;\;\frac{1}{4}\,(\bar{d}u\bar{u}d + \bar{d}d\bar{d}d). \end{aligned}\right\} \qquad \text{(IV.10)}$$

From the standpoint of the suppression factors of eq.(IV.8) and the factor of 1/12 in (IV.9a), we will henceforth ignore X_1 contributions.

TABLE IV.1

X =	2.18	2.79	3.37	4.05	5.00
A_i: $X^p = X^{.264}$ =	1.23	1.31	1.37	1.44	1.52
B_i^A: $X^p = X^{.119}$ =	1.09	1.13	1.15	1.18	1.21

p is derived from the anomalous dimension. The various factors of X that we have chosen are discussed in Section III.2.

$$\delta = X^p - 1$$

IV.2 ESTIMATING MATRIX ELEMENTS

In the preceding section we argued that operators involving only the light quarks, such as $\bar{s}u\bar{u}d$ + $\bar{s}dd\bar{d}$, can be introduced into the effective nonleptonic weak Hamiltonian by the effects of QCD. Depending upon the values of the mixing angle factor, B, as well as the QCD enhancement factors and the Higgson mass, these operators can occur with coefficients of order G_F (and usual Cabbibo angle factors). In the present section we wish to argue that the matrix elements of these light quark operators are actually large compared to the matrix elements of the corresponding light quark operators appearing in the current-current Hamiltonian.

First, it is necessary that we digress to consider the definitions of these operators and their relationship to the renormalization group. The operators occurring in the effective Hamiltonian are normalized at masses of order $m \sim 1$ Gev, i.e., for incoming quark lines carrying momenta of order unity (Gev) these operators will take on their free field theory values (see the discussion of Appendix B on the definition of composite operators). Strictly speaking, this is not a desirable feature as we would prefer to normalize the operators at a mass scale, m_o, which we believe to be a typical value of the quark momentum in a hadron, e.g. 300 Mev to 1 Gev (see III.1). With operators normalized according to the latter prescription we would expect that the matrix elements taken between low-lying mesons and baryons would be of order unity, whereas with the former mass scale, $m \sim 1$ Gev, it is not so clear as to how to compute the matrix elements. However, with the latter mass scale it is not possible to define the operator in perturbation theory since for these lower masses, the effective gluon coupling strength is very large, i.e. κ_M for M of order 500 Mev is effectively infinite (actually, all we really know is that here the perturbation theory does not work). This is effectively a "short blanket" situation; on the one hand with M large we cannot compute matrix elements and on the other, with M small, we cannot compute the operators. This problem arises in any analysis of the gluonic effects upon a flavor interaction and is generally skirted by simply adopting the larger normalization mass scale and assuming that the matrix elements can be estimated approximately. By choosing the larger value of the normalization point, i.e. $M = \mathcal{m}$ or 1 Gev, we will be in a domain where the effective coupling constant is of order 1 to 1/3.

All of the four quark operators that appear in our effective Hamiltonian (e.g. eq.(II.17)) are multiplied by the square of a quark mass. For example, in eq.(IV.6) we see that the light quark operators $\bar{s}u\bar{u}d + \bar{s}d\bar{d}d$ will occur with a coefficient of the form: $fG_F m_c^2$, where fG_F depends only upon physical parameters such as m_H, and mixing angles. Quark masses are not directly physically observable (they are probably not in any sense a physical pole in a propagator) and are functions of the renormalization sliding scale mass M. Since we have now agreed to normalize all quark involving operators at the mass scale of 1 Gev, we must be careful to insure that the values chosen for the quark masses are those appropriate to this normalization point. This poses the problem of determining the values of the quark masses as functions of the renormalization mass.

The ratios for asymptotically large values of the renormalization mass of the light quark masses can be determined by PCAC considerations in terms of physical mass ratios. The renormalization group equations satisfied by the effective masses are readily obtained in analogy to the equation for the effective coupling constant (see Appendix B). In the large M limit the PCAC evaluations are essentially unmodified by the gluon interactions since the effective coupling constant is vanishing, and the ratios obtained will remain relatively constant until M approaches 1 Gev. At this point the behavior cannot be determined since a) the effective coupling constant rapidly rises and b) the light quark approximation is no longer valid. For values of M larger than a Gev or so, we may write the quark masses as:

$$m_q(M) = \mathcal{M}_q/(\ln \frac{M}{\mu})^{4/b_o}. \qquad \text{(IV.11)}$$

The PCAC ratios are then ratios of the coefficients $\mathcal{M}_q$ and are given by:

$$\mathcal{M}_u : \mathcal{M}_d : \mathcal{M}_s = 5 : 10 : 150, \qquad \text{(IV.12)}$$

Prior to the development of the modern QCD picture of strong interactions, specific values of the quark masses were often quoted[40], e.g., one might argue that $m_d - m_u \cong m_n - m_p$ (proton-neutron mass difference). Combined with the ratios given from PCAC, we could state all three masses absolutely, but one recognizes that this is a renormalization group noninvariant prescription. Similarly, PCAC implies relations such as $m_\pi^2 \approx 2m_d m_o$, $m_K^2 \approx 2m_s m_o$ where m_o is again the typical hadronic mass scale of $\sim$1 Gev. It is clearly not possible in these statements to know, a priori, what the normalization mass M should be. Therefore, in the modern language of QCD it is meaningless to state an absolute value for a light quark mass, though for a large range of asymptotic values of $M \gg 1$ Gev, the absolute values of $m_u = 5$ Mev, $m_d = 10$ Mev, $m_s = 150$ Mev are approximately the same as the effective masses. These considerations are developed by Georgi and Politzer in ref.[53].

The story of the charmed quark mass is slightly different. Since the charmed quark mass is large and of order the region at which the effective coupling constant begins to shrink, one is inclined to believe that this mass might be more directly defined. Indeed, it is possible to use an alternative prescription to define this mass without resorting to PCAC, although a rough PCAC estimate seems to be valid. The alternative technique is to recognize that there exists a definite charm threshold in e^+e^- annihilation at about $\sim$4 Gev. We may then use this as a boundary condition on the sliding scale mass by defining: $m_c(M_{threshold}) = \frac{1}{2} M_{threshold}$, hence m_c 1.5 to 2 Gev. This may be regarded as an absolute value of the charm quark mass, though a formula such as eq.(IV.11) still describes the effective charm quark mass at any value of M.

PCAC implies a similar result as described above. In this case a typical charmed meson, such as the D meson, should have a mass given by $m_D^2 \approx 2m_c m_o$, which for $m_D \approx 1.8$ Gev, this agrees roughly with the alternative method of evaluation.

We may use the PCAC evaluation to fix the asymptotic ratios of the charmed quark mass to the other light quarks. For example, we would expect something of the order:

$$\frac{m_s}{m_c} \to \frac{m_K^2}{m_D^2} \quad \text{as} \quad M \to \infty . \tag{IV.13}$$

This method is used by Georgi and Politzer [53] who construct solutions to the renormalization group equations in which the charmed quark is not treated as a light quark. The resulting renormalization group equations are then numerically integrated for the values of M up to 1 Gev, at which point the masses tend to infinity while M tends to μ, the characteristic mass at which $\kappa(M)$ diverges.

The point we have been attempting to emphasize in this discussion is that amplitudes involving the quark masses must be dealt with most carefully. No physical amplitude can depend upon the normalization point, M. A corollary of this statement is that all physical amplitudes must depend only upon dimensionless ratios of quark masses for large values of the normalization point, M. These ratios are known by the PCAC results we have discussed above.

This leads to an important criticism of the work of Shifman, Vainshtein and Zakharov [23] in their approach to the nonleptonic weak Hamiltonian, as has been emphasized by G. Ross [37]. These authors obtain effective "enhancements" of $\Delta I = \frac{1}{2}$ processes of the form $m_K^2/m_d m_s$, and they assume that m_d = 10 Mev, m_s = 150 Mev. It is clear by the discussion we have presented above that this is not a valid representation of any physical amplitude.

In our work, as in any process that is induced by or involves Higgsons, we encounter operators of the form $m_q^2\bar{q}q\bar{q}q$, e.g., $m_c^2\bar{s}u\bar{u}d$. We may use the current divergence relations to rewrite these as:

$$m_c^2\ \bar{s}u\bar{u}d \ = \ \frac{m_c^2}{(m_s - m_u)(m_u - m_d)}(\partial^\mu \bar{s}\gamma_\mu u)(\partial^\nu \bar{u}\gamma_\nu d). \quad \text{(IV.14)}$$

Hence, our work will always involve dimensionless ratios of the quark masses. The operator involving the current divergences is similar to a product of two partially conserved currents, which is not renormalized and is therefore independent of M (this is not strictly true due to operator mixing and the subtleties of defining composite operators, but it is readily seen that the dependence upon M is very slight and reflects the perturbative nature of the calculation). The quark mass ratio is understood to be taken at the normalization point, m, for our operators. Actually, we may take the ratio at any point at which equation (IV.11) is valid, and thus the ratio is given in every case we shall consider by the PCAC arguments reviewed above.

These PCAC ratios are:

$$m_u : m_d : m_s : m_c : \ \cong \ 1 : 2 : 150 : 1500^\dagger \qquad \text{(IV.15)}$$

based upon the mass equations, e.g. $m_K^2 = 2m_s m_o$.

In our estimates of matrix elements we are going to compare the operators such as $\partial^\mu \bar{s}\gamma_\mu u \partial^\nu \bar{u}\gamma_\nu d$ occuring in the Higgson exchange Hamiltonian to the operators such as $\bar{s}\gamma_\mu u\bar{u}\gamma^\mu d$ occuring in the current-current Hamiltonian (we have suppressed the chirality labels L, R in this discussion). The matrix elements we will compare will be those

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†As usual, the n-p mass difference is used to fix m_u-m_d.

the light mesons and baryons, such as $\langle K|H^{p.c.}|\pi\rangle$, relevant to Kaon decay.

For the sake of comparison we employ two very crude approximations, a) the insertion of intermediate vacuum states b) the so-called factorization approximation (for baryon decay and parity violation). We certainly do not believe that these give very credible values for the matrix elements, but for the sake of comparison they may not be so bad.

<u>IV.2A $K \to 2\pi$ Decays.</u> It is, perhaps, the most striking aspect of the $\Delta I = \frac{1}{2}$ rule that the ratio of the decay rates for $K^o \to 2\pi$ to $K^+ \to 2\pi$ is approximately 625, i.e., the amplitude for the pure $\Delta I = \frac{1}{2}$ is enhanced relative to the $\Delta I = \frac{3}{2}$ by about a factor of 25.

The application of PCAC to these decays modes requires a symmetric treatment of the pions [42] and leads to the following result:

$$\begin{aligned}
A(K \to \pi^+\pi^o) &= \frac{1}{2f_\pi}\left[\frac{1}{\sqrt{2}}\, a_{K^+\pi^+} + a_{K^o\pi^o}\right] \\
A(K_S^o \to \pi^+\pi^-) &= \sqrt{2}\, A(K^o \to \pi^+\pi^-) = \frac{1}{\sqrt{2}f_\pi}\, a_{K^+\pi^+} \\
A(K_S^o \to \pi^o\pi^o) &= \sqrt{2}\, A(K^o \to \pi^o\pi^o) = -\frac{1}{\sqrt{2}f_\pi}\, a_{K^o\pi^o}
\end{aligned} \tag{IV.16}$$

where:

$$\begin{aligned}
a_{K^+\pi^+} &= (2\pi)^3\sqrt{2p_o k_o}\ \langle\pi^+|H^{p.c.}|K^+\rangle \\
a_{K^o\pi^o} &= (2\pi)^3\sqrt{2p_o k_o}\ \langle\pi^o|H^{p.c.}|K^o\rangle
\end{aligned} \tag{IV.17}$$

Since our Hamiltonian is pure $\Delta I = \frac{1}{2}$ we will simply set $A(K^+ \to \pi^+\pi^o) = 0$. We must then have:

$$a_{K^o\pi^o} = -\frac{1}{\sqrt{2}}\, a_{K^+\pi^+} \tag{IV.18}$$

Unfortunately, our estimate of the matrix elements will not respect this condition. The estimate that we will make will consist of inserting the vacuum state between the four quark operators as we did in the discussion of the $K_L K_S$ mass difference. This method cannot be regarded as very reliable, but it does seem to indicate in every example that we will consider that the four quark operators appearing in the Higgson exchange Hamiltonian are actually enhanced relative to the current-current operators appearing in the ordinary weak Hamiltonian. It should be noted that our procedure of inserting the vacuum intermediate state is roughly equivalent to using a "wave-function" model, such as the bag model [43] to perform the estimate.

We will presently be concerned with the $\bar{q}\gamma_5 q\bar{q}\gamma_5 q$ piece of the Hamiltonian which is all that contributes in the insertion of the intermediate vacuum states. Furthermore, we need consider only the $\bar{s}\gamma_5 u\bar{u}\gamma_5 d$ piece between K^+ and π^+ since isospin relates this to the K^o, π^o matrix element (they are simply of the same order of magnitude for either $I = \frac{1}{2}$, where the relationship is given in (IV.18), or for $I = \frac{3}{2}$).

We can write, with the help of the current divergence relationships:

$$m_c^2 \bar{s}\gamma_5 u\bar{u}\gamma_5 d = \frac{\partial^\mu \bar{s}\gamma_\mu\gamma_5 u\partial^\nu \bar{u}\gamma_\nu\gamma_5 d}{(m_s + m_d)(m_u + m_d)} \cdot m_c^2. \tag{IV.19}$$

We will compare the matrix elements of this operator between the K^+ and π^+ to the matrix elements of the operator $\bar{s}\gamma_\mu\gamma_5 u\bar{u}\gamma^\mu\gamma_5 d$ which occurs in the current-current Hamiltonian between the same states. Inserting vacuum states we find:

$$\begin{aligned} \langle K^+|\bar{s}\gamma_5 u\bar{u}\gamma_5 d|\pi^+\rangle &= F^{-1} q_K^\mu q_\pi^\nu \langle K^+|\bar{s}\gamma_\mu\gamma_5 u|0\rangle\langle 0|\bar{u}\gamma_\nu\gamma_5 d \quad + \\ &= F^{-1} f_K m_K^2 f_\pi m_\pi^2 \end{aligned} \tag{IV.20}$$

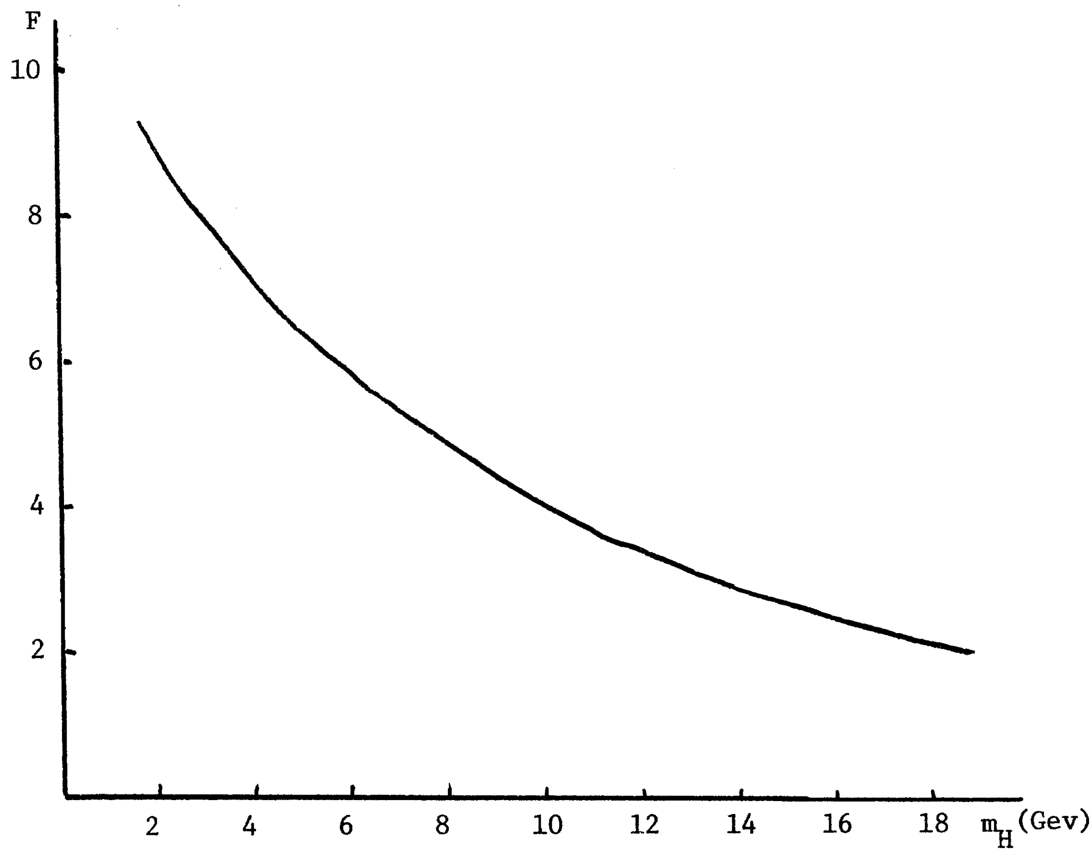
$$\langle K^+|\bar{s}\gamma_\mu\gamma_5 u\bar{u}\gamma^\mu\gamma_5 d|\pi^+\rangle = \langle K^+|\bar{s}\gamma_\mu\gamma_5 u|0\rangle\langle 0|\bar{u}\gamma^\mu\gamma_5 d|\pi^+\rangle$$

$$= \quad f_\pi f_K \; m_K^2/2 \qquad \text{(IV.21)}$$

where we have used $F = (m_s + m_d)(m_u + m_d)$ and the fact that $q_K \cdot q_\pi = m_K^2/2$ since the π is the decay product of the kaon decay, $K \to 2\pi$.

We may use these estimates and return to the full Higgson Hamiltonian, which leads to the renormalization group invariant expression for the matrix element involving the factor $m_c^2 F^{-1}$. We may then compare the Higgson amplitude for the K decay to the current-current amplitude. Actually, we may compare our $\Delta I = \frac{1}{2}$ amplitude to the $\Delta I = \frac{3}{2}$ current-current amplitude. We recall from the work of Lee, et. al.[20] that the $\Delta I = \frac{3}{2}$ amplitudes are actually suppressed by a factor of about 2/3, and there are also Clebsch-Gordon coefficients which cause roughly another factor of two suppression relative to the $\Delta I = \frac{1}{2}$ Higgson contribution (this is simply the fact that the ordinary current-current Hamiltonian is split equally between the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ components).

We may saturate the $K_L K_S$ mass difference bound, $B^2 = 10 m_H/m_c$ obtained in II.5, and use the renormalization group invariant values of the quark masses, as well as the appropriate choice of the strong coupling constant as a function of M to obtain a rough estimate of the Higgson contribution to kaon decay relative to the usual $I = \frac{3}{2}$ current-current contribution. The result is plotted in Fig. IV.1 as a function of the Higgson mass. With this figure we should offer the appropriate apology that it represents a crude estimate.



$$F = \frac{A(\Delta I = 1/2;\ \text{Higgson})}{A(\Delta I = 3/2;\ \text{Current})}$$

for Hyperon and Kaon decays (using Lee & Gaillard assumption of $\kappa_{1\ \text{Gev}} \approx 1$)

Fig.IV.1

In this discussion we have ignored the effects of operators such as $\bar{s}c\bar{c}d$ on the grounds that they will be suppressed due to a) the lack of a substantial number of $\bar{c}c$ pairs in the kaon or pion b) the QCD suppression of the operators describing $s \to d$ + gluon as discussed in II.4 and III.3. (except, of course,for the charge radius corrections which lead to the appearance of the light quark operators).

We have also ignored the effects of other intermediate states, such as two pi states, which will certainly invalidate the above estimates for the absolute values of the matrix elements, but which may not drastically affect the evaluation of the relative values of the matrix elements.

Furthermore, the estimate is always subject to the criticism mentioned earlier that the construction of the operator is carried out for $M \approx m \approx 1$ Gev, where we believe that the effective coupling constant is small, whereas the typical quark momentum in a hadron is really of order 500 Mev or so.

<u>IV.2B Hyperon Decays.</u> As in the preceding example we may estimate the hyperon decay amplitudes by first applying the PCAC to relate the amplitudes for $B_i \to B_j + \pi$ to the matrix elements $<B_i|H|B_j>$. Unfortunately, we must then use baryon intermediate states and the simplest order of magnitude estimates are difficult to perform.

As an alternative, we will use an equally crude method of evaluation, the so-called "factorization approximation" [41]. Consider for concreteness the decay $\Lambda^o \to p\pi^-$, though we could apply this to any hyperon decay and obtain the same result:

$$<\Lambda|\bar{s}_L u_R \bar{u}_R d_L + \bar{s}_L d_R \bar{d}_R d_L|p\pi^-> \; ; \quad <\Lambda|\bar{s}\gamma_\mu u_L \bar{u}\gamma^\mu d_L|p\pi^-> \tag{IV.22}$$

We approximate these amplitudes by:

$$<\Lambda|\bar{s}_L u_R|p><0|\bar{u}_R d|\pi^-> \quad ; \quad <\Lambda|\bar{s}\gamma_\mu u|p><0|\bar{u}\gamma^\mu d|\pi^-> \tag{IV.23}$$

hence, we obtain for the parity conserving (p-wave) and parity violating (s-wave) amplitudes in both cases:

$$\left.\begin{array}{lll} \text{(Higgson)} & \dfrac{m_\Lambda - m_p}{m_s - m_u} g_V(q^2) \dfrac{f_\pi m_\pi^2}{m_u + m_d} & \text{p.c.} \\[2ex] & \dfrac{\chi(q^2)}{m_s + m_u} \dfrac{f_\pi m_\pi^2}{m_u + m_d} & \text{p.v.} \\[4ex] \text{(Current)} & (m_\Lambda - m_p)\, f_\pi g_V(q^2) & \text{p.c.} \\[2ex] & \chi(q^2)\, f_\pi & \text{p.v.} \end{array}\right\} \tag{IV.24}$$

where $g_V(q^2)$ is the vector current form factor and $\chi(q^2)$ is the form factor for the divergence of the axial vector current matrix elements.

Again we see that the Higgson amplitudes are large compared to the current-current case. Inserting into the full Hamiltonian gives a net enhancement of roughly the same order of magnitude as in the preceding example (see Fig. (IV.1)).

We wish to emphasize that we are reluctant to believe that this is a reliable estimate for the absolute value of the matrix elements, but that it may be reliable for the relative sizes. Furthermore, the parameter B need not be so large, in which case the Higgson contribution would be unimportant.

IV.2C Inspection of eq.(II.17) reveals a term of the form $\bar{d}c\bar{c}d$ which will mix via QCD with terms of the form $\bar{d}u\bar{u}d + \bar{d}d\bar{d}d$. These terms are multiplied by a factor of $\sin^2\Theta_c$ and have isospin 0, and 1. Such terms can contribute to parity violation in nuclear processes.

We have evaluated the isospin-1 parity violating π-nucleon vertex which is of the form:

$$c\,\bar{N}(\pi \times \tau)_z N. \qquad \text{(IV.25)}$$

Our evaluation makes use of the factorization approximation and follows that of Schülke [41]. If we use the maximal value of B, and X = 3 and m_H = 6 Gev, we find that the value of c is approximately 3×10^{-8}. This is to be compared with the standard current-current estimate of 4×10^{-8}. Hence, the Higgson effect can be comparable to the current-current effect. With larger values of B we can have Higgson effects dominating those of the standard Hamiltonian.

IV.2D Higgson Decays. We have followed the work of Ellis, Gaillard, and Nanopoulos [45] in studying the expected properties of the single neutral Higgson in the Weinberg model and estimating some of the features of the decays of our charged Higgsons. We expect a total width of order 1 to 10 Mev for masses of order 10 Gev. We are most interested in the properties of the decays that would tell us directly about the parameter B.

A) Decays to Mesons: We expect a dominant decay mode into the charmed mesons (or heavier quarks). An inspection of the Hamiltonians of eqs.(II.8 & II.16) reveals that the mixing angles (corresponding to the B parameter) determine parity violation in the decays. A standard test of parity violation for a scalar that has two and three meson decay modes is to see if the Dalitz distribution for the three body mode vanishes on the perimeter of allowed phase space. Nonvanishing perimeter contributions denote parity violation [46]. With sufficiently many decay modes one could, in principle, determine B.

B) Decays to Baryons: The asymmetry parameters in these decays (polarization measurements) to pairs of hyperons or nucleons would give direct information about a) the value of B b)the values of $g_V(m_H^2)$ and $\chi(m_H^2)$.

C) Decays to Leptons: Though this is not ruled out, it would slightly complicate the simple picture we have presented here. There would then occur additional mixing angles than the parameter B, though we have found that even in these more complicated cases the nonleptonic Hamiltonian is still parameterized by a simple constant B/m_H^2. It is interesting to note that one might be able to account for the recent μe events at SLAC [14] in terms of a three body decay mode of a charged Higgson, via the ordinary weak interactions [47]. Such a Higgson would probably have to be a "hadronic Higgson" of the sort we have been describing or one would observe many more muon events. Assuming it has a mass of order 2 Gev ($= m_c$), and taking $B^2 = 10$, and $X = 2$, we find that the $|\Delta I| = \frac{1}{2}$ Higgson mediated processes are about 8 times enhanced over the ordinary current-current processes. If we supplement this effect with the results of Lee, et.al.[20], we are certainly in the correct order of the observed enhancement. Therefore, we find this suggestion very attractive and suggest definitive tests of the spin of the observed objects.

Appendix A

FACTS ABOUT POTENTIALS

In this section we wish to discuss some of the properties of the Higgs potentials relevant to the models discussed in Chapter II. There is very little literature available on this subject, some of which has errors, though it would appear that the study of Higgs potentials is an exceedingly important subject, especially where general statements about symmetry breaking can be made. We will attempt to deduce some general results about the physical spectra of Higgs potentials, though we will not attempt universal proofs. We will work only with the potentials that occur in the tree approximation, and forego treating the difficult problem of the general properties of the effective potential evaluated in higher order perturbation theory.

Let us consider the most general Higgs potential occurring at the level of the tree approximation in the Weinberg-Salam model that is a polynomial of degree four so as to preserve renormalizability. The most general SU(2)xU(1) invariant polynomial of this kind that is a function of two scalar doublets is:

$$P(\phi_1,\phi_2) = \mu_1{}^2|\phi_1|^2 + \mu_2{}^2|\phi_2|^2 + \frac{\lambda_1}{2}|\phi_1|^4 + \frac{\lambda_2}{2}|\phi_2|^4$$

$$\lambda_3|\phi_1|^2|\phi_2|^2 + \lambda_4|\phi_1^\dagger\phi_2|^2 + \frac{\lambda_5}{2}\{(\phi_1^\dagger\phi_2)^2 e^{i\alpha} + \text{h.c.}\} +$$

$$\left\{\frac{\lambda_6}{2}(\phi_1^\dagger\phi_2|\phi_1|^2 e^{i\beta}) + \frac{\lambda_7}{2}(\phi_1^\dagger\phi_2|\phi_2|^2 e^{i\gamma}) + \text{h.c.}\right\}. \quad (A.1)$$

To find a minimum of the potential, one simply shifts the fields by their vacuum expectation values and then determines the necessary conditions on the variables $\eta, \rho, \chi, \delta, \varepsilon$, such that a) there are no "tadpoles" i.e., linear terms in the fields after shifting b) the resulting mass matrix of all the fields that are leftover after shifting and satisfying (a) must have positive real eigenvalues. There will be at least three massless fields after finding the minimum of the potential; one neutral and two charged. If there are more than three "Goldstone bosons", then we did not choose the most general Higgs potential and the one we did choose has an extra continuous symmetry.

If a solution for the minimum exists of the form (A.2) such that $\eta, \rho, \chi \neq 0$, we will call it "hybrid". In general we will refer to the two other kinds of solutions as follows:

$$\begin{aligned} &\langle 0|\phi_1|0\rangle = \begin{pmatrix} \eta \\ 0 \end{pmatrix} ; \ \langle 0|\phi_2|0\rangle = \begin{pmatrix} \rho \\ 0 \end{pmatrix} \quad \text{(ferromagnetic)} \\ &\langle 0|\phi_1|0\rangle = \begin{pmatrix} \eta \\ 0 \end{pmatrix} ; \ \langle 0|\phi_2|0\rangle = \begin{pmatrix} 0 \\ \chi \end{pmatrix} \quad \text{(antiferromagnetic).} \end{aligned} \tag{A.3}$$

The ferromagnetic solutions are the ones that are physically desirable as they conserve electric charge. i.e., out of the four generators of SU(2)xU(1) we can find a linear combination that annihilates the $\langle 0|\phi_1|0\rangle$ and $\langle 0|\phi_2|0\rangle$ which is a conserved charge. There is no a priori reason why antiferromagnetic solutions that violate all of the four group charges should not occur. One would simply hope that the ferromagnetic solution is of lower energy than the antiferromagnetic one unless a sophisticated symmetry argument exists which would require the ferromagnetic solutions exclusively. Such an argument, as we shall see, may indeed exist for certain potentials.

We may construct the constraint equations for the variables $<0|\phi_1|0>$ and $<0|\phi_2|0>$ for the potential in eq.(A.1) as described above. We obtain:

$$\phi_1 \text{ tadpole:} \quad 0 = \mu_1^2\eta + \lambda_1\eta^3 + \lambda_3\eta(\rho^2 + \chi^2) + \lambda_4\eta\rho^2 + \lambda_6\rho\eta^2 e^{i(\beta+\delta)} + \frac{\lambda_7}{2}\rho(\rho^2 + \chi^2)e^{i(\gamma+\delta)} + \frac{\lambda_6}{2}\rho\eta^2 e^{-i(\beta+\delta)} \quad \text{(A.4a)}$$

$$0 = \chi e^{i\varepsilon}\left\{\lambda_4\eta\rho e^{-i\delta} + \lambda_5\eta\rho e^{i\delta} + \frac{\lambda_6}{2}\eta^2 e^{i\beta} + \frac{\lambda_7}{2}(\rho^2 + \chi^2)e^{i\gamma}\right\} \quad \text{(A.4b)}$$

$$\phi_2 \text{ tadpole:} \quad 0 = \mu_2^2\rho e^{i\delta} + \lambda_2(\rho^2 + \chi^2)\rho e^{i\delta} + \lambda_5\eta^2 e^{i\delta}\rho + \lambda_4\eta^2\rho e^{i\delta} + \lambda_5\eta^2\rho e^{-i\delta} + \frac{\lambda_6}{2}\eta^3 e^{-i\beta} + \frac{\lambda_7}{2}\eta(\rho^2 + \chi^2)e^{-i\gamma} + \frac{\lambda_7}{2}(\eta\rho^2 e^{-(\gamma+\delta)} + \text{h.c.}) \quad \text{(A.4c)}$$

$$0 = \chi e^{i\varepsilon}\left\{\mu_2^2 + \lambda_2(\rho^2 + \chi^2) + \lambda_3\eta^2 + \frac{\lambda_7}{2}(\eta\rho e^{-i(\gamma+\delta)} + \text{h.c.})\right\}. \quad \text{(A.4d).}$$

Superficially, eqs.(A.4a) and (A.4c) by themselves appear to be four separate equations written in complex form. But if we write out the real and imaginary parts we find that there are only three independent equations:

$$0 = \mu_1^2\eta + \lambda_1\eta^3 + \lambda_3\eta(\rho^2 + \chi^2) + \lambda_4\eta\,\rho^2 + \lambda_5\eta\rho^2\cos(2\delta)$$
$$+ \frac{\lambda_6}{2}\rho\eta^2\cos(\beta+\delta) + \frac{\lambda_7}{2}\rho(\rho^2 + \chi^2)\cos(\gamma+\delta) +$$
$$+ \lambda_6\rho\eta^2\cos(\beta+\delta) \tag{A.5a}$$

$$0 = \mu_2^2\rho + \lambda_2\rho(\rho^2 + \chi^2) + \lambda_3\eta^2\rho + \lambda_4\eta^2\rho +$$
$$\lambda_5\eta^2\rho\cos(2\delta) + \frac{\lambda_6}{2}\eta^3\cos(\beta+\delta) + \frac{\lambda_7}{2}\eta(\rho^2 + \chi^2)\cos(\gamma+\delta)$$
$$+ \lambda_7\rho^2\eta\cos(\delta+\gamma) \tag{A.5b}$$

$$0 = \left\{\lambda_5\eta\rho\sin(\delta) + \frac{\lambda_6}{2}\eta^2\sin(\beta+\delta) + \frac{\lambda_7}{2}(\rho^2 + \chi^2)\sin(\delta+\gamma)\right\}\chi. \tag{A.5c}$$

These equations could, in principle, be solved for fixed χ. One would then be forced to find the values of χ that then solved the the additional four constraint equations, (A.4b) and (A.4d). Clearly, $\chi = 0$ is a solution, and note that the phase ε is completely unspecified. With the exception of certain degenerate cases in which the coupling constants take on specific values, this will be the only solution. $\chi = 0$ corresponds to a ferromagnetic solution in which the electric charge is naturally conserved. We therefore seem to have a theorem: <u>The most general Higgs potential involving two doublets conserves electric charge except for certain special values of the Higgs coupling constants</u>.

It is possible to give an example of a case in which the antiferromagnetic solutions are not forbidden, and this is an important case. We have only considered models in this thesis in which the conservation laws of charm and strangeness are upheld by discrete symmetries, i.e., "natural conservation laws". If such discrete symmetries are fundamental in nature, than they must be built into the Higgs couplings as well as the Yukawa interactions. The independent discrete reflection symmetries of the form $\phi_1 \to -\phi_1$ and $\phi_2 \to -\phi_2$ require that the coupling constants λ_6 and λ_7 be zero.

In this case, $\lambda_6 = \lambda_7 = 0$, one readily sees upon studying eqs.(A.5c) that a possible solution to the tadpole equations, (A.5a,b) is $\rho = 0$ and then χ and η must satisfy the remaining two equations, which are no more difficult to solve than were the constraint equations for the ferromagnetic case. We may summarize the two cases by the following pairs of equations which are readily solved:

$$\begin{array}{c}\text{Ferromagnetic:}\\ \chi = 0\end{array}\left\{\begin{array}{l} 0 = \mu_1^2 + \lambda_1\eta^2 + (\lambda_3 + \lambda_4 + \lambda_5)\rho^2 \\ 0 = \mu_2^2 + \lambda_2\rho^2 + (\lambda_3 + \lambda_4 + \lambda_5)\eta^2 \end{array}\right. \tag{A.6a}$$

$$\begin{array}{c}\text{Anti-}\\ \text{ferromagnetic}\\ \rho = 0\end{array}\left\{\begin{array}{l} 0 = \mu_1^2 + \lambda_1\eta^2 + \lambda_3\chi^2 \\ 0 = \mu_2^2 + \lambda_2\chi^2 + \lambda_3\eta^2. \end{array}\right. \tag{A.6b}$$

Note the logic of this peculiar result. If we restrict the symmetry group of the potential to be SU(2)xU(1) and do not allow any additional discrete symmetries, we find that the conservation of electric charge is "natural". If, on the other hand, we allow the independent reflection symmetries of the two scalar doublets, which amounts to allowing one new discrete symmetry since we are free to reflect both scalar fields simultaneously and then perform one of the discrete reflections, we find that the conservation of electric charge ceases to be natural! We are impressed by this fact and believe it to be a special case of a more general theorem about potentials: As one increases the symmetries of a potential, one increases the amount of symmetry breaking that can occur in the solution to the potential's minimum.

When the equations (A.6a, b) are solved and one computes the actual value of the energy of the lowest vacuum state, one finds:

$$E = -\frac{1}{2}\left\{\mu_1^2\eta^2 + \mu_2^2(\rho^2 + \chi^2)\right\}, \tag{A.7}$$

which turns out to be true for any Higgs potential and is easily generalized to any number of Higgs doublets (or multiplets in a more general gauge group):

$$E = -\frac{1}{2}\sum_i \mu_i^2 |\langle 0|\phi_i|0\rangle|^2.$$

It should be emphasized that we are only working in the tree approximation with the classical ground states of quartic potentials and we have not considered the more general problem of the properties of the effective potential.

We have worked out the mass spectrum for the Higgs potential in the case $\lambda_6 = \lambda_7 = 0$. Since we are not interested in CP violating effects we will simply ignore the phase angle α in eq. (A.1). We are also only interested here in the ferromagnetic case, though the antiferromagnetic case is easily constructed as well. The important difference between these cases is that the antiferromagnetic case will have four Goldstone bosons (an extra one gives the photon its mass) whereas the ferromagnetic case has only three. It is extremely important after constructing solutions to the tadpole equations to construct the mass matrix of the leftover Higgsons to make sure that it is positive definite (positive definite eigenvalues) since this is precisely the condition that one has located a minimum of the potential and not a saddle point. The mass spectrum and mass matrix eigenfields are described in Table A.1.

We wish to mention that even in the case $\lambda_6 = \lambda_7 = 0$, there is not, in general, a hybrid solution to the tadpole equations unless the special condition $\lambda_4 + 2\lambda_5 = 0$ is satisfied. Such special cases are probably not renormalization group invariant, i.e., they may not be fixed points of the renormalization group equations, and are therefore not physically acceptable symmetry breaking solutions. Of course, testing to see if these are fixed point conditions begs the question of what are the general properties of the effective potential when one goes beyond the tree approximation?

VECTORLIKE MODEL POTENTIAL

In any vectorlike scheme the minimal Higgsification with respect to the gauge group SU(2)xU(1) requires at least one real triplet and a complex doublet. Singlets of this group cannot have vacuum expectation values without spoiling electric charge conservation since they are charged fields themselves. Of course, one could always extend the gauge group by having additional U(1) structures to incorporate scalar Higgsons that are uncharged SU(2)xU(1) singlets as are needed to completely specify the fermion masses.

The most general potential involving a real triplet and a complex doublet is:

$$P(\Phi^a, \phi) = -\mu_1^2(\Phi^a\Phi_a) - \mu_2^2(\phi^\dagger\phi) + \frac{\lambda_1}{2}(\Phi^2)^2 + \frac{\lambda_2}{2}(\phi^\dagger\phi)^2 + \lambda_3(\Phi^2)\phi^\dagger\phi + \lambda_4\phi^\dagger\tau^a\phi\Phi^a. \tag{A.9}$$

We may always choose the vacuum expectation values to be:

$$<0|\phi|0> = \begin{pmatrix}\eta\\0\end{pmatrix} \equiv \eta \quad <0|\Phi^a|0> = \rho^a \text{ (arbitrary vector)}. \tag{A.10}$$

The resulting tadpole equations are:

$$\left.\begin{aligned} (-\mu_2^2 + \lambda_2\eta^2 + \lambda_3\rho^2)\begin{pmatrix}\eta\\0\end{pmatrix} &= -\lambda_4\rho^a\tau^a\begin{pmatrix}\eta\\0\end{pmatrix} \\ -\mu_1^2\rho^a + \lambda_1\rho^2\rho^a + \lambda_3\eta^2\rho^a + \lambda_4\eta^\dagger\tau^a\lambda &= 0. \end{aligned}\right\} \tag{A.11}$$

One can easily see that the solution to these equations must be of the form:

$$\rho^a = \eta^\dagger \tau^a \eta \tag{A.12}$$

which for our choice of the spinor requires $\rho_x = \rho_y = 0$ and $\rho_z \neq 0$ (let us refer to ρ_z as simply ρ). It follows that electric charge is "naturally conserved". We obtain the resulting two tadpole equations:

$$\left.\begin{aligned} -\mu_2^2 + 2\lambda_2\eta^2 + \lambda_3\rho^2 + \lambda_4\rho &= 0 \\ -\mu_1^2\rho + 2\lambda_1\rho^3 + \lambda_3\rho\eta^2 + \lambda_4\eta^2 &= 0 \end{aligned}\right\} \tag{A.13}$$

(note that this is dimensionally sound as the dimensions of λ_4 are those of a mass).

The mass spectrum is given in Table A.3

An important constraint that the solutions tabulated in these tables be actual minima of the potentials is that the various m^2 values given in the adjacent column for the linear combinations representing the physical fields must be positive. As we have remarked above, if any of the m^2 values vanish, besides those that are already designated as vanishing, then the model contains a hidden symmetry in addition to the SU(2)xU(1) that is a continuous symmetry which is being violated. The massless bosons are simply the Goldstone bosons corresponding to this extra broken symmetry.

If any of the m^2 values are negative, then we are at a saddle point of the potential. If all of the m^2 values are negative we are clearly at a maximum value and the potential is effectively "upside down".

To obtain the physical M_H^2 for a particle in a field theory from the values given in our tables, note that it is necessary to multiply by a factor of $\frac{1}{2}$. We have not included the usual numerical factors that are useful from the standpoint of quantum field theory (e.g. $\frac{1}{2}\, m^2$, or $\frac{1}{4!}\lambda$) simply to avoid the necessity of repetitive printing.

TABLE A.1

$$P(\phi_1,\phi_2) = \mu_1^2|\phi_1|^2 + \mu_2^2|\phi_2|^2 + \frac{\lambda_1}{2}|\phi_1|^4 + \frac{\lambda_2}{2}|\phi_2|^2 + \lambda_3|\phi_1|^2|\phi_2|^2$$

$$+ \lambda_4|\phi_1^\dagger\phi_2|^2 + \frac{\lambda_5}{2}\left\{(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right\}$$

has the following "physical" fields:

I. Ferromagnetic, $\langle 0|\phi_1|0\rangle = \begin{pmatrix}\eta\\0\end{pmatrix}$, $\langle 0|\phi_2|0\rangle = \begin{pmatrix}\rho\\0\end{pmatrix}$

Field	m^2
$\phi_1^\pm = \phi_1^\pm\cos\chi + \phi_2^\pm\sin\chi$	0
$\phi_2^\pm = -\phi_1^\pm\sin\chi + \phi_1^\pm\cos\chi$	$(\lambda_4 + \lambda_5)\kappa^2$
$\phi_3 = \mathrm{Im}\phi_1^o\cos\omega + \mathrm{Im}\phi_2^o\sin\omega$	0
$\phi_4 = -\mathrm{Im}\phi_1^o\sin\omega + \mathrm{Im}\phi_2^o\cos\omega$	$2\lambda_5\kappa^2$
$\phi_5 = \mathrm{Re}\phi_1^o\cos\omega + \mathrm{Re}\phi_2^o\sin\omega$	M^{2+}
$\phi_6 = -\mathrm{Re}\phi_1^o\cos\omega + \mathrm{Re}\phi_2^o\sin\omega$	M^{2-}

where $\tan\chi = \frac{\eta}{\rho}$; $\kappa^2 = (\eta^2 + \rho^2)$; $M^{2\pm} = (A + B) \pm \sqrt{(A - B)^2 - 4C^2}$

$A = \lambda_1\kappa^2\sin^2\chi$, $B = \lambda_2\kappa^2\cos^2\chi$, $C = (\lambda_3 + \lambda_4 + \lambda_5)\kappa^2\sin\chi\cos\chi$

$\tan\omega = -M^{2-}/2C$.

TABLE A.2

II. Antiferromagnetic, $\langle 0|\phi_1|0\rangle = \begin{pmatrix}\eta \\ 0\end{pmatrix}$, $\langle 0|\phi_2|0\rangle = \begin{pmatrix}0 \\ \rho\end{pmatrix}$

Field	m^2
$\phi_1 = \mathrm{Im}\ \phi_1^o$	0
$\phi_2 = \mathrm{Im}\ \phi_2^o$	0
$\phi_3 = \mathrm{Re}\phi_2^o\cos\chi + \mathrm{Re}\phi_1^-\sin\chi$	0
$\phi_4 = -\mathrm{Re}\phi_2^o\sin\chi + \mathrm{Re}\phi_1^-\cos\chi$	$(\lambda_4 + \lambda_5)\kappa^2$
$\phi_5 = \mathrm{Im}\phi_2^o\cos\chi + \mathrm{Im}\phi_1^o\sin\chi$	0
$\phi_6 = -\mathrm{Im}\phi_2^o\sin\chi + \mathrm{Im}\phi_1^o\cos\chi$	$(\lambda_4 - \lambda_5)\kappa^2$
$\phi_7 = \mathrm{Re}\phi_1^o\cos\omega + \mathrm{Re}\phi_2^-\sin\omega$	M^{2+}
$\phi_8 = -\mathrm{Re}\phi_1^o\sin\omega + \mathrm{Re}\phi_2^-\cos\omega$	M^{2-}

where we employ the same definitions as in Table A.1, except:

$$C = (\lambda_3)\kappa^2\sin\chi\cos\chi.$$

TABLE A.3

For the Vectorlike theory Higgs potential of eq.(A.9) we obtain the following physical fields:

$$\langle 0|\phi|0\rangle = \begin{pmatrix}\eta\\0\end{pmatrix} ; \quad \langle 0|\Phi^a|0\rangle = \rho^a\delta^{az}$$

Field	m^2
$\phi_1^\pm = \phi^\pm\cos\chi + \Phi^\pm\sin\chi$	0
$\phi_2^\pm = -\phi^\pm\sin\chi + \Phi^\pm\cos\chi$	$\lambda_4\kappa/\rho$
$\phi_3 = \mathrm{Im}\phi^o$	0
$\phi_4 = \mathrm{Re}\phi^o\cos\omega + \Phi^z\sin\omega$	M^{2+}
$\phi_5 = -\mathrm{Re}\phi^o\sin\omega + \Phi^z\cos\omega$	M^{2-}

where we employ the same definitions as in table A.1, except:

$$A = 2\kappa^2\lambda_2\sin^2\chi \ , \ B = 2\kappa^2\lambda_1\cos^2\chi - \frac{\lambda_4}{2}\kappa\sin\chi\tan\chi$$

$$C = \frac{\lambda_3}{2}\kappa^2\sin 2\chi + \frac{\lambda_4}{2}\sin\chi \ , \ \tan \ = \rho^z/\eta.$$

Appendix B

B.1 OPERATOR PRODUCT EXPANSIONS

Operator product expansions are a conjecture of Wilson [48] about the structure of interacting field theories that has been abstracted from free field theory and perturbation theory and applied to strong coupling constant interacting theories. If $O_1(x)$ and $O_2(x)$ are two local operators, then we assume that the expression:

$$O_1(x)O_2(0) = \sum_n C^n(x)O^n(0) \tag{B.1}$$

holds if x is not too large. Equation (B.1) is assumed in theories that have arbitrary coupling constants and $O^n(x)$ are a complete set of local operators. The definition of "local operators" in interacting field theory is part of the problem. The $C^n(x)$ functions are either singular as x tends to zero, or well behaved depending upon the dimensions of the operators involved and the possible introduction of logarithms by the interactions.

It is most important to realize that the operators, O_1, O_2, O_n, must be renormalized (defined) after the effects of the interactions are included. This implies that the operators depend explicitly upon a renormalization mass, M, which enters as a parameter for each finite local operator. We will be interested in operators defined with $M \cong m$ or 1 Gev, where it is hoped that reliable estimates of matrix elements might be performed and at which point the gluon coupling constant is not large.

This means essentially that when we compute matrix elements of the operators between physical hadrons, the values should be of order unity. More precisely, we require that the single particle irreducible matrix elements of the operators be approximately equal to their free field theory values when the quark momenta are given by $p^2 = (1\ \text{Gev})^2$. Note that we generally define the operator with leg momenta that are "euclidean" so as to avoid possible physical particle poles that might upset the unambiguous definition of the operator for positive p^2.

Operator matrix elements between physical states cannot be determined due to the complexities of hadronic structure. Nonetheless, by defining the operators that appear on the right side of eqn.(B.1) with a renormalization point equal to the typical hadronic quark momentum, we hope that, in principle, naive estimates of the matrix elements might correspond to reality.

Operator product expansions are of interest to us because we are constructing nonleptonic Hamiltonians, such as eq.(II.17), due to Higgson exchange which involves the short distance product of scalar and pseudoscalar quark densities. For example, in analogy with eq.(III.2), we have terms in our Hamiltonian density of the form:

$$\left.\begin{array}{ll}(\mathrm{I}) & \int d^4x\, D_H(x-y)\ (\bar{s}_L c_R(x))(\bar{c}_R d_L(y)) \\ (\mathrm{II}) & \int d^4x\, D_H(x-y)\ (\bar{s}_R c_L(x))(\bar{c}_R d_L(y))\end{array}\right\}, \qquad \text{(B.2)}$$

where $D_H(x - y)$ is the Higgson propagator. $D_H(x - y)$, in configuration space, is peaked around a distance of order $(m_H)^{-1}$, which is small compared to m^{-1}. Therefore, the operator product expansion is justified in evaluating the structure of eqn.(II.17), etc.

We must order the operators appearing in a short distance expansion in terms of increasing mass dimension. For example, in the operator product:

$$\bar{q}q(x)\bar{q}q(0) = C(x) + C_1(x)\bar{q}q(0) + C_2(x)\bar{q}q(0) + C_3(x)\bar{q}\sigma_{\mu\nu}qG^{\mu\nu} + C_4(x)\bar{q}q\bar{q}q(0) + \ldots \tag{B.3}$$

we include first the c-number contribution which will, if present, be associated with the most singular coefficient function. Of course, each term in the series must have the same flavor representation as the left hand side, and hence a c-number piece would not be present in eqn.(B.1) (even if it were, it would merely represent an overall vacuum energy contribution). Next follow the d = 3 operators, d = 5 operators, and d = 6, etc. The functions associated with the d = 7 and higher operators will be well behaved as x tends to zero since they must be associated with positive powers of x to balance the dimensions of both sides of the equation (of course, this statement might be modified by interactions in a strong coupling constant theory, though it certainly remains true in an asymptotically free field theory in which perturbative treatment is valid).

In constructing a Hamiltonian, operators of the form $\bar{q}q$ or $\bar{q}\gamma_5 q$ may be dropped from the discussion as their effects are simply to redefine the quark masses, and they are renormalized away by the inclusion of counterterms. Similarly, the operators of dimension greater than six will not be important as x tends to zero since they require extra powers of x in their coefficient functions. Actually, in practice, this reduces their effect by factors of m_q^2/m_c^2 relative to the d = 5 and d = 6 operators.

Therefore, having completed this preliminary discussion, our next task is to treat the contribution of the dimension five and six operators in the construction of the Hamiltonian. For this purpose, we follow the standard methods[20] of first constructing a renormalization group equation for the coefficient functions of these operators. We can then convert this into a *scaling equation* which gives the short distance behavior of $C^n(x)$ in terms of computable functions in the asymptotically free SU(3) color theory. This problem is discussed in the next section.

B.2 DEFINITION OF COMPOSITE OPERATORS

The definition of composite operators in interacting field theories is highly nontrivial. We will give only an operational definition and supply several examples which are best suited for our computations. We will not enter into the lengthy discussion necessary to clarify potential ambiguities in calculations of higher order or within the context of other applications than the ones we have in mind. We refer the reader to[49] for an extensive discussion of these matters.

We employ the method of dimensional regularization, first introduced by t'Hooft and Veltman[50] as a convenient method of evaluating Feynman integrals in a gauge theory. We will begin by considering the problem of defining a fermionic kinetic term or current to second order in the strong coupling constant in QCD. These operators will not be renormalized when we correctly define the field renormalization constant of the fermions.

Renormalization is a two step process for operators. One begins with the free field theory expression for the operator in question expressed in terms of unrenormalized fields. We then renormalize the fields by multiplying by the appropriate scale factor of $\sqrt{Z_F}$ for each fermion comprising the operator. Then we take the second step of computing the radiative corrections to the operator in renormalized perturbation theory by evaluating diagrams. The diagrams will require subtractions which are performed at the subtraction point, $p^2 = -m^2$. We define the operator for momenta having these values to be equal to the value it would have in free field theory. This completes the renormalization prescription. We have schematically:

$$\left.\begin{array}{lllll} \bar{\psi}_o i\not\partial\psi_o & \rightarrow & Z_2\, \bar{\psi}_r i\not\partial\psi_r & \rightarrow & Z_2 Z'\; :\bar{\psi}_r i\not\partial\psi_r: \\ \bar{\psi}_o \gamma_\mu \psi_o & \rightarrow & Z_2\, \bar{\psi}_r \gamma_\mu \psi_r & \rightarrow & Z_2 Z'\; :\bar{\psi}_r \gamma_\mu \psi_r: \end{array}\right\} . \tag{B.4}$$

Here we have generalized the free field theory notation for normal ordering to include Zimmerman Normal Ordering as discussed in ref.[7]. This is simply the operation of defining a finite operator, though to give a precise statement of this procedure is complicated by the fact that in an interacting field theory a given operator has many different kinds of non-vanishing matrix elements. For example, the operator $\bar{q}q\bar{q}q$ will in general have matrix elements between single quark states as well as quark and gluon states. This is analogous to the phenomena of mixing which we discuss in section B.3 , but is a mixing between operators of different dimensions and will always be accompanied by divergences that are more severe than logarithmic. The process of defining a finite operator, such as $:\bar{q}q\bar{q}q:$, involves the subtraction of these divergences. Unlike logarithmic divergences, the more severe quadratic and quartic divergences that would be associated with the mixing of $\bar{q}q\bar{q}q$ with the operators $\bar{q}q$ and 1, respectively, will be subtracted away for all values of the normalization mass, m. Operators that mix with logarithmic or finite factors will have finite contributions to the single particle irreducible matrix elements of $\bar{q}q\bar{q}q$ for momenta other than $p^2 = -m^2$, in the case of logarithmically divergent mixings, and all momenta in the case of finite mixings.

The current operators, or the kinetic terms, are used to define the values of the fermion wave function renormalization constant, Z_2. Conserved currents, or pieces of the Lagrangian, must not be renormalized since they are conserved quantities and cannot change as the interaction is adiabatically switched on. Therefore, we are led to define:

$$Z_2 = Z_o^{-1}. \tag{B.5}$$

Let us calculate the QCD quark wave function renormalization constant by way of this prescription. Consider the current operator expressed in terms of renormalized fields, but not yet Zimmerman Normal Ordered. The one loop corrections in renormalized perturbation theory are (to this order there is no difference between renormalized and unrenormalized perturbation theory):

$p \rightarrow$ 

$$= \bar{u}\gamma_\mu u - \bar{u}\,\frac{ig^2}{4}\int\frac{d^4k}{(2\pi)^4}\,\frac{\chi^A\gamma_\alpha(\not{k}+\not{p})\gamma_\mu(\not{k}+\not{p})\gamma_\beta\chi^A}{(k+p)^4k^2}\left(g^{\alpha\beta} - \frac{\lambda k^\alpha k^\beta}{k^2}\right)u \tag{B.6}$$

where we have ignored the effects of the fermion masses.

Terms of the form $\bar{u}p_\mu u$ are finite and will be proportional to the mass^2 of the particle (p^2) for on-shell quarks, and lead to the anomalous gluo-magnetic moment by making use of the Gordon decomposition. The terms of the form $\bar{u}\gamma_\mu u$ are divergent, and we find by using the method of dimensional regularization to evaluate the loop integral in $4 - \varepsilon$ dimensions:

$$(\text{III.8}) = \bar{u}\gamma_\mu u \left\{ 1 - \frac{g^2(\lambda - 1)}{16\pi^2} \frac{8}{3} \frac{1}{\varepsilon}\left(1 + \frac{\varepsilon}{2} \ln p^2\right)\right\}. \quad (\text{B.7})$$

Hence, the condition that the current take on its free field theory value for $p^2 = -m^2$ gives the values of the renormalization constants immediately:

$$\left.\begin{aligned} Z' &= 1 + \frac{8g^2(\lambda - 1)}{54\pi^2} + \frac{4g^2(\lambda - 1)\ln{-m^2}}{54\pi^2} \\ Z_2 &= 1 - \frac{8g^2(\lambda - 1)}{54\pi^2} - \frac{4g^2(\lambda - 1)\ln{-m^2}}{54\pi^2} \end{aligned}\right\} \quad (\text{B.8})$$

which satisfy eq.(B.5) to second order in g.

We will now anticipate our discussion of the renormalization group equations in section B.3 and define the anomalous dimensions of the operators and fields. If O^n is a local composite operator made out of n fermion fields, it will receive in the first step of renormalization, eqn.(B.4), a factor of $(Z_2)^{n/2}$. In the second step, it will receive a factor of $Z_0 n$, in accordance with the prescription that the single particle matrix elements of O^n be equal to the free field theory values for $p^2 = -m^2$. We define the "total anomalous dimension" of the operator O^n to be:

$$\tilde{\gamma}_0{}^n = \frac{\partial \ln Z_0{}^n}{\partial \ln m} . \tag{B.9}$$

This quantity is gauge dependent until we subtract the effects of the fermion wave function renormalization. We define the fermion field anomalous dimension by:

$$\gamma_F = (\tfrac{1}{2}) \frac{\partial \ln Z_2}{\partial \ln m} . \tag{B.10}$$

Hence, the proper anomalous dimension for O^n, which will be gauge invariant (λ independent) if O^n is a gauge invariant operator, is defined by:

$$\gamma_0{}^n = \frac{\partial \ln (Z_2)^{n/2} Z_0{}^n}{\partial \ln m} = \tilde{\gamma}_0{}^n + n\gamma_F . \tag{B.11}$$

In particular, we see by this definition that the anomalous dimension of a conserved (or partially conserved) current vanishes, as does the anomalous dimension of a kinetic term or of an entire Lagrangian.

The anomalous dimension of the fermion field may be written down from eqs.(B.10 & B.11)

$$\gamma_F = \frac{4}{3}(1 - \lambda) \frac{g^2}{16\pi^2} . \tag{B.12}$$

We will find as a general rule that the total anomalous dimension of an operator will be simply the <u>negative of the coefficient of the $\frac{1}{\varepsilon}$ divergence of the one loop correction to the operator in renormalized perturbation theory</u>. Of course, we must emphasize that this is only true in the approximation of ignoring the mass corrections to the anomalous dimensions, which will be justified so long as the mass scale of the short distance expansion is sufficiently large compared to the quark masses.

We shall illustrate this rule with the following two examples. First, consider the computation of the anomalous dimension of the quark bilinear, $\delta m\bar{q}q$, which is a simple mass term (this result will also hold for $\bar{q}\gamma_5 q$). The one loop correction to this operator is the diagram:

$p \rightarrow$

$$= \frac{-ig^2}{4}\int \frac{d^4k}{(2\pi)^4}\, \bar{u}(p)\gamma_\mu \chi^A \frac{(\not{p} + \not{k})\,\delta m(\not{p} + \not{k})}{(p + k)^4 k^2}\, \gamma_\nu \chi^A$$

$$\cdot \left(g^{\mu\nu} - \frac{\lambda k^\mu k^\nu}{k^2}\right) u(p), \qquad \text{(B.13)}$$

where we have again ignored the fermion masses, which would not otherwise contribute to the divergence. In $4 - \varepsilon$ dimensions we obtain:

$$= \delta m\left(\frac{16}{3}\right) \frac{-ig^2}{4}\int \frac{d^4k}{(2\pi)^4}\,(4 - \lambda)\,\bar{u}(p)\,(p + k)^{-4}\,u(p)$$

$$= \delta m\,\left(\frac{8}{3}\right)(4 - \lambda)\,\frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\,\bar{u}(p)u(p) \;+\; O(1). \qquad \text{(B.14)}$$

Hence, the total anomalous dimension, which is just the negative of the coefficient of $\frac{1}{\varepsilon}$, is:

$$\tilde{\gamma} = -\frac{8}{3}\frac{g^2}{16\pi^2}2(4 - \lambda) = \gamma_{\bar{q}q} - n\gamma_F, \qquad \text{(B.15)}$$

where we have employed eq.(B.11). Using eq.(B.12) we obtain the (proper) anomalous dimension:

$$= -8 \frac{g^2}{16\pi^2}, \tag{B.16}$$

which is the well known result.

In general, operators of the same dimension will mix under the effects of the interaction which will lead to anomalous dimensions that are matrices instead of simple numbers. To illustrate this phenomenon we will compute the anomalous dimension of the operator $\bar{q}_L q_R \bar{q}_L q_R$. The chiral structure of this operator is such that its mixing is particularly simple; it will only mix with the operator $\bar{q}_L \sigma_{\mu\nu} q_R \bar{q}_L \sigma^{\mu\nu} q_R$, in the limit of neglecting quark masses. If we include the effects of quark masses, we will have mixing with the operator O_2 of section II.4, but this may be ignored as these gluonic operators will only mix amongst themselves.

To evaluate the anomalous dimensions we must evaluate the diagrams of Fig.(III.1). This amounts to an exercise in Fierz transformations after performing the simple four dimensional integrals. The reader is referred to Appendix B for the relevant identities used in this calculation.

Diagrams (1) and (2) of Fig(III.1) are equal and their sum is:

$$(1) + (2) = \frac{-ig^2}{2} \int \left\{ \frac{d^4k}{(2\pi)^4} \, \bar{q}_L \gamma_\mu \chi^A \frac{(\not{k} + \not{p})^2}{(k + p)^4 k^2} \gamma_\nu \chi^A q_R \bar{q}_L q_R \cdot \left(g^{\mu\nu} - \frac{\lambda k^\mu k^\nu}{k^2}\right) \right\}. \tag{B.17}$$

Upon evaluating the integrals we have:

$$(B.17) \quad = \quad \frac{16}{3} \frac{g^2}{16\pi^2} (4 - \lambda) \frac{1}{\varepsilon} \quad + \quad O(1). \qquad (B.18)$$

We find for the remaining diagrams:

$$\left.\begin{aligned} (3) + (4) \quad &= \quad - \frac{ig^2}{2} \int q_L \gamma_\mu \chi^A \not{k} q_R \bar{q}_L \gamma_\nu \not{k} \chi^A q_R \left(g^{\mu\nu} - \frac{\lambda k^\mu k^\nu}{k^2}\right) \frac{1}{k^6} \\ (5) + (6) \quad &= \quad + \frac{ig^2}{2} \int \bar{q}_L \gamma_\mu \chi^A \not{k} q_R \bar{q}_L \not{k} \gamma_\nu \chi^A q_R \left(g^{\mu\nu} - \frac{\lambda k^\mu k^\nu}{k^2}\right) \frac{1}{k^6} \end{aligned}\right\}. \qquad (B.19)$$

The sum of the two terms above leads to:

$$(3) + (4) + (5) + (6) \quad = \quad \frac{2}{3} \bar{q}_L \sigma_{\mu\nu} q_R \bar{q}_L \sigma^{\mu\nu} q_R \frac{1}{\varepsilon} \qquad (B.20)$$

where we have made use of eq.(C.16 & C.17) in performing the Fierz transformations over the Dirac and color indices.

A straightforward calculation gives the proper anomalous dimension of the operator $\bar{q}\sigma_{\mu\nu} q \bar{q} \sigma^{\mu\nu} q$ to be $\frac{g^2}{16\pi^2} \frac{16}{3}$. Hence, we may assemble the results of the above diagrams together to write the anomalous dimension as a matrix for the two four quark operators, $A = \bar{q}_L q_R \bar{q}_L q_R$, $B = q_L \sigma_{\mu\nu} q_R q_L \sigma^{\mu\nu} q_R$, as follows:

$$\gamma \begin{pmatrix} A \\ B \end{pmatrix} \quad = \quad \frac{g^2}{16\pi^2} \begin{bmatrix} -16 & 0 \\ 1/3 & 16/3 \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix}. \qquad (B.21)$$

B.3 RENORMALIZATION GROUP EQUATION

We now return to the operator product expansion of eq. (B.3) and let us assume initially that the operators are renormalized at $p^2 = -m^2$ in accordance with the prescription described in the preceding section. We begin with the operator product expansion for $x = (m)^{-1}$. and consider the effect of scaling x to smaller distances, i.e., $x \to \lambda^{-1}x$. We may write the rescaled short distance expansion as:

$$O_1(x/\lambda)O_2(0) = \sum_n C^n(x/\lambda, m, g(m))O^n(0), \tag{B.22}$$

where we have displayed the dependences upon m and the renormalized coupling constant, g(m), explicitly.

To learn about the behavior of $C^n(x/\lambda)$ as $\lambda \to \infty$ we first obtain the renormalization group equation and then convert it to an equation in the scaling variable, λ [51].

Pure dimensional analysis informs us that C^n must depend upon m in the following way (we ignore the fermion masses in the present discussion):

$$C^n(x/\lambda, m, g(m)) = (m)^{d_1+d_2-d_n} \bar{C}^n(x/\lambda, m, g(m)), \tag{B.23}$$

where d_i is the ordinary mass dimension of the operator O_i. $\bar{C}^n$ is a dimensionless function of the indicated variables. We will be interested in the scaling behavior of $\bar{C}^n$.

Recall from the description of the renormalization process that the definition of the renormalized operator, $:O^n:$, is given by,

$$:O^n: \quad = \quad Z_2^{-n/2} Z_O \, O^n_{un}. \tag{B.24}$$

The unrenormalized operator O^n_{un} is completely independent of the normalization mass, m, used to define the renormalized operator $:O^n:$. This fact is summarized by the statement:

$$m \frac{d}{dm} O^n_{un} \quad = \quad 0. \tag{B.25}$$

We see that this translates into the result:

$$\begin{aligned} m \frac{d}{dm} Z_2^{n/2} \, Z_O :O^n: \quad &= \quad 0 \\ = \quad (m \frac{\partial}{\partial m} \; + \; \beta(g)\frac{\partial}{\partial g} \; &+ \; n\gamma_F + \tilde{\gamma}_O) :O^n: \\ = \quad (m \frac{\partial}{\partial m} \; + \; \beta(g)\frac{\partial}{\partial g} \; &+ \; \gamma_O) :O^n:, \end{aligned} \tag{B.26}$$

which is the familiar renormalization group equation [51] employing the definitions of the anomalous dimensions as well as the usual definition of the $\beta(g)$ function, i.e., $\beta(g) = dg_r/d(\ln m)$. As is well known, the renormalization group equation expresses the invariance of the theory under the choice of a renormalization point, m.

We may derive a renormalization group equation for the coefficient functions appearing in the operator product expansion of eqn.(B.23). We simply apply to both sides of that equation the operator $\frac{\partial}{\partial m} + \beta(g)\frac{\partial}{\partial g}$ and introduce an anomalous dimension for the coefficient function, γ_{C^n}, such that [11]:

$$(m \frac{\partial}{\partial m} + \beta(g) \frac{\partial}{\partial g} \; + \; \gamma_{\bar{C}^n}) \, \bar{C}^n \quad = \quad 0. \tag{B.27}$$

Then one readily obtains:

$$\gamma_{C^n} \quad = \quad \gamma_{O_1} \; + \; \gamma_{O_2} \; - \; \gamma^T_{O^n}. \tag{B.28}$$

Here we have ignored the phenomenon of mixing. If we apply the operator $D = m\frac{\partial}{\partial m} + \beta(g)\frac{\partial}{\partial g}$ to both sides of the operator product expansion we obtain:

$$(\gamma_{O_1} + \gamma_{O_2})O_1(x)O_2(0) = \sum_{n,m} (\gamma_C^{nm}\, C^m(x)O^n(0)) + (C^n(x)\gamma_{On}^{nm}\, O^m(0)), \qquad (B.29)$$

using the renormalization group equations (III.28), suitably generalized for mixing. In this case one obtains the result as a matrix equation:

$$\gamma_{\bar{C}}^{nm} = \delta^{nm}(\gamma_{O_1} + \gamma_{O_2}) - (\gamma_{On})^{mn}. \qquad (B.30)$$

Note that the contribution from the operators, O^n, is the transpose of the anomalous dimension matrix, as computed in eq.(B.21).

To solve the renormalization group equations when mixing occurs, it is necessary to diagonalize the anomalous dimension matrix and work with operators and coefficient functions that are eigenvectors of the matrix. Such eigenvectors, if we are referring to the operators, are known as multiplicatively renormalized operators. It should be noted that if the effects of finite mass are included in this analysis, it is not in general possible to diagonalize the anomalous dimensions for all values of m by the same transformation. Then the solution to the renormalization group equation becomes quite analogous to the solution of a field theory with a Hamiltonian that does not commute with itself for successive times, i.e., we must express the solution in terms of m-ordered exponential integrals, which must be numerically integrated on a computer in general.

As may be seen in equation (B.23), the anomalous dimension for the dimensionless coefficient function, $\bar{C}^n$, will be related to the anomalous dimension of C^n by the form:

$$\gamma_C = \gamma_{\bar{C}}^{nm} + (d_1 + d_2 - d_n)\delta^{nm} \qquad \text{(B.31)}$$

where d_i are the naive "engineering" dimensions of the various operators.

The renormalization group equation is of interest because it allows us to construct a scaling equation [51] in terms of the variable λ. Note that the dimensionless function $\bar{C}^n(x/\lambda,m,g(m))$ can depend upon x only through the dimensionless combination xm and implicitly upon m by the dependence upon g(m) (recall that g(m) also contains a cutoff dependence, but that the cutoff only appears in g(m) after renormalization and the function C^n does not depend explicitly upon the cutoff). Therefore, to rescale x by λ^{-1} is equivalent to rescaling m by λ^{-1} only where it appears <u>explicitly</u> in C^n. Therefore, we have the following relations between the partial derivatives:

$$m \frac{\partial}{\partial m} \bar{C}^n(x/\lambda,m,g(m))\Big|_{g(m)} = -\lambda \frac{\partial}{\partial \lambda} \bar{C}^n(\ldots). \qquad \text{(B.32)}$$

This relationship allows us to write the scaling equation for $\bar{C}^n$ as:

$$(\lambda \frac{\partial}{\partial \lambda} - \beta(g) \frac{\partial}{\partial g}) \bar{C}^n - \sum_m \gamma_{\bar{C}}^{nm} \bar{C}^m = 0 . \qquad \text{(B.33)}$$

The solution to eqn.(B.33) of physical interest is easily verified to be:

$$\bar{C}^n(x/\lambda,\ m,\ g(m)) = \bar{C}^n(x,\ m,\ \bar{g}(\lambda))\exp + \int_0^{\ln\lambda} \gamma_{\bar{C}^n}(\bar{g}(t))dt \tag{B.34}$$

where $\bar{g}(t)$ is defined by:

$$\frac{d\bar{g}}{dt} = \beta(\bar{g}(t)). \tag{B.35}$$

This solution can always be rewritten as:

$$\bar{C}^n(x/\lambda,\ m,\ g(m)) = \bar{C}^n(x,\ m,\ \bar{g}(\lambda))\exp + \int_{g(1)}^{\bar{g}(\lambda)} \frac{\gamma(g)}{\beta(g)}\, dg\,. \tag{B.36}$$

In an asymptotically free theory, such as SU(3) of color with the numbers of quarks prescribed by the models considered in Chapter I, we will always have the forms for the renormalization group functions:

$$\left.\begin{aligned} \gamma_i(g) &= \frac{g^2}{16\pi^2}k_i + O(g^4) \\ \beta(g) &= -\frac{g^3}{16\pi^2}b_o + O(g^5) \end{aligned}\right\}. \tag{B.37}$$

Using these definitions we may rewrite the solutions to the scaling equations in a more convenient form:

$$\bar{C}^n(x/\lambda,\ m,\ g(m)) = \bar{C}^n(x,\ m,\ \bar{g}(\lambda))\left[1 + \frac{\kappa b_o}{2\pi}\ln\lambda\right]^{+\kappa/2b_o} \tag{B.38}$$

where $\kappa = g(m)^2/4\pi$.

The magnificence of this result may be described as follows. The physical coupling constant is a parameter with which we would hope to describe amplitudes involving the strong interactions and is just $g(m)^2/4\pi$, evaluated at the renormalization group invariant mass m. One may solve (B.35) and deduce that:

$$\bar{\kappa}(\lambda) = \frac{\kappa}{1 + \frac{b_o \kappa}{2\pi} \ln \lambda}, \tag{B.39}$$

and since b_o is by definition greater than zero for an asymptotically free theory, we see that $\bar{\kappa}$ (or $\bar{g}$) tend to zero as λ tends to infinity. We are instructed in eqn.(B.38), to evaluate the coefficient on the right hand side, namely $\bar{C}^n(x, m, g(\lambda))$, in a field theory with an effective coupling constant, $\bar{\kappa}(\lambda)$, that is very small. Hence, this coefficient is computable in free field theory for sufficiently large λ. Furthermore, the quantities enclosed in the brackets in eqn.(B.38) are, in principle, determined. Conventional wisdom dictates that the value of κ is of order unity. With only four light quarks we must always take $b_o = 25/3$† [6], and we obtain:

$$\bar{\kappa}(M_H/\mu) \simeq 10^{-1} \tag{B.40}$$

which is reasonably small and would tend to justify the assertion that the coefficient can be estimated in free field theory. Of course, κ is sometimes taken to be of order .33 and this would further reduce the size of the result in (III.42). This question of the

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† $b_o = (11 - \frac{2}{3} n_f)$, where n_f is the number of light quark flavors. Choosing $n = 3$ changes all enhancements, etc., by roughly 8%.

sizes of these parameters will not be so important since the values of the anomalous dimension coefficients, k_i, will be small in our applications. We follow the choices adopted by Gaillard and Lee [20] in their analysis of the short distance corrections to the current-current product. With the mass scale of the Higgsons of order $\sqrt{\alpha}\, M_W$, we should take:

$$\left(1 + \frac{\kappa b_o}{2\pi} \ln M_H/\mu\right) \simeq 5 \text{ to } 6 . \tag{B.41}$$

With this choice for the sizes of the various parameters we can directly compare the Higgson enhancements with the current-current enhancements. The enhancement or suppression of a given operator appearing in the Higgson exchange Hamiltonian will therefore be of order $(X)^{k_i/2b_o}$.

In the Chapter III we treat the operator mixing problem for the operators occurring in the short distance product of scalars and pseudoscalars. We compute the anomalous dimensions of the various operators as a matrix, which we subsequently diagonalize to find the multiplicatively renormalized linear combinations.

Appendix C

DIRACOLOGY AND USEFUL FORMULAE

We employ the representation of the Dirac algebra as defined in the textbook of Bjorken and Drell [52]. It is important to note that the definition of the totally antisymmetric ε-symbol is:

$$\varepsilon_{0123} = +1 \tag{C.1}$$

and therefore:

$$\varepsilon^{0123} = -1; \ \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\mu\nu\rho\lambda} = -6g_\sigma{}^\lambda . \tag{C.2}$$

The γ_5 matrix is defined by:

$$\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \tag{C.3}$$

and we have the trace rule:

$$\mathrm{Tr}\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = 4i\varepsilon_{\mu\nu\rho\sigma} . \tag{C.4}$$

From the usual identity:

$$\mathrm{Tr}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = 4(g_{\mu\nu}g_{\rho\sigma} + g_{\nu\rho}g_{\mu\sigma} - g_{\mu\rho}g_{\nu\sigma}) \tag{C.5}$$

and eqn.(C.4), we obtain:

$$\gamma_\mu\gamma_\nu\gamma_\rho = g_{\mu\nu}\gamma_\rho + g_{\nu\rho}\gamma_\mu - g_{\mu\rho}\gamma_\nu + i\varepsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\sigma \tag{C.6}$$

and:

$$\varepsilon^{\mu\nu\rho\sigma}\gamma_\rho\gamma_\nu\gamma_\mu = 6i\gamma_5\gamma^\sigma . \tag{C.7}$$

From equation (C.6) one readily obtains:

$$\left.\begin{aligned} [\gamma_\alpha\gamma_\beta\gamma_\gamma]_{ij}[\gamma^\alpha\gamma^\beta\gamma^\gamma]_{kl} &= 10[\gamma_\alpha]_{ij}[\gamma^\alpha]_{kl} + 6[\gamma_5\gamma_\alpha]_{ij}[\gamma^5\gamma^\alpha]_{kl} \\ [\gamma_\alpha\gamma_\beta\gamma_\gamma]_{ij}[\gamma^\gamma\gamma^\beta\gamma^\alpha]_{kl} &= 10[\gamma_\alpha]_{ij}[\gamma^\alpha]_{kl} - 6[\gamma_5\gamma_\alpha]_{ij}[\gamma^5\gamma^\alpha]_{kl} \end{aligned}\right\} . \tag{C.8}$$

Henceforth, in writing equations such as (C.8) we will suppress the latin (Dirac) indices i,j,k,... etc.

With the usual definition:

$$\sigma_{\mu\nu} = i/2[\gamma_\mu,\gamma_\nu] \tag{C.9}$$

one obtains:

$$\left.\begin{aligned}\sigma_{\alpha\beta}\gamma_\gamma\gamma_\delta &= i(g_{\beta\gamma}\gamma_\alpha\gamma_\delta - g_{\alpha\gamma}\gamma_\beta\gamma_\delta) - \varepsilon_{\alpha\beta\gamma\rho}\gamma_5\gamma^\rho\gamma_\delta \\ \gamma_\alpha\gamma_\beta\sigma_{\gamma\delta} &= i(g_{\beta\gamma}\gamma_\alpha\gamma_\delta - g_{\beta\gamma}\gamma_\alpha\gamma_\gamma) + \varepsilon_{\beta\gamma\delta\rho}\gamma_5\gamma_\alpha\gamma^\rho\end{aligned}\right\}. \tag{C.10}$$

Combining the above results leads to:

$$\begin{aligned}[\sigma_{\alpha\beta}\gamma_\mu\gamma_\nu][\sigma^{\alpha\beta}\gamma^\mu\gamma^\nu] &= [\gamma_\mu\gamma_\nu\sigma_{\alpha\beta}][\gamma^\mu\gamma^\nu\sigma^{\alpha\beta}] \\ &= -6\left([\gamma_\alpha\gamma_\beta][\gamma^\alpha\gamma^\beta] + [\gamma_5\gamma_\alpha\gamma_\beta][\gamma_5\gamma^\alpha\gamma^\beta]\right) \qquad (C.11) \\ = -24\left([1][1] + [\gamma_5][\gamma_5]\right) &+ 6\left([\sigma_{\mu\nu}][\sigma^{\mu\nu}] + [\gamma_5\sigma_{\mu\nu}][\gamma_5\sigma^{\mu\nu}]\right) ;\end{aligned}$$

$$\begin{aligned}[\sigma_{\alpha\beta}\gamma_\mu\gamma_\nu][\gamma^\nu\gamma^\mu\sigma^{\alpha\beta}] &= [\gamma_\nu\gamma_\mu\sigma_{\alpha\beta}][\gamma^\mu\gamma^\nu\sigma^{\alpha\beta}] \\ &= 6\left([\gamma_\alpha\gamma_\beta][\gamma^\beta\gamma^\alpha] + [\gamma_5\gamma_\alpha\gamma_\beta][\gamma_5\gamma^\alpha\gamma^\beta]\right) \qquad (C.12) \\ = 24\left([1][1] + [\gamma_5][\gamma_5]\right) &- 6\left([\sigma_{\mu\nu}][\sigma^{\mu\nu}] + [\gamma_5\sigma_{\mu\nu}][\gamma_5\sigma^{\mu\nu}]\right) .\end{aligned}$$

Further Dirac identities employed in the text are:

$$\gamma_\mu\sigma_{\alpha\beta}\gamma^\mu = 0; \qquad \sigma_{\alpha\beta}\gamma_\mu\sigma^{\alpha\beta} = 0. \tag{C.13}$$

In our choice of Dirac representation we will also require the Fierz identities. We will define the following operators:

$$\begin{aligned}O_1 = \bar\psi_1\psi_2\bar\psi_3\psi_4;\ O_2 = \bar\psi_1\gamma_\mu\psi_2\bar\psi_3\gamma^\mu\psi_4;\ O_3 = \bar\psi_1\sigma_{\mu\nu}\psi_2\bar\psi_3\sigma^{\mu\nu}\psi_4 \\ O_4 = \bar\psi_1\gamma_5\gamma_\mu\psi_2\bar\psi_3\gamma_5\gamma^\mu\psi_4\ ;\ O_5 = \bar\psi_1\gamma_5\psi_2\bar\psi_3\gamma_5\psi_4 .\end{aligned} \tag{C.14}$$

These operators are ordered according to 1,2,3,4; we define a second set of $\overline{O}_i$, identical to the above, but ordered 1,4,3,2 (equivalently we could order the second set 3,2,1,4). Then the O_i are related to the $\overline{O}_i$ by the Fierz identity:

$$O_i = M_{ij}\overline{O}_j \tag{C.15}$$

where:

$$M_{ij} = -1/8 \begin{bmatrix} 2 & 2 & 1 & -2 & 2 \\ 8 & -4 & 0 & -4 & -8 \\ 24 & 0 & -4 & 0 & 24 \\ -8 & -4 & 0 & -4 & 8 \\ 2 & -2 & 1 & 2 & 2 \end{bmatrix} \tag{C.16}$$

(the overall minus sign is due to the anticommutation property of the field operators).

The Fierz rearrangement on the Dirac spinor indices must always be accompanied by a rearrangement of the color indices when dealing with quarks. The general relations involved in Fierzing color indices follow from:

$$\sum_A \chi^A_{ij}\chi^A_{kl} = 2[\delta_{il}\delta_{kj} - \left(\tfrac{1}{3}\right)\delta_{ij}\delta_{kl}] \tag{C.17}$$

where χ^A are the SU(3) color generators (equivalent to the Gell-Mann λ-matrices). We readily obtain:

$$\sum_A \chi^A\chi^B\chi^A = -\left(\tfrac{2}{3}\right)\chi^B \; ; \quad \sum_A \chi^A\chi^A = (16/3)I. \tag{C.18}$$

REFERENCES

1. E. D. Bloom et. al. SLAC-PUB-796 (1970) report presented at the XVth International Conference on High Energy Physics, Kiev, USSR 1970) See also D.C. Cundy Proceedings of the 17th International Conference on High Energy Physics, London, 1974)

2. J.J.Aubert et.al.,Phys.Rev.Letters 33, 1404 (1974); J.E.Augustan et.al.,Phys.Rev.Letters 33, 1406 (1974); G.S.Abrams et.al.,Phys. Rev.Letters 33, 1435 (1974), A.Benvenuti et.al. Phys.Rev.Letters 34, 419 (1975); E.G.Cazzoli et.al. Phys.Rev.Letters 34, 1125 (1975), G.Goldhaber et.al., Phys.Rev.Letters 37 (1976) I.Perruzi et.al.,Phys.Rev.Letters 37 569 (1976)

3. F.J Hasert, et. al., Phys. Letters 46B, 138 (1973); see also B.C. Barish, Lectures presented at Sixth Hawaii Topical Conference in Particle Physics (1975)

4. C.N Yang, R.Mills, Phys. Rev. 96 (1954) 191

5. H. Fritzsch, M. Gell-Mann, H. Leutwyler, Phys. Letters 47B (1973) 365

6. H.D. Politzer, Phys. Rev. Letters, 30, 1346 (1973); D.J.Gross and F. Wilczek, Phys. Rev. Letters, 30, 1343 (1973)

7. J. M. Cornwall and G. Tiktopoulos, Phys. Rev. Letters 35, 338 (1975), T. Appelquist et. al. Fermilab Pub-76/16-Thy

8. S. Weinberg, Phys. Rev. Letters 19 (1967) 1264, A. Salam, in Elementary Particle Theory, ed. N. Svartholm (Almquist and Forlag, Stockholm, 1968)

9. S.L Glashow, J.Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970)

10. H. Georgi, S.L. Glashow, Phys.Rev.Letters 32, 438 (1974), F. Gursey, P. Sikivie, Phys.Rev. Letters 36 775 (1976), P. Ramond Nuclear Physics B110 (1976) 214

11. P.W. Higgs, Phys. Rev. Letters 12 (1964) 132

12. J.M.Cornwall, R.E.Norton, Phys. Rev. D8, 3338 (1973); J.M. Cornwall, Phys.Rev. D10, 500 (1974); R. Jackiw, K. Johnson Phys. Rev. D8, 2386 (1973)

13. S. Weinberg, Harvard University Preprint, "Implications of Dynamical Symmetry Breaking"

14. M.L.Perl et. al. Phys. Rev. Letters 35 (1975) 1489, see also G.J. Feldman et. al. Phys. Rev. Letters 38 (1977)117

15. T.P. Cheng and L.F. Li, preprint (Dec.1976), S. Weinberg and J.D. Bjorken, SLAC preprint (Dec. 1976)

16. For a review and references, see H. Fritzsch, Proceeding of Intnl. Neutrino Conference, Aachen, (1976), see also A. De-Rujula, H. Georgi, S.L. Glashow, Phys. Rev. Letters 37 (1976) 398

17. See in particular S. Coleman, Secret Symmetry, in Proc. 1973 Int. Summer School of Physics "Ettore Majorana"

18. S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888

19. S. Weinberg, Phys. Rev. Letters 36, (1976) 294

20. M.K Gaillard and B.W. Lee Phys. Rev. Letters 33 (1974) 108; G. Altarelli, L. Maiani, Phys. Letters 52B (1974) 351

21. M. Gell-Mann, Phys. Rev. Letters 12 155 (1964)

22. B.W.Lee, Phys. Rev Letters 12, 83 (1964), H. Sugawara, Prog. Theoretical Phys.(Kyoto) 31, 213 (1964)

23. M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, ITEP-59 (Moscow-1975); G. Ross (private communication)

24. H. Fritzsch, P. Minkowski, Phys. Letters, Phys. Letters 61B 275 (1975); see also DeRujula, et.al. ref. 16.

25. E. Golowich, B. Holstein, Phys. Rev. Letters 35 831 (1975)

26. See H. Fritzsch, ref. 16

27. See M. Barnett, Proc. Intnl. Neutrino Conference, Aachen, (1976)

28. H. Fritzsch,(private communication)

29. F. Wilczek, A. Zee, Nucl. Phys. B106 (1976) 461 (ref. to Witten)

30. e.g. C. Schmid, Cern Preprint, (1977), also ref. 23.; also see Marshak, Riazuddin, Ryan-Theory of Weak Interactions, Wiley Interscience, for older mechanisms

31. Altarelli, et. al., Nucl. Phys. B88 (1975) 215

32. M. Gari, Phys. Reports, 6C (1973) 319

33. S. Weinberg, Phys. Rev. Letters, 37 (1976) 657; see also P. Sikivie, Maryland Preprint, 1976, though this model suffers from off-diagonal neutral vertices.

34. S. Weinberg, S. Glashow, Harvard preprint, A158,"Natural Conservation Laws for Neutral Currents"

35. See P. Ramond, ref. 10

36. M.K. Gaillard, B.W. Lee, Phys. Rev D10 (1974) 897

37. M. Gell-Mann (private communication)

38. See Appelquist et. al. ref. 7.

39. G. Ross (private communication) see ref. 23.

40. H. Leutwyler Nucl. Physics, B76 (1974) 413, and Phys. Letters B48 (1974) 431

41. L. Schülke, Nucl. Physics B40 (1972) 386

42. M. Suzuki, Phys. Rev. 144, 1154 (1966)

43. A. Chodos et. al. Phys. Rev. D9, 3471 (1974), also D10, 2599 (1974)

44. G. Branco, T. Hagiwara, R.N. Mohapatra, Phys. Rev. D13 104 (1976)

45. J. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B106 292 (1976)

46. B.W. Lee, C. Quigg, J.L. Rosner, Fermilab Pub.- 76/63 Thy

47. A. DeRujula, (private communication)

48. K. Wilson, Phys. Rev. 179, 1499 (1969)

49. W. Zimmerman, Lectures, 1970 Brandeis Summer Institute, Vol I., Parental guidance suggested

50. G. t'Hooft and M. Veltman, Nucl. Physics B44, 189 (1972)

51. M. Gell-Mann, F. Low, Phys. Rev. 95 1300, (1954); K. Symansik Commun. Math. Phys. 23, 49 (1971), C.G. Callaen, Phys. Rev. D5 3202, (1972)

52. J.D Bjorken, S.D. Drell,"Relativistic Quantum Fields" McGraw Hill, New York, (1970)

53. H. Georgi & H.D.Politzer, Phys. Rev.D14, 1830 (1976)